Capital Income in an Intangible Economy

(*Preliminary draft*)

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Abstract

Capital formation is increasingly labor intensive, requiring specialized labor (e.g., managers, researchers, etc.). In this paper, we quantify the distributional implications of this shift towards intangible capital. We construct a firm-level dataset (US public firms, 1972-2022) covering capital income and its uses (tangible investments, intangible investments, and payouts). The key novelty is that we unpack the tangible and intangible components of all IPOs, acquisitions, and payments in stocks over our sample. We calibrate a neoclassical model of (partially-intangible) investment using a combination of short- and long-run moments on investments and payouts in the corporate sector. The model predicts that a shift towards intangibles of the magnitude we have seen in the data generates a large redistribution of aggregate income away from production labor and capitalists, towards investment labor. Rising intangibles also imply a more *inelastic* capital supply curve (owing to the inelastic supply of labor), dampening the ability of the corporate sector to absorb saving shocks and exacerbating the response of valuations.

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1 Introduction

Corporate profits can be used either for tangible investments (i.e., produced using goods), intangible investments (i.e., produced using labor) or payouts to owners. The existing literature documents a shift in the post-2000 period towards intangible investments, in particular research and development (Corrado and Hulten, 2010). This paper examines how this shift towards intangibles reshapes the distribution of capital income (who owns future profits?) and impacts the elasticity of capital (how much can firms absorb saving shocks?).

Overview of results. We consider a class of aggregate investment functions of the form:

$$\text{capital formation} = \text{capital}^{\theta} \cdot \left(\text{goods}^{\chi} \cdot \text{labor}^{1-\chi}\right)^{1-\theta}. \tag{1.1}$$

The idea is that capital formation does not only require goods (i.e., machines, computers, plants), but also specialized labor (i.e., researchers, entrepreneurs, early financiers) and accumulated capital (i.e., building on past research, adjustment frictions). The functional form provides a two-parameter departure from the textbook neoclassical growth model: θ is the capital share of investment, and χ is the tangibility of investment.

The paper has three parts. First, we characterize the distributional implications of intangible capital. We embed (1.1) in a standard neoclassical model and characterize how the technology affects the distribution of capital income and the supply elasticity of capital. Second, we map the model to the data (US public firms, 1972–2022), centering our analysis around the corporate sector budget constraint:

$$\text{capital income} = \text{payout} + \text{tangible investments} + \text{intangible investments}. \tag{1.2}$$

The central goal of our accounting exercise is to allocate 100% of aggregate capital income into those three categories. Compared to the BEA's current methodology, we use a more comprehensive definition of intangible investment, seeking to capture *all* payments to nonproduction labor (including entrepreneurs, managers, researchers, etc.)

Third, we calibrate the model by using a combination of long-run facts about the distribution of income and short-run evidence on the supply elasticity of capital. We find that a shift towards intangibles of the magnitude that we have seen in the data implies a significant redistribution of aggregate income away from capitalists (-2 pp. of GDP) and production labor (-5 pp.), towards investment labor ($+7$ pp.). In addition, it makes the economy less elastic: the long-run supply elasticity of capital decreases by 42%. What this means is that saving shocks are increasingly passed through to valuations, not investment (even in the long-run).

Taken as a whole, the model predictions provide a coherent narrative for the evolution of the US corporate sector post-2000, a period marked by stagnating tangible investments, high Tobin Q, and low net payouts to capitalists.

Methodology & literature. In Section 2, we write down a standard neoclassical model of capital accumulation, augmented with the general investment function (1.1) and two upward-sloping labor supply curves (production and investment workers). We characterize the distribution of income as well as the supply elasticity of capital, both in the short- and long-run.

In terms of the long-run distribution of income, we show that the capital share of investment θ governs the amount of investment rents that accrue to owners of capital.[1] The tangibility of investment χ determines how the remainder is split between tangible- and intangible-investment. We benchmark the model predictions against important special cases: Lucas tree (Lucas Jr, 1978), neoclassical growth model (Cass-Koopmans), sweat capital (Bhandari and McGrattan, 2021), and q-theory Hayashi (1982).

In terms of the supply elasticity of capital, we then emphasize the fact that when inelastically supplied labor is an input in capital formation, both the short- and long-run capital supply elasticities are finite. This stands in contrast to the neoclassical growth model, where both the short- and long-run elasticities are infinite (i.e., goods and be turned into capital).

In Section 3, we quantify the distribution of capital income in the US nonfinancial corporate sector. We focus on public firms (Compustat-CRSP) over the 1972–2022 period, where we have full financial statements, which, crucially, include noncash payments (e.g., equity grants, stock options, acquisitions financed with stocks). The empirical counterpart of (1.2) is the cashflow identity, which tracks how much cash is earned (cashflows from operations), how much is investment (minus cashflows from investing), and how much is paid out (minus cashflows from financing). Building on the existing literature, we make three key adjustments. First, we treat nonproduction labor costs as intangible investment, rather than production costs (see, e.g., Corrado, Hulten and Sichel, 2005; Corrado, Hulten and Sichel, 2009; Eisfeldt and Papanikolaou, 2013; Peters and Taylor, 2017; Koh, Santaeulàlia-Llopis and Zheng, 2020). Second, we account for IPOs and acquisitions net of delistings and allocate the implied cashflows to tangible and intangible investments. Third, we account for noncash payments, treating them as economically equivalent to cash payments. This is particularly important for nonproduction labor (researchers, managers) who earn a large share of their compensation in stocks (see, e.g., Eisfeldt, Falato and Xiaolan, 2023).

We obtain an annual dataset on the aggregate (and industry-level) uses of capital income in the nonfinancial corporate sector, which maps into the firm budget constraint (1.2). We emphasize that our accounting is fully consistent with national accounting, which now acknowledges that some form of labor expenses are indeed investment. However, we go further than the BEA and consider any nonproduction labor expenses as investment.[2]

Three long-run facts stand out: (1) the tangible investment share declines, (2) the intangible investment share increases, and (3) the capitalist share is low on average, often negative, and highly countercyclical. Our facts are broadly consistent with separate findings in the literature on investment

[1] We define investment rents in the sense of Ricardo: based on the difference between the average and marginal product of labor.

[2] Corrado, Haskel, Jona-Lasinio and Iommi (2022) discuss this topic in detail, highlighting the fact that the following expenses are not currently considered investment in the national accounts: "market research and branding, operating models, platforms, supply chains, distribution networks, employer-provided training, attributed designs (industrial), and financial product development".

stagnation post-GFC (Gutiérrez and Philippon, 2017; Crouzet and Eberly, 2019), the (rising) importance of intangibles and nonproduction labor (Eisfeldt and Papanikolaou, 2013; Peters and Taylor, 2017; Eisfeldt et al., 2023), and the properties of the aggregate equity payout yield (Fama and French, 2005; Boudoukh et al., 2007; Fried and Wang, 2019). Our dataset offers an opportunity to study these facts jointly, over a consistently-defined sample that covers five decades.

Finally, in Section 4, we combine long-run empirical moments and equilibrium equations to calibrate the model and run counterfactuals. To separately identify the capital share of investment θ from the long-run investment rate (which is not possible using long-run income shares), we use cross-industry evidence on the total-investment-q elasticity.

Equipped with the calibrated model, we simulate the effect of an increase in intangibility from 40% to 70% of total investment (i.e., a decline in the parameter χ), in line with the empirical evidence over the 1972–2022 period. We find a large transformative effect on the long-run composition of GDP, and an important flatening of the long-run capital supply curve. We previously discussed the main quantitative results in "overview of results" on page 1.

2 Model

The economy consists of infinitely-lived firms and workers. Firms maximize the present discounted value of profits by choosing how much to produce and invest. Workers decide how much labor to supply. Both take the discount rate $R > 0$ and the wages $\{w_{Y,t}, w_{H,t}\}_{t=0}^{\infty}$ as given.

2.1 Worker and firm problems

Firm problem. A representative firm maximizes firm value by choosing production labor L_Y, investment labor L_H, tangible investment I:

$$V_0 = \max_{\{L_{Y,t}, L_{H,t}, I_t, K_{t+1}\}} \sum_{t=1}^{\infty} R^{-t} D_t, \qquad \text{(firm value)}$$

$$\text{s.t.} \quad D_t = Y_t - I_t - w_{Y,t} L_{Y,t} - w_{H,t} L_{H,t}, \qquad \text{(budget constraint)}$$

$$Y_t = z_{Y,t} K_t^{\alpha} L_{Y,t}^{1-\alpha}, \qquad \text{(production)}$$

$$G_t = z_{H,t} K_t^{\theta} \left(I_t^{\chi} L_{H,t}^{1-\chi} \right)^{1-\theta}, \qquad \text{(capital formation)}$$

$$K_{t+1} = (1-\delta) K_t + G_t. \qquad \text{(capital accumulation)}$$

There are two departures from the textbook neoclassical growth model (henceforth NGM). First, investment exhibits diminishing marginal products holding constant the existing stock of capital. The parameter $\theta \in (0,1)$ governs the diminishing returns to capital formation. Second, investment requires both tangible inputs I and specialized labor L_H. The parameter $\chi \in (0,1)$ governs the tangibility of capital. The sequences $\{z_{Y,t}, z_{H,t}\}_{t=1}^{\infty}$ represent Hicks-neutral productivity in procuction and investment, TFP and IST for short.

The neoclassical growth model (NGM) can be thought of as a special case, where $\theta = 0$ and $\chi = 1$, which implies that goods can be transformed into capital at a fixed rate. In general, this is not the case, and we denote the marginal value of capital at time t by q_t.[3] The first-order conditions are

$$w_{Y,t} = (1-\alpha)\frac{Y_t}{L_{Y,t}} \qquad \text{(firm foc } L_{Y,t}\text{)}$$

$$w_{H,t} = (1-\theta)(1-\chi)\frac{G_t}{L_{H,t}}q_t \qquad \text{(firm foc } L_{H,t}\text{)}$$

$$1 = (1-\theta)\chi\frac{G_t}{I_t}q_t \qquad \text{(firm foc } I_t\text{)}$$

$$Rq_t = \alpha\frac{Y_{t+1}}{K_{t+1}} + \left(1-\delta+\theta\frac{G_{t+1}}{K_{t+1}}\right)q_{t+1} \qquad \text{(firm foc } K_{t+1}\text{)}$$

Worker problem. Since the focus of the paper is on the firm side, we only provide a brief microfoundation of the labor supply curves for L_H and L_Y, abstracting from potential wealth effects. A representative worker maximizes solves a sequence of static problems where they maximize income net of disutility of work:

$$\max_{L_{Y,t},L_{H,t}} \left\{ w_{Y,t}L_{Y,t} + w_{H,t}L_{H,t} - \frac{1}{1+\sigma}\left((1-\mu)L_{Y,t}^{1+\sigma} + \mu L_{H,t}^{1+\sigma}\right)\right\}. \qquad \text{(worker welfare)}$$

The first-order conditions are

$$w_{Y,t} = (1-\mu)L_{Y,t}^{\sigma} \qquad \text{(worker foc } L_{Y,t}\text{)}$$

$$w_{H,t} = \mu L_{H,t}^{\sigma} \qquad \text{(worker foc } L_{H,t}\text{)}$$

The parameter $\mu \in (0,1)$ governs the relative supply of the two labor types. The parameter $\sigma > 0$ governs the labor supply elasticity.[4]

2.2 Accounting and definitions

Before solving the equilibrium, we introduce the accounting concepts that we will use going forward. From now on, we define capital income as gross output minus production costs:

$$\Pi_t \equiv Y_t - w_{Y,t}L_{Y,t}. \qquad \text{(Capital income)}$$

Note that capital income is $Y - w_Y L_Y$, not $Y - w_Y L_Y - w_H L_H$. The reason is that intangible investments are a form of investment, not a cost of production.

[3] For completeness, the Lagrangian is

$$\mathcal{L}_0 = \sum_{t\geq 1} R^{-t}\left(Y_t - I_t - w_{Y,t}L_{Y,t} - w_{H,t}L_{H,t}\right) + \sum_{t\geq 1} R^{-t}q_t\left((1-\delta)K_t + G_t - K_{t+1}\right)$$

[4] Combining both equations, we have that (μ,σ) pins down the relative wage bill in terms of relative labor and wages:

$$\frac{w_{Y,t}L_{Y,t}}{w_{H,t}L_{H,t}} = \frac{1-\mu}{\mu}\left(\frac{L_{Y,t}}{L_{H,t}}\right)^{1+\sigma} = \left(\frac{1-\mu}{\mu}\right)^{-\frac{1}{\sigma}}\left(\frac{w_{Y,t}}{w_{H,t}}\right)^{1+\frac{1}{\sigma}}. \qquad (2.1)$$

Accounting for income and expenditures. We now define the following concepts in the model: aggregate income, aggregate expenditures, labor income, capital income, consumption, and investment in a way that is consistent with the system national accounts. We mapping between income/expenditure and model objects is given by the GDP identity:

$$\underbrace{\overbrace{w_{Y,t}L_{Y,t} + w_{H,t}L_{H,t}}^{\text{labor income}} + \overbrace{\Pi_t}^{\text{capital income}}}_{\text{income}} = \underbrace{\overbrace{Y_t - I_t}^{\text{consumption}} + \overbrace{I_t + w_{H,t}L_{H,t}}^{\text{investment}}}_{\text{expenditures}}. \tag{2.2}$$

Both sides are equal to GDP, which is $Y + w_H L_H$, not Y. This is because gross output Y does not include the value-added produced by investment labor. We will come back to the composition of GDP in Section 4, where we simulate the effect of a permanent shift towrads intangibles.

The key accounting identity is the the firm budget constraint, which characterizes the *uses* of capital income. I can be expressed as can be written as

$$\underbrace{\Pi_t}_{\text{capital income}} = \underbrace{I_t}_{\text{tangible investments}} + \underbrace{w_{H,t}L_{H,t}}_{\text{intangible investments}} + \underbrace{D_t}_{\text{payouts}}. \tag{2.3}$$

It says that capital income has three uses: tangible investments, intangible investments, and payments to owners of the firm (henceforth "payouts").

Accounting for rents. Capitalists extract production rents due to the fact that the production function has decreasing returns to scale. We define rents in the sense of Ricardo, i.e., as the difference between the average and marginal product of an input. Importantly, (Ricardian) rents are efficient and merely record the role of technology in shaping the sharing of output between the different inputs.

$$\begin{aligned}
\text{production rents} &\equiv L_{Y,t} \cdot \left(\frac{Y_t}{L_{Y,t}} - \frac{\partial Y_t}{\partial L_{Y,t}} \right) = \alpha Y_t \\
\text{investment rents} &\equiv \left(I^{\chi} L_H^{1-\chi} \right) \cdot \left(\frac{G_t}{I^{\chi} L_H^{1-\chi}} - \frac{\partial G_t}{\partial (I^{\chi} L_H^{1-\chi})} \right) = \theta q_t G_t.
\end{aligned}$$

Production rents are the difference between the average and marginal product of production labor. The second equality uses firm optimization. Notice that, in this model, all of capital income is production rents. The idea is that capital is pre-determined, and all of the income that capitalists earn by owning firms at time t is due to the fact that workers are paid below their average product. We define investment rents similarly, but where the input to investment is the bundle of tangible and intangible investments

Accounting for returns and dilution. The definition of returns is the payout yield plus the growth of firm value: $R_{t+1} = \frac{D_{t+1}}{V_t} + \frac{V_{t+1}}{V_t}$. Using the firm budget constraint, we can alternatively write returns as

$$R_{t+1} = \frac{\Pi_{t+1}}{V_t} + \left(1 - \frac{I_{t+1} + w_{H,t+1}L_{H,t+1}}{V_{t+1}} \right) \frac{V_{t+1}}{V_t}, \tag{2.4}$$

where the first term is now the capital income yield: the owner's share of current profits as a share of enterprise value. The second term accounts for the "diluted" share of firm growth. The idea is that, in order to payout all of capital income, a firm would need to give away a share $\frac{I+w_H L_H}{V}$ of its ownership to outside investors in order to finance its growth.

Solving V_0 forward with $R_{t+1} = R$, we obtain

$$V_0 \equiv \sum_{t\geq 1} R^{-t} D_t = \sum_{t\geq 1} R^{-t} \prod_{1\leq s<t} \left(1 - \frac{I_s + w_{H,s} L_{H,s}}{V_s}\right) \Pi_t, \tag{2.5}$$

Equation 2.5 contains an important insight: capitalists at $t = 0$ do not own all of future capital income. They own a share that decays over time, because they must hire labor to maintain the stock of capital.

2.3 Equilibrium

An equilibrium is an initial condition K_0, an allocation $\{L_{Y,t}, L_{H,t}, I_t, K_t, q_t\}_{t\geq 1}$, and prices $\{w_{H,t}, w_{L,t}\}_{t\geq 1}$ that solves the firm problem (firm foc $L_{Y,t}$, firm foc $L_{H,t}$, firm foc I_t, firm foc K_{t+1}) and household problem (worker foc $L_{Y,t}$, worker foc $L_{H,t}$). The discount rate R is taken as given.

Lemma 2.1 (Equilibrium). *The equilibrium $(L_{Y,t}, L_{H,t}, I_t)$, as a function of the state variables $(K_t, z_{Y,t}, z_{H,t})$ and the co-state variable q_t, is given by*

$$\begin{aligned}
L_{Y,t} &= \overline{l_Y} \cdot (z_{Y,t} \cdot K_t^{\alpha})^{\frac{1}{\sigma+\alpha}}, \\
L_{H,t} &= \overline{l_H} \cdot \left(z_{H,t} \cdot q_t \cdot K_t^{\theta}\right)^{\frac{1}{\sigma+\theta-\sigma\chi(1-\theta)}}, \\
I_t &= \bar{\imath} \cdot \left(z_{H,t} \cdot q_t \cdot K_t^{\theta}\right)^{\frac{1+\sigma}{\sigma+\theta-\sigma\chi(1-\theta)}},
\end{aligned}$$

where $\overline{l_Y}, \overline{l_H}, \bar{\imath} > 0$ are defined in Appendix A.2.

As is usual in demand and supply equilibrium systems, the supply σ and demand θ elasticities matter in determining equilibrium quantities. It is instructing to focus on the case $\chi = 1$ at first (all investment is intangible. In this case, the expression for investment labor is $L_{H,t} \propto \left(z_{H,t} \cdot q_t \cdot K_t^{\theta}\right)^{\frac{1}{\sigma+\theta}}$. Investment labor is increasing in capital because L_H is a gross complement with K in the investment process. It is also increasing in q because the marginal product of investment labor has a long duration. Note that, while our equilibrium concept is based on a representative firm, in Appendix A.1, we provide an explicit microfoundation of the aggregate production and capital formation functions.

Now consider the steady-state of the model, where $z_{Y,t} = z_{H,t} = 1$ for all $t \geq 0$. The following lemma expresses the steady state (q, K) as the solution to a supply and demand and supply system.

Lemma 2.2 (Steady-state). *The steady-state level of capital K, and its value q, are determined by the demand and supply of capital:*

$$q = \overline{q_D} \cdot K^{-(1-\alpha)\frac{\sigma}{\sigma+\alpha}}, \tag{Demand}$$

$$q = \overline{q_S} \cdot K^{\frac{1-\chi}{1+\sigma\chi}\sigma}, \tag{Supply}$$

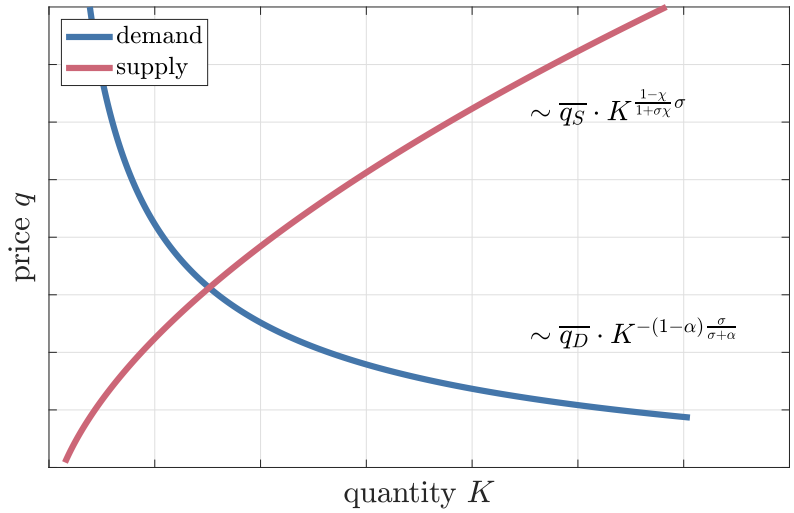


Figure 1: Long-run capital market equilibrium

where $\overline{q_D}, \overline{q_S} > 0$ *are defined in Appendix A.3.*

The demand curve is a valuation equation, it says that investors require a lower valuation of K when the MPK is low. The supply curve is a wage equation, it says that innovators require a higher wage to increase their input. Notice that neither the demand or supply elasticities in the long-run depend on θ. Figure shows a numerical example of how the long-run capital market equilibrium is determined. In Section 2.5, we compare our (upward-sloping) long-run supply curve to the horizontal one in the neoclassical growth model.

Given a steady-state level of capital K, the price of capital q and the dilution rate λ are given by

$$q = \frac{1}{r + (1-\theta)\delta} \cdot \frac{\Pi}{K} \qquad \text{(Value of capital)}$$

It is given by the average return on capital Π/K, times a multiple that depends on the discount rate $r \equiv R - 1$ and the dilution rate $(1-\theta)\delta$.

2.4 Empirical content of the model

We now characterize the empirical content of the model in terms of the distribution of capital income and the elasticity of capital. We break down our analysis in terms of the short- and long-run.

Distribution of capital income. We now characterize how capital income is used. We focus on the firm budget constraint and compute the budget shares of tangible investment, intangible investment, and payouts.

In the long-run, the uses of capital income is pinned down by the steady-state equilibrium described earlier. We define the "short-run" as the deviation from the steady-state. Suppose that the economy is in steady-state at $t = 0$, but is then hit with a sequence of technology shocks $\{dz_{y,t}, dz_{H,t}, dR_t\}_{t\geq 1}$. As a result, both the total amount of capital income will change, but also its distribution. This allows us to characterize the redistributive effect of shocks in the short-run.

Proposition 2.3 (Budget shares)**.** *The distribution of capital income in the long-run is given by:*

$$\frac{I}{\Pi} = \chi \frac{(1-\theta)\delta}{r+(1-\theta)\delta}, \qquad \text{(tangible share, long-run)}$$

$$\frac{w_H L_H}{\Pi} = (1-\chi)\frac{(1-\theta)\delta}{r+(1-\theta)\delta}, \qquad \text{(intangible share, long-run)}$$

$$\frac{D}{\Pi} = \frac{r}{r+(1-\theta)\delta}. \qquad \text{(payout share, long-run)}$$

In the short-run (i.e., conditional on a perturbation $\{\mathrm{d}z_{Y,t}, \mathrm{d}z_{H,t}, \mathrm{d}R_t\}_{t=1}^{\infty}$*), the distribution of capital income, in present value terms, is given by:*

$$\frac{\sum_{t\geq 1} R^{-t}\,\mathrm{d}I_t}{\sum_{t\geq 1} R^{-t}\,\mathrm{d}\Pi_t} = \frac{I}{\Pi}\cdot\mathcal{M}, \qquad \text{(tangible share, short-run)}$$

$$\frac{\sum_{t\geq 1} R^{-t}\,\mathrm{d}(w_{H,t}L_{H,t})}{\sum_{t\geq 1} R^{-t}\,\mathrm{d}\Pi_t} = \frac{w_H L_H}{\Pi}\cdot\mathcal{M}, \qquad \text{(intangible share, short-run)}$$

$$\frac{\sum_{t\geq 1} R^{-t}\,\mathrm{d}D_t}{\sum_{t\geq 1} R^{-t}\,\mathrm{d}\Pi_t} = \frac{D}{\Pi}\cdot\Big(1-\frac{I+w_H L_H}{D}\cdot\mathcal{M}\Big). \qquad \text{(payout share, short-run)}$$

where the object $\mathcal{M}$ *is a sufficient statistic that captures the relative effect of the perturbation of the value of capital formation versus production*

$$\mathcal{M} = \sum_{t\geq 1} R^{-t}\Big(\mathrm{d}\log\left(q_t G_t\right) - \mathrm{d}\log Y_t\Big).$$

Proof. See Appendix A.4 □

First, notice that only four parameters govern the distribution of capital income in the long-run. For the payout share, what matters is the (long-run, required) return r and the dilution rate $\delta(1-\theta)$, which itself depends on depreciation and the capital share of investment θ. The lower the required return r, the lower the share of capital income paid out the capitalists. Given the required return, a higher dilution rate $\delta(1-\theta)$ implies a higher current payout. Then, the tangibility of investment parameter χ pins down how retained capital income is used for tangible versus intangible investment.

In the short-run, the distribution of capital income also depends on a sufficient statistic $\mathcal{M}$, which summarizes the wedge between the income shares *conditional on a shock* and the steady-state income shares. It measures the relative effect of the shock on the value of capital formation versus output.

The logic is straightforward: while the economy has constant income shares of production Y and investment qG individually (both have Cobb-Douglas technologies), technology shocks can differentially affect Y and qG, leading to short-run deviations in the distribution of capital income. In particular, shocks that increase the value of capital formation qG more than output Y lead to a lower payout share, at the expense of a higher total investment share. In contrast, shocks that mostly increase output Y increase the payout share.

The supply elasticity of capital. We now characterize the supply elasticity of capital, which measures how sensitive capital K is to the value of capital q.

Proposition 2.4 (Supply elasticity of capital)**.** *The supply elasticity of capital in the long-run is given by:*

$$\mathrm{d}\log K = \frac{\frac{1}{\sigma}+\chi}{1-\chi}\cdot \mathrm{d}\log q \qquad \text{(Capital elasticity, long-run)}$$

In the short-run (i.e., conditional on a perturbation $\{\mathrm{d}q_t\}_{t=1}^{\infty}$ *aournd the steady-state), it is given by*

$$\mathrm{d}\log K_{T+1} = \delta\frac{(1+\sigma\chi)(1-\theta)}{\sigma+\theta-\sigma\chi(1-\theta)}\cdot\sum_{s=0}^{T}\left(1-\delta\frac{\sigma(1-\theta)(1-\chi)}{\sigma+\theta-\sigma\chi(1-\theta)}\right)^{s}\mathrm{d}\log q_{T-s}.$$

(Capital elasticity, short-run)

Proof. See Appendix A.5. □

The long-run supply elasticity of capital characterizes movements along the supply curve (as defined in Lemma 2.2). Notice that the model allows for the standard result from the NGM of an infinitely-elastic long-run capital supply curve in the case of where investment is fully tangible $\chi = 1$. But when investment requires labor $\chi < 1$, the elasticity of capital is finite and depends on the labor supply elasticity $\frac{1}{\sigma}$. Different combinations of (σ, χ) span long-run capital elasticities from zero to infinity.

The short-run supply is constructed as the counterfactual effect on capital accumulation over $t \in \{0, 1, \ldots, T\}$ of a perturbation $\{\mathrm{d}\log q_t\}_{t\geq 0}$. It characterizes the full term structure of the capital supply elasticity.

While the capital share of investment θ did not matter for the long-run capital elasticity, it is the key determinant of the short-run capital elasticity. Capital being an input in production $\theta > 0$ makes capital less elastic in the short-run. As we discuss shortly, this is exactly the logic behind the q-theory of investment. Similarly, a high depreciation rate δ leads to a high capital formation rate in steady-state G/K, hence affecting the speed of convergence.

The short- and long-run elasticities are related as follows. Setting $\mathrm{d}\log q_{T-s} = \mathrm{d}\log q$ and taking the limit $T \to \infty$, we obtain

$$\lim_{T\to\infty}\mathrm{d}\log K_{T+1} = \frac{\frac{1}{\sigma}+\chi}{1-\chi}\cdot\mathrm{d}\log q,$$

which indeed is the long-run capital supply elasticity.

2.5 Comparison with benchmark models

We now describe four important special cases of the model, in terms of how capital formation G_t is modeled.

Lucas tree model ($\theta = 1$). The first special case is where investment only requires capital ($\theta = 1$). In this case, capital formation becomes $G_t = z_{H,t}K_t$. This is an endowment economy, where capital grows by itself at rate $1 - \delta + z_{H,t}$, in the spirit of Lucas Jr (1978)'s trees.

Sweat capital ($\theta = 0$). The second special case is where investment requires both goods and labor, but not capital itself ($\theta = 0$). In this case, the capital accumulation becomes $G_t = I_t^{\chi}L_{H,t}^{1-\chi}$, as in the Sweat capital model of Bhandari and McGrattan (2021), where entrepreneurial labor is a key input to business

capital formation. A similar idea is at the core of several important models, such as the Melitz (2003) model (firm entry requires labor), the Romer (1986) model (innovation requires labor), and various other models where firm expansion requires labor (e.g., Luttmer, 2011; Eisfeldt and Papanikolaou, 2013). In all these cases, increasing the future productive capacity of the economy requires specialized labor. In other words, investment is partially intangible.

Q-theory. The third special case is where investment requires both goods and capital, but not labor ($\chi = 1$). In this case, the capital accumulation becomes $G_t = K_t^{\theta} I_t^{1-\theta}$. This is equivalent to a q-theory of investment Hayashi (1982), with a (carefully chosen) capital adjustment cost function of the form $\phi(I) = \frac{1}{(1-\theta)(1+\theta)} I^{1+\theta} K^{-\theta}$. The key insight is that the capital share of θ is closely related to the curvature of the adjustment cost function, which governs *short-run* fluctuations in investment. Indeed, with $\chi = 1$, firm optimization (firm foc I_t) implies that the optimal investment rate is $I_t/K_t = (1-\theta)^{\frac{1}{\theta}} q_t^{\frac{1}{\theta}}$: the marginal value of capital q is a sufficient statistic for current investment.

The neoclassical growth model. The last special case is when investment only requires goods ($\chi = 1, \theta = 0$). In this case, we recover the standard capital formation equation from $G_t = I_t$, which implies that goods and capital are effectively the same (they can be transformed into each other frictionlessly). This assumption forms the core of the Neoclassical growth model, and yields the standard result that capital elasticity is infinite in both the short- and long-run, precisely because the relative price of capital q is always one.

Income shares and capital elasticity in benchmark models. Table 1 reports the long-run income shares in each special case.

Table 1: Summary of benchmark models: distribution of capital income.

Model	Constraint		Long-run shares of capital income		
	θ	χ	tangible	intangible	payout
Lucas tree model	$\theta = 1$	–	0	0	1
Sweat capital	$\theta = 0$	–	$\chi \cdot \frac{\delta}{r+\delta}$	$(1-\chi) \cdot \frac{\delta}{r+\delta}$	$\frac{r}{r+\delta}$
Q-theory	–	$\chi = 1$	$\frac{(1-\theta)\delta}{r+(1-\theta)\delta}$	0	$\frac{r}{r+(1-\theta)\delta}$
Neoclassical growth	$\theta = 0$	$\chi = 1$	$\frac{\delta}{r+\delta}$	0	$\frac{r}{r+\delta}$
Baseline	–	–	$\chi \cdot \frac{(1-\theta)\delta}{r+(1-\theta)\delta}$	$(1-\chi) \cdot \frac{(1-\theta)\delta}{r+(1-\theta)\delta}$	$\frac{r}{r+(1-\theta)\delta}$

Notes. α is the production capital share; θ the investment capital share; χ is the tangibility of investment; r is the steady-state return, δ is the steady-state capital formation rate.

As discussed in Section 2.4, the parameters (χ, θ) govern not only the distribution of capital, but also the supply elasticity of capital in the short- and long-run. Table 2 reports the short- and long-run supply elasticity of capital in the different benchmark models. There are two key takeaways. First, in fully-tangible economies ($\chi = 1$), then capital is infinitely elastic in the long-run: the supply curve is

horizontal. This is because the relative price of capital in terms of goods is fixed over time. Adding a positive capital share of investment ($\theta > 0$) leads makes the short-run elasticity finite (see "q-theory"), but does not solve the problem of infinite long-run elasticity.

Second, intangibility $\chi < 0$ makes the supply elasticity of capital finite, both in the short- and long-run (see "sweat capital"). This is because labor supply is finite, which makes the response of capital formation to q restricted by the amount of investment labor that is available. Our baseline model incorporates this force, in addition to

Adding a positive capital share of investment ($\theta > 0$) in addition to intangibility ($\chi < 1$) makes the short-run elasticity finite (as in the baseline model), we have that both the long- and short-run elasticities are finite, but the short-run one is lower, therefore combining the insights of "q-theory" and "sweat capital".

Table 2: Summary of benchmark models: supply elasticity of capital.

Model	Constraint		Elasticity of capital	
	θ	χ	short-run $(\frac{\partial \log K_{t+1}}{\partial \log q_t})$	long-run $(\frac{\partial \log K}{\partial \log q})$
Lucas tree model	$\theta = 1$	$\chi = 1$	0	0
Sweat capital	$\theta = 0$	$-$	$\frac{1}{\sigma}\frac{1+\sigma\chi}{1-\chi}$	$\frac{1}{\sigma}\frac{1+\sigma\chi}{1-\chi}$
Q-theory	$-$	$\chi = 1$	$\delta\frac{1-\theta}{\theta}$	$+\infty$
Neoclassical growth	$\theta = 0$	$\chi = 1$	$+\infty$	$+\infty$
Baseline	$-$	$-$	$\delta\frac{(1-\theta)(1+\sigma\chi)}{\theta+\sigma(1-\chi(1-\theta))}$	$\frac{1}{\sigma}\frac{1+\sigma\chi}{1-\chi}$

Notes. α is the production capital share; θ the investment capital share; χ is the tangibility of investment; r is the steady-state return, δ is the steady-state capital formation rate.

3 Measurement

3.1 Sample construction

We use Compustat-CRSP merged data covering the period from 1972–2022. We use usual screens to focus on the nonfinancial corporate sector. Our goal is to construct industry-level series for: capital income Π, tangible investments I, intangible investments $w_H L_H$, and payments to capitalists D.

3.2 Mapping the model to the data

We start from the statement of cashflows, as reported in Compustat. The key accounting identity is:

$$\text{cf from operations} + \text{cf from financing} + \text{cf from investing} = 0 \tag{3.1}$$

The first term is the cashflows from operations, the second is the cashflows from financing activities, and the third is the cashflow for investing activities. We use the convention that changes in cash balances represent net payments to debt holders. Furthermore, we will later use the fact that cashflows from financing and investing can be decomposed into:

$$\text{cf from financing} = \text{cf from financing (equity)} + \text{cf from financing (debt)} \tag{3.2}$$

$$\text{cf from investing} = \text{cf from investing (capex)} + \text{cf from investing (acquisition)} \tag{3.3}$$

Equation (3.1) allows us to account for 100% of the cash that comes in due to profits (cashflows from operations), the net cash that comes out to pay owners (cashflows from financing activities), and the cash that comes out due to investment (cashflows from investment activities).

Notice that the cashflow identity is the financial accounting counterpart of the firm budget constraint (2.3) in the model. The mapping between model concepts and financial accounting terminology is summarized below:

$$\underbrace{\Pi_t}_{\substack{\text{capital income =} \\ \text{cashflows from operations}}} = \underbrace{(\Pi_t - w_{H,t}L_{H,t} - I_t)}_{\substack{\text{payments to capitalists =} \\ -\text{cashflows from financing}}} + \underbrace{(w_{H,t}L_{H,t} + I_t)}_{\substack{\text{total investment =} \\ -\text{cashflows from investment}}} \tag{3.4}$$

Building on the existing literature, we conduct some data imputations and adjustments, which we describe below.

Adjustment #1: expensed payments to nonproduction labor. One issue with accounting data is that the distinction between an expense (which are subtracted from sales to obtain cashflows from operation) and investment (which are not) can be arbitrary, especially in the case of intangibles.

The existing literature shows that this issue leads to an understatement of cashflows from operations and a corresponding understatement of (minus) cashflows from investment.[5] We follow the existing literature and correct cashflows from operation by adding 30% of SG&A expenses and 60% of R&D expenses. This is meant to account for the incorrect expensing of nonproduction labor expenses, which instead should be treated as a capital expenditure.

$$\text{cf adjustment (nonproduction labor)} \equiv 0.3 \cdot \text{sg\&a} + 0.6 \cdot \text{r\&d}. \tag{3.5}$$

Adjustment #2: net entry in sample. When a private firm becomes public or goes public, it effectively represents a negative cashflow for passive capitalists, who rebalance their portfolio to always own the market. Similarly, when a firm exits or gets acquired, it represents a positive cashflow. We now describe how we measure the contribution of net firm entry to market capitalization growth.

First, we introduce a decomposition of the total market value growth (at the aggregate or within an industry) between time t and $t+1$ as the sum of three components due to stayers, entrants, and exits.

[5] See Peters and Taylor (2017) for evidence from firm-level data and Koh et al. (2020) from aggregate data.

Formally, we have that

$$g_{\text{total}} = g_{\text{stayer}} + g_{\text{entry}} + g_{\text{exit}}, \tag{3.6}$$

where the components are defined in this footnote.[6]

Using these definitions, we define the net cashflow due to net firm entry as

$$\text{cf adjustment (net entry)} \equiv -(g_{\text{entry}} + g_{\text{exit}}) \cdot \text{market capitalization}. \tag{3.7}$$

It corresponds to the net cash that a passive capitalists who owns the market receives due to net firm entry during a period.

Adjustment #3: payments in stocks. How should we record a payment when it is the form of stocks? First, we can decompose the growth in market value of a stayer into growth of price per share and the growth in number of shares. To match the timing of the payment to the timing of the stock issuance, we use the fully-diluted share count (which includes outstanding shares, as well as all possible sources of potential shares such as stock options and reserved shares).

Formally, we can decompose the growth of the market capitalization of stayers (i.e., firms who do not exit the sample) defined in (3.6) into a price and number of shares component:

$$g_{\text{stayer}} = g_{\text{price}} + g_{\text{shares}}, \tag{3.8}$$

where the components are defined in this footnote.[7]

Moreover, we observe the the contribution of the growth in the number of shares issued for cash $g_{\text{shares, cash}}$, and the remainder $g_{\text{shares}} - g_{\text{shares, cash}}$ is due to a combination of stock compensation and stock-financed acquisitions (i.e., "noncash payments"). Using the same logic as for firm entry (see equation 3.7), we define noncash payments as

$$\text{cf adjustment (noncash payments)} \equiv (g_{\text{shares}} - g_{\text{shares, cash}}) \cdot \text{market capitalization}. \tag{3.9}$$

The breakdown between stock compensation and acquisitions is only available after 2011. Therefore, we assume that stock compensation is a constant share $\omega \in (0, 1)$ of noncash payments. Using the average of the aggregate ratio of stock compensation to stock compensation plus stock-financed acquisitions after 2011, we calibrate $\omega = 0.55$.

[6] The formulas are:

$$g_{\text{total}} = \frac{\sum_{i \in \mathcal{F}_{t+1}} V_{i,t+1}}{\sum_{i \in \mathcal{F}_t} V_{i,t}} - 1, \qquad g_{\text{stayer}} = \frac{\sum_{i \in (\mathcal{F}_t \cap \mathcal{F}_{t+1})} V_{i,t+1}}{\sum_{i \in (\mathcal{F}_t \cap \mathcal{F}_{t+1})} V_{i,t}} - 1,$$

$$g_{\text{entry}} = \frac{\sum_{i \in (\mathcal{F}_{t+1} \setminus \mathcal{F}_t)} V_{i,t+1}}{\sum_{i \in \mathcal{F}_t} V_{i,t}}, \qquad g_{\text{exit}} = -\frac{(1 + g_{\text{stayer}}) \sum_{i \in (\mathcal{F}_t \setminus \mathcal{F}_{t+1})} V_{i,t+1}}{\sum_{i \in \mathcal{F}_t} V_{i,t}},$$

where $\mathcal{F}_t$ is the universe of firms at time t and V_i is the market capitalization of firm i.

[7] The definitions of g_{stayer}, g_{price}, and g_{shares} are

$$g_{\text{stayer}} = \frac{P_{i,t+1} N_{i,t+1}}{P_{i,t} N_{i,t}} - 1, \qquad g_{\text{price}} = \frac{P_{i,t+1}}{P_{i,t}} - 1, \qquad g_{\text{shares}} = \left(\frac{N_{i,t+1}}{N_{i,t}} - 1 \right) (1 + g_{\text{price}}),$$

where $P_{i,t}$ is the price per share and $N_{i,t}$ is the number of shares outstanding at firm i period t.

Adjustment #4: imputation of the tangible share of ambiguous investments. How should one proceed to allocate the sum of cashflows due to mergers, acquisitions, and IPOs into intangible investments versus investment. To do so, we use ambiguous-income approach introduced in Cooley et al. (1995). Denoting $\chi \in (0,1)$ the tangiblity of investment,

$$\chi \equiv \frac{\text{cf from investing (capex)}}{\text{cf from investing (capex)} + \text{cf adjustments (nonproduction labor + } \omega \cdot \text{ noncash payments)}} \tag{3.10}$$

which represents the share of unambiguous tangible investment to unambiguous total investment (i.e., including intangible investments). Using the yearly average over the full sample, we obtain $\chi = 0.4$.

3.3 Formulas

Having defined the key variables, we now write down the final formulas:

$$\begin{aligned}
\text{tangible investments} = &- \text{cf from investing (capex)}, \\
&- \chi \cdot \text{cf from investing (acquisition)}, \\
&- \chi \cdot \text{cf adjustments} \Big(\text{net entry + } (1-\omega) \cdot \text{ noncash payments}\Big), \\
\text{intangible investments} = &- \text{cf adjustments} \Big(\text{nonproduction labor + } \omega \cdot \text{ noncash payments}\Big), \\
&- (1-\chi) \cdot \text{cf from investing (acquisition)}, \\
&- (1-\chi) \cdot \text{cf adjustments} \Big(\text{net entry + } (1-\omega) \cdot \text{ noncash payments}\Big), \\
\text{payments to capitalists} = &- \text{cf from investing} \\
&+ \text{cf adjustments} \Big(\text{net entry + noncash payments}\Big).
\end{aligned}$$

Combining these formulas, we obtain an expression for capital income

$$\begin{aligned}
\text{capital income} =& \text{tangible investments} + \text{intangible investments} + \text{payments to capitalists} \\
=& \text{cf from operations} + \text{cf adjustments (nonproduction labor)},
\end{aligned}$$

where the second equality uses the cashflow identity (3.1).

A few remarks are in order. First, that payments to capitalists are invariant to the calibration (χ, ω). This is because they only represent assumptions on how cashflows are distributed between tangible investments and intangible investments. Second, capital income is invariant not only to the calibration (χ, ω), but also the adjustments for noncash payments and net entry. This is because noncash payments and net entry are purely redistributive flows (from capitalists to tangible investments and to innovators). The only adjustment that affects the level of capital income is the adjsutment for expensed nonproduction labor. The idea is that we reclassify line items as capital expenditures, not expenses.

3.4 Consistency with the National Accounts

We now validate the resulting sample against aggregate data. Our definition of capital is meant to be (mostly) constistent withe the national accounts definition. There are two key differences. First, our treatment of noncash payments is more comprehensive than what the BEA does. It is well understood that the BEA undercounts labor income by missing payments or underestimating their true value (see Zwick, 2022 for a detailed discussion on the topic). In contrast, our approach uses the actual growth in the number of (fully-diluted) shares times the market value at the time to impute the value of payments in stocks.

Second, our definition of what an innovator potentially differs from the BEA. The BEA mostly capitalizes expenses on labor in the case of research and development (see Corrado et al., 2009). Instead, we opt for a more comprehensive definition of "non-porduction labor", which includes not only scientists, bu also key managers, entrepreneurs, and early financiers.

Appendix Figure A1 plots capital income and enterprise value, both in our sample and in the the Integrated Macroeconomic Accounts (Table S.5.a). Note that our sample accounts for roughly two-thirds of nonfinancial corporate sector gross capital income over the period, with a slight upward trend.

3.5 Long-run facts

Figure 2 reports the distribution of capital income over time. Two facts stand out: (1) the tangible investment share declines, (2) the intangible investment share increases, and (3) the capitalist share is low on average, often negative, and highly countercyclical.

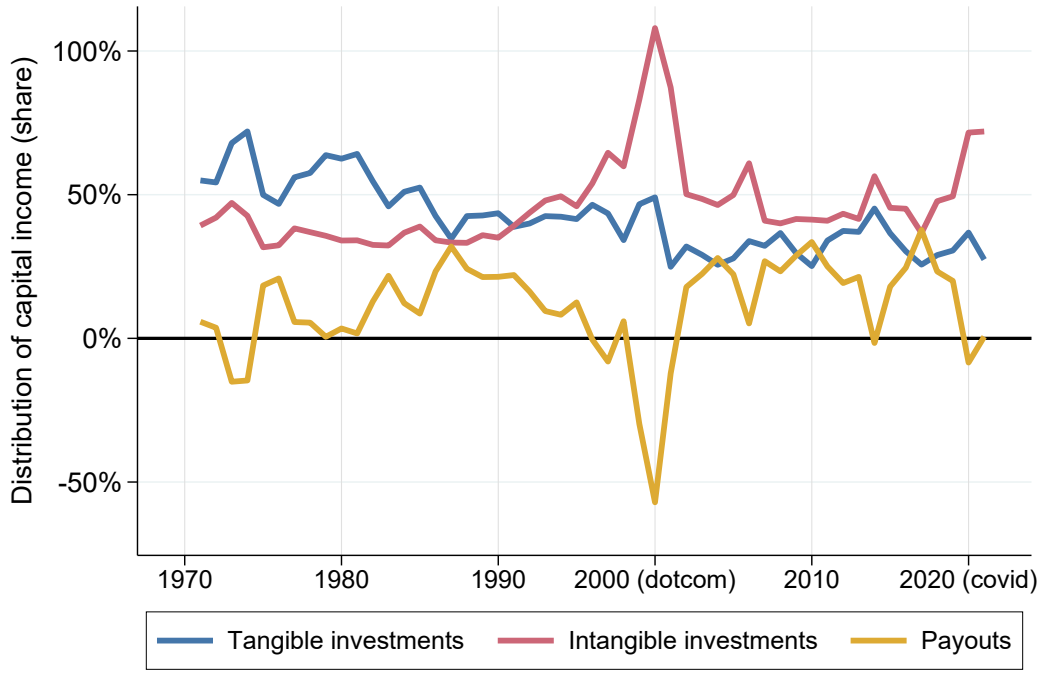


Figure 2: Distribution of capital income (nonfinancial public corporations, 1972–2022).

Those three facts are consistent with findings in the literature relating to the stagnation of capital

expenditures post-GFC (Gutiérrez and Philippon, 2017; Crouzet and Eberly, 2019), the importance of payments to nonproduction labor (Eisfeldt and Papanikolaou, 2013; Peters and Taylor, 2017; Eisfeldt et al., 2023), and the empirical evidence on the aggregate equity payout yield (Fama and French, 2005; Boudoukh et al., 2007; Fried and Wang, 2019). Our dataset offers an opportunity to study these facts jointly, over a consistently-defined sample that covers five decades.

Zooming in on the tangibility of investment, Figure 3 decomposes the empirical tangibility index χ (defined in equation 3.10) into (i) the contribution of cash payments derived from estimates of payments to nonproduction labor (i.e., sg&a and r&d) and (ii) the contribution of noncash payments derived from the growth rate of the share count. Essentially all of the rise in intangibility is accounted for by payments in stocks, in line with the direct evidence from the manufacturing sector put forward in Eisfeldt and Papanikolaou (2013).

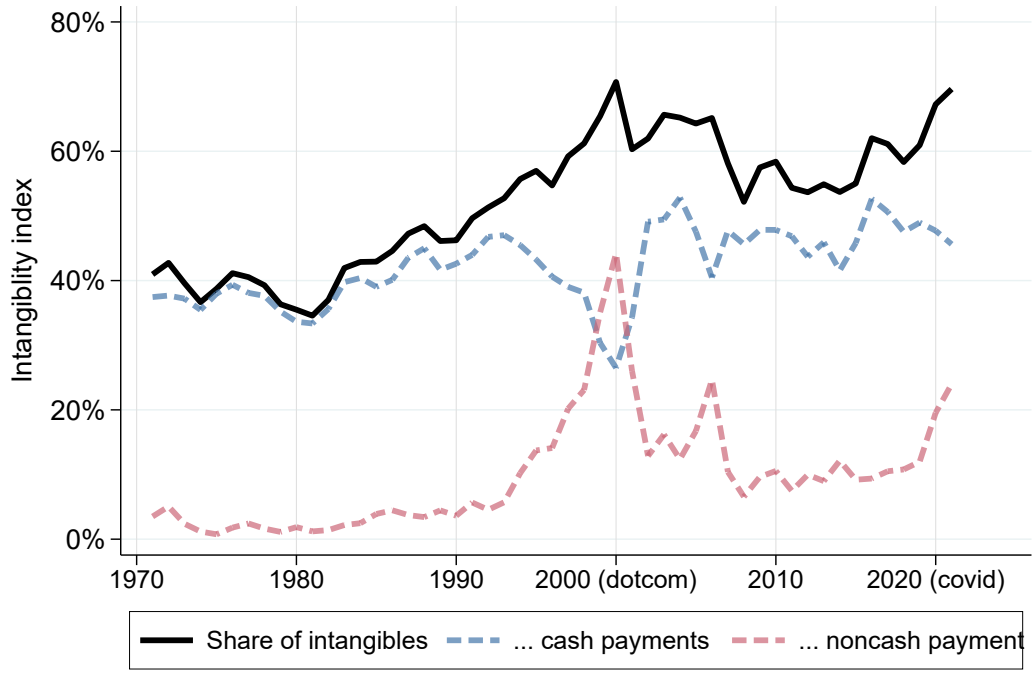


Figure 3: Intangibility of capital $1 - \chi$ (nonfinancial public corporations, 1972–2022).

Table 3 reports the average distribution of capital income over the 1972–2022 period, including a detailed breakdown, and robustness checks. Tangible investments account for 42% of capital over the period, mostly due to direct capital expenditures, but also in part due to acquisition of existing businesses (who themselves are partial tangible). Intangible investments account for 54%. Cash and noncash compensation to nonproduction labor accounts for most of it, but almost 20% comes from payouts associated with cash and noncash payout that arise in acquisitions.

Finally, payments to capitalists account for a mere 4% of capital income over the sample. While net cash payouts to equity holders account for 22% of capital income, but it is offset by a 20% equity dilution. This sample average is skewed by the tech bubble of the 1990s, where payments to capitalists were negative for a full decade (ie, acquisitions and IPOs exceeded dividends and interest payouts, see Fried and Wang, 2019 for a discussion of this fact). Our findings highlight the (growing) importance of

Table 3: Distribution of capital income (1972–2022 average).

Uses of capital income (%)	Baseline	Robustness			
		$\chi = 0$	$\chi = 1$	$\omega = 0$	$\omega = 1$
Tangible investments	42	38	49	45	40
Capital expenditures	38	38	38	38	38
Mergers, acquisitions, and IPOs	5	0	12	8	3
$\chi \cdot$ cash acquisitions	3	0	8	3	3
$\chi(1-\omega) \cdot$ non-cash payments	2	0	4	5	0
$\chi \cdot$ net entry in public universe	-1	0	-1	-1	-1
Intangible investments	47	51	40	44	48
Salaries, bonuses, and stock compensation	40	40	40	32	44
$0.3 \cdot$ selling, general, and admin. expenses	25	25	25	25	25
$0.6 \cdot$ research and development expenses	7	7	7	7	7
$\omega \cdot$ non-cash payments	8	8	8	0	12
Mergers, acquisitions, and IPOs	7	12	0	12	4
$(1-\chi) \cdot$ cash acquisitions	5	8	0	5	5
$(1-\chi)(1-\omega) \cdot$ non-cash payments	3	4	0	7	0
$(1-\chi) \cdot$ net entry in public universe	-0	-1	0	-0	-0
Payout to capitalists	11	11	11	11	11
Net cash equity payout	20	20	20	20	20
− Non-cash payments	-12	-12	-12	-12	-12
− Net entry in public universe	1	1	1	1	1
Net debt payout	3	3	3	3	3

non-cash payments.

4 Counterfactuals

We now combine theory and data to generate counterfactuals. First, we discuss identification, i.e., what features of the data are informative on the values of the two new parameters θ and χ. We then calibrate the model and consider two counterfactuals: a shift towards intangibles and a permanent saving shock. We highlight how the model makes sense of several empirical facts regarding the US corporate sector: the decline in tangible investment despite high valuations, rise in payouts to high-skilled innovative labor, and the low response of total investment to lower interest rates.

4.1 Identification

First, we calibrate the parameters σ (labor supply elasticity) and α (production labor share) to standard values, to separately isolate the effect of our general investment function. The parameter R is set to 4.5%, which is the average total unlevered return in our sample.

Second, we identify the remaining parameters (δ, χ, θ) using two long-run and one short-run model equation. Using (firm foc I_t) and (firm foc $L_{H,t}$), combined with the steady-state condition $G/K = \delta$, we obtain the first two model equations (see Table 5). Basically, it says that, in the long-run, firms invest

on average a fraction $(1-\theta)\delta$ of firm value, a fraction $\chi \in [0,1]$ of which is tangible investment. Note that, in these two equations, the long-run capital formation rate δ is not separately identified from the capital share θ.

For the third equation, we use the fact that the short-run investment-q elasticity is

$$\begin{aligned} \mathrm{d}\log(I_t + w_{H,t}L_{H,t}) &= \mathrm{d}\log q_t + \mathrm{d}\log G_t \\ &= \left(1 + \frac{(1-\theta)(1+\sigma\chi)}{\theta + \sigma(1-\chi(1-\theta))}\right) \mathrm{d}\log q_t. \end{aligned} \tag{Investment-q}$$

The first equality comes from (firm foc I_t) and (firm foc $L_{H,t}$), the second equation uses the short-run capital elasticity (see Table 2).

To estimate (Investment-q), we leverage cross-industry variations. We assume that each industry is small in the aggregate (i.e., they face a flat labor supply curve $\sigma = 0$). In that case, we obtain $\mathrm{d}\log(I_t + w_{H,t}L_{H,t}) = \frac{1}{\theta} \cdot \mathrm{d}\log q_t$. The logic is that if capital is an important input in the investment function (i.e., $\theta > 0$), then the response of investment to q is more muted (in the spirit of adjustment cost functions) and capitalists extract more investment rents (θ governs the amount of Ricardian rents that accrue to capitalists).

Using our industry dataset (i.e., 25 industries over 50 years), we estimate $1/\theta$ using an IV approach:

$$\begin{aligned} \log\left(I_{j,t} + w_{H,j,t}L_{H,j,t}\right) &= \text{controls}_{j,t} + \frac{1}{\theta}\log V_{j,t} + u_{j,t}, \\ \log V_{j,t} &= \text{controls}_{j,t} + \beta \log R^X_{t-h} + v_{j,t}, \end{aligned}$$

where the identification assumption is $\mathrm{E}(u_{j,t}v_{j,t}) = 0$. In words, we regress log total investment on contemporaneous log enterprise value, where the latter is instrumented with return excluding payout at time $t-h$, where $h = 0, 1, \ldots$ refers the instrument horizon. The controls include industry and year fixed effects. The logic is that the annual industry-level returns are mostly drive by changes in the valuation of capital q_t, rather than changes in capital K_t.[8]

Table 4: Industry-level investment-q regressions.

Instrument horizon	Point estimate $\widehat{1/\theta}$	95% Confidence Interval
Contemporaneous	1.59	[0.96 – 2.22]
1-year ahead	2.43	[1.72 – 3.13]
2-years ahead	1.18	[0.77 – 1.59]

Table 4 reports the results for $h = 0, 1, 2$, with confidence intervals constructed using HAC standard errors. First, recall that theory predicts a coefficient weakly above one: $1/\theta \in [1, +\infty)$. The reason is that a higher q increases the cost of investment one-for-one, but there is also an endogenous response of quantities given by the short-run capital supply elasticity. Depending on the instrument horizon,

[8] Violation of the exclusion would arise, for instance, if revaluation $\mathrm{d}\log q_{t-h}$ is related to future IST shocks that would show up in $u_{j,t}$.

we obtain values ranging from 1.2 to 2.4. Given the sampling uncertainty and the potential biases, we consider the range of values from 1 to 3 as equally likely to explain the short-run evidence presented above.

Table 5: Calibration strategy

Moment	Model	Target
Internally calibrated		
Tangible investment / firm value	$(1-\theta)\chi\delta$	0.035
Intangible investment / firm value	$(1-\theta)(1-\chi)\delta$	0.04
Short-run investment-q elasticity	$1/\theta$	[1 – 3]
Externally calibrated		
Labor supply elasticity	$1/\sigma$	0.5
Production capital share	α	0.33
Real unlevered gross return	r	0.045

Table 5 reports the empirical targets and model equations. The solution in the intermediate case $1/\theta = 2$—our preferred calibration—is given by:

$$\chi = 0.45, \qquad \theta = 0.5, \qquad \delta = 0.15. \qquad \text{(Preferred calibration)}$$

Basically, it says that investment is 45% goods and 55% labor. The capital share in investment is 50%, and capital depreciates are rate $\delta = 0.15$ (in line with BEA estimates that account for both tangible and intangible capital, see Corrado et al., 2009).

4.2 Model experiment: shift towards intangibles

We now conduct an experiment where we quantify the long-run effect of a decline in the tangibility of investment χ from 0.6 to 0.3, in line with what we have seen in the data over the 1972-2021 period (see Figure 3).

In Table 6, we now quantify the distributional effects of rising intangibility, not only on the firm budget constraint, but on the composition of GDP (see equation 2.2 for national accounting definitions).

Panel A reports the uses of capital income via the firm budget constraint (expressed in pp. of capital income). We obtain a pure redistribution from tangible (-19 pp.) to intangible investments ($+19$ pp.), which by construction matches our empirical evidence.

On the expenditure side (Panel B), we see taht the rise in the intangible investments ($+4$ pp. of GDP), compensated *more than one-for-one* by a decline in tangible investments (-6 pp. of GDP). To understand why, the long-run intangible share of GDP is

$$\frac{I}{Y + w_H L_H} = \frac{\chi q G}{Y + (1-\chi) q G}. \qquad \text{(Tangible share of GDP)}$$

Table 6: Distributional effect (shift towards intangibles).

Variable	Symbol	Baseline (pp.)	Shock (pp.)
Panel A – Budget constraint			
Tangible investments	I	28	-19
Intangible investments	$w_H L_H$	34	$+19$
Payouts	D	38	0
Panel B – Expenditures			
Consumption	$Y - I$	81	$+2$
Tangible investments	I	8	-6
Intangible investments	$w_H L_H$	10	$+4$
Panel C – Income			
Capital income	$Y - w_Y L_Y$	30	-1
Labor income (production)	$w_Y L_Y$	60	-3
Labor income (investment)	$w_H L_H$	10	$+4$

Using the fact that the long-run value of capital formation relative to production qG/Y is fixed, a decline in χ reduces the numerator (direct effect) and increases the denominator (indirect effect).[9] Consumption *rises* as a share of GDP, owing to the fact that intangible investments don't substract from gross output, since they represent income for households.

On the income side (Panel C), we see decline in the production labor income share (-3 pp.), as well as the capital income share (-1 pp.), compensated by a $+4$ pp. rise in the investment labor income share. In the baseline calibration, labor supply is quite inelastic ($\sigma = 2$), which means that most of this reallocation of incomes between production and investment labor is due to changes in relative wages (i.e., $\mathrm{d}\log\frac{w_H L_H}{w_Y L_Y} = \frac{2}{3}\cdot\mathrm{d}\log\frac{w_H}{w_Y}$).

What this means is that we should observe rising wage polarization in the labor market, where demand for workers with skills that are important for capital formation should lead to higher relative wages. This is reminiscent of an influential literature on US wage inequality post-1980, which finds that occupations requiring "abstract" skills have seen a relative rise in wages relative to those requiring "routing" skills (see, e.g., Autor and Dorn, 2013).

A striking prediction is that the capital income share (-1 pp.) increases. This precisely for the same reason as in (Tangible share of GDP): holding aggregate capital income constant, a rise in $w_H L_H$ leads to a decline of the capital income *share* of GDP. Note, however, that testing this theory requires having aggregate data that captures all payments to labor, including when it takes for form of equity compensation and IPO exits for entrepreneurs and other key employees. Koh et al. (2020) and Eisfeldt et al. (2023) discuss in details the conceptual and measurement challenges associated with national accounting in the presence of intangibles.

[9] In the long-run, we know that $\frac{qG}{Y} = \frac{\alpha\delta}{r+(1-\theta)\delta}$.

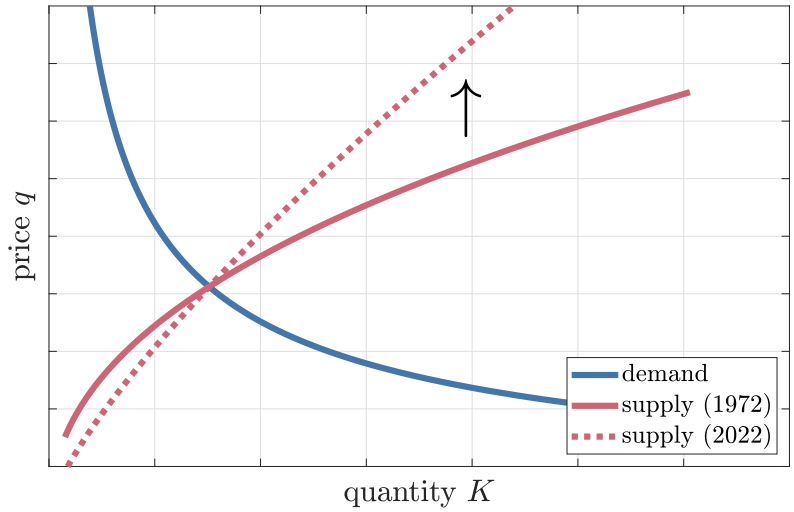


Figure 4: Long-run equilibrium (model experiment, rise of intangibles).

Not only does a shift towards intangibles reshapes the distribution of income, but it also fundamentally changes the supply elasticity of capital. Figure 4 shows the long-run effect of rising intangibles on the capital supply curve, which amounts to a counter-clockwise rotation of the supply curve.[10] Notice that capital supply becomes much less elastic, due to the fact that labor itself is imperfectly elastic (as governed by $1/\sigma$).

Table 7: Capital supply elasticity in intangible economies.

	Formula	Baseline	Q-theory $\chi = 1$	Low intang. $\chi = 0.6$	High intang. $\chi = 0.3$
Short-run	$\frac{\partial \log G_t}{\partial \log q_t}$	0.46	1	0.58	0.36
Long-run	$\frac{\partial \log K}{\partial \log q}$	1.73	$+\infty$	2.75	1.14

Table 7 reports the model-implied elasticities. First, note that the model has low elasticities compared to the case of full tangibility (i.e., setting $\chi = 1$ as in the standard q-theory of investment). In the short-run it is 0.46 versus 1, and in the long-run it is 1.73 versus $+\infty$. Second, a decline in χ in line with what we have seen in the data cuts the short- and long-run elasticities by more than half. Next, we explore the implications of this decline in the elasticity of capital, focusing on the transmission of saving shocks to capital K and prices q.

4.3 Model experiment: saving shock

Over our sample period, the US economy has experience large decline in real interest rates. Gomez and Gouin-Bonenfant (2024) estimate a roughly 2 pp. long-run decline in real expected return R on firm liabilities in the US. What are the implications for investment and the distribution of income? First, we

[10] Note that a change in χ affects the long-run equilibrium Y due to an endowment effect. For visual purposes, we construct Figure 4 by showing a rotation around the initial equilibrium, which amounts to consider a joint change in χ and z_H that ensure no change in the long-run (q, K).

describe the effect of saving shocks in the model, and then quantify the distributional effect of a 2 pp. decline in R.

In the model, we have that $\beta R = 1$ (linear utility), which means that saving shocks $\mathrm{d}\log\beta$ are equivalent to negative $\mathrm{d}\log R$, so we work directly with R. A change in β is a reduced-form way to capture stories such as population aging, global savings glut, convenience yield on USD assets. Figure 5 plots the long-run effect of saving shock in the baseline calibrated model.

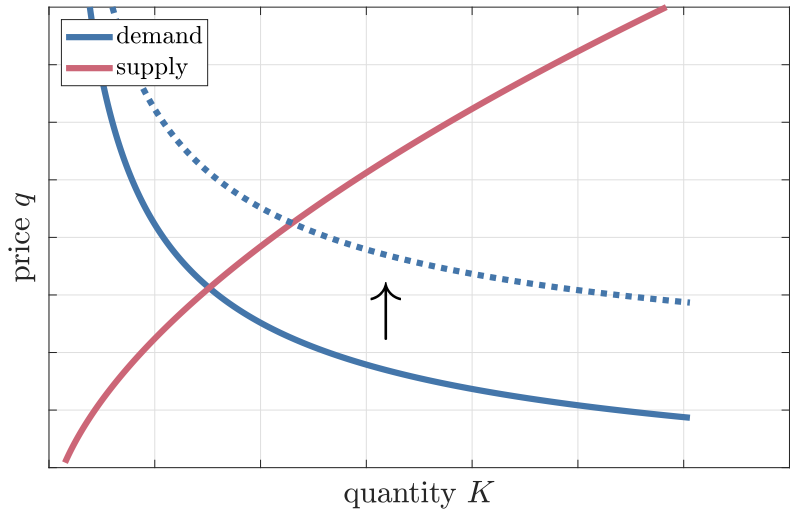


Figure 5: Model experiment: Demand shocks in the long-run.

As usual in supply-demand systems, a positive demand shock will increase prices and quantities. When the supply curve is vertical (as in the ngm), then the demand shock is entirely absorbed by quantities K. But if instead the supply curve horizontal (as in sweat capital with inelastic labor), then the shock is entirely absorbed by prices Q. In Appendix A.6, we derive a sufficient statistic formula for the long-run effect of saving shocks on quantities and prices. Applying them to a hypothetical 2 pp. decline in R, we obtain:

$$\frac{\mathrm{d}\log K}{\mathrm{d}\log R}\cdot(-2\%) = \frac{1}{\text{demand elasticity} + \text{supply elasticity}}\cdot\text{duration} \approx 15\%,$$
$$\frac{\mathrm{d}\log q}{\mathrm{d}\log R}\cdot(-2\%) = \frac{\text{supply elasticity}}{\text{demand elasticity} + \text{supply elasticity}}\cdot\text{duration} \approx 8\%.$$

The model predicts in modest response of capital (15% versus 29% in the ngm), accompanied with a rise in valuations (8% versus 0% in ngm). In words, a shift towards intangibles means that the market increasingly clears via higher valuations, rather than higher investment. It is worth emphasizing that this stands in contrast with standard models, where the long-run capital market always clears via quantities ($q = 1$ in the long-run).

Saving shocks also have important distributional implications. Table 8 reports the long-run effect of a 2% discount rate shock on the composition of GDP and the firm budget constraint.

Focusing on the firm budget constraint (Panel A), the model predicts a large reallocation of capital income away from payouts (-11 pp. of capital income) and towards total investment. This fact is invariant to the choice of χ, but is amplified by the fact that the capital share of investment θ is positive.

Table 8: Changing composition of GDP (model experiment, saving shock).

Variable	Symbol	Baseline (pp.)	Shock (pp.)			
			Baseline	Ngm	$\chi = 0.6$	$\chi = 0.3$
Panel A - Budget constraint						
Tangible investments	I	28	$+4.7$	$+7.9$	$+15$	$+3.2$
Intangible investments	$w_H L_H$	34	$+5.8$	$-$	$+4.2$	$+7.3$
Payouts	D	38	-11	-7.9	-11	-11
Panel B - Expenditures						
Consumption	C	81	-2.8	-2.6	-3.0	-2.7
Tangible investments	I	8	$+1.3$	$+2.6$	$+1.8$	$+0.8$
Intangible investments	$w_H L_H$	10	$+1.5$	$-$	$+1.2$	$+1.9$
Panel C - Income						
Capital income	Π	30	-0.5	0	-0.4	-0.6
Labor income (production)	$w_Y L_Y$	60	-1.0	0	-0.8	-1.2
Labor income (investment)	$w_H L_H$	10	$+1.5$	$-$	$+1.2$	$+1.9$

In particular, the ngm predicts a decline in payouts 25% smaller. Recall that we estimate an average payout share of capital income of 4% (see Table 3), which is much lower than what the model predicts (38%). Our exercises highlights that the model's long-run predictions regarding the payout share are very sensitive to the calibration of R (we do not use it as an empirical target).

We find that saving shocks are strongly redistributive. In terms of expenditures (Panel B), the model predicts a decline in the consumption share (-2.8 pp. of GDP), offset by a rise in total investment. But more interestingly, we find a large reallocation of income towards investment labor ($+$ 1.5, Panel C) at the expense of capital (-0.5 pp.) and production labor (-1.0 pp.). Notice that the redistributive effect of saving shocks is stronger in intangible economies. In the limit where capital is all tangible ($\chi = 1$, ngm) predicts no redistributive effect on income, while the high-intangible economy ($\chi = 0.3$) has a redistribution of $1.9/1.2 \approx 50\%$ larger than the low-intangible economy ($\chi = 0.6$).

Taken as a whole, the model predicts that both the shift towards intangibles and the decline in interest rates have contributed to a rise in the share of income earned by investment labor, at the expense of production labor and capitalists.

5 Conclusion

We provide a parsimonious theory of intangible investment and long-run moments that are informative about the structural elasticities in the model. As a case study, we focus on the US corporate sector over the 1972–2022 period.

We show that intangible economies—where business capital is produced using labor—exhibit a dif-

ferent distribution of income and expenditures. Aggregate income is disproportionately earned by investment labor, rather than production labor and capitalists. The flipside of this is that aggregate expenditures increasingly take the form of intangible investments.

The calibrated model provides a coherent framework in which we can re-assess a number of theories and controversies regarding the macroeconomics of the post-2000: the stagnation of investment despite high valuations, the (mis-)measurement of the labor income share, the presence of Ricardian rents and factorless income, the low interest-rate sensitivity of investment and its effect on the transmission of monetary policy, etc.

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A Proofs

A.1 A microfoundation

We now provide a microfoundation of the aggregate functions Y, G, and q. Suppose that there are $i = 1, \ldots, I$ price-taking firms who differ in their (deterministic) productivity process $\{z_{Y,i,t}, z_{H,i,t}\}_{t \geq 1}$, but otherwise all solve the same firm problem as in the baseline model above. We show that the the model aggregates to the representative firm problem where, except that aggregate technology now depends on the joint distribution of firm-level technology and capital:

$$z_{Y,t} = \left(\sum_{i=1}^{I} \frac{K_{i,t}}{K_t} \cdot z_{Y,i,t}^{\frac{1}{\alpha}} \right)^{\alpha}, \quad z_{H,t} = \left(\sum_{i=1}^{I} \frac{q_{i,t} V_{i,t}}{q_t V_t} \cdot z_{H,i,t}^{\frac{1}{\theta}} \right)^{\theta}.$$

Production function Y. We follow Baqaee and Farhi (2018) and define the aggregate production function as the efficient level of production given (i) the distribution of productivity and capital and (ii) a given amount of labor L_Y.

$$Y \equiv \max_{\{L_{Y,i,t}\}} \sum_{i=1}^{I} Y_{i,t} \quad \text{s.t.} \quad \sum_{i=1}^{I} L_{Y,i,t} = L_{Y,t}$$

Taking the first-order condition and solving for $L_{Y,i,t}$ we obtain

$$L_{Y,i,t} = \left(\frac{1-\alpha}{\lambda_{Y,t}}\right)^{\frac{1}{\alpha}} K_{i,t} z_{Y,i,t}^{\frac{1}{\alpha}},$$

where $\lambda_{Y,t}$ is the marginal value of production labor. Aggregating on both sides and using the labor constraint, we obtain

$$L_{Y,t} = \left(\frac{1-\alpha}{\lambda_{Y,t}}\right)^{\frac{1}{\alpha}} \sum_{i=1}^{I} K_{i,t} z_{Y,i,t}^{\frac{1}{\alpha}} \tag{A.1}$$

Combining both equations, we have that

$$L_{Y,i,t} = \frac{\frac{K_{i,t}}{K_t} \cdot z_{Y,i,t}^{\frac{1}{\alpha}}}{\sum_{i=1}^{I} \frac{K_{i,t}}{K_t} \cdot z_{Y,i,t}^{\frac{1}{\alpha}}} L_{Y,t}. \tag{A.2}$$

The aggregate production function can therefore be calculated as

$$\begin{aligned}
Y_t &= \sum_{i=1}^{I} z_{Y,i,t} K_{i,t}^{\alpha} L_{Y,i,t}^{1-\alpha} \\
&= \sum_{i=1}^{I} z_{Y,i,t} (K_{i,t}/L_{Y,i,t})^{\alpha} \cdot L_{Y,i,t} \\
&= \left(\sum_{i=1}^{I} \frac{K_{i,t}}{K_t} \cdot z_{Y,i,t}^{\frac{1}{\alpha}} \right)^{\alpha} (K_t/L_{Y,t})^{\alpha} \cdot \sum_{i=1}^{I} L_{Y,i,t} \\
&= \left(\sum_{i=1}^{I} \frac{K_{i,t}}{K_t} z_{Y,i,t}^{\frac{1}{\alpha}} \right)^{\alpha} K_t^{\alpha} L_{Y,t}^{1-\alpha}
\end{aligned} \tag{A.3}$$

The key is the third equation, where we use both (A.1) and A.2. Basically, we use the fact that the MPL is equalized across firms (to λ_Y). Aggregate TFP is thus a capital-weighted harmonic mean of individual TFP.

Capital formation function G. The aggregate capital formation function is the optimal allocation resources given the distribution of capital and productivity:

$$\max_{\{L_{H,i,t},I_{i,t}\}} \sum_{i=1}^{I} q_{i,t}G_{i,t} \quad \text{s.t.} \quad \sum_{i=1}^{I} L_{H,i,t} = L_{H,t}, \quad \sum_{i=1}^{I} I_{i,t} = I_t.$$

Following the same steps as for the aggregate production function, the solution is

$$\begin{aligned}
L_{H,i,t} &= \frac{\frac{q_{i,t}K_{i,t}}{q_tK_t} \cdot z_{H,i,t}^{\frac{1}{\theta}}}{\sum_{i=1}^{I} \frac{q_{i,t}K_{i,t}}{q_tK_t} \cdot z_{H,i,t}^{\frac{1}{\theta}}} L_{H,t} \\
I_{i,t} &= \frac{\frac{q_{i,t}K_{i,t}}{q_tK_t} \cdot z_{H,i,t}^{\frac{1}{\theta}}}{\sum_{i=1}^{I} \frac{q_{i,t}K_{i,t}}{q_tK_t} \cdot z_{H,i,t}^{\frac{1}{\theta}}} I_t \\
G_t &= \left(\sum_{i=1}^{I} \frac{q_{i,t}K_{i,t}}{q_tK_t} \cdot z_{H,i,t}^{\frac{1}{\theta}} \right)^{\theta} K_t^{\theta} \left(I_t^{\chi} L_{Y,t}^{1-\chi} \right)^{1-\theta}
\end{aligned}$$

Aggregate IST is a value-weighted harmonic mean of individual IST.

Valuations q. Individual firm valuation is given by

$$Rq_{i,t} = \alpha \frac{Y_{i,t+1}}{K_{i,t+1}} + \left(1 - \delta + \theta \frac{G_{i,t+1}}{K_{i,t+1}}\right) q_{i,t+1}$$

Using the equilibrium allocation, we have that

$$\begin{aligned}
\frac{Y_{i,t+1}}{K_{i,t+1}} &= \frac{z_{Y,i,t}^{\frac{1}{\alpha}}}{\sum_{i=1}^{I} \frac{K_{i,t}}{K_t} \cdot z_{Y,i,t}^{\frac{1}{\alpha}}} \cdot \frac{Y_t}{K_t} \\
\frac{G_{i,t+1}}{K_{i,t+1}} &= \frac{z_{H,i,t}^{\frac{1}{\theta}}}{\sum_{i=1}^{I} \frac{K_{i,t}}{K_t} \cdot z_{H,i,t}^{\frac{1}{\theta}}} \frac{G_t}{K_t}
\end{aligned}$$

Therefore, individual firm valuations can be expressed in terms of aggregates $\{Y_t, G_t, K_t\}$ as well as relative productivity

$$Rq_{i,t} = \left(\frac{z_{Y,i,t}}{z_{Y,t}}\right)^{\frac{1}{\alpha}} \cdot \alpha \frac{Y_{t+1}}{K_{t+1}} + \left(1 - \delta + \left(\frac{z_{H,i,t}}{z_{H,t}}\right)^{\frac{1}{\theta}} \cdot \theta \frac{G_{t+1}}{K_{t+1}}\right) q_{i,t+1}. \tag{A.4}$$

Assuming that $\lim_{T\to\infty} R^{-T} q_{i,T} = 0$, we can easily show that $\sum_{i=1}^{I} \frac{K_{i,t}}{K_t} \cdot q_{i,t} = q_t$.

A.2 Proof of Lemma 2.1

We now provide equilibrium expressions for the choices of the firm $(L_{Y,t}, L_{H,t}, I_t)$ in terms of the state variables $(K_t, z_{Y,t}, z_{H,t})$ and the co-state variable q_t. We only need the the combined foc's for workers and firms (i.e., firm foc $L_{Y,t}$; firm foc $L_{H,t}$; firm foc I_t; worker foc $L_{Y,t}$; worker foc $L_{H,t}$) summarized by three equations:

$$(1-\mu)L_{Y,t}^{\sigma} = (1-\alpha)\frac{Y_t}{L_{Y,t}}, \tag{eq1}$$

$$\mu L_{H,t}^{\sigma} = (1-\theta)(1-\chi)\frac{G_t}{L_{H,t}}q_t, \tag{eq2}$$

$$1 = (1-\theta)\chi\frac{G_t}{I_t}q_t. \tag{eq3}$$

Combining (eq1) and the definition of Y we obtain the solution for $L_{H,t}$

$$L_{Y,t} = \left(\frac{1-\alpha}{1-\mu}\right)^{\frac{1}{\sigma+\alpha}} \cdot (z_{Y,t} \cdot K_t^{\alpha})^{\frac{1}{\sigma+\alpha}},$$

Combining (eq2) and (eq3), we have that

$$I_t = \frac{\chi}{1-\chi}\mu L_{H,t}^{1+\sigma}$$

Plugging into the definition of G_t, we obtain

$$\begin{aligned} G_t &= z_{H,t}K_t^{\theta}\left(\left(\frac{\chi}{1-\chi}\mu L_{H,t}^{1+\sigma}\right)^{\chi} L_{H,t}^{1-\chi}\right)^{1-\theta} \\ &= \left(\frac{\chi}{1-\chi}\mu\right)^{\chi(1-\theta)} z_{H,t}K_t^{\theta}L_{H,t}^{(1+\sigma\chi)(1-\theta)} \end{aligned}$$

Whenever labor supply is elastic $\sigma\chi > 0$, the capital formation elasticity of investment labor is amplified. Plugging this expression for capital formation into (eq2), we obtain

$$L_{H,t} = \left(\frac{1}{\mu}(1-\theta)(1-\chi)\left(\frac{\chi}{1-\chi}\mu\right)^{\chi(1-\theta)}\right)^{\frac{1}{\sigma+\theta-\sigma\chi(1-\theta)}} \cdot \left(z_{H,t} \cdot q_t \cdot K_t^{\theta}\right)^{\frac{1}{\sigma+\theta-\sigma\chi(1-\theta)}}.$$

Finally, we have

$$I_t = \frac{\chi}{1-\chi}\mu\left(\frac{1}{\mu}(1-\theta)(1-\chi)\left(\frac{\chi}{1-\chi}\mu\right)^{\chi(1-\theta)}\right)^{\frac{1+\sigma}{\sigma+\theta-\sigma\chi(1-\theta)}} \cdot \left(z_{H,t} \cdot q_t \cdot K_t^{\theta}\right)^{\frac{1+\sigma}{\sigma+\theta-\sigma\chi(1-\theta)}}$$

A.3 Proof of Lemma 2.2

To solve for the steady-state (q, k), we use two equations:

$$Rq = \alpha \frac{Y}{K} + \left(1 - (1-\theta)\frac{G}{K}\right) q, \tag{eq4}$$

$$K = (1-\delta)K + G. \tag{eq5}$$

Combining (eq4) and (eq5), we have that

$$q = \frac{\alpha \frac{Y}{K}}{r + (1-\theta)\delta},$$

which is a valuation equation: capital income discounted at rate $r + (1-\theta)\delta$, to account for dilution. Using the solution for L_Y from Lemma 2.1, we have that

$$Y = K^{\alpha}\left(\bar{l}_Y K^{\frac{\alpha}{\sigma+\alpha}}\right)^{1-\alpha} \implies Y/K = \bar{l}_Y^{1-\alpha} K^{-(1-\alpha)\frac{\sigma}{\sigma+\alpha}}$$

Therefore, we have that q is a decreasing function of K (a demand curve):

$$q = \frac{\alpha \bar{l}_Y^{1-\alpha}}{r + (1-\theta)\delta} \cdot K^{-(1-\alpha)\frac{\sigma}{\sigma+\alpha}}. \tag{Demand curve}$$

Second, we use the expression for G derived in the proof of Lemma 2.1, and the expression for L_H, we have that

$$\begin{aligned} G &= \left(\frac{\chi}{1-\chi}\mu\right)^{\chi(1-\theta)} K^{\theta}\left(\overline{l_H} q K^{\theta}\right)^{\frac{(1+\sigma\chi)(1-\theta)}{\sigma+\theta-\sigma\chi(1-\theta)}} \\ \implies G/K &= \left(\frac{\chi}{1-\chi}\mu\right)^{\chi(1-\theta)} \overline{l_H}^{(1+\sigma\chi)(1-\theta)} q^{\frac{(1+\sigma\chi)(1-\theta)}{\sigma+\theta-\sigma\chi(1-\theta)}} K^{-(1-\theta)(1-\chi)\frac{\sigma}{\sigma+\theta-\sigma\chi(1-\theta)}} \end{aligned}$$

Imposing $G/K = \delta$, which is equivalent to (eq5), we obtain

$$\delta^{\frac{\sigma+\theta-\sigma\chi(1-\theta)}{1-\theta}} = \left(\left(\frac{\chi}{1-\chi}\mu\right)^{\chi(1-\theta)} \overline{l_H}^{(1+\sigma\chi)(1-\theta)}\right)^{\frac{\sigma+\theta-\sigma\chi(1-\theta)}{1-\theta}} q^{1+\sigma\chi} K^{-(1-\chi)\sigma}$$

Therefore, we have that q is an increasing function of K (a supply curve):

$$q = \delta^{\frac{1}{1+\sigma\chi}\frac{\sigma+\theta-\sigma\chi(1-\theta)}{1-\theta}} \left(\left(\frac{\chi}{1-\chi}\mu\right)^{\chi(1-\theta)} \overline{l_H}^{(1+\sigma\chi)(1-\theta)}\right)^{-\frac{1}{1+\sigma\chi}\frac{\sigma+\theta-\sigma\chi(1-\theta)}{1-\theta}} K^{\frac{1-\chi}{1+\sigma\chi}\sigma}$$

A.4 Proof of Proposition 2.3

Long-run. Evaluating (firm foc I_t) and (firm foc $L_{H,t}$) in steady-state, we have

$$I = \chi(1-\theta)Gq = \chi\frac{(1-\theta)\delta}{r+(1-\theta)\delta}\Pi,$$
$$w_H L_H = (1-\chi)\frac{(1-\theta)\delta}{r+(1-\theta)\delta}\Pi,$$

where the second equalities use the formula for q (see Value of capital) as well as the steady-state condition $G/K = \delta$. Combining with the fact that $\Pi = I + w_H L_H + D$, we obtain the final result

Short-run. We will use two facts. First, (firm foc $L_{Y,t}$) implies that the production labor income moves one-for-one with output and capital income:

$$\mathrm{d}\log(w_{Y,t}L_{Y,t}) = \mathrm{d}\log Y_t = \mathrm{d}\log\Pi_t.$$

Second, (firm foc $L_{H,t}$) implies that the production labor income moves one-for-one with investment and the value of capital formation:

$$\mathrm{d}\log(w_{H,t}L_{H,t}) = \mathrm{d}\log I_t = \mathrm{d}\log(q_t G_t).$$

Using the fact that the initial allocation is in steady-state:

$$\begin{aligned}
\mathrm{d}(w_{H,t}L_{H,t}) &= (w_H L_H)\,\mathrm{d}\log(w_{H,t}L_{H,t}),\\
&= w_H L_H\left(\mathrm{d}\log(w_{Y,t}L_{Y,t}) + \mathrm{d}\log\left(\frac{w_{H,t}L_{H,t}}{w_{Y,t}L_{Y,t}}\right)\right),\\
&= w_H L_H\left(\mathrm{d}\log(w_{Y,t}L_{Y,t}) + \mathrm{d}\log\left(\frac{w_{H,t}L_{H,t}}{w_{Y,t}L_{Y,t}}\right)\right),\\
&= \frac{w_H L_H}{\Pi}\left(\mathrm{d}(\Pi_t) + \mathrm{d}\log\left(\frac{w_{H,t}L_{H,t}}{w_{Y,t}L_{Y,t}}\right)\cdot\Pi\right).
\end{aligned}$$

Taking the present value and re-arranging, we obtain

$$\frac{\sum_{t\geq 1} R^{-t}\,\mathrm{d}(w_{H,t}L_{H,t})}{\sum_{t\geq 1} R^{-t}\,\mathrm{d}(\Pi_t)} = \frac{w_H L_H}{\Pi}\cdot\mathcal{M}$$

where

$$\mathcal{M} = \sum_{t\geq 1} R^{-t}\,\mathrm{d}\log\left(\frac{w_{H,t}L_{H,t}}{w_{Y,t}L_{Y,t}}\right) = \sum_{t\geq 1} R^{-t}\,\mathrm{d}\log\left(\frac{G_t q_t}{Y_t}\right)$$

Using $\mathrm{d}\log I_t = \mathrm{d}\log(w_{H,t}L_{H,t})$, which implies

$$\frac{\sum_{t\geq 1} R^{-t}\,\mathrm{d}I_t}{\sum_{t\geq 1} R^{-t}\,\mathrm{d}(\Pi_t)} = \frac{I}{\Pi}\cdot\mathcal{M}$$

Residually, we have that

$$\frac{\sum_{t\geq 1} R^{-t}\,\mathrm{d}D_t}{\sum_{t\geq 1} R^{-t}\,\mathrm{d}(\Pi_t)} = \frac{D}{\Pi}\cdot\left(1-\frac{w_H L_H + I}{D}\mathcal{M}\right)$$

A.5 Proof of Proposition 2.4

Long-run. The long-run supply elasticity is given by Lemma 2.2, where we express the long-run capital supply curve as

$$q = \overline{q_S}\cdot K^{\frac{1-\chi}{1+\sigma\chi}\sigma}.$$

Taking log and differentiating, we obtain

$$\log K = \frac{\frac{1}{\sigma}+\chi}{1-\chi}\,\mathrm{d}\log q,$$

which characterizes movements along the long-run labor supply curve.

Short-run. Consider a perturbation $\{\mathrm{d}\log q_t\}_{t=1}^{\infty}$. We can characterize the response of K_{t+s}, at any horizon $s\geq 0$, using the following recursion:

$$\begin{aligned}
\mathrm{d}\log K_0 &= 0\\
\mathrm{d}\log K_{t+1} &= (1-\delta)\,\mathrm{d}\log K_t + \delta\,\mathrm{d}\log G_t\\
&= (1-\delta)\,\mathrm{d}\log K_t + \delta\theta\left(1+\frac{(1+\sigma\chi)(1-\theta)}{\sigma+\theta-\sigma\chi(1-\theta)}\right)\mathrm{d}\log K_t + \delta\frac{(1+\sigma\chi)(1-\theta)}{\sigma+\theta-\sigma\chi(1-\theta)}\,\mathrm{d}\log q_t\\
&= \left(1-\delta\frac{\sigma(1-\theta)(1-\chi)}{\sigma+\theta-\sigma\chi(1-\theta)}\right)\mathrm{d}\log K_t + \delta\frac{(1+\sigma\chi)(1-\theta)}{\sigma+\theta-\sigma\chi(1-\theta)}\,\mathrm{d}\log q_t
\end{aligned}$$

Solving forward, we have that

$$\mathrm{d}\log K_{T+1} = \delta\frac{(1+\sigma\chi)(1-\theta)}{\sigma+\theta-\sigma\chi(1-\theta)}\cdot\sum_{s=0}^{T}\left(1-\delta\frac{\sigma(1-\theta)(1-\chi)}{\sigma+\theta-\sigma\chi(1-\theta)}\right)^{s}\mathrm{d}\log q_{T-s}$$

Setting $\mathrm{d}\log q_{T-s} = \mathrm{d}\log q$ and taking the limit, we obtain

$$\lim_{T\to\infty}\mathrm{d}\log K_{T+1} = \frac{\frac{1}{\sigma}+\chi}{1-\chi}\cdot\mathrm{d}\log q,$$

which indeed is the long-run capital supply elasticity.

A.6 Analytics of saving shock

Long-run. Consider the effect of a permanent shock $\mathrm{d}\log R$ in the long-run. Lemma 2.2 states that the equilibrium is determined by

$$\overline{q_D}\cdot K^{-(1-\alpha)\frac{\sigma}{\sigma+\alpha}} = \overline{q_S}\cdot K^{\frac{1-\chi}{1+\sigma\chi}\sigma}$$

Taking a log total derivative on both sides, and using the fact that $\mathrm{d}\log \overline{q_S} = 0$ and $\mathrm{d}\log \overline{q_D} = -\frac{1}{r+(1-\theta)\delta} \cdot \mathrm{d}\log R$, we have that

$$\mathrm{d}\log K = \frac{1}{(1-\alpha)\frac{\sigma}{\sigma+\alpha} + \frac{1-\chi}{1+\sigma\chi}\sigma} \cdot \frac{1}{r+(1-\theta)\delta} \cdot (-\mathrm{d}R)$$

The first term is one over the sum of supply and demand elasticities. The second term is duration, which scales up the shock. The resulting long-run change in q is given by

$$\mathrm{d}\log q = \frac{\frac{1-\chi}{1+\sigma\chi}\sigma}{(1-\alpha)\frac{\sigma}{\sigma+\alpha} + \frac{1-\chi}{1+\sigma\chi}\sigma} \cdot \frac{1}{r+(1-\theta)\delta} \cdot (-\mathrm{d}R).$$

The first-term is a GE adjustment: the equilibrium change in $\frac{\mathrm{d}\log q}{\mathrm{d}R}$ is not the duration $\frac{\partial \log q}{\partial R}$.

A.7 Model equations for calibration

To derive the model equations used for calibraton, we will use (firm foc I_t) and (firm foc $L_{H,t}$):

$$I_t = \chi(1-\theta)q_t G_t,$$
$$w_{H,t}L_{H,t} = (1-\chi)(1-\theta)q_t G_t.$$

Long-run moments. The first two model equations are for the ratio of tangible and intangible investment to firm value in steady-state. First, we use the fact that $G/K = \delta$ in steady-state, as well as the fact that $V = qK$. We obtain:

$$\frac{I}{V} = \chi(1-\theta)\delta,$$
$$\frac{w_H L_H}{V} = (1-\chi)(1-\theta)\delta.$$

Short-run moments. The second set of moment is related to the investment-q elasticity in the case where $\sigma = 0$. In particular, we consider the effect of a hypothetical price shocks $\mathrm{d}\log q_0$. Summing both first-order conditions, we obtain an expression for total investment

$$I_t + w_{H,t}L_{H,t} = (1-\theta)q_t G_t.$$

The log total derivative at $t = 0$ is

$$\mathrm{d}\log(I_0 + w_{H,0}L_{H,0}) = \mathrm{d}\log q_0 + \mathrm{d}\log G_0 = \left(1 + \frac{1-\theta}{\theta}\right)\mathrm{d}\log q_0 = \frac{1}{\theta}\,\mathrm{d}\log q_0$$

The third equality uses the short-run capital supply elasticity in the case where $\sigma = 0$ (q-theory).

In terms of a standard investment-q regression, we can use the log perturbation around the the steady-state

$$\mathrm{d}\Big(\frac{I_0 + w_{H,0}L_{H,0}}{K_0}\Big) = \frac{I + w_H L_H}{qK}\frac{1}{\theta}\,\mathrm{d}q_0 = \delta\frac{1-\theta}{\theta}\,\mathrm{d}q_0$$

Capital taxation. Suppose that firms pay a capital income tax rate $\tau \in [0,1]$ that is rebated lump-sum. The comparative static on V_0 around the steady-state with $\tau = 0$ is

$$\mathrm{d}q_0 = q\,\mathrm{d}\tau.$$

B Model extensions

B.1 Zero-sum investment

Suppose that there is inefficient over investing. For instance, part of investment in customer acquisition is zero sum. In particular, suppose that a share η of capital formation is zero sum. Then the model becomes the same as before, but where $z_{H,t}$ becomes $(1-\eta)z_{H,t}$ (ie, lower aggregate IST).

B.2 Two-sector interpretation

Consider a two-sector model with full capital depreciation ($\delta = 1$), no tangible investment ($\chi = 1$), and no investment rents ($\theta = 0$):

$$\begin{aligned} Y_t &= K_t^{\alpha} L_{Y,t}^{1-\alpha}, && \text{(final good sector)} \\ K_{t+1} &= L_{H,t}. && \text{(capital good sector)} \end{aligned}$$

We also contrast the equilibrium and accounting with an alternative version of the model where capital is an intermediary input.

$$\begin{aligned} Y_t &= K_t^{\alpha} L_{Y,t}^{1-\alpha}, && \text{(final good sector')} \\ K_t &= L_{H,t}. && \text{(capital good sector')} \end{aligned}$$

Equilibrium. The equilibrium is

$$\begin{aligned} (1-\alpha)\frac{Y_t}{L_{Y,t}} &= w_{Y,t} \\ R^{-1}\alpha\frac{Y_{t+1}}{K_{t+1}} &= w_{H,t} \end{aligned}$$

If instead capital was an intermediary input (no lag), the equilibrium would be

$$\begin{aligned} (1-\alpha)\frac{Y_t}{L_{Y,t}} &= w_{Y,t} \\ \alpha\frac{Y_t}{K_t} &= w_{H,t} \end{aligned}$$

In steady-state, both economies are equivalent, except that the initial one has a wedge R (underinvestment).

Accounting. Expenditures and incomes are:

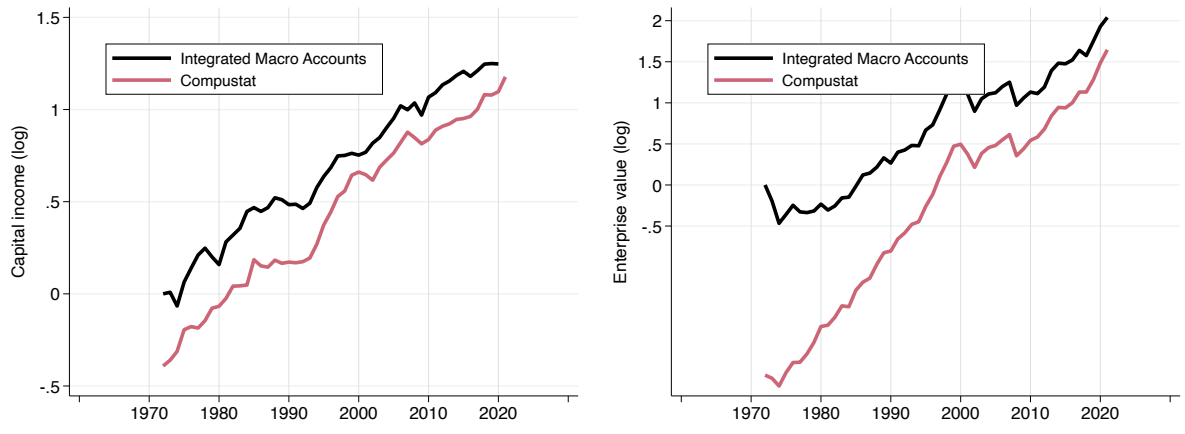
$$\begin{aligned}
\text{consumption}_t &= Y_t \\
\text{investment}_t &= w_{H,t} L_{H,t} \\
\text{agg. expenditures} &= Y_t + w_{H,t} L_{H,t} \\
\text{capital income}_t &= Y_t - w_{Y,t} L_{Y,t} \\
\text{labor income}_t &= w_{Y,t} L_{Y,t} + w_{H,t} L_{H,t} \\
\text{agg. income}_t &= Y_t + w_{H,t} L_{H,t}
\end{aligned}$$

In the economy where capital is an intermediary input, we have that

$$\begin{aligned}
\text{consumption}_t &= Y_t \\
\text{investment}_t &= 0 \\
\text{agg. expenditures} &= Y_t \\
\text{capital income}_t &= Y_t - w_{Y,t} L_{Y,t} - w_{H,t} L_{H,t} \\
\text{labor income}_t &= w_{Y,t} L_{Y,t} + w_{H,t} L_{H,t} \\
\text{agg. income}_t &= Y_t
\end{aligned}$$

C Data

C.1 Compustat sample validation



(a) Capital income

(b) Enterprise value

Figure A1: Sample validation against National accounts data (1970–2022).

C.2 Long-run facts

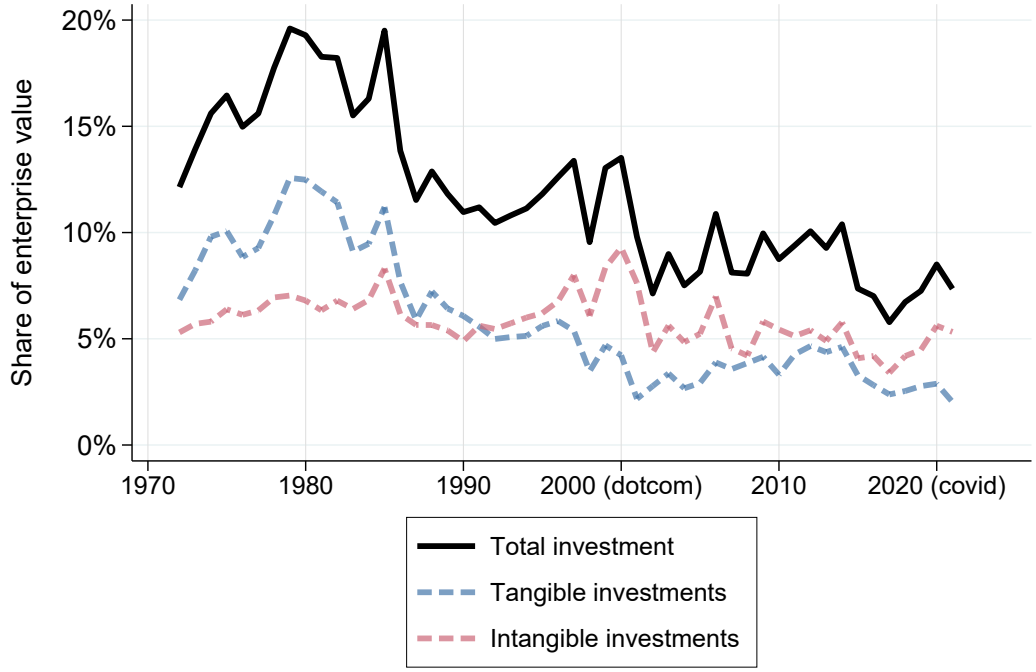


Figure A2: Aggregate dilution rate (nonfinancial public corporations, 1972–2022).