Quants, Strategic Speculation, and Financial Market Quality

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Abstract

We study the effects of quantitative investing, an increasingly popular investment style, on financial market quality. Within a model of strategic speculation with two informed traders, we define discretionary investing as fully strategic and quantitative investing as partially or fully myopic via its reliance on a backtested trading strategy. Growth in quantitative investing is modeled as the introduction of and greater backtest adherence by a quantitative investor. The former generally benefits financial market quality, while the latter worsens (improves) it when leading to less (more) aggressive trading than discretion — especially when endowed with less (more) precise information.

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1 Introduction

Over the last few decades, quantitative investing has become increasingly popular in the financial markets. More generally, quantitative investing takes many forms, from high frequency algorithmic market-making to fundamentally-oriented quantitative strategies. Of particular note is the rapid growth of quantitative ("quant") investing within the active equity investment management sector. While such strategies were previously available primarily through hedge funds to select clients, they are now gaining a wider prominence and are being offered through traditional mutual funds and ETFs to ordinary investors (see Abis (2022) and Beggs et al. (2023) for further evidence).[1] Furthermore, greater data availability and higher computer processing capacity are leading to an institutional push to incorporate systematic approaches within the more traditional discretionary strategies.[2] Despite its growing prevalence, the effects of quant investing on the financial markets are ex-ante unclear. Its strategies are generally based on quantitatively-disciplined trading rules, which results in both benefits, via greater trading discipline and breadth of coverage, and pitfalls, via strategy-crowding and myopia.[3] The goal of this study is to develop a theoretical framework for exploring the strategic interaction between humans and machines in the financial markets and the implications of greater fund automation for their quality.

Many fundamentally-oriented quantitative investment strategies are based on backtests. Backtesting conventionally refers to the development and validation of any trading strategy — from those based on technical and regression analysis to others based on the use

[1] Additionally, the interaction between discretionary and quantitative investing in delegated portfolio management is studied in Harvey et al. (2017) and Grobys et al. (2022) for hedge funds and in Evans et al. (2022) for separately managed accounts.

[2] See, e.g., Michael Lin, "Quantitative vs. Fundamental Equity Investing: Comparing and Contrasting." State Street Global Advisors, October 2019; "Quant Investing — Bridging the Divide." Morgan Stanley Research, October 2017; "Quantamental Investing: The Future is Now." Morgan Stanley Investment Management, May 2018.

[3] Liaudinskas (2023) proposes a form of discipline by algorithmic traders that stems from their insensitivity to the disposition effect. A classic example of quant strategy crowding and myopia is the "quant quake" of August 2007. For additional background see Khandani and Lo (2007) and "Goldman Sachs' lessons from the 'quant quake'." The Financial Times, March 8, 2017.

of large amounts of information ("Big Data") and sophisticated artificial intelligence and machine learning techniques — with historical data realized prior to its actual implementation in the marketplace (e.g., Becker and Reinganum (2017); Huang et al. (2023)). Thus, a defining characteristic of backtested strategies is their reliance on historical data to identify a signal which is predictive of future risk-adjusted returns. The investor then relies on the estimation to translate a real-time signal into a trading rule. An attractive aspect of this approach is its inherent discipline: the trading rule is not as susceptible to human emotion due to its automated nature. However, this greater discipline comes at a cost. Precisely because the trading strategy relies on historical data, it cannot take into account the effect that the implementation of the strategy will have on other market participants. For example, if other fundamental investors know that the machines will be receiving an inaccurate signal, they may trade against them to take advantage of the expected mispricing.[4] It is this inherent myopia of backtested strategies that we focus on as the primary differentiator between quantitative and discretionary investing.

In principle, the effects of quantitative investing on financial market quality are twofold. On the one hand, incremental informed speculators are entering the market. On the other hand, those speculators adhere to quantitative strategies, which may be myopic in nature. We build on the canonical strategic speculator framework of Kyle (1985) to incorporate the two dimensions. First, we use Kyle's economy with a single imperfectly informed speculator, as the benchmark, i.e., before quantitative investing (Economy 1). In this baseline setting, we label the single speculator as the discretionary investor ("DI"), as his fully-strategic trading strategy is meant to resemble the trading strategies of the more traditional fundamental equity investment funds. Economy 1 also serves as the backtesting environment for the quantitative investor ("QI"). She uses the data generating process from Economy 1 to build a profit-maximizing strategy, "the backtest," under the assumption that

[4]See, e.g., Jason Zweig, "The Stock Got Crushed. Then the ETFs Had to Sell." The Wall Street Journal, 31 January, 2020.

the other market participants are not aware of her existence. This is the key assumption of the model, as it captures the notion of myopia inherent in quantitative strategies: by definition, the backtest ignores the strategic response of the discretionary investor and the market maker to the trading strategy of the quant.

Next, we introduce a second differentially informed speculator, the QI, to the market. If the QI is assumed to be fully strategic (as the DI), the resulting economy (Economy 2) captures the effects of incremental informed speculation on the market akin to Foster and Viswanathan (1996). Finally, we introduce quantitative investing by allowing the QI to trade based on either a convex combination of the profit-maximizing strategy of a fully strategic quant in Economy 2 and the backtest (Economy 3) or full automation (i.e., exclusively the backtest; Economy 4). Such a modeling device has been extensively used in the literature on rational deviations from optimal speculation to capture central bank interventions in various financial markets (e.g., Bhattacharya and Weller (1997); Vitale (1999); Pedersen (2018); Pasquariello et al. (2020)) or speculative short-termism (e.g., Pasquariello and Vega (2009); Bhattacharyya and Nanda (2013); Pasquariello and Wang (2023)), among others. In our model, this stylized representation is meant to parsimoniously capture in reduced form unmodeled pecuniary and non-pecuniary benefits of committing, in full or in part, to quantitative trading strategies (e.g., greater fund flows or lower agency costs). The effect of incremental informed speculation on market quality is identified as the change from Economy 1 to Economy 2, while the implications of quantitative investing are identified via changes from Economy 2 to Economy 3 or Economy 4.[5]

Our primary finding is that the direction of the effect of greater automation by the quantitative investor on financial market quality is determined by the sign of the wedge between the trading intensity of a fully-automated quant (Economy 4) and a fully-discretionary

[5]The comparison of the equilibrium properties of economies with and without certain salient features of the financial markets to tease out their effects on equilibrium aggregates of interest is common in the literature on delegated portfolio management; for recent examples, see Basak and Pavlova (2013), Cong and Xu (2016), and Bhattacharya and O'Hara (2018), among others.

quant (Economy 2). The sign of the wedge acts as a sufficient statistic for the net effect of the three primary parameters in the model: information precision of the DI, information precision of the QI, and the correlation of the errors of their signals. A positive wedge, i.e., when the automated quant trades more than the discretionary quant, generally results in positive effects on market quality. Greater automation leads to greater expected trading volume by the quant, generally acts as a deterrent from more trading by the discretionary investor (as he becomes concerned about excess information slippage), and results in a net increase in the expected trading volume for the speculative sector as a whole. Since this net increase is driven by the trading behavior of a QI who adheres less and less to a profit-maximizing strategy, profits for the speculator sector decrease, price informativeness and price volatility rise, market depth rises, and return volatility falls. However, a negative wedge between the trading intensity of a fully-automated and a fully-discretionary quant implies that the quant trades less aggressively with greater automation. Less aggressive trading creates opportunity for the discretionary investor to trade more aggressively at the expense of the quant. As such, the DI trades more with greater automation by the QI, makes greater profits, which more than offsets the declining profits for the quant, and ultimately results in a lower trading volume and greater profits for the speculative sector as a whole. Market quality suffers as market depth, price informativeness, and price volatility fall, while return volatility rises.

We are able to theoretically characterize the determinants of the sign of the wedge, which in turn directly maps to market quality outcomes. We first consider relatively uninformed discretionary investors trading against quants. In the case of disagreement (i.e., either uncorrelated or negatively-correlated signal errors), greater automation by the quant leads to improvements in market quality regardless of the quant's signal precision. With agreement (i.e., positively correlated signal errors), the effect of automation is ambiguous, although the probability of improvement in market quality is rising in the quant's signal precision. Quants with a significant information advantage trading against relatively uninformed discretionary investors leads to definitive improvements in market quality.

We next consider relatively informed discretionary investors trading against quants. With agreement, the quant's automation always hurts market quality since the DI is able to exploit the quant's myopia and his information to his advantage. In the case of disagreement, the effect of quantitative investing is variable. However, the likelihood of deterioration in market quality is rising in the quant's information quality. In particular, the trading activity of highly informed quants trading against highly informed discretionary investors unambiguously hurts market quality since, yet again, the DI exploits the quant's reliance on the backtest to his advantage.

Our theoretical predictions provide a novel, intriguing interpretation for several key findings in the extant empirical literature on quant vs. discretionary investing. For instance, Abis (2022) finds that discretionary investors display higher macro-timing and lower stock-picking ability than quants during recessions. To the extent that recessions coincide with high disagreement, this result is consistent with relatively informed DIs trading against relatively informed quants in the market aggregate and profiting at their expense, ultimately negatively affecting market quality, a common feature of recessions. For stock-picking, the results map to our model as relatively uninformed DIs trading against quants, which under the assumption of disagreement implies unambiguous (i.e., regardless of the quant's signal precision) improvements in market quality and lower profits for the discretionary investor.[6]

Beggs et al. (2023) focus specifically on the market quality effects of quantitative investing and suggest that fire sales by quant funds cause greater market instability than fire sales by discretionary investors. To the extent that fire sales are caused by poor fund performance, one may conjecture that such funds possess signals with low information precision. Within our model, market quality is more likely to deteriorate when discretionary funds trade against relatively uninformed quant funds as opposed to quants trading against rela-

[6]Relatedly, Dyer et al. (2023) demonstrate that quant funds underperform discretionary funds after the implementation of new accounting standards. The results can be mapped to our framework as a decline in quants' signal quality, which may increase the likelihood of a decline in market quality and quant performance. See Section 3.2.2 for details.

tively uninformed DIs. Therefore, the likelihood of market instability is greater for trading initiated by relatively uninformed quant funds, which are the funds more likely to experience fire sales. While the link between the present framework and the existing empirical studies is not exact, our model can serve as a guide for future studies exploring the effects of quantitative investing on financial market quality.

Of institutional relevance is the natural consequence of our model that quantitative strategies are more likely to be successful when their implementation positively affects market quality. Albeit stylized, backtesting in our model captures the dynamic notion of the quant's reliance on historical data to build a profit-maximizing strategy. Ultimately, any backtested strategy has a greater chance of success when its trading wedge is positive. This is more likely to occur in environments of speculative disagreement (e.g., turbulent economic conditions), improving information quality for the quant if trading against relatively uninformed traditional investors (e.g., technological advancements in opaque markets), and declining information quality for the quant if trading against relatively informed fundamental investors (e.g., data abundance in transparent markets). Hence, our model can be used as a guide for identifying those market conditions that are more conducive to quantitative strategies.

Overall our results contribute to an important theoretical literature on price formation in the financial markets. Holden and Subrahmanyam (1992) show how aggressive competition among perfectly informed speculators may lead to speedy incorporation of information into prices (the "rat race"). Foster and Viswanathan (1996) demonstrate that under imperfect information and signals with sufficiently low correlation, the "rat race" may occur only initially, while less aggressive trading, i.e., a "waiting game," may emerge in later rounds of trading. While our framework is static in nature, we capture both the "waiting game" and the "rat race" outcomes via the single-period trading intensity of the speculators in our game. Importantly, we allow for differential information quality, which interacts with signal correlation to dictate market quality outcomes.

Most literature on quantitative investing has focused on the effects of high frequency trading on market liquidity and price discovery. As pointed out by Kirilenko and Lo (2013) a key trade-off exists in greater automation. On the one hand, algorithmic trading lacks human emotion, potentially leading to greater market stability. On the other hand, quantitative investing may be more highly correlated between funds, resulting in greater market instability. The theoretical literature exploring the interaction of quant and discretionary investing is nascent. To our knowledge, only Abis (2022) and Dugast and Foucault (2023) explore the question at hand. The former develops a theoretical framework for exploring the trading behavior of quantitative versus discretionary funds. The author builds on the mutual fund attention allocation model of Kacperczyk et al. (2016) and introduces quantitative investors as those who have unlimited learning capacity only in idiosyncratic shocks.[7] The latter studies a single asset market, within which discretionary investors are assumed to have pre-determined information precision while quants are allowed to search for an optimal signal at a cost, in order to differentiate between lower information processing costs and greater data availability. Importantly, in both studies investors are competitive rather than strategic. We complement these approaches by allowing for strategic trading, a realistic feature of most financial markets, while abstracting from systematic vs. idiosyncratic learning capacity or more advanced information production.

Several studies, including Khandani and Lo (2007), Khandani and Lo (2011), and Beggs et al. (2023), have documented the fact that quantitative funds, whether hedge funds or mutual funds, tend to not consider the behavior of others, consistent with our model's predictions, resulting in violent reversals in periods of unwinding. Many have shown that

[7]Relatedly, Davis (2022) shows that the presence of atomistic quantitative learners, defined as price-taking investors trading on the basis of canonical stochastic discount factor (SDF) models, may lead to higher CAPM alphas (as proxies for mispricing) in counterfactual experiments based on popular factor models in the literature. Both Colliard et al. (2023) and Dou et al. (2023) study equilibrium collusion when, respectively, market-makers or informed speculators rely on machine-learning algorithms to set prices or trade. The former incorporate machine learning by market-makers within an otherwise standard model of trading with asymmetric information based on Glosten and Milgrom (1985), while the latter introduce AI-powered speculators to the Kyle (1985) framework.

algorithmic trading, in particular high frequency trading, leads to improving price efficiency and liquidity (e.g., Hendershott et al. (2011), Hendershott and Riordan (2013), Brogaard et al. (2014), Zhang (2018), and Chakrabarty et al. (2021)), although acquisition of new information may deteriorate (Weller (2018)).

2 Model

We model a three-date financial market with a single risky asset. Participants include informed speculators, noise traders, and a market maker. At time $t = 0$, informed speculators observe their signals with regards to the payoff of the risky asset. At time $t = 1$, speculators and noise traders submit their demands for the risky asset and market markers set prices according to the aggregate order flow. At time $t = 2$, the risky asset payoff is realized. Agents have rational expectations in that the informed speculators and the market maker are aware of the model parameters and each others' price setting and trading behavior respectively. We describe each feature of the model below.

2.1 Model Setup

Economies

There will be four market environments, or economies, indexed by $j \in \{1, 2, 3, 4\}$, which will differ based on the presence and trading strategy of the quantitative investor. The economies will be introduced in greater detail below.

Risky asset

The risky asset has an endogenously determined price P_j set by the market maker in the first period and an exogenous payoff v distributed according to $v \sim N(0, \tau_v^{-1})$, realized in the

second period.[8] The equilibrium price function is indexed by Economy j as it will depend on the presence and the trading strategy of the quantitative investor. The payoff distribution remains the same across all four economies.

Informed speculators

There are two informed speculators indexed by $i \in \{d, q\}$ with different trading behavior: a discretionary investor ($i = d$, "DI") and a quantitative investor ($i = q$, "QI"). Each informed speculator i receives a signal about the final payoff of the risky asset given by:

$$s_i = v + e_i, \tag{1}$$

where $e_i \sim N(0, \tau_{e_i}^{-1})$, $\sigma_{ve_i} = 0$, $\rho = \sigma_{e_d e_q}/(\sigma_{e_d}\sigma_{e_q})$, and $\phi_i = \tau_v^{-1}/(\tau_v^{-1} + \tau_{e_i}^{-1})$ is a measure of information quality.

Discretionary investor The discretionary investor is risk-neutral and behaves akin to the Kyle (1985) insider, choosing a share amount x_{dj} to maximize period two expected profit:

$$x_{dj} = \arg\max_{\tilde{x}_{dj}} \mathbb{E}[\tilde{x}_{dj}(v - P_j)|s_d]. \tag{2}$$

The investor is strategic in that he accounts for the expected effect of his order on the price set by the market maker as well as the information revealed by his signal about the expected trading behavior of the quantitative investor. Within our framework, discretionary behavior is modeled via an investor's ability to take into account the expected behavior of others in the market when developing a trading strategy. A quant, in contrast, either fully or partially lacks this ability.

[8] Mean 0 unconditional payoff expectation is without loss of generality. For any random variables x and y $\sigma_x^2 = \tau_x^{-1}$ refers to the variance of x, τ_x refers to the precision of x, andσ_{xy} refers to the covariance between x and y.

Quantitative investor The quantitative investor develops a profit-maximizing trading rule under the assumption that the discretionary investor and market maker are not aware of her existence. This approach originates from the notion that many quantitative strategies are based on backtested signals, which by definition take as given the behavior of market participants. In other words, when developing trading strategies based on historical market data, quant traders are not able to incorporate the effect that their *presence* may have had on the strategic trading behavior of other market participants present at the time.[9] Even if the effects of quantitative trading were present in historical data, any incremental trading due to new or modified strategies would not be captured in the backtest. In our stylized framework, the backtested strategy, labeled x_b and introduced in greater detail below, is derived from a world in which only the discretionary investor is present.

We model the potentially myopic behavior of the quantitative investor in a reduced form manner. Specifically, we assume that the quant's optimal asset demand x_{qj} is given by the solution to the following optimization:

$$x_{qj} = \arg\max_{\tilde{x}_{qj}} \mathbb{E}[\underbrace{-\gamma(\tilde{x}_{qj} - x_b)^2}_{\text{backtest}} + \underbrace{(1-\gamma)\tilde{x}_{qj}(v - P_j)}_{\text{strategic speculation}} | s_q], \tag{3}$$

where $0 \leq \gamma \leq 1$.[10]

Eq. (3) is based on the observation that a quant's welfare is at least partially determined by her adherence to a backtested, potentially well-known, trading approach (x_b). The degree of adherence, or "automation," is parametrized by γ. Such behavior may be explained by marketing reasons, and the related greater investing discipline and transparency. For example, quantitative fund strategies are less-likely to be affected by human emotion, and the associated discipline may appeal to investors thereby generating greater fund flows.

[9] This concept is known within the macroeconomics literature as the "Lucas Critique" (Lucas (1976))

[10] Our inference is unaffected by replacing Eq. (3) with a weighted average of the speculator's expected profits under the conventional (i.e., fully-strategic) pricing rule and those under the historical pricing rule (i.e., from the backtest); this additional analysis is available on request from the authors.

Furthermore, the adherence to a particular well-known strategy may reduce the moral hazard issues associated with delegated portfolio management.

It is important to note that human judgement continues to play an important role in quantitative strategies. From the choice of investment strategy to its real-world implementation, human decisions are at the core of launching and maintaining the operations of quant funds. Quants may very well be strategic in their trading via, e.g., "feeler orders" to test market liquidity, scaling and turning strategies on and off depending on market conditions, and attending industry conferences to interact with other quant fund managers to understand the behavior of others. We model this "discretionary" aspect of quantitative investing in a reduced form via the loading $1 - \gamma$ of the strategy on the traditional profit maximization objective of the Kyle insider.

An immediate consequence of Eq. (3) is that for $\gamma > 0$ the quant is not maximizing her profits. As noted in the Introduction, this modeling approach is common in the literature on rational deviations from purely speculative motives (e.g., Vitale (1999); Bhattacharyya and Nanda (2013); Pedersen (2018)). Implicit in this assumption are certain un-modeled gains that may arise due to the pursuit of a quantitative versus a discretionary strategy. For example, the lower reliance on human judgement may lower the risk of moral hazard, thereby reducing agency frictions. Furthermore, adherence to a very specific strategy may cater to a particular set of investors, thereby attracting greater fund flows. Lastly, quantitative strategies may have a greater breadth due to their reliance on computing power, thereby potentially allocating to a much higher number of securities. The resulting benefits to diversification, although absent from the model, may also be captured by the backtested trading strategy. Exogenous parameterizations of γ in Eq. (3) therefore also capture in reduced form the average reliance on quantitative strategies among professional investors as it is observed in the marketplace. Such reliance is often overt and its potential benefits are often openly publicized, e.g., in funds' own marketing materials (Abis (2022) and Beggs et al. (2023) provide numerous examples of such disclosures) — consistent with the assumption,

conventional in the aforementioned literature, that quant investors' exogenous adherence coefficient γ is known to all market participants. In addition, if rational (as in our setting), all market participants would always be able to correctly infer any ex-ante optimal γ selected by quant investors, i.e., even if it was not otherwise known to them. Accordingly, Eq. (3) allows us to capture those unmodeled incentives to commit (in full or in part) to such a strategy in a parsimonious and analytically convenient fashion.[11]

Noise traders

Noise traders provide an exogenously given aggregate demand of the risky asset of $z \sim N(0, \tau_z^{-1})$ such that $\sigma_{ze_d} = \sigma_{ze_q} = \sigma_{zv} = 0$. Some examples of noise demand include uninformed retail investors buying or selling shares for liquidity needs, informed asset managers buying or selling securities purely for hedging purposes, or corporations unexpectedly issuing or buying back stock. We will rely on $\eta = \tau_z^{-1}/\tau_v^{-1}$ as a normalized measure of noise trading in the market.

Market makers

Following Kyle (1985), market makers ("MM") are assumed to have risk-neutral preferences and operate in a competitive environment. Therefore, given aggregate order flow $\omega_j = x_{dj} + x_{qj} + z$, the market maker sets a price such that he breaks even in expectation:

$$P_j(\omega_j) = \mathbb{E}[v|\omega_j], \tag{4}$$

i.e., such that the equilibrium asset price is semi-strong from efficient. An underlying assumption in this framework is that the market maker has sufficient inventory (if selling) or liquidity (if buying) to satisfy the net demands of the traders. The market maker attempts

[11] Allowing for multiple, possibly heterogeneously informed speculators (e.g., Foster and Viswanathan (1996); Pasquariello and Vega (2007); Pasquariello and Vega (2009)) and / or quantitative investors would considerably complicate the analysis that follows without materially affecting its main insights.

to discern between the informed and uninformed order flow and to set prices accordingly.

2.2 Model Solution

Our objective is to identify the effect of growth in quantitative investing on financial market quality. Intuitively, this growth may be driven by new quant funds entering the market, incumbent discretionary funds switching to quantitative strategies, or both new entrants and incumbent strategy transitions. We focus primarily on the effects of new quant funds in the financial markets as we believe this to be the primary driver of the growth in quantitative investing. However, our framework can also speak to the effects of automation by incumbent discretionary funds and can be extended to jointly consider new quant funds and incumbent strategy transitions.

Growth in quantitative investing that is driven by new entrants in the financial market has a dual effect on market quality. First, market quality changes due to the incremental informed participants in the financial markets. Second, market quality is impacted by the quantitative investing approach of the new entrants. To speak directly to the effects of quantitative strategies we must untangle the two forces. We do so by considering market quality across four economies:

- Economy 1: DI
- Economy 2: DI + QI ($\gamma = 0$)
- Economy 3: DI + QI ($0 < \gamma < 1$)
- Economy 4: DI + QI ($\gamma = 1$)

Economy 1 resembles a world before quantitative investing was introduced. This is also the quant's backtesting environment, which allows her to back out the DI's trading rule and the MM's pricing function. The backtest will be a regression of prices on fundamentals, which

coupled with observable order flow provides the quant with sufficient information to form her profit-maximizing strategy. The quant will then build her strategy such that profits are maximized under the assumption that the MM and the DI are not aware of her existence.

Economies 2-4 introduce the quantitative investor, but do so in phases in order to distinguish between the effects of an additional informed speculator and the speculator's backtested trading rule. First, Economy 2 isolates the effect of introducing a second informed speculator to the economy. Applying $\gamma = 0$ to Eq. (3), demonstrates that the QI's and DI's objective functions are identical. Therefore, Economy 2 is simply an extension of Kyle (1985) with two informed speculators, a static version of Foster and Viswanathan (1996) but with differential signal precision. Second, Economy 3 "turns on" the quant's strategy. Increasing γ leads to greater adherence to the backtested strategy implying greater "automation" by the quant. Finally, Economy 4 reflects a fully-automated quant. In our baseline theoretical results we only consider Economies 1,2, and 4, however we also conduct a simulation exercise for intermediate values of γ in Economy 3.

We now proceed to solve for the DI's and QI's trading strategies and the MM's pricing rule in Economies 1-4. For each economy j we conjecture that the optimal trading strategy of investor i is linear in his / her signal and is given by

$$x_{ij}(s_i) = \beta_{ij} s_i, \tag{5}$$

and the optimal pricing rule of the market maker is linear in the aggregate order flow such that

$$P_j(\omega_j) = \lambda_j \omega_j. \tag{6}$$

We then derive the coefficients that simultaneously satisfy the investors' and market makers' optimization functions given by Eq. (2), (3), and (4) while taking the price-setting and

trading behavior of others as given. We refer to β_{ij} as the "trading intensity" of speculator i as it reflects how aggressively the speculator reacts to an incremental unit of signal. We identify $1/\lambda_j$ as "market depth," our primary measure of liquidity: the inverse of the market maker's price sensitivity to an incremental share of order flow. Importantly, while prima facie static, this setting naturally and parsimoniously captures the dynamic notion of a quantitative investor developing a backtested trading strategy — i.e., one assessed from historical data prior to her entering the market, when her presence could not have been accounted for by all other market participants — hence it is well suited to our objective of analyzing the extensive margin effects of quantitative investing on the quality of the process of price formation in the affected markets.

2.2.1 Economy 1

Economy 1 is a world with a single informed speculator (DI) who trades strategically to minimize information slippage and maximize profits. This is the traditional Kyle (1985) framework with the sole difference that the informed speculator observes a noisy signal $s_d = v + e_d$ instead of perfect information v about the risky asset payoff. The DI solves Eq. (2) while the MM solves Eq. (4) with $\omega_1 = x_{d1} + z$. Lemma 1 demonstrates the existence and uniqueness of a linear strategy equilibrium.

Lemma 1 (Equilibrium in Economy 1) *Within the class of linear equilibria, there exists a unique linear trading rule for the DI and a unique linear pricing function for the MM specified by Eq.* (5) *and* (6) *respectively such that Eq.* (2) *and* (4) *are satisfied for all realizations of v, e_d, and z.*

Specifically, as derived in Appendix A, the equilibrium trading rule of the DI is

$$x_{d1}(s_d) = \sqrt{\phi_d \eta} s_d, \tag{7}$$

and the pricing rule of the MM is

$$P_1(\omega_1) = \frac{1}{2}\sqrt{\frac{\phi_d}{\eta}}\omega_1. \tag{8}$$

The DI is more aggressive in trading on his signal with more noise trading, a more precise signal, and a greater expected payoff. The MM sets prices such that the expected profits from providing liquidity to the noise traders exactly offset the expected losses from trading against the DI. Therefore, equilibrium liquidity, as measured by $1/\lambda_1 = 2\sqrt{\eta}/\sqrt{\phi_d}$, is decreasing in signal quality and is increasing in the relative amount of noise trading. Both are driven by the market maker's reaction to adverse selection: the more precise the DI's information or the greater the likelihood of informed trading, the more sensitive will the price be to order flow.

2.2.2 Quant's Backtest

A defining characteristic of backtested strategies is their reliance on historical data to identify a signal, which is predictive of future risk-adjusted returns. The quant relies on the strategy to map a real-time signal to a trading rule, which is implemented in the financial markets. By relying on historical data to generate a signal, the quant cannot know how other market participants would have reacted to her presence in the market. Even if similar strategies already exist in the market during the backtest, the quant's strategy will likely be incremental, implying that the backtest does not fully capture the strategic behavior of other market participants. It is this lack of strategic consideration that we focus on as the primary friction associated with quantitative strategies.

We model this inherent myopia of backtested strategies, by endowing the quant with a signal, but assuming that she builds the strategy under the assumption that the other market participants are not aware of her existence. In other words, the quant assumes that the behavior of other market participants does not change in response to her presence.

Prior to launching her strategy, the quant uses historical data from Economy 1 to generate her optimal trading rule. One can think of this as years of trading data that is based on the data generating process of Economy 1, used as the backtest into the quant's trading approach. She gathers an extended time series of this data, and backs out the trading rule of the DI and the pricing strategy of the MM.

The DI's trading strategy, Eq. (7), and a simple manipulation of the MM's pricing rule, Eq. (8), demonstrate that:

$$P_1 = \frac{1}{2}\sqrt{\frac{\phi_d}{\eta}}\omega_1$$
$$P_1 = \frac{\phi_d}{2}v + \frac{\phi_d}{2}e_d + \frac{1}{2}\sqrt{\frac{\phi_d}{\eta}}z$$

Let's assume that Economy 1 plays out over many periods, e.g., days, $t = 1, \ldots, T$ and in each day t the following data is recorded: $\{P_{1t}, \omega_{1t}, v_t\}$. In other words, the QI is able to observe historical prices, trading volume, and fundamentals. The quant can then take this data and run the following ordinary least squares regressions:

$$P_{1t} = a + b\omega_{1t} + \varepsilon_t$$
$$P_{1t} = c + dv_t + \epsilon_t$$

By construction, $\varepsilon_t = 0 \perp \omega_1$ and $\epsilon_t = \frac{\phi_d}{2}e_d + \frac{1}{2}\sqrt{\frac{\phi_d}{\eta}}z \perp v$ implying that the QI will be able to obtain unbiased estimates for $a = c = 0$, $b = \frac{1}{2}\sqrt{\frac{\phi_d}{\eta}}$ and $d = \frac{\phi_d}{2}$. Assuming the QI has a sufficiently long data sample, she will deduce both the DI's signal precision ϕ_d and the normalized amount of noise trading η from the data. Together, these parameters allow her to identify the MM's pricing rule and the DI's trading strategy: we assume that the data sample is sufficiently long such that she obtains highly accurate estimates. This, in turn,

allows her to construct an optimal, profit-maximizing strategy *under the assumption that the DI and MM are not aware of her existence:*

$$\begin{aligned} x_b &= \arg\max_{\tilde{x}_b} \mathbb{E}\Big[\tilde{x}_b\Big(v - P_1\big(x_{d1} + z + \tilde{x}_b\big)\Big)|s_q\Big] \\ &= \frac{1}{2}\sqrt{\frac{\eta}{\phi_d}}\left((2-\phi_d)\phi_q - \rho\sqrt{\phi_d\phi_q(1-\phi_d)(1-\phi_q)}\right)s_q, \end{aligned} \tag{9}$$

derived in Appendix B. The notion of myopia is incorporated by the QI's reliance on the MM's pricing function $P_1(\omega_1)$ and the DI's demand $x_{d1}(s_d)$ from Economy 1. In other words, the QI assumes that the MM will treat her order flow as noise trading and the DI will not internalize the QI's presence.

Similar to the DI's Economy 1 trading strategy, the QI's backtest trades more aggressively with more noise trading and a greater expected payoff. For negative and sufficiently low positive signal error correlation, the quant will always buy more shares with greater expected payoff as she's less concerned about moving the price in the same direction as the DI. However, with high signal error correlation, the QI may trade against the DI since she becomes concerned about excess information slippage. Ultimately, the backtested strategy will depend not only on the quant's signal precision and level of noise trading, but also on the DI's signal precision and the signal error correlation.

2.2.3 Economies 2-4

Armed with the backtested strategy of the quantitative investor, we proceed to derive the equilibrium trading and price setting behavior in Economies 2-4. Economy 2 will be the classic extension of Kyle (1985) to include two informed speculators with correlated signals (e.g., Foster and Viswanathan (1996)) and potentially differential information quality. Economy 3 will incorporate the notion of quantitative investing as the QI will at least partially rely on the backtested strategy ($\gamma > 0$), while Economy 4 will include a fully-automated quant

($\gamma = 1$). The trading behavior of the DI and the QI are determined by the solutions to Eq. (2) and (3) respectively, while the MM continues to set prices to break-even in expectation as specified in Eq. (4). Lemma 2 demonstrates the existence and uniqueness of a linear strategy equilibrium.

Lemma 2 (Equilibrium in Economies 2-4) *Within the class of linear equilibria, there exist unique linear trading rules for the DI and QI and a unique linear pricing function for the MM specified by Eq.* (5) *and* (6) *respectively such that Eq.* (2), (3), *and* (4) *are satisfied for all realizations of* v, e_d, e_q, *and* z.

In Appendix C we derive and demonstrate the existence of linear equilibria for Economies 2-4 and are able to demonstrate uniqueness for Economy 2. For Economies 3 and 4 the demonstration of uniqueness is theoretically intractable, however a simulation exercise presented in Section 3.2 lends support to this notion.

To gain intuition behind the equilibrium it is helpful to analyze the functional forms of the trading rules for the informed speculators and the pricing function of the market maker. For the DI, demand is given by:

$$\begin{aligned} x_{dj} &= \frac{\mathbb{E}[v|s_d] - \lambda_j \mathbb{E}[x_{qj}|s_d]}{2\lambda_j} \\ &= \underbrace{\frac{\mathbb{E}[v|s_d]}{2\lambda_j}}_{\text{Single strategic speculator demand}} - \underbrace{\frac{\mathbb{E}[x_{qj}|s_d]}{2}}_{\text{Adjustment for second speculator}}, \end{aligned} \tag{10}$$

as derived in Eq. (25) and (26). The DI's demand is the sum of his demand function as if he were acting alone (conditional on the market maker's pricing rule) and an adjustment for the expected trading behavior of the QI conditional on his information. The adjustment is equal to the expected price impact of the QI's expected trading activity ($\lambda_j \mathbb{E}[x_{qj}|s_d]$) scaled by $2\lambda_j$, which maps ex-ante expected profits to share demand. The adjustment term is still dependent on λ_j because the expected share demand of the QI is also (inversely) dependent on λ_j. Furthermore, the adjustment will depend on the QI's level of automation γ, known

by both the MM and the DI, since it will factor into both the equilibrium pricing function of the MM via λ_j and the expectation of the demand of the QI via $\mathbb{E}[x_{qj}|s_d]$. Decomposing the numerator of the adjustment even further yields:

$$\mathbb{E}[x_{qj}|s_d] = \beta_{qj}\left(\phi_d + \rho\sqrt{\frac{\phi_d}{\phi_q}(1-\phi_d)(1-\phi_q)}\right)s_d, \tag{11}$$

such that the DI's optimal trading strategy depends on both what he expects v to be and what he expects the QI to know about v and use in her trading strategy. The expected trading strategy of the quant based on the DI's information is the sum of the unconditional expectation of her trading strategy and an information adjustment based on the DI's signal. The signal inference will be jointly determined by the DI's signal, QI's trading intensity, the absolute and relative precision of QI's and DI's signal, and the signal error correlation.

As discussed below, we can apply a similar decomposition, derived in Eq. (52), to understand the trading behavior of the quantitative investor:

$$x_{qj} = \frac{\gamma}{\gamma + (1-\gamma)\lambda_j}x_b + \frac{(1-\gamma)\lambda_j}{\gamma + (1-\gamma)\lambda_j}\frac{\mathbb{E}[v|s_q] - \lambda_j\mathbb{E}[x_{dj}|s_q]}{2\lambda_j}. \tag{12}$$

The exogenously assigned automation parameter γ will determine whether the model represents Economy 2 ($\gamma = 0$), Economy 3 ($\gamma > 0$), or Economy 4 ($\gamma = 1$).

2.2.4 Economy 2: $\gamma = 0$

With no automation the QI becomes a fully-strategic speculator akin to the discretionary investor. The forces driving the behavior of the DI specified above will be symmetric for the QI. Of particular note is the relationship between the trading intensity of the DI and the QI as derived in Eq. (33) since the aggregate trading intensity will determine various market

quality measures introduced in the next section:

$$\begin{aligned}\beta_{d2} &= \frac{2\phi_d - \phi_d\phi_q - \rho\sqrt{\phi_d\phi_q(1-\phi_d)(1-\phi_q)}}{2\phi_q - \phi_d\phi_q - \rho\sqrt{\phi_d\phi_q(1-\phi_d)(1-\phi_q)}}\beta_{q2} \\ &\equiv k\beta_{q2}\end{aligned} \tag{13}$$

It is immediate that $\partial k/\partial\rho > (<)0 \Leftrightarrow \phi_d > (<)\phi_q$, which implies that greater signal error correlation will cause the investor with an information advantage to trade more aggressively. The importance of signal correlation to trading behavior is consistent with the findings of Foster and Viswanathan (1996).

2.2.5 Economy 3: $0 < \gamma < 1$

For a partially automated quant, i.e., intermediate values of γ, the trading rule is an endogenously determined weighted average of the backtested strategy and the fully-strategic trading approach. As evident from Eq. (12), the relative weights are determined by λ_j, which, in turn, identifies the depth of the market. For a fixed level of liquidity λ_j by the market maker, greater automation (higher γ) results in a greater weight being placed on the back-tested strategy versus the strategic rule. For relatively liquid markets ($\lambda_j < 1$), greater γ leads to a more than a one-to-one weight increase on the backtested strategy (e.g., a 1% increase in γ leads to a more than 1% of incremental weight being placed on the back-tested rule). For relatively illiquid markets ($\lambda_j > 1$), greater γ leads to a less than one-to-one increase in the weight placed on the backtested strategy. Finally, for $\lambda_j = 1$, a greater γ results in exactly a one-to-one increase in the weight placed on x_b. The quant is more aggressive in implementing her backtest in more liquid environments.

2.2.6 Economy 4: $\gamma = 1$

A fully-automated quant adheres solely to the back-tested strategy x_b, which is developed under the assumption that the DI and the MM are not aware of her existence. However, both the DI and MM internalize the QI's expected behavior thereby rendering QI's strategy suboptimal from the perspective of profit-maximizing behavior.

2.2.7 Economies 2-4: Market Maker

For the market maker's pricing rule, first note that the aggregate order flow ω_j provides a noisy signal for v:

$$\omega_j = \underbrace{(\beta_{dj} + \beta_{qj})v}_{\text{Signal}} + \underbrace{\beta_{dj}e_d + \beta_{qj}e_q + z}_{\text{Noise}}, \tag{14}$$

where the informativeness of the order flow can be expressed as:

$$\text{Var}(\omega_j|v) = \beta_{dj}^2\sigma_{e_d}^2 + \beta_{qj}^2\sigma_{e_q}^2 + 2\beta_{dj}\beta_{qj}\sigma_{e_d e_q} + \sigma_z^2. \tag{15}$$

A lower conditional variance implies a more informative order flow. Lower trading intensity by either informed speculator, greater signal precision for either informed speculator, lower noise trading variance, and a lower signal covariance all lead to a more informative order flow. This in-turn translates into the following effects on market liquidity as given by $1/\lambda_j$:

$$\begin{aligned}
\lambda_j^{-1} &= \frac{\text{Var}(\omega_j)}{\text{Cov}(v, \omega_j)} \\
&= \beta_{dj} + \beta_{qj} + \frac{\text{Var}(\omega_j|v)}{(\beta_{dj} + \beta_{qj})\sigma_v^2} \\
&= \underbrace{(\beta_{dj} + \beta_{qj})}_{\text{Aggregate trading intensity}} \times \left(1 + \underbrace{\frac{\text{Var}(\omega_j|v)}{\text{Var}(\omega_j|\beta_{dj}e_d + \beta_{qj}e_q + z)}}_{\text{Noisiness of order flow}}\right)
\end{aligned} \tag{16}$$

Holding aggregate trading intensity of the speculators constant, a noisier order flow results in a more liquid price system. Market makers are less concerned about adverse selection and are therefore less sensitive to greater order flow. Less precise signals, greater signal covariance, and more noise trading all lead to a noisier system. Changes to trading intensity for either speculator may either increase or decrease the liquidity provision both through the noise-to-signal ratio and through the aggregate trading intensity measure.

3 Comparative Statics

We explore the effects of the introduction of and the greater automation by the quantitative investor on financial market quality. Ultimately, our goal is to understand the effects of quantitative investing on the process of price formation in the affected financial markets, and, guided by the literature (e.g., Vives (2008)), we rely on measures such as market liquidity (identified via market depth, $1/\lambda_j$), market efficiency (via price informativeness, $\text{Var}(v|P_j)^{-1}$), and the behavior of prices ($\text{Var}(P_j)$) and returns ($\text{Var}(v - P_j)$). To that same end, we also focus on the trading behavior of the speculator sector via such equilibrium aggregates as the expected trading volume of each speculator ($\mathbb{E}[|x_{ij}|]$) and the sector as a whole ($\mathbb{E}[|x_{dj} + x_{qj}|]$), as well as the resulting expected profits for each speculator ($\mathbb{E}[\pi_{ij}] = \mathbb{E}[x_{ij}(v - P_j)]$) and the sector as a whole ($\mathbb{E}[\pi_j] = \mathbb{E}[\pi_{dj}] + \mathbb{E}[\pi_{qj}]$).

The closed-form solutions for each of the measures above utilizing the informed speculators' trading rules and the market maker's pricing function as derived in Appendices A and C are provided in Appendix D. Unfortunately, the theoretical framework does not allow for analytically tractable solutions for the trading rules and pricing functions. Therefore, we split the comparative statics analysis into two sections. The first is a special case of the generalized model, which assumes that the DI has perfect information, and only considers changes to market quality when the QI switches from full discretion ($\gamma = 0$) to full automation ($\gamma = 1$), thereby only exploring changes to market quality for Economies 1,2,

and 4. Within this framework, we are able to theoretically demonstrate the effects of the introduction of and automation by the quant on market quality. The second section conducts a quantitative exercise for the generalized model, documenting changes in market quality due to the addition of and all levels of automation by the QI (Economies 1-4) across a large number of parameter draws. We first discuss the theoretical benchmark and conclude with the market quality outcomes of the generalized model.

3.1 Theoretical Benchmark

The benchmark case assumes the DI has perfect information ($\phi_d = 1$). Inherent in this assumption is the notion that traditional fundamental investors may have superior access to fundamental information via corporate management meetings, industry and market expertise, and deep-dive fundamental analysis. Quants, on the other hand, may look at rough proxies for fundamental value, such as historical multiples or price trends, and hope that on average, across a large sample of securities, such proxies will be indicative of future payoffs. Examples of such quant funds would be smart beta exchange traded funds and statistical arbitrage strategies nested within hedge funds. However, this category does not include fundamental investors who rely on big data as an input into their investment process, since such fundamental investors may actually have superior information. Examples of the latter would be fundamental funds relying on credit card transaction data to accurately estimate retailers' revenues or utilizing airfare pricing datasets to forecast airline revenues.

The introduction of quantitative investing has a dual effect on the financial markets. First, an incremental informed speculator is introduced to the economy. Second, the incremental speculator chooses to pursue an automated strategy. We attempt to distinguish between these two effects by first exploring the implication of adding a second informed strategic speculator to the market (Economy 1 to Economy 2), and, second, analyzing the impact of automation by the incremental strategic speculator (Economy 2 to Economy 4).

Under the assumption that growth in quantitative investing is driven by new entrants in the market, the net effect of quantitative investing will be interpreted as the change in market quality between Economy 1 and Economy 4. However, if the growth is driven by strategy transitions of incumbent funds from discretionary to quantitive, the effects could be viewed through the changes from Economy 2 to Economy 4.

Proposition 1 derived in Appendix E highlights our main theoretical findings regarding the effects of quantitative investing on financial market quality. Fig. 1 demonstrates the comparative statics graphically.

Proposition 1 (Quantitative investing and financial market quality) *Market quality improves due to the introduction of the quantitative investor, but the improvement is dampened by the quant's pursuit of the backtested trading rule. The shift from Economy 1 to Economy 2 results in greater trading volume by the speculator sector, more informative prices, greater market depth, and lower profits for the speculator sector. The transition from Economy 2 to Economy 4 leads to less aggressive trading and lower profits by the QI, more aggressive trading and greater profits by the DI, less informative prices, lower market depth, and greater profits for the speculator sector as a whole. Overall, market quality improves from Economy 1 to Economy 4.*

The introduction of a second strategic speculator to the economy, i.e., the transition from Economy 1 to Economy 2, unequivocally improves financial market quality. As can be seen from Fig. 1a, the introduction of a fully-discretionary quant is internalized by the DI, causing him to trade less aggressively relative to a market in which he's the only informed speculator due to concerns regarding incremental information slippage. However, the decline in the DI's trading, is more than offset by incremental trading by the QI, resulting in a net growth in expected trading volume by the speculator sector. This net growth is increasing in the QI's signal precision, as she trades more aggressively on her information. Fig. 1d highlights that the greater trading by the speculator sector leads to more informative prices

relative to a benchmark world of a single informed speculator, which, in turn is translated into lower profits for the discretionary investor and the speculator sector as a whole (Fig. 1b). In equilibrium, the greater trading volume and more informative prices correspond to greater market depth (Fig. 1c), greater price volatility (Fig. 1e), and lower return volatility (Fig. 1f). The latter two outcomes are directly driven by prices that more accurately track fundamental values.

The primary contribution of the present framework is an analysis of the effects of automation by the QI on the behavior of the DI and the net effects to financial market quality. The quant's backtested rule is less aggressive (i.e., has lower trading intensity) than her trading strategy with full discretion. In part this is due to the foundation of the backtest, which assumes that the DI and the MM are not aware of the QI's existence: they treat the incremental order flow as originating from the noise traders. Once the presence of the quant is internalized by other market participants, the DI trades more cautiously, allowing a discretionary quant to trade more aggressively in response. On the other hand, an automated quant is, by definition, pursuing a strategy that is not profit maximizing. This opens the door for the discretionary investor to trade more aggressively on his information, but the net effect is less expected trading volume by the speculator sector as a whole relative to Economy 2. The decline in trading volume leads to less informative prices and greater profits for the DI and the speculator sector as a whole. The QI earns less in expectation in Economy 4 due to the sub-optimality of her strategy from a profit maximization perspective. Concurrent with the lower price informativeness, liquidity dries up in the form of lower market depth, price volatility declines, and return volatility increases. Overall, Economy 4 market quality decreases relative to Economy 2.

Despite the potentially detrimental effects of automation by the quant, as the shift from Economy 2 to Economy 4 would suggest, we emphasize the overall improvement to market quality from Economy 1 to Economy 4. The beneficial effects from the introduction of an incremental informed speculator to the economy dominate the potential inefficiency

induced by the trading strategy they pursue. To the extent that quantitative investing growth is driven by new entrants in the market, our model would suggest overall improvements in market quality. However, if existing funds are transitioning from discretionary to quantitative strategies, market quality may be negatively impacted.

The presented framework makes strong assumptions in exchange for theoretical tractability. Next, we conduct a numerical exercise to describe the properties of a more general version of the model.

3.2 Generalized Model

In the generalized model, we do not restrict the DI to perfect information, allow for signal error correlation between the informed speculators, and allow for partial automation (i.e., intermediate values of γ). As such, we are able to not only consider the relationships between the levels of market quality in Economies 1,2, and 4, but also to explore the effects of intermediate automation through Economy 3. After all, quantitative funds are ultimately deployed by humans, and inherent in this is a certain level of discretion. Humans may choose to strategically scale, turn off, and alter their investment strategies, even if such strategies full rely on backtested analysis. Furthermore, the framework allows for differential signal precision for the QI and the DI, thereby allowing us to explore the effects of both superior and inferior information for the quant. The latter feature of the model allows us to speak more generally to systematic quantitative strategies, whereby quants may actually have superior information given their ability to process large data to more accurately forecast future earnings.

As the simulation will highlight, the effects of quantitative investing on financial market quality will be determined by whether a fully-automated quant (Economy 4) trades more than a fully-discretionary quant (Economy 2). We identify this as the quant's "trading wedge," $\beta_{q4} - \beta_{q2}$: positive if the backtest is more aggressive, negative if less aggressive. Since

the quant's strategy is (loosely, i.e., Eq. (12)) a weighted average between the Economy 2 and the Economy 4 strategy, growth in automation (γ) will lead to more (less) aggressive trading by the QI if $\beta_{q4} > (<)\beta_{q2}$. The change in trading behavior due to greater automation is internalized by the discretionary investor and the market maker and ultimately determines the effects of quantitative investing on financial market quality. Since the fully-automated quant is by definition not maximizing profits, to the extent that this sub-optimality is accompanied by less aggressive trading, the DI has the opportunity to exploit the greater resulting mis-pricing to his advantage by trading more aggressively without positive implications to market quality. If, however, greater automation leads to greater trading intensity by the quant, the DI is mostly unable to exploit this to his advantage as he's concerned about excess information leakage. The net effect in the latter scenario is an improvement in market quality. The simulation results, provided in Tables 1-3, are summarized in Conclusions 1 and 2.

Conclusion 1 (Effect of the introduction of a fully-discretionary quant) *The introduction of a fully-discretionary quantitative investor leads to improvements in market quality via more informative prices, greater price volatility, and lower return volatility. The effects on market depth, trading volume, and profits for the speculator sector depend on signal correlation and precision for the QI and DI. If the parameters are such that the fully discretionary quant trades more than the fully-automated quant ($\beta_{q2} > \beta_{q4}$), trading volume for the speculator sector increases, profits mostly decrease, the DI trades less and makes less in profits, the QI makes positive profits, and market depth mostly rises. Conversely, if $\beta_{q2} \leq \beta_{q4}$, trading volume for the speculator sector mostly increases, profits always increase, the DI trades less and makes less in profits, the QI makes positive profits, and market depth always decreases.*

Conclusion 2 (Effect of automation by the quantitative investor) *The effect of greater automation γ by the QI on financial market quality depends on whether the fully-automated quant trades more than the fully-discretionary quant. If the fully-automated quant trades*

more ($\beta_{q4} > \beta_{q2}$), market quality broadly improves as market depth, price informativeness, price volatility are mostly rising and return volatility is falling in γ. Furthermore, the speculator sector as a whole trades more, profits are mostly lower, the effects on the DI's trading and profits can be either be increasing, decreasing, or U-shaped in γ, while the QI always trades more and her profits are increasing, decreasing or hump-shaped in γ. If the fully-automated quant trades less ($\beta_{q4} < \beta_{q2}$), market quality deteriorates via lower market depth, less informative prices, lower price volatility, and greater return volatility. The speculator sector as a whole trades less and makes greater profits as the DI trades more aggressively and makes greater profits, while the QI trades less and makes lower profits.

Conclusion 1 is generally consistent with the insights from the extant literature on models of strategic speculation with multiple informed speculators (e.g., Holden and Subrahmanyam (1992); Foster and Viswanathan (1996)), namely that incremental speculation benefits the financial markets although its effects on market depth and trading behavior depend on speculator signal quality and correlation. To gain intuition on Conclusion 2, the main emphasis of the paper, we first *separately* consider the effects of signal error correlation (ρ) and information precision (ϕ_d, ϕ_q) on market quality. Next, we theoretically characterize the net effect of (ϕ_d, ϕ_q, ρ) on the quant's trading wedge, whose sign acts as a sufficient statistic for market quality outcomes. Finally, we arrive at various empirical implications by relying on ρ to speak to the likelihood of improvement in market quality and interpreting ρ as a measure of disagreement in the market.

3.2.1 Isolating the effects of signal error correlation and precision

Fig. 2-3 demonstrate the effect of automation on market quality for different correlation structures for the generalized model within which the DI and QI have equal information precision ($\phi_d = \phi_q$) and other parameter values are held constant. We first consider the trading behavior of the QI in response to different correlation levels, highlighted in Fig. 2e,

as she gradually switches from a fully discretionary strategy ($\gamma = 0$) to a strategy fully reliant on the backtest ($\gamma = 1$). Due to identical signal precision, a fully-strategic quant and the DI have the same trading intensity regardless of the correlation parameter. Conversely, a fully-automated quant's trading intensity is decreasing in correlation, which is a direct outcome of the QI's backtest, as described in Section 2.2.2. The backtest trades more aggressively with lower correlation as the QI is able to more effectively trade against the DI as she becomes less concerned about information slippage. For sufficiently low correlation levels, the trading wedge turns positive and continues to increase with lower correlation. Since the quant's strategy is loosely a weighted average between the backtest and full-discretion (i.e., Eq. (12)), growth in automation (γ) will lead to more (less) aggressive trading by the QI if the wedge is positive (negative).

Pictured in Fig. 2c, the discretionary investor's reaction to the changes in signal correlation is more nuanced. On the one hand, a sufficiently negative signal correlation implies that he can trade more aggressively on his information since the QI is more likely to submit offsetting trades thereby decreasing information slippage. On the other hand, for sufficiently low correlation, the QI is also trading more aggressively, and is acting less strategically, thereby limiting the extent to which the DI can be more aggressive in his trading. The net effect of the two forces can lead to either an increase or a decrease in the trading volume of the DI as can be seen when, e.g., $\rho = -1$ or $\rho = -0.5$. However, if the signals are sufficiently correlated, the QI will always trade less aggressively with greater automation, which opens the door for the DI to be more aggressive in his trading.

As demonstrated in Fig. 2a, the net effect of the trading behavior of the DI and the QI on the aggregate speculator trading volume is lower aggregate trading volume in response to lower correlation levels. This is intuitive, as the negative correlation leads to offsetting orders, causing a decrease in aggregate order flow. However, a more intricate outcome, is that the increase in total trading volume in response to greater γ is greatest for lower correlation levels. In other words, the change in the trading intensity of the QI in response to greater

automation for varying correlation levels dominates the change in the trading intensity of the DI.

As can be seen in Fig. 2f, the trading patterns above generally result in lower profits for the quant with greater automation. However, profits need not always decline with automation since the ex-ante commitment to a backtested strategy may deter the DI from trading more aggressively and leave the QI with greater profits. This strategy cannot be implemented in the strategic equilibrium (Economy 2), since the strategic speculator does not commit ex-ante, and therefore cannot influence the response of the other market participants.[12] Lower correlation levels result in greater profits to the QI as price impact is lower. In Fig. 2d, for the discretionary investor, profits are rising with greater automation by the quant to the extent that he is able to trade more aggressively. However, the relationship with correlation levels is nontrivial as highlighted above. The speculator sector as a whole, Fig. 2b, makes greater expected profits with more negatively correlated signals and aggregate profits are rising in automation if the quant is less aggressive in her trading.

In Fig. 3, we consider the net effects of the speculator sector trading behavior and the market maker's pricing function in response to changing signal correlation and greater automation. A more negative correlation results in more informative prices as the aggregate order flow becomes more reflective of fundamentals (i.e., signal errors offset each other). This, in turn, implies greater price and lower return volatility. For lower correlation values, a more informative order flow results in lower market depth due to greater adverse selection risk for the market maker. Greater quant automation with a positive (negative) trading wedge positively (negatively) affects these market quality measures — a positive (negative) wedge leads to greater (lower) market depth, greater (lower) price informativeness, and lower (greater) return volatility with greater automation.

Fig. 4 and 5 highlight the effects of changing signal quality for the discretionary

[12]Ex-ante commitment to a trading strategy in a Kyle (1985) setting, otherwise known as Stackelberg competition, is discussed further in Bernhardt and Boulatov (2023).

investor for varying levels of automation for the quant, while holding the quant's signal precision and error correlation constant. It is immediate, as seen in Fig. 4e, that the quant trades more as her informational advantage (disadvantage) increases (shrinks), for the fully-strategic strategy, the backtest, and intermediate levels of automation. This is a direct outcome of the quant's backtest, Eq. (9), which for $\rho = 0$ trades more aggressively in response to lower ϕ_d. For sufficiently high relative information precision (i.e., $\phi_q - \phi_d$), the quant's trading wedge turns positive and increases further for lower values of ϕ_d. In Fig. 4c, the DI trades more with greater signal precision, even increasing his trading intensity for higher values of ϕ_d as the quant becomes more automated, which creates the opportunity for greater profits for the DI (Fig. 4d). The net effect of the speculators' trading behavior, Fig. 4a, depends on both the trading wedge and the level of automation. For sufficiently high automation, a positive (negative) trading wedge leads to greater (lower) aggregate trading for the speculator sector. The relative effects on market quality measures in Fig. 5 are ambiguous; however, the sign of the quant's trading wedge dictates the direction of the effect of greater automation. A positive (negative) wedge implies greater (lower) market depth, more (less) informative prices, and lower (greater) return volatility.

Overall, the above examples highlight the importance of signal precision and correlation to the effects of quantitative investing on financial market quality. The sign of the quant's trading wedge, i.e., the difference between the trading intensity of the quant's fully-automated and fully-discretionary strategy, determines the effect of greater automation on the quality of the financial markets. A positive (negative) wedge generally implies greater (lower) market depth, more (less) efficient prices, and less (more) volatile returns as the quant becomes more automated. We proceed to theoretically characterize the determinants of the sign of the wedge.

3.2.2 Determinants of the trading wedge

Given the differential effects of signal precision and information correlation on trading activity and market quality, at first sight it may seem complex to fully describe the effects of the parameter space (ϕ_d, ϕ_q, ρ) on market quality. One may conjecture that for sufficiently low correlation together with sufficiently high information advantage for the quant, market quality would generally benefit from greater automation and vice versa. However, understanding the implications when the parameter effects are offsetting, e.g., high correlation and high information advantage for the quant, appears inherently challenging.

Fortunately, as highlighted by the simulation results in Tables 1-3, a sufficient statistic defined for each triplet (ϕ_d, ϕ_q, ρ) nearly unambiguously determines the effect of the introduction of and greater automation by the quantitative investor on financial market quality. The sign of the wedge between the trading intensity of a fully-discretionary and a fully-automated quant, i.e., $\text{sign}(\beta_{q4} - \beta_{q2})$, captures the net effect of the signal precision for each speculator and the signal error correlation. As derived in Appendix F, the relationship between the trading intensity of a fully-automated and a fully-discretionary quant is jointly determined by (ϕ_d, ϕ_q, ρ), which allows us to fully characterize the parameter space and the effects of any triplet on the sign of the wedge. Lemma 3 summarizes our findings.

Lemma 3 (Effect of signal error correlation on quant's trading wedge) *The quant's trading wedge, $\beta_{q4} - \beta_{q2}$, is either 1) positive $\forall \rho \in [-1, 1]$; 2) negative $\forall \rho \in [-1, 1]$; or 3) decreasing in ρ and positive $\forall \rho \in [-1, \rho^*)$ and negative for $\forall \rho \in (\rho^*, 1]$ for $\rho^* \in (-1, 1)$.*

The key takeaway from Lemma 3 is that for any information quality pair (ϕ_d, ϕ_q), we can find a breakeven signal error correlation parameter ρ^* such that for any $\rho < \rho^*$, the trading wedge is positive, and, consequently, market quality is positively affected by quantitative investing. Therefore, since $\rho \in [0, 1]$, if (ϕ_d, ϕ_q) is such that 1) $\rho^* \leq -1$, the trading wedge is never positive, 2) $\rho^* \geq 1$, the trading wedge is always positive, and 3)

$\rho^* \in (0,1)$, the trading wedge is positive for $\rho \in [-1, \rho^*)$. Furthermore, a higher ρ^* implies a greater likelihood of a positive trading wedge, which in turn corresponds to a higher probability of improvement to market quality due to quantitative investing. We rely on this logic to provide a classification of the parameter space via a contour plot illustrated in Fig. 6. Specifically, for each pair of signal precision parameters (ϕ_d, ϕ_q), we plot the maximum correlation level ρ^* for all values below which the fully-automated quant trades more than the fully-discretionary quant. Bright yellow regions correspond to information precision pairs for which market quality will always improve. Dark red regions reflect information precision pairs for which market quality will always deteriorate. Finally, for pairs located in the intermediate region the effect is not definite, yet higher breakeven correlation levels imply a greater likelihood of market quality improvement.

As apparent from Fig. 6, when trading against relatively uninformed discretionary investors (i.e., for ϕ_d below ≈ 0.52), the likelihood of market quality improvement due to quant investing is increasing in the quant's signal precision. In particular, as demonstrated in Appendix F, if $\phi_q > 3\phi_d$, the trading wedge is always positive, implying that market quality will always improve due to quantitative investing. Conversely, if trading against relatively informed DIs (i.e., for ϕ_d above ≈ 0.52), the likelihood of market quality improvement is falling in the quant's signal precision. Highly informed DIs (e.g., $\phi_d \geq 0.8$) trading against highly informed quants (e.g., $\phi_q \geq 0.8$) unambiguously leads to deterioration in market quality. Similarly, DIs with nearly perfect information (i.e., $\phi_d \approx 1$) cause market quality declines with near certainty regardless of the quant's signal precision.

If we do not have prior knowledge of the DI's information quality and assume that it is uniformly distributed (i.e., $\phi_d \sim U(0,1)$), a more subtle outcome of Fig. 6 is that a higher ϕ_q is more likely to result in improvements in market quality. Conversely, for any fixed information precision for the quant, the likelihood of improvement in market quality is decreasing in ϕ_d. Taken together, these findings are consistent with Beggs et al. (2023) who show that mutual fund fire sales have greater price impact for quant vs. discretionary

funds. To the extent that fire sales are more likely for funds with less-precise information, our model suggests that lower information precision is more likely to have adverse effects on market quality for quant as opposed to discretionary funds.

We now connect the sign of the quant's trading wedge to the results of the simulation highlighted in Tables 1-3. A positive wedge results in mostly increasing market depth, increasing price informativeness and volatility, and decreasing return volatility in the level of automation γ by the quant. These overall improvements in market quality are driven by greater expected trading volume for the quant, mostly lower volume for the DI, and greater volume for the speculator sector as a whole. The effects on the quant's profits are ambiguous, while the DI and the speculator sector as a whole mostly see declining profits. Greater automation, which results in more aggressive trading by the quantitative investor, has a positive impact on financial market quality.

In cases where the quant's trading wedge is negative, market quality generally deteriorates as the lower trading intensity opens the door for the DI to benefit at the expense of the QI. The simulation highlights that market depth, price informativeness, and price volatility unambiguously decrease, while return volatility increases with greater automation by the quant. This is driven by lower trading volume for the quant, greater trading volume for the DI, and lower trading volume for the speculator sector as a whole. Concurrently, profits for the quant unambiguously fall, while profits for the discretionary investor and the speculator sector as a whole increase with greater automation. The DI is able to exploit the quant's greater reliance on the backtest, which is myopic to the DI's strategic behavior. In all, greater automation by the quant, which results in less aggressive trading, leads to deterioration in market quality.

An important takeaway from the simulation above is that the backtest is more likely to perform well when the quant's trading wedge is positive. Even though greater reliance on the backtest implies greater deviation from a profit-maximizing strategy, a positive trading

wedge leads to greater profits for the fully-automated quant (E4>E2) in roughly one-third of the simulations. Furthermore, the profit pattern is either increasing or hump-shaped in roughly two-thirds of the simulations, suggesting that either full automation or an intermediate level of automation can lead to greater profits for the quant relative to the discretionary strategy. This is in stark contrast to the outcomes of a negative trading wedge, whereby the quant's profits always fall with greater automation, while the DI benefits at her expense. Therefore, conditions under which the trading wedge is positive are also the conditions under which the corresponding backtested strategies are more likely to be successful.

3.2.3 Empirical implications

Our theory provides guidance for future empirical research on the effects of quantitative investing on financial market quality, instructs policy makers on the effects of quant investing on financial stability, and directs portfolio managers to market environments ripe for quantitative strategies. We summarize those key, novel insights below.

First, our theory highlights the differential effects of quantitative investing when trading against relatively uninformed vs. relatively informed discretionary investors. In particular, our model suggests that quants are more likely to improve market quality when trading against uninformed vs. informed DIs, a prediction that can be tested both in the time-series and the cross-section. Our model further suggests that error correlation ρ — a measure of disagreement in the financial markets — is an important determinant of these effects. Such a relationship could be investigated in correspondence with macroeconomic cycles: To the extent that recessions correspond to periods of disagreement (i.e., $\rho < 0$), our model predicts that quantitative investors are more likely to benefit market quality (i.e., the trading wedge is more likely to be positive). In addition, when trading against relatively uninformed discretionary investors in periods of disagreement, quantitative investing unambiguously improves market quality, and only highly-informed DIs can realistically adversely

affect market quality. The latter point is consistent with the evidence in Abis (2022) who finds that quants are better stock-pickers and worse market-timers than traditional investors during recessions.

From a policy perspective, our analysis highlights some conditions under which quantitative investing may have detrimental effects on the financial markets. Sophisticated discretionary investors are more likely to take advantage of quant funds by exploiting their reliance on the backtested strategy. Stable and expansionary periods within which agreement is high are also environments within which quant investing is more likely to negatively affect market quality. Finally, uninformed quant funds are more prone to exploitation. The latter point is of particular concern given the proliferation of index funds, many of which (especially the most basic broad-market value-weighted funds), can be viewed as quantitative funds with low signal precision.

Finally, from an investment management perspective, our model highlights conditions under which quantitative investment strategies based on backtests are more likely to be successful. Although the backtested strategy is highly stylized in our framework, it does capture the key dimension of using historical data to develop a profit-maximizing trading rule. The conditions which result in a positive trading wedge for the quant, are precisely the conditions under which the backtested strategy is more likely to perform well. Specifically, as gleamed from the determinants of the quant's trading wedge, markets with high disagreement are generally more likely to be conducive to backtested strategies. Furthermore, the information quality of the traditional investors is important. To the extent that the market is saturated by relatively uninformed investors, the bar for a successful quant strategy is lower, and the likelihood of success rises in the precision of the quant's information. Conversely, if the market is populated by relatively informed investors, the bar for success is higher, and its probability of success is falling in the quant's signal precision.

4 Conclusion

Quantitative investment strategies are playing an increasingly important role in the financial markets. What was previously available primarily via hedge funds to a select clientele is now widely accessible for household investments via mutual funds and smart-beta ETFs. The effects of the growth in quantitative investing on the quality of the financial market is not immediate. Quantitative trading is inherently disciplined by backtesting. Precisely due to the reliance on backtesting, quantitative investing may not fully incorporate the strategic trading behavior of other market participants. We develop a model of strategic speculation that captures the potential myopia of quantitative funds via their reliance on a backtested trading strategy – i.e., one assuming by definition that other market participants are unaware of the quant's existence.

The introduction of the quantitative investor to our model broadly benefits market quality. However, greater automation by the quant (i.e., a greater reliance on the backtest) may have disparate effects on market quality. If greater adherence to backtesting results in greater trading intensity for the quant, market quality improves as the strategic market participants are unable to take advantage of the quant's myopia. Conversely, if greater automation leads to less trading by the quant, the fully-strategic speculators may take advantage of the quant's myopia, leading to worse market quality. Importantly, many of these effects depend on potentially observable fund characteristics, such as the relative precision and correlation of the information of discretionary and quant investors.

Our analysis provides a novel theoretical framework to guide future empirical investigations of the implications of the growth in quantitative investing — and its strategic interactions with more traditional investment styles — for the process of price formation in financial markets. In particular, our model emphasizes the importance of relative signal precision and signal correlation among discretionary and quantitative investors when assessing the effects of quantitative investing on market quality. For example, smart-beta ETFs are

generally deemed to be less informed than proprietary quant traders, and the correlation of their trading strategies may change over time with materially different implications for the liquidity, volatility, and informativeness of the affected markets. Future research bringing our model to the data has the potential to shed light on the empirical effects of quantitative investing on the trading behavior of the financial markets.

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Tables and Figures

	Incr.	Decr.	Hump	U	E1<E2	E1<E4	E2>E4	E3>E1 (%γ)	E3<E2 (%γ)	E3>E4 (%γ)	% Sim.
All	43.9	44.6	0.0	11.5	27.1	24.6	46.8	25.5	48.9	45.2	100.0
$\beta_{q4} > \beta_{q2}$	75.1	5.5	0.0	19.5	0.0	0.0	9.0	0.0	12.7	6.4	58.2
$\beta_{q4} \leq \beta_{q2}$	0.5	99.1	0.0	0.5	64.8	58.8	99.4	60.9	99.5	99.2	41.8

(a) Market depth

	Incr.	Decr.	Hump	U	E1<E2	E1<E4	E2>E4	E3>E1 (%γ)	E3<E2 (%γ)	E3>E4 (%γ)	% Sim.
All	57.1	42.9	0.0	0.0	100.0	100.0	42.9	100.0	42.9	42.9	100.0
$\beta_{q4} > \beta_{q2}$	98.0	2.0	0.0	0.0	100.0	100.0	2.0	100.0	2.0	2.0	58.2
$\beta_{q4} \leq \beta_{q2}$	0.1	99.9	0.0	0.0	100.0	100.0	99.9	100.0	99.9	99.9	41.8

(b) Price informativeness

	Incr.	Decr.	Hump	U	E1<E2	E1<E4	E2>E4	E3>E1 (%γ)	E3<E2 (%γ)	E3>E4 (%γ)	% Sim.
All	57.1	42.9	0.0	0.0	100.0	100.0	42.9	100.0	42.9	42.9	100.0
$\beta_{q4} > \beta_{q2}$	98.0	2.0	0.0	0.0	100.0	100.0	2.0	100.0	2.0	2.0	58.2
$\beta_{q4} \leq \beta_{q2}$	0.1	99.9	0.0	0.0	100.0	100.0	99.9	100.0	99.9	99.9	41.8

(c) Price volatility

	Incr.	Decr.	Hump	U	E1<E2	E1<E4	E2>E4	E3>E1 (%γ)	E3<E2 (%γ)	E3>E4 (%γ)	% Sim.
All	42.9	57.1	0.0	0.0	0.0	0.0	57.1	0.0	57.1	57.1	100.0
$\beta_{q4} > \beta_{q2}$	2.0	98.0	0.0	0.0	0.0	0.0	98.0	0.0	98.0	98.0	58.2
$\beta_{q4} \leq \beta_{q2}$	99.9	0.1	0.0	0.0	0.0	0.0	0.1	0.0	0.1	0.1	41.8

(d) Return volatility

Table 1: Simulation Results for Market Quality Measures

We use 100,000 parameter draws: $\rho \sim U(-1,1), \phi_d \sim U(0,1), \phi_q \sim U(0,1), \eta \sim U(0,2), P_0 = 0, \sigma_v^2 = 1$. For Economy 3, we evaluate market quality for 99 values of γ from 1% to 99% at 1% increments. Numbers represent the percent of simulations such that the condition in the header is satisfied (Col. 1-7) or the average fraction of γ's across all simulations such that the condition satisfied (Col. 8-10). "E" stands for Economy. First four columns document the shape of the market quality measure in Economy 3 for increasing γ.

	Incr.	Decr.	Hump	U	E1<E2	E1<E4	E2>E4	E3>E1 (%γ)	E3<E2 (%γ)	E3>E4 (%γ)	% Sim.
All	57.7	42.1	0.0	0.2	84.3	94.6	42.1	93.2	42.2	42.1	100.0
$\beta_{q4} > \beta_{q2}$	99.2	0.4	0.0	0.4	73.1	90.9	0.6	88.4	0.7	0.5	58.2
$\beta_{q4} \leq \beta_{q2}$	0.0	100.0	0.0	0.0	99.9	99.9	100.0	99.9	100.0	100.0	41.8

(a) Trading volume: speculator sector

	Incr.	Decr.	Hump	U	E1<E2	E1<E4	E2>E4	E3>E1 (%γ)	E3<E2 (%γ)	E3>E4 (%γ)	% Sim.
All	58.8	25.1	0.0	16.0	0.0	1.7	31.4	1.1	34.4	26.9	100.0
$\beta_{q4} > \beta_{q2}$	29.4	43.1	0.0	27.5	0.0	2.9	53.9	1.9	59.1	46.1	58.2
$\beta_{q4} \leq \beta_{q2}$	99.8	0.1	0.0	0.0	0.0	0.0	0.1	0.0	0.1	0.1	41.8

(b) Trading volume: discretionary investor

	Incr.	Decr.	Hump	U	E1<E2	E1<E4	E2>E4	E3>E1 (%γ)	E3<E2 (%γ)	E3>E4 (%γ)	% Sim.
All	57.1	42.9	0.0	0.0	100.0	100.0	42.9	100.0	42.9	42.9	100.0
$\beta_{q4} > \beta_{q2}$	98.0	2.0	0.0	0.0	100.0	100.0	2.0	100.0	2.0	2.0	58.2
$\beta_{q4} \leq \beta_{q2}$	0.1	99.9	0.0	0.0	100.0	100.0	99.9	100.0	99.9	99.9	41.8

(c) Trading volume: quantitative investor

Table 2: Simulation Results for the Speculator Sector Trading Intensity

We use 100,000 parameter draws: $\rho \sim U(-1,1), \phi_d \sim U(0,1), \phi_q \sim U(0,1), \eta \sim U(0,2), P_0 = 0, \sigma_v^2 = 1$. For Economy 3, we evaluate trading volume for 99 values of γ from 1% to 99% at 1% increments. Numbers represent the percent of simulations such that the condition in the header is satisfied (Col. 1-7) or the average fraction of γ's across all simulations such that the condition is satisfied (Col. 8-10). "E" stands for Economy. First four columns document the shape of the market quality measure in Economy 3 for increasing γ.

	Incr.	Decr.	Hump	U	E1<E2	E1<E4	E2>E4	E3>E1 (%γ)	E3<E2 (%γ)	E3>E4 (%γ)	% Sim.
All	44.6	43.9	11.5	0.0	72.9	75.4	53.2	74.5	51.1	54.8	100.0
$\beta_{q4} > \beta_{q2}$	5.5	75.1	19.5	0.0	100.0	100.0	91.0	100.0	87.3	93.6	58.2
$\beta_{q4} \leq \beta_{q2}$	99.1	0.5	0.5	0.0	35.2	41.2	0.6	39.1	0.5	0.8	41.8

(a) Expected profits: speculator sector

	Incr.	Decr.	Hump	U	E1<E2	E1<E4	E2>E4	E3>E1 (%γ)	E3<E2 (%γ)	E3>E4 (%γ)	% Sim.
All	57.0	33.0	0.0	10.0	2.8	7.6	37.9	6.3	39.5	34.3	100.0
$\beta_{q4} > \beta_{q2}$	26.2	56.6	0.0	17.2	4.7	12.9	65.1	10.7	67.8	58.9	58.2
$\beta_{q4} \leq \beta_{q2}$	100.0	0.0	0.0	0.0	0.1	0.1	0.0	0.1	0.0	0.0	41.8

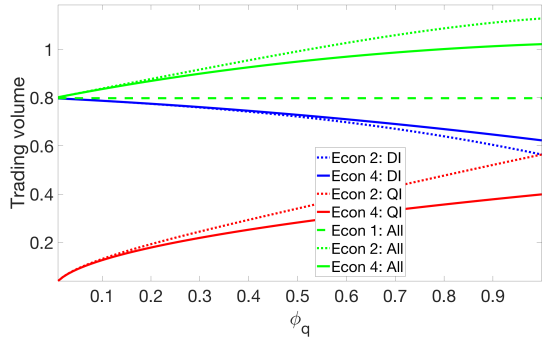
(b) Expected profits: discretionary investor

	Incr.	Decr.	Hump	U	E1<E2	E1<E4	E2>E4	E3>E1 (%γ)	E3<E2 (%γ)	E3>E4 (%γ)	% Sim.
All	13.1	62.4	24.5	0.0	100.0	98.3	80.9	98.9	76.1	85.2	100.0
$\beta_{q4} > \beta_{q2}$	22.4	35.4	42.2	0.0	100.0	97.1	67.3	98.1	59.1	74.7	58.2
$\beta_{q4} \leq \beta_{q2}$	0.1	99.9	0.0	0.0	100.0	100.0	99.9	100.0	99.9	99.9	41.8

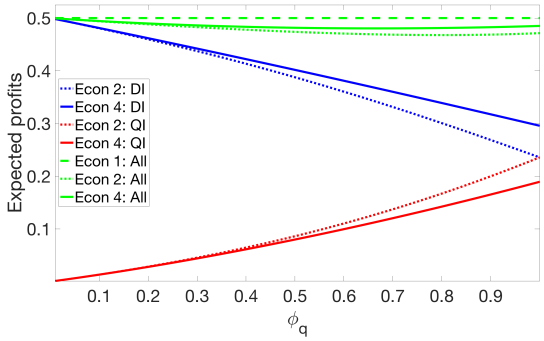
(c) Expected profits: quantitative investor

Table 3: Simulation Results for the Speculator Sector Profits

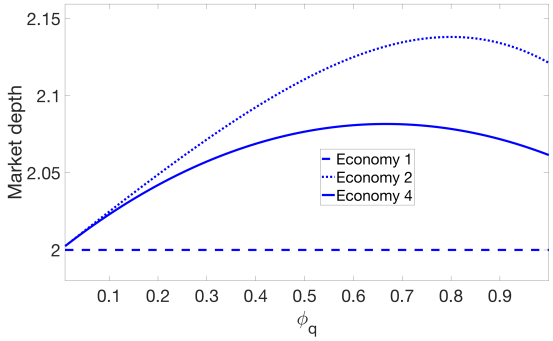
We use 100,000 parameter draws: $\rho \sim U(-1,1), \phi_d \sim U(0,1), \phi_q \sim U(0,1), \eta \sim U(0,2), P_0 = 0, \sigma_v^2 = 1$. For Economy 3, we evaluate expected profits for 99 values of γ from 1% to 99% at 1% increments. Numbers represent the percent of simulations such that the condition in the header is satisfied (Col. 1-7) or the average fraction of γ's across all simulations such that the condition is satisfied (Col. 8-10). "E" stands for Economy. First four columns document the shape of the market quality measure in Economy 3 for increasing γ.



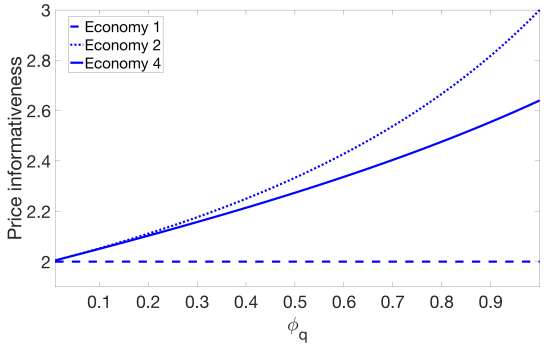
(a) Expected trading volume for speculator sector



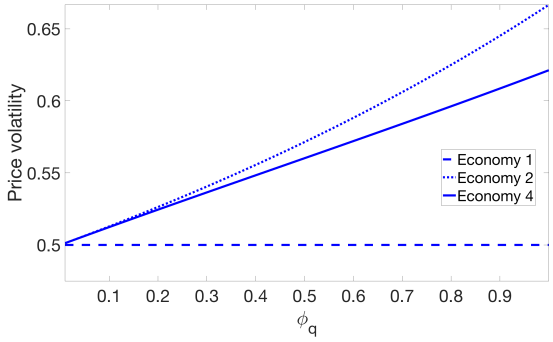
(b) Expected trading profits for speculator sector



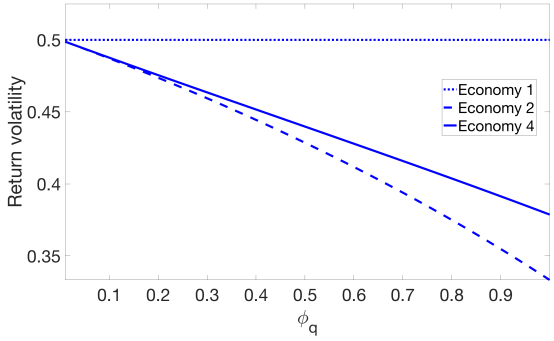
(c) Market depth in Economy j: $1/\lambda_j$



(d) Price informativeness in Economy j: $\text{Var}(v|P_j)^{-1}$

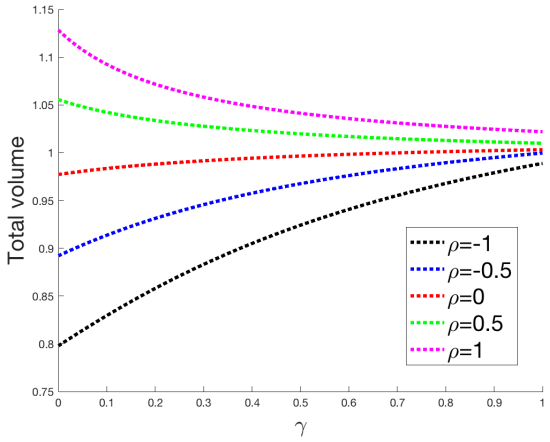


(e) Price volatility in Economy j: $\text{Var}(P_j)$

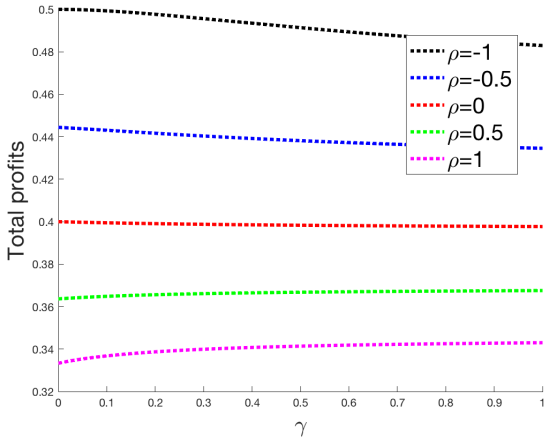


(f) Return volatility in Economy j: $\text{Var}(v - P_j)$

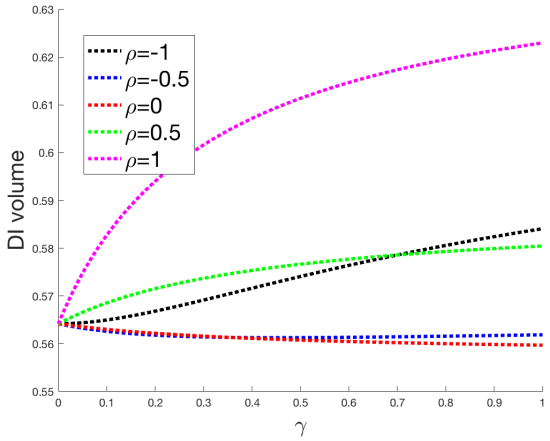
Figure 1: Effect of Quantitative Investing on Market Quality by Quant's Information Quality.



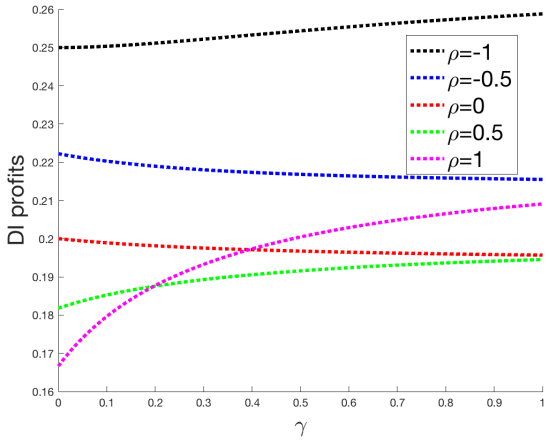
(a) Expected trading volume for speculator sector



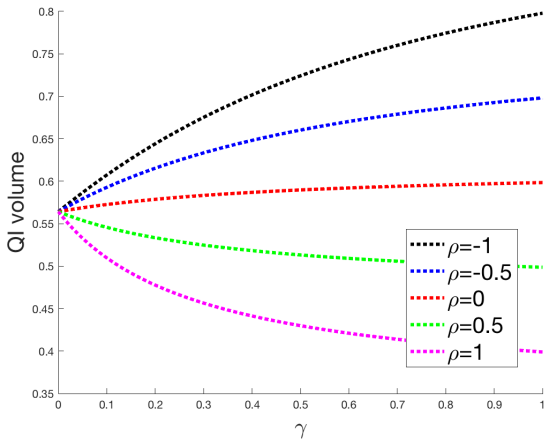
(b) Expected trading profits for speculator sector



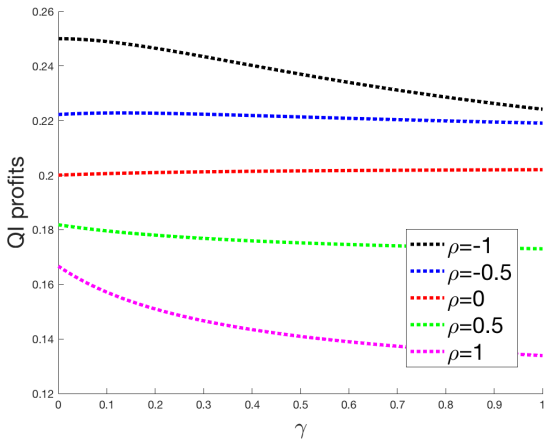
(c) Expected trading volume for DI



(d) Expected trading profits for DI

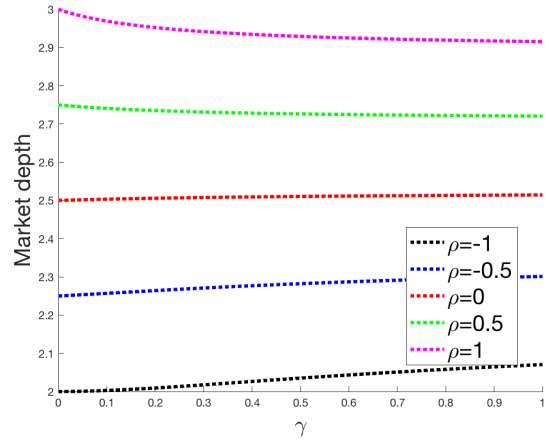


(e) Expected trading volume for QI

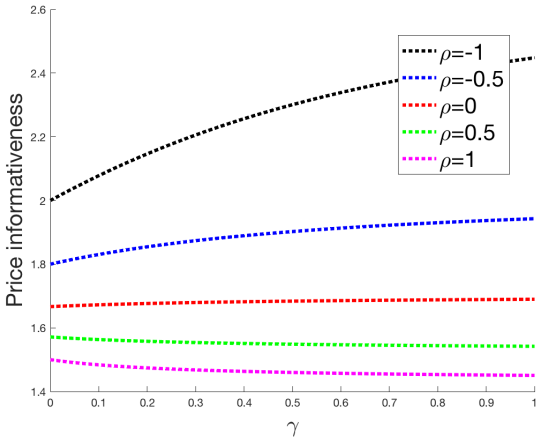


(f) Expected trading profits for QI

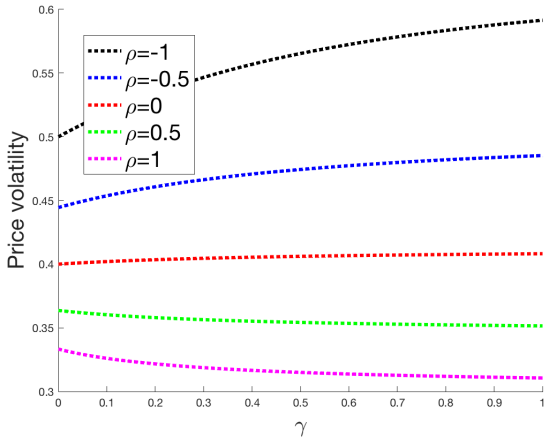
Figure 2: Effect of Signal Correlation on Speculator Sector Trading Volume and Profits. Figures are constructed for the generalized model and assume equal information quality for both the discretionary investor and the quantitative investor: $\phi_d = \phi_q = 0.5$.



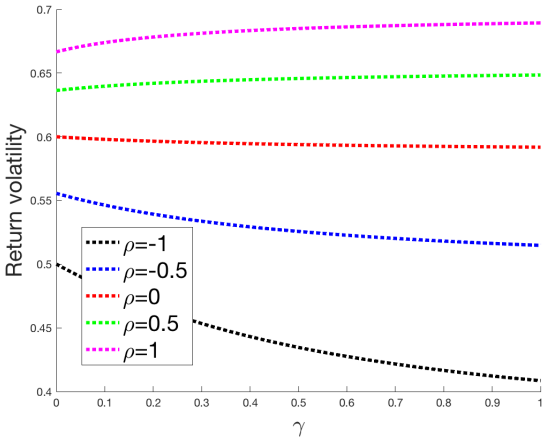
(a) Market depth



(b) Price informativeness

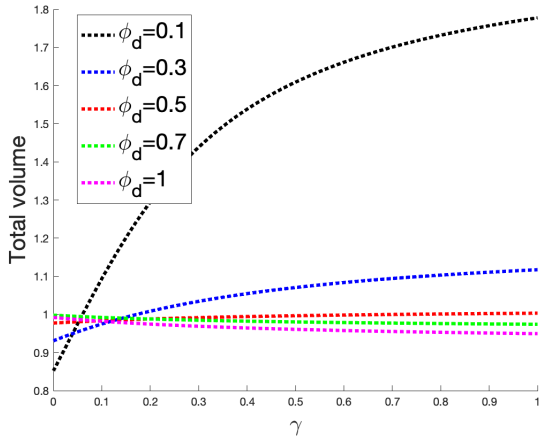


(c) Price volatility

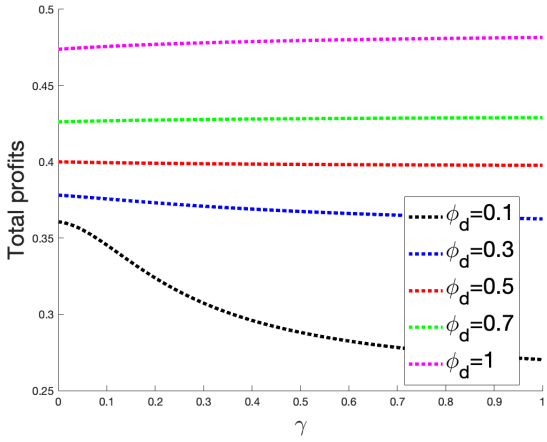


(d) Return volatility

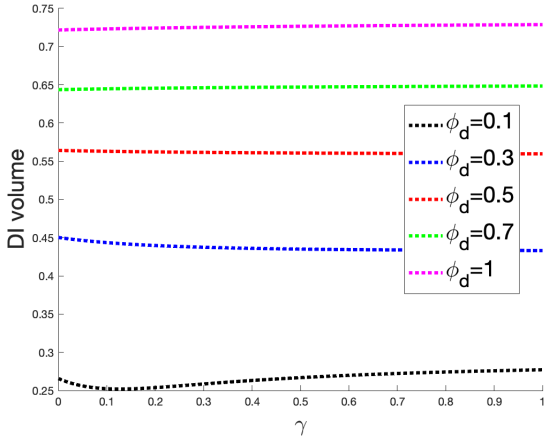
Figure 3: Effect of Signal Correlation on Market Quality.
Figures are constructed for the generalized model and assume equal information quality for both the discretionary investor and the quantitative investor: $\phi_d = \phi_q = 0.5$.



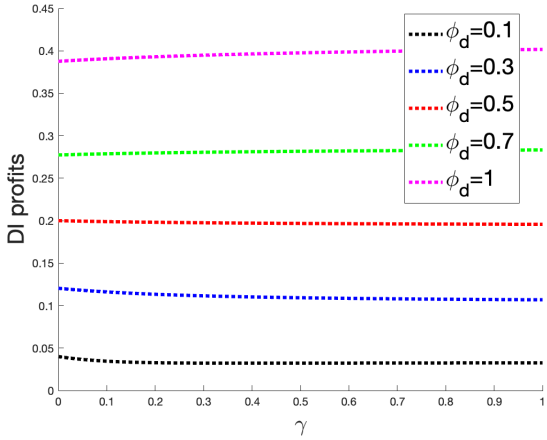
(a) Expected trading volume for speculator sector



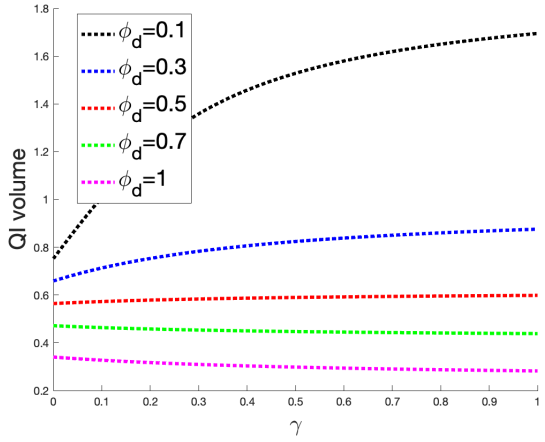
(b) Expected trading profits for speculator sector



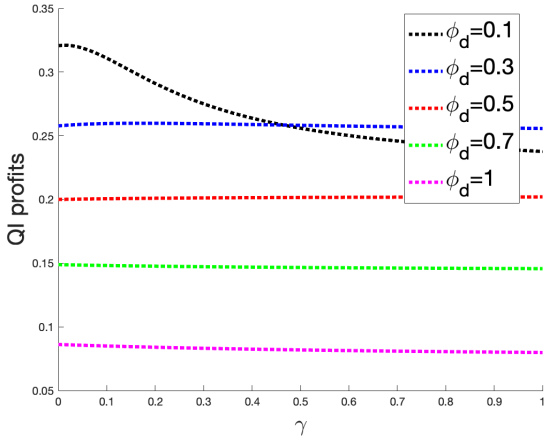
(c) Expected trading volume for DI



(d) Expected trading profits for DI

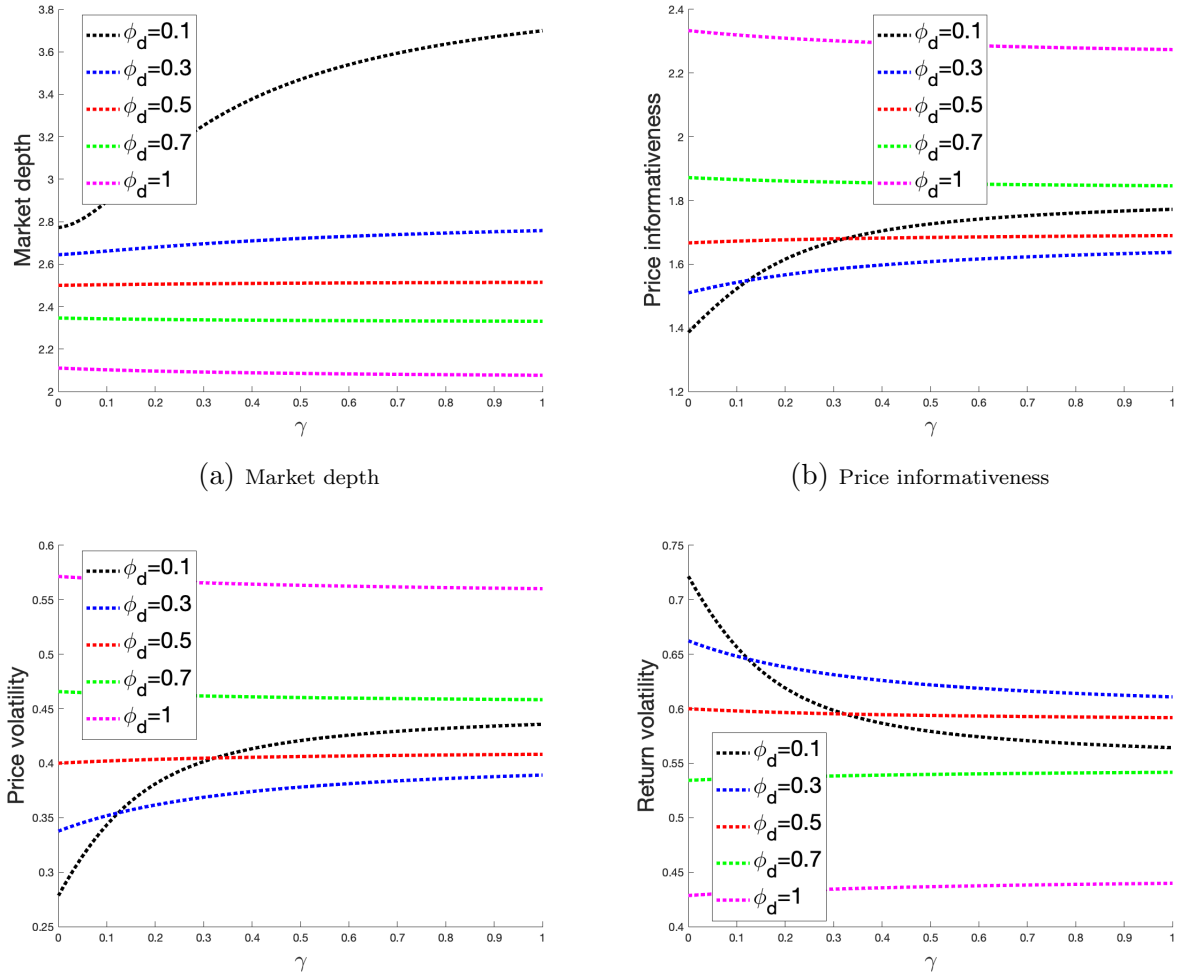


(e) Expected trading volume for QI



(f) Expected trading profits for QI

Figure 4: Effect of Relative Signal Precision on the Speculator Sector.
Figures are constructed for the generalized model, assume uncorrelated signals for the discretionary and the quantitative investor, and allow for potentially inferior or superior information for the discretionary investor: $\phi_q = 0.5, \rho = 0$.



(a) Market depth

(b) Price informativeness

(c) Price volatility

(d) Return volatility

Figure 5: Effect of Relative Signal Precision on Market Quality

Figures are constructed for the generalized model, assume uncorrelated signals for the discretionary and the quantitative investor, and allow for potentially inferior or superior information for the discretionary investor: $\phi_q = 0.5, \rho = 0$.

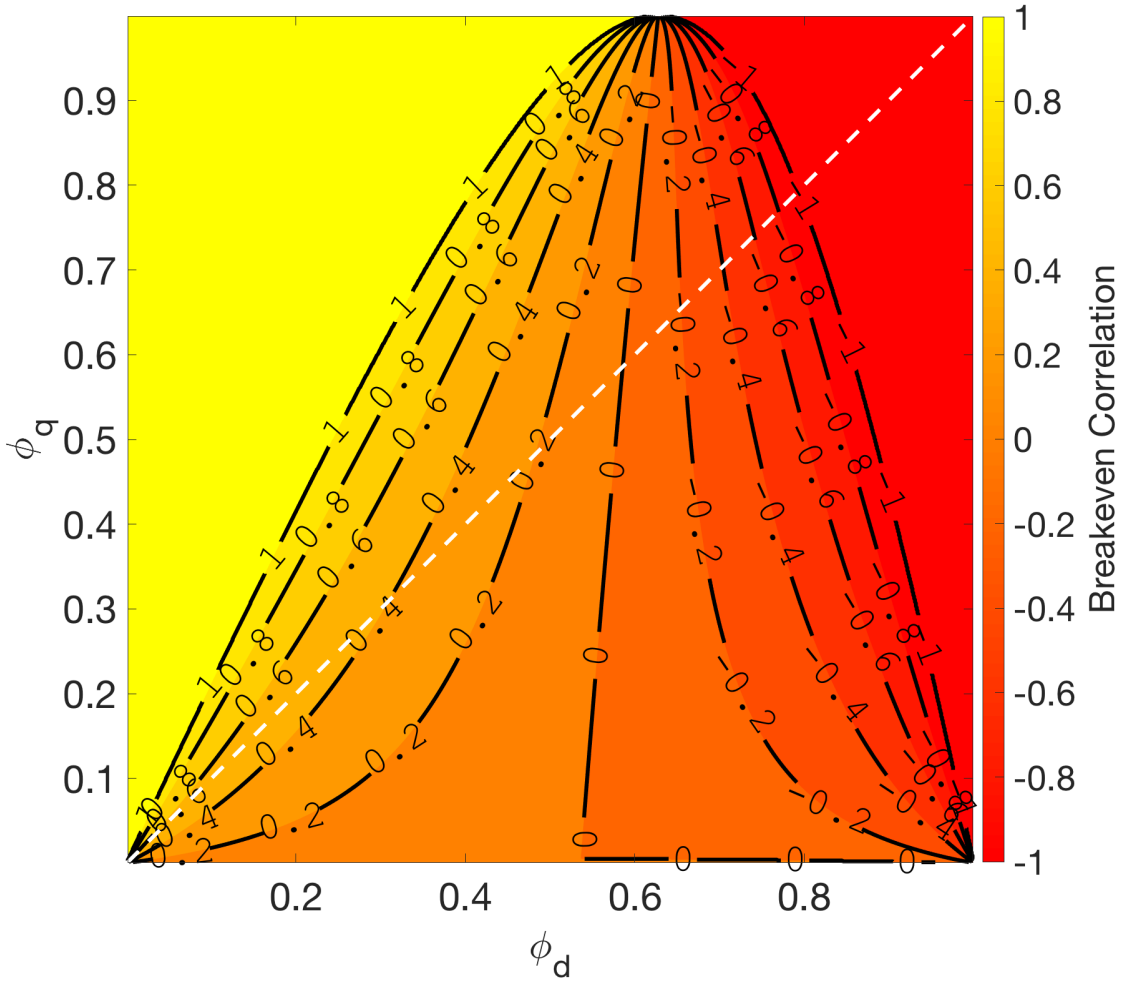


Figure 6: Breakeven Correlation for the Quant's Trading Wedge

The contour plot demonstrates the maximum signal correlation levels such that the fully discretionary quant (Economy 2) trades less than the fully automated quant (Economy 4) for varying levels of signal precision for the DI and QI. The white dotted line highlights equal signal precision for the DI and QI, with the region above signifying an informational advantage for the quant, and the region below an informational advantage for the discretionary investor.

Appendix A Proof of Lemma 1

We conjecture that the DI's demand and MM's price functions are governed by Eq. (5) and (6). Given rational expectations, the DI's problem simplifies to:

$$\begin{aligned} x_d &= \arg\max_{\tilde{x}} \mathbb{E}[\tilde{x}(v - P_1)|s_d] \\ &= \arg\max_{\tilde{x}} \mathbb{E}[\tilde{x}(v - \lambda_1\omega_1)|s_d] \\ &= \arg\max_{\tilde{x}} \mathbb{E}[\tilde{x}(v - \lambda_1(\tilde{x} + z))|s_d] \\ &= \arg\max_{\tilde{x}} \mathbb{E}[\tilde{x}(v - \lambda_1 z) - \lambda_1\tilde{x}^2|s_d] \\ &= \arg\max_{\tilde{x}} \left\{\tilde{x}\mathbb{E}[v|s_d] - \lambda_1\tilde{x}^2\right\} \\ &= \frac{\mathbb{E}[v|s_d]}{2\lambda_1} \\ &= \frac{\phi_d}{2\lambda_1}s_d \\ &\equiv \beta_{d1}s_d \end{aligned} \tag{17}$$

Similarly, the market-maker's problem becomes

$$\begin{aligned} P_1(\omega_1) &= \mathbb{E}[v|\omega_1] \\ &= \mathbb{E}[v|x_d + z] \\ &= \mathbb{E}[v|\beta_{d1}(v + e_d) + z] \\ &= \frac{\beta_{d1}\sigma_v^2}{\beta_{d1}^2(\sigma_v^2 + \sigma_{e_d}^2) + \sigma_z^2}\omega_1 \\ &= \frac{\beta_{d1}\sigma_v^2}{\beta_{d1}^2(\sigma_v^2 + \sigma_{e_d}^2) + \sigma_z^2}\omega_1 \\ &\equiv \lambda_1\omega_1 \end{aligned} \tag{18}$$

We first solve for λ_1 utilizing Eq. (17) and (18). From Eq. (17):

$$\begin{aligned} \beta_{d1} &= \frac{\phi_d}{2\lambda_1} \\ &\Downarrow \\ \frac{1}{\lambda_1} &= \frac{2\beta_{d1}}{\phi_d} \end{aligned} \tag{19}$$

From Eq. (18)

$$\begin{aligned} \lambda_1 &= \frac{\beta_{d1}\sigma_v^2}{\beta_{d1}^2(\sigma_v^2+\sigma_{e_d}^2)+\sigma_z^2} \\ &\Downarrow \\ \frac{1}{\lambda_1} &= \frac{\beta_{d1}}{\phi_d}+\frac{\eta}{\beta_{d1}} \end{aligned} \tag{20}$$

Setting Eq. (19) into (20) equal

$$\begin{aligned} \frac{\beta_{d1}}{\phi_d}+\frac{\eta}{\beta_{d1}} &= \frac{2\beta_{d1}}{\phi_d} \\ &\Downarrow \\ \frac{\eta}{\beta_{d1}} &= \frac{\beta_{d1}}{\phi_d} \\ &\Downarrow \\ \beta_{d1} &= \pm\sqrt{\eta\phi_d} \\ &\Downarrow \\ \lambda_1 &= \pm\frac{1}{2}\sqrt{\frac{\phi_d}{\eta}} \end{aligned}$$

By the second order condition of the DI's optimization function given by Eq. (17), $\lambda_1 > 0$, which, by Eq. (19), implies that $\beta_{d1} > 0$. Therefore:

$$\begin{aligned} \beta_{d1} &= \sqrt{\eta\phi_d} \\ \lambda_1 &= \frac{1}{2}\sqrt{\frac{\phi_d}{\eta}} \end{aligned}$$

Appendix B Derivation of Backtest: Eq. (9)

The QI feeds the DI's trading strategy and the MM's pricing rule from Economy 1 given by Eq. (7) and (8) into her optimization problem:

$$
\begin{aligned}
x_b &= \arg\max_{\tilde{x}_b} \mathbb{E}\Big[\tilde{x}_b\Big(v - P_1\big(x_{d1} + z + \tilde{x}_b\big)\Big)|s_q\Big] \\
&= \arg\max_{\tilde{x}_b} \mathbb{E}\left[\tilde{x}_b\left(v - \frac{1}{2}\sqrt{\frac{\phi_d}{\eta}}\Big(x_{d1} + z + \tilde{x}_b\Big)\right)|s_q\right] \\
&= \arg\max_{\tilde{x}_b} \mathbb{E}\left[\tilde{x}_b\left(v - \frac{1}{2}\sqrt{\frac{\phi_d}{\eta}}\Big(\sqrt{\phi_d\eta}s_d + z + \tilde{x}_b\Big)\right)|s_q\right] \\
&= \arg\max_{\tilde{x}_b} \mathbb{E}\left[\tilde{x}_b\left(v - \frac{1}{2}\sqrt{\frac{\phi_d}{\eta}}\Big(\sqrt{\phi_d\eta}(v + e_d) + z + \tilde{x}_b\Big)\right)|s_q\right] \\
&= \arg\max_{\tilde{x}_b} \left(\mathbb{E}[v|s_q] - \frac{\phi_d}{2}\Big(\mathbb{E}[v|s_q] + \mathbb{E}[e_d|s_q]\Big)\right)\tilde{x}_b - \frac{1}{2}\sqrt{\frac{\phi_d}{\eta}}\tilde{x}_b^2 \\
&= \arg\max_{\tilde{x}_b} \left(\frac{2-\phi_d}{2}\mathbb{E}[v|s_q] - \frac{\phi_d}{2}\mathbb{E}[e_d|s_q]\right)\tilde{x}_b - \frac{1}{2}\sqrt{\frac{\phi_d}{\eta}}\tilde{x}_b^2 \\
&= \sqrt{\frac{\eta}{\phi_d}}\left(\frac{2-\phi_d}{2}\mathbb{E}[v|s_q] - \frac{\phi_d}{2}\mathbb{E}[e_d|s_q]\right) \\
&\text{(...note that } \mathbb{E}[v|s_q]{=}\phi_q s_q \text{ and } \mathbb{E}[e_d|s_q]{=}\mathbb{E}[e_d|v{+}e_q]{=}\sigma_{e_d e_q}/\sigma^2_{s_q}s_q \\
&{=}\rho\sigma_{e_d}\sigma_{e_q}/\sigma^2_{s_q}s_q{=}\rho(\sigma_{e_d}/\sigma_{s_q})(\sigma_{e_q}/\sigma_{s_q})s_q{=}\rho\sqrt{\tfrac{\phi_q(1-\phi_d)(1-\phi_q)}{\phi_d}}s_q\ldots) \\
&= \frac{1}{2}\sqrt{\frac{\eta}{\phi_d}}\left((2-\phi_d)\phi_q - \rho\sqrt{\phi_d\phi_q(1-\phi_d)(1-\phi_q)}\right)s_q
\end{aligned}
$$

Appendix C Proof of Lemma 2

C.1 Economy 2. Fully-strategic Quant, $\gamma = 0$

This is the case of two strategic speculators $i \in \{d, q\}$. The problem is fully symmetric for both strategic speculators, therefore we solve it from the QI's perspective and reverse

notation for the DI. In what follows, note that:

$$\begin{aligned}
\mathbb{E}[v|s_q] &= \phi_q s_q \\
\mathbb{E}[v|s_d] &= \phi_d s_d \\
\mathbb{E}[e_d|s_q] &= \rho\sqrt{\frac{\phi_q(1-\phi_d)(1-\phi_q)}{\phi_d}}s_q \\
&\equiv \nu_q s_q \\
\mathbb{E}[e_q|s_d] &= \rho\sqrt{\frac{\phi_d(1-\phi_d)(1-\phi_q)}{\phi_q}}s_d \\
&\equiv \nu_d s_d
\end{aligned}$$

The QI chooses demand x_q to maximize the following expectation:

$$x_{q2} = \arg\max_{\tilde{x}_q} \mathbb{E}[\tilde{x}_q(v - P_2)|s_q] \tag{21}$$

For both speculators we conjecture a demand function linear in the signal:

$$x_{i2}(s_i) = \beta_{i2} s_i \tag{22}$$

Noise traders supply z shares and market-makers set prices according to the aggregate order flow:

$$\omega_2 = x_{d2} + x_{q2} + z \tag{23}$$

The market makers in the economy are competitive and risk-neutral and are assumed to have a pricing rule linear in the aggregate order flow:

$$\begin{aligned}
P_2(\omega_2) &= \mathbb{E}[v|\omega_2] \\
&= \lambda_2 \omega_2
\end{aligned} \tag{24}$$

We first consider the QI's problem:

$$\begin{aligned}
x_{q2} &= \arg\max_{\tilde{x}_{q2}} \mathbb{E}[\tilde{x}_{q2}(v - P_2(\omega_2))|s_q] \\
&= \arg\max_{\tilde{x}_{q2}} \mathbb{E}[\tilde{x}_{q2}(v - \lambda_2\omega_2)|s_q] \\
&= \arg\max_{\tilde{x}_{q2}} \mathbb{E}[\tilde{x}_{q2}(v - \lambda_2(x_{d2} + \tilde{x}_{q2} + z))|s_q] \\
&= \arg\max_{\tilde{x}_{q2}} \mathbb{E}[\tilde{x}_{q2}(v - \lambda_2 x_{d2} - \lambda_2 z) - \lambda_2\tilde{x}_{q2}^2|s_q] \\
&= \arg\max_{\tilde{x}_{q2}} \{\tilde{x}_{q2}(\mathbb{E}[v|s_q] - \lambda_2\mathbb{E}[x_{d2}|s_q]) - \lambda\tilde{x}_{q2}^2\} \\
&= \frac{\mathbb{E}[v|s_q] - \lambda_2\mathbb{E}[x_{d2}|s_q]}{2\lambda_2} \\
&= \frac{\mathbb{E}[v|s_q] - \lambda_2\beta_{d2}\mathbb{E}[s_d|s_q]}{2\lambda_2} \\
&= \frac{\mathbb{E}[v|s_q] - \lambda_2\beta_{d2}\mathbb{E}[v + e_d|s_q]}{2\lambda_2} \\
&= \frac{(1 - \lambda_2\beta_{d2})\mathbb{E}[v|s_q] - \lambda_2\beta_{d2}\mathbb{E}[e_d|s_q]}{2\lambda_2} \\
&= \left(\frac{\phi_q}{2\lambda_2} - \frac{\phi_q\beta_{d2}}{2} - \frac{\nu_q\beta_{d2}}{2}\right)s_q \\
&\equiv \beta_{q2}s_q
\end{aligned} \tag{25}$$

Similarly, for the DI:

$$\begin{aligned}
x_{d2} &= \arg\max_{\tilde{x}_{d2}} \mathbb{E}[\tilde{x}_{d2}(v - P_2(\omega_2))|s_d] \\
&= \left(\frac{\phi_d}{2\lambda_2} - \frac{\phi_d\beta_{q2}}{2} - \frac{\nu_d\beta_{q2}}{2}\right)s_d \\
&\equiv \beta_{d2}s_d
\end{aligned} \tag{26}$$

Finally, the competitive market maker sets prices according to:

$$\begin{aligned}
P_2(\omega_2) &= \mathbb{E}[v|\omega_2] \\
&= \mathbb{E}[v|x_{d2} + x_{q2} + z] \\
&= \mathbb{E}[v|\beta_{d2}(v + e_d) + \beta_{q2}(v + e_q) + z] \\
&= \frac{(\beta_{d2} + \beta_{q2})\sigma_v^2}{(\beta_{d2} + \beta_{q2})^2\sigma_v^2 + \beta_{d2}^2\sigma_{e_d}^2 + \beta_{q2}^2\sigma_{e_q}^2 + 2\beta_{d2}\beta_{q2}\sigma_{e_de_q} + \sigma_z^2}\omega_2 \\
&\equiv \lambda_2\omega_2
\end{aligned} \tag{27}$$

We have three equations and three unknowns:

$$\lambda_2 = \frac{(\beta_{d2}+\beta_{q2})\sigma_v^2}{(\beta_{d2}+\beta_{q2})^2\sigma_v^2 + \beta_{d2}^2\sigma_{e_d}^2 + \beta_{q2}^2\sigma_{e_q}^2 + 2\beta_{d2}\beta_{q2}\sigma_{e_de_q} + \sigma_z^2}$$

$$\left(\text{ note that } \sigma_{e_de_q}/\sigma_v^2 = \rho(\sigma_{e_d}/\sigma_v)(\sigma_{e_q}/\sigma_v) = \rho\sqrt{\frac{(1-\phi_d)(1-\phi_q)}{\phi_d\phi_q}} = \nu_q/\phi_q = \nu_d/\phi_d \right) \quad (28)$$

$$\Downarrow$$

$$\frac{1}{\lambda_2} = \beta_{d2}+\beta_{q2}+\frac{\beta_{d2}^2}{\beta_{d2}+\beta_{q2}}\frac{1-\phi_d}{\phi_d}+\frac{\beta_{q2}^2}{\beta_{d2}+\beta_{q2}}\frac{1-\phi_q}{\phi_q}+\frac{2\beta_{d2}\beta_{q2}}{\beta_{d2}+\beta_{q2}}\frac{\nu_q}{\phi_q}+\frac{\eta}{\beta_{d2}+\beta_{q2}}$$

$$\beta_{d2} = \frac{\phi_d}{2\lambda_2} - \frac{\phi_d\beta_{q2}}{2} - \frac{\nu_d\beta_{q2}}{2} \quad (29)$$

$$\beta_{q2} = \frac{\phi_q}{2\lambda_2} - \frac{\phi_q\beta_{d2}}{2} - \frac{\nu_q\beta_{d2}}{2} \quad (30)$$

We first utilize the equations 29 and 30 to solve for $1/\lambda_2$ and set the results equal to each other. From equation 29:

$$\begin{aligned} \beta_{d2} &= \frac{\phi_d}{2\lambda_2} - \frac{\phi_d\beta_{q2}}{2} - \frac{\nu_d\beta_{q2}}{2} \\ \Rightarrow \frac{1}{\lambda_2} &= \frac{2\beta_{d2}}{\phi_d} + \beta_{q2} + \frac{\nu_d\beta_{q2}}{\phi_d} \end{aligned} \quad (31)$$

Similarly, from equation 30:

$$\begin{aligned} \beta_{q2} &= \frac{\phi_q}{2\lambda_2} - \frac{\phi_q\beta_{d2}}{2} - \frac{\nu_q\beta_{d2}}{2} \\ \Rightarrow \frac{1}{\lambda_2} &= \frac{2\beta_{q2}}{\phi_q} + \beta_{d2} + \frac{\nu_q\beta_{d2}}{\phi_q} \end{aligned} \quad (32)$$

Equating equations 31 and 32:

$$\begin{aligned} \frac{2\beta_{d2}}{\phi_d} + \beta_{q2} + \frac{\nu_d\beta_{q2}}{\phi_d} &= \frac{2\beta_{q2}}{\phi_q} + \beta_{d2} + \frac{\nu_q\beta_{d2}}{\phi_q} \\ \Rightarrow \beta_{d2} &= \frac{2\phi_d - \phi_d\phi_q - \nu_d\phi_q}{2\phi_q - \phi_d\phi_q - \nu_q\phi_d}\beta_{q2} \\ &\equiv k\beta_{q2} \end{aligned} \quad (33)$$

Substituting β_{d2} from equation 33 into equation 28 and into equation 31, setting the two

equal to each other, we obtain:

$$\beta_{d2}+\beta_{q2}+\frac{\beta_{d2}^2}{\beta_{d2}+\beta_{q2}}\frac{1-\phi_d}{\phi_d}+\frac{\beta_{q2}^2}{\beta_{d2}+\beta_{q2}}\frac{1-\phi_q}{\phi_q}+\frac{2\beta_{d2}\beta_{q2}}{\beta_{d2}+\beta_{q2}}\frac{\nu_q}{\phi_q}+\frac{\eta}{\beta_{d2}+\beta_{q2}}=\frac{2\beta_{d2}}{\phi_d}+\beta_{q2}+\frac{\nu_d\beta_{q2}}{\phi_d}$$

$$\Downarrow$$

$$\frac{\eta}{k+1}\frac{1}{\beta_{q2}}=\frac{2k}{\phi_d}\beta_{q2}+\frac{\nu_d}{\phi_d}\beta_{q2}-k\beta_{q2}-\frac{k^2}{k+1}\frac{1-\phi_d}{\phi_d}\beta_{q2}-\frac{1}{k+1}\frac{1-\phi_q}{\phi_q}\beta_{q2}-\frac{2k}{k+1}\frac{\nu_q}{\phi_q}\beta_{q2}$$

$$\Downarrow$$

$$\frac{\eta}{\beta_{q2}}=\beta_{q2}\left(\frac{2k(k+1)}{\phi_d}+\frac{(k+1)\nu_d}{\phi_d}-k(k+1)-\frac{k^2(1-\phi_d)}{\phi_d}-\frac{1-\phi_q}{\phi_q}-\frac{2k\nu_q}{\phi_q}\right)$$

$$\Downarrow$$

$$\frac{\eta}{\beta_{q2}}=\beta_{q2}\frac{2k(k+1)\phi_q+(k+1)\nu_d\phi_q-k(k+1)\phi_d\phi_q-k^2(1-\phi_d)\phi_q-(1-\phi_q)\phi_d-2k\nu_q\phi_d}{\phi_d\phi_q}$$

$$\Downarrow$$

$$\beta_{q2}=\pm\sqrt{\frac{\eta\phi_d\phi_q}{2k(k+1)\phi_q+(k+1)\nu_d\phi_q-k(k+1)\phi_d\phi_q-k^2(1-\phi_d)\phi_q-(1-\phi_q)\phi_d-2k\nu_q\phi_d}}$$

$$\Downarrow(\text{ note that } \nu_d\phi_q=\nu_q\phi_d)$$

$$\beta_{q2}=\pm\sqrt{\frac{\eta\phi_d\phi_q}{\phi_q k^2+(2\phi_q-\nu_d\phi_q-\phi_d\phi_q)k+\nu_d\phi_q-\phi_d+\phi_d\phi_q}} \tag{34}$$

$$\Downarrow$$

$$\beta_{d2}=\pm\sqrt{\frac{\eta\phi_d\phi_q k^2}{\phi_q k^2+(2\phi_q-\nu_d\phi_q-\phi_d\phi_q)k+\nu_d\phi_q-\phi_d+\phi_d\phi_q}}$$

$$\Downarrow$$

$$\frac{1}{\lambda_2}=\frac{2\beta_{d2}}{\phi_d}+\beta_{q2}+\frac{\nu_d\beta_{q2}}{\phi_d}=\beta_{q2}\left(\frac{2k}{\phi_d}+1+\frac{\nu_d}{\phi_d}\right)=\frac{2k+\phi_d+\nu_d}{\phi_d}\beta_{q2}$$

$$\Downarrow$$

$$\lambda_2=\pm\frac{\phi_d}{2k+\phi_d+\nu_d}\sqrt{\frac{\phi_q k^2+(2\phi_q-\nu_d\phi_q-\phi_d\phi_q)k+\nu_d\phi_q-\phi_d+\phi_d\phi_q}{\eta\phi_d\phi_q}}$$

Furthermore, note that $\lambda_2 > 0$ by the second order condition of the QI and DI as given by Eq. (25) and (26). However, it need not be the case that $\beta_{q2} > 0$ since this is governed by

the sign on $2k + \phi_d + \nu_d$ from Eq. (34), which need not be positive (see below).

$$\begin{aligned}
2k + \phi_d + \nu_d &= \frac{4\phi_d - 2\phi_d\phi_q - 2\nu_d\phi_q}{2\phi_q - \phi_d\phi_q - \nu_q\phi_d} + \phi_d + \nu_d \\
&= \frac{4\phi_d - \phi_d^2\phi_q - \nu_q\phi_d^2 - \phi_d\phi_q\nu_d - \nu_q\nu_d\phi_d}{2\phi_q - \phi_d\phi_q - \nu_q\phi_d} \\
&= \frac{4\phi_d - \phi_d^2\phi_q - 2\phi_d\rho\sqrt{\phi_d\phi_q(1-\phi_d)(1-\phi_q)} - \phi_d\rho^2(1-\phi_d)(1-\phi_q)}{2\phi_q - \phi_d\phi_q - \rho\sqrt{\phi_d\phi_q(1-\phi_d)(1-\phi_q)}} \\
&= \frac{2\phi_d(1 - \rho\sqrt{\phi_d\phi_q(1-\phi_d)(1-\phi_q)}) + \phi_d(1-\phi_d\phi_q) + \phi_d(1-\rho^2(1-\phi_d)(1-\phi_q)}{\phi_q(1-\phi_d) + \phi_q - \rho\sqrt{\phi_d\phi_q(1-\phi_d)(1-\phi_q)}}
\end{aligned} \tag{35}$$

It is immediate that the numerator of equation (35) is always positive since the terms in the parenthesis are positive due to the bounds on ϕ_d, ϕ_q, and ρ. Therefore, the sign of the coefficient linking β_{q2} and λ_2 depends on

$$\begin{aligned}
&\phi_q(2-\phi_d) \vee \rho\sqrt{\phi_d\phi_q(1-\phi_d)(1-\phi_q)} \\
&\Updownarrow \\
f(\phi_q) \equiv \phi_q\big(\phi_d^2(1-\rho^2) + \phi_d(\rho^2-4) + 4\big) - \rho^2\phi_d(1-\phi_d) &\vee 0
\end{aligned} \tag{36}$$

Note that $f(\phi_q)$ is linear in ϕ_q, $f(0) = -\rho^2\phi_d(1-\phi_d) < 0$, and $f(1) = \phi_d^2 + 4(1-\phi_d) > 0$. Therefore the sign of the coefficient linking β_{q2} and λ_2, can be either positive or negative, which implies that we cannot rule out the negative coefficients for β_{q2}.

We have solved for β_{q2}, β_{d2}, and λ_2.

C.1.1 Economy 2: Solution Summary

To summarize the solution, we first solve for β_{q2},β_{d2},λ_2

$$\begin{cases}
\text{If } 2k + \phi_d + \nu_d > 0 \text{ then } \beta_{q2} & = +\sqrt{\frac{\eta\phi_d\phi_q}{\phi_q k^2 + (2\phi_q - \nu_d\phi_q - \phi_d\phi_q)k + \nu_d\phi_q - \phi_d + \phi_d\phi_q}} \\
\text{If } 2k + \phi_d + \nu_d < 0 \text{ then } \beta_{q2} & = -\sqrt{\frac{\eta\phi_d\phi_q}{\phi_q k^2 + (2\phi_q - \nu_d\phi_q - \phi_d\phi_q)k + \nu_d\phi_q - \phi_d + \phi_d\phi_q}}
\end{cases} \tag{37}$$

$$\beta_{d2} = k\beta_{q2} \tag{38}$$

$$\lambda_2 = \frac{\phi_d}{2k + \phi_d + \nu_d}\beta_{q2}^{-1}, \tag{39}$$

where

$$
\begin{aligned}
k &= \frac{2\phi_d - \phi_d\phi_q - \nu_d\phi_q}{2\phi_q - \phi_d\phi_q - \nu_q\phi_d} \\
\nu_q &= \rho\sqrt{\frac{\phi_q(1-\phi_d)(1-\phi_q)}{\phi_d}} \\
\nu_d &= \frac{\phi_d}{\phi_q}\nu_q
\end{aligned}
$$

C.2 Economy 4: Fully-automated Quant, $\gamma = 1$

In this case the QI is fully-automated and applies the trading rule dictated by Equation (9) without taking into consideration the behavior of the DI:

$$
\begin{aligned}
x_{q4} &= x_b \\
&= \frac{1}{2}\sqrt{\frac{\eta}{\phi_d}}\left((2-\phi_d)\phi_q - \rho\sqrt{\phi_d\phi_q(1-\phi_d)(1-\phi_q)}\right)s_q \\
&= \frac{1}{2}\sqrt{\frac{\eta}{\phi_d}}\left((2-\phi_d)\phi_q - \rho\sqrt{\phi_d\phi_q(1-\phi_d)(1-\phi_q)}\right)s_q \\
&\equiv \beta_b s_q
\end{aligned}
\tag{40}
$$

The DI pursues a strategy similar to Economy 2, specified by Eq. (26), but with different inputs for the QI's trading rule and the MM's pricing function:

$$
\begin{aligned}
x_{d4} &= \arg\max_{\tilde{x}_{d4}} \mathbb{E}[\tilde{x}_{d4}(v - P_4(\omega_4))|s_d] \\
&= \left(\frac{\phi_d}{2\lambda_4} - \frac{\phi_d\beta_b}{2} - \frac{\nu_d\beta_b}{2}\right)s_d \\
&\equiv \beta_{d4}s_d
\end{aligned}
\tag{41}
$$

Similarly, the MM's pricing rule is comparable to Economy 2, as given by Eq. (42), with different inputs for the DI's and QI's strategies:

$$
\begin{aligned}
P_4(\omega_4) &= \mathbb{E}[v|\omega_4] \\
&= \mathbb{E}[v|x_{d4} + x_{q4} + z] \\
&= \mathbb{E}[v|\beta_{d4}(v+e_d) + \beta_b(v+e_q) + z] \\
&= \frac{(\beta_{d4}+\beta_b)\sigma_v^2}{(\beta_{d4}+\beta_b)^2\sigma_v^2 + \beta_{d4}^2\sigma_{e_d}^2 + \beta_b^2\sigma_{e_q}^2 + 2\beta_{d4}\beta_b\sigma_{e_de_q} + \sigma_z^2}\omega_4 \\
&\equiv \lambda_4\omega_4
\end{aligned}
\tag{42}
$$

We have two equations and two unknowns:

$$\lambda_4 = \frac{(\beta_{d4}+\beta_b)\sigma_v^2}{(\beta_{d4}+\beta_b)^2\sigma_v^2+\beta_{d4}^2\sigma_{e_d}^2+\beta_b^2\sigma_{e_q}^2+2\beta_{d4}\beta_b\sigma_{e_de_q}+\sigma_z^2}$$

$$\left(\text{note that } \sigma_{e_de_q}/\sigma_v^2=\rho(\sigma_{e_d}/\sigma_v)(\sigma_{e_q}/\sigma_v)=\rho\sqrt{\tfrac{(1-\phi_d)(1-\phi_q)}{\phi_d\phi_q}}=\nu_q/\phi_q=\nu_d/\phi_d\right) \tag{43}$$

$$\Downarrow$$

$$\frac{1}{\lambda_4} =\beta_{d4}+\beta_b+\tfrac{\beta_{d4}^2}{\beta_{d4}+\beta_b}\tfrac{1-\phi_d}{\phi_d}+\tfrac{\beta_b^2}{\beta_{d4}+\beta_b}\tfrac{1-\phi_q}{\phi_q}+\tfrac{2\beta_{d4}\beta_b}{\beta_{d4}+\beta_b}\tfrac{\nu_q}{\phi_q}+\tfrac{\eta}{\beta_{d4}+\beta_b}$$

$$\beta_{d4} = \frac{\phi_d}{2\lambda_4}-\frac{\phi_d\beta_b}{2}-\frac{\nu_d\beta_b}{2} \tag{44}$$

From Eq. (55)

$$\begin{aligned}\beta_{d4} &= \frac{\phi_d}{2\lambda_4}-\frac{\phi_d\beta_b}{2}-\frac{\nu_d\beta_b}{2}\\ \Rightarrow \frac{1}{\lambda_4} &= \frac{2}{\phi_d}\beta_{d4}+\beta_b+\frac{\nu_d}{\phi_d}\beta_b\end{aligned} \tag{45}$$

Setting Eq. (45) and (43) equal:

$$\begin{aligned}
&\beta_{d4}+\beta_b+\tfrac{\beta_{d4}^2}{\beta_{d4}+\beta_b}\tfrac{1-\phi_d}{\phi_d}+\tfrac{\beta_b^2}{\beta_{d4}+\beta_b}\tfrac{1-\phi_q}{\phi_q}+\tfrac{2\beta_{d4}\beta_b}{\beta_{d4}+\beta_b}\tfrac{\nu_q}{\phi_q}+\tfrac{\eta}{\beta_{d4}+\beta_b}=\tfrac{2}{\phi_d}\beta_{d4}+\beta_b+\tfrac{\nu_d}{\phi_d}\beta_b\\
&\Downarrow\\
&\beta_{d4}(\beta_{d4}+\beta_b)+\tfrac{1-\phi_d}{\phi_d}\beta_{d4}^2+\tfrac{1-\phi_q}{\phi_q}\beta_b^2+\tfrac{2\beta_b\nu_q}{\phi_q}\beta_{d4}+\eta-(\tfrac{2}{\phi_d}\beta_{d4}+\tfrac{\nu_d}{\phi_d}\beta_b)(\beta_{d4}+\beta_b)=0\\
&\Downarrow\\
&\left(-\tfrac{1}{\phi_d}\right)\beta_{d4}^2+\left(\beta_b+\tfrac{2\beta_b\nu_q}{\phi_q}-\tfrac{2\beta_b}{\phi_d}-\tfrac{\nu_d\beta_b}{\phi_d}\right)\beta_{d4}+\left(\tfrac{1-\phi_q}{\phi_q}\beta_b^2+\eta-\tfrac{\nu_d\beta_b^2}{\phi_d}\right)=0\\
&\Downarrow\left(\nu_q/\phi_q=\nu_d/\phi_d\right)\\
&\left(-\tfrac{1}{\phi_d}\right)\beta_{d4}^2+\beta_b\left(1+\tfrac{\nu_q}{\phi_q}-\tfrac{2}{\phi_d}\right)\beta_{d4}+\left(\tfrac{1-\phi_q}{\phi_q}\beta_b^2+\eta-\tfrac{\nu_d\beta_b^2}{\phi_d}\right)=0\\
&\Downarrow\\
&\beta_{d4}=\beta_b\left(\tfrac{\phi_d}{2}+\tfrac{\nu_d}{2}-1\right)\pm\sqrt{\beta_b^2\left(\tfrac{\phi_d^2}{4}+\tfrac{\nu_d^2}{4}+1+\tfrac{\phi_d\nu_d}{2}-2\phi_d-2\nu_d+\tfrac{\phi_d}{\phi_q}\right)+\phi_d\eta}
\end{aligned} \tag{46}$$

We can now substitute β_{d4} from above to solve for market depth:

$$\begin{aligned}\frac{1}{\lambda_4} &= \frac{2}{\phi_d}\beta_{d4} + \beta_b + \frac{\nu_d}{\phi_d}\beta_b \\ &= \beta_b\Big(2 + \frac{2\nu_d}{\phi_d} - \frac{2}{\phi_d}\Big) \pm \frac{2}{\phi_d}\sqrt{\beta_b^2\Big(\frac{\phi_d^2}{4} + \frac{\nu_d^2}{4} + 1 + \frac{\phi_d\nu_d}{2} - 2\phi_d - 2\nu_d + \frac{\phi_d}{\phi_q}\Big) + \phi_d\eta} \\ &= \beta_b\Big(2 + \frac{2\nu_d}{\phi_d} - \frac{2}{\phi_d}\Big) \pm \sqrt{\beta_b^2\Big(1 + \frac{\nu_d^2}{\phi_d^2} + \frac{4}{\phi_d^2} + \frac{2\nu_d}{\phi_d} - \frac{8}{\phi_d} - \frac{8\nu_d}{\phi_d^2} + \frac{4}{\phi_d\phi_q}\Big) + \frac{4\eta}{\phi_d}}\end{aligned} \tag{47}$$

C.2.1 Economy 4: Solution Summary

To summarize, we first identify the QI's trading rule:

$$\beta_b = \frac{1}{2}\sqrt{\frac{\eta}{\phi_d}}\left((2-\phi_d)\phi_q - \rho\sqrt{\phi_d\phi_q(1-\phi_d)(1-\phi_q)}\right) \tag{48}$$

We then use this as an input into the DI's trading intensity

$$\beta_{d4} = \beta_b\Big(\frac{\phi_d}{2} + \frac{\nu_d}{2} - 1\Big) \pm \sqrt{\beta_b^2\Big(\frac{\phi_d^2}{4} + \frac{\nu_d^2}{4} + 1 + \frac{\phi_d\nu_d}{2} - 2\phi_d - 2\nu_d + \frac{\phi_d}{\phi_q}\Big) + \phi_d\eta}, \tag{49}$$

which feeds into the equilibrium market depth:

$$\begin{aligned}&\frac{1}{\lambda_4} = \beta_b\Big(2 + \frac{2\nu_d}{\phi_d} - \frac{2}{\phi_d}\Big) \pm \sqrt{\beta_b^2\Big(1 + \frac{\nu_d^2}{\phi_d^2} + \frac{4}{\phi_d^2} + \frac{2\nu_d}{\phi_d} - \frac{8}{\phi_d} - \frac{8\nu_d}{\phi_d^2} + \frac{4}{\phi_d\phi_q}\Big) + \frac{4\eta}{\phi_d}} \\ &\quad\Downarrow \\ &\lambda_4 = \left(\beta_b\Big(2 + \frac{2\nu_d}{\phi_d} - \frac{2}{\phi_d}\Big) \pm \sqrt{\beta_b^2\Big(1 + \frac{\nu_d^2}{\phi_d^2} + \frac{4}{\phi_d^2} + \frac{2\nu_d}{\phi_d} - \frac{8}{\phi_d} - \frac{8\nu_d}{\phi_d^2} + \frac{4}{\phi_d\phi_q}\Big) + \frac{4\eta}{\phi_d}}\right)^{-1}.\end{aligned} \tag{50}$$

C.3 Economy 3: Partially-automated Quant, $0 < \gamma < 1$

The DI's optimal demands are formulaically identical to Economy 2:

$$\begin{aligned}x_{d3} &= \arg\max_{\tilde{x}_{d3}} \mathbb{E}[\tilde{x}_{d3}(v - P_3(\omega_3))|s_d] \\ &= \left(\frac{\phi_d}{2\lambda_3} - \frac{\phi_d\beta_{q3}}{2} - \frac{\nu_d\beta_{q3}}{2}\right)s_d \\ &\equiv \beta_{d3}s_d\end{aligned} \tag{51}$$

The QI now solves the following objective function for this optimal demand x_{q3} where x_b is given by Eq. (9):

$$\begin{aligned}
x_{q3} &= \arg\min_{\tilde{x}_{q3}} \mathbb{E}[\gamma(\tilde{x}_{q3}-x_b)^2+(1-\gamma)\tilde{x}_{q3}(P_3(\omega_3)-v)|s_q] \\
&= \arg\min_{\tilde{x}_{q3}} \mathbb{E}[\gamma(\tilde{x}_{q3}-x_b)^2+(1-\gamma)\tilde{x}_{q3}(\lambda_3(x_{d3}+\tilde{x}_{q3}+z)-v)|s_q] \\
&= \arg\min_{\tilde{x}_{q3}} \mathbb{E}[\gamma\tilde{x}_{q3}^2-2\gamma\tilde{x}_{q3}x_b+\gamma x_b^2+(1-\gamma)\tilde{x}_{q3}(\lambda_3 x_{d3}+\lambda_3 z-v)+(1-\gamma)\lambda_3\tilde{x}_{q3}^2|s_q] \\
&= \arg\min_{\tilde{x}_{q3}} \mathbb{E}[(\gamma+(1-\gamma)\lambda_3)\tilde{x}_{q3}^2-2\gamma\tilde{x}_{q3}x_b+\gamma x_b^2+(1-\gamma)\tilde{x}_{q3}(\lambda_3 x_{d3}+\lambda_3 z-v)|s_q] \\
&= \arg\min_{\tilde{x}_{q3}} \mathbb{E}\left[\Big(\gamma+(1-\gamma)\lambda_3\Big)\tilde{x}_{q3}^2+\Big((1-\gamma)(\lambda_3 x_{d3}+\lambda_3 z-v)-2\gamma x_b\Big)\tilde{x}_{q3}+\gamma x_b^2|s_q\right] \\
&= \arg\min_{\tilde{x}_{q3}} \mathbb{E}\left\{\Big(\gamma+(1-\gamma)\lambda_3\Big)\tilde{x}_{q3}^2+\Big((1-\gamma)(\lambda_3\mathbb{E}[x_{d3}|s_q]-\mathbb{E}[v|s_q])-2\gamma x_b\Big)\tilde{x}_{q3}+\gamma x_b^2|s_q\right] \\
&= \frac{2\gamma x_b+(1-\gamma)(\mathbb{E}[v|s_q]-\lambda_3\mathbb{E}[x_{d3}|s_q])}{2\Big(\gamma+(1-\gamma)\lambda_3\Big)} \\
&= \frac{\gamma}{\gamma+(1-\gamma)\lambda_3}x_b+\frac{(1-\gamma)\lambda_3}{\gamma+(1-\gamma)\lambda_3}\frac{\mathbb{E}[v|s_q]-\lambda_3\mathbb{E}[x_{d3}|s_q]}{2\lambda_3} \\
&= \frac{\gamma}{\gamma+(1-\gamma)\lambda_3}\beta_b s_q+\frac{(1-\gamma)\lambda_3}{\gamma+(1-\gamma)\lambda_3}\left(\frac{\phi_q}{2\lambda_3}-\frac{\phi_q\beta_{d3}}{2}-\frac{\nu_q\beta_{d3}}{2}\right)s_q \\
&= \frac{2\gamma\beta_b+(1-\gamma)\phi_q-(1-\gamma)\lambda_3(\phi_q+\nu_q)\beta_{d3}}{2(\gamma+(1-\gamma)\lambda_3)}s_q \\
&\equiv \beta_{q3}s_q
\end{aligned} \tag{52}$$

The market maker also has a formulaically identical pricing rule to Economy 2:

$$\begin{aligned}
P_3(\omega_3) &= \mathbb{E}[v|\omega_3] \\
&= \frac{(\beta_{d3}+\beta_{q3})\sigma_v^2}{(\beta_{d3}+\beta_{q3})^2\sigma_v^2+\beta_{d3}^2\sigma_{e_d}^2+\beta_{q3}^2\sigma_{e_q}^2+2\beta_{d3}\beta_{q3}\sigma_{e_d e_q}+\sigma_z^2}\omega_3 \\
&\equiv \lambda_3\omega_3
\end{aligned} \tag{53}$$

We have three equations and three unknowns:

$$\lambda_3 = \frac{(\beta_{d3}+\beta_{q3})\sigma_v^2}{(\beta_{d3}+\beta_{q3})^2\sigma_v^2+\beta_{d3}^2\sigma_{e_d}^2+\beta_{q3}^2\sigma_{e_q}^2+2\beta_{d3}\beta_{q3}\sigma_{e_d e_q}+\sigma_z^2}$$

$$\left(\text{note that } \sigma_{e_d e_q}/\sigma_v^2=\rho(\sigma_{e_d}/\sigma_v)(\sigma_{e_q}/\sigma_v)=\rho\sqrt{\tfrac{(1-\phi_d)(1-\phi_q)}{\phi_d\phi_q}}=\nu_q/\phi_q=\nu_d/\phi_d\right) \tag{54}$$

$$\Downarrow$$

$$\frac{1}{\lambda_3} = \beta_{d3}+\beta_{q3}+\frac{\beta_{d3}^2}{\beta_{d3}+\beta_{q3}}\frac{1-\phi_d}{\phi_d}+\frac{\beta_{q3}^2}{\beta_{d3}+\beta_{q3}}\frac{1-\phi_q}{\phi_q}+\frac{2\beta_{d3}\beta_{q3}}{\beta_{d3}+\beta_{q3}}\frac{\nu_q}{\phi_q}+\frac{\eta}{\beta_{d3}+\beta_{q3}}$$

$$\beta_{d3} = \frac{\phi_d}{2\lambda_3}-\frac{\phi_d\beta_{q3}}{2}-\frac{\nu_d\beta_{q3}}{2} \tag{55}$$

$$\beta_{q3} = \frac{2\gamma\beta_b+(1-\gamma)\phi_q-(1-\gamma)\lambda_3(\phi_q+\nu_q)\beta_{d3}}{2(\gamma+(1-\gamma)\lambda_3)} \tag{56}$$

We first use Eq. (55) and (56) to solve for λ_3 and set the results equal:

$$\begin{aligned}
\beta_{d3} &= \frac{\phi_d}{2\lambda_3} - \frac{\phi_d\beta_{q3}}{2} - \frac{\nu_d\beta_{q3}}{2} \\
&\Rightarrow \frac{1}{\lambda_3} = \frac{2\beta_{d3}}{\phi_d} + \beta_{q3} + \frac{\nu_d\beta_{q3}}{\phi_d} \\
\beta_{q3} &= \frac{2\gamma\beta_b + (1-\gamma)\phi_q - (1-\gamma)\lambda_3(\phi_q+\nu_q)\beta_{d3}}{2(\gamma+(1-\gamma)\lambda_3)} \\
&\Rightarrow \frac{1}{\lambda_3} = \frac{2(1-\gamma)\beta_{q3} + (1-\gamma)(\phi_q+\nu_q)\beta_{d3}}{2\gamma\beta_b - 2\gamma\beta_{q3} + (1-\gamma)\phi_q} \\
&\Rightarrow \frac{2}{\phi_d}\beta_{d3} + \Big(1+\frac{\nu_d}{\phi_d}\Big)\beta_{q3} = \frac{2(1-\gamma)\beta_{q3} + (1-\gamma)(\phi_q+\nu_q)\beta_{d3}}{2\gamma\beta_b - 2\gamma\beta_{q3} + (1-\gamma)\phi_q} \\
&\Rightarrow \beta_{d3} = \frac{2\gamma\left(1+\frac{\nu_d}{\phi_d}\right)\beta_{q3}^2 + \left(2(1-\gamma) - 2\gamma\beta_b\left(1+\frac{\nu_d}{\phi_d}\right) - (1-\gamma)\phi_q\left(1+\frac{\nu_d}{\phi_d}\right)\right)\beta_{q3}}{-\frac{4\gamma}{\phi_d}\beta_{q3} + \frac{4\gamma\beta_b}{\phi_d} + \frac{2(1-\gamma)\phi_q}{\phi_d} - (1-\gamma)(\phi_q+\nu_q)} \\
&\equiv \frac{a\beta_{q3}^2 + b\beta_{q3}}{c\beta_{q3}+d}
\end{aligned} \tag{57}$$

Next, we use Eq. (55) to solve for $1/\lambda_3$ and set the result equal to Eq. (54):

$$\begin{aligned}
&\beta_{d3} = \frac{\phi_d}{2\lambda_3} - \frac{\phi_d\beta_{q3}}{2} - \frac{\nu_d\beta_{q3}}{2} \\
&\Rightarrow \frac{1}{\lambda_3} = \frac{2\beta_{d3}}{\phi_d} + \beta_{q3} + \frac{\nu_d\beta_{q3}}{\phi_d} \\
&\Rightarrow \beta_{d3} + \beta_{q3} + \frac{\beta_{d3}^2}{\beta_{d3}+\beta_{q3}}\frac{1-\phi_d}{\phi_d} + \frac{\beta_{q3}^2}{\beta_{d3}+\beta_{q3}}\frac{1-\phi_q}{\phi_q} + \frac{2\beta_{d3}\beta_{q3}}{\beta_{d3}+\beta_{q3}}\frac{\nu_q}{\phi_q} + \frac{\eta}{\beta_{d3}+\beta_{q3}} = \frac{2\beta_{d3}}{\phi_d} + \beta_{q3} + \frac{\nu_d\beta_{q3}}{\phi_d} \\
&\Rightarrow \beta_{d3}^2 + \underbrace{\Big(2-\phi_d-\nu_d\Big)}_{\equiv e}\beta_{q3}\beta_{d3} + \underbrace{\Big(\nu_d+\phi_d-\frac{\phi_d}{\phi_q}\Big)}_{\equiv f}\beta_{q3}^2 - \underbrace{\phi_d\eta}_{\equiv g} = 0 \\
&\Rightarrow (a\beta_{q3}^2 + b\beta_{q3})^2 + e\beta_{q3}(c\beta_{q3}+d)(a\beta_{q3}^2+b\beta_{q3}) + f\beta_{q3}^2(c\beta_{q3}+d)^2 - g(c\beta_{q3}+d)^2 = 0 \\
&\Rightarrow (a^2+ace+c^2f)\beta_{q3}^4 + (2ab+bce+ade+2cdf)\beta_{q3}^3 + (b^2+bde+d^2f-c^2g)\beta_{q3}^2 - 2cdg\beta_{q3} - d^2g = 0
\end{aligned} \tag{58}$$

This is quartic, which has a closed-form solution for β_{q3} resulting in a maximum of four roots. For each root, we can then utilize Eq. (57) to solve for β_{d3} and, in-turn, use Eq. 54 to solve for λ. We have solved for β_{d3}, β_{q3}, and λ_3.

C.3.1 Economy 3: Solution Summary

We first obtain β_{q3} from the solution to

$$(a^2+ace+c^2f)\beta_{q3}^4+(2ab+bce+ade+2cdf)\beta_{q3}^3+(b^2+bde+d^2f-c^2g)\beta_{q3}^2-2cdg\beta_{q3}-d^2g=0$$

where

$$a = 2\gamma\big(1+\frac{\nu_d}{\phi_d}\big)$$
$$b = 2(1-\gamma)-2\gamma\beta_b\big(1+\frac{\nu_d}{\phi_d}\big)-(1-\gamma)\phi_q\big(1+\frac{\nu_d}{\phi_d}\big)$$
$$c = -\frac{4\gamma}{\phi_d}$$
$$d = \frac{4\gamma\beta_b}{\phi_d}+\frac{2(1-\gamma)\phi_q}{\phi_d}-(1-\gamma)(\phi_q+\nu_q)$$
$$e = 2-\phi_d-\nu_d$$
$$f = \nu_d+\phi_d-\frac{\phi_d}{\phi_q}$$
$$g = \phi_d\eta$$

and use the solution to obtain

$$\beta_{d3} = \frac{a\beta_{q3}^2+b\beta_{q3}}{c\beta_{q3}+d}$$

and

$$\lambda_3 = \frac{\beta_{d3}+\beta_{q3}}{(\beta_{d3}+\beta_{q3})^2+\beta_{d3}^2\frac{1-\phi_d}{\phi_d}+\beta_{q3}^2\frac{1-\phi_q}{\phi_q}+2\beta_{d3}\beta_{q3}\frac{\nu_q}{\phi_q}+\eta}$$

Appendix D Derivations for Measures of Market Quality

Market Depth We take the inverse of λ_j as derived for Economies 1-4 in Appendices A and C.

Price Informativeness: Economy 1

$$\begin{aligned} P_1(\omega_1) &= \lambda_1\omega_1 \\ &= \lambda_1(x_{d1}+z) \\ &= \lambda_1(\beta_{d1}(v+e_d)+z) \\ &= \lambda_1(\beta_{d1}v+\beta_{d1}e_d+z) \\ &= \lambda_1\beta_{d1}v+\lambda_1\beta_{d1}e_d+\lambda_1 z \end{aligned} \tag{59}$$

Therefore,

$$\begin{aligned}
\text{Var}(v|P_1) &= \text{Var}(v) - \text{Cov}(v, P_1)\text{Var}(P_1)^{-1}\text{Cov}(P_1, v) \\
&= \sigma_v^2 - \frac{\text{Cov}(v, \lambda_1\beta_{d1}v + \lambda_1\beta_{d1}e_d + \lambda_1 z)^2}{\text{Var}(\lambda_1\beta_{d1}v + \lambda_1\beta_{d1}e_d + \lambda_1 z)} \\
&= \sigma_v^2 - \frac{\lambda_1^2\beta_{d1}^2\sigma_v^4}{\lambda_1^2\beta_{d1}^2\sigma_v^2 + \lambda_1^2\beta_{d1}^2\sigma_{e_d}^2 + \lambda_1^2\sigma_z^2} \\
&= \sigma_v^2 \frac{\beta_{d1}^2\sigma_{e_d}^2 + \sigma_z^2}{\beta_{d1}^2\sigma_v^2 + \beta_{d1}^2\sigma_{e_d}^2 + \sigma_z^2} \\
&\Downarrow \\
\text{Var}(v|P_1)^{-1} &= \frac{1}{\sigma_v^2}\frac{\beta_{d1}^2 + \beta_{d1}^2\frac{1-\phi_d}{\phi_d} + \eta}{\beta_{d1}^2\frac{1-\phi_d}{\phi_d} + \eta}
\end{aligned}$$

Price Informativeness: Economies 2-4 For $j \in \{2, 3, 4\}$

$$\begin{aligned}
P_j(\omega_j) &= \lambda_j\omega_j \\
&= \lambda_j(x_{dj} + x_{qj} + z) \\
&= \lambda_j(\beta_{dj}s_d + \beta_{qj}s_q + z) \\
&= \lambda_j(\beta_{dj}(v + e_d) + \beta_{qj}(v + e_q) + z) \\
&= \lambda_j(\beta_{dj} + \beta_{qj})v + \lambda_j\beta_{dj}e_d + \lambda_j\beta_{qj}e_q + \lambda_j z
\end{aligned} \tag{60}$$

Therefore,

$$\begin{aligned}
&\text{Var}(v|P_j) = \text{Var}(v) - \text{Cov}(v, P_j)\text{Var}(P_j)^{-1}\text{Cov}(P_j, v) \\
&= \sigma_v^2 - \frac{\text{Cov}(v, \lambda_j(\beta_{dj} + \beta_{qj})v + \lambda_j\beta_{dj}e_d + \lambda_j\beta_{qj}e_q + \lambda_j z)^2}{\text{Var}(\lambda_j(\beta_{dj} + \beta_{qj})v + \lambda_j\beta_{dj}e_d + \lambda_j\beta_{qj}e_q + \lambda_j z)} \\
&= \sigma_v^2 - \frac{\lambda_j^2(\beta_{dj} + \beta_{qj})^2\sigma_v^4}{\lambda_j^2(\beta_{dj} + \beta_{qj})^2\sigma_v^2 + \lambda_j^2\beta_{dj}^2\sigma_{e_d}^2 + \lambda_j^2\beta_{qj}^2\sigma_{e_q}^2 + 2\lambda_j^2\beta_{dj}\beta_{qj}\sigma_{e_de_q} + \lambda_j^2\sigma_z^2} \\
&= \frac{\sigma_v^2(\lambda_j^2\beta_{dj}^2\sigma_{e_d}^2 + \lambda_j^2\beta_{qj}^2\sigma_{e_q}^2 + 2\lambda_j^2\beta_{dj}\beta_{qj}\sigma_{e_de_q} + \lambda_j^2\sigma_z^2)}{\lambda_j^2(\beta_{dj} + \beta_{qj})^2\sigma_v^2 + \lambda_j^2\beta_{dj}^2\sigma_{e_d}^2 + \lambda_j^2\beta_{qj}^2\sigma_{e_q}^2 + 2\lambda_j^2\beta_{dj}\beta_{qj}\sigma_{e_de_q} + \lambda_j^2\sigma_z^2} \\
&= \sigma_v^2\frac{\beta_{dj}^2\frac{1-\phi_d}{\phi_d} + \beta_{qj}^2\frac{1-\phi_q}{\phi_q} + 2\beta_{dj}\beta_{qj}\frac{\nu_q}{\phi_q} + \eta}{(\beta_{dj} + \beta_{qj})^2 + \beta_{dj}^2\frac{1-\phi_d}{\phi_d} + \beta_{qj}^2\frac{1-\phi_q}{\phi_q} + 2\beta_{dj}\beta_{qj}\frac{\nu_q}{\phi_q} + \eta} \\
&\Downarrow \\
&\text{Var}(v|P_j)^{-1} = \frac{1}{\sigma_v^2}\frac{(\beta_{dj} + \beta_{qj})^2 + \beta_{dj}^2\frac{1-\phi_d}{\phi_d} + \beta_{qj}^2\frac{1-\phi_q}{\phi_q} + 2\beta_{dj}\beta_{qj}\frac{\nu_q}{\phi_q} + \eta}{\beta_{dj}^2\frac{1-\phi_d}{\phi_d} + \beta_{qj}^2\frac{1-\phi_q}{\phi_q} + 2\beta_{dj}\beta_{qj}\frac{\nu_q}{\phi_q} + \eta}
\end{aligned}$$

Price Volatility From equation 60:

$$\begin{aligned}\text{Var}(P_j) &= \text{Var}(\lambda_j(\beta_{dj}+\beta_{qj})v + \lambda_j\beta_{dj}e_d + \lambda_j\beta_{qj}e_q + \lambda_j z)\\ &= \lambda_j^2\Big((\beta_{dj}+\beta_{qj})^2\sigma_v^2 + \beta_{dj}^2\sigma_{e_d}^2 + \beta_{qj}^2\sigma_{e_q}^2 + 2\beta_{dj}\beta_{qj}\sigma_{e_de_q} + \sigma_z^2\Big)\\ &= \lambda_j^2\sigma_v^2\Big((\beta_{dj}+\beta_{qj})^2 + \beta_{dj}^2\frac{1-\phi_d}{\phi_d} + \beta_{qj}^2\frac{1-\phi_q}{\phi_q} + 2\beta_{dj}\beta_{qj}\frac{\nu_q}{\phi_q} + \eta\Big)\end{aligned}$$

Return Volatility From equation 60:

$$\begin{aligned}&\text{Var}(v-P) = \text{Var}(v - \lambda_j(\beta_{dj}+\beta_{qj})v - \lambda_j\beta_{dj}e_d - \lambda_j\beta_{qj}e_q - \lambda_j z)\\ &= \text{Var}\big((1-\lambda_j(\beta_{dj}+\beta_{qj}))v - \lambda_j\beta_{dj}e_d - \lambda_j\beta_{qj}e_q - \lambda_j z\big)\\ &= (1-\lambda_j(\beta_{dj}+\beta_{qj}))^2\sigma_v^2 + \lambda_j^2\beta_{dj}^2\sigma_{e_d}^2 + \lambda_j^2\beta_{qj}^2\sigma_{e_q}^2 + 2\lambda_j^2\beta_{dj}\beta_{qj}\sigma_{e_de_q} + \lambda_j^2\sigma_z^2\\ &= \sigma_v^2\Big((1-\lambda_j(\beta_{dj}+\beta_{qj}))^2 + \lambda_j^2\beta_{dj}^2\frac{1-\phi_d}{\phi_d} + \lambda_j^2\beta_{qj}^2\frac{1-\phi_q}{\phi_q} + 2\lambda_j^2\beta_{dj}\beta_{qj}\frac{\nu_q}{\phi_q} + \lambda_j^2\eta\Big)\end{aligned}$$

Expected Trading Volume: Aggregate We are after $\mathbb{E}[|x_{dj}+x_{qj}|]$, which is the expected value of the absolute value of a normal random variable. Otherwise known as the "folded normal distribution," given a normally distributed random variable $X \sim N(\mu,\sigma^2)$, the expected value of $|X|$ is given by:

$$\mathbb{E}[|X|] = \sigma\sqrt{\frac{2}{\pi}}\exp\left(-\frac{\mu^2}{2\sigma^2}\right) + \mu\Big(1-2\Phi\big(-\frac{\mu}{\sigma}\big)\Big) \tag{61}$$

where Φ is the standard normal cumulative distribution function. We proceed to derive the distribution of x_1+x_2.

$$\begin{aligned}x_{dj}+x_{qj} = \quad &\underbrace{(\beta_{dj}+\beta_{qj})v}_{\sim N\left(0,(\beta_{dj}+\beta_{qj})^2\sigma_v^2\right)} \quad + \quad \underbrace{\beta_{dj}e_d+\beta_{qj}e_q}_{\sim N\left(0,\beta_{dj}^2\sigma_{e_d}^2+\beta_{qj}^2\sigma_{e_q}^2+2\beta_{dj}\beta_{qj}\sigma_{e_de_q}\right)}\\ &\Downarrow\\ x_{dj}+x_{qj} \sim N\big(&\underbrace{0}_{\equiv\mu_{x_{dj}+x_{qj}}}, \underbrace{(\beta_{dj}+\beta_{qj})^2\sigma_v^2 + \beta_{dj}^2\sigma_{e_d}^2 + \beta_{qj}^2\sigma_{e_q}^2 + 2\beta_{dj}\beta_{qj}\sigma_{e_de_q}}_{\equiv\sigma^2_{x_{dj}+x_{qj}}}\big)\end{aligned}$$

Substituting into equation 61

$$\mathbb{E}[|x_{dj}+x_{qj}|] = \sigma_{x_{dj}+x_{qj}}\sqrt{\frac{2}{\pi}}$$

Expected Trading Volume: by Trader We are after $\mathbb{E}[|x_{ij}|]$, which is the expected value of the absolute value of a normal random variable. We utilize equation 61 to derive

the expectation of this random variable. The distribution of x_{ij} is:

$$\begin{aligned} x_{ij} &= \beta_{ij} s_i \\ &\Downarrow \\ x_{ij} &\sim N\big(\underbrace{0}_{\equiv \mu_{x_{ij}}}, \underbrace{\beta_{ij}^2(\sigma_v^2 + \sigma_{e_i}^2)}_{\equiv \sigma_{x_{ij}}^2}\big) \end{aligned}$$

Substituting into equation 61

$$\mathbb{E}[|x_{ij}|] = \sigma_{x_{ij}} \sqrt{\frac{2}{\pi}}$$

Expected Profits: By Trader and Aggregate Expected profits of trader i in Economy j (π_{ij}) are equal to:

$$\begin{aligned} \mathbb{E}[\pi_{ij}] &= \mathbb{E}[x_{ij}(v - P)] \\ &= \mathbb{E}[(\beta_{ij} s_i)(v - \lambda_j \omega_j)] \\ &= \mathbb{E}[(\beta_{ij} s_i)(v - \lambda_j (x_{dj} + x_{qj} + z))] \\ &= \mathbb{E}[(\beta_{ij}(v + e_i))(v - \lambda_j (\beta_{dj} s_d + \beta_{qj} s_q + z))] \\ &= \mathbb{E}[(\beta_{ij} v + \beta_{ij} e_i)(v - \lambda_j (\beta_{dj}(v + e_d) + \beta_{qj}(v + e_q) + z))] \\ &= \beta_{ij}(1 - \lambda_j \beta_{dj} - \lambda_j \beta_{qj})\sigma_v^2 - \lambda_j \beta_{ij}^2 \sigma_{e_i}^2 - \lambda_j \beta_{dj} \beta_{qj} \sigma_{e_d e_q} \end{aligned}$$

Finally,

$$\begin{aligned} \mathbb{E}[\pi_{dj} + \pi_{qj}] &= \mathbb{E}[\pi_{dj}] + \mathbb{E}[\pi_{qj}] \\ &= (\beta_{dj} + \beta_{qj})(1 - \lambda_j \beta_{dj} - \lambda_j \beta_{qj})\sigma_v^2 - \lambda_j(\beta_{dj}^2 \sigma_{e_d}^2 + \beta_{qj}^2 \sigma_{e_q}^2) - 2\lambda_j \beta_{dj} \beta_{qj} \sigma_{e_d e_q} \end{aligned}$$

Appendix E Comparative Statics for the Benchmark Case: Proposition 1

The simplified version assumes the following model parameters: $\phi_d = 1, \rho = \nu_q = \nu_d = 0, \sigma_v^2 = 1$. The model solution across the four economies is as follows:

- Backtest:

$$\beta_b = \frac{1}{2}\sqrt{\eta}\phi_q$$

- Economy 1:

$$\begin{aligned} \beta_{d1} &= \sqrt{\eta} \\ \lambda_1 &= \frac{1}{2}\sqrt{\frac{1}{\eta}} \end{aligned}$$

- Economy 2:

$$\beta_{d2} = \sqrt{\eta}\sqrt{\frac{(2-\phi_q)^2}{\phi_q^2 - 3\phi_q + 4}}$$
$$\beta_{q2} = \sqrt{\eta}\sqrt{\frac{\phi_q^2}{\phi_q^2 - 3\phi_q + 4}}$$
$$\lambda_2 = \sqrt{\frac{1}{\eta}}\sqrt{\frac{\phi_q^2 - 3\phi_q + 4}{(4-\phi_q)^2}}$$

- Economy 4:

$$\beta_{d4} = -\frac{1}{4}\sqrt{\eta}\phi_q + \frac{1}{4}\sqrt{\eta}\sqrt{-3\phi_q^2 + 4\phi_q + 16}$$
$$\beta_{q4} = \beta_b = \frac{1}{2}\sqrt{\eta}\phi_q$$
$$\lambda_4 = \frac{2}{\sqrt{\eta}}\sqrt{\frac{1}{-3\phi_q^2 + 4\phi_q + 16}}$$

E.1 Market Depth

We first show that market depth increases with the introduction of the quantitative investor, both with and without automation:

$$\begin{aligned}
\frac{1}{\lambda_1} &< \frac{1}{\lambda_2} \\
&\Updownarrow \\
\lambda_1 &> \lambda_2 \\
&\Updownarrow \\
\frac{1}{2}\sqrt{\frac{1}{\eta}} &> \sqrt{\frac{1}{\eta}}\sqrt{\frac{\phi_q^2 - 3\phi_q + 4}{(4-\phi_q)^2}} \\
&\Updownarrow \\
3\phi_q\Big(\frac{4}{3} - \phi_q\Big) &> 0
\end{aligned}$$

where the latter inequality holds for $\phi_q \in (0,1]$

$$\begin{aligned}
\frac{1}{\lambda_1} &< \frac{1}{\lambda_4} \\
&\Updownarrow \\
\lambda_1 &> \lambda_4 \\
&\Updownarrow \\
\frac{1}{2}\sqrt{\frac{1}{\eta}} &> \frac{2}{\sqrt{\eta}}\sqrt{\frac{1}{-3\phi_q^2+4\phi_q+16}} \\
&\Updownarrow \\
3\phi_q\Big(\frac{4}{3}-\phi_q\Big) &> 0
\end{aligned}$$

where the latter inequality holds for $\phi_q \in (0,1]$

We now show that market depth decreases due to automation by quant.

$$\begin{aligned}
\frac{1}{\lambda_2} &> \frac{1}{\lambda_4} \\
&\Updownarrow \\
\lambda_2 &< \lambda_4 \\
&\Updownarrow \\
\sqrt{\frac{1}{\eta}}\sqrt{\frac{\phi_q^2-3\phi_q+4}{(4-\phi_q)^2}} &< \frac{2}{\sqrt{\eta}}\sqrt{\frac{1}{-3\phi_q^2+4\phi_q+16}} \\
&\Updownarrow \\
\frac{\phi_q^2-3\phi_q+4}{(4-\phi_q)^2} &< \frac{4}{-3\phi_q^2+4\phi_q+16} \\
&\Updownarrow \text{ (note that } -3\phi_q^2+4\phi_q+16>0 \text{ for } \phi_q \in (0,1]) \\
f(\phi_q) \equiv -3\phi_q^2+13\phi_q-12 &< 0
\end{aligned}$$

where the latter inequality holds for $\phi_q \in (0,1]$ since the roots of $f(\phi_q)$ are $\phi_q^* \approx 1.33$ and $\phi_q^{**} = 3$.

Finally, we demonstrate that the gap in market depth between Economy 2 and Economy

4 increases with greater signal precision for the quant:

$$\begin{aligned}
&\frac{\partial}{\partial\phi_q}\frac{\lambda_4}{\lambda_2} \vee 0 \\
&\Updownarrow \\
&\frac{\partial}{\partial\phi_q}\frac{2(4-\phi_q)}{\sqrt{(-3\phi_q^2+4\phi_q+16)(\phi_q^2-3\phi_q+4)}} \vee 0 \\
&\Updownarrow \\
&\frac{\partial}{\partial\phi_q}\frac{(4-\phi_q)^2}{(-3\phi_q^2+4\phi_q+16)(\phi_q^2-3\phi_q+4)} \vee 0 \\
&\Updownarrow \\
&(2\phi_q-8)(-3\phi_q^4+13\phi_q^3-8\phi_q^2-32\phi_q+64)-(\phi_q^2-8\phi_q+16)(-12\phi_q^3+39\phi_q^2-16\phi_q-32)\vee 0 \\
&\Updownarrow \\
&f(\phi_q) \equiv 6\phi_q^4-85\phi_q^3+400\phi_q^2-720\phi_q+384 \vee 0
\end{aligned}$$

We first note that $f(0) = 384, f(1) = -15$. We now demonstrate that $f(\phi_q)$ is monotone for $\phi_q \in (0,1]$, implying a single root for $\phi_q \in (0,1]$. Note the following derivatives of $f(\phi_q)$:

$$\begin{aligned}
f'(\phi_q) &= 24\phi_q^3 - 255\phi_q^2 + 800\phi_q - 720 \\
f''(\phi_q) &= 72\phi_q^2 - 510\phi_q + 800
\end{aligned}$$

The two roots of $f''(\phi_q) = 0$ are $\approx 2.3, \approx 4.7$, which implies that $f''(\phi_q) > 0$ for $\phi_q \in (0,1]$. This in turn means that $f'(\phi_q)$ is increasing for $\phi_q \in (0,1]$, which coupled with $f'(0) = -720, f'(1) = -151$ demonstrates that $f''(\phi_q) < 0$ for $\phi_q \in (0,1]$. The latter implies that $f(\phi_q)$ is decreasing for $\phi_q \in (0,1]$, which proves that there exists $\phi_q^* \in (0,1]$ such that $f(\phi_q) > 0$ for $\phi_q \in (0,\phi_q^*)$ and $f(\phi_q) < 0$ for $\phi_q \in (\phi_q^*,1]$. Therefore, the percent decrease in market depth due to automation is hump-shaped in the precision of the QI's signal. The peak of the relative decline in market depth is around $\phi_q^* \approx 0.91$.

E.2 Price Informativeness

$$\begin{aligned}
\mathrm{Var}(v|P_j)^{-1} &= \frac{1}{\sigma_v^2}\frac{(\beta_{dj}+\beta_{qj})^2 + \beta_{dj}^2\frac{1-\phi_d}{\phi_d} + \beta_{qj}^2\frac{1-\phi_q}{\phi_q} + 2\beta_{dj}\beta_{qj}\frac{\nu_q}{\phi_q} + \eta}{\beta_{dj}^2\frac{1-\phi_d}{\phi_d} + \beta_{qj}^2\frac{1-\phi_q}{\phi_q} + 2\beta_{dj}\beta_{qj}\frac{\nu_q}{\phi_q} + \eta} \\
&= \frac{(\beta_{dj}+\beta_{qj})^2 + \beta_{qj}^2\frac{1-\phi_q}{\phi_q} + \eta}{\beta_{qj}^2\frac{1-\phi_q}{\phi_q} + \eta} \\
&= 1 + \frac{(\beta_{dj}+\beta_{qj})^2}{\beta_{qj}^2\frac{1-\phi_q}{\phi_q} + \eta}
\end{aligned}$$

We first show that price informativeness decreases due to automation by quant.

$$
\begin{aligned}
&\mathrm{Var}(v|P_2)^{-1} > \mathrm{Var}(v|P_4)^{-1} \\
&\Updownarrow \\
&\frac{(\beta_{d2}+\beta_{q2})^2}{\beta_{q2}^2\frac{1-\phi_q}{\phi_q}+\eta} > \frac{(\beta_{d4}+\beta_{q4})^2}{\beta_{q4}^2\frac{1-\phi_q}{\phi_q}+\eta} \\
&\Updownarrow \\
&\frac{\left(\sqrt{\eta}\sqrt{\frac{(2-\phi_q)^2}{\phi_q^2-3\phi_q+4}}+\sqrt{\eta}\sqrt{\frac{\phi_q^2}{\phi_q^2-3\phi_q+4}}\right)^2}{\left(\sqrt{\eta}\sqrt{\frac{\phi_q^2}{\phi_q^2-3\phi_q+4}}\right)^2\frac{1-\phi_q}{\phi_q}+\eta} > \frac{\left(-\frac{1}{4}\sqrt{\eta}\phi_q+\frac{1}{4}\sqrt{\eta}\sqrt{-3\phi_q^2+4\phi_q+16}+\frac{1}{2}\sqrt{\eta}\phi_q\right)^2}{\left(\frac{1}{2}\sqrt{\eta}\phi_q\right)^2\frac{1-\phi_q}{\phi_q}+\eta} \\
&\Updownarrow \\
&\frac{\left(\sqrt{\frac{(2-\phi_q)^2}{\phi_q^2-3\phi_q+4}}+\sqrt{\frac{\phi_q^2}{\phi_q^2-3\phi_q+4}}\right)^2}{\left(\sqrt{\frac{\phi_q^2}{\phi_q^2-3\phi_q+4}}\right)^2\frac{1-\phi_q}{\phi_q}+1} > \frac{\left(\frac{1}{4}\phi_q+\frac{1}{4}\sqrt{-3\phi_q^2+4\phi_q+16}\right)^2}{\left(\frac{1}{2}\phi_q\right)^2\frac{1-\phi_q}{\phi_q}+1} \\
&\Updownarrow \\
&\frac{2}{2-\phi_q} > \frac{-\phi_q^2+2\phi_q+8+\phi_q\sqrt{-3\phi_q^2+4\phi_q+16}}{-2\phi_q^2+2\phi_q+8} \\
&\Updownarrow \text{ note that } -2\phi_q^2+2\phi_q+8>0 \text{ for } \phi_q\in(0,1] \\
&-\phi_q^3+8\phi_q+16 > (2-\phi_q)\sqrt{-3\phi_q^4+4\phi_q^3+16\phi_q^2} \\
&\Updownarrow \\
&f(\phi_q) \equiv 4\phi_q^6-16\phi_q^5-4\phi_q^4+16\phi_q^3+256\phi_q+256>0
\end{aligned}
$$

The latter inequality holds for the following reasons. First, note that $f(0)=256$ and $f(1)=512$. Since $f(\phi_q)$ is continuous, if $f(\phi_q)$ is either monotone or hump-shaped for $\phi_q\in(0,1]$ then $f(\phi_q)>0$ in this interval. We now take derivatives of $f(\phi_q)$ and work backwards to demonstrate that $f(\phi_q)$ is monotone over the interval.

$$
\begin{aligned}
f'(\phi_q) &= 24\phi_q^5-80\phi_q^4-16\phi_q^3+48\phi_q^2+256 \\
f''(\phi_q) &= 120\phi_q^4-320\phi_q^3-46\phi_q^2+96\phi_q \\
f'''(\phi_q) &= 480\phi_q^3-960\phi_q^2-92\phi_q+96 \\
f''''(\phi_q) &= 1440\phi_q^2-1920\phi_q-92
\end{aligned}
$$

First note that the zeros of $f''''(\phi_q)$ are $\approx -.05$ and ≈ 1.38 implying that $f''''(\phi_q)<0$ for $\phi_q\in(0,1]$. Since $f'''(0)=96$ and $f'''(1)=-476$ and $f'''(\phi_q)$ is decreasing for $\phi_q\in(0,1]$, there exists $\phi_q^*\in(0,1]$ such that $f'''(\phi_q)>0$ for $\phi_q\in(0,\phi_q^*)$ and $f'''(\phi_q)<0$ for $\phi_q\in(\phi_q^*,1]$. This implies that $f''(\phi_q)$ is hump-shaped in the interval $\phi_q\in(0,1]$, which coupled with $f''(0)=0$ and $f''(1)=-150$ suggests that there exists $\phi_q^{**}\in(0,1]$ such that $f''(\phi_q)>0$ for $\phi_q\in(0,\phi_q^{**})$ and $f''(\phi_q)<0$ for $\phi_q\in(\phi_q^{**},1]$. Therefore $f'(\phi_q)$ is also hump-shaped in the interval $\phi_q\in(0,1]$. Since $f'(0)=256$ and $f'(1)=232$ and $f'(\phi_q)$ is hump-shaped in the interval $\phi_q\in(0,1]$, $f'(\phi_q)>0$ in the interval. Coupled with $f(1)>f(0)>0$, $f(\phi_q)>0$ for $\phi_q\in(0,1]$.

We now show that price informativeness is always greater with the quant, both fully-discretionary and fully-automated.

$$\begin{aligned}
\text{Var}(v|P_2)^{-1} &> \text{Var}(v|P_1)^{-1} \\
&\Updownarrow \\
\frac{(\beta_{d2}+\beta_{q2})^2}{\beta_{q2}^2\frac{1-\phi_q}{\phi_q}+\eta} &> \frac{\beta_{d1}^2}{\eta} \\
&\Updownarrow \\
\frac{2}{2-\phi_q} &> 1
\end{aligned}$$

The latter inequality always holds for $\phi_q \in (0,1]$.

$$\begin{aligned}
\text{Var}(v|P_4)^{-1} &> \text{Var}(v|P_1)^{-1} \\
&\Updownarrow \\
\frac{(\beta_{d4}+\beta_{q4})^2}{\beta_{q4}^2\frac{1-\phi_q}{\phi_q}+\eta} &> \frac{\beta_{d1}^2}{\eta} \\
&\Updownarrow \\
\frac{-\phi_q^2+2\phi_q+8+\phi_q\sqrt{-3\phi_q^2+4\phi_q+16}}{-2\phi_q^2+2\phi_q+8} &> 1 \\
&\Updownarrow \\
\sqrt{-3\phi_q^2+4\phi_q+16} &> -\phi_q
\end{aligned}$$

The latter inequality always holds for $\phi_q \in (0,1]$.

E.3 Return Volatility

$$\text{Var}(v-P_j) = \sigma_v^2\Big((1-\lambda_j(\beta_{dj}+\beta_{qj}))^2 + \lambda_j^2\beta_{dj}^2\frac{1-\phi_d}{\phi_d} + \lambda_j^2\beta_{qj}^2\frac{1-\phi_q}{\phi_q} + 2\lambda_j^2\beta_{dj}\beta_{qj}\frac{\nu_q}{\phi_q} + \lambda_j^2\eta\Big)$$

We first demonstrate the return volatility increases with automation by the quant.

$$\begin{gathered}
\mathrm{Var}(v-P_2) < \mathrm{Var}(v-P_4) \\
\Updownarrow \\
(1-\lambda_2(\beta_{d2}+\beta_{q2}))^2+\lambda_2^2\beta_{q2}^2\frac{1-\phi_q}{\phi_q}+\lambda_2^2\eta < (1-\lambda_4(\beta_{d4}+\beta_{q4}))^2+\lambda_4^2\beta_{q4}^2\frac{1-\phi_q}{\phi_q}+\lambda_4^2\eta \\
\Updownarrow \\
\left(1-\sqrt{\tfrac{1}{\eta}}\sqrt{\tfrac{\phi_q^2-3\phi_q+4}{(4-\phi_q)^2}}\left(\sqrt{\eta}\sqrt{\tfrac{(2-\phi_q)^2}{\phi_q^2-3\phi_q+4}}+\sqrt{\eta}\sqrt{\tfrac{\phi_q^2}{\phi_q^2-3\phi_q+4}}\right)\right)^2 \\
+\left(\sqrt{\tfrac{1}{\eta}}\sqrt{\tfrac{\phi_q^2-3\phi_q+4}{(4-\phi_q)^2}}\sqrt{\eta}\sqrt{\tfrac{\phi_q^2}{\phi_q^2-3\phi_q+4}}\right)^2\tfrac{1-\phi_q}{\phi_q}+\left(\sqrt{\tfrac{1}{\eta}}\sqrt{\tfrac{\phi_q^2-3\phi_q+4}{(4-\phi_q)^2}}\right)^2\eta \\
< \\
\left(1-\tfrac{2}{\sqrt{\eta}}\sqrt{\tfrac{1}{-3\phi_q^2+4\phi_q+16}}\left(-\tfrac{1}{4}\sqrt{\eta}\phi_q+\tfrac{1}{4}\sqrt{\eta}\sqrt{-3\phi_q^2+4\phi_q+16}+\tfrac{1}{2}\sqrt{\eta}\phi_q\right)\right)^2 \\
+\left(\tfrac{2}{\sqrt{\eta}}\sqrt{\tfrac{1}{-3\phi_q^2+4\phi_q+16}}\tfrac{1}{2}\sqrt{\eta}\phi_q\right)^2\tfrac{1-\phi_q}{\phi_q}+\left(\tfrac{2}{\sqrt{\eta}}\sqrt{\tfrac{1}{-3\phi_q^2+4\phi_q+16}}\right)^2\eta \\
\Updownarrow \\
\frac{\phi_q^2-6\phi_q+8}{\phi_q^2-8\phi_q+16} < \frac{1}{2}-\frac{1}{2}\sqrt{\frac{\phi_q^2}{-3\phi_q^2+4\phi_q+16}} \\
\Updownarrow \\
\frac{1}{\phi_q-4} > \sqrt{\frac{1}{-3\phi_q^2+4\phi_q+16}} \\
\Updownarrow \\
\phi_q-4 < \sqrt{-3\phi_q^2+4\phi_q+16}
\end{gathered}$$

The latter inequality always holds for $\phi_q \in (0,1]$ since the left-hand-side is negative and the right-hand-side is positive within the interval.

We now demonstrate that return volatility decreases with the introduction of the

quantitative investor, both with and without automation.

$$\begin{aligned} \mathrm{Var}(v - P_2) &< \mathrm{Var}(v - P_1) \\ &\Updownarrow \\ \frac{\phi_q^2 - 6\phi_q + 8}{\phi_q^2 - 8\phi_q + 16} &< \frac{1}{2} \\ &\Updownarrow \\ \phi_q(\phi_q - 4) &< 0 \end{aligned}$$

The latter inequality always holds for $\phi_q \in (0, 1]$ since the left-hand-side is always negative in the interval.

$$\begin{aligned} \mathrm{Var}(v - P_4) &< \mathrm{Var}(v - P_1) \\ &\Updownarrow \\ \frac{1}{2} - \frac{1}{2}\sqrt{\frac{\phi_q^2}{-3\phi_q^2 + 4\phi_q + 16}} &< \frac{1}{2} \\ &\Updownarrow \\ -\phi_q\sqrt{\frac{1}{-3\phi_q^2 + 4\phi_q + 16}} &< 0 \end{aligned}$$

The latter inequality always holds for $\phi_q \in (0, 1]$ since the left-hand-side is always negative in the interval.

E.4 Price Volatility

We will rely on derivations in the return volatility analysis by noting that

$$\begin{aligned} \mathrm{Var}(v - P_j) &= \sigma_v^2 + \mathrm{Var}(P_j) - 2\mathrm{Cov}(v, P_j) \\ \mathrm{Var}(P_j) &= \mathrm{Var}(v - P_j) - \sigma_v^2 + 2\lambda_j(\beta_{dj} + \beta_{qj}) \end{aligned}$$

We first demonstrate that price volatility increases with automation by the quant.

$$\begin{aligned}
\mathrm{Var}(P_2) &> \mathrm{Var}(P_4) \\
&\Updownarrow \\
\frac{\phi_q^2 - 6\phi_q + 8}{\phi_q^2 - 8\phi_q + 16} + \frac{4}{4 - \phi_q} &> \frac{1}{2} - \frac{1}{2}\sqrt{\frac{\phi_q^2}{-3\phi_q^2 + 4\phi_q + 16}} + 1 + \sqrt{\frac{\phi_q^2}{-3\phi_q^2 + 4\phi_q + 16}} \\
&\Updownarrow \\
\frac{\phi_q}{4 - \phi_q} &> \sqrt{\frac{\phi_q^2}{-3\phi_q^2 + 4\phi_q + 16}} \\
&\Updownarrow \\
4 - \phi_q &< \sqrt{-3\phi_q^2 + 4\phi_q + 16} \\
&\Updownarrow \\
\phi_q(\phi_q - 3) &< 0
\end{aligned}$$

The latter inequality always holds for $\phi_q \in (0, 1]$ since the left-hand-side is always negative in the interval.

We now demonstrate that price volatility increases with the introduction of the quantitative investor, both with and without automation.

$$\begin{aligned}
\mathrm{Var}(P_2) &> \mathrm{Var}(P_1) \\
&\Updownarrow \\
\frac{\phi_q^2 - 6\phi_q + 8}{\phi_q^2 - 8\phi_q + 16} + \frac{4}{4 - \phi_q} &> \frac{1}{2} + 1 \\
&\Updownarrow \\
\phi_q(4 - \phi_q) &> 0
\end{aligned}$$

The latter inequality always holds for $\phi_q \in (0, 1]$ since the left-hand-side is always positive

in the interval.

$$\begin{aligned}
\mathrm{Var}(P_4) &> \mathrm{Var}(P_1) \\
&\Updownarrow \\
\frac{1}{2} - \frac{1}{2}\sqrt{\frac{\phi_q^2}{-3\phi_q^2+4\phi_q+16}} + 1 + \sqrt{\frac{\phi_q^2}{-3\phi_q^2+4\phi_q+16}} &> \frac{1}{2} + 1 \\
&\Updownarrow \\
\phi_q\sqrt{\frac{1}{-3\phi_q^2+4\phi_q+16}} &> 0
\end{aligned}$$

The latter inequality always holds for $\phi_q \in (0, 1]$ since the left-hand-side is always positive in the interval.

E.5 Trading Intensity and Expected Trading Volume

Quantitative investor The QI does not trade in Economy 1. She trades more aggressively as a fully-discretionary than as a fully-automated investor:

$$\begin{aligned}
\beta_{q2} &> \beta_{q4} \\
&\Updownarrow \\
\sqrt{\eta}\sqrt{\frac{\phi_q^2}{\phi_q^2-3\phi_q+4}} &> \frac{1}{2}\sqrt{\eta}\phi_q \\
&\Updownarrow \\
\phi_q^2 - 3\phi_q &< 0 \\
&\Updownarrow \\
\phi_q(\phi_q - 3) &< 0,
\end{aligned}$$

where the latter inequality holds for $\phi_q \in (0, 1]$.

A higher trading intensity implies a greater expected trading volume and vice versa, immediate from Eq. (61), with $\mu_{x_{qj}} = 0$ and $\sigma_{x_{qj}} = \beta_{qj}\sigma_{s_q}$. Therefore the QI trades less as she turns on her strategy.

Discretionary investor We first show that β_{d4} is always positive:

$$\begin{aligned}
\beta_{d4} &> 0 \\
&\Updownarrow \\
-\frac{1}{4}\sqrt{\eta}\phi_q + \frac{1}{2}\sqrt{\eta}\sqrt{\phi_q - \frac{3}{4}\phi_q^2 + 4} &> 0 \\
&\Updownarrow \\
\frac{1}{2}\sqrt{\eta}\sqrt{\phi_q - \frac{3}{4}\phi_q^2 + 4} &> \frac{1}{4}\sqrt{\eta}\phi_q \\
&\Updownarrow \\
-\frac{3}{16}\phi_q^2 + \frac{1}{4}\phi_q + 1 &> \frac{1}{16}\phi_q^2 \\
&\Updownarrow \\
0 &> \phi_q^2 - \phi_q - 4
\end{aligned}$$

The latter inequality holds for $\phi_q \in (0, 1]$ since the right-hand-side is an upwards parabola with roots at ≈ -1.56 and ≈ 2.56.

Next, we show that the strategic speculator trades less once the QI is introduced in the economy both with and without automation:

$$
\begin{aligned}
\beta_{d2} &< \beta_{d1} \\
&\Updownarrow \\
\sqrt{\eta}\sqrt{\frac{(2-\phi_q)^2}{\phi_q^2-3\phi_q+4}} &< \sqrt{\eta} \\
&\Updownarrow \\
(2-\phi_q)^2 &< \phi_q^2-3\phi_q+4 \\
&\Updownarrow \\
-\phi_q &< 0
\end{aligned}
$$

$$
\begin{aligned}
\beta_{d4} &< \beta_{d1} \\
&\Updownarrow \\
-\frac{1}{4}\sqrt{\eta}\phi_q+\frac{1}{2}\sqrt{\eta}\sqrt{\phi_q-\frac{3}{4}\phi_q^2+4} &< \sqrt{\eta} \\
&\Updownarrow \\
-3\phi_q^2+4\phi_q+16 &< (\phi_q+4)^2 \\
&\Updownarrow \\
-4\phi_q(\phi_q+1) &< 0
\end{aligned}
$$

where the latter inequalities hold because $\phi_q \in (0,1]$.

Finally, we show that the DI always trades more aggressively in Economy 4 than

in Economy 2:

$$
\begin{aligned}
\beta_{d4} &> \beta_{d2} \\
&\Updownarrow \\
-\frac{1}{4}\sqrt{\eta}\phi_q + \frac{1}{2}\sqrt{\eta}\sqrt{\phi_q - \frac{3}{4}\phi_q^2 + 4} &> \sqrt{\eta}\sqrt{\frac{(2-\phi_q)^2}{\phi_q^2 - 3\phi_q + 4}} \\
&\Updownarrow \\
\sqrt{-3\phi_q^2 + 4\phi_q + 16} - \phi_q &> \frac{8 - 4\phi_q}{\sqrt{\phi_q^2 - 3\phi_q + 4}} \\
&\Updownarrow \\
-2\phi_q^2 + 4\phi_q + 16 - \sqrt{-12\phi_q^4 + 16\phi_q^3 + 64\phi_q^2} &> \frac{16\phi_q^2 - 64\phi_q + 64}{\phi_q^2 - 3\phi_q + 4} \\
&\Updownarrow \\
(-2\phi_q^2 + 4\phi_q + 16)(\phi_q^2 - 3\phi_q + 4) - 16\phi_q^2 + 64\phi_q - 64 &> (\phi_q^2 - 3\phi_q + 4)\sqrt{-12\phi_q^4 + 16\phi_q^3 + 64\phi_q^2} \\
&\Updownarrow \\
-\phi_q^3 + 5\phi_q^2 - 10\phi_q + 16 &> (\phi_q^2 - 3\phi_q + 4)\sqrt{-3\phi_q^2 + 4\phi_q + 16} \\
&\Updownarrow \\
f(\phi_q) \equiv \phi_q^4 - 8\phi_q^3 + 26\phi_q^2 - 44\phi_q + 33 &> 0
\end{aligned}
$$

We demonstrate that the latter inequality holds for $\phi_q \in [0, 1]$ by proof by contradiction. First, $f(0) = 33$, $f(1) = 8$. Therefore, for the inequality to not hold it must be the case that $f(\phi_q)$ has a local minimum in $\phi_q \in (0, 1)$. However, this, in turn, implies that $f'(\phi_q)$ has a a value of zero in the interval $\phi_q \in (0, 1)$, i.e.:

$$f'(\phi_q) = 4\phi_q^3 - 24\phi_q^2 + 52\phi_q - 44 = 0$$

However, this does not hold since, $f'(0) = -44, f'(1) = -12$, which in-turn implies that $f''(\phi_q)$ must have a local minimum for $\phi_q \in (0, 1)$. Since

$$f''(\phi_q) = 12\phi_q^2 - 48\phi_q + 52 > 0,$$

$f'(\phi_q)$ cannot have a zero in $\phi_q \in (0, 1)$, which implies that $f(\phi_q)$ is always greater than zero.

A higher trading intensity implies a greater expected trading volume and vice versa, immediate from Eq. (61), with $\mu_{x_{dj}} = 0$ and $\sigma_{x_{dj}} = \beta_{dj}\sigma_v$. Therefore, the DI trades more in expectation with automation by the QI.

Speculator sector First, we show that the speculator sector as a whole trades more with the addition of the QI:

$$\begin{aligned}
\beta_{d4} + \beta_{q4} &> \beta_{d1} \\
&\Updownarrow \\
-\frac{1}{4}\sqrt{\eta}\phi_q + \frac{1}{2}\sqrt{\eta}\sqrt{\phi_q - \frac{3}{4}\phi_q^2 + 4} + \frac{1}{2}\sqrt{\eta}\phi_q &> \sqrt{\eta} \\
&\Updownarrow \\
\sqrt{\phi_q - \frac{3}{4}\phi_q^2 + 4} &> -\frac{1}{2}\phi_q
\end{aligned}$$

$$\begin{aligned}
\beta_{d2} + \beta_{q2} &> \beta_{d1} \\
&\Updownarrow \\
\sqrt{\eta}\sqrt{\frac{(2-\phi_q)^2}{\phi_q^2 - 3\phi_q + 4}} + \sqrt{\eta}\sqrt{\frac{\phi_q^2}{\phi_q^2 - 3\phi_q + 4}} &> \sqrt{\eta} \\
&\Updownarrow \\
\frac{4}{\phi_q^2 - 3\phi_q + 4} &> 1 \\
&\Updownarrow \\
0 &> \phi_q(\phi_q - 3)
\end{aligned}$$

$$
\begin{aligned}
\beta_{d4} + \beta_{q4} &< \beta_{d2} + \beta_{q2} \\
&\Updownarrow \\
-\frac{1}{4}\sqrt{\eta}\phi_q + \frac{1}{2}\sqrt{\eta}\sqrt{\phi_q - \frac{3}{4}\phi_q^2 + 4} + \frac{1}{2}\sqrt{\eta}\phi_q &< \sqrt{\eta}\sqrt{\tfrac{(2-\phi_q)^2}{\phi_q^2 - 3\phi_q + 4}} + \sqrt{\eta}\sqrt{\tfrac{\phi_q^2}{\phi_q^2 - 3\phi_q + 4}} \\
&\Updownarrow \\
\frac{1}{4}(\sqrt{-3\phi_q^2 + 4\phi_q + 16} + \phi_q) &< \sqrt{\frac{(2-\phi_q)^2}{\phi_q^2 - 3\phi_q + 4}} + \sqrt{\frac{\phi_q^2}{\phi_q^2 - 3\phi_q + 4}} \\
&\Updownarrow \\
-\phi_q^2 + 2\phi_q + 8 + \phi_q\sqrt{-3\phi_q^2 + 4\phi_q + 16} &< \frac{32}{\phi_q^2 - 3\phi_q + 4} \\
&\Updownarrow \\
(\phi_q^3 - 3\phi_q^2 + 4\phi_q)\sqrt{-3\phi_q^2 + 4\phi_q + 16} &< (\phi_q^2 - 3\phi_q + 4)(\phi_q^2 - 2\phi_q - 8) + 32 \\
&\Updownarrow \\
f(\phi_q) \equiv -\phi_q^5 + 8\phi_q^4 - 22\phi_q^3 + 8\phi_q^2 + 71\phi_q - 96 &< 0
\end{aligned}
$$

We prove the latter inequality for $\phi_q \in (0, 1]$ via proof by contradiction. Let's first consider the first three derivatives of $f(\phi_q)$ and work backwards:

$$
\begin{aligned}
f'(\phi_q) &= -5\phi_q^4 + 32\phi_q^3 - 66\phi_q^2 + 16\phi_q + 71 \\
f''(\phi_q) &= -20\phi_q^3 + 96\phi_q^2 - 132\phi_q + 16 \\
f'''(\phi_q) &= -60\phi_q^2 + 192\phi_q - 132
\end{aligned}
$$

Note that the two roots of $f'''(\phi_q) = 0$ are $\phi_q^* = 1$ and $\phi_q^{**} = 2.2$. Since $f'''(\phi_q)$ is a downwards parabola, $f'''(\phi_q) < 0$ for $\phi_q \in (0, 1)$. This implies that $f''(\phi_q)$ is decreasing for all $\phi_q \in (0, 1)$. Note that $f''(0) = 16$ and $f''(1) = -40$. Hence, there exists an $\phi_q^0 \in (0, 1)$ such that $f'(\phi_q)$ is increasing from $\phi_q \in (0, \phi_q^0)$ and decreasing from $\phi_q \in (\phi_q^0, 1)$, while reaching a local maxima at ϕ_q^0. Together with the fact that $f'(0) = 71$ and $f'(1) = 48$, $f'(\phi_q) > 0$ for $\phi_q \in (0, 1)$. Finally, since $f(0) = -96$ and $f(1) = -32$ and the function is monotone and increasing across the interval, $f'(\phi_q) < 0$ for all $\phi_q \in (0, 1)$. QED.

The speculator sector as a whole trades less with automation by the quant, which follows from Eq. (61), with $\mu_{x_{dj}+x_{qj}} = 0$ and $\sigma_{x_{dj}+x_{qj}} = \sqrt{(\beta_{dj} + \beta_{qj})^2\sigma_v^2 + \beta_{qj}^2\sigma_{e_q}^2}$, since both aggregate trading intensity $\beta_{dj} + \beta_{qj}$ and the QI's trading intensity β_{qj} decrease as the QI turns on the strategy (i.e. from Economy 2 to Economy 4).

E.6 Speculator Profits

$$\mathbb{E}[\pi_{ij}] = \beta_{ij}(1 - \lambda_j(\beta_{dj} + \beta_{qj})) - \lambda_j \beta_{ij}^2 \frac{1-\phi_i}{\phi_i}$$

In what follows, note that:

$$\begin{aligned}
\beta_{d2} + \beta_{q2} &= \sqrt{\eta}\frac{2}{\sqrt{\phi_q^2 - 3\phi_q + 4}} \\
\beta_{d4} + \beta_{q4} &= \sqrt{\eta}\frac{1}{4}\left(\phi_q + \sqrt{-3\phi_q^2 + 4\phi_q + 16}\right) \\
1 - \lambda_2(\beta_{d2} + \beta_{q2}) &= \frac{2-\phi_q}{4-\phi_q} \\
1 - \lambda_4(\beta_{d4} + \beta_{q4}) &= \frac{1}{2}\left(1 - \frac{\phi_q}{\sqrt{-3\phi_q^2 + 4\phi_q + 16}}\right) \\
\lambda_2 \beta_{q2}^2 \frac{1-\phi_q}{\phi_q} &= \sqrt{\eta}\frac{-\phi_q^2 + \phi_q}{(4-\phi_q)\sqrt{\phi_q^2 - 3\phi_q + 4}} \\
\lambda_4 \beta_{q4}^2 \frac{1-\phi_q}{\phi_q} &= \sqrt{\eta}\frac{1}{2}\frac{-\phi_q^2 + \phi_q}{\sqrt{-3\phi_q^2 + 4\phi_q + 16}}
\end{aligned} \tag{62}$$

Quantitative investor

$$\begin{aligned}
\mathbb{E}[\pi_{q2}] &= \beta_{q2}(1 - \lambda_2(\beta_{d2} + \beta_{q2})) - \lambda_2 \beta_{q2}^2 \frac{1-\phi_q}{\phi_q} \\
\mathbb{E}[\pi_{q4}] &= \beta_{q4}(1 - \lambda_4(\beta_{d4} + \beta_{q4})) - \lambda_4 \beta_{q4}^2 \frac{1-\phi_q}{\phi_q}
\end{aligned}$$

The quant's profits decrease with automation by design, since she switches from a profit-maximizing strategy.

Discretionary investor

$$\begin{aligned}
\mathbb{E}[\pi_{d1}] &= \beta_{d1}(1 - \lambda_1 \beta_{d1}) \\
\mathbb{E}[\pi_{d2}] &= \beta_{d2}(1 - \lambda_2(\beta_{d2} + \beta_{q2})) \\
\mathbb{E}[\pi_{d4}] &= \beta_{d4}(1 - \lambda_4(\beta_{d4} + \beta_{q4}))
\end{aligned}$$

First, we show that the profits of the DI increase with the automation by the QI:

$$\begin{gathered}
\mathbb{E}[\pi_{d2}] < \mathbb{E}[\pi_{d4}] \\
\Updownarrow \\
\beta_{d2}(1 - \lambda_2(\beta_{d2} + \beta_{q2})) < \beta_{d4}(1 - \lambda_4(\beta_{d4} + \beta_{q4}))
\end{gathered}$$

We have already demonstrated that $\beta_{d2} < \beta_{d4}$. It is sufficient to show that $0 < 1 - \lambda_2(\beta_{d2} + \beta_{q2}) < 1 - \lambda_4(\beta_{d4} + \beta_{q4})$.

$$\begin{aligned}
1 - \lambda_2(\beta_{d2} + \beta_{q2}) &< 1 - \lambda_4(\beta_{d4} + \beta_{q4}) \\
&\Updownarrow \\
0 < \frac{2 - \phi_q}{4 - \phi_q} &< \frac{1}{2}\left(1 - \phi_q\sqrt{\frac{1}{-3\phi_q^2 + 4\phi_q + 16}}\right) \\
&\Updownarrow \\
\frac{1}{4 - \phi_q} &> \sqrt{\frac{1}{-3\phi_q^2 + 4\phi_q + 16}} \\
&\Updownarrow \\
(4 - \phi_q)^2 &< -3\phi_q^2 + 4\phi_q + 16 \\
&\Updownarrow \\
\phi_q(\phi_q - 3) &< 0
\end{aligned}$$

where the latter inequality holds for $\phi_q \in (0, 1]$. Therefore $\mathbb{E}[\pi_{d2}] < \mathbb{E}[\pi_{d4}]$.

Next, we show that the profits of the DI decrease with the introduction of the QI, both with and without automation. It is sufficient to show that profits in Economy 4 are lower, since, as demonstrated above, Economy 2 profits are greater than Economy 4 profits.

$$\begin{aligned}
\mathbb{E}[\pi_{d4}] &< \mathbb{E}[\pi_{d1}] \\
&\Updownarrow \\
\beta_{d4}(1 - \lambda_4(\beta_{d4} + \beta_{q4})) &< \beta_{d1}(1 - \lambda_1\beta_{d1})
\end{aligned}$$

We have already demonstrated that $\beta_{d4} < \beta_{d1}$. It remains to be shown that

$$\begin{aligned}
1 - \lambda_4(\beta_{d4} + \beta_{q4}) &< 1 - \lambda_1\beta_{d1} \\
&\Updownarrow \\
\frac{1}{2}\left(1 - \phi_q\sqrt{\frac{1}{-3\phi_q^2 + 4\phi_q + 16}}\right) &< \frac{1}{2} \\
&\Updownarrow \\
0 &< \phi_q\sqrt{\frac{1}{-3\phi_q^2 + 4\phi_q + 16}}
\end{aligned}$$

where the latter inequality holds for $\phi_q \in (0, 1]$.

Speculator sector

$$\mathbb{E}[\pi_{d1} + \pi_{q1}] = \beta_{d1}(1 - \lambda_1\beta_{d1})$$
$$\mathbb{E}[\pi_{d2} + \pi_{q2}] = (\beta_{d2} + \beta_{q2})(1 - \lambda_2(\beta_{d2} + \beta_{q2})) - \lambda_2\beta_{q2}^2\frac{1-\phi_q}{\phi_q}$$
$$\mathbb{E}[\pi_{d4} + \pi_{q4}] = (\beta_{d4} + \beta_{q4})(1 - \lambda_4(\beta_{d4} + \beta_{q4})) - \lambda_4\beta_{q4}^2\frac{1-\phi_q}{\phi_q}$$

First, we show that profits for the entire speculator sector increase with automation by the quantitative investor. From Eq. (62):

$$\begin{gathered}
\mathbb{E}[\pi_{d2}] + \mathbb{E}[\pi_{q2}] < \mathbb{E}[\pi_{d4}] + \mathbb{E}[\pi_{q4}] \\
\Updownarrow \\
(\beta_{d2} + \beta_{q2})(1 - \lambda_2(\beta_{d2} + \beta_{q2})) - \lambda_2\beta_{q2}^2\frac{1-\phi_q}{\phi_q} < (\beta_{d4}+\beta_{q4})(1-\lambda_4(\beta_{d4}+\beta_{q4}))-\lambda_4\beta_{q4}^2\frac{1-\phi_q}{\phi_q} \\
\Updownarrow \\
\sqrt{\eta}\frac{2}{\sqrt{\phi_q^2-3\phi_q+4}}\frac{2-\phi_q}{4-\phi_q} - \sqrt{\eta}\frac{-\phi_q^2+\phi_q}{(4-\phi_q)\sqrt{\phi_q^2-3\phi_q+4}} \\
< \\
\sqrt{\eta}^{\frac{1}{4}}\left(\phi_q+\sqrt{-3\phi_q^2+4\phi_q+16}\right)^{\frac{1}{2}}\left(1-\frac{\phi_q}{\sqrt{-3\phi_q^2+4\phi_q+16}}\right)-\sqrt{\eta}^{\frac{1}{2}}\frac{-\phi_q^2+\phi_q}{\sqrt{-3\phi_q^2+4\phi_q+16}} \\
\Updownarrow \\
\frac{\sqrt{\phi_q^2-3\phi_q+4}}{4-\phi_q} < \frac{2}{\sqrt{-3\phi_q^2+4\phi_q+16}} \\
\Updownarrow \\
-3\phi_q^2+13\phi_q-12<0
\end{gathered}$$

The latter inequality holds for $\phi_q \in (0, 1]$ because the two roots of the quadratic are $4/3$ and 3 and the parabola is concave.

We now demonstrate that the profits for the speculator sector as a whole decrease with the introduction of the quant, either with or without automation. It is sufficient to show that profits are lower for Economy 4, since they are greater than Economy 2, as just

demonstrated.

$$\begin{aligned}
\mathbb{E}[\pi_{d4}] + \mathbb{E}[\pi_{q4}] &< \mathbb{E}[\pi_{d1}] + \mathbb{E}[\pi_{q1}] \\
&\Updownarrow \\
(\beta_{d4} + \beta_{q4})(1 - \lambda_4(\beta_{d4} + \beta_{q4})) - \lambda_4 \beta_{q4}^2 \frac{1 - \phi_q}{\phi_q} &< \beta_{d1}(1 - \lambda_1 \beta_{d1}) \\
&\Updownarrow \\
\frac{2}{\sqrt{-3\phi_q^2 + 4\phi_q + 16}} &< \frac{1}{2} \\
&\Updownarrow \\
0 &< \phi_q(1 - \frac{3}{4}\phi_q)
\end{aligned}$$

where the latter inequality holds for $\phi_q \in (0, 1]$.

Appendix F Breakeven Correlation

We are after the conditions under which the fully automated quant (Economy 4) trades more aggressively than the fully discretionary quant (Economy 2). From Eq. (37) and (48), we have:

$$
\begin{aligned}
&\beta_{q4} > \beta_{q2} \\
&\Updownarrow \\
&\tfrac{1}{2}\sqrt{\tfrac{\eta}{\phi_d}}\left((2-\phi_d)\phi_q - \rho\sqrt{\phi_d\phi_q(1-\phi_d)(1-\phi_q)}\right) > \sqrt{\tfrac{\eta\phi_d\phi_q}{\phi_q k^2 + (2\phi_q - \nu_d\phi_q - \phi_d\phi_q)k + \nu_d\phi_q - \phi_d + \phi_d\phi_q}} \\
&\Updownarrow \text{(label } w = \sqrt{\phi_d\phi_q(1-\phi_d)(1-\phi_q)}\text{)} \\
&\frac{1}{2}\sqrt{\frac{\eta}{\phi_d}}(2\phi_q - \phi_d\phi_q - \rho w) > \sqrt{\frac{\eta\phi_d\phi_q}{\phi_q k^2 + \phi_d}} \\
&\Updownarrow \\
&\frac{1}{2}\sqrt{\frac{\eta}{\phi_d}}(2\phi_q - \phi_d\phi_q - \rho w) > \sqrt{\frac{\eta\phi_d\phi_q}{\phi_q\left(\frac{2\phi_d - \phi_d\phi_q - \rho w}{2\phi_q - \phi_d\phi_q - \rho w}\right)^2 + \phi_d}} \\
&\Updownarrow \\
&\frac{1}{4\phi_d} > \frac{\phi_d\phi_q}{\phi_q(2\phi_d - \phi_d\phi_q - \rho w)^2 + \phi_d(2\phi_q - \phi_d\phi_q - \rho w)^2} \\
&\Updownarrow \\
&\phi_q(2\phi_d - \phi_d\phi_q - \rho w)^2 + \phi_d(2\phi_q - \phi_d\phi_q - \rho w)^2 > 4\phi_d^2\phi_q \\
&\Updownarrow \\
&f(\rho) \equiv w^2(\phi_d + \phi_q)\rho^2 + 2w\phi_d\phi_q(\phi_d + \phi_q - 4)\rho + \phi_d\phi_q^2(\phi_d\phi_q - 8\phi_d + \phi_d^2 + 4) > 0
\end{aligned}
$$

To identify parameter values for which the inequality above holds, we solve $f(\rho) = 0$ analytically to obtain:

$$\rho_{\pm}^* = \sqrt{\frac{\phi_d\phi_q}{(1-\phi_d)(1-\phi_q)}}\left(-1 + \frac{4}{\phi_d + \phi_q} \pm \frac{2\sqrt{3 - \phi_q/\phi_d}}{\phi_d + \phi_q}\right) \tag{63}$$

Since $f(\rho)$ is an upward parabola and $\rho \in [-1, 1]$, the inequality above holds for the following conditions:

1. $\forall\rho \in [-1, 1]$ if $\phi_q > 3\phi_d$ or $\rho_-^* > 1$ or $\rho_+^* < -1$
2. $\forall\rho \in [\rho_+^*, 1]$ if $\rho_-^* \leq -1$ and $\rho_+^* \in (0, 1)$
3. $\forall\rho \in [-1, \rho_-^*]$ if $\rho_+^* \geq 1$ and $\rho_-^* \in (0, 1)$
4. $\forall\rho \in [0, \rho_-^*] \cup [\rho_+^*, 1]$ if $\rho_-^* \in (0, 1)$ and $\rho_+^* \in (0, 1)$
5. Never holds if $\rho_-^* < -1$ and $\rho_+^* > 1$

Given the range of $\phi_d \in (0, 1)$ and $\phi_q \in (0, 1)$, a numerical analysis demonstrates that only conditions 1, 3, and 5 are possible.