Reputation and Fragility*

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Abstract

We study a model à la Kyle (1985) in which financial traders care not only about profits but also their reputation, i.e., a recruiter's perception of their skills. The model presents two self-fulfilling equilibria. One resembles Kyle's original equilibrium. In the other, a "reputation-driven" equilibrium, traders trade more aggressively to appear skilled, leading to poorer performance and higher price volatility. We show that (1) labor mobility in finance generates traders' reputation concerns, triggering market fragility and increasing wage inequality, (2) traders with different skill levels adopt different trading styles, and (3) skills in active management may not translate into performance.

JEL classification: G11, G12, G14, G23

Key words: informed trading, market reputation, trader skill, financial fragility, poaching, asset prices

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1 Introduction

Financial firms compete for skilled employees. As the boundaries between investment banking, commercial banking, and asset management blur, this competition intensifies. Following the 2007-2009 crisis, pressures to reduce compensation and the resulting challenges in retaining talent have brought poaching practices into focus.[1] Poaching, or more generally labor mobility, raises concerns among financial specialists about their reputation within the industry. How do these concerns influence their trading strategies and skill acquisition? How do they interact with asset prices and market liquidity?

To address these questions, we study a model inspired by Kyle (1985), where traders, privately informed about their skill, trade risky assets anonymously with market makers. We define skill as the trader-specific ability to uncover more information about asset payoffs than the market. The more skilled a trader is, the larger the potential mispricing she can identify, and thus, the greater the trading profits she can potentially earn. Each trader selects an asset based on her skill and strategically submits a market order to the market maker for that asset, who sets the price after observing the aggregate order flow. A recruiter attempts to infer each trader's skill from the observed price and payoff of the asset chosen by the trader. The trader values not only trading profits but also her reputation, i.e., the recruiter's estimate of her skill. In a simple one-period model presented in Section 2, each trader derives an exogenous benefit from her reputation, which we model as an increasing quadratic function of reputation. In Section 4, we extend the model to a two-period setting, providing a microfoundation for the reputation benefit. In this extension, each trader randomly encounters a recruiter who compensates her for her skill based on his estimate. In equilibrium, the quadratic form assumed

[1] According to *The Wall Street Journal*, poaching is a common practice in the sophisticated capital markets industry, occurring in both bull and bear markets ("Greed is Good," February 2009), and employees at all levels are being poached ("Employee Poaching Returns to Wall Street," May 2010). *The New York Times* ("A Mad Scramble for Young Bankers," July 2014) and *Bloomberg* ("Saturdays Off Lose to Cash as Buyout Firms Poach Junior Bankers," March 2014) highlight the buy-side's (private equity, hedge funds) recruitment of young talent from investment banking amid a changing financial industry.

in Section 2 arises endogenously.

The model yields two stable, self-fulfilling equilibria, one of which is similar to that of Kyle (1985), where reputation is irrelevant. In the other, the "reputation-driven" equilibrium, traders trade more aggressively to appear skilled. This strategy increases trading volume, price volatility, and market liquidity but undermines trading profits by revealing too much private information to the market maker. Intuitively, each trader can boost her reputation by placing a large order, either buy or sell, because such an action makes her appear like a skilled trader who has identified an asset with significant mispricing. In equilibrium, the recruiter accurately anticipates the trader's strategy, so the recruiter's learning is not manipulated; nevertheless, the trader may trade aggressively depending on the market's belief. To illustrate the mechanism, suppose the recruiter believes that the trader trades more aggressively than the profit-maximizing level. If she trades less aggressively to increase profits, the recruiter (incorrectly) infers that she is less skilled. To avoid the reduction in her reputation benefit, she conforms to the recruiter's belief and indeed trades very aggressively.

The self-fulfilling property of our equilibria implies that a mere shift in investor beliefs, unaccompanied by changes in fundamentals, can trigger a transition from the standard Kyle-like equilibrium to the reputation-driven one, where price volatility is high and trading performance is relatively poor. We interpret this as *fragility* in financial markets.[2] Key to our multiple equilibria and fragility results are the trader's conflicting motives for revealing skill: on one hand, she wishes to hide her skill from the market maker to earn trading profits, but on the other hand, she wants to show it off to the recruiter to obtain the reputation benefit. If the trader's concern for reputation is very weak (very strong), the first (second) motive dominates, and the Kyle-like (reputation-driven) equilibrium holds uniquely. However, if the concern is moderately strong, either equilibrium may arise depending on the market's self-fulfilling beliefs. Therefore, the model predicts that fragility may occur

[2] As in Greenwood and Thesmar (2011), we define a market as fragile if it is susceptible to non-fundamental demand shifts, such as those caused by changes in investor beliefs.

when poaching is (moderately) active and there is high demand for financial skill, as these situations increase the likelihood of future recruitment and lead to heightened reputation concerns.

Our results shed light on how traders' underlying skills relate to their trading styles and contribute to financial fragility. The model predicts that high-skilled traders adopt low-intensity, profit-driven strategies, while those with lower skills engage in aggressive, risk-seeking behavior to gain reputation. Traders with intermediate skill levels, facing multiple equilibria, may switch between profit-seeking and reputation-driven strategies based on market beliefs, thereby contributing to market fragility. This behavior aligns with real-world observations during financial crises, such as momentum crashes and sudden reversals, which occur even without fundamental shocks.

The model has implications for compensation in the finance industry. In the reputation-driven equilibrium, traders' wages exhibit greater inequality among high- and low-skilled traders compared to those in the Kyle-like equilibrium, as they trade aggressively and reveal their skill more than they do in the Kyle-like equilibrium. This allows the recruiter to better assess each trader's skill and, therefore, to offer wages that more accurately reflect their abilities. This result is consistent with the findings by Philippon and Reshef (2012) and Célérier and Vallée (2019), who document increasing returns to talent and their significant contribution to income inequality in the finance industry.

The aggressive trade in the reputation-driven equilibrium also causes traders to reveal excessive information to the market maker, resulting in poorer trading performance than in the Kyle-like equilibrium. Consequently, high-skilled traders in the reputation-driven equilibrium may underperform low-skilled traders following the Kyle-like equilibrium. This result offers a novel explanation for the puzzling relationship (or the lack thereof) between trader skill and persistent performance, as documented and analyzed in the literature (e.g., Busse, Goyal, and Wahal, 2010; Berk and Green, 2004).

In Section 5, we study traders' endogenous skill acquisition. We show that, due to the multiplicity of equilibria, each trader may intentionally acquire a "middling" level of skill, which is too high or too low compared to the level she

would acquire if she knew which type of equilibrium would prevail. Ex-post, in the Kyle-like equilibrium, where profits are prioritized over reputation, the acquired skill level is insufficient. Conversely, in the reputation-driven equilibrium, where reputation outweighs profit motives, the skill level is excessive. Ex-ante, without knowing which equilibrium will emerge, the trader optimally chooses a middling skill level to prepare for both scenarios. This finding aligns with real-world observations that some finance professionals tend to pursue broader business qualifications rather than specializing in areas like advanced computer science or pursuing higher academic degrees such as a PhD.

Our paper is related to the literature on fragility in financial markets. Cespa and Vives (2015) is most closely related, as they also find multiple equilibria in an asymmetric information trading model. The key ingredients of their model are short-term investor horizons and persistent liquidity trading, which generate strategic complementarities. In Froot, Scharfstein, and Stein (1992), strategic complementarities among short-term traders are also important for the emergence of multiple equilibria. Unlike these studies, our model does not feature short horizons; instead, we obtain fragility from traders' forward-looking concerns about future recruitment opportunities. In Cespa and Foucault (2014), learning from other assets amplifies shocks and causes liquidity crashes. Although the mechanism is different, our model can also result in liquidity crashes when the economy switches between equilibria due to non-fundamental shocks.

Our paper also connects to the literature on trading frenzies, such as Veldkamp (2006) and Goldstein, Ozdenoren, and Yuan (2013), in which strategic complementarities play a key role. In Veldkamp (2006) frenzies originate from complementarities in information markets, while in Goldstein, Ozdenoren, and Yuan (2013) strategic complementaries, triggered by a feedback effect from financial markets to a firm's real investment, lead to coordinated aggressive trading. By contrast, in our reputation-driven equilibrium, each trader trades aggressively out of fear of being perceived as less skilled by the recruiter who believes that she would trade aggressively.

Lastly, our work contributes to the broader understanding of the role of

asymmetric information about skills in financial markets. DiMaggio (2015), Malliaris and Yan (2021), and Bijlsma, Boone, and Zwart (2018) derive implications for the risk-taking behavior of agents when ability is private information but is reflected in performance. We also link trader skill to private information and asset markets; however, unlike these papers, our focus is on deriving pricing implications. Guerrieri and Kondor (2012) show that career concerns amplify price volatility, similar to our findings. In Dasgupta (2006), career concerns lead to churning (trading without private information), and in Dasgupta and Prat (2008), they lead to herding (ignoring private information and trading like everyone else). These three papers are closely related to ours in that they study career concerns and endogenous asset prices, but they do not analyze fragility, which is the main focus of our paper.

The rest of the paper proceeds as follows. Section 2 presents a baseline model with exogenous reputation concerns, and Section 3 studies the equilibrium. Section 4 endogenizes reputation concerns by introducing a two-period model and presents the implications of our results. Section 5 analyzes traders' skill acquisition. The Online Appendix contains all proofs for the theoretical results.

2 Model

This section presents a one-period trading model à la Kyle (1985). There is a continuum of risk-neutral *traders*, competitive risk-neutral *market makers*, and *liquidity investors*, who trade risky assets. There is also a *recruiter*, whose role is to evaluate each trader's skill (defined below). There is a continuum of risky assets, each generating a stochastic payoff δ at the end of the period. At the beginning of the period, each informed trader picks one of the assets and trades with a market maker who sets the price p.[3] Unless otherwise noted, the following analyses focus on a generic trader without loss of generality.

[3]Assume that, on the equilibrium path, there is one competitive market maker for each asset. As in Kyle (1985), competitively many market makers exist off the equilibrium path, ensuring a break-even price in the equilibrium.

Skill. The traders can uncover more information about asset payoffs than market makers, modeled as follows. At the beginning of the period, each trader independently draws a number s from a normal distribution, $N(0, \omega_s)$, where s is her private information. The trader who draws s selects an asset whose payoff, δ, follows a normal distribution with mean $\hat{\delta} + s$ and variance ω_δ, where $\hat{\delta}$ and ω_δ are publicly known constants. Since the market maker for this asset does not observe s, the prior mean of δ from his perspective is $\mathrm{E}[\delta] = \hat{\delta} + \mathrm{E}[s] = \hat{\delta}$. Therefore, the trader who draws s selects an asset such that the market maker's prior mean, $\hat{\delta}$, is biased by s. As the market maker would set a price $p = \mathrm{E}[\delta] = \hat{\delta}$ for that asset in the absence of the trader, s can be viewed as the asset's "potential mispricing."[4] Indeed, as shown later, the trader profits by buying the asset if $s > 0$ (underpriced) and selling it if $s < 0$ (overpriced); the larger the $|s|$, the larger the profit. Thus, we refer to $|s|$ as the trader's *asset-selection skill*.[5] This formalization of skill, which we believe is new to the theoretical literature, aligns with findings in the empirical studies that attribute fund managers' performance to stock-picking skills (e.g., Wermers, 2000; Kosowski, Timmermann, Wermers, and White, 2006). As $|s|$ is the trader-specific ability, if she were to pick assets multiple times, each time she would systematically select an asset such that the market maker's estimate is biased by the same s. Although this assumption is irrelevant in this section's model, where she picks an asset only once, in the two-period setting in Section 4, it will be critical that the trader can identify the same potential mispricing multiple times (in different assets).

Moreover, in determining the skill, the variance of the skill distribution, $\mathrm{Var}[s] = \omega_s$, controls the shape of the underlying skill distribution from which a trader draws the actual skill. A trader equipped with a large ω_s draws her skill from a distribution with a wider scope, making her more likely to acquire a high skill level, $|s|$. Therefore, we refer to ω_s as the trader's *skill potential*.

[4] s is not the *actual* mispricing because the market maker will take into consideration the presence of the trader; thus, p will deviate from $\hat{\delta}$ in equilibrium.

[5] We do not model each trader's asset choice. Alternatively, we could assume that s is the highest potential mispricing that the trader can identify and that trading more than one asset is costly. She would then trade in the asset with the highest potential mispricing.

In practical terms, ω_s can reflect factors such as the trader's investment in higher education in finance and economics, which broadens the range of skills available to her. In Section 5, we endogenize ω_s by incorporating costly skill acquisition by traders.

Trading. Upon picking an asset, each trader submits a market order x to the market maker for that asset.[6] The liquidity investors submit order $u \sim N(0, \omega_u)$ exogenously, where u is independent across assets and their payoffs. For each asset, the market maker knows whether or not a trader has submitted an order. He observes the aggregate order flow, $x+u$, but not x or u separately, and sets the semi-strong efficient price to earn zero expected profit, that is,

$$p = \mathrm{E}[\delta | x + u]. \tag{2.1}$$

Reputation. The recruiter can track which trader has traded what asset, whereas he cannot observe either the order size, x, or the trader's profits, $\pi = (\delta - p)x$. Upon meeting a trader, he observes the price p and payoff δ of the asset picked by the trader and updates his estimate of s. The recruiter's posterior mean of s is denoted as $\hat{s} \equiv \mathrm{E}[s|\delta, p]$, and $|\hat{s}|$ is referred to as the trader's *reputation*. Our assumption regarding the recruiter's information set is supported by the reality that the trading strategies (i.e., details of portfolios) implemented by active fund managers are not accessible to outsiders, such as recruiters. This limitation arises due to insider trading laws, fiduciary duties, and non-compete agreements.[7,8]

[6] Each asset is never picked by multiple traders, as the probability that the same s is drawn twice is zero.

[7] For example, SEC Rule 10b-5 prohibits the misuse of material non-public information, including tipping confidential trading strategies. Similarly, the Graham-Leach-Bliley Act imposes restrictions on sharing sensitive financial information, and non-compete agreements prohibit managers from disclosing proprietary trading strategies to potential competitors, often for one to two years after leaving their firm.

[8] It is critical for our results that the recruiter cannot *perfectly* observe the traders' strategy, x. However, we could alternatively assume that the recruiter observes not only publicly available information, such as δ and p, but also a noisy signal about the trader's x or π. Such a setting would yield results essentially identical to ours. To see this, suppose that the recruiter observes a noisy signal about x, such as $q = x + m$, where $m \sim N(0, \omega_m)$

Maximization problem. At the end of the period, each trader obtains not only the trading profit, π, but also the "reputation benefit," characterized by $\theta\hat{s}^2$, where $\theta > 0$ is an exogenous constant measuring the strength of her reputation concern. In this section, we assume the quadratic form of the reputation benefits. In Section 4, the quadratic form of the reputation benefit and parameter θ emerge endogenously in a standard two-period setting where each trader randomly meets the recruiter who offers her compensation for her skill; the compensation is based on his expectation of future trading profits.

The trader's problem at the beginning of the period is to choose market order x to maximize the expected trading profit and the reputation benefit, conditional on her s:

$$\max_x \mathrm{E}\left[(\delta - p)x + \theta\hat{s}^2 \middle| s\right]. \tag{2.2}$$

A special case with $\theta = 0$ corresponds to the model of Kyle (1985).

Definition of equilibrium. Let $p(x+u)$ represent the market maker's pricing strategy, $x(s)$ denote each trader's trading strategy, and $\hat{s}(p,\delta)$ denote the recruiter's posterior mean of s. An equilibrium consists of $p(\cdot)$, $x(\cdot)$, and $\hat{s}(\cdot)$ such that $p(\cdot)$ satisfies (2.1) given $x(\cdot)$, $x(\cdot)$ solves (2.2) given $p(\cdot)$ and $\hat{s}(\cdot)$, and $\hat{s}(\cdot)$ is determined by Bayes' rule given $x(\cdot)$ and $p(\cdot)$.

3 Equilibrium

In this section, we search for a linear equilibrium in which each trader's market order x is linear in her skill, s. Proofs for the analytical results are provided in the Online Appendix.

is a noise arising from the difficulty in observing the true x. Since p conveys the order-flow information, $x+u$, observing p and q (in addition to δ) is informationally equivalent to observing a sufficient statistic, $x + km + (1-k)u$, with $k = \frac{\omega_m}{\omega_m + \omega_u}$. Thus, this extension is nested within the baseline analyses, as it can be expressed by replacing the liquidity trading u with $km + (1-k)u$, while appropriately adjusting ω_u in the baseline model.

3.1 Trading Strategy

Conjecture and later verify that each trader's equilibrium trading strategy is

$$x(s) = \beta s, \tag{3.1}$$

where $\beta > 0$ is an endogenous constant determined later. Conjecture (3.1) states that each trader buys (sells) the asset if her s is positive (negative), where the order size is proportional to her skill $|s|$; the more skilled the trader, the more aggressively she trades.[9] The scale factor β is referred to as the *trading intensity*.

3.2 Execution Price

The market maker decides on the price of each asset by extracting information about the mean of the asset payoff, $\bar{\delta} = \hat{\delta} + s$, from the aggregate order flow, $x + u$. Since he believes that each trader follows strategy (3.1), the price in equation (2.1) can be rewritten as

$$p = \hat{\delta} + \lambda(x + u), \tag{3.2}$$

where

$$\lambda \equiv \frac{\beta\omega_s}{\beta^2\omega_s + \omega_u} \tag{3.3}$$

is "Kyle's lambda," a measure of the trader's price impact.[10] Following the literature, we define a market to be *liquid* if λ is small.

[9] This conjecture follows from the original Kyle model where an informed trader trades based on the difference between her belief and that of the market maker, i.e., $\mathrm{E}[\delta|s] - \mathrm{E}[\delta] = s$.

[10] From (2.1), we have $p = \mathrm{E}[\delta|x+u] = \mathrm{E}[\mathrm{E}[\delta|x+u, s]|x+u] = \mathrm{E}[\hat{\delta} + s|x+u]$. Since the market maker believes that the trader follows (3.1), s and $x+u$ are both normally distributed from the market maker's perspective. Thus, $p = \hat{\delta} + \mathrm{E}[s] + \frac{\mathrm{Cov}[s,x+u]}{\mathrm{Var}[x+u]}(x + u - \mathrm{E}[x+u]) = \hat{\delta} + \frac{\mathrm{Cov}[s,\beta s+u]}{\mathrm{Var}[\beta s+u]}(x + u - \mathrm{E}[\beta s + u]) = \hat{\delta} + \frac{\beta\omega_s}{\beta^2\omega_s+\omega_u}(x+u)$.

3.3 Reputation

The recruiter updates his estimate of each trader's skill, $|s|$, based on the realized values of p and δ. The execution price is informative about the skill, as the equilibrium trading and pricing strategies in (3.1) and (3.2) suggest that it is positively influenced by s. For example, a positive and large s induces a trader to buy aggressively, thereby increasing x and p. We denote the recruiter's updated estimate as $|\hat{s}|$, which also represents the trader's reputation.

Lemma 3.1 (Trader's reputation)**.** *Upon observing the asset's price p and payoff δ, the recruiter's updated estimate of the trader's skill is given by $|\hat{s}|$, where*

$$\hat{s} = \xi \left(\gamma \left(\frac{p - \hat{\delta}}{\lambda} \right) + (1 - \gamma)(\delta - \hat{\delta}) \right) \tag{3.4}$$

with

$$\xi \equiv \frac{\omega_s(\beta\omega_\delta + \omega_u)}{\omega_s(\beta^2\omega_\delta + \omega_u) + \omega_u\omega_\delta} \tag{3.5}$$

and

$$\gamma \equiv \frac{\beta\omega_\delta}{\beta\omega_\delta + \omega_u}. \tag{3.6}$$

In the updated estimate, ξ measures how much the recruiter relies on the new information (p and δ) when computing $\hat{s}$. Holding β constant, ξ is large when his prior is unreliable (ω_s is large). γ represents the weight he places on p relative to δ. If the asset payoff is noisy (ω_δ is large), he puts a heavier weight on p; if the price is noisy (ω_u is large), he weighs δ relatively more.

3.4 Trader's Optimization

The trader is aware of the impact of her choice, x, on her reputation, $|\hat{s}|$, because $\hat{s}$ depends on p, which is influenced by x (see [3.2]). Therefore, by plugging (3.2) into (3.4), the trader views $\hat{s}$ as a function of her choice of x:

$$\hat{s} = \xi \left(\gamma \left(x + u \right) + (1 - \gamma)(\delta - \hat{\delta}) \right). \tag{3.7}$$

With this $\hat{s}$, the trader's objective in equation (2.2) can be written as a simple quadratic function of x, as presented in the following lemma.

Lemma 3.2 (Trader's problem)**.** *Each trader's maximization problem is*

$$\max_{x} \; sx - \lambda x^2 + \theta\xi^2 \left(\gamma^2 x^2 + 2\gamma(1-\gamma)sx\right). \tag{3.8}$$

The first term of equation (3.8) is the trader's expected trading revenue. The second term represents the trading cost resulting from her price impact, which is increasing in the size of her trade, $|x|$. The third term corresponds to her reputation benefit, which also increases with $|x|$ because she can potentially inflate her reputation $|\hat{s}|$ by placing a large order (either buy or sell) and influencing the price that the recruiter observes.

The unique solution to (3.8) (given β) is

$$x = F(\beta)s, \tag{3.9}$$

where

$$F(\beta) \equiv \frac{1 + 2\theta\xi^2\gamma(1-\gamma)}{2\left(\lambda - \theta\xi^2\gamma^2\right)}. \tag{3.10}$$

$F(\cdot)$ is a nonlinear function of β because λ, ξ, and γ are functions of β.

3.5 Equilibrium

Each trader optimally chooses (3.9) given that the market maker and the recruiter believe that she follows strategy (3.1). In equilibrium, their beliefs must be correct, that is, (3.1) and (3.9) must be consistent with each other, requiring the following condition:

$$\beta = F(\beta). \tag{3.11}$$

That is, the equilibrium levels of β are determined at the fixed points of $F(\cdot)$. Having identified β, the equilibrium levels of x and p are determined by equations (3.1) and (3.2), respectively. In what follows, we first illustrate the

determination of equilibrium β through a numerical example, followed by an analytical proposition that formally establishes the equilibria.

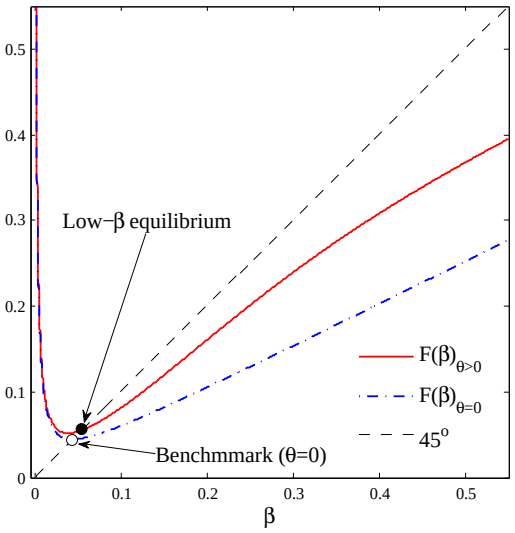
Figure 1 illustrates the determination of β for three different levels of θ. The intersections of $F(\beta)$ (the solid line) and the 45-degree line yield the equilibrium levels of β. As a benchmark, we also plot $F(\beta)$ with $\theta = 0$ (the dash-dotted line), which corresponds to the model of Kyle (1985) where reputation concerns are irrelevant. The trading intensity in the benchmark equilibrium is denoted as $\beta_{\theta=0}$.

Panel (a) of Figure 1 shows that if θ is small, there is a unique "low-β equilibrium" where β is close to $\beta_{\theta=0}$. Panel (b) shows that a large θ leads to a unique "high-β equilibrium," in which β is much larger than $\beta_{\theta=0}$.[11] Panel (c) shows that multiple equilibria arise for an intermediate level of θ. Specifically, the high-β and the low-β equilibria are stable, as $F(\beta)$ crosses the 45-degree line from above, whereas the one with an intermediate β is unstable, as $F(\beta)$ crosses the line from below.[12] In what follows, we focus only on the stable equilibria.

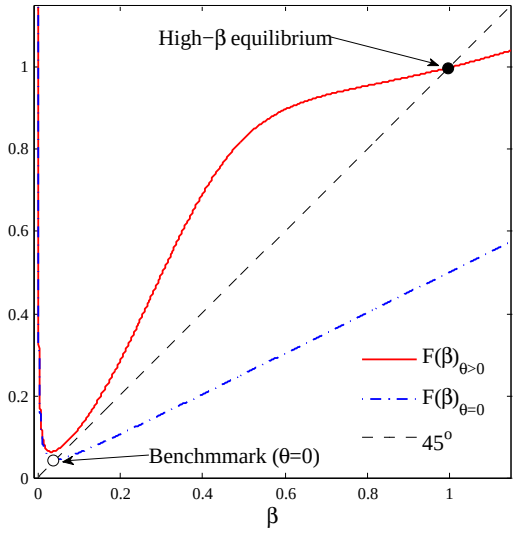
Impact of reputation concerns. To see the intuition behind the different equilibrium patterns presented in Figure 1, note that the trader faces two conflicting motives when trading. On the one hand, she seeks to avoid trading too aggressively on her private information, as doing so would move the price excessively and reduce her trading profits. As in Kyle (1985), this motive leads the trader to decrease β. On the other hand, she wishes to trade intensively to signal her high skill to the recruiter, aiming to establish a strong reputation. This motive encourages her to increase β. In summary, she faces a tradeoff

[11] Formally, we define an equilibrium to be low-β (high-β) if $\beta < \tilde{\beta}$ ($\beta > \tilde{\beta}$), where $\tilde{\beta}$ is defined as square root of the unique solution to equation (E.9) in Lemma E.1 of Online Appendix E. In the context of Figure 1, a low-β equilibrium is given by a fixed point of function F at which F is convex and U-shaped, while a high-β equilibrium is given by a fixed point that is larger than the first inflection point of F. Using this definition, Kyle's original equilibrium ($\theta = 0$) is classified as a low-β equilibrium.

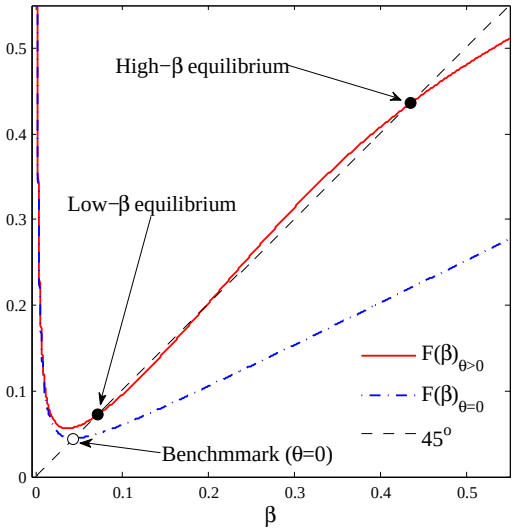
[12] We define an equilibrium to be stable (unstable) if the corresponding β is a stable (unstable) fixed point of $F(\cdot)$, that is, $|F'(\beta)| < 1$ (> 1). A stable equilibrium is robust to a small perturbation to the agents' belief about β, meaning that the economy converges back to the original equilibrium through the best-response dynamics of the agents' beliefs.



(a) Small θ ($\theta = 0.2$) $\Rightarrow$ unique low-β equilibrium.



(b) Large θ ($\theta = 0.55$) $\Rightarrow$ unique high-β equilibrium.



(c) Intermediate θ ($\theta = 0.33$) $\Rightarrow$ multiple high-β & low-β equilibria.

Figure 1: Determination of β

Note: The figure plots $F(\beta)$ for different values of θ. The equilibrium values of β are determined at the fixed points of $F(\cdot)$. The parameter values used in the figures are $\omega_s = 50$, $\omega_\delta = 1$, and $\omega_u = 0.1$.

between "hiding" her skill from the market maker and "showing it off" to the recruiter. The degree of reputation concern, θ, governs this tradeoff.

For small θ (panel (a)), the hiding motive dominates the showing-off motive, leading to a unique low-β equilibrium. In this equilibrium, the recruiter believes that the trader's trading intensity is relatively low. Knowing this, the trader could inflate her reputation by placing an order larger than what the recruiter anticipates. However, she would refrain from doing so because it would cause a large price impact, reducing her trading profits. Consequently, the equilibrium β is close to $\beta_{\theta=0}$, meaning that it is a "Kyle-like equilibrium."

For large θ (panel (b)), the showing-off motive prevails, resulting in a unique high-β equilibrium. In this equilibrium, the trader is willing to move the price substantially, undermining her profits to appear skilled. Importantly, the recruiter's learning is not manipulated, as his belief about the trader's trading strategy is correct. Nonetheless, the trader places a large order because doing otherwise would induce the recruiter to incorrectly estimate that she is less skilled, thereby deteriorating her reputation. She trades aggressively solely to conform to the recruiter's belief. To emphasize this mechanism, we call this equilibrium the "reputation-driven equilibrium."

For intermediate levels of θ (panel (c)), there is no clear dominance between the two motives, hence both the low-β and the high-β equilibria are supported. Each equilibrium is associated with a self-fulfilling belief of the market maker and the recruiter.

The following proposition and Figure 2 summarize the above discussion and provide an analytical characterization of the equilibria (the proof is provided in Online Appendix E as a special case of Proposition 4.2).

Proposition 3.1 (Equilibrium characterization)**.** *There exists a critical value of the skill potential, $\tilde{\omega}_s > 0$, and two thresholds for reputation concern, θ_L and θ_H ($\geq \theta_L$), such that, $\theta \geq \theta_H$ leads to a unique high-β equilibrium, while $\theta \leq \theta_L$ leads to a unique low-β equilibrium. Furthermore,*

(i) if $\omega_s \leq \tilde{\omega}_s$ then $\theta_H = \theta_L$, hence the equilibrium is unique; and

(ii) if $\omega_s > \tilde{\omega}_s$ then $\theta_H > \theta_L$, and $\theta \in (\theta_L, \theta_H)$ leads to multiple equilibria.

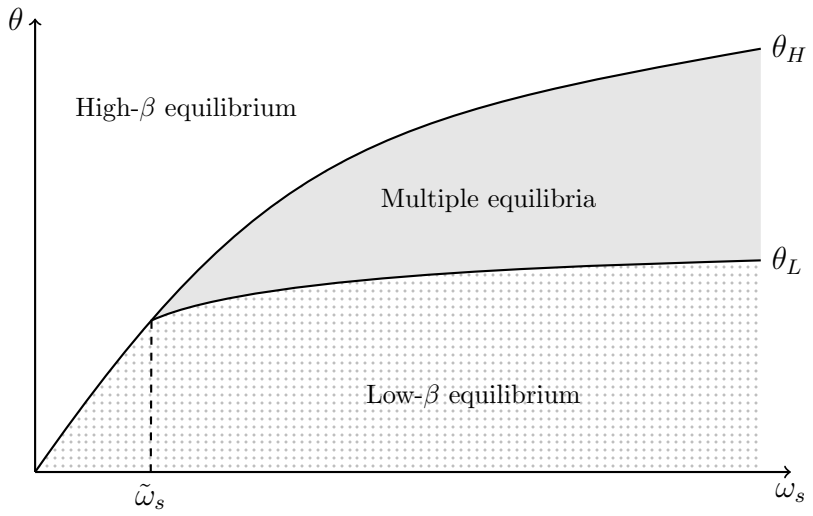


Figure 2: Equilibrium Characterization with Exogenous θ

Note: This figure illustrates the equilibrium states using the reputation concern, θ, and the skill potential, ω_s.

Impact of skill potential. Proposition 3.1 and Figure 2 indicate that, for a given θ, a low skill potential ω_s supports only the high-β equilibrium, while increases in ω_s can generate the low-β equilibrium, potentially leading to multiple equilibria.[13] To understand intuition, the tradeoff between hiding and showing-off motives, as explained above, is key. When ω_s is small, the market maker anticipates that the trader has low skill and, due to limited adverse selection, sets a low price impact λ. In this case, the trader's hiding motive is less important. Consequently, the showing-off motive prevails, prompting her to trade aggressively and leading to the high-β equilibrium. Conversely, when ω_s is large, the market maker sets a high λ, and the hiding motive becomes more influential, making the low-β equilibrium more likely to be realized.

Overall, the type of equilibrium depends on the combination of θ and ω_s, as illustrated in Figure 2. Small θ and large ω_s, both of which amplify the trader's hiding motive, lead to the low-β equilibrium. Conversely, large θ and small ω_s, which both intensify her showing-off motive, result in the high-β

[13]Lemma E.3 in Online Appendix E analytically characterizes the upward-sloping θ_H and θ_L in relation to ω_s, as presented in Figure 2.

equilibrium. However, with a combination of large θ and large ω_s, the result is ambiguous, as the two motives offset each other. In this case, either the high-β or the low-β equilibrium may be realized, depending on the market's self-fulfilling beliefs.

Equilibrium comparison. Comparing the two stable equilibria, does the trader perform better in one of them? What are the implications for market liquidity, asset prices, and learning about asset-selection skills?

Corollary 3.1 (High-β vs. low-β equilibrium)**.** *In the high-β equilibrium, relative to the low-β equilibrium,*

(i) the market liquidity is higher, that is, λ is lower;

(ii) the trader's expected trading profit given the realized skill, $\mathrm{E}[(\delta - p)x|s] = \frac{\beta\omega_u s^2}{\beta^2\omega_s+\omega_u}$, is lower;

(iii) the price informativeness, $\tau \equiv \frac{1}{\mathrm{Var}[\delta|p]} = \frac{\beta^2\omega_s+\omega_u}{\omega_\delta(\beta^2\omega_s+\omega_u)+\omega_u\omega_s}$, is higher;

(iv) the price volatility, $\mathrm{Var}[p] = \frac{\beta^2\omega_s^2}{\beta^2\omega_s+\omega_u}$, is higher; and

(v) the recruiter's estimate of s is more precise, that is, the posterior variance of s, $\hat{\omega}_s = \frac{\omega_s\omega_u\omega_\delta}{\omega_s(\beta^2\omega_\delta+\omega_u)+\omega_u\omega_\delta}$, is lower.

Statement (i) of Corollary 3.1 holds because price impact λ is decreasing in β. Since the high-β equilibrium emerges only if θ is relatively large, the statement also implies that a higher reputation concern leads to higher market liquidity. The intuition is as follows. Relative to the low-β equilibrium, in the high-β equilibrium the trader trades more aggressively and thus reveals more private information on s to the market maker. From the market maker's perspective, the information is "exaggerated" in that the order size is too large relative to the true $|s|$. Thus, the market maker attempts to partially "undo" the trader's information revelation by incorporating less of it into the price. This mechanism results in a reduced price impact.

The intuition for statement (ii) is closely related to that for statement (i). In the high-β equilibrium, the trader earns a lower trading profit due to a

smaller informational advantage over the market maker. This result may appear inconsistent with statement (i) because, ceteris paribus, a lower price impact is typically associated with a higher trading profit. However, the trader's order size $|x|$ in the high-β equilibrium is so large that the *overall* price impact (i.e., λ multiplied by $|x|$) is larger than that in the low-β equilibrium. Overall, the trader moves the price excessively and earns a lower trading profit in this equilibrium. Due to her aggressive trading, more information about s—which is informative about δ—is incorporated into the price in the high-β equilibrium, supporting statement (iii).

Statement (iv) follows, again, from the trader's aggressive trading in the high-β equilibrium. Specifically, she places a large buy (sell) order when $s > 0$ ($s < 0$), even if $|s|$ is small. This causes a large upward (downward) pressure on p, leading to greater price variability. This result aligns with the insight of Guerrieri and Kondor (2012) that reputation concerns amplify price volatility, although their mechanism differs from ours.[14]

In the high-β equilibrium, the trader reveals more information about s, thereby improving the precision of the recruiter's estimation (statement (v)). This makes the reputation benefit more responsive to skill, as the recruiter can assess it more accurately.

4 Endogenous Reputation Concerns

This section shows that the quadratic form of the reputation benefit postulated in Section 2 endogenously arises in a two-period model. Unless otherwise noted, the same assumptions as the baseline model prevail. We discuss economic implications drawn from the equilibrium analyses in Subsection 4.3.

[14] Guerrieri and Kondor (2012) argue that bond price volatility increases because fund managers demand a premium to offset the risk of reputation damage in the event of default.

4.1 Setup

There are two periods, $t = 0, 1$. In each period, the trader with skill $|s|$ picks an asset that yields a payoff $\delta_t \sim N(\hat{\delta}_t + s, \omega_\delta)$ at the end of the period, where $\hat{\delta}_t$ is the market maker's prior expectation about δ_t. Each asset lives only for one period. Although the trader picks different assets in $t = 0$ and $t = 1$, both of them are characterized by the same s because s represents her time-invariant skill. In other words, her skill allows her to *systematically* pick an asset such that the market maker's prior expectation about δ_t is biased by $|s|$. We assume that the market is anonymous, meaning that market makers cannot track which trader selects which assets, while the recruiter can. This assumption, combined with the existence of a continuum of traders with various s, implies that market makers at $t = 1$ cannot update their estimate of each trader's s, whereas the recruiter can.[15] This distinction is critical: it is the market makers' inability to update the estimate of s that allows the recruiter—who can update it—to potentially make profits by recruiting traders.

Timing. The model unfolds as follows.

1. At the beginning of $t = 0$, each trader draws s independently from $N(0, \omega_s)$, which is her private information. She selects an asset and submits market order x_0 to the market maker for that asset, and liquidity investors' demand $u_0 \sim N(0, \omega_{u,0})$ is realized. The market maker sets a price using order-flow information, $p_0 = \mathrm{E}\left[\delta_0 | x_0 + u_0\right]$.
2. At the end of $t = 0$, the asset's payoff δ_0 is realized, and the trader earns the profit, $\pi_0 = (\delta_0 - p_0)x_0$.
3. At the beginning of $t = 1$, each trader meets the recruiter with probability φ. The recruiter observes the past execution price (p_0) and the realized payoff (δ_0) of the asset that the trader has picked in $t = 0$. The trader's x_0 and π_0 are not disclosed. The recruiter updates his estimate

[15] This assumption is justified by the anonymity of the market. Each market maker trades with a different trader in each period and cannot track or identify which traders have traded which assets in the past. Hence, he cannot update traders' skills based on past information. Conversely, recruiters may keep track of traders via direct and individual communications.

of the trader's s, obtaining the posterior mean $\hat{s} = \mathrm{E}[s|p_0, \delta_0]$. The recruiter then employs the trader for a competitive salary:

$$w = \mathrm{E}[\pi_1|p_0, \delta_0], \tag{4.1}$$

where $\pi_1 = (\delta_1 - p_1)x_1$ is the trading profit in $t = 1$. Note that w is determined by the recruiter's expectation about π_1.[16] This is why the trader cares about the recruiter's perception of her $|s|$, leading to the endogenous reputation benefit. With probability $1 - \varphi$, the trader does not meet the recruiter and remains with her current employer.[17] The trader submits market order x_1 to the market maker. Liquidity investors' demand $u_1 \sim N(0, \omega_{u,1})$ is realized ($\omega_{u,1}$ and $\omega_{u,0}$ can be different). The market maker sets the price $p_1 = \mathrm{E}[\delta_1|x_1 + u_1]$.

4. At the end of $t = 1$, δ_1 is realized and π_1 is determined.

The trader's problem at the beginning of $t = 0$ is

$$\max_{x_0} \ \mathrm{E}[\pi_0 + \varphi w + (1 - \varphi)\pi_1|s]. \tag{4.2}$$

4.2 Equilibrium

We first solve the problems in $t = 1$ and obtain the salary, w, which is shown to be a quadratic function of the perceived skill, $|\hat{s}|$. Subsequently, we solve the problems in $t = 0$, which are similar to those of Section 2.

[16] An implicit assumption is that the trader encounters a recruiter upon losing her current job. Consequently, she always accepts the salary offered. A trader might find herself in such a situation due to the closure of a trading desk or the discontinuation of a fund, as described by eFinancialCareers ("How Investment Banks are Now Poaching from Floundering Hedge Funds," May 2014). Also, parameter φ can incorporate labor mobility in the finance industry. For example, non-compete clauses and garden leave may reduce φ.

[17] Alternatively, we could introduce the following setting. In the current firm's compensation, the proportion α is paid as a fixed wage, and the remaining portion is paid based on performance, π_t, while the recruiter's firm pays the α' portion as the fixed wage. The fixed wage paid by the current firm, $\bar{w}$, is already fixed, so that the trader's expected utility is written as $\mathrm{E}[\alpha\bar{w} + (1 - \alpha)\pi_0 + (1 - \varphi)(\alpha\bar{w} + (1 - \alpha)\pi_1) + \varphi(\alpha' w + (1 - \alpha')\pi_1)]$. Appropriately scaling and rewriting the coefficients of this utility, we obtain a maximization problem virtually identical to equation (4.2), which yields all the results presented in this section.

4.2.1 Period 1

In $t = 1$, each trader has no reputation concerns. Hence, she simply chooses x_1 to maximize π_1, resulting in the standard Kyle (1985) equilibrium.

Lemma 4.1 (Equilibrium in $t = 1$)**.** *In $t = 1$, each trader's trading strategy is characterized by $x_1 = \beta_1 s$, and the market maker sets the execution price according to $p_1 = \hat{\delta}_1 + \lambda_1 (x_1 + u_1)$, where $\beta_1 \equiv \sqrt{\frac{\omega_{u,1}}{\omega_s}}$ and $\lambda_1 \equiv \frac{1}{2}\sqrt{\frac{\omega_s}{\omega_{u,1}}}$.*

Lemma 4.1 yields the trader's profit, π_1. Then, the *recruiter's* expectation about π_1 determines the trader's salary when she meets the recruiter.

Lemma 4.2 (Trader's salary)**.** *Let $\hat{s}$ and $\hat{\omega}_s$ be the posterior mean and variance of s from the recruiter's point of view. The salary he offers to the trader is*

$$w = \frac{\beta_1}{2}\left(\hat{s}^2 + \hat{\omega}_s\right). \tag{4.3}$$

Lemma 4.2 gives the competitive wage, w, as a quadratic function of $\hat{s}$, providing a microfoundation for the reputation benefit assumed in Section 2.

4.2.2 Period 0

In $t = 0$, the trader knows that she will potentially obtain w in the form of (4.3). Hence, plugging (4.3) into (4.2), her objective function in $t = 0$ is

$$\mathrm{E}\left[\pi_0 + \frac{\varphi\beta_1}{2}\left(\hat{s}^2 + \hat{\omega}_s\right) + (1-\varphi)\pi_1 \middle| s\right]. \tag{4.4}$$

Since π_1 and $\hat{\omega}_s$ are independent of x_0, her optimization problem in $t = 0$ is characterized by (4.5) in the following proposition.

Proposition 4.1 (Endogenous reputation)**.** *In $t = 0$, each trader's maximization problem is identical to (2.2) of Section 2, that is,*

$$\max_{x_0} \mathrm{E}\left[\pi_0 + \theta\hat{s}^2 \middle| s\right], \tag{4.5}$$

where parameter θ for reputation concerns is given by

$$\theta \equiv \frac{\varphi}{2}\sqrt{\frac{\omega_{u,1}}{\omega_s}}. \tag{4.6}$$

Proposition 4.1 provides a microfoundation to the quadratic form of the reputation benefit in Section 2. Moreover, θ is endogeneized as a composite parameter (4.6). Not surprisingly, it is increasing in φ. It decreases with the trader's skill potential, ω_s, suggesting that a high-potential trader is less concerned about her reputation. This is due to the *future* trading intensity β_1 and the price impact λ_1: a high-potential trader is likely to possess a high skill, which induces a large price impact in the future as well. Since it is difficult for the trader to exploit her skill in $t = 1$, the competitive wage becomes less responsive to $|s|$, weakening the reputation concern.

As problem (4.5) is effectively the same as (2.2), the equilibrium outcomes resemble Proposition 3.1. However, unlike Proposition 3.1, the impact of the skill potential ω_s on the equilibrium characterization is generalized due to the dependence of θ on ω_s. This is summarized by the following proposition, which is graphically presented in Figure 3.

Proposition 4.2 (Equilibrium in $t = 0$)**.** *The equilibrium at $t = 0$ is characterized as follows.*

1. *If φ is large enough, there are two thresholds for the skill potential, denoted as $\underline{\omega}_s > 0$ and $\bar{\omega}_s$ $(\geq \underline{\omega}_s)$, such that*

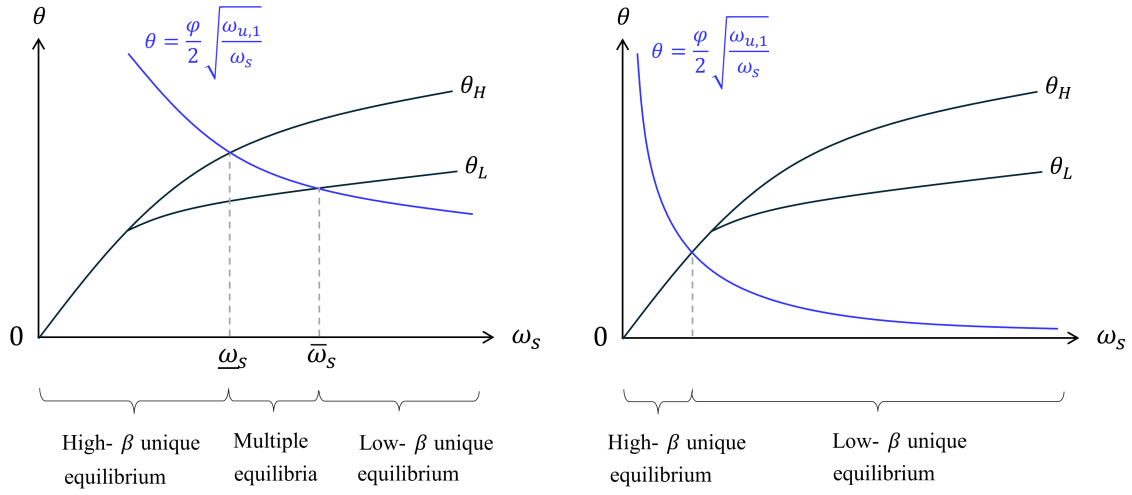
   (i) for $\omega_s \geq \bar{\omega}_s$, the low-$\beta$ equilibrium is uniquely realized;

   (ii) for $\omega_s \in (\underline{\omega}_s, \bar{\omega}_s)$, both the high- and low-β equilibria arise;

   (iii) for $\omega_s \leq \underline{\omega}_s$, the high-$\beta$ equilibrium is uniquely realized.

2. *If φ is small enough, there is a unique equilibrium. For small ω_s, the high-β equilibrium is realized, while for large ω_s, the low-β equilibrium is realized.*

Since the trader's reputation concern θ is endogeneized as (4.6), which is a decreasing function of ω_s, Figure 3 is obtained by overlaying (4.6) on Figure 2.



(a) Large φ (b) Small φ

Figure 3: Equilibrium Characterization with Endogenous θ

Note: These figures illustrate the equilibrium states using the reputation concern, θ, and the skill potential, ω_s. The curves represented by θ_H and θ_L are identical to those in Figure 2 and Proposition 3.1, while θ is endogenous and decreasing in ω_s as equation (4.6) suggests.

The first and the second parts of Proposition 4.2 correspond to panels (a) and (b) of Figure 3, respectively. Proposition 4.2 states that multiple equilibria may arise only when labor mobility is sufficiently high (φ is large), i.e., only when the traders have a large reputation concern, θ.[18] Since the case in panel (a) yields richer implications, while panel (b) is qualitatively similar to Kyle (1985), we focus on the case in panel (a), which arises when φ is relatively large.

Panel (a) of Figure 3 shows that, as a result of endogeneizing θ as a function of ω_s, the conditions characterizing the equilibrium types simplify to a single condition solely on ω_s: a large ω_s yields a unique low-β equilibrium, an intermediate ω generates multiple equilibria, and a small ω_s leads to a unique high-β equilibrium.

The intuition is as follows. When the skill potential is high ($\omega_s \geq \bar{\omega}_s$), the

[18]Equation (E.15) in Online Appendix E provides the definition of the boundary value of φ that classifies cases 1 and 2 in Proposition 4.2.

trader is more likely to have a large informational advantage over the market maker. This leads to a strong price impact, making it costly to inflate the trading strategy to receive high reputation. Consequently, she is more inclined to adopt profit-seeking strategies, and the Kyle-like equilibrium is uniquely realized. As the skill potential decreases, however, the trader's informational advantage and the price impact tend to diminish, making it easier to prioritize her reputation concerns. Therefore, for intermediate levels of $\omega_s \in (\underline{\omega}_s, \bar{\omega}_s)$, the reputation-driven equilibrium arises and coexists with the Kyle-like equilibrium. If the skill potential declines further ($\omega_s \leq \underline{\omega}_s$), the reputation becomes the dominant factor, and the Kyle-like equilibrium disappears.

4.3 Implications

This section discusses economic implications drawn from the equilibrium analyses in Section 4.2.

4.3.1 Skills, Trading Styles, and Financial Fragility

The self-fulfilling nature of multiple equilibria suggests that financial markets may become vulnerable to non-fundamental factors. A shift in investor beliefs alone, without any structural changes, could cause the market to transition from the standard Kyle-like equilibrium to the reputation-driven equilibrium, characterized by high price volatility and poor trading performance. We interpret this phenomenon as a form of *fragility* in financial markets. Our model suggests that fragility may occur when labor mobility is high or demand for skills is high in the finance industry, as these factors heighten traders' reputation concerns in anticipation of future recruitment opportunities.

Proposition 4.2 yields novel implications connecting traders' skills, their trading styles, and financial fragility. First, traders with high skill potential ($\omega_s \geq \bar{\omega}_s$) adopt low-intensity strategies that minimize market impact, prioritizing profits over reputation, whereas those with low skill potential ($\omega_s \leq \underline{\omega}_s$) engage in high-intensity strategies with flashy risk taking, seeking reputation in the industry. The model predicts that these tendencies are more pronounced

for traders with very high or very low ω_s, as the equilibrium is unique at such levels of ω_s (Proposition 4.2).

These results are somewhat consistent with common observations in the real market. For example, skilled HFT firms, such as Citadel Securities or Virtu Financial, are highly profit-driven and focused on consistent performance, leveraging advanced algorithms and technologies. They employ top talent and have a clear performance-based focus, with reputational benefits being regarded as secondary to profitability. Similarly, hedge fund managers often vary in their approach depending on their skill level and reputation. Studies show that highly skilled managers, such as those at Bridgewater or Renaissance Technologies, are more focused on generating long-term performance through data-driven strategies (Schwager, 2012). On the other hand, lesser-known funds sometimes take on highly visible and risky bets that generate headlines but do not always result in sustainable returns. The high-profile blowup involving Archegos Capital in 2021 is the leading example that illustrates how traders seeking visibility through concentrated and flashy positions can underperform and may go bankrupt.

The second implication is drawn from the model's multiple equilibria. Namely, the relation between traders' activities and skills may become blurry when their skill potential is mediocre ($\omega_s \in (\underline{\omega}_s, \bar{\omega}_s)$). Notably, this indeterminacy may lead to financial fragility. Depending on market beliefs, intermediate-skilled traders may shift between profit-seeking and reputation-driven strategies, creating non-fundamental fluctuations. Our result suggests that even relatively skilled traders may follow the crowd due to market beliefs, hopping between safe and risky strategies.

A notable observation in line with this result is momentum crashes (Daniel and Moskowitz, 2016), where traders engaging in risky and visible momentum trades often switch to safer strategies, generating sharp reversals and amplifying price declines. More generally, the rapid shifts between risk-taking and solid performance-based behavior have contributed to market fragility and volatility, as documented in various studies on economic crises, such as Brunnermeier (2009) on the credit crunch during the Global Financial Crisis in

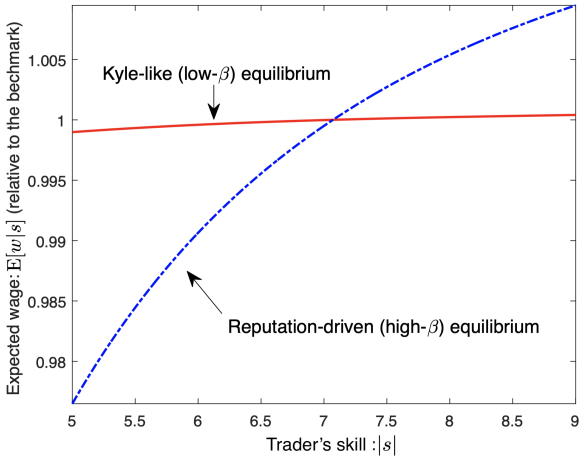


Figure 4: Skill versus Expected Wage

Note: The figure plots the trader's expected wages (relative to the benchmark with $\theta = 0$) at the low-β equilibrium (solid line) and high-β equilibrium (dashed line) for different levels of skill, $|s|$. The parameter values used in the figures are $\omega_s = 50$, $\omega_\delta = 1$, $\omega_{u,0} = 0.1$, $\omega_{u,1} = 60$, and $\theta = \frac{\varphi\beta_1}{2} \approx 0.33$ ($\varphi = 0.6$).

2007. Importantly, large fundamental shifts that support such transitions are rarely identified (Gennotte and Leland, 1999). Our model does not require such fundamental shocks, as a mere change in beliefs can trigger a shift.

4.3.2 Compensation in Finance Industry

Our model provides insights into how skills are reflected in compensation in the financial industry. As shown in statement (v) of Corollary 3.1, the recruiter learns s more precisely in the high-β equilibrium than in the low-β equilibrium, as the trader demonstrates her skill by trading more intensively. Consequently, traders' wages become more sensitive to their skills. As an illustration, Figure 4 plots the traders' expected wages in relation to their skills in the low- and high-β equilibria. In the high-β equilibrium, the recruiter's accurate evaluations result in greater pay inequality: low-skilled ones receive very low wages, while high-skilled ones are very highly paid. This result corroborates empirical observations, such as those provided by Philippon and Reshef (2012) and Böhm, Metzger, and Strömberg (2018), highlighting the comparatively large

wage inequality in the finance industry. Célérier and Vallée (2019) attribute its source to the scalability of skill, while inequality arises from traders' reputation concerns in our model.

Our results also have implications for the wage structure in the finance industry. The model suggests that moving away from performance-tied compensation and assigning more weight to the fixed salary component creates reputation concerns, thereby generating fragility in financial markets. Recall that reputation concerns stem from the dependence of future salaries on current trading, as the recruiter learns about skill from the realized execution prices. Suppose instead that after meeting a trader, the recruiter hires her for a salary that depends entirely on $t = 1$ trading performance. Since trading actions at $t = 0$ bear no consequence on trading profits at $t = 1$, the trader's objective at $t = 0$ is only to maximize her short-run profits. That is, traders have no reputation concerns, and the model collapses to the Kyle (1985) equilibrium. Therefore, our results highlight the importance of traders' forward-looking motivations—stemming from labor mobility within the finance industry—in contributing to financial fragility or pay inequality.

4.3.3 Skill and Performance Persistence

Do high-skilled traders consistently perform well? Not always: skilled traders may not necessarily outperform less skilled ones due to multiple equilibria. Consider a highly skilled trader and a relatively less skilled one. Since trading profits are increasing in traders' skill, $|s|$, the former is expected to achieve higher trading profits than the latter *conditional on* both traders trading at the same intensity, β. However, if the recruiter believes, for whatever reason, that the skilled trader trades more aggressively than the less skilled one, and traders conform to his belief, a high skill level does not translate into performance. This is because the skilled trader, trading at the reputation-driven equilibrium, reveals too much private information to the market maker. Hence, compared to the less skilled trader, who plays the Kyle-like equilibrium, the skilled trader may earn lower expected profits. Figure 5 illustrates such a situation. It plots the expected trading profits at high- and low-β equilibria with different levels

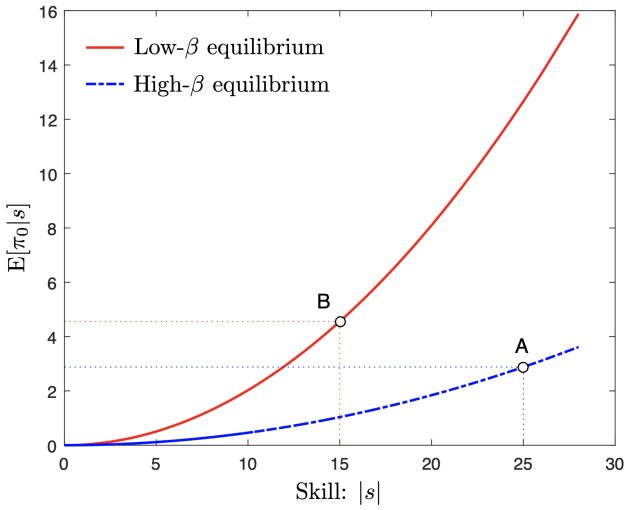


Figure 5: Skill versus Expected Trading Profit

Note: The figure plots the expected trading profit $\mathrm{E}[\pi_0|s]$ at the low-β equilibrium (solid line) and the high-β equilibrium (dashed line) for different levels of skill, $|s|$. The parameter values used in the figure are $\omega_s = 50$, $\omega_\delta = 1$, $\omega_u = 0.1$, and $\theta = \frac{\varphi\beta_1}{2} \approx 0.33$ ($\varphi = 0.6$).

of skill, suggesting that a low-skilled trader in the low-β equilibrium (point B) outperforms a high-skill trader in the high-β equilibrium (point A).

The empirical literature (e.g., Jensen, 1967; Busse, Goyal, and Wahal, 2010) finds that asset managers do not outperform passive benchmarks on average and, in cross-section, those who outperform cannot do so consistently. While the literature attributes the lack of persistent performance to the lack of skill, our result proposes an alternative explanation. Several studies reconcile the existence of skill with the lack of persistent performance. Berk and Green (2004), for example, argue that skill-based alphas eventually disappear due to competition between investors and decreasing returns to scale at the fund level.[19] Our model contributes to the understanding of this matter by showing that the lack of interplay between trader performance and skill could be the result of self-fulfilling multiple equilibria.

[19] Pastor, Stambaugh, and Taylor (2015) document increases in skill in finance and attribute the lack of corresponding increases in performance to decreasing returns to scale at the industry level.

5 Skill Acquisition

This section analyzes endogenous skill acquisition. We find that equilibrium multiplicity at the trading stage leads to middling skill investments and reveals the potential for multiplicity in the optimal skill levels.

5.1 Setup

Before the trading stage in $t = 0$, each trader makes a costly investment to improve her skill potential. This investment could include pursuing an advanced degree in finance, studying for professional exams, or engaging in specialized training to expand her expertise. The trader selects her skill potential, denoted as ω_s^*, by paying the skill-acquisition cost, $C(\omega_s^*) = c\omega_s^*$ with a positive marginal cost $c > 0$.[20] We assume that ω_s^* is not observable to the market makers and the recruiter. Instead, they form a belief about it, denoted by ω_s, and the equilibrium in the trading stage is determined by ω_s following Proposition 4.2. On the equilibrium path, the belief must be consistent with the actual skill potential, $\omega_s^* = \omega_s$. However, at the skill-acquisition stage, the trader cannot influence ω_s, as she does not have a commitment device.[21]

Expected utility of a trader. Given the realized equilibrium in the first trading stage (i.e., β_0), the *ex-ante* expected utility of a trader is characterized by the following lemma:

Lemma 5.1. *For each β_0, the expected utility of a trader at the skill-acquisition stage is given by*

$$V(\omega_s^*, \omega_s, \beta_0) = v(\omega_s, \beta_0)\omega_s^* + \bar{v}, \tag{5.1}$$

[20] The following analysis does not hinge on the functional form of the cost, as long as it is weakly increasing in ω_s^*.

[21] This setting follows the literature on private information acquisition with an unobservable quality (precision) of the private signal. See, for example, Xiong and Yang (2023).

where $\bar{v}$ *summarizes constant terms unrelated to* ω_s^*,[22] *and* $v(\omega_s, \beta_0)$ *is*

$$v(\omega_s, \beta_0) \equiv (1 - \lambda_0\beta_0)\beta_0 + (1 - \psi)\frac{1}{2}\sqrt{\frac{\omega_{u,1}}{\omega_s}} + \theta\xi^2\left[\gamma^2\beta_0^2 + 2\gamma(1-\gamma)\beta_0 + (1-\gamma)^2\right]. \tag{5.2}$$

Equation (5.1) indicates that, for each unit of the selected skill potential (ω_s^*), the trader earns the marginal utility of $v(\omega_s, \beta_0)$. The marginal utility in equation (5.2), in turn, consists of three factors. The first two terms represent the maximized trading profit in $t = 0$ and that in $t = 1$ when she does not meet the recruiter. In the first trading round, the equilibrium profit margin of the skill potential is $(1 - \lambda_0\beta_0)\beta_0$, while the profit margin in the second trading round is identical to that in the standard Kyle (1985) model.[23] The third term is the reaction of the reputation benefit to an increase in the selected skill potential, which emanates from the reputation concerns, θ, and the magnitude of the recruiter's belief updating (see equations [3.7] and [4.3]).

Importantly, $v(\omega_s, \beta_0)$ is characterized by the market's belief about the trader's skill potential (ω_s), as ω_s^* is unobservable to the market. The trader cannot influence ω_s by controlling ω_s^* due to limited commitment. Consequently, the optimality condition for the skill acquisition is characterized by ω_s, as the following discussion elaborates.

Sunspot shock. To formalize the skill-acquisitiom problem based on Lemma 5.1, we introduce an equilibrium-selection device. In particular, following the most common approach, we assume that market participants condition their actions on the realization of an exogenous *sunspot* shock, characterized by the binary random variable $z \in \{0, 1\}$.[24] Namely, when $\omega_s \in (\underline{\omega}_s, \bar{\omega}_s)$, they

[22] The formal expression is given by $\bar{v} = \frac{\psi}{4\lambda_1}\left[\xi^2\gamma^2\omega_{u,0} + \xi^2(1-\gamma)^2\omega_\delta + \omega_{u,0}\frac{\gamma\xi}{\beta_0}\right]$

[23] In the standard Kyle (1985) model, the optimal trading intensity is $\beta_0 = \frac{1}{2\lambda_0}$, so that the equilibrium profit margin of the information advantage (i.e., the skill in our model) is directly captured by $\frac{\beta_0}{2}$. Our model does not have this feature, as the trading intensity is set by incorporating both the price impact and the reputation concern.

[24] The sunspot equilibrium is proposed by Diamond and Dybvig (1983) as an equilibrium selection device in the context of bunk runs and formalized in the equilibrium analyses by

play the high-β equilibrium if a spot appears on the sun ($z = 1$), whereas the low-β equilibrium is realized if no spots are observed ($z = 0$). Accordingly, the high-β equilibrium is realized with probability $\Pr(z = 1) = \rho$, where ρ denotes the exogenous probability of a sunspot. We assume that z is realized at the beginning of the trading stage in $t = 0$, meaning that a trader acquires her skill potential before z is realized. In what follows, the high-β and low-β equilibria are denoted as $\beta_0 = \beta_H$ and β_L, respectively.

By incorporating the equilibrium multiplicity stipulated in Proposition 4.2, the trader's *ex-ante* expected utility from acquiring ω_s^* in equation (5.1) is rewritten as follows:[25]

$$\mathrm{E}_\beta[v(\omega_s, \beta_0)]\omega_s^* = \begin{cases} v(\omega_s, \beta_L)\omega_s^* & \text{if } \omega_s \geq \bar{\omega}_s, \\ \left[\rho v(\omega_s, \beta_H) + (1-\rho)v(\omega_s, \beta_L)\right]\omega_s^* & \text{if } \omega_s \in (\underline{\omega}_s, \bar{\omega}_s), \\ v(\omega_s, \beta_H)\omega_s^* & \text{if } \omega_s \leq \underline{\omega}_s. \end{cases} \tag{5.3}$$

E_β denotes the expectation over $\beta_0 = \beta_H$ and β_L in cases of multiple equilibria and incorporates the sunspot shock with $\rho = \Pr(z = 1) = \Pr(\beta_0 = \beta_H)$.

5.2 Equilibrium

Prior to the trading game, each trader solves the following problem by controlling her skill potential:

$$\max_{\omega_s^*} \mathrm{E}_\beta[v(\omega_s, \beta_0)]\omega_s^* - C(\omega_s^*), \tag{5.4}$$

where $\mathrm{E}_\beta[v(\omega_s, \beta_0)]\omega_s^*$ is given by equation (5.3). As the trader cannot control ω_s, she takes the marginal benefit in (5.3) as given.

In the equilibrium, the marginal benefit balances the marginal cost of skill acquisition, as equation (5.5) below illustrates. Otherwise, the trader has an incentive to increase or decrease ω_s^*. Together with the belief-consistency

the subsequent studies, such as Cooper and Ross (1998).

[25] We omit the constant term, $\bar{v}$, in the following discussion, as it does not affect the optimal skill potential.

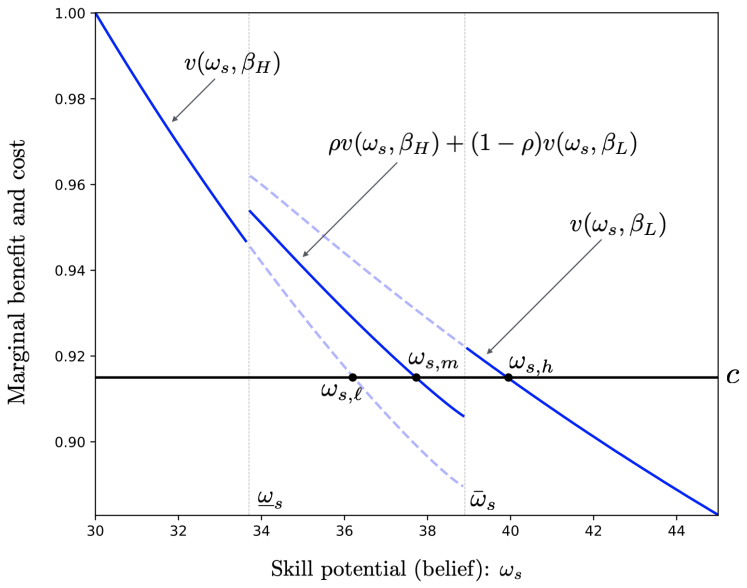


Figure 6: Marginal Benefit and Cost of Skill Acquisition

Note: The figure plots the marginal benefit of skill acquisition, given by $\mathrm{E}_\beta[v(\omega_s, \beta_0)]$, and its marginal cost c against the belief about the skill potential, ω_s. The parameter values are $\omega_\delta = 1$, $\omega_{u,0} = 0.1$, $\omega_{u,1} = 60$, $\theta = \frac{\varphi\beta_1}{2} \approx 0.33$ ($\varphi = 0.6$), and $\rho = 0.5$.

condition, the optimal skill potential ω_s^* is derived as solutions to the following optimality condition with respect to ω_s:

$$\mathrm{E}_\beta[v(\omega_s, \beta_0)] = c. \tag{5.5}$$

Figure 6 plots the marginal benefit (the LHS of [5.5]) in relation to the market's belief about the skill potential, ω_s. Although it is orthogonal to the actual skill potential, ω_s and ω_s^* on the equilibrium path are determined by the intersections of the marginal benefit and the marginal cost curves in Figure 6.

The marginal benefit of skill acquisition globally diminishes as ω_s increases. This result is not obvious because there are two channels through which ω_s affects its marginal benefit. In the first channel, when the market maker believes that the trader has acquired a high ω_s, the price impact grows, and the trading intensity diminishes in both trading rounds at $t = 0$ and $t = 1$, thereby

reducing the benefit of increasing ω_s^*. Conversely, in the second channel, when ω_s is high, the recruiter faces higher prior uncertainty in estimating the realized skill ($|s|$), and the wage becomes more reliant on the price information (p_0) and the asset payoff in $t = 0$ (δ_0). Hence, the wage becomes more sensitive to the trader's skill, making it more valuable for the trader to acquire a higher skill potential ω_s^*. Although the second channel encourages the trader to increase ω_s^*, the negative effect in the first channel is dominant, forming the downward-sloping marginal benefit curve in Figure 6.

Notably, the marginal benefit curve in the skill-acquisition stage has discontinuities due to the multiple equilibria in the trading stage. When the (belief of) skill potential is sufficiently high or low ($\omega_s \geq \bar{\omega}_s$ or $\omega_s \leq \underline{\omega}_s$), β_0 in the trading stage is uniquely determined. However, when the skill potential turns to intermediate levels, the trader starts incorporating the possibility of multiple equilibria in the trading stage, generating jumps in the marginal benefit of skill acquisition. By focusing on the intermediate levels of $\omega_s \in (\underline{\omega}_s, \bar{\omega}_s)$, we obtain the following result.

Lemma 5.2. *With ω_s being fixed, the marginal benefit of increasing the skill potential is smaller in the high-β equilibrium than in the low-β equilibrium. Therefore, the following inequalities hold, and the marginal benefit curve has discontinuities at $\omega_s = \underline{\omega}_s$ and $\omega_s = \bar{\omega}_s$:*

$$v(\omega_s, \beta_H) < \mathrm{E}_\beta[v(\omega_s, \beta_0)] < v(\omega_s, \beta_L). \tag{5.6}$$

Intuitively, the trader in the high-β equilibrium excessively reveals information to the market maker by trading aggressively. Although the recruiter estimates the trader's skill more precisely and increases the wage's sensitivity to the skill potential, it is dominated by the negative impact of a reduction in the trading profit margin. Therefore, the marginal benefit of increasing the skill potential becomes smaller in the high-β equilibrium than in the low-β equilibrium, leading to inequalities (5.6). In what follows, we elaborate on the discontinuities and derive their implications for skill acquisition.

5.2.1 Middling Skill Acquisition

We first propose "middling" skill acquisition due to the multiple trading-stage equilibria by focusing on $\omega_s \in (\underline{\omega}_s, \bar{\omega}_s)$. We refer to it as middling because the acquired skill level is either higher or lower than what she would acquire if she knew which type of equilibrium would be realized. On the equilibrium path, both the selected skill potential (ω_s^*) and the market's belief about it (ω_s) fall in this region if the marginal skill-acquisition cost is intermediate. In this region, the trading stage involves multiple equilibria (i.e., $\beta_0 = \beta_H$ or β_L), whereas the trader at the skill-acquisition stage cannot influence the equilibrium selection. Consequently, she determines her skill potential by anticipating the sunspot shock, which leads to the skill acquisition as follows.

Proposition 5.1 (Middling skill acquisition)**.** *The equilibrium skill potential in the presence of multiple trading-stage equilibria is*

(i) lower than the skill potential that she would acquire in the low-β equilibrium; and

(ii) higher than the skill potential that she would acquire in the high-β equilibrium.

Figure 6 illustrates a numerical example; $\omega_{s,m}$ is the ex-ante optimal skill potential, while the ex-post equilibrium is either $\omega_{s,\ell}$ or $\omega_{s,h}$, depending on the equilibrium realization in the trading stage.

The trader invests in skill potential incorporating the possibility of both high- and low-β equilibria. At the trading stage, however, the acquired skill potential turns out to be insufficient if the low-β equilibrium is realized, as its optimal play involves the profit-driven strategy and requires a relatively high asset-selection skill (i.e., $\omega_{s,h}$ is the *ex-post* equilibrium). Conversely, the acquired skill potential is too high if the high-β equilibrium is realized, as the trader takes the reputation-driven strategy that does not require a high skill potential (i.e., $\omega_{s,\ell}$ is the *ex-post* equilibrium).

This result indicates that individuals or firms may invest in skills that seem oddly intermediate or cautious—neither fully specialized nor overly basic. For

example, it explains why some traders and finance professionals choose to pursue broader business qualifications instead of more specialized academic paths, allowing them to adapt to a wide range of market situations. Traders also learn intermediate-level coding (e.g., Python) for algorithmic trading or data analysis but stop short of deep computer science expertise. A similar trend can be observed with respect to CFA Levels and quantitative skills among hedge fund managers. These skill acquisitions would be difficult to rationalize in an economy with a single equilibrium but become logical under the equilibrium multiplicity and uncertainty regarding equilibrium realization.

5.2.2 Fragile Skill Acquisition and Skill Diversity

By expanding our scope to the entire range of skill potential (ω_s), the discontinuities in Lemma 5.2 imply that the skill-acquisition problem in (5.4) may have multiple solutions.[26]

For example, when the marginal cost of skill acquisition is moderately small, as Figure 6 illustrates, both the high skill potential ($\omega_{s,h}$) and the intermediate skill potential ($\omega_{s,m}$) become optimal for the trader. Corresponding to $\omega_{s,h}$, the equilibrium in the trading stage is unique with $\beta_0 = \beta_L$, where the trader engages in solid profit-driven trading. Conversely, the trading stage with skill potential $\omega_{s,m}$ involves multiple equilibria, and the selected skill potential is middling, as Proposition 5.1 demonstrates. The symmetric argument holds when the marginal cost is relatively high: the intermediate skill potential and the low skill potential show up as multiple optima. This result can explain at least three realistic phenomena without relying on ad-hoc assumptions on the skill-acquisition environment.

Firstly, the coexistence of traders with different skill levels arises not from heterogeneity in skill-acquisition problems (e.g., different marginal costs), but from the multiple trading-stage equilibria and the possibility of mixed strategies in the skill acquisition stage. This reflects a realistic scenario where, due

[26] Although the equilibrium multiplicity in the trading stage (i.e., β_0) is determined by the level of ω_s, as Proposition 4.2 stipulates, it is orthogonal to the existence of multiple solutions to the optimal skill acquisition in equation (5.5).

to advancements in the internet and social media (e.g., online webinars via Zoom and academic courses uploaded on YouTube), opportunities for skill acquisition have become widely and uniformly accessible to all traders, reducing barriers to learning. Despite the increased homogeneity in skill-acquisition costs, we continue to observe heterogeneity in skill levels in the finance industry. Our model offers an explanation for this observation, showing how diverse skill levels can emerge among market participants, without the need to assume inherent differences in their skill acquisition costs.

Secondly, it also captures the possibility of widespread surges in skill acquisition without assuming special conditions for the skill-acquisition environment. For example, we can explain traders and fund managers in competitive environments (e.g., Wall Street) may collectively strive to acquire high-level skills, such as pursuing advanced degrees, driven by the mixed-strategy equilibrium. This surge in skill acquisition could occur even in the absence of significant changes in external factors, and without the need for heterogeneity in skill-acquisition costs, reflecting industry-wide trends in upskilling and degree inflation, as documented by Philippon and Reshef (2012) and Fuller and Raman (2017).

Finally, discontinuities in the marginal benefit curve add another layer of insight into the effect of the skill-acquisition cost on the equilibrium skill potential. Namely, discontinuities introduce the possibility of significant jumps in the optimal skill level triggered by a small change in the marginal skill-acquisition cost. For instance, when marginal technological advancements slightly reduce the marginal cost of learning, the result could be a sizable upward shift in skill acquisition. Conversely, even a small increase in the cost of learning might cause a large decline in the equilibrium skill level. This discontinuity-driven sensitivity can help explain the historical observations stylized by Philippon and Reshef (2012) that certain external changes lead to abrupt shifts in the distribution of skill levels in the finance industry.

6 Conclusion

This paper studies a model à la Kyle (1985) where traders are privately informed about their skill, i.e., the trader-specific ability to identify potentially mispriced assets. The traders care not only about trading profits but also the recruiter's perception of their skill. The model has two stable self-fulfilling equilibria, which we interpret as fragility in financial markets. One of the equilibria is similar to that of Kyle (1985) where reputation is irrelevant. In the other, "reputation-driven" equilibrium, the traders trade more aggressively for fear of being seen as unskilled by the recruiter, causing higher price volatility, higher market liquidity, and lower trading profits. Our model predicts that fragility may arise when poaching is (moderately) active in the finance industry and when there are high demands for financial skill. The model yields implications connecting traders' skills, their trading styles, and financial fragility. Moreover, consistent with empirical observations, our model predicts large pay inequality in the finance industry and reconciles skill in active management and the lack of performance persistence.

As a direction for future research, it would be interesting to analyze various types of wage structures in financial markets, ranging from back-end to front-end compensations. The implications of the forward-looking trading motives introduced in our model can be applied to investigate which types of wage structures and employment systems can mitigate financial market fragility while also improving the welfare of market participants.

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Online Appendix

A Proof of Lemma 3.1

From (3.1) and (3.2), p is observationally equivalent to $\frac{p-\hat{\delta}}{\lambda} = \beta s + u \equiv z_p$, which is the sum of two normally distributed variables from the market maker's perspective. Since $\delta \sim N(\hat{\delta} + s, \omega_\delta)$ is equivalently presented as $\delta = \hat{\delta} + s + \epsilon$ with $\epsilon \sim N(0, \omega_\delta)$, δ is observationally equivalent to $\delta - \hat{\delta} = s + \epsilon \equiv z_\delta$, which is also the sum of normal variables. Thus, given $Z = [z_p,\ z_\delta]^{\mathrm{T}}$, the market maker's posterior mean of s is

$$\mathrm{E}\left[s|Z\right] = \underbrace{\mathrm{E}[s]}_{=0} + \Sigma_{sZ}\Sigma_Z^{-1}(Z - \underbrace{E[Z]}_{=[0\ 0]^{\mathrm{T}}}), \tag{A.1}$$

where

$$\Sigma_{sZ} = \begin{bmatrix}\beta\omega_s, & \omega_s\end{bmatrix} \text{ and } \Sigma_Z = \begin{bmatrix}\beta^2\omega_s + \omega_u, & \beta\omega_s \\ \beta\omega_s, & \omega_s + \omega_\delta\end{bmatrix}. \tag{A.2}$$

Since

$$\begin{aligned}\det[\Sigma_Z] &= (\beta^2\omega_s + \omega_u)(\omega_s + \omega_\delta) - \beta^2\omega_s^2 \\ &= \omega_s(\beta^2\omega_\delta + \omega_u) + \omega_u\omega_\delta,\end{aligned} \tag{A.3}$$

it holds that

$$\begin{aligned}\mathrm{E}\left[s|Z\right] &= \begin{bmatrix}\beta\omega_s, & \omega_s\end{bmatrix}\frac{1}{\det[\Sigma_Z]}\begin{bmatrix}\omega_s + \omega_\delta, & -\beta\omega_s \\ -\beta\omega_s, & \beta^2\omega_s + \omega_u\end{bmatrix}\begin{bmatrix}z_p \\ z_\delta\end{bmatrix} \\ &= \frac{\omega_s}{\omega_s(\beta^2\omega_\delta + \omega_u) + \omega_u\omega_\delta}\left(\beta\omega_\delta z_p + \omega_u z_\delta\right) \\ &= \frac{\omega_s(\beta\omega_\delta + \omega_u)}{\omega_s(\beta^2\omega_\delta + \omega_u) + \omega_u\omega_\delta}\left(\frac{\beta\omega_\delta}{\beta\omega_\delta + \omega_u}z_p + \left(1 - \frac{\beta\omega_\delta}{\beta\omega_\delta + \omega_u}\right)z_\delta\right) \\ &= \xi\left(\gamma z_p + (1-\gamma)z_\delta\right),\end{aligned} \tag{A.4}$$

where

$$\xi \equiv \frac{\omega_s(\beta\omega_\delta + \omega_u)}{\omega_s(\beta^2\omega_\delta + \omega_u) + \omega_u\omega_\delta} \tag{A.5}$$

and

$$\gamma \equiv \frac{\beta\omega_\delta}{\beta\omega_\delta + \omega_u}. \tag{A.6}$$

Re-writing z_p and z_δ in (A.4) in terms of p and δ, we obtain the required result. □

B Proof of Lemma 3.2

Let us present $\delta \sim N(\hat{\delta}+s, \omega_\delta)$ equivalently as $\delta = \hat{\delta}+s+\epsilon$ with $\epsilon \sim N(0, \omega_\delta)$. Then, squaring (3.7), we have

$$\begin{aligned}\hat{s}^2 &= \xi^2 \left(\gamma^2 x^2 + 2\gamma x \left(\gamma u + (1-\gamma)(\delta - \hat{\delta})\right) + \left(\gamma u + (1-\gamma)(\delta - \hat{\delta})\right)^2\right) \\ &= \xi^2 \left(\begin{array}{c} \gamma^2 x^2 + 2\gamma^2 xu + 2\gamma(1-\gamma)(s+\epsilon)x \\ +\gamma^2 u^2 + 2\gamma(1-\gamma)(s+\epsilon)u + (1-\gamma)^2(s+\epsilon)^2 \end{array} \right).\end{aligned} \tag{B.1}$$

Taking the expectation of (B.1) from the trader's perspective,

$$\mathrm{E}[\hat{s}^2|s] = \xi^2 \left(\gamma^2 x^2 + 2\gamma(1-\gamma)sx + A\right), \tag{B.2}$$

where

$$A \equiv \gamma^2\omega_u + (1-\gamma)^2(s^2 + \omega_\delta) \tag{B.3}$$

is the term that the trader is unable to influence by her choice of x. We can omit A in the maximization problem below. Using (B.2), the trader's problem (2.2) is rewritten as

$$\begin{aligned}&\max_x \ \mathrm{E}\left[(\delta - p)x|s\right] + \theta\mathrm{E}[\hat{s}^2|s] \\ \Rightarrow \ &\max_x \ \mathrm{E}\left[\left(\hat{\delta} + s + \epsilon - \left(\hat{\delta} + \lambda(x+u)\right)\right)x \middle| s\right] + \theta\mathrm{E}[\hat{s}^2|s] \\ \Rightarrow \ &\max_x \ sx - \lambda x^2 + \theta\xi^2\left(\gamma^2 x^2 + 2\gamma(1-\gamma)sx\right). \quad \square\end{aligned} \tag{B.4}$$

C Proof of Lemma 4.1

At the beginning of $t = 1$, the trader does not have reputation concerns anymore. Her problem is just to choose x_1 to maximize the expected trading profit, that is,

$$\max_{x_1 \in (-\infty,\infty)} \mathrm{E}[(\delta_1 - p_1)x_1|s]. \tag{C.1}$$

We conjecture and later verify that the trader's equilibrium order in $t = 1$ is

$$x_1 = \beta_1 s, \tag{C.2}$$

where $\beta_1 > 0$ is a constant to be determined. Given (C.2), the market maker sets the price

$$\begin{aligned}
p_1 =& \mathrm{E}\left[\delta_1 | x_1 + u_1\right] \\
=& \mathrm{E}[\mathrm{E}[\delta_1 | x_1 + u_1, s] | x_1 + u_1] \\
=& \mathrm{E}[\hat{\delta}_1 + s | x_1 + u_1] \\
=& \hat{\delta}_1 + \frac{\mathrm{Cov}[s, x_1 + u_1]}{\mathrm{Var}[x_1 + u_1]} \left(x_1 + u_1 - \mathrm{E}[x_1 + u_1]\right) \\
=& \hat{\delta}_1 + \frac{\mathrm{Cov}[s, \beta_1 s + u_1]}{\mathrm{Var}[\beta_1 s + u_1]} \left(x_1 + u_1 - \mathrm{E}[\beta_1 s + u_1]\right) \\
=& \hat{\delta}_1 + \lambda_1 \left(x_1 + u_1\right), \qquad \text{(C.3)}
\end{aligned}$$

where

$$\lambda_1 \equiv \frac{\beta_1 \omega_s}{\beta_1^2 \omega_s + \omega_{u,1}}. \tag{C.4}$$

Given (C.3), the trader's problem (C.1) is rewritten as

$$\max_{x_1} \ \mathrm{E}\left[\left(\delta_1 - \left(\hat{\delta}_1 + \lambda_1(x_1 + u_1)\right)\right) x_1 \middle| s\right] \ \Rightarrow \ \max_{x_1} \ s x_1 - \lambda_1 x_1^2. \tag{C.5}$$

The FOC for x_1 is $s - 2\lambda_1 x_1 = 0$, which determines the trader's optimal action:

$$x_1 = \frac{1}{2\lambda_1} s. \tag{C.6}$$

For the other agents' belief (C.2) to be consistent with (C.6), we need

$$\beta_1 = \frac{1}{2\lambda_1} \iff \beta_1 = \sqrt{\frac{\omega_{u,1}}{\omega_s}}, \tag{C.7}$$

which implies

$$\lambda_1 = \frac{1}{2}\sqrt{\frac{\omega_s}{\omega_{u,1}}}. \quad \square \tag{C.8}$$

D Proof of Lemma 4.2

Let us present $\delta_1 \sim N(\hat{\delta}_1 + s, \omega_\delta)$ equivalently as $\delta_1 = \hat{\delta}_1 + s + \epsilon_1$ with $\epsilon_1 \sim N(0, \omega_\delta)$. Then the trader's trading profit in $t = 1$ is

$$\begin{aligned}
\pi_1 &= (\delta_1 - p_1)x_1 \\
&= \left(\hat{\delta}_1 + s + \epsilon_1 - \left(\hat{\delta}_1 + \lambda_1\left(\beta_1 s + u_1\right)\right)\right)\beta_1 s \\
&= \left(\hat{\delta}_1 + s + \epsilon_1 - \left(\hat{\delta}_1 + \frac{1}{2}\sqrt{\frac{\omega_s}{\omega_{u,1}}}\left(\sqrt{\frac{\omega_{u,1}}{\omega_s}}s + u_1\right)\right)\right)\sqrt{\frac{\omega_{u,1}}{\omega_s}}s \\
&= \frac{1}{2}\sqrt{\frac{\omega_{u,1}}{\omega_s}}s^2 + \left(\sqrt{\frac{\omega_{u,1}}{\omega_s}}\epsilon_1 - \frac{u_1}{2}\right)s.
\end{aligned} \tag{D.1}$$

Taking the expectation of (D.1) from the recruiter's perspective, we obtain the trader's salary:

$$\begin{aligned}
w &= \mathrm{E}[\pi_1 | p_0, \delta_0] \\
&= \frac{1}{2}\sqrt{\frac{\omega_{u,1}}{\omega_s}}\mathrm{E}[s^2 | p_0, \delta_0] \\
&= \frac{1}{2}\sqrt{\frac{\omega_{u,1}}{\omega_s}}\left(\mathrm{E}[s|p_0,\delta_0]^2 + \mathrm{Var}[s|p_0,\delta_0]\right) \\
&= \frac{1}{2}\sqrt{\frac{\omega_{u,1}}{\omega_s}}\left(\hat{s}^2 + \hat{\omega}_s\right),
\end{aligned} \tag{D.2}$$

where $\hat{\omega}_s$ is the posterior variance of s, which is a constant. $\square$

E Proof of Propositions 3.1 and 4.2

In the following analyses, denote

$$a = \frac{\omega_{u,0}}{\omega_s}, \ b = \frac{\omega_{u,0}}{\omega_\delta}, \tag{E.1}$$

so that we rewrite equation (3.11) as

$$\beta = F(\beta) \equiv \frac{1 + 2\theta \left(\frac{1}{a+b+\beta^2}\right)^2 b\beta}{2\left(\frac{\beta}{\beta^2+a} - \theta\left(\frac{\beta}{a+b+\beta^2}\right)^2\right)}. \tag{E.2}$$

By rearranging (E.2), the equilibrium β solves the following condition.

$$2\theta\beta = J(\beta^2), \tag{E.3}$$

where, by denoting $\beta^2 = y$,

$$J(y) = \frac{(y-a)(y+b+a)^2}{(y+a)(y+b)}. \tag{E.4}$$

The first-order derivative of the RHS of (E.3) with respect to β is

$$K^{(1)}(\beta) \equiv 2\beta J^{(1)}(\beta^2), \tag{E.5}$$

with

$$J^{(1)}(y) \equiv \frac{N(y)}{(y+a)^2(y+b)^2}, \tag{E.6}$$

where the numerator is

$$N(y) = y^4 + 2(a+b)y^3 + n_2 y^2 + n_1 y + n_0$$

with coefficients being

$$n_2 = 2a^2 + 6ab + b^2,$$

$$n_1 = 2a\left[3b^2 + 3ab + a^2\right],$$

$$n_0 = a\left(a^2 + ab + 2b^2\right)(a+b).$$

From (E.4) and (E.6), $y = a$ is the unique solution to $J(y) = 0$ in $y > 0$, and $J(y)$ is increasing in y for $y \geq a$. Hence, there is no equilibrium in $y < a$, and we focus on $y \geq a$.

The second-order derivative of the RHS of (E.3) with respect to β is expressed as

$$K^{(2)}(y) \equiv 2\left(J^{(1)}(y) + 2yJ^{(2)}(y)\right), \tag{E.7}$$

where $J^{(2)}$ denotes the second-order derivative of J with respect to y. $K^{(1)}$ and $K^{(2)}$ have the following properties:

Lemma E.1. *(i) $K^{(1)}(\beta)$ takes a U-shaped curve in relation to β.*
(ii) $K^{(2)}(a) < 0$ and $K^{(2)}(\infty) > 0$. There exists a unique solution to $K^{(2)}(y) = 0$.

Proof. Note that point (i) directly follows from point (ii). By denoting that $N^{(1)}(y) = \frac{dN(y)}{dy}$, it holds that

$$J^{(1)}(y)+2yJ^{(2)}(y) = J^{(1)}(y)\frac{-7y^2 - 3(a+b)y + ab}{(y+a)(y+b)}+2y\frac{\left(y^2 + (a+b)y + ab\right)N^{(1)}(y)}{(y+a)^3(y+b)^3}. \tag{E.8}$$

Evaluating at $y = a$,

$$\begin{aligned} J^{(1)}(a) + 2aJ^{(2)}(a) &= -J^{(1)}(a)\frac{5a+b}{a+b} + \frac{N^{(1)}(y)}{2a(a+b)^2} \\ &\propto -2a(a+b)(5a+b)J^{(1)}(a) + N^{(1)}(a) \\ &= -4a^3 - b(4+b)a - b^3 \\ &< 0. \end{aligned}$$

Therefore, $K^{(2)}(a) < 0$ holds.

The numerator of (E.8) is summarized as

$$\begin{aligned} T(y) = &- \left[y^4 + 2(a+b)y^3 + n_2y^2 + n_1y + n_0\right]\left[7y^2 + 3(a+b)y - ab\right] \\ &+ 2y\left(y^2 + (a+b)y + ab\right)\left[4y^3 + 6(a+b)y^2 + 2n_2y + n_1\right]. \end{aligned}$$

The above argument suggests that $T(a) < 0$.

Using Mathematica, we summarize T into the following polynomial:

$$T(y) = \sum_{k=0}^{6} t_k y^k, \tag{E.9}$$

where the coefficients are

$$t_0 = a^2b(a+b)\left(a^2 + ab + 2b^2\right) > 0,$$

$$t_1 = -3a\left(a^4 + (a+b)(a-b)^2b + b^4\right) < 0,$$

$$t_2 = -3a\left(3a^3 + 4a^2b + ab^2 + 5b^3\right) < 0,$$

$$t_3 = -8a^3 - 8a^2b - 9ab^2 + b^3,$$

$$t_4 = 3b\,(a+b) > 0,$$

$$t_5 = 3(a+b) > 0,$$

and $t_6 = 1$. As $t_6 > 0$, $T(\infty) = \infty > 0$. Together with $T(a) < 0$, the intermediate value theorem implies that $T(y) = 0$ has at least one solution in $y > a$.

Suppose that y_{min} is one of the solutions to $T(y) = 0$. Consider the first-order derivative of T evaluated at $y = y_{min}$ and multiplied y_{min}:

$$\mathcal{T}(y_{min}) = y_{min}\frac{dT(y_{min})}{dy} = \sum_{k=1}^{6} k t_k y_{min}^k.$$

Applying $T(y_{min}) = 0$ and eliminating the terms related to t_3,

$$\mathcal{T}(y_{min}) = -3t_0 - 2t_1y_{min} - t_2y_{min}^2 + t_4y_{min}^4 + 2t_5y_{min}^5 + 3t_6y_{min}^6,$$

According to the signs of t_k and $y_{min} > a$, it holds that

$$\begin{aligned}\mathcal{T}(y_{min}) &> -3t_0 - 2t_1 a - t_2 a^2 + t_4 a^4 + 2t_5 a^5 + 3t_6 a^6 \\ &= 3a^2\left(6a^4 + 8a^3 b + b^4 + (a^2 - b^2)^2\right) \\ &> 0.\end{aligned}$$

Therefore, if y_{min} is a solution to $T(y) = 0$, then the slope of $T(y)$ at this point must be positive. Together with $T(a) < 0 < T(\infty)$, y_{min} is the unique solution to $T(y) = 0$, and $T(y) > 0 \Leftrightarrow y > y_{min}$.[27] The argument above implies that $K^{(1)}(\beta)$ takes a U-shaped curve with respect to β, and the minimum point is given by $\beta_{min} \equiv \sqrt{y_{min}}$. Note that $\tilde{\beta}$ in footnote 11 is given by $\tilde{\beta} = \beta_{min}$. □

The RHS of (E.3) is monotonically increasing in β, and Lemma E.1 attests that it is concave in $y \leq y_{min}$, while it becomes convex in $y > y_{min}$. In other words, the slope of the curve is the least steep at $y = y_{min}$, and it is given by

$$K_{min} = 2\beta_{min} J^{(1)}(y_{min}). \tag{E.10}$$

Note that K_{min} is represented only by parameters, a, b, and does not involve θ.

Consider the following condition to characterize equilibrium:

$$J(y_{min}) > K_{min}\beta_{min}. \tag{E.11}$$

Then the following result holds:

Lemma E.2. *(i) If inequality (E.11) holds, then there exist θ_H and θ_L with $\theta_H > \theta_L$. If, and only if, $\theta \in \Theta \equiv (\theta_L, \theta_H)$, then the equilibrium condition (E.3) has three solutions.*
(ii) If inequality (E.11) is violated, then $\theta_H = \theta_L$, and the equilibrium condition (E.3) has a unique solution. The solution is $\beta > \beta_{min}$ ($\beta < \beta_{min}$) if $\theta > \theta_H$

[27] Suppose that $T(y) = 0$ has more than two solutions. For any solutions, denoted as y_0 and y_1, the mean-value theorem implies that there exists at least one $y_2 \in [y_0, y_1]$ such that $\frac{dT(y_2)}{dy} = 0$. This contradicts to the strict positivity or negativity of $\mathcal{T}$.

($\theta < \theta_H$).

Proof. Consider the plot of the curve, $J(\beta^2)$, and the line, $2\theta\beta$, against β. With the curve being fixed, consider a line that goes through the origin and is tangent to the curve. The tangent point is characterized by β that satisfies

$$J(\beta^2) = K^{(1)}(\beta^2)\beta,$$

where the LHS represents the y-coordinate of the curve at β, and the RHS represents that of the line that goes through the origin. Note that their slopes must be identical and $K^{(1)}(\beta^2)$. Applying (E.5), the above condition is rewritten as

$$J(\beta^2) = 2\beta^2 J^{(1)}(\beta^2).$$

Hence, denoting $y = \beta^2$, the tangent point is characterized by the solution to

$$0 = G(y) = 2yJ^{(1)}(y) - J(y).$$

Note that we focus on $y > \sqrt{a}$, and thus y and β have one-to-one mapping.

Function $G(y)$ has the following first-order derivative:

$$\begin{aligned} G^{(1)}(y) &= 2\left(yJ^{(2)}(y) + J^{(1)}(y)\right) - J^{(1)}(y) \\ &= J^{(1)}(y) + 2yJ^{(2)}(y) \\ &= \frac{K^{(2)}(y)}{2}, \end{aligned}$$

where we apply the definition of $K^{(2)}$ in (E.7). Lemma E.1 has established that $K^{(2)}(a) < 0$ and $K^{(2)}(\infty) > 0$. Also, there is a unique $y = y_{min}$ such that $K^{(2)}(y_{min}) = 0$ and $K^{(2)}(y_{min}) > 0 \Leftrightarrow y > y_{min}$. Hence, $G(y)$ takes a U-shaped curve with respect to y and $y = y_{min}$ is the minimizer of the curve.

With $G(y)$ being U-shaped, $G(y) = 0$ has two solutions if and only if its minimum point dips below the x-axis, namely,

$$2y_{min}J^{(1)}(y_{min}) - J(y_{min}) < 0.$$

Applying the definition of K_{min} in (E.10), the above inequality is rewritten as condition (E.11).

When condition (E.11) holds and $G(y) = 0$ has two solutions, we denote them as y_0 and $y_1 (> y_0)$. The corresponding slopes of these tangent lines are $K^{(1)}(y_0)$ and $K^{(1)}(y_1)$, respectively. Then the original line, $2\theta\beta$, and the curve, $J(\beta^2)$, have three intersections if and only if

$$\frac{K^{(1)}(y_1)}{2} \equiv \theta_L < \theta < \theta_H \equiv \frac{K^{(1)}(y_0)}{2}. \tag{E.12}$$

This result concludes the proof of statement (i) of the lemma.

Conversely, if condition (E.11) does not hold, no tangent line that goes through the origin is obtained: function $J(\beta^2)$ becomes the least steep at $\beta_{min} = \sqrt{y_{min}}$, and the tangent line is specified by $K_{min}\beta - K_{min}\beta_{min} + J(\beta^2_{min})$, while the intercept of this line is negative, as condition (E.11) is violated. Hence, the line, $2\theta\beta$, and the curve, $J(\beta^2)$ have a unique intersection. This intersection occurs in $\beta > \beta_{min}$ if, and only if, $\theta > \theta'_H \equiv \frac{J(\beta^2_{min})}{2\beta_{min}}$.

To conclude statement (ii), we must show that θ_H and θ_L in statement (i) converges to θ'_H in statement (ii) at the threshold of these two cases so that we re-define it as $\theta_H = \theta_L = \theta'_H$. This result is shown in Lemma E.3 below. □

Finally, consider the comparative statics with respect to ω_s. The following result holds:

Lemma E.3. *(i) $J(y_{min}) - K_{min}\beta_{min}$ is increasing in ω_s. Hence, there exists a unique $\tilde{\omega}_s > 0$, such that, (E.11) holds with equality, and inequality (E.11) holds if and only if $\omega_s > \tilde{\omega}_s$.*
(ii) When condition (E.11) holds, both θ_H and θ_L increase as ω_s increases. Interval Θ expands, as the following inequalities hold:

$$0 < \frac{d\theta_L}{d\omega_s} < \frac{d\theta_H}{d\omega_s}.$$

(iii) At $\omega_s = \tilde{\omega}_s$, it holds that $\theta_H = \theta_L = \theta'_H$, while at the limit of $\omega_s \to 0$, $\theta'_H \to 0$.

(iv) There exists a unique threshold for φ^*, such that, $\varphi \geq \varphi^* (< \varphi^*)$ leads to case 1 (case 2) in Proposition 4.2.

Proof of Statements (i) and (ii). Note that we are interested in the reaction of the following function when $a = \frac{\omega_{u,0}}{\omega_s}$ changes and, hereafter, we make the dependence of functions on a explicit:

$$2\theta_k = K^{(1)}(y_j, a) = 2\sqrt{y_j} J^{(1)}(y_j, a),$$

where $y_j = y_0, y_1$ denote the solutions to $G(y, a) = 0$, if any, and $k = L, H$ are appropriately determined following (E.12). According to the definition of G, the following partial derivative holds.

$$\frac{\partial G(y,a)}{\partial a} = 2y \frac{\partial J^{(1)}(y,a)}{\partial a} - \frac{\partial J(y,a)}{\partial a}.$$

The first term is summarized by

$$\begin{aligned}
\frac{\partial J^{(1)}(y,a)}{\partial a} &\propto (y+a)\frac{\partial N(y,a)}{\partial a} - 2N(y,a) \\
&= (y+a)\left[2y^3 + n_{2,a}y^2 + n_{1,a}y + n_{0,a}\right] - \left[y^4 + 2(a+b)y^3 + n_2 y^2 + n_1 y + n_0\right] \\
&= \sum_{k=0}^{4} A_k y^k
\end{aligned}$$

where $A_4 = 1$, $A_3 = 4(a+b)$,

$$A_2 = 2a^2 + 5b^2 + 2(3ab + a^2) + 2a\left[3b + 2a\right],$$

$$A_1 = 2a^2\left[3b + 2a\right]\left(a^2 + ab + 2b^2\right)(2a + b) + a\,(2a + b)\,(a + b),$$

$$\begin{aligned}
A_0 &= a\left[(2a+b)\left(a^2 + ab + 2b^2 + a(a+b)\right) - \left(a^2 + ab + 2b^2\right)(a+b)\right] \\
&> \left(a(a+b)\right)^2.
\end{aligned}$$

Hence, $\frac{\partial J^{(1)}(y,a)}{\partial a} > 0$. The second term is

$$(y+b)\frac{\partial J(y,a)}{\partial a} = -2\frac{(y+b+a)\left(y(a+b)+a^2\right)}{(y+a)^2} < 0. \tag{E.13}$$

Therefore, $\frac{\partial G}{\partial a} > 0$. As the previous proof has established that $G(y,a)$ takes a U-shaped curve, $\frac{\partial G}{\partial a} > 0$ implies that its minimum point is monotonically decreasing in ω_s, concluding the first statement of point (i) in the lemma.

As $a \to 0$, it holds that $y_{min} \to 0$ and

$$\lim_{a\to 0} G(y_{min}, a) = \lim_{a\to 0}[2y_{min} - (y_{min}+b)] = -b < 0.$$

Also

$$\lim_{a\to\infty} G(y_{min}, a) = +\infty.$$

Therefore, together with

$$\frac{dG(y_{min},a)}{da} = \frac{dy_{min}}{da}\frac{\partial G}{\partial y} + \frac{\partial G}{\partial a} > 0,$$

the above limits imply the existence of $\bar{a}$, such that, condition (E.11) holds if and only if $a < \bar{a}$. Given $\omega_{u,0}$, this result can be translated into ω_s so that $\tilde{\omega}_s = \frac{\omega_{u,0}}{\bar{a}}$. Noting that $a < \bar{a} \Leftrightarrow \omega_s > \tilde{\omega}_s$, we conclude the second half of point (i) in the lemma.

Suppose that condition (E.11) holds, meaning that y_0 and y_1 exist. For $j \in 0, 1$, $\frac{\partial G}{\partial a} > 0$ implies that $\frac{dy_0}{da} > 0$ and $\frac{dy_1}{da} < 0$. Also, the implicit function theorem leads to

$$\frac{dy_j}{da} = -\frac{1}{G^{(1)}(y_j,a)}\frac{\partial G(y_j,a)}{\partial a}.$$

By taking the total derivative of θ_k,

$$\begin{aligned}\frac{d\theta_k}{da} &= \frac{1}{2}\frac{1}{\sqrt{y_j}}\frac{dy_j}{da}G^{(1)}(y_j,a) + \sqrt{y_j}\frac{\partial J^{(1)}(y_j,a)}{\partial a} \\ &= -\frac{1}{2}\frac{1}{\sqrt{y_j}}\left(2y_j\frac{\partial J^{(1)}(y_j,a)}{\partial a} - \frac{\partial J(y_j,a)}{\partial a}\right) + \sqrt{y_j}\frac{\partial J^{(1)}(y_j,a)}{\partial a} \\ &= \frac{1}{2}\frac{1}{\sqrt{y_j}}\frac{\partial J(y_j,a)}{\partial a} \\ &< 0,\end{aligned} \tag{E.14}$$

where the last inequality follows from (E.13). Therefore, we conclude that the boundary slopes, θ_H and θ_L are both increasing in ω_s.

Consider the magnitude of the reactions. By plugging (E.13) into (E.14),

$$\frac{d\theta_k}{da}| \propto \frac{\sqrt{y_j}}{(y_j+a)^2}\left(1+\frac{a}{y_j+b}\right)\left(1+\frac{1}{y_j}\frac{a^2}{a+b}\right).$$

It is straightforward that the last terms are all decreasing in y_j. Since we compare $y_0 < y_1$ with a being fixed, we obtain $\frac{d\theta_H}{d\omega_s} > \frac{d\theta_L}{d\omega_s}$, thereby concluding the proof.

Proof of Statements (iii) and (iv). Note that $\lim_{\omega_s\to\tilde{\omega}_s} y_j = y_{min}$, and $J(y_{min}) = K_{min}\beta_{min} = 2\beta_{min}J^{(1)}(y_{min})$ holds. Therefore, at this limit,

$$\theta'_H = \frac{J(\beta^2_{min})}{2\beta_{min}} = \beta_{min}J^{(1)}(\beta^2_{min}) = \frac{K_{min}}{2} = \theta_k,$$

where the last equality holds because of (E.5). This concludes the proof of statement (iii).

Finally, consider the limit at $\omega_s \to 0$, which implies $a \to \infty$ (given $\omega_{u,0}$). Since $y_{min} \searrow a$, we can denote it as $y_{min} = a + \epsilon$, where $\epsilon > 0$ is a small positive and $\lim_{\omega_s\to\tilde{\omega}_s}\epsilon = 0$. With this notation, function J is rewritten at the limit as

$$\lim_{\omega_s\to\tilde{\omega}_s} J(y_{min},a) = \lim_{a\to\infty,\epsilon\to 0}\frac{\epsilon(2a+b+\epsilon)}{(2a+\epsilon)(a+b+\epsilon)} = 0.$$

Since $\theta'_H = \frac{J(y_{min})}{2\sqrt{y_{min}}}$, we conclude that $\lim_{\omega_s \to \tilde{\omega}_s} \theta'_H = 0$.

As for statement (iv), case 1 occurs if, and only if, the curve $\theta = \frac{\varphi}{2}\sqrt{\frac{\omega_{u,1}}{\omega_s}}$ exceeds θ_H at $\omega_s = \tilde{\omega}_s$. The above analyses have already established that $\theta_H = \frac{K^{(1)}(y_{min})}{2}$, where y_{min} is implicitly characterized by parameters a and b, we can define the threshold φ^* as

$$\varphi^* \equiv K^{(1)}(y_{min})\sqrt{\frac{\omega_{u,0}}{\bar{a}\omega_{u,1}}}. \tag{E.15}$$

□

Note that Proposition 3.1 follows from points (i) and (ii) of Lemma E.3, whereas Proposition 4.2 is shown by putting monotonically decreasing θ in equation (4.6) together with points (i) and (ii).

F Proof of Lemma 5.2 and Proposition 5.1

The optimal trading strategy satisfies the following condition.

$$\therefore (1 - 2\lambda_0\beta_0) + \frac{\psi}{2\lambda_1}\xi^2(\gamma^2\beta_0 + \gamma(1-\gamma)) = 0. \tag{F.1}$$

The marginal benefit of increasing s^* is given by

$$\mathrm{E}_\beta[v(\omega_s, \beta_0)] = \left[(1 - \lambda_0\beta_0) + \frac{\psi}{4\lambda_1}\xi^2(\gamma^2\beta_0 + 2\gamma(1-\gamma))\right]\beta_0 + \frac{\psi}{4\lambda_1}\xi^2(1-\gamma)^2 + (1-\psi)\frac{1}{2}\sqrt{\frac{\omega_{u,1}}{\omega_s}}. \tag{F.2}$$

Then, it holds that

$$\begin{aligned}\frac{\partial \mathrm{E}_\beta[v(\omega_s, \beta_0)]}{\partial \beta_0} &= \left[(1 - 2\lambda_0\beta_0) + \frac{\psi}{2\lambda_1}\xi^2(\gamma^2\beta_0 + \gamma(1-\gamma))\right] \\ &+ \left[-\beta_0\frac{\partial \lambda_0}{\partial \beta_0} + \frac{\psi}{4\lambda_1}\frac{\partial}{\partial \beta_0}\left[\xi^2(\gamma^2\beta_0 + 2\gamma(1-\gamma))\right]\right]\beta_0 + \frac{\psi}{4\lambda_1}\frac{\partial}{\partial \beta_0}\left(\xi^2(1-\gamma)^2\right).\end{aligned}$$

Note that the first term is zero due to the envelope theorem using (F.1). By using the notations introduced in equations (E.1), the remaining terms are

summarized as follows.

$$-\beta_0 \frac{\partial}{\partial \beta_0} \frac{\beta_0}{\beta_0^2 + a} + \frac{\psi}{4\lambda_1} \left[\beta_0^2 \frac{\partial}{\partial \beta_0} \left(\frac{\beta_0}{a + b + \beta_0^2} \right)^2 + 2\beta_0 \frac{\partial}{\partial \beta} \left(\frac{1}{a + b + \beta_0^2} \right)^2 b\beta_0 + \frac{\partial}{\partial \beta} \left(\frac{b}{a + b + \beta_0^2} \right)^2 \right]. \tag{F.3}$$

From the proof in Appendix E, it holds that $y > a$ in the equilibrium. Therefore, the first term is negative. As for the second term, it holds that

$$\begin{aligned} \text{Second terms of (F.3)} &= 2\beta_0^3 \left(\frac{a + b - \beta_0^2}{(a + b + \beta_0^2)^3} \right) + 2 \frac{a + b - 3\beta_0^2}{(a + b + \beta_0^2)^3} b\beta_0 - 4b^2 \frac{\beta_0}{(a + b + \beta_0^2)^3} \\ &\sim -y^2 + y(a + b - 3b) + b(a - b). \end{aligned}$$

Since $y > a$, the quadratic equation above is negative for all $y > a$ if and only if it is negative at $y = a$. This is true because

$$-a^2 + a(a - 2b) + b(a - b) = -ab - b^2 < 0.$$

Therefore, the marginal benefit in (F.2) is decreasing in β_0, concluding the proof of Lemma 5.2. Proposition 5.1 directly follows from Lemma 5.2 and equation (5.5).