Business Succession and Economic Dynamism: The Role of Aging in Resource Misallocation*

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Abstract

Declining population growth makes it harder for business successions to be accomplished. We develop a model that links population decline and aging to macroeconomy through this "succession channel". Firm heterogeneity arises from managerial ability evolving over the lifecycle, that makes individual occupational choices and firms' decisions age-dependent, thereby shaping the impact of demographic changes on aggregate economy. We validate the models' property by comparing the model-generated firm dynamics with what we actually observe in a massive size of firm-level panel data accompanied by succession records in Japan. The model suggests that, in addition to the natural consequences of lower population growth (e.g., high exit rates and low succession rates), per capita output could possibly become larger due to the accumulation of human capital and the selection of firms through succession process.

Keywords: Succession; managerial ability; entry and exit; macroeconomy
JEL classification: E22, L16, O31, O41

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1 Introduction

Declining population growth rates are the typical phenomena we observe in advanced economies (Bloom et al. (2024)). The consequences of this demographic change have been intensively examined from the viewpoints of shrinking labor supply (e.g., Ferrero (2010); Backus et al. (2014); Sasaki and Hoshida (2017)). Such a decrease in labor supply due to lower population growth rates negatively affects a country's aggregate economy through a direct effect originating from the tightened resource constraint. Recent studies have been further highlighting additional indirect effects of lower population growth rates through, for example, smaller capital accumulation (Cooley et al. (2024)) as well as lower entrepreneurship and innovative activities (Engbom (2019); Peters and Walsh (2021); Hopenhayn et al. (2022); Jones (2022); Karahan et al. (2024); Inokuma and Sanchez (2024)).

In the present paper, we tackle this important question on the relation between demographic changes and macroeconomy by paying a special attention to "business successions" from predecessors (i.e., old top managers) to successors (i.e., new top managers). Such successions become harder to be accomplished under lower population growth rates because more predecessors need to succeed their businesses to potential successors in a smaller pool. The goal of this paper is to examine the qualitative and quantitative natures of this specific "succession channel" linking demography to macroeconomy by developing a theoretical model, that is consistent with the facts on business successions provided by Japanese firm-level data.

Toward this end, first, we construct a general equilibrium model, in which firms produce their output by using capital, labor, and managerial resource of top managers. An important feature of this model is that top managers' ability to operate a firm follows a hump-shaped (exogenous and deterministic) dynamics and thus they would definitely retire at some point in their tenure. In the process toward their retirement, top managers would optimally choose whether they continue to operate the firm or search for their successor by paying costs while those managers might also choose to

close its business without succession. Here, to achieve the business succession, top managers need to be matched with a worker (i.e., a potential successor) who also faces a hump-shaped dynamics of managerial ability that would be effective only when he or she works as a manager. The efficiency of this succession process naturally depends on the population growth rate. To illustrate, under declining population growth, a larger number of current managers would want to find their successor while the size of the pool of potential successors (i.e., the current workers) would become smaller. As a result, the succession rate would be lower, which further lowers an incentive to search for successors. Along with this natural mechanisms, our model incorporates firms' endogenous entry and exit, growth, and succession with heterogeneity in terms of managers' ability and age, all of which allow us to implement various comparative statics with respect to population growth rate.

Given this theoretical model, second, we validate the model's property by comparing the model-generated data of the firm dynamics and aggregate equilibrium objects with the counterparts in actual data accounting for a massive number of Japanese business enterprises over two decades. On the one hand, using the data of model-generated firm dynamics, we measure the causal impacts running from business succession to firms' sales dynamics in the model. Precisely, focusing on the firms owned by the top managers being in the process of searching for their successor, we measure the responses of firm sales to the purely exogenous event of succession. On the other hand, using the detailed firm-level panel data accompanied by succession records, we implement a standard causal inference of the impacts running from succession to firm sales and compare it with the causal parameters obtained from the model-generated data.

Third, using the casually calibrated (temporal) model, we simulate the equilibrium objects and numerically implement various comparative statics. Through this exercise, we aim to examine the qualitative and quantitative natures of the succession channel running from demography to macroeconomy.

The results obtained in the present paper are summarized as follows: First, we have constructed a tractable general equilibrium model in which we can perform various comparative statics of equilibrium objects with respect to population growth rates. We define the competitive equilibrium as the steady state in per capita terms accompanied by stationary distributions of economic objects such as the productivity of firms and managers ages. The change in population growth rates, that are governed by the birth and death rates, affect each individual dynamics as well as the equilibrium objects.

Second, we estimate the causal impacts running from succession to firm sales dynamics by comparing the model-generated sales dynamics of firms in the process of successor search and find their successor with that in the search process but could not make it. The sales moves in a parallel manner before the succession for those two cases, then the sales of the former starts to increase due to the succession, which we also confirm in the actual Japanese data. As an additional exercise, when we simply depict the model-generated sales dynamics of the firms experiencing succession and that do not without focusing on the searching managers, the relative sales size of the former to the latter declines toward the succession, then starts to increase after the succession. Again, we have also confirmed this pattern in the actual Japanese data. The downward trend of sales size of the firms experiencing succession toward the timing of the succession reflects the decay of managers' ability, which is replaced through the succession. As the last exercise, we also check to what extent the model generated age transition patterns of predecessors and successors are consistent with what we observed in the actual Japanese data. Although the last analysis is tentative in the current version of our paper, we have confirmed up to some extent that our model nicely replicates the firm dynamics observed in the data.

Third, by using the model under tentative parameter assignments, we simulate the responses of various endogenously determined variables against the change in population growth rates. As a set of natural consequences of lower population growth rates,

we obtain, for example, the higher closure rates and lower entry rates accompanied by the lower succession rates. Interestingly, under these temporal parameter configurations, we have also observed higher per capita output for lower population growth rates. This result is accompanied by the improvement of managers quality. In a nutshell, the quantitative examination of our model suggest that the lower population growth rates could result in better welfare implication for each individual.

The results obtained in the present paper provide a number of practical implications. First, the evolution of managers' ability and the selection process through succession could result in higher per capita output even under lower population growth rate. While the lower population growth inevitably lead to lower aggregate output, it should be noted that the per capital output could move in a different mechanism. Second, in this mechanism, the accumulation of human capital (i.e., ability evolution) and the matching quality of successors and predecessors are the keys for the economy to improve under lower population growth rates. Given the population growth rates are one of the most difficult structural parameters to control, it would be suggestive to pay more attention to those controllable objects under the lower population growth rates.

Related Literature The underlying mechanism we are considering in the present paper is the misallocation of managerial resources. Same as in the prevalent discussions on the misallocation of tangible assets (Hsieh and Klenow (2009)) and intangible assets (Andrews and de Serres (2012); Caggese and Perez-Orive (2022); Crouzet and Eberly (2023)), top managers' managerial resources might not be efficiently allocated to firms in the way that marginal products of managerial resources are equated among firms due to, for example, the search and matching frictions associated with top managers' appointment. This issue could become more apparent in the process of succession and further aggravated under population decline. Such qualitative nature and the quantitative impacts are exactly what the extant studies have been missing

and we want to address in the present paper.

Theoretically, our model is most closely aligned with Lucas (1978) and Atkeson and Kehoe (2005) in its basic setup of management ability. We extend their frameworks to examine the impact of declining population growth on business successions and, consequently, economic outcomes, by incorporating business successions. Carillo et al. (2019) also extend the model of Lucas (1978) to consider the successions of family and non-family firms in the process of economic development although they assume a constant population size. Engbom (2019) studies the effects of population aging on firm entry in the context of a search model like ours. However, the property of search is different: a search is for the better job by workers in Engbom (2019), while we model a search for successors by managers. Thus, to the best of our knowledge, this study represents the first attempt to theoretically bridge the relationship between population growth and macroeconomy through business successions. Although the aforementioned existing theoretical studies have explored the effects of declining population growth on firm dynamics and aggregate economy, we introduce such a novel channel through the lens of business successions.

On the empirical side, while there is extensive research on business successions, the implications for the macroeconomy—particularly in the context of population aging—have received limited attention. Namely, many empirical studies have been employing various granular datasets to uncover the importance of top managers in the context of firms' business activities and more specifically over the course of business succession (Johnson et al. (1985); Bertrand and Schoar (2003); Bennedsen et al. (2007); Kaplan et al. (2012); Bandiera et al. (2020); Dessein and Prat (2022)). While the role of CEOs and its succession have been attracting a huge attention from these micro point of views, the aforementioned aggregate implication of CEO succession has not been intensively discussed in literature. In the present paper, we are specifically interested in this point, i.e., how a specific demographic change affects macroeconomy through a channel associated with business successions. Such a channel would be more

important under a specific demographic change such as population decline and thus worth to be examined.

Note that there is a substantial body of literature on business successions among family-owned firms (e.g., Bertrand and Schoar (2003)). This literature frequently examines whether business successions within families enhance firm performance, often documenting negative effects (e.g., Burkart et al. (2003); Perez-Gonzalez (2006); Bennedsen et al. (2007); Caselli and Gennaioli (2013)). However, Mehrotra et al. (2013), for example, find that Japanese family firms perform well on average, with firms led by non-consanguineous heirs outperforming those led by blood-related heirs. In the context of Japan, several studies have also empirically analyzed business successions in an aging society, including Saito (2008), Tsuruta (2020); Tsuruta (2021), Kodama et al. (2021), and Wongkaew and Saito (2023). Unlike these studies, our analysis does not differentiate between family and non-family firms, and thus it cannot provide insights into the recent challenges faced by aging Japanese firms with few or no successors within their families. Nonetheless, as Mehrotra et al. (2013) highlight, business successions to non-consanguineous heirs were common even during the post-war period of positive population growth in Japan. The feature that the population growth leads to a difficulty in finding successors holds both for family and non-family firms. Therefore, the lack of distinction between family and non-family firms does not necessarily undermine the validity of our research.

The rest of this paper is organized as follows. Section 2 provides some empirical facts that motivate the model constructed in Section 3. Section 4 describes the empirical analysis and Section 5 presents numerical examples to show the impact of declining population, and Section 6 concludes.

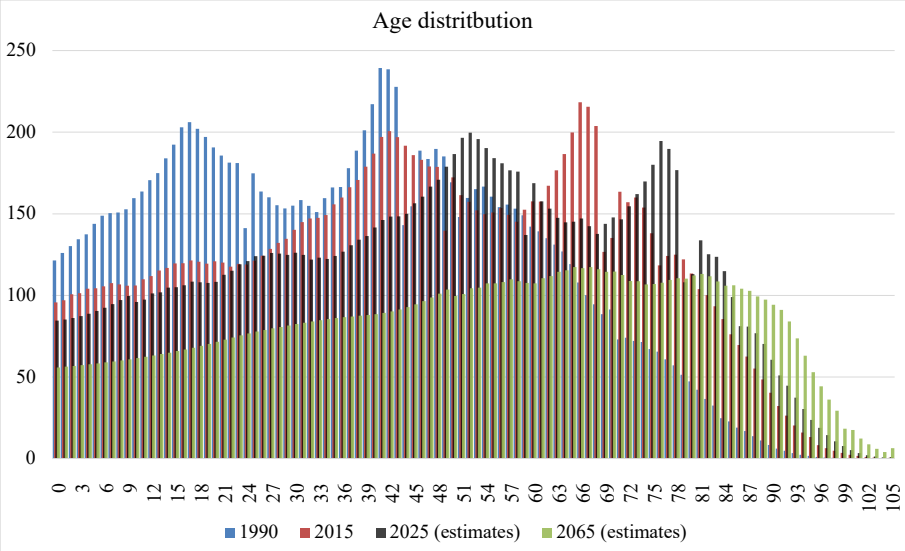


Figure 1: Demographic Change in Japan from 1990 to 2065

Source: Ministry of Health, Labor and Welfare
(https://www.mhlw.go.jp/wp/hakusyo/kousei/17/backdata/xls/2/01-01-01.xls).

2 Motivating Facts

Japan is an ideal research ground for us to study the aggregate implication of a population decline specifically through the succession channel. First, it is the frontrunner of the aging society and thus the business enterprises in Japan has been already started to face the population decline to a large extent. Figure 1 depicts the evolution of the age distribution for Japan from the year of 1990 to that of 2065 (estimated) that suggests the rapid aging with smaller young cohorts.

Second, more directly to the topic on the relation between demographic changes and macroeconomy in the context of business successions, Figure 2 illustrates the evolution of the top managers' age distribution for Japanese business enterprises from the year of 1990 to that of 2020. The figure clearly indicates that top managers are aging. Consistent with these numbers, New York Times (Jan. 3, 2023) and Economist (Dec 4th 2021) as well as SME White Paper in Japan reported the difficultly faced by the top managers of Japanese firms to find their successors. Given these backgrounds,

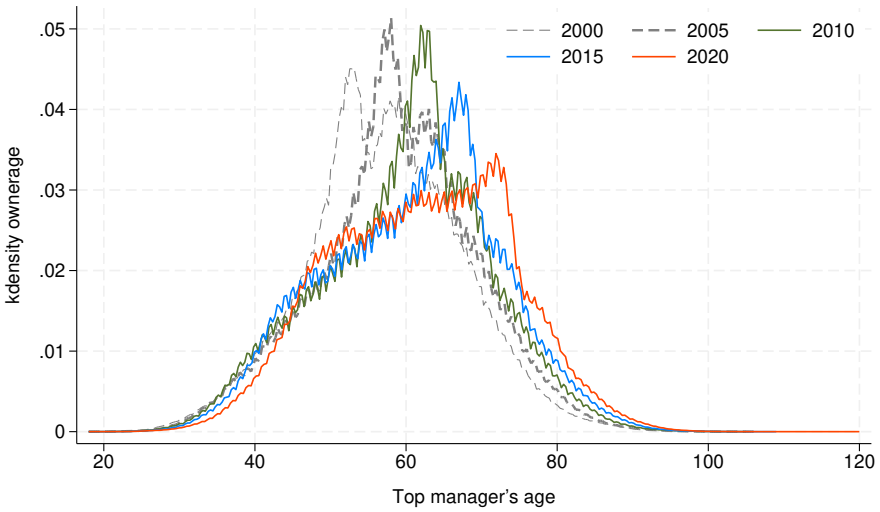


Figure 2: Top Managers' Age Distribution from 2000 to 2020

Source: Tokyo Shoko Research Ltd.

we believe that validating the model's property under the parameter configuration corresponding to population decline and simulate the equilibrium object with calibrating the model parameters by using such Japanese data would provide informative insights for other advanced economies soon to face the same problem.

Third, using the data accounting for a massive number of business enterprises accompanied by sufficient number of succession records for Japanese firms, we can document an important association between managers' age and firms' sales dynamics. Using the data we provide detailed explanation in the later section, we estimate the following equation accounting for the association between firm size and manager's age with controlling for the age of firms, the duration of managers' tenure, and firm-level fixed-effects.

$$\begin{aligned}\log\left(\text{sales}_{f,t}\right) = &\sum_{h=1}^{9} \alpha_h \mathbb{1}\left(10(h+1) > \text{Manager age}_{f,t} \geq 10h\right) \\ &+ \sum_{i=1}^{9} \beta_i \mathbb{1}\left(10(i+1) > \text{Firm age}_{f,t} \geq 10i\right) \\ &+ \sum_{j=1}^{22} \gamma_j \mathbb{1}\left(j+1 > \text{Manager tenure}_{f,t} \geq j\right) + \eta_f + \varepsilon_{j,t}\end{aligned}$$

In this equation, α_h denotes the relative size of the firms managed by a top manger

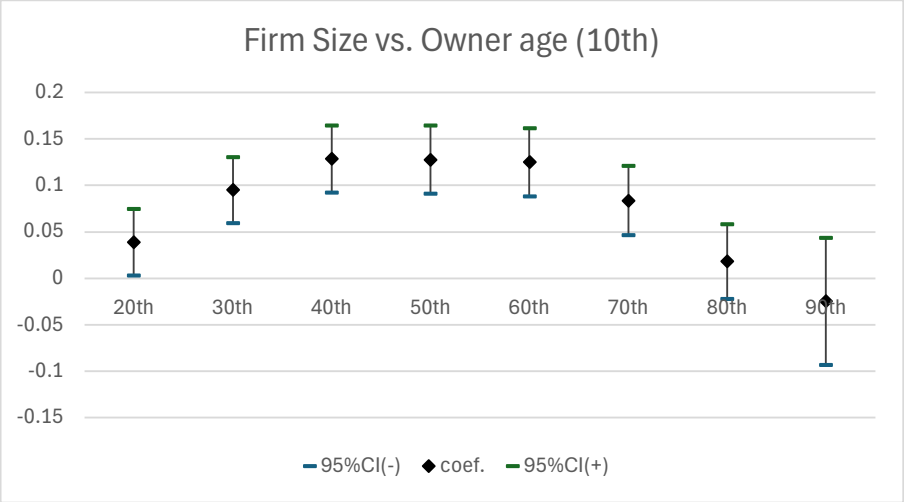


Figure 3: Managers' Age vs. Firm Size

Note: The figure shows coefficient α_h for each h, which captures the relative sales of a firm managed by a top manger with the age equal to or older than $10h$ and younger than $10(h+1)$.

with the age equal to or older than $10h$ ($h = 1, \cdots, 9$) and younger than $10(h+1)$. As we set $h = 1, \cdots, 9$, the sales size is measured in relative to the average of the firms with very young or old top managers. In the result of our estimation controlling for various confounding factors such as firm age, Figure 3 shows the robust association between managers' age and firm size that is consist with our assumption on the hump-shaped evolution of manager's ability. This result is consistent with the setup used in Atkeson and Kehoe (2005) and allows us to assume the hump-shaped evolution of manager's ability in our theoretical model.

3 Model

We develop a model that links demographic changes, such as population decline and aging, to macroeconomic performance through firm dynamics—specifically, entry, closure, and succession. Firm heterogeneity arises from managerial ability x, which evolves exogenously over the lifecycle. This framework makes individual occupational choices and firms' decisions age-dependent, thereby shaping the impact of

demographic changes on aggregate output.

3.1 Demographic and Production Setting

We construct a structural model capable of replicating the observations presented in the previous section to quantify the impact of aging and a declining population on the aggregate economy through the channel of business succession. Let N_t be the total population in period t, which consists of birth-year cohorts, $N_{\tau,t}$, where τ is the period of birth. The size of each cohort is shrinking over time by death. We assume the death rate depends on age, $a = t - \tau$, such that $d_a \leq d_{a+1}$ for any $a \geq 0$, and $\lim_{a\to\infty} d_a = 1$.[1]

$$N_t = \sum_{\tau=-\infty}^{t} N_{\tau,t} = \sum_{\tau=-\infty}^{t} N_{\tau,\tau} \prod_{a=0}^{t-\tau-1} (1 - d_a).$$

The population growth rate, ν, is defined as $N_{t+1,t+1} = (1+\nu)\, N_{t,t}$. $\nu < 0$ implies population decline accompanied by a declining birthrate and an aging population.

We introduce heterogeneous managerial ability for each agent. Let $x_{i,\tau,a}$ be the managerial ability of agent i, born in t, at the age of a. The initial $x_{i,\tau,0}$ is a random draw at birth from a log-normal distribution, $\mathcal{LN}\left(\ln \bar{x} - \frac{\sigma^2}{2}, \sigma^2\right)$, with $\mathbb{E}[x_{i,\tau,0}] = \bar{x}$ for any i and τ. Managerial ability $x_{i,\tau,a}$ evolves deterministically, following

$$x_{i,\tau,a+1} = x_{i,\tau,a} h_{a+1}, \qquad \text{for } a = 0, 1, 2, \ldots, \quad \forall \tau$$

where $h_a > 0$ for all a, and

$$h_a \begin{cases} \geq 1 & \text{for } a \leq \bar{a}, \\ < 1 & \text{for } a > \bar{a}. \end{cases} \tag{1}$$

Define $H_a \equiv \prod_{a'=1}^{a} h_{a'}$ so that $x_{i,\tau,a} = x_{i,\tau,0} H_a$. This setting is similar to Atkeson and Kehoe (2005) except that we assume the agents face randomness only when they are

[1] We describe the model under the assumption of infinite age for notational simplicity. In simulations, we set $d_a = 1$ for finite a to prevent infinite age. There is no essential difference as long as the measure of the unrealistically aged individuals is negligible in the current model.

born. In addition, we assume that $h_{a+1} \leq h_a$ for $a > \bar{a}$ and $\lim_{a\to\infty} H_a = 0$.

There are two occupations: managers and workers, whose measures are represented by M_t and L_t, respectively ($N_t = M_t + L_t$). Each manager runs a single firm that hires workers. We assume that worker's wage, w_t, is independent of managerial ability. The managerial ability affects firm productivity in the form of span-of-control model developed by Lucas (1978). If an agent with ability x becomes a manager, the firm's production function is given by

$$y = x^{\mu} \left[k^{\alpha} l^{1-\alpha}\right]^{1-\mu} \qquad \alpha, \mu \in (0,1), \tag{2}$$

where y is output, k is capital, and l is employment. Assuming perfect competition, the flow profit of the firm is

$$\pi_t(x) = \max_{k,l} x^{\mu} \left[k^{\alpha} l^{1-\alpha}\right]^{1-\mu} - (r_t + \delta)k - w_t l,$$

for given factor prices. Then, the size of the firm is linear in x such that

$$k_t(x) = \bar{k}_t x, \qquad \text{where } \bar{k}_t \equiv \left(\frac{\alpha(1-\mu)}{r_t+\delta}\right)^{\frac{1}{\mu}} \left(\frac{1-\alpha}{\alpha}\frac{r_t+\delta}{w_t}\right)^{\frac{(1-\alpha)(1-\mu)}{\mu}}, \tag{3}$$

$$l_t(x) = \bar{l}_t x, \qquad \text{where } \bar{l}_t \equiv \left(\frac{\alpha(1-\mu)}{r_t+\delta}\right)^{\frac{1}{\mu}} \left(\frac{1-\alpha}{\alpha}\frac{r_t+\delta}{w_t}\right)^{\frac{(1-\alpha)(1-\mu)}{\mu}+1}, \tag{4}$$

$$y_t(x) = \bar{y}_t x, \qquad \text{where } \bar{y}_t \equiv \left(\frac{\alpha(1-\mu)}{r_t+\delta}\right)^{\frac{1-\mu}{\mu}} \left(\frac{1-\alpha}{\alpha}\frac{r_t+\delta}{w_t}\right)^{\frac{(1-\alpha)(1-\mu)}{\mu}}. \tag{5}$$

And the profit is also linear in x,

$$\pi_t(x) = \mu \bar{y}_t x. \tag{6}$$

3.2 Occupational Choice

All agents are workers when they are born and they can switch their occupation to managers in two ways: business succession or entry. Let $V_{\tau,t}(x)$ denote the value of a manager who is born at τ and has ability x in period t. Similarly, $W_{\tau,t}(x)$ is the values of a worker who is born at τ and has ability x in period t. For the later convenience, we also define the gap in those values as

$$G_{\tau,t}(x) \equiv V_{\tau,t}(x) - W_{\tau,t}(x).$$

3.2.1 Successor Search

A manager can search for a successor by incurring a cost of κ each period. A searching manager randomly meets a worker from the current pool of workers. Assume that the measure of searching managers is sufficiently small compared to the measure of workers, ensuring that each searching manager can meet one worker and that the probability of two or more managers meeting the same worker is negligible. For a worker to succeed to an existing business, they must make a transfer of q to the preceding manager after succession. The transfer is determined by a take-it-or-leave-it offer made by the predecessor. Thus, all excess value from being a manager is transferred to the predecessor. If a worker in cohort τ with ability x succeeds to a firm, they pay

$$q = G_{\tau,t+1}(xh_{t-\tau+1}),$$

at the beginning of the next period.

For simplicity, we assume that searching managers accept successor candidates as long as $q \geq 0$. If $G_{\tau,t+1}(xh_{t-\tau+1}) < 0$, then the search fails. The probability of success in successor search, conditional on predecessor's survival, is given by

$$p_t = \sum_{\tau=-\infty}^{t} \frac{(1-d_{t-\tau})L_{\tau,t}}{L_t} \int_{G_{\tau,t+1}(xh_{t+1-\tau})\geq 0} dF^L_{\tau,t}, \tag{7}$$

where $F^L_{\tau,t}(x)$ represents the distribution of managerial ability among workers in cohort τ at period t. The expected transfer, conditional on success in search, is

$$\bar{q}_t = \frac{1}{p_t} \sum_{\tau=-\infty}^{t} \frac{L_{\tau,t}}{L_t} \int_{G_{\tau,t+1}(xh_{t+1-\tau})\geq 0} G_{\tau,t+1}(xh_{t+1-\tau}) dF^L_{\tau,t}. \tag{8}$$

Below, we consider the situation that κ is sufficiently small so that,

$$\exists \tau, t \qquad p_t \bar{q}_t > \tilde{\kappa}_{\tau,t} \equiv \frac{1+r_t}{1-d_{t-\tau}} \kappa, \tag{9}$$

allowing successor search to occur in equilibrium.[2]

3.2.2 Firm Entry

Each worker has a probability $\eta \in (0,1)$ of coming up with an idea to start a new business. A worker in cohort τ with ability x takes this opportunity and starts a new business in the next period if $G_{\tau,t+1}\left(xh_{t-\tau+1}\right) \geq 0$. This entry setting creates an environment where workers with high managerial abilities are available due to a lack of business ideas, thereby providing incumbent managers with sufficient incentives to search for successors among these workers. We assume that the probability that a single worker gets both opportunities of business succession and entry is negligible.

3.2.3 Value Functions and Phase Structure

We formulate the value functions for managers and workers and analyze how their occupational choices and successor search decisions depend on those values. We first focus on the case that satisfy condition (9) and then discuss the case that violates it.

[2]Rigorously speaking, there exists an incentive to wait for another successor candidate if the current matched candidate can pay small q. Waiting is better when

$$q < \mathbb{E}_t \frac{\left(1-d_{t-\tau+1}\right) p_{t+1} \bar{q}_{t+1}}{\left(1+r_{t+1}\right)} - \kappa.$$

By the simplifying assumption above, we assume that the waiting behavior has only an insignificant quantitative impact.

Manager Values First, if a manager closes their business at the end of a period, they become a worker in the following period. In this case, the manager's value satisfies

$$V_{\tau,t}(x) = \pi_t(x) + \frac{1 - d_{t-\tau}}{1 + r_t} W_{\tau,t+1}\left(xh_{t+1-\tau}\right). \tag{10}$$

Second, if a manager does not close the business immediately and conducts a search for a successor, the value function satisfies

$$V_{\tau,t}(x) = \pi_t(x) - \kappa + \frac{1 - d_{t-\tau}}{1 + r_t}\left[V_{\tau,t+1}\left(xh_{t+1-\tau}\right) + p_t\left(\bar{q}_t - G_{\tau,t+1}\left(xh_{t+1-\tau}\right)\right)\right]. \tag{11}$$

Comparing equations (10) and (11), a manager searches a successor instead of closing its business if

$$G_{\tau,t+1}\left(xh_{t+1-\tau}\right) \geq \frac{\tilde{\kappa}_{\tau,t} - p_t\bar{q}_t}{1 - p_t}. \tag{12}$$

Since $G_{\tau,t+1}\left(xh_{t+1-\tau}\right)$ is strictly increasing in x for given cohort, we define the unique threshold $\hat{x}^c_{\tau,t}$ for each cohort at which (12) holds with equality.

Third, if a manager is sufficiently capable, they continues to run their firm without closure or successor search. The value in this case satisfies

$$V_{\tau,t}(x) = \pi_t(x) + \frac{1 - d_{t-\tau}}{1 + r_t} V_{\tau,t+1}\left(xh_{t+1-\tau}\right). \tag{13}$$

Comparing equations (11) and (13), a manager continues its business without successor search if

$$G_{\tau,t+1}\left(xh_{t+1-\tau}\right) \geq \bar{q}_t - \frac{\tilde{\kappa}_{\tau,t}}{p_t}. \tag{14}$$

Define the threshold $\hat{x}^s_{\tau,t}$ for each cohort as the value of x at which (14) holds with equality.

Worker Values Since all excess value obtained from succeeding to a firm is transferred to the predecessor, a worker is indifferent between business succession and remaining a worker. Consequently, the threshold arises solely from the entry deci-

sion.

If a worker ignores an entry opportunity, the worker value satisfies

$$W_{\tau,t}(x) = w_t + \frac{1-d_{\tau,t}}{1+r_t} W_{\tau,t+1}\left(xh_{t+1-\tau}\right). \tag{15}$$

If a worker takes an entry chance,

$$W_{\tau,t}(x) = w_t + \frac{1-d_{t-\tau}}{1+r_t}\left[W_{\tau,t+1}\left(xh_{t+1-\tau}\right) + \eta\left(G_{\tau,t+1}\left(xh_{t+1-\tau}\right)\right)\right]. \tag{16}$$

Hence, a worker takes an entry chance if and only if $G_{\tau,t+1}\left(xh_{t+1-\tau}\right) \geq 0$. We define the entry threshold $\hat{x}^e_{\tau,t}$ for each cohort by $G_{\tau,t+1}\left(xh_{t+1-\tau}\right) = 0$.

Manager-Worker Value Phases The occupational decisions of agents consist of four phases: Phase I, II, III, and IV. Figure 4 draws these phases based on the thresholds, $\hat{x}^c_{\tau,t}$, $\hat{x}^e_{\tau,t}$, and $\hat{x}^s_{\tau,t}$. Note that, as long as condition (9) holds,

$$\frac{\tilde{\kappa}_{\tau,t} - p_t\bar{q}_t}{1-p_t} \leq 0 \leq \bar{q}_t - \frac{\tilde{\kappa}_{\tau,t}}{p_t} \quad \Rightarrow \quad \hat{x}^c_{\tau,t} \leq \hat{x}^e_{\tau,t} \leq \hat{x}^s_{\tau,t}.$$

When x falls in Phase I, managers choose closure, and workers do not take the entry opportunity. Subtracting equation (15) from (10), the gap value satisfies

$$G_{\tau,t}(x) = \pi_t(x) - w_t. \tag{17}$$

In Phase II, managers search for successors, while workers do not take the entry opportunity. From equations (11) and (15),

$$G_{\tau,t}(x) = \pi_t(x) - \kappa - w_t + \frac{1-d_{t-\tau}}{1+r_t}\left[\left(1-p_t\right)G_{\tau,t+1}\left(xh_{t+1-\tau}\right) + p_t\bar{q}_t\right]. \tag{18}$$

In Phase III, managers continue to search for successors, while workers start businesses

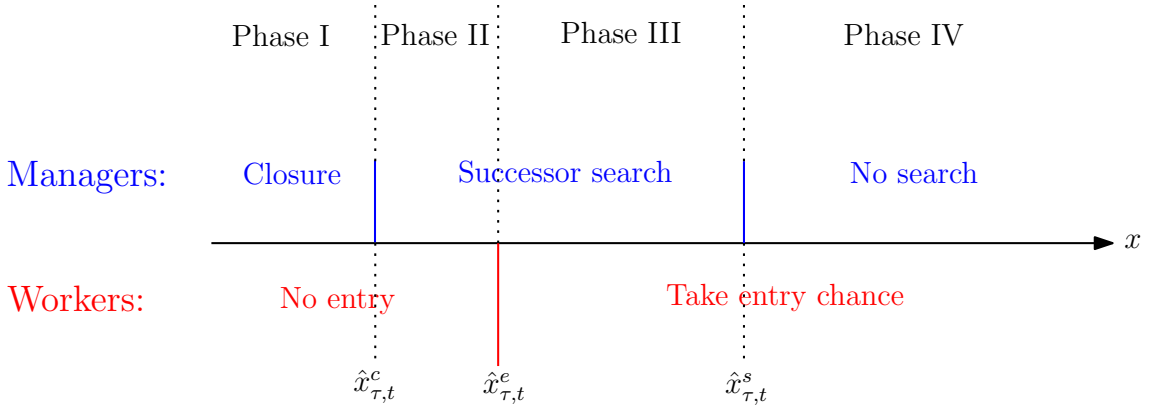


Figure 4: Phases of the value function.

if given the opportunity. From equations (11) and (16), we have

$$G_{\tau,t}(x) = \pi_t(x) - \kappa - w_t + \frac{1-d_{t-\tau}}{1+r_t}\left[(1-\eta-p_t)\, G_{\tau,t+1}\left(xh_{t+1-\tau}\right) + p_t \bar{q}_t\right]. \tag{19}$$

Lastly, in Phase IV, managers continue running their business without searching for successors, while workers take entry opportunities. From equations (13) and (16),

$$G_{\tau,t}(x) = \pi_t(x) - w_t + \frac{1-d_{t-\tau}}{1+r_t}\left[(1-\eta)\, G_{\tau,t+1}\left(xh_{t+1-\tau}\right)\right]. \tag{20}$$

When condition (9) is violated, a high probability of death significantly reduces the expected value of the transfer, making the successor search option unattractive. Consequently, managers with very high $\tilde{\kappa}_{\tau,t}$ do not conduct successor searches. By comparing equations (10) and (13), the threshold for such managers coincides with $\hat{x}^e_{\tau,t}$: managers with values below this threshold close their businesses, while those above it continue as managers without successor searches. In other words, only Phase I and Phase IV are relevant for these older agents.

3.3 Aggregation

We define the managerial ability distributions of managers and workers in cohort τ in period t as $F^M_{\tau,t}(x)$ and $F^L_{\tau,t}(x)$, respectively. The demand for workers in the aggregate economy is

$$L^D_t = M_t \bar{l}_t \bar{\mathbb{E}}^M_t [x],$$

where $\bar{\mathbb{E}}^M_t [x] \equiv \frac{1}{M_t} \sum_{\tau=-\infty}^{t} M_{\tau,t} \int_0^\infty x dF^M_{\tau,t}$ is the cross-sectional average of ability among managers in period t. The labor market clearing condition in each period is given by $L_t = L^D_t$, or

$$L_t = M_t \bar{l}_t \bar{\mathbb{E}}^M_t [x]. \tag{21}$$

The aggregate output is given by

$$Y_t = M_t \bar{y}_t \bar{\mathbb{E}}^M_t [x], \tag{22}$$

which depends on the measure of firms and the average ability of managers. Since firm capacity is constrained by managerial ability, the total measure of firms, M_t, plays a crucial role in determining the aggregate output. Consequently, business closure and the death of managers without succession cause a loss in the aggregate output. The distribution of managerial ability is also critical. A declining population results in a high proportion of middle-aged agents, whose abilities tend to exceed those of young agents. This maturity effect positively contributes to the average ability of managers.

Another channel to enhance average ability under a declining population arises through labor shortage. As the population declines, labor shortages drive up wages. Higher wages increase the minimum x required to become a manager, resulting in an upward shift in average managerial ability. This relationship is reflected in equation (21), where $\bar{\mathbb{E}}^M_t [x]$ is inversely proportional to $\bar{l}_t$, which is decreasing in w_t, for a given pair of L_t and M_t.

The rates of business entry, succession, and closure relative to the current measure

of firms are derived as

$$\text{Entry rate: } \frac{\eta}{M_t}\sum_{\tau=-\infty}^{t} L_{\tau,t}\left[1-F^L_{\tau,t}\left(\hat{x}^e_{\tau,t}\right)\right],$$

$$\text{Succession rate: } \frac{p_t}{M_t}\sum_{\tau=-\infty}^{t} M_{\tau,t}\left[F^M_{\tau,t}\left(\hat{x}^s_{\tau,t}\right)-F^M_{\tau,t}\left(\hat{x}^c_{\tau,t}\right)\right],$$

$$\text{Closure rate: } \frac{1}{M_t}\sum_{\tau=-\infty}^{t} M_{\tau,t}F^M_{\tau,t}\left(\hat{x}^c_{\tau,t}\right).$$

The demographic structure among managers can be obtained by construction. By assumption, $M_{\tau,\tau}=0$ and $L_{\tau,t}=N_{\tau,t}-M_{\tau,t}$ for any $t\geq\tau$, where $N_{\tau,t}$ is determined by exogenous conditions. Given $M_{\tau,t}$, $M_{\tau,t+1}$ follows

$$\begin{aligned}M_{\tau,t+1}=&(1-d_{t-\tau})M_{\tau,t}\left[1-F^M_{\tau,t}\left(\hat{x}^c_{\tau,t}\right)-p_t\left(F^M_{\tau,t}\left(\hat{x}^s_{\tau,t}\right)-F^M_{\tau,t}\left(\hat{x}^c_{\tau,t}\right)\right)\right]\\&+(1-d_{t-\tau})L_{\tau,t}\left(\eta+s_t\right)\left(1-F^L_{\tau,t}\left(\hat{x}^e_{\tau,t}\right)\right),\end{aligned}$$

where s_t is the probability that a worker receives a succession offer,

$$s_t=\frac{1}{L_t}\sum_{\tau=-\infty}^{t} M_{\tau,t}\left(F^M_{\tau,t}\left(\hat{x}^s_{\tau,t}\right)-F^M_{\tau,t}\left(\hat{x}^c_{\tau,t}\right)\right).$$

3.4 Steady State

We assume the homogeneous household sector, with optimization resulting in $r_t=\rho+\nu$, where ρ is the time preference. Then, a steady state equilibrium exists in per-capita terms, characterized by constant factor prices over time, optimal decisions by agents regarding production, entry, closure, and business succession, as well as stationary age compositions and ability distributions of managers and workers. In this steady state, all cohort-dependent variables depend solely on age and are independent of both cohort and calendar time. Accordingly, we denote the managerial ability thresholds between phases as $\hat{x}^c_a$, $\hat{x}^e_a$, and $\hat{x}^s_a$, rather than $\hat{x}^c_{\tau,t}$ and so on. Similarly,

the ability distributions are expressed as F_a^M and F_a^L, instead of $F_{\tau,t}^M$ and $F_{\tau,t}^L$.

We leave the computation steps of gap values, $G_a(x)$, stationary age composition, $m_a \equiv \frac{M_{a,t}}{N_t}$, and stationary ability distribution, F_a^M, to Appendix A. Here, we summarize the key properties of the equilibrium to understand what happens under a declining population. In the current model, per-capita variables, such as the manager share and the aggregate output, are not neutral to the population growth rate through the decisions regarding occupational change and business succession. The next proposition shows how the thresholds of those decisions depend on ages.

Proposition 1. *For given p, $\bar{q}$, r, and w, there exist unique profile of $G_a(x)$ and ability distributions F_a^M, F_a^L for each a. Moreover, the thresholds $\hat{x}_a^c$ and $\hat{x}_a^e$ are increasing in $a > \bar{a}$ while the change in $\hat{x}_a^s$ in response to a is ambiguous.*

With a negative population growth rate, the age composition of agents will become skewed toward higher ages. For $a > \bar{a}$, managerial ability declines with age, while the death rate increases. As a result, managers are more likely to close their businesses without initiating a successor search, leading to an increase in $\hat{x}_a^c$. Additionally, the expected return from starting a new business decreases for ages exceeding $\bar{a}$, resulting in a rise in $\hat{x}_a^e$. The response of $\hat{x}_a^s$ is ambiguous, as both successor search and business continuation become less attractive for older managers. Consequently, the range of x within which successor search is conducted narrows and eventually disappears for sufficiently large a. Proposition 1 implies that there are less frequency of business succession and entry, and more closures in the steady state under a declining population.

The lower entry rate and higher closure rate discussed above exert downward pressure on the share of managers. In contrast, the maturity effect of aging increases the proportion of agents with sufficient ability to become managers. Therefore, the change in the steady-state share of managers in response to a decline in the population growth rate depends on which of these opposing factor prevails.

The aggregate output per capita in the steady state,

$$y = m\bar{y}\bar{\mathbb{E}}^{M}[x], \tag{23}$$

where $y = \frac{Y_t}{N_t}$ and $m = \frac{M_t}{N_t}$, is significantly influenced by the maturity effect. If this effect is sufficiently strong, both the share of managers and the average managerial ability increase under a declining population. However, the general equilibrium effect through $\bar{y}$ would be negative, as $\bar{y}$ is decreasing in w, which rises due to labor shortages.

4 Empirical Validation

In this section, we validate the model's property by comparing the model-generated data of firm dynamics and the actually observed data. First, we show that the association between firm sales dynamics and top managers' age in the massive size of firm-level panel data in Japan exhibit the hump-shape evolution as our theoretical model assumes. Second, we compare the model generated firm sales dynamics with that in the actual data. Third, we also compare the transition matrix accounting for the age of predecessors and that of successors in the actual data with the model-generated transition patterns. Through these empirical studies, we aim to validate the property of our theoretical model.

4.1 Data

For this validation, we use firm-level panel data provided by Tokyo Shoko Research (TSR), which is known as one of the largest credit rating companies in Japan. This company is a counterpart to the Dun & Bradstreet in the United States that is the world largest credit rating agency and data provider. The data we used for our validation study contain the information on firm sales from 2001 to 2019. The number

of firm observations is around 0.8 to 0.9 million per year. According to the Economic Census of 2016, the total number of firms in Japan is 3.9 million; thus, the TSR data cover more than 20% of all firms in Japan.[3]

Regarding the data coverage, it is extremely important for us to get an access to the data including not only listed but also unlisted firms. This is because most of the firms (i.e., more than 99.9%) operating in an economy are private and thus the succession would be highly crucial event (e.g., Carillo et al. (2019)). We take advantage of the access to the dataset, that has been intensively used in academic researches (e.g.,Bernard et al. (2019); Carvalho et al. (2021); Miyauchi (2024)) and covered a large share of the firms and its employees, to precisely model the entire economy. Regarding business succession, we use the information on the top manager's of each firm. The information is stored in the Kigyo-Joho (KJ) file of TSR data and account for the name and the date of birth, which we use to identify each succession.

4.2 Validation

4.2.1 Firm Size Dynamics over Succession

In our theoretical model, managerial ability is one of the inputs for firm production process. Thus, through the succession from an old predecessor to a young successor, the firms experiencing the succession can get an access to the manager not only exhibiting better ability than the predecessor but also facing the improvement of the ability. Thus, the succession causes significant positive impacts on firm growth. To explicitly see this causal impacts running from succession to firm growth, focusing of the firms with the mangers searching for their successor in the model generated data, we estimate the following equation:

$$\log\left(\text{sales}_{f,t}\right) = \sum_{\tau=-10}^{5} \delta_\tau \mathbb{1}\left(\text{Succession}_{f,t+\tau}\right) + \varepsilon_{j,t}$$

[3]Hong et al. (2020) show that the TSR data resemble the Census data in terms of geographic coverage and firm size.

In this equation, $\mathbb{1}\left(\text{Succession}_{f,t+\tau}\right)$ is a dummy variable taking the value of one if the firm f experiences succession at τ periods ahead of the current period (i.e., t) and zero otherwise. When $\tau < 0$, firm experienced succession at τ periods prior to the current period (i.e., t). In the model-generated data, we set one period as five years.

To make the empirical counterpart of this exercise, we first estimate the propensity score of succession by using the lagged firm age, the lagged owner age, the tenure duration of the predecessor, the lagged sales, the lagged sales growth from the second lag to the fourth lag, the dummy variable accounting for the lagged positive profit, the number of industries the firm f belongs to (maximum three industries), the number of products and services the firm f provides (maximum six industries). Then, using the obtained propensity score, we estimate the average treatment effects of succession on sales. In this ATE estimation, we set one period as one year.

The left and right panels of Figure 5 depict the causal effects we compute from the model-generated data of firm dynamics and the estimated ATE from the data, respectively. As we presumed, there is almost no trend in the causal impacts prior to the succession but the positive growth after the succession. Reflecting the unit of time is five times longer in the case of left panel compared to the right panel, the post-succession dynamics of sales exhibits decay over the very long horizon. To summarize, we can confirm the consistency between the model's property in terms of the causal impacts of succession on sales dynamics and what we observe in the data.

As an additional exercise, we also run the equation above without focusing on the firms searching for successor in the case of the model-generated data. We also run the same regression for the actual data without taking care of the selection mechanism of succession by using propensity score. The left and right panels of Figure 6 depict the model-generate data and the estimated coefficients in the case of actual data, respectively. As a natural consequence of neither focusing on the firms in the search process nor taking care of the selection, we obtain the U-shaped dynamics of sales around the timing of succession. Again, reflecting the unit of time is five times

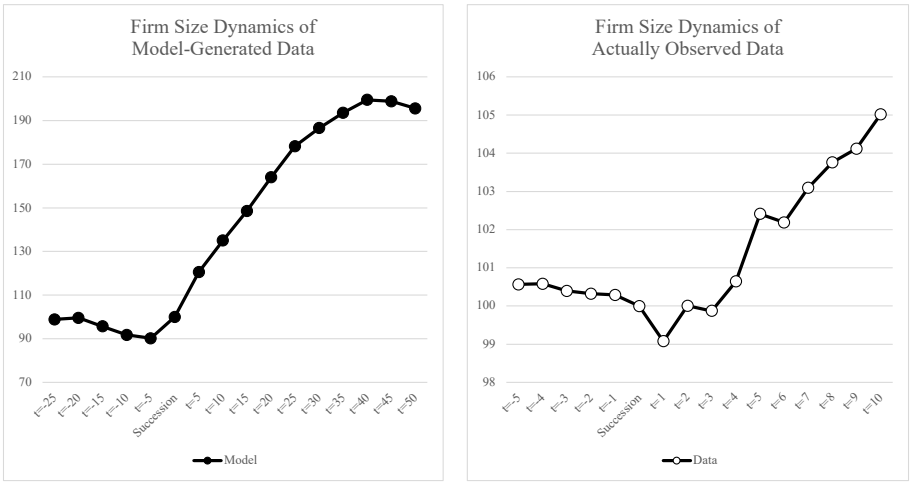


Figure 5: Firm Dynamics over Succession δ_τ

Note: The figure shows coefficient δ_τ with taking care of the selection mechanism of the succession.

longer in the case of left panel compared to the right panel, the post-succession dynamics of sales exhibits hump-shape over the long horizon. These results supports our theoretical model from the viewpoint of empirical validity.

4.2.2 Age Transition Pattern of Predecessors and Successors (Tentative)

As the next attempt to validate our theoretical model, we compare the model-generated age transition pattern of predecessors and successors. First, the upper panel of Figure 7 presents the heat map of the model-generated age pairs for predecessors (horizontal axis) and successors (vertical axis) in the case of zero population growth. Note that the peak age pairs are with predecessors typically around age 10 (i.e., 66 to 70 years old) and successors around age 4 (i.e., 36 to 40 years old). As the empirical counterpart of this transition matrix, we construct the lower panel of Figure 7 by using TSR data. Note that the peak age pairs are with predecessors typically around 70 years old and successors around 40 years old. We also find another typical case of the pairs with predecessors around 60 years old and successors around 50 to 60 years old. The former case accounts for the family firms (i.e., 30 years younger successors to predecessors), which is replicated in the model-generated data up to

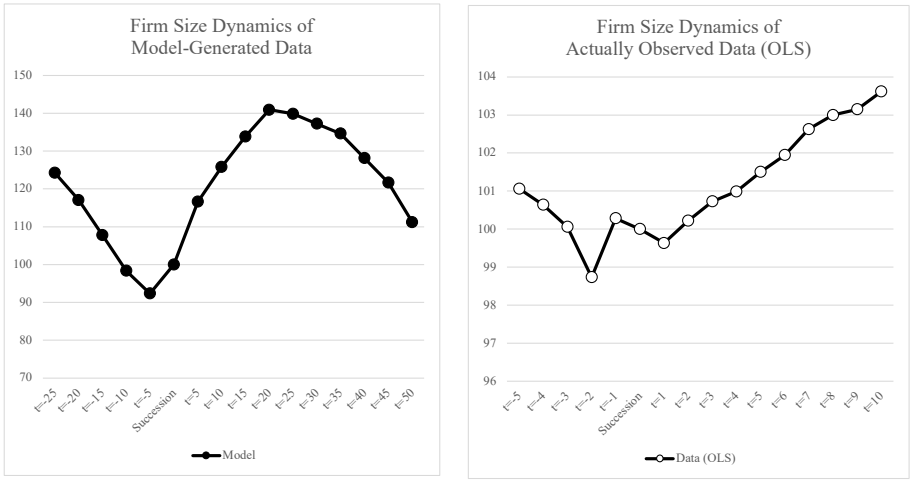


Figure 6: Firm Dynamics over Succession δ_τ

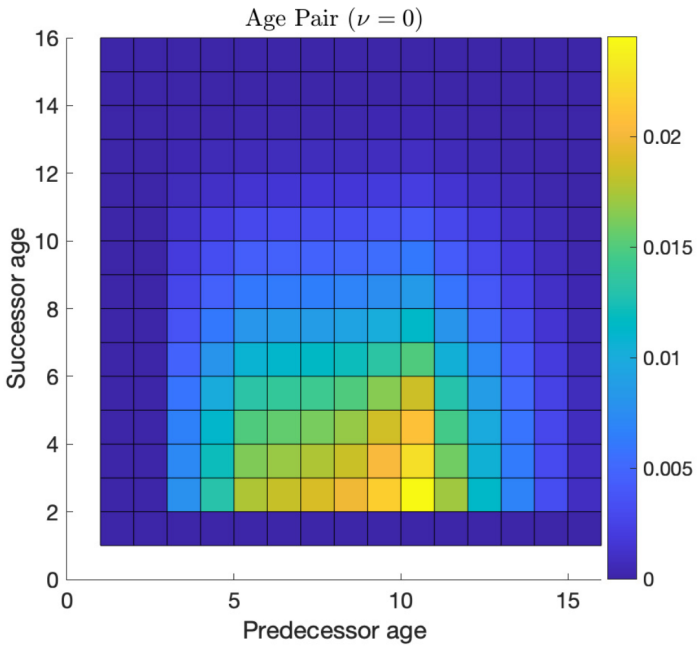
Note: The figure shows coefficient δ_τ without taking care of the selection mechanism of the succussion

some extent, while the latter is for the standard succession pattern in non-family firms. We are planning to further adjust the model parameters so as to confirm the consistency between them.

5 Quantitative Investigation (Tentative)

We simulate the model introduced in the previous section to quantitatively investigate how a declining population impacts on the economy through the channel of business succession. Agents live for a maximum of 16 periods, with each period representing 5 years, assuming that agents work or run businesses from ages 21 to 100. The parameters for the distribution of initial managerial ability are $\bar{x} = 1$ and $\sigma = 1$. The managerial ability dynamics is given by $h_1 = 1.3$ and $h_{a+1} = h_a - 0.05$, so that $\bar{a} = 6$. The death rate is nondecreasing and $d_{16} = 1$.[4] Parameters regarding production are $\alpha = 0.3$ and $\mu = 0.25$; the fixed cost of succession search is $\kappa = 0.05$; the probability of obtaining entry opportunities is $\eta = 0.15$; the time preference is $\rho = 0.15$.

[4]Specifically, we set $d_a = 0.01$ for $a \leq 4$, $d_a = 0.1$ for $5 \leq a \leq 9$, $d_a = 0.3$ for $10 \leq a \leq 14$, $d_a = 0.6$ for $a = 15$, and $d_a = 1$ for $a = 16$.



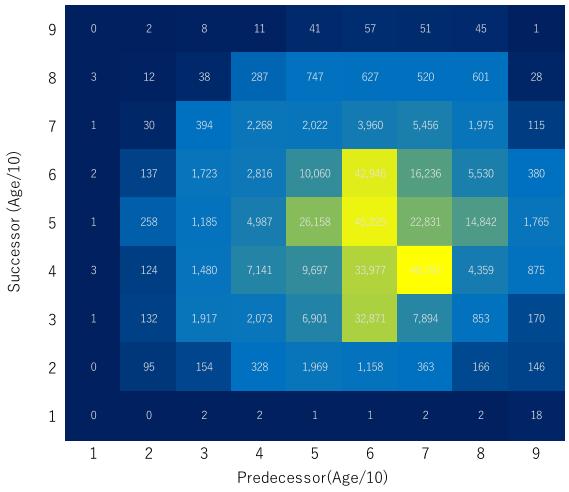


Figure 7: Predecessor's and Successor's Ages at Business Succession: Model-generated

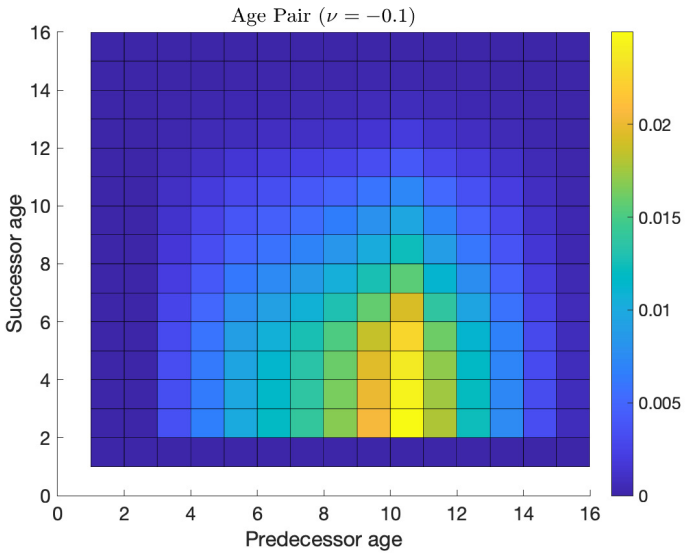


Figure 8: Predecessor's and Successor's Ages at Business Succession: $\nu = -0.1$.

Age Matrix of Predecessors and Successors Figure 8 presents the heat map of age pairs for predecessors (horizontal axis) and successors (vertical axis) in the case of negative population growth ($\nu = -0.1$). Both in the cases of the zero population growth ($\nu = 0$) in the upper panel of Figure 7 and this figure, the peak age pairs are similar, with predecessors typically around age 10 (i.e., 66 to 70 years old) and successors around age 4 (i.e., 36 to 40 years old). However, under negative population growth, business succession is concentrated within a narrower age range. Notably, there is a reduced frequency of middle-aged predecessors in the scenario of negative population growth.

Managers' Ability and Age Distributions Figure 9 shows the ability and age distributions among managers in the steady states under $\nu = 0$ (blue) and $\nu = -0.1$ (red). In an economy with a declining population, the age distribution naturally shifts rightward, reflecting a smaller proportion of young agents. The ability distribution also shifts rightward, albeit more moderately. This occurs because middle-aged managers, who typically have higher abilities, are less likely to search for successors. Instead, they are more inclined to remain as managers, as costly successor searches

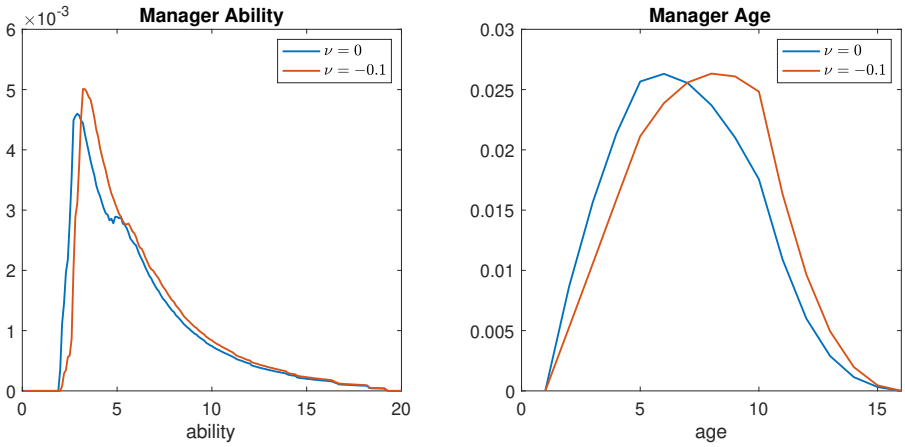


Figure 9: Ability and Age Distributions of Managers.

become less fruitful when the relative size of young cohort is small.

Figure 10 presents the distributions of managerial ability and age, conditional on the various modes of occupational change. The left column presents the ability distributions, while the right column shows the age distribution. In every mode of occupational change, the managerial ability tends to be higher in a declining population, reflecting the maturity effect of aging. Interestingly, the managers in an aging economy tend to quit earlier than in the scenario with a constant population. As observed in the subsequent analysis, an increase in wages due to labor shortage under the declining population is behind the rise in the ability of closing managers.

Comparative Statics Figure 11 demonstrates the numerical comparative statics for aggregate per-capita variables across various population growth rates. As the population growth rate declines, the economy experiences lower rates of entry and succession but a higher closure rate, consistent with the discussion based on Proposition 1. The share of managers increases under a declining population, implying that the maturity effect is sufficiently strong given the current parameter setting. Furthermore, a declining population leads to an increase in the wage rate due to labor shortages. This wage effect further enhances the average managerial ability by raising the minimum managerial ability required to become a manager, thereby

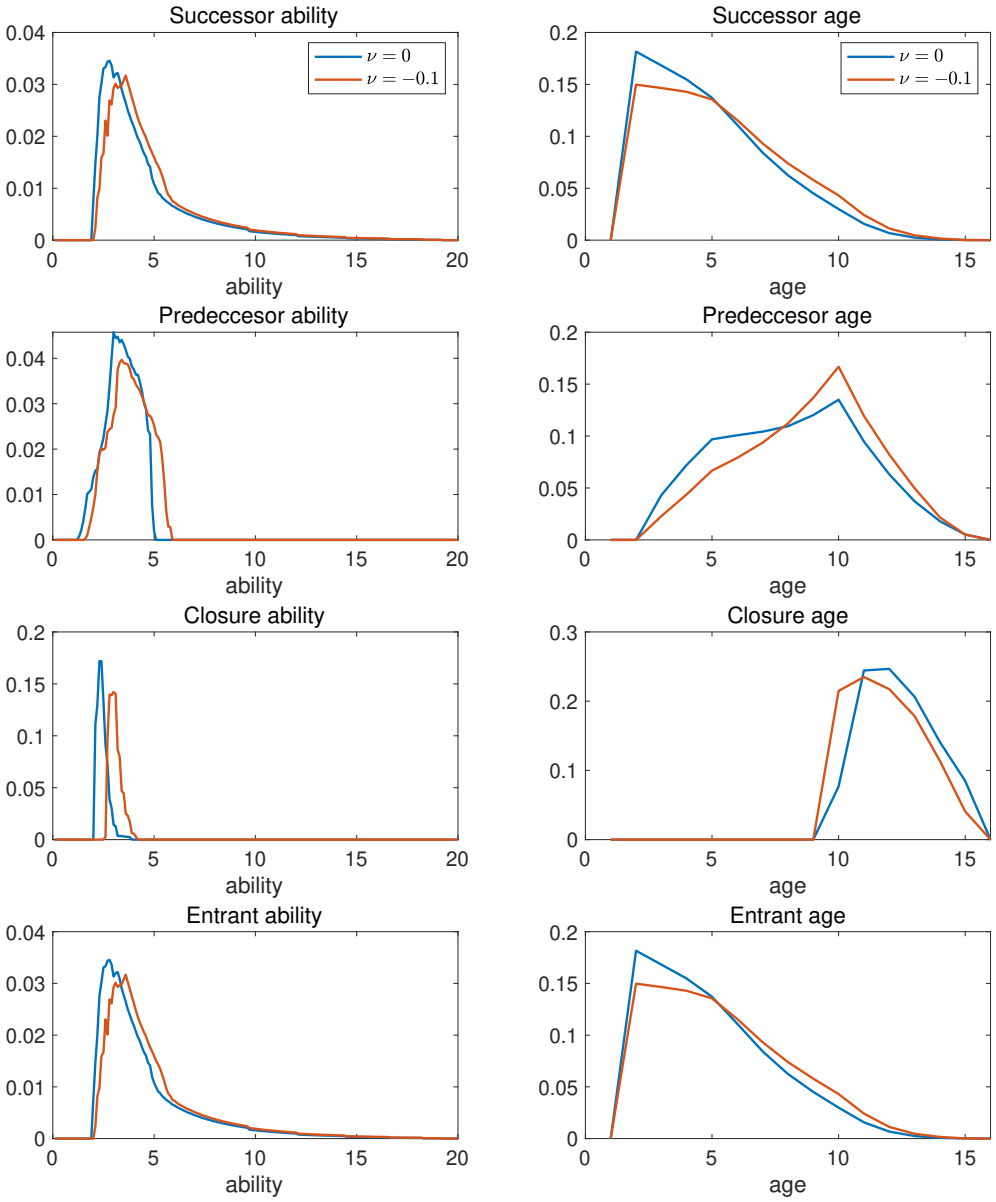


Figure 10: Ability and Age Distributions by Modes of Occupational Change.

strengthening the selection among managers. As a result, as the population growth rate decreases, the average ability of managers and the aggregate output per capita rise.

6 Concluding Remarks

In this study, we develop a general equilibrium model incorporating business succession, that is modeled as the occupation choices of managers and workers. We validate this theoretical model by documenting the consistency of firm dynamics over succession and age transition patterns (tentative) between the model-generated data and the actually observed data. The simulation studies based on the temporarily calibrated model reveal that, on top of the the natural consequences of lower population growth (i.e., high exit rates, low entry, and low succession rates), the per capita output could in fact become higher thanks to the maturity effect of aging and the selection of firms and managers through succession process. The present paper contributes to highlighting a novel channel through which population dynamics affects macroeconomy.

The future avenue of the present study is as follows: First, it would be promising to extend the model to incorporate an additional state variable accounting for a firm-specific asset. Under the current setup, each firm does not have any transferable asset (e.g., brand, technology, human capital) from a predecessor to a successor. Introducing this additional asset allows us to shed light on another inefficiency coming from population decline as such an asset could be lost due to the failure of succession under the population decline. Second, we would put more effort to validate the model by using the empirical facts. Specifically, in addition to the casual comparison of data-generated firm dynamics and what we observe in the data, we would like to implement more formal test for the consistency. Third, careful calibration study would be necessary to obtain practical implications for required policy interventions.

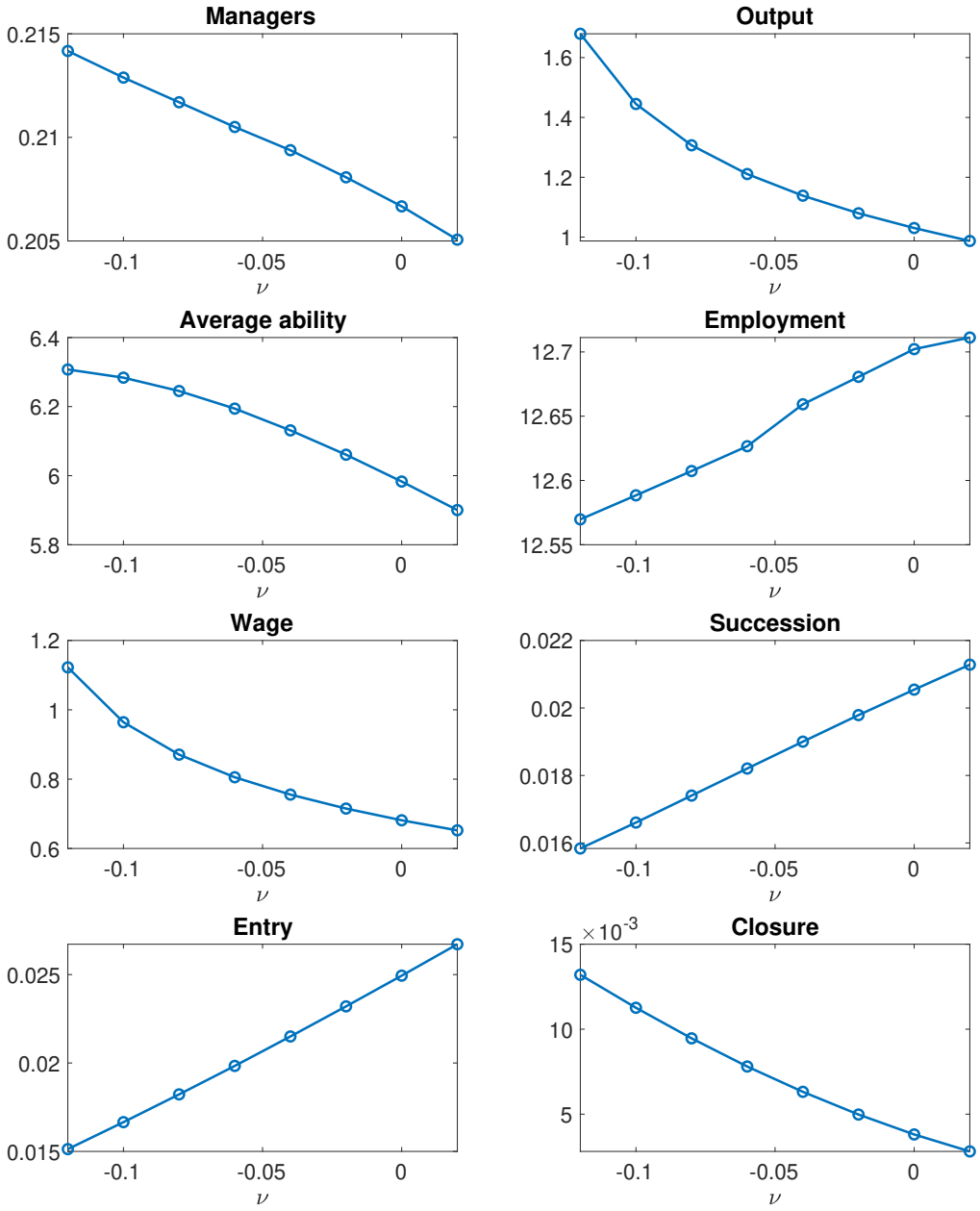


Figure 11: Comparative Statics

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A Model Details

A.1 Demographic Structure

Here we present the demographic structure of the current model economy. Under the current supposition and given the number of newborns, $N_{t,t,}$, the population in each cohort is

$$N_{\tau,t} = N_{t,t} \frac{\prod_{a=0}^{t-\tau-1}(1-d_a)}{(1+\nu)^{t-\tau}},$$

where we suppose that

$$\lim_{\tau \to -\infty} \frac{\prod_{a=0}^{t-\tau-1}(1-d_a)}{(1+\nu)^{t-\tau}} = 0,$$

even with $\nu < 0$ by assuming d_a is sufficiently close to one for high ages.

The total population in t is

$$N_t = \sum_{\tau=-\infty}^{t} N_{\tau,t} = N_{t,t} \sum_{\tau=-\infty}^{t} \frac{\prod_{a=0}^{t-\tau-1}(1-d_a)}{(1+\nu)^{t-\tau}},$$

thus

$$\frac{N_{t+1}}{N_t} = \frac{N_{t+1,t+1}}{N_{t,t}} \frac{\sum_{\tau=-\infty}^{t+1} \frac{\prod_{a=0}^{t-\tau}(1-d_a)}{(1+\nu)^{t-\tau+1}}}{\sum_{\tau=-\infty}^{t} \frac{\prod_{a=0}^{t-\tau-1}(1-d_a)}{(1+\nu)^{t-\tau}}} = 1+\nu.$$

Hence, the population growth rate is equivalent with the growth rate of newborns.

The relative size of age cohorts, n_a,

$$n_a = \frac{\prod_{a'=0}^{a-1}(1-d_{a'})}{(1+\nu)^a} \left[\sum_{a'=0}^{\infty} \frac{\prod_{a''=0}^{a'-1}(1-d_{a''})}{(1+\nu)^{a'}} \right]^{-1},$$

and

$$\frac{n_{a+1}}{n_a} = \frac{1-d_a}{1+\nu}.$$

The change in age-cohort shares in response to a change in the population growth rate is calculated as

$$\frac{dn_a}{d\nu} = -\frac{n_a}{1+\nu} \left[a - \frac{\sum_{a'=0}^{\infty} a' \frac{D_{a'}}{(1+\nu)^{a'}}}{\sum_{a'=0}^{\infty} \frac{D_{a'}}{(1+\nu)^{a'}}} \right], \tag{24}$$

where $D_a \equiv \prod_{a'=0}^{a-1}(1-d_{a'})$. Thus, we have greater share of young people under a higher population growth rate, vice versa.

$$\frac{dn_a}{d\nu} > 0 \quad \Leftrightarrow \quad a < \frac{\sum_{a'=0}^{\infty} a' \frac{D_{a'}}{(1+\nu)^{a'}}}{\sum_{a'=0}^{\infty} \frac{D_{a'}}{(1+\nu)^{a'}}}. \tag{25}$$

For a special case with $d_a = d \in (-\nu, 1)$ for any a, equations (24) and (25) become

$$n_a = \left(\frac{1-d}{1+\nu}\right)^a \frac{\nu+d}{1+\nu}, \qquad \frac{dn_a}{d\nu} > 0 \quad \Leftrightarrow \quad a < \frac{1-d}{\nu+d}.$$

Manager share, m_a, can be calculated as follows. First, the age structure among managers and workers are determined by

$$m_{a+1} = \frac{1-d_a}{1+\nu}\left[m_a\left(1 - F_a^M\left(\hat{x}_a^c\right) - p\left[F_a^M\left(\hat{x}_a^s\right) - F_a^M\left(\hat{x}_a^c\right)\right]\right) + \ell_a\left(\eta + s\right)\left[1 - F_a^L\left(\hat{x}_a^e\right)\right]\right].$$

We can solve this equation as

$$m_a = n_a(\eta+s)\sum_{a'=0}^{a-1}\left[1 - F_{a'}^L\left(\hat{x}_{a'}^e\right)\right]\left[\prod_{a''=a'+1}^{a-1}\left(1 - F_{a''}^M\left(\hat{x}_{a''}^c\right) - p\left[F_{a''}^M\left(\hat{x}_{a''}^s\right) - F_{a''}^M\left(\hat{x}_{a''}^c\right)\right]\right)\right].$$

A.2 Phases and Gap Values in the Steady State

The steady state gap value $G_a(x)$ is computed in the following steps. Taking the macro variables, r, w, p, and $\bar{q}$ as given for individual agents. For the initial ability draw, say x_0, each agent calculates his or her sequence of phases and values over life cycle backwardly. Conditional on survival, all agents eventually reach Phase I. Fix a sufficiently large age $a > \bar{a}$ that satisfies

$$x_0 H_{a+1} < \frac{1}{\mu\bar{y}}\left(w - \frac{p\bar{q} - \tilde{\kappa}_a}{1-p}\right).$$

Note that H_a is decreasing in a for any $a > \bar{a}$ and $\tilde{\kappa}_a$ is increasing in a, so the above inequality holds for any greater a. For this sufficiently large a, the agent is in Phase I and its value gap is $G_a(x) = \mu\bar{y}x - w$. Starting from this terminal value, the agent's

phase at age a is given by the following conditions:

$$G_{a+1}\left(xh_{a+1}\right) \in \begin{cases} \left(-\infty,\ -\frac{p\bar{q}-\tilde{\kappa}_a}{1-p}\right) & \Rightarrow \quad \text{Phase I at } a, \\ \left[-\frac{p\bar{q}-\tilde{\kappa}_a}{1-p},\ 0\right) & \Rightarrow \quad \text{Phase II at } a, \\ \left[0,\ \bar{q}-\frac{\tilde{\kappa}_a}{p}\right) & \Rightarrow \quad \text{Phase III at } a, \\ \left[\bar{q}-\frac{\tilde{\kappa}_a}{p},\ \infty\right) & \Rightarrow \quad \text{Phase IV at } a. \end{cases} \tag{26}$$

And the gap value, $G_a(x)$, for each phase is computed as

$$G_a(x) = \begin{cases} \mu\bar{y}x - w & \text{in Phase I,} \\ \mu\bar{y}x - \kappa - w + \frac{1-d_a}{1+r}\left[(1-p)G_{a+1}\left(xh_{a+1}\right) + p\bar{q}\right] & \text{in Phase II,} \\ \mu\bar{y}x - \kappa - w + \frac{1-d_a}{1+r}\left[\left(1-\eta-p\right)G_{a+1}\left(xh_{a+1}\right) + p\bar{q}\right] & \text{in Phase III,} \\ \mu\bar{y}x - w + \frac{1-d_a}{1+r}\left(1-\eta\right)G_{a+1}\left(xh_{a+1}\right) & \text{in Phase IV.} \end{cases} \tag{27}$$

A.3 Stationary Managerial Ability Distributions

Here we describe how $F_a^M(x)$ and $F_a^L(x)$ are determined. Because we introduce a stochastic factor only at the initial draw of managerial ability, the ability distribution among all agents by age groups, $F_a(x)$, is mechanically determined by the initial distribution, the population growth rate, ν, and the age-dependent death rates, $\{d_a\}$. With the shares of managers and workers in the steady state, we have

$$F_a(x) = m_a F_a^M(x) + (1-m_a)F_a^L(x),$$

and, thus, it is sufficient to derive $F_a^M(x)$ to describe all ability distributions. The managerial ability distribution is determined by construction. First, $F_0^M(x) = 0$ for any $x \geq 0$ because there are no age-0 manager (equivalently, $F_0^L(x) = F_0(x)$ for any $x \geq 0$).

To consider $F_a^M(x)$ for $a \geq 1$, we specify the thresholds of x that divide phases

for managers and workers. Let $\hat{x}_a^c$ be the threshold below which a manger with age a closes its business, $\hat{x}_a^s$ be the threshold above which a manager with age a continues its business without successor search, and $\hat{x}_a^e$ be the threshold above which a worker with age a takes an entry opportunity. From equations (26) and (27), we have

$$\begin{aligned}
\mu\bar{y}\hat{x}_a^c - w &= -\frac{p\bar{q} - \tilde{\kappa}_a}{1-p}, \\
\mu\bar{y}\hat{x}_a^s h_{a+1} - w + \frac{1-d_{a+1}}{1+r}(1-\eta)G_{a+2}\left(\hat{x}_a^s h_{a+1} h_{a+2}\right) &= \bar{q} - \frac{\tilde{\kappa}_a}{p}, \\
\mu\bar{y}\hat{x}_a^e h_{a+1} - \kappa - w + \frac{1-d_{a+1}}{1+r}\left[(1-\eta-p)G_{a+2}\left(\hat{x}_a^e h_{a+1} h_{a+2}\right) + p\bar{q}\right] &= 0,
\end{aligned}$$

which pin down the thresholds $\hat{x}_a^c$, $\hat{x}_a^s$, and $\hat{x}_a^e$. Note that $\hat{x}_a^c \leq \hat{x}_a^e \leq \hat{x}_a^s$ for any a.

Finally, we describe the transition of density of managerial ability of managers, f_a^M, from a to $a+1$.

$$f_{a+1}^M(x) = \begin{cases}
0 & \text{for } x \leq \hat{x}_a^c h_{a+1}, \\
(1+\nu)m_{a+1}(1-p)f_a^M\left(\frac{x}{h_{a+1}}\right) & \text{for } x \in \left(\hat{x}_a^c h_{a+1}, \hat{x}_a^e h_{a+1}\right], \\
(1+\nu)\left[m_{a+1}(1-p)f_a^M\left(\frac{x}{h_{a+1}}\right) + \ell_{a+1}(\eta+s)f_a^L\left(\frac{x}{h_{a+1}}\right)\right] & \text{for } x \in \left(\hat{x}_a^e h_{a+1}, \hat{x}_a^s h_{a+1}\right], \\
(1+\nu)\left[m_{a+1}f_a^M\left(\frac{x}{h_{a+1}}\right) + \ell_{a+1}(\eta+s)f_a^L\left(\frac{x}{h_{a+1}}\right)\right] & \text{for } x > \hat{x}_a^e h_{a+1},
\end{cases}$$

where $\ell_a = 1 - m_a$ and $f_a^L(x) = \frac{1}{1-m_a}\left(f_a(x) - m_a f_a^M(x)\right)$.

A.4 Age-dependent Shifts of the Thresholds (The Latter Part of Proposition 1)

Because the source in the value gap is $\mu\bar{y}x - w$, $G_a(x)$ is increasing in x for any x and decreasing in a for $a > \bar{a}$. We also see that d_a is nondecreasing in a by assumption, leading to $\tilde{\kappa}_a \leq \tilde{\kappa}_{a+1}$. The threshold between Phase I and Phase II is determined by

$$G_{a+1}(xh_{a+1}) = \frac{\tilde{\kappa}_a - p\bar{q}}{1-p},$$

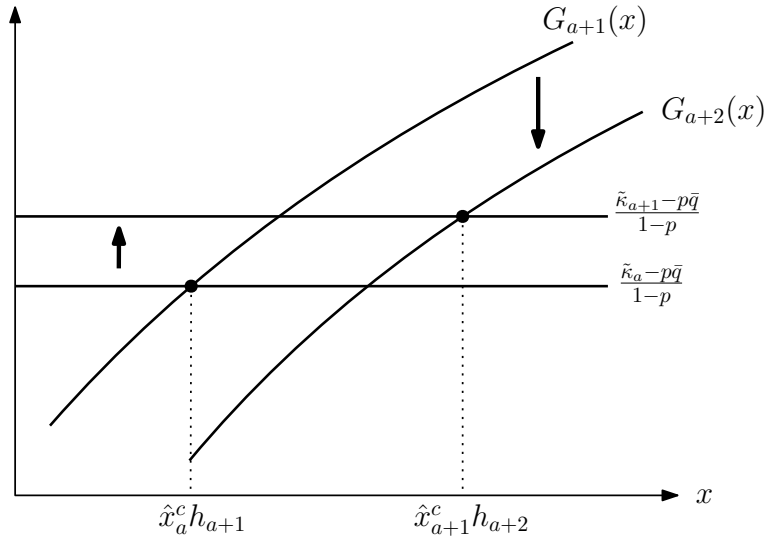


Figure 12: The Shift in $\hat{x}^c_a$.

implying that

$$\hat{x}^c_a h_{a+1} < \hat{x}^c_{a+1} h_{a+2} \quad \Rightarrow \quad \hat{x}^c_a < \hat{x}^c_{a+1} \underbrace{\frac{h_{a+2}}{h_{a+1}}}_{\leq 1} \leq \hat{x}^c_{a+1}.$$

Figure 12 illustrates the change in $\hat{x}^c_a$ in response to aging for $a > \bar{a}$.

Next, the threshold between Phase II and Phase III is determined by $G_{a+1}(xh_{a+1}) = 0$, which also implies that

$$\hat{x}^e_a < \hat{x}^e_{a+1} \qquad \forall a > \bar{a},$$

by a similar argument. The threshold between Phase III and IV is given by

$$G_{a+1}(xh_{a+1}) = \bar{q} - \frac{\tilde{\kappa}_a}{p},$$

resulting in an ambiguous shift in $\hat{x}^s_a$ in response to aging because both left- and right-hand sides shifts downward.