Market Fragmentation and Asset Pricing *

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Abstract

We demonstrate that market fragmentation, the dispersion of trading volume across exchanges, is an important determinant of asset pricing. Fragmentation makes it difficult for investors to locate liquidity supply and, thereby, results in a return premium. We show that for stocks listed on the NYSE, the long-term concentration of share volume on the primary listing exchange—NYSE—lowers expected returns. Meanwhile, short-term deviations in exchange-specific share volumes from their long-term averages require return compensation, regardless of whether they occur in or out of the NYSE. We also link fragmentation to conventional illiquidity measures. While these measures consistently exhibit a significantly positive return premium across different levels of fragmentation, the illiquidity premium rises markedly with fragmentation. To capture this relationship, we construct a new fragmentation-adjusted illiquidity measure. We formalize market fragmentation theoretically using a partially directed search model, wherein uncertainty in liquidity supply reduces both matching efficiency and the price of the asset.

Keywords: Market Fragmentation, Asset Pricing, Liquidity.

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1 Introduction

The U.S. equities market has become increasingly fragmented over the past three decades. Figure 1 illustrates this trend with two interconnected facts. First, the New York Stock Exchange (NYSE) has experienced a significant decline in its market share for its listed stocks. The NYSE executed more than 80% of the consolidated share volume in its listed stocks until early 2005. However, since then, the NYSE has rapidly lost its market share, dropping to approximately 20% by the beginning of 2009 and maintaining this level thereafter. Alongside this shift in market shares, the equities market has become increasingly fragmented. For stocks listed on the NYSE, the Herfindahl–Hirschman Index (HHI)—a commonly used metric for gauging fragmentation—remained around 8,000 until early 2005, after which it experienced a sharp decline over the next four years, eventually settling around 2,500.[1]

In parallel with these developments, a considerable body of theoretical and empirical academic research has emerged within the market microstructure literature, primarily focusing on how market fragmentation relates to various dimensions of market quality.[2] However, fragmentation has not been examined from an asset pricing perspective, as asset pricing studies typically overlook microstructure aspects, as emphasized by O'Hara (2003) in her presidential address. This paper takes a step toward filling this gap in the literature by exploring the asset pricing implications of market fragmentation. We examine stocks listed on the NYSE from 1993 to 2023 and demonstrate that market fragmentation is an important determinant of asset pricing.

We begin our analysis by using HHI to measure fragmentation. Our findings indicate that HHI exhibits a positive return premium over the period 1993–2023, suggesting that higher market concentration—where fewer exchanges dominate trading—tends to result in higher expected returns. However, once we focus on two sub-periods, 1993–2004 versus 2009–2023, each with a more homogeneous and stable market structure, we find that HHI is negatively and positively associated with expected stock returns in these sub-periods, respectively, in-

[1] The triggering point for these declines in the NYSE's market share and the HHI coincides with the U.S. Securities and Exchange Commission's adoption of Regulation NMS in April 2005. Regulation NMS has significantly changed market structure and increased market fragmentation, which will be discussed in detail in Section 2. After Regulation NMS, similar shifts occurred for all U.S.-listed stocks. However, this transformation is most apparent for stocks listed on the NYSE, primarily due to the NYSE's predominant reliance on manual trading until October 2006 (75 FR 3594, p. 3594).

[2] Among others, see Foucault and Menkveld (2008), O'Hara and Ye (2011), and Chen and Duffie (2021).

dicating that investors accept lower expected returns in exchange for lower fragmentation before Regulation NMS, while demanding compensation for lower fragmentation after Regulation NMS. We demonstrate that this conflicting feature of HHI stems from its black-box nature. Regulation NMS has not only dispersed liquidity across numerous exchanges but also significantly increased uncertainty about each exchange's daily share of liquidity. HHI fails to take this uncertainty into account, resulting in seemingly contradictory conclusions regarding how fragmentation affects expected returns before and after Regulation NMS.

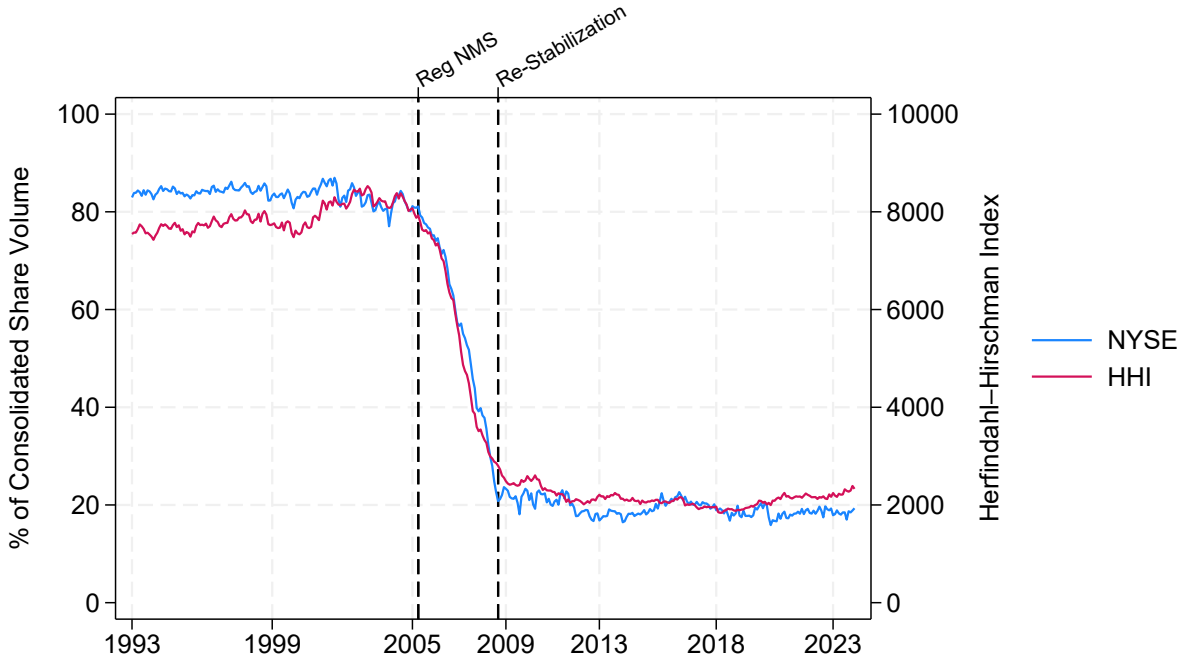


Figure 1: **Market Shares and Herfindahl–Hirschman Index for Stocks Listed on the NYSE.** The sample covers stock-months that have non-missing data from January 1993 to December 2023, encompassing ordinary common shares listed on the NYSE. The left y-axis represents the percentage of monthly consolidated share volume executed on the NYSE for stocks listed on the NYSE. The right y-axis displays the monthly cross-sectional averages of the Herfindahl–Hirschman Index for the same stocks, based on the market shares of each exchange and total share of off-exchanges. The left dashed line corresponds to April 2005, the time of adoption of Regulation NMS. The right dashed line corresponds to September 2008, around when the NYSE's long-term market share stabilized again.

To address this issue and reduce reliance on a black-box approach for gauging fragmentation, we develop a novel fragmentation measure. For each NYSE-listed stock, we begin

by decomposing the daily market share, in terms of share volume, of each exchange into two components. The first component is defined as the average fraction of total share volume executed by an exchange over the past three months. This component, which exhibits highly stable patterns, will be referred to as the "long-term market shares." The second component, defined as the deviation of an exchange's daily market share from its long-term average, is quite noisy and experiences significant day-to-day variation; it will therefore be referred to as "short-term noise." This division is made to account for the fact that, while exchange market shares are relatively stable over longer horizons, they experience significant fluctuations over shorter periods.[3]

We construct our fragmentation measure based on this decomposition. We first calculate the ratio of average daily absolute short-term noises to the long-term market share of the primary listing exchange—the NYSE. This ratio quantifies the extent of uncertainty in a stock's daily liquidity distribution across exchanges relative to its long-term average liquidity on its primary listing exchange. We then use the monthly average of this ratio to measure fragmentation and show that investors consistently require compensation for holding more fragmented stocks. Stocks in the top fragmentation quintile outperform those in the bottom quintile by 0.91% per month in four-factor alpha, after accounting for the three Fama-French factors and the momentum factor. This effect remains both economically and statistically significant after controlling for other risk factors and stock characteristics, including but not limited to traditional illiquidity measures, price impact, idiosyncratic volatility, transaction costs, as well as across sub-samples.

To gain further insight into why market fragmentation is priced, we examine how NYSE's long-term market share and average absolute short-term noise across all exchanges affect asset prices, which are the denominator and numerator of our fragmentation measure, respectively. Our analysis reveals that investors accept lower expected returns in exchange for reduced uncertainty in daily market shares, regardless of whether this reduction arises from a greater long-term market share on the NYSE or smaller short-term market share fluc-

[3]This stability feature of exchange market shares is documented by Budish, Lee, and Shim (2024), both in aggregate and within-symbol, consistent with the market share dynamics observed in our sample. Figure 1 clearly reflects this stability for the NYSE versus other market centers. We will later demonstrate that market shares exhibit considerable volatility over shorter horizons. Budish et al. (2024) also documents that, for the top five exchanges, including the NYSE, an exchange's market share in total consolidated share volume has a one-to-one relationship with its market share in liquidity. In this respect, our decomposition using share volume is equivalent to a decomposition based on liquidity.

tuations across exchanges. Specifically, we show that stocks in the top quintile of NYSE's long-term market share have four-factor alphas that are 0.52% lower per month than those in the bottom quintile. In contrast, the difference in four-factor alphas between the top and bottom quintiles of average absolute short-term noise is 0.75% per month. Furthermore, the impact of average absolute short-term noise on expected returns intensifies as NYSE's long-term market share decreases. While this component generates a significant return premium across different levels of long-term market share of the NYSE, this monthly premium declines from 1.15% for stocks with the lowest NYSE long-term market share to 0.24% for those with the highest.

Market fragmentation also challenges traditional liquidity measures, such as those proposed by Amihud (2002) and Pastor and Stambaugh (2003), because these measures are based on how stock returns respond to trading volume in dollars. However, the interpretation of trading volume differs significantly when it is concentrated in a single exchange versus when it is distributed across multiple exchanges. In this context, we first demonstrate, based on Amihud (2002), that while illiquidity consistently exhibits a significantly positive return premium under varying degrees of fragmentation, this premium monotonically rises from 0.57% per month for the least fragmented stocks to 2.08% per month for the most fragmented ones. To capture this relationship, we construct a new fragmentation-adjusted illiquidity measure. Our measure is a variant of the original Amihud measure, but it captures illiquidity not only in its traditional sense but also its interaction with fragmentation. It generates a 1.22% monthly premium in four-factor alphas between its top and bottom quintile portfolios. This effect remains both economically and statistically significant after controlling for a variety of other risk factors and stock characteristics.

Finally, we formalize the effect of market fragmentation on asset pricing using a search model. In a fragmented market, investors are uncertain about where to locate liquidity. They can only partially direct their search because neither market makers nor investors observe the realized liquidity of the exchanges. The failure of buyers and sellers to coordinate their orders across exchanges generates uncertainty in the fragmented market, reducing both matching efficiency and the price of the asset.

Our paper also holds policy significance. Policy discussions regarding trading and markets generally focus on their impacts on market quality. However, we emphasize that frictions stemming from market microstructure and its regulations can also influence asset prices.

Regulations can have unintended consequences, which can lead to significant changes in market structure and consequently shape investors' risk perceptions, potentially to varying degrees across different stocks. We demonstrate that market fragmentation, an unintended consequence of Regulation NMS, can serve as such an impetus. In this respect, the SEC's recent amendments on September 18, 2024, to reduce exchange access fees and minimum price increments, might affect asset prices, as they are likely to influence competition between market centers and, consequently, fragmentation.

Our paper is related to three strands of literature: market fragmentation, the cross-section of expected stock returns, and liquidity. The literature on market fragmentation typically examines how fragmentation affects market quality, yielding mixed results. For instance, Foucault and Menkveld (2008) shows that fragmentation can improve liquidity and lower trading costs. O'Hara and Ye (2011) finds that fragmentation increases short-term volatility but improves transaction costs, execution speeds, and market efficiency. Chen and Duffie (2021) demonstrates that fragmentation reduces price-impact avoidance, making collective exchange prices more informative. In contrast, Baldauf and Mollner (2021) shows that fragmentation intensifies adverse selection, outweighing its benefit of lowering trading fees through competition. Haslag and Ringgenberg (2023) documents that fragmentation generally benefits large stocks, while small stocks tend to experience poorer market quality. Other studies examining visible or dark fragmentation include, but are not limited to, Bennett and Wei (2006), Degryse, de Jong, and van Kervel (2015), Kwan, Masulis, and McInish (2015), Pagnotta and Philippon (2018), and Guo and Jain (2023).

Departing from these studies on market microstructure, we focus on the implications of market fragmentation for asset prices, which, in turn, relates our paper to the literature examining the cross-section of expected stock returns. Harvey, Liu, and Zhu (2016), Green, Hand, and Zhang (2017), Linnainmaa and Roberts (2018), Feng, Giglio, and Xiu (2020), and Jensen, Kelly, and Pedersen (2023) extensively cover asset pricing factors. This literature typically ignores the evolving dynamics of equity markets and overlooks microstructure aspects (O'Hara (2003)). Our paper takes a step toward filling this gap and establishes a connection between microstructure and asset pricing. Regulation NMS has dispersed trading across numerous exchanges, increasing uncertainty about each exchange's daily liquidity share. Harvey et al. (2016) defines a risk factor as "a variable that has unpredictable variation through time." Our new fragmentation measure meets this criterion, being unpredictable

both in the cross-section and over time, and is robustly priced.

We also contribute to the literature studying the effect of illiquidity on asset pricing: among others, Amihud (2002), Acharya and Pedersen (2005), Pastor and Stambaugh (2003), Brennan, Huh, and Subrahmanyam (2013), Bali, Peng, Shen, and Tang (2014), Lou and Shu (2017), Amihud and Noh (2021). This strand of research intersects with the broader asset pricing literature, as illiquidity is often conceptualized as a priced risk factor. We demonstrate that the impact of illiquidity on asset prices depends on market fragmentation, and to account for this variation, we develop a new fragmentation-adjusted illiquidity metric.

The rest of the paper is organized as follows. Section 2 discusses the institutional details of the U.S. equities market. Section 3 introduces the data and empirical framework. Section 4 examines the effects of fragmentation on asset pricing using HHI and discusses its limitations. Section 5 develops a new fragmentation measure and applies it to asset pricing. Section 6 links fragmentation to illiquidity and introduces a unified measure accounting for both. Section 7 presents our theoretical model. Sections 8 and 9 discuss policy implications and robustness of our results, respectively. Finally, Section 10 concludes.

2 Institutional Details

The U.S. equities market has undergone a significant transformation over the past three decades. Until April 2005, the NYSE executed more than 80% of consolidated share volume for its listed stocks. However, since then, the NYSE has rapidly lost its dominance, reaching a market share of approximately 20% by early 2009 and remaining at this level thereafter. The market has become more fragmented in tandem with this shift. Some trading has shifted to off-exchange market centers, while others have moved to competing national securities exchanges, resulting in a more fragmented market structure. More specifically, for stocks listed on the NYSE, the Herfindahl–Hirschman Index (HHI) fluctuated around 8,000 until April 2005, then experienced a sharp decline over the next four years, stabilizing around 2,500 after January 2009.[4] The U.S. Department of Justice and the Federal Trade Commission consider a change of more than 100 points to be significant according to their guidelines, with a decrease of 5,500 representing a radical drop in market concentration.

[4]HHI is calculated using NYSE TAQ data, where market shares are computed for each exchange and off-exchange as a whole. HHI measures market concentration, and thus $10,000 - \text{HHI}$ represents fragmentation.

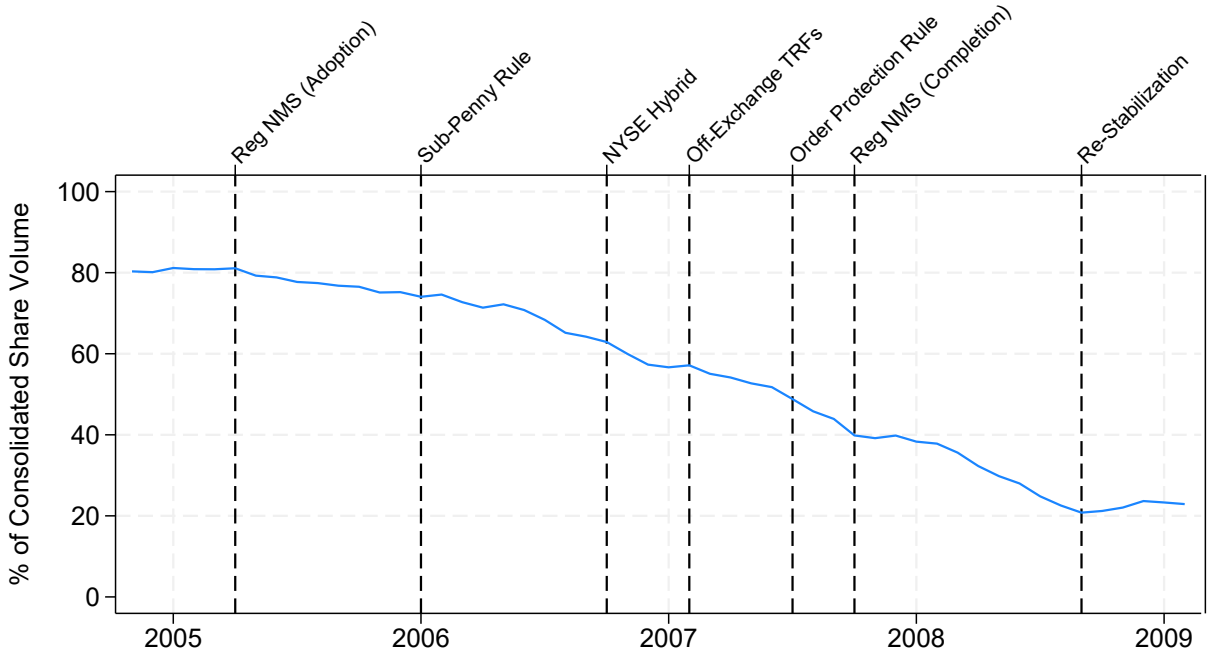


Figure 2: **Timing of Regulatory and Market Structure Changes and Market Shares for Stocks Listed on the NYSE.** The sample covers stock-months with non-missing data from November 2004 to February 2009, encompassing ordinary common shares listed on the NYSE. The left y-axis represents the percentage of monthly consolidated share volume executed on the NYSE. The seven dashed lines correspond to the following events and dates, from left to right: **(1)** the adoption of Regulation NMS in April 2005; **(2)** the implementation of the Sub-Penny Pricing Rule (Rule 612 of Regulation NMS) in January 2006; **(3)** the NYSE's implementation of the NYSE Hybrid in October 2006 to expand its electronic trading; **(4)** the requirement to report off-exchange trades separately from on-exchange trades through trade reporting facilities (TRFs) in March 2007; **(5)** the implementation of the Order Protection Rule (Rule 611 of Regulation NMS) in July 2007; **(6)** the completion of phased-in compliance with Regulation NMS in October 2007; **(7)** the period around when the NYSE's long-term market share stabilized again, September 2008.

The catalyst for these changes in market dynamics corresponds to the adoption of Regulation NMS. The SEC adopted Regulation NMS on April 6, 2005, which encompasses four major interrelated changes. Two of these changes are primarily responsible for the shift in trading from the NYSE to other market centers, resulting in greater fragmentation: namely, Sub-Penny Pricing Rule (Rule 612) and Order Protection Rule (Rule 611). The former has led to trading moving from the NYSE to off-exchange market centers, while the latter has led to trading shifting from the NYSE to other exchanges. Contrary to the common tendency in the literature to use the full implementation date of Regulation NMS, July-August

2007, as a shock to market structure, Figures 1 and 2 clearly demonstrate that the market structure had significantly changed by that time.[5] By the time Regulation NMS was fully implemented, the NYSE had already lost more than 30% of its market share, and the market had become 150% more fragmented.

The primary reason why Regulation NMS began affecting markets even before its implementation is the Sub-Penny Pricing Rule, which set the minimum price increment (i.e., tick size) to \$0.01 for all quotations priced at or above \$1.00. This rule applies to both national securities exchanges and off-exchange market centers—alternative trading systems (ATSs) and non-ATSs. It has often created artificially larger quoted bid-ask spreads, even for actively traded stocks, as quoted spreads frequently remain at or just above a penny (Kwan et al. (2015)). This dynamic, together with exchanges' price-time priority rules according to which limit orders are filled, results in longer queues on exchanges' limit order books, which, in turn, has led investors to off-exchange market centers to jump these long queues. This migration is likely to be reinforced by network externalities: as more investors switch to off-exchange market centers, execution becomes more likely, resulting in further migration.

Larger quoted spreads have also facilitated off-exchange trading by providing more incentive for investors to seek sub-penny price improvements (Bartlett and McCrary (2015)). Although the Sub-Penny Pricing Rule does not prohibit any market center from executing trades in sub-penny increments, it has led to a more frequent occurrence of sub-penny trading on off-exchange market centers, especially non-ATSs, compared to exchanges.[6] Exchanges match multilateral orders on an agency basis in accordance with publicly disclosed rules and procedures, without the discretion of their operators or trading parties, where orders submitted must be in penny increments as per Rule 612. In this environment, sub-penny trading occurs on exchanges only for mid-point or volume-weighted executions or for executions falling under exemptions outlined in Rule 612(c). In contrast, it is easier for non-ATSs to trade in sub-penny increments because they typically trade in a principal capacity and exercise discretion on a per-order basis in determining price improvements, without needing to establish publicly disclosed pre-determined rules and procedures. In this regard, the Sub-

[5]For instance, Haslag and Ringgenberg (2023) compares the market quality of small and large stocks in a difference-in-differences setting, using July-August 2007 to separate pre- and post-policy periods.

[6]For instance, in the first half of 2022, of the total dollar volume of sub-penny trades, excluding mid-point and volume-weighted average price executions, 11% occurred on exchanges while 89% occurred on off-exchange market centers (87 FR 80266, p. 80273).

Penny Pricing Rule has provided non-ATSs with a competitive edge relative to exchanges and helped them attract additional order flow.

The initial compliance date of the Sub-Penny Pricing Rule was set for August 29, 2005, giving investors and their brokers less than five months to adjust their routing matrices. Figure 2 clearly reveals that investors started to adjust their routing immediately after the adoption of the Sub-Penny Pricing Rule instead of waiting for its implementation. This rapid movement in trading is likely due to the fact that making such a significant change in routing and re-optimizing routing matrices requires time, prompting market participants to begin acting sooner rather than later. The compliance date was later extended to January 31, 2006, on August 2, 2005. Figure 2 reveals that the drop in the NYSE's market share accelerated after the official implementation of the Sub-Penny Pricing Rule in January 2006.

The Order Protection Rule has further contributed to the shift of trading from the NYSE to other market centers, especially to other national securities exchanges. It was originally scheduled to be implemented towards mid-2006; however, its implementation was delayed for more than a year because market centers needed additional time to modify their trading systems and conduct quality assurance tests beforehand. The rule began to be implemented for 250 NMS stocks on July 9, 2007, and for all remaining NMS stocks on August 20, 2007. This rule prohibits any market center from executing orders that are worse than protected quotations—lower than a protected bid or higher than a protected offer. It protects only the best bid and best offer at the top of each market center's order book. The remaining quotations lower in a market center's book are not guaranteed to receive price priority, even if they are better than the best available protected prices in other market centers.[7] This situation has created an opportunity for other exchanges to draw order flow from the NYSE, as each exchange has its top quotes protected, consequently intensifying competition against the NYSE. Larger orders have typically begun to be executed across multiple exchanges, resulting in a more fragmented market structure and the decline of the NYSE's previously dominant position as a trading venue (Spatt (2018)).[8]

[7]For instance, in the spirit of Stoll (2006)'s example, suppose there are only two market centers: A and B. A bids \$50.05 for 300 shares and \$50.00 for 200 shares, while market center B bids \$49.95 for 400 shares. The Order Protection Rule protects A's bid of \$50.05 but not its bid of \$50.00. An incoming market order to sell 500 shares would be required to execute at the best available "top of the book" prices, meaning it would sell 300 shares in A at \$50.05 and 200 shares in B at \$49.95.

[8]In its Form 424B3 filing on May 5, 2006, the NYSE mentioned its expectation that Regulation NMS could adversely affect their business by intensifying competition between market centers following the Order

Under Regulation NMS, for a quote to be protected, it must be an automated quotation displayed by an automated trading center. These automation aspects generally require that quotations be immediately accessible and automatically executable without any delay, which, along with increased competition from fully electronic market centers, such as electronic communications networks, has put pressure on the NYSE to automate its trade execution processes.[9] The NYSE officially adopted a new market system, introduced as a mixture of floor-based auction and electronic trading and called the Hybrid Market, on March 22, 2006, and gradually expanded its automatic order routing and execution practices between October 6, 2007 and January 24, 2007. This initiative likely accelerated the NYSE's loss of market share, as it forfeited an important competitive advantage provided by the old specialist system, which featured dominant manual elements (Spatt (2018)), thereby making the NYSE more similar to other exchanges in terms of trade execution processes.

3 Data and Empirical Framework

We use two main data sources: the Center for Research in Securities Prices (CRSP) and NYSE Trades and Quotes (TAQ). CRSP provides daily stock prices, returns, shares outstanding, primary listing exchange information, and other stock characteristics. The main analysis includes all ordinary common stocks in CRSP (CRSP share code: 10-11) that are listed on the NYSE (CRSP primary exchange: N). TAQ contains intraday transactions data (trades and quotes) for all stocks listed on the New York Stock Exchange (NYSE), American Stock Exchange (AMEX), Nasdaq Stock Market (NASDAQ), and SmallCap issues. We obtain exchange-specific daily share volumes from TAQ. We match CRSP and TAQ using the WRDS Daily TAQ CRSP Link. We utilize similar filters as Amihud (2002), requiring the total number of trading days for stock i in month t (D_{it}) to be greater than or equal to 10 and excluding stock-months with monthly average share prices less than \$5.

Our sample spans the period from January 1993 to December 2023. We conduct univariate and bivariate portfolio analyses, as well as perform stock-level regressions following

Protection Rule.

[9] Another key feature for a quote to be protected is that it must be disseminated through the consolidated market data feeds. The dissemination aspect implies that no quotes are protected when the consolidated market data feeds are not in operation. Please refer to the SEC's memorandum on April 30, 2015 for more details regarding the Order Protection Rule and other key elements of a quote required to receive protection.

Fama and MacBeth (1973) to confirm that our results remain robust after accounting for other risk factors and control variables. In Fama-MacBeth regressions, we conduct monthly cross-sectional regressions and report the time-series averages of the coefficients, along with their associated Newey and West (1987) standard errors with six lags. These analyses are primarily conducted for the entire sample, but we occasionally apply them to two particular sub-samples, 1993–2004 and 2009–2023, which represent two distinct yet stable market structures. The former period corresponds to the pre-Regulation NMS environment, where the NYSE, as the primary listing exchange, heavily dominated trading in its listed stocks. The latter period corresponds to the post-Regulation NMS environment, where trading is highly fragmented across exchanges, with no single exchange dominating. Despite these differences, a common feature across the two sub-samples is that the NYSE's long-term market share remains stable in both periods, as shown in Figure 1.

Our dependent variable in Fama-MacBeth regressions is the monthly three-factor adjusted return, as defined in Fama and French (1993).[10] Factor loadings for a stock in month t are estimated using its monthly excess returns over the risk-free rate and the three Fama-French factors from the previous 60 months ($t-60$ to $t-1$), with a minimum requirement of 24 observations for the estimation. Our main independent variables of interest are fragmentation measures in Section 5 and fragmentation-adjusted illiquidity measures in Section 6. These measures are developed in their respective sections.

In addition to these primary independent variables, we include two sets of control variables in our regressions. The first set of variables encompasses those that are correlated with our fragmentation or fragmentation-adjusted illiquidity measures and have been established to affect expected stock returns in the literature. Foucault and Menkveld (2008) show that fragmentation affects liquidity supply, and it has been well established in the literature that liquidity affects expected returns.[11] To this end, we control for the logarithm of the original Amihud illiquidity measure lnA, with A calculated as the monthly average of the daily ratio of absolute return to dollar trading volume in millions, based on daily trading volume data

[10] We select this dependent variable to align our study with existing research on liquidity, facilitating comparisons of our new fragmentation-adjusted illiquidity measure with those commonly employed in such studies. See, for instance, Pastor and Stambaugh (2003), Brennan et al. (2013), and Lou and Shu (2017). Our results are robust to using monthly excess stock returns over the risk-free rate, as adopted in Amihud and Noh (2021).

[11] Among others, for instance, Amihud (2002), Pastor and Stambaugh (2003), Brennan et al. (2013), Lou and Shu (2017) and Amihud and Noh (2021).

from TAQ. Fragmentation is also related to price impact through changing traders' incentives to send more aggressive orders (Chen and Duffie (2021)) and price impact affects expected stock returns (Lou and Shu (2017)). Therefore, we control for monthly average of daily price impact coefficients, where daily values are directly obtained from WRDS Intraday Indicators. WRDS estimates daily price impact coefficients based on regressions of five-minute midpoint changes against the square root of the signed absolute net volume (absolute difference between buyer- and seller-initiated share volume times sign of this difference). In the regression for month t, these two independent variables along with our primary independent variables are taken from month $t-2$.[12]

The second set comprises standard control variables, as discussed in Lou and Shu (2017) and Amihud and Noh (2021), which include idiosyncratic return volatility and past returns. Idiosyncratic volatility is estimated as the standard deviation of the residuals from regressions of daily stock returns against the daily Fama-French three factors over the previous year. Additionally, we include the previous month's stock return to control for short-term price reversals and cumulative stock returns over the past twelve months, excluding the previous month, to control for momentum.

4 A Traditional Way to Measure Fragmentation

In this section, we analyze how the Herfindahl–Hirschman Index (HHI), a widely used measure to gauge fragmentation in the literature, affects expected stock returns and discuss practical considerations in calculating it. We then introduce a simple yet effective framework that decomposes market shares of exchanges into persistent long-term and noisy short-term components. We use this decomposition to discuss what the HHI actually measures in different market structures and demonstrate that the HHI fails to capture important market dynamics—an intricate limitation of the HHI in gauging fragmentation—that is not easily noticeable due to its black-box nature.

[12] We do not include lnA when examining how our new fragmentation-adjusted illiquidity measure affects expected stock returns in Section 6 since our new measure embeds A, as will be demonstrated later.

4.1 Herfindahl-Hirschman Index and Expected Stock Returns

The TAQ data currently identifies share volume for each national securities exchange separately and all off-exchange market centers collectively. Let $j = 1, \ldots, J$ index exchanges, and $J+1$ represent all off-exchange centers as a whole.[13] The total consolidated share volume of stock i on day d in month t, denoted as $\mathrm{V}_{i,d,t}$, is equal to $\sum_{j=1}^{J} \mathrm{V}_{i,d,t}^{j} + \mathrm{V}_{i,d,t}^{J+1}$, with $\mathrm{V}_{i,d,t}^{j}$ and $\mathrm{V}_{i,d,t}^{J+1}$ denoting share volumes executed on exchange j and off-exchange market centers, respectively. In terms of market share, exchange j executes the fraction $\frac{\mathrm{V}_{i,d,t}^{j}}{\mathrm{V}_{i,d,t}}$ of total share volume. The HHI for stock i on day d in month t, denoted as $\mathrm{HHI}_{i,d,t}$, can be calculated as:

$$\mathrm{HHI}_{i,d,t} := \sum_{j=1}^{J+1} \left(\frac{\mathrm{V}_{i,d,t}^{j}}{\mathrm{V}_{i,d,t}} \right)^{2} \tag{1}$$

Calculating market shares, and thus the HHI, is more empirically challenging than it might initially appear due to off-exchange trades. Prior to March 5, 2007, off-exchange market centers reported their trades to exchanges, which then reported these trades to the consolidated tape along with their on-exchange transactions, as described in Bartlett and McCrary (2015). Consequently, off-exchange market shares cannot be separately identified in the data before March 2007.[14] Furthermore, during this period, consolidated data revenue was allocated among exchanges—the only eligible entities—primarily in proportion to their reported trades. This market structure incentivized exchanges to establish arrangements with off-exchange market centers, mainly electronic communication networks (ECNs), to increase their reported trades, effectively boosting data revenue. These arrangements created noticeable jumps in the market shares of certain exchanges, including NASDAQ, Boston, Cincinnati, and Pacific Stock Exchanges, as demonstrated in Caglio and Mayhew (2016). This market structure artificially distorts HHIs prior to March 5, 2007, due to reporting practices rather than actual changes in market concentration.

[13] In the data, the number of exchanges varies across months. For notational simplicity, we omit the time subscript and use J to refer to the time-varying number of exchanges.

[14] In terms of our notations, before March 5, 2007, $\mathrm{V}_{i,d,t}^{J+1}$ is not observed separately in the data. Instead, this total off-exchange share volume is included in the reported exchange volumes, $\left\{ \mathrm{V}_{i,d,t}^{j} \right\}_{j=1}^{J}$. It cannot be determined how much of $\mathrm{V}_{i,d,t}^{j}$ corresponds to exchange j's own trades and how much corresponds to off-exchange trades.

Starting from March 5, 2007, off-exchange transactions began to be reported to designated trade reporting facilities (TRFs) and can therefore be separately identifiable in the data. However, although this change in reporting has made market shares of exchanges free from off-exchange trades, it has complicated the construction of a consistent HHI in samples that span both before and after this date, as in our case, since a market share of an exchange is not directly comparable before and after this date. For the period before March 2007, one has no choice but to overlook these data limitations despite their issues and calculate the HHI among the J exchanges. Afterward, the HHI can either be calculated solely among the J exchanges, as in Chao, Yao, and Ye (2017) and Guo and Jain (2023), or by treating off-exchange market centers as a single entity in addition to the J exchanges, calculating the HHI among $J+1$ entities, as in Haslag and Ringgenberg (2023).

The first approach focuses on national securities exchanges, which share more similar characteristics with each other than with off-exchange market centers. However, this approach excludes a significant portion of total trading volume. Due to the reporting structure, econometricians are forced to incorporate this volume into pre-March 2007 HHI calculations, leading to inconsistencies across periods in terms of trading volume coverage. The second approach captures the entire volume in either period, thereby ensuring a more uniform HHI metric in terms of coverage across different periods. Yet, it groups approximately 200 off-exchange market centers as a single entity and treats this entity like a giant exchange. Ultimately, econometricians relying on TAQ data to calculate the HHI over extended periods face a trade-off: calculating the HHI among more homogeneous entities in each reporting period or ensuring higher consistency across periods in terms of volume coverage. We adopt the second approach to maintain a more comparable metric throughout our sample period.

We now proceed to construct univariate portfolios based on monthly HHI values to examine how market fragmentation affects expected stock returns, where the monthly HHI is calculated as the average of daily HHI values. At the beginning of each month t from 1993 to 2023, we sort stocks into quintiles based on their HHI values from month $t-2$. We report four-factor alphas, calculated using the three Fama-French factors (MKT, SMB, and HML) and the momentum factor (UMD). [15]

[15] Our results are robust to using excess returns over the risk-free rate. We do not report these results for the sake of brevity and to avoid repetition, as both measures produce similar results.

	Low	2	3	4	High	H-L	t-stat	p-val
1993-2023	0.44	0.29	0.36	0.34	0.74	0.29	1.37	0.17
1993-2004	0.90	0.25	0.16	0.09	0.19	-0.70	-2.49	0.01
2009-2023	0.14	0.20	0.34	0.35	1.09	0.95	4.46	0.00

Table 1: **Monthly Excess Portfolio Returns Sorted on HHI**

Portfolios are constructed from January 1993 to December 2023 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. At the beginning of each month t, from January 1993 to December 2023, stocks are sorted into quintile portfolios based on HHI from two months earlier, month $t-2$. We then calculate monthly equal-weighted excess returns over four factors and report time-series averages for each quintile (Fama-French three factors—MKT, SMB, and HML—and the momentum factor—UMD), where the factor loadings are estimated using the past 60 months, with the requirement of having at least 24 observations. The differences between the top and bottom quintiles are also reported, along with the associated t-statistics and p-values. **HHI** is calculated as the monthly average of daily Herfindahl–Hirschman indices, where the daily index is calculated as the sum of the squares of daily market shares on each exchange and off-exchanges as a whole, based on their share volumes.

Table 1 presents equally weighted four-factor alphas of these portfolios. In the entire sample, stocks in the highest HHI quintile have higher alphas than stocks in the lowest HHI quintile, but this difference is not statistically significant. However, once we focus on two sub-periods, 1993–2004 and 2009–2023—each with a more homogeneous and stable market structure—we find that HHI not only has return premiums but also opposing ones. In the former period, alpha typically decreases with HHI, indicating that fragmentation has a positive premium. Stocks in the lowest HHI quintile—the most fragmented—provide 0.70% more alpha per month than stocks in the highest HHI quintile, the most concentrated. In the latter period, alpha monotonically increases in HHI, indicating that fragmentation has a negative premium. Stocks in the lowest quintile earn 0.95% less alpha per month than stocks in the highest quintile.

We also conduct regression analysis to test whether the impact of HHI remains robust after accounting for other control variables. Table 2 presents these results. HHI exhibits a positive premium in the entire sample as before. However, it is now statistically significant. This finding suggests that investors tend to demand higher returns for holding stocks with highly concentrated trading. Specifically, a one standard deviation increase in HHI is associated with a 0.77% higher alpha per month, based on Regression 4. The impacts of HHI during the two sub-samples are also consistent with our previous portfolio analysis. HHI has negative and positive coefficients in 1993-2004 and 2009-2023, respectively, which are both economically and statistically significant.

These results indicate that investors require compensation for holding stocks with higher fragmentation before Regulation NMS, while they demand compensation for holding more concentrated stocks after Regulation NMS. They appear conflicting in terms of how HHI affects asset pricing at first glance. However, this conflicting aspect of HHI stems from its black-box nature. A closer examination of what HHI captures in each sub-sample provides a reconciliation. To this end, we now turn to market share decomposition.

	(1)	(2)	(3)	(4)	(5)	(6)
	1993-2023	1993-2004	2009-2023	1993-2023	1993-2004	2009-2023
HHI	1.154**	-1.643**	3.676***	1.188**	-1.390**	3.421***
	(0.565)	(0.660)	(0.750)	(0.519)	(0.628)	(0.674)
lnA	0.297***	0.162***	0.445***	0.138***	0.068	0.222***
	(0.041)	(0.052)	(0.056)	(0.036)	(0.062)	(0.050)
λ				0.021***	0.005***	0.034***
				(0.004)	(0.001)	(0.006)
Observations	448924	197015	192808	448924	197015	192808
Adjusted R^2	0.045	0.036	0.055	0.049	0.038	0.061
Other Controls	✓	✓	✓	✓	✓	✓

Table 2: **Fama-MacBeth Regressions of Excess Returns on HHI**

Monthly Fama-MacBeth regressions are estimated from January 1993 to December 2023 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. We estimate a cross-sectional regression in each month t and then report the time-series averages of the coefficients and standard errors, as well as the adjusted R^2. The **dependent variable** is the monthly excess return over four factors (Fama-French three factors—MKT, SMB, and HML—and the momentum factor—UMD), where the factor loadings are estimated using the past 60 months, with the requirement of having at least 24 observations. **HHI** is calculated as the logarithm of the monthly average of daily Herfindahl–Hirschman indices, where the daily index is calculated as the sum of the squares of daily market shares on each exchange and off-exchange centers as a whole, based on their share volumes. $\boldsymbol{\lambda}$ is the monthly average of daily price impact coefficients. **lnA** is the natural logarithm of the original Amihud illiquidity measure A, calculated as the monthly average of the daily ratio of absolute return to dollar trading volume in millions. A is calculated using daily trading volume data from TAQ. In the regression for month t, these independent variables are taken from month $t-2$. Additionally, we include other standard control variables, which are defined explicitly in Section 3, with coefficients not presented to save space: previous month's stock return, cumulative stock returns over the past twelve months excluding the previous one, and idiosyncratic return volatility. t-statistics are calculated using Newey-West robust standard errors with six lags. **Standard errors** are in parentheses; * $p < 0.10$ ** $p < 0.05$ *** $p < 0.01$.

4.2 Market Share Decomposition

Budish et al. (2024) document that exchanges have very stable market shares, which is consistent with the market share dynamics in our sample. In this context, the daily market share of exchange j for a stock can be decomposed into a stable long-term component and a noisy short-term component as follows:

$$\frac{\mathrm{V}^{j}_{i,d,t}}{\mathrm{V}_{i,d,t}} = \gamma^{j}_{i,-t} + \frac{\varepsilon^{j}_{i,d,t}}{\mathrm{V}_{i,d,t}} \quad \text{for } j = 0, 1, \ldots, \mathrm{J}. \tag{2}$$

This formulation asserts that each exchange's market share in total trading volume for a stock remains stable over longer periods, while experiencing noisy fluctuations over shorter ones. The parameter $\gamma^{j}_{i,-t}$ represents long-term market share of exchange j for stock i. It is predetermined and reflects stable trading patterns from past periods. It remains constant across the days of a month but can vary from one month to another. Specifically, we compute $\gamma^{j}_{i,-t}$ for month t as the average daily market share of stock i on exchange j using data over the preceding m months, where we use $m = 3$ in our main analyses.

The term $(\varepsilon^{j}_{i,d,t}/\mathrm{V}_{i,d,t})$ captures transient daily fluctuations in exchange j's market share due to external factors, such as traders' preferences, company announcements, or macroeconomic news. It is calculated as the residual of exchange j's daily market share, derived by subtracting $\gamma^{j}_{i,-t}$ from $(\mathrm{V}^{j}_{i,d,t}/\mathrm{V}_{i,d,t})$. The stability of market shares across exchanges implies that the total market share of off-exchange market centers is also stable. This stability arises from the fact that, by construction, $\gamma^{J+1}_{i,-t} = 1 - \sum_{j=1}^{J} \gamma^{j}_{i,-t}$ and $\varepsilon^{J+1}_{i,d,t} = -\sum_{j=1}^{J} \varepsilon^{j}_{i,d,t}$, where $\gamma^{J+1}_{i,-t}$ and $\varepsilon^{J+1}_{i,d,t}$ are defined similarly for off-exchange volumes. Using this decomposition, HHI can be written as:

$$\mathrm{HHI}_{i,d,t} = \sum_{j=1}^{J+1} \left(\gamma^{j}_{i,-t} + \frac{\varepsilon^{j}_{i,d,t}}{\mathrm{V}_{i,d,t}} \right)^{2} \tag{3}$$

In this formula, an exchange j's long-term market share and short-term noise are perfect substitutes, as they are linearly additive. To illustrate when this poses a problem, consider exchange j's contribution to the HHI: $\left(\gamma^{j}_{i,-t}\right)^{2} \left(1 + \left[\frac{\varepsilon^{j}_{i,d,t}}{\mathrm{V}_{i,d,t}}\right] / \gamma^{j}_{i,-t}\right)^{2}$.

If the short-term noise in exchange j's market share is sufficiently small relative to its long-term counterpart, that is, if $\frac{\varepsilon^j_{i,d,t}}{\mathrm{V}_{i,d,t}}/\gamma^j_{i,-t}$ is small, then j's contribution to the HHI is approximately equal to $(\gamma^j_{i,-t})^2$. In this scenario, perfect substitutability is not a major concern. This situation describes the era before Regulation NMS, where short-term noise had negligible impacts on fragmentation, and the HHI was roughly equal to $\sum_{j=1}^{J+1}(\gamma^j_{i,-t})^2$. Equally importantly, this approximate sum primarily reflected the NYSE's long-term market share in this era, as it dominated trading in its listed stocks with a median long-term market share of 88%.[16] To sum up, an increase in HHI during 1993–2004 is almost equivalent to a higher concentration of trading on the primary listing exchange, the NYSE. The correlation of HHI with the NYSE's long-term market share is 0.83 in this period.

However, if the short-term noise in j's market share is relatively large compared to its long-term counterpart, then this short-term noise can also make a meaningful contribution to the HHI. This situation applies to the post-Regulation NMS era. If short-term and long-term components have different implications for asset pricing, perfect substitutability becomes a major concern.[17] Just as crucially, the NYSE lost its dominance in trading its listed stocks, significantly weakening its long-term market share correlation with HHI to 0.42, indicating that an increase in HHI during 2009–2023 does not necessarily reflect a higher concentration on the NYSE in this period. To summarize, in the post-Regulation NMS period, the HHI captures not only the long-term market share of the primary listing exchange but also those of other exchanges and short-term noises, and thus can approximately be expressed as:

$$\mathrm{HHI}_{i,d,t} \approxeq \underbrace{\left(\gamma^{\mathrm{pex}}_{i,-t}\right)^2}_{\mathrm{HHI(LT)}^{\mathrm{pex}}_{i,-t}} + \underbrace{\sum_{j\neq \mathrm{pex}} \left(\gamma^j_{i,-t}\right)^2}_{\mathrm{HHI(LT)}^{\mathrm{other}}_{i,-t}} + \underbrace{\left(\frac{\varepsilon^{\mathrm{pex}}_{i,d,t}}{\mathrm{V}_{i,d,t}}\right)^2}_{\mathrm{HHI(ST)}^{\mathrm{pex}}_{i,d,t}} + \underbrace{\sum_{j\neq \mathrm{pex}} \left(\frac{\varepsilon^j_{i,d,t}}{\mathrm{V}_{i,d,t}}\right)^2}_{\mathrm{HHI(ST)}^{\mathrm{other}}_{i,d,t}} \tag{4}$$

where *pex* and *other* denote primary listing exchange and other exchanges, respectively.

[16] To further illustrate how this domination affects the HHI, consider a stock where $\gamma^{\mathrm{nyse}}_{i,-t} = 0.88$. This approximated sum takes its maximum value when another exchange executes the rest, and all others have zero market shares. In this case, the NYSE's contribution to the HHI is $\frac{0.88^2}{0.88^2+0.12^2} = 98.2\%$, which provides a lower bound for the NYSE's contribution to the HHI. In any other allocation of the 0.12 market share among other exchanges, the NYSE's contribution to the HHI would be higher.

[17] In Appendix B, we provide a numerical example to demonstrate how HHI treats long-term market shares and short-term noises as perfect substitutes, consequently failing to take into account important market dynamics.

Table 3 presents how each of these four components, as well as the total long-term and short-term components, affect expected returns in a univariate monthly portfolio setting. At the beginning of each month t from 1993 to 2023, we sort stocks into quintiles based on those components from month $t-2$ and report their four-factor adjusted returns (alphas).

	Low	2	3	4	High	H-L	t-stat	p-val
HHI(LT)	0.52	0.31	0.33	0.29	0.71	0.19	0.90	0.37
$\text{HHI(LT)}^{\text{pex}}$	0.80	0.40	0.38	0.29	0.28	-0.52	-3.29	0.00
$\text{HHI(LT)}^{\text{other}}$	0.20	0.26	0.24	0.36	1.10	0.91	5.23	0.00
HHI(ST)	0.16	0.26	0.34	0.51	0.90	0.74	6.99	0.00
$\text{HHI(ST)}^{\text{pex}}$	0.20	0.22	0.41	0.53	0.81	0.61	5.26	0.00
$\text{HHI(ST)}^{\text{other}}$	0.16	0.28	0.29	0.53	0.91	0.75	7.89	0.00

Table 3: **Monthly Excess Portfolio Returns Sorted on HHI Components**

Portfolios are constructed from January 1993 to December 2023 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. At the beginning of each month t, from January 1993 to December 2023, stocks are sorted into quintile portfolios based on HHI components from two months earlier, month $t-2$. We then calculate monthly equal-weighted excess returns over four factors and report time-series averages for each quintile (Fama-French three factors—MKT, SMB, and HML—and the momentum factor—UMD), where the factor loadings are estimated using the past 60 months, with the requirement of having at least 24 observations. The differences between the top and bottom quintiles are also reported, along with the associated t-statistics and p-values. Daily HHI is decomposed into long-term (LT) and short-term (ST) components, where each component is defined in Equation 4. $\mathbf{HHI(ST)^{pex}}$ and $\mathbf{HHI(ST)^{other}}$ are calculated as the monthly average of their daily values. $\text{HHI(LT)} = \text{HHI(LT)}^{\text{pex}} + \text{HHI(LT)}^{\text{other}}$ and $\text{HHI(ST)} = \text{HHI(ST)}^{\text{pex}} + \text{HHI(ST)}^{\text{other}}$.

The results demonstrate the intricate nature of HHI. Firstly, without explicit distinction between the primary listing and other exchanges, one might erroneously conclude that long-term market share concentration is of no importance for asset pricing, given that the resulting total sum, HHI(LT), does not produce any return premium. Long-term concentration of share volume on the primary listing exchange is negatively associated with expected stock returns, while long-term concentration on other exchanges is positively correlated with expected stock returns. Alpha monotonically decreases in $\text{HHI(LT)}^{\text{pex}}$ from 0.80% in the lowest quintile, stocks with the lowest concentration on the primary listing exchange, to 0.28% in the highest quintile, the most highly concentrated stocks, yielding a spread of -0.52%. Similarly, $\text{HHI(LT)}^{\text{other}}$ shows that alpha typically increases with long-term market share concentration in exchanges outside the primary listing exchange, a result that aligns with the near-perfect negative correlation between $\text{HHI(LT)}^{\text{pex}}$ and $\text{HHI(LT)}^{\text{other}}$. Secondly,

Table 3 demonstrates that short-term noises are consistently positively associated with expected stock returns, illustrating that long-term market share concentration and short-term noises have different implications for asset pricing.

Before Regulation NMS, HHI is heavily dominated by $\text{HHI(LT)}^{\text{pex}}$, resulting in a negative association between HHI and expected stock returns. After Regulation NMS, HHI primarily reflects $\text{HHI(LT)}^{\text{other}}$ and HHI(ST), both of which are positively associated with expected stock returns, leading HHI to have a positive assification as well. HHI does not explicitly take account these market dynamics, resulting in seemingly contradictory conclusions regarding how fragmentation affects expected stock returns before and after Regulation NMS. To this end, rather than relying on the black-box nature of HHI to gauge fragmentation, we now turn to develop a novel fragmentation measure.

5 Fragmentation Premium

We have demonstrated HHI's limitations in gauging fragmentation due to its treatment of long-term market shares and short-term noises as perfect substitutes. In this section, we first construct a simple but effective alternative measure to gauge fragmentation that circumvents this problem to a great extent. We then analyze how our new fragmentation measure affects expected stock returns.

5.1 A Novel Way to Measure Fragmentation

Using our market share decomposition in Equation 2, an exchange j's market share for stock i on day d in month t is expressed as:

$$\frac{\text{V}^j_{i,d,t}}{\text{V}_{i,d,t}} = \gamma^j_{i,-t} + \frac{\varepsilon^j_{i,d,t}}{\text{V}_{i,d,t}} \tag{5}$$

$\gamma^j_{i,-t}$ represents stock i's long-term market share on exchange j. Specifically, it is calculated as the average daily market share of stock i on exchange j over the past three months $(t-3, t-2, t-1)$. The term $\varepsilon^j_{i,d,t}/\text{V}_{i,d,t}$ captures short-term noise in stock i's daily market share on exchange j, which quantifies the daily deviations in stock i's market share on ex-

change j from its long-term value $\gamma^j_{i,-t}$, due to unpredictable daily market conditions. These conditions include factors, among others, traders' preferences, company announcements, or macroeconomic news. $\varepsilon^j_{i,d,t}/\mathrm{V}_{i,d,t}$ is calculated as the residual value of the market share, obtained by subtracting $\gamma^j_{i,-t}$ from $\mathrm{V}^j_{i,d,t}/\mathrm{V}_{i,d,t}$.

To illustrate how short-term noise affect expected returns, let's start with the primary listing exchange. Let $\varepsilon^{\text{pex}}_{i,t}$ be the monthly average of daily short-term noises for stock i on the NYSE:

$$\varepsilon^{\text{pex}}_{i,t} := \frac{1}{D_{it}} \sum_{d=1}^{D_{it}} \frac{\varepsilon^{\text{pex}}_{i,d,t}}{\mathrm{V}_{i,d,t}} \tag{6}$$

where D_{it} is the total number of days that stock i is traded in month t, with an index $d = 1, 2, \ldots, D_{it}$. At the beginning of each month t from 1993 to 2023, we sort stocks into quintiles based on their ε^{pex} from month $t-2$ and report their four-factor adjusted returns (alphas). Table 4 presents equally weighted four-factor alphas of these portfolios. Short-term noise on the NYSE does not seem to exhibit any excess return over four factors. The difference between the highest and lowest quintile returns is only -0.07%, economically small and statistically insignificant. However, both the lowest and highest quintiles consistently have higher returns than the middle quintile, which includes noises that are close to zero. This result suggests that stocks with larger noises, regardless of their signs, can generate higher returns relative to stocks with small noises.

	Low	2	3	4	High	H-L	t-stat	p-val
ε^{pex}	0.62	0.40	0.30	0.29	0.55	-0.07	-0.86	0.39

Table 4: **Monthly Excess Portfolio Returns Sorted on Short-Term Noise on the Primary Listing Exchange (ε^{pex})**

Portfolios are constructed from January 1993 to December 2023 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. At the beginning of each month t, from January 1993 to December 2023, stocks are sorted into quintile portfolios based on ε from two months earlier, month $t-2$. We then calculate monthly equal-weighted excess returns over four factors and report time-series averages for each quintile (Fama-French three factors—MKT, SMB, and HML—and the momentum factor—UMD), where the factor loadings are estimated using the past 60 months, with the requirement of having at least 24 observations. The differences between the top and bottom quintiles are also reported, along with the associated t-statistics and p-values. $\boldsymbol{\varepsilon}$ is the monthly average of the daily deviations in the NYSE's market share from its long-term value γ^{pex}, and is defined in Equation 6.

This feature of short-term noise is not particular to the primary listing exchange. It is also applicable to other exchanges and overall off-exchange market centers. Let J_{it} be the

number of markets that stock i is traded in during month t, where each exchange and total off-exchange market centers are referred to as separate markets. Abusing notation, let $|\varepsilon_{i,t}|$ denote the monthly average of the *absolute* daily short-term noises for stock i across markets where stock i is traded:

$$|\varepsilon_{i,t}|^{\text{total}} := \frac{1}{D_{it}} \sum_{d=1}^{D_{it}} \left(\frac{1}{J_{it}} \sum_{j=1}^{J_{it}} \left| \frac{\varepsilon^j_{i,d,t}}{\text{V}_{i,d,t}} \right| \right) \tag{7}$$

We use the pair $(|\varepsilon_{i,t}|^{\text{total}}, \gamma^{\text{pex}}_{i,-t})$ to construct a fragmentation measure. Specifically, we use their ratios to measure fragmentation:

$$\varphi_{i,t} := \frac{|\varepsilon_{i,t}|^{\text{total}}}{\gamma^{\text{pex}}_{i,-t}} \tag{8}$$

We can loosely interpret $\gamma^j_{i,-t}$ as the probability of matching with a counterparty on exchange j, which holds true on average in the long term. $|\varepsilon_{i,t}|$ represents the average deviations from these long-term matching probabilities over a month. In this regard, our fragmentation measure quantifies the extent of deviations in actual matching probabilities from their long-term values due to unpredictable daily factors, normalized using the long-term market share of the primary listing exchange, the NYSE. Larger values of $\varphi_{i,t}$ correspond to higher levels of fragmentation. Fragmentation increases when short-term fluctuations become larger or when the NYSE starts executing less trading consistently.[18]

Our fragmentation measure explicitly separates long-term market share from short-term

[18]It is easy to extend our fragmentation measure to include the long-term market shares of other exchanges, such as using a weighted average of the long-term market share in the denominator, $\omega\gamma^{\text{pex}}_{i,-t} + (1-\omega)\bar{\gamma}^{\text{other}}_{i,-t}$ or $(\gamma^{\text{pex}}_{i,-t})^{\omega}(\bar{\gamma}^{\text{other}}_{i,-t})^{(1-\omega)}$, where $(\omega, 1-\omega)$ represent weights, and $\bar{\gamma}^{\text{other}}_{i,-t}$ is defined as:

$$\bar{\gamma}^{\text{other}}_{i,-t} = \begin{cases} \frac{1}{J_{it}-1} \sum_{j \neq \text{pex}} \gamma^j_{i,-t} \text{ if } \gamma^{\text{pex}}_{i,-t} < 1 \\ 0 \text{ if } \gamma^{\text{pex}}_{i,-t} = 1 \end{cases}$$

We opt for our measure over this alternative one for two reasons. Firstly, as we will demonstrate later in this section, $\gamma^{\text{pex}}_{i,-t}$ and $\bar{\gamma}^{\text{other}}_{i,-t}$ affect asset pricing in opposing ways, and thus weighting them reintroduces some limitations of the HHI. Secondly, our measure can be rewritten as $|\varepsilon_{i,t}|/(1-(J_{it}-1)\bar{\gamma}^{\text{other}}_{i,-t})$, thus partially capturing the long-term market shares of other exchanges as well. With $\gamma^{\text{pex}}_{i,-t}$ and $\bar{\gamma}^{\text{other}}_{i,-t}$ having a correlation of -0.65, there is little need to introduce the complexity of determining optimal weights, which would optimally combine the effects of different impacts of $\gamma^{\text{pex}}_{i,-t}$ and $\bar{\gamma}^{\text{other}}_{i,-t}$.

noise. Thus, it avoids both a black-box approach and treating distinct elements as perfect substitutes. These two features of our fragmentation measure circumvent several major limitations of the HHI and provide a cleaner metric for examining how fragmentation affects expected stock returns, offering insights into how different market structures can influence asset pricing in general. The separation of the NYSE from all other market centers for its listed stocks is also consistent with the fact that the NYSE is the only major U.S. stock exchange that has consistently offered designated market makers (DMMs), formerly known as specialists. DMMs are required to maintain price continuity and stabilize prices for their assigned stocks listed on the NYSE, and hold unique responsibilities such as quoting bids and offers throughout the trading day. Although the NYSE has lost its dominance in trading its listed stocks after Regulation NMS, this unique feature still distinguishes the NYSE from other markets in terms of maintaining a certain level of liquidity on a continuous basis.[19][20]

5.2 Fragmentation and Expected Stock Returns

We begin by constructing seven univariate portfolios to examine how fragmentation, as well as its components, affect expected stock returns. At the beginning of each month t from 1993 to 2023, we sort stocks into quintiles based on their φ, γ^{pex}, $|\varepsilon|^{\text{total}}$, number of markets traded J, $\bar{\gamma}_{i,-t}^{\text{other}}$, $|\varepsilon|^{\text{pex}}$, and $|\varepsilon|^{\text{other}}$ from month $t-2$ and report their four-factor adjusted returns (alphas). Table 5 presents equally weighted four-factor alphas of these portfolios, revealing that alpha is monotonic in market fragmentation. It increases from 0.11% in the lowest quintile, the least fragmented stocks, to 1.03% in the highest quintile, the most fragmented stocks, yielding a spread of 0.91%. This result indicates that investors require compensation for holding more fragmented stocks, which contrasts with the conclusion reached using HHI to measure fragmentation.

[19] Clark-Joseph, Ye, and Zi (2017) use two technological glitches as a natural experiment and show that DMMs are still highly useful in the current market structure and significantly enhance liquidity. More information about DMMs can be found in a white paper by Steven W. Poser: "Market Makers in Financial Markets: Their Role, How They Function, Why They are Important, and the NYSE DMM Difference," which can be accessed at: https://www.nyse.com/publicdocs/nyse/NYSE_Paper_on_Market_Making_Sept_2021.pdf. Similarly, Brogaard, Ringgenberg, and Rösch (2024) demonstrate that floor traders are still significant contributors to market quality.

[20] We also construct a fragmentation measure based on only the long-term and short-term components of the primary listing exchange. The results are robust to this specification. We discuss the pros and cons of this alternative and present the results in Appendix D.

	Low	2	3	4	High	H-L	t-stat	p-val
Fragmentation (φ)	0.11	0.18	0.29	0.55	1.03	0.91	7.43	0.00
Long-Term M.S. (γ^{pex})	0.80	0.40	0.38	0.29	0.28	-0.52	-3.29	0.00
Short-Term Noise ($\lvert\varepsilon\rvert^{\text{total}}$)	0.16	0.24	0.32	0.53	0.92	0.75	7.41	0.00
Number of Markets	0.65	0.35	0.35	0.19	0.72	0.18	0.34	0.74
Long-Term M.S. ($\bar{\gamma}^{\text{other}}$)	0.19	0.25	0.31	0.40	1.02	0.83	6.16	0.00
Short-Term Noise ($\lvert\bar{\varepsilon}\rvert^{\text{pex}}$)	0.20	0.20	0.38	0.54	0.84	0.64	5.52	0.00
Short-Term Noise ($\lvert\bar{\varepsilon}\rvert^{\text{other}}$)	0.15	0.27	0.31	0.51	0.93	0.78	8.18	0.00

Table 5: **Monthly Excess Portfolio Returns Sorted on Fragmentation and Its Components**

Portfolios are constructed from January 1993 to December 2023 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. At the beginning of each month t, from January 1993 to December 2023, stocks are sorted into quintile portfolios based on φ, γ^{pex}, $|\varepsilon|^{\text{total}}$, number of markets traded J, $\bar{\gamma}^{\text{other}}_{i,-t}$, $|\varepsilon|^{\text{pex}}$, and $|\varepsilon|^{\text{other}}$ from two months earlier, month $t-2$. We then calculate monthly equal-weighted excess returns over four factors and report time-series averages for each quintile (Fama-French three factors—MKT, SMB, and HML—and the momentum factor—UMD), where the factor loadings are estimated using the past 60 months, with the requirement of having at least 24 observations. The differences between the top and bottom quintiles are also reported, along with the associated t-statistics and p-values. $\boldsymbol{\varphi}$ is our new fragmentation measure, equal to $|\boldsymbol{\varepsilon}|^{\text{total}}/\gamma^{\text{pex}}$, and defined in Equation 8. $\boldsymbol{\gamma}^{\text{pex}}$ is defined as the average daily market share on the NYSE over the past three months, representing the long-term market share of the NYSE. $|\boldsymbol{\varepsilon}|^{\text{total}}$ is the monthly average of the absolute daily deviations in daily market shares from their long-term values, and is defined in Equation 7. **Number of Markets** is the number of markets that a stock is traded in during a month, where each exchange and total off-exchange market centers are referred to as separate markets. $\bar{\gamma}^{\text{other}}_{i,-t}$, $|\varepsilon|^{\text{pex}}$, and $|\varepsilon|^{\text{other}}$ are defined in analogous ways.

An increase in fragmentation can result from lower long-term market share of the primary listing exchange or higher short-term noise; both are compensated, as shown in Table 5. Specifically, alpha monotonically decreases in γ^{pex}, from 0.80% in the lowest quintile to 0.28% in the highest quintile, resulting in a statistically and economically significant return spread of -0.52% per month between the two extreme portfolios. This finding indicates that investors require compensation for holding stocks that are consistently traded away from the NYSE, generating higher returns compared to their counterparts primarily traded on the NYSE. In contrast, short-term deviations in market share drive a positive return premium, indicating that investors value the stability of long-term market shares. There is a monotonic increase in alpha with $|\varepsilon|^{\text{total}}$, ranging from 0.16% in the lowest quintile to 0.92% in the highest quintile, leading to a statistically and economically significant return spread of 0.75% per month. Table 5 also reveals that the number of markets traded has no effect on asset pricing. Instead, the stability of market shares does matter. The portfolios in the last three rows also provide consistent results. Long-term concentration of market share outside the primary listing exchange increases expected stock returns. Similarly, any deviations of

market shares from their long-term averages are positively associated with expected returns.

	γ^{pex}:1	γ^{pex}:2	γ^{pex}:3	γ^{pex}:4	γ^{pex}:5
Short-Term ($\lvert\varepsilon\rvert^{\text{total}}$)					
Low	0.33	0.18	0.17	0.03	0.33
2	0.56	0.29	0.17	0.24	0.15
3	0.54	0.30	0.38	0.22	0.22
4	1.11	0.29	0.53	0.31	0.16
High	1.48	0.97	0.67	0.66	0.56
H-L	1.15	0.78	0.50	0.63	0.24
t-stat	4.27	3.61	3.78	5.74	2.04
p-val	0.00	0.00	0.00	0.00	0.04

Table 6: **Monthly Excess Portfolio Returns Sorted on Long-Term and Short-Term Components of Market Shares**

Portfolios are constructed from January 1993 to December 2023 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. At the beginning of each month t, from January 1993 to December 2023, using values from two months earlier, month $t-2$, stocks are first sorted into quintile portfolios based on γ^{pex}, and then, within each γ^{pex} quintile, sorted on $|\varepsilon|$. We then calculate monthly equal-weighted excess returns over four factors and report time-series averages for each portfolio (Fama-French three factors—MKT, SMB, and HML—and the momentum factor—UMD), where the factor loadings are estimated using the past 60 months, with the requirement of having at least 24 observations. The differences between the top and bottom $|\varepsilon|^{\text{total}}$ quintiles within a given γ^{pex} quintile are also reported, along with the associated t-statistics and p-values. $\boldsymbol{\gamma}^{\text{pex}}$ is defined as the average daily market share on the NYSE over the past three months, representing the long-term market share of the NYSE. $|\boldsymbol{\varepsilon}|^{\text{total}}$ is the monthly average of the absolute daily deviations in daily market shares from their long-term values, and is defined in Equation 7.

We now turn to bivariate portfolio analysis to gain deeper insight into how short-term noise affects expected stock returns under varying levels of concentration of long-term market share on the NYSE. Table 6 presents equally weighted four-factor alphas of these portfolios, revealing two important dynamics. Firstly, regardless of the extent of long-term market share concentration on the NYSE, short-term deviations in market shares from their long-term values are consistently associated with higher returns in both statistical and economic terms. In each γ^{pex} quintile, alpha generally increases with $|\varepsilon|^{\text{total}}$, always resulting in statistically and economically significant return spreads between stocks with the lowest and highest levels of short-term noise. For instance, in the lowest quintile of γ^{pex}, alpha increases monotonically from 0.33% in the lowest $|\varepsilon|^{\text{total}}$ quintile to 1.48% in the highest $|\varepsilon|^{\text{total}}$ quintile, generating a return spread of 1.15%. Secondly, this spread in four-factor alpha between the two extreme portfolios is almost monotonic in γ^{pex}, decreasing from 1.15% in the smallest γ^{pex} quintile

to 0.24% in the largest γ^{pex} quintile, demonstrating that short-term noise is most impactful on expected returns for stocks traded away from the NYSE. In the largest γ^{pex} quintile, the spread is still significant but exhibits a lower p-value compared to other quintiles of γ^{pex}. This further supports the notion that a sufficiently high level of market share on the NYSE weakens the effect of short-term deviations in market shares.

We have so far demonstrated that market fragmentation is positively related to expected stock returns and the underlying drivers of this relationship. We now test these predictions in a regression framework to control for additional variables and conduct further analyses that are impractical in univariate and bivariate portfolio analysis settings. Table 7 presents the results from these regressions. We include standard control variables in all six regressions. Additionally, we control for the Amihud illiquidity measure and price impact starting from the second and third regression, respectively.

We examine how our new fragmentation measure affects expected stock returns in Regressions 1 to 3. The results from these regressions are consistent with our portfolio analyses. Including illiquidity reduces the coefficient of fragmentation from 0.091 to 0.067, while adding price impact causes almost no change. The coefficient of fragmentation remains positive, economically, and statistically significant, demonstrating that fragmentation is not priced solely due to its interaction with illiquidity, price impact, or other controlled risk factors. After accounting for these additional variables and four factors, a one standard deviation increase in fragmentation raises expected returns by 0.26% per month, based on Regression 3.

In Regression 4, we test whether each component of fragmentation—NYSE's long-term market share and average absolute short-term noise—is still priced, which reveals consistent results with our portfolio analyses. Long-term fragmentation reduces expected returns: a one standard deviation increase in γ^{pex} reduces four-factor adjusted returns by 0.67% per month after controlling for illiquidity, price impact, and other aforementioned standard control variables. This finding strongly indicates that investors place substantial value on the centrality of trading on the NYSE, accepting lower returns for holding stocks traded more heavily on the NYSE. Short-term fragmentation is also similarly priced as before. A one standard deviation increase in $|\varepsilon|^{\text{total}}$ raises excess returns by 0.64% per month.

	(1)	(2)	(3)	(4)	(5)	(6)
φ	0.091***	0.067***	0.065***		0.074***	
	(0.014)	(0.013)	(0.011)		(0.012)	
$\mathbb{1}(\varepsilon^{pex} \geq 0) \times \varphi$					-0.016***	
					(0.006)	
γ^{pex}				-0.022***		-0.025***
				(0.007)		(0.007)
$\bar{\lvert\varepsilon\rvert}^{\text{total}}$				0.264***		0.282***
				(0.057)		(0.059)
$\mathbb{1}(\varepsilon^{pex} \geq 0) \times \lvert\varepsilon\rvert^{\text{total}}$						-0.045***
						(0.016)
λ			0.021***	0.022***	0.021***	0.022***
			(0.004)	(0.004)	(0.004)	(0.004)
lnA		0.299***	0.141***	0.131***	0.142***	0.139***
		(0.046)	(0.037)	(0.034)	(0.037)	(0.035)
Observations	448924	448924	448924	448924	448924	448924
Adjusted R^2	0.041	0.044	0.048	0.049	0.048	0.050
Other Controls	✓	✓	✓	✓	✓	✓

Table 7: **Fama-MacBeth Regressions of Excess Returns on Fragmentation and Its Components**

Monthly Fama-MacBeth regressions are estimated from January 1993 to December 2023 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. We estimate a cross-sectional regression in each month t and then report the time-series averages of the coefficients and standard errors, as well as the adjusted R^2. The **dependent variable** is the monthly excess return over four factors (Fama-French three factors—MKT, SMB, and HML—and the momentum factor—UMD), where the factor loadings are estimated using the past 60 months, with the requirement of having at least 24 observations. $\boldsymbol{\varphi}$ is our new fragmentation measure, equal to $|\boldsymbol{\varepsilon}|^{\text{total}}/\gamma^{\text{pex}}$, and defined in Equation 8. $\boldsymbol{\gamma}^{\text{pex}}$ is defined as the average daily market share on the NYSE over the past three months, representing the long-term market share of the NYSE. $|\boldsymbol{\varepsilon}|^{\text{total}}$ is the monthly average of the absolute daily deviations in daily market shares from their long-term values, and is defined in Equation 7. $\mathbb{1}(\boldsymbol{\varepsilon^{pex}} \geq \mathbf{0})$ is a dummy variable that is equal to one when ε^{pex} is non-negative, where $\boldsymbol{\varepsilon^{pex}}$ is the monthly average of the daily deviations in the NYSE's market share from its long-term value γ^{pex}, and is defined in Equation 6. $\boldsymbol{\lambda}$ is the monthly average of daily price impact coefficients. **lnA** is the natural logarithm of the original Amihud illiquidity measure A, calculated as the monthly average of the daily ratio of absolute return to dollar trading volume in millions. A is calculated using daily trading volume data from TAQ. In the regression for month t, these independent variables are taken from month $t-2$. Additionally, we include other standard control variables, which are defined explicitly in Section 3, with coefficients not presented to save space: previous month's stock return, cumulative stock returns over the past twelve months excluding the previous one, and idiosyncratic return volatility. t-statistics are calculated using Newey-West robust standard errors with six lags. **Standard errors** are in parentheses; * $p < 0.10$ ** $p < 0.05$ *** $p < 0.01$.

The last two regressions in Table 7 test whether short-term noise has an asymmetric effect, examining whether positive and negative noise on the primary listing exchange have hetero-

geneous effects on expected stock returns. Regression 5 indicates that stocks experiencing short-term noises that increase the NYSE's market share achieve approximately 22% lower fragmentation premium (0.016/0.074) compared to stocks experiencing short-term noises in the opposite direction. However, the fragmentation premium remains economically and statistically significant regardless of the direction of short-term noises, indicating that investors require compensation for any deviations in market shares, whether they occur in or out of the NYSE. Regression 6 provides similar conclusions.

6 Illiquidity and Fragmentation: A Unifying Measure

As previously illustrated in Figure 1, for stocks listed on the NYSE, there has been a significant shift in trading from the NYSE to other market centers over the period from 1993 to 2023. This transition moved the market from a state where the NYSE dominated trading with more than 80% of market share, to a more fragmented market structure where the NYSE is responsible for roughly 20% of trading for its listed stocks. This change in market structure poses a challenge to traditional liquidity measures, such as those proposed by Amihud (2002) and Pastor and Stambaugh (2003), which do not account for the evolving nature of the U.S. equities market. They rely on trading volumes that are now fragmented across numerous market centers. The core principle behind these measures is to assess how trading volume of a stock influences its return. When a stock achieves a significant absolute return with minimal trading activity, it suggests a lack of liquidity, as a small number of trades had a substantial impact on its price. However, the interpretation of trading volume and its impact on returns can differ significantly when trading is concentrated in a single dominant market center versus when it is distributed across many market centers.

This interaction between illiquidity and fragmentation can easily be observed in a bivariate portfolio setting analyzing how illiquidity affects expected stock returns under varying levels of fragmentation. Table 8 presents equally weighted four-factor alphas of these portfolios. Regardless of the extent of fragmentation, illiquidity is consistently associated with higher returns in both statistical and economic terms. In each φ quintile, alpha typically increases with illiquidity, and illiquidity consistently leads to statistically and economically significant return spreads between stocks with the lowest and highest levels of illiquidity. However, this spread level heavily depends on fragmentation, increasing monotonically from

0.32% in the portfolio containing the least fragmented stocks to 2.10% in the portfolio with the most fragmented stocks. This result demonstrates that illiquidity has more significant impact on expected returns for stocks with higher fragmentation. The difference in the illiquidity premium between the lowest and highest quintiles of fragmentation is 1.78% per month, resulting in an annual excess return of 23.6% over four-factor adjusted returns.

	φ:1	φ:2	φ:3	φ:4	φ:5
Illiquidity (A)					
Low	-0.01	-0.02	0.01	0.17	0.36
2	0.13	0.06	0.17	0.15	0.43
3	0.13	0.19	0.21	0.39	0.78
4	0.02	0.19	0.47	0.70	1.14
High	0.31	0.48	0.58	1.37	2.46
H-L	0.32	0.50	0.57	1.20	2.10
t-stat	2.44	2.99	3.08	5.69	10.44
p-val	0.02	0.00	0.00	0.00	0.00

Table 8: **Monthly Excess Portfolio Returns Sorted on Fragmentation and Illiquidity**

Portfolios are constructed from January 1993 to December 2023 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. At the beginning of each month t, from January 1993 to December 2023, using values from two months earlier, month $t-2$, stocks are first sorted into quintile portfolios based on γ^{pex}, and then, within each γ^{pex} quintile, sorted on A. We then calculate monthly equal-weighted excess returns over four factors and report time-series averages for each portfolio (Fama-French three factors—MKT, SMB, and HML—and the momentum factor—UMD), where the factor loadings are estimated using the past 60 months, with the requirement of having at least 24 observations. The differences between the top and bottom $|\varepsilon|$ quintiles within a given γ^{pex} quintile are also reported, along with the associated t-statistics and p-values. $\boldsymbol{\varphi}$ is our new fragmentation measure, equal to $|\boldsymbol{\varepsilon}|^{\text{total}}/\gamma^{\text{pex}}$, and defined in Equation 8. **A** is the original Amihud illiquidity measure, calculated as the monthly average of the daily ratio of absolute return to dollar trading volume in millions.

These results demonstrate that the impact of a stock's trading volume on its return depends on whether its volume is concentrated or fragmented. Thus, trading volume has different meanings across markets with varying levels of fragmentation. To address this limitation embedded in traditional volume-based liquidity measures, we construct a new illiquidity metric that is a variant of the original Amihud (2002) measure, taking into account both illiquidity and fragmentation.[21] The original Amihud (2002) illiquidity measure is defined for stock i in month t as:

[21] A similar measure could be constructed using Pastor and Stambaugh (2003). We choose Amihud (2002) for its simplicity and lesser data requirement.

$$\mathrm{A}_{i,t} := \frac{1}{D_{it}} \sum_{d=1}^{D_{it}} \frac{|r_{i,d,t}|}{\mathrm{V}^{\$}_{i,d,t}}, \tag{9}$$

where $r_{i,d,t}$ is the return of stock i on day d in month t measured as a decimal, and $\mathrm{V}^{\$}_{i,d,t}$ represents the total dollar trading volume for stock i on day d in month t, calculated as the closing price times the number of shares traded, expressed in millions of dollars. D_{it} is the total number of days that stock i is traded in month t, with an index $d = 1, 2, \ldots, D_{it}$.[22]

We aim to construct a new liquidity measure that accounts for how returns respond to aggregate volume as well as long-term and short-term fragmentation. The existing illiquidity measures use dollar trading volume. To this end, we multiply each volume term in Equation 5 by stock i's closing price on day d to derive a similar equation for dollar trading volume:[23]

$$\frac{\mathrm{V}^{\$,j}_{i,d,t}}{\mathrm{V}^{\$}_{i,d,t}} = \gamma^{j}_{i,-t} + \frac{\varepsilon^{\$,j}_{i,d,t}}{\mathrm{V}^{\$}_{i,d,t}} \tag{10}$$

In the construction of fragmentation-adjusted illiquidity measure, we focus on two categories of share volume: primary listing exchange versus all other market centers (other exchanges and off-exchange market centers). Consider a market devoid of short-term noise in market shares, implying a permanently stable distribution of share volume across markets. Equation 10 implies that $\mathrm{V}^{\$}_{i,d,t} = \mathrm{V}^{\$,\mathrm{pex}}_{i,d,t}/\gamma^{\mathrm{pex}}_{i,-t} = \mathrm{V}^{\$,\mathrm{other}}_{i,d,t}/\left(1-\gamma^{\mathrm{pex}}_{i,-t}\right)$. These terms are always well-defined since $\gamma^{\mathrm{pex}}_{i,-t}$ is strictly interior for all stocks. In the absence of any short-term noise, the response of stock i's return to its dollar trading volume on day d can be equivalently written as:

$$\frac{|r_{i,d,t}|}{\mathrm{V}^{\$,\mathrm{other}}_{i,d,t}/(1-\gamma^{\mathrm{pex}}_{i,-t})} = \frac{|r_{i,d,t}|}{\mathrm{V}^{\$,\mathrm{pex}}_{i,d,t}/\gamma^{\mathrm{pex}}_{i,-t}} \left(= \frac{|r_{i,d,t}|}{\mathrm{V}^{\$}_{i,d,t}} \right)$$

[22] The turnover version of the Amihud (2002) illiquidity measure is also used in the literature. Both fragmentation results in Section 5 and the illiquidity results presented in this section are robust to this alternative specification of illiquidity. These results are presented in Appendix C and E.

[23] This specification implies that market shares are exactly the same for share trading volume and dollar trading volume, as we use daily stock prices and share volumes to calculate dollar trading volumes. This practice is quite standard in the empirical asset pricing literature studying liquidity, as seen in studies such as Amihud (2002), Pastor and Stambaugh (2003), Lou and Shu (2017), and Amihud and Noh (2021).

However, the presence of short-term frictions leads to a discrepancy between these terms. Let $\delta_{i,d,t}$ denote this discrepancy with a slight modification:

$$\delta_{i,d,t} := (1 - \gamma^{\text{pex}}_{i,-t}) \times \left(\frac{|r_{i,d,t}|}{\text{V}^{\$,\text{other}}_{i,d,t}/(1 - \gamma^{\text{pex}}_{i,-t})} - \frac{|r_{i,d,t}|}{\text{V}^{\$,\text{pex}}_{i,d,t}/\gamma^{\text{pex}}_{i,-t}} \right) \tag{11}$$

$\delta_{i,d,t}$ quantifies daily variation in a stock's illiquidity when traded on market centers other than the NYSE compared to the NYSE itself. As short-term noise diminishes, $\delta_{i,d,t}$ approaches zero. Positive values of $\delta_{i,d,t}$ indicate that trading on non-NYSE market centers exhibits greater illiquidity than on the NYSE, whereas negative values suggest that non-NYSE markets are more liquid than the NYSE on a given day d. The long-term market shares in the denominators normalize $\text{V}^{\$,\text{pex}}_{i,d,t}$ and $\text{V}^{\$,\text{other}}_{i,d,t}$, making sure that the return responses of a particular stock are comparable across the NYSE and non-NYSE market centers. Otherwise, one return response could appear larger simply due to a lower dollar volume. The multiplicative term $(1 - \gamma^{\text{pex}}_{i,-t})$ makes $\delta_{i,d,t}$ more comparable across stocks. It ensures that a stock receives a low discrepancy score if it is primarily traded on the NYSE.

We use monthly averages of absolute daily discrepancies, $|\delta_{i,d,t}|$, to gauge liquidity in conjunction with fragmentation. We denote this new measure as $|\text{LDA}_{i,t}|$, standing for the Liquidity Discrepancy of A, where A represents the original Amihud (2002) illiquidity measure defined in Equation 9. We specifically define it as:

$$|\text{LDA}_{i,t}| := \frac{1}{D_{it}} \sum_{d=1}^{D_{it}} |\delta_{i,d,t}| \tag{12}$$

We now demonstrate that $\delta_{i,d,t}$ is closely related to our fragmentation measure, as well as the original Amihud (2002) illiquidity measure. When short-term noise is non-zero, $\delta_{i,d,t}$ can be rewritten in terms of $\varepsilon^{\$,\text{pex}}_{i,d,t}$ using Equation 10 and the fact that $\varepsilon^{\$,\text{pex}}_{i,d,t} + \varepsilon^{\$,\text{other}}_{i,d,t} = 0$. Substituting these terms, taking a first-order Taylor approximation around $\varepsilon^{\$,\text{pex}}_{i,d,t} = 0$, and using $\varepsilon^{\$,\text{pex}}_{i,d,t}/\text{V}^{\$}_{i,d,t} = \varepsilon^{\text{pex}}_{i,d,t}/\text{V}_{i,d,t}$ yields:

$$\delta_{i,d,t} \approx \frac{|r_{i,d,t}|}{\text{V}^{\$}_{i,d,t}} \times \frac{\varepsilon^{\text{pex}}_{i,d,t}}{\text{V}_{i,d,t}} \times \frac{1}{\gamma^{\text{pex}}_{i,-t}} \tag{13}$$

$|\text{LDA}_{i,t}|$ can be approximated then:

$$|\text{LDA}_{i,t}| \approx \left(\frac{1}{D_{it}} \sum_{d=1}^{D_{it}} \frac{|r_{i,d,t}|}{\text{V}^{\$}_{i,d,t}} \times \left| \frac{\varepsilon^{\text{pex}}_{i,d,t}}{\text{V}_{i,d,t}} \right| \right) \times \frac{1}{\gamma^{\text{pex}}_{i,-t}} \tag{14}$$

where we use the fact that both dollar trading volume and long-term market shares are always positive, and $\gamma^{\text{pex}}_{i,-t}$ does not change in a given month t. To better illustrate what it measures and how it captures both traditional illiquidity and fragmentation, we perform one more decomposition. The correlation between $|r_{i,d,t}| / \text{V}^{\$}_{i,d,t}$ and $\left|\varepsilon^{\text{pex}}_{i,d,t}/\text{V}_{i,d,t}\right|$ is relatively weak in our sample (0.33). Therefore, we can separate the multiplication inside the summation and approximately express $|\text{LDA}_{i,t}|$ as:

$$|\text{LDA}_{i,t}| \approx \underbrace{\left[\frac{1}{D_{it}} \sum_{d=1}^{D_{it}} \frac{|r_{i,d,t}|}{\text{V}^{\$}_{i,d,t}} \right]}_{\text{Amihud Illiquidity } (\text{A}_{i,t})} \times \underbrace{\left[\frac{1}{D_{it}} \sum_{d=1}^{D_{it}} \left| \frac{\varepsilon^{\text{pex}}_{i,d,t}}{\text{V}_{i,d,t}} \right| \right]}_{\text{Short-Term Noise } (\varepsilon^{\text{pex}}_{i,t})} \times \underbrace{\left[\frac{1}{\gamma^{\text{pex}}_{i,-t}} \right]}_{\text{Long-Term Market Share}} \tag{15}$$

Thus, the fragmentation-adjusted (il)liquidity measure of stock i in month t is a function of the original Amihud (2002) illiquidity measure, the average absolute noise in NYSE's daily market shares reflecting short-term noise, and the reciprocal of the NYSE's long-term market share, representing long-term fragmentation. This formulation establishes that the impact of traditional illiquidity is amplified by higher market fragmentation.

We start with a univariate portfolio analysis as usual. At the beginning of each month t from 1993 to 2023, we sort stocks into quintiles based on their LDA and |LDA| from month $t-2$ and report their four-factor adjusted returns (alphas). Table 9 presents equally weighted four-factor alphas of these portfolios. LDA follows a similar pattern to ε. LDA does not exhibit any excess return over four factors: the difference between the highest and lowest quintile returns is -0.14%, but it is statistically insignificant. However, both of these quintiles consistently have higher returns than the middle quintiles. The portfolios sorted on |LDA| clearly reveal this dynamic. Alpha is monotonic in |LDA|, increasing from 0.04% in the lowest quintile, representing stocks with the smallest liquidity discrepancy, to 1.22% in the highest quintile, representing stocks with the larges liquidity discrepancy, yielding a spread of 1.22%.

	Low	2	3	4	High	H-L	t-stat	p-val
LDA	0.89	0.13	0.14	0.25	0.75	-0.14	-1.47	0.14
|LDA|	0.04	0.13	0.25	0.49	1.25	1.22	8.72	0.00

Table 9: **Monthly Excess Portfolio Return Sorted on LDA and |LDA|**

Portfolios are constructed from January 1993 to December 2023 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. At the beginning of each month t, from January 1993 to December 2023, stocks are sorted into quintile portfolios based on LDA and |LDA| from two months earlier, month $t-2$. We then calculate monthly equal-weighted excess returns over four factors and report time-series averages for each quintile (Fama-French three factors—MKT, SMB, and HML—and the momentum factor—UMD), where the factor loadings are estimated using the past 60 months, with the requirement of having at least 24 observations. The differences between the top and bottom quintiles are also reported, along with the associated t-statistics and p-values. **LDA** (|**LDA**|) is the (absolute) illiquidity spread of A, defined as the product of $1-\gamma^{\text{pex}}$ and the monthly average of the (absolute) difference between two ratios: the daily ratio of absolute return to dollar trading volume, in millions, outside the primary listing exchange—NYSE—divided by $1-\gamma^{\text{pex}}$, and a similar measure on the primary exchange divided by γ^{pex}. $\boldsymbol{\gamma}^{\text{pex}}$ is defined as the average daily market share on the NYSE over the past three months, representing the long-term market share of the NYSE.

We conduct regression analysis to check further if the impact of |LDA| remains robust after controlling for additional variables. Table 10 presents the results from these regressions. We include standard control variables in all three regressions and price impact in Regressions 2 and 3. Including price impact reduces the coefficient of |LDA| from 1.414 to 0.953. This reduction is due to the correlation of illiquidity with price impact (0.42), as fragmentation and price impact have weaker correlation (0.17). The coefficient of |LDA| remains positive, and economically and statistically significant in all three regressions, demonstrating that |LDA| is not priced solely due to its interaction with price impact or other controlled variables. Based on Regression 2, a one standard deviation increase in |LDA| raises expected returns by 0.62% per month.

The last regression in Table 10 tests whether liquidity discrepancy has an asymmetric effect, examining whether positive and negative LDAs have heterogeneous effects on expected stock returns. Regression 3 indicates that stocks experiencing negative illiquidity discrepancy, i.e., higher illiquidity on the NYSE relative to other market centers, achieve approximately 44% higher fragmentation-adjusted illiquidity premium compared to stocks experiencing illiquidity discrepancy in the opposite direction. However, the illiquidity discrepancy premium remains economically and statistically significant regardless of the direction of the discrepancy.

	(1)	(2)	(3)
|LDA|	1.414***	0.953***	1.145***
	(0.233)	(0.159)	(0.164)
$\mathbb{1}(\text{LDA} \geq 0) \times \text{LDA}$			-0.352***
			(0.116)
λ		0.021***	0.021***
		(0.003)	(0.003)
Observations	448924	448924	448924
Adjusted R^2	0.043	0.047	0.048
Other Controls	✓	✓	✓

Table 10: **Fama-MacBeth Regressions of Excess Returns on |LDA|**

Monthly Fama-MacBeth regressions are estimated from January 1993 to December 2023 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. We estimate a cross-sectional regression in each month t and then report the time-series averages of the coefficients and standard errors, as well as the adjusted R^2. The **dependent variable** is the monthly excess return over four factors (Fama-French three factors—MKT, SMB, and HML—and the momentum factor—UMD), where the factor loadings are estimated using the past 60 months, with the requirement of having at least 24 observations. **LDA** (**|LDA|**) is the (absolute) illiquidity spread of A, defined as the product of $1 - \gamma^{\text{pex}}$ and the monthly average of the (absolute) difference between two ratios: the daily ratio of absolute return to dollar trading volume, in millions, outside the primary listing exchange—NYSE—divided by $1 - \gamma^{\text{pex}}$, and a similar measure on the primary exchange divided by γ^{pex}. $\boldsymbol{\gamma}^{\text{pex}}$ is defined as the average daily market share on the NYSE over the past three months, representing the long-term market share of the NYSE. $\mathbb{1}(\text{LDA} \geq 0)$ is a dummy variable that is equal to one when LDA is non-negative. $\boldsymbol{\lambda}$ is the monthly average of daily price impact coefficients. In the regression for month t, these independent variables are taken from month $t-2$. Additionally, we include other standard control variables, which are defined explicitly in Section 3, with coefficients not presented to save space: previous month's stock return, cumulative stock returns over the past twelve months excluding the previous one, and idiosyncratic return volatility. t-statistics are calculated using Newey-West robust standard errors with six lags. **Standard errors** are in parentheses; * $p < 0.10$ ** $p < 0.05$ *** $p < 0.01$.

7 A Theory of Fragmentation and Asset Pricing

Consider a fragmented secondary market where investors trade a single stock. There are N exchanges where the stock is traded. Each exchange is run by a single market maker. There are some liquidity providers and a unit measure of buyers. Liquidity providers submit their sell-orders to one of the exchanges non-strategically. The number of sell orders in exchange i is S_i. S_i is an exogenous random variable that follows distribution G_i with expected value $\mathbb{E}_i S_i = \int S dG_i(S)$. Their value over a share is V_S. A buyer seeks to buy a unit share. Her value over a share is V_B. $V_B > V_S > 0$. Buyers choose optimally where to submit their orders. Sellers are liquidity traders who submit their orders to exchanges non-strategically.

Buyers do not observe sellers' order flow when they submit their orders.

Market makers match buy-orders and sell-orders, offer bid prices to sellers, and offer ask prices to buyers. A matching function $m(B, S)$ determines the number of matches between sell-orders and buy-orders. We assume that the matching function has constant returns to scale and is increasing in both the number of buy-orders B and the number of sell-orders S. A matching function with minimal friction is $m(B, S) = \min(B, S)$. In this case, the market maker rations the order flows from the long side of the market. A more general matching function would capture the mismatch in the timing of order flows and delay in order flow execution. Market makers offer to buy shares from the sellers at their reservation value V_S since they do not choose strategically where to submit their orders.

The market maker in exchange i posts price P_i and coordinate with buyers in determining the number of buy-orders submitted to the exchange, B_i. She sets the price and the number of buy-orders before observing the realized volume of sell orders S_i. Thus the optimization problem of the market maker in exchange i is

$$\max_{P_i, B_i} \mathbb{E}_i m(B_i, S_i)(P_i - V_S), \tag{16}$$

subject to buyers' participation constraint, that the expected payoff for a buyer to choose exchange i is no less than her equilibrium payoff. Denote the buyer's expected value from trade W_B. The buyer's participation constraint in exchange i is

$$\mathbb{E}_i \left(\frac{m(B_i, S_i)}{B_i} \right) (V_B - P_i) \geq W_B, \tag{17}$$

where $\frac{m(B_i,S_i)}{B_i}$ is the realized probability that a buyer is rationed a share given the realized sell-orders, S_i, and $V_B - P_i$ is the trade surplus for the buyer rationed a share.

A symmetric equilibrium is buyers' order allocation B, prices P, buyer's reservation value W_B such that, for all exchanges $i \in \{1, \ldots, N\}$,

1. prices $P_i = P$ and buyer orders $B_i = B$ maximize market maker i's profit subject to buyers' participation constraint in exchange i, Eq. (17);
2. buyers' equilibrium payoff $W_B = \mathbb{E}_i \left(\frac{m(B_i,S_i)}{B_i} \right) (V_B - P_i)$;
3. buyers' orders sum up to one so that $B_i = 1/N$.

The equilibrium price

$$P = P_i = (1-\theta)V_B + \theta V_S, \text{ where} \tag{18}$$

$$\theta \equiv \frac{\mathbb{E}_i\left(\frac{\partial m}{\partial B}(1/N, S_i)\right)}{\mathbb{E}_i\left(Nm(1/N, S_i)\right)}. \tag{19}$$

The price is a weighted average of the buyer's reservation value, V_B, and the seller's, V_S. The weight on the seller's value is θ. The weight depends on the dispersion in liquidity supply. The expected equilibrium trade volume is

$$\text{volume} = N\mathbb{E}_i m(\frac{1}{N}, S_i). \tag{20}$$

Suppose the matching function $m(B,S) = \min(B,S)$. We can show that a mean-preserving spread in the distribution of S_i that satisfies the single-crossing property at $1/N$ thus increases θ and decreases the asset price P_i. This is because, in this example,

$$\mathbb{E}_i \frac{m(B_i, S_i)}{B_i} = N\int_0^{1/N} \widetilde{G}(S_i)dS_i$$

$$\mathbb{E}_i\left(\frac{\partial m}{\partial B}(B_i, S_i)\right) = \widetilde{G}(1/N)$$

where $\widetilde{G}(S) = 1 - G(S)$ is the inverse distribution function. So, the equilibrium weight on the seller's value,

$$\theta = \frac{\widetilde{G}(\frac{1}{N})}{N\int_0^{\frac{1}{N}} \widetilde{G}(S_i)dS_i}.$$

and the equilibrium expected volume is

$$\text{volume} = N\int_0^{1/N} \widetilde{G}(S_i)dS_i.$$

Figure 3 illustrates the effects of liquidity uncertainty on asset prices and trading volume. As the uncertainty increases, both asset prices and the expected volume decrease. Thus, our model predictions are consistent with our empirical findings. These comparative statistics are invariant to the number of exchanges N, because we keep constant the expected number

of sell orders, $\mathbb{E}(\sum_i S_i) = \sum_i B_i = 1$. In other words, fragmentation affects the asset price and trade volume only through the uncertain realization of liquidity supply, S_i. The current model is static and thus illustrative. We can extend the model to a dynamic setting for quantitative exercises.

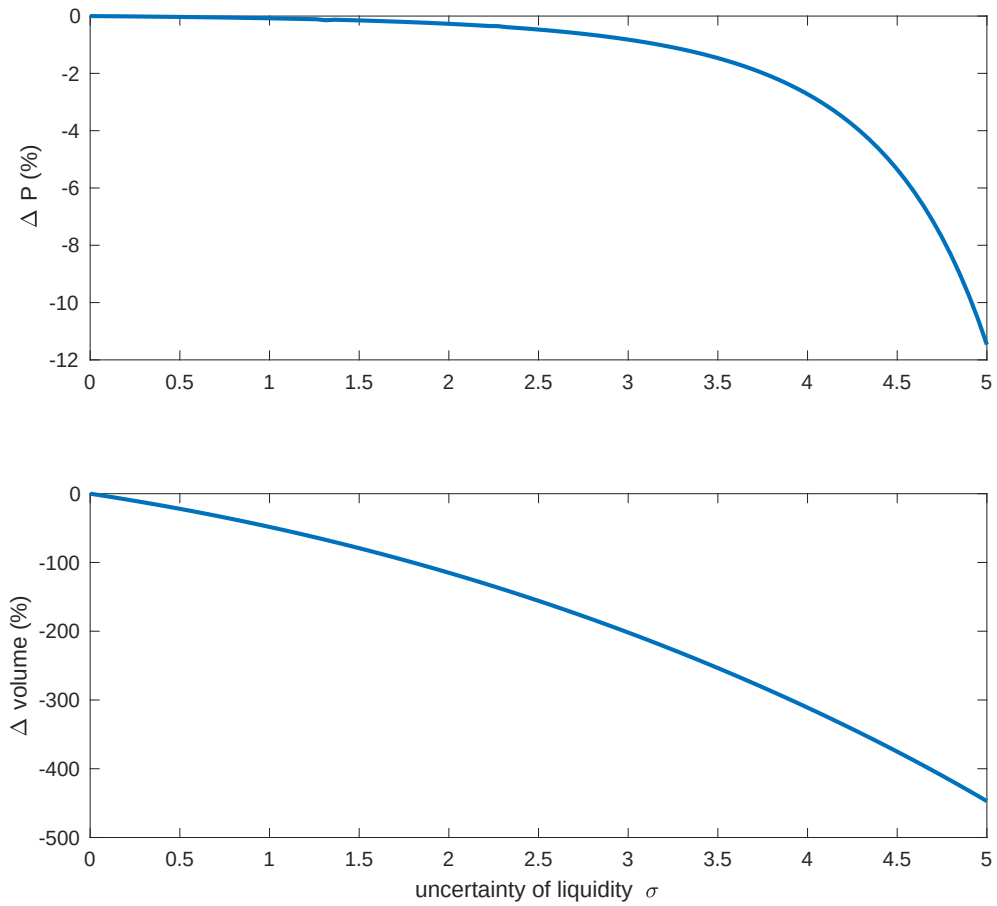


Figure 3: **Liquidity uncertainty decreases asset prices and trading volume.** We assume that S_i follows a lognormal distribution with standard deviation parameter σ and mean parameter $-\log(N) - \sigma^2/2$ so that the expected value of S_i equals the equilibrium number of buy orders $1/N$. We use the matching function with minimal frictions, $m(B, S) = \min(B, S)$.

Market fragmentation could cause the uncertainty about the non-strategic liquidity supply S_i to increase, although, in the current model, we take the dispersion as an exogenous parameter. Even if the aggregate liquidity provision is deterministic, liquidity could be randomly distributed across exchanges when non-strategic liquidity providers fail to coordinate their order flows.

8 Policy Relevance

We demonstrate that investors require compensation for holding stocks with higher levels of fragmentation. Investors place substantial value on the centrality of trading on the NYSE. They accept lower returns for stocks with higher long-term market share, those of consistently traded more heavily on the NYSE, while requiring compensation for short-term deviations in daily market shares from their long-term levels. Policy discussions or academic studies on fragmentation generally focus on how it affects market quality. Our paper highlights that fragmentation is an important determinant of asset prices and, thus, regulations intensifying fragmentation can have long-term repercussions for publicly listed companies, as higher expected returns typically translate into a higher cost of capital.

In terms of both HHI and our fragmentation measure, Regulation NMS has significantly increased fragmentation without any doubt. However, as shown in Figure 1, except for a transition period, market shares are quite stable under a market structure where the NYSE executes more than 80% of trades, as well as under a structure where trading is dispersed across numerous exchanges with no single exchange dominating. We now illustrate that fragmentation has become more significant for asset pricing in the second market structure. To this end, we conduct regression analysis in two sub-samples, 1993–2004 versus 2009–2023, representing these stable periods before and after Regulation NMS.

We start by acknowledging that these samples are relatively short from an asset pricing perspective. Our purpose is to conduct only a comparative analysis before and after Regulation NMS, which can offer valuable insights into how recent regulations, such as SEC's latest adoption of reducing access fee caps for exchanges, can affect expected returns. Table 11 presents results from these regressions. The coefficient on fragmentation increases three to four times from 1993–2004 to 2009–2023. One standard deviation increase in fragmentation results in a 0.12% alpha per month during 1993–2004 compared to an alpha of 0.36% during 2009–2023. Thus, one standard deviation increase in fragmentation requires three times more compensation after Regulation NMS.

This result is consistent with the idea that when a single market center dominates trading, seeking liquidity outside this platform is more dependent on trader preferences rather than a necessity. In fragmented markets, like the current U.S. equities markets, due to significant uncertainty about daily liquidity distribution across market centers, seeking liquidity became

a necessity rather than a choice. Table 11 indicates that this type of uncertain and required liquidity search is compensated more than a liquidity search resulting from trader preferences rather than necessity.

	(1)	(2)	(3)	(4)
	1993-2004	2009-2023	1993-2004	2009-2023
φ	0.033***	0.104***	0.037***	0.120***
	(0.010)	(0.017)	(0.010)	(0.020)
$\mathbb{1}(\varepsilon^{pex} \geq 0) \times \varphi$			-0.012	-0.022***
			(0.010)	(0.007)
λ	0.006***	0.035***	0.006***	0.035***
	(0.001)	(0.006)	(0.001)	(0.006)
lnA	0.011	0.265***	0.013	0.265***
	(0.067)	(0.040)	(0.067)	(0.040)
Observations	197015	192808	197015	192808
Adjusted R^2	0.036	0.060	0.036	0.061
Other Controls	✓	✓	✓	✓

Table 11: **Fama-MacBeth Regressions of Excess Returns on Fragmentation Before and After Regulation NMS**

Monthly Fama-MacBeth regressions are estimated from January 1993 to December 2023 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. We estimate a cross-sectional regression in each month t and then report the time-series averages of the coefficients and standard errors, as well as the adjusted R^2. The **dependent variable** is the monthly excess return over four factors (Fama-French three factors—MKT, SMB, and HML—and the momentum factor—UMD), where the factor loadings are estimated using the past 60 months, with the requirement of having at least 24 observations. $\boldsymbol{\varphi}$ is our new fragmentation measure, equal to $|\boldsymbol{\varepsilon}|^{\text{total}}/\gamma^{\text{pex}}$, and defined in Equation 8. $\boldsymbol{\gamma}^{\text{pex}}$ is defined as the average daily market share on the NYSE over the past three months, representing the long-term market share of the NYSE. $|\boldsymbol{\varepsilon}|^{\text{total}}$ is the monthly average of the absolute daily deviations in daily market shares from their long-term values, and is defined in Equation 7. $\mathbb{1}(\boldsymbol{\varepsilon^{pex}} \geq \mathbf{0})$ is a dummy variable that is equal to one when ε^{pex} is non-negative, where $\boldsymbol{\varepsilon^{pex}}$ is the monthly average of the daily deviations in the NYSE's market share from its long-term value γ^{pex}, and is defined in Equation 6. $\boldsymbol{\lambda}$ is the monthly average of daily price impact coefficients. **lnA** is the natural logarithm of the original Amihud illiquidity measure A, calculated as the monthly average of the daily ratio of absolute return to dollar trading volume in millions. A is calculated using daily trading volume data from TAQ. In the regression for month t, these independent variables are taken from month $t-2$. Additionally, we include other standard control variables, which are defined explicitly in Section 3, with coefficients not presented to save space: previous month's stock return, cumulative stock returns over the past twelve months excluding the previous one, and idiosyncratic return volatility. t-statistics are calculated using Newey-West robust standard errors with six lags. **Standard errors** are in parentheses; * $p < 0.10$ ** $p < 0.05$ *** $p < 0.01$.

The SEC approved a rule to lower access fees that exchanges charge brokers on September 18, 2024. While this rule applies to all exchanges, it is likely to impact smaller or newer

exchanges more significantly. Larger exchanges, such as the NYSE, are less reliant on access fees due to their higher listing fees and market data revenues from consolidated tape plans. Reflecting these concerns, some financial entities highlighted potential competitive issues in their comments to the SEC. For example, Virtu stated that "limitation on exchange rebates and fee tiers would reduce incentives to provide liquidity and harm competition between exchanges, especially new exchanges." Similarly, Themis Trading argued that "[l]ower access fees will not harm the major exchanges." This rule change could increase the NYSE's market power for its listed stocks, thereby potentially mitigating the adverse effects of fragmentation on asset prices for stocks listed on the NYSE to some extent.[24]

9 Robustness of Results

We conduct two main sets of robustness analyses. The first set checks the robustness of our results in Section 5.2 regarding the relationship between fragmentation and asset prices. In this regard, we first analyze whether fragmentation is priced due to its relation with transaction costs. We use the effective spread or quoted spread to measure transaction costs.[25] Second, instead of using the four-factor adjusted returns as the dependent variable, we utilize excess stock returns over the risk-free rate and separately control for four factors in the regression, as in Amihud and Noh (2021). Under all these specifications, our results regarding how fragmentation affects expected returns remain robust.

The second set includes robustness analysis for our fragmentation-adjusted illiquidity measure developed in Section 6. We derive two alternative measures of liquidity that accounts for fragmentation and demonstrate that our results are robust to these alternative specifications. First, we develop a measure without adjusting daily liquidity discrepancies with $1-\gamma_{i,-t}^{\text{pex}}$ and define $\delta_{i,d,t}$ in Equation 11 without $1-\gamma_{i,-t}^{\text{pex}}$. This setup results in a liquidity measure very similar to that of Equation 12, except the last term is $1/(\gamma_{i,-t}^{\text{pex}} \times (1-\gamma_{i,-t}^{\text{pex}}))$ instead of $1/\gamma_{i,-t}^{\text{pex}}$. This alternative measure is highly correlated with our main measure and

[24]The comment letters are available at the following links: Virtu and Themis Trading. The SEC also approved another rule on September 18, 2024, to reduce tick sizes for many stocks, which will likely further affect fragmentation. A summary of both changes can be found in SEC Press Release, 2024-137, with the full text available in 89 FR 81620.

[25]Fragmentation can positively affect transaction costs by intensifying competition between market centers or negatively affect them by reducing liquidity in each market center.

also shows significant pricing. However, there is an important nuance concerning the aspect of long-term fragmentation it captures. Our main measure reflects the extent to which long-term market share is consolidated on a stock's primary listing exchange, the NYSE, while this alternative captures how equally long-term market shares are distributed across the NYSE and other market centers. Second, we construct a measure without absolute values. We directly use monthly averages of $\delta_{i,d,t}$, the approximated daily liquidity discrepancies given in Equation 13, to gauge liquidity with fragmentation. The advantage of this approach is that the correlation between $|r_{i,d,t}| / \mathrm{V}^{\$}_{i,d,t}$ and $\varepsilon^{\mathrm{pex}}_{i,d,t}/\mathrm{V}_{i,d,t}$ is nearly zero (0.0175), allowing for more precise decomposition. The downside is that $\varepsilon^{\mathrm{pex}}_{i,d,t}$ can frequently alternate in sign for some stocks, leading to cancellations between positive and negative terms when calculating $\frac{1}{D_{it}} \sum_{d=1}^{D_{it}} \frac{\varepsilon^{\mathrm{pex}}_{i,d,t}}{\mathrm{V}_{i,d,t}}$. Both of these alternative measures are provided in Appendix E in detail.

We also use the turnover version of the Amihud illiquidity measure (AT), rather than the originally proposed version, as a control variable in the regressions of Section 5.2, as well as develop our fragmentation-adjusted illiquidity measure in Section 6 with this turnover version. Our results are also robust against this alternative specification. AT is calculated as the monthly average of the daily ratio of absolute return to share turnover.

10 Conclusion

We show that market fragmentation can reduce market liquidity by making it difficult for investors to locate liquidity across exchanges. Increased market fragmentation decreases asset prices, reduces trading volume, and worsens market efficiency. By connecting market microstructure to asset prices, we demonstrate that frictions in the market structure are as important as ever, if not more so, even though trading technology is becoming increasingly efficient and sophisticated. The stock market has become increasingly fragmented, especially after the introduction of Regulatory NMS. While the regulation was intended to increase market transparency and competition between exchanges, fragmentation has become an unintended consequence, affecting asset prices in addition to market quality.

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A Summary Statistics

	Mean	St.D.	1st	25th	Median	75th	99th
A	0.0300	0.1369	0.0000	0.0003	0.0014	0.0083	0.6073
HHI	0.5136	0.2878	0.1641	0.2131	0.5475	0.7998	0.9537
φ	6.5191	3.9407	1.0787	3.5306	5.9937	8.5507	19.4323
$\lvert\bar{\varepsilon}\rvert^{\text{total}}$	3.1580	2.4420	0.8737	1.6733	2.3435	3.6929	12.7810
γ^{pex}	57.7407	30.5285	14.5236	24.9766	70.3600	87.5952	96.0925
ε^{pex}	-0.3476	3.9806	-11.5731	-2.4390	-0.1991	1.9057	9.6239
$\lvert\varepsilon\rvert^{\text{pex}}$	6.5895	3.7788	1.8784	3.9331	5.5605	8.1911	19.5133
λ	37.3142	73.7072	-5.9349	4.7947	13.8710	37.6968	379.4380
LDA	0.1437	1.4087	-2.0594	-0.0022	0.0008	0.0252	4.9093
|LDA|	0.5687	2.6320	0.0002	0.0036	0.0186	0.1428	11.3494
ME (bn)	9.4506	28.5812	0.0580	0.6320	1.9360	6.1070	149.6110
B/M	0.6151	0.5359	-0.2237	0.3025	0.5193	0.8089	2.5078
Observations	448924						

Table 12: **Summary Statistics (1993-2023)**

Summary statistics are constructed from January 1993 to December 2023 for stock-months that have non-missing data, encompassing ordinary common shares listed on the NYSE. **A** is the original Amihud (2002) illiquidity measure, calculated as the monthly average of the daily ratio of absolute return to dollar trading volume in millions. **AT** is the turnover version of the original Amihud (2002) illiquidity measure, which is calculated as the monthly average of the daily ratio of absolute return to share turnover. Share turnover is calculated as the daily share volume divided by the number of shares outstanding. A and AT are calculated using daily trading volume data from TAQ. **HHI** is the monthly average of daily Herfindahl–Hirschman indices, where the daily index is calculated as the sum of the squares of the daily market shares of each exchange and off-exchanges as a whole, based on their share volumes. $\boldsymbol{\varphi}$ is our new fragmentation measure, equal to $|\boldsymbol{\varepsilon}|^{\text{total}}/\gamma^{\text{pex}}$, and defined in Equation 8. $\boldsymbol{\gamma}^{\text{pex}}$ is defined as the average daily market share on the NYSE over the past three months, representing the long-term market share of the NYSE. $|\boldsymbol{\varepsilon}|^{\text{total}}$ is the monthly average of the absolute daily deviations in daily market shares from their long-term values, and is defined in Equation 7. $\boldsymbol{\varepsilon}^{\text{pex}}$ denotes short-term noise in market share, calculated as the monthly average of the daily deviation of the primary exchange's market share from its long-term market share γ^{pex}. $|\boldsymbol{\varepsilon}|^{\text{pex}}$ is the monthly average of the absolute daily deviation of the primary exchange's market share from its long-term market share γ^{pex}. $\boldsymbol{\lambda}$ is the monthly average of daily price impact coefficients. **LDA** (|**LDA**|) is the (absolute) illiquidity spread of A, defined as the monthly average of the (absolute) difference between two ratios: the daily ratio of absolute return to dollar trading volume, in millions, outside the primary listing exchange—NYSE—divided by $1-\gamma^{\text{pex}}$, and a similar measure on the primary exchange divided by γ^{pex}. **LDAT** and |**LDAT**| are constructed in analogous ways using AT. **ME (bn)** is market capitalization, expressed in billions of dollars, calculated using the closing prices and shares outstanding on the last trading day of the previous year. **B/M** is the ratio of book value of equity to the market value of equity, where the ratio in months from July of year $y+1$ to June of year $y+2$ is calculated using the book value from the fiscal year ending in calendar year y and the market value at the end of year y.

	A	HHI	φ	$\lvert\bar{\varepsilon}\rvert^{\text{total}}$	γ^{pex}	ε^{pex}	$\lvert\varepsilon\rvert^{\text{pex}}$	λ	LDA	|LDA|	lnME
A	1.00										
HHI	0.20	1.00									
φ	0.23	-0.45	1.00								
$\lvert\bar{\varepsilon}\rvert^{\text{total}}$	0.46	0.39	0.49	1.00							
γ^{pex}	0.16	0.97	-0.51	0.37	1.00						
ε^{pex}	0.02	0.09	-0.01	-0.11	-0.04	1.00					
$\lvert\varepsilon\rvert^{\text{pex}}$	0.33	0.33	0.41	0.86	0.34	-0.16	1.00				
λ	0.42	0.26	0.17	0.46	0.25	0.04	0.38	1.00			
LDA	0.03	0.12	0.07	0.20	0.09	0.23	0.17	0.29	1.00		
|LDA|	0.61	0.13	0.34	0.47	0.09	0.08	0.43	0.43	0.30	1.00	
lnME	-0.36	-0.31	-0.33	-0.61	-0.29	-0.02	-0.57	-0.49	-0.13	-0.34	1.00
Observations	448924										

Table 13: **Correlations Between (Il)Liquidity and Fragmentation Measures (1993-2023)**

Correlations are constructed from January 1993 to December 2023 for stock-months that have non-missing data, encompassing ordinary common shares listed on the NYSE. **A** is the original Amihud (2002) illiquidity measure, calculated as the monthly average of the daily ratio of absolute return to dollar trading volume in millions. **AT** is the turnover version of the original Amihud (2002) illiquidity measure, which is calculated as the monthly average of the daily ratio of absolute return to share turnover. Share turnover is calculated as the daily share volume divided by the number of shares outstanding. A and AT are calculated using daily trading volume data from TAQ. **HHI** is the monthly average of daily Herfindahl–Hirschman indices, where the daily index is calculated as the sum of the squares of the daily market shares of each exchange and off-exchanges as a whole, based on their share volumes. $\boldsymbol{\varphi}$ is our new fragmentation measure, equal to $|\boldsymbol{\varepsilon}|^{\text{total}}/\gamma^{\text{pex}}$, and defined in Equation 8. $\boldsymbol{\gamma}^{\text{pex}}$ is defined as the average daily market share on the NYSE over the past three months, representing the long-term market share of the NYSE. $|\boldsymbol{\varepsilon}|^{\text{total}}$ is the monthly average of the absolute daily deviations in daily market shares from their long-term values, and is defined in Equation 7. $\boldsymbol{\varepsilon}^{\text{pex}}$ denotes short-term noise in market share, calculated as the monthly average of the daily deviation of the primary exchange's market share from its long-term market share γ^{pex}. $|\boldsymbol{\varepsilon}|^{\text{pex}}$ is the monthly average of the absolute daily deviation of the primary exchange's market share from its long-term market share γ^{pex}. $\boldsymbol{\lambda}$ is the monthly average of daily price impact coefficients. **LDA** (|**LDA**|) is the (absolute) illiquidity spread of A, defined as the monthly average of the (absolute) difference between two ratios: the daily ratio of absolute return to dollar trading volume, in millions, outside the primary listing exchange—NYSE—divided by $1-\gamma^{\text{pex}}$, and a similar measure on the primary exchange divided by γ^{pex}. **LDAT** and |**LDAT**| are constructed in analogous ways using AT. **lnME** is the logarithm of market capitalization, calculated using the closing prices and shares outstanding on the last trading day of the previous year.

	A (CRSP)	A (TAQ)	AT (CRSP)	AT (TAQ)
A (CRSP)	1.00			
A (TAQ)	0.99	1.00		
AT (CRSP)	0.71	0.71	1.00	
AT (TAQ)	0.69	0.70	0.99	1.00
Observations	687754			

Table 14: **Correlations Between CRSP and TAQ Amihud Illiquidity Measures (1993-2023)**

Correlations are constructed from January 1993 to December 2023 for stock-months that have non-missing data, encompassing ordinary common shares listed on the NYSE. **A** is the original Amihud (2002) illiquidity measure, calculated as the monthly average of the daily ratio of absolute return to dollar trading volume in millions. **AT** is the turnover-version of the original Amihud (2002) illiquidity measure, which is calculated as the monthly average of the daily ratio of absolute return to share turnover. Share turnover is calculated as the daily share volume divided by the number of shares outstanding. **CRSP** and **TAQ** indicate that daily trading volume data are obtained from CRSP and TAQ, respectively.

B Black-Box Nature of HHI: An Illustrative Example

This section provides a hypothetical numerical example to demonstrate how HHI treats long-term market shares and short-term noises as perfect substitutes and consequently fails to take into account an important market dynamic. Consider a simple example with two stocks traded over three exchanges. Stock 1 has no short-term noise ($\varepsilon^j_{i,d,t} = 0$ for each j) and long-term market shares as follows: $\gamma^1_{i,-t} = 35\%$, $\gamma^2_{i,-t} = 56.10\%$, and $\gamma^3_{i,-t} = 8.90\%$. Stock 2 also has no short-term noise, with a long-term market share of 60% in exchange 1 ($\varepsilon_{i,d,t} = 0$ and $\gamma^1_{i,-t} = 60\%$). However, its total daily market shares in exchanges 2 and 3 fluctuate as follows: on any given trading day, one market center has a daily market share of 15%, and the other has 25%. Each of these two exchanges experiences these market shares equally across a month in a random manner. Suppose these patterns remain the same for an extended period of time. The average daily market share for Stock 2 in exchanges 2 and 3 over the past three months can then be calculated as 20%, which is assumed to be their long-term market share. Consequently, in exchanges 2 and 3, short-term noise is $\pm 5\%$ half of the time each month. The HHI for Stock 1 is 4,451, which is almost the same as for Stock 2, whose HHI is 4,450. If we use HHI to measure fragmentation, we would need to accept that both stocks are equally fragmented, yet they exhibit entirely distinct characteristics.

If we loosely interpret long-term market shares as the probability of matching with a counterparty in exchanges, for Stock 1, a trader knows the probability of matching in each exchange in advance and can accordingly divide their order across exchanges. However, for Stock 2, submitting an order to exchange 2 or 3 on a given day results in a match with either a 15% or 25% probability, without prior knowledge of which probability will apply. This introduces additional uncertainty when splitting an order between exchanges, highlighting a significant practical difference in trading dynamics despite similar HHI values. Although this is a hypothetical example to illustrate a point, there are numerous examples of stocks in our sample with similar HHIs but very distinct short-term and long-term market share characteristics.

C Fragmentation Premium with Turnover-Based Amihud Illiquidity Measure

	(1)	(2)	(3)	(4)	(5)	(6)
	1993-2023	1993-2004	2009-2023	1993-2023	1993-2004	2009-2023
HHI	1.406**	-1.735**	4.109***	1.188**	-1.464**	3.459***
	(0.604)	(0.676)	(0.785)	(0.522)	(0.660)	(0.645)
lnAT	0.352***	0.257***	0.458***	0.093	0.103	0.074
	(0.060)	(0.079)	(0.103)	(0.068)	(0.091)	(0.117)
λ				0.024***	0.005***	0.040***
				(0.004)	(0.001)	(0.006)
Observations	448924	197015	192808	448924	197015	192808
Adjusted R^2	0.044	0.035	0.054	0.049	0.037	0.061
Other Controls	✓	✓	✓	✓	✓	✓

Table 15: **Fama-MacBeth Regressions of Stock Returns on Herfindahl–Hirschman Index (HHI)**

Monthly Fama-MacBeth regressions are estimated from January 1993 to December 2023 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. We estimate a cross-sectional regression in each month t and then report the time-series averages of the coefficients and standard errors, as well as the adjusted R^2. The **dependent variable** is the monthly excess return over the Fama-French three-factor model, where the factor loadings are estimated using the past 60 months, with the requirement of having at least 24 observations. **HHI** is the monthly average of daily Herfindahl–Hirschman indices, where the daily index is calculated as the sum of the squares of the daily market shares of each exchange and off-exchanges as a whole, based on their share volumes. **lnAT** is the logarithm of AT, where **AT** is the turnover-version of the original Amihud (2002) illiquidity measure, which is calculated as the monthly average of the daily ratio of absolute return to share turnover. Share turnover is calculated as the daily share volume divided by the number of shares outstanding. AT is calculated using daily trading volume data from TAQ. $\boldsymbol{\lambda}$ is the monthly average of daily price impact coefficients. In the regression for month t, these independent variables are taken from month $t-2$. Additionally, we include other standard control variables, which are defined explicitly in Section 3, with coefficients not presented to save space: previous month's stock return, cumulative stock returns over the past twelve months excluding the previous one, and idiosyncratic return volatility. t-statistics are calculated using Newey-West robust standard errors with six lags. **Standard errors** are in parentheses; * $p < 0.10$ ** $p < 0.05$ *** $p < 0.01$.

	(1)	(2)	(3)	(4)	(5)	(6)
φ	0.091***	0.079***	0.067***		0.077***	
	(0.014)	(0.014)	(0.012)		(0.013)	
$\mathbb{1}(\varepsilon^{pex} \geq 0) \times \varphi$					-0.017***	
					(0.006)	
γ^{pex}				-0.011*		-0.012*
				(0.006)		(0.007)
$\bar{\lvert\varepsilon\rvert}^{\text{total}}$				0.360***		0.375***
				(0.070)		(0.071)
$\mathbb{1}(\varepsilon^{pex} \geq 0) \times \lvert\varepsilon\rvert^{\text{total}}$						-0.033**
						(0.015)
λ			0.024***	0.024***	0.024***	0.025***
			(0.004)	(0.004)	(0.004)	(0.004)
lnAT		0.379***	0.112*	0.044	0.118*	0.052
		(0.062)	(0.066)	(0.066)	(0.066)	(0.066)
Observations	448924	448924	448924	448924	448924	448924
Adjusted R^2	0.041	0.043	0.047	0.049	0.048	0.050
Other Controls	✓	✓	✓	✓	✓	✓

Table 16: **Fama-MacBeth Regressions of Stock Returns on Short-Term and Long-Term Fragmentation**

Monthly Fama-MacBeth regressions are estimated from January 1993 to December 2023 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. We estimate a cross-sectional regression in each month t and then report the time-series averages of the coefficients and standard errors, as well as the adjusted R^2. The **dependent variable** is the monthly excess return over four factors (Fama-French three factors—MKT, SMB, and HML—and the momentum factor—UMD), where the factor loadings are estimated using the past 60 months, with the requirement of having at least 24 observations. $\boldsymbol{\varphi}$ is our new fragmentation measure, equal to $|\boldsymbol{\varepsilon}|/\gamma^{\text{pex}}$, and defined in Equation 8. $\boldsymbol{\gamma}^{\text{pex}}$ is defined as the average daily market share on the NYSE over the past three months, representing the long-term market share of the NYSE. $|\boldsymbol{\varepsilon}|$ is the monthly average of the absolute daily deviations in the NYSE's market share from its long-term value γ^{pex}, and is defined in Equation 7. $\boldsymbol{\mathbb{1}(\varepsilon^{pex} \geq 0)}$ is a dummy variable that is equal to one when ε^{pex} is non-negative, where $\boldsymbol{\varepsilon^{pex}}$ is the monthly average of the daily deviations in the NYSE's market share from its long-term value γ^{pex}, and is defined in Equation 6. $\boldsymbol{\lambda}$ is the monthly average of daily price impact coefficients. **lnAT** is the logarithm of AT, where **AT** is the turnover-version of the original Amihud (2002) illiquidity measure, which is calculated as the monthly average of the daily ratio of absolute return to share turnover. Share turnover is calculated as the daily share volume divided by the number of shares outstanding. AT is calculated using daily trading volume data from TAQ. In the regression for month t, these independent variables are taken from month $t-2$. Additionally, we include other standard control variables, which are defined explicitly in Section 3, with coefficients not presented to save space: previous month's stock return, cumulative stock returns over the past twelve months excluding the previous one, and idiosyncratic return volatility. t-statistics are calculated using Newey-West robust standard errors with six lags. **Standard errors** are in parentheses; * $p < 0.10$ ** $p < 0.05$ *** $p < 0.01$.

	(1)	(2)	(3)	(4)
	1993-2004	2009-2023	1993-2004	2009-2023
φ	0.035***	0.112***	0.039***	0.129***
	(0.013)	(0.016)	(0.011)	(0.019)
$\mathbb{1}(\varepsilon^{pex} \geq 0) \times \varphi$			-0.012	-0.023***
			(0.011)	(0.007)
λ	0.006***	0.042***	0.006***	0.042***
	(0.001)	(0.006)	(0.001)	(0.006)
lnAT	0.020	0.164	0.024	0.171
	(0.092)	(0.113)	(0.091)	(0.111)
Observations	197015	192808	197015	192808
Adjusted R^2	0.035	0.060	0.035	0.061
Other Controls	✓	✓	✓	✓

Table 17: **Fama-MacBeth Regressions of Excess Returns on Fragmentation Before and After Regulation NMS**

Monthly Fama-MacBeth regressions are estimated from January 1993 to December 2023 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. We estimate a cross-sectional regression in each month t and then report the time-series averages of the coefficients and standard errors, as well as the adjusted R^2. The **dependent variable** is the monthly excess return over four factors (Fama-French three factors—MKT, SMB, and HML—and the momentum factor—UMD), where the factor loadings are estimated using the past 60 months, with the requirement of having at least 24 observations. $\boldsymbol{\varphi}$ is our new fragmentation measure, equal to $|\boldsymbol{\varepsilon}|/\gamma^{\text{pex}}$, and defined in Equation 8. $\boldsymbol{\gamma}^{\text{pex}}$ is defined as the average daily market share on the NYSE over the past three months, representing the long-term market share of the NYSE. $|\boldsymbol{\varepsilon}|$ is the monthly average of the absolute daily deviations in the NYSE's market share from its long-term value γ^{pex}, and is defined in Equation 7. $\mathbb{1}(\boldsymbol{\varepsilon^{pex}} \geq \mathbf{0})$ is a dummy variable that is equal to one when ε^{pex} is non-negative, where $\boldsymbol{\varepsilon^{pex}}$ is the monthly average of the daily deviations in the NYSE's market share from its long-term value γ^{pex}, and is defined in Equation 6. $\boldsymbol{\lambda}$ is the monthly average of daily price impact coefficients. **lnAT** is the logarithm of AT, where **AT** is the turnover-version of the original Amihud (2002) illiquidity measure, which is calculated as the monthly average of the daily ratio of absolute return to share turnover. Share turnover is calculated as the daily share volume divided by the number of shares outstanding. AT is calculated using daily trading volume data from TAQ. In the regression for month t, these independent variables are taken from month $t-2$. Additionally, we include other standard control variables, which are defined explicitly in Section 3, with coefficients not presented to save space: previous month's stock return, cumulative stock returns over the past twelve months excluding the previous one, and idiosyncratic return volatility. t-statistics are calculated using Newey-West robust standard errors with six lags. **Standard errors** are in parentheses; * $p < 0.10$ ** $p < 0.05$ *** $p < 0.01$.

D Fragmentation Measure Based on Only Primary Listing Exchange

This fragmentation measure offers an advantage. Although it is easy to extend this measure and include long-term market shares of other exchanges and their short-term deviations, this alternative measure better mitigate problems of HHI resulting from off-exchange reporting regulations. The NYSE market shares are less susceptible to off-exchange reporting practices and can therefore be estimated in a more consistent manner throughout the entire TAQ data period. This stability results from the NYSE's lesser inclination to make arrangements with ECNs, given its longstanding tradition of manual floor trading until the implementation of the NYSE Hybrid in October 2006.[26] Even though the NYSE's market share prior to March 2007 includes some off-exchange transactions with utmost certainty, Figure 2 illustrates that the changes in off-exchange reporting regulations appear to cause only a slight decline in the NYSE's market share, not a jump, when passing from February to March 2007. Moreover, this decline is relatively modest compared to the decreases observed in other exchanges, such as NASDAQ. The downside of this alternative is that it ignores short-term market share deviations in other markets, which are important in fragmented market structures.

	Low	2	3	4	High	H-L	t-stat	p-val
Fragmentation (φ^{pex})	0.18	0.16	0.35	0.46	1.01	0.83	6.69	0.00

Table 18: **Monthly Excess Portfolio Returns Sorted on Alternative Fragmentation Measure**

[26] For instance, Anderson and Dyl (2007) discuss differences in reported volumes of NYSE and NASDAQ, noting no arrangements between the NYSE and ECNs, but reporting NASDAQ's acquisitions of two ECNs. Similarly, Caglio and Mayhew (2016) examine arrangements between exchanges and ECNs and their effects on exchanges' market shares, but do not report any such arrangements for the NYSE.

	(1)	(2)	(3)	(4)	(5)	(6)
φ^{pex}	0.042***	0.029***	0.029***		0.034***	
	(0.005)	(0.005)	(0.004)		(0.005)	
$\mathbb{1}(\varepsilon^{pex} \geq 0) \times \varphi^{\text{pex}}$					-0.007**	
					(0.003)	
γ^{pex}				-0.029***		-0.034***
				(0.007)		(0.008)
$\lvert\varepsilon\rvert^{\text{pex}}$				0.067***		0.080***
				(0.014)		(0.016)
$\mathbb{1}(\varepsilon^{pex} \geq 0) \times \lvert\varepsilon\rvert^{\text{pex}}$						-0.024***
						(0.007)
λ			0.022***	0.022***	0.022***	0.022***
			(0.004)	(0.004)	(0.004)	(0.004)
lnA		0.295***	0.134***	0.157***	0.134***	0.164***
		(0.045)	(0.038)	(0.037)	(0.037)	(0.037)
Observations	448924	448924	448924	448924	448924	448924
Adjusted R^2	0.040	0.044	0.047	0.049	0.048	0.049
Other Controls	✓	✓	✓	✓	✓	✓

Table 19: **Fama-MacBeth Regressions of Excess Returns on Alternative Fragmentation Measure and Its Components**

Monthly Fama-MacBeth regressions are estimated from January 1993 to December 2023 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. We estimate a cross-sectional regression in each month t and then report the time-series averages of the coefficients and standard errors, as well as the adjusted R^2. The **dependent variable** is the monthly excess return over four factors (Fama-French three factors—MKT, SMB, and HML—and the momentum factor—UMD), where the factor loadings are estimated using the past 60 months, with the requirement of having at least 24 observations. $\boldsymbol{\varphi}$ is our new fragmentation measure, equal to $|\boldsymbol{\varepsilon}|/\gamma^{\text{pex}}$, and defined in Equation 8. $\boldsymbol{\gamma}^{\text{pex}}$ is defined as the average daily market share on the NYSE over the past three months, representing the long-term market share of the NYSE. $|\boldsymbol{\varepsilon}|$ is the monthly average of the absolute daily deviations in the NYSE's market share from its long-term value γ^{pex}, and is defined in Equation 7. $\mathbb{1}(\boldsymbol{\varepsilon^{pex}} \geq \mathbf{0})$ is a dummy variable that is equal to one when ε^{pex} is non-negative, where $\boldsymbol{\varepsilon^{pex}}$ is the monthly average of the daily deviations in the NYSE's market share from its long-term value γ^{pex}, and is defined in Equation 6. $\boldsymbol{\lambda}$ is the monthly average of daily price impact coefficients. **lnA** is the natural logarithm of the original Amihud illiquidity measure A, calculated as the monthly average of the daily ratio of absolute return to dollar trading volume in millions. A is calculated using daily trading volume data from TAQ. In the regression for month t, these independent variables are taken from month $t-2$. Additionally, we include other standard control variables, which are defined explicitly in Section 3, with coefficients not presented to save space: previous month's stock return, cumulative stock returns over the past twelve months excluding the previous one, and idiosyncratic return volatility. t-statistics are calculated using Newey-West robust standard errors with six lags. **Standard errors** are in parentheses; * $p < 0.10$ ** $p < 0.05$ *** $p < 0.01$.

E Illiquidity and Fragmentation

We derive two alternative measures of (il)liquidity that also accounts for how returns respond to aggregate volume as well as long-term and short-term fragmentation, and demonstrate that our results are robust to these alternative specifications. Even though these alternative measures are highly correlated with our main measure that we develop in Equation 12, they slightly capture different aspects of short- or long-term fragmentation.

E.1 Measure I

We develop a measure that does not adjust for differences in long-term market shares across stocks when defining $\delta_{i,d,t}$. Instead, we re-define $\delta_{i,d,t}$ in Equation 11 without incorporating $1-\gamma_{i,-t}$ as follows:

$$\hat{\delta}_{i,d,t} := \left(\frac{|r_{i,d,t}|}{\mathrm{V}^{\$,\mathrm{other}}_{i,d,t}/(1-\gamma_{i,-t})} - \frac{|r_{i,d,t}|}{\mathrm{V}^{\$,\mathrm{nyse}}_{i,d,t}/\gamma_{i,-t}} \right) \tag{21}$$

Following similar steps, this setup results in an equation very similar to our main measure that we develop in Equation 12. We call this new measure $\widehat{|\mathrm{LDA}_{i,t}|}^{*}$:

$$\widehat{|\mathrm{LDA}_{i,t}|}^{*} = \left[\frac{1}{D_{it}} \sum_{d=1}^{D_{it}} \frac{|r_{i,d,t}|}{\mathrm{V}^{\$}_{i,d,t}} \times \left| \frac{\varepsilon_{i,d,t}}{\mathrm{V}_{i,d,t}} \right| \right] \times \frac{1}{\gamma_{i,-t}(1-\gamma_{i,-t})}. \tag{22}$$

which can be further decomposed as

$$\widehat{|\mathrm{LDA}_{i,t}|}^{*} \approx \underbrace{\left[\frac{1}{D_{it}} \sum_{d=1}^{D_{it}} \frac{|r_{i,d,t}|}{\mathrm{V}^{\$}_{i,d,t}} \right]}_{\text{Amihud Illiquidity } (\mathrm{A}_{i,t})} \times \underbrace{\left[\frac{1}{D_{it}} \sum_{d=1}^{D_{it}} \left| \frac{\varepsilon_{i,d,t}}{\mathrm{V}_{i,d,t}} \right| \right]}_{\text{Short-Term Fragmentation } (|\varepsilon_{i,t}|)} \times \underbrace{\left[\frac{1}{\gamma_{i,-t}(1-\gamma_{i,-t})} \right]}_{\text{Long-Term Fragmentation}} \tag{23}$$

It is not surprising that this alternative measure is highly correlated with our main measure and also exhibits significant pricing. It is identical to our main measure except for the last term. However, there is an important nuance concerning the aspect of long-term fragmentation it captures. Our main measure reflects the extent to which long-term market share is consolidated on a stock's primary listing exchange, the NYSE. In contrast, this alternative measure captures how equally long-term market shares are distributed across the NYSE and other market centers. *Ceteris paribus*, $\widehat{|\mathrm{LDA}_{i,t}|}$ takes its minimum value when $\gamma_{i,-t} = 1/2$—indicating that the NYSE and other market centers hold equal amounts of

long-term market shares. If we loosely interpret long-term market share as the probability of matching with a counterparty in a market center, this alternative measure suggests that long-term fragmentation is least harmful to liquidity when market centers are more alike in terms of execution probabilities.

	(1)	(2)	(3)
|LDA|	1.063***	0.703***	0.794***
	(0.202)	(0.135)	(0.144)
$\mathbb{1}(\text{LDA} \geq 0) \times \text{LDA}$			-0.204**
			(0.094)
λ		0.021***	0.022***
		(0.003)	(0.003)
Observations	448924	448924	448924
Adjusted R^2	0.043	0.047	0.048
Other Controls	✓	✓	✓

Table 20: **Fama-MacBeth Regressions of Stock Returns on |LSA|**

Monthly Fama-MacBeth regressions are estimated from January 1993 to December 2023 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. We estimate a cross-sectional regression in each month t and then report the time-series averages of the coefficients and standard errors, as well as the adjusted R^2. The **dependent variable** is the monthly excess return over the Fama-French three-factor model, where the factor loadings are estimated using the past 60 months, with the requirement of having at least 24 observations. **LDA** (**|LDA|**) is the (absolute) illiquidity spread of A, defined as the monthly average of the (absolute) difference between two ratios: the daily ratio of absolute return to dollar trading volume, in millions, outside the primary listing exchange—NYSE—divided by $1 - \gamma^{\text{pex}}$, and a similar measure on the primary exchange divided by γ^{pex}. $\mathbb{1}(\text{LDA} \geq 0)$ is a dummy variable that is equal to one when LSA is non-negative. $\boldsymbol{\lambda}$ is the monthly average of daily price impact coefficients. In the regression for month t, these independent variables are taken from month $t-2$. Additionally, we include other standard control variables, which are defined explicitly in Section 3, with coefficients not presented to save space: previous month's stock return, cumulative stock returns over the past twelve months excluding the previous one, and idiosyncratic return volatility. t-statistics are calculated using Newey-West robust standard errors with six lags. **Standard errors** are in parentheses; * $p < 0.10$ ** $p < 0.05$ *** $p < 0.01$.

E.2 Measure II

We develop a measure that directly uses monthly averages of approximated $\delta_{i,d,t}$ in Equation 13 to measure liquidity with fragmentation, instead of first taking its absolute values. Following similar steps, this setup results in an equation very similar to our main measure that we develop in Equation 12. We call this new measure $\text{LDA}_{i,t}$:

$$\text{LDA}_{i,t} := \left[\frac{1}{D_{it}}\sum_{d=1}^{D_{it}}\frac{|r_{i,d,t}|}{\text{V}^{\$}_{i,d,t}} \times \frac{\varepsilon_{i,d,t}}{\text{V}_{i,d,t}}\right] \times \frac{1}{\gamma_{i,-t}} \tag{24}$$

which can be further decomposed as

$$\text{LDA}_{i,t} \approx \underbrace{\left[\frac{1}{D_{it}}\sum_{d=1}^{D_{it}}\frac{|r_{i,d,t}|}{\text{V}^{\$}_{i,d,t}}\right]}_{\text{Amihud Illiquidity } (\text{A}_{i,t})} \times \underbrace{\left[\frac{1}{D_{it}}\sum_{d=1}^{D_{it}}\frac{\varepsilon_{i,d,t}}{\text{V}_{i,d,t}}\right]}_{\text{Short-Term Fragmentation } (\varepsilon_{i,t})} \times \underbrace{\left[\frac{1}{\gamma_{i,-t}}\right]}_{\text{Long-Term Fragmentation}} \tag{25}$$

This measure is the same as our main measure, except that the second term takes averages of $\frac{\varepsilon_{i,d,t}}{\text{V}_{i,d,t}}$ instead of $\left|\frac{\varepsilon_{i,d,t}}{\text{V}_{i,d,t}}\right|$. The advantage of this approach is that the correlation between $|r_{i,d,t}|/\text{V}^{\$}_{i,d,t}$ and $\varepsilon_{i,d,t}/\text{V}_{i,d,t}$ is nearly zero (0.0175), allowing for a more precise approximation when switching from Equation 24 to 25. The downside is that $\varepsilon_{i,d,t}$ can frequently alternate in sign for some stocks, leading to cancellations between positive and negative terms when calculating $\frac{1}{D_{it}}\sum_{d=1}^{D_{it}}\frac{\varepsilon_{i,d,t}}{\text{V}_{i,d,t}}$, which can underestimate short-term fragmentation in some cases.

	(1)	(2)	(3)
|LDA|	1.989***	1.207***	1.494***
	(0.337)	(0.213)	(0.232)
$\mathbb{1}(\text{LDA} \geq 0) \times \text{LDA}$			-0.287
			(0.182)
λ		0.022***	0.022***
		(0.004)	(0.004)
Observations	448924	448924	448924
Adjusted R^2	0.042	0.046	0.047
Other Controls	✓	✓	✓

Table 21: **Fama-MacBeth Regressions of Stock Returns on |LSA|**

Monthly Fama-MacBeth regressions are estimated from January 1993 to December 2023 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. We estimate a cross-sectional regression in each month t and then report the time-series averages of the coefficients and standard errors, as well as the adjusted R^2. The **dependent variable** is the monthly excess return over the Fama-French three-factor model, where the factor loadings are estimated using the past 60 months, with the requirement of having at least 24 observations. **LDA** (**|LDA|**) is the (absolute) illiquidity spread of A, defined as the monthly average of the (absolute) difference between two ratios: the daily ratio of absolute return to dollar trading volume, in millions, outside the primary listing exchange—NYSE—divided by $1 - \gamma^{\text{pex}}$, and a similar measure on the primary exchange divided by γ^{pex}. $\mathbb{1}$**(LDA** $\geq$ **0)** is a dummy variable that is equal to one when LSA is non-negative. $\boldsymbol{\lambda}$ is the monthly average of daily price impact coefficients. In the regression for month t, these independent variables are taken from month $t-2$. Additionally, we include other standard control variables, which are defined explicitly in Section 3, with coefficients not presented to save space: previous month's stock return, cumulative stock returns over the past twelve months excluding the previous one, and idiosyncratic return volatility. t-statistics are calculated using Newey-West robust standard errors with six lags. **Standard errors** are in parentheses; * $p < 0.10$ ** $p < 0.05$ *** $p < 0.01$.

F Figures

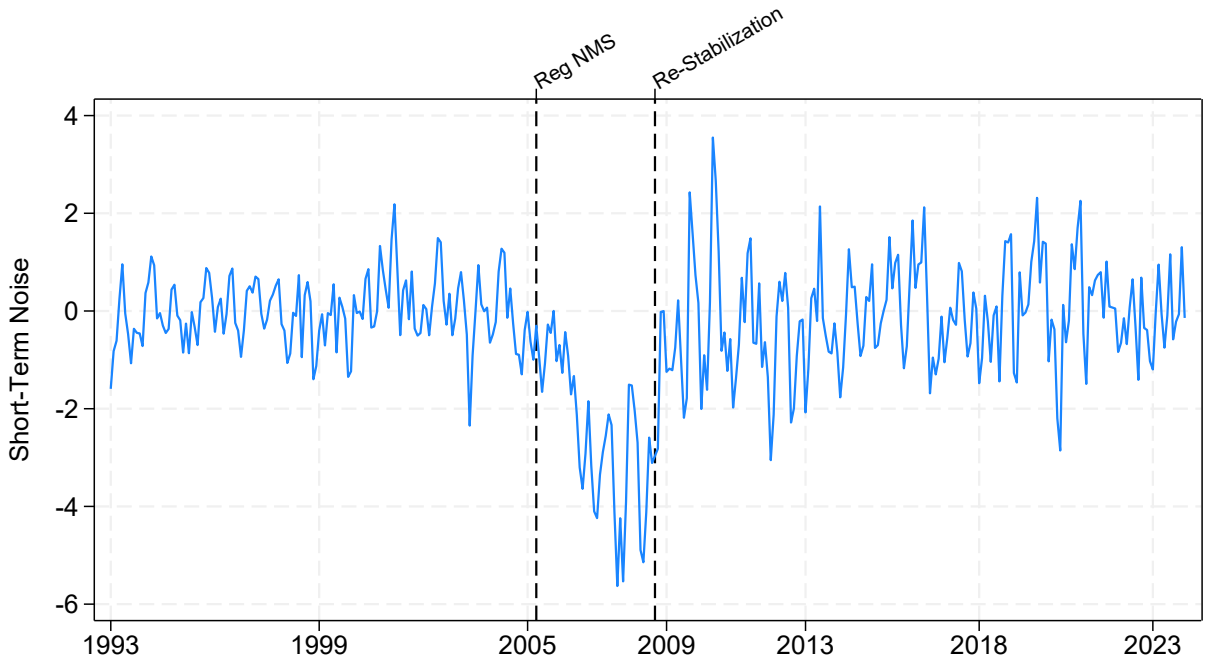


Figure 4: Short-Term Deviations in Market Share from NYSE to Other Market Centers (ε^{pex})

This figure displays the monthly cross-sectional averages of ε^{pex} from January 1993 to December 2023 for stock-months that have non-missing data, encompassing ordinary common shares listed on the NYSE. We decompose market shares (in share volumes) of the primary listing exchange—NYSE—into long-term (γ^{pex}) and short-term (ε^{pex}) components. $\boldsymbol{\gamma}^{\text{pex}}$ is defined as the average daily market share on the NYSE over the past three months, representing the long-term market share of the NYSE. $\boldsymbol{\varepsilon}^{\text{pex}}$ denotes short-term noise in market share, calculated as the monthly average of the daily deviation of the primary exchange's market share from its long-term market share γ^{pex}. The left dashed line corresponds to April 2005, the time of adoption of Regulation NMS. The right dashed line corresponds to September 2008, around when the NYSE's long-term market share stabilized again.

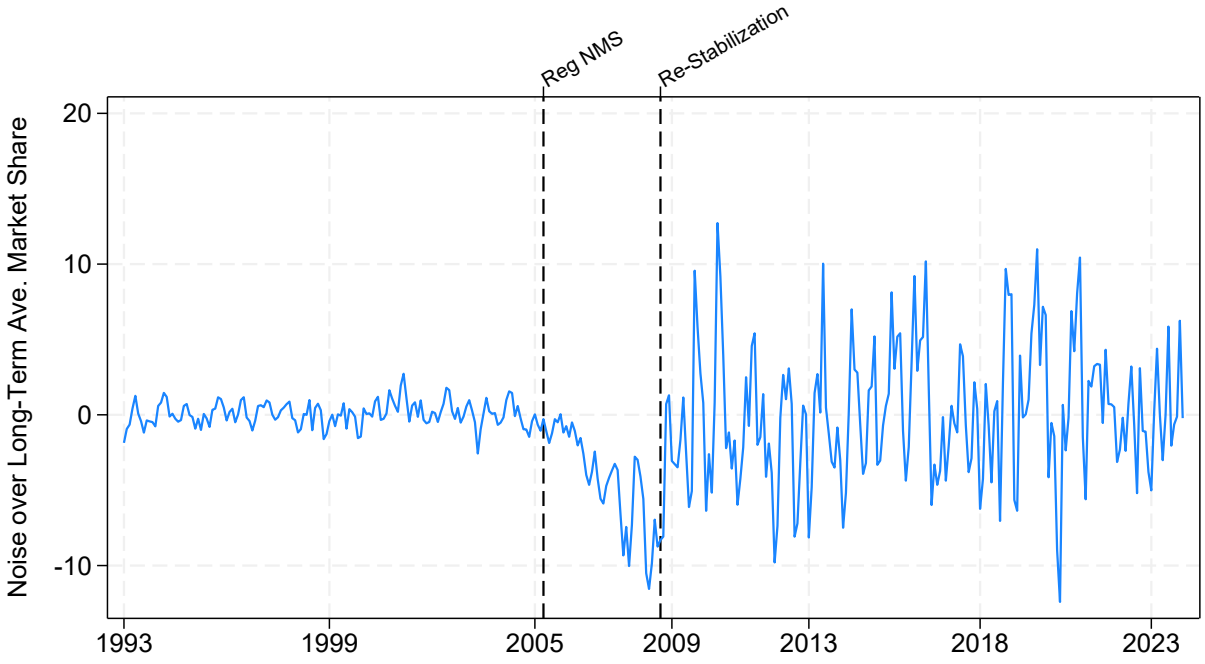


Figure 5: Short-Term Noise over Long-Term Market Share of NYSE ($\varepsilon^{\text{pex}}/\gamma^{\text{pex}}$)

This figure displays the monthly cross-sectional averages of the ratio of ε^{pex} to $\boldsymbol{\gamma}^{\text{pex}}$ from January 1993 to December 2023 for stock-months with non-missing data, encompassing ordinary common shares listed on the NYSE. We decompose market shares (in share volumes) of the primary listing exchange—NYSE—into long-term (γ^{pex}) and short-term (ε^{pex}) components. $\boldsymbol{\gamma}^{\text{pex}}$ is defined as the average daily market share on the NYSE over the past three months, representing the long-term market share of the NYSE. $\boldsymbol{\varepsilon}^{\text{pex}}$ denotes short-term noise in market share, calculated as the monthly average of the daily deviation of the primary exchange's market share from its long-term market share γ^{pex}. The left dashed line corresponds to April 2005, the time of adoption of Regulation NMS. The right dashed line corresponds to September 2008, around when the NYSE's long-term market share stabilized again.

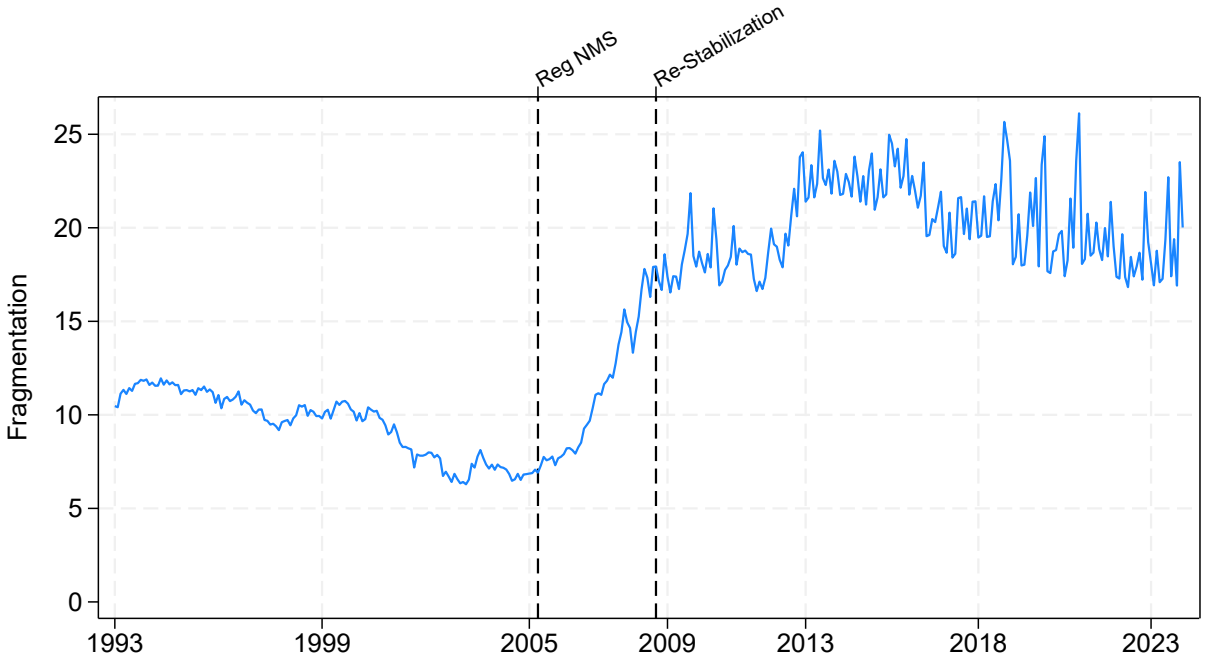


Figure 6: Market Fragmentation (φ)

This figure displays the monthly cross-sectional averages of our new fragmentation measure: the ratio of $|\varepsilon|^{\text{total}}$ to $\boldsymbol{\gamma}^{\text{pex}}$ from January 1993 to December 2023 for stock-months that have non-missing data, encompassing ordinary common shares listed on the NYSE. We decompose market shares (in share volumes) of each exchange into long-term (γ) and short-term (ε) components. $\boldsymbol{\gamma}^{\text{pex}}$ is defined as the average daily market share on the NYSE over the past three months, representing the long-term market share of the NYSE. $|\boldsymbol{\varepsilon}|^{\text{total}}$ is the monthly average of the absolute daily deviations in daily market shares of exchanges from their long-term values, and is defined in Equation 7. The left dashed line corresponds to April 2005, the time of adoption of Regulation NMS. The right dashed line corresponds to September 2008, around when the NYSE's long-term market share stabilized again.

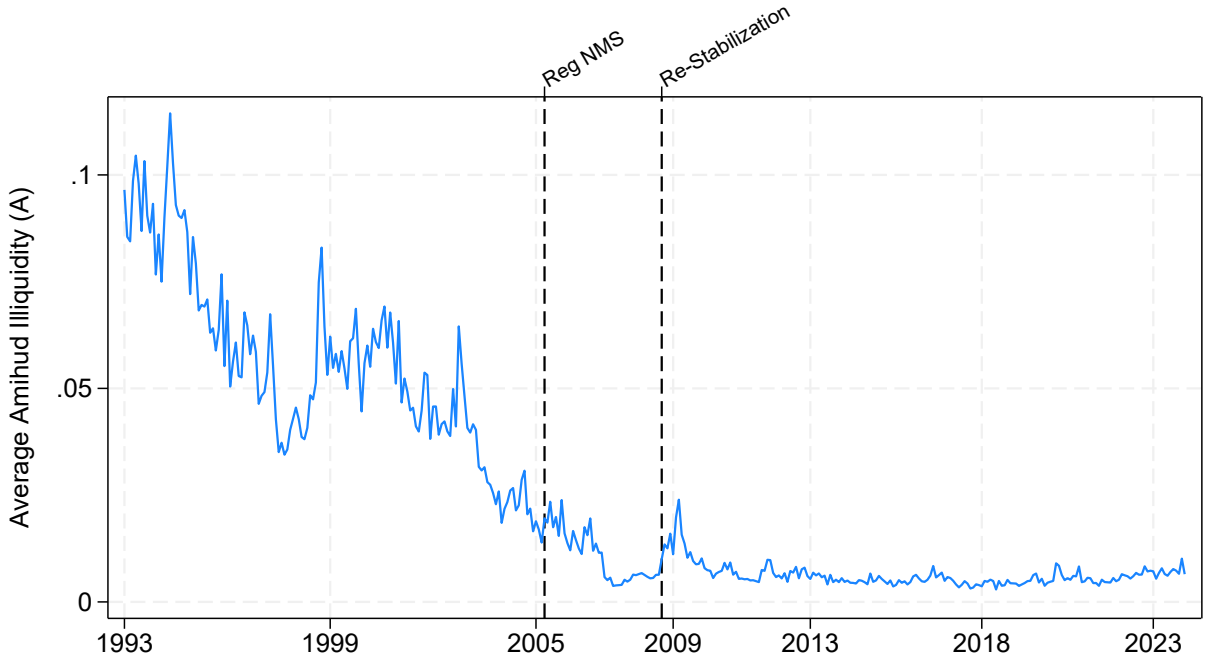


Figure 7: Average Amihud Illiquidity (A)

This figure displays the monthly cross-sectional averages of the original Amihud (2002) illiquidity measure A from January 1993 to December 2023 for stock-months that have non-missing data, encompassing ordinary common shares listed on the NYSE. **A** of a stock for a specific month is calculated as the monthly average of the daily ratio of absolute return to dollar trading volume in millions. The left dashed line corresponds to April 2005, the time of adoption of Regulation NMS. The right dashed line corresponds to September 2008, around when the NYSE's long-term market share stabilized again.

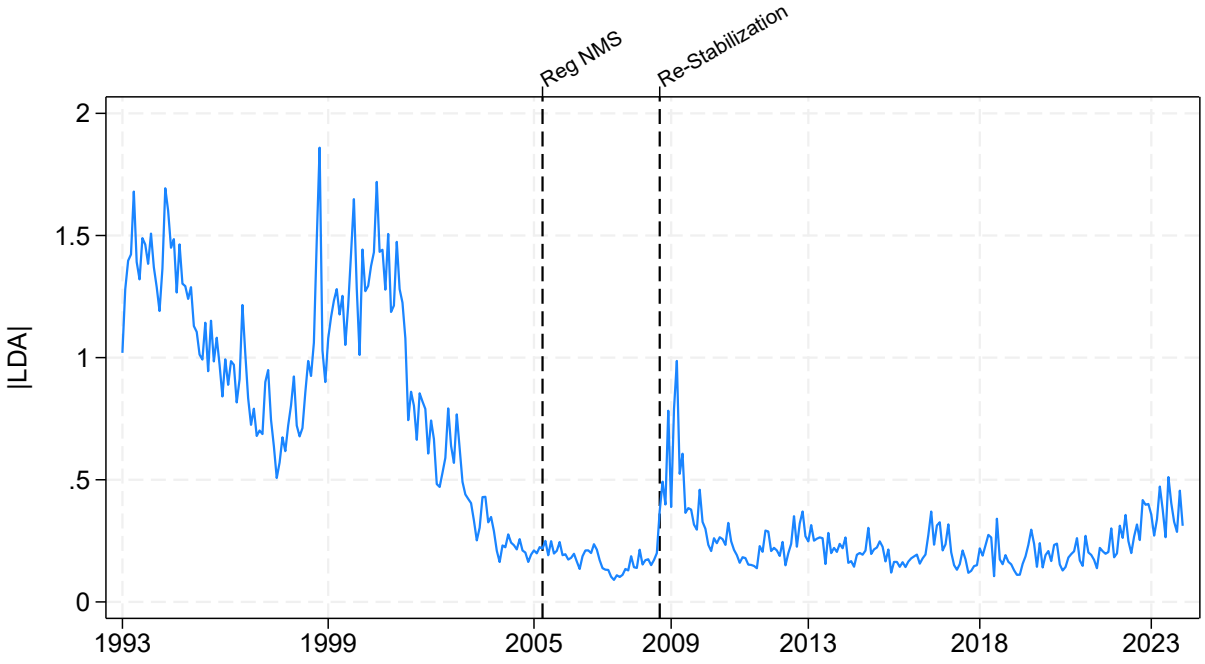


Figure 8: Absolute Illiquidity Discrepancy of A (|LDA|)

This figure displays the monthly cross-sectional averages of |LDA| from January 1993 to December 2023 for stock-months that have non-missing data, encompassing ordinary common shares listed on the NYSE. **LDA** (|**LDA**|) is the (absolute) illiquidity spread of A, defined as the product of $1 - \gamma^{\text{pex}}$ and the monthly average of the (absolute) difference between two ratios: the daily ratio of absolute return to dollar trading volume, in millions, outside the primary listing exchange—NYSE—divided by $1 - \gamma^{\text{pex}}$, and a similar measure on the primary exchange divided by γ^{pex}. $\boldsymbol{\gamma}^{\text{pex}}$ is defined as the average daily market share on the NYSE over the past three months, representing the long-term market share of the NYSE.