Helicopter Drops and Liquidity Traps*

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Abstract

We propose a theory of helicopter money during a liquidity trap. We develop a simple analytically tractable New Keynesian monetary model in which the monetary authority must remit its profits to the fiscal authority and cannot receive capital injections. Contrary to standard irrelevance results, we show that helicopter drops can stabilize the economy at the zero lower bound when the monetary authority lacks commitment. However, if the central bank can commit, helicopter drops remain at best irrelevant, even when facing balance sheet constraints.

Keywords: Zero lower bound, helicopter drops, liquidity traps, central bank independence, monetary policy, time inconsistency

JEL: E31, E52, E58, E61, E63

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1 Introduction

During the 2008 global recession and the COVID-19 pandemic, nominal interest rates in the developed world dropped to zero, leading central banks to resort to alternative policies such as quantitative easing and forward guidance to stabilize the economy. Yet, the severity of downturns at the zero lower bound has led to the consideration of more unorthodox proposals. One that has received substantial attention is "helicopter drops," a metaphor coined by Friedman (1969) to describe a situation where the central bank prints money and transfers it directly to the public.[1]

However, standard economic theory suggests that helicopter drops are irrelevant in a liquidity trap (see, e.g., Krugman, 1998, 2015). At zero nominal interest rates, households become satiated with money balances, rendering money and bonds perfect substitutes. According to Ricardian equivalence, a helicopter drop of money (or bonds) has no effects on nominal or real variables as households increase their savings, leaving aggregate demand unchanged. The standard irrelevance of helicopter drops mirrors the classic irrelevance of open market operations at the zero lower bound (Wallace, 1981; Eggertsson and Woodford, 2003). Thus, despite its unorthodox nature, a helicopter drop is as ineffective as conventional open market operations in a liquidity trap.

We propose a theory of how helicopter drops can be effective during a liquidity trap, contrary to the standard irrelevance results. When the central bank faces balance sheet constraints, a helicopter drop hinders future tightening of monetary policy. This is because the increase in unbacked monetary liabilities reduces the central bank's net worth, limiting its ability to reduce the monetary liabilities in the future. In the absence of commitment, we argue that helicopter drops offer a credible promise of higher future inflation and output, helping stabilize the economy during a liquidity trap. However, if the central bank can credibly commit to future policies, helicopter drops become at best irrelevant, even when balance sheet constraints are present.

We develop a simple New Keynesian monetary model in which the central bank is subject to balance sheet constraints. As in practice, the central bank must remit its profits to the fiscal authority and cannot receive capital injections. Additionally, the central bank is limited in its ability to borrow using interest-bearing securities. Besides these features, the model is an otherwise textbook New Keynesian model (Woodford, 2001; Galí, 2015). In this environment, we fully characterize the Markov perfect equilibrium, compare with the optimal policy under full commitment, and assess the effectiveness of helicopter drops of money as a policy tool.

Our characterization starts by considering the set of private sector equilibria for a given set of policies. We show that in this environment, any private sector equilibrium that arises from a policy with a helicopter drop remains an equilibrium with an alternative policy without the

[1] See, for example, Bernanke (2002, 2016); Bartsch, Boivin, Fischer, Hildebrand and Wang (2019); Gali (2020).

helicopter drop, even in the presence of balance sheet constraints. Hence, when the central bank can commit to future policies, there is no scope for helicopter drops. Under commitment, the central bank can already credibly promise expansionary policies in the future to help mitigate the recession originating from the liquidity trap (Krugman, 1998; Eggertsson and Woodford, 2003; Jung, Teranishi and Watanabe, 2005; Werning, 2011). Resorting to helicopter drops would only inefficiently limit the central bank's ability to stabilize output and inflation in the future. Thus, helicopter drops should not be used under commitment.

We then examine the policy equilibria when the central bank cannot commit, which is our main focus. We study a Markov perfect equilibrium in which the central bank at each point in time chooses the best available policy, taking as given its inherited states (including its balance sheet) and how the private sector's expectations react to its policy choices.

We characterize analytically this Markov equilibrium and show that there exists a unique piece-wise linear equilibrium. We show that a lower net worth pushes the central bank towards a more expansionary policy (i.e., higher output and inflation). The reason is as follows. To reduce monetary liabilities, the monetary authority has to either sell assets or reduce the transfers it makes to the fiscal authority. Because the central bank is constrained to transfer at a minimum its excess earnings and has a limited ability to borrow, it faces a limit to how much it can reduce its monetary liabilities. A lower net worth implies that the central bank needs to keep higher nominal balances to satisfy the balance sheet constraints, and this results in lower interest rates and higher levels of output and inflation.

We argue that helicopter drops during a liquidity trap are a credible way for the central bank to commit to an expansionary monetary policy in the future. We highlight that open market operations remain irrelevant in a liquidity trap in our environment. However, when the central bank expands the money supply without a corresponding increase in assets, this generates expectations of lower interest rates going forward, which imply high output and inflation in the future that help mitigate the recession today. In a simulation, the optimal helicopter drop achieves a degree of stabilization that comes close to the optimal policy under commitment. The downside, however, is that helicopter drops induce a more protracted period of inflation.

Related literature. A seminal paper on monetary and fiscal policy interactions is Sargent and Wallace (1981). They show that fiscal policy imposes constraints on the central bank's ability to control inflation. In particular, fiscal deficits force the central bank to eventually collect higher seigniorage revenues to balance the consolidated government budget constraint.

A more recent literature has unbundled the budget constraints of the government and the

central bank and studied the implications for monetary policy.[2] Sims (2004) is an early paper exploring the implications of different institutional configurations for price level determinacy. Hall and Reis (2015) take as given that the central bank is committed to an inflation target and evaluate when the central bank faces the risk of becoming insolvent, in the sense of facing an explosive path for liabilities. Del Negro and Sims (2015) study how the central bank balance sheet can determine whether price determinacy can be achieved and the extent to which fiscal support is needed. Bassetto and Messer (2013) examine the fiscal consequences of paying interest on reserves, arguing that although it increases the flexibility of the balance sheet, it can increase the risk of insolvency. Benigno and Nisticò (2020) explore the neutrality of changes in the central bank's balance sheet, given transfer policies for the central bank and treasury, and highlight how lack of fiscal support may constrain the central bank's choices when it can suffer losses. We instead focus on the case where the Central Bank lacks commitment, and ask whether the Central Bank will voluntarily weaken its net worth through a helicopter drop.

There is a large literature on central bank policies during a liquidity trap. A key insight from this literature (Krugman, 1998; Eggertsson and Woodford, 2003; Jung et al., 2005; Werning, 2011) is that the Central Bank should commit to delivering high inflation and output in the future (after the liquidity trap is over) to help mitigate the contraction in output when the zero lower bound constraint binds. Without commitment, however, this policy is not feasible, as the government would find it optimal to stabilize output and inflation when the liquidity trap is over.

Our paper is more directly related to those exploring the role of helicopter drops during liquidity traps. Auerbach and Obstfeld (2005) and Galí (2019) show that money-financed stimulus can be expansionary at the zero lower bound.[3] However, it is assumed that money supply remains high beyond the period of the liquidity trap, a policy that is not necessarily time-consistent. Benigno and Nisticò (2022) consider a model where the fiscal and the monetary authorities are separated and where the fiscal theory of the price level is operative. They study how, under commitment, helicopter drops can reflate the economy. Also in a setup with commitment, Michau (2022) analyzes the effects of helicopter drops in a model with secular stagnation and where households have a utility preference for wealth.[4] In contrast to these contributions, our paper focuses on the time-consistency problem for the central bank in a liquidity trap and shows that helicopter drops of money, by constraining the central bank's future behavior, can be a credible

[2] See Bassetto and Sargent (2020) for a review of the research on the interactions between monetary and fiscal policy.

[3] For a survey of the literature on helicopter drops, see Reis and Tenreyro (2022).

[4] In the context of models with heterogeneous agents, studies such as Buera and Nicolini (2020) show that a transfer to constrained agents financed by printing money can have stimulative effects at the zero lower bound. However, the same effects can be achieved with issuing debt, a manifestation of the classic irrelevance of open market operations at the zero lower bound. Bilbiie and Ragot (2021) find small quantitative differences between helicopter drops and open market operations, outside the zero lower bound.

way to raise expectations of higher inflation and output in the future.

Finally, our paper is related to the literature exploring the connection between the maturity structure of the government debt and the time consistency of optimal fiscal and monetary policy. Lucas and Stokey (1983) argue that a carefully chosen maturity structure for government debt can make optimal fiscal and monetary policies time consistent when taxation is costly (see also Debortoli, Nunes and Yared, 2021). Alvarez, Kehoe and Neumeyer (2004) study an economy with nominal debt and show that a portfolio can be chosen to deliver time-consistent policies if the Friedman rule is optimal. Calvo and Guidotti (1992) show that a lower level of nominal government debt can help reduce the temptation to inflate ex post. Closer to our paper is the work of Bhattarai, Eggertsson and Gafarov (2022), who study quantitative easing in a liquidity trap where the problem is that the lack of commitment generates deflationary bias. They show that by shortening the maturity of the consolidated government debt, quantitative easing provides incentives for the central bank to slow down the exit from the liquidity trap because it is averse to losses in its balance sheet. In our model, the central bank does not suffer losses on its balance sheet but intentionally engineers a reduction in net worth through a helicopter drop.[5]

The paper is organized as follows. Section 2 presents the model. Sections 3 and 4 present the analysis of optimal monetary policy with and without commitment. Section 5 concludes.

2 Model

We consider a deterministic closed economy New Keynesian model with a representative household, a monetary authority, and a fiscal authority.

2.1 Main Elements

Monetary Authority. We start by considering the balance sheet of the monetary authority. The monetary authority issues monetary liabilities M_t, accumulates a nominal risk-free asset A_t, which pays a nominal interest rate ι_t, and makes nominal transfers τ_t to the fiscal authority.[6] The budget constraint of the monetary authority is given by

$$\frac{A_t}{1+\iota_t} + M_{t-1} + \tau_t = M_t + A_{t-1}. \tag{1}$$

[5]Also related is Jeanne and Svensson (2007), which shows how purchases of foreign currency assets can help avoid a liquidity trap when the central bank faces a lower bound on the level of capital. Amador, Bianchi, Bocola and Perri (2016) show how a large currency mismatch in a central bank can provoke an early abandonment of a floor on the exchange rate, because of the risk of facing losses.

[6]Alternatively, we could allow the monetary authority to make transfers directly to households. Because the model features Ricardian properties in terms of debt by the fiscal authority, this alternative is equivalent to the current one.

Define the beginning-of-period nominal net worth of the monetary authority as $N_t \equiv A_t - M_t$. Using this definition, we have that,

$$N_t = N_{t-1} + \frac{\iota_t}{1+\iota_t} A_t - \tau_t.$$

Let $i_t = \log(1+\iota_t)$ be the corresponding instantaneous rate of interest. We define

$$\tau^\star(A, i) \equiv A(1 - e^{-i}).$$

Then, $N_t < N_{t-1}$ if and only if $\tau_t > \tau^\star(A_t, i_t)$. The value of $\tau^\star$ denotes the nominal net gains from holding financial assets ($\iota \times A$) relative to the nominal cost of liabilities ($0 \times M$). When the monetary authority remits more than $\tau^\star$ to the fiscal authority, the net worth of the monetary authority falls in nominal terms. Conversely, if remittances are lower than $\tau^\star$, the net worth increases in nominal terms.

We assume that the monetary authority is required to remit at least $\tau^\star$ every period to the fiscal authority:

$$\tau_t \geq \tau^\star(A_t, i_t), \text{ for all } t, \tag{2}$$

which implies that net worth cannot increase. In effect, this also means that the monetary authority cannot count on equity injections by the fiscal authority. We also assume that the monetary authority cannot issue bonds.[7]

$$A_t \geq 0, \text{ for all } t. \tag{3}$$

These two constraints, (2) and (3), imply that the central bank faces a lower bound on monetary liabilities given by

$$M_t \geq -N_{t-1}. \tag{4}$$

According to (4), lower net worth forces the monetary authority to keep a higher level of monetary liabilities. The reason is as follows. If the monetary authority would like to reduce M_t, it has to either sell assets or reduce the transfers it makes to the fiscal authority. Because the monetary authority is constrained to make the minimum transfer and maintain a minimum amount of assets,

[7] We are also assuming that currency is the only monetary liability. In practice, central banks that pay interest on reserves can effectively issue interest-bearing liabilities. While this grants them more flexibility to set nominal rates and monetary aggregates, paying interest on reserves entails fiscal costs (Hall and Reis, 2015) and can also give rise to a contraction in financial intermediation (see e.g., Bianchi and Bigio, 2022).

it faces a limit on how much it can reduce its monetary liabilities.[8] In particular, by selling all its assets and paying the minimum transfer $\tau^\star$, the monetary authority can reduce its monetary liabilities to a maximum level of $-N_{t-1}$.

We refer to constraints (2) and (3) as the "balance sheet constraints", and they constitute our addition to the New Keynesian model. Absent either of these two constraints on the monetary authority, the model would feature a consolidated budget constraint for the government and the net worth of the central bank would be irrelevant. Following early work by Stella (1997, 2005), there is extensive evidence that balance sheet constraints on central banks can actually have implications for monetary policy. As they note, countries generally have different accounting rules that govern the required level of capital for the central bank, as well as the rules for dividends and remittances. These factors, in turn, determine the central bank's ability to operate with a low level of capital or manage operating losses. For example, the Federal Reserve Act requires the Reserve Banks to remit every year excess earnings after expenses to the U.S Treasury.[9] Other central banks, such as Bank of England and Netherlands are able to retain a fraction of the net earnings (see Chaboud and Leahy, 2013 for a summary of central bank practices). Our modeling assumptions attempt to capture in a simple way the institutional features that characterize the interaction between fiscal and monetary authorities. In the terminology of Benigno and Nisticò (2020), constraint (2) implies that the monetary authority is *financially independent*, and balance sheet policies are not necessarily neutral. We also note that some recent literature has focused on the effects of central bank losses (e.g., Hall and Reis, 2015). In our analysis, realized losses or concerns about losses do not play a role, as we abstract from long-term or other risky assets.

Fiscal Authority. There is a fiscal authority that collects the revenue received from the monetary authority. In addition, it issues bonds and makes lump-sum transfers/taxes to households. Let B_t denote the level of debt of the fiscal authority. The fiscal authority budget constraint is

$$\frac{B_t}{1+\iota_t} + \tau_t = T_t + B_{t-1}, \tag{5}$$

where T_t represents the lump-sum transfers to households.

[8] Notice that we could have allowed for a borrowing limit different from zero in eq. (3) (i.e., we could have written it as $A_t/P_t \geq \bar{a}$ for given $\bar{a}$). In that case, the constraint (4) would be $M_t \geq -N_{t-1} + \bar{a}P_t$ and the results would be similar, although the reduction in net worth necessary for the constraint to bind would be larger.

[9] When these excess earnings turn negative, however, the Federal Reserve does not receive a transfer from the Treasury. Instead, it accumulates a so-called "deferred asset." This means that future excess earnings are credited against the deferred asset and the Federal Reserve resumes remitting to the Treasury once the deferred asset is extinguished. As highlighted by Greenlaw, Hamilton, Hooper and Mishkin (2013), the accumulation of a deferred asset for a significant amount of time can bring monetary policy decisions under additional public scrutiny, and potentially threaten the independence of the Federal Reserve. For some discussion in the financial press, see Explainer: Why huge European Central Bank losses matter in Reuters.

Households. There is a representative household that has the following preferences over consumption and real money balances:

$$\sum_{t=0}^{\infty} \beta(t) \left[u(C_t) + v\left(\frac{M_t}{P_t}\right) \right]$$

with $\beta(t) = \Pi_{s=0}^{t} e^{-(\rho+\xi_{s-1})} < 1$ the discount factor from period 0 to t. The value of ξ_t is a time-varying exogenous disturbance to the discount rate (and, as in Werning, 2011, this is the only non-stationary parameter and the reason for a liquidity trap). The function u is assumed to be strictly increasing, strictly concave, and differentiable in $\mathbb{R}^+$. The function v is differentiable in $\mathbb{R}^+$; strictly increasing, and strictly concave in $[0, \overline{m}]$; reaching a satiation point at $\overline{m}$: $v(m) = v(\overline{m})$ for $m > \overline{m}$.

The household faces the following sequential budget constraint:

$$P_t C_t + M_t + \frac{W_t}{1+\iota_t} \leq M_{t-1} + W_{t-1} + T_t + P_t Y_t, \tag{6}$$

where P_t is the price level, C_t is the households' real consumption, Y_t is the households' real income (composed of real labor income and profits), and W_t is the financial wealth net of money holdings.

The household is subject to the following No-Ponzi condition:[10]

$$\lim_{t\to\infty} \frac{W_t + M_t}{\prod_{s=0}^{t}(1+\iota_s)} \geq 0. \tag{7}$$

The household's problem is to maximize utility subject to the budget constraint, (6), and the No-Ponzi condition, (7), taking prices and transfers as given. Letting $\pi_t \equiv \log(P_t/P_{t-1})$, we have the following sufficiency result.[11]

Lemma 1 (Sufficiency). *Take a sequence of prices and transfers $\{P_t, \iota_t, T_t\}$ and initial conditions P_{-1}, W_{-1}, ι_{-1}. Let $\{C_t, M_t, W_t\}$ be such that (i) $C_t > 0$ for all $t \geq 0$, (ii) the household budget constraint (6) holds, (iii) the No-Ponzi condition (7) holds, (iv) the following first-order conditions*

[10] Buiter and Sibert (2007) propose an alternative No-Ponzi condition that does not include money holdings. Part of their argument is a potential difficulty with obtaining sufficiency results for the household problem. We show in Lemma 1 that this is not an issue.

[11] We will impose the sufficient conditions of this Lemma as requirements for household optimality. For the necessity of the transversality condition, we refer the reader to Kamihigashi (2002).

(an Euler equation, and a money demand condition) hold

$$u'(C_t) = e^{i_t - \rho - \xi_t - \pi_{t+1}} u'(C_{t+1}),$$
$$v'\left(\frac{M_t}{P_t}\right) = u'(C_t)\frac{\iota_t}{1+\iota_t},$$

and (v) the following transversality condition holds

$$\lim_{t\to\infty} \frac{W_t + M_t}{\prod_{s=0}^{t}(1+\iota_s)} \leq 0.$$

Then the sequence $\{C_t, M_t, W_t\}$ solves the household's problem.

Proof. In Appendix A.1. □

Note that we can collapse the No-Ponzi condition and the transversality condition into just one condition at an optimal solution:

$$\lim_{t\to\infty} \frac{W_t + M_t}{\prod_{s=0}^{t}(1+\iota_s)} = 0, \tag{8}$$

Firms. The firms' side of the model follows the New Keynesian tradition. The model features monopolistic competition and Calvo-style sticky prices. Because these features are standard, we work directly with the log-linearized Phillips curve, which describes the optimal price-setting behavior of firms:

$$\pi_t = \beta\pi_{t+1} + \kappa y_t, \tag{9}$$

where $\beta \equiv e^{-\rho}$, $y_t \equiv \log(Y_t/\bar{Y})$ and $\bar{Y} > 0, \kappa \geq 0$.

Market Clearing. In an equilibrium, the goods market clears $Y_t = C_t$. Assuming that

$$u(C) = \frac{C^{1-1/\sigma}}{1-1/\sigma},$$

the household's Euler equation can be written as:

$$y_t = y_{t+1} - \sigma(i_t - \pi_{t+1} - \rho - \xi_t). \tag{10}$$

The first-order condition for money holdings requires that $\iota_t \geq 0$. Let h denote the inverse of

the first derivative of v, we let

$$L(c, i) \equiv h\left(u'(e^c)(1 - e^{-i})\right),$$

which is defined for $i \geq 0$, and is increasing in c, decreasing in i, and where $L(c, 0) = \overline{m}$.

Thus, the money market clearing condition, together with $C_t = Y_t$, is equivalent to

$$\frac{M_t}{P_t} \geq L(y_t, i_t); \text{ with equality if } i_t > 0. \tag{11}$$

2.2 Private Sector Equilibrium

Let us define a private sector equilibrium. The economy starts at $t = 0$ with the monetary authority's balance sheet given by A_{-1} and M_{-1}, an initial fiscal authority debt level B_{-1}, and an initial price level P_{-1}.[12]

Definition 1. A *private sector equilibrium* is a sequence $\{y_t, \pi_t, i_t, P_t, M_t, A_t, B_t, \tau_t, T_t\}$ such that for all $t \geq 0$ and

(i) households optimize and markets clear; that is, (6), (8), (10), and (11) hold for $W_t = B_t - A_t$ for all $t \geq -1$.

(ii) firms optimize; that is, (9) holds.

(iii) the zero lower bound constraint holds, $i_t \geq 0$.

(iv) the budget constraint of the monetary authority, (1), holds.

(v) the budget constraint of the fiscal authority, (5), holds.

Equations (9) and (10), together with the zero lower bound, $i_t \geq 0$, constitute the basic equations of the New Keynesian model. Suppose that we take a sequence $\{y_t, \pi_t, i_t\}$ that satisfies (9) and (10), and the zero lower bound. Based on this sequence, we can construct a private sector equilibrium as follows. We can use the initial price level to solve for the corresponding $\{P_t\}$ and use the condition (11) with equality to recover a sequence for $\{M_t\}$. With this, we can then obtain $\{A_t, B_t\}$ by setting any combination such that $(A_t - B_t) = M_t$ for all t large enough, which guarantees that the transversality condition, (8), holds. Finally, the implied transfers and taxes from (1) and (3) make sure that all the budget constraints hold. This justifies the common approach to focus on just the interest rate policy (satisfying the zero bound) and two equations, the Euler equation (10)

[12] In what follows, we use the notation $\{a_t, b_t, ...\}$ to refer to the sequences $\{a_t\}_{t=0}^{\infty}$, $\{b_t\}_{t=0}^{\infty}$, etc.

and the Phillips curve (9), when studying optimal policy.

Introducing Balance Sheet Constraints and Helicopter Drops. In our definition of a private sector equilibrium we did not yet introduce the restriction that the balance sheet constraints of the monetary policy must hold. We proceed to do this now:

Definition 2. A private sector equilibrium $\{y_t, \pi_t, i_t, P_t, M_t, A_t, B_t, \tau_t, T_t\}$ *is consistent with balance sheet constraints* if equations (2) and (3) hold for all t.

To see the effects of the balance sheet constraints, let us first rewrite the budget constraint of the monetary authority, (1), as follows:

$$M_t - M_{t-1} = (A_t - A_{t-1}) + (\tau_t - \tau^\star(A_t, i_t)).$$

This equation shows that an increase in the monetary base arises as a result of an open market operation where the monetary authority purchases bonds (that is, $A_t - A_{t-1} > 0$) or a remittance to the fiscal authority τ in excess of $\tau^\star$. We refer to the latter as a *helicopter drop*:

Definition 3 (Helicopter Drops)**.** A private sector equilibrium features *a helicopter drop* at time t if $\tau_t > \tau^\star(A_t, i_t)$.

Recall that $N_t = A_t - M_t$; thus, helicopter drops necessarily reduce the nominal net worth of the monetary authority as they increase M_t without a corresponding increase in A_t. A first straightforward result is that the presence of helicopter drops do not enlarge the set of allocations that constitute a private sector equilibrium consistent with balance sheet constraints.

Lemma 2 (Irrelevance of Helicopter Drops)**.** *Consider a private sector equilibrium* $\{y_t, \pi_t, i_t, P_t, M_t, A_t, B_t, \tau_t, T_t\}$ *that is consistent with balance sheet constraints. Let* $\{\hat{\tau}_t, \hat{A}_t, \hat{B}_t\}$ *be such that* $\hat{\tau}_t = \tau^\star(\hat{A}_t, i_t)$, $\hat{A}_t = M_t - M_{-1} + A_{-1}$, *and* $\hat{B}_t = \hat{A}_t - A_t$. *Then* $\{y_t, \pi_t, i_t, P_t, M_t, \hat{A}_t, \hat{B}_t, \hat{\tau}_t, T_t\}$ *is also a private sector equilibrium consistent with balance sheet constraints.*

Proof. Given $\hat{\tau}_t$ and $\hat{A}_t$, the budget constraint of the monetary authority is

$$\hat{A}_t = M_t - M_{t-1} + \hat{A}_{t-1} = M_t - M_{-1} + A_{-1},$$

where the last follows from repeated substitution of the value of $\hat{A}$.

For the balance sheet constraints, letting $\hat{A}_{-1} = A_{-1}$, we have

$$\hat{A}_t - \hat{A}_{t-1} = M_t - M_{t-1} \geq M_t - M_{t-1} - (\tau_t - \tau^\star(A_t, i_t)) = A_t - A_{t-1},$$

where we use that the original sequence satisfies $\tau_t \geq \tau^\star(A_t, i_t)$. Thus $\hat{A}_t - \hat{A}_{t-1} \geq A_t - A_{t-1}$. Given that $\hat{A}_{-1} = A_{-1}$, it follows that $\hat{A}_t \geq A_t$ and thus condition (3) holds at the new sequence given that it holds at the old. For the new sequence, condition (2) holds with equality. Thus, the balance sheet constraints are satisfied.

The budget constraint of the fiscal authority holds, given the construction of $\hat{B}_t$. Given that $B_t - A_t = \hat{B}_t - \hat{A}_t$. the transversality condition (8) holds given that it was holding at the initial equilibrium.

The new sequence then satisfies all of the conditions for a private sector equilibrium. □

The above Lemma tells us that if there is a helicopter drop at some t (that is, $\tau_t > \tau^\star$), then it is possible to find an alternative policy with $\tau_t = \tau^\star$ that is also consistent with all equilibrium conditions and where the balance sheet constraints are satisfied. For example, suppose that starting from an original equilibrium, the monetary authority has increased M_t at some time t while simultaneously setting $\tau_t > \tau^\star$. Such a policy reduces the monetary authority's net worth at time t (this is a "helicopter drop"). Consider instead an alternative in which the monetary authority conducts an open market operation at time t; that is, it increases M_t by the same amount but uses the proceeds to purchase assets, A_t, while leaving $\tau_t = \tau^\star$. The lemma above establishes that this new policy is also consistent with the same original equilibrium outcome for $\{y_t, \pi_t, i_t, P_t, M_t\}$ and consistent with balance sheet constraints.[13]

We highlight that the reverse of Lemma 2 is not true. That is, the result that a helicopter drop can be introduced without affecting the equilibrium allocation does not generally hold. This is because a helicopter drop generates a reduction in net worth, which may lead to a violation of the balance sheet constraint (4), thus implying that the original allocation cannot longer be part of an equilibrium.

We have established that the ability to implement helicopter drops does not expand the set of private sector equilibria, even in the presence of balance sheet constraints. But do balance sheet

[13] In the Lemma, the fiscal authority changes B_t to adjust the budget. Alternatively, we could have the fiscal authority to arbitrarily change T_t. This would also work as long as the transversality condition of the households remains satisfied after the change.

constraints matter at all for the set of private sector equilibria?[14] Take any allocation $\{y_t, \pi_t, i_t\}$ that satisfies the Euler equation, the Phillips curve, and the zero lower bound constraint. Then, as long as the monetary authority is well capitalized, we can find a policy under which this allocation constitutes a private sector equilibrium consistent with balance sheet constraints.

Lemma 3 (Balance-sheet irrelevance with sufficient initial net worth)**.** *Suppose that* $N_{-1} = A_{-1} - M_{-1} \geq 0$*. Consider a sequence* $\{y_t, \pi_t, i_t\}$ *that satisfies the Euler equation,* (10)*, the price setting equation,* (9)*, and the zero lower bound constraint,* $i_t \geq 0$*. Let* $\{P_t\}$ *be the corresponding price level given* $\{\pi_t\}$ *and* P_{-1}*. Then, there exists a policy* $\{M_t, A_t, B_t, \tau_t, T_t\}$ *such that* $\{y_t, \pi_t, i_t, P_t, M_t, A_t, B_t, \tau_t, T_t\}$ *is a private sector equilibrium consistent with balance sheet constraints.*

Proof. The proof is constructive. For $t \geq 0$, let

$$
\begin{aligned}
M_t &= P_t L(e^{y_t}, i_t) \\
A_t &= M_t + A_{-1} - M_{-1} \\
\tau_t &= \tau^\star(A_t, i_t) \\
B_t &= A_t - M_t \\
T_t &= \frac{B_t}{1 + \imath_t} + \tau_t - B_{t-1}
\end{aligned}
$$

Condition (i) of the private sector equilibrium definition holds, as the household budget constraint, (6), holds with $W_t = B_t - A_t$, the Euler equation (10) holds by assumption, by construction the money demand condition, (11), holds as well, and the private sector total net financial wealth (inclusive of money holdings) $M_t + B_t - A_t = 0$ for all $t \geq 0$, implying that the transversality condition (8) holds. Conditions (ii) and (iii) hold. And by construction, the budget constraints of the monetary and the fiscal authority hold as well. The sequence constitutes a private sector equilibrium. Given that $A_{-1} - M_{-1} \geq 0$, it follows that $A_t \geq 0$, as $M_t \geq 0$ and (3) holds. Finally, by construction condition (2) holds with equality. Thus, the balance sheet constraints are satisfied. □

Lemma 3 thus tells us that, as long as the monetary authority initial net worth is non-negative, any sequence that satisfies the Euler equation, the Phillips curve, and the zero lower bound

[14] Although related, our analysis is different from Benigno and Nisticò (2020) analysis of the neutrality of the monetary authority's balance sheet. They consider an environment where the transfer policy is fixed and the monetary authority chooses an interest on reserves and an asset portfolio. In this environment, they analyze whether a sequence $\{\pi_t, i_t\}$ is a private sector equilibrium for a given transfer/balance sheet policy. In our case, we examine an endogenous transfer (while abstracting from interest on reserves and a portfolio decision), and we ask instead whether the sequence $\{y_t, \pi_t, i_t\}$ constitutes a private sector equilibrium for at least one such policy.

constitutes a private sector equilibrium for some policy. Lemma 2 in addition tells us (and the proof of Lemma 3 explicitly shows) that it suffices to consider policies without helicopter drops.

2.3 Policy Objectives

In the previous section, we presented irrelevance results that require changes to the policies followed by the monetary and fiscal authorities. However, we have not yet discussed how these policies are decided and whether the monetary and fiscal authorities will actually follow them. Our next goal is to analyze how policies are chosen. Towards this, we first describe the policy objectives.

Monetary Policy Objective. We assume that the monetary authority seeks to minimize departures from zero for both inflation and the output gap. Specifically, the monetary authority evaluates welfare according to the following objective function:

$$\sum_{t=0}^{\infty} e^{-\rho t} \mathcal{U}(\pi_t, y_t) \tag{12}$$

where $\mathcal{U}(\pi_t, y_t)$ encapsulates the objective of keeping inflation and output close to the target and is strictly maximized at $\mathcal{U}(0, 0)$.

Fiscal Policy Objective. The fiscal authority is assumed to select a path of $\{B_t, T_t\}$ such that its budget constraint holds at all times, and the consolidated debt of the government satisfies the following condition:

$$\lim_{t \to \infty} \frac{B_t + M_t - A_t}{\prod_{s=0}^{t}(1 + \iota_s)} = 0, \tag{13}$$

This implies that the transversality condition of the households will be satisfied. And also implies that the sequence of taxes chosen by the fiscal authority does not affect the rest of the equilibrium.[15]

In our analysis of the equilibrium without commitment, we will impose the additional restriction that the real *assets* of the fiscal authority do not explode:

[15] Under just this assumption, and without Condition 1 below, *the fiscal theory of the price level* would not play a role in our analysis; as the consolidated government sector follows a "Ricardian" fiscal policy in the terminology of Benhabib, Schmitt-Grohé and Uribe (2001a). With regard to the fiscal theory of the price level, see Woodford (2001) and Kocherlakota and Phelan (1999) for summaries. For a game-theoretical analysis of the equilibrium selection issues that arise, see Bassetto (2002). For an analysis of helicopter drops under commitment, operating via the fiscal theory of the price level, see Benigno and Nisticò (2022).

Condition 1. *In a private sector equilibrium, the fiscal authority does not accumulate unbounded assets on the private sector:*

$$\lim_{t\to\infty} \frac{B_t}{\prod_{s=0}^{t}(1+\iota_s)} \geq 0.$$

As we will see below, in our environment, this restriction rules out the deflationary trap equilibria of Benhabib, Schmitt-Grohé and Uribe (2001a, 2001b, 2002) that otherwise will be pervasive in the analysis without commitment.

To see the role that this constraint can play, let us iterate forward on the budget constraint of the monetary authority. Using Condition 1, together with the household's transversality condition, we get that in a private sector equilibrium, we must have that

$$N_{t-1} + \sum_{s=t}^{\infty} \frac{\frac{\iota_s}{1+\iota_s} M_s}{\prod_{\tau=t}^{s-1}(1+\iota_\tau)} \geq \sum_{s=t}^{\infty} \frac{\tau_s}{\prod_{\tau=t}^{s-1}(1+\iota_\tau)} \tag{14}$$

The left-hand side can be seen as the "equity value" of the monetary authority: its net worth plus the discounted future value of its "profits" (i.e. seigniorage revenue). This value cannot be lower than the present discounted value of the "dividends," (i.e., the transfers made by the monetary authority).

Note that without Condition 1, the monetary authority's net worth could explode to minus infinity in a private sector equilibrium, even in the presence of balance sheet constraints. In that case, the fiscal authority must accumulate assets to compensate for the ever increasing monetary authority's net liabilities. That is, exploding negative net worth can be part of a private sector equilibrium if the fiscal authority effectively *provides the demand for the monetary authority liabilities*, which is intermediated, in equilibrium, through the household sector. Condition 1 restricts this fiscal behavior, and as a result, inequality (14) generates a lower bound on the monetary authority net worth given that $\tau_s \geq 0$ (and assuming that the present value of seigniorage revenue has an upper-bound).

Note that if transfers τ_t were equal to or larger than the seigniorage revenue at all times, then (14) would imply that net worth could not be negative. However, the minimum remittance constraint (2) operates on a nominal mark-to-market rule, as in practice.[16] This leaves space for $\tau_s < \iota_s M_s/(1+\iota_s)$, and thus a monetary authority that operates with negative net worth can, in principle, be part of a private sector equilibrium.

It is interesting to note that the irrelevance of the helicopter drops, stated in Lemma 2, holds

[16] See Hall and Reis (2015) for more details on this.

even in the presence of Condition 1. Inspection of the proof of the lemma shows that the alternative allocation without helicopter drops features a fiscal debt that is weakly higher than the original allocation ($\hat{B}_t \geq B_t$ for all t), and thus if Condition 1 held at the original, it remains to hold as well in the alternative.

Finally, our subsequent analysis in the case without commitment can be done without imposing Condition 1, by focusing on equilibrium without deflationary traps.

3 Commitment Solution

Having specified the policy objectives, we start by analyzing the optimal monetary policy under commitment. The monetary authority's problem is as follows: the monetary authority maximizes its objective function by choosing a sequence $\{y_t, \pi_t, i_t, M_t, A_t, \tau_t\}$ while the fiscal authority selects a sequence of $\{B_t, T_t\}$ such that $\{y_t, \pi_t, i_t, M_t, A_t, \tau_t, B_t, T_t\}$ constitutes a private sector equilibrium consistent with balance sheet constraints. In this case under commitment, we are assuming that the monetary authority chooses the equilibrium outcome as long as it satisfies the required equilibrium conditions. We are left to guarantee that the path of the fiscal authority's real debt remains bounded, as specified in Condition 1. We will discuss this last point at the end of this section.

Lemma 3 implies that if $A_{-1} \geq M_{-1}$, we can ignore balance sheets *at all times* and focus on $\{y_t, \pi_t, i_t\}$ when solving for the optimal policy under commitment. In the rest of the paper, we will narrow attention to this case of $A_{-1} \geq M_{-1}$, as it starkly showcases the difference between the commitment and the no-commitment solutions.

Liquidity Trap under Commitment

We consider a situation where a liquidity trap may arise in period $t = 0$. Towards this, we let the discount factor disturbance, ξ_t, be

$$\xi_t = \begin{cases} \tilde{\xi} & \text{if } t = 0, \\ 0 & \text{otherwise.} \end{cases} \tag{15}$$

The monetary authority's objective is

$$\mathcal{U}(\pi, y) = -\left[\pi^2 + \varphi y^2\right],$$

for $\varphi \in [0, \infty)$.[17]

Assuming that the initial net worth is positive (so that the balance sheet constraints do not bind), the problem under commitment is in effect, a discrete-time version of Werning (2011), with the liquidity trap lasting for one period. (See also Eggertsson and Woodford (2003) and Jung et al. (2005) for related setups).

Numerical Results. Each period represents a quarter. We set the discount rate to $\rho = 0.01$ and the intertemporal elasticity of substitution to $\sigma = 0.5$. The value for the slope of the Phillips curve is set to $\kappa = 0.35$, which is in the middle range of those typically used in calibrations of New Keynesian models. The coefficient on the output gap in the loss function is set to $\varphi = 0.05$.[18] In addition, we set the shock to the natural rate to $\tilde{\xi} = -0.12$. In the solution without helicopter drops, this shock will generate a fall in output of 6%. Finally, we let money demand be approximated by

$$L(y, i) = \theta e^{y - \eta i}, \tag{16}$$

(although, as we discussed above in Lemma 3, this does not affect the commitment solution for y_t, π_t, i_t). We set the interest rate elasticity of money demand to $\eta = 0.5$, a standard value (see, e.g., Benati, Lucas, Nicolini and Weber, 2021), and the intercept θ to match a ratio of currency to GDP of 10%.

Figure 1 presents the numerical results. The figure shows the path for output, inflation, and the nominal interest rate. In line with existing results in the literature, the economy experiences deflation and a negative output gap at $t = 0$; to optimally manage the liquidity trap, the monetary authority maintains nominal interest rates low beyond the duration of the liquidity trap. In the example presented, lift-off occurs after 4 periods, while ξ goes back to zero for $t \geq 1$.

To complete the analysis, we can specify the balance sheets and transfers. Ricardian equivalence means that these are not pinned down, but it suffices to use the sequence described in the proof of Lemma 3, which guarantees that the fiscal authority debt position satisfies Condition 1.

[17] This objective can be derived from a second-order approximation to welfare around an efficient steady-state with zero inflation under a cashless limit, following Woodford (2011). Efficiency requires a subsidy on production to undo the monopolistic distortion.

[18] The small weight on output is standard in the literature. A second-order approximation of the welfare function in the canonical New Keynesian model delivers a relative weight of κ divided by the elasticity of substitution between differentiated inputs. Given the value of κ, our choice of φ would be consistent with an elasticity of substitution of 7, which is within the range of the values used in the literature.

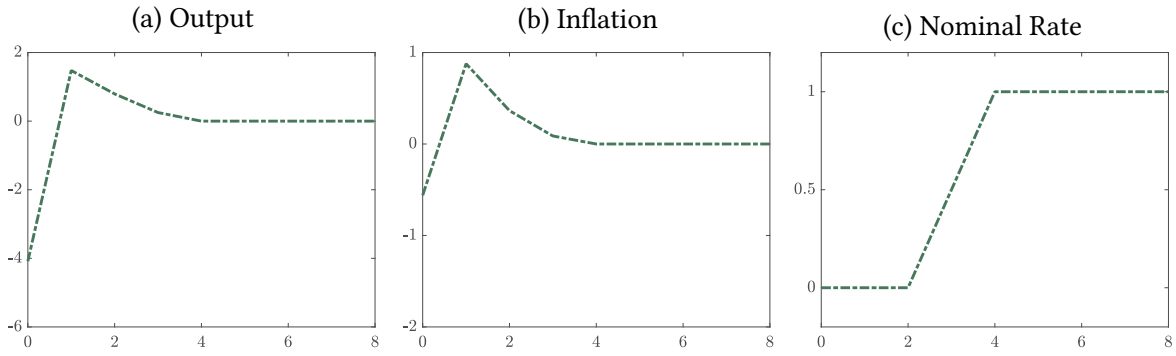


Figure 1: Simulation under Commitment

Note: Output gap, inflation, and the nominal interest rate are expressed as percentages. The x-axes represent time.

4 Policy without Commitment

We now proceed to the main analysis of the paper: the study of equilibria in which the monetary authority cannot commit. We focus on Markov equilibria. This is the equilibrium concept used in several previous papers, such as Albanesi, Chari and Christiano (2003); Eggertsson (2003); Adam and Billi (2007); Armenter (2018); Nakata and Schmidt (2019). In what follows, we also narrow attention to the monetary authority's problem and assume that the fiscal authority selects a path of debt and taxes consistent with its budget constraint and the consolidated government's transversality condition. We rule out as an equilibrium outcome any allocations where the fiscal path violates Condition 1.

4.1 The Stationary Case: Markov Equilibria

First, we consider equilibria after the discount rate disturbance has passed, that is $\xi_t = 0$ at all times. Given that all exogenous variables are constant, we refer to this as the "stationary case". But as we will see below, even in this stationary case, there are dynamics induced through the policy choices.

The state variables at time t are the inherited balance sheet positions, M_-, A_-, and the previous period price level, P_-. As usual in monetary models, it suffices to carry $M_-/P_- = m_-$ and $A_-/P_- = a_-$ as state variables.

In a Markov equilibrium, the monetary authority takes as given the private sector expectation functions of next-period's output and inflation. At the beginning of the period, the monetary authority inherits its balance sheet, m_- and a_-, and chooses y, π, i, m, a, and $\tilde{\tau} = e^{\pi}\tau/P_-$ to

maximize its objective function, subject to the Euler equation, the Phillips curve, the zero lower bound constraint, the money market clearing condition; its budget constraint, and the two balance sheet constraints. The budget constraint and balance sheet constraints (1)-(3) can be written as

$$e^{\pi-i}a + m_- + \tilde{\tau} = e^{\pi}m + a_-; \quad a \geq 0; \quad \text{and } \tilde{\tau} \geq (1 - e^{-i})a.$$

Defining $n \equiv a_- - m_-$, and $n' \equiv a - m$, the two balance sheet constraints are equivalent to

$$m \geq -n'; \quad n \geq e^{\pi}n'.$$

The next step is to notice that the inherited values of m_- and a_- do not affect any other constraint, and thus we can simply use real net worth, n, as the only state variable.

We let $V(n)$ denote the value function of the monetary authority as a function of its net worth and Ω its domain set. The functions $\mathcal{Y}(n)$ and $\Pi(n)$ denote the private sector expectations of future output and inflation. The monetary authority's problem can be formulated as follows.

Problem 1 (The Stationary Case)**.**

$$V(n) = \max_{(y,\pi,i,n' \in \Omega)} \mathcal{U}(\pi, y) + \beta V(n')$$

subject to:

$$y = \mathcal{Y}(n') - \sigma(i - \Pi(n') - \rho) \tag{17}$$

$$\pi = \beta\Pi(n') + \kappa y \tag{18}$$

$$i \geq 0, \tag{19}$$

$$L(e^y, i) \geq -n' \text{ if } i > 0 \tag{20}$$

$$n' \leq e^{-\pi}n. \tag{21}$$

The first three constraints are standard ones: (17) and (18) are the Euler and Phillips curve, respectively, while (19) is the zero lower bound on the nominal interest rate. Constraint (20) is the balance sheet constraint ($a \geq 0$) with the money market equilibrium. In writing this constraint, we use that if $i = 0$ then, it is always possible to find an m' large enough such that the money market clearing condition holds, $m' \geq L(e^y, i)$, and $n' \geq -m$; and thus these constraints on m' can be ignored. If $i > 0$, however, then we must have that $m' = L(e^y, i) \geq -n'$. Finally, constraint (21) is the bound on net worth implied by the lack of fiscal support ($\tau \geq \tau^\star$).

The last two constraints of Problem 1 capture the key innovation of our model. Together they imply $e^{\pi}L(e^y, i) \geq -n$. Because L is decreasing in i, this means that the monetary authority cannot

increase the nominal interest rate unless it has enough net worth.

Further inspection of the monetary authority's problem shows that the inherited net worth, n, only appears in the last constraint. It follows then that increases in n relax the monetary authority's problem, which guarantees that the value function $V(n)$ must be (weakly) increasing in net worth. The result that the value function is monotonic in n raises the question of why would the monetary authority ever conduct a helicopter drop that would deplete its net worth and thus lower its continuation value. The answer has to do with how, during a liquidity trap, a low net worth induces higher inflation and output in the future, affecting private sector expectations, as captured by the $\mathcal{Y}$ and Π functions. We will return to this point in the section that follows.

Let $\mathcal{N}(n)$ denote the corresponding policy functions for the net worth evolution. With this, we are ready to define a Markov equilibrium for this case:

Definition 4. A *Markov equilibrium* (in the stationary case) is given by a closed interval $\Omega \subset \mathbb{R}$ and functions $V : \Omega \to \Omega, \mathcal{Y} : \Omega \to \mathbb{R}, \Pi : \Omega \to \mathbb{R}, \mathcal{N} : \Omega \to \Omega$ such that V solves the monetary authority's problem given $\mathcal{Y}$ and Π for all $n \in \Omega$; and $\mathcal{Y}$, Π, $\mathcal{N}$ are optimal policy functions for output, inflation, and real net worth.

We can now show that given any initial $n_{-1} \in \Omega$ and some initial B_{-1}, the resulting sequence of aggregates constitutes a private sector equilibrium. To see this, iterate the net worth policy function, $\mathcal{N}$, to obtain a sequence of $\{n_t\}$ starting from n_{-1}. Let $\{y_t, \pi_t\}$ denote the associated sequence of output and inflation given the policies $\mathcal{Y}, \Pi$. We let $\{i_t\}$ be uniquely defined by $\mathcal{Y}(n_t) = \mathcal{Y}(n_{t+1}) - \sigma(i_t - \Pi(n_{t+1}) - \rho)$. For $i_t > 0$, we let $M_t = P_t L(e^{y_t}, i_t)$. For $i_t = 0$, we let $M_t = P_t \max\{L(e^{y_t}, i_t), -n_{t+1}\}$. We let $A_t = P_t n_t - M_t$. Finally $\tau_t = M_t - P_{t-1} n_{t-1} - \frac{1}{1+\iota_t} A_t$. We can then set $B_t = A_t - M_t$, and $T_t = \frac{B_t}{1+\iota_t} + \tau_t - B_{t-1}$, with B_{-1} given. It is straightforward to check that the sequence $\{y_t, \pi_t, i_t, P_t, M_t, A_t, B_t, \tau_t, T_t\}$ we have just constructed *constitutes a private sector equilibrium consistent with balance sheet constraints.*

As we discussed previously, there are many sequences of $\{B_t, T_t\}$ that also constitute an equilibrium, as long as (13) holds. But do any of them satisfy Condition 1? The answer relies on the long-run behavior of the sequence $\{n_t\}$. To see this, suppose that the sequence $\{n_t \prod_{s=0}^{t} e^{\pi_s - i_s}\}$ does not converge to a non-negative number. Then, we have that

$$0 > \lim_{t\to\infty} -n_t \prod_{s=0}^{t} e^{\pi_s - i_s} = \lim_{t\to\infty} \frac{M_t - A_t}{P_t} \prod_{s=0}^{t} \frac{P_s/P_{s-1}}{1+\iota_s} = \lim_{t\to\infty} \frac{M_t - A_t}{P_{-1} \prod_{s=0}^{t}(1+\iota_s)}.$$

For the household transversality condition to hold, it would be necessary that

$$\lim_{t\to\infty} \frac{B_t}{P_{-1}\prod_{s=0}^{t}(1+\iota_s)} < 0,$$

a contradiction of Condition 1. If however, the sequence $\{n_t \prod_{s=0}^{t} e^{\pi_s - i_s}\}$ converges to a non-negative number, then it is possible to construct a fiscal policy that generates a private sector equilibrium where Condition 1 holds. We will restrict attention to Markov equilibria consistent with this:

Definition 5. A Markov equilibrium $(\Omega, V, \mathcal{Y}, \Pi, \mathcal{N})$ is *fiscally consistent* if for all $n_0 \in \Omega$, the resulting equilibrium path of $\{n_t, i_t, \pi_t\}$ is such that

$$\lim_{t\to\infty} n_t \prod_{s=0}^{t} e^{\pi_s - i_s} \geq 0. \tag{22}$$

Ruling Out Deflationary Traps. The environment we are analyzing has the potential for multiple Markov equilibria, a characteristic that has been highlighted by Albanesi et al. (2003); Armenter (2018); Nakata and Schmidt (2019). In particular, Armenter (2018) and Nakata and Schmidt (2019) show that there exists a "deflationary trap": in our case $\pi = -\rho, y = -\rho(1-\beta)/\kappa$. If the private sector expects these levels of inflation and output tomorrow, then setting $i = 0$ delivers them as outcomes today per equations (17) and (18). And with these expectations, there is not much the monetary authority can do, as raising the nominal interest rate above zero further lowers inflation and reduces output (and thus welfare).[19] The next question is whether this deflationary trap is consistent with the Condition 1.

Using equation (21), we can see that in this expectational trap (as $i = 0$),

$$n_t \leq e^{-\pi} n_{t-1} = e^{i-\pi} n_{t-1} \leq \ldots \leq e^{(i-\pi)(t+1)} n_{-1}$$
$$e^{(\pi-i)(t+1)} n_t \leq n_{-1} \Rightarrow \lim_{t\to\infty} e^{(\pi-i)(t+1)} n_t \leq n_{-1}$$

And thus, if $n_{-1} < 0$, the deflationary trap is not fiscally consistent, as it necessarily violates (22). Note that we can use the above to also rule out transitional paths that converge to the deflationary

[19] The existence of this deflationary trap is anticipated by the self-fulfilling liquidity traps of Benhabib et al. (2001b; 2002) in their analysis of a monetary authority that adheres to a (non-optimizing) Taylor rule but faces a zero lower bound constraint on nominal rates.

trap.[20] This argument does not work if $n_{-1} \geq 0$, but in this case, given that we know that the value function is monotonic, reducing n' suffices to also eliminate the trap as an equilibrium outcome, as we discuss below.

The Best Possible Equilibrium. Once we have ruled out the trap outcome, the next question is whether the best allocation ($\pi_t = 0$ and $y_t = 0$ at all t) constitutes an equilibrium. Note that in this case, $i_t = \rho > 0$, and thus the zero lower bound constraint is automatically satisfied. Let us define $n^\star \equiv -L(1, \rho)$. We assume that $n^\star \in \text{interior}(\Omega)$. Note that $n^\star < 0$.

When $n = n^\star < 0$, the deflationary trap is not an equilibrium, as we discussed above. In addition, the $\{\pi_t = 0, y_t = 0\}$ allocation can be implemented as an outcome of a Markov equilibrium in which the monetary authority maintains a constant level of net worth, $n_t = n^\star$, and chooses $\pi = 0, y = 0$ every period. Given that this allocation achieves the maximum welfare possible, there are no incentives for the monetary authority to deviate from it at any time if future monetary authorities follow this strategy. We assume that this is indeed the equilibrium at $n = n^\star$. For values of $n > n^\star$, the deflationary trap cannot be an equilibrium even if $n > 0$. This follows from the monotonicity of the value function. Alternatively, one can see that setting $n' = n^\star$, $\pi = 0$, and $y = 0$ is a feasible choice for $n > n^\star$ that guarantees the best equilibrium outcome as the continuation. We restrict attention to Markov equilibria that satisfy the following condition:

Definition 6. An equilibrium is *good* if for $n \geq n^\star$, $\mathcal{Y}(n) = \Pi(n) = 0$ and $\mathcal{N}(n) \geq n$.

This condition—namely, that the monetary policy implements the good equilibrium outcome—is similar to the assumption in Werning (2011) that the monetary authority implements the zero output gap and zero inflation outcome once such outcome becomes feasible in his environment. In our case, however, it is necessary that the monetary authority networth be larger than $n^\star$. When $n < n^\star$, the $\pi = 0, y = 0$ allocation is no longer a feasible choice because of the balance sheet constraints: the monetary authority lacks sufficient net worth to implement the allocation. To shed light on the trade-offs faced by the monetary policy in this case, consider the following feasible policy. At $t = 0$ (given that $n < 0$), the monetary authority chooses a sufficiently high level of inflation to guarantee that $n' = n^\star$ is a feasible choice. In this way, the monetary authority can guarantee that the first-best allocation is achieved from period $t = 1$ onward at the cost of a positive inflation rate in $t = 0$. However, the monetary authority may choose a less drastic policy

[20] Benigno (2020) shows that a policy commitment by the monetary authority to remit its profits, as well as Condition 1 holding with equality, can lead to price level determinacy in a flexible price model where the monetary policy follows a Taylor rule. In his analysis, it is an exploding positive net worth by the monetary authority that is ruled out by the policy, while in ours, is an exploding negative one. In our model, the monetary authority is operating without commitment and it is not following a Taylor rule, but the basic insight that eliminates the deflationary trap is the same as in his model.

by setting inflation above zero at $t = 0$, increasing n' above n, but not enough to get to $n^\star$. Doing so may result in an equilibrium where inflation and output are above their first-best levels for longer than just the initial period.

The Sationary Case: Log-linear Equilibria

When money demand takes the form in (16), the model allows for an analytical characterization of equilibria. Towards this, we restrict attention to good equilibria that are log-linear:

Definition 7. A good equilibrium is a *log-linear equilibrium* in a closed interval Ω if $n^\star \in \text{interior}(\Omega)$ and for $n \in \Omega$ and $n < n^\star$,

$$\begin{aligned} \mathcal{Y}(n) &= a(\log(-n) - \log(-n^\star)), \\ \Pi(n) &= b(\log(-n) - \log(-n^\star)), \text{ and} \\ \log(-\mathcal{N}(n)) - \log(-n^\star) &= \phi_n(\log(-n) - \log(-n^\star)), \end{aligned}$$

for some scalars a, b, and $\phi_n \in [0, 1]$.

Before discussing existence and uniqueness of linear equilibria, let us define the following value $x^\star$:

Definition 8. Let $x^\star$ be the value $x \in (0, 1)$ such that

$$1 = x\left(\frac{\eta(1+\sigma\kappa)+\sigma\kappa}{\eta+\alpha\sigma} + 1 + \beta\right) - x^2\left(\frac{\eta(1+\sigma\kappa+\beta)}{\eta+\alpha\sigma} + \beta\right) + x^3\left(\frac{\beta\eta}{\eta+\alpha\sigma}\right).$$

And define $\underline{n}$ to be

$$\underline{n} \equiv n^\star \exp\left(\frac{\rho\kappa\sigma}{(1-x^\star)[(1-x^\star(1+\beta(1-x^\star)+\sigma\kappa)]}\right).$$

It is straightforward to see that there exists such a value of $x^\star \in (0, 1)$ and that it is indeed unique.[21] Given that $n^\star < 0$, it follows that $\underline{n} < n^\star$. With this, we have the following result:

[21] To see this note that the right-hand side takes a value of zero at $x = 0$ and a value strictly larger than 1 for $x = 1$, and thus by continuity a root exists. Uniqueness follows from showing that the right-hand side is concave in x for $x \in (0, 1)$.

Proposition 1. *Suppose* $\Omega = [n_0, n_1]$ *with* $\underline{n} \leq n_0 < n^\star < n_1$*, and money demand is given by* (16)*. Then, there exists a unique log-linear equilibrium in* Ω*, where*

$$\begin{aligned} a &= (1 - x^\star)(1 - \beta x^\star)/\kappa, \\ b &= (1 - x^\star), \\ \phi_n &= x^\star. \end{aligned}$$

Proof. In Appendix A.2. □

Let us briefly discuss the proposition. The lower bound on networth, $\underline{n}$ is there as it makes sure that the zero lower bound is not binding in a linear equilibrium. The unique value of $x^\star$ determines the rate at which networth converges to $n^\star$. The coefficients a and b are both strictly positive, and thus imply that higher net worth leads to lower output and lower inflation in equilibrium. This result highlights that a sufficiently large helicopter drop (a reduction in n) leads to higher inflation and output in the future. We can define the helicopter drops that occur in equilibrium by HD as follows:

$$HD(n) \equiv \mathcal{N}(n) - e^{-\Pi(n)} n.$$

There is a helicopter drop if $HD(n) > 0$, or equivalently, nominal net worth falls.

In the log-linear equilibrium, helicopter drops are not used:

Corollary 1. *Suppose* $\Omega = [n_0, n_1]$ *with* $\underline{n} \leq n_0 < n^\star < n_1$*, and money demand is given by* (16)*. In the unique log-linear equilibrium in* Ω*, there are no helicopter drops:*

$$HD(n) = 0.$$

Proof. This result is proved in the proof of Proposition 1. □

The intuition for this result is as follows. Given that $n < n^\star$, in equilibrium, the monetary authority necessarily chooses positive inflation. If the monetary authority were to engineer a helicopter drop, it could achieve the same level of output with a lower helicopter drop (increasing its continuation value) and a lower interest rate. This satisfies the balance sheet constraints, generates less inflation, and is feasible as long as $i > 0$. Thus the alternative policy constitutes an improvement relative to the solution with a helicopter drop.

A final analytical result we would like to highlight here is that in the log-linear equilibrium,

the nominal interest rate,

$$i = \rho + \Pi(\mathcal{N}(n)) + \frac{1}{\sigma}(\mathcal{Y}(\mathcal{N}(n)) - \mathcal{Y}(n))$$

is strictly increasing in n for $n < n^\star$. That is, a lower net worth generates a lower nominal interest rate.[22]

Extending the domain below $\underline{n}$ requires us to move beyond the class of log-linear equilibria. Although we are not able to analytically characterize the equilibrium for this larger domain, it is indeed possible to characterize it numerically.

Figure 2 presents the results that arise of such a simulation. (We use the same parameter values as in the previous section.) The x-axis in each panel is a transformation of networth, $-\log(-n)$. In each panel, there are two vertical lines. The first one represents the value of $\underline{n}$, and the second one represents $n^\star$. Panels (a), (b), and (c) display output, inflation, and the nominal interest rate as a function of net worth, respectively. Panels (e), (f), and (g) display the policy function for net worth, the value function, and the helicopter drop, respectively.

The simulation is solved without imposing log-linearity (solid blue line), but the figure also displays the log-linear equilibrium prediction (dashed red line). For values of $n \geq n^\star$ the good equilibrium is attained, as expected, and all policies are constant. For values of n between $\underline{n} < n < n^\star$, the equilibrium is indeed the log-linear one (the linearity can be visually observed in the plots and the solid lines coincide with the dashed ones). For values $n < \underline{n}$, the equilibrium is no longer log-linear (the solid lines deviate from the dashed lines), but it inherits the same monotonic properties of the latter.

Notice that for $n < \underline{n}$, welfare is *lower* under the projected log-linear equilibrium compared to the actual solution. This occurs because the log-linear equilibrium is solved under t binding balance sheet constraint (20). That is, for $i > 0$, the monetary authority must ensure that monetary liabilities do not fall short of negative net worth. However, once the monetary authority sets $i = 0$, which occurs at $n = \underline{n}$, the constraint (20) no longer binds because households are now indifferent between any combination of bonds and money. The monetary authority is still subject to the remittance constraint (21)), but it faces a more relaxed problem. As a result, it reduces inflation and achieves a level of output closer to the efficient level. In other words, in the stationary environment, $i_t \geq 0$ is not a binding constraint.

[22] Using the log-linear equilibrium solution, we have that for $n < n^\star$, the interest rate is

$$i = -(1 - \phi_n^\star)\frac{1 - \phi_n^\star(1 + \beta(1 - \phi_n^\star) + \sigma\kappa)}{\sigma\kappa}\log(-n) + \text{ a constant}$$

which is strictly increasing in n.

Panel (g) highlights another key result from the simulation: helicopter drops are never utilized in this stationary environment. However, this will change when we examine the liquidity trap scenario, as we will see next.

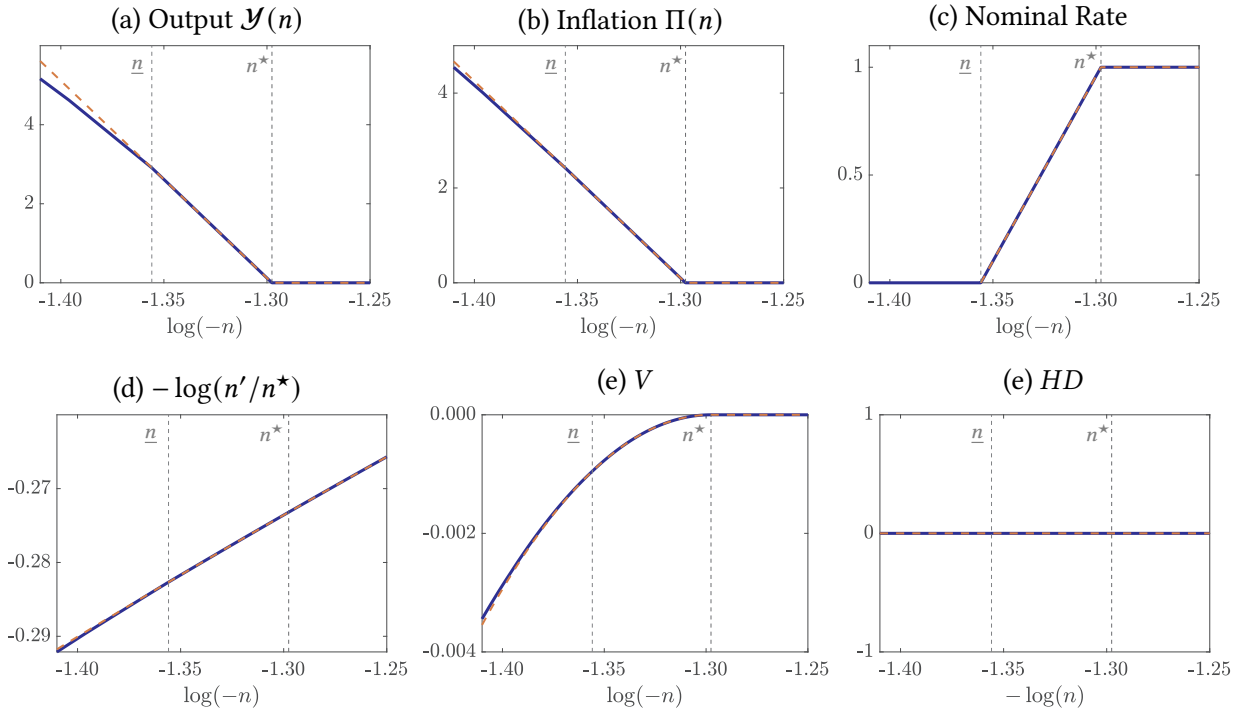


Figure 2: Stationary Environment and Markov Equilibrium

Note: The blue solid line in the figure shows the equilibrium policies y, π, i, n' and value V as a function of net worth, n. The red-broken line denotes the values from the log-linear equilibrium. The two vertical lines denote $\underline{n}$ and $n^\star$. Output, inflation, and the nominal rate are expressed as percentages.

4.2 Markov Equilibrium: A Liquidity Trap

We now return to the liquidity trap environment of Section 3 and let the path of ξ be as in (15): temporarily low at $t = 0$ and zero afterwards. Note that from period $t = 1$ onward, the equilibrium is as described in the stationary environment section, and thus we can use the previously computed $\mathcal{Y}(n)$, $\Pi(n)$, and $V(n)$ as the continuation policies and value function.

We are interested in the Markov equilibrium starting from period $t = 0$, the period of the liquidity trap. In this period, the problem of the monetary authority is given by:

Problem 2 (Liquidity Trap Problem)**.**

$$
\begin{aligned}
V_0(n) = \max_{(y,\pi,i,n' \in \Omega)} \; & \mathcal{U}(\pi, y) + \beta V(n') \\
\textit{subject to:} \\
& y = \mathcal{Y}(n') - \sigma(i - \Pi(n') - \rho - \tilde{\xi}), \\
& \pi = \beta \Pi(n') + \kappa y, \\
& i \geq 0, \\
& L(e^y, i) \geq -n' \; \textit{if} \; i > 0, \\
& n \geq e^{\pi} n'
\end{aligned}
$$

Notice that the only difference with Problem 1 from the stationary case is the presence of the discount factor shock $\tilde{\xi}$ in the Euler equation.

We will assume, as in the commitment analysis, that the initial net worth is sufficiently high so that the balance sheet constraints do not bind at time $t = 0$. (For simplicity, we set net worth to zero.) Given that the last two constraints in Problem 2 do not bind, we obtain the following optimality condition with respect to n_1

$$
\mathcal{U}_{\pi}\big(\beta \Pi'(n_1) + \kappa(\mathcal{Y}'(n_1) + \sigma \Pi'(n_1))\big) + \mathcal{U}_y(\mathcal{Y}'(n_1) + \sigma \Pi'(n_1)) = -\beta V'(n_1), \tag{23}
$$

This condition outlines the key trade-off the monetary authority faces during a liquidity trap. A helicopter drop (i.e., a lower n_1) raises expectations of inflation and output, as discussed in the previous section, which in turn boosts current inflation and output. To the extent that in a liquidity trap $\pi < 0$ and $y < 0$, a helicopter drop generates positive benefits today. However, as shown in the previous section, the continuation value $V(n_1)$ increases with net worth, indicating that there are costs associated with conducting a helicopter drop. Condition (23) illustrates how the monetary authority, at the optimum, balances the marginal benefits of improving macroeconomic outcomes today with the marginal costs of worse future outcomes.

Using the same numerical parameters of previous sections, we proceed to compute the equilibrium policy. Figure 3 shows the equilibrium outcomes that would result in period $t = 0$ as functions of the level of n' chosen in period 0 (i.e., the starting value of 0 minus the size of the helicopter drop). The vertical line represents $n^{\star}$, and the solid dots represent the optimal policy chosen by the monetary authority.

The top panels of Figure 3 show the effects on output and inflation of a helicopter drop in period $t = 0$. A larger helicopter drop generates higher inflation and output in period 0, relative to the case without helicopter drops (which corresponds to the levels above $n^{\star}$, at the flat region).

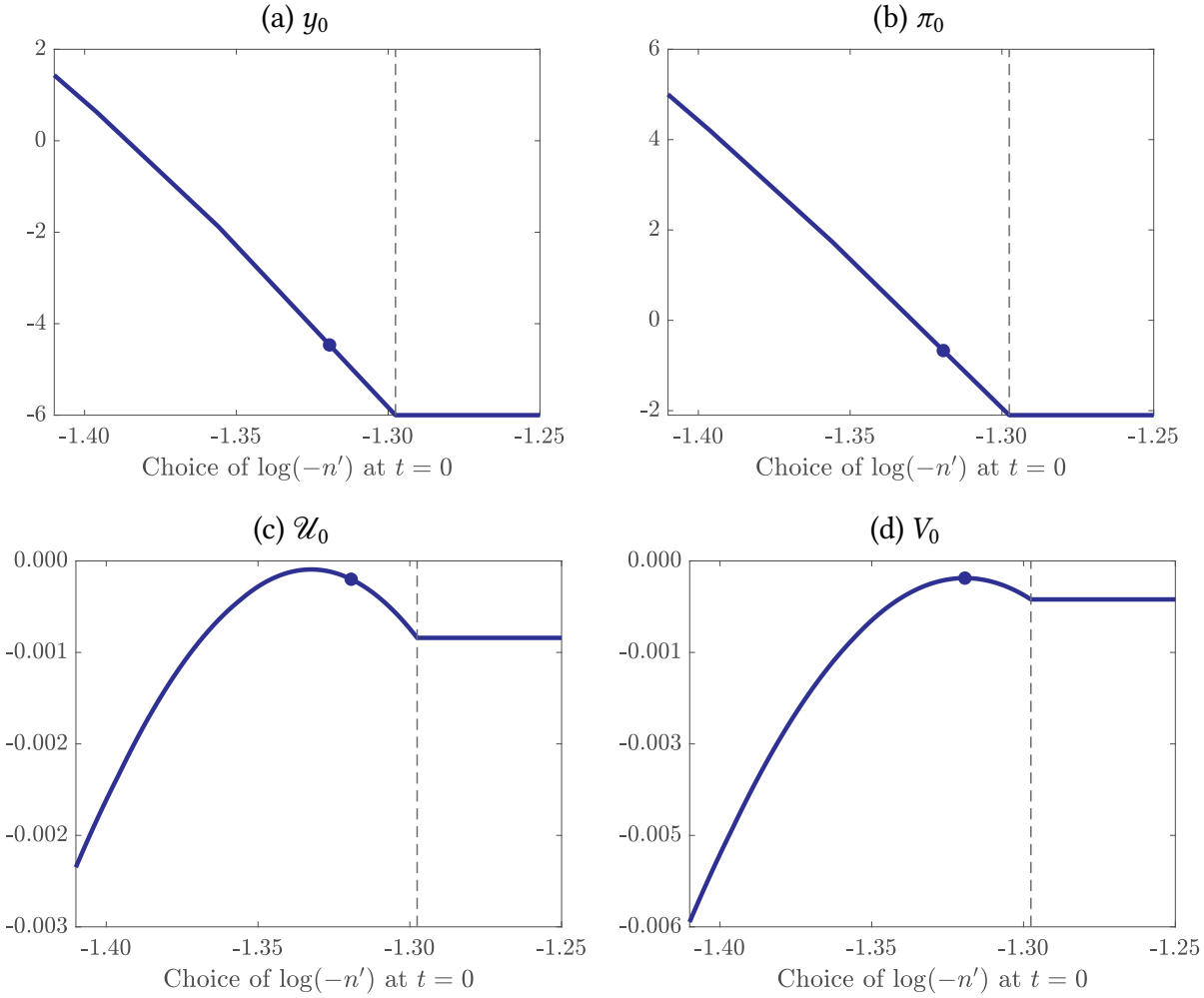


Figure 3: Liquidity Trap and the Markov Equilibrium

Note: This figure shows how different levels of net worth chosen at $t = 0$ affect allocations and welfare at $t = 0$. All variables in the y-axes are expressed as percentages. The solid dot represents the optimal choice of the helicopter drop.

As shown before in Figure 2, a lower net worth starting in period 1 implies higher inflation and output in the future, and this increases current output and inflation.

The bottom panels show the trade-off faced by the monetary authority. In the bottom right panel, one can see the helicopter drop that maximizes the objective function during the liquidity trap. The bottom left panel shows that it is possible to further increase the current value W_0 with an even larger helicopter drop, but this would come at the expense of raising future losses. The optimal helicopter drop turns out to be roughly 2.5 percentage points of annual GDP.

Comparison with commitment solution. We now compare the simulation outcomes over time of the economy under commitment with the simulations for the Markov perfect equilibrium with and without helicopter drops. In all cases, we initialize the economy at $t = 0$ with a high enough net worth so that balance sheet constraints do not initially bind, and we feed the economy with the same shock $\tilde{\xi}$ used above.

Figure 4 presents the results. The red dashed line represents the Markov perfect equilibrium *without helicopter drops*. This economy features the standard outcome of $\pi_t = 0, y_t = 0$ from $t = 1$ onward. One can see that this economy experiences a much larger recession and deflation

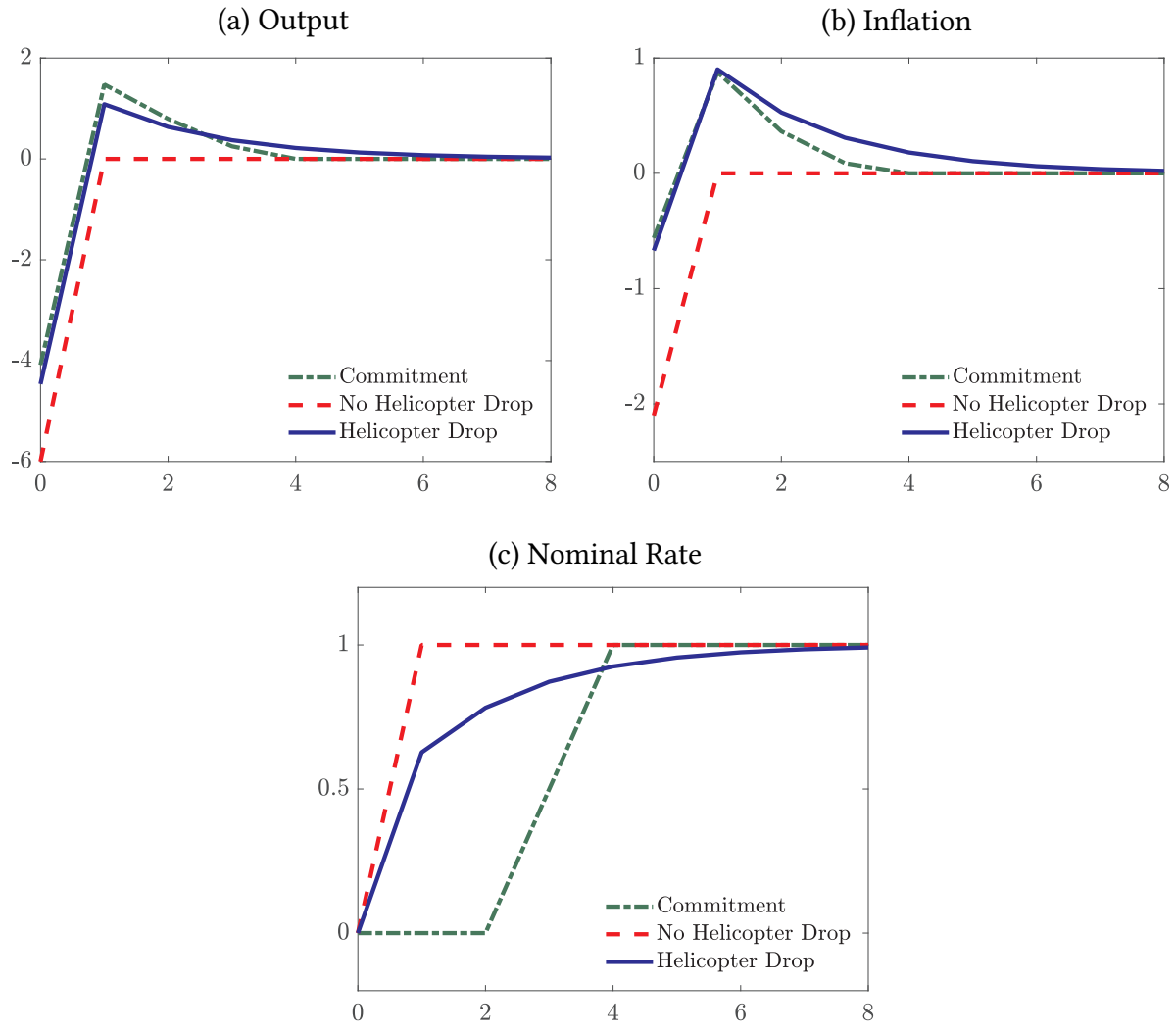


Figure 4: Simulation Comparison

Note: This figure shows the simulations for the three economies starting from a liquidity trap at $t = 0$. All variables in the y-axes are expressed as percentages. The x-axis represents time.

compared to the commitment solution. This is because when the government can commit, it promises inflation and an output boom from $t = 1$ onward, which mitigates the recession at $t = 0$.

The Markov perfect equilibrium with helicopter drops is represented by the solid blue line. As the figure shows, helicopter drops are quite effective at alleviating the liquidity trap. Relative to the equilibrium without helicopter drops, the monetary authority is able to reduce the output gap from -6% to -4% and deflation from -2.2% to -0.5%. As explained above, this is because the use of helicopter drops implies that the monetary authority will keep interest rates lower for longer after the liquidity trap is over. It is worth highlighting that the optimal use of helicopter drops achieves almost as much output stabilization as the commitment solution. The downside, however, lies in the evolution of inflation. While the amount of inflation is more short-lived under commitment, the economy with helicopter drops experiences a more protracted period of inflation.

4.3 Discussion and Policy Implications

The prevalence of ZLB episodes in advanced economies has led to numerous proposals suggesting that central banks resort to helicopter drops of money. These proposals implicitly assume that once the central bank increases the money supply, it will remain elevated, thereby stimulating inflation in the present and alleviating the liquidity trap. Our paper provides a theory of how helicopter drops can serve as a stabilization tool by articulating how the presence of balance sheet constraints can make permanent increases in money supply credible.

Before we conclude, we would like to highlight a few caveats and observations. In terms of implementation, the use of helicopter drops raises of course several challenges. While acknowledging the challenges, Bernanke (2016) argues that it would be premature to rule out helicopter drops as a stabilization tool.[23] One challenge is that central banks do not currently have the legal authority to provide direct transfers to households. An implication of our analysis, however, is that a transfer from the monetary authority to the fiscal authority has identical effects to a transfer from the monetary authority to households. To the extent that central banks have some discretion in remitting funds beyond their operating profits, this would effectively make helicopter drops easier to implement.

One caveat from our analysis is that we have taken as given the institutional features that give rise to the balance sheet constraints. In particular, we have assumed that such institutional

[23] Bernanke (2016) highlights that"...helicopter drops, are very unlikely to be needed in the United States in the foreseeable future. They also present a number of practical challenges of implementation, including integrating them into operational monetary frameworks and assuring appropriate governance and coordination between the legislature and the central bank. However, under certain extreme circumstances—sharply deficient aggregate demand, exhausted monetary policy, and unwillingness of the legislature to use debt-financed fiscal policies—such programs may be the best available alternative. It would be premature to rule them out."

features prevent the fiscal authority from recapitalizing the monetary authority, leading to a strict limit on central banks' net worth. Although our theoretical insights are likely to extend in the case where there are costly recapitalizations, a quantitative analysis on implementation of helicopter drops—and an integration to standard monetary policy frameworks—would require a more precise measure of balance sheet constraints.

Finally, like any commitment device, helicopter drops may involve a tradeoff between commitment and flexibility. In particular, while helicopter drops can help to credibly raise inflation expectations, it does so by reducing the flexibility of future monetary policy. This may become costly for the central bank if it later needs to reverse the monetary expansion in a response to a shock.

5 Conclusion

We develop an analytically tractable New Keynesian monetary model in which the central bank faces balance sheet constraints. Transfers from the monetary authority to the fiscal authority cannot be lower than the operating profits. Additionally, the central bank is constrained in its issuance of interest-bearing securities. These two balance sheet constraints imply that there is a lower bound on monetary liabilities, determined by the central bank's net worth.

We show that while conventional open market operations are irrelevant in a liquidity trap, helicopter drops are not. Crucially, we show that helicopter drops are desirable during a liquidity trap when the central bank lacks commitment. This is because helicopter drops reduce the central bank's net worth, making it more difficult to reverse the initial increase in the money supply later. Thus, when interest rates are at the zero lower bound, a helicopter drop enables the central bank to credibly commit to a more expansionary policy in the future, thereby mitigating the recessionary effects of a liquidity trap. However, if the central bank has the ability to commit, a helicopter drop becomes undesirable, as it limits the central bank's flexibility to reduce inflation.

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