Inelastic Capital in Intangible Economies*

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Abstract

Capital accumulation in modern economies increasingly takes the form of intangible investment. This form of investment relies heavily on the contributions of specialized workers—such as inventors, managers, and entrepreneurs. To examine the macroeconomic implications of the rising share of intangibles, we develop and calibrate a general neoclassical model where capital formation requires both investment goods (tangible investments) and specialized labor (intangible investments). We show that rising intangibles renders the supply of capital more inelastic owing to the limited supply of specialized labor. Rising intangibles also change the incidence of capital taxation: whereas in traditional neoclassical models the tax burden falls entirely on production workers, in intangible economies, it is borne primarily by specialized workers and capital owners.

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1 Introduction

The standard neoclassical growth model treats capital formation as a frictionless process: final goods can be converted into new capital at no cost. As a result, the standard model predicts an infinitely elastic supply curve of capital: shifts in the supply or demand for capital — arising from changes in household saving or changes in the productivity of investment — result in large movements in the capital stock rather than its price.

This prediction, however, stands in stark contrast to the empirical data, as asset prices typically exhibit significant volatility relative to real quantities (e.g., the capital stock). While models with capital adjustment costs yield a short-run inelastic capital supply (Hayashi, 1982), they still imply an infinitely elastic capital supply in the long run and thus cannot account for the persistent gaps between market and book values in the corporate sector (Gutiérrez and Philippon, 2017, Crouzet and Eberly, 2023).

This paper takes a different route: we emphasize that transforming final goods into productive capital requires specialized labor—entrepreneurs, inventors, and managers—whose aggregate supply is inherently limited. Because more capital cannot simply be constructed from final goods but must also be conceptualized, designed, and implemented by these scarce workers, the supply curve for capital slopes upward over any horizon.

We show that this more realistic view of capital formation provides a natural explanation for recent macro-finance trends in modern economies. It explains why large "savings gluts" (Bernanke et al., 2007, Mian et al., 2020) tend to generate higher capital prices rather than being absorbed through higher investment. It also emphasizes the link between financial and labor markets, as an influx of savings raises the wages of specialized investment workers relative to those of production workers, contributing to rising wage polarization (Autor and Dorn, 2013). Finally, this perspective on capital formation has crucial implications for the incidence of corporate taxation: while, in the standard neoclassical model, the tax burden falls entirely on production workers, it is borne primarily by specialized workers and capital owners in intangible economies.

Our paper proceeds as follows. We first develop a parsimonious extension of the standard neoclassical growth model that features a more realistic investment function. Our key departure is that investment goods (i.e., machines, computers, plants) must be paired with specialized labor (i.e., inventors, entrepreneurs, financiers) and accumulated capital (i.e., building on past research, adjustment frictions) in order to produce new productive capital. We formalize this idea through the following aggregate investment function:

$$\text{capital formation} = \Phi\left(\text{investment goods}, \text{investment labor}, \text{accumulated capital}\right).$$

In the baseline, we parametrize this investment function using a constant return to scale Cobb-Douglas function and close the model with inelastic labor supply (i.e., there is a fixed number of investment workers). Our general specification nests and generalizes several important models

used in the literature. For instance, the neoclassical growth model only requires investment good, the Q-theory of investment requires goods and existing capital, while models of "sweat capital" require both existing capital and labor.

We then characterize the capital supply elasticity, which governs how responsive firms are to capital demand shocks.[1] In the neoclassical growth model, capital supply is infinitely elastic, since the final good can be frictionlessly transformed into capital. In that case, small demand-side shocks (e.g., interest rate) have large and immediate effect on capital accumulation. In contrast, when investment requires labor in fixed supply, the supply elasticity is finite, even in the long-run. We show that a higher degree of intangibility in capital formation leads to a lower capital supply elasticity, both in the short-run and in the long-run.

In Section 3, we then map the firm budget constraint in the model to the data. We construct a dataset on the uses of capital income in the US nonfinancial corporate sector, based around the identity:

$$\text{capital income} = \text{tangible investments} + \text{intangible investments} + \text{capital payout}.$$

We focus on public firms (Compustat-CRSP) over the 1972-2022 period, where we have full financial statements, which include noncash payments (e.g., stock options, acquisitions financed with stocks). Building on the existing literature, we make three key adjustments to the accounting data. First, we treat nonproduction labor costs as intangible investment, rather than production costs (see, e.g., Corrado, Hulten and Sichel, 2005; Corrado, Hulten and Sichel, 2009; Eisfeldt and Papanikolaou, 2013; Peters and Taylor, 2017;Koh, Santaeulàlia-Llopis and Zheng, 2020). The idea is that if an employee dose not contribute to the production process, then they must contribute to capital formation, broadly defined. Second, we account for IPOs and acquisitions net of de-listings and allocate the implied cashflows to tangible and intangible investments. Third, we account for noncash payments, treating them as economically equivalent to cash payments. This is particularly important for nonproduction labor (inventors, managers) who earn a large share of their compensation in stocks (see, e.g., Eisfeldt, Falato and Xiaolan, 2023).

We obtain an annual dataset on the aggregate (and industry-level) uses of capital income, which maps exactly into the firm budget constraint above. Note that our accounting is fully consistent with national accounting, which now acknowledges that some form of labor expenses are indeed investment. However, we go further than the BEA and consider any nonproduction labor expenses as investment. Corrado, Haskel, Jona-Lasinio and Iommi (2022) discuss this topic in detail, highlighting the fact that the following expenses are not currently considered investment in the national accounts: "market research and branding, operating models, platforms, supply chains, distribution networks, employer-provided training, attributed designs (industrial), and financial product development".

[1] To be precise, the supply elasticity of capital in the model is the elasticity of the capital stock (at some horizon) with respect to a change in the value of capital (i.e., the marginal q).

In Section 4, we explore the quantitative effect of this observed shift towards intangibles. We calibrate the key elasticities of the model using data on the uses of capital income and valuations (intangible share, total investment yield, and aggregate capital share), as well as a short-run tax elasticity of investment taken from Chodorow-Reich, Smith, Zidar and Zwick (2024). Using the calibrated model we simulate a large (25 pp.) rise in the share of intangible investment, consistent with our evidence from the US corporate sector over 1972-2022. We find that a shift towards intangibles of the magnitude that we have seen in the data implies a significant redistribution of aggregate income away from production labor towards investment labor (-6 pp. of GDP). The model also predicts that the economy becomes more inelastic: the supply elasticity declines by roughly half in the long-run and one-third in the short-run.

Finally, we use the calibrated model to quantify the incidence of corporate taxes in our economy. In the neoclassical growth model, the capital supply elasticity is infinite. As a result, capital tax cuts have a large effect on investment, and end up being born by production workers via higher wages. In intangible economies, however, tax cut have a weaker effect on investment, hence a lower effect on production worker wages. As a result, capitalists and investment labor absorb more of the shock via revaluation gains (for capitalists) and higher wages (for investment labor). In the calibrated model, the incidence of capital taxes is roughly half for capitalists, a quarter for production labor, and a quarter for investment labor.

Literature review. A growing literature documents a rise in intangible capital (see, e.g., Eisfeldt and Papanikolaou, 2013, Peters and Taylor, 2017, Eisfeldt, Falato and Xiaolan, 2023, and Corrado, Haskel, Jona-Lasinio and Iommi, 2022). Beyond being harder to measure, intangible capital may exhibit distinct economic properties relative to tangible capital. In Eisfeldt and Papanikolaou (2013), intangible capital is unique because it is embedded in key talents. In Crouzet, Eberly, Eisfeldt and Papanikolaou (2022), intangible capital tends to be non-rival—allowing it to be used simultaneously in different production streams—and is characterized by limited excludability, which prevents firms from capturing all the associated benefits or rents. In this paper, we shift our focus away from the economic properties of intangible capital to its distinct inputs, namely, that it is created by specialized labor in finite supply.

Luttmer (2018) studies an economy in which households supply both managerial and production labor—with managerial labor contributing to both production and investment. Like our paper, Luttmer (2018) highlights that organizational capital is produced using a specialized input by discussing the implications of a fixed supply of managerial capital. Another closely related paper is Bhandari and McGrattan (2021), which highlights that a significant portion of small firms' value stems from organizational capital accumulated through the owner's effort. The paper further explores the implications of this finding for the taxation of non-corporate businesses. Relative to these papers, we provide new data and moments to calibrate a flexible theory of capital supply.

In our model, the revenues from capital formation accrue to three distinct factors: investment

goods, investment workers, and existing capital. This is similar to Kogan et al. (2020), which assume that investment workers generate blueprints, which are then implemented by firms through a decreasing return to scale function in investment goods. The key difference is that, in our model, the distribution of investment rents between investment workers and existing capital is determined by their respective Cobb-Douglas exponents in the capital formation production function, rather than being exogenous.[2]

Finally, our paper offers an alternative explanation for investment stagnation despite high Tobin's Q inferred from the data. The existing literature focuses on markups and market power (e.g., Gutiérrez and Philippon, 2017; Crouzet and Eberly, 2019; Barkai, 2020; Ball and Mankiw, 2023; De Ridder, 2024). In contrast, we emphasize the fact that capital formation requires specialized labor (i.e., entrepreneurs, inventors, etc.) which constrains the extent to which investment goods can be transformed into productive capital. Put differently, in our paper, the "fixed factor" that limits the ability (or willingness) of firms to scale is not the fact that it would lower their price, but, rather, that it would increase their labor cost.

2 Stylized model

2.1 Setup

Output production. We focus on a representative firm. Output Y_t (i.e., the final good) is produced using capital K_t and production labor $L_{Y,t}$ through the standard Cobb-Douglas production function

$$Y_t = z_{Y,t} K_t^{\alpha} L_{Y,t}^{1-\alpha}, \qquad \text{(output production)}$$

where $\alpha \in (0,1)$ denotes the importance of capital relative to labor and $z_{Y,t}$ represent Hicks-neutral productivity in production (TFP).

Capital formation. Our key departure from the standard neoclassical growth model is that the final good cannot be frictionlessly transformed into productive capital: producing and installing new productive units of capital also requires specialized labor (i.e., inventors, entrepreneurs, managers) as well as accumulated capital (i.e., building on past research, adjustment frictions). We represent this with a standard capital accumulation equation and a general capital formation function:

$$K_{t+1} = (1-\delta)K_t + H_t \quad \text{where} \quad H_t \equiv z_{H,t} K_t^{\theta} \left(I_t^{\chi} L_{H,t}^{1-\chi} \right)^{1-\theta}. \qquad \text{(capital formation)}$$

The capital formation function is a Cobb-Douglas function of three inputs: existing level of capital K_t, tangible investment I_t, and intangible investment $L_{H,,t}$. The parameter $\chi \in (0,1)$ governs

[2]This assumption ensures that factors are paid their marginal products, and so that the decentralized equilibrium implements the first best (this is similar to the Hosios (1990)'s condition in matching models).

the importance of tangible investment relative to intangible investment while the parameter $\theta \in (0,1)$ governs the importance of existing capital relative to investment (tangible or intangible). Finally, $z_{H,t}$ represent Hicks-neutral productivity in capital formation (IST shock).[3] We provide a microfoundation for this aggregate capital formation function in Appendix A.1.

Our specification of the investment function nests and generalizes several important models used in the literature. One such special case is where capital formation only requires investment goods ($\theta = 0, \chi = 1$); that is, $H_t = z_{H,t} I_t$. This setup corresponds to the neoclassical growth model (NGM).[4] Another important special case is where investment requires a mix of investment goods and existing capital ($\chi = 1$); that is, $H_t = z_{H,t} K_t^{\theta} I_t^{1-\theta}$. This setup corresponds to the q-theory of investment (Uzawa, 1969, Hayashi, 1982).[5] The parameter θ governs the curvature of the adjustment cost function. Another important special case is where investment requires both capital and specialized labor, but not goods themselves ($\chi = 0$); that is $H_t = z_{H,t} K_t^{\theta} L_{H,t}^{1-\theta}$. This corresponds to models of "sweat capital", where firm expansion requires the creation of organizational capital or blueprints by specialized workers (e.g., Luttmer, 2011; Bhandari and McGrattan, 2021). The idea that investment costs requires labor inputs rather than final goods is also present in Melitz (2003) or Romer (1986) models.

Firm optimization. The rest of the model is standard. The representative firm takes as given the interest rate r_t, as well as wage rates for production and investment labor $(w_{Y,t}, w_{H,t})$. It chooses production labor $L_{Y,t}$, investment labor $L_{H,t}$, and tangible investment I_t to maximize the present value of future payouts:

$$\begin{aligned} V_0 = & \max_{\{L_{Y,t}, L_{H,t}, I_t, K_{t+1}\}} \sum_{t=1}^{\infty} R_{0\to t}^{-1} D_t, \\ \text{s.t.} \quad & D_t = Y_t - I_t - w_{Y,t} L_{Y,t} - w_{H,t} L_{H,t}, \qquad \text{(firm budget constraint)} \end{aligned}$$

where $R_{0\to t} = \prod_{s=0}^{t}(1 + r_s)$ denotes the cumulative return of one dollar invested between 0 and t. This maximization is subject to the equations for production, capital formation, and capital accumulation above. The corresponding Lagrangian is

$$\mathcal{L}_0 = \sum_{t\geq 1} R_{0\to t}^{-1} \Big(Y_t - I_t - w_{Y,t} L_{Y,t} - w_{H,t} L_{H,t} \Big) + \sum_{t\geq 1} R_{0\to t}^{-1} q_t \Big((1-\delta) K_t + H_t - K_{t+1} \Big).$$

[3] Because we choose a Cobb-Douglas production function, this can alternatively represent technology shocks that make it easier to install capital, shocks that make investment good more productive (as in Greenwood et al., 1997), or shocks that make investment labor more productive.

[4] Note that, even in this case, the efficiency $z_{H,t}$ at which the final good can be transformed into capital can change over time as in Greenwood et al. (1997).

[5] Hayashi (1982) assumes capital accumulation takes the form $K_{t+1} = (1-\delta) + K_t \phi(I_t/K_t)$, where $\phi(\cdot)$ is a concave function; hence, our theory corresponds to the case in which $\phi(I_t/K_t) = (I_t/K_t)^{1-\theta}$. Since this seminal paper, the q-theory of investment has evolved away from modeling capital formation as concave in I/K (like we do) towards cost functions that are convex in I/K. The key difference is that adjustment costs appear in the budget constraint while capital formation appear in the capital accumulation equation.

The Lagrange multiplier q_t can be interpreted as the marginal value of one capital unit. Given our assumption that the production and capital formation functions exhibit linear returns to scale, the market value of the representative firm is given by $V_t = q_t K_{t+1}$; that is, the marginal and average values of capital units are equal (Hayashi, 1982).

Solving the representative firm problem yields the following set of optimality conditions:

$$w_{Y,t} L_{Y,t} = (1-\alpha) \cdot Y_t, \qquad \text{(firm foc } L_{Y,t}\text{)}$$
$$w_{H,t} L_{H,t} = (1-\theta)(1-\chi) \cdot q_t H_t, \qquad \text{(firm foc } L_{H,t}\text{)}$$
$$I_t = (1-\theta)\chi \cdot q_t H_t, \qquad \text{(firm foc } I_t\text{)}$$
$$(1+r_{t+1}) q_t K_{t+1} = \alpha \cdot Y_{t+1} + q_{t+1}\left((1-\delta)K_{t+1} + \theta \cdot H_{t+1}\right), \qquad \text{(firm foc } K_{t+1}\text{)}$$

with the transversality conditions $\lim_{T\to\infty} R_{0\to t}^{-1} q_T K_{T+1} = 0$. Note that, due to our Cobb-Douglas assumption, the three inputs of capital formation — existing capital, investment goods, and investment workers — earn a constant share of the revenues from capital formation $q_t H_t$. More specifically, a share θ of revenues is paid to existing capital holders, a share $(1-\theta)(1-\chi)$ is paid to investment workers, and a share $(1-\theta)\chi$ is paid to purchase investment goods.

Note that capital owners earn two types of income: they receive a constant share α of revenues from output production Y_t and a constant share θ of revenues from capital formation $q_t K_{t+1}$.

Household optimization. Since the focus of the paper is on the firm side, we only provide a stylized microfoundation of the household problem. The representative household takes as given the sequence of prices $\{w_{Y,t}, w_{H,t}, r_t\}_{t=1}^{\infty}$ and chooses a sequence of consumption and wealth $\{C_t, V_t\}_{t=1}^{\infty}$ to maximize welfare:

$$U_0 = \max_{\{C_t, V_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma}.$$
$$\text{s.t.} \quad C_t + V_t = w_{Y,t} L_{Y,t} + w_{H,t} L_{H,t} + (1+r_t) V_{t-1},$$

where V_0, the initial level of wealth, is given. For simplicity, we assume in this section that the representative household supplies a fixed quantity of production labor $L_{Y,t} = 1-\mu$ and investment labor $L_{H,t} = \mu$; we will consider the more general case in which labor is elastically supplied in Section 4, when taking the model to the data. The household optimality with respect to consumption gives the usual Euler equation:

$$C_t^{-\gamma} = \beta(1+r_{t+1}) C_{t+1}^{-\gamma}. \qquad \text{(worker foc } C_t\text{)}$$

Equilibrium. An equilibrium is an initial condition K_0, an allocation $\{L_{Y,t}, L_{H,t}, I_t, K_t\}_{t\geq 1}$, and prices $\{w_{H,t}, w_{L,t}, r_t, q_t\}_{t\geq 1}$ that solve the firm problem (firm foc $L_{Y,t}$, firm foc $L_{H,t}$, firm foc I_t, firm foc K_{t+1}) and household problem (worker foc C_t), with $L_{Y,t} = 1-\mu$ and $L_{H,t} = \mu$.

2.2 Steady-state

We now characterize the steady-state of the model, assuming that the productivity of the production and capital formation functions remain fixed over time $z_{Y,t} = z_Y$ and $z_{H,t} = z_H$. We consider the more general case of a balanced growth path where the productivity of investment and production labor grow at some exogenous rate in Section 4.

It is useful to think of the steady-state (or long-run) quantity of capital as determined by an equilibrium between the demand and supply of capital. The demand for capital directly obtains by combining firm foc K_{t+1} with the steady state condition $H = \delta K$:

$$z_Y \alpha K^{-(1-\alpha)} (1-\mu)^{1-\alpha} = q\left(r + (1-\theta)\delta\right). \qquad \text{(capital demand)}$$

This equation can be seen as a market pricing equation, which pins down the price of capital q given the flow of payments to capital holders and the interest rate r. Another, equivalent, interpretation of (capital demand) is that firms demand capital until the marginal productivity of capital (the left-hand side) coincides with its user cost (the right-hand side), as in Hall and Jorgenson (1967).[6] This equation traces a downward slopping demand for capital: a higher capital price q increases its user cost (since a firm would need to borrow more to purchase one unit of capital), reducing the quantity of capital demanded by firms.

The supply of capital obtains by plugging optimal investment (firm foc I_t) into the steady-state version of (capital formation):

$$z_H K^{\theta} \Big(\left((1-\theta)\chi q \delta K\right)^{\chi} \mu^{1-\chi} \Big)^{1-\theta} = \delta K. \qquad \text{(capital supply)}$$

This equation says that, in steady-state, the quantity of new capital goods created every period H (the left-hand side) must equal the quantity of capital goods that depreciate every period (the right-hand side). A higher shadow value of capital q pushes firms to invest more (firm foc I_t), which increases the steady-state supply of capital (capital formation). In the particular case where investment is purely tangible ($\chi = 1$), the supply of capital is perfectly elastic and this equation pins down q. As soon as capital formation requires investment labor ($\chi < 1$), however, this equation traces an upward slopping supply curve for capital. Solving for K in (capital supply) and (capital demand) gives the long-run demand and supply curve of capital, respectively. The capital demand curve says that a higher capital price q increases the user cost of capital, decreasing the firm's demand for capital (capital demand). The capital supply curve says that a higher capital price q increases firm investment, increasing the steady-state quantity of capital (capital supply). The equilibrium is determined by the intersection between these two curves.

[6] The analogy provided by Hall-Jorgenson is that "the firm may be treated as accumulating assets in order to supply capital services to itself".

2.3 Implications

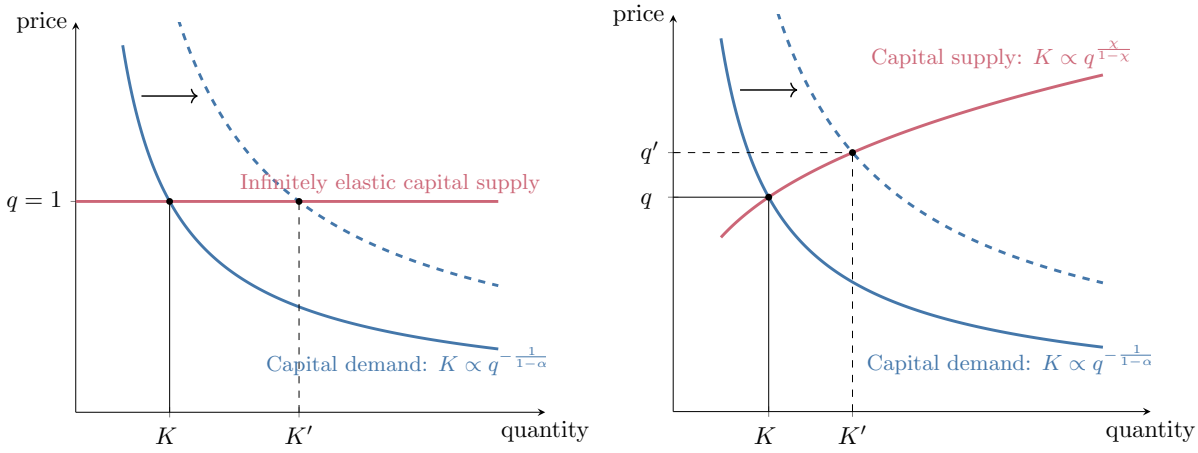
2.3.1 Supply elasticity of capital

In contrast with the NGM, our model generates an inelastic supply of capital, both in the short-run and in the long-run.

Long-run capital supply. Differentiating (capital supply) implies that the long-run elasticity of capital supply to q is

$$\frac{\partial \log K}{\partial \log q} = \frac{\chi}{1-\chi}. \qquad \text{(long-run elasticity)}$$

This elasticity spans values from zero (when investment is fully intangible) to infinity (when capital is fully tangible, as in the neoclassical growth model). Notice that the long-run capital supply elasticity does not depend on θ; as we discuss below, the parameter θ only determines the short-run capital supply elasticity.



(a) Neoclassical growth model ($\chi = 1$) (b) Model with intangibles ($\chi < 1$)

Figure 1: Long-run equilibrium in the capital market

The fact that, in the presence of intangibles, the supply of capital is inelastic has key implications for the long-run effect of a shift in the demand of capital — say, due to a permanent increase in the household's time preference parameter β or a permanent increase in output productivity z_Y. To visualize this fact, Figure 1 plots the steady-state demand and supply for capital. Panel (a) corresponds to the neoclassical growth model ($\chi = 1$), while Panel (b) corresponds to an intangible economy ($\chi < 1$ in Panel b). In the neoclassical growth model (Panel a), the supply of capital is perfectly elastic, so an outward shift in capital demand results in a higher steady-state capital stock with no change in the rental rate of capital. In contrast, in a model with intangibles (Panel b), the supply of capital is inelastic, so a shift in the demand for capital is only partially absorbed by a rise in capital, and part of the adjustment takes the form of a higher capital price (i.e., a higher firm value relative to its capital stock).

Short-run capital supply. Capital is also inelastic in the short-run, due to the joint presence of specialized labor ($\chi < 1$) and existing capital ($\theta > 0$) in the capital formation function. Differentiating (capital formation) and (firm foc I_t) implies

$$\frac{\partial \log K_{t+1}}{\partial \log q_t} = \delta \frac{\chi(1-\theta)}{1-\chi(1-\theta)}. \qquad \text{(short-run elasticity)}$$

While the capital share of investment θ did not matter for the long-run capital elasticity, it is the key determinant of the short-run capital elasticity. Capital being an input in production $\theta > 0$ makes capital less elastic in the short-run. This is exactly the logic behind the q-theory of investment, which emphasizes the presence of capital adjustment costs in the short-run. As in the long-run, the capital supply elasticity increases with the level of tangibility χ.

More generally, for a given sequence of perturbation $(\mathrm{d}\log q_s)_{s\leq t}$, we have:

$$\mathrm{d}\log K_{t+1} = \delta \frac{\chi(1-\theta)}{1-\chi(1-\theta)} \sum_{s=0}^{\infty} \left(1 - \delta \frac{(1-\chi)(1-\theta)}{1-\chi(1-\theta)}\right)^s \mathrm{d}\log q_{t-s}.$$

See Appendix A.4 for the derivation. For a constant price perturbation $\mathrm{d}\log q_{t-s} = \mathrm{d}\log q$, we recover the long-run supply elasticity $\chi/(1-\chi)$.

Table 1 compares the short- and long-run supply elasticity of capital in our model relative to different benchmark models. There are two key takeaways. First, in fully-tangible economies ("NGM" and "q-Theory"), capital is infinitely elastic in the long-run. This is because the relative price of capital in terms of goods must be one since they are the same thing. Adding a positive capital share of investment ($\theta > 0$) makes the short-run elasticity finite (see "q-theory"), but does not solve the problem of infinite elasticity in the long-run. Second, intangibility (i.e., $\chi < 0$) makes the supply elasticity of capital finite, both in the short- and long-run (see "sweat capital"). This is because labor supply is finite (zero in our case), which makes the response of capital formation to q constrained by the fixed pool of investment worker. In other words, investment booms in intangible economies raise the wages of investment labor, hence dampening the size of the boom.

Table 1: Comparing the supply elasticity of capital across benchmark models

Model	Constraint		Elasticity of capital	
	θ	χ	short-run	long-run
Neoclassical growth	$\theta = 0$	$\chi = 1$	$+\infty$	$+\infty$
Q-theory	$\theta \in (0,1)$	$\chi = 1$	$\delta\frac{1-\theta}{\theta}$	$+\infty$
Sweat capital	$\theta \in (0,1)$	$\chi = 0$	0	0
Stylized model	$\theta \in (0,1)$	$\chi \in (0,1)$	$\delta\frac{\chi(1-\theta)}{1-\chi(1-\theta)}$	$\frac{\chi}{1-\chi}$

Notes. θ the investment capital share; χ is the tangibility of investment; δ is the depreciation rate. The short-run supply elasticity of capital is $\partial \log K_{t+1}/\partial \log q_t$ while the long-run is $\partial \log K_t/\partial \log q$.

2.3.2 Wage gap between investment and production workers

Capital supply is inelastic in our model because capital formation requires an input that is fixed at the aggregate level: investment labor. In equilibrium, after a shift in the demand for capital, firms respond by increasing investment, which bids up the wages of investment workers who are in limited supply. Hence, the flip-side of inelastic capital is that shifts in the demand for capital will affect the wage gap between investment and production workers.

To see this, one can combine (firm foc $L_{Y,t}$) and (firm foc $L_{H,t}$) to express the ratio of payments between investment and production workers:

$$\frac{w_H L_H}{w_Y L_Y} = \frac{(1-\theta)(1-\chi)}{1-\alpha}\frac{q_t H_t}{Y_t} = \frac{(1-\theta)(1-\chi)}{1-\alpha}\frac{\delta\alpha}{r+\delta(1-\theta)}. \tag{2.1}$$

This first equality reflects that, due to our Cobb-Douglas assumptions for our production functions, investment and production workers receive a fixed share of the revenues from their respective sectors. The second equality uses (capital demand), which pins down the ratio between output and the value of the capital stock in steady state.

Therefore, shocks that disproportionately affect the investment sector will increase the wage gap w_H/w_Y. Consider, for instance, an increase in the household's time preference β generating a decline in r. As shown in (2.1), the decline in r increases the steady-state value of capital relative to output, which increases the wage gap between investment and production workers. Similar, a rise in intangibles (a decrease in χ) increases the wage gap between the two type of workers.

Put differently, while all workers are paid their marginal products, investment workers produce cash-flows that have a longer duration than production workers. As a result, as interest rates decline, the marginal product of investment workers increases relative to production workers, and so the wage gap between the two workers increases as a result.

Our theory for the wage gap between investment and production workers provides an alternative view of the "skill-biased" view of technical change (see, for instance, Griliches, 1969, Krusell et al., 2000, Berlingieri et al., 2024). In our model, investment workers benefit from investment booms because they are key inputs in capital formation, not because they are complements with capital in production. One key difference between the two theories is that, in our model, any shock that make capital more valuable increases the wage of investment workers — whether or not this higher capital value ultimately translates into a higher capital stock. We will return to this idea when quantifying the effect of the rise in intangibility on the wage gap between investment and production workers in Section 4.

3 Empirical evidence

We now measure the importance of tangible and intangible investment in the U.S. corporate sector.

3.1 Measuring tangible and intangible investment

Model accounting. GDP in our economy is $Y_t + w_{H,t}L_{H,t}$, not Y_t. This captures the fact that the economy produces a quantity Y_t of goods (used for consumption or for tangible investment) as well as a quantity $w_{H,t}L_{H,t}$ of intangible investment. While this logic is fully consistent with the *System of National Accounts*, distinguishing between payments to production versus investment labor is difficult in practice and the BEA's methodology is known to have limitations in that regard (Corrado et al., 2022).

Capital income is then defined as GDP minus labor costs, which gives $\Pi_t \equiv Y_t - w_{Y,t}L_{Y,t}$.[7] Plugging this equality into (firm budget constraint), we obtain the following accounting identity, which describes the *uses* of capital income (i.e., how do firms spend their profits):

$$\underbrace{\Pi_t}_{\text{capital income}} = \underbrace{I_t}_{\text{tangible investments}} + \underbrace{w_{H,t}L_{H,t}}_{\text{intangible investments}} + \underbrace{D_t}_{\text{payouts}}. \tag{3.1}$$

It says that capital income has three uses: tangible investments, intangible investments, and payments to owners of the firm (henceforth "payouts").

Cashflow accounting. We use Compustat-CRSP merged data covering the period from 1972–2022. We use usual screens to focus on the nonfinancial corporate sector. Our goal is to construct industry-level series for: capital income Π, tangible investments I, intangible investments $w_H L_H$, and payments to capitalists D.

To measure the use of capital income in the data, we start from the statement of cashflows, as reported in Compustat. The key accounting identity is:

$$\text{cf from operations} + \text{cf from financing} + \text{cf from investing} = 0. \tag{3.2}$$

The first term is the cashflows from operations, the second is the cashflows from financing activities, and the third is the cashflow for investing activities. We use the convention that changes in cash balances represent net payments to debt holders. Notice that this cashflow identity is the financial accounting counterpart of the firm budget constraint (3.1) in the model. The mapping between model concepts and financial accounting terminology is summarized below:

$$\underbrace{\Pi_t}_{\substack{\text{capital income =}\\ \text{cashflows from operations}}} = \underbrace{(w_{H,t}L_{H,t} + I_t)}_{\substack{\text{tangible and intangible investment =}\\ -\text{cashflows from investment}}}. + \underbrace{D_t}_{\substack{\text{payouts =}\\ -\text{cashflows from financing}}}$$

[7] In our economy, the standard GDP identity (i.e., income equals expenditures) can then be written as:

$$\underbrace{\overbrace{w_{Y,t}L_{Y,t} + w_{H,t}L_{H,t}}^{\text{labor income}} + \overbrace{\Pi_t}^{\text{capital income}}}_{\text{income}} = \underbrace{\overbrace{Y_t - I_t}^{\text{consumption}} + \overbrace{I_t + w_{H,t}L_{H,t}}^{\text{investment}}}_{\text{expenditures}}.$$

Furthermore, we will later use the fact that cashflows from financing and investing can be decomposed into:

$$\text{cf from financing} = \text{cf from financing (equity)} + \text{cf from financing (debt)}$$
$$\text{cf from investing} = \text{cf from investing (capex)} + \text{cf from investing (acquisition)}.$$

These equations allow us to account for 100% of the cash that comes in due to profits (cashflows from operations), the net cash that comes out to pay owners (cashflows from financing activities), and the cash that comes out due to investment (cashflows from investment activities).

Adjustments. Building on the existing literature, we conduct some data imputations and adjustments. We make four main adjustments to ensure that our measures accurately capture both tangible and intangible investments. We provide a brief description below, with the detailed methodology in Appendix B.1.

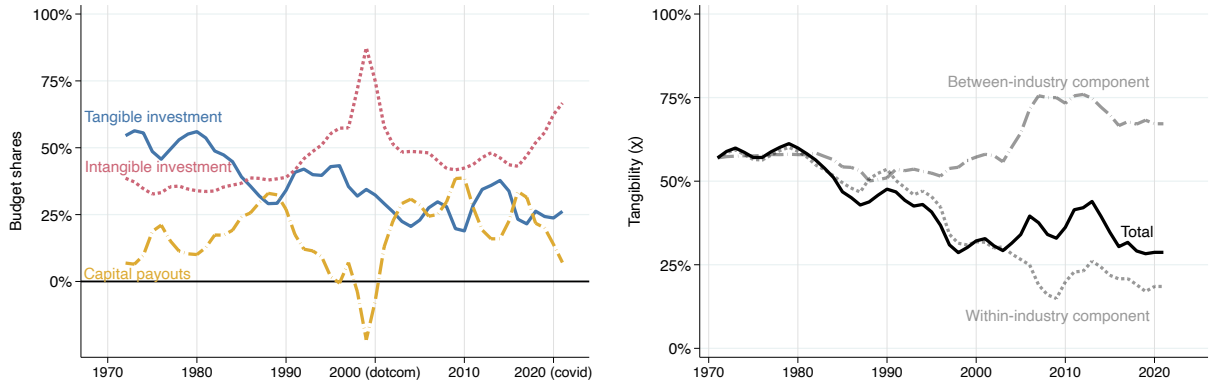
Our first adjustment corrects the undercounting of intangible investments by reclassifying a portion of SG&A (selling, general, and administrative expenses) and all R&D expenditures from operating expenses to capital expenditures. The reason is that thes line items typically include labor costs associated with intangible capital formation. Expensing these investment labor costs (as is currently done) lowers measured operating cashflows and overstates current profits. Our second adjustment reflects net entry of firms into and out of the public market. We account for the cash outflows (for instance due to new listings) and inflows (for instance due to acquisitions of public targets) experienced by passive investors who rebalance their portfolio to maintain a market-neutral investment. Our third adjustment deals with payments made in the form of new shares rather than cash by splitting the growth in share count into a cash component and a "noncash" component, thereby correctly incorporating stock-based compensation and stock-financed acquisitions as economic costs. The breakdown between stock compensation and acquisitions is only available after 2011. In prior years, we assume that acquisition is a constant share $\omega \in (0, 1)$ of noncash payments, where ω is computed at the industry year on post 2011 data. Finally, our fourth adjustment imputes the intangible share of "ambiguous" investments, such as acquisitions and IPOs, by assuming they contain the same tangibility, $\tilde{\chi}$, as other forms of investment within each industry and year.

Results. Figure 2a reports the distribution of capital income over time. Three facts stand out: (1) the tangible investment share declines, (2) the intangible investment share increases, and (3) the payout share is low on average and highly countercyclical.

Those three facts are consistent with findings in the literature relating to the stagnation of capital expenditures post-GFC (Gutiérrez and Philippon, 2017; Crouzet and Eberly, 2019), the importance of payments to investment labor (Eisfeldt and Papanikolaou, 2013; Peters and Taylor,

2017; Eisfeldt et al., 2023), and the empirical evidence on the aggregate equity payout yield (Fama and French, 2005; Boudoukh et al., 2007; Fried and Wang, 2019). Zooming in on the tangibility of investment,

Figure 2b shows a large decline in the tangibility of investment, from roughly 60% in the 1970s to 35% post-2000. We further decompose the decline in the tangibility of investment using a shift-share approach. More specifically, we quantify how much the decline in aggregate tangibility is driven by a decline in tangibility within each industry (a *within* component) rather than intangible industries growing bigger relative to intangible industries (a *between* component). We find that most of the effect is driven by the within component, implying that the effect is not driven by changes in the industry composition of public firms.



(a) Budget shares

(b) Tangibility Index χ

Figure 2: Uses of income in the corporate sector (1972–2022)

Notes: Aggregate tangibility is $\chi \equiv \sum_i w_i \chi_i$ where χ_i denotes tangibility in industry i and w_i denotes the fraction of aggregate investment accounted by that industry. The yearly change in aggregate tangibility can be decomposed into a within and a between components: $\Delta\left(\sum_i w_i \chi_i\right) = \sum_i \left(w_i + \frac{\Delta w_i}{2}\right) \Delta\chi_i + \sum_i \left(\chi_i + \frac{\Delta \chi_i}{2}\right) \Delta w_i$. To obtain the two components plotted in the figure, we start from the aggregate tangibility at time $t = 1971$, and add the cumulative sum of the within and between components, respectively.

Table 2 reports the average distribution of capital income over the 1972–2022 period, including a detailed breakdown, and robustness checks. Tangible investments account for 42% of capital over the period, mostly due to direct capital expenditures, but also in part due to acquisition of existing businesses (who themselves are partial tangible). Intangible investments account for 44%. Cash and noncash compensation to investment labor accounts for most of it, but almost 20% comes from payouts associated with cash and noncash payout that arise in acquisitions.

Finally, payments to capitalists account for a mere 14% of capital income over the sample. While net cash payouts to equity holders account for 19% of capital income, they were offset by a 12% equity dilution (i.e., noncash payments). This sample average is skewed by the tech bubble of the 1990s, where payments to capitalists were negative for a few years (i.e., acquisitions and IPOs exceeded dividends and interest payouts, see Fried and Wang, 2019 for a discussion of this fact).

Table 2: Distribution of capital income (1972–2022 average).

Uses of capital income (%)	Baseline	Robustness			
		$\widetilde{\chi}=0$	$\widetilde{\chi}=1$	$\omega=0$	$\omega=1$
Tangible investments	36	37	45	37	42
Tangible capital expenditures	37	37	37	37	37
Mergers, acquisitions, and IPOs	-1	0	8	0	5
$\chi\cdot$ cash acquisitions	3	0	8	3	3
$\chi(1-\omega)\cdot$ non-cash payments	2	0	5	0	5
$\chi\cdot$ net entry in public universe	-6	0	-6	-3	-3
Intangible investments	46	46	38	42	46
Intangible capital expenditure	38	38	38	39	35
$0.3\cdot$ selling, general, and admin. expenses	24	24	24	24	24
research and development expenses	12	12	12	12	12
$\psi\cdot\omega\cdot$ non-cash payments	2	2	2	4	0
Mergers, acquisitions, and IPOs	8	8	0	3	10
$(1-\chi)\cdot$ cash acquisitions	5	8	0	5	5
$(1-\chi)(1-\omega)\cdot$ non-cash payments	3	5	0	0	7
$(1-\chi)\cdot$ net entry in public universe	1	-6	0	-2	-2
Payout to capitalists	18	18	18	18	18
Net cash equity payout	19	19	19	19	19
− Non-cash payments	-13	-13	-13	-13	-13
− Net entry in public universe	6	6	6	6	6
Net debt payout	6	6	6	6	6

Comparison with the National Accounts. In Appendix B.2, we contrast our results with aggregate data from the National Accounts. Our definition of capital is meant to be (mostly) consistent with the national accounts definition. There are two key differences. First, our treatment of non-cash payments is more comprehensive than what the BEA does. It is well understood that the BEA undercounts labor income by missing payments or underestimating their true value (see Zwick, 2022 for a detailed discussion on the topic). In contrast, our approach uses the actual growth in the number of (fully-diluted) shares times the market value at the time to impute the value of payments in stocks. Second, our definition of what an innovator potentially differs from the BEA. The BEA mostly capitalizes expenses on labor in the case of research and development (see Corrado et al., 2009). Instead, we opt for a more comprehensive definition of "investment labor", which includes not only scientists, bu also key managers, entrepreneurs, and early financiers. Appendix Figure A1 plots capital income and enterprise value, both in our sample and in the the Integrated Macroeconomic Accounts. Appendix Figure A2 compares the time variation in the use of capital income in the national accounts versus Compustat.

3.2 Model validation

We now use this data on the uses of capital income to test some implications of our model.

Time series evidence. One important prediction of our model is that an increase in the valuation of the corporate sector relative to output should be associated with a rise in the wage gap between investment and production workers (2.1). We now test this time-series prediction.

Figure 3a plots the ratio between the valuation of public firms in Compustat and their capital income. We find a rising disconnect between the market value of the firms and their revenues, which is consistent with a vast array of evidence (see, e.g., Gomez and Gouin-Bonenfant, 2024).

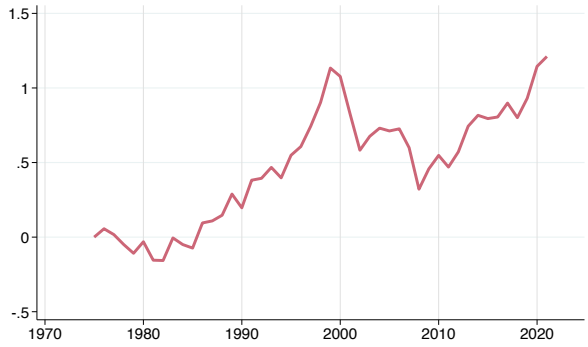
We then plot the wage gap between investment and production workers. We first use data from the Current Population Survey to measure the average wage gap between investment and production workers. We define "investment labor" as occupations in managerial and professional specialty occupations, whereas"production labor" includes occupations in technical, sales, and administrative support occupations, service occupations, and production occupations. We focus on the annual labor income reported in the March survey. We compute the growth in the average wage within occupation class — investment or production labor — with and without adjusting for composition changes. To control for composition changes within each class, we first residualize wages across a set of interacted worker characteristics and construct the composition-adjusted wage as the average residualized wage adjusted for the average wage within the occupation across years.[8]

Figure 3b shows the evolution of the wage gap — the ratio between the wage of investment and production workers — over time. We find that the wage premium increases by 10 log points on an unadjusted basis, and by 25 log points once we account for composition changes within occupations over time. The larger premium after controlling for composition implies that characteristics associated with lower wages have become relatively more common in investment occupations than in production occupations. This pattern reflects that, as the demand for investment labor expands, workers switching from production to investment roles are, on average, less skilled than the workers already in these positions— thereby increasing the wage gap once composition is taken into account. Note that, while our estimates control for changes in observed characteristics, similar dynamics may be at play for unobservable ones, suggesting that our results may underestimate the actual rise in the wage gap on a fixed-composition basis.

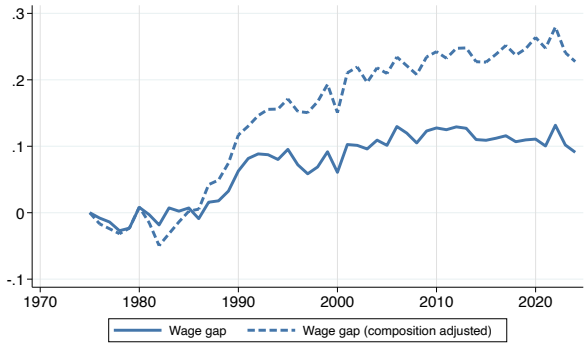
The overall pattern for a rising gap between investment and production workers aligns with findings from the literature on U.S. wage inequality since the 1980s, which document that occupations requiring "abstract" skills have seen a relative increase in wages compared to those relying on "routine" skills (see, e.g., Autor and Dorn, 2013). Finally, this pattern is also related to the rise

[8]The set of characteristics includes sex (two groups: male and female), race (two groups: White and non-White), age (five groups: 16–25, 26–35, 36–45, 46–55, and 56–64 years), and education (five groups: some high school, high school diploma, some college, bachelor's degree, and more than a bachelor's degree.

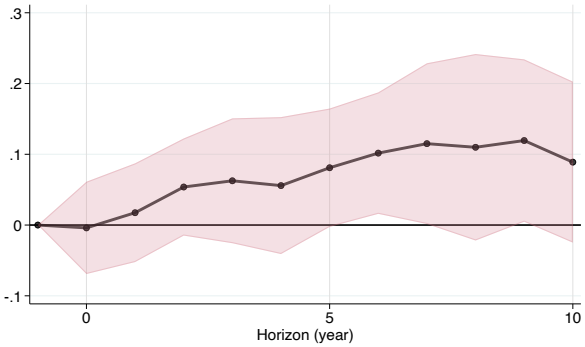
in the CEO wage gap over the same period, as documented by Gabaix and Landier (2008), as we classify managers as investment workers.



(a) Enterprise value / capital income (log)



(b) Wage gap (log)



(c) Local projections of wage gap on valuation shock

Figure 3: Relation between corporate valuations and the investment-production wage gap

We can more formally test the relationship between the two quantities by estimating local projections of the wage gap between production and investment workers on shocks in corporate sector valuations:

$$\log\left(\frac{w_{H,t+h}}{w_{Y,t+h}}\right) = \alpha_h + \beta_h \Delta \log\left(\frac{V_t}{Y_t}\right) + \gamma_h \log\left(\frac{w_{H,t-1}}{w_{Y,t-1}}\right) + \epsilon_{t+h} \tag{3.3}$$

Figure 3c presents the results $\{\widehat{\beta}_h\}_{h=1}^{10}$ with a 95% confindence interval. We find that a rise in corporate valuations predicts an increase in the wage gap over the following years. The response of wages is gradual, consistent with the presence of adjustment costs in the short-run (i.e., $\theta > 0$ in the model). Note that the coefficient β_h is not estimated from the overall trends in the wage gap and in valuations, but, rather, from the deviations of these two series around their respective trends.

Cross-sectional evidence. Our model predicts that more intangible economies have a lower elasticity of capital — see the expression (short-run elasticity). A similar implication would arise in

the cross-section in a multi-sector model. As long as there is an imperfectly elastic supply of labor across industries and occupations, we should expect that industries with a higher share of intangibility tend to have a lower capital supply elasticity.

As a model validation exercise, we now examine the relationship between short-run capital elasticity and tangibility at the industry-level. To estimate the short-run investment-q elasticity $\frac{\partial \log(I_t + w_{H,t} L_{H,t})}{\partial \log q_t}$, we leverage cross-industry variation in total investment and valuations. We focus on $J = 33$ broad industry groups over 1972-2022. As an empirical proxy for $\log q_{j,t}$ we use log enterprise value $\log V_{j,t}$. The idea is that annual industry-level fluctuations in market values are mostly driven by changes in the valuation of capital q_t, rather than changes in capital K_t. We estimate an industry-specific "investment-q elasticity" β_j via:

$$\Delta \log(I_{j,t} + w_{j,t} L_{H,j,t}) = \alpha_j + \beta_j \Delta \log V_{j,t} + \gamma_j \log(I_{j,t-1} + w_{j,t-1} L_{H,j,t-1}) + \epsilon_t. \tag{3.4}$$

Figure 4 plots the average tangibility of investment over the sample for each industry, as well as its investment-q elasticity. Despite the limited sample, we find a significant positive relationship, consistent with the prediction from the model.

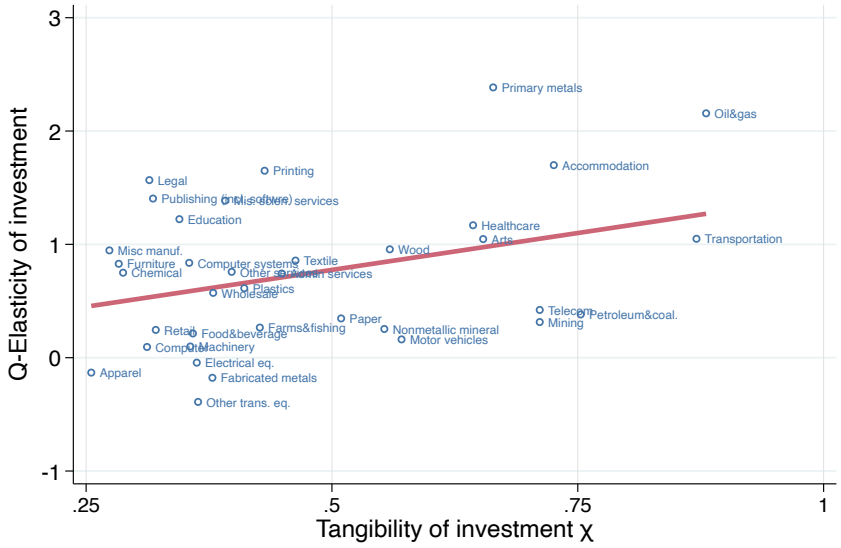


Figure 4: Investment-q elasticity and tangibility of investment.

Notes: On the x-axis, the industry-level tangibility of investment is the (unweighted) 1972-2022 average χ_j. On the y-axis, we report estimated coefficient φ_j, which captures the elasticity of total investment to firm value (a proxy for q). The red line reports the predicted relationship using all data points (unweighted).

4 Quantitative analysis

We now develop a more quantitative model that we calibrate using our micro-data on the uses of capital income. We then use the model for two experiments (i) the effect of a shift towards

intangible capitals on the economy, and (ii) the effect of a corporate tax cut.

4.1 Full model

We first extend our stylized model to make it more realistic. We add four ingredients : (i) corporate taxes (ii) an elastic labor supply in and out of the labor force and between between production and investment occupations, and (iii) an exogenous growth π in the productivity of production and investment labor.

Production side. The production side is the same as in the stylized model except we now add corporate taxes, that are rebated to the household The firm budget constraint then becomes:

$$D_t = (1 - \tau_{K,t})(Y_t - w_{Y,t}L_{Y,t}) - I_t - w_{H,t}L_{H,t}, \qquad \text{(firm budget constraint')}$$

where $\{\tau_{K,t}\}_{t=1}^{\infty}$ is a sequence of capital income tax rates, or more precisely taxes on payouts.

Household side. On the household side, we now allow the quantity of production and investment labor to be elastically supplied. More precisely, we assume that the representative household chooses a sequence of consumption, production labor, and investment labor to maximize welfare:

$$U_0 = \max_{\{C_t, L_{Y,t}, L_{H,t}, V_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^t \left\{ \frac{1}{1-\gamma} \left(C_t - \frac{L^{1+\sigma}}{1+\sigma} \right)^{1-\gamma} \right\}$$
$$\text{where } L_t \equiv \left((1-\mu)^{-\rho} L_{Y,t}^{1+\rho} + \mu^{-\rho} L_{H,t}^{1+\rho} \right)^{\frac{1}{1+\rho}}.$$

The quantity $1/\sigma$ represents the elasticity of aggregate labor supply while $1/\rho$ represents the elasticity of substitution between production and investment labor. The stylized model presented in Section 2 corresponds to the special case $\sigma = \rho \to \infty$, in which case the supply of production and investment labor is fixed. The household first-order conditions with respect to $L_t, L_{Y,t}$ and $L_{H,t}$ gives:

$$L_t = w_t^{\frac{1}{\sigma}}; \qquad L_{Y,t} = (1-\mu) L_t \left(\frac{w_{Yt}}{w_t} \right)^{\frac{1}{\rho}}; \qquad L_{H,t} = \mu L_t \left(\frac{w_{H,t}}{w_t} \right)^{\frac{1}{\rho}}, \qquad \text{(worker foc } L_t\text{)}$$

where w_t is defined similarly to L_t. In the case $\sigma = \rho \to \infty$ (inelastic labor supply) we get $L_t = 1, L_{Y,t} = 1 - \mu, L_{H,t} = \mu$ as in the stylized model from Section 2. Conversely, in the case $\rho \to 0$ (perfect substitution), the wages between production and investment workers are equalized, $w_{H,t} = w_{Y,t}$.

Equilibrium. An equilibrium is an initial condition K_0, an allocation $\{L_{Y,t}, L_{H,t}, L_t, I_t, K_t\}_{t\geq 1}$, and prices $\{w_{H,t}, w_{L,t}, w_t, r_t, q_t\}_{t\geq 1}$ that solve the firm problem (firm foc $L_{Y,t}$, firm foc $L_{H,t}$, firm foc I_t,

firm foc K_{t+1}) and household problem (worker foc C_t, worker foc L_t).

4.2 Calibration

We now calibrate the model. First, we set two parameters externally. We set $\sigma = 2$ to match an aggregate labor supply elasticity of 0.5, consistent with the macro evidence at the business cycle frequency. We then set $\rho = \sigma$, which corresponds to a linearly separable labor supply for production workers and investment workers. The parameter β is chose to match a roughly 7% log return (unlevered), which is the average over our sample. In Appendix C.1, we show that, along a balanced growth path with growth rate π, the log return is constant at $\log R = -\log\beta + \gamma\pi$, where γ is the household's EIS. Assuming $\gamma = 1$ and $\pi = 2\%$, we get $\beta = 0.95$. Second, we calibrate the parameter $\chi = 0.41$, which governs the importance of tangible inputs in capital formation using our evidence on the tangibility of investment (2b) .

Third, we calibrate the remaining parameters four parameters (α, θ, δ) internally by targeting two long-run moments (the total investment yield and the labor income share) and one short-run moment (the tax elasticity of investment). The total investment yield is the sum of tangible and intangible investments, as a share of enterprise value. In Appendix A.2, we discuss how this moment has the interpretation of a dilution rate: it measures how much a capitalists that would consume all the capital income would get diluted over time. The second moment is the labor share, which is defined as total payments to labor (to both production and investment labor) as a share of aggregate income.

Table 3: Internally calibrated parameters

Moment	Notation	Formula	Target
Total investment yield	$\frac{I+w_H L_H}{V}$	$(1-\theta)\delta$	13%
Capital share	$\frac{\Pi}{GDP}$	$\frac{\alpha}{1+(1-\chi)\alpha\frac{(1-\theta)(\delta+\pi)}{r-\pi+(1-\theta)(\delta+\pi)}}$	33%
Tax elasticity of investment	$\frac{\partial\log(I_0+w_{H,0}L_{H,0})}{\partial\log(1-\tau_K)}$	$\frac{r+(1-\theta)\delta}{r+\phi}\frac{1}{\theta}$	4

Notes. α is the capital share in production; θ is the production share in investment; χ is the tangibility of investment; δ is the depreciation rate; r is the return; ϕ is the annual decay of the tax cut.

Note that, together, these two long-run moments can not separately identify $1-\theta$ from δ. The reason is that a high investment yield can be due to the fact that capitalists extract little rents from investment (θ is low) or that the quantity of investment is high (δ is high). Income shares alone can not distinguish between these two cases. Guided by the earlier intution that θ effectively governs the elasticity of capital supply in the short-run (as in the q-theory of investment), we use a short-run moment for our last empirical target.

We draw on evidence from Chodorow-Reich, Smith, Zidar and Zwick (2024) on the empirical

response of US firms to the 2017 Tax Cuts and Jobs Act. We focus on their result that uses tax files for a sample of roughly $N = 7000$ (nonfinancial, non-passthrough domestic firms). The variable of interest is log investment change (post-TCJA versus pre-TCJA). The authors estimate that a 1% decline in the corporate income tax rate leads to a roughly 4% decline in investment (see their Table 3, columns 2 and 3). We interpret their estimates as a partial equilibrium short-run response in the model (i.e., holding prices w_Y, w_H, R constant).

In Appendix 4, we map their regression coefficient to a simple formula that is the product of two terms. We show that a tax cut (with an annual decay rate of τ) implies a short run response of log total investment of $\frac{r+(1-\theta)\delta}{r+\phi}\frac{1}{\theta}$ (see Table 3). The first terms accounts for the fact that (i) capital payout is levered to the tax rate and (ii) the tax cut is not fully permanent. The second term is $\frac{1}{\theta}$, which governs the short-run elasticity of capital supply in partial equilibrium. Table 4 reports the model calibration using these short-run and long-run targets.

Table 4: Calibrated parameters

Parameters	Description	Value
σ	Inverse labor supply elasticity (aggregate)	2
ρ	Inverse labor supply elasticity (sectoral)	2
β	Subjective discount factor	0.95
α	Capital share in production	0.38
θ	Capital share in investment	0.3
δ	Capital depreciation rate	0.19
χ	Tangibility index	0.41

Given the Cobb-Douglas assumption on capital formation, our calibration implies that the revenues from capital formation are distributed as follows: a share $\theta = 30\%$ is paid to capital owners, a share $(1-\theta)\chi = 29\%$ is spent on investment goods and a share $(1-\theta)(1-\chi) = 41\%$ is spend on investment workers.[9]

4.3 Quantifying the effect of a rise in intangibles

We now conduct an experiment where we quantify the long-run effect of a decline in the tangibility of investment χ from 0.54 to 0.29 (i.e., a 25 pp. decline around the baseline calibration), in line with what we have seen in the data over the 1972-2022 period (see Figure 2b).

First, Table 5 reports the effect of rising intangibility on the composition of GDP (see Section 3.1 for national accounting definitions). On the expenditure side (Panel A), there is a rise in intangible investments (6 pp. of GDP) offset by a decline in tangible investments (-6 pp. of GDP). On the

[9]Using an indirect inference method, Kogan et al. (2020) estimates that the revenues from capital formation are distributed as follows: a share $\alpha = 45\%$ is paid to investment goods, a share $(1-\eta)(1-\alpha) = 13\%$ is paid to capital owners, and a share $\eta(1-\alpha) = 42\%$ is paid to investment workers. Hence, compared to this paper, our estimates imply that a higher share of investment rents accrue to capital owners relative to investment workers.

Table 5: Distributional effect of a rise in intangibles

Variable	Symbol	Baseline (pp.)	1972→2022
Panel A – Expenditures			
Consumption	$Y - I$	76	-0
Tangible investments	I	10	-6
Intangible investments	$w_H L_H$	14	6
Panel B – Income			
Capital income	$Y - w_Y L_Y$	33	0
Labor income (production)	$w_Y L_Y$	53	-6
Labor income (investment)	$w_H L_H$	14	6

income side (Panel B), we see a decline in the production labor income share 53 pp. compensated by a rise in the investment labor income share (53 pp.).

Effect on capital elasticity. Figure 5 shows the long-run effect of rising intangibles on the capital supply curve, which amounts to a counter-clockwise rotation of the supply curve.[10] Notice that capital supply becomes much less elastic, due to the growing importance of imperfectly-elastic labor in capital formation.

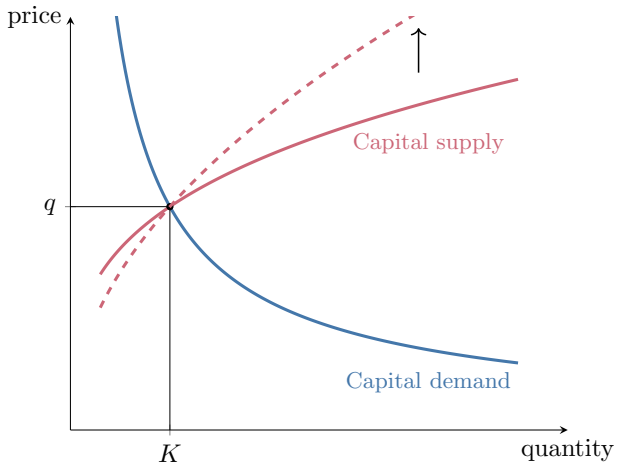


Figure 5: Effect of a rise in intangibles on the market for capital

Quantitatively, varying the value of χ in the model, while holding all the other parameters constant, has a large effect on both the short- and long-run capital supply elasticities. Table 6 reports the model-implied elasticities. The baseline model has low elasticities compared to the case of full tangibility (i.e., setting $\chi = 1$ as in the standard q-theory of investment). In the short-run it is 0.14 versus 0.43, and in the long-run it is 1.54 versus $+\infty$. Simulating the transition from

[10]Note that a change in χ affects the long-run equilibrium Y due to an endowment effect. For visual purposes, we construct Figure 5 by showing a rotation around the initial equilibrium, which amounts to consider a joint change in χ and z_H that ensure no change in the long-run (q, K).

1972 to 2022 (i.e., rise in intangibles), we obtain a decline in both the long-run elasticity by more than half.

Table 6: Capital supply elasticity.

	Baseline	Q-theory ($\chi=1$)	1972 ($\chi=0.54$)	2022 ($\chi=0.29$)
Short-run	0.14	0.43	0.17	0.11
Long-run	1.54	$+\infty$	2.23	1.1

Notes. The short-run supply elasticity of capital is $\partial \log K_{t+1} / \partial \log q_t$ while the long-run is $\partial \log K_t / \partial \log q$.

Effects on the wage gap. A decline in χ corresponds to a rise in labor demand for one type of labor (i.e., investment labor). How does that translate into changes in relative wages? In the counterfactual, we obtain a large response of the wage premium by $\Delta(w_H/w_Y)/(w_H/w_Y) = +33\%$. To understand the sources of the rise in the wage premium, we use the following decomposition:

$$\mathrm{d}\log\frac{w_H}{w_Y} = \frac{\rho}{1+\rho}\,\mathrm{d}\log\frac{w_H L_H}{w_Y L_Y} = -\frac{\rho}{1+\rho}\frac{\mathrm{d}\chi}{1-\chi}.$$

The first equality uses labor supply (worker foc L_t) to express the change in relative wage in terms of the change in wage bills, which depends on the occupation-level labor supply elasticity ρ. If it was perfectly elastic ($\rho = 0$), we would see no change in relative wages. In the baseline calibration, labor is moderately elastic, with $\frac{\rho}{1+\rho} = 0.67$. The second equality uses the long-run relative marginal products of labor (2.1) $\frac{-\mathrm{d}\chi}{1-\chi} = 54\%$. Quantitatively, the rise in the wage gap implied by the model is consistent with the actual increase in the wage gap between investment and production workers observed in the data since 1970, as plotted in Figure 3.

4.4 Quantifying the effect of a corporate tax cut

We now use the calibrated model to consider the allocative and redistributive effects of a corporate tax cut.

Theory. A corporate tax cut can be seen as a shift in the demand for capital, as represented in Figure 5. As usual in supply-demand systems, a positive demand shock will increase prices and quantities. When the supply curve is horizontal (as in the NGM), then the demand shock is entirely absorbed by quantities K. But in the baseline, long-run capital supply is imperfectly elastic due to finite labor supply. As a result, a demand shock, in this case a tax cut, leads to a rise of both prices q_t and quantities K_t.

To set ideas, consider a permanent decline in τ_K. Using a standard comparative statics approach and the long-run demand and supply elasticities in the model, we obtain

$$\frac{\mathrm{d}\log K}{\mathrm{d}\log(1-\tau_K)} = \frac{1}{\text{demand elasticity} + \text{supply elasticity}} = 0.9,$$
$$\frac{\mathrm{d}\log q}{\mathrm{d}\log(1-\tau_K)} = \frac{\text{supply elasticity}}{\text{demand elasticity} + \text{supply elasticity}} = 0.6.$$

The shock is absorbed through both a higher valuation of capital q *and* a rise in capital formation K. As a benchmark, in the NGM, the response of capital would be roughly twice as high (1.9) with no response of valuations.

More generally, a shift towards intangibles means that the market increasingly clears via higher valuations rather than actual capital formation. As we discuss next, this imperfect pass-through of demand shocks to quantities has important effects on the incidence of capital taxes (i.e., who wins and loses in terms of welfare). Intuitively, a jump q induced by a tax cut will benefit investment labor (via higher wages) and initial capitalists (via wealth effect).

We first consider an arbitrary perturbation $\{\mathrm{d}\tau_{K,t}\}_{t=0}^{\infty}$ around the undistorted steady-state. Which factors of production win and/or lose in response to this shock? We follow Fagereng et al. (2024) and apply the envelope condition on the household value function to obtain the total welfare effect of the change in prices induced by the tax shock.

Proposition 4.1. *The equilibrium welfare effect, in units of $t = 0$ consumption, associated with the perturbation $\{\mathrm{d}\tau_{K,t}\}_{t=1}^{\infty}$ around the undistorted steady-state, is given by*

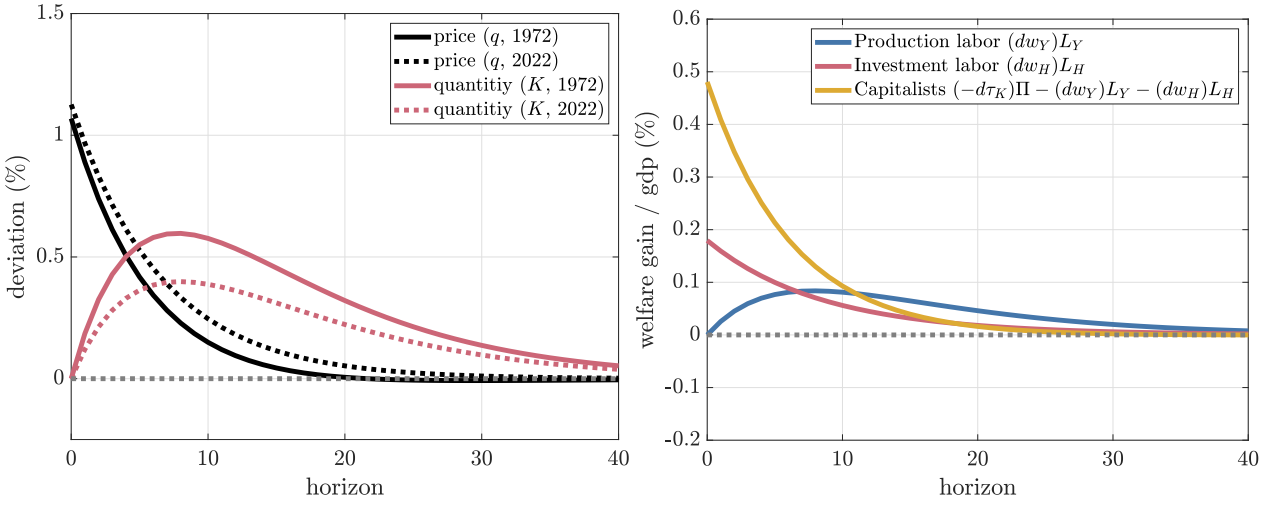
$$\textit{Welfare Gain} = \underbrace{\sum_{t=1}^{\infty} R^{-t} L_Y \cdot \mathrm{d}w_{Y,t}}_{\textit{welfare gain (production workers)}} + \underbrace{\sum_{t=1}^{\infty} R^{-t} L_H \cdot \mathrm{d}w_{H,t}}_{\textit{welfare gain (investment workers)}} + \underbrace{\mathrm{d}V_0 + \sum_{t=1}^{\infty} R^{-t} V \cdot \mathrm{d}r_t}_{\textit{welfare gain (capitalists)}}.$$

Proposition (4.1) decomposes the welfare effect into the contribution of changes in prices for the three factors of production: production labor, investment labor, and capital. For labor, the welfare effect captures the fact that the tax shock affects the path of wages $\{\mathrm{d}w_{Y,t}, \mathrm{d}w_{H,t}\}_{t=1}^{\infty}$. For capitalists, there is an initial wealth gain $\mathrm{d}V_0$ as well as the contribution of changes in forward returns $\{\mathrm{d}r_t\}_{t=1}^{\infty}$. The total (private sector) welfare effect is equal to the present value of the change in the tax rate times capital income (i.e., the mechanical effect of taxes on profits holding everything else constant): Welfare Gain $= \sum_{t=1}^{\infty} R_{0\to t}^{-1} \Pi_t \cdot \mathrm{d}\tau_{K,t}$.

Quantitative experiment. We now simulate a 1% cut around the steady-state, which decays at an annual rate of $1 - \phi = 0.9$. We consider the case of a small-open economy, where forward returns are constant (i.e., $\mathrm{d}r_t = 0$). Throughout this section, our baseline environment is a small open economy, where we simulate deviations induced by taxes while imposing $\mathrm{d}r_t = 0$ (the interest

rate is not affected by the shock, but wages are).[11] This simplifies the formula for the welfare effect for capitalists in Proposition (4.1), which becomes only the initial wealth gain dV_0 (i.e., the rise in their wealth due to the fact that future post-tax capital income has increased).

Capital accumulation. Figure 6a plots the response of capital accumulation and valuations over a 40 year period after the shock. We show the dynamics implied by the model using the 1972 and 2022 calibrations to describe how the shift towards capital affects the response of the economy. In both cases, the value of capital jumps on impact, but it takes several years for the stock of capital to peak. Notice, however, that in the intangible economy (2022 calibration), the stimulative effect of the tax cut is much lower, consistent with the earlier calculations regarding the supply elasticity of capital (see Table 6).



(a) Capital and valutations

(b) Welfare effects

Figure 6: Model-implied effect of a cut in corporate taxes

How does the general equilibrium (GE) response of investment respond to the partial equilibrium (PE) target used for calibration? In PE, we target a high elasticity, but in a GE experiment where the tax applies to all of capital income (which represents 4% of GDP in the baseline), we expect a much weaker response due to rising wages and interest rates.

Table 7 reports the elasticity of total investment to the tax shock in PE (our calibration target), as well as the corresponding elasticity in a small open economy (the baseline), and in GE. We find that rising wages reduce the tax elasticity of investment nearly fourfold. The second column reports the tax elasticity of investment wages: compared to the PE environment (where wages are fixed), in a small open economy the elasticity is large at 0.64. Finally, the GE case, where interest rates dr_t also respond, has an even lower tax elasticity of investment. The response of interest rates is governed by the household EIS $\frac{1}{\gamma}$. Intuitively, in the case where $\gamma \to \infty$ (Leontief

[11] In terms of the implied dynamics dq_t, dK_t, assuming a "small open economy" is this is equivalent to setting the EIS to $\gamma = 0$, which implies $1 + r_t = \beta^{-1}$.

Table 7: Short-run response to a tax shock.

Tax elasticity of:	Investment $(I + w_H L_H)$	Investment wages (w_H)	Valuation (q)
Partial equilibrium	4	0	1.1
Small open economy (baseline)	0.96	0.64	0.55
General equilibrium	0.36	0.24	0.21

Notes. The short-run tax elasticity of a variable X_t is the response $\mathrm{d}\log X_1$ in response to a tax shock $\mathrm{d}\tau_{k,t} = (1-\phi)^t \mathrm{d}\tau_K$ ($\phi = 0.1$ annually). "Partial equilibrium" is the equilibrium deviation with the restriction $\mathrm{d}w_Y = \mathrm{d}w_H = \mathrm{d}R = 0$; "small open economy" is the equilibrium deviation with the restriction $\mathrm{d}w_Y = \mathrm{d}w_H = 0$; "general equilibrium" is the equilibrium deviation where all prices adjust (EIS is set to $\gamma = 1$)).

preferences), aggregate capital does not adjust because household are not willing to substitute across time periods. We report the case $\gamma = 1$ (log utility) and the resulting tax elasticity of investment nearly ten times lower than in PE.

Welfare effects. The welfare effect for workers works through changes in their wage, which are themselves fully pinned down by the path of the state and co-state variables $\{q_t, K_t\}_{t=0}^{\infty}$. A rise in capital K benefits both types of workers, via their complementarity with existing capital, but the value of capital q directly affects the marginal product of investment labor. Figure 6b plots the resulting path of (undiscounted) welfare effects for both types of labor, expressed as a share of steady-state GDP. While the tax shock appears to benefit both types of labor equally, it is worth pointing out that production labor accounts for roughly 4 times more labor income than investment labor in steady-state (see Table 5).

The welfare effect for capitalists can be decomposed as the present value of higher future payouts:

$$\mathrm{d}V_0 = \sum_{t=1}^{\infty} R^{-t}\Big(- \Pi \cdot \mathrm{d}\tau_{K,t} - L_Y \cdot \mathrm{d}w_{Y,t} - L_H \cdot \mathrm{d}w_{H,t}\Big).$$

Notice that those higher payouts are very front-loaded, especially compared to the back-loaded increase in production worker wages (see Figure 6b). This is because wages rise slowly with capital accumulation, while the benefit for capitalists on post-tax capital income is immediate.

Table 8 reports the share of private welfare gains that accrue to each factor of production. In the baseline, 20% of the tax incidence falls on production workers, while 26% falls on investment labor and 54% on capitalists. Note that this stands in sharp contrast with the NGM, where 100% of the incidence falls on productions workers. This is because, when capital is fully elastic ($\theta = 0$), there is no wealth effect since $q = 1$ at all times.

To understand the difference between the baseline and the NGM, it is useful to consider an intermediary model (i.e., the q-theory special case), where we impose ful tangiblity $\chi = 1$ while keeping a positive capital share $\theta = 0.3$ as in the baseline calibration. In that case, we a slightly lower capitalist share. We also run the same model experiment in the low- and high-intangible

Table 8: Incidence of capital taxes (welfare gain as a share of private sector welfare gains).

Factor	Baseline	NGM (χ=1; θ=0)	Q-Theory ($\chi=1$)	1972 ($\chi=0.54$)	2022 ($\chi=0.29$)
Production workers	20	100	50	26	17
Investment workers	26	0	0	22	30
Capitalists	54	0	50	52	53

calibrations that correspond roughly to 1972 and 2022. Notice that the shift towards intangibles has led to a reallocation of the incidence of capital taxes, away from production labor and towards investment labor and capitalists.

5 Conclusion

In this paper, we enrich the standard neoclassical growth framework by introducing a more realistic investment process that pairs tangible inputs—machinery, computers, plants—with both accumulated capital and specialized labor, such as inventors, entrepreneurs, and financiers. This key departure recognizes that the formation of new capital hinges on skilled workers who are in finite supply. As a result, the supply curve for capital slopes upward even in the long run, reconciling the empirical observation of large disconnect between corporate sector valuations and investment quantities.

After calibrating our model to the U.S. data, we show that a growing importance of intangible investment can explain several key macroeconomic trends over the past two decades: persistently weak tangible investment despite high valuations, increasing factorless income, and widening wage disparities across occupations.

For tractability, our model parametrizes capital formation using a Cobb-Douglas function. This specification implies that tangible and intangible investments are neither complements or substitutes, and so they each represent a constant expenditure share of capital formation. Alternatively, we could consider an investment function in which tangible and intangible investments have an arbitrary elasticity of substitution. In the realistic case in which the two inputs are complement, the secular decline of equipment prices (investment goods) could have contributed to the rise in the expenditure share of intangible investment — a version of Baumol's cost disease. Estimating the elasticity of substitution between tangible and intangible investments is an important avenue for future research.

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Appendix

A Appendix for Section 2

A.1 A microfoundation for aggregate capital formation

We now discuss one particular way of micro-founding the stylized model in Section 2 along the lines of Kogan et al. (2020). Assume that the representative firm can hire investment workers to create blueprints according to a Cobb-Douglas production function $N_t = K_t^{1-u} L_{H,t}^u$, where N_t denotes the number of blueprints, K_t denotes the amount of capital, and $L_{H,t}$ denotes the amount of investment workers. The parameter $u < 1$ encapsulates the idea that the creation of new blueprints exhibits diminishing returns to scale, holding constant the amount of existing capital.

The firm then implements new blueprints using investment goods. For each new blueprint j, the firm can purchase a quantity of investment goods $I_{j,t}$ to create a quantity of project-specific capital $K_{j,t} = z_{H,t} I_{j,t}^v$. The parameter $v < 1$ encapsulates the idea that investment in new project is subject to decreasing returns to scale. Once implemented with a quantity of project-specific capital $K_{j,t}$, the project yields an output $Y_{j,t} = z_{Y,t} K_{j,t}^\alpha L_{Y,j,t}^{1-\alpha}$ every period, where $L_{Y,j,t}$ denotes the amount of project-specific production workers. Finally, the project-specific capital $K_{j,t}$ depreciates at rate δ.

Denote $Y_t = \int Y_{j,t} \, \mathrm{d}j, K_t = \int K_{j,t} \, \mathrm{d}j$ and $L_{Y,t} = \int L_{j,t} \, \mathrm{d}j$ the aggregate amount of output, capital, and production workers across existing projects in the economy. Since all projects can hire production workers at the same wage, total output in the economy can be represented using the aggregate production function $Y_t = z_{Y,t} K_t^\alpha L_{Y,t}^{1-\alpha}$. The total amount of capital in the economy then evolves as

$$K_{t+1} = (1-\delta) K_t + N_t \cdot z_{H,t} \left(\frac{I_t}{N_t} \right)^v,$$

where $N_t = K_t^{1-u} L_{H,t}^u$ denotes the aggregate number of new blueprints and I_t denotes the aggregate quantity of investment goods used to implement new blueprints — so that $z_{H,t} \left(\frac{I_t}{N_t} \right)^v$ represents the quantity of capital per blueprint.

Note that this system of equations for output production and capital formation in this economy is equivalent to the one discussed in Section 2, with $\theta \equiv (1-u)(1-v)$ and $\chi \equiv v/(v + u(1-v))$, so the two economies are isomorphic.

A.2 Capital rents

We now describe the income (or rents) associated with capital ownership. We use this income to write the value of firms in different ways.

Rents. Capital extracts *Ricardian* rents in the model (i.e., workers are paid their marginal product but below their average product). We define production and investment rents associated to capital ownership as:

$$\text{production rents} \equiv L_{Y,t} \cdot \left(\frac{Y_t}{L_{Y,t}} - \frac{\partial Y_t}{\partial L_{Y,t}} \right) = \alpha Y_t$$

$$\text{investment rents} \equiv \left(I_t^{\chi} L_{H,t}^{1-\chi} \right) \cdot \left(\frac{H_t}{I_t^{\chi} L_{H,t}^{1-\chi}} - \frac{\partial H_t}{\partial (I_t^{\chi} L_{H,t}^{1-\chi})} \right) = \theta q_t H_t.$$

Production rents are the difference between the average and marginal product of production labor. Because of the Cobb-Douglas assumption for the production function, production rents represent a constant share α of gross output Y_t, as in the NGM. We define investment rents similarly, but where the input to investment is the bundle of tangible and intangible investments. Because of the Cobb-Douglas assumption for the capital formation function, investment rents represent a constant θ of the revenues from capital formation $q_t H_t$.

Present-value of capital. The previous paragraph says that capital owners earn rents from both the production and the capital formation processes. In a present-value sense, this implies that the value of capital can be decomposed into the present value of production rents and the present value of investment rents. To see this formally, re-arrange (firm foc K_{t+1}) to obtain:

$$(1 + r_{t+1}) q_t K_{t+1} = \underbrace{\alpha \cdot Y_{t+1}}_{\text{production rents}} + \underbrace{\theta \cdot q_{t+1} H_{t+1}}_{\text{investment rents}} + (1 - \delta) q_{t+1} K_{t+1}$$

Solving forward gives the following present value equation for the value of a unit of capital q_0:

$$q_0 = \sum_{t=1}^{\infty} R_{0 \to t}^{-1} (1 - \delta)^{t-1} \left(\alpha \cdot \frac{Y_t}{K_t} + \theta \cdot \frac{q_t H_t}{K_t} \right).$$

Re-arranging gives the value of a unit of capital q_0 as the sum of the present-value of production rents associated to that unit of capital and the present-value of its investment rents:[12]

$$q_0 = \underbrace{\sum_{t=1}^{\infty} R_{0 \to t}^{-1} (1 - \delta)^{t-1} \alpha \cdot \frac{Y_t}{K_t}}_{\text{present value of production rents}} + \underbrace{\sum_{t=1}^{\infty} R_{0 \to t}^{-1} (1 - \delta)^{t-1} \theta \cdot \frac{q_t H_t}{K_t}}_{\text{present-value of investment rents}}$$

Present-value of firm. We now examine another present-value identity, which focuses on the cash flows associated with owning a financial claim on the firm, rather than owning a unit of capital (in our model, the two approaches are equivalent because of constant return to scale). The

[12] Kogan et al., 2020 calls this second term the present value of future growth opportunities.

valuation equation for the representative firm is

$$(1 + r_{t+1})V_t = D_{t+1} + V_{t+1}.$$

This equation says that the return of owning a firm is the sum of a flow component — the firm payout $D_{t+1} = Y_{t+1} - w_{Y,t+1}L_{Y,t+1} - I_{t+1} - w_{H,t+1}L_{H,t+1}$ — and a capital gain component — the value of the firm next period V_{t+1}. We can re-express this equation as:

$$(1 + r_{t+1})V_t = \Pi_{t+1} + \left(1 - \frac{I_{t+1} + w_{H,t+1}L_{H,t+1}}{V_{t+1}}\right) V_{t+1},$$

where $\Pi_t \equiv Y_{t+1} - w_{Y,t+1}L_{Y,t+1}$ denotes capital income (see Section 3). Relative to the previous equation, the flow component is now the total amount of capital income Π_t while the capital gain component is the "diluted" value of next-period firm value $\left(1 - \frac{I_{t+1}+w_{H,t+1}L_{H,t+1}}{V_{t+1}}\right) V_{t+1}$. The intuition is that, while a firm could pay out all of its capital income to capital owners, it would have to finance its investment through share issuance: the dilution rate $(I_t + w_{H,t}L_{H,t})/V_t$ then corresponds to the rate at which existing owners get diluted over time.[13]

Solving this equality forward and using the transversality condition gives that the value of the firm can be written as the present value of future dividends, or, alternatively, as the present value of diluted capital income:

$$V_0 = \sum_{t=1}^{\infty} R_{0\to t}^{-1} D_t = \sum_{t=1}^{\infty} R_{0\to t}^{-1} \left(\prod_{s=1}^{t-1} \left(1 - \frac{I_{t+1} + w_{H,t+1}L_{H,t+1}}{V_{H,t+1}}\right)\right) \Pi_t.$$

It says that existing capitalists (those that own capital at $t = 0$) do not own all of future capital income; rather, they own a share that decays over time because they must continuously re-invest to maintain their stocks of capital. This is an accounting identity implied by the definition of a firm return.

Using the firm first-order condition for (firm foc I_t), we can rewrite the dilution rate as:

$$\frac{I_{t+1} + w_{H,t+1}L_{H,t+1}}{V_{H,t+1}} = (1 - \theta) \cdot \frac{H_t}{K_{t+1}}.$$

In steady-state, $H_s = \delta K_{s+1}$ and so the dilution rate is exactly $(1 - \theta)\delta$. Note that the dilution rate decreases with θ: the fact that capital owners earn some rents associated with capital formation ($\theta > 0$) makes capital formation less dilutive for capital holders relative to the neoclassical model.

[13] Our concept of dilution rate is related to Gârleanu et al. (2016), who emphasize the gap between the growth of aggregate dividends distributed by public firms and the growth of dividends per share as a result of IPOs.

A.3 Equivalence to multi-sector model

We now show that the production technology (see Section 2.1) can be re-written as a multi-sector economy that produces that has three factors of production (K, L_Y, L_H) and three sectors that produce, respectively, a final good consumption C and a capital good H. We start with three equations from the model:

$$\begin{aligned} C &= Z_Y K^{\alpha} L_Y^{1-\alpha}, \\ I &= Z_Y K^{\alpha} L_Y^{1-\alpha}, \\ H &= Z_H K^{\theta} I^{\chi(1-\theta)} L_H^{(1-\chi)(1-\theta)}. \end{aligned}$$

We can instead express the system as

$$\begin{aligned} C &= Z_Y K^{\alpha} L_Y^{1-\alpha}, \\ H &= Z_H Z_Y^{\chi(1-\theta)} K^{\theta+\alpha\chi(1-\theta)} L_Y^{(1-\alpha)\chi(1-\theta)} L_H^{(1-\chi)(1-\theta)}. \end{aligned}$$

Notice that the capital good production function is constant return to scale in (K, L_Y, L_H). A decline in tangibility χ implies a greater importance of investment labor L_H at the expense of production labor and capital (K, L_Y).

A.4 Log-linearized model

We know log-linearize the stylized model around its steady-state to obtain the log deviation in capital and prices as a function of the sequence of productivity shocks.

We first solve for the perturbation in K_{t+1} in terms of backward-looking variables. Log-linearizing (capital formation) and (firm foc I_t) gives

$$\begin{aligned} \mathrm{d}\log K_{t+1} &= (1-\delta)\,\mathrm{d}\log K_t + \delta\,\mathrm{d}\log H_t \\ \mathrm{d}\log H_t &= \mathrm{d}\log z_{H,t} + \theta\,\mathrm{d}\log K_t + (1-\theta)\chi\,\mathrm{d}\log I_t \\ \mathrm{d}\log I_t &= \mathrm{d}\log q_t + \mathrm{d}\log H_t. \end{aligned}$$

Combining the last two equations gives

$$\mathrm{d}\log H_t = \frac{1}{1-(1-\theta)\chi}\left(\mathrm{d}\log z_{H,t} + \theta\,\mathrm{d}\log K_t + (1-\theta)\chi\,\mathrm{d}\log q_t\right). \tag{A.1}$$

Plugging into the first equation gives the following log-linearized backward-looking equation for

K_t

$$\begin{aligned}\mathrm{d}\log K_{t+1} &= (1-\delta)\,\mathrm{d}\log K_t + \frac{\delta}{1-(1-\theta)\chi}\left(\mathrm{d}\log z_{H,t} + \theta\,\mathrm{d}\log K_t + (1-\theta)\chi\,\mathrm{d}\log q_t\right)\\ &= \left(1-\delta\frac{(1-\theta)(1-\chi)}{1-(1-\theta)\chi}\right)\mathrm{d}\log K_t + \delta\frac{\theta}{1-(1-\theta)\chi}\,\mathrm{d}\log z_H + \delta\frac{(1-\theta)\chi}{1-(1-\theta)\chi}\,\mathrm{d}\log q_t.\end{aligned}$$

We now find the log-linearized forward equation for q_t. Start from (firm foc K_{t+1})

$$(1+r_t)K_{t+1}q_t = \alpha Y_{t+1} + ((1-\delta)K_{t+1} + \theta H_{t+1})q_{t+1},$$

which gives

$$\begin{aligned}\mathrm{d}\log(1+r_t) + \mathrm{d}\log q_t + \mathrm{d}\log K_{t+1} = \frac{r+(1-\theta)\delta}{1+r}\,\mathrm{d}\log Y_{t+1} + \frac{1-(1-\theta)\delta}{1+r}\,\mathrm{d}\log q_{t+1}\\ + \frac{1-\delta}{1+r}\,\mathrm{d}\log K_{t+1} + \frac{\theta\delta}{1+r}\,\mathrm{d}\log H_{t+1}.\end{aligned}$$

Substituting out $\mathrm{d}\log Y_{t+1}$ and $\mathrm{d}\log H_{t+1}$ using (output production) and (A.1), respectively, yields

$$\begin{aligned}\mathrm{d}\log(1+r_t) + \mathrm{d}\log q_t + \mathrm{d}\log K_{t+1} = \frac{r+(1-\theta)\delta}{1+r}\left(\mathrm{d}\log z_{Y,t+1} + \alpha\,\mathrm{d}\log K_{t+1}\right) + \frac{1-(1-\theta)\delta}{1+r}\,\mathrm{d}\log q_{t+1}\\ + \frac{1-\delta}{1+r}\,\mathrm{d}\log K_{t+1}\\ + \frac{\theta\delta}{1+r}\frac{1}{1-(1-\theta)\chi}\left(\mathrm{d}\log z_{H,t+1} + \theta\,\mathrm{d}\log K_{t+1} + (1-\theta)\chi\,\mathrm{d}\log q_{t+1}\right).\end{aligned}$$

Log-linearizing (worker foc C_t) with a potentially time varying impatience parameter β_t, we get

$$\mathrm{d}\log(1+r_{t+1}) = -\,\mathrm{d}\log\beta_{t+1} + \gamma\,\mathrm{d}\log C_{t+1}/C_t.$$

We consider the case $\gamma \to 0$ to simplify (or open economy). Re-arranging give the forward-looking equation for q_t:

$$\begin{aligned}\mathrm{d}\log q_t = \mathrm{d}\log\beta_{t+1} + \frac{r+(1-\theta)\delta}{1+r}\,\mathrm{d}\log z_{Y,t+1} + \frac{\theta\delta}{1+r}\frac{1}{1-(1-\theta)\chi}\,\mathrm{d}\log z_{H,t+1}\\ - \left(\frac{r+(1-\theta)\delta}{1+r}(1-\alpha) + \frac{\theta\delta}{1+r}\left(1-\frac{\theta}{1-(1-\theta)\chi}\right)\right)\mathrm{d}\log K_{t+1}\\ + \frac{1-\delta\frac{(1-\theta)(1-\chi)}{1-(1-\theta)\chi}}{1+r}\,\mathrm{d}\log q_{t+1}.\end{aligned}$$

Hence, the log-linearized equilibrium can be written as a system of ODEs:

$$\begin{pmatrix} \mathrm{d}\log K_{t+1} \\ \mathrm{d}\log q_t \end{pmatrix} = (I + A) \begin{pmatrix} \mathrm{d}\log K_t \\ \mathrm{d}\log q_{t-1} \end{pmatrix} + B \begin{pmatrix} \mathrm{d}\log \beta_t \\ \mathrm{d}\log z_{Y,t} \\ \mathrm{d}\log z_{H,t} \end{pmatrix},$$

where the matrix A and B are defined as follows

$$A \equiv \begin{pmatrix} -\delta\frac{(1-\theta)(1-\chi)}{1-(1-\theta)\chi} & \delta\frac{(1-\theta)\chi}{1-(1-\theta)\chi} \\ \frac{(r+(1-\theta)\delta)(1-\alpha)+\theta\delta\left(1-\frac{\theta}{1-(1-\theta)\chi}\right)}{1-\delta\frac{(1-\theta)(1-\chi)}{1-(1-\theta)\chi}} & \frac{1+r}{1-\delta\frac{(1-\theta)(1-\chi)}{1-(1-\theta)\chi}} - 1 \end{pmatrix}$$

$$B \equiv \begin{pmatrix} 0 & 0 & \delta\frac{\theta}{1-(1-\theta)\chi} \\ -\frac{1+r}{1-\delta\frac{(1-\theta)(1-\chi)}{1-(1-\theta)\chi}} & -\frac{r+(1-\theta)\delta}{1-\delta\frac{(1-\theta)(1-\chi)}{1-(1-\theta)\chi}} & -\frac{\frac{\delta\theta}{1-(1-\theta)\chi}}{1-\delta\frac{(1-\theta)(1-\chi)}{1-(1-\theta)\chi}} \end{pmatrix}.$$

The eigenvalues of the matrix A discipline the speed at which the system converge to its new steady-state after a permanent change in the time preference parameter β, or TFP shocks z_Y and z_H.

B Appendix for Section 3

B.1 Methodology

We now describe the list of adjustments we make to the cash-flow accounting statements of firms in Compustat in order to estimate the uses of capital income (the model equation (3.1)).

Adjustment #1: expensed payments to investment labor. One issue with accounting data is that the distinction between an expense (which are subtracted from sales to obtain cashflows from operation) and investment (which are not) can be arbitrary, especially in the case of intangibles.

The existing literature shows that this issue leads to an understatement of cashflows from operations and a corresponding understatement of (minus) cashflows from investment.[14] We follow the existing literature and correct cashflows from operation by adding 30% of SG&A expenses and 100% of R&D expenses. This is meant to account for the incorrect expensing of investment labor expenses, which instead should be treated as a capital expenditure.

$$\text{cf adjustment (investment labor)} \equiv 0.3 \cdot \text{SG\&A} + \text{R\&D}.$$

Adjustment #2: net entry in sample. When a private firm becomes public or goes public, it effectively represents a negative cashflow for passive capitalists, who re-balance their portfolio to

[14] See Peters and Taylor (2017) for evidence from firm-level data and Koh et al. (2020) from aggregate data.

always own the market. Similarly, when a firm exits or gets acquired, it represents a positive cash-flow. We now describe how we measure the contribution of net firm entry to market capitalization growth.

First, we introduce a decomposition of the total market value growth (at the aggregate or within an industry) between time t and $t+1$ as the sum of three components due to stayers, entrants, and exits. Formally, we have that

$$g_{\text{total}} = g_{\text{stayer}} + g_{\text{entry}} + g_{\text{exit}}, \tag{B.1}$$

where the components are defined in this footnote.[15]

Using these definitions, we define the net cashflow due to net firm entry as

$$\text{cf adjustment (net entry)} \equiv -(g_{\text{entry}} + g_{\text{exit}}) \cdot \text{market capitalization}. \tag{B.2}$$

It corresponds to the net cash that a passive capitalists who owns the market receives due to net firm entry during a period.

Adjustment #3: payments in stocks. How should we record a payment when it is the form of stocks? First, we can decompose the growth in market value of a stayer into growth of price per share and the growth in number of shares. To match the timing of the payment to the timing of the stock issuance, we use the fully-diluted share count (which includes outstanding shares, as well as all possible sources of potential shares such as stock options and reserved shares).

Formally, we can decompose the growth of the market capitalization of stayers (i.e., firms who do not exit the sample) defined in (B.1) into a price and number of shares component:

$$g_{\text{stayer}} = g_{\text{price}} + g_{\text{shares}},$$

where the components are defined in this footnote.[16]

Moreover, we observe the the contribution of the growth in the number of shares issued for cash $g_{\text{shares, cash}}$, and the remainder $g_{\text{shares}} - g_{\text{shares, cash}}$ is due to a combination of stock compensa-

[15] The formulas are:

$$g_{\text{total}} = \frac{\sum_{i\in\mathcal{F}_{t+1}} V_{i,t+1}}{\sum_{i\in\mathcal{F}_t} V_{i,t}} - 1; \qquad g_{\text{stayer}} = \frac{\sum_{i\in(\mathcal{F}_t\cap\mathcal{F}_{t+1})} V_{i,t+1}}{\sum_{i\in(\mathcal{F}_t\cap\mathcal{F}_{t+1})} V_{i,t}} - 1,$$
$$g_{\text{entry}} = \frac{\sum_{i\in(\mathcal{F}_{t+1}\setminus\mathcal{F}_t)} V_{i,t+1}}{\sum_{i\in\mathcal{F}_t} V_{i,t}}; \qquad g_{\text{exit}} = -\frac{(1+g_{\text{stayer}})\sum_{i\in(\mathcal{F}_t\setminus\mathcal{F}_{t+1})} V_{i,t+1}}{\sum_{i\in\mathcal{F}_t} V_{i,t}},$$

where $\mathcal{F}_t$ is the universe of firms at time t and V_i is the market capitalization of firm i.

[16] The definitions of g_{stayer}, g_{price}, and g_{shares} are

$$g_{\text{stayer}} = \frac{P_{i,t+1}N_{i,t+1}}{P_{i,t}N_{i,t}} - 1; \qquad g_{\text{price}} = \frac{P_{i,t+1}}{P_{i,t}} - 1; \qquad g_{\text{shares}} = \left(\frac{N_{i,t+1}}{N_{i,t}} - 1\right)(1+g_{\text{price}}),$$

where $P_{i,t}$ is the price per share and $N_{i,t}$ is the number of shares outstanding at firm i period t.

tion and stock-financed acquisitions (i.e., "noncash payments"). Using the same logic as for firm entry (see equation B.2), we define noncash payments as

$$\text{cf adjustment (noncash payments)} \equiv (g_{\text{shares}} - g_{\text{shares, cash}}) \cdot \text{market capitalization}.$$

The breakdown between stock compensation and acquisitions is only available after 2011. Therefore, we assume that acquisition is a constant share $\omega \in (0,1)$ of noncash payments (stock issuance not associated with cash). We compute ω at the industry level after 2011, and use it to split stock compensation before 2011.

Adjustment #4: imputation of the tangible share of ambiguous investments. How should one proceed to allocate the sum of cashflows due to mergers, acquisitions, and IPOs into tangible versus intangible investment? To do so, we use the "ambiguous income" approach introduced in Cooley et al. (1995). In our setup, the idea will be to assume that acquisitions have the same intangible content as other forms of investment. Denoting $\widetilde{\chi} \in (0,1)$ the tangibility of investment in the rest of investment, we have that

$$\widetilde{\chi} \equiv \frac{\text{cf from investing (capex)}}{\text{cf from investing (capex)} + \text{cf adjustments}\Big(\text{investment labor} + \psi \cdot (1-\omega) \cdot \text{noncash payments}\Big)},$$

where we discuss the role of ψ shortly. We compute $\widetilde{\chi}$ at the industry year-level. Equipped with this last estimate, we are ready to construct our final variables.

Formulas. Having defined the key variables, we now write down the final formulas:

$$\begin{aligned}
\text{tangible investments} = &- \text{cf from investing (capex)}, \\
&- \widetilde{\chi} \cdot \text{cf from investing (acquisition)}, \\
&- \widetilde{\chi} \cdot \text{cf adjustments}\Big(\text{net entry} + \omega \cdot \text{noncash payments}\Big), \\
\text{intangible investments} = &- \text{cf adjustments}\Big(\text{investment labor} + \psi \cdot (1-\omega) \cdot \text{noncash payments}\Big), \\
&- (1-\widetilde{\chi}) \cdot \text{cf from investing (acquisition)}, \\
&- (1-\widetilde{\chi}) \cdot \text{cf adjustments}\Big(\text{net entry} + \omega \cdot \text{noncash payments}\Big), \\
\text{payments to capitalists} = &- \text{cf from financing} \\
&+ \text{cf adjustments}\Big(\text{net entry} + \text{noncash payments}\Big).
\end{aligned}$$

Combining these formulas, we obtain an expression for capital income

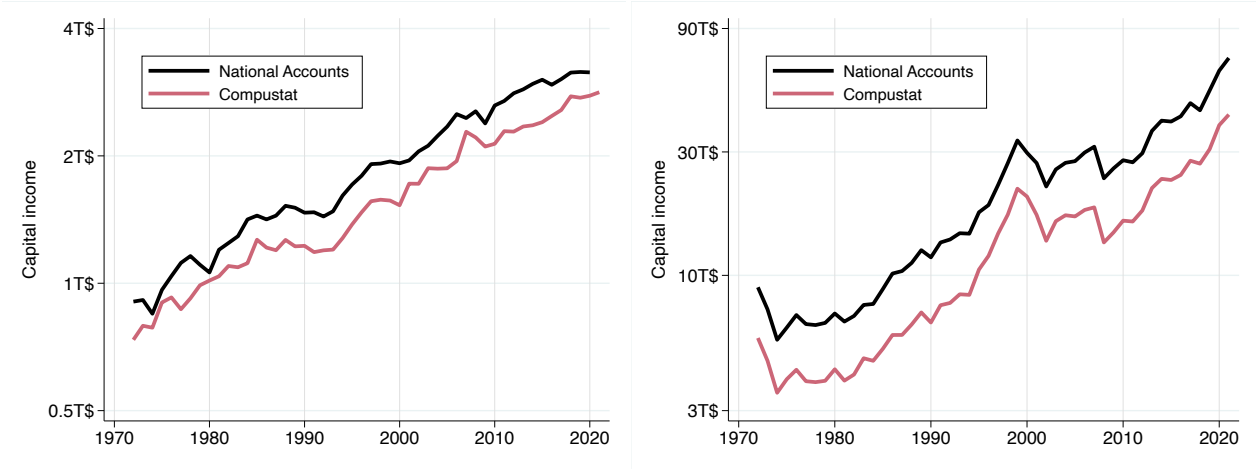
$$\begin{aligned}\text{capital income} =& \text{tangible investments} + \text{intangible investments} + \text{payments to capitalists} \\ =& \text{cf from operations} + \text{cf adjustments (investment labor)} \\ & - (1-\psi)\cdot(1-\omega)\cdot\text{noncash payments},\end{aligned}$$

where the second equality uses the cashflow identity (3.2). The number $\psi(1-\omega)$ corresponds to the share of noncash payments that is unreported in R&D and SG&A. Following changes in accounting standards in 2006, we set $\psi = 0.5$ before 2006 and $\psi = 0$ afterwards.

A few remarks are in order. First, that payments to capitalists are invariant to the calibration $(\widetilde{\chi}, \omega)$. This is because they only represent assumptions on how cashflows are distributed between tangible investments and intangible investments. Second, capital income is invariant not only to the calibration $(\widetilde{\chi}, \omega)$, but also the adjustments for noncash payments and net entry. This is because noncash payments and net entry are purely redistributive flows (from capitalists to tangible investments and to innovators). The only adjustment that affects the level of capital income is the adjustment for expensed investment labor. The idea is that we reclassify line items as capital expenditures, not expenses.

B.2 Comparison with the National Accounts

Appendix Figure A1 plots capital income and enterprise value, both in our sample and in the the Integrated Macroeconomic Accounts (Table S.5.a). Note that our sample accounts for roughly two-thirds of nonfinancial corporate sector gross capital income over the period, with a slight upward trend.

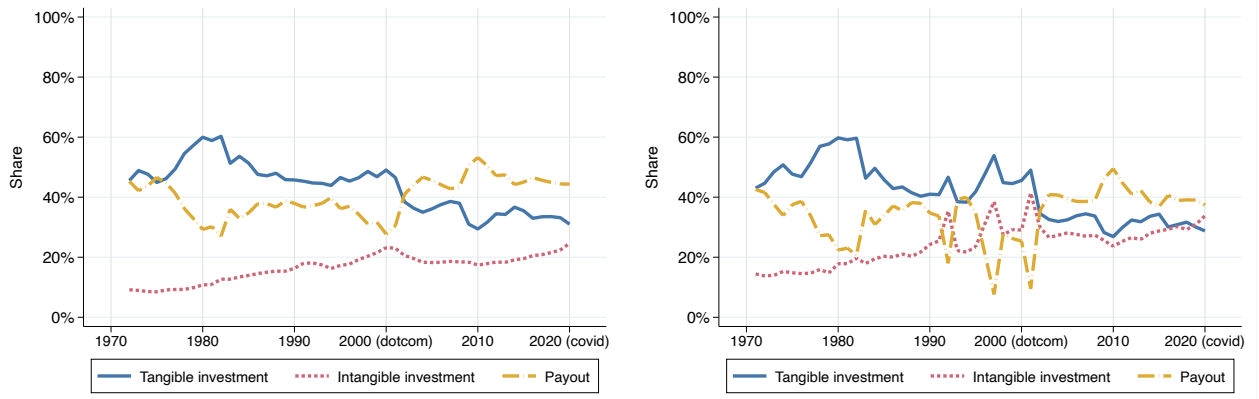


(a) Capital income (b) Enterprise value

Figure A1: Sample validation against National accounts data (1970–2022).

Figure A2 reports the use of capital income for entire U.S. economy using data from the na-

tional accounts (BEA). While Panel A2a reports the data aggregated across all industries, Panel A2b reweights the BEA data within industry cells to mimic the distribution of capital in Compustat, showing that part of the difference with the distribution of capital income in Compustat (as reported in Figure 2a) comes from the difference in industry composition across the two samples. Still, the figure due to the difference between public firms and private firms, as well as differences in the definition of intangible investment in our paper relative to the BEA classification.



(a) Economy-wide

(b) Reweighting industries to match public firms

Figure A2: Distribution of capital income (BEA)

C Appendix for Section 4

C.1 Calibration details

We first extend the concept of "long-run" to a balanced growth path. Denote $\pi_X \equiv \log(X_{t+1}/X_t)$ the growth rate associated to a variable X. Differentiating the production functions for output and investment gives:

$$\pi_Y = \pi_{Z_Y} + \alpha\pi_K + (1-\alpha)\pi_{L_Y},$$
$$\pi_H = \pi_{Z_H} + \theta\pi_K + (1-\theta)\chi\pi_I + (1-\theta)(1-\chi)\pi_{L_H}.$$

To obtain a balanced growth path with constant labor inputs, we assume that the productivity of production labor and investment labor grow at the same rate π. Expressed in terms of Hicks-neutral productivity, this is equivalent to assuming $\pi_{Z_Y} = (1-\alpha)\pi$, $\pi_{Z_H} = (1-\theta)(1-\chi)\pi$. In this case, all variables (K, Y, C, I, w_Y, w_H) grow at the same rate π. Using the optimality conditions,

we obtain

$$\begin{aligned} 1+r &= \beta^{-1}(1+\pi)^{\gamma} \\ \frac{H}{K} &= \delta+\pi \\ q &= \frac{1}{r-\pi+(1-\theta)(\delta+\pi)}\frac{\alpha Y}{K}. \end{aligned}$$

Taking the log of the first equation implies that we can calibrate β as the difference between log returns minus $\gamma\pi$.

The total investment yield is:

$$\frac{I+w_H L_H}{V} = (1-\theta)\frac{qH}{V} = (1-\theta)\frac{H}{K} = (1-\theta)(\delta+\pi).$$

As explained in Appendix A.2, the total investment yield corresponds to capitalists' dilution rate; that is, the extent to which an investor drawing down total capital income every period from the representative firm would get diluted over time. Consistently with this idea, note that the equation for q given above can be interpreted as the value of a income flow initially equal to αY (capital income) and growing at rate $\pi-(1-\theta)(\delta+\pi)$, which corresponds to the growth rate of the economy minus the dilution rate of capitalists.

Manipulating the first-order conditions of the firm implies that, in the long run, the uses of capital income in the economy are given by:

$$\begin{aligned} \frac{I}{\Pi} &= \chi\frac{(1-\theta)(\delta+\pi)}{r-\pi+(1-\theta)(\delta+\pi)} \\ \frac{w_H L_H}{\Pi} &= (1-\chi)\frac{(1-\theta)(\delta+\pi)}{r-\pi+(1-\theta)(\delta+\pi)} \\ \frac{D}{\Pi} &= \frac{r-\pi}{r-\pi+(1-\theta)(\delta+\pi)}. \end{aligned}$$

This equation expresses the share of tangible investment, the share of intangible investment, and the payout share in steady state in terms of five parameters.

The capital share of income is

$$\frac{\Pi}{Y+w_H L_H} = \frac{\alpha Y}{Y+(1-\chi)(1-\theta)qH} = \frac{\alpha}{1+(1-\chi)(1-\theta)\frac{qH}{Y}} = \frac{\alpha}{1+(1-\chi)\alpha\frac{(1-\theta)(\delta+\pi)}{r-\pi+(1-\theta)(\delta+\pi)}},$$

The last moment is the short-run, partial equilibrium tax elasticity. Consider a capital income tax shock $\mathrm{d}\tau_{K,t} = (1-\phi)^t\,\mathrm{d}\tau_K$ at $t=0$ around the undistorted steady-state. Using (firm foc I_t) and (firm foc $L_{H,t}$):

$$I_t + w_{H,t}L_{H,t} = (1-\theta)q_t H_t.$$

The log total derivative at $t = 0$ is

$$\mathrm{d}\log(I_0 + w_{H,0}L_{H,0}) = \mathrm{d}\log q_0 + \mathrm{d}\log H_0 = \left(1 + \frac{1-\theta}{\theta}\right)\mathrm{d}\log q_0,$$

where the second equality is obtained using the fact that, in particular equilibrium, wages are taken as given. In that case, we obtain

$$\mathrm{d}\log(I_0 + w_{H,0}L_{H,0}) = \frac{1}{\theta}\,\mathrm{d}\log q_0,$$

Using the envelope theorem on the firm value function, and using the fact that $\mathrm{d}w_{Y,t} = \mathrm{d}w_{H,t} = 0$, we obtain

$$\mathrm{d}\log q_0 = -\frac{r}{r+\phi}\frac{r+(1-\theta)\delta}{r}\,\mathrm{d}\tau_K$$

The first term accounts for the duration of the tax change, where ϕ represents an annual rate of decay. The second term accounts for the ratio of the tax base (capital income) to the capital payout. Putting together, we obtain

$$\mathrm{d}\log(I_0 + w_{H,0}L_{H,0}) = -\frac{r+(1-\theta)\delta}{r+\phi}\frac{1}{\theta}\,\mathrm{d}\tau_K.$$

C.2 Proofs

Proof of Proposition 4.1. The Lagrangian associated with the household problem is

$$\mathcal{L}_0 = \sum_{t=1}^{\infty}\beta^t U(C_t, L_t) - \sum_{t=1}^{\infty}\lambda_t\left(C_t + V_t - w_{Y,t}L_{Y,t} - w_{H,t}L_{H,t} - (1+r_t)V_{t-1}\right)$$
$$\text{where } U(C,L) \equiv \frac{1}{1-\gamma}\left(C_t - \frac{L_t^{1+\sigma}}{1+\sigma}\right)^{1-\gamma}.$$

Applying the envelope theorem gives:

$$\begin{aligned}
\mathrm{d}U_0 &= \frac{\partial\mathcal{L}_0}{\partial V_0}\,\mathrm{d}V_0 + \sum_{t=1}^{\infty}\left(\frac{\partial\mathcal{L}_0}{\partial w_{Y,t}}\,\mathrm{d}w_{Y,t} + \frac{\partial\mathcal{L}_0}{\partial w_{Y,t}}\,\mathrm{d}w_{H,t} + \frac{\partial\mathcal{L}_0}{\partial r_t}\,\mathrm{d}r_t\right)\\
&= \lambda_1(1+r_1)\,\mathrm{d}V_0 + \sum_{t=1}^{\infty}\lambda_t\left(L_{Y,t}\,\mathrm{d}w_{Y,t-1} + L_{H,t}\,\mathrm{d}w_{H,t-1} + V_{t-1}\,\mathrm{d}r_t\right)\\
&= \beta\partial_C U(C_1,L_1)(1+r_1)\,\mathrm{d}V_0 + \sum_{t=1}^{\infty}\beta^t\partial_C U(C_t,L_t)\left(L_{Y,t}\,\mathrm{d}w_{Y,t-1} + L_{H,t}\,\mathrm{d}w_{H,t-1} + V_{t-1}\,\mathrm{d}r_t\right),\\
&= \beta\partial_C U(C_1,L_1)\left((1+r_1)\,\mathrm{d}V_0 + \sum_{t=1}^{\infty}R_{0\to t}^{-1}\left(L_{Y,t}\,\mathrm{d}w_{Y,t-1} + L_{H,t}\,\mathrm{d}w_{H,t-1} + V_{t-1}\,\mathrm{d}r_t\right)\right),
\end{aligned}$$

where the second and third equalities use the household's focs with respect to C_t and V_t, respectively. In the case where $\gamma = 0$ (small open economy), we have that $1 + r_t = \beta^{-1}$ which means that $\mathrm{d}r_t = 0$. □