Data Redundancies and Cyber Attacks

Linda M. Schilling* and Rodney J. Garratt†

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Abstract

Payments transaction data observed by both sender and receiver entities create data redundancies. Redundancies serve as partial, complementary backups if entities agree to share mutual transaction data in the event of a cyber attack, restoring their records and avoiding default. We analyze how redundancies impact private and socially optimal data security investments. Data-sharing agreements are both privately and socially preferable to data autarky. But redundancies encourage free-riding, leading to underinvestment. A regulated financial infrastructure that monitors and protects all data can bring outcomes closer to the social optimum.

Key words: cyber attacks, data security, data sharing, backups, free riding, redundancy, DLT

JEL codes: D8, G28

*corresponding author. email: lindas@wustl.edu; Olin Business School at Washington University in St Louis, CEPR and FTG. Olin Business School, 1 Snow Way Dr, St. Louis, MO 63130, USA. We thank Cyrus Mevorach (Olin) for excellent research assistance. We thank Bruno Biais, Lucy White, and Xavier Vives as well as conference participants at the FTG meeting in Barcelona, Boston University, and Gerzensee for great comments and insights.

†email: rodney.garratt@bis.org, Bank for International Settlements

1 Introduction

Data is a valuable resource for financial institutions (FIs). It is essential for their operations and FIs have both legal and financial incentives to prevent data loss and protect data privacy. However, there is substantial evidence that firms consistently underinvest in data security, often overlooking the broader societal costs of such vulnerabilities. According to the Online Trust Alliance, up to 65% of banks in the United States are "extremely" vulnerable to cyber attacks (IBS Intelligence, 2017). For instance, the 2017 NotPetya cyber attack, which resulted in widespread data destruction across numerous global organizations, is estimated to have caused over $10 billion in damages.[1]

Numerous studies have suggested that cyber attacks could have severe impacts on the functioning of the economic systems, and in particular, payment systems. Duffie and Younger (2019) perform a comprehensive study of the ability of systemically important U.S. financial institutions to withstand wholesale funding runoffs that would likely be associated with a cyber attack and conclude that there is no guarantee that the payment system would continue to process payments sufficiently rapidly to avoid damage to the real economy. Eisenbach, Kovner, and Lee (2022) estimate that a cyber attack on a top-five United States bank would have spillover effects via liquidity hording that would disrupt up to 31 percent of the network.[2] Kotidis and Schreft (2022) look at a cyber attack on a technology service provider that prevented customers from making payments through Fedwire in their usual way, resulting in propagation effects and payment slowdowns. Further evidence of the cost of cyber risk is found in Florackis, Louca, Michaely, and Weber (2023) who estimate the impact of cyber risk on stock prices.

As the main contribution of this paper we provide the first economic analysis of optimal investment in data security when data is not only valuable but also observed by multiple parties. Independent observers of the same data may mutually serve as backups, creating redundancies that effectively insure each party against the risk of cyber attacks. Our model, involving multiple observers of the same data, is particularly relevant to payment transactions, where each transaction is inherently observed by both a sender and a receiver. While we focus on transacting entities such as banks,

[1] Greenburg, Andy (22 August 2018). "The Untold Story of NotPetya, the Most Devastating Cyberattack in History". Wired.

[2] See also McAndrews and Potter (2002) who describe the impact on United States payment systems of the 9/11 attacks.

credit card companies, or payment processors, our analysis is broadly applicable in any context where information is observed by multiple parties.

We define an attack on a given entity, A, as an event where A's data is compromised, meaning A loses access to any correct or reliable backup of its data, rendering it unable to generate revenue and leading to default. We refer to the way data is stored and backed up across the financial system as an information system.[3] We focus on information system that are exogenously shaped by payment flows. Hence the data structure depends on how banks interact in terms of payments and the system through which payments are made. Every entity keeps a record of its own transaction data, meaning each entity knows everything about the transactions they make and part of what there is to know about transactions others make. We think of the existing banking system as a complete network of a finite number of banks that are symmetric in data size and transact with one another to an equal extent. We exclude on-us transactions (where customers of the same bank transact internally) to focus on the economic impact of data redundancies, and because we show that institutions protect data of which they are the sole observer very differently than data that is observed by multiple parties, see our section 3.

To formalize the concept of redundancies in our paper, we introduce *Data sharing agreement (DSA)* between entities. DSAs enable entities to cross-insure each other against data loss by agreeing to share mutual transaction data in the event of an attack. Under DSAs, each entity's data is backed up in two ways. First, each entity keeps a complete record ("security cover") of its own transactions. Second, because each transaction involves both a sender and a receiver, an additional complete record of any entity's transactions can be reconstructed by combining records from all its transaction partners. This second record is created by consolidating the partial yet complementary records shared by each partner with the attacked entity. DSAs formalize this arrangement by establishing a claim that an attacked entity has on its non-attacked transaction partners. If an entity is attacked and loses access to its data, it can recover its data by contacting transaction partners, who must provide copies of truthful, mutual transaction data as a backup. DSAs thus create reliable, independent backups in the financial system, since banks are attacked independently of one another, and because truth-telling can be enforced through cryptographic methods such as hash functions. With DSAs, entity A's data is compromised only if A and at least one of its transaction, and

[3]Throughout the paper we use the terms *data* and *information* synonymously.

thus, backup partners, say entity B, are attacked simultaneously. In such a case, the transaction data between A and B is irretrievably lost, and we adopt an all-or-nothing assumption that A defaults due to reputational damage.

Data Sharing Agreements (DSAs) create an interesting externality among firms. By investing in data security, an entity can lower its risk of being attacked, thereby reducing the chance of its data being compromised. The greater the investment in security, the lower the likelihood of an attack on that entity. Importantly, by enhancing its data security, entity A also decreases the risk of compromised data for other entities, as A acts as a backup for them through the DSAs. At first glance, DSAs appear to reduce each entity's risk of data compromise relative to a benchmark scenario of data autarky, where every entity is the sole observer of its data. However, DSAs also introduce a free-rider problem: each party may rely on others to safeguard the data, and fail to internalize how its own underinvestment in security weakens the overall data protection across entities.

We analyze the data economy with DSAs and find two types of private equilibria. There exists a symmetric, interior equilibrium where all entities invest in security, but there also exist asymmetric corner equilibria where one entity shirks by investing nothing (total free-riding), while the remaining entities invest in security and, through DSAs, indirectly provide protection to the shirking entity. We show that the social optimum always involves symmetric investment across all entities, which rules out the asymmetric corner equilibrium as a socially optimal outcome.

We contrast the data economy with DSAs to a data economy in autarky, where each entity protects its data as if it were the sole observer, facing complete data loss and default if attacked, as it cannot rely on data sharing with others. We demonstrate that entering into DSAs is ex-ante both socially and privately optimal across all possible private equilibria, compared to the autarky outcome. Specifically, individual and aggregate profits are higher with DSAs than in autarky, even in the private asymmetric shirking equilibrium. In this sense, autarky profits serve as a lower bound for profits in the DSA environment.

However, even in the symmetric private equilibrium with DSAs, where every entity invests to protect its data, the entities do not fully internalize the security benefits of the DSA agreements. They invest less in data security than in the autarky case, and, in particular, invest less than would be optimal under a planner's solution. Both types of private DSA equilibria yield underinvestment.

In such circumstances, achieving a social optimum may require government intervention or policies that reflect private incentives to free ride. However, regulating investments in data security is complex. These investments may not be fully observable to regulators, and entities might have an incentive to understate the value of their data, which affects optimal investment decisions. Consequently, we adopt an indirect regulatory approach and investigate whether the inception of a regulated financial market infrastructure (FMI) that observes the precise transaction data of all entities in the economy can enhance security outcomes and bring them closer to the social optimum. This FMI can be conceptualized as a payment infrastructure through which all transaction data is processed, such as Fedwire in the United States, CHIPS, or ACH, or as a central securities depository that facilitates the transfer of securities across financial institutions via book entry. We, in particular, choose this indirect implementation method because there is recent interest in the private sector surrounding the creation of a common data layer (analogous to a global state machine in blockchain environments) that records all transactions in traditional economic systems. The analysis presented in this paper would be directly applicable to the security of this additional data layer.

With the FMI and the DSAs, there is a third method for recovering all data: the records maintained by the FMI. We assume that the FMI is a public entity run by the government, which aims to maximize the social welfare of the system by optimally choosing its level of data security while internalizing the impact its security decisions have on those made by private entities. Furthermore, we assume that the FMI faces the same probability of attacks and costs for data security as the private entities.

We analyze whether the introduction of this additional socially-minded FMI can enhance overall security and move it closer to the social optimum. If the FMI opts to provide and protect this third layer of security, we demonstrate that private entities may respond by further reducing their investments away from the social optimum (as compared to the scenario without the FMI). However, the overall impact on welfare, when accounting for the FMI's security expenditures, remains positive. In a regulated environment with the FMI, the outcome can never be worse than in an unregulated setting, as the FMI can choose to walk away by investing zero if the private entities engage in excessive misbehavior. In such a scenario, the third layer of security would not be established, and the FMI framework endogenously reverts to the unregulated environment. This is due to the particular design of our attack probability function. Entities must not only determine whether to protect their data but also to what extent.

If an entity or the Financial Market Infrastructure (FMI) invests nothing in security, it is guaranteed to face an attack, which effectively represents a decision to abandon any protective measures.

We simulate the FMI environment using two different attack probability functions and demonstrate that, despite shirking, the environment with the FMI can achieve greater overall security than the social planner in an unregulated setting. This is possible for two primary reasons related to the information environment provided by the FMI, which significantly differs from that of the unregulated environment.

First, the regulated FMI environment can offer up to three covers of security if the FMI invests, while the social planner can only provide two covers. Second, the FMI consolidates its security coverage of all data through a single investment, whereas the social planner must invest across all entities, incurring the cost function multiple times.

1.1 Literature

This paper is related to the literature on the economics of information security. This literature examines public good aspects of security provision from three perspectives outlined in Hirshleifer (1983): total effort, weakest link and best shot. Crucially, each of these perspectives models security as a joint effort toward a common goal. For instance, the likelihood of a successful attack on a city may depend on the thickness of the wall, the height in the place where the wall is the shortest, or, if there are several walls, the height of the highest wall. Hirshleifer (1983) demonstrates that each of these scenarios leads to the under-provision of security relative to the social optimum; see also Cornes (1993). Meanwhile, Varian (2004) relates outcomes specifically to expenditures on computer system reliability and security where the likelihood of avoiding an attack depends on expenditures. Our work shares this emphasis on costs, benefits, and probability of failure, however, the public goods problem we examine does not fit any of the cases considered by Varian or the previous studies.

In our setup, contributors to security are not protecting the same thing. They are protecting their own proprietary data sets. The public good aspect is introduced only because there is overlap between these data sets, in particular data relating to transactions that are common to both parties is the same, and aggregation of others' data sets can create a substitute for one's own data set. Our analysis is therefore more closely related to the literature on production externalities (Laffont, 2008). That literature ex-

plains why output that results from competition among private firms whose output impacts the output of others is likely to be suboptimal. In this case, the output we are considering is security and its production by one firm, in our case a bank, contributes to the security of others. It is, however, not a standard externality arrangement in that, if we define security with respect to all of a bank's transactions, then the positive externality we are modeling is generated only by the combined effort of all the other banks. We know of no previous literature that considers this type of externality generation. The literature on network effects comes closest, in that this literature (eg. chapter 17 of Easley and Kleinberg (2010)), considers situations where participation by a certain fraction of the population is required to generate a positive externality.

Very much related, our paper contributes to the growing literature on the economic implications of data or information sharing. Kühn, Vives, et al. (1994); Vives, Durlauf, and Blume (2008) study how information sharing among firms affects competition. Vives (1984) and Ziv (1993) study the incentives to share information depending on the competition structure of firms. Vives (1990b, 2006) study incentives to share information, and its impact on welfare. Kirpalani and Philippon (2020) analyze the impact of consumer data sharing with a two-sided platform on match quality and market power. In the context of open banking, where borrowers may share their data with financial institutions, Parlour, Rajan, and Zhu (2022); He, Huang, and Zhou (2023) and Goldstein, Huang, and Yang (2022) study the impact on lending market competition and allocative efficiency. Our focus substantially differs from these papers since we study how information sharing creates data redundancies which gives rise to free-riding with regard to the protection of data with implications for cyber risk.

Externalities related to security expenditures also arise in contagion models where security expenditures determine the likelihood a threat will pass through one party and on to others (see Acemoglu et al. (2016). In that work, the level of security expenditures by each participant affects the possibility of cascading failures. Interestingly, in this setting when the attack location is endogenous externalities can lead to over-provision of security relative to the social optimum, as agents seek to "push" the attack elsewhere. These externalities are not present in our model as there is no contagion across banks' proprietary data systems.

This paper also contributes to the literature on the value of information (Feltham, 1968; Hirshleifer, 1978; Morris and Shin, 2002; Angeletos and Pavan, 2007), analyzing how the presence of information backups creates free-riding with regard to the protection of said information.

We add to the economic literature on incentives involved with attacking information ledgers such as blockchains (Biais, Bisiere, Bouvard, and Casamatta, 2019; Ebrahimi, Routledge, and Zetlin-Jones; Budish, 2018; Schilling, 2019; Huberman, Leshno, and Moallemi, 2021). Unlike that literature, the information systems we consider are not constrained to systems where every entity observes everything, as in the case of Bitcoin. Rather, in our setting complex free-riding incentives, and thus risk of data compromise occurs because parties observe only part of what other parties observe.

We contribute to the literature that analyzes the economic value of data as a source of revenue to firms (Begenau, Farboodi, and Veldkamp, 2018; Farboodi, Mihet, Philippon, and Veldkamp, 2019; Farboodi and Veldkamp, 2021). Unlike these papers, we analyze how redundant observations of data give rise to free-riding, and thus suboptimal protection of data despite its value.

Lastly, we contribute to the computer science literature on backups and redundancies (Ghaffarzadegan, 2008; Littlewood and Strigini, 2004; Jia, Xin, Wang, Guo, and Wang, 2018; AlZain, Soh, and Pardede, 2012) as well as the literature on data security and privacy (Denning and Denning, 1979; Sun et al., 2014; Aldossary and Allen, 2016; Yu et al., 2016; Wang et al., 2019; Garagad et al., 2020) by considering the economic incentives of agents to protect their data.

2 Model

Initially, we consider one-period simultaneous-move games. Later, we consider two-stage games when introducing the FMI.

Let $\mathcal{N}$ denote the finite set of all agents ("entities") in the economy with $N = |\mathcal{N}|$. In general we think of these entities as banks that observe information on payment transactions of their customers. We do not model bank customers. All entities have deep pockets.

2.1 Data and Information sets

We assume that every entity transacts with every other entity. Therefore, one may think of these entities as being connected in the form of a complete network, where transactions across entities cause network links, see Figure 1.

Let I denote the set of all transaction data in the economy, i.e. across these N entities. This can include payment information, such as transaction amounts, names

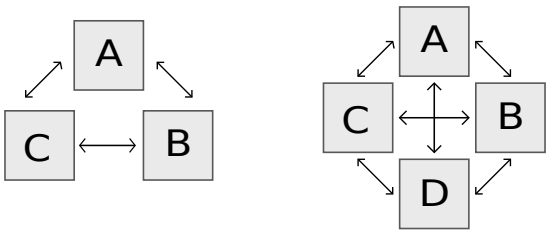


Figure 1: Symmetric information sets: Left: $N = 3$, $|I_i| = 2$ and $|I_i \cap I_j| = 1$ for $i, j \in \{A, B, C\}, i \neq j$. Right: $N = 4$ with $|I_i| = 3$ and $|I_i \cap I_j| = 1$ for $i, j \in \{A, B, C, D\}, i \neq j$

of transactors, time and date stamps and any other customer information that might be recorded by the financial institutions.

Let $I_i \subseteq I$ denote the information on transaction data observed by entity $i \in \mathcal{N}$. All entities jointly observe the full data set I, $I \subseteq \bigcup_{j=1}^{N} I_j$.

A transaction τ is observed by bank $i \in \mathcal{N}$ if $\tau \in I_i$. We assume that entities are symmetric in the quantity of transaction data they observe, $|I_i| = |I_j|$ for all $i, j \in \mathcal{N}$, but not with regard to the content, generically $I_i \neq I_j$. Observed information I_i is exogenous to i, that is, there is no data acquisition. One can think of I_i as all transactions observed by entity i up until now.

Data sharing and Redundancy A payment transaction (data point) $\tau \in I$ requires a sending and a receiving entity where sending and receiving happens in the same time period. We abstract from cases where transactions happen within one institution to focus on the economic impact of data redundancies, and because we show that institutions protect data of which they are the sole observer very differently than data that is observed by multiple parties, see our analysis in section 3. Therefore, every transaction τ in our model is observed by exactly two entities, implying that entities observe partially overlapping information on transactions $\tau \in I_i \cap I_j \neq \emptyset$. These partial overlaps are important for our analysis since they create redundancy of observed data.

All transactions are truthful. All entities are symmetric with regard to the intensity they transact with one another, i.e., the quantity of transactions that are sent or received across any two entities i, j, $|I_i \cap I_j|$, is the same across all entities $i, j \in \mathcal{N}$. For examples of this symmetric case, see Figure 1.

We now draw on algebraic topology to define an important concept that we use throughout the paper to create data security backups ("covers")[4] and, thus, redun-

[4]In algebraic topolgy one would say that the (information) sets of entities $k_1, \ldots k_m$ form a topological cover of set I_i.

dancies of data via the observed information of other entities.

Definition 2.1 (Security Cover: Creating redundancy via shared transaction data)**.** *Consider the information of entity i, I_i. Entities $k_1, \dots k_m \in \mathcal{N}/\{i\}$, $m < N$, form a cover of information I_i if their joint information sets include all the information in I_i, $I_i \subseteq \bigcup_{j=1}^{m} I_{k_j}$.*

Because we rule out "on-us" transactions so that every transaction is across entities, and because every entity transacts with every other entity, a security cover and thus a backup of data I_i is generated by the union of all the information sets of those entities i transacts with, $\{1, \dots, N\}/\{i\}$.

$$I_i \subseteq \bigcup_{j \in \mathcal{N}/\{i\}}^{m} I_j. \tag{1}$$

Further, and by the same reason, every bank i is necessarily part of a security cover of every other bank j. That is, banks mutually serve as security covers and this is important for our analysis.

Transaction example *In the case of 4 banks, the assumption of a complete network and symmetry in transaction size requires, for instance: Bank A does one transaction with B, B does one transaction with C, C does one transaction with D, D does one transaction with A, B does one transaction with D, and A does one transaction with C. Then $|I_i| = 3$ since every bank makes exactly three transactions, and $|I_i \cap I_j| = 1$ since every pair of banks interacts exactly once, $i \neq j$, see Figure 1.*

2.2 Value of Information

An entity $i \in \mathcal{N}$ requires access to its uncompromised data I_i to generate revenue $R(|I_i|)$. Therefore, information I_i is valuable to every entity $i \in \mathcal{N}$, and worth protecting. We define data I_i as compromised if i's data is lost (destroyed by an attack) *and* every backup of i's data in the economy is attacked. If i's data is compromised, i no longer generates revenue and defaults. Note, the risk that entity i's data is compromised differs from the risk of privacy loss, which we do not consider in this analysis. We assume revenue is independent of the number of entities that observe information I_i. Moreover, we assume that revenue $R(|I_i|)$ only depends on the quantity of $i's$ data $|I_i|$, meaning all data points are equally valuable. Because all entities

observe the same quantity of data, they are symmetric with regard to their revenue, $R(|I_i|) = R(|I_j|)$, $i, j \in \mathcal{N}$.

2.3 Attacks and Data Security

We model attacks in a reduced form without modeling attackers as separate agents.

A successful attack on entity i occurs with probability $\alpha(c_i) \in [0, 1]$, where the exogenous attack probability function $\alpha(\cdot)$ is twice differentiable, strictly decreasing and strictly convex in i's endogenous investment in data security $c_i \geq 0$. We model individual attack probabilities on different entities as independent. We think of an attack on entity i as the event where a malicious actor hacks the database of entity i and destroys its data or blocks access to data (malware). Each entity chooses investment in data security $c_i \geq 0$ to protect its information I_i and therefore to reduce the chance of an individual attack. By the choice of c_i, each entity's attack probability $\alpha(c_i)$ becomes endogenous.

If an entity is attacked, privacy is always lost but its data is not necessarily compromised. Data I_i is compromised only if i is attacked *and* every backup of i's data in the economy is attacked. Therefore, individual security investment choices give rise to an important positive spill-over effect which is at the heart of the paper: entity i's transaction data is also observed by other entities with which i's customers transact. The security investments of other entities, thus, benefit the data security of i, reducing its probability of data loss, and vice versa. This is very much in the spirit of a public goods game where the investment of one entity benefits all, which ultimately gives rise to free-riding, see below.

Costs of data security Data security is costly. Let f_I denote the cost of one unit investment in data security. By investing c_i in data security, each entity incurs the cost $-c_i\, f_I$ to reduce its individual attack probability to $\alpha(c_i)$ and, thus, the chance of its data becoming compromised. All entities share the same cost and attack function which is common knowledge.

We make the following assumptions on the individual attack probability function

Assumption 2.1. *The function $\alpha(\cdot)$ is strictly convex.*

Assumption 2.2. *The function $\alpha(\cdot)$ satisfies*
(i). $\alpha(0) = 1$ *and* $\lim_{c\to\infty} \alpha(c) = 0$
(ii). $-\alpha'(0) > \frac{f_I}{R(|I_i|)}$
(iii) The function $h(x) = \alpha(x)/\alpha'(x)$ is weakly decreasing.

Assumption 2.2 (i) says that a zero security investment leads to an attack for sure whereas infinite investment reduces the chance of an attack to zero. Assumption 2.2 (ii) says that the chance of an attack declines sufficiently fast for a small investment in security. Assumption 2.2 (iii) plays a role in ensuring the absence of asymmetric interior or corner equilibria. Note that if 2.2 (i) holds, then $-\lim_{c\to\infty}\alpha'(c)(2\alpha(c)-\alpha^2(c))=0<\frac{f_I}{R(|I_i|)}$ follows.

These assumptions can be relaxed, and we do so later, but they lead to cases which are not empirically interesting. The assumptions ensure that the cost of security is not so large that it never makes sense for banks to protect their data and not so infinitesimally small that they would choose an infinite amount of protection.

2.4 Data Recovery, data sharing and chance of data loss

We can write each entity's profit as

$$\Pi_i(I_i, c_i, \alpha(\cdot), c_1, \ldots, c_N) = (1 - \mathbb{P}(I_i \text{ compromised}|c_1, \ldots, c_N))R(|I_i|) - c_i\, f_I \qquad (2)$$

where the probability that i's data is compromised equals the probability that all covers of i's data are successfully attacked, and thus depends on the investments in data security of all transacting counterparties.

We assume that if i loses access to some share of its data, i defaults due to reputational damage. In that sense, we take an all-or-nothing approach here.

At this point we cannot pin down the probability of data loss further since it varies depending on how much information an entity observes and by how many parties that information is observed. We therefore next proceed to consider concrete information systems.

For the equilibrium concept, since this is a simultaneous move game, each entity plays an optimal response in terms of investment given the investment choices by all other entities (Nash equilibrium). Later, when we add a third party entity (FMI) and an additional time step, our equilibrium concept becomes subgame-perfect Nash.

3 The Data Economy in Autarky (Benchmark)

In this section we analyze the real world setting without data sharing. In data autarky, each entity i_j, $j = 1, \ldots, N$ observes its data I_{i_j} and is aware of the fact that all of the

entities it transacts with have a copy of mutual transaction data. However, the entities have no contracts with one another to share mutual data in the case of an attack. No entity can, therefore, rely on the other entities for data protection since there is no data sharing. Thus, each entity i_j, $j = 1, \ldots, N$ has access to only one single copy of its data, namely its own transaction data I_i, which it protects optimally. If entity i_j, $j = 1, \ldots, N$ gets attacked, it loses access to its data, and defaults. Therefore, each entity's probability of data loss equals $\alpha(c_{i_j})$, and every entity i_j maximizes the profit function

$$\Pi^a_{i_j}(c_{i_j}) = (1 - \alpha(c_{i_j}))\, R_{i_j} - c_{i_j} f_I \tag{3}$$

through its investment $c_{i_j} \geq 0$ in data security. The social planner, on the other hand, maximizes the joint entities' profits in autarky:

$$\Pi_P(c_{i_1}, \ldots, c_{i_N}) = \sum_{j=1}^{N} \Pi_{i_j}(c_{i_j}) = \sum_{j=1}^{N} \left[(1 - \alpha(c_{i_j}))\, R_{i_j} - c_{i_j} f_I\right] \tag{4}$$

Since the institutions do not cross-insure each other by writing data sharing agreements, an entity's investment in data security has no effect on another entity's probability of data loss, and thus, profits. Therefore, the planner's optimality conditions and solutions coincide with the private optimality conditions and solutions, and we obtain,

Proposition 3.1 (Autarky: Private and social optimum)**.** *Let assumption 2.2 hold. In autarky, the privately and socially optimal investments in data security coincide, are positive, finite, and given as the solution $c^a_{i_j} \in (0, \infty)$ to the equation*

$$-\alpha'(c_{i_j}) = \frac{f_I}{R_{i_j}}, \text{ for all } j = 1, \ldots, N. \tag{5}$$

Private profits in the autarky equilibrium are given as $\Pi^a_{i_j}(c^a_{i_j}) = (1 - \alpha(c^a_{i_j}))\, R_{i_j} - c^a_{i_j} f_I$ whereas welfare in the autarky equilibrium equals $\Pi^a_P = \sum_{j=1}^{N} \Pi_{i_j}(c^a_{i_j})$. If Assumption 2.2 fails, and $-\alpha'(0) \leq f_I/R$, the privately and socially optimal investment is zero, $c^a_{i_j} = 0$, and profits and welfare equal zero.

The proof is straightforward: The first order condition of entity i_j is

$$\frac{\partial}{\partial c_{i_j}} \Pi_{i_j}(c_{i_j}) = -\alpha'(c_{i_j})\, R_{i_j} - f_I = 0 \tag{6}$$

With assumption 2.2, $-\alpha'(0) > f_I/R$ implies that (6) has a finite, interior solution $c^a_{i_j} > 0$ because $\alpha(c)$ is convex, continuous and because $\alpha(c) \to 0$ for $c \to \infty$.[5] Moreover, because $\alpha(c)$ is strictly convex, it crosses f_I/R only once, so that the solution $c^a_{i_j} > 0$ is unique. If $-\alpha'(0) \leq f_I/R$, then $\frac{\partial}{\partial c_{i_j}}\Pi_{i_j}(c_{i_j}) \leq 0$ for all $c_{i_j} \geq 0$, implying that no investment in data security is optimal, $c^a_{i_j} = 0$ in autarky. The same holds for the social planner since the first order conditions coincide.

4 Data Economy with Data Sharing Agreements and Redundancies

In this section, we introduce data redundancies through mutual data sharing agreements (DSAs). All entities know that they transact with one another, implying that there are exactly two observing entities for every transaction data point. Relative to the autarky case, we assume, that all entities $i_j,\ j = 1, \ldots, N$ mutually insure each other against the risk of data loss by writing the following data-sharing contract:

Assumption 4.1 (Data Sharing Agreements (DSAs))**.** *All entities write Data Sharing Agreements (DSAs) with one another, to truthfully share mutual transaction data at zero cost in the case where one entity gets attacked, and loses access to its data.*

We prove the ex ante social and private optimality of writing such DSAs relative to the autarky case in section 5. The DSAs create a legal claim an attacked entity has on the entities it transacts with. The literature indicates that incentives for truthful data sharing are influenced by firms' competition structure, as discussed by Vives (1984) and Ziv (1993). Here, we assume that truthful information sharing can be enforced through cryptographic methods, such as hash functions. Alternatively, the attacked entity can verify truthfulness by reviewing recovered data and ensuring that all payment senders' account balances are positive and within budget constraints at the time of each transaction. A negative balance would indicate a discrepancy, potentially revealing an erroneous or missing transaction. In such cases, the attacked entity could pursue legal action against the other entities.

We believe that writing such insurance contracts to share data will be legal in the real world, since the hacked entity observed that same data already before the attack, and only mutual transaction data is shared. Therefore, in the event of an attack and

[5] It holds $\lim_{c\to\infty} \alpha'(c) = \lim_{c\to\infty} \lim_{h\to 0} \frac{\alpha(c+h)-\alpha(c)}{h} = 0$.

subsequent data sharing, the attacked institution solely restores its knowledge about the transactions of its customer base.

In a nutshell, writing these DSA's has the effect that every entity i_j is provided with an additional security backup (cover) of its data, *if* all entities invest in data security.

As an important implication, data I_i is compromised only if *both* covers of I_i are jointly attacked. If at least one cover of I_i is not attacked, entity i retains access to its data I_i, and thus prevents default. For instance, if one entity i_j gets attacked and loses access to its data I_{i_j}, but the remaining entities $\{i_1, \ldots, i_N\}/\{i_j\}$ are not attacked, these remaining entities provide the hacked entity i_j with a perfect copy of its mutual transaction data $I_{i_j} \cap \bigcup_{\substack{k=1, \\ k\neq j}}^{N} I_{i_k}$ at zero costs.

The second security cover, $\bigcup_{\substack{k=1, \\ k\neq j}}^{N} I_{i_k}$, is formed by aggregating the mutual transaction data from all entities with which i interacts. This cover is thus constructed from $N-1$ partial backups, each complementing the others, as each transaction is observed by no more than two entities.

Not only entity i knows that it benefits from the investment in data security by all other institutions. Vice versa, every entity including i serves as part of the DSA cover to every other institution (it transacts with). Therefore, i's investment in data security also reduces the chance that all other institutions' data becomes compromised.

If more than one entity, say i_j and $i_{\hat{j}}$, get attacked, the data $I_{i_j} \cap I_{i_{\hat{j}}}$ cannot be recovered, the DSA between these entities is worthless, and entity i_j and $i_{\hat{j}}$ both default. In particular, the DSA cover to entity i_j, formed by the remaining institutions $\mathcal{N}/\{i_j\}$, $I_{i_j} \cap \bigcup_{\substack{k=1, \\ k\neq j}}^{N} I_{i_k}$ is successfully attacked if at least one among the entities $\mathcal{N}/\{i_j\}$ that form the cover is attacked. This occurs for sure, if one of the entities $\mathcal{N}/\{i_j\}$ does not invest in security, by $\alpha(0) = 1$. In that sense, our model takes an all or nothing approach to model a severe loss of reputation, and an entity's default if that entity loses access to some of its data.

Writing the DSAs creates up to two security covers for each entity, and creates two covers only if all entities invest $c_i > 0$ for all $i \in \mathcal{N}$. If one institution shirks to zero, this has serious consequences for all other entities. If an entity does not invest in data security, it is attacked for sure by assumption 2.2 and $\alpha(0) = 1$. It, therefore, destroys its own, say i_j's, security cover. Further, because every institution is part of the DSA covers of all of the other institutions, the shirking entity renders all but its own DSA cover worthless since those DSA covers are attacked for sure. The shirking entity, therefore, destroys the second security cover for all other institutions. The remainder

of this section analyzes how the mutual DSAs impact the entities' incentives to protect their data.

4.1 Analysis

To ease the exposition, we initially focus on the case with three entities $\mathcal{N} = \{A, B, C\}$ that transact with one another and observe transaction data. All of the results hold for N entities and are provided in its general form in section 9. The data structure is as depicted in Figure 2.

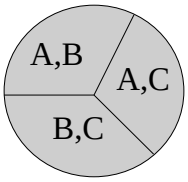


Figure 2: Each transaction in I is observed by exactly two entities, the sender and the receiver.

4.2 The private optimum with data sharing

We focus on Bank A. Given our symmetric structure the same calculations can be applied to banks B and C. There are three scenarios under which the data of entity A is lost: A and B are attacked, A and C are attacked or all three are attacked. Hence, the probability of a data loss for bank A is given as the probability that A is attacked and at least one out of B and C is attacked. Individual attack probabilities are independent.

Thus, A's probability of a data loss equals

$$\alpha(c_A)[\alpha(c_B) + \alpha(c_C) - \alpha(c_B)\alpha(c_C)]. \tag{7}$$

We need to subtract $\alpha(c_B)\alpha(c_C)$ once because the probability that both B and C are attacked is included twice in the probability that B is attacked and that C is attacked. In equation (7), one can see very nicely the spill-over effect of investment in data security: A's probability of a data loss very much depends on B's and C's investment in data security and vice versa since they mutually serve as backups. This is related to public goods provision by private parties.

The expected profits of entity A given expenditure c_A equals

$$\pi_A(c_A, c_B, c_C) = (1 - \alpha(c_A)[\alpha(c_B) + \alpha(c_C) - \alpha(c_B)\alpha(c_C)])\ R(|I_A|) - c_A f_I. \tag{8}$$

where $(1 - \alpha(c_A)[\alpha(c_B) + \alpha(c_C) - \alpha(c_B)\alpha(c_C)])$ equals the probability that entity A retains access to its data and can thus earn revenue $R(|I_A|)$. See that $\pi_A(c_A, c_B, c_C)$ is symmetric in c_B, c_C. Likewise, we define $\pi_B(c_A, c_B, c_C)$ and $\pi_C(c_A, c_B, c_C)$.[6] We abstract from limited liability. All entities invest simultaneously. Each entity i sets investment c_i to maximize profits, given the optimal investment responses by all other entities (Nash), and taking into account how others' investments reduce the chance of i's data becoming compromised. We have the following result.

Proposition 4.1 (DSA Data Economy - Private Equilibrium)**.** *Suppose that assumptions 2.1 and 2.2 hold. Let* $|I_A| = |I_B| = |I_C|$.
(i) There exists a unique interior, finite private equilibrium, and this equilibrium is symmetric, $0 < c_A^* = c_B^* = c_C^* < \infty$*, characterized as the solution to*

$$-\alpha'(c_A^*)\alpha(c_A^*)[2 - \alpha(c_A^*)] = \frac{f_I}{R(|I_i|)}. \tag{9}$$

(ii) Asymmetric corner equilibrium: If

$$-\alpha'(0)\alpha(c^a)(2 - \alpha(c^a)) < \frac{f_I}{R} \tag{10}$$

where c^a *is the autarky solution to* $-\alpha'(c) = \frac{f}{R}$*, there exist three asymmetric corner equilibria* $(0, c^a, c^a)$*. In that equilibrium, one entity does not invest and the remaining entities invest the autarky level,* $c^a > 0$*, thus, indirectly providing cover for the shirking institution via the DSAs.*
(iii) If assumption 2.2(ii) fails, and $-\alpha'(0) \leq \frac{f_I}{R(|I_A|)}$*, "no investment" arises as the unique private equilibrium, i.e.,* $c_i^* = 0$ *for all* i*.*

By the Proposition, as long as assumption 2.2 holds, there exist two types of private equilibria, a symmetric interior equilibrium and 3 asymmetric corner equilibria. The asymmetric corner equilibria are interesting since they involve extreme free-riding by one entity at the expense of all other entities. Because the free-riding institution, call it A, shirks by investing zero, that entity gets hacked for sure. The shirking of one

[6] That is, e.g., $\pi_B(c_A, c_B, c_C) = (1 - \alpha(c_B)[\alpha(c_A) + \alpha(c_C) - \alpha(c_A)\alpha(c_C)])\ R(|I_B|) - c_B f_I$.

institution, thus, changes the security problem for all other entities since it cannot provide a backup of mutual data if other entities get attacked. As a consequence, the DSAs with all other entities become worthless.

By investing, every entity participates in establishing a second security cover for every other entity. If one entity fully shirks, all the remaining entities lose the second cover, and have one cover only if they invest on their own. If one entity fully shirks, the remaining entities play a best response. Since they cannot rely on the data sharing agreements, every entity optimizes as if it were the only observer of its data, and behaves as in the autarky economy, investing the autarky amount. This response makes them a non-shirking entity, enjoying a single security cover established through their own investments, see (6), and earning autarky profits.

$$\pi_B(c_B^a) = (1 - \alpha(c_B^a))\ R(|I_B|) - c_B^a\ f \tag{11}$$

In particular, B's profits become independent of C's investment.

The shirking entity A, likewise enjoys a single security cover but indirectly, relying on the data sharing agreement with the other entities because it does not invest and thus gets hacked for sure. Most importantly the defecting entity continues to make profits

$$\pi_A(0) = \Big[1 - (\alpha(c_B^a) + \alpha(c_C^a) - \alpha(c_B^a)\alpha(c_C^a))\Big]\ R(|I_A|) \tag{12}$$

A's profit depends on the investments of B and C that establish and protect the DSA cover. Because A does not invest, it does not pay investment fees f_I. We show below that an asymmetric corner equilibrium is never socially optimal. Note, as long as $\alpha'(0) > f_I/R$, i.e. if assumptions 2.1 and 2.2 hold, there cannot exist an equilibrium where more than one entity shirks to zero. This is because the two shirking institutions get hacked and lose access to their data for sure. Therefore, there exists no backup of the mutual data of the two shirking institutions, and both entities default. This is, however, not optimal since every entity would optimally protect its data in autarky, $c^a > 0$.

In the symmetric interior equilibrium, all entities invest the same amounts $c_i^* = c_A^* = c_B^* = c_C^*$, and thus enjoy two security covers, their direct cover established by positive investment and indirect cover through the DSAs. Profits are symmetric, depend on all entities' investment levels, and equal

$$\pi_A(c_A^*) = \Big[1 - \alpha(c_A^*)(\alpha(c_B^*) + \alpha(c_C^*) - \alpha(c_B^*)\alpha(c_C^*))\Big]\ R(|I_A|) - c_A^*\ f_I \tag{13}$$

Note that in every private equilibrium, each entity's profit is bounded from above, $\pi_A(c_A, c_B, c_C) < R(|I_A|) - c_A f_I$, because the probability of no data loss is bounded by one. Therefore, every entity invests a finite amount in data security, if at all.

One may think of the private symmetric equilibrium as the "good" equilibrium with DSAs. Every entity invests in security, and thus contributes to establishing a second security cover to all other entities via the DSAs. The private corner equilibria present the "bad" equilibria, where one entity fully free-rides on the security investments of the remaining entities and the DSAs. We however show in Proposition 5.1 below that even the symmetric private equilibrium involves free-riding.

If $\alpha'(0) \leq f_I/R$, i.e. if assumption 2.2 fails, the private equilibrium is unique and equals symmetric zero investment. Therefore, the outcome is the same as in autarky.

If assumption 2.2 (iii) does not hold, i.e., if $h(x) = \alpha(c)/\alpha'(x)$ is not weakly decreasing, asymmetric interior private equilibria can exist.

4.3 The social optimum with data sharing

Now consider the social planner problem. The planner takes into account that the expenditure on data protection by any entity impacts the likelihood that another entity will suffer an unrecoverable data loss. In particular, the planner jointly chooses c_A, c_B, and c_C to maximize the sum of bank profits[7],

$$\pi_P(c_A, c_B, c_C) = \pi_A(c_A, c_B, c_C) + \pi_B(c_B, c_A, c_C) + \pi_C(c_C, c_B, c_A) \tag{14}$$

where π_A is given in (8), likewise π_B, π_C. We then have the following result.

Proposition 4.2 (DSA Data Economy - Social Optimum)**.** *Suppose that assumptions 2.1 and 2.2 hold. The social planner equilibrium is symmetric, interior, finite,* $0 < \hat{c}_A = \hat{c}_B = \hat{c}_C < \infty$*, and is among the solutions to*

$$-\alpha'(\hat{c}_A)\alpha(\hat{c}_A)(4 - 3\alpha(\hat{c}_A)) = \frac{f_I}{R(|I_A|)}. \tag{15}$$

Only if assumption 2.2 fails, and $-\alpha'(0) \leq f_I/R$*, the social planner equilibrium can (but does not have to) be symmetrically at zero investment,* $\hat{c}_A = 0$*.*

[7]This calculation assumes that all consumer costs to data loss are reflected in bank profits. This would be true, for example, if banks were required to pay restitution to consumers equal to their damages.

First, see that the social optimum is never asymmetric unlike the private optimum. This yields the important insight, that the social planner always provides both security covers for all institutions, if at all. The planner never provides solely one cover by having one institution not invest (asymmetric corner equilibrium).

Generically, a solution to (15) may not be unique since the planner's profit function may not be globally concave, and can become convex for small levels of investment c. However, at least one solution to (15) always exists and the social optimum is always among the solutions. If assumption 2.2 fails, and $-\alpha'(0) \leq f_I/R$, a solution to (15) might not exist. However, unlike in the private case, the social planner investment level might be positive even for $-\alpha'(0) < f_I/R$ which yields an improvement relative to the autarky case. Only if assumption 2.2 (iii) does not hold, i.e., $h(x) = \alpha(x)/\alpha'(x)$ is not monotonically decreasing, the social planner equilibrium can be asymmetric-interior or can be given as the asymmetric corner equilibrium $(0, c_B, c_C)$, $c_B, c_C > 0$, where the social planner sets investment by one institution to zero and provides cover to all institutions by letting the remaining two institutions invest.

5 The Optimality of writing DSAs: Autarky versus Data sharing

In this section, we argue that entering into the data-sharing agreements is optimal for all entities relative to the autarky case, but raises novel challenges.

Proposition 5.1. *[Autarky versus Economy with DSAs] Let assumption 2.2 hold.*

1. *Optimal Investment: In every possible private optimum of the DSA economy, the security investment c^* undercuts the autarky level of investment in data security, c^a. However, the socially optimal level of investment in the DSA economy, $\hat{c}$, exceeds the autarky level of investment in data security $c^*_{i_1} \leq c^a_{i_1} < \hat{c}_{i_1}$.*

2. *Profits:*

   (a) *In every possible private optimum with DSAs, each entity's profit is at least as high as in autarky, $\Pi_{i_j}(c^*_{i_j}|c^*_{i_1}, \ldots, c^*_{i_N}) \geq \Pi_{i_j}(c^a_{i_j})$.*

   (b) *Welfare in every private equilibrium with DSAs exceeds welfare in autarky*

To see Proposition 5.1 (1), recall that by Proposition 4.1, there are two possible private equilibria, one is symmetric, and interior whereas the other equilibrium is the

asymmetric corner equilibrium where one entity k shirks, investing zero, $c^*_{i_k} = 0$, and the remaining entities invest the autarky amount $c^*_i = c^a$ and thus protect entity k indirectly via the DSAs, where the autarky amount is as described in section 3. The proof shows that in the symmetric private equilibrium, it holds $c^*_{i_1} < c^a_{i_1} < c^s_{i_1}$ whereas in the asymmetric private corner equilibrium, $c^*_{i_1} \leq c^a_{i_1} < c^s_{i_1}$. In the asymmetric corner equilibrium, the shirking institution that invests zero clearly invests less than in autarky, $c^*_{i_1} = 0 < c^a_{i_1}$, whereas the remaining institutions invest the autarky amount $c^*_{i_1} = c^a_{i_1}$ to protect themselves, and indirectly protect the shirking institution through the DSAs.

For intuition on Proposition 5.1, let us first discuss the private optimum versus autarky. Both the symmetric private equilibrium and the autarky optimum yield symmetric investment. Because in the symmetric private equilibrium all entities invest, thus establishing two covers, individual profits are higher than in the single cover autarky case, even though the autarky investments are higher. The result that profits in the symmetric private equilibrium are higher than in autarky is natural because the autarky profit is always available to an entity by investing the autarky amount. However, the result that entities invest less than in autarky shows that the entities do not fully exploit the security benefit of the DSAs since they scale back on their investment rather than scale up, relative to autarky, free-riding on one another's investment.

Now comparing the asymmetric private corner equilibrium to autarky, profits are equal in the private and the autarky optimum for the non-shirking entities since in either case they invest the autarky amount and the DSAs are worthless. For the shirking entity, though, profits must be higher in the private optimum than in the autarky optimum, $\Pi_{i_k}(0|c^*_{i_j} = c^a_{i_j}) > \Pi_{i_k}(c^a_{i_k})$, since, again, the autarky profit acts like a lower bound on profits for all entities, however, the shirking entity decided to forego it.

Now let us compare individual profits in the social optimum to profits in the private optimum with DSAs. The average individual profit at the social optimum exceeds the average individual profit in every private optimum simply by the definition of welfare. Moreover, because one private equilibrium is symmetric as the social optimum, this implies that in this private equilibrium, also every entity's individual profit is higher at the social rather than at the symmetric private equilibrium. For the private symmetric equilibrium, we thus have the profit ranking $\Pi_{i_j}(\hat{c}_{i_j}) > \Pi_{i_j}(c^*_{i_j}) > \Pi_{i_j}(c^a_{i_j})$.

The same is however not true for the asymmetric private corner equilibrium. The shirking entity's individual profit at the private asymmetric corner equilibrium may exceed the individual profit at the social optimum. For the not shirking institutions $j \in \{1, \ldots, N\}/\{k\}$, they earn the autarky profit in the asymmetric private equilibrium,

implying higher profits in the social optimum $\Pi_{i_j}(\hat{c}_{i_j}) > \Pi_{i_j}(c^*_{i_j}) = \Pi_{i_j}(c^a_{i_j})$.

As a consequence of Proposition 5.1, (2), we obtain

Corollary 5.1 (Optimality of data sharing)**.** *For the entities it is ex ante both socially and privately optimal to write DSAs with one another to share data at zero costs in the event of an attack.*

Note that Proposition 5.1 (2)(a) is stronger than and implies Proposition 5.1 (2)(b) since welfare is utilitarian. Since in every possible private equilibrium, individual profits and welfare exceed individual profits and welfare in autarky, entering the DSAs is privately and socially optimal ex ante. Note, that ex post, entities may find it optimal to deviate for reasons outside of the model, e.g. due to competition between entities they might not want to share data, see for instance Vives (1990a); Ziv (1993). However, we assume that the DSAs are written ex ante, and create legal claims entities have against one another. Moreover, cryptographic methods such as those described at the beginning of section 4 can enforce truthtelling even when an entity would want to lie about the content of data.

Proposition 5.1, (1), also states

Corollary 5.2 (Sub-Optimality)**.** *In every private optimum with DSAs, there is strict under-investment in security relative to the social optimum with DSAs.*

One might now consider a standard regulatory approach to tax the entities depending on their security investment, to attain investment levels closer to the social optimum. Alternatively, one may consider private solutions in the sense of Coase (1937), and price the DSAs depending on an entity's externality on other entities. This paper chooses a different, indirect approach that is more in the current spirit of the industry, namely by adding a third party regulatory entity that observes and protects all data, but takes into account how private entities respond in terms of their investment, and levies the costs of its data protection on the private entities through symmetric lump-sum taxation.

6 Regulated Data Economy with Data Sharing

By the section above, absent a regulator the private entities under-invest in data security relative to the social optimum. Direct regulation of security investment is tricky

in the real world since the required investments to improve security might not be perfectly observable or verifiable. Further, the firms could understate the value of their data to the regulator or misstate the required technology for security enhancement to reduce costs.

Therefore, we take an indirect implementation approach. We are interested in whether the regulatory provision of a government layer of security improves overall data security in a (privately) incentive-compatible way. The addition of a third-party FMI (eg. Fedwire) changes the information environment, because every transaction is observed by the sender, the receiver, and the FMI. We let the FMI optimally protect its data, where optimal means that we consider how the addition of the FMI changes the privately optimal behavior of the entities, and in the worst case, allow the FMI to abstain from the provision of this security layer if the private investment incentives lead to an overall undesired outcome.

Our approach is further motivated by a possible future implementation of an account-based central bank digital currency (CBDC) which aims to use private banks as a front-end for CBDC transactions, and route payments from a sender bank to the central bank on to the receiver bank. In that environment, the FMI corresponds to the central bank, observing all CBDC transactions, whereas the private banks observe only CBDC transactions across their customers.

We next analyze whether the addition of an FMI improves overall data security given the optimal responses of all other private entities.

6.0.1 FMI model

To introduce a socially-minded, third-party FMI, assume that, as before, each transaction is observed by the payor and the payee and that every transaction is now additionally observed by the FMI. That is, the FMI observes all payments whereas the private entities observe only a share of all data. The FMI can invest $c_F \geq 0$ in data security, and occurs the same cost function as the private entities. Importantly, the FMI can also be attacked and faces the same attack probability function $\alpha(c)$ as the private entities. If the FMI invests a positive amount in data security, $c_F > 0$, the FMI additional provides one full cover, and that cover's probability of a (successful) attack equals $\alpha(c_F)$. Therefore, entity A's probability of data loss changes relative to the unregulated case, and becomes equal to the chance that A and the FMI gets attacked and

that at least one out of B or C gets attacked,

$$\alpha(c_A)\alpha(c_F)(\alpha(c_B)+\alpha(c_C)-\alpha(c_B)\alpha(c_C))) \tag{16}$$

If $c_F = 0$ (the FMI abstains), then by $\alpha(0) = 1$, the FMI gets attacked for sure which is equivalent to the FMI's choice to not provide a third security cover. Observe, that in this case A's probability of data loss equals the probability of data loss in the unregulated environment, given in (7). The FMI finances the expenditures for data security $c_F f$ by symmetric lump-sum taxation of all private entities. The profits of entity A in the FMI environment, therefore, equal

$$\pi_A^F(c_A|c_B,c_C,c_F) = (1-\alpha(c_A)\alpha(c_F)(\alpha(c_B)+\alpha(c_C)-\alpha(c_B)\alpha(c_C)))\ R(|I_A|)-c_A\ f_I-\frac{c_F f}{3} \tag{17}$$

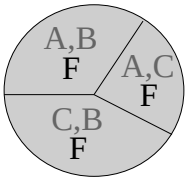


Figure 3: Each transaction is observed by two entities and the Fed.

For timing, we consider a two-stage game: the FMI sets its security $c_F \geq 0$ first and perfectly observably to all entities. Then the entities follow by simultaneously choosing their security levels in stage two.

We consider subgame perfect equilibria where given the choice of the FMI in stage 1, the entities choose their privately optimal level of data security in stage 2. The FMI's objective is to maximize social welfare, i.e. joint profits, by optimally choosing c_F, accounting for its own data security costs and the tax income from levying those costs on the three entities via taxation, taking as given the privately optimal security choices that follow by the entities in stage 2:

$$\begin{aligned}
&\pi_P^F(c_A^{*,F},c_B^{*,F},c_C^{*,F},c_F)\\
&=\pi_A^F(c_A^{*,F},c_B^{*,F},c_C^{*,F},c_F)+\pi_B^F(c_A^{*,F},c_B^{*,F},c_C^{*,F},c_F)+\pi_C^F(c_A^{*,F},c_B^{*,F},c_C^{*,F},c_F)-c_F\ f_I+3\times\frac{c_F\ f_I}{3}\\
&=\pi_A^F(c_A^{*,F},c_B^{*,F},c_C^{*,F},c_F)+\pi_B^F(c_A^{*,F},c_B^{*,F},c_C^{*,F},c_F)+\pi_C^F(c_A^{*,F},c_B^{*,F},c_C^{*,F},c_F)
\end{aligned} \tag{18}$$

Definition 6.1 (Equilibrium with an FMI)**.** *An equilibrium in the FMI environment is a tuple* $(c_F^*, c_A^*, c_B^*, c_C^*)$ *where:*

(i) Given the FMI's choice c_F^* in $t = 1$, the investment levels (c_A^*, c_B^*, c_C^*) are privately optimal in $t = 2$.
(ii) Given the privately optimal choices $(c_A^*(c_F), c_B^*(c_F), c_C^*(c_F))$ for every possible FMI choice c_F, the FMI choice c_F^* and the following private choices $(c_A^*(c_F^*), c_B^*(c_F^*), c_C^*(c_F^*))$ jointly maximize the FMI's objective (18).

The FMI equilibrium is the subgame perfect Nash equilibrium of the FMI environment that maximizes the FMI's objective function (18). Note, the FMI equilibrium is the constrained optimum, and is not the social planner equilibrium in the FMI environment because the private entities set privately and not socially optimal investments in $t = 2$. Recall that subgame perfection means, that if the FMI plays the FMI equilibrium $(c_F^*, c_A^*, c_B^*, c_C^*)$ and invests c_F^*, it will have no incentive to deviate from its decision ex post in $t = 2$ after the private entities have invested. This holds because if the FMI deviated say from c_F^* to $\hat{c}_F$, also the private entities would have an incentive to change their investments, playing an optimal response $(c_A^*(\hat{c}_F), c_B^*(\hat{c}_F), c_C^*(\hat{c}_F))$. But this outcome cannot be more desirable than $(c_F^*, c_A^*, c_B^*, c_C^*)$ since otherwise $(c_F^*, c_A^*, c_B^*, c_C^*)$ is not the FMI equilibrium. This is important because the same FMI investment c_F may give rise to two different private responses, one being symmetric and one being the private asymmetric corner equilibrium. If the private asymmetric corner equilibrium was an undesired response to the FMI investment but the symmetric response was desired, the asymmetric response cannot be part of an FMI equilibrium, since the FMI would have an incentive to deviate from c_F ex post.

Lemma 6.1 (3 possible covers in the FMI environment with DSAs)**.** *Consider the regulated environment where every transaction is observed by the sender, the receiver and the FMI. If in the FMI equilibrium all private entities invest positive amounts, $c_{i_j}^* > 0, j = 1, 2, 3$, and the FMI optimally invests $c_F^* > 0$, each data set I_{i_j} has exactly three security covers: itself I_{i_j}, the DSA cover $I_{i_j} \subset \bigcup_{k \neq j}^{N} I_{i_k}$, and the FMI cover $I_{i_j} \subset I_F = I$, for all $j \in \mathcal{N}$.*

Recall that if an entity does not invest, this entity is attacked for sure $\alpha(0) = 1$. Therefore, if the FMI does not invest, the third cover I_F is essentially not established. This is an important feature of this regulated information environment, it nests the unregulated environment: For $c_F = 0$, the information environment endogenously reduces to the unregulated environment analyzed in the section above. The FMI may choose to invest zero if it anticipates bad behavior that follows by the private entities in stage 2. Likewise, if a private entity does not invest, it does not cover itself, and simultaneously destroys the DSA covers to all other institutions but itself.

We are interested in whether this regulated information environment can attain levels of data security that are closer to the social optimum, analyzed in section 4.3 above. Note, however, that the FMI environment is substantially different from the unregulated social planner environment. This holds, first, since an FMI cannot prescribe the investments of the individual firms but can only determine its own investment optimally. Second, an FMI provides up to three layer of security whereas the social planner environment provides two layers for sure. More precisely, we show below that the provision of the third layer by an FMI further exacerbates the free-riding problem by the private entities, which may lead these entities to either not invest at all (then only the FMI cover exists) or may cause the FMI to optimally walk away, not providing the third layer.

6.0.2 FMI analysis Stage 2: Private security investment

We begin in stage 2, solving for the private equilibrium of the subgame that is played by entities A,B and C taking as given the security choice by the FMI, c_F, in stage 1.

Proposition 6.1 (FMI stage 2: Private Optimum)**.** *Let assumptions 2.1 and 2.2 hold.*
(i) For every FMI security investment choice $c_F \geq 0$, there exist a unique, symmetric private equilibrium $c_A^{*,F} = c_B^{*,F} = c_C^{*,F}$ in $[0, \infty)$. If given c_F it holds $-\alpha'(0) > \frac{1}{\alpha(c_F)} \frac{f_I}{R(|I_i|)}$, the symmetric private equilibrium is interior, $c_A^{*,F} > 0$, and is characterized as the solution to

$$-\alpha'(c_A^{*,F})\alpha(c_A^{*,F})[2 - \alpha(c_A^{*,F})] = \frac{1}{\alpha(c_F)} \frac{f_I}{R(|I_A|)} \tag{19}$$

If given c_F it holds $-\alpha'(0) \leq \frac{1}{\alpha(c_F)} \frac{f_I}{R(|I_A|)}$, the unique symmetric private equilibrium is the "no investment" equilibrium, no entity invests, $c_i^{*,F} = 0$.
(ii) Given any $c_F \geq 0$, there do not exist any asymmetric interior private equilibria.
(iii) If c_F satisfies $-\alpha'(0) > \frac{1}{\alpha(c_F)} \frac{f_I}{R(|I_i|)}$, and if the solution $c_{i_j}^*$ to $-\alpha'(c_{i_j}) = \frac{f}{R\,\alpha(c_F)}$ jointly with c_F satisfy

$$-\alpha'(0) \times \left[1 - (1 - \alpha(c_{i_j}^*))^2\right] < \frac{f}{R\,\alpha(c_F)}, \tag{20}$$

there exist 3 asymmetric corner equilibria, in which one entity $i_k \in \{A, B, C\}$ shirks by investing zero, $c_{i_k} = 0$, whereas the other institutions $i_j \in \{A, B, C\}/\{i_k\}$ invest the symmetric positive amount $c_{i_j}^* > 0$, thus, providing cover for the shirking entity through the DSAs. If c_F is such that $-\alpha'(0) \leq \frac{1}{\alpha(c_F)} \frac{f_I}{R(|I_A|)}$, the asymmetric corner equilibrium cannot exist, and the symmetric "no investment" equilibrium is the only equilibrium.

We see, that the asymmetric corner equilibria also exist in the FMI environment. Given any c_F, the unique symmetric private equilibrium cannot be at $c^* = \infty$ because profits are bounded whereas costs become infinite. From equation (19) it is apparent that the symmetric private equilibrium varies in the FMI's choice of data security c_F. Via its choice c_F, the FMI can therefore steer the symmetric equilibrium $c_i^{*,F}$ that follows in the subgame.

Lemma 6.2 (Private shirking in the FMI environment)**.** *Let assumptions 2.1 and 2.2 hold. Assume c_F is such that $-\alpha'(0) > \frac{1}{\alpha(c_F)} \frac{f_I}{R(|I_i|)}$, giving rise to the symmetric private equilibrium $c_A^{*,F} = c_B^{*,F} = c_C^{*,F} > 0$. Then the change of the symmetric equilibrium due to a marginal change in the FMI investment c_F is given as*

$$\frac{\partial c_A}{\partial c_F} = \frac{\frac{\alpha'(c_F)}{\alpha^2(c_F)}}{\alpha''(c_A)\alpha(c_A)[2-\alpha(c_A)] + 2[\alpha'(c_A)]^2(1-\alpha(c_A))]} \frac{f_I}{R(|I_A|)} < 0, \text{ for all } c_f \in [0,\infty) \tag{21}$$

The addition of the FMI, hence, exacerbates the free-riding problem among the private entities. The private institutions free-ride on the FMI's investment in data security, gradually reducing their private investment as the FMI scales up the security of the third layer. The shirking formula for the case of N entities can be found in Lemma 9.2 of section 12.2.

6.0.3 FMI analysis Stage 1: Optimal security choice of the FMI

In stage 1, the FMI seeks to maximize social welfare (18) by optimally choosing c_F, taking as given the privately optimal security choices that follow by the banks in stage 2. First, let us consider corner equilibria: Consider the case where assumption 2.2(ii) is violated, where $-\alpha'(0) < \frac{f_I}{R(|I_i|)}$.

Theorem 6.1 (Impossibility result)**.** *Let assumptions 2.1, 2.2(i) and (iii) hold.*
If $-\alpha'(0) < \frac{f_I}{R(|I_A|)}$, no FMI investment choice $c_F \in [0,\infty]$ in $t = 1$ can deter the unique private "no investment" equilibrium at $c^* = 0$ that follows in $t = 2$. "No investment" is the unique private equilibrium in the setting without the FMI and remains the unique private equilibrium in the regulated setting with the FMI, for every possible choice $c_F \geq 0$.

Theorem 6.1 says that if "no-investment" is the unique equilibrium in the setting without the FMI, then "no-investment" remains the unique private equilibrium in the setting with the FMI, whatever investment choice c_F. Also recall from Proposition 3.1,

that if $-\alpha'(0) < \frac{f_I}{R(|I_A|)}$ holds, the entities invest zero in the autarky setting. However, the social planner in the unregulated environment can find it worthwhile to invest, see Proposition 4.2 and the example in section 12.4. This Theorem is the first indicator that adding the FMI is not always a powerful tool to enhance overall investment in data security. The condition $-\alpha'(0) < \frac{f_I}{R(|I_A|)}$ means that the attack probability is rather inelastic in the investment in data security relative to its costs and revenue at stake.

The theorem is simple to prove, since $\frac{f_I}{R(|I_A|)} \leq \frac{f_I}{R(|I_A|)\alpha(c_F)}$ for every $c_F \geq 0$ by the properties of the function $\alpha(c) \in [0,1]$. With Proposition 6.1(i)-(iii), Theorem 6.1 follows.

The next Theorem summarizes candidates for the FMI equilibria for different elasticities of the attack probability function.

Theorem 6.2 (Regulated Data Economy stage 1: FMI Optimum)**.** *Suppose that assumption 2.1 and 2.2(i) and (iii) hold.*
(i) Assume $-\alpha'(0) \leq \frac{f_I}{R(|I_A|)}$
(ia) If $(-\alpha'(0)) \in (0, \frac{f_I}{3R(|I_A|)}]$*, the FMI optimally abstains from providing a security cover,* $c_F^* = 0$*, and the unique FMI equilibrium is given as* $(c_A^*, c_B^*, c_C^*, c_F^*) = (0,0,0,0)$*.*
(ib) If $(-\alpha'(0)) \in (\frac{f_I}{3R(|I_A|)}, \frac{f_I}{R(|I_A|)}]$*, the FMI optimally provides a security cover, and the unique FMI equilibrium is given as* $(c_A^*, c_B^*, c_C^*, c_F^*) = (0,0,0,c_F^*)$ *where* $c_F^* \in (0,\infty)$ *is given as the unique solution to*

$$-\alpha'(c_F) = \frac{f_I}{3\,R(|I_A|)} \tag{22}$$

(ii) Assume $-\alpha'(0) > \frac{f_I}{R(|I_A|)}$*. Define* $\bar{c}_F > 0$ *as the unique solution to* $-\alpha'(0) = \frac{1}{\alpha(\bar{c}_F)}\frac{f_I}{R(|I_A|)}$*.*
(iia) Subgames $c_F \geq \bar{c}_F$*: If the solution* c_F^* *to* $-\alpha'(c_F) = \frac{f_I}{3\,R(|I_A|)}$ *satisfies* $c_F^* \geq \bar{c}_F$*, a candidate for an FMI equilibrium is given by* $(c_A^*, c_B^*, c_C^*, c_F^*) = (0,0,0,c_F^*)$*. If the solution satisfies* $c_F^* < \bar{c}_F$*, no* $c_F^* \geq \bar{c}_F$ *is part of an FMI equilibrium.*
(iib) Consider $c_F \in [0, \bar{c}_F)$*. Let* $c_i^* > 0$ *the private, interior, symmetric equilibrium absent the FMI, given as the solution to the FOC (9). A necessary condition for FMI- optimality of* $c_F^* > 0$ *is*

$$\frac{\alpha'(0)}{\alpha'(c_i^*)/\alpha(c_i^*)} - \frac{1}{3} > 0. \tag{23}$$

There exists an interior candidate for an FMI equilibrium $(c_A^*(c_F), c_B^*, c_C^*, c_F^*)$ *with* $c_A^*(c_F) = c_B^* = c_C^* > 0$ *and* $c_F > 0$ *if at* $c_F = 0$ $\frac{d}{dc_F}\pi_P(c_i^*, c_i^*, c_i^*, 0) > 0$ *and* $\frac{d}{dc_F}\pi_P(c_A^*(c_F), c_A^*(c_F), c_A^*(c_F), c_F) = 0$ *for some* $c_F < \bar{c}_F$ *where* $c_A^*(c_F)$ *is the private symmetric equilibrium given as the solution to (19).*

In section 12.2.4, Theorem 9.1 and its proof describe in detail how to find all can-

didates for the FMI equilibrium for the general setting with N banks. There can exist three types of equilibria in the FMI environment. There can be an equilibrium where the FMI enters and invests $c_F^* > 0$ but the private entities abstain. An example of such an equilibrium is given in the appendix section 12.4. The second type of an equilibrium is where both the FMI and the private entities invest simultaneously, $c_F^* > 0, c_A^* > 0$. An example of this case is given in section 8. As the third type of equilibrium, both the FMI and the private entities may invest zero. This occurs if the elasticity of the attack probability $-\alpha'(0)$ is too small relative to the costs of security and revenue R. The FMI equilibrium considers all subgame-perfect equilibria and picks the best that maximizes the FMI objective 18. For our search of FMI equilibria in (ii), we focus on the symmetric private equilibrium that follows the FMI's choice c_F, and abstract from the possibility that the asymmetric private corner equilibrium may arise as part of the FMI equilibrium.

Recall that an FMI choice $c_F^* = 0$ means that the setting endogenously reduces to the unregulated case analyzed in section 4. It means, the conditions of the information system and its costs are such that the FMI optimally does not provide the third cover, implying the private entities are on their own when it comes to protecting their data.

The intuition behind the Proposition is as follows. The private entities and the FMI adjust their optimal behavior depending on how fast the attack probability declines in investment in data security. On (i), if the attack probability $\alpha(c)$ declines slowly for small levels of investment in data security all private entities decide to optimally not invest, independently of what the FMI does. The FMI faces the same attack probability $\alpha(c)$ as the private entities. If the attack probability declines particularly slow in the investment in data security, (ia), then also the FMI abstains from investment since the monetary costs of investing do not justify the benefits. If the attack probability declines slightly faster in investment, the private entities still do not invest but the FMI does. In that case, the monetary costs of investing exceed the benefits of providing the third cover, and thus more security to all entities. The FMI may invest "sooner," i.e. for lower attack elasticities than the social planner in the unregulated environment because the FMI protects all data through a single investment whereas the social planner can protect all data only by investment via at least $N - 1 = 2$ institutions, and thus incurs the cost function at least twice. As the appendix in the proof shows, the FMI's marginal change in profit due to a marginal increase in data security investment consists of two terms

$$\frac{d}{dc_F}\pi_P^F(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F) = \underbrace{3f\left(\frac{\alpha(c_A)(-\alpha'(c_F))}{\alpha(c_F)(-\alpha'(c_A))}\right)}_{\text{Profit increase from additional security}} - \underbrace{f_I}_{\text{costs of security}} \tag{24}$$

$$- \underbrace{6\alpha'(c_A)\alpha(c_A)\alpha(c_F)\frac{\partial c_A}{\partial c_F}(1-\alpha(c_A))\ R(|I_A|)}_{\text{indirect costs of additional security by free-riding}} \tag{25}$$

The first line in (24) captures the change in expected net profits to all three firms as the FMI increases security by providing the third cover. The added security makes attacks less likely which increases expected profits. But additional security comes at a unit cost f_I. The first line in (24) can, thus, be positive or negative depending on how fast an investment in security reduces the chance of an attack. This speed at which the attack probability falls is captured by the derivative $-\alpha'(c_F)$. In addition to the direct cost of security, the FMI's provision of a third cover entails an indirect cost since its security provision causes free-riding by all private entities. The entities reduce their private investment in response to the FMI's investment, $\frac{\partial c_A}{\partial c_F} < 0$ by Lemma 6.2, thus, the term in the second line of (24) is negative.

The FMI optimally provides the third cover only if the monetary costs and the extent of free-riding are not too intense relative to the benefit of the increased security and resulting higher profits. If the costs are too high, the FMI walks away (laissez-faire) by investing zero, $c_F = 0$. A necessary condition for the FMI's investment is when inequality (23) holds. In that case, the profits from additional security outweigh the direct costs of the FMI's security investment. As we show in the proof, the inequality holds if c_i^* is sufficiently small.

As an important Corollary we obtain

Corollary 6.1. *In cases where the FMI chooses $c_F^* > 0$ their presence in the system is welfare improving relative to the unregulated case where the FMI is absent. In cases where the FMI chooses $c_F^* = 0$ welfare is not lower relative to the unregulated case.*

The FMI environment can never to worse than the unregulated environment since the FMI maximizes welfare and can choose to leave private incentives unchanged by walking away (laissez-faire).

7 Welfare comparison across information systems

In this section, we compare the equilibrium outcomes of the regulated and the unregulated environment with data sharing agreements.

Proposition 7.1 (Double Covers versus FMI: Free-riding on the FMI)**.** *When adding the FMI as an additional cover, all banks respond by shirking. They reduce their symmetric private equilibrium investment in data security, $c_i^* \geq c_i^{*,F}$ for all $i \in \{A, B, C\}$, relative to the setting without the FMI, whatever the choice of c_F.*

Proposition (7.1) follows from Lemma 6.2 because the privately optimal investment in data security without the FMI corresponds to the privately optimal investment in data security with the FMI, but where the FMI decides not to invest, $c_A^{*,F}(0) = c_A^*$. This is by design of our environment. Lemma 6.2 shows that $c_A^{*,F}(c_F) \leq c_A^{*,F}(0)$. When additionally considering the insights of Proposition 5.1, we obtain the following ordering of optimal investments:

$$c_A^{*,F}(c_F) \leq c_A^{*,F}(0) = c_A^* < c_A^{*,soc}, \quad \text{for any } c_F > 0 \tag{26}$$

The privately optimal investment in the setting with the FMI is not only below the privately optimal investment in the setting without the FMI but, in particular, below the socially optimal investment in the setting without the FMI. This means, as the FMI enters it needs to take into account how much security its additional cover adds while internalizing that the two security covers provided by the private entities become weaker as they invest less.

Next, we investigate whether the regulated FMI environment can achieve a level of welfare close to the social optimum in the unregulated environment. Note, while the FMI is socially minded, the FMI equilibrium is generically not the social optimum of the FMI environment because the private entities in stage 2 play privately and not socially optimal best responses to the FMI's investment level. In addition, the FMI environment is informationally different from the unregulated environment because in the FMI case each transaction is observed by three parties, and the FMI can provide an additional third security cover whereas in the unregulated environment each transaction is observed by only two parties and only up to two security covers are established. As a consequence of this difference, welfare in the FMI environment can possibly exceed the optimum in the unregulated environment.

Welfare in the FMI case exceeds welfare at the socially optimal investment in the unregulated case without the FMI if and only if

$$3\,\Pi_A(c_A^{*,soc}, c_B^{*,soc}, c_C^{*,soc}) \leq 3\,\Pi_A^F(c_F^*, c_A^{*,F}, c_B^{*,F}, c_C^{*,F}) \tag{27}$$

Recall, if assumption 2.2 holds, every planner equilibrium is symmetric via $c_A^{*,soc} = c_B^{*,soc} = c_C^{*,soc}$. Plugging into (14), (8), (17) and (18), and imposing symmetry for the private FMI equilibrium[8], we can rewrite this inequality as

$$\underbrace{3[(1-\alpha(c_A)^2+\alpha(c_A)^3)R - c_A f]}_{=\Pi_P^{DC,Soc}(c_i^{*,soc})} \leq \underbrace{3\left[(1-2\alpha^2(c_A^F)\alpha(c_F)+\alpha^3(c_A^F)\alpha(c_F))R(|I_A|) - c_A^F f_I\right] - c_F\, f_I}_{=\Pi_P^{Fed}(c_F^*, c_i^{*,F})} \tag{28}$$

The evaluation of this condition is analytically tricky. Therefore, we consider the welfare comparison numerically in two example, available in section 8 and 12.4.

8 Example I

Consider the attack probability function $\alpha(c) = \frac{1}{\sqrt{\beta c+1}}$, $\beta > 0$. This function is positive, decreasing, convex, and can represent a probability since $\alpha(0) = 1$, $\alpha(c) \to 0$ as $c \to \infty$, and $\alpha(c) \in [0,1]$ for all $c \geq 0$. For larger β, the attack probability is more elastic in a small security investment, meaning the attack probability declines faster. Therefore, different values β change the trade-off between the costs and benefits of investment in data security. We set the return to $R = 2$, and per unit costs of security to $f_I = 1$.

We solve this example analytically in the appendix, section 12.3. Note, for the simulation we optimize over all possible private equilibria in $t = 2$.

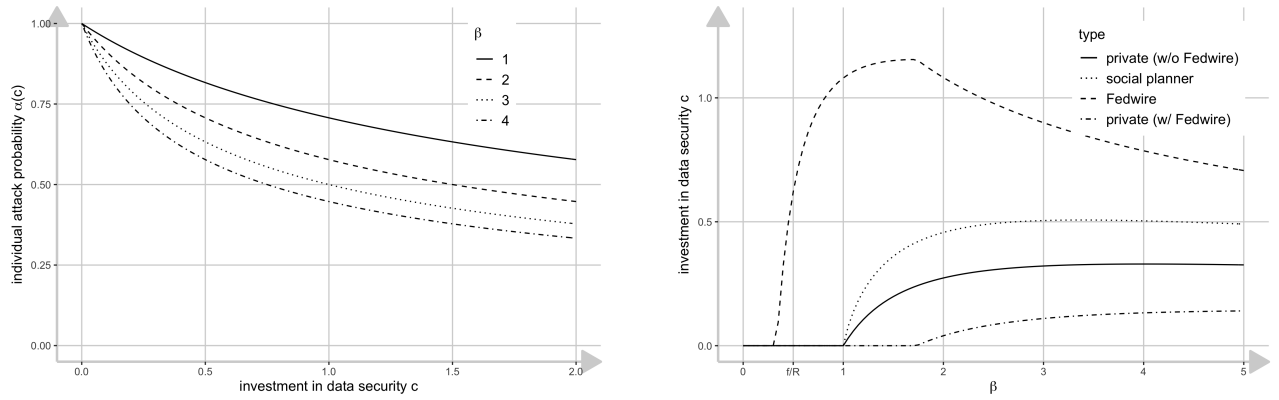
8.1 Simulation and discussion

In Figure 4, we show the optimal investment in data security across varying individual attack probabilities $\alpha(c) = \frac{1}{\sqrt{\beta c+1}}$, with $R = 2$, and per unit costs of security to $f_I = 1$. The elasticity of the attack probability function equals $-\alpha'(0) = \beta/2$. As β increases along the x-axis, the rate at which the attack probability diminishes accelerates for marginal investments in data security, see Figure (4). One can thus think of

[8]There exists a condition in the FMI case similar to (10), if this condition fails to hold the private equilibrium must be symmetric.

β as the elasticity of the attack probability for a marginal investment in security. This means that as β attains larger values, investing in data security yields increasingly high "security returns".

In the simulation, we let entities maximize freely. That is, we do not impose a particular type of private equilibrium in stage 2. When considering the investments in Figure 4, as expected, private institutions that operate in the unregulated environment invest less compared to the socially optimal benchmark. Furthermore, and as anticipated, the private institutions that operate in the regulated environment next to the FMI invest less relative to the case where the FMI is absent which is in line with our free-riding result, Lemma 6.2. However, the FMI invests carefully in a way that shirking by the private institutions is not too extreme - the private institutions do not shirk to zero, meaning they provide two security covers next to the FMI cover. This result can fail to hold under different attack probabilities $\alpha(c)$. Example II in the appendix shows that with an exponential attack probability, the FMI's strategic decision to provide the third cover and to protect it much, triggers a cascade effect leading all private institutions to curtail their investments to zero. In that case, the FMI environment endogenously provides only one cover because private entities refuse to provide their share of effort, see section 12.4 and Figure 7.



Different attack probabilities $\alpha(c) = \frac{1}{\sqrt{\beta c+1}}$. Optimal individual investment in data security.

Figure 4: The larger β, the faster the attack probability $\alpha(c) = \alpha(c) = \frac{1}{\sqrt{\beta c+1}}$ drops for a marginal investment in security c.

At $\beta = 1$ a significant shift in the behavior of all institutions is evident for the unregulated environment (no FMI) in Figure 4. This transition arises due to the fact that for smaller values of β, i.e. if $-\alpha'(0) = \beta/2 < f_I/R = 0.5$, that is, $\beta < 1$, both pri-

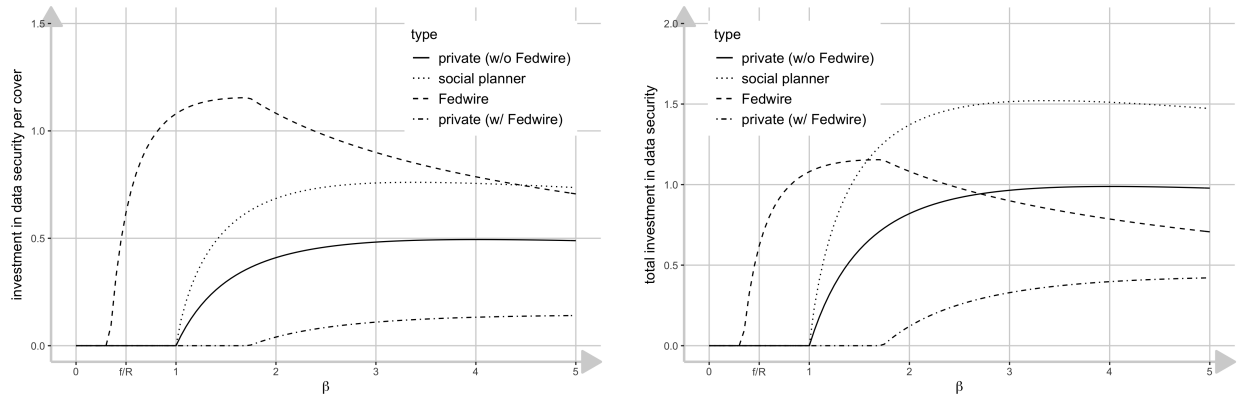
vate and socially optimal investments equal zero, see the analytical solution in section 12.3. This occurs because a smaller β implies that the attack probability diminishes too slow relative to the per-unit costs f_I associated with data investment and the revenue R generated when data is secure. These simulated values are in line with our analytical solution in section 12.3.2. Conversely, for $\beta > 1$, optimal investments in security become strictly positive. As β surpasses the threshold of 1, the cost-benefit trade-off faced by all institutions undergoes alteration. Consequently, as β increasingly exceeds 1, all institutions systematically reduce their investments. This likewise holds for the exponential example in section 12.4 of the Appendix.

Furthermore, the graph illustrates that the FMI invests for smaller values β than the private institutions and the social planner in the unregulated environment. This is in line with our analytical results in section 12.3, and is related to the fact that the FMI information environment is distinct from the unregulated environment the social planner or the private institutions face. In the unregulated environment, the social planner distributes data security investments across all three institutions to provide two security covers, incurring the cost function for security investment threefold. In contrast, to the social planner the FMI provides an entire security cover for all data via one single investment, thus incurring the cost only once.

It is worth noting, that the social planner in this scenario optimally chooses to provide both covers. Hence, it is not socially optimal to set the investment of one institution to zero and optimize welfare through security investment by the remaining two institutions, thereby providing only one cover akin to the FMI.

Figure 4 shows optimal individual investment. These numbers are however tricky to compare since the social planner invests via three instutions and incurs the cost function thrice, providing two covers whereas the FMI invests via one institution, providing one cover. To enhance comparability and provide a reasonable performance measure between the FMI's investment and the social planner's investment, Figure 5 plots the total investments per provided security cover and Figure 5 plots the total investment per institution. For the FMI, the individual investment, total investment and investment per cover coincides. To the social planner, the total cost attributed equal $3 \times c_A^*$, where c_A^* represents the individual investment. The social planner's adjustment for investment per cover equals $(3/2) \times c_A^*$.

Figure shows that the social planner in the unregulated environment invests more per cover and more in total than the FMI for elastic attack probabilities, i.e. high β.

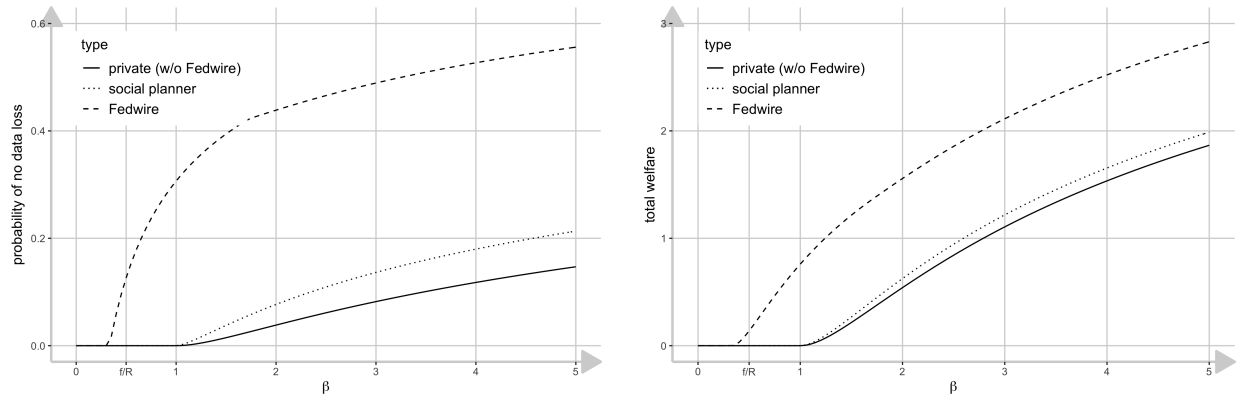


Equilibrium investment per security cover Total Investment in equ.

Figure 5: Investment per cover: The private institutions, respectively the social planner invest symmetrically via 3 institutions to provide two security covers, incurring the cost function three times. Investment per cover thus equals individual investment in Figure (4) times $2/3$. The the FMI environment is different since the FMI invests once and provides one full cover, incurring the cost function once. The FMI's investment per cover thus equals the total investment in Figure (4). Total Investment: The three private institutions jointly invest $3c_A$. Likewise, the social planner invests via three institutions $3c_A$. The FMI makes only one large investment c_F. We see that the social planner may invest more in total than the FMI since the planner needs to invest through multiple institutions to obtain a security cover, thus incurring the cost function, several times whereas the FMI invests only once to establish one security cover. The FMI cannot establish more than one cover. but by investing smartly it takes case that the private institutions do not shirk too much, and protect their two security covers as well.

This provides further evidence for the distinct cost-benefit trade-offs regarding an investment in security faced by the social planner and the FMI.

Figure 6 illustrates the endogenous levels of data security resulting from optimal investment strategies. In the scenario involving only private institutions, all entities collectively provide two covers, resulting in a level of security (probability of no data loss) quantified by $(1 - \alpha(c)^2(2 - \alpha(c)))$. The social planner adheres to the same security function but evaluates it at a higher investment level c, thereby enhancing overall security. As the FMI enters the economy, the overall security function changes to $(1 - \alpha(c)\alpha(c_F)(2\alpha(c) - \alpha(c)^2))$, where c_F represents the FMI's investment, and c denotes investment via the remaining institutions. Because the regulated environment altogether provides 3 security covers, overall security in the regulated the FMI environment is much higher than in the social planner case for the unregulated environ-



Probability of no data loss Total welfare

Figure 6: The probability of no data loss equals the probability that each institution's data is not compromised (at least one security cover is not successfully attacked). In terms of welfare, the FMI environment with three security covers dominates the social planner environment that can only establish two security covers.

ment.

Welfare is depicted in Figure 6, according to the analysis in section 7. Evidently, welfare achieves higher levels when the planner invests in security compared to private institutions. When comparing welfare in the regulated FMI environment relative to the unregulated social planner solution one needs to be cautious because the welfare functions are different. As mentioned above, it is essential to recognize that the planner invests in security through all three institutions, incurring the cost function threefold. Through its investment strategy, the planner establishes two covers of security. Conversely, the FMI invests solely once, incurring the cost function once, but provides only one cover. Jointly with the private institutions, the regulated FMI environment provides three security covers, one more than the social planner which contributes to the high levels of welfare relative to the unregulated social planner solution.

Section 12.4 in the appendix analytically solves and simulates a second example for the attack probability $\alpha(c) = e^{-\beta c}$. The example is interesting since it demonstrates the existence of an FMI equilibrium, where the FMI optimally provides the additonal cover even though all private entities shirk to zero. That is, in equilibrium there is overall only one security cover, and it is provided by the FMI.

9 General Case for N banks with DSAs

In this section we develop a general model for the unregulated and the regulated FMI case with N banks.

As before, we exclude transactions within one bank and assume a complete, and symmetric network among the banks, where network links are formed via transactions. Generically, we define the unregulated case as a symmetric information environment, where each transaction in the full set of all transaction data $I = \bigcup_{i=1}^{N} I_i$ is observed by exactly two entities (banks) i, j, and, by symmetry, all entities in $\{1, \dots N\}$ observe the same amount of data $|I_i|$. Every entity transacts with every other bank. Transactions across any two banks are not observed by the remaining banks. Therefore, each transaction in i' information set I_i is observed by exactly one other entity $j \in \{1, \dots N\}/\{i\}$. Therefore, every bank can fully recover its data via asking the remaining entities to share the part of their transaction data they have in common with that bank. In short, there are two covers of I.

How can we generalize the attack probability for the Double Covers case with N banks?

Lemma 9.1. *Let entities $i_1, \dots i_n \in \mathcal{N}$, $n \geq 1$ form a cover of entity i_{n+1}'s information set $I_{i_{n+1}}$, $I_{i_{n+1}} \subseteq \bigcup_{j=1}^{n} I_{i_j}$. Assume that no transaction in $I_{i_{n+1}}$ is observed by more than one other entity in $\{i_1, \dots i_n\}$ so that every entity in $\{i_1, \dots i_n\}$ is necessary to form a cover of $I_{i_{n+1}}$, $I_{i_{n+1}} \not\subset \bigcup_{\substack{j=1, \\ j \neq k}}^{n} I_{i_j}, i_k \in \{i_1, \dots, i_n\}$. Then the probability of a successful attack on this cover of $I_{i_{n+1}}$ equals*

$$\mathbb{P}(\textit{at least one out of } \{i_1, \dots i_n\} \textit{ is attacked}) = 1 - \prod_{j=1}^{n} (1 - \alpha(c_{i_j})) \tag{29}$$

where $\prod_{j=1}^{n}(1 - \alpha(c_{i_j}))$ equals the probability that no entity out of $\{i_1, \dots i_n\}$ is attacked.

Important for formula (29) is that we assume that attacks on banks happen independently of one another. All of the remaining proofs on the generalized case of N banks can be found in the Mathematical Appendix. For computations, it turns out useful that the formula (29) is symmetric in j.

Corollary 9.1 (Attack probability for N banks)**.** *Consider the unregulated case with DSAs*

and N entities. The chance that entity i_j's data is compromised, $i_j \in \{i_1, \ldots i_N\}$ equals

$$\alpha(c_{i_j}) \times \left(1 - \prod_{\substack{k=1, \\ k \neq j}}^{N} (1 - \alpha(c_{i_k})) \right) \tag{30}$$

In the unregulated case, all entities mutually form covers for each other. Therefore, each entity i_j's probability of data loss equals the probability that i_j is successfully attacked and at least one out of $\{i_1, \ldots, i_N\}/\{i_j\}$ is also attacked. To gain some intuition for the formula, we include here the cases for $N = 3, 4$ banks.

Case: N=3 Consider the symmetric entities $\{A, B, C\}$, $I = I_A \cup I_B \cup I_C$. What is the probability that A's data is compromised? It equals the probability that A is successfully attacked and at least one out of B and C is attacked. However, the chance that one out of B and C is attacked equals one minus the chance that none of them is attacked. Therefore, A's data is compromised with probability

$$\alpha(c_A)\Big[1 - (1 - \alpha(c_B))(1 - \alpha(c_C))\Big] = \alpha(c_A)\Big[\alpha(c_B) + \alpha(c_C) - \alpha(c_B)\alpha(c_C)\Big] \tag{31}$$

Case: N=4 Consider the symmetric entities $\{A, B, C, D\}$, $I = I_A \cup I_B \cup I_C \cup I_D$. The probability that A's data is compromised equals the probability that A is successfully attacked and at least one out of B,C and D is attacked. The chance that one out of B,C and D is attacked equals one minus the chance that none of them is attacked. Therefore, A's data is compromised with probability

$$\begin{aligned} \alpha(c_A)\Big[1 - (1 - \alpha(c_B))(1 - \alpha(c_C))(1 - \alpha(c_D))\Big] = \alpha(c_A)\Big[&\alpha(c_B) + \alpha(c_C) + \alpha(c_D) \\ &- \alpha(c_B)\alpha(c_C) - \alpha(c_B)\alpha(c_D) - \alpha(c_C)\alpha(c_D) \\ &+ \alpha(c_B)\alpha(c_C)\alpha(c_D)\Big] \end{aligned} \tag{32}$$

9.1 Unregulated DSA economy: Private Equilibrium

With N entities in total, entity i_j's profit function, $i_j \in \{i_1, \ldots, i_N\}$, is given by

$$\Pi_{i_j}(c_{i_j}|c_{i_1}, \ldots, c_{i_N}) = \left(1 - \alpha(c_{i_j}) \times \left[1 - \prod_{\substack{k=1,\\ k\neq j}}^{N}(1 - \alpha(c_{i_k})\right]\right) R_{i_j} - c_{i_j} f_I \tag{33}$$

Proposition 9.1 (Private Equilibrium: unregulated DSA economy with N entities)**.** *Consider the case where*
(A) Assume assumptions 2.1 and 2.2 hold.
(Ai) There exists a unique, interior equilibrium. This unique interior equilibrium is symmetric, i.e. there are no asymmetric interior equilibria. The (symmetric) interior equilibrium is characterized as the unique solution to

$$-\alpha'(c_{i_N}) \times \left[1 - (1 - \alpha(c_{i_N}))^{N-1}\right] = \frac{f_I}{R_{i_N}}. \tag{34}$$

(Aiia) Corner equilibrium: Consider the solution $c_{i_1} > 0$ of the equation $-\alpha'(c_{i_1}) = f_I/R$. If the solution c_{i_1} satisfies

$$-\alpha'(0) \times \left[1 - (1 - \alpha(c_{i_1}))^{N-1}\right] < \frac{f_I}{R} = -\alpha'(c_{i_1}), \tag{35}$$

there exist N asymmetric corner equilibria, where one out of the N entities defects by investing zero, e.g. $c_{i_N} = 0$, and all of the other entities invest the symmetric, positive amount $c_{i_1} > 0$ that solves $-\alpha'(c_{i_1}) = f_I/R$, e.g. $(c_{i_1}, c_{i_1}, \ldots, c_{i_1}, 0) \in \mathbb{R}^N$ is one asymmetric corner equilibrium.
(Aiib) There are no asymmetric corner equilibria where one entity invests zero, and the remaining entities invest different, positive amounts.

The condition $-\alpha'(0) > f_I/R$ is essential for positive investment in data security. If and only if $-\alpha'(0) \leq f_I/R$, the unique private equilibrium is symmetric and located at $c = 0$ (no institution invests), even under autarky, see section 5. If assumption 2.2 (iii) fails, $h(x) = \alpha(x)/\alpha'(x)$ is not weakly decreasing, then there can exist asymmetric interior private equilibria.

The existence of the asymmetric corner equilibrium $(c_{i_1}, c_{i_1}, \ldots, c_{i_1}, 0) \in \mathbb{R}^N$ at which one entity defects by investing zero, and yet attains positive profits, shows that

the data sharing insurance can be exploited, giving rise to a "bad equilibrium." We call this a bad equilibrium because the defect of one institution causes the loss of one security cover. In the asymmetric corner equilibrium $(c_{i_1}, c_{i_1}, \ldots, c_{i_1}, 0) \in \mathbb{R}^N$, the investing institutions are just as well off as under autarky, i.e. they do not lose profits relative to the autarky case, see the discussion in section 5.

As long as $-\alpha'(0) > f_I/R$, there cannot be an equilibrium where more than one entity invests zero. This holds, because the two entities with zero investment would have zero covers, and are thus attacked for sure $\alpha(0) = 1$, losing access to their data, and profits. But if $-\alpha'(0) > f_I/R$, zero investment is not optimal, and every institution is better off setting the autarky level of investment, as described in section 5.

9.2 Unregulated DSA economy: Social Planner equilibrium

The social planner maximizes

$$\pi_P(c_{i_1}, c_{i_2}, \ldots c_{i_N}) = \sum_{j=1}^{N} \pi_{i_j}(c_{i_1}, c_{i_2}, \ldots c_{i_N}) \tag{36}$$

$$= \sum_{j=1}^{N} \left(1 - \alpha(c_{i_j}) \times \left[1 - \prod_{\substack{k=1,\\ k\neq j}}^{N} (1 - \alpha(c_{i_k}) \right] \right) R_{i_j} - c_{i_j} f_I \tag{37}$$

Proposition 9.2 (Social Planner Equilibrium)**.** *Consider the case where*
(A) assumptions 2.1 and 2.2 hold, where in particular $-\alpha'(0) > f_I/R$.
(Ai) The social planner equilibrium is interior, symmetric, $c_{i_1} = c_{i_2} = \cdots = c_{i_N} > 0$, *and is among the solutions to*

$$(-\alpha'(c_{i_1})) \left[1 - (1 - \alpha(c_{i_1}))^{N-2} [1 - N\,\alpha(c_{i_1})] \right] = \frac{f_I}{R} \tag{38}$$

(Aii) There cannot exist a corner social planner equilibrium at which one out of the N *entities does not invest,* $c_{i_N} = 0$, *whereas all of the remaining entities invest positive amounts* $c_{i_1} = \cdots = c_{i_{N-1}} > 0$.
(Aiii) No asymmetric interior social planner equilibrium exists, at which all of the entities invest positive but distinct amounts.

If $(-\alpha(0)) < f_I/R$, the social planner equilibrium can but does not have to be

interior, and symmetric zero investment, $c_{i_1} = 0$, becomes a candidate for the planner equilibrium.

If $-\alpha'(0) > f_I/R$, i.e., if assumption 2.2 holds, there cannot exist an asymmetric social planner corner equilibrium where two or more institutions do not invest. This holds, because the two institutions that do not invest enjoy zero cover, and thus make zero profits. However, with $-\alpha'(0) > f/R$, a single entity will choose to protect itself, and invest the autarky level of data security, running positive profits rather than not investing and earning zero, see section 5.

The comparison results between the DSA and the autarky economy, and the private and social planner solution of Proposition 5.1 go through for N entities, see the general proof in section 11.2 in the appendix.

9.3 Regulated DSA economy: FMI equilibria

In $t = 1$, the FMI sets c_F to maximize

$$\pi_P^F(c_F|c_{i_1}^F(c_F), c_{i_2}^F(c_F), \ldots, c_{i_N}^F(c_F)) = \sum_{k=1}^{N} \pi_{i_k}^F(c_{i_1}^F, c_{i_2}^F, \ldots, c_{i_N}^F, c_F) - c_F\ f_I + N \times \frac{c_F f_I}{N} \tag{39}$$

$$= \sum_{k=1}^{N} \left[\left(1 - \alpha(c_{i_k})\alpha(c_F) \times \left[1 - \prod_{\substack{j=1,\\ j\neq k}}^{N} (1 - \alpha(c_{i_k})) \right] \right) R_{i_k} - c_{i_k} f(I_{i_k}) \right] - c_F\ f_I, \tag{40}$$

taking as given the private investment choices $c_{i_1}, \ldots, c_{i_N}$ that are set in $t = 2$. In $t = 2$, each private entity i_j, $j = 1, \ldots, N$ maximizes its profit given c_F,

$$\Pi_{i_j}^F(c_{i_j}|c_{i_1}, \ldots, c_{i_N}) = \left(1 - \alpha(c_{i_j})\alpha(c_F) \times \left[1 - \prod_{\substack{k=1,\\ k\neq j}}^{N} (1 - \alpha(c_{i_k})) \right] \right) R_{i_j} - c_{i_j} f_I - \frac{c_F f_I}{N} \tag{41}$$

where $\frac{c_F f_I}{N}$ is the tax the FMI levies on the entities to finance its investment in data security $-c_F f_I$. We solve for subgame perfect equilibria. The analysis of the FMI equilibria is available in the appendix, section 12.2.

Proposition 9.3 (Private FMI Equilibrium with N entities (stage 2))**.** *Consider the FMI environment with N entities. Suppose that assumptions 2.1 and 2.2 hold.*
(i) Given the FMI's choice c_F, there exists a unique, finite, symmetric private equilibrium

$c_{i_1}^{*,F} = c_{i_2}^{*,F} = \cdots = c_{i_N}^{*,F}$ *in* $[0,\infty)$. *If* c_F *is such that* $-\alpha'(0) > \frac{1}{\alpha(c_F)}\frac{f_I}{R(|I_i|)}$, *the unique symmetric private equilibrium is interior, and characterized as the solution to*

$$-\alpha'(c_{i_N}) \times \left(1-(1-\alpha(c_{i_N}))^{N-1}\right) = \frac{1}{\alpha(c_F)}\frac{f_I}{R_{i_N}}. \tag{42}$$

If c_F *is such that* $-\alpha'(0) \leq \frac{1}{\alpha(c_F)}\frac{f_I}{R(|I_A|)}$, *the unique symmetric private equilibrium is the "no investment" equilibrium where no bank invests,* $c^* = 0$.
(ii) Given any $c_F \geq 0$, *there cannot exist any asymmetric interior private equilibria.*
(iii) Given c_F *satisfies* $-\alpha'(0) > \frac{1}{\alpha(c_F)}\frac{f_I}{R(|I_i|)}$, *there can exist asymmetric corner equilibria, where one entity shirks by investing zero,* $c_{i_N} = 0$, *whereas the other institutions invest symmetric positive amounts* $c_{i_1} = \cdots = c_{i_{N-1}} > 0$. *This equilibrium exists if* c_F *and* $c_{i_{N-1}}$ *jointly satisfy*

$$-\alpha'(0) \times \left[1-(1-\alpha(c_{i_{N-1}}))^{N-1}\right] < \frac{f_I}{R\,\alpha(c_F)} \tag{43}$$

where $c_{i_{N-1}}$ *is given as the solution to* $-\alpha'(c_{i_{N-1}}) = \frac{f_I}{R\,\alpha(c_F)}$. *If* c_F *is such that* $-\alpha'(0) \leq \frac{1}{\alpha(c_F)}\frac{f_I}{R(|I_A|)}$, *the asymmetric corner equilibrium cannot exist, and the symmetric equilibrium is the only equilibrium.*

By assumption 2.2 and given c_F, the unique symmetric private equilibrium cannot be at $c^* = \infty$ since profits are bounded whereas for $c \to \infty$, the costs become infinite.

Lemma 9.2. *Consider the FMI environment with* N *entities. Suppose that assumptions 2.1 and 2.2 hold. Assume,* c_F *satisfies* $-\alpha'(0) > f_I/(R\,\alpha(c_F))$ *so that the symmetric interior private equilibrium exists,* $c_{i_1}^{*,F} = \cdots = c_{i_N}^{*,F} > 0$. *The change of the interior symmetric equilibrium* c_{i_N} *due to a change in the FMI's investment choice* c_F *is given as*

$$\frac{\partial c_{i_N}}{\partial c_F} = \frac{\frac{\alpha'(c_F)}{\alpha^2(c_F)}}{\alpha''(c_{i_N}) \times (1-(1-\alpha(c_{i_N}))^{N-1}) + (N-1)\alpha'(c_{i_N})^2 \times (1-(1-\alpha(c_{i_N}))^{N-2})}\,\frac{f_I}{R(|I_A|)} < 0, \tag{44}$$

The following Theorem characterizes all possible candidates for an FMI equilibrium. The FMI equilibrium is found by plugging all possible FMI candidate equilibria into the FMI objective function, given in equation (152).

Theorem 9.1 (Optimal FMI provision with N entities)**.** *Suppose that assumption 2.1 and 2.2(i) and (iii) hold.*
(i) Assume $-\alpha'(0) \leq \frac{f_I}{R(|I_A|)}$
(ia) If $(-\alpha'(0)) \in (0, \frac{f_I}{NR(|I_A|)}]$, *the FMI optimally abstains from providing a security cover,*

and the unique FMI equilibrium is given as $c_F^* = 0$ *and* $c_{i_j}^{F,*} = 0, j = 1, \ldots, N$.
(ib) If $(-\alpha'(0)) \in (\frac{f_I}{NR(|I_A|)}, \frac{f_I}{R(|I_A|)}]$, *the FMI optimally invests and thus provides a security cover. The unique FMI equilibrium is given as* $c_{i_j}^{F,*} = 0, j = 1, \ldots, N$ *and* $c_F^* \in (0, \infty)$ *where* c_F^* *is defined as the solution to*

$$-\alpha'(c_F) = \frac{f_I}{NR(|I_A|)} \tag{45}$$

(ii) Assume $-\alpha'(0) > \frac{f_I}{R(|I_A|)}$. *Define* $\bar{c}_F > 0$ *as the unique solution to* $-\alpha'(0) = \frac{1}{\alpha(\bar{c}_F)} \frac{f_I}{R(|I_A|)}$.
(iia) Case $c_F \geq \bar{c}_F$: *Consider the solution* $c_F^* > 0$ *to the equation* $-\alpha'(c_F) = \frac{f}{NR}$. *If the solution satisfies* $c_F^* \geq \bar{c}_F$, *there exists a candidate for an FMI equilibrium, given by* $c_F^* > \bar{c}_F$ *and* $c_{i_j}^{F,*} = 0$ *where the FMI invests and provides the public security layer but all private entities shirk. If the solution* c_F^* *satisfies* $c_F^* < \bar{c}_F$, *there exists no FMI equilibrium with* $c_F \geq \bar{c}_F$.
(iib) Consider any $c_F \in [0, \bar{c}_F)$, *and let* $c_{i_j}^{F,*}(c_F) > 0$ *the corresponding interior, private symmetric equilibrium played by the entities. Consider*

$$\frac{d}{dc_F}\pi_P^F(c_{i_1}^F, c_{i_2}^F, \ldots, c_{i_N}^F, c_F) = -N\ \alpha'(c_F)\alpha(c_{i_k}^*(c_F)) \times \left[1 - (1 - \alpha(c_{i_k}^*(c_F)))^{N-1}\right] R_{i_k} - \ f_I \tag{46}$$

$$- N(N-1)\frac{\partial c_{i_k}^*}{\partial c_F}\ \alpha(c_F)\alpha(c_{i_k}^*(c_F))\alpha'(c_{i_k}^*(c_F))(1 - \alpha(c_{i_k}^*(c_F)))^{N-2} R_{i_k} \tag{47}$$

If $\frac{d}{dc_F}\pi_P^F(c_{i_1}^F, c_{i_2}^F, \ldots, c_{i_N}^F, 0) \leq 0$, $c_F = 0$ *jointly with* $c_{i_j}^{F,*}(0) > 0$ *is a candidate for an FMI equilibrium. If* $\frac{d}{dc_F}\pi_P^F(c_{i_1}^F, c_{i_2}^F, \ldots, c_{i_N}^F, c_F) = 0$, *for some* $c_{F,d} \in (0, \bar{c}_F)$, $c_{F,d} = 0$ *jointly with* $c_{i_j}^{F,*}(c_{F,d}) > 0$ *is a candidate for an FMI equilibrium. If* $\frac{d}{dc_F}\pi_P^F(c_{i_1}^F, c_{i_2}^F, \ldots, c_{i_N}^F, c_F) > 0$ *for all* $c_F \in [0, \bar{c}_F)$, *no* $c_F \in [0, \bar{c}_F)$ *is part of an FMI equilibrium.*

10 Conclusion

Existing payment and banking networks were established before the information age. Their adaptation or restructuring in terms of the extent of information centralization and complementarity of entities is costly and requires a thorough analysis of possible subsequent equilibrium effects. The analysis in this paper contributes to that effort by providing an economic model of data security that features redundancies (backups) and data segmentation.

At first sight, redundancies appear to reduce the chance of data compromise. However, we observe that data redundancy causes free-riding among profit-maximizing

entities, so redundancies do not necessarily increase security when all parties internalize the number of backups.

Redundancies occur in any system where transacting entities keep records since a complete collection of any institution's counterparties knows all the transactions of that institution. A third party FMI might also provide an additional data security cover. The provision of an FMI that observes and protects all transaction data can, but does not have to, increase overall security. This holds because the provision of a third backup causes additional free-riding, so the FMI might actually make the system less secure unless it optimally chooses not to provide an additional cover, as we show. In our model, we allow the FMI to abstain in cases where it would make things worse, and "no provision" of a public FMI security cover can arise as an optimal outcome.

Our model of a third party FMI that observes transactional data is meant to capture aspects of current publicly provided payment infrastructures that operate in all countries. In some instances these infrastructures do not observe all payment details and they may not observe on-us transactions that involve customers of the same bank. Nevertheless, the data they observe does serve as a valuable record of system-wide transactional data. Moreover, recent initiatives to build payment infrastructures using distributed ledger technologies are gaining traction for both domestic and cross border applications. For scalability and privacy reasons, these initiatives propose limiting the distribution of the ledger, which would represent an ideal complete security cover, to a few or possibly even one entity (eg. the central bank). Our analysis is useful for understanding the data security implications of these new proposals, a topic that has barely been addressed.

Our analysis focuses on data security in the context of cyber attacks where the data is destroyed by a malevolent third party. We recognize that data may also be captured and used for financial gain or publicly released.

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11 Appendix

11.1 Data Economy with DSAs: 3 and General case with N entities

11.1.1 Private optimum

Proof. [Proposition 4.1 and Proposition 9.1]

Given is the profit function

$$\Pi_{i_j}(c_{i_j}|c_{i_1},\ldots,c_{i_N}) = \left(1-\alpha(c_{i_j})\times\left[1-\prod_{\substack{k=1,\\k\neq j}}^{N}(1-\alpha(c_{i_k})\right]\right)R_{i_j}-c_{i_j}f_I \tag{48}$$

Therefore, entity i_j's marginal profit equals

$$\frac{\partial}{\partial c_{i_j}}\Pi_{i_j}(c_{i_j}|c_{i_1},\ldots,c_{i_N}) = -\alpha'(c_{i_j})\times\left[1-\prod_{\substack{k=1,\\k\neq j}}^{N}(1-\alpha(c_{i_k})\right]R_{i_j}-f_I \tag{49}$$

Clearly, the profit function is concave, meaning a unique maximum exists, because $\alpha(\cdot)$ is convex, and thus

$$\frac{\partial^2}{\partial c_{i_j}^2}\Pi_{i_j}(c_{i_j}|c_{i_1},\ldots,c_{i_N}) = -\alpha''(c_{i_j}) \times \underbrace{\left[1 - \prod_{\substack{k=1,\\ k\neq j}}^{N}(1-\alpha(c_{i_k})\right]}_{>0} R_{i_j} < 0. \tag{50}$$

The first order condition for an interior private equilibrium equals

$$-\alpha'(c_{i_j}) \times \left[1 - \prod_{\substack{k=1,\\ k\neq j}}^{N}(1-\alpha(c_{i_k})\right] = \frac{f_I}{R_{i_j}} \tag{51}$$

See that the FOC's are symmetric across institutions $j = 1,\ldots,N$. The first order condition for an interior symmetric private equilibrium with $c_{i_1} = \cdots = c_{i_N}$ equals

$$-\alpha'(c_{i_N}) \times \left[1 - (1-\alpha(c_{i_N}))^{N-1}\right] = \frac{f_I}{R_{i_N}}. \tag{52}$$

To show existence of an interior symmetric private equilibrium, we first show the existence of a candidate for an interior symmetric private equilibrium $c = c_{i_1} = \cdots = c_{i_N}$ at which the FOC (51) holds. Conditional on all entities investing symmetric amounts c_{i_N}, the left hand side of (52) goes to $-\alpha'(0) > 0$ for $c_{i_N} \to 0$, by $\alpha(0) = 1$. Further, the left hand side of (52) goes to zero for $c_{i_N} \to \infty$, by $\alpha(c) \to 0$ and since $\alpha(c)$ is decreasing and convex. The left hand side of (52) is continuous in c. Thus, if $-\alpha'(0) > f_I/R$, the left hand side of (52) as a function of c crosses f/R at least once for some $c^* \in (0,\infty)$, and by concavity of the profit function, c^* is a maximizer, guaranteeing existence of a candidate for an interior symmetric private equilibrium $c = c_{i_1} = \cdots = c_{i_N}$ at which the FOC (52) holds.

This candidate is indeed an equilibrium: Assume that entity i_N chooses a lower investment level than the other institutions, deviating from their symmetric investment $c_{i_N} < c = c_{i_1} = c_{i_2} \cdots = c_{i_{N-1}}$. Then, by convexity of $\alpha(c)$,

$$\begin{aligned}\frac{\partial}{\partial c_{i_N}}\Pi_{i_N}(c_{i_N}|c_{i_1},\ldots,c_{i_{N-1}}) &= -\alpha'(c_{i_N}) \times \left[1-(1-\alpha(c))^{N-1}\right] R_{i_N} - f_I &(53)\\ &> -\alpha'(c) \times \left[1-(1-\alpha(c))^{N-1}\right] R_{i_N} - f_I = 0 &(54)\end{aligned}$$

Thus, $c_{i_N} < c$ is not optimal. By the same argument, $c_{i_N} > c$ is not optimal because $\frac{\partial}{\partial c_{i_N}}\Pi_{i_N}(c_{i_N}|c_{i_1},\ldots,c_{i_{N-1}} = c) < 0$, showing that $c = c_{i_1} = \cdots = c_{i_N}$ is an equilibrium.

To show uniqueness, we need to show that the left hand side of (52) single-crosses f/R.

Consider the function $g(c) = \alpha'(c) \times \left[1 - (1 - \alpha(c))^{N-1}\right]$. This function is strictly increasing in c because $\left[1 - (1 - \alpha(c))^{N-1}\right] > 0$, and thus, $g'(c) = \alpha''(c)\left[1 - (1 - \alpha(c))^{N-1}\right] + (N - 1)\,\alpha'(c)^2\,(1 - \alpha(c))^{N-2} > 0$, by convexity of $\alpha(c)$. Because the function $-g(c)$ is strictly decreasing and continuous, it crosses f/R only once, if at all. Therefore, if $-\alpha'(0) > f/R$, there exists a unique interior symmetric private equilibrium, characterized as the solution to (52). Assumption 2.2 comprises $-\alpha'(0) > f/R$. If $-\alpha'(0) \leq f/R$, then there exists no crossing of f/R, the interior symmetric equilibrium does not exist, and the symmetric equilibrium is at $c = 0$ instead.

Moreover, if $-\alpha'(0) > f/R$, then symmetric zero investment $c_{i_1} = \cdots = c_{i_N} = 0$ cannot be an equilibrium: Given that all other institutions do not invest, entity i_1 is in the autarky case, see section (5). It has a cover only if it invests to protect itself from an attack. It's profit function equals

$$Pi(c_{i_j}) = (1 - \alpha(c_{i_j}))R - c_{i_j} f_I \tag{55}$$

with first order condition $-\alpha'(c_{i_j}) = f_I/R$. If $-\alpha'(0) > f/R$, this FOC has an interior solution, $c_{i_j} > 0$. Thus, $c_{i_1} = \cdots = c_{i_N} = 0$ is not an equilibrium. It holds that $c_{i_1} = \cdots = c_{i_N} = 0$ is an equilibrium, only if $-\alpha'(0) \leq f/R$.

Excluding asymmetric interior equilibria Assume there exists an equilibrium $(c_{i_1}, \ldots, c_{i_N})$ with all $c_i > 0$ and at least two arguments unequal $c_{i_N} \neq c_{i_{N-1}}$. We show, if $h(x) = \alpha(x)/\alpha'(x)$ is weakly decreasing, then such an asymmetric interior equilibrium cannot exist. Assume, it exists. Define the FOC function

$$g(c_{i_j}|c_{i_1}, \ldots, c_{i_{j-1}}, c_{i_{j+1}}, \ldots c_{i_N}) \equiv -\alpha'(c_{i_j}) \times \left[1 - \prod_{\substack{k=1,\\ k\neq j}}^{N} (1 - \alpha(c_{i_k})\right] \tag{56}$$

In an interior asymmetric equilibrium, we require $g(c_{i_N}|c_{i_1}, \ldots c_{i_{N-1}}) = g(c_{i_j}|c_{i_1}, \ldots, c_{i_{j-1}}, c_{i_{j+1}}, \ldots c_{i_N}) = f/R$ for all $j = 1, \ldots N-1$. In an interior asymmetric equilibrium $(c_{i_1}, \ldots, c_{i_N})$, all entitie's first order conditions need to hold despite the asymmetric investment levels. That is, we require $g(c_{i_N}|c_{i_1}, \ldots c_{i_{N-1}}) = g(c_{i_j}|c_{i_1}, \ldots, c_{i_{j-1}}, c_{i_{j+1}}, \ldots c_{i_N}) = f/R$ for all $j = 1, \ldots N-1$. We next exploit the symmetry of the first order condition (56) across entities to exclude such asymmetric interior equilibria. Consider the equation $g(c_{i_N}|c_{i_1}, \ldots c_{i_{N-1}}) = g(c_{i_{N-1}}|c_{i_1}, \ldots, c_{i_{N-2}}, c_{i_N})$ which is equivalent to which is equivalent to

$$0 = \left[\alpha'(c_{i_{N-1}}) - \alpha'(c_{i_N})\right]\left(1 - \prod_{k=1}^{N-2}(1 - \alpha(c_{i_k}))\right) + \left[\alpha'(c_{i_{N-1}})\alpha(c_{i_N})) - \alpha'(c_{i_N})\alpha(c_{i_{N-1}})\right] \times \prod_{k=1}^{N-2}(1 - \alpha(c_{i_k})) \tag{57}$$

Assume wlog that $c_N \geq c_{N-1}$, implying $\alpha'(c_{i_{N-1}}) - \alpha'(c_{i_N}) \leq 0$. If $h(x) = \alpha(x)/\alpha'(x)$ is weakly decreasing, then also $\alpha'(c_{i_{N-1}})\alpha(c_{i_N}) - \alpha'(c_{i_N})\alpha(c_{i_{N-1}}) \leq 0$. But then equation (57) implies $c_N = c_{N-1}$ because $\prod_{k=1}^{N-2}(1-\alpha(c_{i_k})) \in (0,1)$. Likewise, for all equations $g(c_{i_N}|c_{i_1}, \ldots c_{i_{N-1}}) = g(c_{i_j}|c_{i_1}, \ldots, c_{i_{j-1}}, c_{i_{j+1}}, \ldots c_{i_N}) = f/R$ for all $j = 1, \ldots N-1$. Thus, there cannot exist an asymmetric interior private equilibrium.

Asymmetric corner equilibria Assume there was an equilibrium where entity N invests zero, $c_{i_N} = 0$, and all other entities invest positive, interior amounts, $c_{i_j} > 0$. A sufficient condition for optimality of $c_{i_N} = 0$ is

$$g(c_{i_N}|c_{i_1}, \ldots c_{i_{N-1}}) \equiv -\alpha'(c_{i_N}) \times \left[1 - \prod_{k=1}^{N-1}(1-\alpha(c_{i_k}))\right] < \frac{f}{R}, \text{ for all } c_{i_N} \geq 0 \tag{58}$$

whereas a necessary condition for optimality of $c_{i_N} = 0$ is

$$\lim_{c_{i_N} \to 0} g(c_{i_N}|c_{i_1}, \ldots c_{i_{N-1}}) \equiv -\alpha'(0) \times \left[1 - \prod_{k=1}^{N-1}(1-\alpha(c_{i_k}))\right] < \frac{f}{R} \tag{59}$$

However, by convexity of $\alpha(c)$, $-\alpha'(0) > -\alpha'(c)$ for all $c > 0$, and because the edgy bracket is positive, the necessary condition is also sufficient.

Given $c_{i_N} = 0$, the profit functions of all the other institutions change to the autarky case from section 5, $\Pi(c) = (1-\alpha(c))R - cf_I$. This holds because by setting $c_{i_N} = 0$, none of the institutions $i_1, \ldots, i_{N-1}$ is provided with a second cover. That is, the risk of each entity i_j's data loss equals $\alpha(c_{i_j})$, and no longer depends on what the remaining other institutions do, besides i_N. Wlog, consider the FOC of institution $N-1$,

$$-\alpha'(c_{i_{N-1}}) = f/R \tag{60}$$

Because the FOC is the same for all entities $i_1, \ldots, i_{N-1}$, it follows $c_{i_1} = \ldots c_{i_{N-1}}$, Moreover, given assumption 2.2 holds, i.e. $-\alpha'(0) > f/R$, then $c_{i_1} > 0$, implying that indeed, all the remaining entities invest in security although i_N defects by investing zero. If on the other hand $-\alpha'(0) \leq f/R$, $c_{i_1} = \ldots c_{i_{N-1}} = 0$, and the symmetric no investment equilibrium arises. That is, $-\alpha'(0) \leq f/R$ is sufficient to exclude an asymmetric corner equilibrium. From here on, assume, $-\alpha'(0) > f/R$ holds. By the symmetry of investment, the optimality condition for institution i_N changes to

$$-\alpha'(0) \times \left[1 - (1-\alpha(c_{i_1}))^{N-1}\right] < \frac{f}{R} = -\alpha'(c_{i_1}) \tag{61}$$

where c_{i_1} is given as the solution to (60). This condition can possibly hold. We have $-\alpha'(0) >$

$-\alpha'(c_{i_1})$ whereas $c_{i_1} > 0$, implying $\left[1-(1-\alpha(c_{i_1}))^{N-1}\right] < 1$. Thus an asymmetric private corner equilibrium with one entity defecting $(c,\ldots,c,0)$ can exist.

□

11.1.2 Proof: Social Planner Equilibrium: $N = 3$ and General case

Proof. [Proposition 4.2 and Proposition 9.2] The planner's objective is to maximize cumulative profits across all entities,

$$\pi_P(c_{i_1},c_{i_2},\ldots c_{i_N}) = \sum_{j=1}^{N}\pi_{i_j}(c_{i_1},c_{i_2},\ldots c_{i_N}) = \sum_{j=1}^{N}\left(1-\alpha(c_{i_j})\times\left[1-\prod_{\substack{k=1,\\k\neq j}}^{N}(1-\alpha(c_{i_k})\right]\right)R_{i_j} - c_{i_j}f_I \tag{62}$$

The first-order conditions for the planner with regard to interior maximizers c_{i_m}, $m\in\{1,\ldots N\}$ is given as

$$\frac{\partial}{\partial c_{i_m}}\pi_P(c_{i_1},c_{i_2},\ldots c_{i_N}) \tag{63}$$
$$= (-\alpha'(c_{i_m}))\left[\left(1-\prod_{\substack{k=1,\\k\neq m}}^{N}(1-\alpha(c_{i_k}))\right)+\sum_{\substack{j=1,\\j\neq m}}^{N}\alpha(c_{i_j})\prod_{\substack{k=1,\\k\neq j,k\neq m}}^{N}(1-\alpha(c_{i_k}))\right]R_{i_m} - f_I = 0, \text{for all } m=1,\ldots,N$$

In the asymmetric case, the social planner objective function $\pi_P(c_{i_1},c_{i_2},\ldots c_{i_N})$ is concave in c_{i_m}, $m=1,\ldots N$ by convexity of $\alpha(\cdot)$ and because the edgy bracket in (**??**) is always positive,

$$\frac{\partial^2}{\partial c_{i_m^2}}\pi_P(c_{i_1},c_{i_2},\ldots c_{i_N}) \tag{64}$$
$$= (-\alpha''(c_{i_m}))\left[\left(1-\prod_{\substack{k=1,\\k\neq m}}^{N}(1-\alpha(c_{i_k}))\right)+\sum_{\substack{j=1,\\j\neq m}}^{N}\alpha(c_{i_j})\prod_{\substack{k=1,\\k\neq j,k\neq m}}^{N}(1-\alpha(c_{i_k}))\right]R_{i_m} < 0 \tag{65}$$

Therefore, a solution to $\frac{\partial}{\partial c_{i_m}}\pi_P(c_{i_1},c_{i_2},\ldots c_{i_N}) = 0$ is indeed a valid candidate for the social planner maximizer. In the symmetric case $c_{i_1} = \ldots, c_{i_N} = c_{i_m}$, and with $R_N = R_{i_j}$, the first

order condition of the planner simplifies to

$$\frac{\partial}{\partial c_{i_m}}\pi_P(c_{i_1}, c_{i_2}, \dots c_{i_N}) \tag{66}$$

$$= (-\alpha'(c_{i_m}))\ \left[1-(1-\alpha(c_{i_m}))^{N-1}+(N-1)\alpha(c_{i_m})(1-\alpha(c_{i_m}))^{N-2}\right] R_{i_m} - f_I \tag{67}$$

$$= (-\alpha'(c_{i_m}))\Big[1-(1-\alpha(c_{i_m}))^{N-2}[1-N\,\alpha(c_{i_m})]\Big] R_{i_m} - f_I \tag{68}$$

While we will work with the simpler, last line of $\frac{\partial}{\partial c_{i_m}}\pi_P(c_{i_1}, c_{i_2}, \dots c_{i_N})$, equation (67) shows that the edgy bracket is always positive and is, later, useful for a comparison between the social and the privately optimal investment.

Existence of interior symmetric equilibrium We look for interior symmetric equilibria first. Set $m = N$ to ease notation. This is wlog due to the symmetry of the first and second order conditions. Assume, there exists a symmetric solution $c^* = c_{i_1} = \cdots = c_{i_N}$ to the first order condition given in (67). Assume all other entities but i_N play c^*, $c_{i_1} = c_{i_2} = \cdots = c_{i_{N-1}} = c^*$ and $c_{i_N} < c^*$. Then by concavity, $\frac{\partial}{\partial c_{i_N}}\pi_P(c_{i_1}, c_{i_2}, \dots c_{i_N}) > 0$, implying that $c_{i_N} < c^*$ is not optimal. Likewise, if all other entities play c^*, but $c_{i_N} > c^*$, then $\frac{\partial}{\partial c_{i_N}}\pi_P(c_{i_1}, c_{i_2}, \dots c_{i_N}) < 0$ and $c_{i_N} > c^*$ is not optimal. Thus, given that c^* exists as a solution to (67), and all other entities follow c^*, it is indeed optimal for i_N to also set c^*.

To guarantee the existence of an interior social planner equilibrium, see first that the symmetric first order condition $\frac{\partial}{\partial c_{i_N}}\pi_P(c_{i_N}, c_{i_N}, \dots, c_{i_N}) = 0$ must have at least one solution if $\frac{\partial}{\partial c_{i_N}}\pi_P(0, \dots, 0) = -\alpha'(0)R - f_I > 0$, i.e. if assumption 2.2 holds. This holds by the Intermediate value Theorem. The term $\frac{\partial}{\partial c_{i_N}}\pi_P(c_{i_N}, c_{i_N}, \dots, c_{i_N})$ is continuous in c_{i_N}, and $\frac{\partial}{\partial c_{i_N}}\pi_P(c_{i_N}, \dots, c_{i_N}) \to -f < 0$ for $c \to \infty$ by $\alpha(c) \to 0$, which again holds by assumption 2.2.[9] Therefore, $\frac{\partial}{\partial c_{i_N}}\pi_P(c_{i_N}, c_{i_N}, \dots, c_{i_N})$ crosses zero at least once. Further, because $\pi_P(0, \dots, 0) = 0$, and $\pi_P(c_{i_N}, c_{i_N}, \dots, c_{i_N})$ increases for c_{i_N} small and declines for c_{i_N} large there must be at least one maximum among the solutions to $\frac{\partial}{\partial c_{i_N}}\pi_P(c_{i_N}, c_{i_N}, \dots, c_{i_N}) = 0$. If $\frac{\partial}{\partial c_{i_N}}\pi_P(c_{i_N}, c_{i_N}, \dots, c_{i_N}) = 0$ has only one solution, that solution must be a maximum. Because the function $\pi_P(c_{i_N}, c_{i_N}, \dots c_{i_N})$ is generically not concave in c_{i_N}, there can be multiple solutions to the first order condition $\frac{\partial}{\partial c_{i_N}}\pi_P(c_{i_N}, c_{i_N}, \dots, c_{i_N}) = 0$ among which there can be minimizers. However, $\pi_P(c_{i_N}, c_{i_N}, \dots c_{i_N})$ becomes concave for

[9] Because the individual entity's profits are bounded from above by $\pi_A < R - c_A f$, also the planner's profit is bounded from above, $\pi_P < 3(R - c_{soc} f)$. If c becomes too large, the planner's profit becomes negative, implying that $c_{soc} = \infty$ is never optimal, meaning, $\frac{\partial}{\partial c_{i_N}}\pi_P(c_{i_N}, c_{i_N}, \dots, c_{i_N}) < 0$ for c_{i_N} large.

c_{i_N} large.[10] The maximum that attains the highest value $\pi_P(c_{i_N}, c_{i_N}, \ldots, c_{i_N})$ is the social planner equilibrium, as we show below when we exclude other types of equilibria.

When is the social planner equilibrium symmetrically at zero? It holds $\Pi_P(0, \ldots, 0) = 0$. Because $\pi_P(c_{i_N}, c_{i_N}, \ldots c_{i_N})$ can be convex for c_{i_N} small, $\frac{\partial}{\partial c_{i_N}}\pi_P(0, \ldots, 0) = -\alpha'(0)R - f < 0$ is not a sufficient condition for the social planner equilibrium to be at zero, unlike in the private optimal case. It can be that $-\alpha'(0) < f_I/R$, and yet, the planner equilibrium is interior if $\frac{\partial}{\partial c_{i_N}}\pi_P(c_{i_N}, c_{i_N}, \ldots, c_{i_N})$ becomes positive sufficiently fast, pushing the planner's profit $\pi_P(c_{i_N}, c_{i_N}, \ldots, c_{i_N})$ in a positive range before $\pi_P(c_{i_N}, c_{i_N}, \ldots, c_{i_N})$ becomes concave. If $\pi_P(c_{i_N}, c_{i_N}, \ldots, c_{i_N}) < 0$ for all $c_{i_N} > 0$, the planner equilibrium is symmetric at zero.

Excluding asymmetric interior equilibria Assume there exists an equilibrium $(c_{i_1}, \ldots, c_{i_N})$ with all $c_i > 0$ and at least two arguments unequal $c_{i_N} \neq c_{i_{N-1}}$. We show, if $h(x) = \alpha(x)/\alpha'(x)$ is weakly decreasing, then such an asymmetric interior equilibrium cannot exist. Assume, it exists. Using (63), define the FOC function

$$g(c_{i_N}|c_{i_1}, \ldots c_{i_{N-1}}) \equiv (-\alpha'(c_{i_N})) \left[\left(1 - \prod_{k=1}^{N-1} (1 - \alpha(c_{i_k})) \right) + \sum_{j=1}^{N-1} \alpha(c_{i_j}) \prod_{\substack{k=1, \\ k\neq j}}^{N-1} (1 - \alpha(c_{i_k})) \right] \tag{71}$$

and accordingly via (63),

$$g(c_{i_{N-1}}|c_{i_1}, \ldots c_{i_{N-2}}, c_{i_N}) \tag{72}$$

$$\equiv (-\alpha'(c_{i_{N-1}})) \left[\left(1 - \prod_{\substack{k=1, \\ k\neq N-1}}^{N} (1 - \alpha(c_{i_k})) \right) + \sum_{\substack{j=1, \\ j\neq N-1}}^{N} \alpha(c_{i_j}) \prod_{\substack{k=1, \\ k\neq j, k\neq N-1}}^{N} (1 - \alpha(c_{i_k})) \right] \tag{73}$$

Every entity $i_m, m = 1, \ldots N$ faces the same first order condition, described by $g(c_{i_m}|c_{i_1}, \ldots c_{i_{m-1}}, c_{i_{m+1}}, \ldots, c_{i_N})$ where in an interior social planner equilibrium $(c_{i_1}, \ldots, c_{i_N})$

[10]

$$\frac{\partial^2}{\partial c_{i_N}^2}\pi_P(c_{i_N}) = (-\alpha''(c_{i_N}))R_{i_N} \left[1 - (1 - \alpha(c_{i_N}))^{N-2}[1 - N\,\alpha(c_{i_N})] \right] \tag{69}$$

$$- \alpha'(c_{i_N})^2(1 - \alpha(c_{i_N}))^{N-3}R_{i_N} \left[(N-2)[1 - N\,\alpha(c_{i_N})] + N(1 - \alpha(c_{i_N})) \right] \tag{70}$$

The first term is always negative since $\alpha(\cdot)$ is convex and since the bracket is positive, see (67). The second term is negative if the bracket is positive. This occurs if $1 - N\,\alpha(c_{i_N}) > 0$, that is, for c large because $\alpha(\cdot)$ is decreasing and in $[0, 1]$ with $\alpha(c) \to 0$ for $c \to \infty$. However, the bracket can become negative for c small since $\alpha(0) = 1$ which is why $\pi_P(c_{i_N}, c_{i_N}, \ldots c_{i_N})$ is generically not concave.

it must hold

$$g(c_{i_1}|c_{i_2},\dots c_{i_N}) = g(c_{i_2}|c_{i_1},c_{i_3}\dots c_{i_N}) = \dots = g(c_{i_N}|c_{i_1},\dots c_{i_{N-1}}) = f_I/R \tag{74}$$

Consider the equation $g(c_{i_N}|c_{i_1},\dots c_{i_{N-1}}) = g(c_{i_{N-1}}|c_{i_1},\dots,c_{i_{N-2}},c_{i_N})$ which is equivalent to which is equivalent to

$$\begin{aligned}
&(-\alpha'(c_{i_N}))\left[\left(1-(1-\alpha(c_{i_{N-1}}))\underbrace{\prod_{k=1}^{N-2}(1-\alpha(c_{i_k}))}_{\equiv B}\right)+\alpha(c_{i_{N-1}})\underbrace{\prod_{k=1}^{N-2}(1-\alpha(c_{i_k}))}_{\equiv B}+(1-\alpha(c_{i_{N-1}}))\underbrace{\sum_{j=1}^{N-2}\alpha(c_{i_j})\prod_{\substack{k=1,\\k\neq j}}^{N-2}(1-\alpha(c_{i_k}))}_{\equiv C}\right]\\
&=(-\alpha'(c_{i_{N-1}}))\left[\left(1-(1-\alpha(c_{i_N}))\underbrace{\prod_{k=1}^{N-2}(1-\alpha(c_{i_k}))}_{\equiv B}\right)+\alpha(c_{i_N})\underbrace{\prod_{k=1}^{N-2}(1-\alpha(c_{i_k}))}_{\equiv B}+(1-\alpha(c_{i_N}))\underbrace{\sum_{j=1}^{N-2}\alpha(c_{i_j})\prod_{\substack{k=1,\\k\neq j}}^{N-2}(1-\alpha(c_{i_k}))}_{\equiv C}\right]
\end{aligned} \tag{75}$$

where B and C are independent of both c_{i_N} and $c_{i_{N-1}}$. This equation is equivalent to

$$\begin{aligned}
0 &= (\alpha'(c_{i_{N-1}})-\alpha'(c_{i_N}))\,(1-B+C)+(\alpha'(c_{i_{N-1}})\alpha(c_{i_N})-\alpha'(c_{i_N})\alpha(c_{i_{N-1}}))\,(2B-C)\\
&= (\alpha'(c_{i_{N-1}})-\alpha'(c_{i_N}))\,(1-B) && (76)\\
&+\left[\alpha'(c_{i_{N-1}})\alpha(c_{i_N})-\alpha'(c_{i_N})\alpha(c_{i_{N-1}})\right]2B && (77)\\
&+\left[(\alpha'(c_{i_{N-1}})-\alpha'(c_{i_N}))-(\alpha'(c_{i_{N-1}})\alpha(c_{i_N})-\alpha'(c_{i_N})\alpha(c_{i_{N-1}}))\right]C && (78)
\end{aligned}$$

It holds $B, C > 0$. Moreover, $B < 1$ since $\alpha(c) < 1$ for $c > 0$. WLOG further assume that $c_{i_N} > c_{i_{N-1}}$ (If $c_{i_N} \leq c_{i_{N-1}}$, all inequalities hold with opposite sign, and the same final contradiction obtains). Then $(\alpha'(c_{i_{N-1}}) - \alpha'(c_{i_N})) < 0$, and $\alpha(c_{i_N}) \leq \alpha(c_{i_{N-1}})$. Thus, the first term in (76) is negative. If $h(x) = \alpha(x)/\alpha'(x)$ is decreasing, then $\alpha'(c_{i_{N-1}})\alpha(c_{i_N}) - \alpha'(c_{i_N})\alpha(c_{i_{N-1}}) \leq 0$, and also the second term in (76) is negative. To see that the third term must also be negative, note that

$$\begin{aligned}
0 &= (\alpha'(c_{i_{N-1}})-\alpha'(c_{i_N}))\,C-(\alpha'(c_{i_{N-1}})-\alpha'(c_{i_N}))\,C && (79)\\
&> (\alpha'(c_{i_{N-1}})-\alpha'(c_{i_N}))\,C-\alpha(c_{i_N})\,(\alpha'(c_{i_{N-1}})-\alpha'(c_{i_N}))\,C && (80)\\
&> (\alpha'(c_{i_{N-1}})-\alpha'(c_{i_N}))\,C-\left(\alpha(c_{i_N})\,\alpha'(c_{i_{N-1}})-\alpha(c_{i_{N-1}})\alpha'(c_{i_N})\right)C && (81)
\end{aligned}$$

because $\alpha(\cdot)$ is decreasing. Since all three terms are negative, they cannot jointly equal zero, showing a contradiction. Therefore, an interior asymmetric equilibria cannot exist.

Asymmetric corner equilibria We next look for a planner equilibrium where one entity N invests zero, $c_{i_N} = 0$, and all other entities invest positive, interior amounts, $c_{i_j} > 0$. If one entity does not invest, $c_{i_N} = 0$, while the other entities $j = 1,\dots,N-1$ do, $c_{i_j} > 0$, entity

i_N does not provide a security cover for itself. Instead, it free-rides on all other entities that create a cover for i_N by their positive investment $c_{i_j} > 0$. A sufficient condition for optimality of $c_{i_N} = 0$ at $c_{i_j} > 0$, $j = 1, \ldots, N-1$ is

$$g(c_{i_N}|c_{i_1}, \ldots c_{i_{N-1}}) \equiv -\alpha'(c_{i_N}) \left[\left(1 - \prod_{k=1}^{N-1} (1 - \alpha(c_{i_k})) \right) + \sum_{j=1}^{N-1} \alpha(c_{i_j}) \prod_{\substack{k=1, \\ k \neq j}}^{N-1} (1 - \alpha(c_{i_k})) \right] \leq \frac{f_I}{R}, \text{ for all } c_{i_N} \geq 0 \tag{82}$$

A necessary condition for the optimality of $c_{i_N} = 0$, on the other hand, is

$$\lim_{c_{i_N} \to 0} g(c_{i_N}|c_{i_1}, \ldots c_{i_{N-1}}) \equiv -\alpha'(0) \left[\left(1 - \prod_{k=1}^{N-1} (1 - \alpha(c_{i_k})) \right) + \sum_{j=1}^{N-1} \alpha(c_{i_j}) \prod_{\substack{k=1, \\ k \neq j}}^{N-1} (1 - \alpha(c_{i_k})) \right] \leq \frac{f_I}{R} \tag{83}$$

By the convexity of $\alpha(\cdot)$, it holds $-\alpha'(c_{i_N}) \leq -\alpha'(0)$ for any $c_{i_N} > 0$. Thus, because the edgy bracket in (82) is positive, the necessary condition in (83) is also sufficient, implying (82).

On the other hand, given $c_{i_N} = 0$, the FOC's of the social planner changes since the remaining institutions $j = 1, \ldots, N-1$ have no second cover. These institutions maximize profits as under autarky, given in (3). Institution i_N, on the other hand, does not protect itself but enjoys the cover established by the other institution's investment.

With (62) the planner's objective function becomes

$$\pi_P(c_{i_1}, c_{i_2}, \ldots c_{i_{N-1}}, 0) = \sum_{j=1}^{N-1} \left(1 - \alpha(c_{i_j})\right) \, R_{i_j} - c_{i_j} f_I + \left(1 - \alpha(0) \times \left[1 - \prod_{k=1}^{N-1} (1 - \alpha(c_{i_k}) \right] \right) \, R_{i_N} \tag{84}$$

$$= \sum_{j=1}^{N-1} \left(1 - \alpha(c_{i_j})\right) \, R_{i_j} - c_{i_j} f_I + \left(\prod_{k=1}^{N-1} (1 - \alpha(c_{i_k}) \right) \, R_{i_N} \tag{85}$$

Next, consider the first order condition for an optimal interior investment by any of the remaining institutions $j = 1, \ldots, N-1$. Wlog, and to ease notation, choose $j = N-1$. Plugging in the symmetry in revenue, $R \equiv R_{i_j} = R_{i_N}$, given $c_{i_N} = 0$, i_{N-1}'s first order condition equals

$$\frac{\partial}{\partial c_{i_{N-1}}} \pi_P(c_{i_1}, c_{i_2}, \ldots c_{i_{N-1}}, 0) = (-\alpha'(c_{i_{N-1}})) \left[1 + \prod_{k=1}^{N-2} (1 - \alpha(c_{i_k})) \right] = \frac{f_I}{R} \tag{86}$$

We have continuity of the planner's first order condition $\frac{\partial}{\partial c_{i_{N-1}}} \pi_P(c_{i_1}, c_{i_2}, \ldots c_{i_{N-1}}, c_N)$ as

$c_{i_N} \to 0$: From (86), it holds

$$\lim_{c_{i_N}\to 0} g(c_{i_{N-1}}|c_{i_1},\dots c_{i_{N-2}},c_{i_N}) \tag{87}$$

$$= \lim_{c_{i_N}\to 0}(-\alpha'(c_{i_{N-1}}))\left[\left(1-\prod_{\substack{k=1,\\ k\neq N-1}}^{N}(1-\alpha(c_{i_k}))\right)+\sum_{\substack{j=1,\\ j\neq N-1}}^{N}\alpha(c_{i_j})\prod_{\substack{k=1,\\ k\neq j,k\neq N-1}}^{N}(1-\alpha(c_{i_k}))\right] \tag{88}$$

$$= (-\alpha'(c_{i_{N-1}}))\left[1+\prod_{k=1}^{N-2}(1-\alpha(c_{i_k}))\right] = \frac{\partial}{\partial c_{i_{N-1}}}\pi_P(c_{i_1},c_{i_2},\dots c_{i_{N-1}},0) \tag{89}$$

The symmetry of the planner's first order conditions for all $j = 1,\dots,N-1$ imply symmetric socially optimal investment $c_{i_1} = \cdots = c_{i_{N-1}}$: To see this, consider the planner's FOC for entity i_{N-2}'s optimal investment

$$\frac{\partial}{\partial c_{i_{N-2}}}\pi_P(c_{i_1},c_{i_2},\dots c_{i_{N-1}},0) = (-\alpha'(c_{i_{N-2}}))\left[1+\prod_{\substack{k=1,\\ k\neq N-2}}^{N-1}(1-\alpha(c_{i_k}))\right] = \frac{f_I}{R} \tag{90}$$

Joint optimality of interior $c_{i_{N-1}}$ and $c_{i_{N-2}}$ requires

$$(-\alpha'(c_{i_{N-1}}))\left[1+(1-\alpha(c_{i_{N-2}}))\underbrace{\prod_{k=1}^{N-3}(1-\alpha(c_{i_k}))}_{\equiv D}\right] = (-\alpha'(c_{i_{N-2}}))\left[1+(1-\alpha(c_{i_{N-1}}))\underbrace{\prod_{k=1}^{N-3}(1-\alpha(c_{i_k}))}_{\equiv D}\right] \tag{91}$$

or

$$0 = (\alpha'(c_{i_{N-2}})-\alpha'(c_{i_{N-1}})) \tag{92}$$

$$+ D\ \left(\alpha'(c_{i_{N-2}})(1-\alpha(c_{i_{N-1}}))-\alpha'(c_{i_{N-1}})(1-\alpha(c_{i_{N-2}}))\right) \tag{93}$$

Assume $c_{i_{N-1}} \neq c_{i_{N-2}}$, i.e. wlog, $c_{i_{N-1}} > c_{i_{N-2}}$. By convexity of $\alpha(c)$ if follows $\alpha'(c_{i_{N-2}}) < \alpha'(c_{i_{N-1}})$ so that the first term in (92) is negative. Further, $D \in (0,1)$, and by convexity of $\alpha(c) \in (0,1)$ for $c > 0$ it holds that $\hat{h}(x) = \frac{\alpha'(x)}{1-\alpha(x)}$ is strictly increasing. Thus, $\alpha'(c_{i_{N-2}})(1-\alpha(c_{i_{N-1}})) < \alpha'(c_{i_{N-1}})(1-\alpha(c_{i_{N-2}}))$, and also the second term in (92) is negative. But that is a contradiction. Likewise, for $c_{i_{N-1}} < c_{i_{N-2}}$. Thus, interior optimal $c_{i_{N-1}}$ and $c_{i_{N-2}}$ require $c_{i_{N-1}} = c_{i_{N-2}}$, and by symmetry of the FOC's $c_{i_1} = \cdots = c_{i_{N-1}}$. Given this symmetry of the optimal interior $c_{i_1} = \cdots = c_{i_{N-1}}$, the social planner's first order condition for entity i_{N-1}

simplifies further. With (86), we obtain

$$\frac{\partial}{\partial c_{i_{N-1}}}\pi_P(c_{i_1}, c_{i_2}, \dots c_{i_{N-1}}, 0) = (-\alpha'(c_{i_{N-1}}))\ \left[1 + (1-\alpha(c_{i_{N-1}}))^{N-2}\right] = \frac{f_I}{R} \tag{94}$$

We can further plug in the symmetry of the solutions $c_{i_{N-1}} = c_{i_1} = \cdots = c_{i_{N-2}}$ into the optimality condition of institution i_N, (83), simplifying to:

$$\frac{f_I}{R} > -\alpha'(0)\ \left[\left(1-(1-\alpha(c_{i_{N-1}}))^{N-1}\right) + (N-1)\alpha(c_{i_{N-1}})(1-\alpha(c_{N-1}))^{N-2}\right] \tag{95}$$

See that the edgy bracket is positive, since $\alpha(c) \in (0,1)$ for $c > 0$. If assumption 2.2 holds, $h(x) = \alpha(x)/\alpha'(x)$ is decreasing in x. If $c_{i_N} = 0$, and $c_{i_{N-1}} = c_{i_1} = \cdots = c_{i_{N-2}} > 0$ is a social planner equilibrium, then due to $\alpha(c), 1-\alpha(c) \in (0,1)$ for $c > 0$ the following inequality must be true:

$$\frac{f_I}{R} > -\alpha'(0)\ \left[\left(1-(1-\alpha(c_{i_{N-1}}))^{N-1}\right) + (N-1)\alpha(c_{i_{N-1}})(1-\alpha(c_{i_{N-1}}))^{N-2}\right] \tag{96}$$

$$> -\alpha'(0)\ \left[1-(1-\alpha(c_{i_{N-1}}))^{N-1}\right] = -\frac{\alpha'(0)}{\alpha(0)} \times \left[1-(1-\alpha(c_{i_{N-1}}))^{N-1}\right] \tag{97}$$

$$> -\frac{\alpha'(c_{i_{N-1}})}{\alpha(c_{i_{N-1}})} \times \left[1-(1-\alpha(c_{i_{N-1}}))^{N-1}\right] > -\alpha'(c_{i_{N-1}}) \times \left[1-(1-\alpha(c_{i_{N-1}}))^{N-1}\right] \tag{98}$$

$$> -\alpha'(c_{i_{N-1}}) \times \left[1-(1-\alpha(c_{i_{N-1}}))^{N-2}\right] = \frac{f}{R} \tag{99}$$

which is a contradiction. Hence, there cannot exist social planner equilibria where one out of the N institutions defects $c_{i_N} = 0$ while the remaining institutions invest, providing cover for the defecting institution, $(c_{i_1}, \dots, c_{i_{N-1}}, 0)$ with $c_{i_1}, \dots, c_{i_{N-1}} > 0$.

Last, if $-\alpha'(0) > f_I/R$, i.e., if assumption 2.2 holds, there cannot exist an asymmetric social planner corner equilibrium where two or more institutions do not invest. This holds, because the two institutions that do not invest enjoy zero cover, and thus make zero profits. However, with $-\alpha'(0) > f_I/R$, a single entity will rather choose to protect itself, investing the autarky level of data security, and running positive profits in expectation, see section 5.

□

11.2 Optimality of writing DSAs

Proof. [Proposition 5.1] **Optimal investment (i)** Let assumption 2.2 hold so that $-\alpha'(0) > f_I/R$, implying that the autarky solution is positive $c_{ij}^a > 0$. We show that writing the data sharing agreements (DSAs) to share data in the case of a hack is optimal, making all entities at least as well off as in autarky:

Step 1: We show, in every possible private optimum, investment with DSAs undercuts investment in autarky. Assume all entities write the DSAs with one another. Then, the risk of data loss changes, see Corollary (9.1), and from (33) private profits become

$$\Pi_{i_j}(c_{i_j}|c_{i_1},\ldots,c_{i_N}) = \left(1-\alpha(c_{i_j})\times\left[1-\prod_{\substack{k=1,\\k\neq j}}^{N}(1-\alpha(c_{i_k}))\right]\right)R_{i_j}-c_{i_j}f_I \tag{100}$$

with first order condition

$$\frac{\partial}{\partial c_{i_j}}\Pi_{i_j}(c_{i_j}|c_{i_1},\ldots,c_{i_N}) = -\alpha'(c_{i_j})\times\left[1-\prod_{\substack{k=1,\\k\neq j}}^{N}(1-\alpha(c_{i_k}))\right]R_{i_j}-f_I=0 \tag{101}$$

By Proposition 9.1, assumption 2.2 implies that there exists a symmetric interior private equilibrium $c^*_{i_j}>0$, $j=1,\ldots,N$ that solves (101). Since in that equilibrium, all entities $\{i_1,\ldots,i_N\}$ invest in data security, it holds $(1-\alpha(c_{i^*_k}))<1$ for all k, implying $1-\prod_{\substack{k=1,\\k\neq j}}^{N}(1-\alpha(c_{i^*_k}))<1$ and thus with $-\alpha'>0$,

$$\begin{aligned}0 &= -\alpha'(c^*_{i_j})\times\left[1-\prod_{\substack{k=1,\\k\neq j}}^{N}(1-\alpha(c^*_{i_k}))\right]R_{i_j}-f_I\\ &< -\alpha'(c^*_{i_j})\,R_{i_j}-f_I.\end{aligned} \tag{102}$$

The autarky solution, on the other hand, satisfies

$$0=-\alpha'(c^a_{i_j})\,R_{i_j}-f_I, \tag{103}$$

by Proposition 3.1. Combining inequality (102) with equation (103), yields $-\alpha'(c^a_{i_j})<-\alpha'(c^*_{i_j})$ which jointly with the convexity of $\alpha(c)$ implies that with autarky, an entity optimally invests more in data security than in the private optimum with DSAs, given as the symmetric interior equilibrium, $c^a_{i_j}>c^*_{i_j}$.

Proposition 4.1 also says that the private optimum can be given as an asymmetric corner equilibrium where one entity k shirks, investing zero,$c^*_{i_k}=0$, and all the remaining entities invest the autarky amount, $c^*_{i_j}=ci^a$. Then clearly, $c^*_{i_k}=0<c^a_i$, and again, an entity optimally invests (weakly) more in data security in autarky than in the private optimum with DSAs in the asymmetric corner equilibrium, $c^*_{i_j}\leq c^a_i$, for all $j\in\{1,\ldots,N\}$.

We conclude, in every possible private optimum, investment with DSAs undercuts investment in autarky, $c^*_{i_j} \leq c^a_i$.

Step 2: We show, in the social optimum investment with DSAs exceeds investment in autarky. If assumption 2.2 holds, the socially optimal investment level $c^s_{i_1} > 0$ is interior, symmetric, and given as a solution to

$$(-\alpha'(c^s_{i_1}))\left[1-(1-\alpha(c^s_{i_1}))^{N-2}[1-N\,\alpha(c^s_{i_1})]\right] = \frac{f_I}{R} \tag{104}$$

In contrast, the autarky level of investment $c^a_{i_j}$ solves (6). Because the edgy bracket in (104) is always positive, i.e., see equation (67), the following inequality holds,

$$-\alpha'(c^a_{i_1}) = \frac{f_I}{R} = (-\alpha'(c^s_{i_1}))\left[1-(1-\alpha(c^s_{i_1}))^{N-2}[1-N\,\alpha(c^s_{i_1})]\right] \tag{105}$$

$$> -\alpha'(c^s_{i_1}) \tag{106}$$

The convexity of $\alpha(\cdot)$ implies $c^a_{i_1} < c^s_{i_1}$. Jointly, with step 1, $c^*_{i_j} \leq c^a_i < c^s_{i_1}$, that ism the private entities always underinvest relative the the social planner solution.

Profits (ii) Step 3: We show, that the entitie's profits in autarky always undercut the entitie's profits in the private optimum with DSAs, no matter what type of private optimum arises. First, consider the symmetric private equilibrium, $c^*_{i_j} > 0$. An entity i_j's profit in the symmetric private equilibrium equals

$$\Pi_{i_j}(c^*_{i_j}|c_{i_1},\ldots,c_{i_N}) = \left(1-\alpha(c^*_{i_j})\times\left[1-\prod_{\substack{k=1,\\k\neq j}}^{N}(1-\alpha(c^*_{i_k}))\right]\right)R_{i_j} - c^*_{i_j}f_I \tag{107}$$

$$> \left(1-\alpha(c^a_{i_j})\times\left[1-\prod_{\substack{k=1,\\k\neq j}}^{N}(1-\alpha(c^a_{i_k}))\right]\right)R_{i_j} - c^a_{i_j}f_I \tag{108}$$

$$> \left(1-\alpha(c^a_{i_j})\right)R_{i_j} - c^a_{i_j}f_I = \Pi^a_{i_j}(c_{i_j}) \tag{109}$$

where the first inequality holds because conditional on a symmetric interior solution, private profits in the setting with DSAs are maximized at the symmetric equilibrium $c^*_{i_k}$, not at the higher (symmetric) autarky level of investment $c^a_i > c^*_{i_k}$. Thus, c^a_i must yield lower profits. The second inequality follows because $c^a_{i_j} > 0$, and thus $\left[1-\prod_{\substack{k=1,\\k\neq j}}^{N}(1-\alpha(c^a_{i_k}))\right] < 1$.

Now assume, that the private optimum with DSAs was given by the asymmetric, private corner equilibrium, where one entity, say i_N, shirks by investing zero, $c^*_N = 0$, and the

remaining entities invest the autarky level c_i^a. Then, clearly, profits in the private asymmetric corner equilibrium and profits in autarky coincide for the not-shirking institutions. This holds, because as one entity shirks, the DSAs become worthless since the second cover for all the remaining institutions is not established, meaning those remaining institutions are exactly back in autarky, and get the same profits: $\Pi_{i_j}(c_i^a|(c_1^a,\ldots,c_{i_{N-1}}^a,0)) = \Pi_{i_j}^a(c_i^a)$.

$$\Pi_{i_j}(0|c_{i_1}^*,\ldots,c_{i_{N-1}}^*) = \left(1 - \left[1 - \prod_{k=1}^{N-1}(1-\alpha(c_{i_k}^*))\right]\right) R_{i_j} \tag{110}$$

$$> \left(1-\alpha(c_{i_j}^a)\right) R_{i_j} - c_{i_j}^a f_I = \Pi_{i_j}^a(c_{i_j}^a) \tag{111}$$

For the shirking institution, c_{i_N}, the profit in the private asymmetric corner equilibrium must be strictly higher than in autarky, since the autarky profit is always available by setting $c_{i_N} = c_{i_j}^a > 0$. The autarky profit corresponds to a lower bound on private profits, because by investing the autarky level, an entity can always attain a profit that is at least as high as the autarky profit. Overall, in any private equilibrium the profits of every institution weakly exceed autarky profits. Thus, entering into the data sharing agreements is privately optimal.

Step 4: We show, that welfare in every possible private equilibrium with DSAs exceeds welfare in autarky This step follows directly from step 3. We consider the symmetric private equilibrium $c_{i_j}^*$ first. Welfare in the symmetric private equilibrium equals

$$\Pi_P(c_{i_1}^*,\ldots,c_{i_N}^*) = \sum_{j=1}^{N}\Pi_{i_j}(c_{i_j}^*|c_{i_1}^*,\ldots,c_{i_N}^*) > \sum_{j=1}^{N}\left[\left(1-\alpha(c_{i_j}^a)\right) R_{i_j} - c_{i_j}^a f_I\right] = \Pi_P^a(c_{i_j}^a), \tag{112}$$

using step 3 at the inequality. Now consider the asymmetric private corner equilibrium where one entity shirks, $c_N^* = 0$, investing zero, whereas the other entities invest the autarky amount $c_{i_j}^* = c^a, j = 1,\ldots,N-1$. Welfare in the asymmetric private corner equilibrium equals

$$\Pi_P(c_{i_j}^*,\ldots,c_{i_j}^*,0) = \Pi_{i_N}(0) + \sum_{j=1}^{N-1}\Pi_{i_j}(c_{i_j}^*) = \Pi_{i_N}(0) + (N-1)\Pi_{i_j}(c_{i_j}^a) > \sum_{j=1}^{N}\Pi_{i_j}(c_{i_j}^a) = \Pi_P(c_{i_j}^a) \tag{113}$$

The inequality holds because profits for the shirking institution i_N when investing zero must be at least as high as the profits when investing the autarky level. This holds, because shirking is optimal given others invest the autarky amount, and since the autarky profit is always available for every entity by investing the accooding amount. Overall, welfare in any private equilibrium exceeds welfare in the autarky case. Thus, entering into the data sharing agreements is socially optimal. □

12 Internet Appendix

12.1 Proofs: Regulated Data Economy with DSAs: FMI for $N = 3$

Proof. [Proposition 6.1]
See the proof to Proposition 9.3. □

Proof. [Lemma 6.2] To proof Lemma 6.2, consider the function

$$F(c_A, c_F) = \frac{1}{\alpha(c_F)} \frac{f_I}{R(|I_A|)} + \alpha'(c_A)[2\alpha(c_A) - \alpha(c_A)^2] \tag{114}$$

Consider condition (19) for an interior private equilibrium in the FMI case. For every subgame c_F, the zeros to the implicit function $F(c_A(c_F), c_F) = 0$ describe the symmetric equilibrium. Via the implicit function theorem, the change of the symmetric equilibrium due to a change in the subgame c_F is given as $\frac{\partial c_A}{\partial c_F} = -\frac{\frac{\partial F}{\partial c_F}}{\frac{\partial F}{\partial c_A}}$. □

Proof. [Theorem 6.1] See text below the Theorem. □

12.1.1 Proof FMI Optimization

Proof. [Theorem 9.1] Consider the the FMI objective function from (18),

$$\begin{aligned}&\pi_P^F(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F)\\ &= \pi_A^F(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F) + \pi_B^F(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F) + \pi_C^F(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F)\end{aligned} \tag{115}$$

The optimal private choices $c_A^{*,F}, c_B^{*,F}, c_C^{*,F}$ are all functions of c_F. When calculating the first-order derivative with respect to c_F, we need to take into account that a change in the subgame c_F causes a change of behavior by A,B,C. In addition, we use the symmetry $|I_A| = |I_B| = |I_C|$ to calculate

$$\frac{d}{dc_F}\pi_P^F(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F) \tag{116}$$

$$= \frac{\partial}{\partial c_F}\left(\pi_A^F(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F) + \pi_B^F(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F) + \pi_C^F(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F)\right) \tag{117}$$

$$+ \frac{\partial}{\partial c_A}\pi_A^F(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F)\frac{\partial c_A}{\partial c_F} + \frac{\partial}{\partial c_B}\pi_B^F(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F)\frac{\partial c_B}{\partial c_F} + \frac{\partial}{\partial c_C}\pi_C^F(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F)\frac{\partial c_C}{\partial c_F} \tag{118}$$

$$+ \left[\left(\frac{\partial}{\partial c_B}\pi_A^F\frac{\partial c_B}{\partial c_F} + \frac{\partial}{\partial c_C}\pi_A^F\frac{\partial c_C}{\partial c_F}\right) + \left(\frac{\partial}{\partial c_A}\pi_B^F\frac{\partial c_A}{\partial c_F} + \frac{\partial}{\partial c_C}\pi_B^F\frac{\partial c_C}{\partial c_F}\right) + \left(\frac{\partial}{\partial c_A}\pi_C^F\frac{\partial c_A}{\partial c_F} + \frac{\partial}{\partial c_B}\pi_C^F\frac{\partial c_B}{\partial c_F}\right)\right] \tag{119}$$

See that line (118) is always zero. This is because, if $c_A^* = 0$ is at the corner, then $\frac{\partial c_A}{\partial c_F} = 0$ whereas if $c_A^* > 0$ is interior, then c_A^* must satisfy the first order conditions of a private equilibrium for entity A, implying that $\frac{\partial}{\partial c_A}\pi_A^F(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F) = 0$. Likewise for B and C.

We can further simplify the first and third line of the derivative using the symmetry of the problem and the solution.

In the symmetric equilibrium, $c_A = c_B = c_C$ and $\frac{\partial c_A}{\partial c_F} = \frac{\partial c_B}{\partial c_F} = \frac{\partial c_C}{\partial c_F}$. In addition, $\pi_A^F(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F)$ is symmetric in c_B and c_C, implying $\frac{\partial}{\partial c_B}\pi_A^F = \frac{\partial}{\partial c_C}\pi_A^F$. Likewise, $\pi_B^F(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F)$ is symmetric in c_A and c_C and $\pi_C^F(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F)$ is symmetric in c_A and c_B. Further, $\frac{\partial}{\partial c_F}\pi_A^F(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F) = \frac{\partial}{\partial c_F}\pi_B^F(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F) = \frac{\partial}{\partial c_F}\pi_C^F(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F)$. Therefore,

$$\frac{d}{dc_F}\pi_P^F(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F) = 3\frac{\partial}{\partial c_F}\pi_A^F(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F) + 6\left(\frac{\partial}{\partial c_B}\pi_A^F\right)\frac{\partial c_B}{\partial c_F} \tag{120}$$

Plugging in the profit functions yields

$$\begin{aligned}\frac{d}{dc_F}\pi_P^F(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F) = &-3\alpha^2(c_A)\alpha'(c_F)\ R(|I_A|)\Big(2-\alpha(c_A)\Big) - f_I \\ &- 6\alpha'(c_A)\alpha(c_A)\alpha(c_F)\frac{\partial c_A}{\partial c_F}(1-\alpha(c_A))\ R(|I_A|)\end{aligned} \tag{121}$$

Case 1: Assume $-\alpha'(0) \leq \frac{f_I}{R(|I_A|)}$. Then for any $c_F \geq 0$, $-\alpha'(0) \leq \frac{f_I}{R(|I_A|)\ \alpha(c_F)}$, and Proposition 6.1 implies that the unique private equilibrium following any $c_F \geq 0$ is the zero investment equilibrium, $c_A^* = c_B^* = c_C^* = 0$. We can further follow, $\frac{\partial c_A}{\partial c_F} = \frac{\partial c_B}{\partial c_F} = \frac{\partial c_C}{\partial c_F} = 0$. With $\alpha(0) = 1$, the FMI's FOC simplifies to

$$\frac{d}{dc_F}\pi_P^F(0,0,0,c_F) = -3\alpha'(c_F)R(|I_A|) - f_I \tag{122}$$

To see whether adding the FMI, i.e. setting $c_F > 0$, creates value, a necessary condition for $c_F^* = 0$ is

$$\frac{d}{dc_F}\pi_P^F(0,0,0,0) = -3\alpha'(0)R(|I_A|) - f_I \leq 0 \tag{123}$$

whereas a sufficient condition for $c_F^* = 0$ is $\frac{d}{dc_F}\pi_P^F(0,0,0,c_F) = -3\alpha'(c_F)R(|I_A|) - f_I \leq 0, \quad$ for all $c_F \geq 0$. However, since $\alpha(c)$ is convex, $-\alpha(c_F) \leq -\alpha(0)$ for every $c_F \geq 0$, implying that the necessary condition is also sufficient. We conclude, if

$$-\alpha'(0) \leq \frac{1}{3}\frac{f_I}{R(|I_A|)} \tag{124}$$

$c_F^* = 0$ is optimal to the FMI. Thus, for all $-\alpha'(0) \in \left(0, \frac{1}{3}\frac{f_I}{R(|I_A|)}\right]$, the FMI equilibrium is given as $(c_A^*, c_B^*, c_C^*, c_F^*) = (0,0,0,0)$, and the FMI optimally abstains from investing in data security. For $(-\alpha'(0)) = \frac{f_I}{3R(|I_A|)}$ the benefits of providing the FMI equal the costs. In that case, we break ties by letting the FMI abstain. For $-\alpha'(0) \in \left(\frac{1}{3}\frac{f_I}{R(|I_A|)}, \frac{f_I}{R(|I_A|)}\right]$, the FMI equilibrium is given as $(c_A^*, c_B^*, c_C^*, c_F^*) = (0,0,0,c_F^*)$, where c_F^* is the unique, interior solution to $-3\alpha'(c_F)R(|I_A|) = f_I$. The solution is unique by the convexity of $\alpha(c)$ and exists by continuity of α, and assumption 2.1 and 2.2.

Case 2: Assume $-\alpha'(0) > \frac{f_I}{R(|I_A|)}$. Then, absent the FMI the banks play the interior symmetric private equilibrium $c_i^* > 0$, $i = A, B, C$. There exists an interior $\bar{c}_F > 0$ such that $-\alpha'(0) = \frac{1}{\alpha(\bar{c}_F)}\frac{f_I}{R(|I_A|)}$. For $c_F < \bar{c}_F$, it holds $-\alpha'(0) > \frac{1}{\alpha(\bar{c}_F)}\frac{f_I}{R(|I_A|)}$ whereas for $c_F > \bar{c}_F$ $-\alpha'(0) < \frac{1}{\alpha(\bar{c}_F)}\frac{f_I}{R(|I_A|)}$.

Case $c_F > \bar{c}_F$: The private equilibrium following c_F equals $c_A^* = 0$, and $\frac{\partial c_{i_k}}{\partial c_F} = 0$. The FMI thus faces the FOC as in case 1,

$$\frac{d}{dc_F}\pi_P^F(0,0,0,c_F) = -3\alpha'(c_F)R(|I_A|) - f_I \tag{125}$$

If this FOC is negative for all $c_F \geq \bar{c}_F$, then no $c_F > \bar{c}_F$ can be optimal, and part of an equilibrium. Indeed, if $-\alpha'(\bar{c}_F) \leq \frac{f}{3R}$, the convexity of $\alpha(c)$ implies $-\alpha'(c_F) \leq \frac{f}{3R}$ for all $c_F \geq \bar{c}_F$, and no $c_F \geq \bar{c}_F$ is optimal. A necessary condition for some $c_F \geq \bar{c}_F$ to be optimal is that $\bar{c}_F$ satisfies $-\alpha'(\bar{c}_F) > \frac{f}{3R}$, or equivalently $-\frac{\alpha'(\bar{c}_F)}{\alpha(\bar{c}_F)} > -\frac{1}{3}\alpha'(0)$. In that case, and if $-\alpha'(c) \to 0$ for $c \to \infty$, there exists a $c_F^* > \bar{c}_F$ with $-3\alpha'(c_F^*)R(|I_A|) = f_I$, which jointly with $c_A^* = c_B^* = c_C^* = 0$ is a candidate for the FMI equilibrium.

Case $c_F < \bar{c}_F$: Then private entities invest $c_A^*(c_F) > 0$, and as the FMI invests more, the entities shirk, $\frac{\partial c_{i_k}}{\partial c_F} < 0$. The FMI's FOC is given by

$$\begin{aligned}
\frac{d}{dc_F}\pi_P^F(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F) &= -3\alpha^2(c_A)\alpha'(c_F)\; R(|I_A|)\Big(2 - \alpha(c_A)\Big) - f_I \\
&\quad - 6\alpha'(c_A)\alpha(c_A)\alpha(c_F)\frac{\partial c_A}{\partial c_F}(1-\alpha(c_A))\; R(|I_A|) && (126)\\
&= 3f_I\left(\frac{\alpha(c_A)(-\alpha'(c_F))}{\alpha(c_F)(-\alpha'(c_A))} - \frac{1}{3}\right) && (127)\\
&\quad - 6\alpha'(c_A)\alpha(c_A)\alpha(c_F)\frac{\partial c_A}{\partial c_F}(1-\alpha(c_A))\; R(|I_A|) && (128)
\end{aligned}$$

where at the second equality sign we have rewritten the first term replacing $\alpha(c_A)(2-\alpha(c_A))$, via the equilibrium FOC (19). We can further replace $\frac{\partial c_{i_k}}{\partial c_F} < 0$ via Lemma 6.2. The first term

weighs the increased profits due to higher security against the direct costs of investment in data security. It can have either sign. See that the second term (128) is negative, and represents the shirking of the private instutions in response to a marginal increase in the FMI investment c_F. If the derivative $\frac{d}{dc_F}\pi_P^F(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F)$ is negative in $c_F = 0$ and in the corresponding private responses $c_A^*(0) = c_B^*(0) = c_C^*(0) > 0$, then $(c_A^*(0), c_A^*(0), c_A^*(0), 0)$ is a candidate for an FMI equilibrium.

Thus, the positivity of the first term in $c_F = 0$ is a necessary condition for (some) FMI investment to be optimal. Plugging in $c_F = 0$, and using $\alpha(0) = 1$, the first term is positive if

$$\frac{-\alpha'(0)}{-\alpha'(c_A)/\alpha(c_A)} - \frac{1}{3} > 0. \tag{129}$$

Because $c_i^* > 0$ is the private, interior equilibrium absent the FMI, it is given as the solution to the FOC, $-\alpha'(c_A^*)\alpha(c_A^*)(2-\alpha(c_A^*)) = \frac{f_I}{R(|I_A|)}$. Plugging in, noting that $-\alpha'(0) > \frac{f_I}{R(|I_A|)}$, yields

$$\frac{-\alpha'(0)}{-\alpha'(c_A)/\alpha(c_A)} > -\frac{\frac{f_I}{R(|I_A|)}}{\alpha'(c_A)/\alpha(c_A)} = \alpha(c_A^*)^2(2-\alpha(c_A^*)) \tag{130}$$

That is, for $\alpha(c_A^*)^2(2-\alpha(c_A^*)) > \frac{1}{3}$, the term (127) is positive. This holds if $\alpha(c_A^*) \in (0,1)$ is sufficiently close to one, that is, for c_A^* small and close to zero.

If $\frac{d}{dc_F}\pi_P^F(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, 0) > 0$, the FMI will gradually raise c_F away from zero, which causes the private entities to shirk via $\frac{\partial c_A}{\partial c_F} < 0$. There are two possibilities. First, if $\frac{d}{dc_F}\pi_P(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F) > 0$ for all $c_F \leq \bar{c_F}$, no $c_F < \bar{c_F}$ is optimal. Moreover, the private $c_A^{*,F}$ hit zero in $c_F = \bar{c_F}$. Thus, the optimum must be located at $c_F \geq \bar{c}_F$.

Second, if $\frac{d}{dc_F}\pi_P^F(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F^*) = 0$ for some $c_F^* < \bar{c_F}$, we have found an interior candidate for an FMI equilibrium that involves $c_A^{*,F} > 0$ and $0 < c_F^* < \bar{c_F}$.

We give a more general procedure to find all candidates for a subgame-perfect equilibrium with N banks in section 12.2.4. □

12.2 Proof: FMI with N private entities

12.2.1 Stage 2: private equilibrium

Proof. [Proposition 9.3]

Also with N entities, the FMI moves first in $t = 1$, setting c_F observably to all private entities. The private entities move in $t = 2$ after observing c_F. With the FMI and N private

entities in total, entity i_j's profit function given c_F equals

$$\Pi^F_{i_j}(c_{i_j}|c_{i_1},\ldots,c_{i_N}) = \left(1-\alpha(c_{i_j})\alpha(c_F)\times\left[1-\prod_{\substack{k=1,\\k\neq j}}^{N}(1-\alpha(c_{i_k}))\right]\right)R_{i_j} - c_{i_j}f_I - \frac{c_F f_I}{N} \quad (131)$$

For simplicity, set $j=N$. Entity i_N's marginal profit equals

$$\frac{\partial}{\partial c_{i_N}}\Pi_{i_N}(c_{i_N}|c_{i_1},\ldots,c_{i_{N-1}}) = -\alpha'(c_{i_N})\alpha(c_F)\times\left[1-\prod_{k=1}^{N-1}(1-\alpha(c_{i_k}))\right]R_{i_N} - f_I \quad (132)$$

Clearly, the profit function is concave in i_N's investment c_{i_N}, by convexity of $\alpha(c)$,

$$\frac{\partial^2}{\partial c^2_{i_N}}\Pi_{i_N}(c_{i_N}|c_{i_1},\ldots,c_{i_{N-1}}) = -\alpha''(c_{i_N})\alpha(c_F)\times\left[1-\prod_{k=1}^{N-1}(1-\alpha(c_{i_k}))\right]R_{i_N} < 0. \quad (133)$$

Therefore, a solution to the first order condition is unique if it exists, and is a maximizer. The equilibrium condition for an interior private FMI equilibrium equals

$$-\alpha'(c_{i_N})\times\left[1-\prod_{k=1}^{N-1}(1-\alpha(c_{i_k})\right] = \frac{1}{\alpha(c_F)}\frac{f_I}{R_{i_N}}. \quad (134)$$

Existence and uniqueness of a symmetric equilibrium To show that a symmetric equilibrium exists, assume, all entities invest symmetric amounts $c_{i_1}=\cdots=c_{i_N}$. Then, the first order conditions for an interior symmetric private equilibrium equals

$$-\alpha'(c_{i_N})\times\left(1-(1-\alpha(c_{i_N}))^{N-1}\right) = \frac{1}{\alpha(c_F)}\frac{f_I}{R_{i_N}}. \quad (135)$$

Define the function $g(c_{i_N}) = \alpha'(c_{i_N})\times\left(1-(1-\alpha(c_{i_N}))^{N-1}\right)$. We know, that $g(0)=\alpha'(0)$, and $\lim_{c_{i_N}\to\infty}g(c_{i_N})=0$ since $\lim_{c\to\infty}\alpha(c)=0$. Further, $g(c)$ is continuous and is strictly increasing in c, by the convexity of $\alpha(c)$ and since $\alpha(c)\in(0,1)$. Thus, if c_F is such that $-\alpha'(0) > \frac{1}{\alpha(c_F)}\frac{f_I}{R_{i_N}}$, the function $-g(c)$ must cross $\frac{1}{\alpha(c_F)}\frac{f_I}{R_{i_N}}$, and crosses exactly once at some $c^*\in(0,\infty)$. Thus, there exists a unique candidate for a symmetric interior equilibrium following c_F. This candidate is an equilibrium: assume, all entities invest c^* except for i_N, choosing $c_{i_N} < c^*$. Then convexity of $\alpha(c)$ implies that the profit function of i_N is increasing in c_{i_N}:

$$\frac{\partial}{\partial c_{i_N}}\Pi_{i_N}(c_{i_N}|c^*,\ldots,c^*) = -\alpha'(c_{i_N})\alpha(c_F)\times\left[1-(1-\alpha(c^*))^{N-1}\right]R_{i_N} - f_I \quad (136)$$

$$> -\alpha'(c^*)\alpha(c_F)\times\left[1-(1-\alpha(c^*))^{N-1}\right]R_{i_N} - f_I = 0. \quad (137)$$

Thus, $c_{i_N} < c^*$ cannot be optimal. By the same argument, $c_{i_N} > c^*$ implies $\frac{\partial}{\partial c_{i_N}} \Pi_{i_N}(c_{i_N}|c^*, \ldots, c^*) < 0$ so that $c_{i_N} > c^*$ is not optimal. Thus, c^* is a symmetric equilibrium.

Assume now, that c_F is such that $-\alpha'(0) \leq \frac{1}{\alpha(c_F)} \frac{f_I}{R_{i_N}}$. Consequently, the function $-g(c)$ does not cross $\frac{1}{\alpha(c_F)} \frac{f_I}{R_{i_N}}$, so that no interior symmetric equilibrium exists. Moreover, $c^* = \infty$ is never an equilibrium because profits are bounded whereas for $c \to \infty$, the costs become infinite. To see that zero investment arises as the unique symmetric equilibrium, see that whatever the entitites $i_1, \ldots, i_{N-1}$ do, for every $c_{i_N} \geq 0$ it holds

$$\frac{\partial}{\partial c_{i_N}} \Pi_{i_N}(c_{i_N}|c_{i_1}, \ldots, c_{i_{N-1}}) = -\alpha'(c_{i_N})\alpha(c_F) \times \left[1 - \prod_{k=1}^{N-1}(1 - \alpha(c_{i_k}))\right] R_{i_N} - f_I \tag{138}$$

$$< -\alpha'(0)\alpha(c_F) \times \left[1 - \prod_{k=1}^{N-1}(1 - \alpha(c_{i_k}))\right] R_{i_N} - f_I \tag{139}$$

$$< -\alpha'(0)\alpha(c_F) \times R_{i_N} - f_I \tag{140}$$

$$< 0. \tag{141}$$

since $1 - \prod_{k=1}^{N-1}(1 - \alpha(c_{i_k})) < 1$.

Excluding asymmetric interior equilibria There cannot exist asymmetric interior equilibria for the same reason as in the proof to Proposition 9.1.

Asymmetric corner equilibria

Given c_F, assume there was a private equilibrium where entity N invests zero, $c_{i_N} = 0$, and all other entities invest positive, interior amounts, $c_{i_j} > 0$. A sufficient condition for optimality of $c_{i_N} = 0$ is

$$g(c_{i_N}|c_{i_1}, \ldots c_{i_{N-1}}, c_F) \equiv -\alpha'(c_{i_N}) \times \left[1 - \prod_{k=1}^{N-1}(1 - \alpha(c_{i_k}))\right] < \frac{f_I}{R\ \alpha(c_F)}, \text{ for all } c_{i_N} \geq 0 \tag{142}$$

whereas a necessary condition for optimality of $c_{i_N} = 0$ is

$$\lim_{c_{i_N} \to 0} g(c_{i_N}|c_{i_1}, \ldots c_{i_{N-1}}, c_F) \equiv -\alpha'(0) \times \left[1 - \prod_{k=1}^{N-1}(1 - \alpha(c_{i_k}))\right] < \frac{f_I}{R\ \alpha(c_F)} \tag{143}$$

Again, by convexity of $\alpha(c)$, $-\alpha'(0) > -\alpha'(c)$ for all $c > 0$, and because the edgy bracket is positive, the necessary condition is also sufficient.

The shirking of one institution, $c_{i_N} = 0$, causes the profits of all of the other institutions to change. However, with the FMI, profits are not back to the autarky setting of section 5, as is

the case when the FMI is absent. Rather, the profit function (41) changes to

$$\Pi_{i_j}(c_{i_j}|c_{i_1},\ldots,c_{i_N}) = \left(1-\alpha(c_{i_j})\alpha(c_F)\right) R_{i_j} - c_{i_j} f_I \tag{144}$$

This holds because by setting $c_{i_N} = 0$, none of the institutions $i_1,\ldots,i_{N-1}$ is provided with a second private cover, however, the FMI cover exists if $c_F > 0$. That is, the risk of each entity i_j's data loss equals $\alpha(c_{i_j})\alpha(c_F)$, and no longer depends on what the remaining other private institutions do, besides i_N. Wlog, consider the FOC of institution $N-1$,

$$-\alpha'(c_{i_{N-1}}) = \frac{f_I}{R\ \alpha(c_F)} \tag{145}$$

Because this FOC has a unique solution, and is the same for all entities $i_1,\ldots,i_{N-1}$, it follows $c_{i_1} = \ldots c_{i_{N-1}}$.

If $-\alpha'(0) > f_I/(R\alpha(c_F))$, then convexity and continuity of $\alpha(c)$, jointly with assumotion 2.2 guarantee that an interior solution to (145) exists, and is unique, $c_{i_{N-1}} > 0$. That is, all the remaining entities $i_1,\ldots,i_{N-1}$ invest in security although i_N defects by investing zero. If on the other hand $-\alpha'(0) \leq f_I/(R\alpha(c_F))$, $c_{i_1} = \ldots c_{i_{N-1}} = 0$, and the symmetric no investment equilibrium arises. Therefore, $-\alpha'(0) \leq f_I/(R\alpha(c_F))$ is sufficient to exclude an asymmetric corner equilibrium.

From here on, assume, $-\alpha'(0) > f_I/(R\alpha(c_F))$ holds. By the symmetry of investment, the optimality condition for institution i_N changes to

$$-\alpha'(0) \times \left[1-(1-\alpha(c_{i_{N-1}}))^{N-1}\right] < \frac{f_I}{R\ \alpha(c_F)} = -\alpha'(c_{i_{N-1}}) \tag{146}$$

where $c_{i_{N-1}}$ is given as the solution to (145). This condition can possibly hold. We have $-\alpha'(0) > -\alpha'(c_{i_{N-1}})$ whereas $c_{i_{N-1}} > 0$, implying $\left[1-(1-\alpha(c_{i_1}))^{N-1}\right] < 1$. Thus an asymmetric private corner equilibrium with one entity defecting $(c,\ldots,c,0)$ can exist.

□

12.2.2 Lemma: Private shirking when the FMI invests more

Proof. Lemma 9.2 Consider the function

$$F(c_F, c_{i_N}) = -\alpha'(c_{i_N}) \times \left(1-(1-\alpha(c_{i_N}))^{N-1}\right) - \frac{1}{\alpha(c_F)}\frac{f_I}{R_{i_N}}, \tag{147}$$

and consider the implicit function $c_{i_N}(c_F)$ defined as the solution to the equation

$$F(c_F, c_{i_N}(c_F)) = 0 \tag{148}$$

Note, the function $c_{i_N}(c_F)$ exists by the existence and uniqueness proof of a private interior equilibrium. As the FMI alters its investment c_F, the function $c_{i_N}(c_F)$ describes the change in the symmetric private equilibrium played by the entities in stage 2. It holds

$$\frac{\partial}{\partial c_F} F(c_F, c_{i_N}) = \frac{\alpha'(c_F)}{\alpha^2(c_F)} \frac{f_I}{R_{i_N}} < 0 \tag{149}$$

and

$$\frac{\partial}{\partial c_{i_N}} F(c_F, c_{i_N}) = -\alpha''(c_{i_N}) \times \big(1 - (1 - \alpha(c_{i_N}))^{N-1}\big) - (N-1)\alpha'(c_{i_N})^2 \times \big(1 - (1 - \alpha(c_{i_N}))^{N-2}\big) < 0 \tag{150}$$

Thus, by the implicit function theorem, $c'_{i_N}(c_F) = -\frac{\partial}{\partial c_F} F / \frac{\partial}{\partial c_{i_N}} F < 0$, meaning that Lemma 6.2 is robust.

□

12.2.3 Stage 1: FMI equilibrium

The FMI seeks to maximise social welfare by optimally choosing c_F in $t = 1$, taking as given the privately optimal security choices that follow by the banks in stage 2. The FMI chooses c_F to maximize joint profits accounting for its own data security costs when providing the third cover:

$$\pi_P^F(c_{i_1}^{*,F}, \ldots, c_{i_N}^{*,F}, c_F) = \sum_{j=1}^{N} \pi_{i_j}^F(c_{i_1}^{*,F}, \ldots, c_{i_N}^{*,F}, c_F) - c_F \; f_I + N \times \frac{c_F f_I}{N} = \sum_{j=1}^{N} \pi_{i_j}^F(c_{i_1}^{*,F}, \ldots, c_{i_N}^{*,F}, c_F) \tag{151}$$

Definition 12.1 (FMI equilibrium)**.** *An equilibrium in the FMI environment is a tuple* $(c_F^*, c_{i_1}^{*,F}, \ldots, c_{i_N}^{*,F})$ *where:*
(i) Given the FMI's choice c_F^* *in* $t = 1$*, the investment levels* $(c_{i_1}^{*,F}, \ldots, c_{i_N}^{*,F})$ *are privately optimal in* $t = 2$*.*
(ii) Given the privately optimal responses $(c_{i_1}^{*,F}, \ldots, c_{i_N}^{*,F})$ *following the FMI's choice (subgame)* c_F*, the choice* c_F^* *and the following private choices* $(c_{i_1}^{*,F}(c_F), \ldots, c_{i_N}^{*,F}(c_F))$ *jointly maximize the FMI objective (151).*

As in the case $N = 3$, the FMI equilibrium is the subgame perfect equilibrium of the FMI environment that maximizes the FMI objective function (151). The next subsection describes how to find all subgame perfect equilibria of the FMI environment.

12.2.4 Subgame-perfect Nash equilibria in the FMI environment

Proof. [Theorem 9.1]

The proof is as in the case $N = 3$. The FMI maximizes

$$\begin{aligned}
&\pi_P^F(c_F|c_{i_1}^F(c_F), c_{i_2}^F(c_F), \ldots, c_{i_N}^F(c_F)) \\
&= \sum_{k=1}^{N} \pi_{i_k}^F(c_{i_1}^F, c_{i_2}^F, \ldots, c_{i_N}^F, c_F) && (152) \\
&= \sum_{k=1}^{N} \left[\left(1 - \alpha(c_{i_k})\alpha(c_F) \times \left[1 - \prod_{\substack{j=1, \\ j\neq k}}^{N} (1 - \alpha(c_{i_j})) \right] \right) R_{i_k} - c_{i_k} f(I_{i_k}) \right] - c_F \; f_I && (153)
\end{aligned}$$

subject to the contraint that the private choices $c_{i_1}^F(c_F), c_{i_2}^F(c_F), \ldots, c_{i_N}^F(c_F)$ that follow in $t = 2$ must be optimal given c_F, that is, they are all functions of c_F. When calculating the first-order derivative with respect to c_F, we need to take into account that a change in the subgame c_F causes a change in the behavior of $i_1, \ldots, i_N$, and thus $c_{i_1}^F(c_F), c_{i_2}^F(c_F), \ldots, c_{i_N}^F(c_F)$. Exploiting the symmetry of the problem, we calculate

$$\begin{aligned}
\frac{d}{dc_F}\pi_P^F(c_{i_1}^F, c_{i_2}^F, \ldots, c_{i_N}^F, c_F) &= \frac{\partial}{\partial c_F}\left(\sum_{k=1}^{N} \pi_{i_k}^F(c_{i_1}^F, c_{i_2}^F, \ldots, c_{i_N}^F, c_F) \right) && (154) \\
&+ \sum_{k=1}^{N} \frac{\partial}{\partial c_{i_k}} \pi_{i_k}^F(c_{i_1}^F, c_{i_2}^F, \ldots, c_{i_N}^F, c_F) \frac{\partial c_{i_k}}{\partial c_F} && (155) \\
&+ \sum_{k=1}^{N} \sum_{\substack{m=1 \\ m\neq k}}^{N} \frac{\partial}{\partial c_{i_m}} \pi_{i_k}^F(c_{i_1}^F, c_{i_2}^F, \ldots, c_{i_N}^F, c_F) \frac{\partial c_{i_m}}{\partial c_F} && (156)
\end{aligned}$$

Note, the difference between line (155) and line (156): (155) describes the change in entity i_k's profit due to a change of its own investment choice c_{i_k} in response to a change in c_F, whereas (156) describes the change in entity i_k's profit due to a change of the other entities' investment choices c_{i_m}, $m \neq k$. As in the case $n = 3$, line (155) is always zero, because either the privately optimal investment is at the corner, $c_{i_k}^* = 0$, implying $\frac{\partial c_{i_k}}{\partial c_F} = 0$ or privately optimal investment is interior, $c_{i_k}^* > 0$, implying it must satisfy the first order conditions of a private optimum, $\frac{\partial}{\partial c_{i_k}} \pi_{i_k}^F(c_{i_1}^F, c_{i_2}^F, \ldots, c_{i_N}^F, c_F) = 0$. Thus, the optimization problem simplifies to

$$\frac{d}{dc_F}\pi_P^F(c_{i_1}^F, c_{i_2}^F, \ldots, c_{i_N}^F, c_F) \tag{157}$$

$$= \frac{\partial}{\partial c_F}\left(\sum_{k=1}^{N}\pi_{i_k}^F(c_{i_1}^F, c_{i_2}^F, \ldots, c_{i_N}^F, c_F)\right) + \sum_{k=1}^{N}\sum_{\substack{m=1\\m\neq k}}^{N}\frac{\partial}{\partial c_{i_m}}\pi_{i_k}^F(c_{i_1}^F, c_{i_2}^F, \ldots, c_{i_N}^F, c_F)\frac{\partial c_{i_m}}{\partial c_F} \tag{158}$$

$$= -\alpha'(c_F)\sum_{k=1}^{N}\alpha(c_{i_k}) \times \left[1 - \prod_{\substack{j=1,\\j\neq k}}^{N}(1-\alpha(c_{i_j}))\right] R_{i_k} - \ f_I \tag{159}$$

$$- \alpha(c_F)\sum_{k=1}^{N}\alpha(c_{i_k})\sum_{\substack{m=1\\m\neq k}}^{N}\alpha'(c_{i_m})\frac{\partial c_{i_m}}{\partial c_F}\prod_{\substack{j=1,\\j\neq k, j\neq m}}^{N}(1-\alpha(c_{i_j}))R_{i_k} \tag{160}$$

Case 1: Assume $-\alpha'(0) < \frac{f_I}{R(|I_A|)}$, i.e. assumption 2.2 fails. In this case, $-\alpha'(0) < \frac{1}{\alpha(c_F)}\frac{f_I}{R(|I_A|)}$ for any $c_F \geq 0$. Proposition 9.3 implies that the unique symmetric private equilibrium equals the no investment corner equilibrium $c_{i_j}^F = 0, j = 1, \ldots, N$. With $\alpha(0) = 1$, the FMI optimization thus simplifies to

$$\frac{d}{dc_F}\pi_P^F(0, 0, \ldots, 0, c_F) = -\alpha'(c_F)\sum_{k=1}^{N}\alpha(0) \times \left[1 - \prod_{\substack{j=1,\\j\neq k}}^{N}(1-\alpha(0))\right] R_{i_k} - \ f_I \tag{161}$$

$$- \alpha(c_F)\sum_{k=1}^{N}\alpha(0)\sum_{\substack{m=1\\m\neq k}}^{N}\alpha'(0)\prod_{\substack{j=1,\\j\neq k, j\neq m}}^{N}(1-\alpha(0))R_{i_k} \tag{162}$$

$$= -\alpha'(c_F)N\,R - \ f_I. \tag{163}$$

We know $f/(NR) < f/R$ for all considered $N \geq 3$ and $-\alpha'(0) < f/R$ by assumption in this particular case. If $-\alpha'(0) < f/(NR)$, the convexity of $\alpha(\cdot)$ implies $-\alpha'(0) \geq -\alpha'(c_F)$, thus, there exists no c_F with $-\alpha'(c_F) \geq f/(NR)$. Thus, $c_F^* = 0$ is optimal. If $-\alpha'(0) > f/(NR)$, $c_F^* > 0$ and is characterized as the unique solution to

$$- \alpha'(c_F)N\,R - \ f_I = 0 \tag{164}$$

This c_F exists and is unique because $-\alpha(c) \to 0$ for $c \to \infty$, $\alpha(c)$ is continuous, and strictly convex. Overall, there exists an interior $c_F^* > 0$ and $c_{i_j}^{F,*} = 0$.

Case 2: Assume $-\alpha'(0) > \frac{f_I}{R(|I_A|)}$ so that absent the FMI the banks play the interior sym-

metric private equilibrium $c^*_{i_k} > 0$. Because $\alpha(c) \in (0,1]$, there exists an interior $\bar{c}_F > 0$ such that

$$-\alpha'(0) = \frac{1}{\alpha(\bar{c}_F)} \frac{f_I}{R(|I_A|)}. \tag{165}$$

This holds by continuity of the $\alpha(\cdot)$ function, and since the right hand side of (165) goes to infinity as $c_F \to \infty$, and goes to $\frac{f_I}{R(|I_A|)}$ for $c_F \to 0$ by $\alpha(0) = 1$.

For $c_F < \bar{c}_F$, it holds $-\alpha'(0) > \frac{1}{\alpha(\bar{c}_F)} \frac{f_I}{R(|I_A|)}$ whereas for $c_F > \bar{c}_F$, $-\alpha'(0) < \frac{1}{\alpha(\bar{c}_F)} \frac{f_I}{R(|I_A|)}$.

Case A: Consider an arbitrary subgame with $c_F \geq \bar{c}_F$: By Proposition 6.1, the private symmetric equilibrium following such c_F equals $c^F_{i_j} = 0, j = 1, \ldots, N$. The FMI thus faces the same FOC (161) as in case 1. Moreover, by $-\alpha'(0) > \frac{f_I}{R(|I_A|)} > \frac{f_I}{NR(|I_A|)}$, there always exists a unique interior $c^*_F > 0$ solving the FMI's FOC (164). If this c^*_F satisfies $c^*_F > \bar{c}_F$, we have found a candidate for an FMI equilibrium $(c^*_F, 0)$. If $c^*_F \leq \bar{c}_F$, there exists no FMI equilibrium with $c_F > \bar{c}_F$ since at every $c_F > \bar{c}_F$, the FMI'sprofit function given $c^F_{i_j} = 0, j = 1, \ldots, N$ is downwards sloping in c_F.

Case B: Consider an arbitrary subgame with $c_F < \bar{c}_F$ and we still have $-\alpha'(0) > \frac{f_I}{R(|I_A|)}$: For every such c_F, the private entities optimally invest interior, symmetric amounts $c^{F,*}_{i_j} > 0$. If the FMI invests more within the range $c_F \in (0, \bar{c}_F)$, the private entities optimally respond by shirking symmetrically, $\frac{\partial c_{i_j}}{\partial c_F} < 0$, see Lemma 9.2. The FMI's FOC is given by

$$\frac{d}{dc_F}\pi^F_P(c^F_{i_1}, c^F_{i_2}, \ldots, c^F_{i_N}, c_F) = -\alpha'(c_F) \sum_{k=1}^{N} \alpha(c_{i_k}) \times \left[1 - \prod_{\substack{j=1,\\ j\neq k}}^{N} (1 - \alpha(c_{i_j}))\right] R_{i_k} - f_I \tag{166}$$

$$- \alpha(c_F) \sum_{k=1}^{N} \alpha(c_{i_k}) \sum_{\substack{m=1\\ m\neq k}}^{N} \alpha'(c_{i_m}) \frac{\partial c_{i_m}}{\partial c_F} \prod_{\substack{j=1,\\ j\neq k, j\neq m}}^{N} (1 - \alpha(c_{i_j})) R_{i_k} \tag{167}$$

$$= -N\, \alpha'(c_F)\alpha(c_{i_k}) \times \left[1 - (1 - \alpha(c_{i_k}))^{N-1}\right] R_{i_k} - f_I \tag{168}$$

$$- N(N-1) \frac{\partial c_{i_k}}{\partial c_F}\, \alpha(c_F)\alpha(c_{i_k})\alpha'(c_{i_k})(1 - \alpha(c_{i_k}))^{N-2} R_{i_k} \tag{169}$$

Every choice $c_F \in (0, \bar{c}_F)$ corresponds to an interior private symmetric equilibrium $c^*_{i_k}(c_F) > 0$ played by the private entities. Starting with $c_F = 0 < \bar{c}_F$, consider the private interior symmetric equilibrium, $c^*_{i_k}(0) > 0$. Now, evaluate the FMI's marginal profit at $c_F = 0$ using Lemma

9.2,

$$\frac{d}{dc_F}\pi_P^F(c_{i_1}^F, c_{i_2}^F, \ldots, c_{i_N}^F, c_F) = -N\ \alpha'(0)\alpha(c_{i_k}^*(0)) \times \left[1-(1-\alpha(c_{i_k}^*(0)))^{N-1}\right] R_{i_k} - \ f_I \tag{170}$$

$$- N(N-1)\frac{\partial c_{i_k}^*(c_F)}{\partial c_F}|_{c_F=0}\ \alpha(c_{i_k}^*(0))\alpha'(c_{i_k}^*(0))(1-\alpha(c_{i_k}^*(0)))^{N-2}R_{i_k} \tag{171}$$

If this derivative is (weakly) negative, the pair $c_F^* = 0$ and $c_{i_k}^*(0) > 0$ is a candidate for an FMI equilibrium in the subgame $c_F < \bar{c}_F$. If this derivative is positive, $c_F = 0$ cannot be part of an FMI equilibrium. Regardless of whether the derivative is positive or negative at $c_F = 0$, we need to consider all cases, and increase c_F away from zero towards $\bar{c}_F$. We need to do so because we do not know anything about the monotonicity of $\frac{d}{dc_F}\pi_P^F(c_{i_1}^F, c_{i_2}^F, \ldots, c_{i_N}^F, c_F)$. The increase in c_F will cause a decline in $c_{i_k}^*(c_F)$ towards zero due to shirking, i.e. $\frac{\partial c_{i_k}^*(c_F)}{\partial c_F} < 0$. Then reevaluate the sign of the derivative. There are two possible scenarios. If the derivative remains positive for all $c_F \in [0, \bar{c}_F)$, then no $c_F < \bar{c}_F$ is part of an FMI equilibrium, and the private entities invest zero in the FMI equilibrium $c_{i_k}^* = 0$. An example for this case is given in section 12.4.

If the FMI derivative becomes zero at some $c_{F,d} < \bar{c}_F$, the FMI has reached a point of indifference. Moreover, since $c_{F,d} < \bar{c}_F$, $c_{i_k}^*(c_{F,d}) > 0$. Thus, we have found an interior equilibrium candidate $(c_{F,d}, c_{i_k}^{F,*}(c_{F,d})) > (0,0)$ of the subgame $c_F < \bar{c}_F$. An example for this case is given in section 8.

We have found several candidates for the subgame perfect equilibrium. Plugging all of them back in the FMI objective function (151), and choosing the maximum reveals the FMI equilibrium.

□

12.3 Example I: Solving for equilibria analytically

We now solve explicitly for the equilibria depending on the given attack probability function $\alpha(c) = \frac{1}{\sqrt{\beta c+1}}$, parametrized by beta.

Corollary 12.1 (FMI equilibria of the example)**.** *Consider the attack probability function* $\alpha(c) = \frac{1}{\sqrt{\beta c+1}}$, $R = 2$, $f_I = 1$.
(i) If $\beta \in (0, \frac{2}{3}\frac{f_I}{R})$, $(c_A, c_F) = (0,0)$ *is the unique equilibrium.*
(ii) If $\beta \in (\frac{2}{3}\frac{f_I}{R}, 1)$, *the unique equilibrium is given by* $c_A^* = 0$ *and* $c_F^* = \frac{1}{\beta}\left(\left(\frac{3}{2}\frac{R}{f_I}\beta\right)^{2/3} - 1\right)$.
(iii) Assume $\beta > 1$. *Define* $\bar{c}_F = \frac{\beta^2-1}{\beta}$.

1. *If* $\beta \in (1, \sqrt{\frac{3}{2}\frac{R}{f_I}})$*, one candidate for the FMI equilibrium is given by* $c_F^* = \frac{1}{\beta}\left(\left(\frac{3}{2}\frac{R}{f_I}\beta\right)^{2/3} - 1\right) > \bar{c}_F$ *and* $c_A^* = 0$*. If* $\beta > \sqrt{\frac{3}{2}\frac{R}{f_I}}$*, there cannot exist an equilibrium with* $c_F^* > \bar{c}_F$*. The cut-off* $\bar{c}_F$ *is an upper bound for the FMI investment for large* β*.*

2. *If the privately optimal investment absent the FMI (*$c_F = 0$*) yields* $c_A^*(0) > \bar{c}_F$*, there exists an interior candidate for an equilibrium* $(c_A^*, c_F^*) > (0,0)$ *where* $c_A^* \in (0, \bar{c}_F)$ *and* $c_F^* \in (0, \bar{c}_F)$*. If* $c_A^*(0) << \bar{c}_F$*, a candidate for an equilibrium is given by* $c_F^* = 0$*,* $c_A^*(0) > 0$*. That is, the FMI walks away and the private entities secure themselves.*

We include the solution to the example, i.e., the proof of the Corollary in the main text for expositional purposes. Details of the solution are available in the appendix section 12.3.1.

Case 1: $-\alpha'(0) = \frac{\beta}{2} \in (0, \frac{f_I}{R(|I_i|)}]$. That is, $\beta \in (0,1)$

In this case it follows that $-\alpha'(0) \leq \frac{1}{\alpha(c_F)}\frac{f_I}{R(|I_i|)})$ for all $c_F \geq 0$. This further implies $\beta < \frac{1}{\alpha(c_F)}$, thus $\beta < \sqrt{\beta c_F + 1}$. Then by the instructions above, $c_A^* = 0$, and $\frac{\partial c_A^F}{\partial c_F} = 0$ for any $c_F \geq 0$. Therefore, $\frac{d}{dc_F}\pi_P(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F) = \frac{3}{2}\beta\frac{1}{\sqrt{\beta c_F+1}^3}R - f$ It holds $\frac{d}{dc_F}\pi_P(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F) \geq 0$ if and only if $c_F \in \left(0, \frac{1}{\beta}\left(\left(\frac{3}{2}\frac{R}{f_I}\beta\right)^{2/3} - 1\right)\right]$. Moreover, $\frac{1}{\beta}\left(\left(\frac{3}{2}\frac{R}{f_I}\beta\right)^{2/3} - 1\right) > 0$ if and only if $\beta > \frac{2}{3}\frac{f_I}{R}$. Thus, for $\beta \in (\frac{2}{3}\frac{f_I}{R}, 1)$, $c_F^* = \frac{1}{\beta}\left(\left(\frac{3}{2}\frac{R}{f_I}\beta\right)^{2/3} - 1\right)$ is optimal, and $(c_A, c_F) = \left(0, \frac{1}{\beta}\left(\left(\frac{3}{2}\frac{R}{f_I}\beta\right)^{2/3} - 1\right)\right)$ is an equilibrium. If $\beta \in (0, \frac{2}{3}\frac{f_I}{R})$, then $\frac{d}{dc_F}\pi_P(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F) < 0$ for all $c_F > 0$ so that $c_F^* = 0$ is optimal. $(c_A, c_F) = (0,0)$ is the equilibrium.

Case 2: $-\alpha'(0) = \frac{\beta}{2} > \frac{f_I}{R(|I_i|)}$, $\beta > 1$. In this case we can follow that there exists a cut-off $\bar{c}_F$ such that

$$\frac{\beta}{2} = \frac{1}{\alpha(\bar{c}_F)}\frac{f_I}{R(|I_i|)}) \tag{172}$$

With $\frac{f_I}{R(|I_i|)}) = 1/2$, it holds $\beta < \frac{1}{\alpha(c_F)}$ for $c_F > \bar{c}_F$ and $\beta > \frac{1}{\alpha(c_F)}$ if $c_F < \bar{c}_F$. Plugging in the square root function for $\alpha(c) = \frac{1}{\sqrt{\beta c+1}}$, it holds

$$\bar{c}_F = \frac{\beta^2 - 1}{\beta} \tag{173}$$

Case 2a: Assume $c_F \geq \bar{c}_F$, and thus $\beta \leq \frac{1}{\alpha(c_F)}$. That is, $c_F > \frac{\beta^2-1}{\beta}$.

Then, $c_A^* = 0$ and $\frac{\partial c_A^F}{\partial c_F} = 0$. Therefore, as above, $\frac{d}{dc_F}\pi_P(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F) = \frac{3}{2}\beta\frac{1}{\sqrt{\beta c_F+1}^3}R - f_I$. From above, $\frac{d}{dc_F}\pi_P(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F) \geq 0$ if and only if $c_F \in (0, \frac{1}{\beta}\left(\left(\frac{3}{2}\frac{R}{f_I}\beta\right)^{2/3} - 1\right)]$. Be-

cause $\beta > 1$, and $R/f_I = 2$ we always have $\frac{1}{\beta}\left(\left(\frac{3}{2}\frac{R}{f_I}\beta\right)^{2/3} - 1\right) > 0$. Moreover, $\frac{\beta^2-1}{\beta} < \frac{1}{\beta}\left(\left(\frac{3}{2}\frac{R}{f_I}\beta\right)^{2/3} - 1\right)$ if and only if $\beta < \sqrt{\frac{3}{2}\frac{R}{f_I}}$. Therefore, if $\beta \in (1, \sqrt{\frac{3}{2}\frac{R}{f_I}})$, an equilibrium is given by $c_F^* = \frac{1}{\beta}\left(\left(\frac{3}{2}\frac{R}{f_I}\beta\right)^{2/3} - 1\right) > \frac{\beta^2-1}{\beta}$ and $c_A^* = 0$. If $\beta > \sqrt{\frac{3}{2}\frac{R}{f_I}}$, then for every $c_F > \frac{\beta^2-1}{\beta}$ it holds $\frac{d}{dc_F}\pi_P(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F) < 0$. Thus, there exists no equilibrium with $c_F > \frac{\beta^2-1}{\beta}$. Thus, for large beta, we have derived an upper bound on the FMI investment.

Case 2b (interior case): Fix any positive $c_F < \bar{c}_F = \frac{\beta^2-1}{\beta}$, and thus $\beta > \frac{1}{\alpha(c_F)}$. Then $c_A^* > 0$ and is given as the solution to

$$\frac{\beta}{2}\frac{1}{(\beta c_A + 1)^2}\frac{1}{\sqrt{\beta c_F + 1}}\left[2 - \frac{1}{\sqrt{\beta c_A + 1}}\right] = \frac{1}{2}. \tag{174}$$

Note, the solution $c_A^* > 0$ depends on the choice of c_F. Moreover, as the FMI increases its investment the private entities shirk, $\frac{\partial c_A^F}{\partial c_F} < 0$ where $\frac{\partial c_A^F}{\partial c_F} = -\frac{2}{\beta}\frac{f_I}{R}\frac{(\beta c_A+1)^3}{\sqrt{\beta c_F+1}(8-5\frac{1}{\sqrt{\beta c_A+1}})} < 0$. The FMI's FOC changes to (185), that is,

$$\frac{d}{dc_F}\pi_P(c_A^{*,F}, c_F) = 3f_I\left[\frac{\beta c_A + 1}{\beta c_F + 1}\left(1 - 2\,\frac{1 - \frac{1}{\sqrt{\beta c_A+1}}}{8 - 5\frac{1}{\sqrt{\beta c_A+1}}}\right) - \frac{1}{3}\right] \tag{175}$$

This equation only depends on both c_A and c_F. By $\frac{1}{\sqrt{\beta c+1}} \in (1, \infty]$, it holds

$$\left(1 - 2\,\frac{1 - \frac{1}{\sqrt{\beta c_A+1}}}{8 - 5\frac{1}{\sqrt{\beta c_A+1}}}\right) \geq \frac{1}{3} \tag{176}$$

Moreover, $\frac{1}{\beta c_F+1} > \frac{1}{\beta^2}$. Thus,

$$\frac{d}{dc_F}\pi_P(c_A^{*,F}, c_F) \geq f_I\left[\frac{\beta c_A + 1}{\beta c_F + 1} - 1\right] > f_I\left[\frac{1}{\beta^2}(\beta c_A + 1) - 1\right] \tag{177}$$

We see that $\frac{d}{dc_F}\pi_P(c_A^{*,F}, c_F) > 0$ if the interior private solution c_A^* is large enough, $c_A^* > \frac{\beta^2-1}{\beta} = \bar{c}_F$. To calculate the equilibrium, consider any $c_F \in (0, \bar{c}_F) = (0, \frac{\beta^2-1}{\beta})$. Then $c_A^*(c_F) > 0$ is optimal, given as the solution to the private FOC's.

Case 2bi: If $c_A^*(0) > \frac{\beta^2-1}{\beta}$, then $\frac{d}{dc_F}\pi_P(c_A^*, c_F) > 0$, and the FMI should marginally increase c_F towards $\bar{c}_F$. This increase in c_F will cause a reduction in $c_A^*(c_F) > 0$ down towards $\frac{\beta^2-1}{\beta}$. This results in an interior candidate for an equilibrium $(c_A, c_F) > 0$. To see why, note that $c_A^*(c_F) > 0$ drops down to $\frac{\beta^2-1}{\beta}$ before c_F reaches $\bar{c}_F$. This is because for $c_F = \bar{c}_F$, $c_A^* = 0$ is

optimal, that is, as c_F increases towards $\bar{c}_F$, c_A^* declines from a level above $\frac{\beta^2-1}{\beta}$ to zero, and therefore must cross $\frac{\beta^2-1}{\beta}$. That is, c_F cannot reach $\bar{c}_F$ before $c_A^*(c_F) > 0$ crosses $\frac{\beta^2-1}{\beta}$, then $c_A^* = 0$. Therefore, there exists a level $c_F^* < \bar{c}_F$ at which $c_A^*(c_F^*) \in (0, \frac{\beta^2-1}{\beta})$ such that the FMI attains indifference $\frac{d}{dc_F}\pi_P(c_A^*, c_F) = 0$.

Case 2bii: If for $c_F = 0 < \frac{\beta^2-1}{\beta}$ it holds that $c_A^*(0)$ is sufficiently below $\frac{\beta^2-1}{\beta}$, then $\frac{d}{dc_F}\pi_P(c_A^*, c_F) < 0$, and $c_F^* = 0$, $c_A^*(0) > 0$ is optimal. That is, the FMI abstains from investing.

Overall, the example, and the general setting considered in section 12.2.4 show that the FMI equilibrium can be of three types. There can be an equilibrium where the FMI enters and invests $c_F^* > 0$ but the private entities abstain. An example for such an equilibrium is given in the appendix section 12.4. The second type of an equilibrium is where both the FMI and the private entities invest simultaneously, $c_F^* > 0, c_A^* > 0$. The simulation in the next section shows that the square root attack probability produces such an equilibrium. The third type of equilibrium is where the FMI sets $c_F^* = 0$ and leaves the data security to the private institutions $c_A^* > 0$.

12.3.1 Example I: Details

12.3.2 Unregulated: Private and Social planner equilibrium

Absent the FMI, the unique symmetric private equilibrium c_A^* is given by

- If $(-\alpha'(0)) = \frac{\beta}{2} > f_I/R$, that is for $\beta > 1$, c_A^* is given as the solution to (9), that is,

$$\frac{\beta}{2}\frac{1}{(\beta c_A + 1)^2}\,[2 - \frac{1}{\sqrt{\beta c_A + 1}}] = \frac{1}{2}. \tag{178}$$

- If $(-\alpha'(0)) \leq f_I/R$, $c_A^* = 0$.

The equilibrium condition for the social planner

- Unregulated case (No FMI): The function $h(c) = \alpha(c)/\alpha'(c) = -\frac{2}{\beta}(\beta c + 1)$ is monotonically decreasing. Therefore, the social planner equilibrium is symmetric and interior, given as the unique solution c_A^* to

$$\frac{\beta}{2}\frac{1}{(\beta c_A + 1)^2}\,(4 - 3\frac{1}{\sqrt{\beta c_A + 1}}) = \frac{1}{2} \tag{179}$$

The solution is unique since in this example the function $-g(c) = -\alpha'(c)\alpha(c)(4 - 3\alpha(c))$ is strictly decreasing, see Proposition 4.2. Therefore, $-\alpha'(0) > f_I/R$ if and only if the social planner equilibrium is interior.

12.3.3 Regulated: FMI equilibrium

12.3.4 Case of interior private optimum (stage 2)

- Assume the FMI's investment c_F satisfies, $-\alpha'(0) = \frac{\beta}{2} > \frac{1}{\alpha(c_F)} \frac{f_I}{R(|I_i|)}$. Indeed this is true if and only if $\beta > \sqrt{\beta c_F + 1}$ which holds iff c_F is small, $c_F < \frac{\beta^2 - 1}{\beta}$.
- Then, the privately optimal investment $c_A^F > 0$ is interior by Proposition 6.1, and given as the unique solution to

$$\frac{\beta}{2} \frac{1}{(\beta c_A + 1)^2} \frac{1}{\sqrt{\beta c_F + 1}} \left[2 - \frac{1}{\sqrt{\beta c_A + 1}}\right] = \frac{1}{2}. \tag{180}$$

- Since at the given c_F the private optimum is interior $c_A^* > 0$, the equilibrium change in the privately optimal investment c_A^F due to a marginal change in c_F is given by Lemma 6.2 as

$$\frac{\partial c_A^F}{\partial c_F} = -\frac{2}{\beta} \frac{f_I}{R} \frac{(\beta c_A + 1)^3}{\sqrt{\beta c_F + 1}(8 - 5\frac{1}{\sqrt{\beta c_A + 1}})} < 0 \tag{181}$$

- Because at the given c_F the private optimum is interior $c_A^* > 0$, the first order condition for the optimal FMI choice c_F^* as a function of c_A^* is given by (122) and becomes

$$\frac{d}{dc_F} \pi_P(c_A^{*,F}, c_F) = 3 f_I \left(\frac{\alpha(c_A)(-\alpha'(c_F))}{\alpha(c_F)(-\alpha'(c_A))} - \frac{1}{3} \right) \tag{182}$$

$$- 6\alpha'(c_A)\alpha(c_A)\alpha(c_F) \frac{\partial c_A}{\partial c_F} (1 - \alpha(c_A))\ R(|I_A|) \tag{183}$$

$$= 3 f_I \left(\frac{\beta c_A + 1}{\beta c_F + 1} - \frac{1}{3} \right) - 6 f \frac{\beta c_A + 1}{\beta c_F + 1} \frac{1 - \frac{1}{\sqrt{\beta c_A + 1}}}{8 - 5\frac{1}{\sqrt{\beta c_A + 1}}} \tag{184}$$

$$= 3 f_I \left[\frac{\beta c_A + 1}{\beta c_F + 1} \left(1 - 2 \frac{1 - \frac{1}{\sqrt{\beta c_A + 1}}}{8 - 5\frac{1}{\sqrt{\beta c_A + 1}}} \right) - \frac{1}{3} \right] \tag{185}$$

since $\frac{\alpha'(c_F)/\alpha(c_F)}{\alpha'(c_A)/\alpha(c_A)} = \frac{\beta c_A + 1}{\beta c_F + 1}$. See that this derivative depends on both c_A and c_F.

Corner case of private equilibrium (stage 2)

- Assume the FMI's investment c_F satisfies $-\alpha'(0) = \frac{\beta}{2} < \frac{1}{\alpha(c_F)} \frac{f_I}{R(|I_i|)}$.
- Then, the privately optimal investment is at zero $c_A^F = 0$ by Proposition 6.1. This holds in particular, if $-\alpha'(0) < \frac{f_I}{R(|I_i|)}$.

- If for given c_F the private optimum is at zero $c_A^* = 0$, the equilibrium change in the privately optimal investment c_A^F due to a marginal change in c_F is also zero since all banks shirk and cannot invest less than zero.

$$\frac{\partial c_A^F}{\partial c_F} = 0 \tag{186}$$

- Because the private optimum is at the corner $c_A^* = 0$, the first order condition for the optimal The FMI's choice c_F^* as a function of c_A^* is given by (121). With $\frac{\partial c_A^F}{\partial c_F} = 0$ and $\alpha(0) = 1$ it becomes

$$\begin{aligned}\frac{\partial}{\partial c_F}\pi_P(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F) &= -3\alpha^2(c_A)\alpha'(c_F)\ R(|I_A|)\Big(2-\alpha(c_A)\Big) - f_I \\ &= -3\alpha'(c_F)\ R(|I_A|) - f_I \qquad (187)\\ &= \frac{3}{2}\beta\,\frac{1}{\sqrt{\beta c_F + 1}^3}\ R(|I_A|) - f_I \qquad (188)\end{aligned}$$

Note that this derivative now depends only on c_F.

12.4 Example II

Consider the attack probability function $\alpha(c) = e^{-\beta c}$. This function is positive, decreasing, convex, with $\alpha(0) = 1$, $\alpha(c) \to 0$ as $c \to \infty$, and $\alpha(c) \in [0, 1]$ for all $c \geq 0$. It holds $-\alpha'(0) = \beta$ so that for different $\beta \in (0, \infty)$ we can scale the speed at which the attack probability falls in the data security investment. We set $R = 1,\ f = 0.5$.

12.4.1 Private and Social planner equilibrium: Unregulated case

The unique symmetric private equilibrium c_A^* in the unregulated case is given by

- If $(-\alpha'(0)) = \beta > f_I/R$, c_A^* is given as the solution to (9), that is,

$$\beta e^{-2\beta c_A^*}[2 - e^{-\beta c_A^*}] = \frac{f_I}{R(|I_i|)}. \tag{189}$$

- If $(-\alpha'(0)) = \beta \leq f_I/R$, $c_A^* = 0$.

The equilibrium condition for the social planner becomes

- Double Covers (No Fedwire): If $(-\alpha'(0)) = \beta > f_I/R$, socially optimal investment c_A^* solves

$$\beta\, e^{-2\beta c_A}(4 - 3e^{-\beta c_A}) = \frac{f_I}{R(|I_A|)} \tag{190}$$

As long as $(-\alpha'(0)) = \beta > f/R$, the social planner equilibrium is uniquely determined as the solution to (190) because the function $-g(c) = -\alpha'(c)\alpha(c)(4 - 3\alpha(c))$ crosses the value f/R only once, see Proposition 4.2.[11] For $(-\alpha'(0)) = \beta < f/R$, equation (190) can have two solutions, or no solution. In the case of two solutions, the larger solution is the social optimum. In the case of no solution, the social optimum is at zero. In particular, $(-\alpha'(0)) = \beta < f/R$ is not sufficient for the social optimum to be at zero, unlike in the privately optimal case.

FMI equilibrium

Case of interior private optimum

- Given the FMI's investment c_F satisfies, $-\alpha'(0) = \beta > \frac{1}{\alpha(c_F)} \frac{f_I}{R(|I_i|)}$, the privately optimal investment c_A^F is interior by Proposition 6.1, and given as the solution to

$$\beta\, e^{-2\beta c_A^F}(2 - e^{-\beta c_A^F})e^{-\beta c_F} = f_I/R(|I_A|) \tag{191}$$

- Since at the given c_F the private optimum is interior $c_A^* > 0$, the equilibrium change in the privately optimal investment c_A^F due to a marginal change in c_F is given by Lemma 6.2 as

$$\frac{\partial c_A^F}{\partial c_F} = \frac{-e^{\beta\, c_F}}{\beta\, e^{-2\beta\, c_A^F}(4 - 3e^{-\beta\, c_A^F})} \frac{f}{R} \tag{192}$$

- Because at the given c_F the private optimum is interior $c_A^* > 0$, the first order condition

[11] To see this, note that $g(c) = -\beta e^{-2\beta c}(4-3e^{-\beta c})$. Thus $g'(c) = \beta^2 e^{-2\beta c}(8-9e^{-\beta c})$. The factor $\beta^2 e^{-2\beta c}$ is always positive. The bracket is positive if and only if $c > \frac{1}{\beta} \ln\left(\frac{9}{8}\right)$. Thus, $-g(c)$ decreases if and only if $c > \frac{1}{\beta} \ln\left(\frac{9}{8}\right)$. Because $-g(0) = \alpha'(0) > f/R$ by assumption, and because $-\lim_{c\to\infty} g(c) = 0 < f/R$, the function $-g(c)$ starts at a value above f/R at zero, increases strictly away from that value until it reaches its maximum at $c = \frac{1}{\beta} \ln\left(\frac{9}{8}\right)$, then decreases strictly for all $c > \frac{1}{\beta} \ln\left(\frac{9}{8}\right)$, eventually crossing f/R once, and approaching zero as c gets large.

for the optimal FMI choice c_F^* as a function of c_A^* is given by (122) and becomes

$$\frac{d}{dc_F}\pi_P(c_A^{*,F}, c_F) = 3f\left(\frac{\alpha(c_A)(-\alpha'(c_F))}{\alpha(c_F)(-\alpha'(c_A))} - \frac{1}{3}\right) \tag{193}$$

$$- 6\alpha'(c_A)\alpha(c_A)\alpha(c_F)\frac{\partial c_A}{\partial c_F}(1-\alpha(c_A))\ R(|I_A|) \tag{194}$$

$$= 2f - 6(-\beta)e^{-2\beta c_A}e^{-\beta c_F}(1-e^{-\beta c_A})\ R(|I_A|)\frac{\partial c_A}{\partial c_F} \tag{195}$$

$$= 2f - 6fe^{-2\beta c_A}e^{-\beta c_F}(1-e^{-\beta c_A})\ \frac{\frac{e^{-\beta c_F}}{e^{-2\beta c_F}}}{e^{-2\beta c_A}(2-e^{-\beta c_A}) + 2e^{-2\beta c_A}(1-e^{-\beta c_A})} \tag{196}$$

$$= 2f\left(1 - \frac{(1-e^{-\beta c_A})}{\frac{4}{3}-e^{-\beta c_A})}\right) > 0, \tag{197}$$

since $\frac{(1-e^{-\beta c_A})}{\frac{4}{3}-e^{-\beta c_A})} \in (0,1)$ and since $\alpha'(c)/\alpha(c) = -\beta$, $\frac{\alpha'(c_F)/\alpha(c_F)}{\alpha'(c_A)/\alpha(c_A)} = 1$. Note that this derivative is independent of c_F.

12.4.2 Case of private corner equilibrium

- If the FMI's investment c_F satisfies $-\alpha'(0) = \beta < \frac{1}{\alpha(c_F)}\frac{f_I}{R(|I_i|)}$, the privately optimal investment is at zero $c_A^F = 0$ by Proposition 6.1. This holds in particular, if $-\alpha'(0) = \beta < \frac{f_I}{R(|I_i|)}$.
- If for given c_F the private optimum is at zero $c_A^* = 0$, the equilibrium change in the privately optimal investment c_A^F due to a marginal change in c_F is also zero since all banks shirk and cannot invest less than zero.

$$\frac{\partial c_A^F}{\partial c_F} = 0 \tag{198}$$

- Because the private optimum is at the corner $c_A^* = 0$, the first order condition for the optimal FMI choice c_F^* as a function of c_A^* is given by (121). With $\frac{\partial c_A^F}{\partial c_F} = 0$ and $\alpha(0) = 1$ it becomes

$$\frac{\partial}{\partial c_F}\pi_P(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F)$$

$$= -3\alpha^2(c_A)\alpha'(c_F)\ R(|I_A|)\Big(2-\alpha(c_A)\Big) - f_I \tag{199}$$

$$= -3\alpha'(c_F)\ R(|I_A|) - f_I \tag{200}$$

Note that this derivative now depends on c_F.

12.4.3 Solving the example for different attack probabilities beta

We now solve explicitly for the equilibria depending on the given attack probability function $\alpha(c) = e^{-\beta c}$, parametrized by beta.

Corollary 12.2 (FMI equilibria of the example)**.** *Consider the attack probability function* $\alpha(c) = e^{-\beta c}$.
(i) If $\beta \in \left(0, \frac{f_I}{3R(|I_i|)}\right]$, *the private symmetric equilibrium equals* $c_A^* = 0$ *and the FMI optimally sets* $c_F^* = 0$.
(ii) If $\beta > \frac{f_I}{3R(|I_i|)}$, *the private symmetric equilibrium equals* $c_A^* = 0$ *and the FMI optimally sets* $c_F^* = \left(-\frac{1}{\beta}\right) \ln\left(\frac{f}{3R\beta}\right) > 0$.

We include the solution to the example, i.e., the proof of the Corollary in the main text for expositional purpose.

Case 1: $-\alpha'(0) = \beta \in (0, \frac{f_I}{R(|I_i|)}]$.

In this case it follows that $\beta \leq \frac{1}{\alpha(c_F)} \frac{f_I}{R(|I_i|)})$ for all $c_F \geq 0$.

- Then by the instructions above, $c_A^* = 0$, and $\frac{\partial c_A^F}{\partial c_F} = 0$.
- Therefore, $\frac{\partial}{\partial c_F} \pi_P(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F) = 3\beta e^{-\beta c_F}\ R(|I_A|) - f_I$.

  It holds $\frac{\partial}{\partial c_F} \pi_P(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F) > 0$ if and only if $c_F < \left(-\frac{1}{\beta}\right) \ln\left(\frac{f}{3R\beta}\right)$. If $\beta \in (0, \frac{f}{3R})$, then $\ln\left(\frac{f}{3R\beta}\right) > 0$. Thus, there exists no positive c_F with $c_F < \left(-\frac{1}{\beta}\right) \ln\left(\frac{f}{3R\beta}\right)$, implying $\frac{\partial}{\partial c_F} \pi_P(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F) < 0$ for all $c_F \geq 0$. Thus, $c_F^* = 0$. If $\beta \in (\frac{f}{3R}, \frac{f}{R})$, then $\ln\left(\frac{f}{3R\beta}\right) < 0$, and $c_F^* = \left(-\frac{1}{\beta}\right) \ln\left(\frac{f}{3R\beta}\right) > 0$.
- To verify that we have indeed found an equilibrium, see that for the case $\beta \in (0, \frac{f}{3R})$ with $c_F^* = 0$, respectively for the $\beta \in (\frac{f}{3R}, \frac{f}{R})$ with $c_F^* = \left(-\frac{1}{\beta}\right) \ln\left(\frac{f}{3R\beta}\right)$ it holds $\beta < \frac{1}{\alpha(c_F*)} \frac{f_I}{R(|I_i|)})$, and thus $c_A^* = 0$ is optimal in either case. We conclude, if $\beta \in (0, \frac{f}{3R})$, the equilibrium equals $(c_A^*, c_F^*) = (0, 0)$ whereas in the case $\beta \in (\frac{f}{3R}, \frac{f}{R})$ the equilibrium equals $(c_A^*, c_F^*) = (0, \left(-\frac{1}{\beta}\right) \ln\left(\frac{f}{3R\beta}\right))$.

Case 2: $-\alpha'(0) = \beta > \frac{f_I}{R(|I_i|)}$.

In this case we can follow that there exists a cut-off $\bar{c}_F$ such that

$$\beta = \frac{1}{\alpha(\bar{c}_F)} \frac{f_I}{R(|I_i|)}) \tag{201}$$

Therefore, $\beta < \frac{1}{\alpha(c_F)} \frac{f_I}{R(|I_i|)})$ for $c_F > \bar{c}_F$ and $\beta > \frac{1}{\alpha(c_F)} \frac{f_I}{R(|I_i|)})$ if $c_F < \bar{c}_F$. Plugging in the exponential function for $\alpha(c)$, it holds

$$\bar{c}_F = \frac{1}{\beta} \ln\left(\frac{\beta R}{f_I}\right) \tag{202}$$

Case 2a: Assume $c_F \geq \bar{c}_F$, and thus $\beta \leq \frac{1}{\alpha(c_F)} \frac{f_I}{R(|I_i|)})$.

- Then, $c_A^* = 0$ and $\frac{\partial c_A^F}{\partial c_F} = 0$.
- Therefore, as above, $\frac{\partial}{\partial c_F} \pi_P(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F) = 3\beta e^{-\beta c_F}\ R(|I_A|) - f_I$.

- It holds $\frac{\partial}{\partial c_F} \pi_P(c_A^{*,F}, c_B^{*,F}, c_C^{*,F}, c_F) > 0$ if and only if $c_F < \left(-\frac{1}{\beta}\right) \ln\left(\frac{f_I}{3R\beta}\right)$.

  Because $\beta > f_I/R$, it follows $\ln\left(\frac{f_I}{3R\beta}\right) < \ln\left(\frac{1}{3}\right) < 0$, and $c_F^* = \left(-\frac{1}{\beta}\right) \ln\left(\frac{f_I}{3R\beta}\right) > 0$.

- To verify whether we have indeed found an equilibrium, we need to verify that $c_F^* = \left(-\frac{1}{\beta}\right) \ln\left(\frac{f}{3R\beta}\right) \geq \bar{c}_F$. If that is the case, then indeed, $c_A^* = 0$ is optimal and $(c_A^*, c_F^*) = (0, \left(-\frac{1}{\beta}\right) \ln\left(\frac{f_I}{3R\beta}\right))$ is an equilibrium for $\beta > f_I/R$.

  It holds $c_F^* \geq \bar{c}_F$ if and only if

$$\left(-\frac{1}{\beta}\right) \ln\left(\frac{f_I}{3R\beta}\right) \geq \frac{1}{\beta} \ln\left(\frac{\beta R}{f_I}\right) \tag{203}$$

  which is equivalent to $\ln(3\beta R) \geq \ln(\beta R)$. But this is always true because the logarithm is monotone increasing. Thus, $(c_A^*, c_F^*) = (0, \left(-\frac{1}{\beta}\right) \ln\left(\frac{f_I}{3R\beta}\right))$ is an equilibrium for $\beta > f_I/R$.

Case 2b (interior case): Fix any $c_F < \bar{c}_F$, and thus $\beta > \frac{1}{\alpha(c_F)} \frac{f_I}{R(|I_i|)})$.

- Then $c_A^* > 0$ and is given as the solution to

$$\beta\, e^{-2\beta c_A^F}(2 - e^{-\beta c_A^F}) e^{-\beta c_F} = f_I/R(|I_A|) \tag{204}$$

  Note, the solution $c_A^* > 0$ depends on the choice of c_F.

- Moreover, as the FMI increases its investment the private entities shirk, $\frac{\partial c_A^F}{\partial c_F} < 0$ where

$$\frac{\partial c_A^F}{\partial c_F} = \frac{-e^{\beta\, c_F}}{\beta\, e^{-2\beta\, c_A^F}(4 - 3e^{-\beta\, c_A^F})} \frac{f_I}{R} < 0 \tag{205}$$

- the FMI's FOC changes to (197), that is,

$$\frac{d}{dc_F}\pi_P(c_A^{*,F}, c_F) = 2f_I \left(1 - \frac{(1 - e^{-\beta c_A})}{\frac{4}{3} - e^{-\beta c_A})}\right) > 0 \tag{206}$$

  This equation only depends on c_A and is strictly positive for any $c_A \geq 0$, i.e. cannot become zero or negative.

- To calculate the equilibrium, we plug the private solution $c_A^* > 0$ from (204) into (206), and verify that $\frac{\partial}{\partial c_F}\pi_P(c_A^{*,F}, c_F) > 0$. But this means that $c_F < \bar{c}_F$ is not optimal. We need to marginally increase c_F. As c_F increases towards $\bar{c}_F$, this will cause c_A^* to decline by $\frac{\partial c_A^F}{\partial c_F} < 0$. Eventually c_A^* hits zero as $c_F = \bar{c}_F$. The decline in c_A^* will not impact the sign of $\frac{\partial}{\partial c_F}\pi_P(c_A^{*,F}, c_F) > 0$, thus $c_F = \bar{c}_F$ is optimal. But $c_F = \bar{c}_F$ implies $c_A^* = 0$. Thus, we are back to the case 2a.

Overall, we have shown that there exists no equilibrium with $c_F < \bar{c}_F$ when $\beta > f_I/R$. We have also shown that for any attack probability $\alpha(c) = exp(-\beta c)$, $\beta > 0$ there exists no FMI equilibrium where the private entities invest. There exists no interior private equilibrium with the FMI because the FMI always finds it beneficial to protect, $\frac{\partial}{\partial c_F}\pi_P(c_A^{*,F}, c_F) > 0$, independently of what the private entities do. The private entities respond by shirking down to zero, see Figure 7. . This is related to the particular shape of our attack probability function. Note, however, that in the case $\beta > f_I/R$ the private entities do invest, $c_A^* > 0$, if the FMI is absent (Double Covers case), see Figure 7.

12.4.4 Simulation and discussion: exponential example

In Figure 7, the optimal investment in data security is depicted across varying individual attack probabilities $\alpha(c) = e^{-\beta c}$. As β increases along the x-axis, the rate at which the attack probability diminishes accelerates for marginal investments in data security, see Figure 7. This means that as β attains larger values, investing in data security yields increasingly high "security returns".

Generically, private institutions that operate without the support of the FMI (Double Covers) invest less compared to the socially optimal benchmark. Furthermore, and as anticipated, the private institutions that operate next to the FMI invest less, effectively zero, relative to

the case where the FMI is absent. Recall that the setting with private institutions but without the FMI can be interpreted as the setting with private institutions and the FMI but where the FMI commits to not invest, thus, abstaining from providing the third cover. Consequently, the FMI's strategic decision to provide the third cover, triggers a cascade effect leading all private institutions to curtail their investments to zero.

Furthermore, the graph illustrates an intriguing observation: the social planner may allocate a greater investment in data security compared to the FMI when the parameter β is approximately $f/R = 0.5$. However, for $\beta \geq f/R$, the FMI's investment surpasses that of the social planner. This phenomenon may initially seem counter intuitive; however, it is important to recall that in the symmetric optimum, the social planner distributes data security investments across all three institutions to provide two covers, incurring the cost function for security investment threefold. In contrast, the FMI provides an entire cover for all data via one single investment, thus incurring the cost only once.

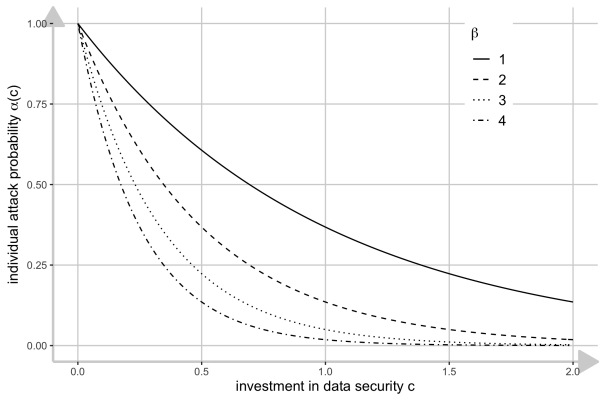
It is worth noting, as demonstrated earlier in the analysis of the Double Covers case, that the social planner in this scenario optimally chooses to provide both covers. Hence, it is not socially optimal to set the investment of one institution to zero and optimize welfare through the remaining two institutions, thereby providing only one cover akin to the FMI.

The relative investments of the FMI versus the social planner hinges on the attack probability, parameterized by β, in relation to the cost function of data security, denoted by $-c\,f$. Depending on this relationship, either the FMI or the social planner may exhibit a greater willingness to invest.

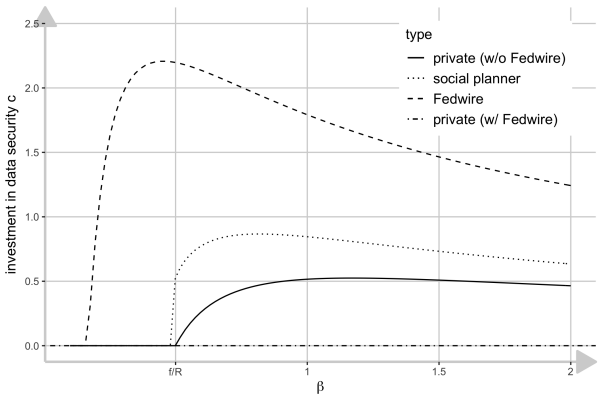
Indeed, a larger investment by the FMI relative to the social planner is further justified by the fact that the entire system with the FMI relies solely on a single cover—the the FMI cover. This is due to the collective decision of all private institutions to abstain from protection, thus leaving the FMI to compensate for the absence of two additional covers by fortifying the single cover extensively. In contrast, the social planner, when investing in security via all three institutions, provides two covers.

To enhance comparability between the FMI's investment and the social planner's investment, Figure 8 plots the total investments per provided cover and Figure 8 plots the total investment per institution. These are reasonable performance comparisons because the FMI invests only once, providing one cover whereas the social planner invests via three institutions to provide two covers. For the FMI, the individual investment, total investment and investment per cover, thus, coincides. But the total cost attributed to the social planner equal $3 \times c_A$, where c_A represents the individual investment whereas the planner's adjustment for investment per cover equals $(3/2) \times c_A$.

At $f/R = 0.5$, a significant shift in the behavior of all institutions is evident in the plot. This transition arises due to the fact that for smaller values of β, both private and socially optimal

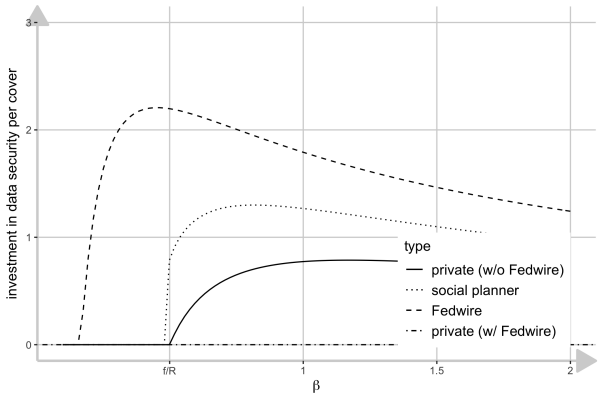


Different attack probabilities $\alpha(c) = exp(-\beta c)$.

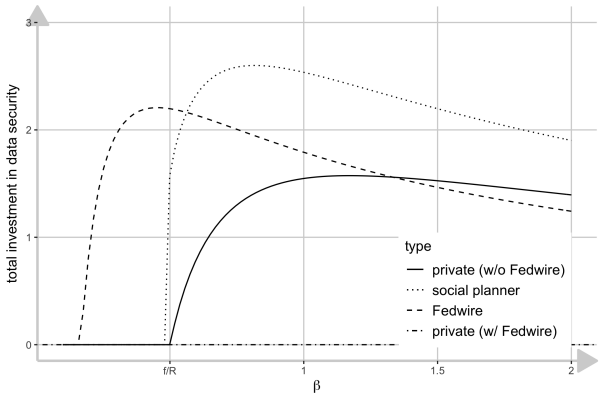


Optimal individual investment in data security.

Figure 7: The larger β, the faster the attack probability $\alpha(c) = exp(-\beta c)$ drops for a marginal investment in security c.



investment per cover



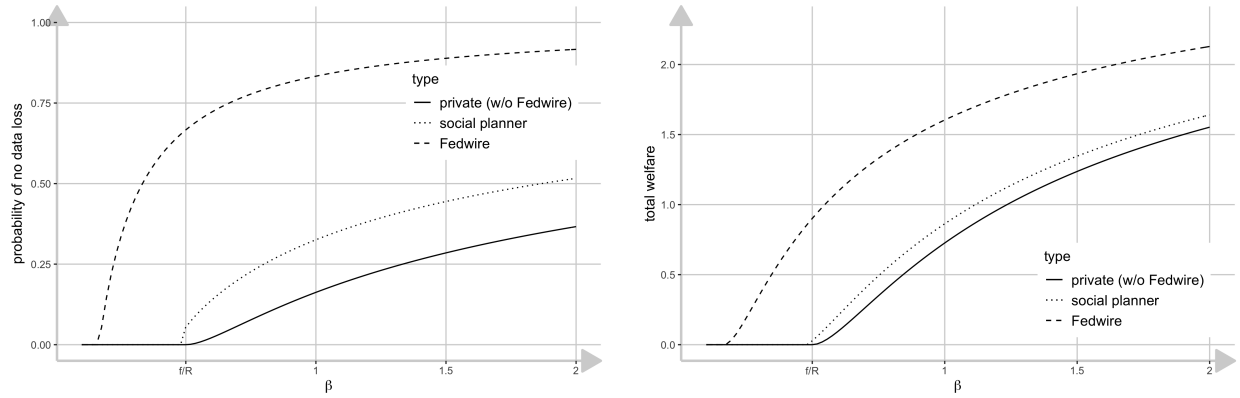
Total Investment

Figure 8: Equilibrium investment per cover: The private institutions, respectively the social planner invests symmetrically via 3 institutions to provide two covers. FMI invests once to provide one cover. Total Investment: The three private institutions jointly invest $3c_A$. Likewise, the social planner invests via three institutions $3c_A$. FMI makes only one large investment c_F.

investments equal zero. This occurs because a smaller β implies that the attack probability diminishes too slow relative to the per-unit costs f associated with data investment and the revenue R generated when data is secure.

Conversely, for β values above f/R, optimal investments in security become strictly positive. As β surpasses the threshold of f/R, the cost-benefit trade-off faced by all institutions undergoes alteration. Specifically, investment in data security becomes increasingly effective in reducing individual attack probabilities, while the cost function for data security remains unchanged. Consequently, as β increasingly exceeds f/R, all institutions systematically re-

duce their investments.



probability of no data loss welfare

Figure 9

Figure 9 illustrates the endogenous levels of data security resulting from optimal investment strategies. In the scenario involving only private institutions, all entities collectively provide two covers, resulting in a level of security quantified by $(1-\alpha(c)\alpha(c)(2-\alpha(c)))$. Similarly, the social planner adheres to the same security function but evaluates it at a higher investment level c, thereby enhancing overall security. As the FMI enters the economy, the overall security function changes to $(1-\alpha(c)\alpha(c_F)(2\alpha(c)-\alpha(c)^2))$, where c_F represents the FMI's investment, and c denotes investment via the remaining institutions. However, due to the collective shirking of all institutions upon the FMI's entry, resulting in their zero investment, the security function for the FMI adapts endogenously to $(1-\alpha(c_F))$ since $\alpha(0)=1$.

Welfare is depicted in Figure 9. Evidently, welfare achieves higher levels when the planner invests in security compared to private institutions. When comparing welfare with the FMI relative to the social planner one needs to be cautious because the welfare functions are different. As mentioned above, it is essential to recognize that the planner invests in security through all three institutions, incurring the cost function threefold. Through its investment strategy, the planner establishes two covers of security. Conversely, the FMI invests solely once, incurring the cost function once, but provides only one cover. This discrepancy stems from the fact that all private entities shirk, thereby abstaining from providing the two additional covers that could be provided. Therefore, while comparing welfare between the FMI and the social planner, it is vital to acknowledge the differing investment and security coverage constraints inherent to their respective strategies.