Business Cycles with Pricing Cascades *

Mishel Ghassibe
CREi, UPF & BSE

Anton Nakov
European Central Bank

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Abstract

How do *large* aggregate and sector-specific shocks affect the macroeconomy? To answer this question, we develop a non-linear dynamic general equilibrium model featuring a disaggregated production economy with networks and *optimal* decisions on the timing and size of price adjustments. The interaction of our model ingredients creates equilibrium *cascades*: large movements in aggregates trigger additional price adjustment decisions at the extensive margin. Crucially, networks may dampen or amplify cascades, depending on the *type of shock* driving the business cycle. When faced with large *demand* shocks, such as monetary interventions, networks *dampen* cascades, thus slowing down price adjustment decisions and giving central banks substantial power to stimulate the real economy with limited inflationary consequences. In contrast, under aggregate or sector-specific *supply* shocks, networks *amplify* cascades, leading to fast increases in the frequency of repricing and large inflationary swings. Applied to the Euro Area data, we show that it is the novel interaction of networks with pricing cascades that allows us to quantitatively match the surges in inflation and the repricing frequency in the post-Covid era.

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1 Introduction

Large and granular shocks are the "new normal". Understanding their transmission to macroeconomic aggregates in a world with increasingly complex supply chains is challenging and requires a substantial expansion of our modelling toolkit. In fact, established frameworks used to analyze shock transmission to real and nominal variables almost exclusively focus on linearized single-sector setups with aggregate shocks only (Woodford, 2004; Galí, 2015). In this paper, we take a step towards developing a framework suitable for the "new normal". In particular, we build a dynamic general equilibrium model that combines three ingredients that have so far been treated separately in the literature. First, our model features a multi-sector setup, which allows for both aggregate, sector-specific, and firm-level shocks, as well as a fully general input-output architecture across firms in the economy. Second, firms are making pricing decisions in *optimal* state-dependent manner subject to menu costs, such that the *intensive* and the *extensive* margins of pricing matter for aggregate dynamics. Third, we solve the model fully non-linearly to study the dynamics of aggregates following arbitrarily large and granular shocks.

A rapidly growing literature studies the transmission of both real and monetary shocks in economies with production networks and nominal rigidities in price setting. However, the near-exclusive focus has been on *linearized* setups with *random* timing of price changes as in Calvo (1983). In such settings, the existence of networks is shown to slow down the transmission of demand shocks to prices and, hence, result in larger non-neutrality of money (Ghassibe, 2021; Rubbo, 2023). Much less is known about how *large shocks*, both demand- and supply-side, propagate through production networks, especially when the timing of price changes is chosen in *optimal* state-dependent fashion. This is particularly important, since the recent post-Covid episode featured large inflationary swings, which came hand in hand with substantial increases in the frequency of price adjustment, both in the US (Montag and Villar, 2022) and in the Euro Area (Cavallo et al., 2024), stressing the relevance of the *extensive* margin of pricing. Unexplored is also the role networks can play in the size- and sign-dependence of inflation and output dynamics in response to large shocks. The model we develop aims to address the above issues in a unified general equilibrium setup.

Our framework generates a novel interaction between large shocks, networks and pricing decisions at the extensive margin, leading to endogenous fluctuations in the frequency of price adjustment, which is also quantitatively important for the dynamics of macroeconomic aggregates. In particular, networks interact with pricing *cascades*: large movements in aggregate or sectoral variables trigger possibly self-reinforcing adjustment decisions at the extensive margin. Whether networks amplify or dampen such pricing cascades depends fundamentally on the type of shock that hits the economy. For demand shocks, such as central bank monetary interventions, production networks shrink the magnitudes of desired price changes at the firm level, which in turn compresses the sizes of price gaps for all firms. As a result, the presence of networks *dampens* pricing cascades,

lowering the response of the aggregate repricing frequency, *ceteris paribus*. Consequently, much larger monetary stimuli are feasible in equilibrium, as much larger shocks are required to reach the limit where all firms choose to adjust prices and the economy becomes money-neutral. Quantitatively, in a model estimated to match sectoral pricing moments in the Euro Area, under production networks a one-time monetary intervention can stimulate aggregate GDP by a maximum of 5%; meanwhile, removing input-output linkages shrinks the maximum possible stimulus to just over 2%.

In contrast, large aggregate and sector-specific *supply* shocks interact with the production network in a manner that is completely opposite to that under demand shocks. In particular, production networks amplify the firm-level desired price changes following supply shocks, hence expanding the price gaps. As a result, they *amplify* cascades in pricing, making the decision to adjust more likely, *ceteris paribus*. As a result, large negative TFP shocks can lead to very fast increases in the aggregate repricing frequency, leading to substantial inflationary spikes. Quantitatively, following an aggregate TFP shock of -10%, production networks double the equilibrium fraction of adjusting firms and more than triple the impact response of inflation (from 5% to 17% monthly). Moreover, large TFP shocks to sectors with a high degree of network centrality, corresponding to important suppliers to other producers in the economy, can also drive up the aggregate repricing frequency, and thus create large aggregate inflationary responses, evolving non-linearly in the size of the shock.

As a further quantification of the role played by the interaction of networks with pricing cascades, we subject our model to the key structural shock series experienced by the Euro Area economy in the (post-)Covid years (2020-2024), and compare the model-implied dynamics of aggregate inflation and repricing frequency to that observed in the data. In particular, we feed in four shock series, corresponding to aggregate nominal demand, aggregate labor wedge, as well as the dynamics of energy and food prices. We find that the model successfully matches the five percentage point increase in the aggregate repricing frequency, as well as the aggregate inflation surge up to 11% at the peak. In contrast, an otherwise identical model without networks generates at most a one percentage point increase in aggregate repricing frequency, as well as an aggregate inflation surge to only 5% at the peak. These results highlight the quantitative importance of our novel theoretical channel – the interaction of networks with pricing cascades – for explaining the aggregate business cycle dynamics.

Literature review Our paper contributes to at least two broad strands of the literature. First, we add to the the vast literature on state-dependent pricing in macroeconomics; see Costain and Nakov (2024) for a recent survey.[1] Under state-dependent pricing, the probability of a price change is affected by idiosyncratic and aggregate shocks, in contrast with time-dependent models such as Taylor (1979) or Calvo (1983). Our main contribution is to the literature on general equilibrium im-

[1]See also Nakamura and Steinsson (2013) and Klenow and Malin (2010) for earlier surveys.

plication of state-dependent pricing, marked by seminal works of Golosov and Lucas (2007), Gertler and Leahy (2008) and Midrigan (2011) in the context of a single-sector model with small aggregate shocks and fixed menu costs.[2] This framework has been further explored analytically by Alvarez and Lippi (2022) among others, whose results provide model-based sufficient statistics linking the dynamics of macro aggregates to moments that can be measured in firm-level data. Subsequent work also considers one-sector models with state-dependent pricing subjected to large aggregate shocks, such as the papers of Karadi and Reiff (2019), Cavallo et al. (2024) and Blanco et al. (2024). As for multi-sector models with state-dependent pricing, the seminal work by Nakamura and Steinsson (2010) studies the transmission of monetary shocks in a setup with heterogeneous pricing and roundabout production. More recent work by Carvalho and Kryvtsov (2021) studies a multi-sector framework with heterogeneous state-dependent pricing, but without networks, to rationalize aggregate price adjustment facts.

Relative to the aforementioned papers, we contribute by developing the first general equilibrium model with a fully unrestricted input-output structure that we solve fully non-linearly for any aggregate or sector-specific shocks, either demand- or supply-side. We also introduce a novel theoretical channel that come out of the *inetraction* of networks with pricing *cascades*.

Second, our paper is related to the growing literature on production networks and aggregate fluctuations. The seminal work by Acemoglu et al. (2012) considers a flexible-price setup and shows how production networks can amplify sector- or firm-specific shocks to create aggregate fluctuations. Subsequent work by Baqaee and Farhi (2020) considers aggregation properties in inefficient economies with networks. A separate strand of this literature studies linearized models with production networks and time-dependent pricing, both positively (Pasten et al., 2020; Ghassibe, 2021; Afrouzi and Bhattarai, 2023) and normatively (Rubbo, 2023; La'O and Tahbaz-Salehi, 2022).

We contribute to this literature by studying non-linear aggregate fluctuations in a setup with state-dependent pricing and arbitrarily large aggregate or sector-specific shocks.

Roadmap The remainder of the paper is structured as follows. Section 2 outlines the optimization problem faced by each type of agent in the economy and the numerical strategy to solve the equilibrium dynamics. Section 3 explains the key model mechanisms in a simplified version of our setup. Section 4 outlines our procedure to estimate the structural parameters of the model to match key sectoral micro pricing moments for the Euro Area. Section 5 shows our quantitative results for monetary shocks. Section 6 turns to quantitative results for aggregate and sector-specific TFP shocks. Section 7 describes our quantification exercise, where we assess the ability of our model to explain the aggregate dynamics of inflation and repricing frequency in the Euro Area. Section 8 concludes.

[2]Several papers have developed "second generation" SDP models in which the price change probability is a *smoothly* increasing function of the gain from adjustment, e.g. Caballero and Engel (1992), Nakamura and Steinsson (2008), Costain and Nakov (2011), instead of the *step function* it is in the fixed menu cost model.

2 Model

2.1 Overview

Time is discrete and indexed by $t \in \{0, 1, 2, ...\}$. There is a continuum of identical households, each consuming output and supplying labor. Firms are subdivided into N sectors, indexed by $i \in \{1, 2, ..., N\}$, each sector containing a continuum of monopolistically competitive firms of measure one; we use Φ_i to denote the set of all firms in sector i. The government consists of the central bank, which conducts policy by setting money supply, and the fiscal authority, which collects taxes from firms and rebates them to households in a lump-sum fashion.

2.2 Households

The representative households chooses a sequence of consumption, labor supply, and one-period nominal bond holdings to maximize expected lifetime utility:

$$\max_{\{C_t, L_t, B_t\}_{t \geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t), \tag{1}$$

subject to the period-by-period budget constraint

$$P_t^C C_t + \mathbb{E}_t\{\Lambda_{t,t+1} B_{t+1}\} \quad = \quad B_t + W_t L_t + \sum_{i=1}^{N} \int_0^1 D_{i,t}(j) dj + T_t, \tag{2}$$

where C_t is consumption, L_t is labor supply, B_t is the level of nominal bond holdings, T_t is the level of lump-sum transfers from the government, $D_{i,t}(j)$ are the dividends received lump-sum from firm j in sector i at time t, $\Pi_t^C = \left(P_t^C / P_{t-1}^C\right)$ is the gross CPI inflation rate, W_t is the nominal wage and $\Lambda_{t,t+1}$ is the nominal stochastic discount factor of the household.

Total final consumption C_t is given by an aggregator over sector-specific varieties:

$$C_t = \mathcal{C}(C_{1,t}, ..., C_{N,t}) \tag{3}$$

where $\mathcal{C}(\cdot)$ is homogeneous of degree one and non-decreasing in each of the arguments. The household chooses consumption of each of the sector-specific varieties to minimize total expenditure $\sum_i P_{i,t} C_{i,t}$, subject to the aggregator in (3). The minimal cost of assembling such a basket of sectoral varieties aggregating to $C_t = 1$ pins down the consumption price index as $P_t^C = \mathcal{P}^C(P_{1,t}, ..., P_{N,t})$, where $\mathcal{P}^C$ is homogeneous of degree one and non-decreasing in each of the arguments.

Sectoral final consumption $C_{i,t}$ is in turn given by the following aggregator over firm-specific varieties:

$$C_{i,t} = \left\{ \int_0^1 \left[\zeta_{i,t}(j) C_{i,t}(j)\right]^{\frac{\epsilon-1}{\epsilon}} dj \right\}^{\frac{\epsilon}{\epsilon-1}}, \tag{4}$$

where $\epsilon > 1$ is the within-sector elasticity of substitution, $C_{i,t}(j)$ is the final demand for the output of firm $j \in [0,1]$ in sector i at time t, and $\zeta_{i,t}(j)$ is a firm-specific idiosyncratic *quality* process. The quality process follows a random walk in logs:

$$\log \zeta_{i,t}(j) = \log \zeta_{i,t-1}(j) + \sigma_i \varepsilon_{i,t}(j), \tag{5}$$

where $\varepsilon_{i,t}(j)$ is an *i.i.d.* Gaussian innovation with mean zero and standard deviation of one. The final demand for firm j in sector i is given by:

$$C_{i,t}(j) = \zeta_{i,t}(j)^{\epsilon-1} \left(\frac{P_{i,t}(j)}{P_{i,t}} \right)^{-\epsilon} C_{i,t}, \tag{6}$$

and the sectoral price index of sector i is given by:

$$P_{i,t} = \left[\int_0^1 \left(\frac{P_{i,t}(j)}{\zeta_{i,t}(j)} \right)^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}}. \tag{7}$$

The representative household is also subject to a cash-in-advance constraint, which requires that the nominal money holdings are sufficient to cover the aggregate nominal final demand:

$$P_t^C C_t \leq M_t. \tag{8}$$

The aggregate money supply process $\{M_t\}_{t \geq 0}$ is set by the central bank, and agents treat this process as exogenous.

We now specify the functional forms for household preferences. First, for household preferences over aggregate consumption and labor supply, we use the log-linear preferences of Golosov and Lucas (2007):

Assumption 1 (Golosov-Lucas preferences)**.** *The utility function over consumption and labor supply is log-linear:* $u(C_t, L_t) = \log C_t - L_t$.

Under such preferences, we obtain the following intra-temporal labor supply condition: $\frac{W_t}{P_t^C} = C_t$. When combined with the cash-in-advance constraint (8), it implies that the nominal wage equals money supply in every period: $W_t = M_t$. In addition, the nominal stochastic discount factor satisfies: $\Lambda_{t,t+1} = \beta \frac{P_t^C C_t}{P_{t+1}^C C_{t+1}} = \beta \frac{M_t}{M_{t+1}}$.

As for aggregation across final consumption of sectoral varieties, in the baseline model we assume it to take the Cobb-Douglas form:

Assumption 2 (Consumption aggregation)**.** *The consumption aggregator* $\mathcal{C}(\cdot)$ *is given by:*

$$\mathcal{C}(C_{1,t},...,C_{N,t}) = \iota^C \prod_{i=1}^{N} C_i^{\overline{\omega}_{i,t}^C}, \tag{9}$$

where $\iota^C \equiv \prod_{i=1}^{N} \overline{\omega}_i^{C^{-\overline{\omega}_i^C}}$ *is a normalization term and* $\sum_i \overline{\omega}_i^c = 1$, $\overline{\omega}_i^c \geq 0, \forall i$.

Under this assumption, the equilibrium sectoral final consumption shares are constant over time: $\omega_{i,t}^C \equiv \frac{P_{i,t}C_{i,t}}{P_t^C C_t} = \overline{\omega}_i^C$.

2.3 Firms: production

The production function of firm j in sector i is given by:

$$Y_{i,t}(j) = \frac{1}{\zeta_{i,t}(j)} \times A_{i,t} \times \mathcal{F}_i \left[L_{i,t}(j), X_{i,1,t}(j), ..., X_{i,N,t}(j)\right], \tag{10}$$

where $\mathcal{F}_i(\cdot)$ is homogeneous of degree one and non-decreasing in inputs; $L_{i,t}(j)$ is the labor used by firm j in sector i at time t, $X_{i,k,t}(j)$ is intermediate inputs bought by firm j in sector i from sector k at time t. In addition, $A_{i,t}$ is an exogenous sector-specific total factor productivity process, while $\zeta_{i,t}(j)$ is the firm-level idiosyncratic quality process introduced in (5).

The intermediates demand $X_{i,k,t}(j)$ is in turn an aggregator over intermediates bought from each firm in sector k:

$$X_{i,k,t}(j) = \left\{ \int_0^1 \left[\zeta_{k,t}(j') X_{i,k,t}(j,j')\right]^{\frac{\epsilon-1}{\epsilon}} dj' \right\}^{\frac{\epsilon}{\epsilon-1}}, \tag{11}$$

where $X_{i,k,t}(j,j')$ is intermediates bought by firm j in sector i from firm j' in sector k, which satisfies the following demand condition in equilibrium: $X_{i,k,t}(j,j') = \zeta_{k,t}(j')^{\epsilon-1} \left(\frac{P_{k,t}(j')}{P_{k,t}}\right)^{-\epsilon} X_{i,k,t}(j)$.

Each firm chooses its labor and intermediate inputs in order to minimize the total cost of production, subject to the production technology in (10). The latter delivers the following marginal cost function for firm j in sector i at time t:

$$MC_{i,t}(j) = \zeta_{i,t}(j) \times \mathcal{Q}_i(W_t, P_{1,t}, ..., P_{N,t}; A_{i,t}) \tag{12}$$

where $\mathcal{Q}_i(\cdot)$ is the common component of the marginal cost index for all firms within a sector, which strictly falls in $A_{i,t}$ and is homogeneous of degree one and non-decreasing in the prices of all inputs.

In our baseline model, we assume that production technology takes the Cobb-Douglas form for all firms in all sectors:

Assumption 3 (Production technology)**.** *The production technology $\mathcal{F}_i(\cdot)$ for a firm j in sector i is given by:*

$$\mathcal{F}_i[L_{i,t}(j), X_{i,1,t}(j), ..., X_{i,N,t}(j)] = \iota_i L_{i,t}(j)^{\overline{\alpha}_i} \prod_{k=1}^{N} X_{i,k,t}(j)^{\overline{\omega}_{ik}}, \tag{13}$$

where $\iota_i \equiv \overline{\alpha}_i^{-\overline{\alpha}_i} \prod \overline{\omega}_{ik}^{-\overline{\omega}_{ik}}$ is a normalization term and $\overline{\alpha}_i + \sum_i \overline{\omega}_{ik} = 1$, $\overline{\alpha}_i$, $\overline{\omega}_{ik} \geq 0, \forall i$.

Under this assumption, the equilibrium labor cost shares and the input-output cost shares are constant over time and the same for all firms within a sector: $\alpha_{i,t} \equiv \frac{W_t L_{i,t}(j)}{MC_{i,t}(j) Y_{i,t}(j)} = \overline{\alpha}$, $\omega_{i,k,t} \equiv \frac{P_{k,t} X_{i,k,t}(j)}{MC_{i,t} Y_{i,t}(j)} = \overline{\omega}_{ik}$.

2.4 Firms: equilibrium size

The goods market clearing condition for firm j in sector i is given by:

$$Y_{i,t}(j) = C_{i,t}(j) + \sum_{k=1}^{N} \int_0^1 X_{k,i,t}(j', j) dj'. \tag{14}$$

Aggregating up to the level of sectors, multiplying both sides by P_i and dividing by aggregate final nominal demand $P_t^C C_t$, one can express the sectoral sales share (Domar weight) $\lambda_i \equiv \frac{P_{i,t} Y_{i,t}}{P_t^C C_t}$ as:

$$\lambda_{i,t} = \omega_{i,t}^C + \sum_{k=1}^{N} \omega_{ki,t} \lambda_{k,t} \times \mu_{k,t}^{-1}, \tag{15}$$

where μ_k^{-1} is the sales-weighted harmonic average of firm-level markups in a sector k : $\mu_{k,t}^{-1} = \int_0^1 \frac{1}{\mu_{k,t}(j')} \times \frac{P_{k,t}(j) Y_{k,t}(j)}{P_{k,t} Y_{k,t}} dj$. Using the downward sloping demand condition for each firm, one can rewrite $\mu_{k,t}^{-1}$ as:

$$\mu_{k,t}^{-1} = \frac{\Delta_{k,t}}{\mathcal{M}_{k,t}}, \qquad \Delta_{k,t} \equiv (P_{k,t}/M_t)^{\epsilon_k} \int_0^1 \left(\frac{P_{k,t}(j')}{\zeta_{k,t}(j') M_t} \right)^{-\epsilon_k} dj', \qquad \mathcal{M}_{k,t} \equiv \frac{P_{k,t}}{\mathcal{Q}_{k,t}}, \tag{16}$$

where $\Delta_{k,t}$ is a measure of price dispersion within the sector and $\mathcal{M}_{k,t}$ is a measure of sectoral markup. Stacking the equation for sales shares across sectors, we can write it as:

$$\boldsymbol{\lambda}_t = \boldsymbol{\omega}_{\boldsymbol{C},t} + \tilde{\Omega}_t^T \boldsymbol{\lambda}_t \quad \Longrightarrow \quad \boldsymbol{\lambda}_t = (I - \tilde{\Omega}_t^T)^{-1} \boldsymbol{\omega}_{\boldsymbol{C},t} \tag{17}$$

where $\tilde{\Omega}_t$ is a $N \times N$ matrix whose $[i, j]$ entry is given by $[\tilde{\Omega}_t]_{i,j} = \omega_{ij,t} \left\{ \frac{\Delta_{i,t}}{\mathcal{M}_{i,t}} \right\}$. Having calculated the sectoral sales shares, one obtains the sectoral total output as $Y_{i,t} = \lambda_{i,t} \times M_t / P_{i,t}$ and then the size of an individual firm as $Y_{i,t}(j) = \zeta_{i,t}(j)^{\epsilon - 1} \left(\frac{P_{i,t}(j)}{P_{i,t}} \right)^{-\epsilon} Y_{i,t}$.

2.5 Firms: pricing

The nominal profit of firm j in sector i at time t is given by:

$$D_{i,t}(j) = [(1-\tau_{i,t})P_{i,t}(j) - MC_{i,t}(j)] \times Y_{i,t}(j), \tag{18}$$

where $\tau_{i,t}$ is an exogenous sector-specific and time-varying sales tax levied by the government.[3] Denoting by $\tilde{P}_{i,t}(j) \equiv \frac{P_{i,t}(j)}{\zeta_{i,t}(j)M_t}$ the firm's quality-adjusted *real* price and by $\tilde{P}_{i,t} \equiv \frac{P_{i,t}}{M_t}$ the sectoral *real* price index, we can write the firm-level *real* profits $\tilde{D}_{i,t}(j) \equiv \frac{D_{i,t}(j)}{M_t}$ as:

$$\tilde{D}_{i,t}(j) = \left(\frac{P_{i,t}}{M_t}\right)^{\epsilon_i - 1} \times \left[(1-\tau_{i,t})\frac{P_{i,t}(j)}{\zeta_{i,t}(j)M_t} - \frac{\tilde{\mathcal{Q}}_{i,t}}{M_t}\right] \times \left(\frac{P_{i,t}(j)}{\zeta_{i,t}(j)M_t}\right)^{-\epsilon_i} \times \lambda_{i,t}$$

$$= \tilde{D}\left(\tilde{P}_{i,t}(j), \tau_{i,t}, \left\{\tilde{P}_{k,t}, \Delta_{k,t}, A_{k,t}\right\}_{k=1}^{N}\right). \tag{19}$$

Note that keeping track of the firm-level real profits requires knowing the firm's real quality-adjusted price, the own sectoral sales tax, as well as the real sectoral prices, price dispersions and productivities of all sectors in the economy.

Resetting the *nominal* price $P_{i,t}(j)$ involves the firm paying a sector-specific and possibly time-varying menu cost $\kappa_{i,t}$ measured in units of labor. The optimal reset price maximizes the firm's value, taking into account that this new price may not change for some period of time. In particular, when the nominal price does not change, the log of quality-adjusted real price $p_{i,t}(j) \equiv \log \tilde{P}_{i,t}(j)$ evolves according to

$$p_{i,t}(j) = p_{i,t-1}(j) + \log\left(\frac{P_{i,t-1}(i)}{\zeta_{i,t}(j)M_t}\right) - \log\left(\frac{P_{i,t-1}(j)}{\zeta_{i,t-1}(j)M_{t-1}}\right) = p_{i,t-1}(j) - \sigma_i \varepsilon_{i,t} - m_t, \tag{20}$$

where $m_t \equiv \Delta \log M_t$.

Without loss of generality, let $\eta_{i,t}(p)$ denote the probability that a firm in sector i with a quality-adjusted log relative price p resets its price at t . Consider a firm with a real quality adjusted price p at the end of period t, and let $p_+ \equiv (p - \sigma_i \varepsilon_{i,t+1}(j) - m_{t+1})$. Then this firm's real value at the end of period t is given by the following Bellman equation:

$$V_{i,t}(p) = \tilde{D}_{i,t}(p) + \beta \mathbb{E}_t\left[\{1 - \eta_{i,t+1}(p_+)\} V_{i,t+1}(p_+) + \eta_{i,t+1}(p_+)\left(\max_{p'} V_{i,t+1}(p') - \kappa_{i,t+1}\right)\right], \tag{21}$$

which consists of the current period real profits $\tilde{D}_{i,t}(p)$, as well as the discounted expected contin-

[3] The proceeds of these taxes are then rebated to households as a lump-sum transfer $T_t = \sum_{i=1}^{N} \tau_{i,t} \int_0^1 P_{i,t}(j) Y_{i,t}(j) dj$.

uation value. The latter is computed taking into account that at time $t+1$ the nominal price does not change with probability $1-\eta_{i,t+1}(\cdot)$, whereas with probability $\eta_{i,t+1}(\cdot)$ the firm pays the menu cost and optimally resets the nominal price.

Our formulation of the pricing problem covers a wide range of existing models of price setting, corresponding to the different functional forms of $\eta_{i,t}(\cdot)$. In the baseline setup of our model, we consider a specific functional form for the probability of adjustment function $\eta_{i,t}(\cdot)$. In particular, following Golosov and Lucas (2007), we assume that a firm adjusts if and only if the value gain from adjustment in a given period exceeds the menu cost:

Assumption 4 (*Ss* pricing)**.** *Consider a firm in sector i with the quality adjusted log relative price p at time t. Then the probability that this firm adjusts its nominal price is given by:*

$$\eta_{i,t}(p) = \mathbf{1}(L_{i,t}(p) > 0) \tag{22}$$

where $\mathbf{1}(\cdot)$ is the indicator function, and

$$L_{i,t}(p) = \max_{p'} V_{i,t}(p') - V_{i,t}(p) - \kappa_{i,t} \tag{23}$$

is the gain from adjustment (or loss from inaction), net of the menu cost.

Note that although here we specify a problem of price setting under nominal rigidities, our setup can automatically handle rigidities in nominal wage setting as well by appropriately parameterizing the input-output structure. In particular, consider a setup with a sector i, which we call the labor union, such that it only uses labor in production ($\overline{\alpha}_i = 1$) and moreover it the only sector purchasing labor directly from households ($\overline{\alpha}_{-i} = 0$). Other sectors instead purchase labor indirectly from the labor union as an intermediate input, such that $\overline{\omega}_{ki}, k \neq i$ represents the empirical cost share of labor for sector k. Then any rigidities in the price setting of the labor union sector are isomorphic to nominal wage rigidities.

2.6 Equilibrium definition and solution method

In addition to the goods market clearing condition in (14), the equilibrium in our economy is also characterized by market clearing in the labor market:

$$L_t = \sum_{i=1}^{N} \int_0^1 L_{i,t}(j)dj + \sum_{i=1}^{N} \kappa_{i,t} \int_0^1 \eta_{i,t}(p_{i,t}(j))dj, \tag{24}$$

as well as in the market for bonds that are in zero net supply: $B_t = 0$.

Having specified the optimality and market clearing conditions, we can now formally define the decentralized equilibrium in our economy:

Definition 1 (Equilibrium). *The equilibrium is a collection of prices* $\{P_{i,t}(j)|j \in \Phi_i\}_{i=1}^N$, *allocations* $\left\{Y_{i,t}(j), L_{i,t}(j), C_{i,t}(j), \{X_{i,r,t}(j,j')|j' \in \Phi_r\}_{r=1}^N | j \in \Phi_i\right\}_{i=1}^N$, *wage* W_t *and bond holdings* B_t, *which given the realizations of firm-level quality process* $\{\zeta_{i,t}(j)|j \in \Phi_i\}_{i=1}^N$, *sectoral productivities* $\{A_{i,t}\}_{i=1}^N$, *sectoral sales tax rates* $\{\tau_{i,t}\}_{i=1}^N$ *and money supply* M_t *satisfy agent optimization and market clearing conditions in every period.*

We now briefly outline our solution strategy, which we use to compute equilibrium prices and quantities given the realizations of exogenous processes. Full details of the numerical strategy are given in the Appendix C.

As a first step, we compute the steady-state of our economy, defined as the equilibrium evaluated at the point where money supply growth and sectoral TFPs are at their unconditional mean values, and the firm-level prices are in their stationary distribution. In particular, for each sector we numerically solve the stationary Bellman equation and firms' price distribution on an evenly spaced grid of log quality adjusted real prices with step size Δp, $p_j \in \left[\underline{p}, \underline{p} + \Delta p, ..., \overline{p}\right]$, $j = 1, .., J$ grid points, so that $V_j = V(p_j)$. In the algorithm we introduce in Appendix C, we jointly search across firm-level prices in each sector and sector-specific sales taxes $\{\overline{\tau}_i\}_{i=1}^N$, so that we satisfy the equilibrium conditions and obtain steady-state real sectoral price indices equal to one.

Next, we compute fully non-linear responses to a sequence of monetary and TFP shocks. We operate under the assumption of perfect foresight over aggregate and sectoral exogenous shocks, while maintaining uncertainty over the idiosyncratic innovations. To compute the responses, we first assume that there exists a finite period T, at which the economy is back to steady state. Then, starting from a guess for the sequences of sectoral and aggregate variables, we iterate *backward* from $t = T$ to $t = 0$ to solve for the micro value functions. Having obtained the micro value functions, we iterate *forward* from $t = 0$ to $t = T$, and numerically aggregate to obtain sectoral and aggregate variables. We repeat this backward-forward iteration until convergence. Appendix C formally details the algorithm to perform the backward-forward iteration.

3 Pricing cascades: formal results in a simplified setup

3.1 Static economy

In order to obtain intuition regarding the transmission of large shocks in our model, we now consider a simplified setup obtained under two additional assumptions. First, we assume the setup to be static in the sense that agents fully discount the future:

Assumption 5 (Myopia). *Agents fully discount the future in their objective function, so that* $\beta = 0$.

In particular, this setting implies that any firm's value function is simply given by contemporaneous profits, and hence the optimal quality-adjusted real reset price for any firm in a sector i is

given by:

$$\tilde{P}^*_{i,t} = \frac{1}{1-\tau_{i,t}} \frac{\epsilon}{\epsilon - 1} \times \frac{1}{A_i} \prod_{k=1}^{N} \tilde{P}_{i,t}^{\overline{\omega}_{ik}} = \Gamma_{i,t} \times \tilde{\mathcal{Q}}_{i,t}. \tag{25}$$

where $\Gamma_{i,t} \equiv \frac{1}{1-\tau_{i,t}} \frac{\epsilon}{\epsilon-1}$ is the (exogenous) desired markup, whose variation across time and sectors is fully pinned down by the movements in the sectoral tax rates $\tau_{i,t}$.

Second, we assume a specific form of time-variation of the sector-specific menu cost $\kappa_{i,t}$:

Assumption 6 (Sectoral menu costs)**.** *The sector-specific menu cost follows the following process:* $\kappa_{i,t} = \overline{\kappa}_i(1-\tau_{i,t})[\tilde{P}_{i,t}/\tilde{P}^*_{i,t}]^{\epsilon-1}\lambda_{i,t}$*, where* $\overline{\kappa}_i$ *is a sector-specific constant.*

The above two assumptions allow us to derive closed-form results regarding the interaction between networks, price adjustment decisions at the extensive margin, and the type of shocks hitting the economy.

The decision to change prices is based on whether the value gain from adjustment exceeds the menu cost. In the static setup, we can obtain a tractable approximation for the gain from adjustment as a function of the *price gap*, or the difference between the current and the optimal reset price:

Lemma 1 (Adjustment gains)**.** *Suppose Assumptions 1-5 hold. Let* $\tilde{p}_{i,t}(j) \equiv \log \tilde{P}_{i,t}(j) - \log \tilde{P}^*_{i,t}$ *be the price gap for a firm* j *in sector* i *at time* t*. Then the profit gain from price adjustment satisfies:*

$$\tilde{D}^*_{i,t}(j) - \tilde{D}_{i,t}(j) \quad = \quad \frac{1}{2}(\epsilon-1)(1-\tau_{i,t})[\tilde{P}_{i,t}/\tilde{P}^*_{i,t}]^{\epsilon-1}\lambda_{i,t} \times [\tilde{p}_{i,t}(j)]^2 \quad + \quad \mathcal{O}[\tilde{p}_{i,t}(j)]^3 \tag{26}$$

where $\tilde{D}^*_{i,t}(j)$ *is profits at the optimal reset price,* $P_{i,t}$ *is the real sectoral price index and* $\lambda_{i,t}$ *is the sectoral sales share (Domar weight).*

To illustrate the interaction between networks and price adjustment decisions, consider the initial period ($t = 0$) in our economy. If the firm chooses to not adjust its nominal price, then the quality-adjusted real price in the initial period is given by:

$$\log \tilde{P}_{i,0}(j) = p_{i,-1}(j) - \sigma_i \varepsilon_{i,0}(j) - m_0 \tag{27}$$

where $p_{i,-1}(j)$ is the initial (exogenous) quality-adjusted real price of firm j in sector i, $\varepsilon_{i,0}(j)$ is the realization of the firm-level quality shock in period $t = 0$, and $m_0 \equiv \log(M_0/M_{-1})$ is the realization of money growth at $t = 0$. Given the expression for the optimal reset price in (25), we can write the firm-level price gap in the initial period as:

$$\tilde{p}_{i,0}(j) = -\sigma_i \varepsilon_{i,0}(j) - m_0 - \gamma_{i,0} + a_{i,0} - \sum_{k=1}^{N} \overline{\omega}_{ik} \log \tilde{P}_{k,0} + (p_{i,-1}(j) - \overline{\gamma}_i). \tag{28}$$

where $\overline{\gamma}_i \equiv \log \frac{\epsilon}{\epsilon-1}\frac{1}{1-\overline{\tau}_i}$, $a_{i,0} \equiv \log A_{i,0}$ and $\gamma_{i,0} \equiv \log \Gamma_{i,0} - \overline{\gamma}_i$.

Without loss of generality, normalize $p_{i,-1}(j) = \log \frac{\epsilon}{\epsilon-1}$; then given the realizations of aggregate and sectoral variables, the magnitude of the price gap of the specific firm j is pinned down by the realization of its idiosyncratic quality innovation $\varepsilon_{i,0}(j)$. We can use the approximate profit gain in Lemma 1 to determine the sector-specific *inaction regions*, defining the ranges for idiosyncratic innovations under which the firm will choose not to adjust:

Lemma 2 (Inaction region)**.** *Suppose Assumptions 1-6 hold. Given the realizations of aggregate and sectoral variables and normalizing* $p_{i,-1}(j) = \log \frac{\epsilon}{\epsilon-1}\frac{1}{1-\overline{\tau}_i}$*, let* $\underline{\varepsilon}_{i,0}$ *and* $\overline{\varepsilon}_{i,0}$ *be thresholds such that a firm in sector* i *will not adjust the price if it draws an innovation in* $[\underline{\varepsilon}_{i,0}, \overline{\varepsilon}_{i,0}]$*. Then,*

$$[\sigma_i \underline{\varepsilon}_{i,0}, \quad \sigma_i \overline{\varepsilon}_{i,0}] \quad = \quad -m_0 - \gamma_{i,0} + a_{i,0} - \sum_{k=1}^{N} \overline{\omega}_{ik} \log \tilde{P}_{k,0} \quad \pm \quad \sqrt{\frac{2\overline{\kappa}_i}{\epsilon - 1}}, \tag{29}$$

where $m_0 \equiv \log(M_0/M_{-1})$, $\gamma_{i,0} \equiv \log \Gamma_{i,0} - \overline{\gamma}_i$, $a_{i,0} \equiv \log A_{i,0}$ *and* $\overline{\gamma}_i \equiv \log \frac{\epsilon}{\epsilon-1}\frac{1}{1-\overline{\tau}_i}$.

Given the realizations of monetary, productivity, and desired markup shocks, which are independent of the presence of input-output linkages, we can now derive the effect of removing input-output linkages ($\overline{\omega}_{i,k} = 0, \forall i, k$) on the firm-level decision to adjust its price.

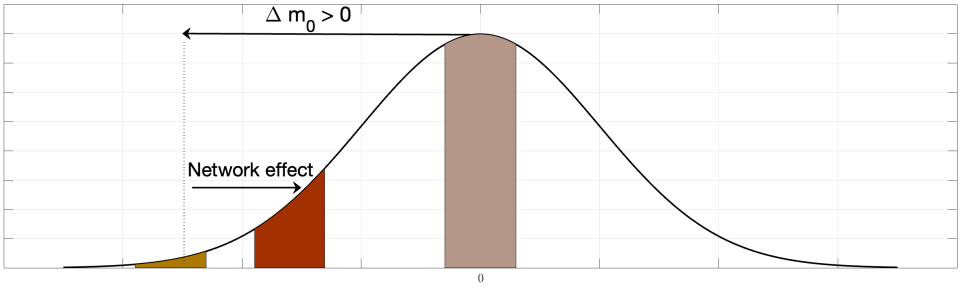
Consider an increase in money supply $m_0 > 0$. According to Lemma 2, this increase in money supply, *ceteris paribus*, to a leftward shift of the inaction region. In other words, more extreme (negative) realizations of idiosyncratic innovations are needed to prevent adjustment. At the same time, Lemma 2 also implies that as long as the pass-through of the money supply to sectoral prices is incomplete ($\log \tilde{P}_{k,0} < 0, \forall k$), the presence of networks attenuates the leftward shift of the inaction region for all firms that have a non-zero cost share of intermediate inputs. As a result, this weakly lowers the probability of price adjustment for any firm, creating *dampening* in price changes. Panel (a) of Figure 1 provides a graphical illustration of this mechanism. In the following, we formalize the notion that networks create *attenuation* in price adjustment decisions after monetary shocks:

Proposition 1 (Cascades and demand shocks)**.** *Suppose Assumptions 1-6 hold. Consider an increase in the money supply* $m_0 > 0$*. Then, as long as the pass-through of the money supply to sectoral prices is incomplete* ($\log \tilde{P}_{k,0} < 0, \forall k$)*, production networks (weakly) lower the probability of adjustment for any firm following the monetary shock.*

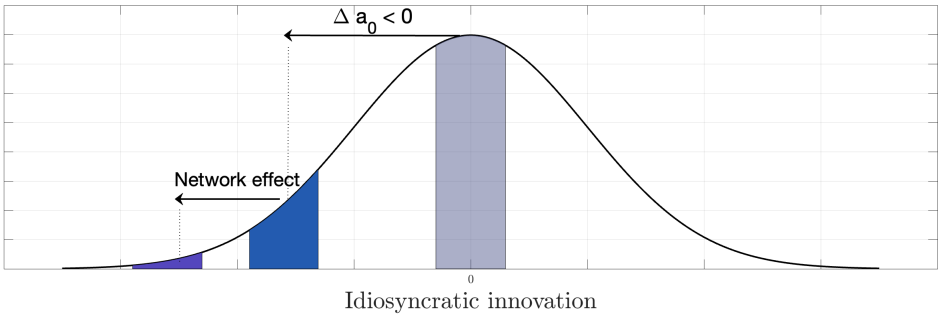
In contrast, consider a sectoral productivity deterioration $a_{i,0} < 0$. According to Lemma 2, this productivity improvement creates a leftward shift of the inaction region for all firms in sector i. Moreover, as long as this productivity improvement leads to a rise in price indices of other sectors ($\log \tilde{P}_{k,0} > 0, \forall k$), then Lemma 2 also implies that networks will further amplify the leftward shift in the inaction region for all firms in sector i, as long as the cost share of intermediates in that sector

Figure 1: **Networks and inaction regions**

(a) Dampening under monetary shocks



(b) Cascades under TFP/markup shocks



Notes: *the figure shows the contribution of production networks to the movements in the inaction region follows monetary and aggregate TFP shocks, respectively.*

is non-zero. In other words, even more extreme (negative) realizations of idiosyncratic innovations are needed to justify non-adjustment. As a result, contrary to the case of monetary shocks, the presence of networks weakly raises the probability of price adjustment for any firm in sector i, thus creating *cascades* in price changes. Panel (b) of Figure 1 illustrates this mechanism graphically.

Note that an identical mechanism of cascades also applies in the case of shocks to desired markups. Following an increase in desired markups, $\gamma_{i,0} > 0$, there is a leftward shift in the inaction region, which is further moved to the left as long as the markup shock is inflationary in the aggregate ($\log \tilde{P}_{k,0} > 0, \forall k$). In the following, we formalize the notion that networks create *cascades* in price adjustment decisions after TFP and markup shocks:

Proposition 2 (Cascades and supply shocks)**.** *Suppose Assumptions 1-6 hold. Consider a decrease in sectoral TFP $a_{i,0} < 0$ or an increase in sectoral desired markup $\gamma_{i,0} > 0$. Then, as long as such shocks lead to a rise in price indices of other sectors ($\log \tilde{P}_{k,0} > 0, \forall k$), production networks (weakly) increase the probability of adjustment for any firm in any other sector.*

In addition, note that a productivity or desired markup shock in a sector i can, in principle,

increase the probability of price adjustment for firms in any other sector i'. This is true as long as the price indices of sectors used as suppliers by sector i' ($k : \overline{\omega}_{i'k} > 0$) rise following the productivity deterioration or markup increase in sector i.

3.2 Simple examples

We now turn to the dynamic version of our model. First, we consider several simple network arrangements in order to clarify the intuition behind key mechanisms. To facilitate further comparability between monetary and TFP shocks, for the remainder of this section we assume that both follow AR(1) in levels with persistence $\rho \in (0, 1)$.

***Example 1*: roundabout production economy**

First, we consider a one-sector roundabout economy ($N = 1$), where firms trade intermediate inputs with other firms in the same sector. Figure 2(a) illustrates such an arrangement graphically. Naturally, in the limit where we set the cost share of labor at one ($\overline{\alpha}_1 = 1$), the economy collapses to the standard arrangement, where firms only use labor and there is no network.

We use this simple example to illustrate how the presence of the network affects the response of the aggregate fraction of adjusting firms to monetary and productivity shocks of different sizes. As can be seen in the right panel of Figure 2(a), when there are no networks ($\overline{\alpha}_1 = 1$), monetary and productivity shocks are exactly isomorphic in their effect on aggregate frequency. However, as soon as we add the roundabout production structure ($\overline{\alpha}_1 < 1$), the aggregate adjustment frequency responds much faster to productivity shocks relative to monetary shocks. This is because under monetary shocks, the network structure generates *dampening* in price adjustment decisions, manifesting itself in slower increases in the aggregate fraction of adjusters. In contrast, under TFP shocks, networks create *cascades* in adjustment decisions, leading to faster increases in the aggregate proportion of firms that choose to adjust.

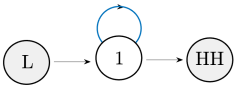
***Example 2*: two-sector vertical chain economy**

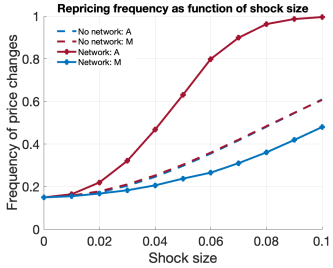
Second, we consider a two-sector example to illustrate how the position in the supply chain affects the transmission of sectoral productivity shocks to aggregate frequency. As can be seen in the left panel of Figure 2(b), the first sector is the Upstream one, and it uses labor to produce inputs for the second, Downstream sector, which sells the final output to households. Importantly, the two sectors have the same size in equilibrium.

We now consider sector-specific productivity shocks of different sizes and record their effect on the aggregate adjustment frequency. In the right panel of Figure 2(b) one can see that large shocks to the Upstream sector deliver faster increases in the aggregate frequency, relative to equally-sized shocks to the Downstream sector. This is because shocks to the Upstream sector additionally

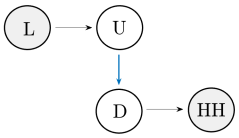
Figure 2: **Three example economies**

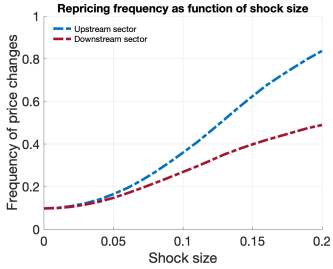
(a) Roundabout production economy



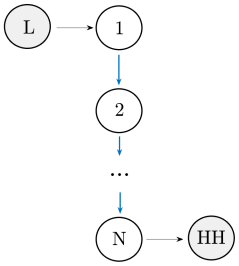


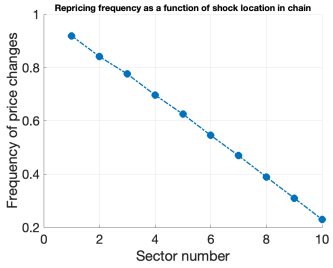
(b) Two-sector vertical chain economy





(c) N-sector vertical chain economy





***Notes**: the figure shows three example economies, as well as the responses of aggregate frequency of adjustment to monetary and TFP shocks.*

affect the optimal reset price, and hence the price gaps of the Downstream sector. Hence, shocks to the Upstream sector also trigger additional extensive margin price adjustment decisions in the Downstream sector. The opposite, however, is not true: shocks to the Downstream sector only affect price gaps in the Downstream sector itself, and do not affect price adjustment decisions in the Upstream sector.

Example 3: N-sector vertical chain economy

Third, we would like to illustrate that as sectors become increasingly more "central", in terms of their importance as suppliers to the rest of the firms in the economy, large shocks to them can deliver increasingly larger increases in the aggregate fraction of adjusting firms. In the left panel

of Figure 2(c), we illustrate a vertical chain production economy consisting of N sectors. Sector 1 has the largest "centrality", as it supplies, directly or indirectly, to the rest of the economy. As one moves down the chain, sectors' centrality falls, with sector N being the least central, as it only supplies final output to consumers.

In the right panel of Figure 2(c), we set $N = 10$ and plot the aggregate frequency response to large (-20%) sector-specific productivity shocks in each sector. One can see that the shock to the most central sector 1 delivers the largest increase in aggregate frequency. Moreover, the aggregate frequency response falls monotonically as we move down the supply chain and consider increasingly less central sectors.

4 Full model with Euro Area data

4.1 Parameterization and Calibration

We discipline the structural parameters of our model at monthly frequency for Euro Area data. Table 1 summarizes our calibration.

For the aggregate parameters, the households' discount factor is set to $\beta = 0.96^{1/12}$ as in Golosov and Lucas (2007). The within-sector elasticity of substitution across varieties is $\epsilon = 3$ as in Midrigan (2011). We assume that aggregate money supply follows a random walk with drift:

$$\log M_t = \overline{\pi} + \log M_{t-1} + \varepsilon_t^M, \tag{30}$$

where $\overline{\pi}$ is the trend growth rate for money supply, which is also the equilibrium level of trend inflation; ε_t^M is an i.i.d. mean zero money growth innovation. The steady-state money growth rate is $\pi = 2\%$ per year, in line with the inflation target of the European Central Bank (ECB). As for the sectoral total factor productivities, we assume those to follow an AR(1) process;

$$\log A_{i,t} = \rho \log A_{i,t-1} + \varepsilon_{i,t}^A, \tag{31}$$

where $\rho \in (0, 1)$ is the persistence parameter and $\varepsilon_{i,t}^A$ is an i.i.d. mean zero sector-specific productivity innovation. We set the persistence of TFP processes equal to $\rho = 0.9$.

We calibrate our economy to 38 production sectors of the Euro Area economy, following the classification in the World Input-Output Database (WIOD). The final consumption shares $\{\overline{\omega}_i^C\}_{i=1}^N$ and the input-output cost shares $\{\overline{\omega}_{ik}\}_{i,k=1}^N$ are taken from the 2014 input-output tables for the Euro Area based on WIOD. As for the sectoral cost shares of labor $\{\overline{\alpha}_i\}_{i=1}^N$, they are taken from 2014 National Income Accounts for the Euro Area, published by the EU KLEMS database. In order to capture the possibility the wages are also sticky, we introduce an auxiliary labor union sector. In particular, we assume that the labor union sector is the only one that directly purchases labor

Table 1: Parameter values (Euro Area, monthly)

		Aggregate parameters	
β	$0.96^{1/12}$	Discount factor (monthly)	Golosov and Lucas (2007)
ϵ	3	Goods elasticity of substitution	Midrigan (2011)
$\overline{\pi}$	0.02/12	Trend inflation (monthly)	ECB target
ρ	0.90	Persistence of the TFP shock	Half-life of seven months
		Sectoral parameters	
N	39	Number of sectors	Data from Gautier et al. (2024)
$\{\overline{\omega}_i^C\}_{i=1}^N$		Sector consumption weights	World IO Tables
$\{\overline{\omega}_{ik}\}_{i,k=1}^N$		Sector input-output matrix	World IO Tables
$\{\overline{\alpha}_i\}_{i=1}^N$		Sector labor weights	World IO Tables
		Firm-level pricing parameters	
$\{\overline{\kappa}_i\}_{i=1}^N$		Menu costs	Estimated to fit frequency, std dev.
$\{\sigma_i\}_{i=1}^N$		Std. dev. of firm-level shocks	of Δp from Gautier et al. (2024)

from households and then sells it to the rest of the sectors as an intermediate input. In general, we work with $N = 39$ sectors: 38 production sectors, and the auxiliary labor union sector.

Unlike in Section 3.1 above, we do not allow time variation in the sectoral menu costs. Instead, we consider the more conventional fixed menu cost setup (Golosov and Lucas, 2007), letting the menu costs to vary in the cross-section only:

Assumption 6′ (Fixed menu costs)**.** *The sector-specific menu cost follows the following process:* $\kappa_{i,t} = \overline{\kappa}_i$*, where* $\overline{\kappa}_i$ *is a sector-specific constant.*

This leaves us with two parameters per sector to estimate: the menu cost $\overline{\kappa}_i$, and the standard deviation of firm-level shocks σ_i. In line with evidence in Gautier et al. (2023), we assume that the sectors "Coke and Petroleum Products" and "Mining and Quarrying" have fully flexible prices at monthly frequency. We calibrate the price setting parameters in the labor union sector to match the frequency and standard deviation of nominal wage changes in Costain et al. (2022). For the remaining 36 sectors, we estimate the parameters $\{\overline{\kappa}_i\}_{i=1}^N$ and $\{\sigma_i\}_{i=1}^N$ to match the frequency and standard deviation of price changes in each sector in the Euro area, taken from Gautier et al. (2024), in steady state.

4.2 Sectoral characteristics

In order to better understand the cross-sectional properties of the sectors we consider in our quantitative setup, we introduce two different measures of sectoral *centrality*. First, in order to capture the full degree to which a sector is important as a buyer of intermediate inputs from the rest of the

economy, we use the following *customer centrality* metric:

$$\text{Customer Centrality}_i \quad \equiv \quad \sum_{j=1}^{N} (I - \overline{\Omega})^{-1}_{ij} - 1 \tag{32}$$

Intuitively, the customer centrality measure captures the total reliance of a sector on intermediate inputs, both direct and indirect. Naturally, if a sector only uses labor in production, the customer centrality measure collapses to zero.

Second, in order to capture the degree to which a sector is important as a provider of intermediate inputs to the rest of the economy, we introduce the following *supplier centrality* metric:

$$\text{Supplier Centrality}_i \quad \equiv \quad \sum_{j=1}^{N} (I - \overline{\Omega}^T)^{-1}_{ij} - 1 \tag{33}$$

The supplier centrality measure capture the total importance of a sector as a seller, either directly or indirectly, of intermediate inputs to the rest of the economy.

5 Quantitative results: monetary shocks

5.1 Aggregate dynamics

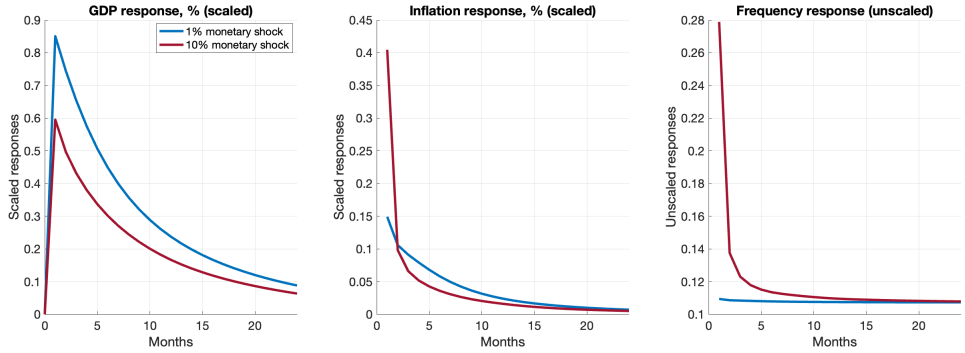
Figure 3(a) shows the normalized responses of aggregate CPI inflation, aggregate GDP, as well as the unscaled fraction of adjusting firms to monetary shocks of two different magnitudes: 1% and 10%. Two key features are apparent. First, the normalized response of inflation increases in the size of the monetary shock, which represents a strong *size effect*. As can be seen in the frequency panel, this happens as the fraction of adjusting firms increases rapidly with larger shocks, reaching almost 30% for the 15% monetary shock.

Second, as shown in Figure 3(b), the contribution of production networks to the magnitudes of responses differs markedly between shocks of different sizes. For the small 1% shock, networks dampen the response of inflation and, as a result, amplify the response of aggregate GDP. This is the effect of production networks known from previous papers, which employ linearized models with time-dependent pricing: input-output linkages create pricing complementarities, dampening inflation and amplifying the consumption response. At the same time, for the large 10% shock, the amplification of the aggregate GDP response due to networks is much greater. Importantly, this is because the 10% monetary shock delivers a markedly smaller increase in the repricing frequency relative to the "no-network" scenario, which is the *dampening* effect introduced earlier.

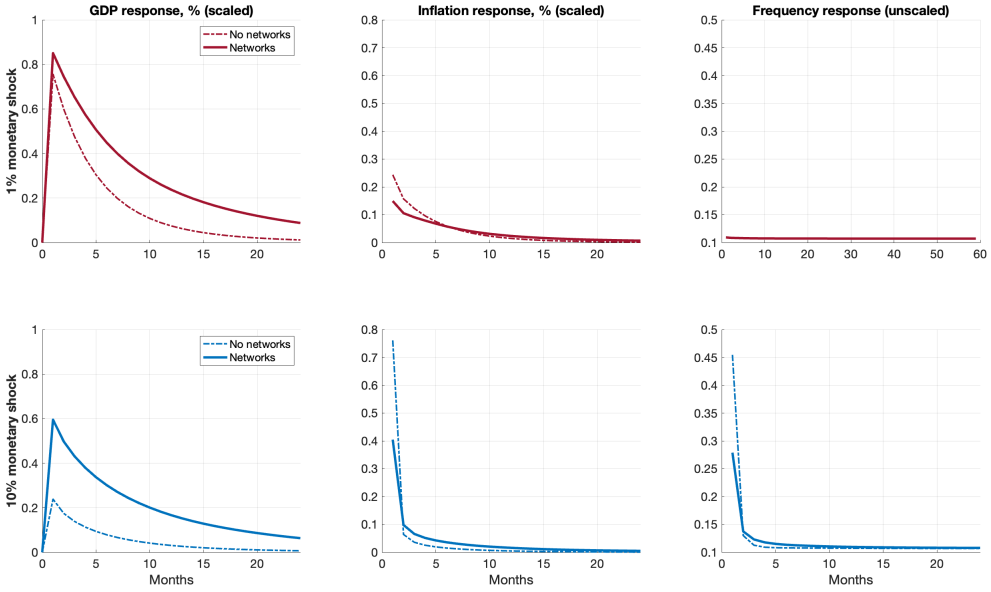
In Figure 4(a), we further investigate the interaction between networks and the response of repricing frequency to monetary shocks. One can see that networks consistently dampen the response of aggregate repricing frequency to monetary shocks of all sizes that we consider. For

Figure 3: **Effect of networks on aggregate responses**

(a) Responses of aggregates to monetary shocks



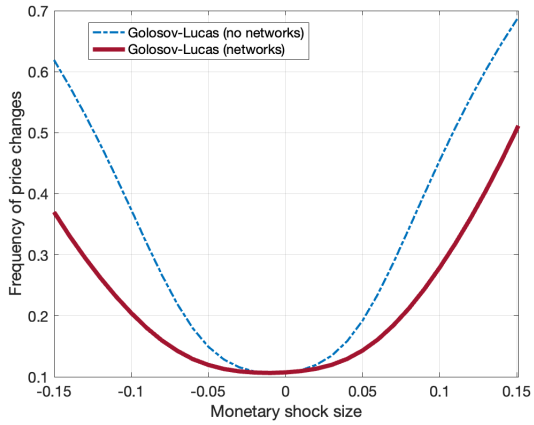
(b) Network contribution to aggregates' responses



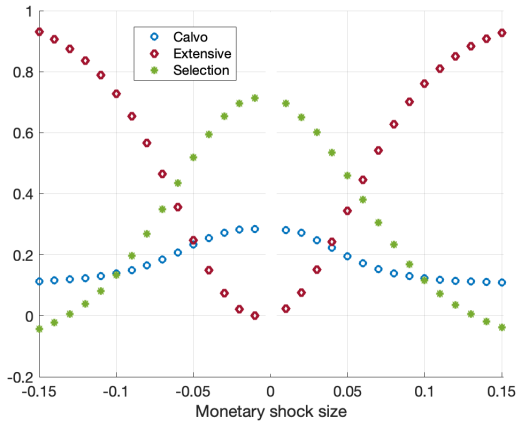
***Notes**: the figure shows the responses of aggregate GDP, inflation and frequency of adjustment in response to monetary shocks of different sizes.*

Figure 4: **Aggregate frequency of adjustment after monetary shocks**

(a) Aggregate frequency response



(b) Contribution of the extensive margin



***Notes**: the figure shows the impact responses of aggregate frequency of adjustment to monetary shocks of different sizes, with and without networks.*

example, following a 10% monetary expansion, the aggregate frequency rises close to 45% in the multi-sector economy without networks, but increases only to 27% in an otherwise identical economy with input-output linkages.

In order to further quantify the contribution of shock *dampening* to aggregate inflation responses, we decompose the effect of the money shock on inflation into three channels, following Blanco et al. (2024) and Costain and Nakov (2011). Note that, up to a first-order approximation, inflation in the absence of the shock is equal to

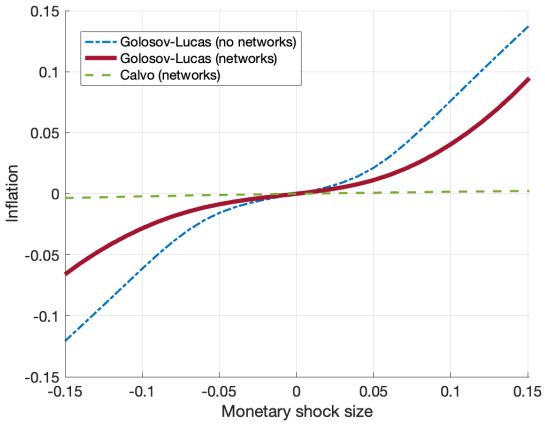
$$\pi = \int \tilde{p}\eta(\tilde{p})\mathrm{d}g(\tilde{p}) \tag{34}$$

where $\tilde{p}$ is the desired log price change, $\eta(\tilde{p})$ is the adjustment hazard, and $g(\tilde{p})$ is the ergodic distribution of desired price changes across firms and sectors. The money shock increases all firms' desired price changes to $\tilde{p}+\alpha$, where $\alpha = p_1^* - p^* + \Delta m$ and where p_1^* is the average across sectors of the log reset price in the first period after the money shock and p^* is the average across sectors of the log reset price in the absence of the shock. The money shock changes the inflation rate to $\pi_1 = \int(\tilde{p}+\alpha)\eta_1(\tilde{p})\mathrm{d}g(\tilde{p})$ where $\eta_1(\tilde{p})$ is the new adjustment hazard after the shock. Following Blanco et al. (2024), the change in inflation can then be decomposed into

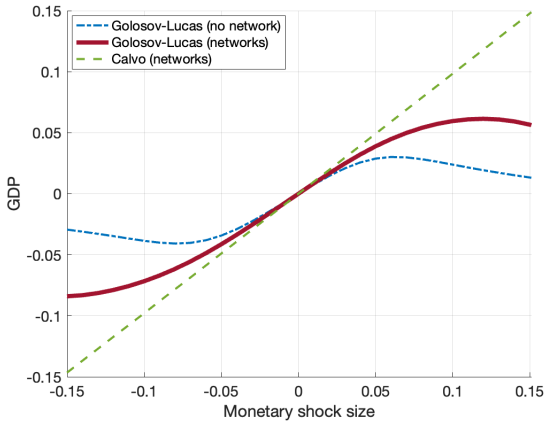
$$\Delta\pi = \underbrace{\alpha\int \eta(\tilde{p})\mathrm{d}g(\tilde{p})}_{\text{Calvo}} + \underbrace{\alpha\int(\eta_1(\tilde{p})-\eta(\tilde{p}))\mathrm{d}g(\tilde{p})}_{\text{extensive}} + \underbrace{\int\tilde{p}(\eta_1(\tilde{p})-\eta(\tilde{p}))\mathrm{d}g(\tilde{p})}_{\text{selection}}. \tag{35}$$

Figure 5: **Inflation and GDP responses to monetary shocks**

(a) Impact inflation response



(b) Impact GDP response

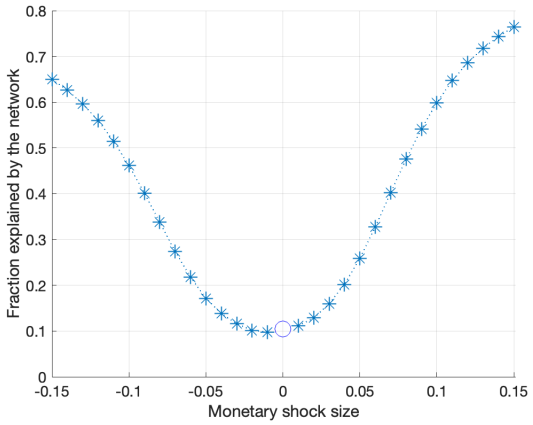


***Notes**: the figure shows the impact responses of inflation and GDP to monetary shocks of different sizes, in different configurations of the model.*

In Figure 4b we show the resulting decomposition of the differential inflation responses in our model with networks compared to that without networks. The difference between the network and no-network cases is explained mainly by the selection effect for smaller shocks, and by the extensive margin component for shocks greater than 5 percent in absolute value. Therefore, for large shocks, most of the network contribution to the slowing down of the inflation response works through the *dampening* in pricing.

The dampening effect of networks on the repricing frequency has important implications for the responses of CPI inflation and aggregate consumption to large monetary interventions. In Figure 5 (a), we show that as the size of the monetary shocks increases, inflation in our baseline economy rises in a non-linear fashion: a 5% shock delivers 2% inflation on impact, whereas tripling the shock to 15% delivers a five-fold increase of inflation to 10%. At the same time, the figure also shows that an otherwise identical economy without networks features inflation rising even faster with larger monetary shocks. The fact that inflation rises relatively more slowly in the economy with networks reflects mainly the slower response of the fraction of adjusters, as documented in Figure 4. In order to quantify the importance of non-linearities and state-dependent pricing, in Figure 5 (a) we also consider a version of our model with time-dependent pricing (Calvo, 1983), calibrated to match the sectoral frequencies of adjustment in steady state. Under such a time-dependent setup, even when solved fully non-linearly, inflation is rising more slowly as the monetary shock gets larger. The latter reflects the contribution of both the selection effect (for smaller shocks) and the extensive margin effect (for larger shocks) in delivering faster pricing increases in the state dependent pricing model.

Figure 6: **Output amplification**



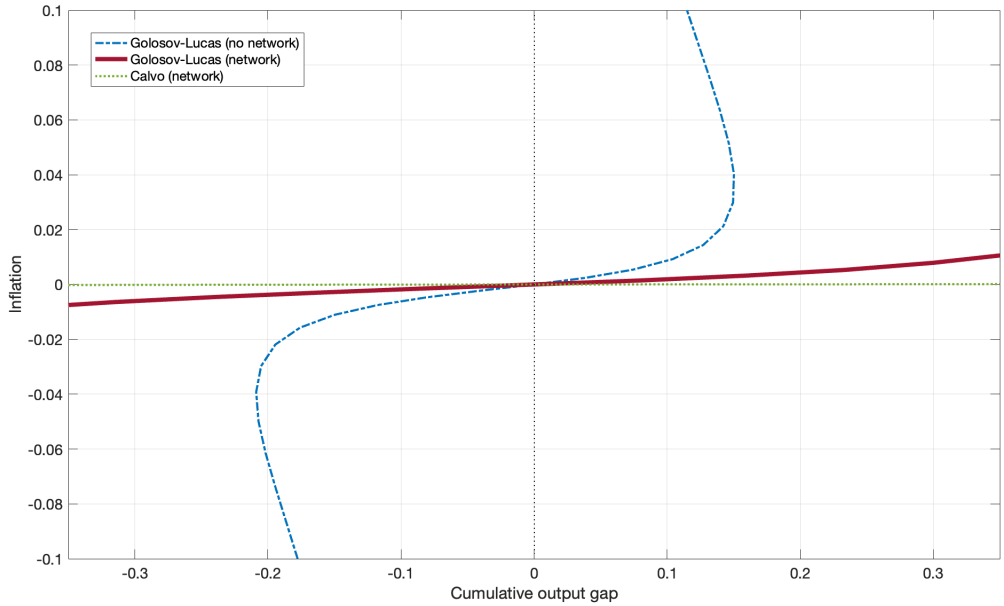
***Notes**: the figure shows the contribution of networks to the impact responses of GDP to monetary shocks of different sizes.*

Figure 5 (b) shows that in the baseline economy with networks, the aggregate consumption response is hump-shaped in the size of monetary shocks and is maximized following a 12% monetary expansion, delivering an increase of almost 6%. At the same time, the equivalent economy without networks has its consumption response maximized following a 5% monetary shock, corresponding to a smaller increase of just over 3%. The higher maximal response of consumption under networks, as well as the fact that it occurs following a larger monetary shock, reflect the slower response of the fraction of adjusters, as documented in Figure 4. Figure 5 (b) (a) also shows the responses in the alternative setup with time-dependent Calvo (1983) pricing. With time-dependent pricing, one can see that even for very large shocks and fully non-linear solution, the time-dependent setup has aggregate consumption rise quasi-linearly in the size of the monetary shock. Moreover, the fully non-linear time-dependent pricing results deviate substantially from the fully non-linear state-dependent solutions.

In order to formally measure the contribution of networks to the output response, in Figure 6 we construct, for each size of the monetary shock, the difference between the output response with and without networks, as a fraction of the former. One can see that for small monetary shocks, the contribution is in the neighborhood of 10-20%. Such magnitudes are consistent with prior estimates of network contributions in linearized time-dependent setups: for example, Ghassibe (2021) estimates the contribution to be just below 30% in such a setup. At the same time, one can also see that as the size of the shock increases, the contribution of the network increases dramatically, reaching almost 80% for a 15% monetary expansion. This reflects the fact that for such large shocks, the fraction of adjusters rises much less in the economy with networks, delivering a much larger aggregate pass-through to inflation and hence a larger consumption response.

Figure 7 illustrates the trade-off between output stimulus and inflation under monetary in-

Figure 7: **Phillips curves**



***Notes**: the figure shows the fully non-linear Phillips curves in the various configurations of the model.*

terventions of different sizes. In particular, the figure traces a non-linear "Phillips curve" in the cumulative output gap–CPI inflation space, under different model configurations. In the network-based baseline economy, a cumulative output stimulus up to 5% or so can be achieved with little inflationary response, reflecting a locally flat Phillips curve. However, in a counterfactual economy without networks, the Phillips curve is steeper for small shocks and low output gap values. This suggests "flattening" of the Phillips curve due to networks, documented in previous studies under time-dependent pricing. Moreover, once the shocks are sufficiently large, the Phillips curve without networks becomes backward bending, with a maximum possible cumulative output stimulus of around 15%. This happens because, under very large shocks, the fraction of adjusters increases much faster in the economy without networks, as documented in Figure 4.

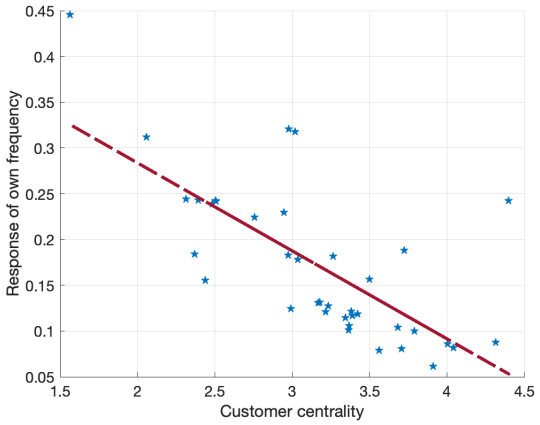
5.2 Disaggregated dynamics

Having analyzed the behavior of macroeconomic aggregates, we now move to studying sector-level behavior following monetary shocks of different sizes.

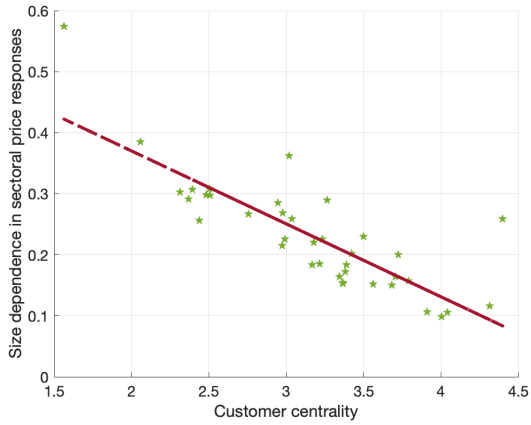
To better understand the role production networks play in dampening frequency responses, in Figure 8 (a) we plot a linear relationship between the sectoral frequency responses and a measure of *customer centrality*, given by the sum of rows of the Leontief inverse matrix, representing the total (direct and indirect) exposure of a sector to purchases of intermediate inputs from other sectors. As

Figure 8: **Sectoral responses vs total intermediates exposure**

(a) Sectoral frequency responses



(b) Size-dependence of sectoral prices



***Notes**: the figures plots the responses of frequencies and sectoral price indices to a large monetary shock.*

can be seen, a higher centrality of customers is associated with a smaller increase in the frequency following the 10% monetary shock.

Similarly, we also investigate the association between customer centrality and the degree of size dependence in sectoral price responses to small vs. large monetary shocks. In Figure 8 (b) we plot a measure of size dependence in sectoral price responses, given by the difference in *normalized* responses to 10% and 0.1% monetary shocks, against the measure of customer centrality. Consistently with our mechanism, a higher total exposure to intermediate inputs lowers size dependence in the sectoral price response.

6 Quantitative results: TFP shocks

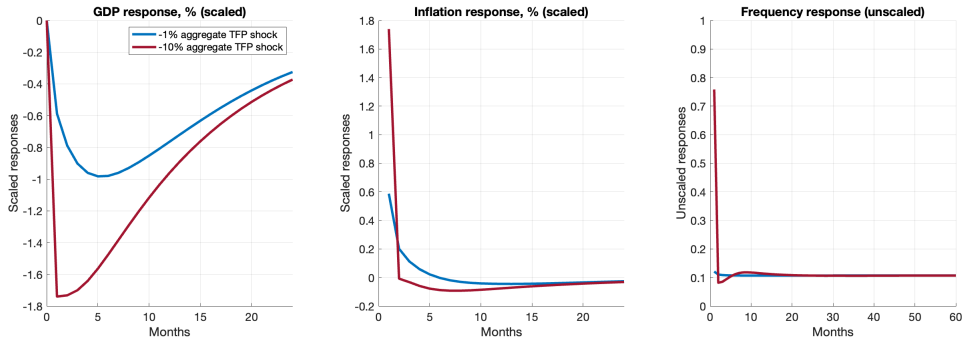
6.1 Aggregate TFP shocks

6.1.1 Aggregate dynamics

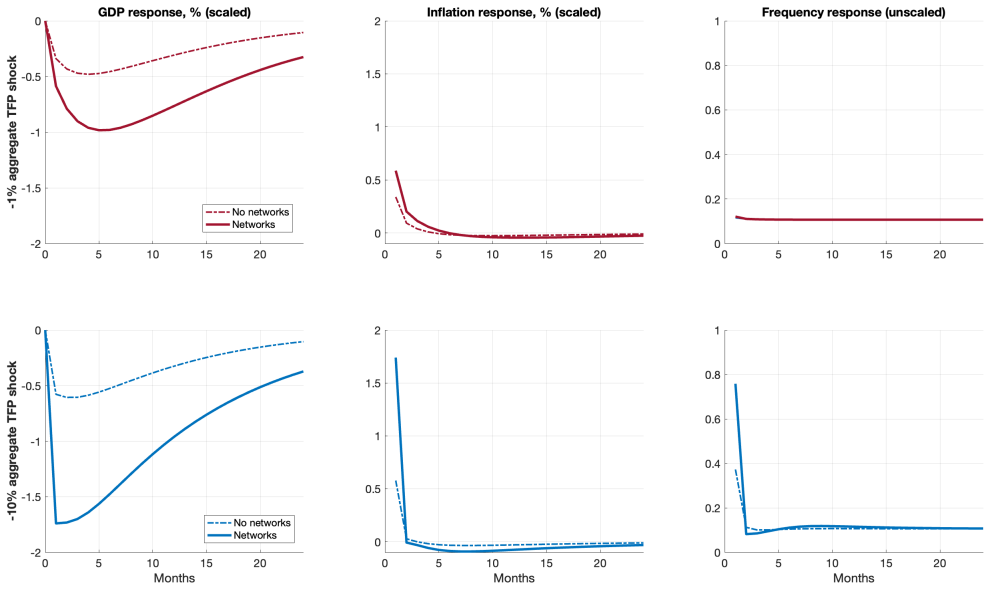
In Figure 9 (a), we report normalized responses of aggregate GDP and CPI inflation, as well as the aggregate fraction of adjusters in response to two negative aggregate TFP shocks: -1% and -10%. Just as with monetary shocks in the previous section, there is substantial *size dependence*: for the -1% shock the normalized response of GDP is -1%, whereas it is just under -2% for the -10% shock. At the same time, the normalized response of CPI inflation increases in the magnitude of the aggregate TFP shock, implying that the aggregate price changes rise more than proportionally in the size of the innovation. Quantitatively, the -1% shock generates a normalized impact response

Figure 9: **Responses to aggregate TFP shocks**

(a) Responses of aggregates to aggregate TFP shocks shocks



(b) Network contribution to aggregates' responses



***Notes**: the figure shows the responses of aggregate GDP, inflation and frequency of adjustment in response to aggregate TFP shocks of different sizes.*

of CPI inflation of 0.6%, whereas the -10% corresponds to a normalized response of almost 1.7% on impact. Key to the observed size dependence is the endogenous response of the fraction of adjusters: for the -1% shock it remains unchanged, whereas the larger -10% shock brings the fraction of adjusters to almost 80%.

In order to understand the contribution of networks to the observed size dependence, in Figure 9 (b) we additionally document the responses to the same aggregate TFP in an otherwise identical economy without networks. For the -1% shock, networks amplify the response of aggregate GDP by a factor 2, while the normalized response of CPI inflation is nearly 0.3% without networks, compared to 0.6% under networks. Significantly, for the larger shocks of -10%, the network amplification of both aggregate consumption and CPI inflation is greater than under the small -1% shock. When it comes to the response of inflation, this is the opposite of what we observed under monetary shocks, where the amplification of inflation response was weakening as shocks became larger. To understand the difference, it is instructive to look at the response of adjustment frequencies. One can see that for the -10% shock, the fraction of adjusters increases substantially more in the economy with networks. This is the exact opposite of what is observed under monetary shocks, in which networks dampen the response of adjustment frequencies, and which represents the *cascades* effect.

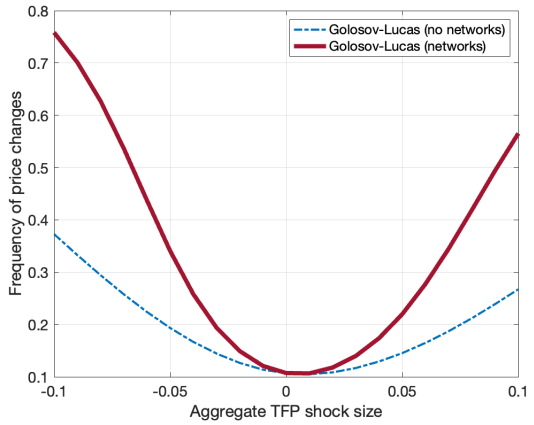
In Figure 10(a), we further investigate the interaction between the aggregate repricing frequency and the size of the aggregate TFP shock. Contrary to what we established for monetary shocks, networks consistently and substantially amplify the response of the aggregate fraction of adjusters to aggregate TFP shocks. For example, following a -10% aggregate TFP shock, the economy with networks predicts a rise in the fraction of adjusters to 75%, while in an otherwise identical model without networks the corresponding increase in the aggregate adjustment frequency is up to just below 40%.

In Figure 10(b) we show the resulting decomposition of the differential inflation responses in our model with networks compared to the one without networks. The difference between the network and no-network cases is explained mainly by the selection effect for smaller shocks, and by the frequency component for aggregate TFP shocks greater than 3 percent in absolute value.

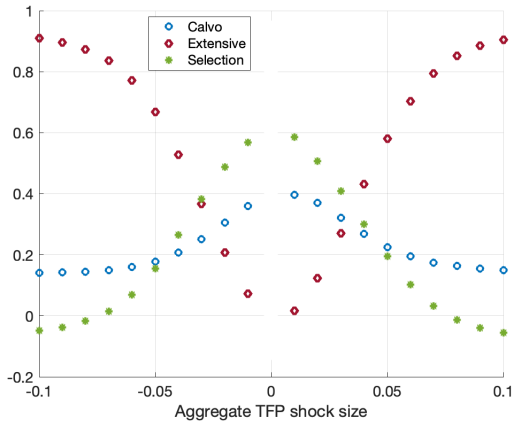
The fact that networks amplify the response of adjustment frequency to aggregate TFP shocks also has notable implications for the dynamics of aggregate CPI inflation and GDP. In Figure 11 (a), we plot the impact response of aggregate CPI inflation to aggregate TFP shocks of different signs and sizes. The inflation response rises much faster in the economy with networks relative to the no-network benchmark, being almost 3.5 times higher after a -10% shock. We also report the inflation responses in an economy with networks, but with time-dependent (Calvo, 1983) pricing, matching the same sectoral frequencies of adjustment in steady state. One can see that the economy with time-dependent pricing predicts dramatically smaller inflation responses.

Figure 10: **Aggregate frequency responses to aggregate TFP shocks**

(a) Aggregate frequency response



(b) Contribution of the extensive margin



***Notes**: the figure shows the impact responses of aggregate frequency of adjustment to aggregate TFP shocks of different sizes, with and without networks.*

6.1.2 Disaggregated dynamics

We now turn to analyzing the responses of individual sectors to aggregate TFP shocks of different sizes.

In Figure 15(a), we plot a relationship between the sectoral frequency responses to the -10% aggregate TFP shock and the measure of customer centrality. There is a clear positive relationship between the two: *ceteris paribus*, increasing a sector's total exposure to intermediate inputs increases the fraction of firms in that sector that choose to adjust following the large aggregate TFP shock.

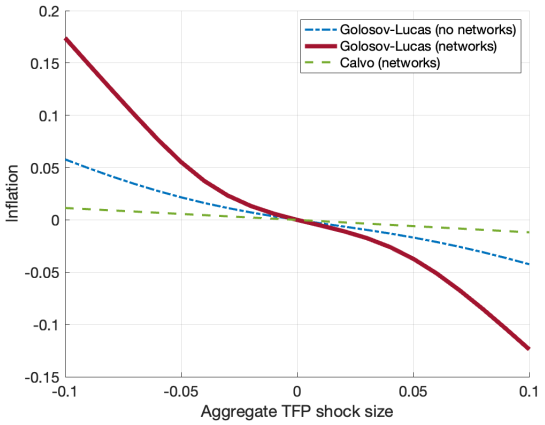
The established relationship between frequency response and exposure to intermediate inputs also has implications for the degree of size dependence in sectoral price dynamics. To see that, in Figure 15(b) we plot the linear relationship between a measure of sectoral price size dependence, given by the difference between normalized responses to -10% and -0.1% aggregate TFP shocks, and the sectoral customer centrality. The estimated relationship is positive, implying that a higher total exposure to intermediate inputs is associated with a greater degree of size dependence in the sectoral price response to a large aggregate TFP shock.

6.2 Sectoral TFP shocks

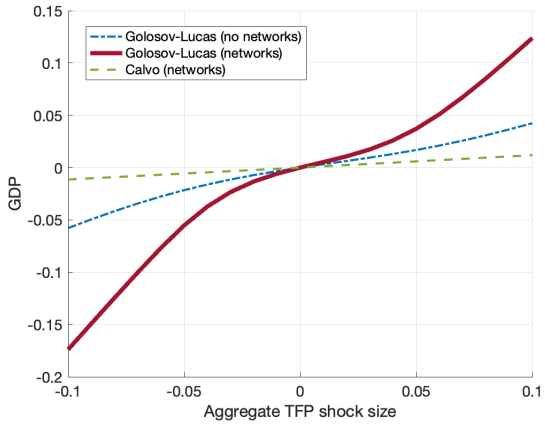
We now turn to the study of the transmission of sector-specific TFP shocks. We focus on the transmission of large contractionary sectoral shocks, modeled as a -20% reduction in sector-specific TFP.

Figure 11: **Inflation and output responses to aggregate TFP shocks**

(a) Impact inflation response



(b) Impact GDP response



***Notes**: the figure shows the responses of aggregate inflation and GDP to large aggregate TFP shocks under different configurations of the model.*

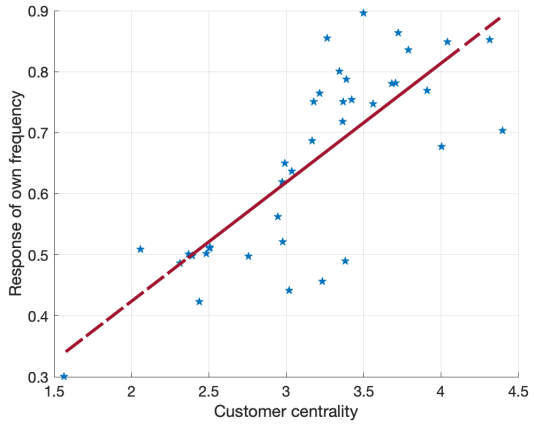
In Figure 13(a), we show the responses of the aggregate fraction of adjusting firms to sector-specific TFP shocks. First, for all sectors, networks amplify the aggregate frequency response to sector-specific TFP shock. Second, for the majority of sectors, the effect of their own shock on aggregate frequency is relatively modest and not much bigger than in the otherwise identical economy without networks. Third, for a number of sectors, such as "Food and Beverages", "Chemicals and Chemical Products" and "Warehousing", networks deliver a substantial amplification of their own TFP shocks on aggregate adjustment frequency.

We also study the response of aggregate CPI inflation to sectoral TFP shocks. Specifically, in Figure 13(b) we depict normalized impact responses of aggregate CPI inflation to large (-20%) negative sectoral TFP shocks. First, networks amplify the aggregate CPI inflation responses for all sectoral TFP shocks. Second, just as with aggregate frequencies, for the majority sectors, the network amplification is relatively modest. Third, some sectors are an exception: "Food and Beverages", "Mining and Quarrying" and "Chemicals and Chemical Products" have their own TFP shocks affect the aggregate CPI inflation response substantially.

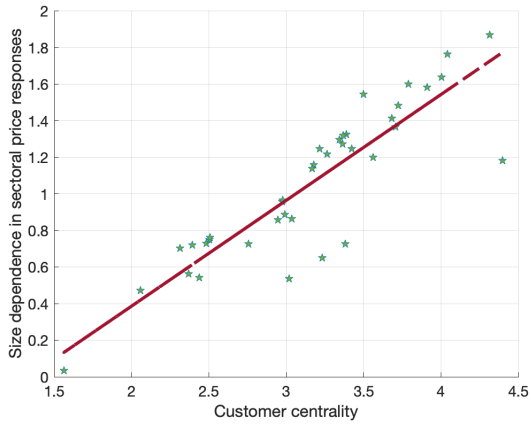
In Figure 14, we investigate the relationship between a sector's *supplier centrality* and the aggregate consequences of that sector's TFP shocks. In Figure 14(a), one sees a very strong positive relationship between the effect of a sectoral TFP shock on the aggregate frequency of adjustment and the centrality measure of that sector.

Figure 12: **Sectoral responses to aggregate TFP shock vs total intermediates exposure**

(a) Sectoral frequency responses



(b) Size-dependence of sectoral prices



***Notes**: the figure shows the responses of sectoral frequency and sectoral price indices to large aggregate TFP shocks.*

7 Application: post-COVID inflation in the Euro Area

7.1 Matching selected time series

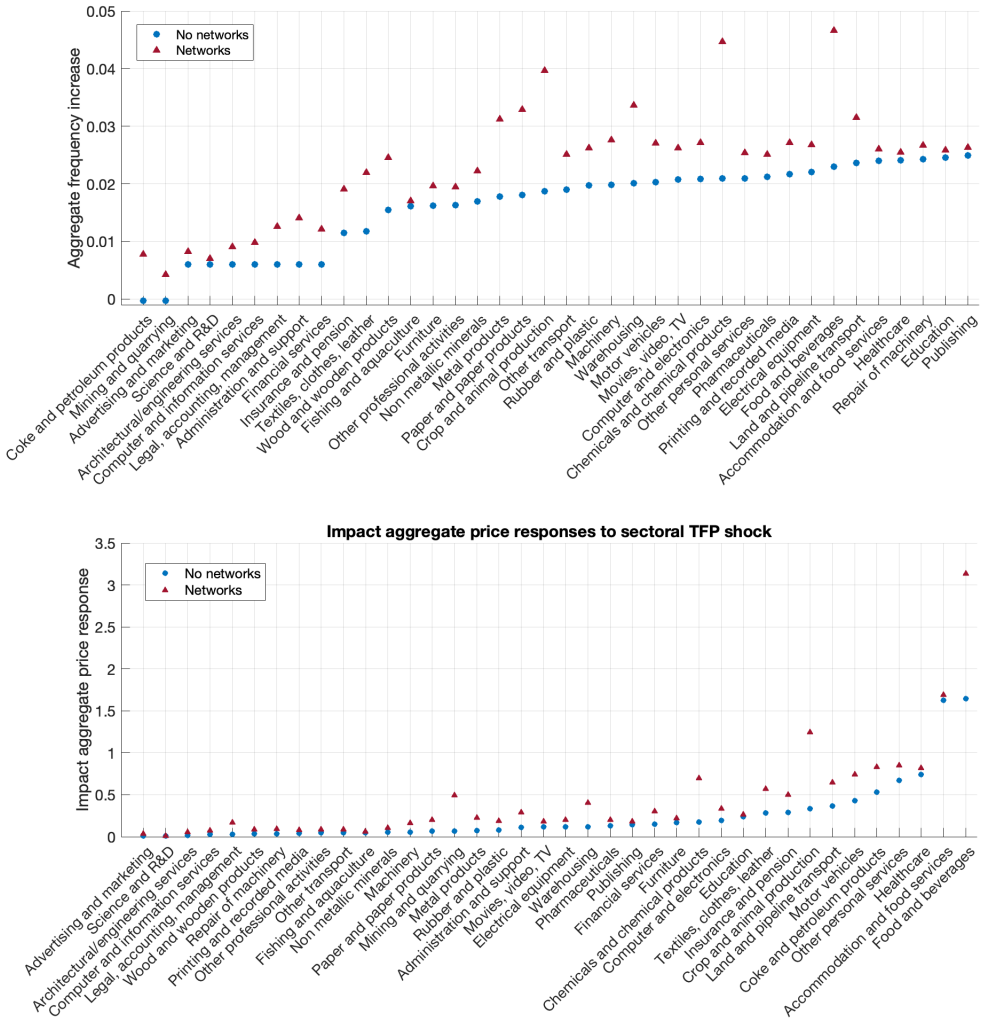
In our exercise, we are going to consider four exogenous shock series. First, we feed in the Euro Area nominal GDP in order to approximate the aggregate money supply series $\{M_t\}_{2019:1}^{2024:6}$. Second, we are going to estimate an exogenous TFP process in the labor union sector $\{A_t^{\text{UNION}}\}_{2019:1}^{2024:6}$ in order to exactly match the Euro Area nominal hourly earnings series in equilibrium. Third, we fit an exogenous TFP process in the "Mining and Quarrying" sector $\{A_t^{\text{ENERGY}}\}_{2019:1}^{2024:6}$ in order to exactly match the real IMF Energy Price Index in equilibrium. Finally, we also feed in an exogeneous TFP series in the "Crop and Animal Production" sector $\{A_t^{\text{FOOD}}\}_{2019:1}^{2024:6}$ in order to exactly match the real IMF Food Price Index.

7.2 Explaining the surge in frequency and inflation

Figure 15 shows the actual vs model-implied changes in aggregate frequency of adjustment and aggregate CPI inflation in the Euro Area. In panel (a), we can see that the baseline fully non-linear model with state-dependent pricing and production networks successfully produces a surge in the aggregate frequency of adjustment of the magnitude observed in the Euro Area microdata. Moreover, the baseline model is also able to generate an increase in the aggregate CPI inflation of the same value as observed in the actual data.

At the same time, as can be seen in panel (b) of Figure 15, an otherwise identical model without production networks fails to match the data, both when in comes to the frequency of adjustment,

Figure 13: **Aggregate responses to sectoral TFP shocks**



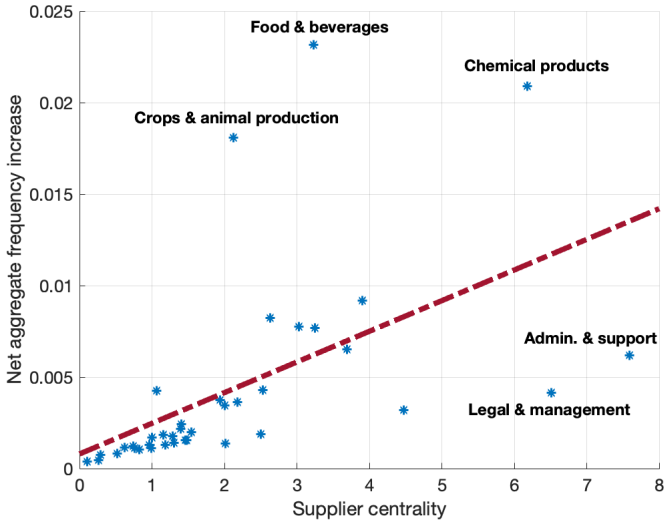
***Notes**: the figure shows the responses of sectoral frequency and sectoral price indices to large aggregate TFP shocks.*

and when it comes to CPI inflation. One can see that the aggregate frequency of adjustment remains essentially flat in the model without networks, apart from a minor uptick at the very beginning. At the same time, the model without networks generates an inflation increase only up to 4% annualized level, whereas in the data it increased all the way to 10%.

8 Conclusions

In this paper, we develop and solve a novel quantitative dynamic general equilibrium model that allows us to study the transmission of large aggregate and sector-specific shocks in economies with realistic input-output linkages and state-dependent pricing decisions as in Golosov and Lucas

Figure 14: **Aggregate responses to sectoral TFP shock vs sector centrality**



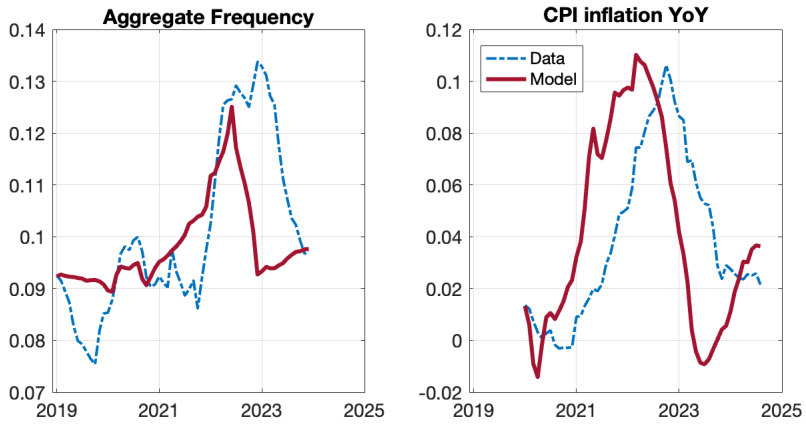
Notes: *the figure shows the responses of aggregate adjustment frequency and aggregate inflation to TFP shocks to different sectors.*

(2007). Our framework makes predictions about the interactions of production networks with pricing decisions at the extensive margin, which is also quantitatively relevant for the dynamics of macroeconomic aggregates.

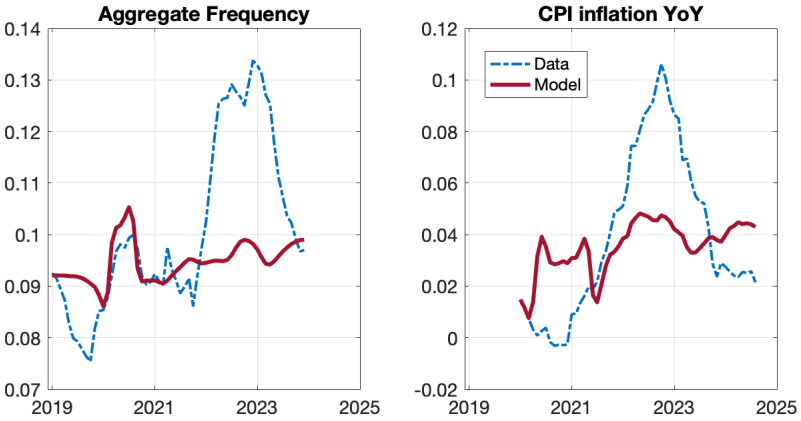
For aggregate nominal shocks, such as a change in the money supply, production networks shrink the magnitudes of desired price changes at the firm level, which in turn compresses the sizes of price gaps for all firms. As a result, the presence of networks slows the extensive margin of price adjustment decisions, lowering the response of the aggregate repricing frequency. At the same time, large aggregate and sector-specific total factor productivity (TFP) shocks interact with the production network in a manner opposite to that under monetary shocks. In particular, production networks amplify firm-level desired price changes following TFP shocks, expanding the price gaps, and making the decision to adjust more likely, *ceteris paribus*. As a result, large negative TFP shocks can lead to fast increases in the aggregate repricing frequency, and to rapid inflationary surges.

Figure 15: **Explaining the observed surge in frequency and inflation surge**

(a) Model with production networks



(b) Model without production networks



Notes: *the figure shows the model-implied changes in aggregate frequency of adjustment and aggregate CPI inflation versus the actual observed values in the Euro Area. Panel (a) considers the baseline model with production networks, whereas panel (b) considers an an otherwise identical economy without networks.*

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Appendix

Business Cycles and Pricing Cascades

Mishel Ghassibe, *CREi, UPF & BSE*
Anton Nakov, *European Central Bank*

A Proofs

Proof of Lemma 1. We want to find a second-order approximation of the firm-level profit function $\tilde{D}_{i,t}(j)$ in the log quality-adjusted real price of that firm $\log \tilde{P}_{i,t}(j)$ near the optimum $\log \tilde{P}^*_{i,t}$. By definition of the optimal reset price, $\frac{\partial \tilde{D}_{i,t}(j)}{\partial \log \tilde{P}_{i,t}(j)}|_{\tilde{P}_{i,t}(j)=\tilde{P}^*_{i,t}} = 0$. As for the second derivative, one can show that:

$$\frac{\partial^2 \tilde{D}_{i,t}(j)}{\partial \log \tilde{P}_{i,t}(j)^2} = \left[(1-\tau_{i,t})e^{(1-\epsilon)\log \tilde{P}_{i,t}(j)}(1-\epsilon)^2 - \epsilon^2 \tilde{\mathcal{Q}}_{i,t} e^{-\epsilon \log \tilde{P}_{i,t}(j)}\right] \times \tilde{P}^{\epsilon}_{i,t} Y_{i,t}. \tag{36}$$

Evaluating the second derivative at $\log \tilde{P}^*_{i,t}$, and after some algebra one obtains:

$$\frac{\partial^2 \tilde{D}_{i,t}(j)}{\partial \log \tilde{P}_{i,t}(j)^2}|_{\tilde{P}_{i,t}(j)=\tilde{P}^*_{i,t}(j)} = -(\epsilon-1)(1-\tau_{i,t})\left[\tilde{P}_{i,t}/P^*_{i,t}\right]^{\epsilon-1}\lambda_{i,t}. \tag{37}$$

Therefore, one can write the second-order approximation as:

$$\tilde{D}_{i,t} = \tilde{D}^*_{i,t} + \frac{1}{2}\frac{\partial^2 \tilde{D}_{i,t}(j)}{\partial \log \tilde{P}_{i,t}(j)^2}|_{\tilde{P}_{i,t}(j)=\tilde{P}^*_{i,t}(j)} \times [\tilde{p}_{i,t}(j)]^2 + \mathcal{O}[\tilde{p}_{i,t}(j)^3], \tag{38}$$

where $\tilde{p}_{i,t}(j) \equiv [\log \tilde{P}_{i,t}(j) - \log \tilde{P}^*_{i,t}]$ is the firm-level price gap. Inserting the expression for the second derivative, one obtains:

$$\tilde{D}^*_{i,t} - \tilde{D}_{i,t} = \frac{1}{2}(\epsilon-1)(1-\tau_{i,t})\left[\tilde{P}_{i,t}/\tilde{P}^*_{i,t}\right]^{\epsilon-1}\lambda_{i,t} \times [\tilde{p}_{i,t}(j)]^2 + \mathcal{O}[\tilde{p}_{i,t}(j)^3]. \tag{39}$$

□

Proof of Lemma 2. Focusing on period $t = 0$, a firm adjusts its price if the profit gain from adjustment exceeds the menu cost:

$$\tilde{D}_{i,0}(j)^* - \tilde{D}_{i,0}(j) \geq \kappa_{i,0} \tag{40}$$

Using the approximation for the profit gain in Lemma 1, as well as the menu cost form in Assumption 6, one can further rewrite the adjustment condition as:

$$\frac{1}{2}(\epsilon-1)(1-\tau_{i,0})\left[\tilde{P}_{i,0}/\tilde{P}^*_{i,0}\right]^{\epsilon-1}\lambda_{i,0} \times [\tilde{p}_{i,0}(j)]^2 \geq \overline{\kappa}_i(1-\tau_{i,0})[\tilde{P}_{i,0}/\tilde{P}^*_{i,0}]^{\epsilon-1}\lambda_{i,0}, \tag{41}$$

$$\implies \quad [\tilde{p}_{i,0}(j)]^2 \geq \frac{2\overline{\kappa}_i}{\epsilon-1}. \tag{42}$$

Using the expression for the price gap in (28), as well as the normalization $p_{i,-1}(j) = \log \frac{\epsilon}{\epsilon-1}\frac{1}{1-\overline{\tau}_i}$, the adjustment condition becomes:

$$\left| -\sigma_i \varepsilon_{i,0}(j) - m_0 - \gamma_{i,0} + a_{i,0} - \sum_{k=1}^{N} \overline{\omega}_{ik} \log \tilde{P}_{k,0} \right| \geq \sqrt{\frac{2\overline{\kappa}_i}{\epsilon - 1}}. \tag{43}$$

Therefore, the inaction region is given by:

$$[\sigma_i \underline{\varepsilon}_{i,0}, \quad \sigma_i \overline{\varepsilon}_{i,0}] \quad = \quad -m_0 - \gamma_{i,0} + a_{i,0} - \sum_{k=1}^{N} \overline{\omega}_{ik} \log \tilde{P}_{k,0} \quad \pm \quad \sqrt{\frac{2\overline{\kappa}_i}{\epsilon - 1}}, \tag{44}$$

where $m_0 \equiv \log(M_0/M_{-1})$, $\gamma_{i,0} \equiv \log \Gamma_{i,0} - \overline{\gamma}_i$, $a_{i,0} \equiv \log A_{i,0}$ and $\overline{\gamma}_i \equiv \log \frac{\epsilon}{\epsilon-1}\frac{1}{1-\overline{\tau}_i}$ □

Proof of Proposition 1. Before providing a proof for Proposition 1, it is useful to formally establish an auxiliary technical result:

Lemma A1. *Define $f_+(x;c) \equiv \Phi(c+x) - \Phi(-c+x)$ and $f_-(x;c) \equiv \Phi(c-x) - \Phi(-c-x)$, where $c > 0$ is a parameter and $\Phi(.)$ is standard normal CDF. Then both $f_+(x;c)$ and $f_-(x;c)$ are decreasing in x for all $x > 0$.*

Proof. First, consider $f_+(x;c)$. Notice that $f_+'(x) = \Phi'(c+x) - \Phi'(-c+x) = \phi(c+x) - \phi(-c+x)$, where $\phi(.)$ is standard normal PDF. Hence, $f_+'(0) = \phi(c) - \phi(-c) = 0$. As for any $x \in (0, c]$, one can deduce that $f_+'(x) = \underbrace{\phi(c+x)}_{<\phi(c)} - \underbrace{\phi(-c+x)}_{>\phi(-c)} < 0$. Further, for any $x > c$ it follows that $f_+'(x) = \phi(c+x) - \phi(-c+x) < 0$, since standard normal PDF is decreasing in positive inputs. All in all, we conclude that $f_+'(x) < 0$ for all $x > 0$.

Similarly, $f_-'(x) = -\Phi'(c-x) + \Phi'(-c-x) = -\phi(c-x) + \phi(-c-x)$. As before, $f_-'(0) = -\phi(c) + \phi(-c) = 0$. For any $x \in (0, c]$, $f_-'(x) = -\underbrace{\phi(c-x)}_{>\phi(c)} + \underbrace{\phi(-c-x)}_{<\phi(-c)} < 0$. As for any $x > c$, $f_-'(x) = -\phi(c-x) + \phi(-c-x) < 0$, since standard normal PDF is increasing in negative inputs. In total, we conclude that $f_-'(x) < 0$ for all $x > 0$. □

Armed with the additional result in Lemma A1, we are now ready to prove Proposition 1. Consider a monetary expansion $m_0 > 0$. The probability that a firm draws an idiosyncratic innovation

that lies in the inaction region following the monetary expansion is given by:

$$\begin{aligned} Pr(\underline{\varepsilon}_{i,0} \leq \varepsilon_{i,0}(j) \leq \overline{\varepsilon}_{i,0}) &= \Phi\left(\frac{1}{\sigma_i}\sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}} - \frac{1}{\sigma_i}\left\{m_0 + \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0}\right\}\right) \\ &\quad - \Phi\left(-\frac{1}{\sigma_i}\sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}} - \frac{1}{\sigma_i}\left\{m_0 + \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0}\right\}\right) \\ &= f_-\left(\frac{1}{\sigma_i}\left\{m_0 + \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0}\right\}; \quad \frac{1}{\sigma_i}\sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}}\right), \end{aligned} \tag{45}$$

where $f_-(.)$ is defined in Lemma A1. Now, as long as the pass-through of the monetary expansion to sectoral prices is incomplete, $\log\tilde{P}_{k,0} < 0, \forall k$, it follows that $m_0 + \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0} < m_0$. Moreover, since $f_-(.)$ is falling in its positive inputs, it immediately follows that:

$$f_-\left(\frac{1}{\sigma_i}\left\{m_0 + \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0}\right\}; \quad \frac{1}{\sigma_i}\sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}}\right) > f_-\left(\frac{1}{\sigma_i}m_0; \quad \frac{1}{\sigma_i}\sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}}\right). \tag{46}$$

Hence, *ceteris paribus*, the probability of drawing a shock in the inaction region following a monetary expansion is higher in the economy with networks.

Similarly, consider a monetary contraction $m_0 < 0$. The probability that a firm draws an idiosyncratic innovation that lies in the inaction region following the monetary contraction is given by:

$$\begin{aligned} Pr(\underline{\varepsilon}_{i,0} \leq \varepsilon_{i,0}(j) \leq \overline{\varepsilon}_{i,0}) &= \Phi\left(\frac{1}{\sigma_i}\sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}} + \frac{1}{\sigma_i}\left\{-m_0 - \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0}\right\}\right) \\ &\quad - \Phi\left(-\frac{1}{\sigma_i}\sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}} + \frac{1}{\sigma_i}\left\{-m_0 - \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0}\right\}\right) \\ &= f_+\left(\frac{1}{\sigma_i}\left\{-m_0 - \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0}\right\}; \quad \frac{1}{\sigma_i}\sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}}\right), \end{aligned} \tag{47}$$

where $f_+(.)$ is defined in Lemma A1. Now, as long as the pass-through of the monetary contraction to sectoral prices is incomplete, $\log\tilde{P}_{k,0} > 0, \forall k$, it follows that $-m_0 - \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0} < -m_0$. Moreover, since $f_+(.)$ is falling in its positive inputs, it immediately follows that:

$$f_+\left(-\frac{1}{\sigma_i}\left\{m_0 + \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0}\right\}; \quad \frac{1}{\sigma_i}\sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}}\right) > f_+\left(-\frac{1}{\sigma_i}m_0; \quad \frac{1}{\sigma_i}\sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}}\right). \tag{48}$$

Hence, *ceteris paribus*, the probability of drawing a shock in the inaction region following a monetary contraction is higher in the economy with networks. □

Proof of Proposition 2. Consider a productivity deterioration $a_{i,0} < 0$ and/or a rise in desired markups $\gamma_{i,0} > 0$ in sector i. The probability that a firm in sector i' (which may or may not be the same as i) draws an idiosyncratic innovation that lies in the inaction region following productivity deterioration/markup increase in sector i is given by:

$$\begin{aligned} Pr(\underline{\varepsilon}_{i',0} \leq \varepsilon_{i',0}(j) \leq \overline{\varepsilon}_{i',0}) &= \Phi\left(\frac{1}{\sigma_{i'}}\sqrt{\frac{2\overline{\kappa}_{i'}}{\epsilon-1}} - \frac{1}{\sigma_{i'}}\left\{-a_{i',0} + \gamma_{i',0} + \sum_{k=1}^{N}\overline{\omega}_{i'k}\log\overline{P}_{k,0}\right\}\right) \\ &\quad - \Phi\left(-\frac{1}{\sigma_{i'}}\sqrt{\frac{2\overline{\kappa}_{i'}}{\epsilon-1}} - \frac{1}{\sigma_{i'}}\left\{-a_{i',0} + \gamma_{i',0} + \sum_{k=1}^{N}\overline{\omega}_{i'k}\log\overline{P}_{k,0}\right\}\right) \\ &= f_{-}\left(\frac{1}{\sigma_{i'}}\left\{-a_{i',0} + \gamma_{i',0} + \sum_{k=1}^{N}\overline{\omega}_{i'k}\log\overline{P}_{k,0}\right\}; \quad \frac{1}{\sigma_{i'}}\sqrt{\frac{2\overline{\kappa}_{i'}}{\epsilon-1}}\right), \quad (49) \end{aligned}$$

where $f_{-}(.)$ is defined in Lemma A1. Now, as long as the productivity deterioration/markup increase in sector i leads to a rise in sectoral prices of all other sectors, $\log\tilde{P}_{k,0} > 0, \forall k$, it follows that $-a_{i',0} + \gamma_{i',0} + \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0} > -a_{i',0} + \gamma_{i',0}$, $\forall i'$. Moreover, since $f_{-}(.)$ is falling in its positive inputs, it immediately follows that:

$$f_{-}\left(\frac{1}{\sigma_{i'}}\left\{-a_{i',0} + \gamma_{i',0} + \sum_{k=1}^{N}\overline{\omega}_{i'k}\log\overline{P}_{k,0}\right\}; \frac{1}{\sigma_{i'}}\sqrt{\frac{2\overline{\kappa}_{i'}}{\epsilon-1}}\right) < f_{-}\left(\frac{1}{\sigma_{i'}}\left\{-a_{i',0} + \gamma_{i',0}\right\}; \frac{1}{\sigma_{i'}}\sqrt{\frac{2\overline{\kappa}_{i'}}{\epsilon-1}}\right). \quad (50)$$

Hence, *ceteris paribus*, the probability that a firm in sector i' draws shock in the inaction region following a productivity deterioration/markup increase in sector i is lower in the economy with networks.

Similarly, consider a productivity improvement $a_{i,0} > 0$ and/or a fall in desired markups $\gamma_{i,0} < 0$ in sector i. The probability that a firm in sector i' (which may or may not be the same as i) draws an idiosyncratic innovation that lies in the inaction region following productivity improvement/markup decrease in sector i is given by:

$$\begin{aligned} Pr(\underline{\varepsilon}_{i',0} \leq \varepsilon_{i',0}(j) \leq \overline{\varepsilon}_{i',0}) &= \Phi\left(\frac{1}{\sigma_{i'}}\sqrt{\frac{2\overline{\kappa}_{i'}}{\epsilon-1}} + \frac{1}{\sigma_{i'}}\left\{a_{i',0} - \gamma_{i',0} - \sum_{k=1}^{N}\overline{\omega}_{i'k}\log\overline{P}_{k,0}\right\}\right) \\ &\quad - \Phi\left(-\frac{1}{\sigma_{i'}}\sqrt{\frac{2\overline{\kappa}_{i'}}{\epsilon-1}} + \frac{1}{\sigma_{i'}}\left\{a_{i',0} - \gamma_{i',0} - \sum_{k=1}^{N}\overline{\omega}_{i'k}\log\overline{P}_{k,0}\right\}\right) \\ &= f_{+}\left(\frac{1}{\sigma_{i'}}\left\{a_{i',0} - \gamma_{i',0} - \sum_{k=1}^{N}\overline{\omega}_{i'k}\log\overline{P}_{k,0}\right\}; \quad \frac{1}{\sigma_{i'}}\sqrt{\frac{2\overline{\kappa}_{i'}}{\epsilon-1}}\right), \quad (51) \end{aligned}$$

where $f_{+}(.)$ is defined in Lemma A1. Now, as long as the productivity improvement/markup decrease in sector i leads to a fall in sectoral prices of all other sectors, $\log\tilde{P}_{k,0} < 0, \forall k$, it follows that $-a_{i',0} + \gamma_{i',0} + \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0} < -a_{i',0} + \gamma_{i',0}$, $\forall i'$. Moreover, since $f_{+}(.)$ is falling in its positive

inputs, it immediately follows that:

$$f_+\left(-\frac{1}{\sigma_{i'}}\left\{a_{i',0}-\gamma_{i',0}-\sum_{k=1}^{N}\overline{\omega}_{i'k}\log\overline{P}_{k,0}\right\};\frac{1}{\sigma_{i'}}\sqrt{\frac{2\overline{\kappa}_{i'}}{\epsilon-1}}\right)<f_+\left(-\frac{1}{\sigma_{i'}}\left\{a_{i',0}-\gamma_{i',0}\right\};\frac{1}{\sigma_{i'}}\sqrt{\frac{2\overline{\kappa}_{i'}}{\epsilon-1}}\right). \tag{52}$$

Hence, *ceteris paribus*, the probability that a firm in sector i' draws shock in the inaction region following a productivity deterioration/markup increase in sector i is lower in the economy with networks.

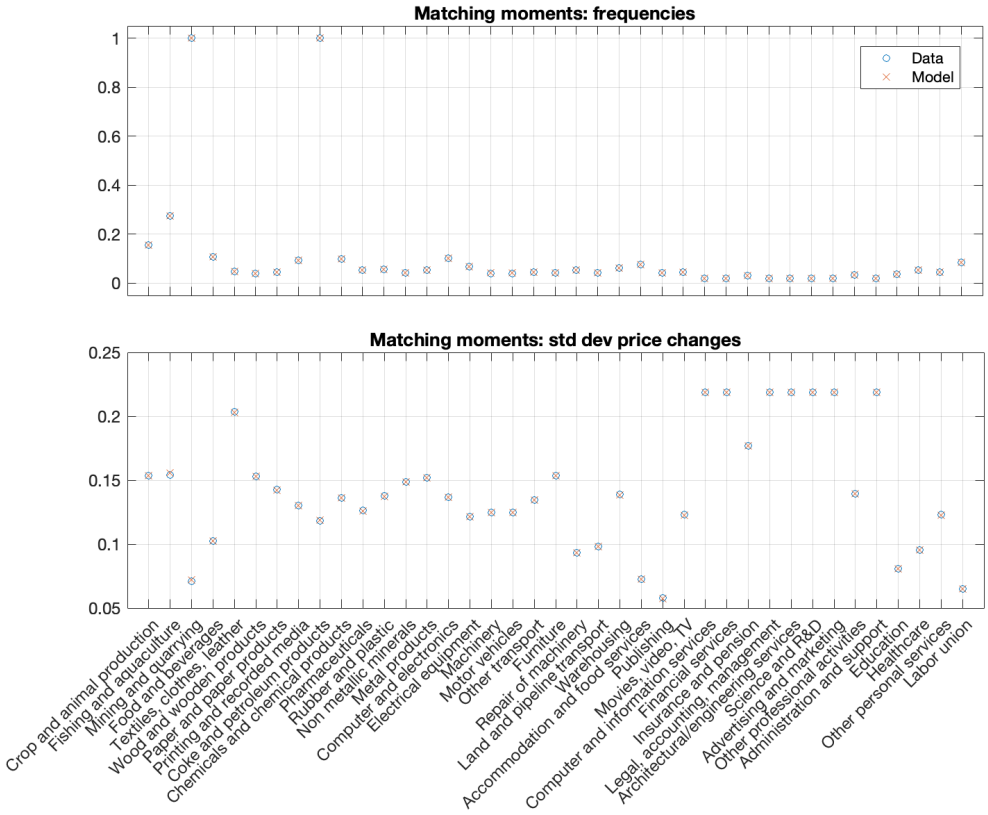
Note that while we provide a proof for sector-specific productivity/markup shocks, results are equivalent for aggregate productivity/markup shocks. This is the case since an aggregate productivity shock a is merely a combination of equally-sized sector-specific productivity shocks $a_i = a, \forall i$, and similarly for an aggregate markup shocks. □

B Additional calibration details (Euro Area)

Table 2: Sectors with Consumption Weights, Suppliers, and Customers Centrality

Sector Name	*Consumption Share*	*Supplier Centrality*	*Customer Centrality*
Crop and animal production	0.0290	2.1266	3.5007
Fishing and aquaculture	0.0039	0.1070	3.2638
Mining and quarrying	0.0051	2.5263	3.2343
Food and beverages	0.1430	3.2264	4.0024
Textiles, clothes, leather	0.0403	1.4829	3.7247
Wood and wooden products	0.0044	1.3072	3.6832
Paper and paper products	0.0079	2.6301	4.0412
Printing and recorded media	0.0035	1.0667	3.3900
Coke and petroleum products	0.0398	3.0308	4.3996
Chemicals and chemical products	0.0162	6.1838	4.3165
Pharmaceuticals	0.0140	0.7374	3.5602
Rubber and plastic	0.0088	2.0056	3.7898
Non metallic minerals	0.0066	0.9865	3.4223
Metal products	0.0081	3.2480	3.1769
Computer and electronics	0.0175	1.9379	3.3442
Electrical equipment	0.0115	1.4046	3.3690
Machinery	0.0066	2.1823	3.3636
Motor vehicles	0.0514	1.5453	3.9103
Other transport	0.0057	0.8266	3.7066
Furniture	0.0224	0.6181	3.1662
Repair of machinery	0.0030	1.1591	2.9738
Land and pipeline transport	0.0398	3.6878	3.0361
Warehousing	0.0125	3.9047	3.2174
Accommodation and food services	0.1475	1.0026	2.9790
Publishing	0.0138	0.7571	3.0197
Movies, video, TV	0.0131	1.2814	2.9922
Computer and information services	0.0069	2.5026	2.4842
Financial services	0.0391	4.4781	2.7573
Insurance and pension	0.0502	1.4619	3.3833
Legal, accounting, management	0.0079	6.5099	2.3130
Architectural/engineering services	0.0037	2.0129	2.3928
Science and R&D	0.0020	0.2637	2.5040
Advertising and marketing	0.0020	1.1808	2.9466
Other professional activities	0.0069	0.9650	2.4393
Administration and support	0.0293	7.5903	2.5066
Education	0.0261	0.5206	1.5639
Healthcare	0.0743	0.2930	2.0575
Other personal services	0.0765	1.3971	2.3677

Figure 16: Matching pricing moments for each sector



***Note**: the figure shows the sector-specific frequencies and sizes of price adjustment, as well as the model-based equivalents under our calibration.*

C Details of the numerical algorithm

C.1 Steady state computation on a grid

For each sector, we solve the stationary Bellman equation and price distribution on an evenly spaced grid of log prices $\mathbf{\Gamma}$ with step size Δp, $p_j \in \left[\underline{p}, \underline{p} + \Delta p, ..., \overline{p}\right]$, $j = 1, .., J$ grid points, so that $V_j = V(p_j)$. The expectation $\mathbb{E}\left[V(p - \sigma\varepsilon_{t+1} - \pi)|p = p_j\right]$ is calculated as $\boldsymbol{TV}$ where we define transition matrix

$$\boldsymbol{T} = \begin{bmatrix} \mathcal{T}_{1,1} & \mathcal{T}_{1,2} & \cdots & \mathcal{T}_{1,J} \\ \mathcal{T}_{2,1} & \mathcal{T}_{2,2} & \cdots & \mathcal{T}_{2,J} \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{T}_{J,1} & \mathcal{T}_{J,2} & \cdots & \mathcal{T}_{J,J} \end{bmatrix}.$$

with elements

$$\mathcal{T}_{j,k} = \int_{p=p_{k-1/2}}^{p_{k+1/2}} \psi\left(\frac{p-(p_j-\pi)}{\sigma}\right) dp = \Psi\left(\frac{p_{k+1/2}-(p_j-\pi)}{\sigma}\right) - \Psi\left(\frac{p_{k-1/2}-(p_j-\pi)}{\sigma}\right),$$

and where $p_{k-1/2} \equiv (p_{k-1}+p_k)/2$, $p_{k+1/2} \equiv (p_k+p_{k+1})/2$, $\psi(\cdot)$ is the standard normal probability density function, and $\Psi(\cdot)$ is the standard normal cumulative distribution function.

We also define the vectors

$$\boldsymbol{\phi} = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_J \end{bmatrix}, \ \eta = \begin{bmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_J \end{bmatrix}, \ \boldsymbol{V} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_J \end{bmatrix}, \ \boldsymbol{D} = \begin{bmatrix} D_1 \\ D_2 \\ \vdots \\ D_J \end{bmatrix}$$

The Bellman equation in matrix notation is then given by

$$\boldsymbol{V} \quad = \quad \boldsymbol{D} + \beta\left[\boldsymbol{T}\left((1-\eta)\cdot\boldsymbol{V}\right) + \boldsymbol{T}\left(\eta\cdot\left(\boldsymbol{\phi}'\boldsymbol{V} - \kappa w\right)\right)\right]$$

where $\cdot$ denotes element-by-element multiplication. Vector $\boldsymbol{\phi}$ distributes unit probability mass to grid points adjacent to p^* according to the logit formula

$$\boldsymbol{\phi} = \frac{\exp(\boldsymbol{V}/\chi)}{\sum_\Gamma \exp\left(\boldsymbol{V}/\chi\right)}$$

with $\chi = 0.0005$. Note that $\boldsymbol{\phi}'\boldsymbol{V}$ performs smooth maximization as in eq.(21).

To solve the problem for N sectors with input-output linkages, we use the following algorithm. Start with a guess for the vector of steady-state price dispersions Δ_k and sectoral taxes τ_k, then:[4]

1. Given $\pi = \overline{\pi}$, compute the transition matrix $\boldsymbol{T}$
2. Using $\frac{W}{M} \equiv w = 1$, compute[5] $\omega_{ik} = \overline{\omega}_{ik} \times \frac{(P_k/M)^{1-\theta_i}}{\overline{\alpha}_i + \sum_{k'=1}^{N} \overline{\omega}_{ik'}(P_{k'}/M)^{1-\theta_i}} = \overline{\omega}_{ik}$
3. λ is given by eq.(15) and η by eq.(22)
4. With that, construct the profit matrix $\boldsymbol{D}$ as in eq.(19)
5. Iterate backward on the value function $\boldsymbol{V}$ above to convergence

[4] We start with the guess $\Delta_k = 1$ and $\tau_k = -1/\epsilon$

[5] We are searching for taxes τ_k such that the steady state equilibrium is symmetric in sectoral prices, $P_k/M = 1$

6. To compute the distribution, iterate forward on

$$\boldsymbol{g} = (1-\eta)\cdot(\boldsymbol{T}'\boldsymbol{g}) + \boldsymbol{\phi}\eta'\left(\boldsymbol{T}'\boldsymbol{g}\right). \tag{53}$$

   until convergence of $\boldsymbol{g}$.

7. Given the distribution, compute the residual vectors $resid_1$ and $resid_2$ as in

$$resid_1 = \Delta_k - (P_k/M)^{\epsilon_k}\int_0^1\left(\frac{P_k(j')}{\zeta_k(j')M}\right)^{-\epsilon_k}dj', \tag{54}$$

$$resid_2 = P_k/M - \int_0^1\left(\frac{P_k(j')}{\zeta_k(j')M}\right)^{1-\epsilon_k}dj' \tag{55}$$

8. Search for a vector of sectoral price dispersions and taxes such that $resid \to 10^{-14}$.

C.2 Solving for impulse-responses in sequence space

We compute fully non-linearly the responses to a MIT shock in the space of sequences, iterating backward in time on the value function and forward in time on the law of motion of the distribution, under the assumption of perfect foresight. The steps are similar to those for computing the steady-state, only this time we keep track of the sequences over time. We start by guessing sequences for time t from 1 to $T = 500$ months, for sectoral prices and price dispersions (our starting guess simply equals the steady-state value for these variables). The key assumption is that all stationary variables must return to steady-state by period T. Given this initial guess, we compute the price of the final good and consumption over time using their definitions. We compute ω_t^C and $\omega_{i,k,t}$ as per eq.(**??**). Given that, we calculate λ_t as in eq.(15). We compute the profits D_t as in eq.(19). Iterating backward in time from $t = T$ to $t = 0$, we solve for the value function as in eq.(21). Given the value function, we can compute the gain from adjustment L_t and the adjustment hazard η_t. Once the backward iteration on the value function reaches period 0, we start from the steady-state distribution and iterate forward in time on the law of motion of the price distribution from period 1 until period T. Given the distribution, we can compute via eq.(7) the sectoral price indices, and by

$$\Delta_{k,t} \equiv (P_{k,t}/M_t)^{\epsilon_k}\int_0^1\left(\frac{P_{k,t}(j')}{\zeta_{k,t}(j')M_t}\right)^{-\epsilon_k}dj'$$

the sectoral price dispersions. This provides us with an updated guess, with which we repeat the previous steps until the change in the sequences (of sectoral prices and price dispersions) becomes near zero.

D Additional results, extensions and robustness

D.1 Additional figures for main text

In Figure 17 (a) we report the impact responses of sectoral fractions of adjusters to a 10% monetary shock. First, for all sectors, the fraction of adjusters increases by more in the economy without networks, relative to the baseline economy with input-output linkages. For some sectors, such as "publishing", the percentage of adjusters increases to almost 100%.

Beyond frequency responses, in Figure 17 (b) we additionally document impact responses of sectoral price indices, normalized by the size of the monetary shock (10%). Like with the frequency responses, the baseline economy with networks features a smaller response relative to an otherwise identical economy without networks.

In Figure 18(a) we report responses of sector-specific fractions of adjusting firms to a -10% aggregate TFP shock. First, for all sectors the fraction of adjusters is larger in the baseline economy with networks, relative to the otherwise identical economy without networks. Second, for a number of sectors, such as "Warehousing" and "Land and Pipeline Transport" the fraction rises almost to 100% following the shock. At the same time, for some other sectors, such as "Computer and Information Services", the frequency rises relatively modestly.

In Figure 18(b) we additionally report the impact responses of sector-specific prices to a -10% aggregate TFP shock. Importantly, for all sectors, the overall price response is larger in an economy with networks, relative to the otherwise identical economy without networks.

D.2 Random menu costs

D.3 CES aggregation across sectors

D.4 Cashless limit

The representative household chooses a sequence of consumption, labor supply and one-period nominal bond holdings to maximize expected lifetime utility:

$$\max_{\{C_t, L_t, B_t\}_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t), \tag{56}$$

subject to the period-by-period budget constraint

$$P_t^C C_t + B_t \quad = \quad (1 + i_{t-1}) B_{t-1} + W_t L_t + \sum_{i=1}^{N} \int_0^1 D_{i,t}(j) dj + T_t, \tag{57}$$

where C_t is consumption, L_t is labor supply, B_t is the level of nominal bond holdings, T_t is the level of lump-sum transfers from the government, $D_{i,t}(j)$ are the dividends received lump-sum from firm j in sector i at time t, $\Pi_t^C = \left(P_t^C/P_{t-1}^C\right)$ is the gross CPI inflation rate, W_t is the nominal wage and i_t is the nominal interest rate set by the central bank.

The nominal interest rate is set by the central bank according to the following Taylor rule:

$$\log(1+i_t) = \log(\overline{\Pi}/\beta) + \phi_\pi \log(\Pi_t^C/\overline{\Pi}) + \varepsilon_t^i, \tag{58}$$

where $\overline{\Pi}$ is the steady-state level of CPI inflation, $\phi_\pi > 1$ pins down the strength of the central bank's reaction to inflation deviations from target and ε_t^i is the monetary policy shock.

We assume the following form of households' preferences:

$$u(C_t, L_t) = \frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{L_t^{1+\varphi}}{1+\varphi}. \tag{59}$$

Note that the Golosov-Lucas log-linear preferences which we use in the main text arise as a special case when $\sigma \to 1$ and $\varphi = 0$.

Given the presence of possibly non-zero steady-state inflation and the non-stationarity of the quality processes, we appropriately normalize our variables. Unlike in the main text, where we normalize by money supply, in the current cashless setting, we instead normalize by the aggregate CPI price level P_{t-1}^C. In particular, we let $\tilde{P_{i,t}}(j) \equiv \frac{P_{i,t}(j)}{\zeta_{i,t}(j)P_{t-1}^C}$ be the quality-adjusted real price, $\tilde{P}_{i,t} \equiv \frac{P_{i,t}}{P_{t-1}^C}$ be the real sectoral price, and $\tilde{W}_t \equiv \frac{W_t}{P_{t-1}^C}$ be the real wage. Then the equilibrium conditions for the aggregate real variables are given by:

$$C_t^{-\sigma} = \beta \mathbb{E}_t \left[\frac{1+i_t}{\Pi_{t+1}^C} C_{t+1}^{-\sigma}\right] \tag{60}$$

$$C_t^\sigma L_t^\varphi = \tilde{W}_t/\Pi_t^C \tag{61}$$

$$L_t = \Pi_t^C \frac{C_t}{\tilde{W}_t}\left[1 - \sum_{i=1}^N \lambda_{i,t}\left(1 - \frac{\Delta_{i,t}}{\mathcal{M}_{i,t}}\right)\right] + \sum_{i=1}^N \kappa_{i,t}\int_0^1 \eta_{i,t}(j)dj. \tag{62}$$

where $\lambda_{i,t}$ is the sectoral Domar weight (sales) share, $\Delta_{i,t}$ is the within-sector dispersion of real prices and $\mathcal{M}_{i,t}$ is the sectoral markup, which are given by:

$$\lambda_{i,t} = \omega_{i,t}^C + \sum_{k=1}^N \omega_{k,i,t}\lambda_{k,t}\frac{\Delta_{i,t}}{\mathcal{M}_{i,t}}, \qquad \Delta_{i,t} \equiv \tilde{P}_{i,t}^\epsilon \int_0^1 \tilde{P}_{i,t}(j)^{-\epsilon}dj, \qquad \mathcal{M}_{i,t} \equiv \frac{\tilde{P}_{i,t}}{\tilde{\mathcal{Q}}_{i,t}}. \tag{63}$$

The real sectoral price indices and marginal costs in turn satisfy:

$$\tilde{P}_{i,t}^{1-\epsilon} = \int_0^1 \tilde{P}_{i,t}(j)^{1-\epsilon} dj, \qquad \tilde{\mathcal{Q}}_{i,t} = \mathcal{Q}_i \left[\tilde{W}_t, \tilde{P}_{1,t}, ..., \tilde{P}_{N,t}; A_{i,t}\right], \qquad \Pi_t^C = \mathcal{P}^{\mathcal{C}} \left[\tilde{P}_{1,t}, ..., \tilde{P}_{N,t}\right]. \quad (64)$$

If the nominal price is not adjusted, then the quality-adjusted real price evolves according to:

$$p_{i,t}(j) = p_{i,t-1}(j) - \sigma_i \varepsilon_{i,t}(j) - \pi_{t-1}^C, \quad (65)$$

where $\pi_{t-1}^C \equiv \log \Pi_{t-1}^C$.

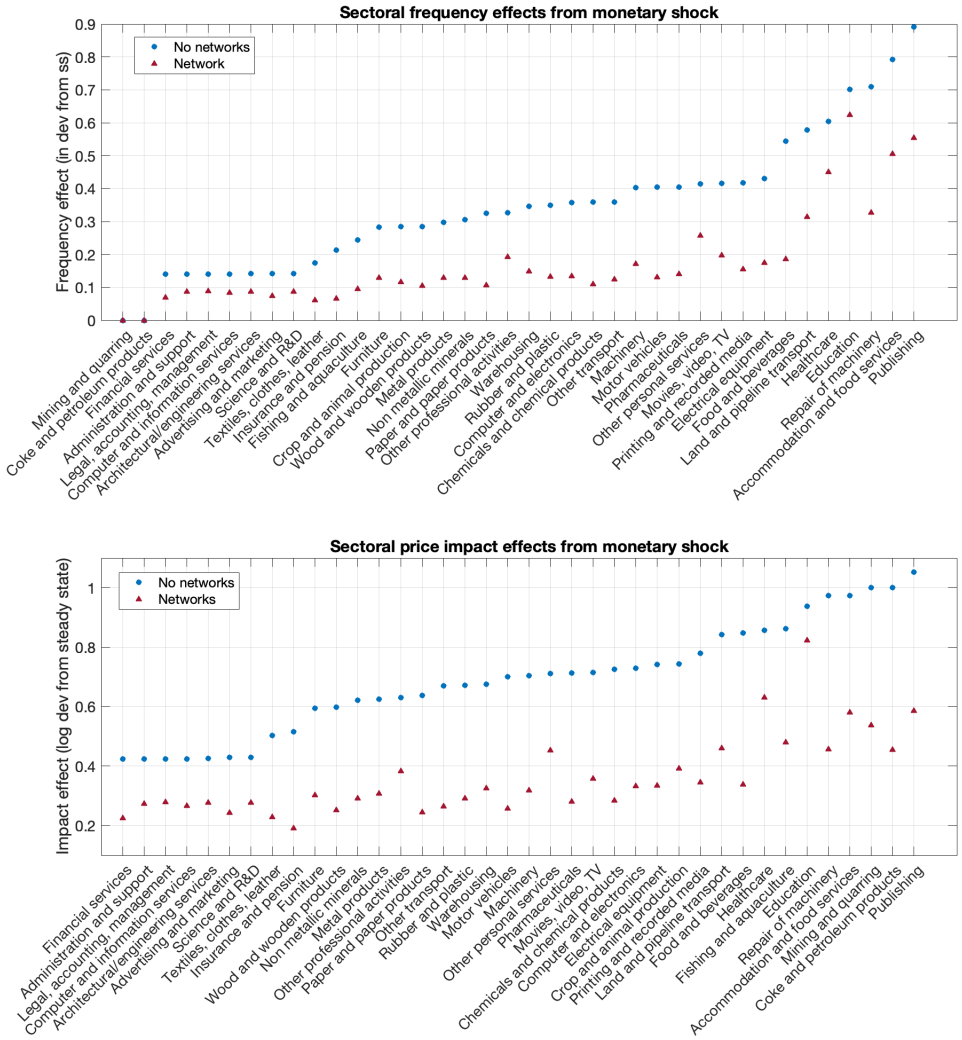
The per-period real profits of a firm are given by:

$$\tilde{D}_{i,t}(j) = \tilde{P}_{i,t}^{\epsilon-1} \left[(1-\tau_{i,t})\tilde{P}_{i,t}(j) - \tilde{\mathcal{Q}}_{i,t}\right] \tilde{P}_{i,t}(j)^{-\epsilon} \times \lambda_{i,t} \times C_t \times \Pi_t^C. \quad (66)$$

Finally, consider a firm with real quality-adjusted price p at the end of period t, and let $p_+ \equiv (p - \sigma_i \varepsilon_{i,t+1}(j) - \pi_t^C)$, where $\pi_t^C \equiv \log \Pi_t^C$. Then this firm's real value at the end of period t is given by the following Bellman equation:

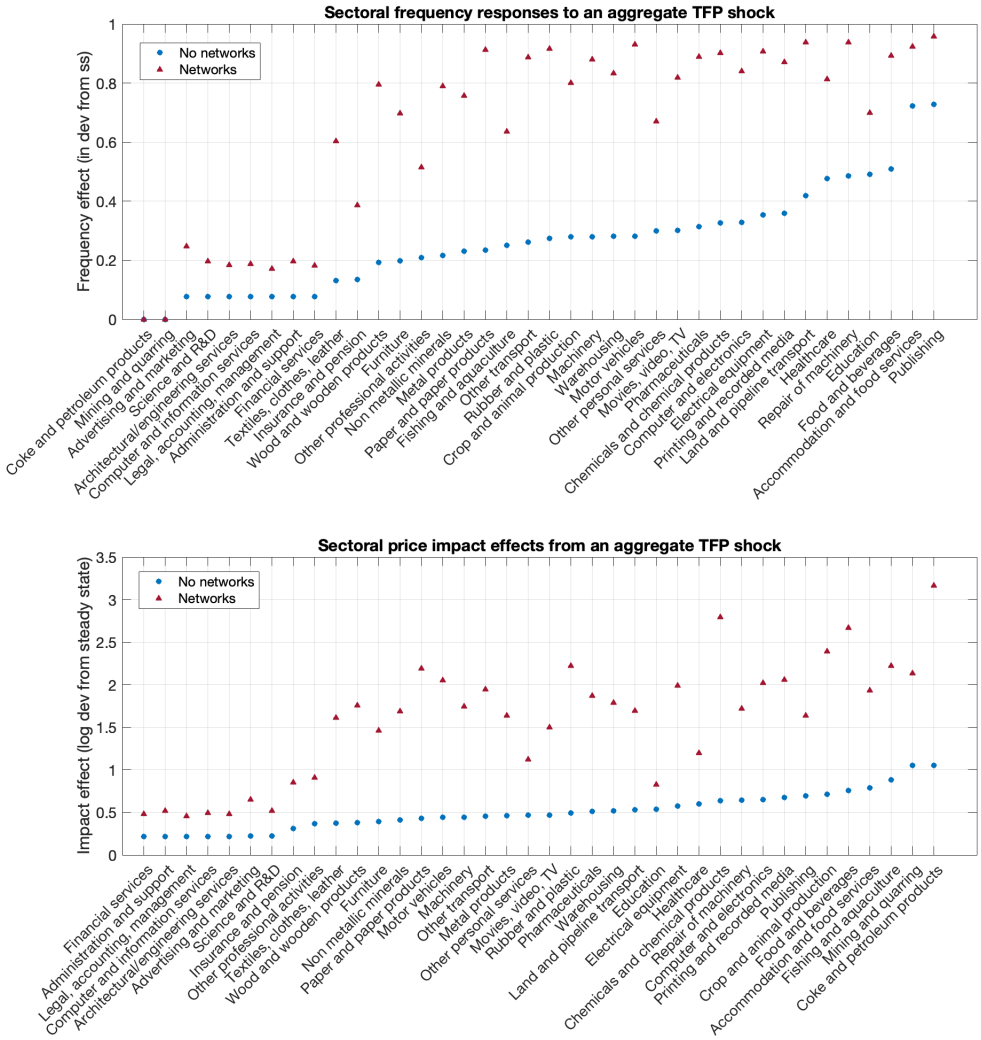
$$\begin{aligned} V_{i,t}(p) \quad &= \quad \tilde{D}_{i,t}(p) \quad + \\ &+ \beta \mathbb{E}_t \left[\{1 - \eta_{i,t+1}(p_+)\} \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{\Pi_t^C}{\Pi_{t+1}^C} V_{i,t+1}(p_+) \right] + \\ &+ \beta \mathbb{E}_t \left[\eta_{i,t+1}(p_+) \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{\Pi_t^C}{\Pi_{t+1}^C} \left(\max_{p'} V_{i,t+1}(p') - \kappa_{i,t+1} \tilde{W}_{t+1} \right) \right]. \end{aligned}$$

Figure 17: **Sectoral responses to a monetary shock**



***Notes**: the figure shows the responses of sectoral frequency and sectoral price indices to large aggregate TFP shocks.*

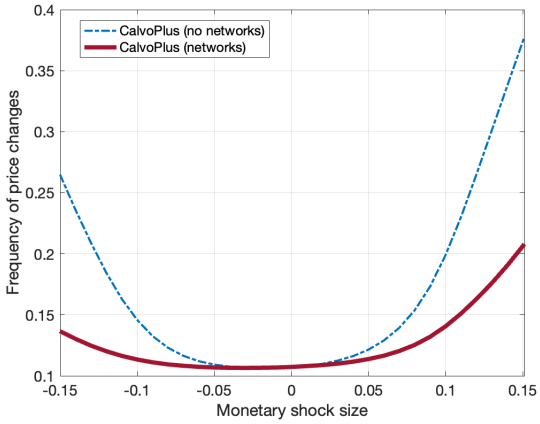
Figure 18: **Sectoral responses to a aggregate TFP shock**



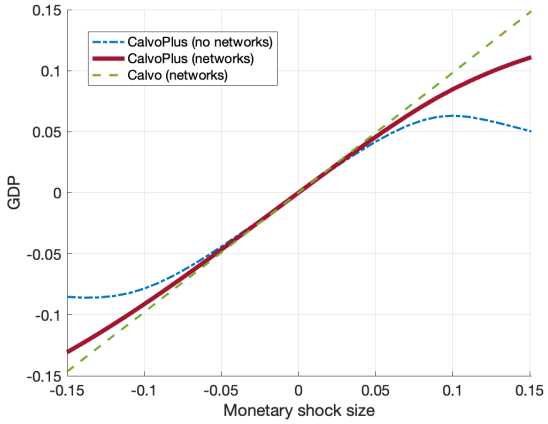
***Notes**: the figure shows the responses of sectoral frequency and sectoral price indices to large aggregate TFP shocks.*

Figure 19: **Frequency and GDP responses to monetary shocks: CalvoPlus**

(a) Aggregate frequency response



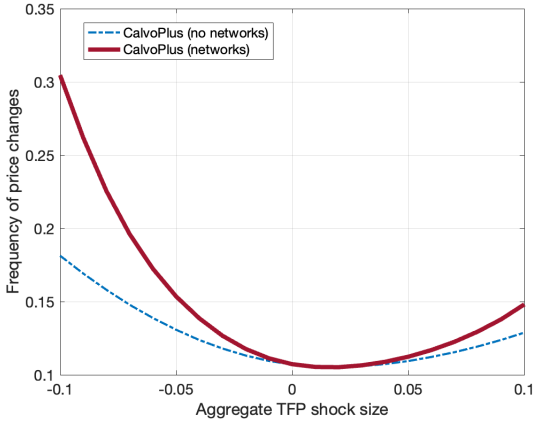
(b) Impact GDP response



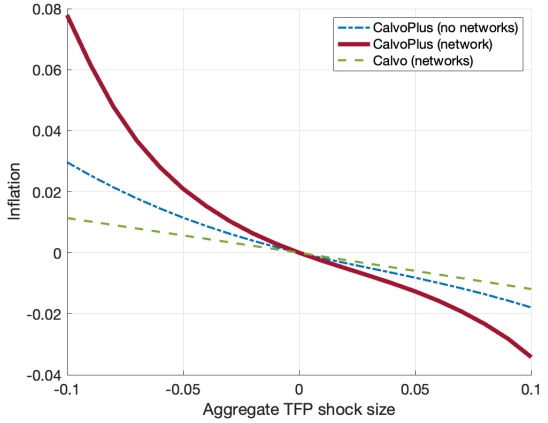
Notes: *the figure shows the impact responses of aggregate frequency of adjustment to monetary shocks of different sizes, with and without networks.*

Figure 20: **Frequency and inflation responses to aggregate TFP shocks: CalvoPlus**

(a) Aggregate frequency response



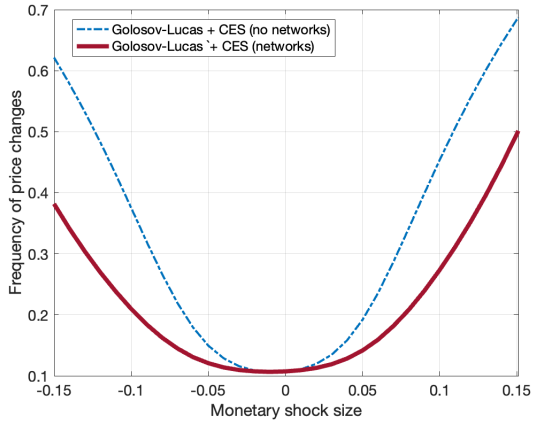
(b) Impact inflation response



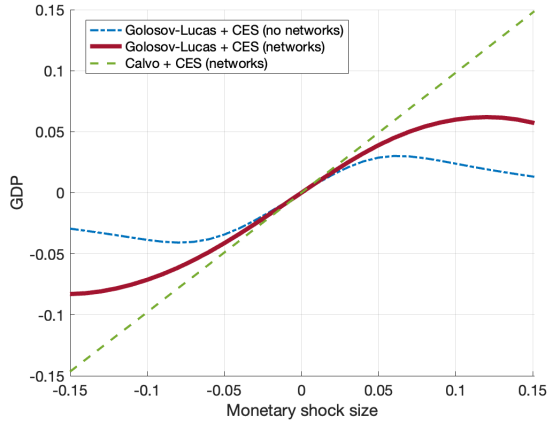
Notes: *the figure shows the impact responses of aggregate frequency of adjustment to monetary shocks of different sizes, with and without networks.*

Figure 21: **Frequency and GDP responses to monetary shocks: CES aggregation**

(a) Aggregate frequency response



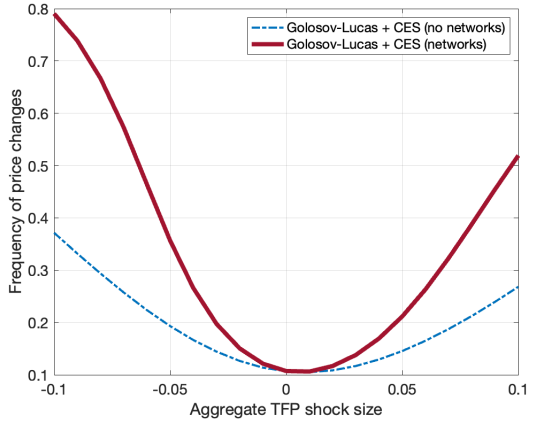
(b) Impact GDP response



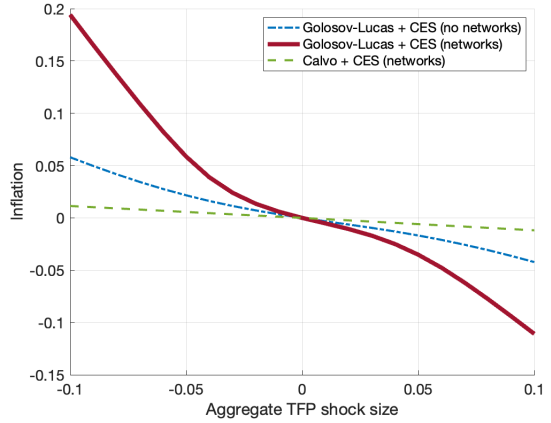
Notes: *the figure shows the impact responses of aggregate frequency of adjustment to monetary shocks of different sizes, with and without networks.*

Figure 22: **Frequency and inflation responses to aggregate TFP shocks: CES aggregation**

(a) Aggregate frequency response



(b) Impact inflation response



Notes: *the figure shows the impact responses of aggregate frequency of adjustment to monetary shocks of different sizes, with and without networks.*