ON WELFARE IN OVER-THE-COUNTER AND CENTRALIZED MARKETS*

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Abstract. This paper studies the welfare implications of accessing centralized versus decentralized over-the-counter (OTC) secondary markets. We develop an informed principal contract design model with a bi-dimensional signalling device and endogenous signalling costs. Good borrowers can choose to signal their type by liquidating assets that cannot be posted as collateral in a centralized market or exchanging them in an OTC market for pledgeable assets. Equilibrium prices and haircuts determine signaling costs endogenously. In the optimal separating contract, the cheapest market in terms of signaling costs is accessed. We establish conditions for equilibria in which different markets are accessed - centralized, OTC-only, and dual-market - and rank them by the welfare they provide to borrowers. We show that OTC-only equilibria offer the highest welfare, followed by dual-market and centralized-market equilibria.

Keywords: Collateral, OTC Markets, Asymmetric Information, Informed Principal, Multidimensional Signalling

JEL Codes: C78, D53, D82, G1

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1. Introduction

Secondary markets are essential for the efficient functioning of the financial system. The U.S. repurchase agreement (repo) market, for instance, enables financial institutions to leverage assets by borrowing against them, enhancing liquidity and monetary policy implementation. Centralized markets (e.g., tri-party and general collateral finance (GCF) repo) handle approximately $2.7 trillion[1] in daily transactions, while decentralized markets (e.g., bilateral repo) average about $4.6 trillion.

Despite their scale, repo markets face inefficiencies. Centralized repos, constrained by limited flexibility, high standardization, and clearing costs, are ill-suited for non-standard collateral. Institutions holding low-quality or illiquid assets must either endure the high costs of raising cash in centralized markets or negotiate in decentralized OTC markets. While decentralized repos offer greater flexibility, they are plagued by counterparty risk, transparency issues, and liquidity constraints during periods of stress. In OTC repos, lower-quality collateral is exchanged for high-quality assets (e.g., U.S. Treasuries) at a "haircut" with terms often favoring dealers in liquidity-scarce environments. To address these inefficiencies, regulators have proposed measures to enhance investor participation in centralized markets (Board, 2022), but the feasibility and welfare impact of these policies remain uncertain. A robust theoretical framework is needed to better understand the economic mechanisms and guide effective policy design.

In this paper, we develop a novel theory to examine the welfare implications of accessing centralized versus decentralized markets. We model a lending market with asymmetric information, where only a good borrower (e.g., a bank or hedge fund) has a profitable project but lacks sufficient cash to invest. To secure financing, the borrower must signal her type to the lender. The borrower, unable to distinguish herself

[1]Daily outstanding amounts for Tri-Party and GCF Repo markets are approximately $2.7 trillion, while decentralized markets (e.g., bilateral repo) account for $4.6 trillion on average.

from a bad borrower through cash alone, possesses low-quality collateral that is legally non-pledgeable[2]. To address this, she can access either a centralized or decentralized OTC secondary market (or both). In centralized markets (e.g., tri-party or general collateral finance repos), she liquidates low-quality assets to raise cash, incurring a cost reflected in equilibrium prices, interpreted as transaction fees or balance sheet costs. In decentralized OTC markets (e.g., bilateral repos), the borrower exchanges low-quality assets for high-quality ones at a haircut by matching with a dealer[3]. This exchange creates surplus by enabling the borrower to use the high-quality collateral to signal her type. The surplus is split via asymmetric Nash bargaining, with the haircut determined by bargaining frictions in decentralized markets.

Our contribution is twofold. First, we establish the conditions under which equilibria with varying usage of secondary markets exist, whereas past literature assumes their existence. We prove the existence of a CM-only equilibrium, where the good borrower exclusively uses the centralized market. For moderate dealer bargaining power, we show that OTC-only equilibria also exist. Furthermore, we identify a class of dual-market equilibria, where both markets are accessed. These dual-market equilibria require less stringent thresholds for dealer bargaining power than OTC-only equilibria and vary based on the relative intensity of market usage.

Second, our model enables a coherent comparison of signaling costs across market access scenarios, ranking equilibria by the welfare they provide to the good borrower. OTC-only equilibria deliver the highest welfare, while CM-only equilibria offer the lowest. Dual-market equilibria perform better than CM-only but do not match the welfare of OTC-only outcomes. This welfare-ranking result depends on the terms of trade in each market and their structural protocols. Specifically, in OTC-only

[2]This maps on to regulation that only unemcumbered high-quality liquid assets can be pledged as collateral. See Basel III: The Liquidity Coverage Ratio and liquidity risk monitoring tools https://www.bis.org/publ/bcbs238.pdf

[3]Jeremy Stein explains this as "collateral transformation." See Jeremy Stein's speech https://www.federalreserve.gov/newsevents/speech/stein20130207a.htm

equilibria, the signaling cost implied by the haircut is lower than the cost implied by the price in a centralized market with no borrower participation. Convex balance sheet costs in centralized markets create a downward-sloping demand curve, increasing cash-raising costs as market usage intensifies. Consequently, the signaling cost in OTC-only equilibria is always lower than in CM-only equilibria. A similar intuition holds for the dual-market equilibria.

We first analyze a benchmark economy without secondary markets, where borrowers unable to signal through cash face a pooling equilibrium, potentially leading to market breakdown if the project returns are too low. In contrast, when secondary markets are accessible, we show that a welfare-improving separating menu of contracts can emerge. Our focus is on equilibria where the good borrower selects her welfare-maximizing contract based on equilibrium prices in the secondary markets. To identify such equilibria, we use a two-step approach. First, we characterize the equilibria in the centralized and OTC markets. Then, given the centralized market price and OTC haircut, we derive a separating menu of contracts optimal for the good borrower and acceptable to the lender. This contract aligns with the least-costly separating equilibrium (the "Riley outcom") in a game with two-dimensional signals, where the signaling costs are endogenously determined by the equilibrium prices and haircuts in the secondary markets.

We then show that the good borrower's optimal separating contract requires accessing only the market with the lowest signaling cost. The contract involves using both markets only if the centralized market price equals the discounted value of low-quality assets implied by the OTC haircut. The first main set of results pertains to the existence of equilibria in the lending game where secondary markets are accessible. We find that a CM-only equilibrium always exists, where the good borrower signals her type by selling assets at a discount to their expected revenue. This discount reflects

the liquidity cost borne by outside investors purchasing the assets, which is increasing and convex in the amount of cash required for the transaction.

Second, in addition to the CM-only equilibrium, we find OTC-only equilibria[4], which exist only if the dealer's bargaining power is not too high. Unlike the CM-only equilibrium, these equilibria involve the borrower acquiring high-quality assets for collateral by conceding a haircut to the dealer in one-to-one bargaining. The haircut reflects the cost of obtaining pledgeable assets, which increases with the dealer's bargaining power.

Third, we identify a set of equilibria where the good borrower accesses both markets to execute the contract, termed "dual-market equilibria." In these equilibria, the borrower's relative use of the centralized and decentralized markets adjusts such that the resulting equilibrium prices make both markets equally costly. While dual-market equilibria also require the dealer's bargaining power to be below a certain threshold, this threshold is significantly higher than for OTC-only equilibria. This is because the borrower raises some cash in the centralized market, reducing her need for pledgeable assets and thereby leading to smaller haircuts in negotiations with the dealer.

Given the equilibrium multiplicity, our final main result ranks the equilibria based on the resulting welfare of the good borrower. This welfare is equal to the net present value of the good project minus the signaling cost—i.e., the cost of obtaining the necessary cash and pledgeable assets to execute the separating contract. In all equilibria, welfare is strictly positive, reflecting the efficiency gain compared to the market breakdown scenario. We find that whenever an OTC-only equilibrium exists, it always provides higher welfare than CM-only and dual-market equilibrium. Moreover, when they exist, dual-market equilibria always dominate the CM-equilibrium and are ranked by the intensity of centralized market use: the less the good borrower relies

[4]The multiplicity of OTC-only equilibria arises because the centralized market price is indeterminate when no borrower participates.

on the centralized market, the higher her welfare. This ranking is driven entirely by the signaling costs implied by secondary market prices. Lower signaling costs lead to higher welfare. In OTC-only equilibria, the signaling cost from the haircut is lower than the cost implied by the centralized market price when no borrower accesses it. Moreover, due to convex costs, the cost of raising cash in the centralized market increases with its usage. As a result, the signaling cost in OTC-only equilibria is always lower than in CM-only equilibria. A similar logic applies to dual-market equilibria, where the costs of swapping collateral in the OTC market and raising cash in the centralized market are equal. However, because centralized market costs rise with intensity of use, signaling costs in dual-market equilibria increase with greater reliance on the centralized market, peaking in the CM-only equilibrium where the borrower exclusively uses the centralized market.

Literature Review. Our paper contributes to several strands of literature. First, we relate to the literature comparing centralized versus decentralized OTC markets. Colliard et al. (2021); Geromichalos and Herrenbrueck (2016); Glode and Opp (2020) compare the costs and benefits associated with centralized and decentralized trading structures without endogenous participation decision while Dugast et al. (2019); Miao (2006); Rust and Hall (2003); Yoon (2018) consider models with endogenous participation and study the welfare implications of searching for OTC trading counterparties or trade with market makers in a centralized market. Differently from these papers, we cast the participation decision into a model of asymmetric information in the lending market and first derive conditions under which equilibria with different degree of market access exists. Only after that, we rank the equilibria according to the resulting welfare and relate that to the participation decision.

Our paper also speaks to the vast work on corporate finance under asymmetric information (Tirole, 2010). We build on Besanko and Thakor (1987); Bester (1985,

1987); Chan and Kanatas (1985) who study the possibility of signaling by pledging collateral building on the idea that collateral is valued less highly by the lender than by the borrower. From a theoretical perspective, our paper falls within the literature on contract design by an informed principal, initiated by Myerson (1983) and Maskin and Tirole (1990, 1992). In particular, we focus on the RSW (Rothschild-Stiglitz-Wilson) allocation of the contract design game, which in our case is equivalent to the least-costly separating equilibrium ("Riley outcome") of the corresponding signalling game. The main difference from Maskin and Tirole (1992)'s framework is that in our model the informed principal has access to two different signalling activities instead of only one. Moreover, the cost of each signalling activity is not exogenous, but endogenously determined by the prices arising in equilibrium in the secondary markets.

Lastly, our paper relates to the literature on the theory and applications of multi-dimensional signalling game (see for example Quinzii and Rochet (1985) and Engers (1987)). In particular, since in our model there are multiple signalling activities that the informed party can undertake, the least-costly separating equilibrium also involves characterizing the cost effective way of combining the two dimensions given the endogenous signalling costs.

Roadmap. We describe the framework in Section 2. Section 3 formalizes the benchmark without secondary markets. Section 4 defines the equilibria in each secondary market. Section 5 describes the design of the optimal contract. Section 6 formally characterizes the equilibria of the lending game and its properties. Section 7 considers the impact of frictions on borrowers' welfare. Section 8 concludes the paper. All proofs can be found in Appendix A.

2. The Model

The economy is populated by continuous populations of agents: borrowers, lenders, outside investors and dealers. We assume that all agents are risk-neutral and do not

discount the future. Borrowers can invest in a project which costs $I > 0$ units of cash and yields a return of R if successful. However, borrowers do not have enough cash A to fund the project: $A < I$. Therefore, they need to borrow an amount from the lender $D \geq I - A$. We assume asymmetric information on the probability of success of the project. We denote by $\Theta \coloneqq \{G, B\}$ the type space of borrowers and let μ be the fraction of type G in the population of borrowers. μ is common knowledge. If a borrower is of type $\theta \in \Theta$, she can invest in the project with probability of success π_θ, where $1 > \pi_G > \pi_B > 0$. Finally, there are two types of financial assets in the economy. The high-quality, riskless asset yields one unit of money next period with probability one. However, the low-quality asset has a potentially risky return in the next period. We denote the returns on high-quality and low-quality assets by X_H and X_L, respectively. In order to abstract from differences in their fundamental values, we assume that *ex-ante* these two assets look the same to a risk-neutral investor: $\mathbb{E}[X_L] = 1 = \mathbb{E}[X_H]$. We assume that both types of borrowers have a large quantity of low-quality assets K_L at their disposal, while dealers hold large quantities of high-quality assets.

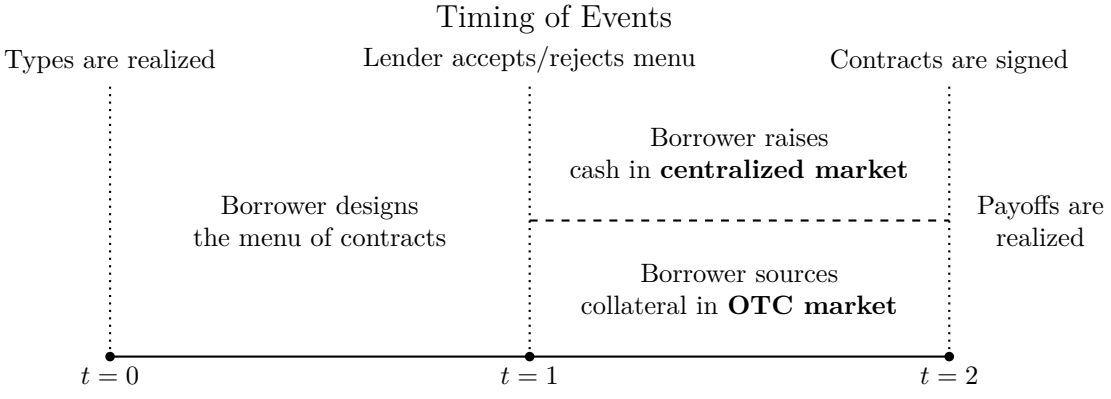


FIGURE 1. In the figure we illustrate the timing of the events happening in the lending game with secondary markets.

As illustrated in Figure 2, there are three periods $t \in \{0, 1, 2\}$:

(i) In period $t = 0$, the lending game with asymmetric information and pledgeable (high-quality) assets takes place.

(ii) In period $t = 1$, the borrower needs to gather the resources necessary to fulfill the terms of the contract stipulated with the lender before signing it. To this end, she can access a centralized and competitive secondary market (CM) to sell collateral in exchange for cash or an over-the-counter secondary market (OTC) where she can exchange L for H asset with dealers at a haircut. At the end of the period, the contract is signed conditional on the borrower having obtained the requisite amounts of cash and collateral. After signing the contract, its terms become legally binding for the borrower.

(iii) In period $t = 2$, if the contract is signed, its terms are executed, and, upon investment, the outcome of the project is realized. At the end of the period, all players receive their payoffs.

We now describe in detail the lending game in period $t = 0$ and the properties of the centralized market for cash and the OTC market for collateral.

2.1. **Time t=0: The Lending Game.** In period $t = 0$, borrowers and lenders play a lending game with asymmetric information and pledgeable assets.[5] We later use "she" for the borrower and "he" for lender. We make the following assumption:.

ASSUMPTION 1: *Only high-quality asset X_H can be pledged as collateral in a financing contract.*

This assumption reflects that regulatory authorities often require the lender to accept only high-quality assets as collateral.[6]

We model the lending game as a problem of contract design by an informed principal.[7] We focus on direct revelation mechanisms, so that the borrower, who in our case

[5]See, for example, Tirole (2010).
[6]See, for example, Basel III: The Liquidity Coverage Ratio and liquidity risk monitoring tools.
[7]See seminal work by Myerson (1983) and Maskin and Tirole (1992) for an overview of the subject.

is the informed party, offers a menu of contracts to the lender, one for each borrower's type. In particular, a contract of type θ specifies the probability of investing in the project $y_\theta \in [0,1]$, the amount of resources borrowed from the lender d_θ, the payment to the lender in case the project is successful r_θ, and the amount of high-quality asset posted as collateral c_θ, which can be seized by the lender in case the project is not successful. Therefore, a contract of type θ is a vector $(y_\theta, d_\theta, r_\theta, c_\theta) \in [0,1] \times \mathbb{R}^3_+$. We assume throughout the paper that the borrower is protected by limited liability so that $r_\theta \leq R$ for each type $\theta \in \Theta$. After the borrower has proposed a menu of contracts $((y_\theta, d_\theta, r_\theta, c_\theta))_{\theta\in\Theta}$, the lender decides whether to accept or reject it. If the lender rejects the menu, he gets his reservation utility, normalized to be 0. If he accepts, the borrower announces a report of her type and offers the corresponding contract by the lender. In order to sign the contract she is offered, the borrower first needs to acquire the resources to fulfill the contract terms. In particular, a contract involving a positive probability of investment requires the borrower to obtain the requisite amounts of cash and collateral. Therefore, in order to evaluate *ex ante* the expected payoff of each contract in the menu she is offering, the borrower will form expectations on the price p^e that will emerge in the centralized market and haircut δ^e in the OTC market.[8] Hence, the expected payoff to borrower θ from executing contract $\tilde{\theta}$ from menu $((y_\theta, d_\theta, r_\theta, c_\theta))_{\theta\in\Theta}$ given her expectations on the competitive market price $p^e \in (0,1)$ and the OTC haircut $\delta_e \in [0,1]$ is

$$\mathcal{U}(\tilde{\theta};\theta|p^e,\delta^e) = \mathcal{V}(\tilde{\theta};\theta|p^e,\delta^e) - \mathcal{C}(\tilde{\theta};\theta|p^e,\delta^e) + A + y_{\tilde{\theta}}c_{\tilde{\theta}}\mathbb{E}X_H + K_L\mathbb{E}X_L \ ,$$

where $K_L\mathbb{E}X_L$ is the expected value of the borrower's low-quality asset holdings, $c_{\tilde{\theta}}\mathbb{E}X_H$ is the expected value of the holding of high-quality assets acquired in the

[8] In equilibrium, we will impose rational expectations; that is, these expectations turn out to be correct.

OTC market when the contract involves investment with positive probability $y_{\tilde{\theta}} > 0$.

$$\mathcal{V}(\tilde{\theta};\theta|p^e,\delta^e) = y_{\tilde{\theta}}\left[\pi_\theta(R - r_{\tilde{\theta}}) + \pi_\theta c_{\tilde{\theta}}\mathbb{E}X_H + \max\{0, d_{\tilde{\theta}} - (I - A)\} - (A + c_{\tilde{\theta}}\mathbb{E}X_H)\right]$$

is the expected net present value of executing the contract, and $\mathcal{C}(\tilde{\theta};\theta|p^e,\delta^e)$ is the expected cost of gathering the high-quality assets and extra cash needed to execute the contract. In particular, if $y_{\tilde{\theta}} = 0$, then the contract prescribes no investment, and, therefore, the borrower neither acquires extra cash nor enters into a collateral swap with a dealer to obtain high-quality assets. On the other hand, if the probability of investment is positive, the borrower must obtain an amount of cash equal to $\max\{0, I - A - d_{\tilde{\theta}}\}$[9] by selling her low-quality assets in the centralized market. Given her expectations on the centralized market price, to obtain such quantity of cash, she has to sell $\frac{\max\{0,I-A-d_{\tilde{\theta}}\}}{p^e}$ units of low-quality assets. Similarly, the borrower also needs to obtain a quantity $c_{\tilde{\theta}}$ of high-quality collateral in the OTC market in exchange for her low-quality assets. In order to do so, given her expectations regarding the OTC haircut δ^e, she expects to forgo $\frac{c_{\tilde{\theta}}}{1-\delta^e}$ units of low-quality assets in the collateral swap with the dealer. Since low-quality assets returns are given by X_L in period 2, the expected cost for the borrower of executing the contract is thus given by:

$$\mathcal{C}(\tilde{\theta};\theta|p^e,\delta^e) = y_{\tilde{\theta}}\left(\frac{\max\{0, I - A - d_{\tilde{\theta}}\}}{p^e}\mathbb{E}X_L + \frac{c_{\tilde{\theta}}}{1-\delta^e}\mathbb{E}X_L\right) .$$

We can further decompose this cost into the expected cost of obtaining the cash needed for investment, $\mathcal{C}_A(\tilde{\theta};\theta|p^e,\delta^e) = y_{\tilde{\theta}}\frac{\max\{0,I-A-d_{\tilde{\theta}}\}}{p^e}$ and the expected cost of obtaining the high-quality asset to be posted as collateral, $\mathcal{C}_C(\tilde{\theta};\theta|p^e,\delta^e) = y_{\tilde{\theta}}\frac{c_{\tilde{\theta}}}{(1-\delta^e)}$.

[9]We can assume without loss of generality that the borrower does not acquire more cash than what is needed to execute the contract. Indeed, if her expectations are consistent with the market price emerging in the centralized market, the asset garnering cost implies that $p^e < 1 = \mathbb{E}X_L$, therefore the borrower incurs a loss when selling low-quality assets.

Given a pair of expected centralized market price p^e and OTC market haircut δ^e, the appropriate solution concept for this lending game is perfect Bayesian equilibrium (PBE), as defined in Maskin and Tirole (1992), where the borrower truthfully announces her type when the menu of contracts is accepted by the lender and signs the corresponding contract. Clearly, there are many PBEs of the game. In this paper, we will focus on the good borrower's preferred separating PBE, that is, the separating PBE that maximizes the good borrower's expected payoff. Denote by $\mathcal{U}(0;\theta|p^e,\delta^e)$ the payoff to borrower $\theta \in \Theta$ of not signing the contract and taking her outside option. This is the best course of action between either doing nothing or financing the project herself by obtaining the needed cash $I - A$ in the centralized secondary market. In order to ease notation, and whenever no confusions can arise, we will stop making explicit the dependence of the borrower's payoffs on her expectations about the centralized market price and the OTC haircut.

2.2. **Time t=1: Accessing Secondary Markets.** In period $t = 1$, if the lender accepts the menu of contracts, each borrower $\theta \in \Theta$ must fulfill the terms of their chosen contract before signing it. This includes securing the resources to invest in the project and acquiring the required high-quality assets as collateral. The borrower can liquidate low-quality assets for cash in a competitive secondary market or obtain high-quality assets through collateral swaps in an OTC market. Borrowers might need to use both markets to meet the contract's terms, especially if additional resources or high-quality collateral are required. We now describe how these two assets markets work.

Competitive Secondary Market. The competitive asset is centralized and populated by a continuous, unit mass of outside investors who are willing to buy the low-quality assets L supplied by the borrowers in exchange for cash. We assume that the external investors have strictly monotone and risk-neutral preferences over monetary

outcomes. In order to acquire a quantity $q \in \mathbb{R}_+$ of asset L at price p, an external investor needs to raise a quantity pq of funds. Following Allen and Gale (1994), we assume that liquidity is scarce and, therefore, the investor incurs an additional cost $g(pq)$ in gathering the requisite amount of funds. We assume that the function $g \in \mathbf{C}^2(\mathbb{R}_+)$ has a strictly positive derivative everywhere and is convex. Borrowers access the competitive market to obtain cash $(A_\theta)_{\theta\in\Theta}$. As it will be explained later, such quantities will result from the terms of the contracts, the relative magnitude of the centralized market price versus the OTC haircut, and the borrower's liquidity constraint.

Definition 1: An equilibrium in the centralized, competitive market for asset L given cash requirements $(A_\theta)_{\theta\in\Theta} \in \mathbb{R}^2_+$ consists of demand $\hat{q}$ and price $\hat{p}$ such that:

(i) $\hat{q}$ solves the maximization problem of the external investor given $\hat{p}$:

$$\hat{q} \in \arg\max_{q\in\mathbb{R}_+} \mathbb{E}[qX_L - \hat{p}q - g(\hat{p}q)];$$

(ii) market clearing holds:

$$\mu A_G + (1-\mu)A_B = \hat{p}\hat{q} \ .$$

OTC Market. Let us now turn to analyze the OTC market. The OTC market is populated by a continuum of identical risk-neutral dealers, whose measure ν_{dl} is determined in equilibrium. Dealers hold large quantities of the high-quality pledgeable assets. The borrower accessing the OTC market is able to exchange asset L for asset H with the dealers at the cost of suffering a haircut $\delta \in (0,1)$. This means that even if both borrowers and dealers are risk-neutral and, therefore, evaluate both assets at the same value of 1, the borrower needs to forego $\frac{1}{1-\delta} > 1$ units of asset L in order to obtain from the dealer a unit of asset H. This haircut results endogenously from the OTC market protocol, which we now describe.

Suppose that for each type $\theta \in \Theta$, there is a measure ν_θ of borrowers of type θ that access the OTC market to obtain C_θ units of collateral. Each carries a surplus $S_\theta \geq 0$, the additional expected payoff from a lender-accepted contract with positive collateral, compared to self-financing the investment when profitable. Then, the total mass of borrowers accessing the market is given by $\nu := \nu_G + \nu_B$. We assume that upon entering the market, a borrower θ is immediately and randomly matched to a dealer with probability $\min\{1, \frac{\nu_{dl}}{\nu}\}$. If the borrower does not match with a dealer, she leaves the market without obtaining the collateral required to sign the contract. Conversely, a dealer is matched to a borrower with probability $\min\{1, \frac{\nu}{\nu_{dl}}\}$. Once a dealer enters a match with a borrower, he observes the type θ of his partner, and hence the surplus S_θ brought by the borrower, as well as her collateral needs C_θ. In order to determine the haircut on the value of the low-quality asset and, hence, the cost for the borrower θ of obtaining the required amount of collateral C_θ, they need to decide how to divide the surplus between themselves. We assume asymmetric Nash bargaining as the adopted bargaining protocol.[10] We denote by $\alpha \in (0, 1)$ the bargaining power of the dealer, so that the bargaining power of the borrower will be $(1 - \alpha)$. We normalize the outside option for the dealer of not participating in the OTC collateral swap to 0. Given how we define the surplus carried in a match by the borrower, her outside option when she does not enter the market for collateral — or she exits the market without having obtained the requisite amount of collateral — is also equal to 0. Finally, we assume that if the borrower or the dealer wishes to leave the current partner to whom they are matched, they can obtain a rematch, but the rematch might be delayed with probability $\beta \in (0, 1)$. If the match is delayed, the party that requests it is unable to make the change on time and, therefore, receives the outside option 0. Then, the discounted probabilities of being re-matched for a borrower θ or a dealer leaving a match are, respectively, $\tilde{\beta}_\theta := \beta \min\{1, \frac{\nu_{dl}}{\nu}\}$ and $\tilde{\beta}_{dl} := \beta \min\{1, \frac{\nu}{\nu_{dl}}\}$.

[10] See definition B.1 in appendix B for a definition of this bargaining protocol and solution concept.

Finally, conditional on being matched to a borrower, the probability that a dealer is matched to type $\theta \in \Theta$ is given by $\xi_\theta := \frac{\nu_\theta}{\nu}$.[11]

Let W_θ^{dl} be the dealer's value of matching with a borrower of type θ. Similarly, let W_θ be the value of a match to a borrower of type θ. Then, the disagreement points of the dealer and borrower θ are given respectively by:

$$o_{dl} = \max\{0, \tilde{\beta}_{dl}[\xi_G W_G^{dl} + (1-\xi_G) W_B^{dl}]\}$$

$$o_\theta = \max\{0, \tilde{\beta}_\theta W_\theta\}$$

We now characterize equilibrium in the OTC collateral market.

Definition 2: Fix collateral needs and surplus of borrowers entering the OTC collateral market $((C_\theta, S_\theta))_{\theta\in\Theta}$ and dealers' bargaining power $\alpha \in (0,1)$. An equilibrium in the OTC collateral market is given by the masses of dealers $\hat{\nu}_{dl}$ and borrowers $(\hat{\nu}_\theta)_{\theta\in\Theta}$, match values $(\hat{W}_\theta^{dl}, \hat{W}_\theta)_{\theta\in\Theta}$, disagreement points $(\hat{o}_{dl}, (\hat{o}_\theta)_{\theta\in\Theta})$, bargaining allocations $((\hat{x}_\theta^{dl}, \hat{x}_\theta))_{\theta\in\Theta}$ and haircut $\hat{\delta}$ such that:

(i) $\hat{\nu}_G = \mu \mathbf{1}\{\hat{C}_G > 0\}$ and $\hat{\nu}_B = (1-\mu)\mathbf{1}\{\hat{C}_B > 0\}$;

(ii) $\hat{\nu}_{dl} = \hat{\nu}_G + \hat{\nu}_B$;

(iii) the disagreement points are given by:

$$\hat{o}_{dl} = \max\{0, \beta[\xi_G \hat{W}_G^{dl} + (1-\xi_G)\hat{W}_B^{dl}]\}$$

$$\hat{o}_\theta = \max\{0, \beta \hat{W}_\theta\} \quad \text{for each } \theta \in \Theta;$$

(iv) $(\hat{W}_\theta^{dl}, \hat{W}_\theta)_{\theta\in\Theta}$ solves for each $\theta \in \Theta$:

$$\hat{W}_\theta^{dl} = \max\{\alpha(S_\theta - \hat{o}_\theta) + (1-\alpha)\hat{o}_{dl},\ \hat{o}_{dl}\}$$

$$W_\theta = \max\{(1-\alpha)(S_\theta - \hat{o}_{dl}) + \alpha\hat{o}_\theta,\ \hat{o}_\theta\}$$

(v) for all $\theta \in \Theta$, if $o_{dl} + o_\theta \leq S_\theta$ then $(\hat{x}_\theta^{dl}, \hat{x}_\theta)$ is the Nash solution to the asymmetric bargaining problem $(S_\theta, \alpha, (\hat{o}_{dl}, \hat{o}_\theta))$; otherwise $(\hat{x}_\theta^{dl}, \hat{x}_\theta) = (0,0)$.

[11] With the convention that if $\nu = 0$, then $\xi_\theta = 0$ for each $\theta \in \Theta$.

(vi) haircut $\hat{\delta}$ satisfies:

$$\hat{x}_\theta^{dl} = \frac{\hat{\delta}}{1-\hat{\delta}} C_\theta \quad \text{for each } \theta \in \Theta$$

whenever $C_G + C_B > 0$ and $\hat{\delta} = 1$ otherwise.

The interpretation of point (vi) in the definition is as follows. If $\hat{\delta}$ is the haircut that emerges in the OTC market, the dealer profits from swapping an amount C_θ of high-quality asset with borrower θ is

$$\frac{C_\theta \mathbb{E}X_L}{1-\hat{\delta}} - C_\theta \mathbb{E}X_H = \frac{\hat{\delta}}{1-\hat{\delta}} C_\theta.$$

Therefore, condition (vi) requires that the equilibrium profits to the dealer from the exchange are consistent with the share of the surplus she is receiving in the Nash bargaining solution. Note that when $C_G = C_B = 0$, the OTC market for collateral does not open at all and, therefore, any haircut would vacuously clear the market. In the previous definition, if the OTC market for collateral does not open, we let $\hat{\delta} = 1$. This convention may be justified in the following way: an individual borrower has mass 0 in the economy and. If she unilaterally decided to access the OTC market, she would carry a measure 0 demand in the market which would be met by a measure-0 supply of high-quality collateral asset. Therefore, an individual borrower is not able to unilaterally open the OTC market and, thus, she would not be able to obtain any high-quality collateral for each unit of low-quality assets she forgoes; that is, she is effectively faced by a haircut equal to 1.

2.3. **Equilibrium.** Given an accepted contract menu $((y_\theta, d_\theta, r_\theta, c_\theta))_{\theta\in\Theta}$,

$$S(\tilde{\theta};\theta|p^e,\delta^e) \coloneqq \mathbf{1}\{c_{\tilde{\theta}} > 0\} \left[\mathcal{V}(\tilde{\theta};\theta|p^e,\delta^e) - \mathcal{C}_A(\tilde{\theta};\theta|p^e,\delta^e) + A + K_L\mathbb{E}X_L - \mathcal{U}(0;\theta|p^e,\delta^e)\right],$$

is the (expected) surplus to type $\theta \in \Theta$ from entering the OTC market to obtain the requisite collateral needed to sign contract $\tilde{\theta}$, as opposed to the value of her outside

option. This is the increase in value over the outside option from being able to sign the contract $\tilde{\theta}$, before considering the net cost of performing the asset swap in the OTC market to obtain the high-quality asset to be posted as collateral.

Definition 3: An equilibrium of the lending market with secondary markets is given by a menu of contracts $((\hat{y}_\theta, \hat{d}_\theta, \hat{r}_\theta, \hat{c}_\theta))_{\theta\in\Theta}$, cash needs $(\hat{A}_\theta)_{\theta\in\Theta}$ and demands for collateral $(\hat{c}_\theta)_{\theta\in\Theta}$, competitive market demand and price $(\hat{q}, \hat{p})$, OTC market dealers' and borrowers' masses $(\hat{\nu}_{dl}, (\hat{\nu}_\theta)_{\theta\in\Theta})$, match values $(\hat{W}^{dl}_\theta, \hat{W}_\theta)_{\theta\in\Theta}$, disagreement points $(\hat{o}_{dl}, (\hat{o}_\theta)_{\theta\in\Theta})$, bargaining allocations $((\hat{x}^{dl}_\theta, \hat{x}_\theta))_{\theta\in\Theta}$ and haircut $\hat{\delta}$ such that:

(i) $((\hat{y}_\theta, \hat{d}_\theta, \hat{r}_\theta, \hat{c}_\theta))_{\theta\in\Theta}$ is the good borrower-preferred separating PBE of the lending game given prices $(\hat{p}, \hat{\delta})$;

(ii) for each $\theta \in \Theta$, $\hat{S}_\theta = S(\theta; \theta|\hat{p}, \hat{\delta})$;

(iii) for each $\theta \in \Theta$, the choices of cash and collateral acquisition maximize borrower θ's expected utility given the contract and prices $(\hat{p}, \hat{\delta})$ and under the constraint that the terms of the contract are fulfilled; i.e. if $\hat{y}_\theta > 0$, $\hat{A}_\theta \geq I - (A + \hat{d}_\theta)$ and $\hat{C}_\theta \geq \hat{c}_\theta$.

(iv) $(\hat{q}, \hat{p})$ is an equilibrium in the competitive market given cash needs $(\hat{A}_\theta)_{\theta\in\Theta}$;

(v) $\left((\hat{\nu}_{dl}, (\hat{\nu}_\theta)_{\theta\in\Theta}), (\hat{W}^{dl}_\theta, \hat{W}_\theta)_{\theta\in\Theta}, (\hat{o}_{dl}, (\hat{o}_\theta)_{\theta\in\Theta}), ((\hat{x}^{dl}_\theta, \hat{x}_\theta))_{\theta\in\Theta}, \hat{\delta}\right)$ is an equilibrium in the OTC collateral market given demands and surpluses $((\hat{C}_\theta, \hat{S}_\theta))_{\theta\in\Theta}$.

Thus, in equilibrium we require that the contract menu is a PBE and cash and collateral acquisition decisions are optimal given the expectations the borrower has formed over the prices and that, in the end, such expectations are correct. That is, the expected competitive market price and OTC haircut exactly match those that will arise in equilibrium in the two markets.

3. Benchmark: No Secondary Markets

Before analyzing the equilibria of the model described in the previous section, we first consider a benchmark case, where borrowers have no access to secondary markets.

Then, we study the case with secondary markets, characterize the optimal contract and high-quality collateral and cash acquisition strategies. Throughout the main text of the paper, we make the following assumption.

ASSUMPTION 2: $\pi_B R < I < \pi_G R$.

This assumption states that only the good borrower is credit-worthy. In particular, consider the first-best case where the borrower's type is observable. An optimal contract would be that type G-borrower is financed exactly the amount she needs to invest in the project $d_G^* = I - A$. The good borrower (type-G) invests in the project $y_G^* = 1$, pays $r_G^* \coloneqq \frac{I-A}{\pi_G}$ to the lender in case of success and posts no collateral $c_G^* = 0$, while the bad borrower (type-B) is not financed due to negative net present value (NPV) of her project: $R < \frac{I}{\pi_B}$, i.e. her contract is given by $(y_B^*, d_B^*, r_B^*, c_B^*) = (0, 0, 0, 0)$.

As mentioned, consider first the benchmark case in the absence of secondary markets. The model reduces to a project-financing game with asymmetric information, where the good borrower cannot pledge her low-quality collateral, so that $c_G^{NC} = 0$. Since we assume the borrower has a positive amount of cash on hand, she can in principle use it to signal her type. If this is the case, we can without loss of generality set $d_G^{NC} = I - A$ so that $r_G^{NC} = \frac{I-A}{\pi_G}$ and $y_G^{NC} = 1$. The bad borrower has no incentives to deviate by disguising herself as the good type if and only if:

$$\pi_B(R - r_G^{NC}) - A \leq 0$$

which is satisfied as long as:

$$A \geq \frac{\pi_B}{\pi_G - \pi_B}(\pi_G R - I) \ .$$

In other words, when the good borrower has enough cash, she can invest it to partially finance the project and distinguish herself from the bad borrower, who prefers to just

keep her cash instead of investing it in her unprofitable project. Since in this case there would be no incentive for the good borrower to engage in costly collateral swap even if the secondary markets exist, we focus on the opposite case.

ASSUMPTION 3 (No Signalling with Own Cash): $A < \frac{\pi_B}{\pi_G - \pi_B}(\pi_G R - I)$.

Under Assumption 3, the only possibility of financing investment when borrowers cannot access asset markets is a pooling contract menu $((y^P, D^P, R^P, C^P))_{\theta \in \Theta}$, which guarantees to each type the required funding $d^P = I - a$ for $a \in [0, A]$, requires the borrowers to invest in the project $y^P = 1$, pays r_P to the lender in case of success and no collateral is posted $c^P = 0$. In this case, the expected profits to the lender are given by:

$$\mu(\pi_G r_P - d^P) + (1-\mu)(\pi_B r_P - d^P) = [\mu\pi_G + (1-\mu)\pi_B]r_P - (I - a)$$

and he would be willing to accept the pooling contract only when they are positive. On the other hand, it is individually rational for the borrowers to actually invest in the projects only if for each $\theta \in \Theta$,

$$\pi_\theta(R - r^P) - a \geq 0 \iff r^P \leq R - \frac{a}{\pi_\theta} .$$

This constraint is more stringent for the bad borrower; therefore, the best we can do to guarantee positive profits to the lender and still ensure individual rationality of the pooling menu is to set $r^P = R - \frac{a}{\pi_B}$. Then, the expected profits of the lender would be:

$$\begin{aligned}[\mu\pi_G + (1-\mu)\pi_B]r_P - (I - a) &= [\mu\pi_G + (1-\mu)\pi_B]R - I - \frac{\mu(\pi_G - \pi_B)}{\pi_B}a \\ &\leq [\mu\pi_G + (1-\mu)\pi_B]R - I\end{aligned}$$

for any $a \in [0, A]$. We will make the following assumption.

ASSUMPTION 4 (Market Breakdown): $R < \frac{I}{\mu\pi_G + (1-\mu)\pi_B}$.

Under Assumption 4, in any pooling menu of contracts that is individually rational for both borrower's types, the lender would lose money on average, should he choose to accept the pooling menu of contracts. Then, the market breaks down, and neither type is financed. Note that in this case, Assumption 2 implies that there would be a welfare loss equal to $\pi_G R - I > 0$ compared to the first best scenario.

4. EQUILIBRIA IN THE SECONDARY MARKETS

We proceed backwards. First, we characterize the equilibria in the centralized and OTC asset markets. We analyze the net costs for a borrower to obtain cash and high-quality collateral in both centralized and OTC markets, considering the expected price and haircut.Then, given these acquisition costs, we characterize a separating menu of contracts optimal for the good borrower and accepted by the lender.

4.1. **Equilibrium in the Centralized Market.**

Let us first analyze the equilibrium on the collateral market when there is a demand for cash given by the profile $(A_\theta)_{\theta\in\Theta}$.

LEMMA 1: *Fix a profile of cash needs $(A_\theta)_{\theta\in\Theta}$. Then, $(\hat{q}, \hat{p})$ is an equilibrium in the competitive market for asset L if and only if:*

- *when $(A_\theta)_{\theta\in\Theta} = (0,0)$, $\hat{q} = 0$ and $\hat{p} \geq \frac{1}{1+g'(0)}$;*
- *when $(A_\theta)_{\theta\in\Theta} > (0,0)$, then:*

$$\hat{q} = [1 + g'(\mu A_G + (1-\mu)A_B)][\mu A_G + (1-\mu)A_B]$$
$$\hat{p} = \frac{1}{1 + g'(\mu A_G + (1-\mu)A_B)} .$$

The centralized market for assets is competitive. For a given price, the external investor will buy assets as long as the marginal benefit, equal to the fundamental value of the assets she buys, is larger than the marginal cost of acquiring such assets, which is given by the market price plus the incremental cost of acquiring the necessary

liquidity. When there is positive demand for cash, market clearing requires that the competitive market price is exactly equal to the marginal benefit of acquiring an additional unit of low-quality assets; that is, its fundamental value deflated by the incremental liquidity cost. This is exactly the point where the external investor has no incentives to buy more or less assets. Note that since the liquidity cost function g has strictly positive first derivative and is convex, the price will be strictly between 0 and 1 and weakly decreasing in the aggregate cash needs. In particular, when g is affine, so that $g' = \kappa > 0$, the price is constant and equal to $\frac{1}{1+\kappa}$ for any positive aggregate demand for cash. On the other hand, when there is no aggregate need for liquidity in the competitive market, the price cannot be exactly pinned down. Indeed, any price higher than the liquidity cost-deflated marginal benefit of acquiring the first unit of assets will make the external investor optimally choose to demand no low-quality assets, so that the market-clearing condition is satisfied.

4.2. **Equilibrium in the OTC Market.** We will now turn to analyze the OTC market equilibrium. Define the expected value for a dealer who has been matched to a borrower before learning her type by $\hat{W}^{dl} \coloneqq \xi_G W_G + (1-\xi_B)W_B$. We have the following proposition.

PROPOSITION 2: *If* $\left((\hat{W}^{dl}_\theta, \hat{W}_\theta)_{\theta\in\Theta}, (\hat{o}_{dl}, (\hat{o}_\theta)_{\theta\in\Theta}), \hat{\nu}_{dl}, (\hat{\nu}_\theta)_{\theta\in\Theta}, (\hat{x}^{dl}_\theta, \hat{x}_\theta)_{\theta\in\Theta}, \hat{\delta}\right)$ *is an equilibrium in the OTC market such that*

$$\frac{\alpha\beta\hat{\xi}_G}{1-\alpha\beta(1-\hat{\xi}_G)} S_G > S_B \geq 0 \ , \tag{1}$$

then it must be the case that:

$$\hat{W}^{dl}_G = \frac{\alpha(1-\beta(1-\hat{\xi}_G))}{1-\alpha\beta(1-\hat{\xi}_G)} S_G$$

$$\hat{W}^{dl}_B = \frac{\alpha\beta\hat{\xi}_G}{1-\alpha\beta(1-\hat{\xi}_G)} S_G$$

$$\hat{W}_G = \frac{1-\alpha}{1-\alpha\beta(1-\hat{\xi}_G)} S_G$$

$$\hat{W}_B = 0$$

and allocations:

$$\hat{x}_G^{dl} = \frac{\alpha(1-\beta(1-\hat{\xi}_G))}{1-\alpha\beta(1-\hat{\xi}_G)} S_G \ , \quad \hat{x}_B^{dl} = \ 0 \ ,$$

$$\hat{x}_G = \frac{1-\alpha}{1-\alpha\beta(1-\hat{\xi}_G)} S_G \ , \qquad \hat{x}_B = \ 0 \ .$$

The proposition above states that if the surplus carried in the OTC market by the good borrower is sufficiently larger than that by the bad borrower, a dealer that is initially matched to a bad borrower is better off not performing the collateral exchange and waiting for a rematch, in the hope of meeting a good borrower to bargain over a larger pie. Thus, if the condition in the above proposition is satisfied, matches between bad borrowers and dealers are always dissolved without the collateral swap happening, so that the actual value of the match, and hence its split, is 0. On the other hand, good borrowers and dealers immediately execute the swap, splitting the surplus according to the Nash bargaining solution. Then, notice that if $S_B = 0$, we can only have an equilibrium where the good borrower enters the OTC market with a positive surplus if $C_B = 0$. Indeed, if the good borrower wants to acquire a positive amount of collateral $C_G > 0$ in the OTC market, the last condition in the OTC equilibrium definition requires a positive haircut to make the profits to the dealer in the collateral swap, $\frac{\hat{\delta}}{1-\hat{\delta}} C_G$, equal to her allocation in the Nash bargaining solution. On the other hand, given that $\hat{x}_B^{dl} = 0$, this same condition would require a haircut equal to 0 if the bad borrower also demands a positive quantity of high-quality assets in the OTC market, which would lead to a contradiction. The above argument leads to the following corollary.

Corollary 3: *Suppose that $C_G > 0$, $S_G > 0$, and $S_B = 0$. Then, an equilibrium in the OTC market exists only if $C_B = 0$. If this is the case, the unique equilibrium*

in the OTC market is given by $\hat{\nu}_G = \hat{\nu}_{dl} = \mu$, $\hat{\nu}_B = 0$, *match values*

$$\hat{W}_G^{dl} = \alpha S_G, \qquad \hat{W}_B^{dl} = \beta\alpha S_G,$$
$$\hat{W}_G = (1-\alpha)S_G, \quad \hat{W}_B = 0 \ ,$$

disagreement points

$$\hat{o}_{dl} = \beta\alpha S_G, \quad \hat{o}_G = \beta(1-\alpha)S_G, \quad \hat{o}_B = 0 \ ,$$

bargaining allocations

$$\hat{x}_G^{dl} = \alpha S_G, \qquad \hat{x}_B^{dl} = 0 \ ,$$
$$\hat{x}_G = (1-\alpha)S_G, \quad \hat{x}_B = 0 \ ,$$

and the equilibrium haircut is given by the unique solution to:

$$\frac{\hat{\delta}}{1-\hat{\delta}} = \alpha\frac{S_G}{C_G} \ .$$

4.3. **Net Expected Costs of Acquiring Cash and Collateral.** Suppose the contract menu $((y_\theta, d_\theta, r_\theta, c_\theta))_{\theta\in\Theta}$ has been accepted by the lender and consider a borrower $\theta \in \Theta$ who announces $\tilde{\theta}$ and wants to execute the corresponding contract, given her expectations on the centralized market price $p^e \in (0,1)$ and on the OTC haircut $\delta_e \in (0,1)$. In order to obtain one unit of cash on the centralized secondary market, the borrower expects she will have to sell $\frac{1}{p^e}$ units of her low-quality assets. Therefore, the net expected cost of obtaining one unit of cash in the centralized market is given by:

$$\gamma^e \coloneqq \frac{\mathbb{E}[X_L]}{p^e} - 1 = \frac{1-p^e}{p^e} \ .$$

Notice after imposing rational expectations, if there is a positive demand for cash in the centralized secondary markets, Lemma 1 implies that $p^e < 1$, so that the net

expected cost will be positive $\gamma^e > 0$. Similarly, in order to obtain a unit of high-quality asset that can be posted as collateral, the borrower expects she will have to give up $\frac{1}{1-\delta^e)}$ units of her low-quality assets in the swap with the dealer. Then, the net expected cost for the borrower of obtaining one unit of collateral in the OTC secondary market is given by:

$$\rho^e := \frac{\mathbb{E}[X_L]}{1-\delta^e} - \mathbb{E}[X_H] = \frac{\delta^e}{1-\delta^e} \ .$$

After imposing rational expectations, if there is a positive demand for collateral in the OTC market, Corollary 3 implies that $\delta^e > 0$. Therefore, the borrower suffers a positive cost in obtaining the high-quality asset $\rho^e > 0$ (possibly infinite).

These are the expected costs that the good borrower will take into account when designing the optimal separating contract, as will be studied in more detail in the next subsection.

5. Optimal Contract Design

We now study the contract design problem faced by the borrower, given her expectations $p^e < 1$ and $\delta^e \in (0, 1)$ on the equilibrium competitive market price and OTC haircut. In particular, we seek a separating contract maximizing the expected payoff of the good borrower and such that for each type is incentive-compatible to truthfully report her own type and individually rational to sign and execute the corresponding contract. After this menu of contracts is offered, the lender is able to distinguish the good borrower from the bad borrower

First, we characterize the outside option of the borrower. Suppose that the menu of contracts is not accepted by the lender, then a borrower θ has two options. She can either just receive in period 2 expected utility $A + K_L\mathbb{E}[X_L]$, or she can sell her low-quality collateral in the competitive market to obtain the requisite cash to finance the project herself. This yields an expected utility of $\pi_\theta R + [K_L - \frac{I-A}{p^e}]\mathbb{E}[X_L]$. Notice

that since $p^e < 1$, the risk-neutral borrower will sell only enough low-quality assets to meet the project's cash needs. Thus, the reservation expected utility to borrower $\theta \in \Theta$ is given by

$$\begin{aligned}\mathcal{U}(0;\theta|p^e,\delta^e) &= \max\left\{A + K_L, \pi_\theta R + \left[K_L - \frac{I-A}{p^e}\right]\right\} \\ &= (A + K_L) + \max\{0, (\pi_\theta R - I) - \gamma^e(I-A)\}\ .\end{aligned}$$

and the second term is 0 if and only if

$$\gamma^e \geq \frac{\pi_\theta R - I}{(I-A)}\ . \tag{2}$$

Notice that, by Assumption 2, this condition is always satisfied for the bad borrower for any $\gamma^e > 0$. It follows that $\mathcal{U}(0; B|p^e, \delta^e) = A + K_L$.

Suppose again that the contract menu $((y_\theta, d_\theta, r_\theta, c_\theta))_{\theta\in\Theta}$ has been accepted by the lender. From now on, we assume that in any feasible contract menu, $d_\theta \leq I - A$. This assumption requires that the borrower will borrow only up to the level needed to finance the project without obtaining additional cash than what she originally needed. The intuition is that the lender is willing to lend money to the borrower only in order to appropriate a portion of the profits produced by investing in the project. Therefore, there is no reason the lender would be willing to lend any further resources than those that the borrower needs to make investment possible. With this assumption, we can rewrite the expected payoff of borrower θ from executing contract $\tilde{\theta}$ in terms of the next expected costs of acquiring cash and collateral. We have:

$$\begin{aligned}\mathcal{U}(\tilde{\theta};\theta|p^e,\delta^e) &= \mathcal{V}(\tilde{\theta};\theta|p^e,\delta^e) - \mathcal{C}(\tilde{\theta};\theta|p^e,\delta^e) + A + y_{\tilde{\theta}}c_{\tilde{\theta}} + K_L \\ &= y_{\tilde{\theta}}\left[\pi_\theta(R - r_{\tilde{\theta}}) + \pi_\theta c_{\tilde{\theta}} - A - c_{\tilde{\theta}}\right] - y_{\tilde{\theta}}\left(\frac{I - A - d_{\tilde{\theta}}}{p^e} + \frac{c_{\tilde{\theta}}}{1-\delta^e}\right) + A + y_{\tilde{\theta}}c_{\tilde{\theta}} + K_L \\ &= y_{\tilde{\theta}}\left[\pi_\theta(R - r_{\tilde{\theta}}) - (1-\pi_\theta)c_{\tilde{\theta}} - \rho^e c_{\tilde{\theta}} - \gamma^e(I - A - d_{\tilde{\theta}}) - (I - d_{\tilde{\theta}})\right] + A + K_L.\end{aligned}$$

Note that in any separating contract menu, we must have that the bad borrower is not financed and does not invest in the project; that is,

$$(y_B, d_B, r_B, c_B) = (0, 0, 0, 0)$$

since, by Assumption 2, the bad borrower is not creditworthy. This contract is clearly individually rational for the bad borrower, as it delivers the same expected payoff as $\mathcal{U}(0; B|p^e, \delta^e)$, and it is individually rational for the lender as well, since it yields him expected utility equal to 0. Then, in the good borrower-optimal separating contract, the borrower chooses (y_G, d_G, r_G, c_G) in order to maximize $\mathcal{U}(G; G|p^e, \delta^e)$ such that the feasibility constraints are satisfied, it is individually rational for the good borrower to sign contract G, the lender is willing to participate in the contract and neither the bad borrower nor the good borrower have incentives to misreport; that is, $\mathcal{U}(G; B|p^e, \delta^e) \leq \mathcal{U}(B; B|p^e, \delta^e) = \mathcal{U}(0; B|p^e, \delta^e)$ and $\mathcal{U}(B; G|p^e, \delta^e) \leq \mathcal{U}(G; G|p^e, \delta^e)$. Thus, the good borrower-optimal separating contract menu solves the following program:

(3)

$$\max_{(y_G, d_G, r_G, c_G)} \mathcal{U}(G; G|p^e, \delta^e)$$

subject to

$$\mathcal{U}(G; G|p^e, \delta^e) \geq \mathcal{U}(0; G|p^e, \delta^e) \qquad (IC_G - IR_G)$$

$$y_G \left\{\pi_G r_G + (1 - \pi_G)c_G - d_G\right\} \geq 0 \qquad (IR_L)$$

$$y_G \left[\pi_B(R - r_G) - (1 - \pi_B)c_G - \gamma^e(I - A - d_G) - \rho^e c_G - (I - d_G)\right] \leq 0 \qquad (IC_B)$$

$$0 \leq y_G \leq 1,\ 0 \leq d_G \leq I - A,\ 0 \leq r_G \leq R,\ c_G \geq 0$$

The first constraint $(IC_G - IR_G)$ incorporates both the incentive compatibility and the participation constraint of the good borrower. Indeed, it implies:

$$\mathcal{U}(G; G|p^e, \delta^e) \geq \mathcal{U}(0; G|p^e, \delta^e) \geq A + K_L = \mathcal{U}(B; G|p^e, \delta^e).$$

This requires that signing the financing contract with the lender should not make her worse off than her outside option and should be more profitable than signing the bad borrower contract. The second constraint (IR_L) is the participation constraint of the lender. This means that it is individually rational for the lender to execute the good borrower's contract. The third constraint (IC_B) is the incentive compatibility constraint for the bad borrower. Such constraint requires that the bad borrower has no incentive to misreport her type after the menu is accepted by the lender. The last line summarizes the constraints identifying the choice set in the maximization program; in particular, the probability of investment should be positive and smaller than one, the payment and collateral pledged to the lender should be positive, the amount of debt required should not be higher than what is needed to invest, and we also assume limited liability on the part of the borrower.

Definition 4: Given expected price and haircuts p^e and δ^e, the good borrower-optimal separating contracts menu $((\hat{y}^e_\theta, \hat{d}^e_\theta, \hat{r}^e_\theta, \hat{c}^e_\theta))_{\theta\in\Theta} \coloneqq ((\hat{y}_\theta(p^e,\delta^e), \hat{d}_\theta(p^e,\delta^e), \hat{r}_\theta(p^e,\delta^e), \hat{c}_\theta(p^e,\delta^e)))_{\theta\in\Theta}$ is defined as follows:

(i) $(\hat{y}^e_G, \hat{d}^e_G, \hat{r}^e_G, \hat{c}^e_G)$ is the solution to program (3),

(ii) $(\hat{y}^e_B, \hat{d}^e_B, \hat{r}^e_B, \hat{c}^e_B) = (0,0,0,0)$.

In the next proposition, we characterize the good borrower-optimal separating contract menu given the expectations formed by the borrower regarding the competitive market price and haircut p^e and δ^e.[12]

Theorem 4: *Fix expected competitive market price p^e and OTC haircut δ^e. We have that the solution $(\hat{y}^e_G, \hat{d}^e_G, \hat{r}^e_G, \hat{c}^e_G)$ to program (3) is given by:*

[12]In the following, hatted, expected quantities always refer to the respective quantities evaluated at the good-borrower optimal separating contract solution given the expected competitive market price and OTC haircut.

(i) if $p^e > 1 - \delta^e$,

$$\begin{aligned}
\hat{y}_G^e &= 1 \\
\hat{d}_G^e &= \frac{\pi_G(I - \pi_B R) + \pi_G \gamma^e (I - A)}{(1 + \gamma^e)\pi_G - \pi_B} \\
\hat{r}_G^e &= \frac{(I - \pi_B R) + \gamma^e (I - A)}{(1 + \gamma^e)\pi_G - \pi_B} \\
\hat{c}_G^e &= 0 \ .
\end{aligned}$$

(ii) if $p^e < 1 - \delta^e$,

$$\begin{aligned}
\hat{y}_G^e &= 1 \\
\hat{d}_G^e &= I - A \\
\hat{r}_G^e &= \frac{(1 - \pi_G)(I - \pi_B R) + (\pi_G - \pi_B + \rho^e)(I - A)}{(1 + \rho^e)\pi_G - \pi_B} \\
\hat{c}_G^e &= \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{(1 + \rho^e)\pi_G - \pi_B}
\end{aligned}$$

(iii) if $p^e = 1 - \delta^e$, *any convex combination of the solutions in point* (i) *and* (ii) *is a solution to program* (3).

Given expected competitive market price p^e and OTC haircut δ^e, the good-borrower optimal separating contract will imply expected cash needs

$$\hat{A}_\theta^e \coloneqq \hat{A}_\theta^e(p^e, \delta^e) = \hat{y}_\theta^e \left[(I - A) - \hat{d}_\theta^e \right] + (1 - y_{\tilde{\theta}}) \mathbf{1}\{(\pi_\theta R - I) - \gamma^e (I - A) > 0\}(I - A).$$

for each type $\theta \in \Theta$ executing her own contract, where the second term is the cash needs when the contract does not prescribe investment, but it is still profitable for the borrower to invest in the project on her own. We then have the following corollary.

Corollary 5: *Given the good borrower-optimal separating contract menu, we have the following expected cash needs in the competitive market:*

- *If* $p^e > 1 - \delta^e$, $\hat{A}_B^e = 0$ *and*

$$\hat{A}_G^e = \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{(1 + \gamma^e)\pi_G - \pi_B} \ .$$

- *If* $p^e < 1 - \delta^e$, $(\hat{A}_G^e, \hat{A}_B^e) = (0, 0)$.

Clearly, since in any separating contract menu, the bad borrower does not invest, the equilibrium demands for cash of the bad borrower will always be zero. As far as the good borrower, we have two cases: if it is cheaper to raise cash in the centralized market than obtaining collateral in the OTC market, the good borrower's equilibrium cash needs will be exactly equal to the quantity needed to discourage the bad borrower from misreporting her type. In case it is cheaper to obtain collateral than cash, the opposite holds. The good borrower will signal her type by just posting collateral and, therefore, she will demand no cash in the centralized market.

6. Equilibria of the Lending Game

We now combine all the results of the previous sections to characterize the equilibria of the lending game with secondary markets. There are three classes of equilibria; one where only the centralized market is open, one where only the OTC market is open, and one where the good borrower enters both markets at the same time.

6.1. **Equilibria where Only One Secondary Market is Open.**

First, there always exists an equilibrium where the optimal separating-contract is implemented by the good borrower by accessing only the competitive market.

Theorem 6: *There exists a unique equilibrium with secondary markets where the good borrower (type-G) enters only the centralized market. In such equilibrium:*

(i) the competitive market equilibrium price $\hat{p}^{cm}$ *is the unique solution to the equation:*

$$\hat{p}^{cm} = \left[1 + g'\left(\mu \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{\frac{\pi_G}{\hat{p}^{cm}} - \pi_B}\right)\right]^{-1} ,$$

(ii) $((\hat{y}_\theta^{cm}, \hat{d}_\theta^{cm}, \hat{r}_\theta^{cm}, \hat{c}_\theta^{cm}))_{\theta\in\Theta}$ *is the optimal separating contract menu, where:*

$$\hat{y}_G^{cm} = 1$$

$$\hat{d}_G^{cm} = \frac{\pi_G(I - \pi_B R) + \pi_G \frac{1-\hat{p}^{cm}}{\hat{p}^{cm}}(I - A)}{\frac{\pi_G}{\hat{p}^{cm}} - \pi_B}$$

$$\hat{r}_G^{cm} = \frac{(I - \pi_B R) + \frac{1-\hat{p}^{cm}}{\hat{p}^{cm}}(I - A)}{\frac{\pi_G}{\hat{p}^{cm}} - \pi_B}$$

$$\hat{c}_G^{cm} = 0 ,$$

and $(\hat{y}_B^{cm}, \hat{d}_B^{cm}, \hat{r}_B^{cm}, \hat{c}_B^{cm}) = (0, 0, 0, 0)$,

(iii) cash needs are given by $\hat{A}_B^{cm} = 0$ *and*

$$\hat{A}_G^{cm} = \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{\frac{\pi_G}{\hat{p}^{cm}} - \pi_B} ,$$

and the competitive market equilibrium demand is:

$$\hat{q}^{cm} = \left[1 + g'\left(\mu \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{\frac{\pi_G}{\hat{p}^{cm}} - \pi_B}\right)\right] \mu \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{\frac{\pi_G}{\hat{p}^{cm}} - \pi_B}$$

(iv) $\hat{\nu}_B^{cm} = \hat{\nu}_G^{cm} = \hat{\nu}_{dl}^{cm} = 0$ *and* $\hat{C}_G^{cm} = \hat{C}_G^{cm} = 0$ *and the equilibrium haircut is* $\hat{\delta}^{cm} = 1$

We now want to see whether there are in fact equilibria where the good borrower accesses the OTC market. Intuitively, this will only be possible when the bargaining power of the dealer is not to high. To this end, consider the following upper bound

on the dealer's bargaining power:

$$\bar{\alpha} \coloneqq \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{\pi_G(\pi_G R - I) + (\pi_G - \pi_B)(I - A)} \ .$$

Note that given Assumptions 2 and 3, we have that this upper bound satisfies $\bar{\alpha} \in (0, 1)$. The following theorem states that whenever the bargaining power of the dealer is not too large, there is always an equilibrium in which OTC markets are accessed.

THEOREM 7: *If the dealer's bargaining power $\alpha \in (0, \bar{\alpha})$, then there is an equilibrium with secondary markets where the good borrower (type-G) only enters the OTC market. In any such equilibrium,*

(i) $\hat{q}^{otc} = \hat{A}_G^{otc} = \hat{A}_B^{otc} = 0$ *and* $\hat{p}^{otc} \geq \frac{1}{1+g'(0)}$,

(ii) $\hat{S}_B^{otc} = 0$, $\hat{\nu}_B^{otc} = 0$, $\hat{\nu}_G^{otc} = \hat{\nu}_{dl}^{otc} = \mu$,

(iii) either $\hat{p}^{otc} \leq \frac{I-A}{\pi_G R - A}$, *and the good borrower's contract is given by*

$$\begin{aligned}
&\hat{y}_G^{otc} = 1 \ , \quad \hat{d}_G^{otc} = I - A \ , \\
&\hat{r}_G^{otc} = \frac{(I - \pi_B R) + [\alpha(1 - \pi_G) + \pi_B](\pi_G R - I) - (\pi_G - \pi_B)A}{\pi_G - \pi_B} \ , \\
&\hat{c}_G^{otc} = \frac{(\pi_B - \alpha\pi_G)(\pi_G R - I) - (\pi_G - \pi_B)A}{\pi_G - \pi_B} \ ,
\end{aligned}$$

the surplus $\hat{S}_G^{otc} = \pi_G R - I$ *and the equilibrium haircut is given by*

$$\hat{\delta}^{otc} = \frac{\alpha(\pi_G - \pi_B)(\pi_G R - I)}{(1 - \alpha)\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A} \ ,$$

(iv) or $\hat{p}^{otc} > \frac{I-A}{\pi_G R - A}$ *and the good borrower's contract is given by:*

$$\hat{y}_G^{otc} = 1 \ , \quad \hat{d}_G^{otc} = I - A \ ,$$
$$\hat{r}_G^{otc} = \frac{(I - \pi_B R) + \pi_B(\pi_G R - I) + \alpha(1-\pi_G)\hat{\gamma}^{otc}(I-A) - (\pi_G - \pi_B)A}{\pi_G - \pi_B} \ ,$$
$$\hat{c}_G^{otc} = \frac{\pi_B(\pi_G R - I) - \alpha\pi_G\hat{\gamma}^{otc}(I-A) - (\pi_G - \pi_B)A}{\pi_G - \pi_B} \ ,$$

the surplus is $\hat{S}_G^{otc} = \hat{\gamma}^{otc}(I-A)$ *and the equilibrium haircut is:*

$$\hat{\delta}^{otc} = \frac{\alpha(\pi_G - \pi_B)\hat{\gamma}^{otc}(I-A)}{\pi_B(\pi_G R - I) - \alpha\pi_B\hat{\gamma}^{otc}(I-A) - (\pi_G - \pi_B)A} \ .$$

The equilibrium where the good borrower exclusively participates in the OTC market is not unique solely due to the indeterminate nature of the competitive market price. Provided the dealers' bargaining power is not excessively high, an equilibrium exists in the economy. In this equilibrium, a good borrower can signal her type in the lending market by pledging high-quality collateral, which she obtains through a collateral swap in the OTC market.

In both equilibria in Theorems 6 and 7, the optimal separating contract menu is implemented, so that the good borrower is financed and invests in her profitable project. On the other hand, the bad borrower is not financed since the lender is now able to infer her type, and her project is therefore not remunerative. Compared to the benchmark case with no assets markets, where the lending market breaks down, both types of equilibria achieve an efficiency gain in the lending market equal to the net present value of the good project $\pi_G R - I > 0$, which is equal to the total market surplus in the first best case where the borrower's types are directly observable by the lender. Finally, the good borrower appropriates the whole efficiency gain in the lending market and, then, either transfers part of it to the external investors to compensate their liquidity costs (Theorem 6) or bargains over it with the dealer she

performs the collateral swap with (Theorem 7). In the latter case, if the dealer's bargaining power is not too large, the good borrower and the dealer are able to implement the Nash bargaining split of the efficiency gain by stipulating an appropriate haircut on the low-quality assets in the collateral swap.

6.2. **Equilibria where Both Secondary Markets are Open.** In the previous section, we focused on equilibria where the good borrower, to sign the optimal separating contract, either enters only the centralized market to obtain cash or enters only the OTC market to obtain pledgeable assets. In this section, we look at dual-market equilibria; that is, equilibria in which the good borrower enters contemporaneously both the centralized and OTC market to obtain the cash and high quality assets needed for the contract. Recall that Theorem 4 implies that this is only possible when the equilibrium centralized market price is equal to the equilibrium OTC haircut. In such case, any convex combination of the solutions in points *(i)* and *(ii)* of Theorem 4 is also a borrower-optimal separating contract. In the following, we assign weight λ to the solution in point *(i)* of Theorem 4 in the previous section. We can interpret this λ as the intensity at which agents enter into the centralized market.[13] If $\lambda = 0$, then the good borrower only accesses the OTC market. If $\lambda = 1$, then the good borrower only accesses the centralized market. When $\lambda \in (0, 1)$, the good borrower accesses both markets at the same time. In the dual-market equilibrium, rational expectations and Lemma 1 imply that for any given equilibrium λ, the equilibrium net cost of obtaining cash is the solution to the following fixed point problem in γ

$$\gamma = g'\left(\lambda\mu\frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{(1+\gamma)\pi_G - \pi_B}\right).$$

[13]Note that when $\hat{p} = \hat{\delta}$, the requisite amount of cash and that of collateral - needed to achieve separation in the two cases where only the centralized market or the OTC market are entered - are equal to each other. Therefore, λ represents the fraction of this amount to be obtained as cash in the centralized market (respectively, $1 - \lambda$ is the fraction to be obtained as collateral in the OTC market).

As shown in the proof of Theorem 8, there is a unique solution $\hat{\gamma}(\lambda)$, (weakly) increasing in $\lambda \in [0,1]$ and such that $\hat{\gamma}(0) = g'(0)$ and $\hat{\gamma}(1) = \hat{\gamma}^{cm}$. By Corollary 3, for each $\lambda \in (0,1)$, the equilibrium net cost of acquiring collateral in the OTC is the unique solution $\hat{\rho}(\lambda)$ to the equation in ρ:

$$\rho = \alpha \frac{\hat{S}(\lambda|\rho, \hat{\gamma}(\lambda)}{\hat{C}(\lambda|\rho, \hat{\gamma}(\lambda))}.$$

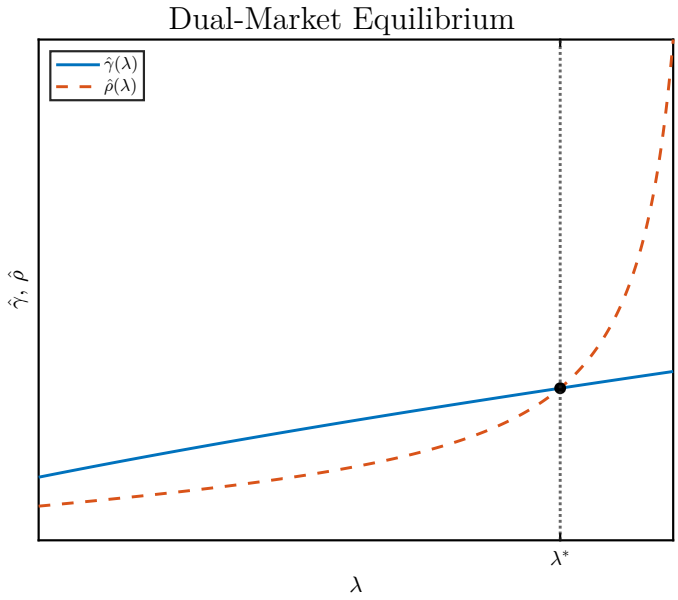


FIGURE 2. Parameters used in computations: $\pi_G = 0.8$, $\pi_B = 0.6$, $I = 80$, $A = 0.1$, $R = 120$, $\mu = 0.3$, $\alpha = 0.25$, $g(x) = e^{0.4x}$. The blue solid line is the equilibrium net cost of cash in the centralized market as a function of λ and the red dotted line is the equilibrium net cost of collateral in the OTC market. The intersection of the lines is the dual-market equilibrium where both markets are open.

Therefore, the existence of equilibria where the good borrower enters contemporaneously the centralized market for cash and the OTC market reduces to the existence of $\lambda^* \in (0,1)$ such that $\hat{\rho}(\lambda^*) = \hat{\gamma}(\lambda^*)$, as we graphically illustrate in Figure 6.2. In this view, we refer in the following to equilibrium λ^* to mean the corresponding dual-market equilibrium. Define the following threshold for the bargaining power of

the dealer:

$$\tilde{\alpha} = \max\left\{ \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{\pi_G g'(0)(I-A) + (\pi_G - \pi_B)(I-A)},\ \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{\pi_G(\pi_G R - I) + (\pi_G - \pi_B)\frac{\pi_G R - I}{g'(0)}} \right\}.$$

Then, we have the following theorem.

THEOREM 8:

(i) Suppose $\hat{\gamma}^{cm} \leq \frac{\pi_G R - I}{I - A}$. *Then, there exists an equilibrium* $\lambda^* \in (0,1)$ *solving* $\hat{\rho}(\lambda) = \hat{\gamma}(\lambda)$ *if and only if* $\alpha < \tilde{\alpha}$. *Moreover, such equilibrium* λ^* *is unique.*

(ii) Suppose $\hat{\gamma}^{cm} > \frac{\pi_G R - I}{I - A}$. *Then, if* $\alpha < \tilde{\alpha}$, *there exists an equilibrium* $\lambda^* \in (0,1)$ *solving* $\hat{\rho}(\lambda) = \hat{\gamma}(\lambda)$.

In Figure 6.2, we graphically illustrate the existance of equilibria for different values of the bargaining power $\alpha \in (0,1)$. It is worth noting that in all the three cases, $\tilde{\alpha}$ is always lower than the threshold $\bar{\alpha}$ we found for the OTC only equilibrium.[14] This means that for values of the bargaining power $\alpha \in (\bar{\alpha}, \tilde{\alpha})$, a dual-market equilibrium exists even if the OTC-only equilibrium does not. As we will see in the next session, this is important because the equilibrium welfare of the good borrower (type-G) in the different equilibria will be decreasing in λ.

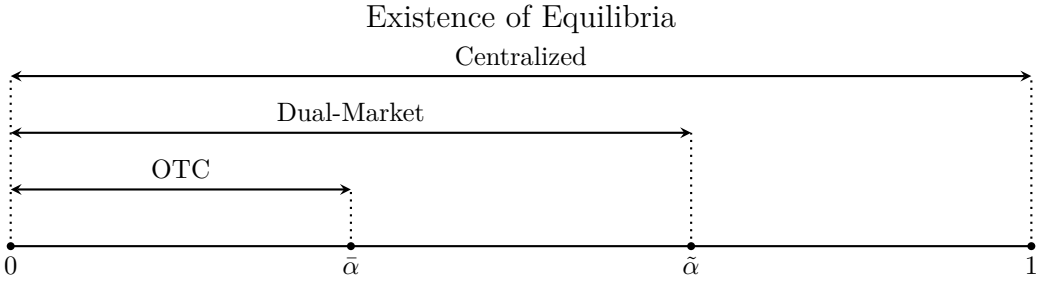


FIGURE 3. In the figure we illustrate the equilibria of the game with secondary markets for different values of the bargaining power α.

[14] Notice that in case *(i)*, $\tilde{\alpha}$ is strictly lower than $\bar{\alpha}$ if $\hat{\gamma}^{cm} < \frac{\pi_G R - I}{I - A}$ or if g is strictly convex; in case *(ii)* if $g'(0) > \frac{\pi_G R - I}{I - A}$; and in case *(iii)* if, again, g is strictly convex.

7. Welfare Analysis

In the centralized market and OTC-only equilibria, the good borrower's equilibrium welfare is given respectively by $\hat{V}_G^{cm} := \hat{\mathcal{U}}(G; G|\hat{p}^{cm}, \hat{\delta}^{cm})$ and $\hat{V}_G^{otc} := \hat{\mathcal{U}}(G; G|\hat{p}^{otc}, \hat{\delta}^{otc})$. Similarly, if we let Λ^* be the set of the dual-market equilibria, for each equilibrium $\lambda^* \in \Lambda^*$, the good borrower's welfare is given by $V_G^{\lambda^*} := \hat{\mathcal{U}}(G; G|\hat{p}(\lambda^*), \hat{\delta}(\lambda^*))$. In the following proposition, we rank the different equilibria in terms of their respective good borrower's welfare.

Proposition 9: *Suppose $\alpha \in (\bar{\alpha}, \tilde{\alpha})$. Then, for all $\lambda_1^*, \lambda_2^* \in \Lambda^*$, $\lambda_1^* < \lambda_2^*$, we have $V_G^{cm} \leq V_G^{\lambda_2^*} \leq V_G^{\lambda_1^*}$. Furthermore, if $\alpha < \bar{\alpha}$, we always have $V_G^{cm} \leq V_G^{\lambda_2^*} \leq V_G^{\lambda_1^*} < V_G^{otc}$, no matter what market-clearing price $\hat{p}^{otc} \geq \frac{1}{1+g'(0)}$ we consider in the OTC-only equilibrium.*

The result in the proposition allows us to rank equilibria in terms of the intensity of entry in the centralized market λ. In particular, the dual-market equilibria provide the good borrower with higher welfare than the centralized market-only equilibrium, and more so, the smaller is λ^*. In addition, whenever the OTC market-only equilibrium exists, it dominates all other equilibria. We illustrate the welfare comparison across equilibria in Figure 7.

The intuition for the result goes as follows. The difference in good borrower's welfare across different equilibria only depends on the net cost associated with sourcing the requisite amounts of cash and collateral in the two secondary markets. As trade in the OTC market happens bilaterally between the good borrower and the dealer, the implied equilibrium haircut $\hat{\delta}^{otc}$ only depends on the primitives of the model, i.e. the rules of the bargaining protocol and the associated parameters, and not on the mass of borrowers accessing the market. However, this is not true for the centralized market price $\hat{p}^{otc}$: by the competitive nature of the market and because we assume the cost function to be weakly convex ($g''(\cdot) \geq 0$), $\hat{p}^{otc}$ increases as the centralized

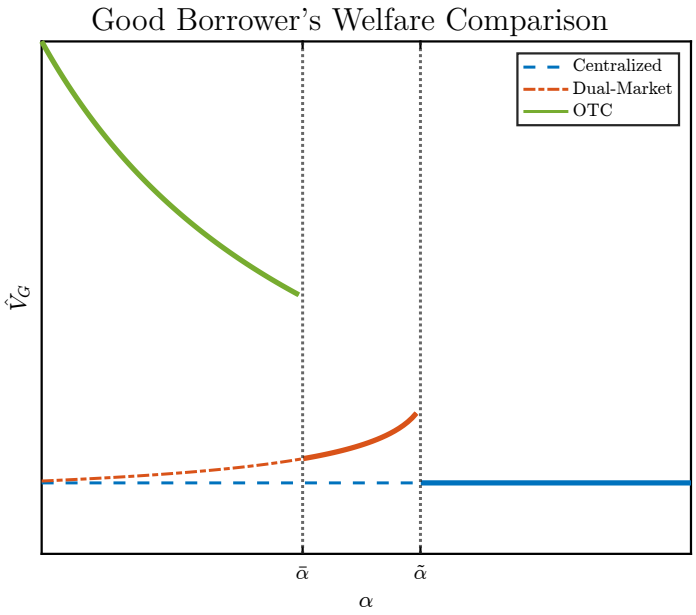


FIGURE 4. Parameters used in computations: $\pi_G = 0.8$, $\pi_B = 0.6$, $I = 80$, $A = 0.1$, $R = 120$, $\mu = 0.3$, $g(x) = e^{0.4x}$. Centralized market's welfare is in dashed blue, the (unique) dual-market equilibrium's welfare is dashed-dotted red, and OTC market's welfare is in solid green. The bold portion of the lines highlights the upper envelope of the welfare functions across the three markets. The dotted vertical lines denote the two thresholds $\bar{\alpha}$ and $\tilde{\alpha}$.

market is accessed more often. Ultimately, this implies

$$\hat{\rho}^{otc} < g'(0) \leq \hat{\gamma}(\lambda^*) \leq g'\left(\mu \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{\frac{\pi_G}{\hat{p}^{cm}} - \pi_B}\right) = \hat{\gamma}^{cm} ,$$

where $\hat{\rho}^{otc}$ is the cost of accessing the OTC market in the OTC-only equilibrium, while $\hat{\gamma}(\lambda^*)$ is the cost of accessing both market in the dual-market equilibrium and $\hat{\gamma}^{cm}$ is the cost of accessing the centralized market in the CM-only equilibrium. Therefore, in any OTC-only equilibrium, the good borrower receives a higher expected payoff than in the equilibrium where she only enters the centralized market or access both. However, if the OTC-only equilibrium does not exist, then the dual-market equilibrium is the best equilibrium in terms of the good borrower's welfare.

8. Conclusions

We propose a theoretical framework to understand the welfare benefits of accessing centralized versus decentralized markets in a lending context with asymmetric information. In our model, only the good borrower has a profitable project but lacks the cash to invest, needing to signal her type to a lender. She can either liquidate low-quality collateral in a centralized market, incurring transaction costs, or trade this collateral for high-quality assets in a decentralized OTC market, where the trade surplus is split through Nash bargaining. Without secondary markets, pooling equilibria may lead to market breakdowns if project returns are low. However, with secondary markets, we find that a welfare-improving, separating equilibrium can be achieved, where the borrower maximizes her welfare given the equilibrium prices in these markets. We characterize equilibria in both market types and derive an optimal contract for the borrower, akin to the Riley outcome, with signaling costs determined endogenously by market prices and OTC haircuts.

We prove the existence of three types of equilibria: CM-only, OTC-only, and dual-market. The CM-only equilibrium involves the good borrower selling assets at a discount due to liquidity costs. The OTC-only equilibrium exists when the dealer's bargaining power is moderate, allowing the borrower to obtain high-quality assets at a haircut. The dual-market equilibrium requires the borrower to use both markets in a way that equalizes the costs of obtaining cash and pledgeable assets. We show that OTC-only equilibria provide the highest welfare for the good borrower, followed by dual-market equilibria, with the CM-only equilibrium yielding the lowest welfare. This ranking is driven by the signaling costs, which are lowest in the OTC-only equilibrium and highest in the CM-only equilibrium. The findings underscore the importance of market access and dealer bargaining power in determining the efficiency and welfare outcomes in lending markets with asymmetric information.

Our theory highlights the importance of measuring properly the underlying frictions characterizing different markets to understand welfare. In particular, the bargaining power of dealers is key to expand the set of possible markets the good borrower can access and so the resulting welfare. Hence, further work should be devoted to empirically assess the extent of frictions in centralized versus OTC markets (Allen and Wittwer, 2023).

Given the existence of equilibria with different welfare properties, in ongoing work we are exploring the role of a large player (e.g. a government) in implementing the desired equilibrium in a more general setting. For example, assume there is a government that wants to select the equilibrium which is more favorable to the good borrower. Suppose the government announces that it will take action so that the OTC haircut will be kept below some level $\bar{\delta}$. If the government is credible, it will manage to steer the borrowers to believe that the OTC haircut is low enough that in the CM-only equilibrium they would profit by deviating to entering the OTC market. Therefore, the equilibrium where only the competitive market opens would cease to exist, and the credible government would be able to select the Pareto-efficient OTC market equilibrium.

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Appendix

Appendix A. Proofs

Proof of Lemma 1: Fix a price $p > 0$ and consider the problem of the external investor. Since $\mathbb{E}[X_L] = 1$, we can rewrite his maximization problem as:

$$\max_{q \in \mathbb{R}_+} q - pq - g(pq)$$

Since g is convex and differentiable, FOC are necessary and sufficient for a maximum. They read:

$$1 - p - g'(pq)p + \lambda = 0$$

$$\lambda \, q = 0$$

where $\lambda \geq 0$ is the multiplier associated to the non-negativity constraint and the second condition is the usual complementary slackness. If $1 - p - g'(pq)p > 0$ for any q, the investor would demand unbounded quantities of low-quality asset and this would violate market clearing. Thus, assume that p is such that this is not the case. If $1 - p - g'(0)p < 0$, g' increasing implies that $\lambda > 0$ and, therefore $q = 0$ by complementary slackness. On the other hand, suppose that $1 - p - g'(0)p \geq 0$. Then, by continuity we can find $q \geq 0$ such that $1 - p - g'(pq)p = 0$ and, therefore, $\lambda = 0$ so that we can write:

$$p = \frac{1}{1 + g'(pq)} \, .$$

Imposing market clearing, we have that if $\mu A_G + (1-\mu)A_B > 0$, the unique equilibrium price is given by $\hat{p} = \frac{1}{1+g'(\mu A_G + (1-\mu)A_B)}$. On the other hand, if $\mu A_G + (1-\mu)A_B = 0$, any $\hat{p} \geq \frac{1}{1+g'(0)}$ supports the equilibrium allocation $\hat{q} = 0$. ■

Proof of Proposition 2: Suppose that

$$\left((\hat{W}^{dl}_\theta, \hat{W}_\theta)_{\theta \in \Theta}, (\hat{o}_{dl}, (\hat{o}_\theta)_{\theta \in \Theta}), \hat{\nu}_{dl}, (\hat{\nu}_\theta)_{\theta \in \Theta}, (\hat{x}^{dl}_\theta, \hat{x}_\theta)_{\theta \in \Theta}, \hat{\delta} \right)$$

is an equilibrium in the OTC secondary market that satisfies condition (1). Then, it must be the case that $\nu_{dl} = \nu_G + \nu_B$ and, therefore, the probability of getting the first match is one for all players and $\tilde{\beta}_\theta = \tilde{\beta}_{dl} = \beta$. Therefore, the disagreement points must be:

$$\hat{o}_{dl} = \max\{0, \beta\hat{W}^{dl}\}$$

$$\hat{o}_\theta = \max\{0, \beta\hat{W}_\theta\}$$

Then, using the characterization of the Nash Bargaining solution to bargaining problem $(S_\theta, \alpha, (\hat{o}_{dl}, \hat{o}_\theta))$, we have that for each $\theta \in \Theta$, the match values satisfy:

$$\hat{W}_\theta^{dl} = \max\left\{\alpha(S_\theta - \max\{0, \beta\hat{W}_\theta\}) + (1-\alpha)\max\{0, \beta\hat{W}^{dl}\}, \max\{0, \beta\hat{W}^{dl}\}\right\}$$
$$\hat{W}_\theta = \max\left\{(1-\alpha)(S_\theta - \max\{0, \beta\hat{W}^{dl}\}) + \alpha\max\{0, \beta\hat{W}_\theta\}, \max\{0, \beta\hat{W}_\theta\}\right\}$$

We can notice right away that $\hat{W}_\theta^{dl} \geq 0$ and $\hat{W}_\theta \geq 0$, so that we can rewrite the system above as:

$$\hat{W}_\theta^{dl} = \max\left\{\alpha(S_\theta - \beta\hat{W}_\theta) + (1-\alpha)\beta\hat{W}^{dl}, \beta\hat{W}^{dl}\right\} \ ,$$
$$\hat{W}_\theta = \max\left\{(1-\alpha)(S_\theta - \beta\hat{W}^{dl}) + \alpha\beta\hat{W}_\theta, \beta\hat{W}_\theta\right\} \ .$$

We cannot have both $\hat{W}_G = \hat{W}_B = 0$ since $\alpha, \beta \in (0,1)$ and since $S_G > S_B$, then $\hat{W}_G > 0$. Similarly, we also have that $\hat{W}_G^{dl} \geq \hat{W}^{dl} > 0$. Then it holds that:

$$\hat{W}_G = (1-\alpha)(S_G - \beta\hat{W}^{dl}) + \alpha\beta\hat{W}_G = \frac{1-\alpha}{1-\alpha\beta}(S_G - \beta\hat{W}^{dl})$$

and, then,

$$\begin{aligned}
\hat{W}_G^{dl} &= \alpha(S_G - \beta\hat{W}_G) + (1-\alpha)\beta\hat{W}^{dl} \\
&= \alpha S_G - \alpha\beta\frac{1-\alpha}{1-\alpha\beta}(S_G - \beta\hat{W}^{dl}) + (1-\alpha)\beta\hat{W}^{dl} \\
&= \frac{\alpha(1-\beta)}{1-\alpha\beta}S_G + \frac{(1-\alpha)\beta}{1-\alpha\beta}\hat{W}^{dl} \ .
\end{aligned}$$

Note that $\hat{W}_B^{dl} = \beta\hat{W}^{dl}$ and $\hat{W}_B = \beta\hat{W}_B = 0$ solve the value functions as long as:

$$\beta\hat{W}^{dl} \geq \alpha(S_B - \beta\hat{W}_B) + (1-\alpha)\beta\hat{W}^{dl} = \alpha S_B + (1-\alpha)\beta\hat{W}^{dl}$$

$$0 = \beta\hat{W}_B \geq (1-\alpha)(S_B - \beta\hat{W}^{dl}) + \alpha\beta\hat{W}_B = (1-\alpha)(S_B - \beta\hat{W}^{dl})$$

which are both satisfied if $\beta\hat{W}^{dl} \geq S_B$. Assume the latter and note that:

$$\hat{W}^{dl} = \hat{\xi}_G\hat{W}_G^{dl} + (1-\hat{\xi}_G)\hat{W}_B^{dl} = \frac{\hat{\xi}_G\hat{W}_G^{dl}}{1-\beta(1-\hat{\xi}_G)}$$

and, therefore, we also have:

$$\begin{aligned}
\hat{W}_G^{dl} &= \frac{\alpha(1-\beta)}{1-\alpha\beta}S_G + \frac{(1-\alpha)\beta}{1-\alpha\beta}\hat{W}^{dl} \\
&= \frac{\alpha(1-\beta)}{1-\alpha\beta}S_G + \frac{(1-\alpha)\beta}{1-\alpha\beta}\frac{\hat{\xi}_G\hat{W}_G^{dl}}{1-\beta(1-\hat{\xi}_G)} \\
&= \frac{\alpha(1-\beta(1-\hat{\xi}_G))}{1-\alpha\beta(1-\hat{\xi}_G)}S_G
\end{aligned}$$

and, thus,

$$S_B \leq \beta\hat{W}^{dl} = \beta\frac{\hat{\xi}_G}{1-\beta(1-\hat{\xi}_G)}\frac{\alpha(1-\beta(1-\hat{\xi}_G))}{1-\alpha\beta(1-\hat{\xi}_G)}S_G = \frac{\alpha\beta\hat{\xi}_G}{1-\alpha\beta(1-\hat{\xi}_G)}S_G$$

which is true given the assumptions of the proposition. Finally,

$$\begin{aligned}
\hat{W}_G &= \frac{1-\alpha}{1-\alpha\beta}(S_G - \beta\hat{W}^{dl}) \\
&= \frac{1-\alpha}{1-\alpha\beta}\left(S_G - \frac{\alpha\beta\hat{\xi}_G}{1-\alpha\beta(1-\hat{\xi}_G)}S_G\right) \\
&= \frac{1-\alpha}{1-\alpha\beta}\frac{1-\alpha\beta}{1-\alpha\beta(1-\hat{\xi}_G)}S_G \\
&= \frac{1-\alpha}{1-\alpha\beta(1-\hat{\xi}_G)}S_G \ .
\end{aligned}$$

To sum up, we have that:

$$\hat{W}_G^{dl} = \frac{\alpha(1-\beta(1-\hat{\xi}_G))}{1-\alpha\beta(1-\hat{\xi}_G)} S_G$$

$$\hat{W}_B^{dl} = \frac{\alpha\beta\hat{\xi}_G}{1-\alpha\beta(1-\hat{\xi}_G)} S_G$$

$$\hat{W}_G = \frac{1-\alpha}{1-\alpha\beta(1-\hat{\xi}_G)} S_G$$

$$\hat{W}_B = 0$$

Now, given that $S_B < \beta\hat{W}^{dl}$, we have that $\hat{x}_B^{dl} = \hat{x}_B = 0$ and, using again the formulae for the Nash bargaining solution:

$$\begin{aligned}
\hat{x}_G^{dl} &= \alpha(S_G - \beta\hat{W}_G) + (1-\alpha)\beta\hat{W}^{dl} \\
&= \alpha\left(1 - \beta\frac{1-\alpha}{1-\alpha\beta(1-\hat{\xi}_G)}\right) S_G + (1-\alpha)\frac{\alpha\beta\hat{\xi}_G}{1-\alpha\beta(1-\hat{\xi}_G)} S_G \\
&= \frac{\alpha(1+\alpha\beta\hat{\xi}_G - \beta) + (1-\alpha)(\alpha\beta\hat{\xi}_G)}{1-\alpha\beta(1-\hat{\xi}_G)} S_G \\
&= \frac{\alpha[1-\beta(1-\hat{\xi}_G)]}{1-\alpha\beta(1-\hat{\xi}_G)} S_G
\end{aligned}$$

and

$$\begin{aligned}
\hat{x}_G &= (1-\alpha)(S_G - \beta\hat{W}^{dl}) + \alpha\beta\hat{W}_G \\
&= (1-\alpha)\left(1 - \frac{\alpha\beta\hat{\xi}_G}{1-\alpha\beta(1-\hat{\xi}_G)}\right) S_G + \alpha\beta\frac{1-\alpha}{1-\alpha\beta(1-\hat{\xi}_G)} S_G \\
&= \frac{(1-\alpha)(1-\alpha\beta) + \alpha\beta(1-\alpha)}{1-\alpha\beta(1-\hat{\xi}_G)} S_G \\
&= \frac{1-\alpha}{1-\alpha\beta(1-\hat{\xi}_G)} S_G \ .
\end{aligned}$$

■

Proof of Corollary 3: Suppose that the conditions of the corollary are satisfied; that is, $C_G > 0$, $S_G > 0$, and $S_B = 0$. Suppose now that $C_B > 0$. Then, using

Proposition 2 and the OTC equilibrium condition:

$$\frac{\hat{\delta}}{1-\hat{\delta}}C_B = \hat{x}_B^{dl} = 0$$

which implies that $\hat{\delta} = 0$. But, then, in equilibrium:

$$0 = \frac{\hat{\delta}}{1-\hat{\delta}}C_G = \hat{x}_G^{dl} = \frac{\alpha(1-\beta(1-\hat{\xi}_G))}{1-\alpha\beta(1-\hat{\xi}_G)}S_G > 0$$

which is an impossibility, implying there is no equilibrium in the OTC market. Hence, an equilibrium in the OTC market exists only if $C_B = 0$. Suppose this is the case. Then, in any equilibrium, it must be the case that $\hat{\nu}_B = 0$, $\hat{\nu}_G = \hat{\nu}_{dl} = \mu$, and, therefore, $\hat{\xi}_G = 1$. Thus, the Nash bargaining allocation to the dealer in the match with a good borrower becomes

$$\hat{x}_G^{dl} = \frac{\alpha(1-\beta(1-\hat{\xi}_G))}{1-\alpha\beta(1-\hat{\xi}_G)}S_G = \alpha S_G$$

and, therefore, the equilibrium haircut is the unique solution to:

$$\frac{\hat{\delta}}{1-\hat{\delta}} = \alpha\frac{S_G}{C_G}\ .$$

It is then easy to see that $\hat{x}_G = (1-\alpha)S_G$ and $\hat{x}_B^{dl} = \hat{x}_B = 0$. Moreover,

$$\hat{W}_G^{dl} = \alpha S_G, \qquad \hat{W}_B^{dl} = \beta\alpha S_G,$$
$$\hat{W}_G = (1-\alpha)S_G, \quad \hat{W}_B = 0,$$

and

$$\hat{o}_{dl} = \beta\alpha S_G, \quad \hat{o}_G = \beta(1-\alpha)S_G, \quad \hat{o}_B = 0$$

are the unique solutions to the equations in points (iii) and (iv) of Definition 2.

■

PROOF OF THEOREM 4: Suppose that $y_G = 1$. Then, $d_G = 0$ can only be optimal if condition $\gamma^e \leq \frac{\pi_G R - I}{I_A}$. In such case, it is then optimal to set $r_G = c_G = 0$. This clearly satisfies IR_L, IC_L, and IR_G, yielding a value equal to the reservation expected utility $\mathcal{U}(0; \theta | p^e, \delta^e)$. Suppose now that it is optimal to choose $d_G > 0$. Then, at the optimum, we need that the IR_L and IC_B are both binding. Indeed, suppose not. If IR_L is not binding, then $\max\{r_G, c_G\} > d_G \geq 0$. Wlog, suppose that $r_G > 0$. If $d_G = I - A$, then choose $\varepsilon \in (0, \min\{\pi_B r_G, (1+\gamma^e)(I-A)\})$ small enough so that the contract

$$(\tilde{y}_G, \tilde{d}_G, \tilde{r}_G, \tilde{c}_G) = \left(1, r_G - \frac{\varepsilon}{\pi_B}, c_G, d_G - \frac{\varepsilon}{1+\gamma^e}\right)$$

still satisfies IR_L. Then:

$$\tilde{\mathcal{U}}(G; B | p^e, \delta^e) - \mathcal{U}(G; B | p^e, \delta^e) = -\pi_B \left(-\frac{\varepsilon}{\pi_B}\right) + (1+\gamma^e)\left(-\frac{\varepsilon}{1+\gamma^e}\right) = 0$$

so that the new contract still satisfies IC_B and

$$\tilde{\mathcal{U}}(G; G | p^e, \delta^e) - \mathcal{U}(G; G | p^e, \delta^e) = -\pi_G \left(-\frac{\varepsilon}{\pi_B}\right) + (1+\gamma^e)\left(-\frac{\varepsilon}{1+\gamma^e}\right) = \left(\frac{\pi_G}{\pi_B} - 1\right)\varepsilon > 0$$

so that the new contract yields a strictly higher value, a contradiction. On the other hand, suppose that $d_G < I - A$. Then, choose $\varepsilon \in (0, (1+\gamma^e)(I - A - d_G))$ small enough so that

$$(\tilde{y}_G, \tilde{d}_G, \tilde{r}_G, \tilde{c}_G) = \left(1, r_G, c_G + \frac{\varepsilon}{(1-\pi_B) + \rho^e}, d_G + \frac{\varepsilon}{1+\gamma^e}\right)$$

still satisfies IR_L. Then:

$$\tilde{\mathcal{U}}(G; B | p^e, \delta^e) - \mathcal{U}(G; B | p^e, \delta^e) = -[(1-\pi_B) + \rho^e]\left(\frac{\varepsilon}{(1-\pi_B)+\rho^e}\right) + (1+\gamma^e)\left(\frac{\varepsilon}{1+\gamma^e}\right) = 0$$

and

$$\tilde{\mathcal{U}}(G;G|p^e,\delta^e)-\mathcal{U}(G;G|p^e,\delta^e)=-[(1-\pi_G)+\rho^e]\left(\frac{\varepsilon}{(1-\pi_B)+\rho^e}\right)+(1+\gamma^e)\left(\frac{\varepsilon}{1+\gamma^e}\right)$$
$$=\left(1-\frac{(1-\pi_G)+\rho^e}{(1-\pi_B)+\rho^e}\right)\varepsilon>0$$

so that the new contract satisfies IC_B and yields a higher objective, reaching again a contradiction. Suppose now that IR_L is binding but IC_B is not. First of all, notice that $r_G=R$ cannot be a solution, as otherwise IR_G would be violated. Thus, restrict attention to $r_G\in(0,R)$ and assume first that $d_G=I-A$. Then, IR_L and IC_B reduce to the following system:

$$\pi_G r_G+(1-\pi_G)c_G=I-A;$$
$$\pi_B(R-r_G)-(1-\pi_B+\rho^e)c_G-A<0.$$

Substituting the binding IR_L in the IC_B we can rewrite the constraint as

$$\pi_B R-\rho^e c_G-I+(\pi_G-\pi_B)(r_G-c_G)<0.$$

Suppose that $c_G=0$. Then, $r_G=\frac{I-A}{\pi_G}$ and, therefore,

$$\pi_B R-\rho^e c_G-I+(\pi_G-\pi_B)(r_G-c_G)=\pi_B R-I+\frac{\pi_G-\pi_B}{\pi_G}(I-A)$$
$$=\frac{\pi_B(\pi_G R-I)-(\pi_G-\pi_B)A}{\pi_G}$$

which is strictly positive by Assumption 3, a violation of IC_B. Then, it must be the case that $c_G>0$. Then, pick $\varepsilon\in(0,\max\{\frac{1-\pi_G}{\pi_G}c_G,R-r_G\})$ small enough so that

$$(\tilde{y}_G,\tilde{d}_G,\tilde{r}_G,\tilde{c}_G)=\left(1,r_G+\varepsilon,c_G-\frac{\pi_G}{1-\pi_G}\varepsilon,d_G\right)$$

still satisfies IC_B. By construction, it also satisfies IR_L with equality. However,

$$\tilde{\mathcal{U}}(G;G|p^e,\delta^e) - \mathcal{U}(G;G|p^e,\delta^e) = -\rho^e\left(-\frac{\pi_G}{1-\pi_G}\varepsilon\right) > 0$$

so that the new contract improves the objective, reaching again a contradiction. Suppose finally that $d_G < I - A$. Then, choose $\varepsilon \in (0, \pi_G(R - r_G))$ small enough so that

$$(\tilde{y}_G, \tilde{d}_G, \tilde{r}_G, \tilde{c}_G) = \left(1, r_G + \frac{\varepsilon}{\pi_G}, c_G, d_G + \varepsilon\right)$$

still satisfies IC_B. By construction, the new contract satisfies IR_L with equality. Moreover,

$$\tilde{\mathcal{U}}(G;G|p^e,\delta^e) - \mathcal{U}(G;G|p^e,\delta^e) = -\pi_G\frac{\varepsilon}{\pi_G} + (1+\gamma^e)\varepsilon = \gamma^e\varepsilon > 0$$

so that again the new contract yields a higher objective, reaching again a contradiction. Therefore, we can conclude that both IR_L and IC_B need to be binding at the optimum.

Now, we characterize the solution to the program given these two binding constraints. Afterwards, we will show that in this solution we, indeed, have $d_G > 0$, and that such solution satisfies IR_G strictly. In particular, this also implies that this solution achieves an higher value than setting $d_G = 0 = r_G = c_G$. This verifies that this solution is, indeed, the solution to the program.

Fix $c_G = c \geq 0$. Since both IR_L and IC_B are binding, we can rewrite them as a square system of equations:

$$\begin{bmatrix} \pi_G & -1 \\ -\pi_B & 1+\gamma^e \end{bmatrix}\begin{bmatrix} r_G \\ d_G \end{bmatrix} = \begin{bmatrix} -(1-\pi_G)c \\ -\pi_B R + (1-\pi_B+\rho^e)c + (1+\gamma^e)I - \gamma^e A \end{bmatrix}.$$

Notice that since $\pi_G > \pi_B$ and $\gamma^e > 0$, the matrix in the LHS in non-singular. Therefore, we can invert it in order to obtain the solution to the system of equations as a function of c.

$$\begin{bmatrix} r_G(c) \\ d_G(c) \end{bmatrix} = \frac{1}{\pi_G - \pi_B + \gamma^e \pi_G} \begin{bmatrix} 1+\gamma^e & 1 \\ \pi_B & \pi_G \end{bmatrix} \begin{bmatrix} -(1-\pi_G)c \\ -\pi_B R + (1-\pi_B+\rho^e)c + (1+\gamma^e)I - \gamma^e A \end{bmatrix}$$

$$= \frac{1}{\pi_G - \pi_B + \gamma^e \pi_G} \begin{bmatrix} c[\pi_G - \pi_B + \rho^e - \gamma^e(1-\pi_G)] + (I - \pi_B R) + \gamma^e(I-A) \\ c[\pi_G - \pi_B + \rho^e \pi_G] + \pi_G(I - \pi_B R) + \pi_G \gamma^e (I - A) \end{bmatrix}$$

First of all, we can take derivatives to study how such solution changes as c varies:

$$\frac{\partial r_G(c)}{\partial c} = \frac{1}{\pi_G - \pi_B + \gamma^e \pi_G}[\pi_G - \pi_B + \rho^e - \gamma^e(1-\pi_G)];$$
$$\frac{\partial d_G(c)}{\partial c} = \frac{1}{\pi_G - \pi_B + \gamma^e \pi_G}[\pi_G - \pi_B + \rho^e \pi_G] \ .$$

Then, taking the derivative of the objective function with respect to c and using the expressions just found we obtain:

$$\begin{aligned}
\frac{\partial \mathcal{U}(G;G|p^e,\delta^e)}{\partial c} &= -\pi_G \frac{\partial r_G(c)}{\partial c} - (1+\rho^e-\pi_G) + (1+\gamma^e)\frac{\partial d_G(c)}{\partial c} \\
&= \frac{-\pi_G[\pi_G - \pi_B + \rho^e - \gamma^e(1-\pi_G)] + (1+\gamma^e)[\pi_G - \pi_B + \rho^e \pi_G]}{\pi_G - \pi_B + \gamma^e \pi_G} - (1+\rho^e-\pi_G) \\
&= \frac{(1-\pi_G)(\pi_G - \pi_B + \gamma^e \pi_G) + \gamma^e(\pi_G - \pi_B + \rho^e \pi_G)}{\pi_G - \pi_B + \gamma^e \pi_G} - (1+\rho^e-\pi_G) \\
&= \frac{\gamma^e(\pi_G - \pi_B + \rho^e \pi_G) - \rho^e[\pi_G - \pi_B + \gamma^e \pi_G]}{\pi_G - \pi_B + \gamma^e \pi_G} \\
&= \frac{(\gamma^e - \rho^e)(\pi_G - \pi_B)}{\pi_G - \pi_B + \gamma^e \pi_G}.
\end{aligned}$$

Then, we have that

$$\begin{aligned}\frac{\partial \mathcal{U}(G;G|p^e,\delta^e)}{\partial c} > 0 &\iff \frac{(\gamma^e-\rho^e)(\pi_G-\pi_B)}{\pi_G-\pi_B+\gamma^e\pi_G} > 0 \\ &\iff (\gamma^e-\rho^e)(\pi_G-\pi_B) > 0 \\ &\iff \gamma^e > \rho^e \ .\end{aligned}$$

Note that the objective is affine in c. Therefore, if $\gamma^e > \rho^e$, so that $\frac{\partial}{\partial d}\mathcal{U}(G;G|p^e,\delta^e) > 0$, it is optimal to set c as high as possible, within the limits of the feasibility constraints of r_G and d_G. Conversely, if $\gamma^e < \rho^e$, it is optimal to choose c as low as possible, again within the limits of the feasibility constraints. The intuition is that the agent would only ask for collateral if the cost of collateral is lower than the cost of obtaining cash in the competitive market. It is also worth noticing that

$$\begin{aligned}\gamma^e > \rho^e &\iff \frac{1-p^e}{p^e} > \frac{\delta^e}{1-\delta^e} \\ &\iff (1-p^e)(1-\delta^e) > p^e\delta^e \\ &\iff 1-\delta^e > p^e \ .\end{aligned}$$

Therefore, in order to determine the optimal value of c, we need to check what bounds are implied by the constraints:

$$\begin{aligned}0 \le r_G(c) \le R \\ 0 < d_G(c) \le I - A \ .\end{aligned}$$

We know that $d_G(c) > 0$ for any $c \ge 0$, since

$$d_G(c) = \underbrace{\frac{1}{\pi_G-\pi_B+\gamma^e\pi_G}}_{>0}\left(\underbrace{c\left[\pi_G-\pi_B+\rho^e\pi_G\right]}_{>0} + \underbrace{\pi_G(I-\pi_B R)}_{>0} + \underbrace{\pi_G\gamma^e(I-A)}_{>0}\right) \ .$$

Moreover, we have that $d_G(c) \leq I - A$ as long as:

$$
\begin{aligned}
& c\left[\pi_G - \pi_B + \rho^e \pi_G\right] + \pi_G(I - \pi_B R) + \pi_G \gamma^e (I - A) \leq (I - A)[\pi_G - \pi_B + \gamma^e \pi_G] \\
\iff & c\left[\pi_G - \pi_B + \rho^e \pi_G\right] \leq (I - A)[\pi_G - \pi_B] - \pi_G(I - \pi_B R) \\
\iff & c \leq \frac{(I - A)(\pi_G - \pi_B) - \pi_G(I - \pi_B R)}{\pi_G - \pi_B + \rho^e \pi_G} =: \bar{C}_0.
\end{aligned}
$$

Note also that:

$$
\bar{C}_0 = \frac{(I - A)(\pi_G - \pi_B) - \pi_G(I - \pi_B R)}{(1 + \rho^e)\pi_G - \pi_B} = \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{(1 + \rho^e)\pi_G - \pi_B}
$$

and, therefore, $\bar{C}_0$ is always positive by Assumption (3).

Next, we want to check when $r_G(c) \geq 0$. This is true as long as:

$$
\begin{aligned}
& c[\pi_G - \pi_B + \rho^e - \gamma^e(1 - \pi_G)] + (I - \pi_B R) + \gamma^e (I - A) \geq 0 \\
\iff & [(1 + \gamma^e)\pi_G - \pi_B + (\rho^e - \gamma^e)]c \geq -[(I - \pi_B R) + \gamma^e (I - A)] \ .
\end{aligned}
$$

If $(1+\gamma^e)\pi_G - \pi_B + (\rho^e - \gamma^e) \geq 0$ this inequality is satisfied for any $c \geq 0$. Otherwise, if $(1 + \gamma^e)\pi_G - \pi_B + (\rho^e - \gamma^e) < 0$, we have the following upper bound for c:

$$
c \leq \frac{(I - \pi_B R) + \gamma^e (I - A)}{-[(1 + \gamma^e)\pi_G - \pi_B + (\rho^e - \gamma^e)]} =: \bar{C}_1 \ .
$$

Finally, $r_G(c) \leq R$ as long as:

$$
\begin{aligned}
& \frac{c[\pi_G - \pi_B + \rho^e - \gamma^e(1 - \pi_G)] + (I - \pi_B R) + \gamma^e (I - A)}{\pi_G - \pi_B + \gamma^e \pi_G} \leq R \\
\iff & c[(1 + \gamma^e)\pi_G - \pi_B + (\rho^e - \gamma^e)] + (I - \pi_B R) + \gamma^e (I - A) \leq [(1 + \gamma^e)\pi_G - \pi_B]R \\
\iff & c[(1 + \gamma^e)\pi_G - \pi_B + (\rho^e - \gamma^e)] \leq (1 + \gamma^e)(\pi_G R - I) + \gamma^e A \ .
\end{aligned}
$$

Now, if $(1+\gamma^e)\pi_G - \pi_B + (\rho^e - \gamma^e) \leq 0$ this inequality is satisfied for any $c \geq 0$. Otherwise, if $(1+\gamma^e)\pi_G - \pi_B + (\rho^e - \gamma^e) > 0$, we have a different upper bound for c:

$$c \leq \frac{(1+\gamma^e)(\pi_G R - I) + \gamma^e A}{(1+\gamma^e)\pi_G - \pi_B + (\rho^e - \gamma^e)} =: \bar{C}_2 \ .$$

We are now able to determine the optimal value of c. If $\rho^e > \gamma^e$ then it is, indeed, optimal to set $c = 0$ and, therefore, the solution would be:

$$\begin{aligned}
\hat{d}_G &= d_G(0) = \frac{\pi_G(I - \pi_B R) + \pi_G \gamma^e (I - A)}{\pi_G - \pi_B + \gamma^e \pi_G} \\
\hat{r}_G &= r_G(0) = \frac{(I - \pi_B R) + \gamma^e (I - A)}{\pi_G - \pi_B + \gamma^e \pi_G} \\
\hat{c}_G &= 0 \ .
\end{aligned}$$

On the other hand, suppose that $\rho^e < \gamma^e$. We need to determine what is the tighter upper bound $\bar{C}$ for c. We have three cases:

(i) if $\pi_G(1+\gamma^e) - \pi_B + (\rho^e - \gamma^e) = 0$, then both the $\bar{C}_1$ and $\bar{C}_2$ upper bounds are irrelevant and, therefore, the only upper bound for c is:

$$\bar{C} = \bar{C}_0 \ .$$

(ii) if $\pi_G(1+\gamma^e) - \pi_B + (\rho^e - \gamma^e) < 0$, the relevant upper bound for c is $\bar{C} = \min\{\bar{C}_0, \bar{C}_1\}$. Now, $\bar{C}_0 \leq \bar{C}_1$ if and only:

$$\frac{(I - A)(\pi_G - \pi_B) - \pi_G(I - \pi_B R)}{(1+\rho^e)\pi_G - \pi_B} \leq \frac{(I - \pi_B R) + \gamma^e (I - A)}{-[(1+\gamma^e)\pi_G - \pi_B + (\rho^e - \gamma^e)]} \ .$$

Note that by Assumption 2 and since $\pi_G > 0$, the condition above holds if the following, more stringent inequality is satisfied:

$$\frac{(I - A)(\pi_G - \pi_B)}{(1+\rho^e)\pi_G - \pi_B} \leq \frac{\gamma^e (I - A)}{-[(1+\gamma^e)\pi_G - \pi_B + (\rho^e - \gamma^e)]}$$

Given the fact that $I > A$, this second inequality holds as logs as:

$$
\begin{aligned}
&-[(1+\gamma^e)\pi_G - \pi_B + (\rho^e - \gamma^e)](\pi_G - \pi_B) \leq \gamma^e[(1+\rho^e)\pi_G - \pi_B] \\
\iff &-[(1+\gamma^e)\pi_G - \pi_B + \rho^e](\pi_G - \pi_B) \leq \gamma^e \rho^e \pi_G
\end{aligned}
$$

and given the assumptions on the parameters, we can see that the latter inequality is always satisfied, since the LHS is always negative, while the RHS is always positive. It follows that the upper bound for c is $\bar{C} = \bar{C}_0$.

(iii) if $\pi_G(1+\gamma^e) - \pi_B + (\rho^e - \gamma^e) > 0$, the relevant upper bound for c is $\bar{C} = \min\{\bar{C}_0, \bar{C}_2\}$. In this case, we have that $\bar{C}_0 \leq \bar{C}_2$ as long as:

$$
\frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{(1+\rho^e)\pi_G - \pi_B} \leq \frac{(1+\gamma^e)(\pi_G R - I) + \gamma^e A}{(1+\gamma^e)\pi_G - \pi_B + (\rho^e - \gamma^e)}.
$$

Since $\gamma^e > 0$, $\pi_G > \pi_B$, and $A \geq 0$, this is true if the following more stringent condition is satisfied:

$$
\frac{\pi_B(\pi_G R - I)}{(1+\rho^e)\pi_G - \pi_B} \leq \frac{(1+\gamma^e)(\pi_G R - I)}{(1+\gamma^e)\pi_G - \pi_B + (\rho^e - \gamma^e)} \ .
$$

Given Assumption 2, this condition is satisfied as long as:

$$
\begin{aligned}
&\pi_B[(1+\gamma^e)\pi_G - \pi_B + (\rho^e - \gamma^e)] \leq (1+\gamma^e)[(1+\rho^e)\pi_G - \pi_B] \\
\iff &(\pi_B - 1)(1+\gamma^e)\pi_G - \pi_B^2 + \pi_B(\rho^e - \gamma^e) \leq (1+\gamma^e)\rho^e \pi_G - \pi_B - \gamma^e \pi_B \\
\iff &\underbrace{(\pi_B - 1)}_{<0}\underbrace{[(1+\gamma^e)\pi_G - \pi_B]}_{>0} + \underbrace{\rho^e}_{>0}\underbrace{(\pi_B - \pi_G)}_{<0} \leq \underbrace{\gamma^e \rho^e \pi_G}_{>0}
\end{aligned}
$$

and the latter inequality is always satisfied given the assumptions on the parameters, since the LHS is always negative while the RHS is always positive. It follows that also in this case the upper bound for c is $\bar{C} = \bar{C}_0$.

Hence, we can conclude that in any case the tighter, and therefore relevant, upper bound for c is always $\bar{C} = \bar{C}_0$. Then, it is optimal to set $c = \bar{C}_0$ and the solution

would, therefore, be equal to:

$$\hat{c}_G^e = \bar{C}_0 = \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{(1+\rho^e)\pi_G - \pi_B} > 0 \ ,$$

$$\begin{aligned}
\hat{d}_G^e = d_G(\bar{C}_0) &= \frac{\bar{C}_0[\pi_G - \pi_B + \rho^e \pi_G] + \pi_G(I - \pi_B R) + \pi_G \gamma^e (I - A)}{\pi_G - \pi_B + \gamma^e \pi_G} \\
&= \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A + \pi_G(I - \pi_B R) + \pi_G \gamma^e (I - A)}{\pi_G - \pi_B + \gamma^e \pi_G} \\
&= \frac{(\pi_G - \pi_B)(I - A) + \pi_G \gamma^e (I - A)}{\pi_G - \pi_B + \gamma^e \pi_G} \\
&= I - A \ ,
\end{aligned}$$

and, finally,

$$\begin{aligned}
\hat{r}_G^e = r_G(\bar{C}_0) &= \frac{\bar{C}_0[\pi_G - \pi_B + \rho^e - \gamma^e(1 - \pi_G)] + (I - \pi_B R) + \gamma^e (I - A)}{\pi_G - \pi_B + \gamma^e \pi_G} \\
&= \frac{\frac{(I-A)(\pi_G - \pi_B) - \pi_G(I - \pi_B R)}{(1+\rho^e)\pi_G - \pi_B}[\pi_G - \pi_B + \rho^e - \gamma^e(1 - \pi_G)] + (I - \pi_B R) + \gamma^e (I - A)}{\pi_G - \pi_B + \gamma^e \pi_G} \\
&= (I - A)\frac{(\pi_G - \pi_B)[(1+\gamma^e)\pi_G - \pi_B] + (\rho^e - \gamma^e)(\pi_G - \pi_B) + \gamma^e(\pi_G - \pi_B) + \rho^e \gamma^e \pi_G}{[(1+\rho^e)\pi_G - \pi_B][(1+\gamma^e)\pi_G - \pi_B]} \\
&\quad + (I - \pi_B R)\frac{-\pi_G[(1+\gamma^e)\pi_G - \pi_B] - \pi_G(\rho^e - \gamma^e) + (\pi_G - \pi_B) + \rho^e \pi_G}{[(1+\rho^e)\pi_G - \pi_B][(1+\gamma^e)\pi_G - \pi_B]} \\
&= \frac{(I - A)[(1+\gamma^e)\pi_G - \pi_B](\pi_G - \pi_B + \rho^e) + (I - \pi_B R)(-\pi_G + 1)[\pi_G - \pi_B + \pi_G \gamma^e]}{[(1+\rho^e)\pi_G - \pi_B][(1+\gamma^e)\pi_G - \pi_B]} \\
&= \frac{(1 - \pi_G)(I - \pi_B R) + (\pi_G - \pi_B + \rho^e)(I - A)}{(1+\rho^e)\pi_G - \pi_B} \ .
\end{aligned}$$

We still need to verify that the solution we have characterised is strictly individually rational for the good borrower. First of all, note that since by Assumption 2 either $\hat{r}^e q_G < R$ or $\hat{c}_G^e > 0$ (or both) and $\pi_G > \pi_B$ we have that:

$$\pi_G(R - \hat{r}_G^e) - (1 - \pi_G)\hat{c}_G^e > \pi_B(R - \hat{r}_G^e) - (1 - \pi_B)\hat{c}_G^e$$

and, therefore, given that at the solution IC_B is binding we conclude that:

$$\hat{\mathcal{U}}(G;G|p^e,\delta^e) > \hat{\mathcal{U}}(G;B|p^e,\delta^e) = \mathcal{U}(0;B|p^e,\delta^e) = (A+K_L) \ .$$

This implies that the solution to the program has, indeed, $\hat{y}^e_G = 1$. Moreover, the contract is always strictly individually rational for the good borrower whenever condition (2) is satisfied. Conversely, suppose condition (2) is not true, so that the good borrower outside option is given by:

$$\mathcal{U}(0;G|p^e,\delta^e) = (A+K_L) + (\pi_G R - I) - \gamma^e(I-A) \ .$$

Now, using the fact that IR_L is binding at the solution, the value of the contract can be rewritten as:

$$\begin{aligned}\hat{\mathcal{U}}(G;G|p^e,\delta^e) &= \pi_G R - \pi_G \hat{r}^e_G - (1-\pi_G)\hat{c}^e_G - \gamma^e(I-A-\hat{d}^e_G) - \rho^e \hat{c}_G - I + \hat{d}^e_G + (A+K_L) \\ &= (\pi_G R - I) - \gamma^e(I-A-\hat{d}^e_G) - \rho^e \hat{c}^e_G + (A+K_L) \ .\end{aligned}$$

Let us focus first on the case when $\gamma^e < \rho^e$. Recalling that in this case $\hat{c}^e_G = 0$, we have $\hat{\mathcal{U}}(G;G|p^e,\delta^e) > \mathcal{U}(0;G|p^e,\delta^e)$ as long as:

$$-\gamma^e(I-A-\hat{d}^e_G) > -\gamma^e(I-A)$$

or, in other words, as long as $\hat{d}^e_G > 0$. But this is always true by Assumption 2. Therefore, also in this case the contract does satisfy IR_G strictly.

Finally, consider the case when $\gamma^e > \rho^e$. In this case, $\hat{d}^e_G = I - A$, and therefore $\hat{\mathcal{U}}(G; G|p^e, \delta^e) > \mathcal{U}(0; G|p^e, \delta^e)$ as long as:

$$
\begin{aligned}
& -\rho^e \hat{c}^e_G > -\gamma^e (I - A) \\
\iff & \rho^e \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{(1+\rho^e)\pi_G - \pi_B} < \gamma^e (I - A) \\
\iff & \frac{\rho^e}{\gamma^e}[\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A] < [(1+\rho^e)\pi_G - \pi_B](I - A) \\
\iff & \left(\frac{\rho^e}{\gamma^e} - 1\right)[\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A] < \pi_G(I - \pi_B R) + \rho^e \pi_G (I - A)
\end{aligned}
$$

which is again always true given Assumption 3, so that also in this case the contract is strictly individually rational for borrower G.

Hence, since in any case the solution to the program does strictly better than the reservation expected utility and does satisfy IR_G strictly, we have that the solution to the program is, indeed, $(\hat{y}^e_G, \hat{d}^e_G, \hat{r}^e_G, \hat{c}^e_G)$.

It just remains to note that given the linearity of the program in each of the choice variables, for $\gamma^e = \rho^e$ any convex combination of the two solutions found above is another solution to the program. ■

Proof of Corollary 5: Fix (p^e, δ^e). First of all, we know that in the good borrower-optimal separating contract menu, $(\hat{y}^e_B, \hat{d}^e_B, \hat{r}^e_B, \hat{c}^e_B) = (0, 0, 0, 0)$. Moreover, it is never profitable for the bad borrower to finance the project by herself. Hence, in equilibrium we know that type B never enters the competitive market: $\hat{A}^e_B = 0$. On the other hand, consider the good borrower. Recall that by Theorem 4, $\hat{y}^e = 1$.

Then, if $\gamma^e < \rho^e$:

$$\begin{aligned}\hat{A}_G^e &= I - A - \hat{d}_G^e \\ &= \frac{[(1+\gamma^e)\pi_G - \pi_B](I-A) - \pi_G(I - \pi_B R) - \pi_G\gamma^e(I-A)}{(1+\gamma^e)\pi_G - \pi_B} \\ &= \frac{(\pi_G - \pi_B)(I-A) - \pi_G(I-\pi_B R)}{(1+\gamma^e)\pi_G - \pi_B} \\ &= \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{(1+\gamma^e)\pi_G - \pi_B} \ .\end{aligned}$$

If, instead, $\gamma^e > \rho^e$, we have:

$$\hat{A}_G^e = I - A - \hat{d}_G^e = I - A - (I - A) = 0 \ .$$

■

Proof of Theorem 6: First of all, we know that in the good borrower-optimal separating contract, $(\hat{y}_B^{cm}, \hat{d}_B^{cm}, \hat{r}_B^{cm}, \hat{c}_B^{cm}) = (0,0,0,0)$. Therefore, $\hat{C}_B^{cm} = \hat{c}_B^{cm} = 0$ by market clearing, so that $\hat{\nu}_B^{cm} = 0$. Moreover, by Corollary 5, $\hat{A}_B^{cm} = 0$.

Suppose $\hat{\gamma}^{cm} < \hat{\rho}^{cm}$. Then, by Theorem 4, the borrower G executing the good borrower-optimal separating contract has no collateral needs, $\hat{C}_G^{cm} = \hat{c}^{cm} = 0$, and does not enter the OTC collateral market. Then, $\hat{\nu}_G^{cm} = 0$, and, therefore, $\hat{\nu}_{dl}^{cm} = 0$. Then, the equilibrium haircut in the OTC market is $\hat{\delta}^{cm} = 1$. Moreover, by Corollary 5, we know that:

$$\hat{A}_G^{cm} = \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{(1+\gamma^e)\pi_G - \pi_B} \ .$$

Then, by Lemma 1, we can pin down the competitive market price:

$$\begin{aligned}\hat{p}^{cm} &= \left[1 + g'\left(\mu \hat{A}_G^{cm}\right)\right]^{-1} \\ &= \left[1 + g'\left(\mu \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{(1+\hat{\gamma}^{cm})\pi_G - \pi_B}\right)\right]^{-1} \\ &= \left[1 + g'\left(\mu \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{\frac{\pi_G}{\hat{p}^{cm}} - \pi_B}\right)\right]^{-1}\end{aligned}$$

Define:

$$\Psi(p) \coloneqq p - \left[1 + g'\left(\mu \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{\frac{\pi_G}{p} - \pi_B}\right)\right]^{-1}.$$

Note that since g is continuously differentiable, Ψ is continuous on $(0,1)$. Moreover:

$$\begin{aligned}\lim_{p\to 0} \Psi(p) &= -\frac{1}{1+g'(0)} < 0\ , \\ \lim_{p\to 1} \Psi(p) &= 1 - \frac{1}{1 + g'\left(\mu \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{\pi_G - \pi_B}\right)} > 0\end{aligned}$$

given that $g' > 0$ everywhere. By the mean-value theorem, we have proved the existence of $\hat{p}^{cm} \in (0,1)$ such that $\Psi(\hat{p}^{cm}) = 0$. Moreover, since g is convex, g' is increasing and, therefore, Ψ is strictly increasing. This shows that $\hat{p}^{cm}$ is also unique. It is also easy to see, since $g' > 0$, that $\hat{p}^{cm} > 0$. But, then, $\hat{p}^{cm} > 1 - \hat{\delta}^{cm} = 0$ and, therefore, the good borrower, indeed, does not enter the OTC market, but only in the competitive market.

Now, the equilibrium price in the competitive market $\hat{p}^{cm}$ is given by the unique solution to $\Psi(p) = 0$, $\hat{q}^{cm}$ is given by Lemma 1, and Theorem 4 gives the good borrower-optimal separating contract quantities.

■

Proof of Theorem 7: Again, we know that $(\hat{y}_B^{otc}, \hat{d}_B^{otc}, \hat{r}_B^{otc}, \hat{c}_B^{otc}) = (0,0,0,0)$ and, therefore, $\hat{A}_B^{otc} = 0$, $\hat{C}_B^{otc} = 0$, $\hat{\nu}_B^{otc} = 0$, and $\hat{S}_B^{otc} = 0$. Suppose that $\hat{\gamma}^{otc} > \hat{\rho}^{otc}$, so that borrower G has no cash needs $\hat{A}_G^{otc} = 0$ by Corollary 5, and she enters only the OTC

market to execute the good borrower-optimal separating contract. Then, by market clearing, we know that the equilibrium quantity in the centralized market must be $\hat{q}^{otc} = 0$ and, therefore, by Lemma 1, we know that the equilibrium competitive market price must be $\hat{p}^{otc} \geq \frac{1}{1+g'(0)}$. Moreover, by Theorem 4 and market clearing:

$$\hat{C}_G^{otc} = \hat{c}_G^{otc} = \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{(1+\hat{\rho})\pi_G - \pi_B} > 0$$

and, therefore, $\hat{\nu}_{dl}^{otc} = \hat{\nu}_G^{otc} = \mu$ and, thus, $\hat{\xi}_G^{otc} = 1$. Define $\hat{\mathcal{U}}^{otc}(0;G) \coloneqq \mathcal{U}(0;\theta|\hat{p}^{otc},\hat{\delta}^{otc})$. We have two cases.

Case (i): $\frac{1}{1+g'(0)} \leq \hat{p}^{otc} \leq \frac{I-A}{\pi_G R - A} \implies \hat{\gamma}^{otc} \geq \frac{\pi_G R - I}{I-A} \implies \hat{\mathcal{U}}^{otc}(0;G) = A + K_L$.

Then, equilibrium implies that

$$\begin{aligned}
\hat{S}_G^{otc} &= \hat{S}_G(\hat{p},\hat{\delta}) = \hat{\mathcal{V}}(G|\hat{p},\hat{\delta}) \\
&= \pi_G(R - \hat{r}_G) - (1-\pi_G)\hat{c}_G - A \\
&= \pi_G R - (I - A) - A = \pi_G R - I
\end{aligned}$$

where in the second to last equality we use the fact that IR_L must be binding. Then, $\hat{S}_G^{otc} > 0 = \hat{S}_B^{otc}$ and, therefore, the condition of Proposition 2 is satisfied and by Corollary 3, we have that the equilibrium haircut needs to satisfy:

$$\hat{\rho}^{otc} \coloneqq \frac{\hat{\delta}^{otc}}{1-\hat{\delta}^{otc}} = \alpha \frac{\hat{S}_G^{otc}}{\hat{C}_G^{otc}} = \alpha \frac{\pi_G R - I}{\hat{C}_G^{otc}}$$

and, substituting in the expression for $\hat{C}_G^{otc}$ at the equilibrium haircut, we obtain:

$$\hat{\rho}^{otc} \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{(1+\hat{\rho}^{otc})\pi_G - \pi_B} = \alpha(\pi_G R - I)$$

which we can solve for $\hat{\rho}^{otc}$ to obtain:

$$\hat{\rho}^{otc} = \frac{\alpha(\pi_G - \pi_B)(\pi_G R - I)}{(\pi_B - \alpha\pi_G)(\pi_G R - I) - (\pi_G - \pi_B)A}$$

which is always positive under the assumption that

$$\alpha < \bar{\alpha} < \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{\pi_G(\pi_G R - I)} \ .$$

Moreover, we also have that

$$\begin{aligned}
&\hat{\rho}^{otc} < \frac{\pi_G R - I}{I - A} \\
\iff &\frac{\alpha(\pi_G - \pi_B)(\pi_G R - I)}{(\pi_B - \alpha\pi_G)(\pi_G R - I) - (\pi_G - \pi_B)A} < \frac{\pi_G R - I}{I - A} \\
\iff &\alpha(\pi_G - \pi_B)(I - A) < (\pi_B - \alpha\pi_G)(\pi_G R - I) - (\pi_G - \pi_B)A \\
\iff &\left[\pi_G(\pi_G R - I) + (\pi_G - \pi_B)I - (\pi_G - \pi_B)A\right]\alpha < \pi_B(\pi_G R - I) - (\pi_G - \pi_B)A
\end{aligned}$$

which is true if $\alpha < \bar{\alpha}$. This means that for any $\frac{1}{1+g'(0)} \leq \hat{p}^{otc} \leq \frac{I-A}{\pi_G R - A}$, $\hat{\rho}^{otc} < \hat{\gamma}^{otc}$, and, therefore, the good borrower does, indeed, only enter the OTC market. Then,

$$\begin{aligned}
\hat{\delta}^{otc} &= \frac{\hat{\rho}^{otc}}{1 + \hat{\rho}^{otc}} \\
&= \frac{\alpha(\pi_G - \pi_B)(\pi_G R - I)}{(\pi_B - \alpha\pi_G)(\pi_G R - I) - (\pi_G - \pi_B)A + \alpha(\pi_G - \pi_B)(\pi_G R - I)} \\
&= \frac{\alpha(\pi_G - \pi_B)(\pi_G R - I)}{(1 - \alpha)\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}
\end{aligned}$$

which is in $(0, 1)$ if $\alpha \in (0, \bar{\alpha})$. Then, we can also compute the equilibrium quantity of pledged collateral:

$$\begin{aligned}
\hat{C}_G^{otc} &= \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{(\pi_G - \pi_B) + \pi_G \hat{\rho}^{otc}} \\
&= \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{(\pi_G - \pi_B) + \pi_G \frac{\alpha(\pi_G - \pi_B)(\pi_G R - I)}{(\pi_B - \alpha\pi_G)(\pi_G R - I) - (\pi_G - \pi_B)A}} \\
&= \frac{[\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A][(\pi_B - \alpha\pi_G)(\pi_G R - I) - (\pi_G - \pi_B)A]}{[(\pi_B - \alpha\pi_G)(\pi_G R - I) - (\pi_G - \pi_B)A](\pi_G - \pi_B) + \pi_G[\alpha(\pi_G - \pi_B)(\pi_G R - I)]} \\
&= \frac{(\pi_B - \alpha\pi_G)(\pi_G R - I) - (\pi_G - \pi_B)A}{\pi_G - \pi_B}
\end{aligned}$$

which is strictly positive whenever $\alpha \in (0, \bar{\alpha})$. Then, using the fact that IR_L must be binding in the equilibrium, we can finally solve for the optimal payment to the lender in the good borrower-optimal contract:

$$\begin{aligned}
\hat{r}_G^{otc} &= \frac{I - A - (1-\pi_G)\hat{C}_G^{otc}}{\pi_G} \\
&= \frac{(\pi_G - \pi_B)(I-A) - (1-\pi_G)[(\pi_B - \alpha\pi_G)(\pi_G R - I) - (\pi_G - \pi_B)A]}{\pi_G(\pi_G - \pi_B)} \\
&= \frac{[\pi_G - \alpha\pi_G - \pi_G\pi_B + \alpha\pi_G^2]I - (1-\pi_G)(\pi_B - \alpha\pi_G)\pi_G R - \pi_G(\pi_G - \pi_B)A}{\pi_G(\pi_G - \pi_B)} \\
&= \frac{(I - \pi_B R) + [\alpha(1-\pi_G) + \pi_B](\pi_G R - I) - (\pi_G - \pi_B)A}{\pi_G - \pi_B} > 0 \ .
\end{aligned}$$

Case (ii): $\hat{p}^{otc} > \max\left\{\frac{1}{1+g'(0)}, \frac{I-A}{\pi_G R - A}\right\} \implies \hat{\gamma}^{otc} < \frac{\pi_G R - I}{I - A} \implies \hat{\mathcal{U}}^{otc}(0; G) = (A + K_L) + (\pi_G R - I) - \hat{\gamma}^{otc}(I - A)$.

In this case, we have that the good borrower's surplus is given by:

$$\begin{aligned}
\hat{S}_G^{otc} &= \hat{S}_G(\hat{p}, \hat{\delta}) = \hat{\mathcal{V}}(G|\hat{p}, \hat{\delta}) - \Big[(\pi_G R - I) - \hat{\gamma}^{otc}(I - A)\Big] \\
&= \pi_G(R - \hat{r}_G) - (1-\pi_G)\hat{c}_G - A - \Big[(\pi_G R - I) - \hat{\gamma}^{otc}(I - A)\Big] \\
&= \pi_G R - (I - A) - A - \Big[(\pi_G R - I) - \hat{\gamma}^{otc}(I - A)\Big] = \hat{\gamma}^{otc}(I - A)
\end{aligned}$$

where again in the second to last equality we use the fact that IR_L must bind. Again, this means that $\hat{S}_G^{otc} > 0 = \hat{S}_B^{otc}$, so that the condition in Proposition 2 is satisfied and we can proceed as in the previous point:

$$\hat{\rho}^{otc} = \alpha \frac{\hat{\gamma}^{otc}(I - A)}{\hat{C}_G^{otc}}$$

which we can rewrite as:

$$\hat{\rho}^{otc} \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{(1 + \hat{\rho}^{otc})\pi_G - \pi_B} = \alpha \hat{\gamma}^{otc}(I - A)$$

and we can solve for $\hat{\rho}$ to obtain:

$$\hat{\rho}^{otc} = \frac{\alpha(\pi_G - \pi_B)\hat{\gamma}^{otc}(I - A)}{\pi_B(\pi_G R - I) - \alpha\pi_G\hat{\gamma}^{otc}(I - A) - (\pi_G - \pi_B)A}$$

and note that this is positive if $\alpha \in (0, \bar{\alpha})$ given that by the assumption of case (ii) we have for any such α:

$$\alpha\pi_G\hat{\gamma}^{otc}(I - A) < \alpha\pi_G(\pi_G R - I) < \pi_B(\pi_G R - I) - (\pi_G - \pi_B)A \ .$$

Moreover, $\hat{\rho}^{otc} < \hat{\gamma}^{otc}$ if and only if

$$\begin{aligned}
&\frac{\alpha(\pi_G - \pi_B)\hat{\gamma}^{otc}(I - A)}{\pi_B(\pi_G R - I) - \alpha\pi_G\hat{\gamma}^{otc}(I - A) - (\pi_G - \pi_B)A} < \hat{\gamma}^{otc} \\
\iff &\alpha(\pi_G - \pi_B)(I - A) < \pi_B(\pi_G R - I) - \alpha\pi_G\hat{\gamma}^{otc}(I - A) - (\pi_G - \pi_B)A \\
\iff &\alpha\left[\pi_G\hat{\gamma}^{otc}(I - A) + (\pi_G - \pi_B)(I - A)\right] < \pi_B(\pi_G R - I) - (\pi_G - \pi_B)A
\end{aligned}$$

Note that given the assumptions of this case:

$$\pi_G\hat{\gamma}^{otc}(I - A) + (\pi_G - \pi_B)(I - A) < \pi_G(\pi_G R - I) + (\pi_G - \pi_B)(I - A)$$

and, therefore, if $\alpha \in (0, \bar{\alpha})$ it is, indeed, true that $\hat{\rho}^{otc} < \hat{\gamma}^{otc}$ and, thus, that the good borrower only enters the OTC market. It just remains to compute the equilibrium haircut, the optimal payment, and the collateral pledged in the good borrower-optimal separating contract.

$$\begin{aligned}
\hat{\delta}^{otc} &= \frac{\hat{\rho}^{otc}}{1 + \hat{\rho}^{otc}} \\
&= \frac{\alpha(\pi_G - \pi_B)\hat{\gamma}^{otc}(I - A)}{\pi_B(\pi_G R - I) - \alpha\pi_G\hat{\gamma}^{otc}(I - A) - (\pi_G - \pi_B)A + \alpha(\pi_G - \pi_B)\hat{\gamma}^{otc}(I - A)} \\
&= \frac{\alpha(\pi_G - \pi_B)\hat{\gamma}^{otc}(I - A)}{\pi_B(\pi_G R - I) - \alpha\pi_B\hat{\gamma}^{otc}(I - A) - (\pi_G - \pi_B)A} \ .
\end{aligned}$$

Moreover, using the expressions from Theorem 4:

$$\begin{aligned}
\hat{C}_G^{otc} &= \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{(1+\hat{\rho}^{otc})\pi_G - \pi_B} \\
&= \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{(\pi_G - \pi_B) + \pi_G \frac{\alpha(\pi_G - \pi_B)\hat{\gamma}_L(I-A)}{\pi_B(\pi_G R - I) - \alpha\pi_G\hat{\gamma}^{otc}(I-A) - (\pi_G - \pi_B)A}} \\
&= \frac{[\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A][\pi_B(\pi_G R - I) - \alpha\pi_G\hat{\gamma}^{otc}(I-A) - (\pi_G - \pi_B)A]}{[\pi_B(\pi_G R - I) - \alpha\pi_G\hat{\gamma}^{otc}(I-A) - (\pi_G - \pi_B)A](\pi_G - \pi_B) + \pi_G[\alpha(\pi_G - \pi_B)\hat{\gamma}^{otc}(I-A)]} \\
&= \frac{[\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A][\pi_B(\pi_G R - I) - \alpha\pi_G\hat{\gamma}^{otc}(I-A) - (\pi_G - \pi_B)A]}{[\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A](\pi_G - \pi_B)} \\
&= \frac{\pi_B(\pi_G R - I) - \alpha\pi_G\hat{\gamma}^{otc}(I-A) - (\pi_G - \pi_B)A}{\pi_G - \pi_B} > 0
\end{aligned}$$

and using the fact that IR_L is binding in the optimal contract:

$$\begin{aligned}
\hat{r}_G &= \frac{I - A - (1-\pi_G)\hat{C}_G^{otc}}{\pi_G} \\
&= \frac{(\pi_G - \pi_B)(I-A) - (1-\pi_G)[\pi_B(\pi_G R - I) - \alpha\pi_G\hat{\gamma}^{otc}(I-A) - (\pi_G - \pi_B)A]}{\pi_G(\pi_G - \pi_B)} \\
&= \frac{(\pi_G - \pi_G\pi_B)I - (1-\pi_G)\pi_B\pi_G R + (1-\pi_G)\alpha\pi_G\hat{\gamma}^{otc}(I-A) - \pi_G(\pi_G - \pi_B)A}{\pi_G(\pi_G - \pi_B)} \\
&= \frac{(I - \pi_B R) + \pi_B(\pi_G R - I) + \alpha(1-\pi_G)\hat{\gamma}^{otc}(I-A) - (\pi_G - \pi_B)A}{\pi_G - \pi_B} > 0 \ .
\end{aligned}$$

■

Proof of Theorem 8: Suppose there is an equilibrium with secondary markets where each good borrower accesses both the centralized and the OTC markets. For this to be possible, it must be the case that the equilibrium price and haircut are such that $\hat{p} = 1 - \hat{\delta}$. By Theorem 1, any convex combination of the solutions in point *(i)* and *(ii)* is a good borrower-optimal separating contract. Denote by $\lambda \in (0,1)$ the weight assigned to the solution in point *(i)*. Then, we have that the optimal contract as a function of λ is given by:

$$\hat{y}_G = 1;$$

$$\hat{d}_G = \lambda \frac{\pi_G(I - \pi_B R) + \pi_G \gamma^e (I - A)}{(1+\gamma^e)\pi_G - \pi_B} + (1-\lambda)(I - A)$$
$$= (I - A) - \lambda \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{(1+\hat{\gamma}(\lambda))\pi_G - \pi_B}$$

$$\hat{r}_G = \lambda \frac{(I - \pi_B R) + \gamma^e (I - A)}{(1+\gamma^e)\pi_G - \pi_B} + (1-\lambda) \frac{(1-\pi_G)(I - \pi_B R) + (\pi_G - \pi_B + \rho^e)(I - A)}{(1+\rho^e)\pi_G - \pi_B}$$
$$= \frac{(I - \pi_B R) + \gamma (I - A) + (1-\lambda)[\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A]}{(1+\hat{\gamma}(\lambda))\pi_G - \pi_B}$$

$$\hat{c}_G = (1-\lambda) \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{(1+\hat{\gamma}(\lambda))\pi_G - \pi_B}$$

where we use the fact that in equilibrium it must be the case that $\hat{\rho} = \rho^e = \gamma^e = \hat{\gamma} \equiv \hat{\gamma}(\lambda)$, and the last equivalence is just to remark that the equilibrium value of $\hat{\gamma}$ is a function of λ.

Recall that the cash needs are given by $\hat{A}_B = 0$ and

$$\hat{A}_G = (I - A) - \hat{d}_G$$
$$= \lambda \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{(1+\hat{\gamma}(\lambda))\pi_G - \pi_B}.$$

Then, by Lemma 1, equilibrium in the competitive market implies:

$$\hat{\gamma}(\lambda) = g'(\mu \hat{A}_G)$$
$$= g' \left(\lambda \mu \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{(1+\hat{\gamma}(\lambda))\pi_G - \pi_B} \right).$$

Now, define

$$\Gamma(\gamma; \lambda) \coloneqq \gamma - g' \left(\lambda \mu \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{(1+\gamma)\pi_G - \pi_B} \right).$$

By an analogous proof to that of Theorem 6, we know that there is a unique solution $\hat{\gamma}(\lambda)$ to the equation $\Gamma(\gamma; \lambda) = 0$ in $\gamma > 0$ for each $\lambda \in (0, 1)$. Moreover, by the

implicit function theorem, we have that

$$\begin{aligned}\hat{\gamma}'(\lambda) &= -\frac{\frac{\partial}{\partial\lambda}\Gamma(\hat{\gamma}(\lambda);\lambda)}{\frac{\partial}{\partial\gamma}\Gamma(\hat{\gamma}(\lambda);\lambda)} \\ &= -\frac{-g''\left(\lambda\mu\frac{\pi_B(\pi_G R-I)-(\pi_G-\pi_B)A}{(1+\hat{\gamma}(\lambda))\pi_G-\pi_B}\right)\mu\frac{\pi_B(\pi_G R-I)-(\pi_G-\pi_B)A}{(1+\hat{\gamma}(\lambda))\pi_G-\pi_B}}{1-g''\left(\lambda\mu\frac{\pi_B(\pi_G R-I)-(\pi_G-\pi_B)A}{(1+\hat{\gamma}(\lambda))\pi_G-\pi_B}\right)\left(-\pi_G\frac{\lambda\mu[\pi_B(\pi_G R-I)-(\pi_G-\pi_B)A]}{[(1+\hat{\gamma}(\lambda))\pi_G-\pi_B]^2}\right)} \\ &= \frac{g''\left(\lambda\mu\frac{\pi_B(\pi_G R-I)-(\pi_G-\pi_B)A}{(1+\hat{\gamma}(\lambda))\pi_G-\pi_B}\right)\mu\frac{\pi_B(\pi_G R-I)-(\pi_G-\pi_B)A}{(1+\hat{\gamma}(\lambda))\pi_G-\pi_B}}{1+g''\left(\lambda\mu\frac{\pi_B(\pi_G R-I)-(\pi_G-\pi_B)A}{(1+\hat{\gamma}(\lambda))\pi_G-\pi_B}\right)\left(\pi_G\frac{\lambda\mu[\pi_B(\pi_G R-I)-(\pi_G-\pi_B)A]}{[(1+\hat{\gamma}(\lambda))\pi_G-\pi_B]^2}\right)} \geq 0\end{aligned}$$

which is positive given that g is convex.

On the other hand, using the fact that IR_L must be binding in equilibrium, the good borrower's surplus is:

$$\begin{aligned}\hat{S}(G;G|\hat{p},\hat{\delta}) &= [\pi_G(R-\hat{r}_G)+\pi_G\hat{c}_G-A-\hat{c}_G]-\frac{I-A-\hat{d}_G}{\hat{p}}+A+K_L-\mathcal{U}(0;G|\hat{p},\hat{\delta}) \\ &= \pi_G R-I-\hat{\gamma}(\lambda)(I-A-\hat{d}_G)-\max\{0,(\pi_G R-I)-\hat{\gamma}(\lambda)(I-A)\}.\end{aligned}$$

which then becomes:

$$\begin{aligned}\hat{S}(G;G|\hat{p},\hat{\delta}) &= \begin{cases}\hat{\gamma}(\lambda)(I-A)-\hat{\gamma}(\lambda)\lambda\frac{\pi_B(\pi_G R-I)-(\pi_G-\pi_B)A}{(1+\gamma)\pi_G-\pi_B} & \text{if } \hat{\gamma}(\lambda)\leq\frac{\pi_G R-I}{I-A} \\ (\pi_G R-I)-\hat{\gamma}(\lambda)\lambda\frac{\pi_B(\pi_G R-I)-(\pi_G-\pi_B)A}{(1+\gamma)\pi_G-\pi_B} & \text{if } \hat{\gamma}(\lambda)>\frac{\pi_G R-I}{I-A}\end{cases} \\ &= \begin{cases}=\hat{\gamma}(\lambda)(I-A)-\hat{\gamma}(\lambda)\frac{\lambda}{1-\lambda}\hat{c}_G & \text{if } \hat{\gamma}(\lambda)\leq\frac{\pi_G R-I}{I-A} \\ =(\pi_G R-I)-\hat{\gamma}(\lambda)\frac{\lambda}{1-\lambda}\hat{c}_G & \text{if } \hat{\gamma}(\lambda)>\frac{\pi_G R-I}{I-A}.\end{cases}\end{aligned}$$

Recall that the equilibrium haircut is given by the equation:

$$\hat{\rho}\hat{c}_G = \alpha\hat{S}(G;G|\hat{p},\hat{\delta}).$$

Since $\hat{c}_G > 0$ for $\lambda \in (0,1)$, we can divide across to obtain that

$$\begin{aligned}
\hat{\rho}(\lambda) &= \alpha \frac{\hat{S}(G;G|\hat{p},\hat{\delta})}{\hat{c}_G} \\
&= \alpha \times \begin{cases} \frac{\hat{\gamma}(\lambda)(I-A)}{\hat{c}_G} - \hat{\gamma}(\lambda)\frac{\lambda}{1-\lambda} & \text{if } \hat{\gamma}(\lambda) \leq \frac{\pi_G R - I}{I-A} \\ \frac{\pi_G R - I}{\hat{c}_G} - \hat{\gamma}(\lambda)\frac{\lambda}{1-\lambda} & \text{if } \hat{\gamma}(\lambda) > \frac{\pi_G R - I}{I-A} \end{cases} \\
&= \alpha \times \begin{cases} \frac{\hat{\gamma}(\lambda)(I-A)}{(1-\lambda)\frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{(1+\hat{\gamma}(\lambda))\pi_G - \pi_B}} - \hat{\gamma}(\lambda)\frac{\lambda}{1-\lambda} & \text{if } \hat{\gamma}(\lambda) \leq \frac{\pi_G R - I}{I-A} \\ \frac{\pi_G R - I}{(1-\lambda)\frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{(1+\hat{\gamma}(\lambda))\pi_G - \pi_B}} - \hat{\gamma}(\lambda)\frac{\lambda}{1-\lambda} & \text{if } \hat{\gamma}(\lambda) > \frac{\pi_G R - I}{I-A} \end{cases} \\
&= \hat{\gamma}(\lambda) \times \begin{cases} \alpha \frac{(1+\hat{\gamma}(\lambda))\pi_G - \pi_B}{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}\frac{I-A}{1-\lambda} - \alpha\frac{\lambda}{1-\lambda} & \text{if } \hat{\gamma}(\lambda) \leq \frac{\pi_G R - I}{I-A} \\ \alpha \frac{(1+\hat{\gamma}(\lambda))\pi_G - \pi_B}{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}\frac{\pi_G R - I}{\hat{\gamma}(\lambda)(1-\lambda)} - \alpha\frac{\lambda}{1-\lambda} & \text{if } \hat{\gamma}(\lambda) > \frac{\pi_G R - I}{I-A} \end{cases}
\end{aligned}$$

Then, existence of mixed equilibria boils down to finding $\lambda \in (0,1)$ such that $\rho(\lambda) = \gamma(\lambda)$. Let $\hat{\gamma}(1^-) \coloneqq \lim_{\lambda \uparrow 1} \hat{\gamma}(\lambda) = \hat{\gamma}^{cm}$ and $\hat{\gamma}(0^+) \coloneqq \lim_{\lambda \downarrow 0} \hat{\gamma}(\lambda) = g'(0)$.

Case 1. $\hat{\gamma}(1^-) \leq \frac{\pi_G R - I}{I-A}$.

In this case, $\hat{\gamma}(\lambda) \leq \frac{\pi_G R - I}{I-A}$ for all $\lambda \in (0,1)$. Then, from the previous expression for ρ, we obtain that

$$\hat{\rho}(\lambda) = \hat{\gamma}(\lambda) \iff \alpha \frac{(1+\hat{\gamma}(\lambda))\pi_G - \pi_B}{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}\frac{I-A}{1-\lambda} - \alpha\frac{\lambda}{1-\lambda} = 1.$$

This holds if and only if

$$\frac{(1+\hat{\gamma}(\lambda))\pi_G - \pi_B}{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}(I-A) + \frac{1-\alpha}{\alpha}\lambda = \frac{1}{\alpha}.$$

Define for all $\lambda \in (0,1)$,

$$F(\lambda) \coloneqq \frac{(1+\hat{\gamma}(\lambda))\pi_G - \pi_B}{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}(I-A) + \frac{1-\alpha}{\alpha}\lambda$$

and note that F is differentiable with derivative

$$F'(\lambda) = \frac{\pi_G(I-A)\hat{\gamma}'(\lambda)}{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A} + \frac{1-\alpha}{\alpha} > 0$$

for all $\lambda \in (0,1)$. Moreover,

$$F(0^+) \coloneqq \lim_{\lambda \downarrow 0} F(\lambda) = \frac{(1+g'(0))\pi_G - \pi_B}{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}(I-A)$$
$$F(1^-) \coloneqq \lim_{\lambda \uparrow 1} F(\lambda) = \frac{(1+\hat{\gamma}^{cm})\pi_G - \pi_B}{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}(I-A) + \frac{1-\alpha}{\alpha}$$

Given that F is monotonically strictly increasing and continuous, an application of the intermediate value theorem implies that $\lambda \in (0,1)$ solving the equation $F(\lambda) = 1$ exists if and only if $F(0^+) < \frac{1}{\alpha} < F(1^-)$. Moreover, it is unique if it exists. Note that it is always true that $\frac{1}{\alpha} < F(1^-)$, while $F(0^+) < \frac{1}{\alpha}$ if and only if

$$\alpha < \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{\pi_G g'(0)(I-A) + (\pi_G - \pi_B)(I-A)} \coloneqq \tilde{\alpha}_1.$$

Thus, if $\alpha \in (0, \tilde{\alpha}_1)$, there is a unique $\hat{\lambda} \in (0,1)$ such that $F(\hat{\lambda}) = \frac{1}{\alpha}$.

Case 2. $\hat{\gamma}(0^+) \geq \frac{\pi_G R - I}{I - A}$.

In this case, $\hat{\gamma}(\lambda) \geq \frac{\pi_G R - I}{I - A}$ for all $\lambda \in (0,1)$. Then, from the previous expression for $\hat{\rho}$, we obtain that

$$\hat{\rho}(\lambda) = \hat{\gamma}(\lambda) \iff \alpha \frac{(1+\hat{\gamma}(\lambda))\pi_G - \pi_B}{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}\left(\frac{\pi_G R - I}{\hat{\gamma}(\lambda)(1-\lambda)}\right) - \alpha\frac{\lambda}{1-\lambda} = 1.$$

This holds if and only if

$$\frac{(1+\hat{\gamma}(\lambda))\pi_G - \pi_B}{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}\frac{\pi_G R - I}{\hat{\gamma}(\lambda)} + \frac{1-\alpha}{\alpha}\lambda = \frac{1}{\alpha}.$$

Define for all $\lambda \in (0,1)$,

$$H(\lambda) \coloneqq \frac{\frac{(\pi_G - \pi_B)}{\hat{\gamma}(\lambda)} + \pi_G}{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}(\pi_G R - I) + \frac{1-\alpha}{\alpha}\lambda$$

and note that H is differentiable with derivative

$$H'(\lambda) = -\frac{(\pi_G - \pi_B)(\pi_G R - I)}{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A} \times \frac{\hat{\gamma}'(\lambda)}{\hat{\gamma}(\lambda)^2} + \frac{1-\alpha}{\alpha}$$

for all $\lambda \in (0,1)$ and note that $H'(\lambda) > 0$ if and only if

$$\begin{aligned}\frac{1}{\alpha} &> 1 + \frac{(\pi_G - \pi_B)\frac{\pi_G R - I}{\hat{\gamma}(\lambda)}}{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A} \times \frac{\hat{\gamma}'(\lambda)}{\hat{\gamma}(\lambda)} \\ &= \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A + \frac{\hat{\gamma}'(\lambda)}{\hat{\gamma}(\lambda)}\left[(\pi_G - \pi_B)\frac{\pi_G R - I}{\hat{\gamma}(\lambda)}\right]}{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}\end{aligned}$$

and therefore $H'(\lambda) > 0$ if and only if

$$\alpha < \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A + \frac{\hat{\gamma}'(\lambda)}{\hat{\gamma}(\lambda)}\left[(\pi_G - \pi_B)\frac{\pi_G R - I}{\hat{\gamma}(\lambda)}\right]} \coloneqq \alpha^*(\lambda).$$

Moreover,

$$\begin{aligned}H(0^+) &\coloneqq \lim_{\lambda \downarrow 0} H(\lambda) = \frac{\frac{(\pi_G - \pi_B)}{g'(0)} + \pi_G}{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}(\pi_G R - I) \\ H(1^-) &\coloneqq \lim_{\lambda \uparrow 1} H(\lambda) = \frac{\frac{(\pi_G - \pi_B)}{\hat{\gamma}^{cm}} + \pi_G}{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}(\pi_G R - I) + \frac{1-\alpha}{\alpha}\end{aligned}$$

First of all, note that it is always true that $H(1^+) > \frac{1}{\alpha}$. Moreover, we also have that $H(0^+) < \frac{1}{\alpha}$ if and only if

$$\alpha < \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{\pi_G(\pi_G R - I) + (\pi_G - \pi_B)\frac{\pi_G R - I}{g'(0)}} =: \tilde{\alpha}_2$$

Observe that both $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ are larger than $\bar{\alpha}$ under the respective cases.

Case 3. There exists $\lambda^\dagger \in (0,1)$ such that $\hat{\gamma}(\lambda^\dagger) = \frac{\pi_G R - I}{I - A}$.

Then, since $\hat{\gamma}(\lambda)$ is monotonically increasing, $\hat{\gamma}(\lambda) \leq \frac{\pi_G R - I}{I - A}$ if $0 < \lambda < \lambda^\dagger$ and $\hat{\gamma}(\lambda) \geq \frac{\pi_G R - I}{I - A}$ if $\lambda^\dagger < \lambda < 1$. Now, define for all $\lambda \in (0,1)$,

$$J(\lambda) = \begin{cases} \frac{(1+\hat{\gamma}(\lambda))\pi_G - \pi_B}{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}(I - A) + \frac{1-\alpha}{\alpha}\lambda & \text{if } \lambda \leq \lambda^\dagger \\ \frac{(1+\hat{\gamma}(\lambda))\pi_G + \pi_B}{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}\frac{\pi_G R - I}{\hat{\gamma}(\lambda)} + \frac{1-\alpha}{\alpha}\lambda & \text{if } \lambda > \lambda^\dagger \end{cases}$$

and note that we have $\hat{\rho}(\lambda) = \hat{\gamma}(\lambda)$ if and only if $J(\lambda) = \frac{1}{\alpha}$. Moreover, J is continuous for $\lambda \in (0,1)$ and

$$\forall \lambda \in (0, \lambda^\dagger), \quad H'(\lambda) = \frac{\pi_G(I - A)\hat{\gamma}'(\lambda)}{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A} + \frac{1-\alpha}{\alpha}$$
$$\forall \lambda \in (\lambda^\dagger, 1), \quad H'(\lambda) = -\frac{(\pi_G - \pi_B)(\pi_G R - I)}{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A} \times \frac{\hat{\gamma}'(\lambda)}{\hat{\gamma}(\lambda)^2} + \frac{1-\alpha}{\alpha}$$

Moreover,

$$J(0^+) \coloneqq \lim_{\lambda \downarrow 0} J(\lambda) = \frac{(1 + g'(0))\pi_G - \pi_B}{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}(I - A)$$
$$J(\lambda^\dagger) = \frac{(1 + \hat{\gamma}(\lambda^*))\pi_G - \pi_B}{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}(I - A) + \frac{1-\alpha}{\alpha}\lambda^\dagger$$
$$J(1^-) \coloneqq \lim_{\lambda \uparrow 1} J(\lambda) = \frac{\frac{(\pi_G - \pi_B)}{\hat{\gamma}^{cm}} + \pi_G}{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}(\pi_G R - I) + \frac{1-\alpha}{\alpha}$$

Note that $J(0^+) < \frac{1}{\alpha}$ if and only if $\alpha < \tilde{\alpha}_1$, while it is always true that $J(1^-) > \frac{1}{\alpha}$. Since J is continuous, an application of the intermediate value theorem tells us that if $\alpha < \tilde{\alpha}_1$, then there exists $\lambda^* \in (0,1)$ such that $J(\lambda^*) = \frac{1}{\alpha}$. ■

Proof of Proposition 9: Denote $\hat{V}_G^{cm} \coloneqq \hat{\mathcal{U}}(G; G|\hat{p}^{cm}, \hat{\delta}^{cm})$ and $\hat{V}_G^{otc} \coloneqq \hat{\mathcal{U}}(G; G|\hat{p}^{otc}, \hat{\delta}^{otc})$. Suppose that $\alpha \in (0, \bar{\alpha})$ so that there exist both the equilibrium where the good borrower only enters the centralized market and the equilibria where she enters only the OTC market. First of all, notice that the equilibrium expected payoff of the good

borrower in the equilibrium where she only enters the centralized market is given by:

$$\begin{aligned}\hat{V}_G^{cm} &= \pi_G(R - \hat{r}_G^{cm}) - \hat{\gamma}^{cm}(I - A - \hat{d}_G^{cm}) - (I - \hat{d}_G^{cm}) + A + K_L \\ &= \pi_G R - \hat{\gamma}^{cm}(I - A - \hat{d}_G^{cm}) - I + A + K_L\end{aligned}$$

where in the second equality we use the fact that in equilibrium, IR_L holds with equality and, therefore, $\pi_G \hat{r}_G^{cm} = \hat{d}_G^{cm}$. On the other hand, the expected equilibrium payoff of the good borrower in the equilibrium where she only enters the OTC market:

$$\begin{aligned}\hat{V}_G^{otc} &= \pi_G(R - \hat{r}_G^{otc}) - (1 - \pi_G)\hat{c}_G^{oct} - \hat{\rho}^{otc}\ \hat{c}_G^{otc} - (I - \hat{d}_G^{otc}) + A + K_L \\ &= \pi_G R - \hat{\rho}^{otc}\ \hat{c}_G^{otc} - I + A + K_L\end{aligned}$$

where, again, in the second equality we use the fact that in equilibrium IR_L must bind and, therefore, $\pi_G \hat{r}_G^{otc} + (1 - \pi_G)\hat{c}_G^{otc} = \hat{d}_G^{otc}$. Therefore,

$$\hat{V}_G^{cm} < V_G^{otc} \iff \hat{\rho}^{otc}\ \hat{c}_G^{otc} < \hat{\gamma}^{cm}(I - A - \hat{d}_G^{cm}).$$

Using the expression for the equilibrium cash needs from Corollary 5 and collateral needs, we see that the above expression holds if and only if

$$\begin{aligned}&\hat{\rho}^{otc}\frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{(1 + \hat{\rho}^{otc})\pi_G - \pi_B} < \hat{\gamma}^{cm}\frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{(1 + \hat{\gamma}^{cm})\pi_G - \pi_B} \\ \iff &\hat{\rho}^{otc}[(1 + \hat{\gamma}^{cm})\pi_G - \pi_B] < \hat{\gamma}^{cm}[(1 + \hat{\rho}^{otc})\pi_G - \pi_B] \\ \iff &\hat{\rho}^{otc}[\pi_G - \pi_B] < \hat{\gamma}^{cm}[\pi_G - \pi_B]\end{aligned}$$

and since $\pi_G > \pi_B$, we can conclude that $\hat{V}_G^{cm} < V_G^{otc}$ if and only if $\hat{\rho}^{otc} < \hat{\gamma}^{cm}$. Now, we know that in the OTC-only equilibrium, it must be the case that $\hat{p}^{otc} \geq \frac{1}{1+g'(0)}$ in order to ensure market clearing in the centralized market. This implies that $\hat{\gamma}^{otc} \leq g'(0)$. Moreover, in the equilibrium where the good borrower enters only the OTC market, it must be the case that $\hat{\rho}^{otc} < \hat{\gamma}^{otc}$. Then, since g is assumed to be convex and,

therefore, g' is increasing, we can conclude that

$$\hat{\rho}^{otc} < \hat{\gamma}^{otc} \leq g'(0) \leq g'\left(\mu \frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{\frac{\pi_G}{\hat{p}^{cm}} - \pi_B}\right) = \hat{\gamma}^{cm}.$$

Therefore, any OTC-only equilibrium, no matter what $\hat{p}^{otc}$ we choose, always provides the good borrower with a higher equilibrium expected payoff than the equilibrium where she only enters the centralized market.

Consider, on the other hand an equilibrium $\lambda^* \in (0,1)$ where both secondary markets are active. Clearly, in this case we need to have $\hat{p}(\lambda^*) = 1 - \hat{\delta}(\lambda^*)$. In this case, the expected payoff of the good borrower is given by

$$\begin{aligned}
\hat{V}_G^{\lambda^*} &= \pi_G(R - \hat{r}_G^{\lambda^*}) - (1-\pi_G)\hat{c}_G^{\lambda^*} - \hat{\rho}(\lambda^*)\hat{c}_G^{\lambda^*} - \hat{\gamma}(\lambda^*)(I - A - \hat{d}_G^{\lambda^*}) - (I - \hat{d}_G^{\lambda^*}) + A + K_L \\
&= \pi_G R - \hat{\rho}(\lambda^*)\hat{c}_G^{\lambda^*} - \hat{\gamma}(\lambda^*)(I - A - \hat{d}_G^{\lambda^*}) - I + A + K_L \\
&= \pi_G R - \hat{\gamma}(\lambda^*)\frac{\pi_B(\pi_G R - I) - (\pi_G - \pi_B)A}{(1 + \hat{\gamma}(\lambda^*)))\pi_G - \pi_B} - I + A + K_L.
\end{aligned}$$

Then, since

$$\hat{\rho}^{otc} \leq g'(0) = \hat{\gamma}(0) \leq \hat{\gamma}(\lambda^*) \leq \hat{\gamma}(1) = \hat{\gamma}^{cm}$$

we can conclude that $\hat{V}_G^{cm} \leq \hat{V}_G^{\lambda^*} < V_G^{otc}$.

■

Appendix B. A Detour into Asymmetric Bargaining

Definition B.1: An asymmetric bargaining problem is a profile $(\mathcal{S}, \alpha, (d_1, d_2))$ where

- $\mathcal{S} \subseteq \mathbb{R}$ is a compact, convex set of allocations;
- $\alpha, (1-\alpha) \in (0,1)$ are the bargaining power of, respectively, agent 1 and 2;
- $(d_1, d_2) \in \mathcal{S}$ are the disagreement points of, respectively, agent 1 and 2.

The *asymmetric Nash bargaining solution* to the asymmetric bargaining problem $(\mathcal{S}, \alpha, (d_1, d_2))$ is given by:

$$\arg \max_{(x_1,x_2)\in\mathcal{S}} \left\{(x_1 - d_1)^{\alpha}(x_2 - d_2)^{1-\alpha} \text{ s.t. } x_1 \geq d_1,\, x_2 \geq d_2\right\} \tag{4}$$

LEMMA B.1: *Given an asymmetric bargaining problem* $(\mathcal{S}, \alpha, (d_1, d_2))$ *with* $\mathcal{S} = \{x \in \mathbb{R}^2_+ : x_1 + x_2 \leq S\}$ *for some* $S > 0$*, the shares* (x_1^*, x_2^*) *to players 1 and 2 according to the asymmetric Nash Bargaining solution are:*

$$\begin{aligned} x_1^* &= \alpha(S - d_2) + (1 - \alpha)d_1 \\ x_2^* &= (1 - \alpha)(S - d_1) + \alpha d_2 \ . \end{aligned} \tag{5}$$

PROOF: We may as well solve the problem:

$$\max_{(x_1,x_2)\in\mathbb{R}^2_+} \{\alpha \ln(x_1 - d_1) + (1 - \alpha)\ln(x_2 - d_2) \text{ s.t. } x_1 \geq d_1,\, x_2 \geq d_2, \text{ and } x_1 + x_2 \leq S\}$$

Notice that since the objective is strictly increasing in x_1 and x_2, at the optimum it must be true that $x_1 + x_2 = S$. Then, substituting inside the objective the latter constraint, we can rewrite:

$$\max_{x_1\in\mathbb{R}_+} \{\alpha \ln(x_1 - d_1) + (1 - \alpha)\ln(S - x_1 - d_2) \text{ s.t. } d_1 \leq s_1 \leq S - d_2\} \ .$$

Clearly, the solution must be interior and, therefore, must satisfy the FOC:

$$\frac{\alpha}{x_1 - d_1} = \frac{1 - \alpha}{S - x_1 - d_2}$$

which are also sufficient by concavity of the objective function in x_1. From the previous expression, we can solve for x_1 to obtain that $x_1^* = \alpha(S - d_2) + (1 - \alpha)d_1$ and then, substituting in the constraint $S = x_1 + x_2$, we obtain $x^* = \alpha d_2 + (1 - \alpha)(S - d_1)$ as we wanted to show. ■