Investment Demand Shocks and Procyclical Inflation: A Theory*

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Abstract

This paper attributes the origin of aggregate fluctuations to the stochastic synchronization of firms' investments, which we call investment avalanches. In a monopolistic competition model with idiosyncratic productivity shocks, firms' decisions on lumpy investments exhibit complementarity in equilibrium due to aggregate demand externality. A firm's lumpy investment can cause other firms' lumpy investments, which in turn causes other firms' and so on. The size of this investment avalanche depends on the distribution of firms' states on productivity and capital. We show that this variation of avalanche size generates sizable fluctuations in aggregate investment demand in a model with no exogenous aggregate shocks. We incorporate the investment demand shocks in an otherwise standard New Keynesian business cycle model to show that the shocks naturally give rise to a procyclical response in inflation.

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1 Introduction

The term "animal spirits" haunts business cycle discussions persistently. In a regular business up and down, the investors' collective mood appears to drive the fluctuations of aggregate investment demand. In a standard business cycle literature, this origin of aggregate fluctuations is captured by exogenous shocks on preference or technology, such as a time preference shock, a productivity shock in the investment goods sector, or a shock on the volatility of underlying uncertainty. Attributing the cause of animal spirits to an exogenous aggregate shock is consequential, as it implies that the welfare loss of business cycles does not directly originate from animal spirits. Instead, welfare loss arises from inefficiency elsewhere, hindering the economy's response to shocks in preference or technology.

In this paper, we develop a business cycle model without exogenous aggregate shocks. We consider a monopolistic competition model, in which firms' investment decisions are complementary in equilibrium due to aggregate demand externality. We assume that capital is indivisible up to a lumpiness parameter. Thus, how many firms invest in a period, i.e., the extensive margin of capital adjustments, determines aggregate investment demand. When the economy consists of a finite large number n of firms, the aggregate investment demand deviates from the steady-state level due to the finiteness. The deviation is amplified by the aggregate demand externality in equilibrium. We will show that this amplification effect, which we call an investment avalanche, leads to a qualitatively different distribution from what the central limit theorem predicts and is quantitatively significant even when n is very large.

We analyze the propagation effect on macro variables by incorporating the investment demand shocks in an otherwise standard New Keynesian business cycle model. We are particularly interested in the response of inflation rates to investment demand shocks. We examine to what extent the investment demand shocks help explain the procyclical response of inflation rates.

Related literature This paper is part of a long list of literature on the origin of business cycles and the role of investment demand. Our model is closest to the business cycle literature on investment shocks. Fisher (2006) found empirical evidence for technological shocks

in the investment goods sector and explored its implications in business cycles. Justiniano, Primiceri, and Tambalotti (2010) established an important role played by an investment technological shock in an estimated business cycle model and interpreted it as a shock originating from the financial sector that transforms investment into effective capital. Christiano, Motto, and Rostagno (2014) took the view further and found a shock on the volatility of underlying risk, such as in Bloom (2009), plays an essential role in an estimated New Keynesian model.

While these studies attribute the aggregate fluctuation to an exogenous aggregate shock to production technology or the financial environment that converts investment expenditures to capital increases, we focus on many firms' simultaneous investment decisions. Whereas an investment technological shock expands capital without costing a resource, an investment demand shock in our model crowds out consumption. In this sense, our model is related to Galí and Rabanal (2004) who took a more direct approach in which a shock on time preference affects the representative agents' demand toward current or future consumption. Our model can be interpreted as a microfoundation for the time preference shock, although welfare implications will differ.

Our model is in the tradition of sectoral business cycles of Long and Plosser (1983), which feature realistic technological shocks at the sectoral level. It has been pointed out that the aggregate fluctuations generated from idiosyncratic shocks suffer a diversification effect by which the volatility of aggregate fluctuations decreases fast as the number of sectors increases, as discussed by Dupor (1999) and Horvath (2000). An introduction of non-linear behavior at the micro level, such as in (S,s) models, may open the possibility of avoiding the diversification effect. However, previous research has concentrated on the case where non-linear behaviors generate weak aggregate fluctuations (Caplin and Spulber, 1987; Caballero and Engel, 1991; Thomas, 2002; Khan and Thomas, 2003, 2008).

This paper instead illustrates that the (S,s) behavior can cause aggregate fluctuations. Our attempt is related to recent studies on the role of interest elasticity of investment demand in achieving the diversification effect (Auclert, Rognlie, and Straub, 2020; Winberry, 2021; Zwick and Mahon, 2017). In our model, firms' investments exhibit strategic complementarity in equilibrium. In an environment with a finite number of firms rather than a continuum of

firms, we show that the complementarity in lumpy investments can generate sizable aggregate fluctuations.

Our model shares the spirit of endogenous business cycle studies, such as sunspot models pursued by Galí (1994) and Wang and Wen (2008) among others. Our model includes micro-level independent shocks but no exogenous aggregate shocks. In that sense, our model is capable of generating aggregate fluctuations endogenously. Our model differs from the sunspot and indeterminacy models in that our model produces a locally unique equilibrium rather than a continuum of equilibria. We select a locally unique equilibrium closest to an equilibrium of a "smooth" economy in which the lumpiness of investments is irrelevant to aggregates and show that the equilibrium exhibits significant fluctuations. The point of the local uniqueness is that the equilibrium of a smooth economy cannot be generically realized as an equilibrium of an (S,s) economy with finite firms, and the equilibrium of the (S,s) economy deviates from the smooth counterpart quantitatively significantly.

Such authors have studied the departure from the diversification effect as Durlauf (1993), Gabaix (2011), and Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012), among others. Our paper's underlying analytics of the fluctuation mechanism are based on power-law distributions. Unlike the granular hypothesis of Gabaix (2011), our model features a stochastic synchronization called avalanches. The so-called self-organized criticality model that generates avalanches has been explored by Bak, Chen, Scheinkman, and Woodford (1993), Nirei (2006), and Nirei, Stachurski, and Watanabe (2020). The investment avalanche employed in this paper has been proposed by Nirei (2015) in an environment of real business cycles, and a mathematically complete treatment of avalanches in an (S,s) model has been recently provided by Nirei and Scheinkman (2024).

Lumpy investments Many authors, including Cooper and Haltiwanger (1996) and Gourio and Kashyap (2007), have emphasized the vital role that lumpy investments play in business cycles. We quickly recapitulate this point using a Japanese business survey (BSJBSA).[1]

[1]Basic Survey of Japanese Business Structure and Activities is conducted by the Ministry of Economy, Trade and Industry and covers firms with 50 or more employees and 30 million yen or more capital. The response rate in 2022 was 90.2%. In our database, the survey consists of an unbalanced panel of 31197 firms from 2007 to 2021. See Nirei (2024) for details.

We define a lumpy investment as an incident of a firm's annual gross investment exceeding 20% of capital. In our dataset, the lumpy investment occurs at a rate of 13.5% with a standard deviation of 21.6%.

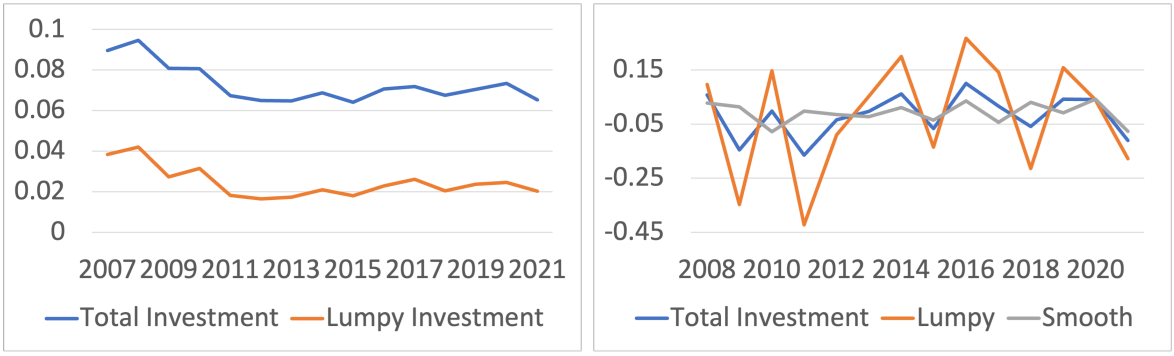


Figure 1: Aggregate total investments and lumpy investments in Japanese business survey. The left panel plots the ratio of the aggregate investments to aggregate assets held by the firms. The right panel plots growth rates. Source: BSJBSA.

The left panel of Figure 1, adapted from Nirei (2024), shows the sizable portion of lumpy investments in aggregate investments. Total investment indicates the aggregate investments (tangible and intangible) divided by the total assets held by the firms in each year. The lumpy investment shows the investments aggregated over firms that exhibited an investment spike in the year, divided by the total assets. The average ratio of lumpy investment to total investment is 33%. In the right panel, we observe a clear comovement of the lumpy and total investments growth rates with the coefficient of correlation exceeding 95%. When we define the total minus the lumpy investment as smooth investments, the correlation coefficient between the smooth and total growth rates is only 29%. In what follows, we will explicitly incorporate the fluctuations of lumpy investments in a business cycle model.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 analyzes the mechanism of an investment avalanche that causes the aggregate investment demand shock. Section 4 shows the impulse-response analysis of the investment demand shock. Section 5 concludes. An appendix contains proofs and extensions of the model.

2 Model

The model consists of representative final good producers, representative wholesalers, intermediate good producers, the central bank, worker households, and capitalist households. Worker households supply labor and cannot borrow or lend because of credit constraints. Capitalist households own firms and consume out of dividend income. Intermediate producers produce differentiated goods monopolistically using capital and labor. Wholesalers aggregate intermediate goods, incur price adjustment costs and sell them to final good producers. Final goods are used for consumption and investment. Intermediate producers own capital and make investment decisions. Our model features indivisible capital where intermediate producers can choose the capital level only discretely. In the remainder of this section, we present parametric specifications and define an equilibrium.

2.1 Production

Final goods Final good Y_t is produced by a CES aggregator function,

$$Y_t = \left(\int y_{it}^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}},$$

where $\epsilon > 1$, and competitively supplied. Denoting the price of y_{it} by p_{it}, competitive price of Y_t is $P_t = \left(\int p_{it}^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}$ and minimized cost is $\int p_{it} y_{it} di = P_t Y_t$. Derived demand for y_{it} is $y_{it} = \left(\frac{p_{it}}{P_t} \right)^{-\epsilon} Y_t$.

Wholesale $i \in [0,1]$ is a monopolistic supplier of y_i with quadratic price adjustment costs à la Rotemberg. They are symmetric across i. Production function of the wholesale is $y_{it} = M_{it}$ where M_{it} is a CES aggregate of intermediate goods as $M_{it} = \left(\int m_{ijt}^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}}$ with $\eta > 1$.

The real value of wholesale i is the expected discounted sum of its future dividends $\max \mathbb{E}\left[\sum_t \Lambda_t \Omega_{it} \right]$ where

$$\Omega_{it} := \frac{p_{it} M_{it} - \int q_{jt} m_{ijt} dj}{P_t} - \frac{\psi_P}{2} \left(\frac{p_{it}}{p_{i,t-1}} - 1 \right)^2 Y_t,$$

subject to production and demand functions. Minimized unit cost is $Q_t := \left(\int q_{jt}^{1-\eta} dj \right)^{\frac{1}{1-\eta}}$. Derived demand for intermediate good j is $m_{ijt} = \left(\frac{q_{jt}}{Q_t} \right)^{-\eta} M_{it}$.

We consider a symmetric equilibrium where $p_{it} = P_t$ and $q_{it} = Q_t$ for all i. The markup charged by the wholesale sector is $\mu_t := P_t/Q_t$ and the inflation rate is $\pi_t := P_t/P_{t-1} - 1$. Using the notations, the equilibrium aggregate profits of wholesale are $\Omega_t := \int \Omega_{it} di = (1 - \mu_t^{-1} - (\psi_P/2)\pi_t^2) Y_t$.

Wholesales maximize their value by choosing their prices. We obtain the following New Keynesian Phillips curve after usual derivations[2]

$$\pi_t(1+\pi_t) = \frac{1 - \epsilon + \epsilon/\mu_t}{\psi_P} + \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \frac{Y_{t+1}}{Y_t} \pi_{t+1}(1+\pi_{t+1}) \right]. \tag{1}$$

We set the stationary inflation rate at zero. The steady-state markup is obtained as $\mu = \epsilon/(\epsilon - 1)$. Symmetry across wholesaler i implies $m_{jt} = m_{ijt}$ and $M_t := \int M_{it} di = M_{it}$ for any i.

Intermediate goods Intermediate goods are differentiated, and each intermediate good $j \in [0,1]$ is supplied by a monopolist firm j using production function $m_{jt} = a_{jt}(k_{jt}^{\alpha} l_{jt}^{1-\alpha})^{\theta}$, where $\theta \leq 1$ denotes the returns to scale.

Our model features a lumpy investment behavior. For simplicity, we assume that the capital is indivisible up to a lumpiness parameter λ. Specifically, we assume that an intermediate producer is subject to discrete capital constraint $k_{jt} \in \{k_{j,t-1}(1-\delta)\lambda^s\}_{s=0,\pm 1,\dots}$, where the lumpiness parameter satisfies $(1-\delta)\lambda > 1$. It is possible to endogenize λ by considering a non-convex cost of adjustments (e.g., see Nirei and Scheinkman (2024) in menu-cost pricing). We adopt the indivisibility assumption for the sake of expositional simplicity.

Intermediate firm j faces demand function from wholesales as $m_{jt} = \int m_{ijt} di = \left(\frac{q_{jt}}{Q_t}\right)^{-\eta} M_t$. The real value of intermediate producer j is $\max \mathbb{E}\left[\sum_t \Lambda_t(\varphi_{jt} - x_{jt})\right]$ where φ_{jt} denotes the operating surplus $\varphi_{jt} := \frac{q_{jt}}{P_t} m_{jt} - w_t l_{jt}$ and x_{jt} denotes investment demand $x_{j,t} :=$

[2]First-order condition with respect to p_{it}:

$$0 = \Lambda_t Y_t \left[\frac{(1-\epsilon) p_{it}^{-\epsilon}}{P_t^{1-\epsilon}} + \epsilon \frac{Q_t}{P_t} \frac{p_{it}^{-\epsilon-1}}{P_t^{-\epsilon}} - \psi_P \left(\frac{p_{it}}{p_{i,t-1}} - 1 \right) \frac{1}{p_{i,t-1}} \right] - \mathbb{E}_t \left[\Lambda_{t+1} Y_{t+1} \psi_P \left(\frac{p_{i,t+1}}{p_{i,t}} - 1 \right) \frac{-p_{i,t+1}}{p_{i,t}^2} \right].$$

We imposed the symmetry $p_{it} = P_t$ and denoted $\pi_t = p_{it}/p_{i,t-1} - 1$. Then, we have

$$0 = \Lambda_t Y_t \left[\frac{1-\epsilon}{P_t} + \epsilon \frac{Q_t}{P_t^2} - \psi_P \frac{\pi_t}{P_{t-1}} \right] + \mathbb{E}_t \left[\Lambda_{t+1} Y_{t+1} \psi_P \pi_{t+1} \frac{1+\pi_{t+1}}{P_t} \right].$$

$k_{j,t+1} - (1-\delta)k_{j,t}$. Solving for labor demand and aggregating, we obtain aggregate goods supply and labor demand functions as follows,[3]

$$Y_t = M_t = \left(\frac{(1-1/\eta)(1-\alpha)\theta}{w_t\mu_t}\right)^{\frac{(1-\alpha)\theta}{1-(1-\alpha)\theta}} K_t^{\frac{\alpha\theta}{1-(1-\alpha)\theta}}, \tag{2}$$

$$L_t := \int l_{jt}dj = \left(\frac{(1-1/\eta)(1-\alpha)\theta}{w_t\mu_t}\right)^{\frac{1}{1-(1-\alpha)\theta}} K_t^{\frac{\alpha\theta}{1-(1-\alpha)\theta}}, \tag{3}$$

where $K_t := \left(\int (a_{jt}^{\frac{1}{\alpha\theta}} k_{jt})^\rho dj\right)^{1/\rho}$ and $\rho := \frac{(1-1/\eta)\alpha\theta}{1-(1-1/\eta)(1-\alpha)\theta}$.

At the optimal factor demands, the operating surplus is written as a function of productivity and capital $\varphi_t(a_{jt}, k_{jt})$ as

$$\varphi_t(a_{jt}, k_{jt}) = (a_{jt}^{1/(\alpha\theta)} k_{jt})^\rho c_\varphi \left(w_t^{(1-\alpha)\theta}\mu_t\right)^{\frac{-1}{1-(1-\alpha)\theta}} K_t^{\frac{(\alpha\theta/\eta)/(1-(1-\alpha)\theta)}{1-(1-1/\eta)(1-\alpha)\theta}} \tag{4}$$

where $c_\varphi := (1-(1-1/\eta)(1-\alpha)\theta)\left((1-1/\eta)(1-\alpha)\theta\right)^{\frac{(1-\alpha)\theta}{1-(1-\alpha)\theta}}$. Aggregating (4) yields an expression for aggregate operating surplus:

$$\int \varphi(a_i, k_i)di = c_\varphi (w_t^{(1-\alpha)\theta}\mu_t)^{\frac{-1}{1-(1-\alpha)\theta}} K_t^{\frac{\alpha\theta}{1-(1-\alpha)\theta}}.$$

2.2 Lumpy investment

An intermediate producer's capital choice is subject to indivisibility constraint, which plays a central role in our analysis. The operating surplus function φ_t in (4) is strictly concave in k_{jt}. Since the capital choice of an intermediate firm is constrained by a discrete set, the firm optimally chooses a threshold of capital for an investment spike by which the firm's capital increases by a factor of λ. The firm can choose an investment spike in t and no spike in $t+1$, or no spike in t and a spike in $t+1$, which leaves the firm at the same level of capital in $t+2$. At the threshold, firm j is indifferent between the two alternative plans. Thus, the optimal

[3]The derivation is straightforward. The nominal sales is written as $q_{jt}m_{jt} = m_{jt}^{1-1/\eta} M_t^{1/\eta} Q_t$ using the demand function for m_{jt}. The first-order condition for cost minimization with respect to l_{jt} is $(1-1/\eta)(m_{jt}/M_t)^{-1/\eta}(1/\mu_t)(\partial m_{jt}/\partial l_{jt}) = w_t$. Thus, the labor cost share is constant: $w_t l_{jt}/(m_{jt}q_{jt}/P_t) = (1-1/\eta)(1-\alpha)\theta$. Using this and production function to substitute out l_t and aggregating across j yields (2). Aggregate labor demand is $L_t := \int l_{jt}dj = \int (1-1/\eta)(1-\alpha)\theta m_{jt}^{1-1/\eta} M_t^{1/\eta}/(w_t\mu_t) = (1-1/\eta)(1-\alpha)\theta M_t/(w_t\mu_t)$ and hence (3) obtains.

threshold of investment spike, $k^* = k_{j,t+1}$, satisfies the following indifference condition:

$$\mathbb{E}_t\Lambda_{t+1}(\varphi_{t+1}(a,k^*)+(1-\delta)k^*)-\Lambda_t k^* = \mathbb{E}_t\Lambda_{t+1}(\varphi_{t+1}(a,\lambda k^*)+(1-\delta)\lambda k^*)-\Lambda_t\lambda k^*.$$

Solving for k^*, we obtain

$$k^*_{j,t+1} = a_{j,t+1}^{\frac{1-1/\eta}{1-\theta+\theta/\eta}} K_{t+1}^{\frac{(\alpha\theta/\eta)/(1-(1-\alpha)\theta)}{1-\theta+\theta/\eta}} \Phi_t$$

where Φ_t summarizes the expected factor prices

$$\Phi_t := \left(c_\varphi \frac{\lambda^\rho - 1}{\lambda - 1}\mathbb{E}_t\left[\Lambda_{t+1}\left(w_{t+1}^{(1-\alpha)\theta}\mu_{t+1}\right)^{\frac{-1}{1-(1-\alpha)\theta}}\right]\mathbb{E}_t\left[\Lambda_t - \Lambda_{t+1}(1-\delta)\right]^{-1}\right)^{\frac{1}{1-\rho}}.$$

The optimal inaction region for firm j in t is $k_{j,t} \in [k^*_{j,t+1}/(1-\delta), \lambda k^*_{j,t+1}/(1-\delta))$. The support of a stationary distribution is $k_{j,t} \in [k^*_{j,t}, \lambda k^*_{j,t})$.

We define a state variable s_{jt} of firm j that denotes the distance of log capital from the threshold level as:

$$s_{jt} := \frac{\log k_{jt} - \log k^*_{jt}}{\log \lambda}.$$

Let F_t denote a joint distribution of (a_{jt}, s_{jt}) and F the stationary distribution. Aggregate capital is written by using F_t as $K_t = \left(\int (a_{jt}^{1/(\alpha\theta)} k_{jt})^\rho dj\right)^{1/\rho} = \mathbb{E}^{F_t}\left[(a_{jt}^{1/(\alpha\theta)} k^*_{jt}\lambda^{s_{jt}})^\rho\right]^{1/\rho}$. Substituting $k^*_{j,t+1}$ into this expression for K_{t+1} yields an equilibrium condition for factor prices Φ_t:

$$1 = \mathbb{E}^{F_{t+1}}\left[\lambda^{s\rho} a^{\left(\frac{1}{\alpha\theta}+\frac{1-1/\eta}{1-(1-1/\eta)\theta}\right)\rho}\right]^{1/\rho} \Phi_t K_{t+1}^{\frac{(\alpha\theta/\eta)/(1-(1-\alpha)\theta)}{1-\theta+\theta/\eta}-1}. \tag{5}$$

Under the equilibrium condition, the threshold simplifies to

$$k^*_{jt} = B_t K_t a_{jt}^{\frac{1-1/\eta}{1-\theta+\theta/\eta}} \tag{6}$$

where

$$B_t := \mathbb{E}^{F_t}[\lambda^{s\rho} a^{\left(\frac{1}{\alpha\theta}+\frac{1-1/\eta}{1-\theta+\theta/\eta}\right)\rho}]^{-1/\rho}.$$

2.3 Households

There are two types of households in the model: capitalists and workers.

Capitalists Capitalists own monopolistic firms and receive profits of wholesale Ω_t and intermediate producers $\int \varphi(a_{it}, k_{it}) - x_{it}\ di$. They have access to risk-free nominal asset A_t at risk-free nominal rate i_t. Capitalists' budget constraint is

$$A_t + P_t c_t^K = (1 + i_{t-1})A_{t-1} + P_t \left(\Omega_t + \int \varphi(a_{it}, k_{it}) - x_{it}\ di \right).$$

Capitalists instruct their stochastic discount factor to the firms they own, and take the dividends and investments X_t as given and exogenous for their decision. Capitalists' utility function is $\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t^K) \right]$. The choice of A_{t+1} leads to the Euler equation

$$u'(c_t^K) = \beta \mathbb{E}_t \left[\frac{1 + i_t}{1 + \pi_{t+1}} u'(c_{t+1}^K) \right]. \tag{7}$$

Capitalists own firms, so the stochastic discount factor is $\Lambda_{t+1}/\Lambda_t = \beta u'(c_{t+1}^K)/u'(c_t^K)$. Using this stochastic discount factor for Φ_t in (5), we obtain a New Keynesian IS relationship in the first-order approximation.

Workers We introduce hand-to-mouth households, which we call workers. Workers supply labor and consume final goods to maximize their preference $u(c^W, N) = \frac{(c^W)^{1-\sigma}}{1-\sigma} - \chi \frac{N^{1+\gamma}}{1+\gamma}$. We assume that workers do not have access to financial markets, and therefore, their budget constraint is

$$c_t^W = w_t N_t. \tag{8}$$

Then, the labor supply function is

$$N_t = w_t^{\frac{1-\sigma}{\gamma+\sigma}} \chi^{\frac{1}{\gamma+\sigma}}. \tag{9}$$

2.4 Monetary policy and market clearing

We assume that monetary policy sets a risk-free nominal rate at

$$1 + i_t = (1 + r_{ss})(1 + \pi_t)^{\phi} \tag{10}$$

with Taylor principle $\phi > 1$. A risk-free asset is supplied at net zero: $A_t = 0$.

The market-clearing conditions for labor and final goods are

$$N_t = L_t \tag{11}$$

$$Y_t \left(1 - \frac{\psi_P}{2} \pi_t^2 \right) = c_t^W + c_t^K + X_t. \tag{12}$$

Aggregate capital is accumulated as

$$K_{t+1} = (1-\delta)K_t + B_X X_t. \tag{13}$$

The adjustment constant B_X is needed because K is a productivity-weighted average of capital.[4]

2.5 Recursive equilibrium

The recursive equilibrium of $(Y_t, K_{t+1}, X_t, L_t, N_t, c_t^W, c_t^K, w_t, \mu_t, i_t, \pi_t)$ with a continuum of intermediate firms $j \in [0,1]$ is determined by (1,2,3,5,7,8,9,10,11,12,13). In particular, we write $K_{t+1} = \Xi(K_t)$ for a mapping of aggregate capital which the recursive equilibrium determines.

The recursive equilibrium requires that B_t in (5) to be constant. To ensure this, we assume that, at $t = 0$, the distribution of a_{it} achieves stationarity and that s_{it} is uniformly distributed over $[0,1)$. Since s_{it} rotates on a circle, adding to s_{it} independent random shocks or aggregate shifts leaves the uniform distribution of s_{it} unchanged (see, e.g., Caballero and Engel (1991)). Thus, with this initial distribution, $F_t(a,s)$ stays at the stationary distribution F. Thus, the firms' threshold rule is $k_{jt}^* = BK_t a_{jt}^{\frac{1-1/\eta}{1-\theta+\theta/\eta}}$ where B is evaluated at F as $B = \mathbb{E}^F\big[\lambda^{s\rho} a^{\left(\frac{1}{\alpha\theta}+\frac{1-1/\eta}{1-\theta+\theta/\eta}\right)\rho}\big]^{-1/\rho}$. This assumption leads (5) to implying $\Phi_t = BK_{t+1}^{-\frac{(\alpha\theta/\eta)/(1-(1-\alpha)\theta)}{1-\theta+\theta/\eta}+1}$ and leading to the New Keynesian IS relationship.

[4] B_X is derived under the premise of s following a uniform distribution (Nirei, 2015). Let s_{it}^* denote the state corresponding to $k_{i,t+1}^*$. Under $\theta = 1$, we obtain $B_X = \left(\frac{\lambda-1}{\log\lambda} B\mathbb{E}[a_{it}^{\eta-1}]\right)^{-1}$ as follows.

$$X_t = \int\int_0^{s_{it}^*} (\lambda-1)(1-\delta)k_{it}\,ds_{it}\,di = (1-\delta)(\lambda-1)\int\int_0^{s_{it}^*} \lambda^{s_{it}} k_{it}^*\,ds_{it}\,di = \frac{(1-\delta)(\lambda-1)}{\log\lambda}\int(\lambda^{s_{it}^*}-1)k_{it}^*\,di$$
$$= \frac{(1-\delta)(\lambda-1)}{\log\lambda}\int((1-\delta)^{-1}k_{i,t+1}^* - k_{it}^*)\,di = \frac{\lambda-1}{\log\lambda} B\mathbb{E}[a_{it}^{\eta-1}](K_{t+1}-(1-\delta)K_t)$$

2.6 Equilibrium Dynamics when $\theta = 1$

From now on, we work on the case of constant returns to scale, $\theta = 1$. The recursive equilibrium system that determines $(Y_t, K_{t+1}, X_t, L_t, N_t, c_t^W, c_t^K, w_t, \mu_t, \pi_t)$ is written as follows.[5]

$$Y_t = M_t = \left(\frac{(1-1/\eta)(1-\alpha)}{w_t \mu_t}\right)^{\frac{1-\alpha}{\alpha}} K_t \qquad \text{(GDP)}$$

$$L_t = \left(\frac{(1-1/\eta)(1-\alpha)}{w_t \mu_t}\right)^{\frac{1}{\alpha}} K_t \qquad \text{(Labor Demand)}$$

$$N_t = w_t^{\frac{1-\sigma}{\gamma+\sigma}} \chi^{\frac{1}{\gamma+\sigma}} \qquad \text{(Labor supply)}$$

$$N_t = L_t \qquad \text{(Labor market)}$$

$$c_t^W = w_t N_t \qquad \text{(Workers consumption)}$$

$$\left(1 - \frac{\psi_P}{2}\pi_t^2\right) Y_t = c_t^W + c_t^K + X_t \qquad \text{(Goods market)}$$

$$K_{t+1} = (1-\delta)K_t + B_X X_t \qquad \text{(Capital accumulation)}$$

$$u'(c_t^K) = \beta \mathbb{E}_t \left[\frac{(1+r_{ss})(1+\pi_t)^\phi}{1+\pi_{t+1}} u'(c_{t+1}^K)\right] \qquad \text{(Euler equation)}$$

$$B^{1-\rho} = c_\varphi \frac{\lambda^\rho - 1}{\lambda - 1} \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \left(w_{t+1}^{1-\alpha} \mu_{t+1}\right)^{\frac{-1}{\alpha}}\right] \mathbb{E}_t \left[1 - \frac{\Lambda_{t+1}}{\Lambda_t}(1-\delta)\right]^{-1} \qquad \text{(Factor prices)}$$

$$\pi_t(1+\pi_t) = \frac{1 - \epsilon + \epsilon/\mu_t}{\psi_P} + \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \frac{Y_{t+1}}{Y_t} \pi_{t+1}(1+\pi_{t+1})\right] \qquad \text{(Phillips curve)}$$

2.7 Steady state

The steady state of the above system of equations satisfies $\pi_t = 0$, $\Lambda_{t+1}/\Lambda_t = \beta$, and $X_t = K_t \delta / B_X$. The steady-state wage is determined by:

$$\left(\frac{1}{\beta} - 1 + \delta\right)^\alpha w^{1-\alpha} = \mathbb{E}^F \left[\lambda^{s\rho} a^{\eta-1}\right]^{1/(\eta-1)} \left(\frac{\lambda^\rho - 1}{\lambda - 1} c_\varphi\right)^\alpha \frac{\epsilon - 1}{\epsilon}.$$

It is straightforward to see that our model nests the case of divisible capital as a limit $\lambda \to 1$. By solving the model with smoothly adjusting capital, we obtain the steady-state

[5] In the equations, the stochastic discount factor satisfies $\Lambda_{t+1}/\Lambda_t = \beta u'(c_{t+1}^K)/u'(c_t^K)$. Constants are $\rho = \frac{(1-1/\eta)\alpha}{1-(1-1/\eta)(1-\alpha)}$, $c_\varphi = (1-(1-1/\eta)(1-\alpha))\left((1-1/\eta)(1-\alpha)\right)^{\frac{1-\alpha}{\alpha}}$, and $B = \mathbb{E}^F[\lambda^{s\rho} a^{\eta-1}]^{-1/\rho}$. To evaluate B and B_X, we normalize the productivity by $\mathbb{E}[a^{\eta-1}] = 1$ and use uniformly distributed s, resulting in $\mathbb{E}[\lambda^{s\rho}] = (\lambda^\rho - 1)/(\rho \log \lambda)$ and $B_X = \left(\frac{\lambda-1}{\log \lambda} \frac{\lambda^\rho - 1}{\rho \log \lambda}\right)^{-1}$. Capitalists' budget constraint is omitted by Walras's law. Note that $\mu_t = P_t/Q_t$ (inverse of m in the Dynare code)).

wage w_{smooth} to be determined by

$$(1/\beta - 1 + \delta)^{\alpha} w_{smooth}^{1-\alpha} = \mathbb{E}[a^{\eta-1}]^{1/(\eta-1)} \alpha^{\alpha} (1-\alpha)^{1-\alpha} (1 - 1/\eta)(1 - 1/\epsilon),$$

which coincides with the equation above when $\lambda \to 1$.

3 Investment demand shocks

3.1 Finite number of firms and deviations from a continuum model

In the previous section, we present a model with a continuum of intermediate producers and derive a recursive equilibrium that pins down the aggregate capital path as $K_{t+1} = \Xi(K_t)$. The intermediate producers incur idiosyncratic productivity shocks but cannot generate aggregate stochastic fluctuations because a continuum of independent shocks always integrates to a deterministic value. This section considers a large but finite n number of intermediate producers. Due to the finiteness, the idiosyncratic shocks can generate aggregate stochastic fluctuations around the deterministic path determined by Ξ. Let $X_t^e = (K_{t+1} - \Xi(K_t))/B_X$ denote the aggregate investment expected in the continuum model, and $u_t := X_t/X_t^e - 1$ a deviation of actual aggregate investment from X_t^e. We call u_t an investment demand shock.

It is commonly thought that idiosyncratic productivity shocks would generate little aggregate fluctuations in u_t due to the diversification effect. According to the central limit theorem, the standard deviation of an average of independent n shocks declines as fast as $1/\sqrt{n}$. Thus, for example, one million firms' idiosyncratic shocks of 10% standard deviation generate aggregate volatility of $10\%/\sqrt{10^6} = 0.01\%$ standard deviation, which does not amount to the order of magnitude for business cycle fluctuations.

The diversification argument for u_t does not hold as described above in our model with finite n firms. First, a firm's investment is a non-linear function of aggregate capital due to the indivisibility. Second, firms' investment decisions are complementary to each other in equilibrium. A firm's decision to invest increases the aggregate capital, which raises the demand for all firms and pushes up the threshold of their investment decisions. Hence, a firm's lumpy investment increases the likelihood of the other firms' investments due to aggregate demand externality.

We will show in the next section that the aggregate investment in the finite economy can be quantitatively substantial. In particular, we will show that the convergence speed of the volatility of u_t to zero deviates from $1/\sqrt{n}$ using the methodology developed in Nirei (2015) and Nirei and Scheinkman (2024). Moreover, we will show that the investment demand shock u_t exhibits autocorrelation in an extended model with heterogeneous time-to-build. In Section 4, we incorporate the shock term u_t in the full model and investigate its consequences on other macro variables, including the inflation rate. Thus, the present section provides a microfoundation for the aggregate investment demand shock u_t.

With n firms and an aggregate shock u_t, the model economy features the distribution of (a, s) as a state. To deal with the curse of dimensionality in the sense of Krusell and Smith, Jr. (1998), we make a behavioral assumption by which intermediate firm j follows their threshold policy in a continuum economy when the distribution of (a, s) by the stationary distribution F. Namely, firm j follows a threshold (6) with $B_t = B$. Note that we maintain that the initial distribution of s_{it} is uniform. Hence, the future distribution of s is also uniform even though aggregate shocks u_t exist. The behavioral assumption we make addresses the issue of finiteness. Since n is finite, the actual profile $(s_{it})_i$ necessarily deviates from any continuous distribution. The assumption is that agents approximate the average of the exact profile by an expected value using the uniform distribution when they form expectations utilizing the model.

We set the timing of the investment demand shock as follows. At the end of $t-1$, aggregate capital K_t is already installed, and the recursive equilibrium pins down the expected capital of the beginning of $t+1$ as $K^e_{t+1} = \Xi(K_t)$. Thus, the investment of t expected at the end of $t-1$ is $X^e_t = (K^e_{t+1} - (1-\delta)K_t)/B_X$. At the beginning of t, the investment demand shock u_t is realized, and the actual aggregate investment is determined as $X_t = X^e_t(1+u_t)$. After u_t is realized, production and consumption in t occur.

3.2 Investment avalanche

Due to the nonlinearity arising from the indivisible capital, the model can have multiple equilibria. We select an equilibrium that is close to the equilibrium determined by the model with a continuum of firms. To do so, we use the best response dynamics of firms

following the threshold policy $k_i^* = \gamma(a_i, K^e) := a_i^{\eta-1} B K^e$. The selection algorithm is the following.

1. Capital is depreciated as $k_i^o = (1-\delta)k_{i,t}$.
2. Productivity profile $(a_i)_i$ realizes. K^e is set at $K_{t+1}^e = \Xi(K_t)$.
3. Firms apply the threshold rule $\gamma(a_i, K^e)$ to set (k_i).
4. Compute the aggregate capital K using k_i. Stop if $K = K^e$ or if $k_i \neq k_i^o$ for all i. Otherwise, reset K^e to K and repeat step 3.

This process stops in finite time by construction in step 4. The selected equilibrium is close to K^e in the sense that there is no equilibrium aggregate capital with $k_i \in [k_i^*, \lambda k_i^*)$ between the selected K and K_{t+1}^e. This property holds because of the characteristic of the best response dynamics (Vives, 1990). We will show in the next section that the case of all firms adjusting (i.e., $k_i \neq k_i^o$ for all i) occurs with probability zero when $n \to \infty$ if a and s are orthogonal.

The best response dynamic starts at the realization of idiosyncratic productivity shocks and capital depreciation. Initial expectation of future capital K_{t+1}^e sets the extensive margin of firms who adjust capital due to productivity shocks and depreciation. The equilibrium is reached if the resulting aggregate capital is equal to K_{t+1}^e. Otherwise, successive adjustments of capital follow until an equilibrium is reached. We call this successive adjustment within a period an investment avalanche.

3.3 Non-neglifible fluctuations of investment avalanches

Non-negligible fluctuations of the investment avalanche are demonstrated analytically as well as numerically. In the analysis, the degree of complementarity plays a key role, which we define shortly.

Consider a steady-state equilibrium of a continuum model. Let Δ denote a forward difference of a variable concerning t. The state variable $s_{it} = (\log k_{it} - \log k_{it}^*)/\log\lambda$ evolves as

$$s_{i,t+1} = s_{it} + \frac{\log(1-\delta) - \Delta \log K_t - (\eta-1)\Delta \log a_{it}}{\log \lambda}$$

if $s_{i,t+1} > 0$. In an avalanche process, depreciation $1-\delta$ and productivity shocks $\Delta \log a_{it}$ are realized first. At the steady state of a continuum model, $\Delta \log K_t = 0$. Thus, firms with $s_{it} \leq -\frac{\log(1-\delta)-(\eta-1)\Delta \log a_{it}}{\log \lambda}$ invest and transit to $s_{i,t+1} = 1 + s_{it} + \frac{\log(1-\delta)-(\eta-1)\Delta \log a_{it}}{\log \lambda}$, whereas other firms' $s_{i,t+1}$ is positive and determined by the equation above. We denote the threshold state for investment as $s^*_{it} := -\frac{\log(1-\delta)-(\eta-1)\Delta \log a_{it}}{\log \lambda}$.

Now we suppose that firms in $[s^*_{it}, s^*_{it} + \nu]$ for a small value $\nu > 0$ also invest. Since s is uniformly distributed, ν is the measure of firms located in this interval. Their investments increase K_{t+1} and ΔK_{t+1}, reducing s_{jt} of other firms by ν'. We define the degree of complementarity as $\vartheta := \lim_{\nu \to 0} \nu'/\nu$. The complementarity ϑ measures the number of firms induced to invest by a firm's investment. Then, we obtain the following property.

Proposition 1 *Suppose that s_{it} is distributed uniformly over $[0,1)$ and orthogonal to a_{it}. Also suppose that $a_{i,t+1} = a_{i,t}$ for all i. Then, the degree of complementarity is $\vartheta = 1$.*

The proof is provided in the appendix. Orthogonality between a and s is assumed. A simple case where this assumption is satisfied is when the productivity a is constant.

We use ϑ to characterize the asymptotic distribution of the number of firms involved in an investment avalanche. We consider the following experiment in a model with a finite number n of firms. As in Proposition 1, we impose that s_i follows the uniform distribution over $[0,1)$ and a_i and s_i are orthogonal at the distribution F. First, we draw a profile $(a_i, s_i)_{i=1}^n$ from F^n. This profile corresponds to a capital profile $(k_i)_i$ where $k_i = \lambda^{s_i} k^*_i$ where $k^*_i = a_i^{\eta-1} BK$ and K solves $K = (\sum_{i=1}^n (a_i^{1/\alpha} k_i)^\rho / n)^{1/\rho}$. Firms expect K^e, the next period aggregate capital, to be the same as K.

Then, we let capital be depreciated at rate δ, and find a next-period capital profile by the selection algorithm. In the first round of step 3, firms with state $s_i \leq s^* := -(1-\delta)/\log \lambda$ hit the threshold and decide to invest. In step 4, suppose that $\log K = \log K^e + \epsilon^n_0$ with $\epsilon^n_0 > 0$ occurs. That is, the gross investments of firms that go below the threshold by the direct effect of depreciation are greater than expected. This increases the threshold, and firms with state with $s^* < s_i \leq s^* + \epsilon_0/\log \lambda =: s^n_0$ go below the threshold in the second round of step 3. Let M^n_0 denote the set of those firms and $m^n_0 := |M^n_0|$ the number of those firms. If $m^n_0 > 0$ occurs, the aggregate capital is increased in step 4. The increment is

denoted by ϵ_1^n. Then, a new set of firms with $s_0^n < s_i \leq s_0^n + \epsilon_1^n =: s_1^n$ are induced to invest in the third round of step 3. Let M_1^n denote the sef of those firms and $m_1^n := |M_1^n|$. We repeat this process by defining ϵ_u^n, s_u^n, M_u^n, and m_u^n for $u = 0, 1, \ldots, U$ accordingly, where U denotes the iteration times where the algorithm stops. The size of an investment avalanche is defined as $L^n := \sum_{u=0}^{U} m_u^n$. When $\epsilon_0^n < 0$, we can similarly define a process of retracting investment decisions. We are interested in the distribution of L^n, which characterizes the distribution of $\Delta \log K$ in this experiment.

Since s_i are i.i.d. random variables and the sets of newly investing firms M_u^n are mutually exclusive, m_u^n is a stochastic process. A probability function of m_{u+1}^n conditional on m_u^n is given by a binomial distribution with probability ϵ_u^n and population $n - \sum_{v=1}^{u} m_v^n$. Thus, $(m_u^n)_u$ is a branching process in which each of "parents" m_u^n bears a "child" with probability $(n - \sum_{v=1}^{u} m_v^n)\epsilon_u^n / m_u^n$. Thus, L^n is a cumulative sum of the branching process with initial parents m_0^n until no children are born.

We show in the appendix that the mean of m_{u+1}^n, which follows the binomial with mean $(n - \sum_{v=1}^{u} m_v)\epsilon_u^n$, converges to ϑm_u^n as $n \to \infty$. Thus, as $n \to \infty$, the binomial distribution $Bin(\epsilon_u^n, n - \sum_{v=1}^{u} m_v^n)$ converges to a Poisson distribution with mean ϑm_u, which is equivalent to m_u-convolution of a Poisson distribution with mean ϑ. Let L denote a cumulative sum of a Poisson branching process $(m_u)_u$ in which each parent bears a random number of children following a Poisson distribution with mean ϑ. Thus, we can show that L^n converges to L in total variation (Nirei and Scheinkman, 2024, Proposition 6).

The limit distribution of L conditional on m_0 follows the Borel-Tanner distribution with parameter ϑ (Nirei and Scheinkman, 2024, Proposition 8a). Since m_0 also follows a Poisson distribution, an unconditional distribution of L is represented as a Generalized Poisson Distribution (Nirei and Scheinkman, 2024, Proposition 8b). Complementarity ϑ determines the shape of the tail part of distributions. In particular, when $\vartheta \to 1$, the index of dispersion of L diverges to infinity (Nirei and Scheinkman, 2024, Proposition 8d).

Unlike L, the distribution of m_0^n obeys the diversification effect. In the initial step of an avalanche, m_0^u follows a binomial distribution with probability $s^* = -(1-\delta)/\log\lambda$ and population n. By the central limit theorem, $m_0^n/\sqrt{n}$ asymptotically follows a normal distribution with finite variance $s^*(1-s^*)$. From this observation, we can draw a heuristic inspection of the

diversification effect for L^n. Conditional on $m_0^n = 1$, the asymptotic distribution of L^n follows a power-law-tailed distribution with tail index 0.5 (Nirei and Scheinkman, 2024, Corollary 5). Thus, the variance of $L^n/n \mid m_0^n = 1$ declines as $\int_1^n (L^n/n)^2 (L^n)^{-1.5} dL^n \sim 1/\sqrt{n}$. Since the distribution of $m_0^n/\sqrt{n}$ is normal with a finite variance, the mean of $|m_0^n|$ scales as $\sqrt{n}$. This implies that the unconditional variance of L^n/n scales as n^0, a constant (Nirei, 2015).

The experiment considered here differs from the dynamic equilibrium model in some regards. First, the orthogonality of a and s is not generally obtained. It can happen, for example, when the total variation of a tends to infinity (Caballero and Engel, 1991), but in that case, a may not achieve a stationary distribution. We inspect the assumption by numerical simulations of the model in Section 3.5.

3.4 Time to build

It is straightforward to introduce time-to-build, i.e., an investment gestation lag is more than 1 period. Time-to-build helps the model to generate an autocorrelated series of aggregate investment. This is for a simple reason. Let $J \geq 1$ denote the time to build. We suppose that the investment is decided in J periods in advance, and the investment purchase is spread out over the $J-1$ periods. In each period, the investment decisions among firms cause an investment avalanche regarding the J-period ahead capital. Thus, an aggregate investment is the weighted average of the past investment avalanches, generating an autocorrelation.

Let $s_{it} := k_{i,t+J} - (1-\delta)k_{i,t+J-1}$ denote the increment of capital that materializes in J periods later. The investment s_{it} is spent over J periods from t to $t+J-1$ with weight for each period ζ_τ, $\tau = 0, 1, \ldots, J-1$. Thus, firm i's total investments x_{it} in period t is a weighted sum of past s_{it} as $x_{it} = \sum_{\tau=0}^{J-1} \zeta_\tau s_{i,t-\tau}$. The real value of intermediate firm i is $\mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda_t(\varphi_t(a_{it}, k_{it}) - x_{it})$.

Let Λ_t^J denote a J-period rolling average as $\Lambda_t^J := \sum_{\tau=0}^{J-1} \zeta_\tau \Lambda_{t+\tau}$. Then, the terms relevant to the choice of k_{t+J} is:

$$\mathbb{E}_t \Lambda_{t+J} \varphi_{t+J}(a_{i,t+J}, k_{i,t+J}) + (1-\delta)\Lambda_{t+1}^J k_{t+J} - \Lambda_t^J k_{t+J}$$

The firm's threshold for lumpy investment is solved as in the previous section. First, the

indifference condition between an investment spike in $t+J$ or $t+J+1$ yields

$$k^*_{i,t+J} = a_{i,t+J}^{\frac{1-1/\eta}{1-\theta+\theta/\eta}} K_{t+J}^{\frac{(\alpha\theta/\eta)/(1-(1-\alpha)\theta)}{1-\theta+\theta/\eta}} \Phi_t^J$$

where Φ_t^J summarizes the expected factor prices

$$\Phi_t^J := \left(c_\varphi \frac{\lambda^\rho - 1}{\lambda - 1} \mathbb{E}_t \left[\Lambda_{t+J} \left(w_{t+J}^{(1-\alpha)\theta} \mu_{t+J} \right)^{\frac{-1}{1-(1-\alpha)\theta}} \right] \mathbb{E}_t \left[\Lambda_t^J - \Lambda_{t+1}^J (1-\delta) \right]^{-1} \right)^{\frac{1}{1-\rho}}. \tag{14}$$

By aggregating the threshold across i, we obtain

$$k^*_{it} = B_{t+J} K_t a_{it}^{\frac{1-1/\eta}{1-\theta+\theta/\eta}}.$$

With the stationarity of F_t, the coefficient B_{t+J} is constant over time at B.

The steady-state values are affected by the change in stochastic discount factors in (14). At the steady state, $\Lambda_t^J/\Lambda_{t+J}$ becomes $\sum_{\tau=0}^{J-1} \zeta_\tau \Lambda_{t+\tau}/\Lambda_{t+J} = \sum_{\tau=0}^{J-1} \zeta_\tau \beta^{\tau-J} =: (\beta_J)^{-1}$. Also, at the steady state, $\Lambda_t^J/\Lambda_{t+1}^J = \beta^{-1}$. The equation at the steady state becomes:

$$\Phi^J := \left(c_\varphi \frac{\lambda^\rho - 1}{\lambda - 1} \left[\frac{\beta_J}{\beta} \left(w^{(1-\alpha)\theta} \frac{\epsilon}{\epsilon - 1} \right)^{\frac{-1}{1-(1-\alpha)\theta}} \right] \left[\frac{1}{\beta} - (1-\delta) \right]^{-1} \right)^{\frac{1}{1-\rho}}. \tag{15}$$

3.5 Numerical simulations

We investigate the property of investment avalanches by numerical simulations. First, we compute the steady state of an economy with a continuum of firms. Then, we simulate the path of an economy with a finite n number of firms. The unit of time is a quarter. The expected aggregate capital is determined by a recursive equilibrium $K_{t+1}^e = \Xi(K_t)$. Using a computation result of the recursive equilibrium explained in the next section, and applying a first-order approximation of the dynamics, we set that $\log K_{t+1}^e$ follows an AR(1) process with persistent 0.975, i.e., the half-life period of a deviation from the steady state is 20 quarters. We assume constant returns to scale, $\theta = 1$. Time-to-build is set to $J = 8$ quarters. The lumpiness parameter is $\lambda = 1.2$ which corresponds to the benchmark value of 20% used in the literature on lumpy investments (Cooper, Haltiwanger, and Power, 1999; Gourio and Kashyap, 2007). The firm-level productivity process is specified as a logarithmic AR(1) process with persistence $\rho_a = 0.9$ and an i.i.d. shock of standard deviation $\sigma_a = 0.03$. We set the final wholesale sector as almost perfectly competitive ($\epsilon = 100$) and the intermediate

sector fairly competitive $\eta = 30$. The number of firms n is set at the level that Japanese business survey covers as in Figure 1. The rest of the parameter values are calibrated as shown in Table 1.

α	β	δ	ϵ	η	σ	Ψ	χ	λ	σ_a	ρ_a	J	n
0.36	0.995	0.025	100	30	1.5	1	500	1.2	0.03	0.9	8	30000

Table 1: Calibrated values of parameters

Figure 2 shows the simulated time series of aggregate investments. The standard deviation of the fluctuation is 2.4% and the autocorrelation coefficient is 0.83. Note that the exogenous shocks given to the model are firm-level independent productivity shocks with $\sigma_a = 0.03$. Thus, the simple average of productivity shocks yields $\sigma_a/\sqrt{n} \approx 0.017\%$ standard deviation, which cannot generate the 2.4% standard deviation of aggregate investment. The true source of aggregate fluctuations is the evolution of capital profile which determines the size of investment avalanches.

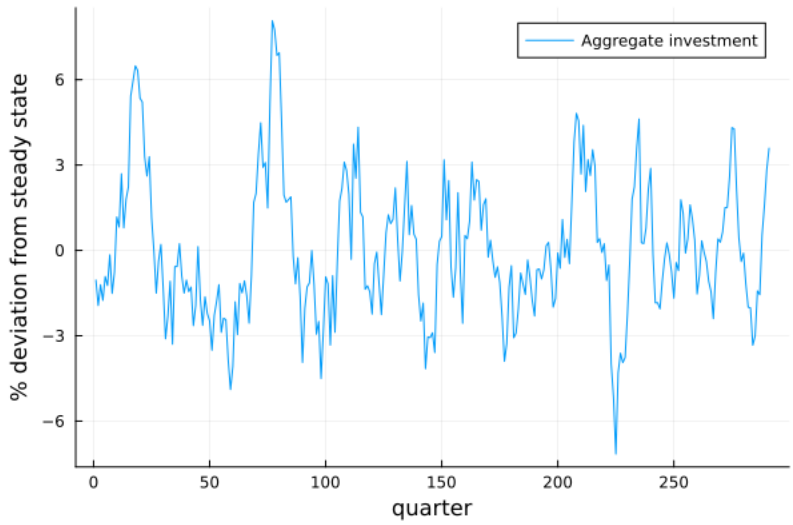


Figure 2:

The size of investment avalanches is measured by the number of firms involved in the avalanche after the initial response to the capital depreciation and productivity shocks occurred. Figure 3 shows a distribution of the investment avalanche measured by the number of firms in this way. The plot shows a counter-cumulative distribution conditional on whether

the avalanche resulted in a positive growth of aggregate capital or a negative growth. We observe that the size of investment avalanches exhibits a sizable fluctuation.

The number of firms in an investment avalanche and the lumpiness λ determines the magnitude of aggregate investment decided by firms in a period. In this simulation, the standard deviation of the aggregate investment amount decided is 10.5%. The decided amount is equally spread over the time-to-build periods J. The aggregate investment in a period is a sum of the overlapping investments decided in the current and the past $J-1$ periods. Hence, by diversification, the volatility of aggregate investments is smaller than that of aggregate investment amounts decided in a period.

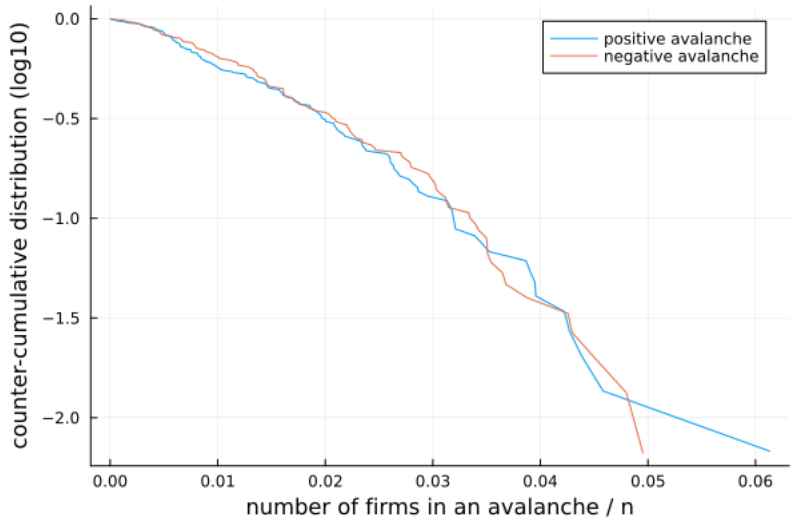


Figure 3:

Finally, we inspect the distributions of s_{it} over periods. In an analytical characterization, we assume that the distribution of s deviates little from the uniform distribution. Simulation results support this assumption. We computed the first four non-centered moments of $(s_{it})_i$ for each t. In a run for 300 quarters, the maximum deviation of the first moment from that of the uniform distribution is 1.1%, and less than 2.5% for all four moments. To check the convergence of moments, Figure 4 inspects if the processes of simulated moments are stationary and converging to their true moments. The simulation shows that the deviation process is stationary and that the time averages of the simulated moments seem to show convergence to the true moments of the uniform distribution and the assumed orthogonality

with a_{it}.

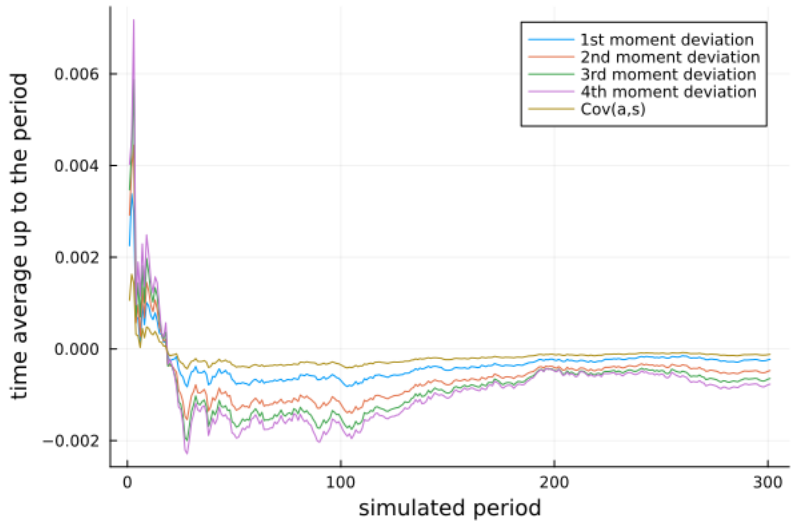


Figure 4: Time-averages of simulated moments $\mathbb{E}[a_{it}]-1/2$, $\mathbb{E}[a_{it}^2]-1/3$, $\mathbb{E}[a_{it}^3]-1/4$, $\mathbb{E}[a_{it}^4]-1/5$, and $Cov(a_{it}, s_{it})$.

3.6 Why does an avalanche occur? An intuition

To understand the investment avalanche intuitively, we contrast with an economy with smoothly adjusted capital without indivisibility. An intermediate good is monopolistically supplied with production function $y_i = k_i^{\alpha} l_i^{1-\alpha}$. The monopolist faces demand function $y_i = p_i^{-\eta} Y$ where $Y = (\int y_i^{(\eta-1)/\eta})^{\eta/(\eta-1)}$. Aggregate labor demand function is $L = (1-1/\eta)(1-\alpha)Y/w$, and labor supply $v'(L) = wC^{-\sigma}$. Goods market clearing condition is $Y = C + X$. Operating profit is $(1-1/\eta)(1-\alpha)y_i^{1-1/\eta}Y^{1/\eta}$. Substituting l, the profit function is $\varphi(k_i, K, w)$. Maximizing monopolist's value leads to $\Lambda_t/\Lambda_{t+1} - 1 + \delta = \partial\varphi/\partial k_i$. By aggregating this expression across i, k_i and K cancel out and we are left with the following

factor price condition[6]

$$w_t^{1-\alpha}(R_t - 1 + \delta)^{\alpha} = \alpha^{\alpha}(1-\alpha)^{1-\alpha}(1 - 1/\eta)\left(\int a_i^{\eta-1} di\right)^{\frac{1}{\eta-1}}.$$

As long as the factor prices must satisfy this equation, the optimal k_i is linear in K; hence, the equilibrium level of K is unaffected. As is often the case in a constant-returns-to-scale economy, the level of aggregate capital is determined by the households' decisions. Firms' decisions do constrain the factor prices, but they do not put a constraint on aggregate capital.

As in our model, the aggregate capital adjustment is made at the firm's extensive margin when capital is discrete. In this case, it no longer holds that any level of aggregate capital is consistent with firms' decisions. In other words, firms' decisions do constrain equilibrium aggregate capital. As a result, a socially efficient level of aggregate capital may not be supported as an equilibrium since it may violate the firm's decision at the extensive margin. When the firm at the extensive margin invests, it raises aggregate capital and incentivizes other firms to follow the firm in the smoothly adjusting model. The same incentive operates in a discrete adjusting model, which results in an avalanche.

4 Impulse-response functions for investment demand shocks

In this section, we incorporate the investment demand shocks in the recursive equilibrium defined in Section 2 and solve for impulse-response functions for a shock on aggregate investment demand.

[6]The derivation is straightforward.

$$\begin{aligned}\left(\int a_i^{\eta-1} di\right)^{\frac{1}{\eta-1}}\left(\frac{\Lambda_t}{\Lambda_{t+1}} - 1 + \delta\right) &= \rho(1-(1-1/\eta)(1-\alpha))\left(\frac{(1-1/\eta)(1-\alpha)}{w_t}\right)^{\frac{(1-\alpha)(1-1/\eta)+(1-\alpha)/(\alpha\eta)}{1-(1-1/\eta)(1-\alpha)}} \\ &= \alpha(1-1/\eta)\left(\frac{(1-1/\eta)(1-\alpha)}{w_t}\right)^{\frac{1-\alpha}{\alpha}}\end{aligned}$$

where $\rho = \alpha(1-1/\eta)/(1-(1-1/\eta)(1-\alpha))$.

4.1 The intuition in the simplest model

First, we formulate the model in the representative agent case to provide an intuition of the model's mechanism. The time-to-build is set at $J = 8$ quarters. We consider an investment demand shock that increases K_{t+J} by one percentage point from the steady-state level. The investment purchase is equally spent over $t, t+1, \ldots, t+J-1$. This simplification is motivated by introducing capital and avalanche dynamics, which is already an important deviation compared to the case of the no-capital economy. Second, to clarify the mechanisms further, we assume an exogenous labor supply and focus only on capital's effect. With the representative households and exogenous labor supply, the structure of the model is given by $L_t = L_{ss}$ and the representative households' Euler equation:

$$u'(c_t) = \beta \mathbb{E} \frac{1+i_t}{1+\pi^P_{t+1}} u'(c_{t+1})$$

with $1 + i_t = (1 + r_{ss})(1 + \pi_t)^{\phi}$. The goods market clearing condition is modified to

$$Y_t = c_t + \frac{\psi_P}{2}(\pi_t)^2 Y_t + X_t(1 + u_t).$$

The rest of the equations are the same as the ones in Section 2.6.

In this simple setup, we can easily compute the post-avalanche dynamics after period $t + J$. To understand the dynamics, we consider a first-order perturbation of the model. We denote as $\tilde{X}_t$ the proportional deviations of the variable X_t to its steady state value. Note that at period $t + J$, the economy starts with a higher capital stock: The economy is richer, and labor productivity is higher. As a consequence, the dynamics exhibit a positive supply shock compared to steady-state, generating a dynamics of inflation and consumption, such that $\tilde{c}_{t+J} > 0$ and $\pi_{t+J} < 0$.

The dynamics before the avalanche are also easy to derive. From the resource constraint, we obtain $c_t = K^{\alpha}_{ss} L^{1-\alpha}_{ss} - X_{ss}(1 + u_t) - \frac{\psi_P}{2}(\pi_t)^2 Y_t$. Thus, we get the first-order approximation,

$$\tilde{c}_t = -\frac{X_{ss}}{c_{ss}} u^{av},$$

where u^{av} is the constant proportional increase in investment during the time-to-build. In plain words, consumption decreases as investment increases (and output is fixed) during the time-to-build.

From $u'(c_t) = \beta \mathbb{E} \frac{1+i_t}{1+\pi_{t+1}} u'(c_{t+1})$ and $1 + i_t = (1 + r_{ss})(1 + \pi_t)^{\phi}$, we obtain

$$\tilde{c}_{t+1} - \tilde{c}_t = \frac{1}{\sigma}(\phi \pi_t - \pi_{t+1}).$$

Thus, in the period before the capital is increased, $t + J - 1$, we have

$$\pi_{t+J-1} = \frac{1}{\phi}\left(\sigma\left(\tilde{c}_{t+J} + \frac{X_{ss}}{c_{ss}} u^{av}\right) + \pi_{t+J}\right).$$

As a consequence, since it is likely that $\tilde{c}_{t+J} + \frac{X_{ss}}{c_{ss}} u^{av} > 0$ is high and dominates $\pi_{t+J} < 0$, we can have a positive inflation in period $t + J - 1$ when u^{av} is sufficiently high.

For $t \leq t + J - 1$, we have $\tilde{c}_{t+1} = \tilde{c}_t$ and, as consequence, $\pi_t = \frac{1}{\phi}\pi_{t+1}$. To summarize this analysis, inflation increases at the end of the increased investment demand as consumption rises sharply. During the buildup of investments, inflation slowly grows due to the increase in future expected inflation.

The following graphs represent the dynamics of this economy for a standard calibration $\beta = 0.99$, $\sigma = 1$, $\delta = 0.025$, $\alpha = 0.33$, $\psi_P = 100$, $\epsilon = 6$, and $\phi = 1.5$. We obtain the following dynamics as provided in Figure 5.

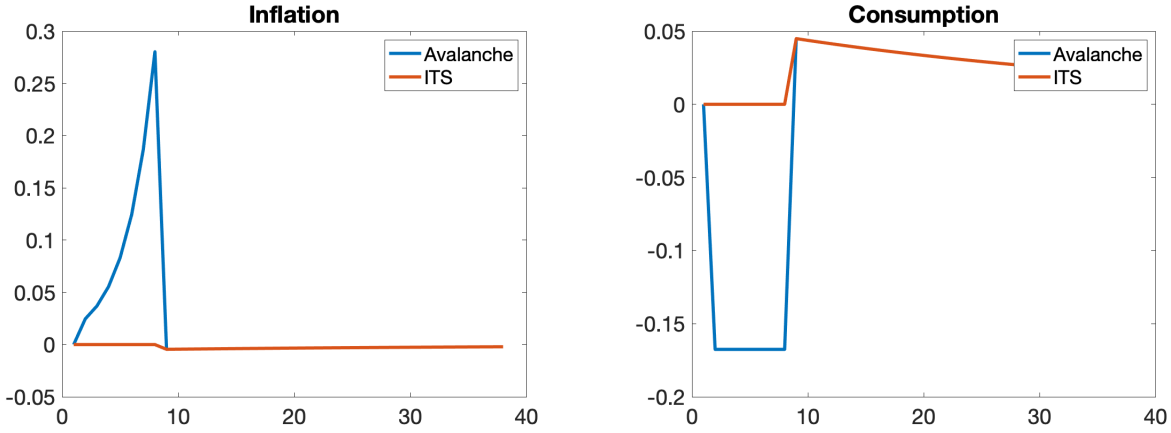


Figure 5: Representative agent case with exogenous labor. The blue line shows the responses when an investment demand shock hits. The orange line shows the responses when an investment technology shock hits. Plots show percentage deviations from steady states against quarters after the shock.

We use the same model for comparison but replace the investment demand shock with an exogenous aggregate shock on investment technology u_t^{ITS} as in Justiniano, Primiceri,

and Tambalotti (2010). In this alternative setup, the invested amount X_t is expanded by a shock when it is installed into capital as $K_{t+J} = (1-\delta)K_{t+J-1} + X_t(1+u_t^{ITS})$. We set u_t^{ITS} to increase K_{t+J} by one percent, just like for the investment demand shock. The difference from the investment demand shock is that the shock does not require any resources before $t+J$.

In this setup, the equilibrium with the investment demand shock and the investment technology shock does not differ after K_{t+J}. The two impulse response functions coincide. Because of the time to build, no change occurs in K before $t+J$. In period $t+J$, the capital jumps up by one percent and then gradually declines to the steady state afterward. The speed of convergence in terms of half-life is about 20 quarters.

In Figure 5, one can observe that the investment demand shock changes consumption, inducing very different inflation dynamics from the investment technology shock.

4.2 The two-agent economy

We now simulate the economy of the baseline model with two types of households. To highlight the effect of inflation, we here focus on the case $J = 2$. Consequently, we will observe the inflation movement only before and after the capital increase.

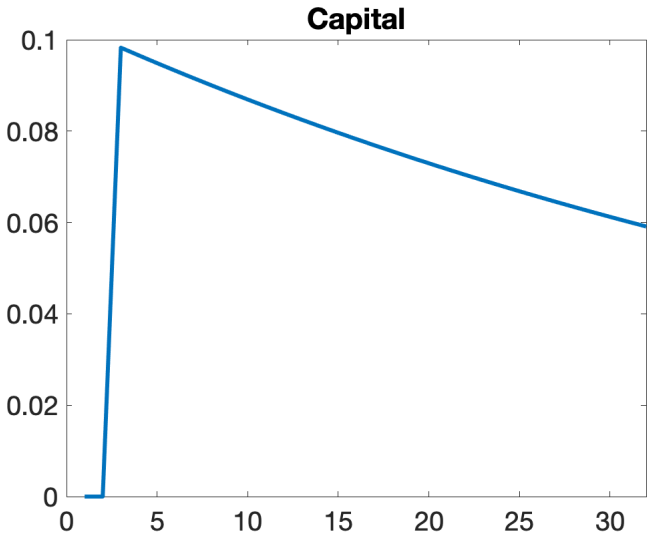


Figure 6: Impulse response function in a two-agent model

Figure 6 plots the path of capital. Figure 7 plots the impulse response functions for inflation rates (left panel) and consumption (right panel). The left panel of Figure 7 shows a sizable positive response in inflation when an investment demand shock occurs, whereas an investment technological shock does not generate such a positive response. This pattern can be understood by examining the consumption path first.

The consumption path is simple to understand. For an investment technological shock, no change occurs before $t+J$. In $t+J$, the shock enlarges the investment goods installed into capital, and the capital increases by 1%. The consumption increases in $t+J$ and declines with capital. For an investment demand shock, the consumption path is identical to the case of an investment technological shock after $t+J$. Before $t+J$, the avalanche shock reduces the dividend payments, while labor income is unaffected because labor supply and labor productivity are constant. Hence, consumption decreases by increased investment in the periods between t and $t+J$.

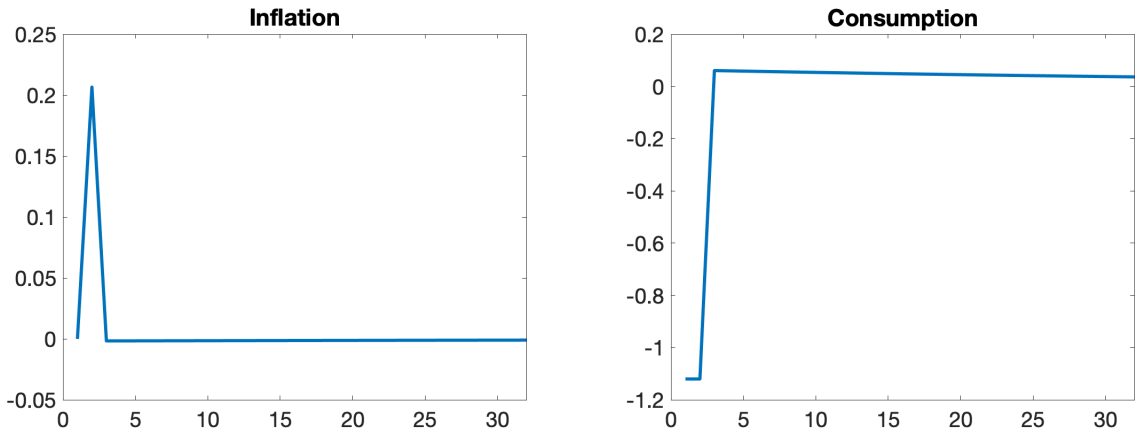


Figure 7: Impulse response functions for inflation rates and consumption (%)

In $t+J$, the inflation rate goes slightly under the steady-state level (zero) for both shocks. This is because the consumption after $t+J$ is decreasing. Since the marginal utility of consumption increases in these periods, the real risk-free rate is lower than the steady state, as shown in the households' Euler equation. The low real rate is achieved by deflation under the monetary policy guided by the Taylor principle. An investment avalanche shock also leads to the same path after $t+J$.

The difference occurs before $t + J$. In the investment avalanche case, the consumption jumps in $t+J$ when the investment is completed, and the installed capital generates income. The boom in consumption is associated with a reduction in the real risk-free rate by the households' Euler equation, which is accompanied by positive inflation. One can interpret it as that the investment avalanche shock crowds out consumption demand, whereas the anticipated consumption boom after the investment avalanche induces loan demand and drives up the equilibrium real interest rate, leading to positive inflation under the Taylor principle.

5 Conclusion

This paper presents a model in which aggregate investment demand shocks arise. We consider that the firm's capital is indivisible, and hence, the investment at the firm level is lumpy. We also consider a monopolistically competitive economy where an increase in aggregate demand incentivizes firms' investments. In this setup, a firm's lumpy investment induces another firm's lumpy investment, inducing an investment avalanche. How far the investment avalanche continues depends on firms' capital profile and idiosyncratic productivity. Hence, as the capital and productivity profile evolves, the avalanche size also varies. This generates fluctuations in investment avalanche size, which we call investment demand shocks.

The paper analyzes the fluctuations of the investment avalanche analytically and numerically. Our analysis shows that the investment demand shocks generated by the avalanche are quantitatively substantial. Also, by incorporating time-to-build, the investment demand shocks exhibit autocorrelation. Thus, the investment avalanche provides a microfoundation for the animal spirits that drive investment demands in business cycles.

We incorporate the mechanism of investment avalanches in an otherwise standard New Keynesian business cycle model. The standard software readily solves the model with the micro-founded investment demand shocks. We investigate impulse-response dynamics of an investment demand shock in a simplified setup of our model. The analysis shows that the investment demand shock can generate a positive comovement of inflation and output in the business cycle frequency under a reasonable parameter alignment.

Appendix

A Proof of Proposition 1

Firm i's capital is written as $k_{it} = \lambda^{s_{it}} k^*_{it}$ and $k^*_{it} = a_{it}^{\eta-1} BK_t$. Using this, the evolution of the aggregate capital is written as follows.

$$\begin{aligned} K_{t+1}^{\rho} &= \int \left[\int_0^{s^*+\nu} (a_{it}^{1/\alpha} \lambda (1-\delta) \lambda^{s_{it}} a_{it}^{\eta-1} BK_t)^{\rho} ds + \int_{s^*+\nu}^{1} (a_{it}^{1/\alpha} (1-\delta) \lambda^{s_{it}} a_{it}^{\eta-1} BK_t)^{\rho} ds \right] da \\ &= (\lambda^{\rho} - 1) \int \left[\int_0^{s^*+\nu} (a_{it}^{1/\alpha} (1-\delta) \lambda^{s_{it}} a_{it}^{\eta-1} BK_t)^{\rho} ds \right] da + K_t^{\rho} \end{aligned}$$

Thus, we have $\partial \rho \log K_{t+1} / \partial \nu = (\lambda^{\rho} - 1) \int (a_{it}^{1/\alpha} (1-\delta) \lambda^{s^*+\nu} a_{it}^{\eta-1} BK_t)^{\rho} da / K_{t+1}^{\rho}$. This expression is evaluated at $\nu = 0$ by using $s^* = (-\log(1-\delta) + (\eta-1)\Delta \log a_{it}) / \log \lambda$ as

$$\begin{aligned} \left. \frac{\partial (\rho \log K_{t+1})}{\partial \nu} \right|_{\nu=0} &= \frac{(\lambda^{\rho} - 1) \int (a_{it}^{1/\alpha} (1-\delta) \lambda^{s^*} a_{it}^{\eta-1} BK_t)^{\rho} da}{K_{t+1}^{\rho}\big|_{\nu=0}} \\ &= \frac{(\lambda^{\rho} - 1) \int (a_{it}^{1/\alpha} a_{i,t+1}^{\eta-1} BK_t)^{\rho} da}{K_t^{\rho}}. \end{aligned}$$

We impose by assumption that $a_{i,t+1} = a_{it}$. Note that $(1/\alpha + \eta - 1)\rho = \eta - 1$. Also, $B^{-\rho} = \mathbb{E}^F[\lambda^{s\rho} a^{\eta-1}] = \frac{\lambda^{\rho}-1}{\rho \log \lambda} \mathbb{E}^F[a^{\eta-1}]$ under uniformly distributed s. Using these, the decline of $s_{i,t+1}$ by the increase of K_{t+1}, which is $\Delta \log K_t / \log \lambda$, is evaluated as

$$\left. \frac{d\nu'}{d\nu} \right|_{\nu=0} = \frac{1}{\log \lambda} \left. \frac{\partial \log K_{t+1}}{\partial \nu} \right|_{\nu=0} = \frac{\int a_{it}^{\eta-1} da}{\mathbb{E}^F[a^{\eta-1}]} = 1.$$

Hence, we obtain $\vartheta = \lim_{\nu \to 0} (\nu'/\nu) = \lim_{\nu \to 0} d\nu'/d\nu = 1$.

B Mean of m_u^n in Section 3.3

In a model with n firms, aggregate capital is $K = \left(\sum_{i=1}^n (a_i^{1/\alpha} k_i)^{\rho} / n \right)^{1/\rho}$. Firm i's lumpy investment increases K^{ρ} by $(\lambda^{\rho} - 1) a_i^{\rho/\alpha} \lambda^{\rho s_1} (k_i^*)^{\rho} / n$. Since s_i and k_i are bounded, the increase in $\log K$ due to i's investment is $O(1/n)$. Provided that m_u^n is finite, ϵ_u^n is also $O(1/n)$. Hence $|s_u^n - s^*|$ is $O(1/n)$, where $s^* = -\log(1-\delta)/\log \lambda$. Since s_i is located in $[s_{u-1}^n, s_u^n)$, we have $s_i - s^* = O(1/n)$. Moreover, $\Delta \log K = \Delta(K^{\rho})/(\rho K^{\rho}) + O(\Delta \log K)^2$. Now, the increase of K due to firm i's investment reduces all the other firms' state s_j. The probability of those firms

going below the threshold is equal to $\Delta \log K / \log \lambda$ under the uniform distribution of s_i, and there are $n - \sum_{v=0}^{u} m_u^n$ firms that potentially hit the threshold. Then the mean number is $(n-1)\Delta \log K / \log \lambda$. Combining the results above, we obtain $\lim_{n\to\infty}(n-1)\frac{\Delta \log K}{\log \lambda} = \vartheta$. When there are m_u^n firms investing in u, the resulting $\Delta \log K$ is a sum of the increase of $\log K$ by each firm in M_u^n up to order $1/n$. Hence, $\lim_{n\to\infty}(n-1)\frac{\Delta \log K}{\log \lambda} = \vartheta m_u^n$.

C Heterogeneous time-to-build

In what follows, we extend the time-to-build model to the case where there are multiple types of intermediate firms with different time-to-build. We show that the heterogeneous time-to-build model augments the autocorrelation in the series of aggregate demand shocks u_t.

C.1 1-period and 2-period types

Suppose that ν_τ $(\tau = 1, 2)$ fraction firms are type-τ with τ-period time-to-build. For type $\tau = 2$, the investment decision occurs in t, two periods before the capital generates output in $t+2$. The investment purchase occurs one period before, in $t+1$.

Let $\varphi_t(k)$ denote the operating profit function in period t. At the threshold of investment spike, $k^* = k_{j,t+2}$, firm j of 2-period type is indifferent between deciding a spike in $t+1$ and a spike in $t+2$. Thus,

$$E_t\left[\Lambda_{t+2}(\varphi_{t+2}(k^*) + (1-\delta)k^*) - \Lambda_{t+1}k^* - (\Lambda_{t+2}(\varphi_{t+2}(\lambda k^*) + (1-\delta)\lambda k^*) - \Lambda_{t+1}\lambda k^*)\right] = 0.$$

For simplicity, we impose perfect foresight. Solving for k^* yields the similar threshold rule as 1-period type:

$$k_j^* = a_{j,t+2}^{\eta-1} K_{t+2} \left(\frac{\lambda^\rho - 1}{\lambda - 1}\frac{1-(1-1/\eta)(1-\alpha)}{\frac{\Lambda_{t+1}}{\Lambda_{t+2}} - 1 + \delta}\right)^{\frac{1}{1-\rho}} \left(\frac{(1-1/\eta)(1-\alpha)}{w_{t+2}}\right)^{\frac{1-\alpha}{\alpha}\frac{1}{1-\rho}}$$

Thus, the optimal policy for the 2-period type is the same as the 1-period type except that the investment decision is made two-period before. The aggregate capital in a stationary equilibrium is unaltered from the homogeneous case.

Let n denote the total number of firms and $n_\tau = \nu_\tau n$ the number of type-τ firms. The type-1 firms are indexed by $i_1 = 1, \ldots, n_1$ and the type-2 $i_2 = n_1 + 1, \ldots n_1 + n_2$. Aggregate investment X_t is the sum, $X_t = X_t^1 + X_t^2$, of investments aggregated for each type, X_t^τ for $\tau = 1, 2$, where

$$X_t^\tau = \sum_{i_\tau : (1-\delta)k_{i_\tau,t-1} < k^*_{i_\tau,t}} (\lambda - 1 + \delta) k_{i_\tau,t}.$$

The investment of type-2 firms, X_t^τ, is committed in $t-1$. The aggregate committed amount of type-2 in period t is denoted by S_t^2. Thus, S_t^2 is an aggregate of decision variables in t, and determines the investment in the next period as $X_{t+1}^2 = S_t^2$.

The aggregate investment in a stationary equilibrium is the same as the homogeneous case. An investment avalanche process differs from the homogeneous case. Suppose that there is a gap in the initial realization of K_{t+1} from the expected capital K_{t+1}^e. This gap impacts K_{t+1} and K_{t+2}, hence both types of firms adjust their investments in t. The effect on the threshold for type-1 is the same as the homogeneous case. The effect for type-2 is different: an increase ΔK_{t+1} induces an increase of expected aggregate capital in $t+2$, which leads to an avalanche of type-2 firms. We formulate the avalanche process as follows.

Let K_{t+1}^e and K_{t+2}^e denote the future aggregate capital expected at the beginning of t. K_{t+1}^e consists of $(1-\delta)K_t$, the investment commitment S_t^2 of type-2 decided in $t-1$, and the expected investment of type-1. For K_{t+2}^e, there is no predetermined portion of capital. Provided with K_{t+1}, firms and households can form expectation on K_{t+2} using the general equilibrium model as an expectation system. Let $K_{t+2}^e = \Xi(X_{t+1}^e)$ denote the expectation system.

If the actual investment decisions at the beginning of t is consistent with the expected aggregate capital K_{t+1}^e, K_{t+2}^e, the process stops. Otherwise, the threshold is updated and the process continues. Since K_{t+1} does not depend on S_t^2, the investment of type-1 is determined without considering S_t^2.

Consider the threshold of type-1:

$$\begin{aligned}
k^*_{i,t+1} =& a_{i,t+1}^{\eta-1} K_{t+1} B_{t+1} \\
B_{t+1} :=& \left(\frac{\lambda^\rho - 1}{\lambda - 1} \frac{1 - (1 - 1/\eta)(1-\alpha)}{\frac{\Lambda_t}{\Lambda_{t+1}} - 1 + \delta} \right)^{\frac{1}{1-\rho}} \left(\frac{(1-1/\eta)(1-\alpha)}{w_{t+1}} \right)^{\frac{1-\alpha}{\alpha}\frac{1}{1-\rho}}
\end{aligned}$$

The threshold of type-2 is $k^*_{i,t+1} = a^{\eta-1}_{i,t+1} E_{t-1}[K_{t+1}B_{t+1}]$.

By taking a weighted sum, we obtain

$$\int_0^1 (a^{1/\alpha}_{i,t+1}\lambda^{s_{i,t+1}}k^*_{i,t+1})^\rho di = \int_0^{\nu_1} (a^{\eta-1+1/\alpha}_{i,t+1}\lambda^{s_{i,t+1}})^\rho di (K_{t+1}B_{t+1})^\rho + \int_{\nu_1}^1 (a^{\eta-1+1/\alpha}_{i,t+1}\lambda^{s_{i,t+1}})^\rho di E_{t-1}(K_{t+1}B_{t+1})^\rho$$

Hence

$$1 = \int_0^{\nu_1} (a^{\eta-1+1/\alpha}_{i,t+1}\lambda^{s_{i,t+1}})^\rho di B^\rho_{t+1} + \int_{\nu_1}^1 (a^{\eta-1+1/\alpha}_{i,t+1}\lambda^{s_{i,t+1}})^\rho di \left(\frac{E_{t-1}[K_{t+1}B_{t+1}]}{K_{t+1}}\right)^\rho$$

At the beginning of t, K^e_{t+1} coincides with $E_{t-1}[K_{t+1}]$ applied to form $k^*_{i,t+1}$ of type-2. We also assume that firms expect B_{t+1} to be constant that satisfies the above equation.

By taking a partial weighted sum up to ν_1, we obtain

$$\begin{aligned}\int_0^{\nu_1} (a^{1/\alpha}_{i,t+1}\lambda^{s_{i,t+1}}k^*_{i,t+1})^\rho di &= \int_0^{\nu_1} (a^{\eta-1+1/\alpha}_{i,t+1}\lambda^{s_{i,t+1}})^\rho di (K_{t+1}B_{t+1})^\rho \\ &= \int_0^{\nu_1} (a^{\eta-1+1/\alpha}_{i,t+1}\lambda^{s_{i,t+1}})^\rho di \left(\int_0^{\nu_1} (a^{1/\alpha}_{i,t+1}\lambda^{s_{i,t+1}}k^*_{i,t+1})^\rho di + \int_{\nu_1}^1 (a^{1/\alpha}_{i,t+1}k_{i,t+1})^\rho di\right) B^\rho_{t+1}\end{aligned}$$

Hence,

$$\int_0^{\nu_1} (a^{1/\alpha}_{i,t+1}\lambda^{s_{i,t+1}}k^*_{i,t+1})^\rho di = \frac{\int_0^{\nu_1} (a^{\eta-1+1/\alpha}_{i,t+1}\lambda^{s_{i,t+1}})^\rho di B^\rho_{t+1}}{1 - \int_0^{\nu_1} (a^{\eta-1+1/\alpha}_{i,t+1}\lambda^{s_{i,t+1}})^\rho di B^\rho_{t+1}} \int_{\nu_1}^1 (a^{1/\alpha}_{i,t+1}k_{i,t+1})^\rho di$$

Thus,

$$K^\rho_{t+1} = \frac{1}{1 - \int_0^{\nu_1} (a^{\eta-1+1/\alpha}_{i,t+1}\lambda^{s_{i,t+1}})^\rho di B^\rho_{t+1}} \int_{\nu_1}^1 (a^{1/\alpha}_{i,t+1}k_{i,t+1})^\rho di \tag{16}$$

We assume that, at the beginning of t, firms expect that $s_{i,t+1}$ will follow a stationary distribution. Then, the above equation provides the initial expectation K^e_{t+1} using the capital information of type-2 firms only.

Using this K^e_{t+1} as the initial expectation of type-1, we can define an avalanche process that determines equilibrium K_{t+1}.

Provided with the equilibrium K_{t+1}, we update $K^e_{t+2} = \Xi(K_{t+1})$. Suppose a gap in K_{t+2}. Then, the thresholds of type 2 are updated. If some of them undercut the threshold, they decide to invest and increase S_t. The increase in S_t, in turn, affects the threshold of type-2 firms.

In contrast, suppose that there is a gap in K_{t+1}. This affects the thresholds of type-1. If some decide to invest, that increases X_t, K_{t+1}, and K_{t+2}. The avalanche process continues for both types. We formulate the process as an algorithm.

1. K^e_{t+1}, K^e_{t+2}, S^2_{t-1}, and X^2_t are given.

2. k^*_t is determined and investment decisions X_t and S_t are computed. The investments update K_{t+1} and K_{t+2}.

3. If $K_{t+1} \neq K^e_{t+1}$, k^*_{1t} is updated.

4. Compute X_t, K_{t+1}, and K_{t+2}. If these values are updated, go to Step 3. If these values are unchanged, proceed to Step 5.

5. If $K_{t+2} \neq K^e_{t+2}$, k^*_{2t} is updated.

6. Compute S^2_t and K_{t+2}. If these values are updated, go to Step 5. Otherwise, stop.

In this equilibrium, an autocorrelation of aggregate investment may arise. This is because an investment decision S^2_t affects the prospect of K_{t+2} expected in $t+1$ and affects the initial expectation of type-1 in $t+1$. Thus, investment avalanches propagate from S^2_t to X^1_t. We will generalize this propagation in a multi-period environment below.

C.2 Multi-period time-to-build: Computational details

There are $\bar{\tau} \in \mathcal{Z}$ types of firms, where $\bar{\tau}$ does not exceed $-\log\lambda/\log(1-\delta)$. Let $\mathcal{N}_\tau$ denote the set of type-τ firms. The cardinality of $\mathcal{N}_\tau$ is denoted by n_τ, and the fraction of the firms is $\nu_\tau := n_\tau/n$. In computation, firm index i is set in the ascending order of type τ, i.e., firms $i = 1, \ldots, n_1$ are type-1, $i = n_1 + 1, \ldots, n_1 + n_2$ are type-2, and so forth.

A type-τ firm has τ periods of time-to-build. The firm's investment decision in period t materializes in new capital in $t+\tau$. For simplicity, we assume that the purchase of investment goods occurs in $t + \tau - 1$, i.e., one period before the new capital materializes.

The state of firm profiles in period t is a triple of length n vectors, (a_t, k_t, s_t), where a_t and k_t denote the firm-level productivity and capital respectively, whereas s_t denotes the state of committed future investments. For a type-τ firm i, s_{it} takes a value from $\{0, 1, \ldots, \tau - 1\}$. State $s_{it} = 0$ indicates that firm i at the end of period t is not committed to investment in the future. State $s_{it} = u$ means that i is committed to investing in period $t + u$, and its new capital materializes in $t + u + 1$. For example, a type-2 firm with $s_{it} = 1$ will invest in

$t+1$. A type-1 firm's state is always zero because its investment decision and purchasing investment goods occur in the same period.

Type-τ firms make decision on investment using the threshold $k^{*\tau}_{i,t+\tau} = a^{\eta-1}_{i,t+\tau} B K_{t+\tau}$ where B is a constant defined previously. In period t, a type-τ firm in state $s_{i,t-1} = 0$ decides to invest if $(1-\delta)^\tau k_{i,t} < k^{*\tau}_{i,t+\tau}$ and transits to $s_{i,t} = \tau - 1$. After $t+1$, the state s_i decreases by one in every period until period $t+\tau-1$. In $t+\tau-1$, the firm invests an amount $x_{i,t+\tau-1} = (\lambda-1)(1-\delta)^{\tau-1} k_{i,t}$, and its capital in $t+\tau$ becomes $k_{i,t+\tau} = \lambda(1-\delta)^\tau k_{i,t}$.

We define a decision function of a type-τ firm as $\gamma_\tau : \mathcal{R}^4 \mapsto \mathcal{R}$ where

$$\gamma_\tau(a_i, k_i, s_i, K^e) = \begin{cases} \lambda(1-\delta)^\tau k_i & \text{if } (1-\delta)^\tau k_i < k^{*\tau}_i(a_i, K^e) \text{ and } s_i = 0 \\ \lambda^{-1}(1-\delta)^\tau k_i & \text{if } (1-\delta)^\tau k_i > \lambda k^{*\tau}_i(a_i, K^e) \text{ and } s_i = 0 \\ (1-\delta)^\tau k_i & \text{otherwise} \end{cases}$$

and $k^{*\tau}_i(a_i, K^e) = a^{\eta-1}_i B K^e$.

Firms form expectations on aggregate capital K^e. The relevant aggregate capital for type-τ firms' decisions in t is $K_{t+\tau}$. In period t, type-τ ($\tau < \bar{\tau}$) firms observe $(S^{\tau+1}_{t-1}, \ldots, S^{\bar{\tau}}_{t-(\bar{\tau}-\tau)})$, which determines $(X^{\tau+1}_{t+\tau-1}, \ldots, X^{\bar{\tau}}_{t+\tau-1})$ and capital $k_{i,t+\tau}$ of type τ' firms with $\tau' > \tau$. Thus, using an argument similar to (16), type-$\tau < \bar{\tau}$ firms form $K^e_{t+\tau}$ at the beginning of t as

$$K^e_{t+\tau} = \left(1 - \int_{\bigcup_{\tau' \leq \tau} \mathcal{N}_{\tau'}} (a^{\eta-1+1/\alpha}_{i,t+\tau} \lambda^{s_{i,t+\tau}})^\rho di B^\rho\right)^{-1/\rho} \left(\int_{\bigcup_{\tau' > \tau} \mathcal{N}_{\tau'}} (a^{1/\alpha}_{i,t+\tau} k_{i,t+\tau})^\rho di\right)^{1/\rho} \tag{17}$$

with positing $s_{i,t+\tau}$ to follow the stationary distribution.

Type-$\bar{\tau}$ firms do not observe relevant S^τ in t which directly affects $K_{t+\bar{\tau}}$. Provided with $K^e_{t+\bar{\tau}-1}$ in (17), firms and households form expectations for $K_{t+\bar{\tau}}$ equilibrium recursive mapping Ξ (defined by the expectation system) as $K^e_{t+\bar{\tau}} = \Xi(K^e_{t+\bar{\tau}-1})$.

C.3 Investment avalanches with heterogenous time-to-build

Among the decisions revealed until period t, the directly relevant decisions for type-τ firms are those of type-τ' with $\tau' \geq \tau$, which is exploited as in (17). Thus, the decisions within t allow for sequential solution in the ascending order of types. First, type-1 firms decide given the committed investments of other types in t. Subsequently, type-τ decides provided with the committed investments of types $\tau' \geq \tau$. Lastly, type-$\bar{\tau}$ decides given $K^e_{t+\tau} = \Xi(K^e_{t+\tau-1})$.

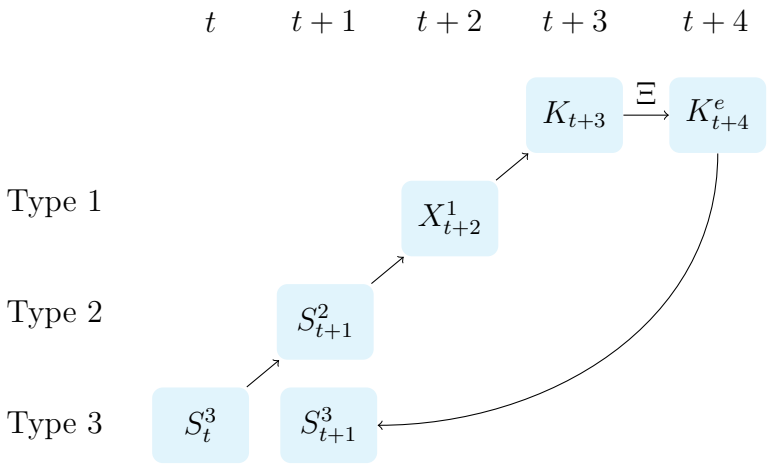


Figure 8: Cascading avalanches across heterogeneous types of time-to-build

The algorithm proceeds as follows.

1. Initialize $\tau = 1$. Set $\tilde{k}_t = k_t$.
2. Compute $\tilde{k}_{i,t+\tau}$ for $i \in \bigcup_{\tau' > \tau} \mathcal{N}_{\tau'}$ by applying depreciations $(1-\delta)^\tau$ and committed investments to $\tilde{k}_{i,t}$.
3. Compute $K^e_{t+\tau}$ by substituting $\tilde{k}$ in (17), if $\tau < \bar{\tau}$. If $\tau = \bar{\tau}$, set $K^e_{t+\bar{\tau}} = \Xi(K^e_{t+\bar{\tau}-1})$.
4. Compute $\hat{k}_i = \gamma_\tau(a_{i,t+\tau}, \tilde{k}_{i,t+\tau}, s_{i,t+\tau}, K^e_{t+\tau})$ for $i \in \mathcal{N}_\tau$.
5. If $\hat{k}_i = \tilde{k}_i$ for all $i \in \mathcal{N}_\tau$, proceed to the next step. Otherwise, set $\tilde{k}$ to $\hat{k}$ and go back to the previous step.
6. Set $K_{t+\tau} = K^e_{t+\tau}$. Increase τ to $\tau + 1$ and go to step 2, if $\tau < \bar{\tau}$. Otherwise, stop.

The avalanche of investment decisions of type-τ causes another avalanche of investment decisions of type $\tau - 1$ in the next period, amplifying the fluctuation of investment amounts in τ periods ahead. Suppose that type-$\bar{\tau}$ firms decide to invest more than expected in t. This increase of $S^{\bar{\tau}}_t$ raises the expectation for $K^e_{t+\bar{\tau}}$ formed in $t+1$, which leads to an increase of the threshold of type $\bar{\tau} - 1$ firms in $t+1$. The increased threshold shifts up the likelihood of a greater value of $S^{\bar{\tau}-1}_{t+1}$. If a large value of $S^{\bar{\tau}-1}_{t+1}$ realizes, it increases the threshold of type $\bar{\tau} - 2$ in $t+2$. This process continues up to type-1 firms. Thus, the series $(S^{\bar{\tau}}_t, S^{\bar{\tau}-1}_{t+1}, \ldots, S^2_{t+\bar{\tau}-2}, X^1_{t+\bar{\tau}-1})$ shows an autocorrelation.

Also, the investment avalanche produces an autocorrelation of X_t by the following mechanism. Suppose again that the aggregate investment of type-$\bar{\tau}$ firms, $S_t^{\bar{\tau}}$, is greater than expected. The aggregate capital $K_{t+\bar{\tau}}^e$ becomes large through the mechanism described in the previous paragraph. Hence, $K_{t+\bar{\tau}+1}^e = \Xi(K_{t+\bar{\tau}}^e)$ becomes large, leading to a greater $S_{t+1}^{\bar{\tau}}$. Thus, a shock in $S_t^{\bar{\tau}}$ causes autocorrelation in $K_{t+\bar{\tau}}, K_{t+\bar{\tau}+1}, \ldots$.

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