Redistribution, distortions, and the welfare effects of Social Security*

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December 27, 2024

Abstract

How should the funding problems of Social Security be addressed? Academic literature offers diverging advice. There is a well-known result that the optimal size of Social Security is zero, implying it is best to phase the program out. Other studies argue that much can be gained by re-designing the program given its current size. We provide a unified analysis by asking how the optimal size of Social Security depends on the key features of its design? We first do a theoretical decomposition tracing welfare effects of the program to (i) income redistribution, (ii) distortions on the optimal annuitization level; (iii) intertemporal distortion. We then quantitatively assess the role of these channels. We show that the zero-optimal-size result is due to the fact that Social Security is too distortive and not redistributive enough. Once these design flaws are corrected, it is even optimal to increase the size of the program.

Keywords: Social Security, Pensions, Annuities, Consumption and Saving, Life-Cycle Models
JEL Classification Codes: D15, E60, H55

*We thank Hamish Low, and all the participants at the McMaster University and Fudan University seminars for comments and suggestions.

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1 Introduction

Social Security is the largest U.S. government program in terms of expenditure, and is the primary source of income after retirement for many Americans (Dushi et al., 2017). However, the program faces serious funding problems: absent any changes, after 2035, Social Security can pay only around 83% of scheduled benefits using its tax income (SSA, 2024).

This makes reforming Social Security a pressing policy question, and various ways to make the program sustainable in the long-run are actively discussed (CBO, 2015; Duggan, 2023; Kitao, 2014). However, one question is essential for this discussion: should the program be smaller or larger in the long-run?

Academic literature gives almost an unanimous answer to this question: starting from Auerbach and Kotlikoff (1987) and Hubbard and Judd (1987), many studies conclude that in the long-run, the optimal size of Social Security is zero. Building on these results, the policy discussion should focus on how best to phase out Social Security without hurting current generations.

Importantly, the zero-optimal-size result is conditional on keeping the design of Social Security as it is. At the same time, a number of studies have shown that there is much room to improve the design of the program (e.g., Golosov et al., 2013; Huggett and Parra, 2010; O'Dea, 2018). Building on these results, the policy discussion should focus on how to implement these improvements.

These diverging policy implications arise because the questions of optimal size and of optimal design of Social Security were studied in isolation. We have gained much understanding of what is the optimal size of Social Security *given* its current design; and how best to design Social Security *given* its current size. However, it is the integrated answer to these two questions that ultimately informs the policy debate, and that is where we see a big gap in the literature. In this paper, we thus ask how the optimal size of Social Security depends on the key features of its design?

Our starting point is the trade-off between protection and distortions in the Social Security system. The optimal-zero-size result suggests that this trade-off is poor. Hence, an important question is whether this trade-off can be improved so that the existence of Social Security is optimal in the long-run, and if so, how this can be done.

We first turn to the positive side of this trade-off, which is that Social Security insures lifetime income risk. Pension benefits are redistributive providing higher replacement rates for low (lifetime) earners. How meaningful is the implied redistribution has been a focus of much research. Empirical studies find that taking income-mortality correlation into account, Social Security benefits become much less progressive (Coronado et al., 2011; Goda

et al., 2011). Normative studies argue that increasing redistribution through the system can substantially improve welfare (Golosov et al., 2013; Huggett and Parra, 2010; Jones and Li, 2022).

On the distortions side, much emphasize in prior work was given to that on labor supply (Diamond, 2002; Krueger and Pischke, 1992) and the *level* of savings (Feldstein, 1974). We emphasize another two distortions: that on the *timing* of savings and on the annuitization levels. Being in essence a forced saving program, Social Security dictates not only how much but also *when* people must contribute into the system, producing intertemporal distortions (Hubbard and Judd, 1987; Hurst and Willen, 2007; Pries, 2007).

In addition, Social Security wealth is subject to full mandatory annuitization. This is often considered in the positive light given that private annuity markets are small (Eckstein et al., 1985). However, compulsory insurance schemes are associated with costs, among them, the mismatch between people's demand for an insurance product and its provision (Diamond, 1977). We argue that if there is strong unwillingness to hold annuities, the mandatory annuitization can be viewed as another distortion, that on optimal asset allocation.

We thus examine how the optimal size of Social Security depends on (i) the degree of redistribution embedded in the program; (ii) the distortions on intertemporal choice and on asset allocation.

We proceed in several steps. We start by reviewing several empirical facts that are suggestive of non-trivial intertemporal and annuitization distortions created by Social Security. What points out to possibly large intertemporal distortions is the large number of the so-called hand-to-mouth workers. We document that around a third of working men below the age of 45 can be classified into this category.[1] This group has almost no regular savings: their median wealth is equal to zero. Yet, they are forced to save for retirement by paying payroll taxes.

What points out to possibly large annuitization distortions is the low annuity demand. People in the US can increase their annuity income by purchasing private annuities or by delaying claiming their Social Security benefits. Neither option is very popular. Among 70-year old men, only 5% own private annuities. Among the cohorts who are currently fully retired, around two-third of men claimed their benefits early, that is to say, before reaching the full retirement age (FRA). Moreover, around 40% of men in each cohort claimed their benefits at the earliest eligibility age (age 62).[2]

We next provide a theoretical analysis of positive and negative aspects of Social Security.

[1] Own calculations based on the Panel Study of Income Dynamics (PSID) and using the definition of the hand-to-mouth in Zeldes (1989). See Section 2 for details.

[2] Own calculations based on the Health and Retirement Study (HRS). See Section 2 for details.

We use a tractable overlapping-generations (OLG) model to construct two economies. The first economy has a stylized version of Social Security. The program mandates young people to contribute a fixed fraction of their income every period towards Social Security wealth, which is fully annuitized upon retirement. In addition, Social Security embeds some degree of income redistribution. The second is a hypothetical undistorted economy where people have the same saving/portfolio allocation instruments as in the economy with Social Security, but without the accompanying mandates. In the undistorted economy, people freely decide on (i) what fraction (if any) of income to contribute to their pension wealth every period, (ii) what fraction of pension wealth to annuitize upon retirement. There is also no income redistribution in the undistorted economy. The comparison between the two economies provides an analytical framework to study the (positive) effects of redistribution and the (negative) effects of distortions of Social Security, and to map these effects to the key design features of the program.

In the final part of our analysis, we quantitatively evaluate how changing the key design features of Social Security affects its optimal size. To do this, we construct and estimate a general equilibrium model where individuals differ in many characteristics including income and mortality. We estimate the model using several micro datasets. Our estimated model matches many features of the data, including the demand for Social Security annuities and the fraction of hand-to-mouth workers at different ages.

Our key results can be summarized as follows.

In our theoretical analysis, we show that the welfare effects of Social Security in partial equilibrium can be decomposed into four distinct components. The first component is the positive effect of redistribution: as there is some pooling of lifetime income when determining pensions, there is less consumption inequality after retirement. The second component is due to mandatory annuitization. This component can be positive or negative depending on whether Social Security mandates people to buy too much or too little annuities compared to their optimal choice. The third component is due to intertemporal distortions, and it can be represented as the weighted sum of distortions on optimal savings over the working stage of the life-cycle. This (negative) component increases in absolute value in the number of periods an agent is borrowing constrained. This is because, in those periods, payroll taxes directly reduce an agent's consumption. The final component is due to dynamic (in)efficiency. We represent this component as the difference in lifetime pension wealth accumulated through Social Security (with the rate of return equal to population growth) and through the private market (with the market rate of return).

Turning to our quantitative analysis, we start by re-examining the question of the optimal size of Social Security given its current design. Consistent with prior research, we find that

the optimal size is zero, however, our decomposition analysis offers some newer insights.

Prior studies emphasize that an important channel through which Social Security reduces welfare is the crowding-out of aggregate capital. Without Social Security, people save more to fund their retirement years, which leads to higher aggregate capital, and hence higher wages and welfare. In our model, the response of aggregate capital to the Social Security elimination is small. This is due to the strong estimated bequest motives: since every dollar saved for bequests can also be used to finance consumption after retirement, there is only a small change in savings once Social Security is removed (see also Fuster et al., 2003, for the discussion of this point).

Our results thus emphasize that much of the negative welfare effects of Social Security is not a mechanical effect of the change in aggregate capital, but can be linked to its design. In our subsequent analysis, we re-examine the question of the optimal size of Social Security when changing its key design features.

We start with the mandatory annutization. To understand its role, we consider the modified Social Security program where only a fraction of pension wealth is annuitized. Specifically, people receive part of their pensions not as lifelong annuities but as an equivalent (in present value terms) lump-sum payment. We first evaluate what fraction of Social Security wealth should be annuitized to maximize ex-ante welfare in the baseline economy (keeping the size of the program fixed). We find that this fraction is zero: people prefer a complete elimination of mandatory annuitization. Such policy results in the welfare gains equivalent to 0.5% of annual consumption. However, even with this policy in place, ex-ante welfare is still higher in the economy without Social Security. In other words, while mandatory annuitization is welfare-decreasing, this is not the main culprit for the negative role of Social Security.

We next examine the role of income redistribution through the Social Security program. We start by asking what level of redistribution maximizes ex-ante welfare in the baseline economy (keeping the size of Social Security fixed). Consistent with prior work, we find that pension benefits should be a flat function of average lifetime earnings (e.g., Golosov et al., 2013; Jones and Li, 2022). Implementing this change results in the welfare gains equal to 1.85% of annual consumption. However, increasing income redistribution does not change the optimal size of Social Security, which is still equal to zero.

We next evaluate the importance of the intertemporal distortions. Young workers are mandated to contribute to Social Security even though many of them are liquidity-constrained and would optimally delay saving for retirement. We consider a policy that exempts workers below a certain age from payroll taxation, while increasing the tax rate for older groups to balance the Social Security budget. We set the maximum exemption age to 41 years old

as this policy maximizes welfare in the baseline economy. We find that this age-dependent payroll tax policy not only results in large welfare gains given the current size of Social Security (3.5% of annual consumption), but also justifies its existence. While it is optimal to reduce the size of Social Security to 60% of its baseline value, it is no longer optimal to eliminate it altogether.

Finally, we consider various combinations of the policies described above and show that there are substantial complementarities. In particular, the best results are achieved when all policies are implemented at the same time: when Social Security is more redistributive, does not have mandatory annuitization, and features age-dependent payroll taxation. In this case, it is optimal not only to keep the program but also to increase its size by 60% compared to its size in the baseline economy.

Our results thus emphasize that the main reason for the welfare-decreasing role of Social Security lies in its design flaws: the program provides too much longevity insurance, too little insurance against lifetime income risk, and creates too much intertemporal distortions. Correcting these design flaws overturns the well-know result that the optimal size of Social Security is zero, and even implies it is optimal to increase the size of the program.

Our findings have important implications for the active debate about reforming Social Security. We argue that the reform directions should differ depending on whether the key design flaws of Social Security are addressed. If the design of Social Security is left unchanged, it is best to gradually phase the program out as it reduces welfare in the long-run. In contrast, correcting these design flaws makes the program welfare-improving and even justifies expanding its size in the long-run.

Related literature Our paper is related to several strands of literature. The first strand studies the long-run welfare effects of Social Security. Auberbach and Kotlikoff (1987) and Hubbard and Judd (1987) are the two early studies showing that the long-run welfare effects of Social Security are negative. With few exceptions, a large body of literature illustrates that this result is robust when taking into consideration many additional features of the economic/institutional environments. In particular, Social Security still reduces ex-ante welfare in the environments with explicit modeling of aggregate risk (Krueger and Kubler, 2006), land as a factor of production (Imrohoroglu et al., 1999), annuity or life insruance markets (Hong and Rios-Rull, 2007), housing wealth (Chen, 2010), home production (Dotsey et al., 2015), or time inconsistency in agents' behavior (Imrohoroglu et al., 2003). However, Social Security can also increase ex-ante welfare in several situations: when the economy without Social Security is dynamically inefficient (Imrohoroglu et al., 1995), when there are strong altruistic saving motives (Fuster et al., 2003), when both idiosyncratic and aggregate

risks are taken into account (Harenberg and Ludwig, 2019), or when people have self-control preferences (Kumru and Thanopoulos, 2011).

The second strand of related literature studies the optimality of Social Security as a provider of insurance against lifetime income risk. Huggett and Parra (2010) examine Social Security and the tax system jointly and find that there is much room for improvements: they show that one way to improve the system is by making the Social Security benefit formula more redistributive. Golosov et al. (2013) focus on the way to optimally modify Social Security benefit formula and also argue it should incorporate more redistribution. Several other studies come to a similar conclusion when considering richer frameworks with health and medical expense uncertainty (Bagchi and Jung, 2021), mortality-income correlation (Jones and Li, 2023), or heterogeneity in job stability (Nam, 2023). O'Dea (2018) also argues that a more progressive pension system can improve welfare, but in his approach, the increase in progressivity is achieved by expanding means-tested support at old age.

We also relate to the literature studying the welfare effects of distortions created by Social Security. Much of this literature focuses on labor supply distortions (Fuster et al., 2007; Imrohoroglu and Kitao, 2013). While payroll tax is a not a pure tax but rather a form of retirement savings, it can still be distortive due to imperfect link between benefits and contributions (French et al., 2022). In our study, we focus on another two types of distortions: that on intertemporal choice and on annuitization levels. While several studies point out the importance of intertemporal distortions for the welfare effects of Social Security (Hubbard and Judd, 1987; Hurst and Willen, 2007; Pries, 2007), to the best of our knowledge, the mandatory annuitization feature of the program has not yet been studied at the same angle. At the same time, there is survey evidence that people would like to substitute at least a part of their pensions with a lump-sum (Maurer et al., 2018, 2021).

The rest of the paper is organized as follows. Section 2 presents data facts. Section 3 contains our theoretical analysis. Sections 4 and 5 introduces our quantitative model and describe its estimation. Section 6 presents the quantitative results and Section 7 concludes.

2 Data

In this section, we document several empirical facts related to annuity demand of older people and intertemporal choices of the young. Our goal is to understand whether there is any indication that the annuitization/intertemporal distortions created by Social Security may be important.

In the U.S., people have access to two types of annuities: those provided by Social Security (or public annuities) and those that can be purchased in private markets. We start with the

demand for public annuities. While the size of one's pension (or public annuity) is largely predetermined by one's lifetime earnings, there is still some room to adjust this amount. This can be done by choosing the age at which one claims Social Security benefits. The earliest age one can claim benefits is 62. People who claim benefits before the full retirement age (FRA), have their pensions permanently reduced, while people who claim after the FRA, receive larger pensions.

Because of these penalties/rewards for early/late claiming, by choosing the age of claiming, people can change the amount of annuities obtained from Social Security: with each year of claiming delay, people get entitled to higher annuity income. Thus, claiming delay can be viewed as a "purchase" of public annuity with the price being equal to the year of forgone benefits.

To understand the demand for public annuities, we consider the distribution of people by age of claiming in the HRS data. The left top panel of Figure 1 plots the distribution among men in four cohorts with different FRA. We restrict our sample to non-disabled people. Around two thirds of men in all four cohorts claim their benefits before reaching the FRA (65-66 years old depending on cohort). Moreover, more than 40% claim at the earliest eligibility age (age 62), as shown in the right top panel of Figure 1. This shows that most retirees forgo an option to increase their public annuity income, moreover, many choose the lowest available annuity income by claiming as early as possible.

While it is possible that some of early claiming can be due to financial difficulties, the empirical evidence does not support this: early claimers typically do not have financial problems or liquidity constraints (see Goda et al., 2011; Shepard, 2011). Another explanation can be that these public annuities are too expensive. However, Pashchenko and Porapakkarm (2024) show that the Social Security annuity prices are better than actuarially fair at ages 62-64 for men with average life expectancy. We thus can consider claiming as at least to some extent reflecting people's annuitizaztion preferences.[3]

We next turn to the demand for private annuities. For this analysis, we consider the same sample described above: non-disabled men in four cohorts. The bottom panel of Figure 1 (solid color bars) plots the percentage of men in each cohort who report owning private annuities at age 70. In each cohort, only around 4-5% of respondents are annuity owners. This is consistent with the well-known annuity puzzle, a well-documented phenomenon that people are unwilling to annuitize (Pashchenko, 2013).

The low demand for private annuities stands in sharp contrast with the demand for life

[3] In fact, the number of early claimers may overestimate the true demand public annuities because of the Social Security earnings test. The test withholds pension benefits of working early claimers with relatively high earnings and "returns" the withheld benefits only at the FRA. This incentivizes some people to delay claiming in order to avoid the earnings test, artificially increasing the demand for public annuities.

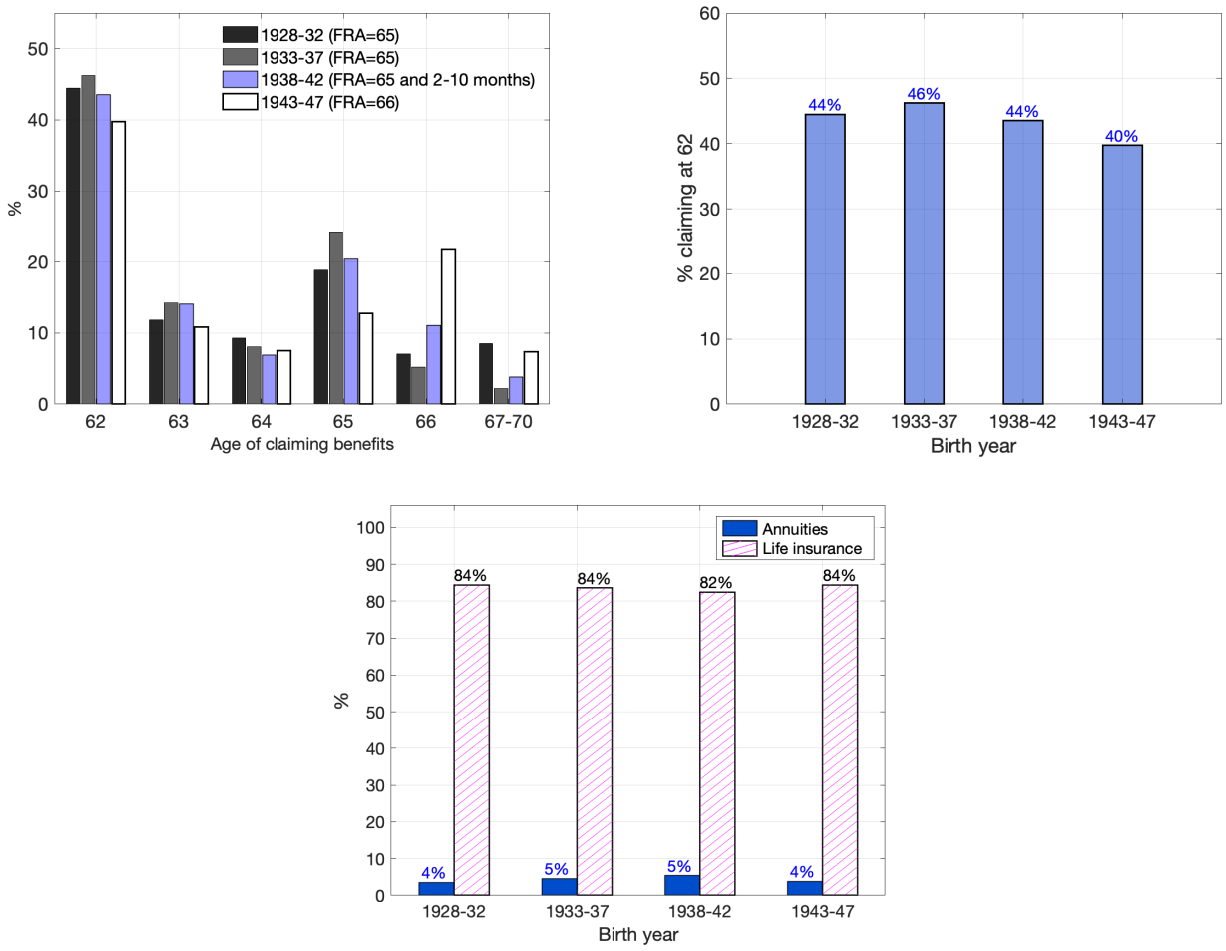


Figure 1: Top left panel: distribution of non-disabled men by age of claiming Social Security benefits. Top right panel: percentage of non-disabled men in each cohort who claim at the earliest eligibility age of 62. Bottom panel: percentage of non-disabled men who report having life insurance or private annuities at age 70. Source: HRS.

insurance, an insurance product that can be thought of as the opposite of an annuity. The hatched bars on the same graph show that more than 80% of 70-year-old non-disabled men in each cohort report having life insurance, which exceeds the corresponding number for annuity ownership by more than tenfold.

Taking together, the data facts presented above can indicate that people have low annuity demand. In the subsequent analysis, we examine to what extent the low annuity demand matters for the welfare effects of Social Security.

We next turn to the importance of intertemporal distortions. These distortions affect people who are liquidity constrained or the so-called hand-to-mouth. For this group, consumption closely tracks income, and thus is reduced almost one-for-one by payroll taxation. To understand the prevalence of this problem, we use the PSID to evaluate the number of

men who are likely liquidity constrained. To do this, we use the definition of the hand-to-mouth based on Zeldes (1989): a working respondent is classified as hand-to-mouth if his total net worth is below two months of his labor earnings.[4]

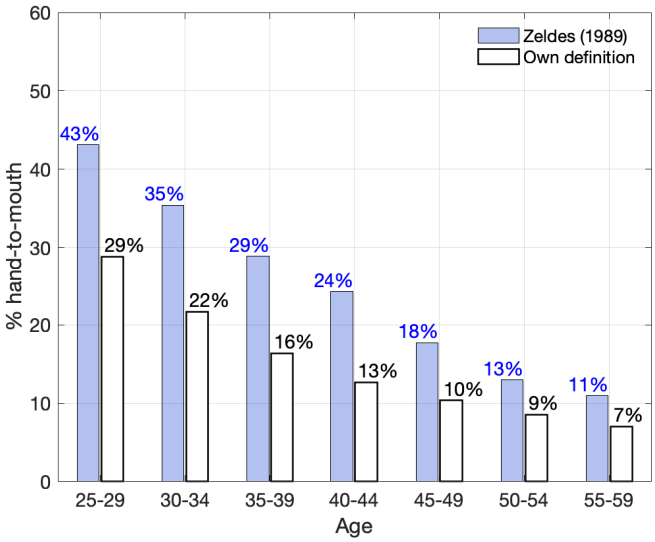


Figure 2: Percentage of the hand-to-mouth by age group. Shaded bars: Zeldes (1989) definition of hand-to-mouth. Non-shaded bars: own definition based on $100 net worth threshold.

The shaded bars in Figure 2 plot the resulting percentage of the hand-to-mouth among working men. The number of people in this category decreases with age: while among 24-29 year-old, 40% are hand-to-mouth, the corresponding number for 60-64 years old is less than 10%. Overall, 24-43% of people younger than age 45 can be considered liquidity constrained. Importantly, the median wealth of the hand-to-mouth below the age of 45 is equal to zero.

We also consider another definition of the liquidity constrained household. Specifically, we classify a respondent as being hand-to-mouth if his net worth is less than $100. The resulting percentage of people falling into this category is plotted as non-shaded bars in Figure 2. Despite being a more narrow definition of the liquidity-constrained compared to that of Zeldes (1989), this category still contains a non-trivial number of people. Overall, for men below the age of 45, this number varies between 13% and 29%.

Overall, this analysis shows that many young workers have not yet started actively building their assets and are likely liquidity-constrained. The fact that Social Security still mandates them to save around a tenth of their income for retirement can be an important distortion, which we will explore in our subsequent analysis.

[4] Since net worth is measured at the household level, we normalize it using the OECD household equivalence scale.

3 Theoretical decomposition

In this section, we construct a tractable overlapping-generation (OLG) model. We consider two versions of our model: a laissez-faire economy and an economy with Social Security. We compare the two economies to understand the various channels through which the program affects welfare. We consider a small open economy thus focusing on the partial equilibrium effects of Social Security. We incorporate the general equilibrium effects in our quantitative analysis in the next section.

3.1 Environment

Consider an OLG model with population growth n. Each individual lives for T periods: for the first R periods, an agent works, and between periods $R+1$ and T, an agent is retired. At the working stage, each individual receives labor earnings $y_{it} = \epsilon_i \lambda_t$ where ϵ_i is permanent productivity differing across individuals with $\epsilon_i \sim F(\epsilon_i)$, and λ_t is a deterministic function of age.

We assume agents survive with probability one to the first age of their retirement period, $t = R + 1$. After that age, agents face survival uncertainty, and we denote the probability to survive from age t to $t + 1$ as θ_i. Survival probability θ_i differs across individuals with $\theta_i \sim G(\theta_i)$, and the average survival probability is defined as $\overline{\theta} = \int \theta_i \, dG(\theta_i)$. We denote the joint distribution of θ_i and ϵ_i as $H(\theta, \epsilon)$.

Agents discount the future at the rate β, and can transfer resources intertemporarily at the interest rate r. We denote savings made at time t by an agent i as a_{it+1}. We assume agents cannot borrow, so that $a_{it+1} \geq 0$. Agents derive utility from consumption c_{it} and from leaving bequests beq_{it} after they die, where $beq_{it} = a_{it}(1 + r)$. Consumption is valued based on the utility function $u(\cdot)$ and bequests - based on the bequest function $v(\cdot)$. We assume that both $u(\cdot)$ and $v(\cdot)$ are strictly increasing, weakly concave and twice continuously differentiable.

The lifetime utility of an agent i can be represented as follows:

$$V_i = \underbrace{\sum_{t=1}^{R} \beta^{t-1} u(c_{it})}_{\text{working stage}} + \underbrace{\beta^R \left[u(c_{iR+1}) + \sum_{t=R+2}^{T} (\beta\theta_i)^{t-R-1} \left(u(c_{it}) + \frac{1-\theta_i}{\theta_i} v(beq_{it}) \right) + \beta^{T-R} \theta_i^{T-R-1} v(beq_{iT+1}) \right]}_{\text{retirement stage}} \tag{1}$$

3.2 Laissez-faire case

We start our analysis with the lassez-faire case. Our goal is to construct an environment with the same choice set as Social Security provides but without the accompanying distortions.

Setup The Social Security program mandates people to contribute to the pension system, and these contributions can be thought of as illiquid pension wealth. Upon retiring, contributions are converted into lifelong pensions, thus illiquid pension wealth is subject to mandatory annuitization.

To conserve these features of the program while removing the distortions, we construct an environment where people also have access to illiquid pension accounts and can annuitize its balance at retirement. The two key differences from the Social Security system are that agents can choose (i) how much to contribute to the pension account every period; (ii) what fraction of pension wealth to annuitize upon retiring.

Specifically, every working period, an agent decides on the fraction γ_{it} of his income y_{it} to contribute to the illiquid saving account. We can thus denote the sequence of contributions over working period, from age $t = 1$ to R, as $\{\gamma_{it} y_{it}\}_{t=1}^{R}$. Denoting the rate of return on the illiquid savings as r^c, we can denote the total wealth accumulated by an agent i over his entire working period in the laissez-faire case as PW_i^{lf}:

$$PW_i^{lf} = \sum_{t=1}^{R} \gamma_{it} y_{it} (1 + r^c)^{R+1-t} \tag{2}$$

PW_i^{lf} is available to an agent at the first age of retirement ($t = R + 1$).

At the start of retirement (age $t = R + 1$), an agent decides on the fraction $\alpha_i \in [0, 1]$ of his pension wealth PW_i^{lf} to annuitize at the annuity price q. We denote the corresponding annuity income as d_i:

$$d_i = \frac{\alpha_i \, PW_i^{lf}}{q} \tag{3}$$

We assume that an annuity starts paying out right away, so that an agent receives his first payment d_i at age $R + 1$.

We can thus write the individual maximization problem in the following way:

$$\max_{\{c_{it}\}_{t=1}^{T}, \{a_{it+1}\}_{t=1}^{T}, \{\gamma_{it}\}_{t=1}^{R}, \alpha_i} V_i$$

subject to

$$c_{it} = \begin{cases} y_{it}(1-\gamma_{it}) + a_{it}(1+r) - a_{it+1} & ;\ \text{if } t \leq R \\ (1-\alpha_i)PW_i^{lf} + d_i + a_{it}(1+r) - a_{it+1} & ;\ \text{if } t = R+1 \\ d_i + a_{it}(1+r) - a_{it+1} & ;\ \text{if } t > R+1 \end{cases} \tag{4}$$

$$0 \leq \gamma_{it} \leq 1, \quad 0 \leq \alpha_i \leq 1,$$

where V_i is given in Eq (1), PW_i^{lf} - in Eq (2), and d_i - in Eq (3).

Optimality conditions Before proceeding, we introduce several new notations. First, we use the following notations for the marginal utilities of consumption and bequests:

$$\frac{\partial\, u(c_{it})}{\partial\, c_{it}} \equiv u_{it} \qquad \frac{\partial\, v(beq_{it})}{\partial\, beq_{it}} \equiv v_{it}$$

Second, we denote the present value of the marginal utility of consumption during the entire retirement period (from age $R+1$ to T) as MU_i:

$$MU_i \equiv u_{iR+1} + \sum_{t=R+2}^{T} (\beta\theta_i)^{t-R-1} u_{it} \tag{5}$$

Finally, for each at t, we denote the optimal asset holdings and the optimal fraction of income contributed towards illiquid pension wealth as a_{t+1}^* and γ_t^*, respectively. We denote the optimal fraction of pension wealth annuitized upon retirement as α_i^*.

Using these notations, we can write the first-order conditions (FOCs) as follows.
FOCs for **savings** (Euler's equations):

$$u_{it} \geq \beta(1+r)u_{it+1} \quad \text{for} \quad t < R+1 \tag{6}$$

$$u_{it} \geq \beta(1+r)(\theta_i u_{it+1} + (1-\theta_i)v_{it+1}) \quad \text{for} \quad t \geq R+1 \tag{7}$$

FOC for the choice of **the fraction of annuitized pension wealth** α_i:

$$u_{iR+1}\, q \begin{cases} > MU_i & ;\ \text{when } \alpha_i^* = 0 \\ = MU_i & ;\ \text{when } 0 < \alpha_i^* < 1 \\ < MU_i & ;\ \text{when } \alpha_i^* = 1 \end{cases} \tag{8}$$

FOCs for **pension contributions** γ_{it}:

$$u_{it}y_{it} \geq (\beta(1+r^c))^{R+1-t} y_{it}\left((1-\alpha_i^*)u_{iR+1} + \alpha_i^* \frac{MU_i}{q}\right) \quad \text{for} \quad t = 1, ..., R \tag{9}$$

Defining wedges Our goal is to measure to what extent the rules imposed by the Social Security program distort agents' optimal choices. To do this, we use the concept of wedges, that is the difference between agents' marginal costs and marginal benefits of making certain decisions. We formally define the wedges below.

Definition 1 Consider an agent i who at age $t < R+1$, saves $a_{it+1} \geq 0$. We define an age-t saving wedge, $wedge_{it}^s$, as the difference between the marginal costs and the marginal benefits of making this choice:

$$wedge_{it}^s(a_{it+1}) \equiv u_{it} - \beta(1+r)u_{it+1} \tag{10}$$

It is worth noting that $wedge_{it}^s(a_{it+1}^*) = 0$ when $a_{it+1}^* > 0$. However, when the borrowing constraint binds, the saving wedge is positive, $wedge_{it}^s(0) > 0$.

We next define the wedge for the annuitization choice.

Definition 2 Consider an agent i who annuitizes a fraction α_i of his pension wealth and thus gets annuity income d_i. We define as the annuitization wedge, $wedge_i^a$, the difference between the marginal costs and the marginal benefits of this decision:

$$wedge_i^a(\alpha_i) \equiv u_{iR+1}\, q - MU_i, \tag{11}$$

where MU_i is given in Eq (5).

If an agent optimally sets $0 < \alpha_i^* < 1$, $wedge_i^a(\alpha_i^*) = 0$. If $wedge_i^a(0) > 0$ an agent optimally chooses not to buy any annuities. If $wedge_i^a(1) < 0$ an agent would like to annuitize more than just his pension wealth.

Finally, we define the wedges for the contribution into the pension account.

Definition 3 Consider an agent i who at age $t < R+1$ contributes a fraction $\gamma_{it} \geq 0$ of his income to the illiquid pension account with return r^c. We define an age-t pension contribution wedge, $wedge_{it}^{pc}$, as the difference between the marginal costs and the marginal benefits of making this choice:

$$wedge_{it}^{pc}(\gamma_{it}, \alpha_i) \equiv u_{it}y_{it} - (\beta(1+r^c))^{R+1-t} y_{it}\left((1-\alpha_i)u_{iR+1} + \alpha_i \frac{MU_i}{q}\right) \tag{12}$$

The pension contribution wedge also depends on the fraction of annuitized pension wealth, α_i. If $wedge^{pc}_{it}(0, \alpha_i) > 0$, an agent optimally chooses not to contribute to the pension account at age t given that fraction α_i of his pension wealth will be annuitized.

Decomposing the pension contribution wedge We next provide a more detailed analysis of the pension contribution wedge. In particular, we establish the relationship between the pension contribution wedge on the one hand, and saving and annuitization wedges on the other. In our analysis, we assume an agent optimally chooses his savings while the fraction of income he contributes to pension wealth, γ_{it}, and the fraction of wealth he will annuitize at retirement, α_i are not necessary optimal. We do this because in the economy with Social Security discussed in the next section, both γ_{it} and α_i are not chosen optimally but are mandated by the system. Our results are summarized in Lemma 1.

Lemma 1 The wedge in pension contribution decisions at age t, $wedge^{pc}_{it}$, can be decomposed into three components: (i) the discounted sum of saving wedges from age t to the last working age (R); (ii) the difference in return to regular savings r and illiquid savings r^c weighted by the marginal utility at the age when pension wealth becomes liquid ($t = R+1$); (iii) the annuitization wedge.

$$wedge^{pc}_{it}(\gamma_{it}, \alpha_i) = \underbrace{y_{it}\left(\sum_{k=t}^{R}(\beta(1+r))^{k-t} wedge^{s}_{ik}(a^{*}_{it+1})\right)}_{\text{sum of saving wedges}} + \underbrace{\beta^{R+1-t} u_{iR+1}\left[(1+r)^{R+1-t} - (1+r^c)^{R+1-t}\right]}_{\text{difference in returns}}$$
$$+ \underbrace{(\beta(1+r^c))^{R+1-t} y_{it}\alpha_i \frac{wedge^{a}_{i}(\alpha_i)}{q}}_{\text{annuitization wedge}}$$

Proof We start by rewriting the pension contribution wedge in Eq (12) as follows:

$$wedge^{pc}_{it}(\gamma_{it}, \alpha_i) = u_{it}y_{it} - (\beta(1+r^c))^{R+1-t} y_{it}\left(u_{iR+1} - \alpha_i \underbrace{\left(u_{iR+1} - \frac{MU_i}{q}\right)}_{wedge^{a}_{i}(\alpha_i)/q}\right) \tag{13}$$

Note that the term inside the last bracket is the annuitization wedge evaluated at α_i and divided by the annuity price q.

We next proceed proving the lemma for age $t = 1$, and the proof for other ages is analogous. By adding and subtracting $\beta(1+r)u_{i2}y_{i1}$, $(\beta(1+r))^2 u_{i3}y_{i1}$ and so forth in Eq

(13), we can transform the wedge for pension contributions at $t = 1$ as follows:

$$wedge^{pc}_{i1} = y_{i1}\Bigg((u_{i1} - \beta(1+r)u_{i2}) + \beta(1+r)(u_{i2} - \beta(1+r)u_{i3}) + ... \\ + \beta^R(1+r)^R u_{iR+1} - \beta^R(1+r^c)^R u_{iR+1} + (\beta(1+r^c))^R \alpha_i \frac{wedge^a_i(\alpha_i)}{q} \Bigg)$$

The resulting expression directly yields the statement of the lemma when using the definitions of the saving wedges.

To better understand the intuition for the statement of the lemma, consider first the situation when all the saving wedges and the annutization wedge are zero. In this case, the size of the pension contribution wedge is determined solely by the difference between the return on illiquid savings r^c and regular savings r. When $r > r^c$, pension wedge is positive so that an agent never voluntary contributes to the illiquid account since it brings lower return.

Consider a situation when the saving wedge is positive at some age t, so that the borrowing constraint is binding ($a^*_{it+1} = 0$). The positive saving wedge increases the size of the pension contribution wedge. This is because illiquid savings that cannot be accessed when the borrowing constraint binds increase the marginal costs of pension contributions.

Consider next the annuitization wedge. When the fraction of the annuitized wealth is not set optimally ($\alpha_i \neq \alpha^*_i$), the annuitization wedge can be positive or negative. The negative (positive) annuitization wedge means that an agent is restricted to buy too few (too many) annuities compared to his optimal choice. For example, when an agent is mandated to buy too many annuities, this increases the pension contribution wedge. This is because the illiquid pension wealth becomes less attractive given that it is used to purchase undesirable annuities.

Lemma 1 gives raise to the following corollary that we will use in the subsequent analysis.

Corollary 1 The discounted sum of the pension contribution wedges, $\sum_{t=1}^R \beta^{t-1} wedge^{pc}_{it}$, can be expressed as follows:

$$\sum_{t=1}^R \beta^{t-1} wedge^{pc}_{it} = \sum_{t=1}^R (\beta(1+r))^{t-1} wedge^s_{it}(a^*_{it+1})\, PE_{it} \\ + \beta^R u_{iR+1}\Big(LW_i(r) - LW_i(r^c) \Big) + \beta^R\, \alpha_i \frac{wedge^a_i(\alpha_i)}{q} LW_i(r^c),$$

where PE_{it} is the present value of labor income up to age t:

$$PE_{it} = \sum_{k=1}^{t} \frac{y_{ik}}{(1+r)^{k-1}}, \tag{14}$$

and $LW_i(\hat{r})$ is the hypothetical lifetime wealth corresponding to the situation when an agent i saves his entire income every working period ($t = 1, ..., R$) at the interest rate $\hat{r}$:

$$LW_i(\hat{r}) = \sum_{t=1}^{R} y_{it}(1+\hat{r})^{R+1-t} \tag{15}$$

3.3 Social Security

We consider a stylized version of Social Security where each agent contributes into the system a fraction τ of his earnings y_{it} every working period. Individuals' contributions are used to finance pensions of the old. We denote pension income of an agent i as Δ_i.

Social Security pension wealth Within the framework of the previous section, we can think of Social Security as a program that allows agents to invest in illiquid pension wealth with return n. We can formally define the Social Security pension wealth as follows:

$$PW_i^{ss} = \sum_{t=1}^{R} \tau y_{it}(1+n)^{R+1-t} = \tau LW_i(n) \tag{16}$$

The last equality relates pension wealth with the lifetime wealth defined in Eq (15).

Importantly, Social Security restricts the agents' choice set. First, the contributions into the system are fixed and age-independent: $\gamma_{it} = \tau$ for all i and $t = 1, ..., R$. Second, the entire contributions are converted into annuities upon retirement: $\alpha_i = 1$ for all i.

Social Security budget To write down the Social Security balance equation, we denote as N the initial size of the cohort who is currently of age T (the oldest cohort). Given the survival uncertainty, the current size of this cohort is $\int_{\theta} \theta^{T-R-1} N d\theta$. The size of the cohort of age $t = 1$ (the youngest cohort) is $(1+n)^{T-1}N$. We summarize the size of each currently living cohorts in Table 1.

We can thus write the pension system balance equation as follows:

$$N \int_{\epsilon} \int_{\theta} \Delta \sum_{t=1}^{T-R} \theta^{t-1}(1+n)^{T-R-t} H(\theta, \epsilon)\, d\theta\, d\epsilon = N \int_{\epsilon} \sum_{t=1}^{R} \tau y_{it}(1+n)^{T-t} dF(\epsilon)$$

	Working period				Retirement period			
Age	$t=1$	$t=2$	...	$t=R$	$t=R+1$	$t=R+2$	...	$t=T$
Cohort size	$N(1+n)^{T-1}$	$N(1+n)^{T-2}$	...	$N(1+n)^{T-R}$	$N(1+n)^{T-R-1}$	$\int_\theta \theta N(1+n)^{T-R-2}d\theta$	...	$\int_\theta \theta^{T-R-1}Nd\theta$

Table 1: Size of each cohort in the model.

Dividing both sides by $N(1+n)^{T-R-1}$, we arrive at the following expression:

$$\int_\epsilon \int_\theta \Delta \underbrace{\sum_{t=1}^{T-R} \left(\frac{\theta}{1+n}\right)^{t-1}}_{\text{cost of pension annuity}} H(\theta,\epsilon)\, d\theta\, d\epsilon = \int_\epsilon \underbrace{\sum_{t=1}^{R} \tau y_{it}(1+n)^{R+1-t}}_{\text{SS pension wealth}} dF(\epsilon) \tag{17}$$

On the left hand side, we have the total costs of paying pension income to all currently alive retired cohorts. We can think of the expression $\sum_{t=1}^{T-R}\left(\frac{\theta_i}{1+n}\right)^{t-1}$ on the left-hand side as the cost of delivering a unit stream of lifelong annuity income to an agent i through the pay-as-you-go system. We denote this expression as q_i. On the right hand side, we have the total contributions from all working-age cohorts, which can also be thought of as the aggregate pension wealth.

Using the definition of Social Security pension wealth in Eq (16) and the cost of pension annuity q_i, we can rewrite the pension system balance equation in the following form:

$$\int_\epsilon \int_\theta \Delta\, q\, H(\theta,\epsilon)\, d\theta\, d\epsilon = \int_\epsilon PW^{ss} dF(\epsilon) \tag{18}$$

Pension benefits We next consider how pension benefits Δ_i are determined. While in essence, pension benefits represent the annuitized value of one's contributions into the system, the actual conversion rate is rather complex. Two key features of the pension-contributions link in the U.S. Social Security are as follows. First, there is pooled annuitization, that is to say, pension annuities are independent of one's life expectancy. Second, there is a redistributive component. To captures this in our stylized framework, we explicitly model the conversion of one's contributions into pensions as a two-step arrangement.

At the first step, the agent's total contributions are subject to redistribution. We denote the ex-post (after the redistribution took place) Social Security pension wealth as $\widehat{PW}_i^{ss}$, where

$$\widehat{PW}_i^{ss} = A \cdot PW_i^{ss} + (1-A)\cdot \overline{PW}^{ss}, \tag{19}$$

where $\overline{PW}^{ss}$ is average Social Security pension wealth:

$$\overline{PW}^{ss} \equiv \int_{\epsilon} PW^{ss} dF(\epsilon)$$

In Eq (19), the parameter A determines the extent of redistribution, with $A = 1$ corresponding to absence of any redistribution, and $A = 0$ corresponding to a complete pooling of resources (full redistribution).

At the second step, the ex-post contribution $\widehat{PW}_i^{ss}$ is annuitized. The conversion of one's pension wealth into pensions is independent of individual mortality, hence we assume that the conversion is done at the same rate q^{pub} for all agents. Thus, pension income of an agent i, Δ_i, can be represented as follows

$$\Delta_i = \frac{\widehat{PW}_i^{ss}}{q^{pub}} \tag{20}$$

Here q^{pub} is such that the Social Security budget is balanced (Eq 18). Formally, the conversion rate of pension wealth into pension annuities, q^{pub}, is described in Lemma A1 in Appendix A.

3.4 Changing the size of the pension system

We now consider the effects of the pension system's size on individual and aggregate welfare. Given the balanced budget, the size of the pension system is determined by the tax rate τ, and in what follows, we study how welfare changes in response to τ.

Change in individual welfare We start with individual welfare. Consumption of an agent i when there is a pension system can be written as follows:

$$c_{it} = \begin{cases} y_{it}(1-\tau) + a_{it}(1+r) - a_{it+1} & ; \text{ if } t \leq R \\ \Delta_i + a_{it}(1+r) - a_{it+1} & ; \text{ if } t > R \end{cases}$$

where Δ_i is defined in Eq (20).

Consider how the agent-i's lifetime utility in Eq (1) changes with the marginal change in the size of the program τ. In our analysis, we assume that bequest motive is strong enough

until borrowing constraint never binds after retirement.[5]

$$\frac{\partial V_i}{\partial \tau} = \underbrace{-\sum_{t=1}^{R} \beta^{t-1} u_{it} y_{it} \; + \; \frac{\partial \Delta_i}{\partial \tau} \, \beta^R MU_i}_{\text{direct tax effects}} \; \underbrace{- \sum_{t=1}^{R} \beta^{t-1} wedge_{it}^{s} \frac{\partial a_{it+1}}{\partial \tau}}_{\text{effects on decisions}} \tag{21}$$

Here, the first two terms are the direct effects of the change in the tax rate on agents' after tax income (first term) and pensions benefits (second term). The last term comes from the effects of the tax change on saving decisions. It is worth noting that if an agent is never borrowing constrained, the latter effects will be absent since all saving wedges are zero.[6]

We can rewrite the change in individual welfare in terms of wedges defined earlier, as shown in the following lemma.

Lemma 2 The change in individual welfare in response to the marginal change in the size of the pension system can be represented as follows:

$$\begin{aligned} \frac{\partial V_i}{\partial \tau} = & -\sum_{t=1}^{R} (\beta(1+r))^{t-1} wedge_{it}^{s} \widetilde{PE}_{it} - \beta^R u_{iR+1} \Big(LW_i(r) - LW_i(n) \Big) \\ & - \beta^R \frac{LW_i(n)}{q^{pub}} wedge_i^a + \beta^R \frac{(1-A)(\overline{LW} - LW_i(n))}{q^{pub}} MU_i \end{aligned} \tag{22}$$

In this expression,

$$\widetilde{PE}_{it} = PE_{it} + \frac{1}{(1+r)^{t-1}} \frac{\partial a_{it+1}}{\partial \tau},$$

where PE_{it} is the present value of earnings up to age t defined in Eq (14). $\overline{LW}(n)$ is the average lifetime wealth:

$$\overline{LW}(n) = \int_{\epsilon} LW(n) dF(\epsilon)$$

Proof See Appendix A.

Change in aggregate welfare We next consider the effects of the size of the pension system on aggregate welfare, W, defined as follows:

$$W = \int V_i \, dG\left(\theta_i\right)$$

[5] This assumption does not change the key components of our decomposition but makes the expressions less cluttered.

[6] The last term in Eq (21) only includes the saving wedges up to age R because of our assumption that the bequest motive is strong enough until borrowing constraint is never binding after retirement.

The distinct role of redistribution and distortions for the welfare effects of the pension system is described in the following proposition.

Proposition 1: The change in aggregate welfare in response to the marginal change in the size of the pension system can be decomposed into four parts: the effect of redistribution, the effect of dynamic (in)efficiency, distortions in optimal annuitization choice, and the intertemporal distortions:

$$\frac{\partial W}{\partial \tau} = -\frac{\beta^R}{q^{pub}} \underbrace{(1-A)cov(LW_i, MU_i)}_{\text{redistribution}} - \beta^R \underbrace{\int u_{iR+1}\Big(LW_i(r) - LW_i(n)\Big)\, dH\left(\theta_i, \epsilon_i\right)}_{\text{dynamic (in)efficiency}} \tag{23}$$

$$- \frac{\beta^R}{q^{pub}} \underbrace{\int LW_i(n) wedge_i^a \, dH\left(\theta_i, \epsilon_i\right)}_{\text{annuitization distortions}} - \underbrace{\int \sum_{t=1}^{R} (\beta(1+r))^{t-1} wedge_{it}^s \, \widetilde{PE}_{it} \, dH\left(\theta_i, \epsilon_i\right)}_{\text{intertemporal distortions}}$$

Proof: The statement of the proposition directly follows when taking the integral of Eq (22) and using the definition of the covariance.

Consider the intuition of Eq (23). The first term captures inequality in consumption after retirement arising from the difference in lifetime wealth. Since high-income agents have higher consumption and thus lower marginal utility, the correlation $cov(LW_i, MU_i)$ is negative. Through redistribution, this correlation decreases in absolute value. For example, if the pension system was able to equalize consumption after retirement, the correlation would be zero.

The second term shows the effect of dynamic (in)efficiency. Social Security mandates people to save at the rate n, while private savings bring return r. The difference $LW_i(r) - LW_i(n)$ represents opportunity costs for people of having accumulating pension wealth with low return, weighted by the marginal utility at the time of retirement since that is when pension wealth can be used.

The third term represents the average wedge in the annuitization choice. Since Social Security mandates high-income people to purchase more annuities in absolute terms due to proportional taxation, the average wedge is weighted by the lifetime wealth. The third term can be positive or negative depending on the willingness to annuitize. Social Security is the only way to obtain annuities in this framework. If people would like to have more annuities than the mandated amount, the wedge is negative, and the third term is positive. In this case, enlarging the Social Security program improves aggregate welfare by allowing people to get more annuities. The opposite is trues if people would like to have less annuities than what they are required to obtain through Social Security.

The last term captures intertemporal distortions arising from the mandatory contributions into the pension system. Social Security mandates people to contribute a fixed fraction of their income τ into the system every period. This creates distortions when borrowing constraint is binding. When the constraint never binds, saving wedges are zero every period and the intertemporal distortions are absent. In this case, people can offset the effect of mandated pension contributions on their consumption. However, when borrowing constraint binds, some saving wedges are positive. In those periods, consumers behave as the hand-to-mouth, setting their consumption equal to income. Mandating them to contribute to the pension system at those periods directly translates into lower consumption, reducing welfare of Social Security.

It is worth noting that when $A = 1$ (no redistribution), the first term in the decomposition in Eq (23) disappears. In this case, Social Security is just a forced savings program accompanied by a mandatory annuitization feature, and its effects depend on whether the economy is dynamically efficient and how much distortions the mandated savings and mandated annuitization introduce.

4 Baseline Model

The theoretical model described in the previous section highlights three important sources of the welfare effects of Social Security: the positive effect of income redistribution and the negative effects of intertemporal and annuitization distortions. To understand the quantitative importance of these channels, in this section we construct a rich life-cycle model. The model includes many important features of reality that were omitted from our theoretical analysis for the reasons of tractability. In particular, we allow for endogenous labor supply and claiming decisions, and also introduce several additional sources of uncertainty. We incorporate our life-cycle model into a general equilibrium framework so that we can also take into account the effects of Social Security on aggregate capital and the distribution of bequests.

4.1 Demographics and preferences

We consider an OLG economy with the population growth n. Agents enter the model at age 25 and can live at most until age 99. Agents younger than age R^E work and make consumption/saving decisions. Between ages R^E and R^D, they also decide when to claim Social Security benefits, while also choosing whether to continue working or to retire. After age R^D, agents no longer work. We denote the health-dependent probability to survive from

age t to $t+1$ as θ_t^h.

We allow for flexible preferences where the coefficient of risk aversion may differ from the inverse of the intertemporal elasticity of substitution (IES). Specifically, we use Epstein-Zin preferences (Epstein and Zin, 1989) by assuming that an individual's utility over consumption (c_t), leisure ($\widetilde{l}_t$), and bequeathed assets (k_{t+1}) can be represented in the following recursive form:

$$U_t^{1-\gamma} = \underbrace{\left(c_t^{\chi}\widetilde{l}_t^{1-\chi}\right)^{1-\gamma}}_{\text{current flow utility}} + \beta\left(\theta_t^h \underbrace{E_t U_{t+1}^{1-\psi}}_{\text{future utility when alive}} + (1-\theta_t^h)\underbrace{\eta\left(k_{t+1}+\phi_{Beq}\right)^{1-\psi}}_{\text{bequest utility}}\right)^{\frac{1-\gamma}{1-\psi}}$$

Here χ is a parameter determining the relative weight of consumption in the consumption-leisure composite, ψ is the risk-aversion, $1/\gamma$ is the IES, β is the discount factor, η is the strength of the bequest motive, and ϕ_{Beq} is a shift parameter that controls to what extent bequest is a luxury good. In this formulation of bequest motives, we follow De Nardi (2004).

4.1.1 Health and medical expenses

Agents differ ex-ante in their fixed productivity type (ξ) which affects labor earnings and health evolution. Fixed productivity type can take two values: $\xi_1 < \xi_2$. This is a stylized representation of heterogeneity in fixed ex-ante characteristics (childhood circumstances, genetics, etc.) that affects both health and labor market outcomes.

Health status, h_t, is stochastic and takes two values: good or bad, $h_t \in \{G, B\}$. It evolves according to a type- and age-dependent Markov process, $\mathcal{H}_t(h_t|h_{t-1}, \xi)$. Health affects productivity, medical expenses, and survival probability.

Each period, an agent receives an out-of-pocket medical expense shock x_t^h which depends on his age and health. We denote the probability distribution of medical shocks as $\mathcal{G}_t(x_t^h)$. Starting from age R^D there is an additional risk, that of needing long-term care. We denote the age- and health-dependent probability of experiencing a long-term care shock as pn_t^h. An agent who experiences this shock has to pay an out-of-pocket nursing home cost xn_t.

4.1.2 Labor supply, leisure, and labor income

Each agent is endowed with one unit of time that can be used for either leisure or work. Labor supply (l_t) is indivisible, so that agents either do not work ($l_t = 0$) or work full-time ($l_t = \bar{l}$). Work brings disutility modeled as a fixed cost of leisure ϕ_w. In addition, agents who claim Social Security benefits early (before reaching the FRA) while continuing to work,

incur extra leisure costs ϕ_{ss}.[7]

The leisure $\widetilde{l_t}$ can thus be expressed as follows:

$$\widetilde{l_t} = 1 - l_t - \phi_w \mathbf{1}_{\{l_t>0\}} - \phi_{ss} \mathbf{1}_{\left\{l_t>0 \,\cap\, i_t^C=1 \,\cap\, t<FRA\right\}}$$

Here $\mathbf{1}_{\{.\}}$ is an indicator function equal to one if its argument is true, and i_t^C is a variable equal to one if an agent claims Social Security benefits at age t, and is equal to zero otherwise.

Earnings y_t are determined as:

$$y_t = w z_t^h l_t$$

where w is marginal product of labor in the economy and z_t^h is the idiosyncratic productivity, which consists of the deterministic age- and health-dependent component, a persistent shock, and the fixed productivity effect. More details on labor productivity are given in Section 5.1.4.

4.1.3 Taxes, transfers, and Social Security

Income taxation: There are three types of taxes in the model economy. First is the progressive income tax $\mathcal{T}\left(y^{tax}\right)$, where taxable income y^{tax} is based on labor, asset income, and a taxable part of Social Security benefits $y^{ss_{tax}}$. Second is the Social Security (τ_{ss}) payroll tax, which is set to zero for earnings above $\overline{y}_{ss}$. We set $\overline{y}_{ss}$ to \$76,200 (corresponding to year 2000). Third is the Social Security Earnings tax, paid by some workers who already claimed benefits. The latter differs from other taxes since the amount taxed away is paid back in terms of higher future pensions.

Mean-tested transfers: The government provides a means-tested transfer T_t^{SI} which guarantees each individual the minimum consumption level $\underline{c}$. Hence, individuals who have low earnings and high medical expenses can receive additional income to maintain their consumption at the level $\underline{c}$. This allows us to capture in a parsimonious way several U.S. safety net programs such as food stamps, Supplemental Security Income, disability insurance, Medicaid, and uncompensated care.

Social Security program The earliest age an individual can start receiving benefits (R^E) is 62 years old, while the latest age the benefits must be claimed (R^D) is 70. Pension income

[7] This is a reduced form way to capture the spike of claiming at the FRA observed in the data. This spike is due to the rules of the Social Security earnings test as shown by Pashchenko and Porapakkarm (2024). To model all the details of the earnings test, we need to introduce an additional state variable. To reduce computational costs, we instead use the extra utility costs for working early claimers.

$ss(AE, j^R)$ is a concave function of the average lifetime earnings (AE) and the age when the benefits were claimed ($j^R \geq R^E$).

One's average lifetime earnings evolve over the life-cycle, and we approximate this evolution as follows:

$$AE_{t+1} = \begin{cases} AE_t + \dfrac{y_t}{35} & ;\ \text{if } t < 60 \\ AE_t + \dfrac{1}{35}\max\{0, y_t - AE_t\} & ;\ \text{otherwise} \end{cases} \tag{24}$$

where

$$y_t = \min\left\{wz_t^h l_t, \overline{y}_{ss}\right\}.$$

Over the 35-year period from age 25 to 60, AE_t is updated every period, while after age 60, it is updated only if the current earnings exceed the average of previous earnings.[8] Once agents claim benefits, AE_t an be updated only due to the earnings test as described below.

People who claim Social Security benefits at the FRA (R^F) receive full benefits, which are calculated as follows:

$$ss(AE, j^R = R^F) = \begin{cases} 0.9AE & ;\ \text{if } AE < B_1 \\ 0.9B_1 + 0.32(AE - B_1) & ;\ \text{if } B_1 \leq AE < B_2 \\ 0.9B_1 + 0.32(B_2 - B_1) + 0.15(AE - B_2) & ;\ \text{if } AE \geq B_2, \end{cases} \tag{25}$$

where B_1 and B_2 are the bend points, i.e., the levels of AE when the replacement rate changes first from 0.9 to 0.32, then from 0.32 to 0.15. Social Security rules regarding benefits calculations change for each cohort; we use individuals born in 1936-1938 as our base cohort. The FRA for our base cohort is 65 years.[9] We set the bend points B_1 to \$6,372 and B_2 to \$38,424 based on the Social Security benefits formula for 2000.[10]

People who claim earlier or later than the FRA have their pension income adjusted in the following way. The benefits of early claimers are reduced by 6.7% per year for ages between 62 and 65. Individuals who claim benefits after the full retirement age get their basic benefits increased by 6.5% for every year up to age 70. The full schedule of benefits/rewards for early/late claiming is shown in the first row of Table 2.

Individuals who are younger than the FRA, and who receive Social Security benefits

[8] The Social Security benefits are a function of the average earnings of the 35 years with the highest earnings. We use a simplified version of this rule because otherwise we have to keep track of the entire previous earnings history as additional state variables, which makes our computation infeasible.

[9] For individuals born in 1936 and 1937 the full retirement age is 65 years, for individuals born in 1938 it is 65 years and 2 months.

[10] These numbers correspond to the annual benefits, they are derived by multiplying the bend points corresponding to monthly benefits by 12.

Age	$62(R^E)$	63	64	65 (FRA)	66	67	68	69	$70(R^D)$
% of full benefits	80%	86.7%	93.3%	100%	106.5%	113%	119.5%	126%	132.5%

Table 2: Reduction (increase) in benefits for early (late) claiming as a percentage of the benefits received at the full retirement age (1937 cohort).

but continue to work are subject to the Social Security earning tax t^{earn}[11] The earnings tax withholds pension benefits from people with relatively high earnings based on the following schedule:

$$t^{earn} = \begin{cases} \min\left\{ss(AE, j^R),\ \dfrac{wz_t^h l_t - \$10,080}{2}\right\} & ;\ \text{if} \quad t < R^F \\ 0 & ;\ \text{otherwise} \end{cases} \tag{26}$$

The withheld benefits go towards increasing the future benefits. More specifically, the Social Security earning tax partially offsets the penalty for early claiming. For example, if an individual has all of his benefits withheld for the entire year, his benefits will be adjusted as if he claimed them one year later. To avoid keeping track of withheld benefits as an additional state variable, we approximate these rules as follows. If more than 50% of one's benefits are withheld due to the earning tax we increase j^R by one year. Otherwise, we do not make any adjustments.

Social Security benefits are subject to the income taxation. We denote the taxable portion of benefits as $y^{ss_{tax}}$ and it is equal to Social Security pension net of the earnings tax: $y^{ss_{tax}} = ss\left(AE, j^R\right) - t^{earn}$.

4.1.4 Timing in the model

The timing in the model is as follows. In the beginning of the period, individuals learn their productivity and health status. Based on this information, an individual decides his labor supply (l_t). An individual who is older than age R^E also decides whether to claim Social Security benefits ($i_t^C = 1$) or not ($i_t^C = 0$). Afterward, the out-of-pocket medical shock (x_t^h) is realized; for individuals older than age R^D the long-term care shock (xn_t) is realized. At the very end of the period, consumption/saving decisions are made. After age R^D, individuals make only consumption/saving decisions.

[11] Starting from 2000, the Social Security earning tax for individuals who reach the full retirement age was abolished.

4.1.5 Optimization problem

Individuals younger than age R^E The state variables for this age group at the beginning of each period are asset (k_t), health ($h_t \in \{G, B\}$), fixed productivity type ($\xi \in \{\xi_1, \xi_2\}$), idiosyncratic labor productivity $\left(z_t^h\right)$, and average lifetime earnings (AE_t). We denote the vector of state variables of an individual of age t as $\mathbb{S}_t$.

The individual optimization problem takes the following form:

$$V_t(\mathbb{S}_t) = \max_{l_t} \left\{ \sum_{x_t^h} \mathcal{G}_t\left(x_t^h\right) W_t(\mathbb{S}_t; l_t, x_t^h)^{1-\psi} \right\}^{\frac{1}{1-\psi}} \tag{27}$$

where

$$W_t(\mathbb{S}_t; l_t, x_t^h) = \max_{c_t, k_{t+1}} \left\{ \begin{array}{c} \left(c_t^{\chi} \widetilde{l}_t^{1-\chi}\right)^{1-\gamma} + \\ \beta \left(\theta_t^h E_t \left(V_{t+1}(\mathbb{S}_{t+1})\right)^{1-\psi} + (1-\theta_t^h)\eta \left(k_{t+1} + \phi_{Beq}\right)^{1-\psi}\right)^{\frac{1-\gamma}{1-\psi}} \end{array} \right\}^{\frac{1}{1-\gamma}} \tag{28}$$

subject to

$$k_t\left(1+r\right) + w z_t^h l_t + T_t^{SI} + Beq(\xi) = k_{t+1} + c_t + x_t^h + Tax \tag{29}$$

$$T_t^{SI} = \max\left(0, \underline{c} + x_t^h + Tax - k_t\left(1+r\right) - w z_t^h l_t - Beq(\xi)\right) \tag{30}$$

$$Tax = \mathcal{T}\left(y_t^{tax}\right) + \tau_{ss} \min\left(w z_t^h l_t, \overline{y}_{ss}\right) \tag{31}$$

$$y_t^{tax} = k_t r + w z_t^h l_t \tag{32}$$

The evolution of AE_t is described in Eq (24). The conditional expectation on the right-hand side of Eq (28) is over z_{t+1}^h and h_{t+1}. Eq (29) is the budget constraint, where $Beq(\xi)$ is the amount of bequests received by each living agent of type ξ. Eq (30) describes the means-tested transfers that provide the minimum consumption guarantee $\underline{c}$. In Eq (31), the first term is the income tax and the last term is the payroll tax. Eq (32) describes the taxable income.

Individuals between the ages of R^E and R^D and who has *yet to claim benefits.* An individual in this age range has to decide whether to claim Social Security benefits or not: $i_t^C \in \{0,1\}$. An individual at age R^D must claim benefits if he still didn't do so. The individual maximization problem can be written as follows:

$$V_t(\mathbb{S}_t) = \max_{l_t, i_t^C} \left\{ \sum_{x_t^h} \mathcal{G}_t\left(x_t^h\right) W_t^E(\mathbb{S}_t; l_t, i_t^C, x_t^h)^{1-\psi} \right\}^{\frac{1}{1-\psi}} \tag{33}$$

The interim value functions $W_t^E(\cdot)$ vary depending on whether an individual decides to claim benefits or not. For the former group, there will be an additional state variables next period, the age at which one claims $\left(j^R\right)$:

$$W_t^E(\mathbb{S}_t; l_t, i_t^C = 0, x_t^h) = \max_{c_t, k_{t+1}} \left\{ \begin{array}{c} \left(c_t^{\chi} \widetilde{l}_t^{1-\chi}\right)^{1-\gamma} + \\ \beta \left(\theta_t^h E_t \left(V_{t+1}(\mathbb{S}_{t+1})\right)^{1-\psi} + (1-\theta_t^h)\eta \left(k_{t+1} + \phi_{Beq}\right)^{1-\psi}\right)^{\frac{1-\gamma}{1-\psi}} \end{array} \right\}^{\frac{1}{1-\gamma}}$$

$$W_t^E(\mathbb{S}_t; l_t, i_t^C = 1, x_t^h) = \max_{c_t, k_{t+1}} \left\{ \begin{array}{c} \left(c_t^{\chi} \widetilde{l}_t^{1-\chi}\right)^{1-\gamma} + \\ \beta \left(\theta_t^h E_t \left(V_{t+1}^C \left(\mathbb{S}_{t+1}, j^{R,}\right)\right)^{1-\psi} + (1-\theta_t^h)\eta \left(k_{t+1} + \phi_{Beq}\right)^{1-\psi}\right)^{\frac{1-\gamma}{1-\psi}} \end{array} \right\}^{\frac{1}{1-\gamma}}$$

subject to

$$k_t\left(1+r\right) + wz_t^h l_t + ss(AE_t, t)\mathbf{1}_{\{i_t^C=1\}} + T_t^{SI} + Beq(\xi) = k_{t+1} + c_t + x_t^h + Tax \tag{34}$$

The amount of means-tested transfers is determined based on one's net resources Res_t:

$$T_t^{SI} = \max\left(0, Res_t\right),$$

where

$$Res_t = \underline{c} + x_t^h + Tax - k_t\left(1+r\right) - wz_t^h l_t - Beq(\xi) - ss(AE_t, t)\mathbf{1}_{\{i_t^C=1\}} \tag{35}$$

The taxes are determined as follows:

$$Tax = \mathcal{T}\left(y_t^{tax}\right) + \tau_{ss} \min\left(wz_t^h l_t, \overline{y}_{ss}\right) + t^{earn}\mathbf{1}_{\{i_t^C=1\}} \tag{36}$$

where

$$y_t^{tax} = k_t r + wz_t^h l_t + y_t^{ss_{tax}}\mathbf{1}_{\{i_t^C=1\}} \tag{37}$$

For working individuals who claimed in the current period and had most of their benefits withheld by the Social Security earning tax, the claiming age can be increased by one year:

$$j^R = \begin{cases} t & ; \text{ if } t^{earn} < 0.5 \times ss(AE_t, t) \\ t+1 & ; \text{ otherwise} \end{cases} \tag{38}$$

In the individual problem, Eq (34) now includes Social Security benefits $ss(AE_t, t)$ for individuals who claimed in the current period ($i_t^C = 1$). Eq (36) now includes a Social Security earning tax for individuals who are younger than the full retirement age ($t < R^F$) and who choose to claim benefits but continue working. The taxable income in Eq (37) includes a taxable part of Social Security income $y^{ss_{tax}}$ for those who claimed pension benefits.

Individuals between the ages of R^E and R^D and who already claimed benefits. An individual in this category has an additional state variable, the claiming age j^R. Even though he claimed benefits, he still may continue to work. This, in turn, can change his future pension benefits due to the earnings test. His optimization problem can be written as follows:

$$V_t^C(\mathbb{S}_t, j^R) = \max_{l_t} \left\{ \sum_{x_t^h} \mathcal{G}_t\left(x_t^h\right) W_t^C(\mathbb{S}_t, j^R; l_t, x_t^h)^{1-\psi} \right\}^{\frac{1}{1-\psi}} \tag{39}$$

$$W_t^C(\mathbb{S}_t, j^R; l_t, x_t^h) = \max_{c_t, k_{t+1}} \left\{ \begin{array}{c} \left(c_t^{\chi} \widetilde{l}_t^{1-\chi}\right)^{1-\gamma} + \\ \beta \left(\theta_t^h E_t \left(V_{t+1}^C(\mathbb{S}_{t+1}, \widehat{j}^R)\right)^{1-\psi} + \left(1-\theta_t^h\right) \eta \left(k_{t+1} + \phi_{Beq}\right)^{1-\psi}\right)^{\frac{1-\gamma}{1-\psi}} \end{array} \right\}^{\frac{1}{1-\gamma}}$$

subject to

$$k_t\left(1+r\right) + wz_t^h l_t + ss(AE, j^R) + T_t^{SI} + Beq(\xi) = k_{t+1} + c_t + x_t^h + Tax \tag{40}$$

$$T_t^{SI} = \max\left(0, \underline{c} + x_t^h + Tax - k_t\left(1+r\right) - wz_t^h l_t - Beq - ss(AE, j^R)\right)$$

$$Tax = \mathcal{T}\left(y_t^{tax}\right) + \tau_{ss} \min\left(wz_t^h l_t, \overline{y}_{ss}\right) + t^{earn} \tag{41}$$

$$y_t^{tax} = k_t r + wz_t^h l_t + y_t^{ss_{tax}} \tag{42}$$

$$\widehat{j}^R = \begin{cases} j^R & ; \text{ if } \quad t^{earn} < 0.5 \times ss(AE, j^R) \\ j^R + 1 & ; \text{ otherwise} \end{cases} \tag{43}$$

Note that the age at which an individual first claimed benefits $\left(j^R\right)$ affects his pension income $ss(AE, j^R)$, but for those younger than the FRA this age is adjusted upward if most of their benefits are withheld because of the earnings tax, as described in Eq (43).

Individuals after age R^D. An individual older than age R^D only makes consumption/saving decisions. He also faces nursing home shocks. His state variables are asset (k_t), health (h_t), average lifetime earnings (AE), and claiming age (j^R). Denoting the vector of state variables as $\mathbb{S}_t^R$, we can write the value function of an individual in this age range as follows:

$$V_t^R(\mathbb{S}_t^R) = \left\{ \sum_{x_t} \sum_{xn_t} \mathcal{G}_t\left(x_t^h\right) pn_t^h W_t^R(\mathbb{S}_t^R; x_t^h, xn_t)^{1-\psi} \right\}^{\frac{1}{1-\psi}}$$

where

$$W_t^R(\mathbb{S}_t^R; x_t^h, xn_t) = \max_{c_t, k_{t+1}} \left\{ \begin{array}{c} \left(c_t^{\chi} \widetilde{l}_t^{1-\chi}\right)^{1-\gamma} + \\ \beta \left(\theta_t^h E_t \left(V_{t+1}^R(\mathbb{S}_{t+1}^R)\right)^{1-\psi} + (1-\theta_t^h)\eta \left(k_{t+1} + \phi_{Beq}\right)^{1-\psi}\right)^{\frac{1-\gamma}{1-\psi}} \end{array} \right\}^{\frac{1}{1-\gamma}} \tag{44}$$

subject to:

$$k_t\left(1+r\right) + ss(AE, j^R) + T_t^{SI} + Beq(\xi) = k_{t+1} + c_t + \mathcal{T}\left(y_t^{tax}\right) + x_t^h + xn_t$$

$$T_t^{SI} = \max\left(0, \underline{c} + \mathcal{T}\left(y_t^{tax}\right) + x_t^h + xn_t - k_t\left(1+r\right) - Beq(\xi) - ss(AE, j^R)\right)$$

$$y_t^{tax} = k_t r + y_t^{ss_{tax}}$$

Here, the interim value function W_t^R is conditional on the realization of the out-of-pocket medical spending shock x_t^h and the nursing home shock xn_t.

4.2 Production sector

The representative firm has the following production function:

$$Y = AK^{\alpha}L^{1-\alpha},$$

where K is aggregate capital, and L is aggregate labor input. We thus can write the factor prices as follows:

$$r = \alpha A \left(\frac{K}{L}\right)^{1-\alpha} - \delta, \tag{45}$$

$$w = (1-\alpha) A \left(\frac{K}{L}\right)^{\alpha}, \tag{46}$$

where δ is the depreciation rate.

4.3 Government budget

We assume the government runs two balanced budgets: the regular government budget and the Social Security budget. To simplify the notation, let $\mathbb{S}^P = \mathbb{S} \cup \mathbb{S}^{\mathbb{R}}$ define the space of agents' state variables, and let $\mathbf{s} \in \mathbb{S}^{\mathbb{P}}$. Then the regular government budget balance takes the form:

$$\int \left(\mathcal{T}\left(y_t^{tax}\right) - T_t^{SI}\right) \mathcal{M}(\boldsymbol{s}_t) = G, \tag{47}$$

where G is the exogenous government spending, and $\mathcal{M}(\boldsymbol{s}_t)$ is the measure of population with state variables $\boldsymbol{s}_t$.

The Social Security budget takes the following form:

$$\int \left(\tau_{ss} \min\left(w_t z_t^h l_t, \overline{y}_{ss}\right) + t^{earn}\right) \mathcal{M}(\boldsymbol{s}_t) = b_{adj} \times \int ss \mathcal{M}(\boldsymbol{s}_t) \tag{48}$$

The left-hand side is the total tax revenue from payroll taxes and from the Social Security earnings tax. The right-hand side is the total spending on pension benefits multiplied by the adjustment factor b_{adj}. In the baseline economy, the adjustment factor is set to one.

To balance the Social Security budget in the baseline economy, we adjust the payroll tax τ_{ss}. We do this because currently Social Security runs with a surplus, and to obtain a balanced budget, we set the payroll tax at a slightly lower value. In all the counterfactual economies where we keep the size of Social Security unchanged, we fix the payroll tax and change the adjustment factor b_{adj} to balance the budget. In all the counterfactual experiments where we phase out Social Security, we gradually decrease the payroll tax and also gradually decrease the adjustment factor to balance the budget.

4.4 Competitive equilibrium

The stationary competitive equilibrium for our economy can be defined as follows.

Definition Given the set of government policies $\{\underline{c}, \overline{y}_{ss}, t^{earn}, \mathcal{T}(\cdot), \tau_{ss}\}$ and Social Security rules, the competitive equilibrium of this economy is a set of prices $\{w, r\}$, decision rules and value functions of agents $\left\{k', c, l, i^C, V, V^C, V^R\right\}$, and aggregate variables $\{K, L\}$ such that the following conditions are satisfied:

1. Given prices and the tax functions, the decision rules solve the agents' optimization problem.

2. Factor prices w and r satisfy the firms' first-order conditions in Eqs (45)-(46), and

$$K = \int k \ \mathcal{M}(\boldsymbol{s}), \tag{49}$$

$$L = \int z^h l \ \mathcal{M}(\boldsymbol{s}) \tag{50}$$

3. The income tax $\mathcal{T}(\cdot)$ and Social Security payroll tax and τ_{ss} are such that the government and the Social Security budget constraints in Eqs (47) and (48) are satisfied.

4. Aggregate resource constraint is satisfied

$$Y = G + \int \left(c + x_t^h + xn\right)\mathcal{M}(\boldsymbol{s}) + \delta K \tag{51}$$

5. Assets of all deceased agents are allocated in a lump-sum manner within each fixed productivity group ξ:

$$\frac{Beq(\xi)}{1+n} = \int \left(1 - \theta_t^h\right) k \ \mathcal{M}(\boldsymbol{s}\,|\xi)$$

6. The distribution of agents over states is stationary

$$\mathcal{M}' = \Gamma(\mathcal{M}),$$

where $\Gamma(\cdot)$ is the law of motion of the distribution.

5 Model estimation

To estimate our model, we use three datasets: the Medical Expenditure Panel Survey (MEPS), the Health and Retirement Study (HRS) and the Panel Study of Income Dynamics (PSID). In each dataset, we select a sample of male individuals. We use 2002 as the base year, and all level variables are normalized to the base year using the Consumer Price Index (CPI). Appendix B.1 contains more details about the data and our sample selection.

We follow a two-step estimation strategy (Cagetti, 2003; Gourinchas and Parker, 2002). In the first step, we set or estimate a number of parameters directly from the data. Then we implement the Method of Simulated Moments to estimate our remaining model parameters.

5.1 First step estimates

At the first step, we estimate the parameters related to demographics, taxes and transfers, survival probabilities, health, medical expenses, and labor productivity. In this section, we explain how we set all our first-step parameters. For estimates of health transitions, nursing home shocks, and labor productivity we only provide an outline of our approach, and refer a reader to a more detailed explanation in Appendix B.2.

5.1.1 Health transitions and survival probabilities

To construct our health measure, we use the self-reported health status, which has five categories: excellent, very good, good, fair, and poor. We aggregate these five categories into two: a person is considered healthy or in good health ($h_t = G$) if he reports being in the first three categories, and he is considered unhealthy or in bad health ($h_t = B$) otherwise.

We estimate health transitions from the PSID in two steps. We first estimate the fixed productivity type ξ using the data on labor income. We then estimate a logit model where the probability to move to health status h_{t+1} conditional on surviving depends on current health h_t, age, estimated fixed productivity type, and cohort dummy variables.

To estimate health-dependent survival probabilities, we start by using the unconditional survival probabilities from the Social Security Life Tables for our base year from Bell and Miller (2005). We then use the HRS to estimate the logit model where the survival probability depends on a cubic polynomial of age and health. Using these estimates, we compute the survival premium, the difference between the estimated survival probabilities of the healthy and the unhealthy for each age. Combining information on average survival from the Social Security Life Tables, our estimated survival premium and the health transition matrix, we can recover survival probabilities for healthy and unhealthy people at each age.

5.1.2 Medical expenses and nursing home shocks

To estimate out-of-pocket medical spending shocks, we use the MEPS dataset. We approximate these shocks by a 3-state discrete health- and age-dependent stochastic process. For each age and health status, these three states correspond to the average out-of-pocket medical expenses for the three groups: those with out-of-pocket medical spending below the

50th, between the 50th and 95th, and above the 95th percentiles, respectively.[12]

We estimate the probability of incurring a nursing home shock (pn_t^h) from the HRS. To do this, we compute the fraction of people reporting nursing home stays at each age and health group. To compute out-of-pocket nursing home costs, we compute the average number of nights people report spending in a nursing home (conditional on reporting a stay), and multiply it by the average daily rate for a semiprivate room from Metlife (2003).

5.1.3 Income taxation and Social Security earnings cap

We use Gouveia and Strauss's (1994) functional form when parametrizing the tax function $\mathcal{T}(y)$:

$$\mathcal{T}(y) = a_0 \left[y - (y^{-a_1} + a_2)^{-1/a_1} \right] + \tau_y y \tag{52}$$

As in Gouveia and Strauss (1994), we set a_0 and a_1 to 0.258 and 0.768, respectively. We set the parameter a_2 to 0.616 following Pashchenko and Porapakkarm (2013). We use the proportional tax τ_y to balance the government budget in Eq (47).

5.1.4 Labor productivity process

We specify the individual productivity as following:

$$z_t^h = \lambda_t^h \exp(v_t) \exp(\xi) \tag{53}$$

where λ_t^h is the deterministic component that depends on age and health; while the stochastic component consists of the persistent shock v_t and a fixed productivity type ξ:

$$v_t = \rho v_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \tag{54}$$

$$\xi \sim N(0, \sigma_\xi^2)$$

For the stochastic productivity, we set the parameters based on the incomplete market literature (Storesletten et al., 2004). For the AR(1) part, we set ρ and σ_ε^2 to 0.984 and 0.022, respectively. The fixed productivity ξ has a normal $N(0, \sigma_\xi^2)$ distribution with σ_ξ^2 equal to 0.242. We draw v_1 in Eq (54) from the $N(0, 0.352^2)$ distribution following Heathcote et al. (2010).

[12] The MEPS tends to underestimate aggregate medical expenditures as reported in the National Health Expenditure Account (NHEA) (Pashchenko and Porapakkarm, 2016a). To bring aggregate medical expenses computed from the MEPS in line with the corresponding statistics in the NHEA, we multiply our estimated medical expenses by 1.60.

We estimate the deterministic component of productivity λ_t^h from the PSID. We first estimate the regression of labor income on a set of age interacted with health and cohort dummy variables using a sample of men who report working at least 520 hours per year and earnings at least the minimum wages. Using these estimates, we compute the labor income for our base cohort.

5.1.5 Remaining first-step parameters

We set the population growth rate n equal to 0.952%. We set the consumption share in the utility function (χ) to 0.5, which is within the range estimated by French (2005). The labor supply of working individual $\bar{l}$ is set to 0.4.

For the production side of the economy, we set α to 0.36 based on the capital income share. For the government budget, we fix the government spending at 20% of aggregate output, so that $\frac{G}{Y} = 0.20$.

5.2 Second step estimation

At the second stage, we estimate the following parameters: disutility from work, additional disutility when working and claiming early, discount factor, IES, risk aversion, bequest parameters, the consumption minimum floor, and the depreciation rate: $\{\phi_w, \phi_{ss}, \beta, \gamma, \psi, \eta, \phi_{Beq}, \underline{c}, \delta\}$. In our estimation, we minimize the unweighted sum of squared differences between the simulated and data moments. Our targeted moments are described below.

Labor market outcomes We use the PSID to estimate the percentage of working people by 5-year age group. The target moments in our estimation are the fraction of unhealthy workers at two age groups: 30-34 and 60-64. This profile is informative about the minimum consumption guarantee $\underline{c}$ and the disutility to work ϕ_w.

Wealth moments We use the PSID to construct the mean, the 25th and 75th percentiles of wealth distribution over 5-year age bins. These wealth profiles are informative about the bequest parameters, risk aversion, and IES.

Claiming behavior To construct the distribution by claiming age, we use a sample of men born between 1936-1938 in the HRS who do not receive disability benefits. We target the fraction of people claiming at the earliest eligibility age of 62, and the fraction of people claiming at the FRA (65 years old). The first moment is informative about the discount

factor, β. The second moment is informative about the disutility from work when claiming early, ϕ_{ss}.

Equilibrium interest rate We target the equilibrium interest rate r of 2%, and for this, we use the capital depreciation rate δ.

5.3 Second step estimation results

Table 3 reports our estimated preference parameters, the consumption floor, and the capital depreciation rate. Our estimated discount factor is equal to 0.948. This parameter is primarily identified from the percentage of people who claim their Social Security benefits at age 62 (Pashchenko and Porapakkarm, 2024, provide a detailed discussion of the connection between claiming behavior and the discount factor). Our estimates of the inverse of the IES (1.403) and the risk aversion (3.847) are quite different from each other.

Parameters		Estimates
Discount factor	β	0.948
1/IES	γ	1.403
Risk aversion	ψ	3.847
Bequest strength		
Threshold	$\{\phi_{Beq}, \eta\}$	\$4,172
MPB		0.97
Consumption floor	$\underline{c}$	\$2,401
Disutility from work		
All	ϕ_w	0.109
Working early claimers	ϕ_{ss}	0.144
Depreciation rate	δ	0.071

Table 3: Preference parameters, the consumption floor, and the depreciation rate from the second step.

We report the estimated strength of bequest motives using two metrics: the bequest threshold and the Marginal Propensity to Bequeath (MPB). In a one-period consumption-saving model, these estimates imply that the bequest motive becomes operational at an asset level of \$4,172 and the marginal propensity to bequeath (MPB) is 0.97. In other words, people with assets below \$4,172 would not leave bequests, while people with assets above this level would bequeath 97 cents out of every additional dollar. These are comparable with bequest parameters estimated in other studies that include wealth decumulation after retirement among the targeted moments, e.g., Lockwood (2018) and De Nardi et al. (2016). The estimates in the former study imply the threshold and MPB of \$14,665 and 0.96, respectively; while in the latter study, these numbers are \$3,268 and 0.78, respectively. The

conversion of the estimates in each study to comparable bequest threshold and MPB values is provided in Pashchenko and Porapakkarm (2024). Our bequest parameters, together with the IES and risk aversion, are jointly identified from wealth profiles.

Our estimated consumption floor is \$2,401, which is in line with estimates from other structural life-cycle models with uncertain medical expenses and endogenous labor supply: Capatina's (2015) estimate of the consumption floor is \$4,114 (in 2006 USD), and De Nardi et al.'s (2022) estimate is \$3,505 (in 2013 USD). In our estimation, this parameter, together with the disutlity from work, is identified from the employment among the unhealthy.

5.4 Model fit

Figure 3 compares the wealth profiles from our model (solid line) with that from the PSID (dots). Our model well tracks the average (left panel), as well as the 25th and 75th (right panel) of the wealth distribution over the entire life-cycle.

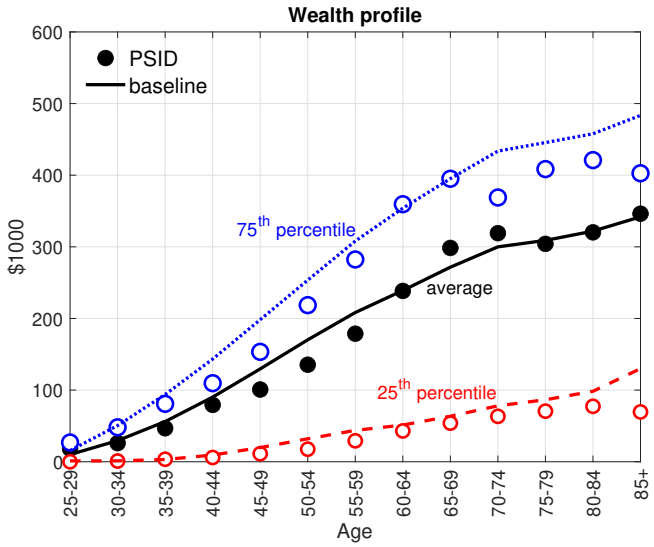


Figure 3: Wealth profiles: baseline model vs PSID

The left panel of Figure 4 displays the employment profiles by health status in the data (dots) and in the model (solid line). We target only two moments on this graph: employment among the unhealthy in the age groups 30-34 and 60-64. Our model thus captures labor supply of the unhealthy well, but somewhat underestimates that of the healthy.

The right panel of Figure 4 compares the distribution of people by age when they claimed their Social Security benefits. The distribution by claiming age is captured well by our model, even though we target only claiming at two ages: 62 and 65 (FRA).

As an external validation, we consider how well our model captures the percentage of the hand-to-mouth, a non-targeted moment. Figure 5 compares the percentage of people in

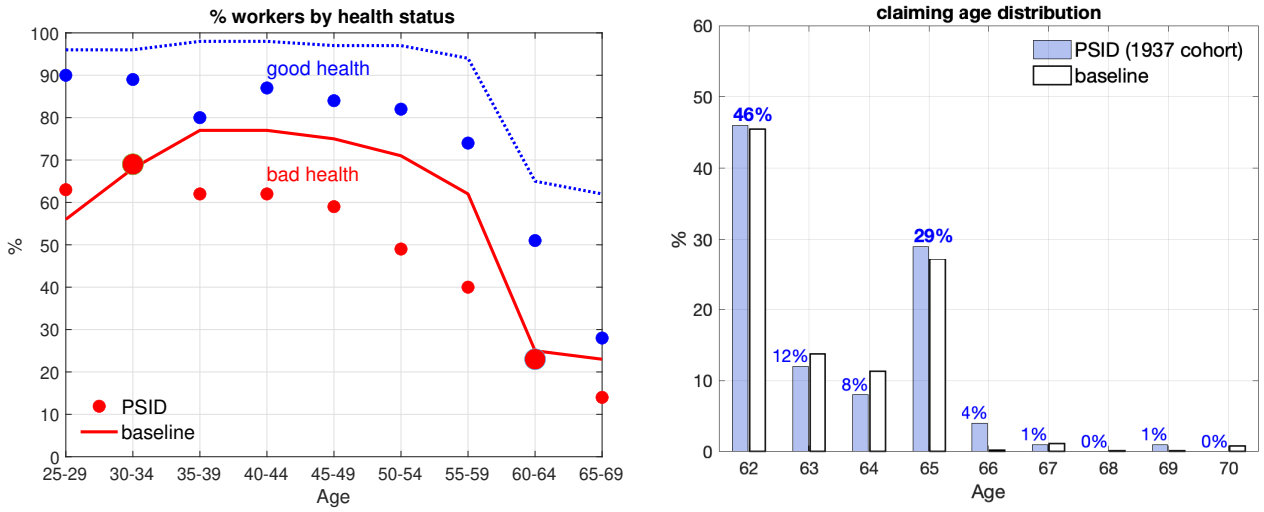


Figure 4: Left panel: percentage of working individuals by health status, model vs data (PSID). Right panel: distribution by claiming age, model vs data (HRS).

this category by age in the model (dark-colored bars) and the data (light-colored bars). The left panel uses the definition of the hand-to-mouth based on Zeldes (1989), and the right panel - our own definition based on $100 threshold for net worth. In each case, our model capture the large number of the hand-to-mouth in the data, and the fact that this number monotonically declines with age.

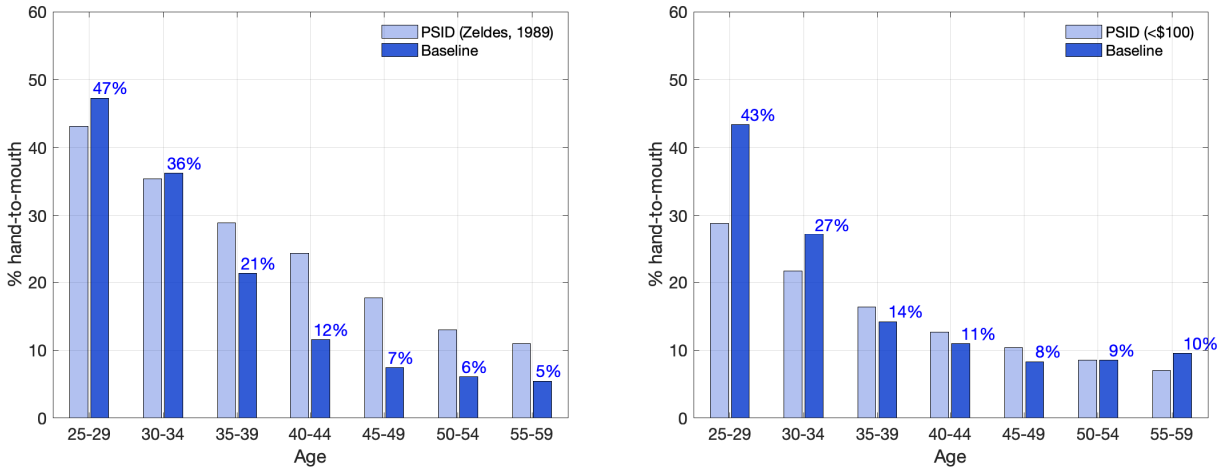


Figure 5: Percentage of the hand-to-mouth by age group, model vs data (PSID). Left panel: Zeldes's (1989) definition of hand-to-mouth. Right panel: own definition based on $100 net worth threshold.

Our estimation results thus show that our model captures several features of the data that are of a particular importance for our study. First is the strong unwillingness to annutiize, which manifests itself in the large number of people claiming as early as possible (age 62).

Second is the unwillingness to decumulate wealth after retirement, which is usually taken as an evidence of a strong intentional bequest motive. Third is the large number of liquidity-constrained or hand-to-mouth people among the young.

6 Results

In this section, we use our estimated life-cycle model to deliver several results. We first examine the welfare effects of the Social Security elimination in our framework, both in general and partial equilibrium. Consistent with prior research, we find that the optimal size of Social Security is zero in the long-run. We then examine how the optimal size of Social Security depends on its key design features. Specifically, we evaluate how reducing annuitization and intertemporal distortions, or increasing income redistribution changes our conclusions about the optimal size of Social Security.

6.1 Welfare effects of Social Security in the baseline economy

We start by examining the ex-ante welfare effects of changing the size Social Security in our settings. Specifically, we incrementally decrease the Social Security payroll tax τ_{ss} to zero. In each case, we proportionally reduce pension benefits so that the Social Security budget in Eq (48) holds.

We use the following way to measure the long-run welfare effects of each policy in our analysis. We first compute the monetary compensation that makes a newborn individual indifferent between being born in the baseline or in the counterfactual economy. The positive number means he is better off in the counterfactual economy. We then report this number as a percentage of average consumption, so it can be thought of as the consumption equivalent variation (CEV).

The solid line in Figure 6 plots the resulting CEV as a function of the size of the Social Security program. The value of one on the horizontal axis corresponds to the size of Social Security in the baseline economy, and the value of zero corresponds to its complete elimination.

The overall welfare monotonically increases as Social Security gets smaller, and its complete removal results in the highest gains equivalent to around 4% of annual consumption. This result is consistent with many other studies showing that Social Security reduces the steady-state welfare (e.g., Auerbach and Kotlikoff, 1987; Hubbard and Judd, 1987; Imrohoroglu et al., 1999, 2003; Storesletten et al., 1999).

Our theoretical analysis in Section 3 emphasizes several channels through which Social

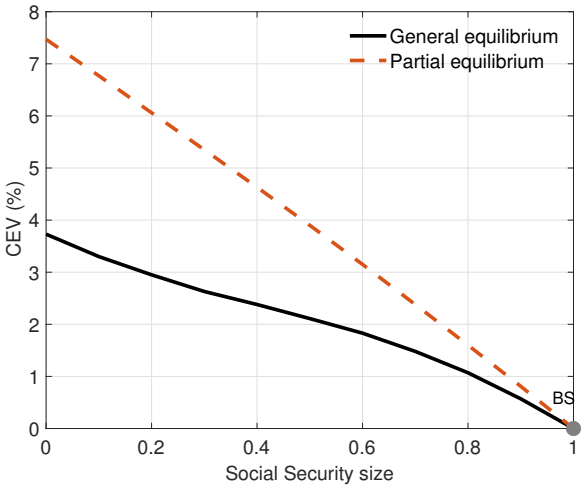


Figure 6: Ex-ante welfare of changing the size of Social Security.

Security changes aggregate welfare in *partial* equilibrium. In *general* equilibrium, there are additional effects due the change in aggregate capital, factor prices, taxes and bequests. Hence, an important question is how important are these general equilibrium effects in accounting for the zero-optimal-size result.

To understand this, we incrementally reduce the size of Social Security in the partial equilibrium environment, that is to say, when factor prices, accidental bequests and taxes are fixed at their baseline level. The resulting welfare effects are plotted as the dashed line in Figure 6. An important observation from Figure 6 is that in partial equilibrium, eliminating Social Security produces even larger welfare gains compared to the results in general equilibrium (more than 8% of annual consumption compared to around 4% in general equilibrium).

To better understand this result, in Table 4, we compare the aggregate variables in economies with and without Social Security. Importantly, Social Security has a small effect on aggregate capital in our model: eliminating the program increases K only by about 4% (from 3.979 to 4.095). Because of this small change, the wage rate stays unchanged. Similarly, Social Security has small and negative effect on the amount of bequest income received by all living agents.

	K	r	w	Bequests		τ_y
				ξ_1	ξ_2	
Baseline	3.979	2%	1.18	0.033	0.166	0.13
No Social Security	4.095	2%	1.180	0.027	0.133	0.16

Table 4: Aggregate variables in the economy with and without Social Security

Both of these effects (small change in aggregate capital and the negative response of bequest income to the removal of Social Security) are due to the same cause: strong bequest motives that we uncover in our estimation (see Section 5.3).

In a model without bequest motive, the elimination of Social Security usually results in a large increase in aggregate capital (e.g. Imrohoroglu et al., 1995; Storesletten et al., 1999). This is because without Social Security, people have to save more to finance consumption after retirement. This effect is much smaller when people also save for bequest motives. This is because wealth accumulated for bequests can also be used to finance retirement consumption (see also Fuster et al., 2003).

We next turn to the effects on bequest income. In a model without bequest motives, all bequests are *accidental*. Social Security reduces accidental bequests by providing longevity insurance, hence its elimination substantially increases income from bequests (see Caliendo et al., 2014; Eckstein et al., 1985). However, in a model with bequest motives, bequests are *intentional* and are much less sensitive to the elimination of Social Security (this point is also discussed in Caballe and Fuster, 2003).

Overall, our results in this section show that the negative welfare effects of Social Security are not a mechanical outcome of the crowding out of aggregate capital or bequest income. Instead, much of the welfare effects can be attributes to the program's design flaws. In the next sections, we explore how changing the design features of Social Security impacts its long-run welfare effects.

6.2 The effects of mandatory annuitization

In this section, we investigate the role of mandatory annuitization for the welfare effects of Social Security. Our theoretical analysis in Section 3 highlights that Social Security negatively affects welfare when people are mandated to have more annuities compared to what they would optimally choose. An additional consideration is that mandatory annutization creates mortality-regressivity: everything else equal, short-lived people receive less lifetime pensions (see Jang et al., 2023, for an extensive discussion). Because short-lived people also tend to have lower income, this partially undermines the progressive feature of Social Security (see Coronado et al., 2011; Goda et al., 2011).

To quantify these effects, we construct a counterfactual economy where people receive part of their Social Security pensions not as lifelong annuities but as a one-time lump-sum payment. The size of the payment depends on pension benefits one is entitled to, $ss(AE, j^R)$. Since pensions in the baseline economy depend on one's AIME and the age of claiming, the lump-sum payment depends on these variables as well, and is denoted as $LS(AE, j^R)$.

We denote the fraction of pension benefits converted into a lump-sum payment as γ^{LS}. Then the lump-sum payment $LS\left(AE, j^R\right)$ is computed as the present value of the corresponding part of benefits $\gamma^{LS} ss(AE, j^R)$ using average life expectancy:

$$LS\left(AE, j^R\right) = \gamma^{LS} ss\left(AE, j^R\right) \sum_{m=j^R}^{T} \frac{\overline{\theta}_{m|j^R}}{(1+r)^{m-j^R}}. \tag{55}$$

Here $\overline{\theta}_{m|j^R}$ is the cohort-average probability to survive to age m conditional on being alive at the claiming age j^R.

We gradually decrease the fraction of annuitized Social Security wealth by increasing γ^{LS} from zero to one. In each case, we keep the size of the Social Security program unchanged. To do this, we use the scaling parameter b_{adj} in the Social Security balance equation in Eq (48).[13]

The left panel of Figure 7 displays the welfare effects of changing γ^{LS} in the baseline economy (keeping the size of Social Security fixed). The welfare changes are not monotone because of the effects of reducing annuitization on aggregate variables in equilibrium.[14] Welfare gains are maximized when $\gamma^{LS} = 1$, hence it is best to fully eliminate mandatory annutization of the Social Security wealth. This result is due to the strong unwillingness to hold annuities uncovered by our estimated preference parameters (strong bequest motives and low discount factor). In addition, eliminating mandatory annuitization undoes the mortality regressivity of Social Security. Converting pensions into a lump-sum payment based on average mortality eliminates the difference in lifetime pensions arising form the difference in life expectancy.

The overall welfare gains of the complete elimination of mandatory annuitization constitute 0.70%. However, as shown in the first row of Table 5, these gains are unequally distributed across productivity types: while low-productivity people have welfare gains of 2.59%, high-productivity people's losses are equal to -2.67% of annual consumption. This largely due to the fact that high-income group tends to have higher life expectancy and hence benefits from the mortality-regressivity. Once this feature is removed, their lifetime pensions go down.

We next consider what will be the effect of reducing the size of the Social Security program in these counterfactual settings. We repeat the same exercise as in Section 6.1 by gradually

[13] As in the baseline economy, the calculation of the Social Security earnings tax in Eq (26) and the adjustment of claiming age due to the earnings tax are still based on the full size of pension benefits $ss\left(AE, j^R\right)$.

[14] In partial equilibrium, reducing the fraction of the annuitized Social Security wealth monotonically increases welfare.

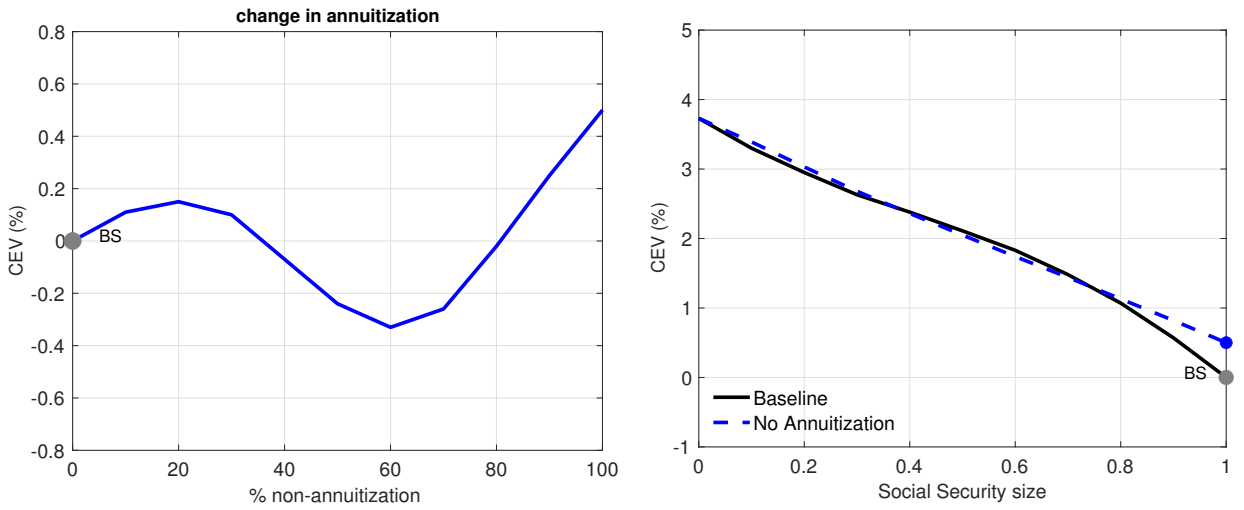


Figure 7: Left panel: welfare effects when changing the degree of mandatory annuitization keeping the size of Social Security fixed as in the baseline. Right panel: welfare effects of changing the size of Social Security, baseline versus the economy without mandatory annuitization.

	All	Low ξ	High ξ
No mandatory annuitization	0.50	2.23	-2.80
More redistribution	1.85	4.21	-0.63
No intertemporal distortions	3.50	5.09	6.12

Table 5: Ex-ante welfare of removing the three distortions given the current size of Social Security.

decreasing the payroll tax to zero. We plot the resulting welfare change for the economy without mandatory annuitization alongside the corresponding welfare change in the baseline economy in the right panel of Figure 7. The results are similar in both cases: even without mandatory annuitization, removing Social Security increases steady-state welfare. Hence, while mandatory annuitization is a welfare-reducing feature of Social Security, this is not the main reason why people are better off without the program.

6.3 Changing the degree of income redistribution

An important feature of Social Security is redistribution based on lifetime income. One's pension is computed as a concave function of past earnings as described in Eq (25) in Section 4.1.3. As a result, pensions replace a higher fraction of past income for low-earners compared to that of high-earners.

While this provides some insurance against low lifetime income realizations, a number of studies show that making Social Security more redistributive improves welfare. Golosov et al. (2003), Bagchi and Jung (2021), Jones and Li (2022) find that the aggregate welfare is maximized when benefits are constant, that is to say, independent of past earnings. Huggett

and Parra (2010) find that making benefits a decreasing function of one's lifetime earnings can produce even higher welfare gains.

We thus start by evaluating the optimal degree of income redistribution through Social Security in our settings (fixing the current size of the program). To do this, we consider a set of reforms when we gradually increase the progressivity of pensions. In particular, consider an individual with average lifetime earnings AE and who claims at the FRA. In the baseline economy, he is entitled to receive benefits $ss\left(AE, j^R = R^F\right)$ given in Eq (25). In the counterfactual economy with more redistribution, he instead receives benefits $ss^R\left(AE, j^R = R^F\right)$, which are determined as follows:

$$ss^R(AE, j^R = R^F) = A \cdot ss\left(AE, j^R = R^F\right) + (1 - A) \cdot \overline{ss}$$

where $\overline{ss}$ is average pension benefits in the baseline economy:

$$\overline{ss} = \frac{\int ss\mathcal{M}\left(\boldsymbol{s}_t\right)}{\int \mathcal{M}\left(\boldsymbol{s}_t\right)},$$

and the parameter A determines the additional degree of redistribution. When $A = 1$, we have the same degree of redistribution as in the baseline economy. As A decreases, the degree of redistribution increases, and at $A = 0$, pension benefits are the same for everyone and are equal to $\overline{ss}$.

It is worth noting that people can still receive higher or lower benefits than $ss^R(AE, j^R = R^F)$ depending on when they decide to claim. We keep the schedule of penalties/rewards for early/late claiming the same as in the baseline economy.

The left panel of Figure 8 displays the welfare results of this set of reforms: CEV monotonically increases as A decreases, with $A = 0$ corresponding to the highest welfare gains. In other words, aggregate welfare is maximized when pension benefits are the same for all income groups. Making pensions more progressive results in substantial welfare gains equivalent to 1.85% of annual consumption. People with low fixed productivity has welfare gains of 4.2%, while those with high productivity experience welfare losses equivalent to -0.63% (the second row of Table 5).

We next examine welfare effects of incrementally decreasing the size of Social Security in the environment where pensions are independent of lifetime income ($A = 0$). The results are plotted in the right panel of Figure 8. Despite the large gains this policy produces in the baseline economy, even with more progressive pensions, people are still better off without Social Security. In other words, increasing insurance against lifetime income risk provided by Social Security in itself is not enough to overturn the zero-optimal-size result.

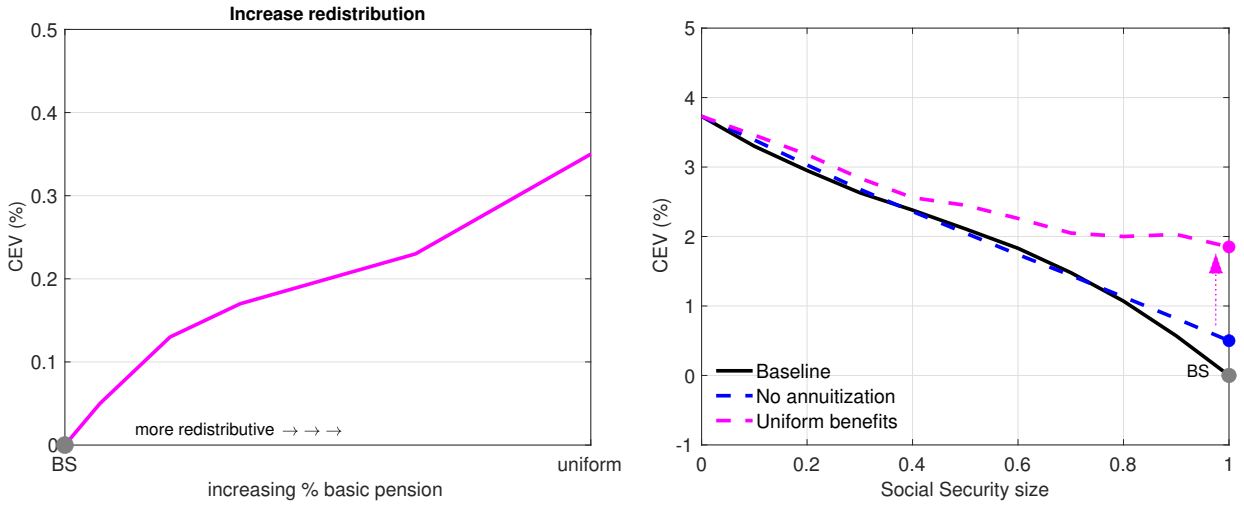


Figure 8: Left panel: ex-ante welfare effects when changing the degree of income redistribution through Social Security (reducing A from 1 to 0) while fixing its size as in the baseline. Right panel: ex-ante welfare effects of changing the size of Social Security, baseline versus the economy with constant pension benefits ($A = 0$).

6.4 Reducing intertemporal distortions

Another important feature of Social Security we consider is related to its financing. By paying payroll taxes one is mandated to save around a tenth of one's lifetime income for retirement. This number is unlikely to be a binding constraint as people typically save more (see Cagetti, 2003; Gourinchas and Parker, 2003; Pashchenko and Porapakkarm, 2021, for quantitative evaluations of retirement savings). The problem lies in the timing of savings: one has to pay payroll taxes immediately on entering the labor market. However, for younger people, it may be optimal to delay retirement saving given their relatively high expected income growth and the presence of borrowing constraints. As we show in our theoretical analysis in Section 3, the more often the borrowing constraint binds, the larger is the negative impact of Social Security on welfare.

To understand how important are intertemporal distortions for the welfare effects of Social Security, we consider a more flexible policy when young workers are exempt form paying payroll taxes. We start by choosing the maximum exemption age that maximizes ex-ante welfare given the baseline size of Social Security. Specifically, we consider changing the maximum exemption age from 25 (baseline) to 45 years old. As we increase the age at which people start paying payroll taxes, we also increase the payroll tax rate so that the Social Security budget is unchanged.

The CEV for each maximum exemption age is plotted in the left panel of Figure 9, while the right panel of Figure 9 reports the corresponding equilibrium payroll tax rates. Overall, exempting young people from paying payroll taxes is always welfare improving,

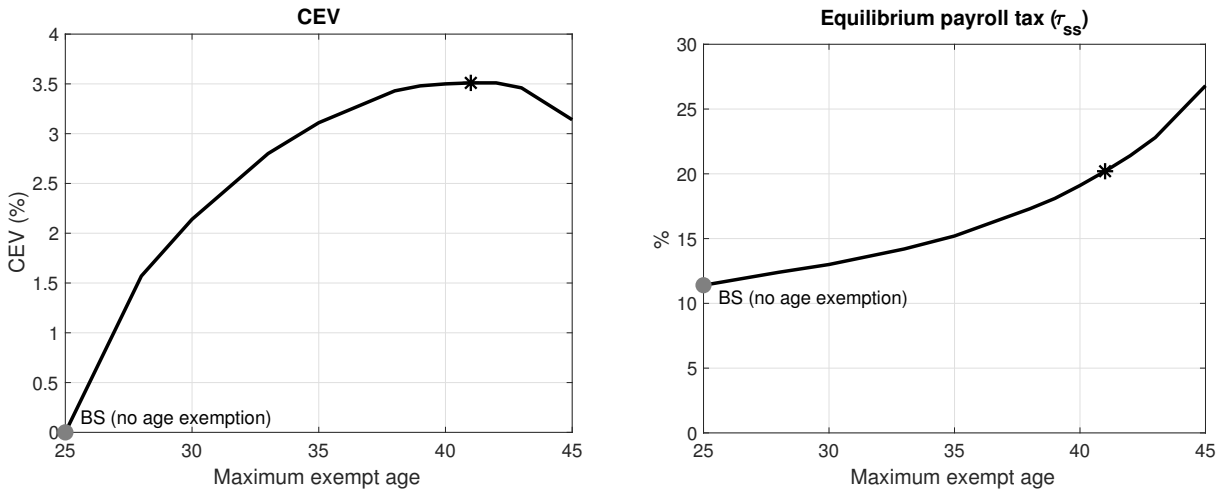


Figure 9: Ex-ante welfare (left) and resulting τ_{ss} (right) when increasing the maximum age at which people are exempt from paying payroll taxes.

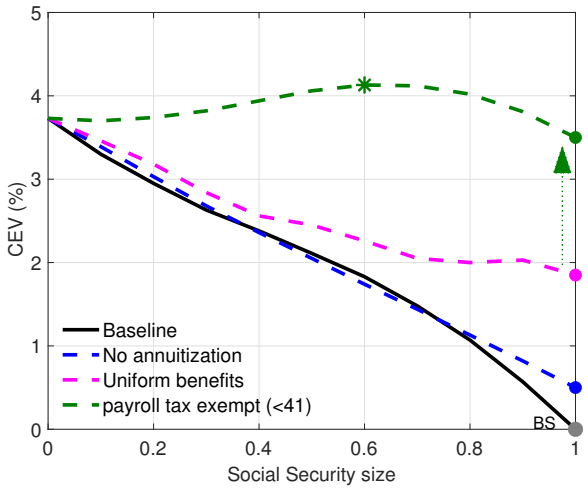


Figure 10: Ex-ante welfare of changing the size of Social Security, baseline versus the economy without intertemporal distortions.

and the highest welfare gains are achieved when the maximum exemption age is set to 41. Introducing this policy in the baseline economy results in the CEV equal to 3.5% of annual consumption (the third row of Table 5).

We next consider the welfare effects of eliminating Social Security in the economy where people younger than 41 years old do not pay payroll taxes. Figure 10 shows that with this policy in place, the Social Security elimination no longer produces welfare gains. While it is optimal to reduce the size of Social Security to 60% of its baseline value, the optimal size is still substantially above zero. This analysis shows that the intertemporal distortions created by the mismatch between young people's optimal savings and "forced" contributions into the pension system is the most quantitatively important design flaw of the system.

6.5 Policy interactions

Our analysis above evaluates the effects of the three policies, one-at-a-time, for the optimal size of Social Security: removing mandatory annuitization, increasing the redistribution, and reducing intertemporal distortions. Introducing each policy in the baseline economy produces large welfare gains. This suggests that insufficient insurance against lifetime income risks, over-annuitization and intertemporal distortions can all be considered as important design flaws of the system. We next study the interaction between the considered policies.

To do this, in Figure 11 we compare the ex-ante welfare effects of changing the size of Social Security for six economies: the baseline and the five counterfactual economies differing with different combinations of policies.

The first counterfactual economy is when we exempt people below the age of 41 to from the payroll taxation. As discussed earlier, in such an economy, the optimal size of Social Security represents 60% of the baseline size. Next, keeping the payroll tax exemption for young people in place, we consider two additional policies, one-at-a-time: we either remove mandatory annuitization or make pension benefits independent of lifetime income. In each case, we observe important complemenarities between policies. In the first combination (payroll tax exemption and no mandatory annuitization), the optimal size of Social Security is equal to its baseline value. In the second combination (payroll tax exemption and more income redistribution), the optimal size is 30% higher than its baseline value.

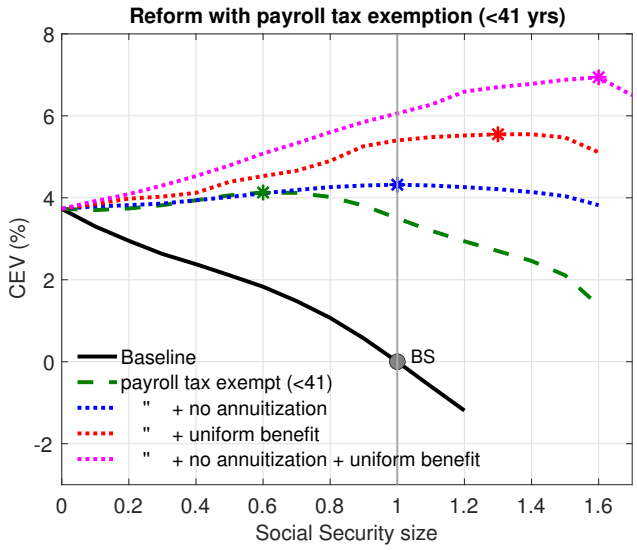


Figure 11: Ex-ante welfare effects of changing the size of Social Security, baseline versus five counterfactual economies differing in the combination of policies.

As a final exercise, we consider the effect of combining all three policies (payroll tax exemption, no mandatory annuitization, and more redistributive pensions). In this case, it

is optimal to increase the size of the program by 60% compared to its baseline level. This will result in large ex-ante welfare gains equivalent to 7% of annual consumption.

7 Conclusion

In this paper, we revisit the question of what is the optimal size of Social Security. A well-know result in the literature is that given the current design of the program, it is optimal to eliminate it in the long-run. We thus investigate how changing the key features of the Social Security design changes our conclusion about its optimal size.

We start with the theoretical analysis and show that the welfare effects of Social Security can be decomposed into four distinct parts. First is a (positive) effect of redistribution due to the reduction in consumption inequality after retirement. The second effect is due to mandatory annuitization which can be positive or negative depending on the differnce between the mandated and the optimal levels of annuitization. The third effect is due to the weighted sum of intertemporal distortions over the life-cycle. This effect arises if agents are borrowing constrained and thus prefer not to save at all, yet are forced to contribute to Social Security through the payroll taxation. The fourth is the negative effect due to dynamic inefficiency, which can be thought of as opportunity costs of saving at the rate of return equal to the population growth rate as opposed to the market rate of return.

We next construct a general equilibrium model with the aim to quantitatively evaluate the importance of different design features of Social Security for its overall welfare effects. Our model features a rich heterogeneity among individuals, including in the dimension of income and mortality, and is estimated using three micro datasets. The estimated model matches many features of the data, including the percentage of people claiming pension benefits early, and wealth deccumulation after retirement. Matching these data features allow us to identify individual preferences, among them, willingness to annuitize and to bequeath, which as we show, are important for the welfare assessment of pension programs.

We start our quantitative analysis by re-establishing the result that the optimal size of Social Security is zero. We also show that this result is not a mechanical effect of the change in aggregate capital, factor prices, and accidental bequests. Instead, much of these welfare effects can be traced to the design of Social Security.

In the next steps of our analysis, we examine how the three key design features of Social Security affect its optimal size: mandatory annuitization, income redistribution through pensions, and intertemporal distortions. We show that changing these design features given the current size of Social Security substantially improves welfare. Among these, reducing intertemporal distortions by exempting young workers form payroll taxation produces the

largest welfare effects. We show that jointly improving these design feature of Social Security by increasing the embedded income redistribution, and reducing distortions on intertemporal choice and annuitization level overturns the zero-optimal-size result of Social Security. Moreover, it is even optimal to increase the size of the program by 60% compared to its baseline value.

Our results stress the importance of the comprehensive approach when assessing various Social Security reforms. We argue that the key question is not what policy can improve welfare given the current size of the program, but what policy can justify the program's existence in the long-run. Many studies point out that the optimal log-run size of Social Security is zero. We show that this result is robust to many policy changes, however, it is possible to improve Social Security to the point when it is even optimal to increase it size in the log-run. This represents an important consideration that should play the key role in the ongoing debate about the long-run sustainability of Social Security.

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Appendix

A Additional lemmas and proofs

This section contains a new lemma, Lemma A1, and the proof of Lemma 2, that were omitted from Section 3.

Lemma A1 The conversion rate of agents' contributions into pensions q^{pub} can be expressed as follows:

$$q^{pub} = A\,\overline{q}^{y} \;+\; (1-A)\,\overline{q},$$

where q_i is the cost of delivering a unit stream of lifelong annuity income to an individual i through the pension system:

$$q_i \equiv \sum_{t=1}^{T-R}\left(\frac{\theta_i}{1+n}\right)^{t-1}$$

Here we distinguish between two ways to compute the average of q_i. First is the plain average:

$$\overline{q} \equiv \int q\,dG\,(\theta),$$

and second is lifetime-wealth weighted average:

$$\overline{q}^{y} \equiv \;\frac{\int qLW\,dG\,(\theta)}{\overline{LW}(n)},$$

where $\overline{LW}(n)$ is the average lifetime wealth:

$$\overline{LW}(n) = \int_{\epsilon} LW(n)dF(\epsilon)$$

Proof

Using the fact that $PW_i^{ss} = \tau LW_i(n)$, we can represent pension benefits as follows:

$$\Delta_i = \tau\frac{A\cdot LW_i(n) + (1-A)\cdot\overline{LW}(n)}{q^{pub}}, \tag{56}$$

Using Eq (56), we can rearrange the pension system balance equation in the Social Security budget in Eq (18) as follows:

$$\int_{\epsilon}\int_{\theta}\tau\frac{A\cdot LW(n) + (1-A)\cdot\overline{LW}}{q^{pub}}\,q\,H(\theta,\epsilon)\,d\theta\,d\epsilon = \tau\int_{\epsilon} LW(n)dF(\epsilon) = \tau\overline{LW}(n)$$

From here it follows

$$\frac{A \int\limits_{\epsilon} \int\limits_{\theta} LW(n)\, q\, dH(\theta,\epsilon) + (1-A)\, \overline{LW}(n)\, \overline{q}}{\overline{LW}(n)} = q^{pub}$$

From here, using the definition of $\overline{q}^y$, the statement of the proposition follows directly.

The intuition for Lemma A1 is as follows. The conversion rate of (redistributed) pension wealth into a pension annuity, q^{pub}, is the weighted-average between the average annuity price $\overline{q}$ and the income-weighted annuity price $\overline{q}^y$. If there is full redistribution ($A = 0$), the conversion is based just on the average annuity price. This is because in this case, every agent gets the same pension income regardless of his pension wealth. When there is partial redistribution ($A > 0$), the conversion rate is a weighted average between $\overline{q}$ and $\overline{q}^y$. The latter term is higher than the former if income and longevity are positively correlated. In this case, q^{pub} is higher compared to the no-redistribution case. People who live longer have higher lifetime wealth and thus are entitled to higher lifelong pensions. This increases the overall costs of the pension system to finance their annuities, driving q^{pub} up.

Lemma 2 The change in individual welfare in response to the marginal change in the size of the pension system can be represented as follows:

$$\begin{aligned}\frac{\partial V_i}{\partial \tau} = &- \sum_{t=1}^{R} (\beta(1+r))^{t-1} wedge^s_{it}\, \widetilde{PE}_{it} - \beta^R u_{iR+1} \Big(LW_i(r) - LW_i(n) \Big) \\ &- \beta^R \frac{LW_i(n)}{q^{pub}} wedge^a_i + \beta^R \frac{(1-A)(\overline{LW} - LW_i(n))}{q^{pub}}\, MU_i,\end{aligned} \tag{57}$$

where

$$\widetilde{PE}_{it} = PE_{it} + \frac{1}{(1+r)^{t-1}} \frac{\partial a_{it+1}}{\partial \tau}$$

and PE_{it} is the present value of earnings up to age t defined in Eq (14), and $\overline{LW}(n)$ is the average lifetime wealth:

$$\overline{LW}(n) = \int\limits_{\epsilon} LW(n) dF(\epsilon)$$

Proof Consider first the change in V_i due to direct effects of the tax change, the first two terms in Eq (21):

$$\left.\frac{\partial V_i}{\partial \tau}\right|_{\text{tax}} \equiv - \sum_{t=1}^{R} \beta^{t-1} u_{it} y_{it} \; + \; \frac{\partial \Delta_i}{\partial \tau}\, \beta^R MU_i$$

Using the fact that $PW_i^{ss} = \tau LW_i(n)$, we can represent pension benefits in Eq (20) as a function of the lifetime wealth:

$$\Delta_i = \tau \frac{A \cdot LW_i(n) + (1-A) \cdot \overline{LW}(n)}{q^{pub}}, \tag{58}$$

We can thus rewrite $\left.\frac{\partial V_i}{\partial \tau}\right|_{\text{tax}}$ as follows:

$$\left.\frac{\partial V_i}{\partial \tau}\right|_{\text{tax}} = -\sum_{t=1}^{R} \beta^{t-1} u_{it} y_{it} \; + \; \beta^R MU_i \frac{LW_i(n) + (1-A)(\overline{LW}(n) - LW_i(n))}{q^{pub}} \tag{59}$$

Next, using the definition of the pension contribution wedge in Eq (12) with $r^c = n$ and $\alpha_i = 1$, we can express $\sum_{t=1}^{R} \beta^{t-1} u_{it} y_{it}$ as follows:

$$\sum_{t=1}^{R} \beta^{t-1} u_{it} y_{it} = \sum_{t=1}^{R} \beta^{t-1} wedge_{it}^{pc} + \beta^R \frac{LW_i(n)}{q^{pub}} MU_i$$

Plugging this into Eq (59) we have:

$$\begin{aligned}\left.\frac{\partial V_i}{\partial \tau}\right|_{\text{tax}} = &-\sum_{t=1}^{R} \beta^{t-1} wedge_{it}^{pc} - \beta^R \frac{LW_i(n)}{q^{pub}} MU_i \; + \\ &\beta^R \frac{LW_i(n)}{q^{pub}} MU_i + \beta^R \frac{(1-A)(\overline{LW}(n) - LW_i(n))}{q^{pub}} MU_i\end{aligned}$$

Canceling the second and third terms, we have:

$$\left.\frac{\partial V_i}{\partial \tau}\right|_{\text{tax}} = -\sum_{t=1}^{R} \beta^{t-1} wedge_{it}^{pc} + \beta^R \frac{(1-A)(\overline{LW}(n) - LW_i(n))}{q^{pub}} MU_i \tag{60}$$

Using Corollary 1 to Lemma 1, we can substitute $-\sum_{t=1}^{R} \beta^{t-1} wedge_{it}^{pc}$ out (using $r^c = n$ and $\alpha_i = 1$), and transform this expression as follows:

$$\begin{aligned}\left.\frac{\partial V_i}{\partial \tau}\right|_{\text{tax}} = &\sum_{t=1}^{R} (\beta(1+r))^{t-1} wedge_{it}^{s} \, PE_{it} + \beta^R u_{iR+1} \Big(LW_i(r) - LW_i(n) \Big) \\ &+ \beta^R \frac{wedge_i^a}{q^{pub}} LW_i(n) + \beta^R \frac{(1-A)(\overline{LW}(n) - LW_i(n))}{q^{pub}} MU_i\end{aligned} \tag{61}$$

Adding the effects of the tax change on saving decisions, $\sum_{t=1}^{R} \beta^{t-1} wedge_{it}^{s} \frac{\partial a_{it+1}}{\partial \tau}$, and

combining coefficients in front of the saving wedges, we arrive to expression in Eq (57). This finishes the proof of the lemma.

B Additional data and estimation details

In this section, we provide additional details about our data and our estimation of the first-step parameters.

B.1 Data and samples

In this section, we provide additional details about our data and our estimation of the first-step parameters.

For our estimation, we use three data sets: the Panel Study of Income Dynamics (PSID), the Health and Retirement Study (HRS), and the Medical Expenditure Panel Survey (MEPS). All three are nationally-representative surveys. PSID and MEPS survey households of all ages (even though MEPS top-codes age at 80 year old), while the HRS focuses on people above the age of 50.

We use the PSID to construct data moments related to labor market outcomes, health status, and wealth accumulation.[15] Since health status is not available in earlier waves, our main sample includes individuals without missing records on health status from 1984 onward.

We use the HRS (RAND version 2018) to construct moments related to claiming behavior, and to estimate survival probability and out-of-pocket nursing home costs. For claiming moments used in our estimation, we use males born in years 1936-1938 and who were not receiving Disability Insurance (DI) benefits. To estimate out-of-pocket nursing home costs, we use a larger sample by pooling waves 2002-2018 of the HRS. We use a sample of individuals older than 70 who do not have missing information on nursing home use, health or age.

We us MEPS (17 waves from 1999 to 2017) to estimate the out-of-pocket medical spending. This does not includ nursing home costs because the MEPS only samples the non-institutionalized population and thus excludes nursing home residents.

B.2 Additional estimation details

Health transitions We estimate health transitions from the PSID. Since in our model, health transitions depend on the productivity type ξ, we start by estimating the fixed pro-

[15] The information on net worth is not available in every wave before 1999. We use 1994 wave and every waves after 1999 to construct wealth profile, which results in 36,392 individual-wave observations.

ductivity. We do this by running a fixed effect regression of log labor income on a set of age dummy variables interacted with health. We define the two productivity types in our sample based on the median of the estimated fixed effects distribution. We then model the probability to move to health status h_{t+1} conditional on surviving as a logit model which depends on: (i) age polynomial degree three interacted with the dummy variable for the current health status, (ii) dummy variables for the two productivity types, (iii) cohort dummy variables, where cohort is defined based on a 5-year interval for birth year.

Nursing home shocks We estimate the risk of incurring a nursing home shock (pn_t^h) from the HRS as follows. First, we compute the probabilities of entering a nursing home for selected ages: 67, 72, 77, 82, 87, and 95. In each case, we use a sample within a 5-year age bracket. That is, we compute the percentage of individuals who report staying in a nursing home in each interview round for the following age groups: 65-69, 70-74, 75-79, 80-84, 85-89, and older than 90. Since the HRS is a biennnial survey, we convert these numbers into annual probabilities under the assumption that the probability to stay in a nursing home over the two-year interval is equal to the product of the annual probabilities. We then extrapolate the probability to stay in a nursing home at other ages using polynomial degree three approximation. We do this separately for healthy and unhealthy males. The HRS also reports the number of nights for all nursing home stays. To compute the average nursing home costs, we multiply the number of nights by the average daily rate for a semiprivate room in a nursing home, which was $158.26 in 2003 according to Metlife (2003).

Deterministic labor productivity To estimate the deterministic productivity profile λ_t^h we proceed as follows. We use a sample of working males in PSID, defined as people working at least 520 hours per year and earning at least the minimum wages, and estimate the following regression:

$$y_{it} = d^y_{age} D^{age}_{it} \times D^h_{it} + d^y_c D^c_i + \epsilon^y_{it}, \tag{62}$$

where y_{it} is labor income; D^{age}, D^h, and D^c is the set of age, health, and 5-year birth cohort dummy variables, and ϵ^y_{it} is the component orthogonal to age, health and cohort. Using our estimates, we compute the labor income for our base cohort:

$$\hat{y}_{it} = \hat{d}^y_{age} D^{age}_{it} \times D^h_{it} + \hat{d}^y_{j^c}(D^c_i = 1937) + \hat{\epsilon}^y_{it},$$

After computing the average $\hat{y}_{it}$ for each age and health group, we take the log and use the age polynomial degree two to smooth our estimates. To obtain the profile, we normalize the productivity of the healthy at age 30 to zero.