Transaction Costs and Slow-Moving Capital in Risk-On and Risk-Off Arbitrage Regimes*

Won-chang Choi†

November 26, 2024

Abstract

I investigate asset price dynamics and market efficiency in a competitive economy where risk-averse arbitrageurs face price impact costs and gradually share hedgers' liquidity risks in segmented markets. The asset spread varies with the level of hedgers' liquidity risks and aggregate arbitrage capital. While the spread increases with hedgers' liquidity risks, its relationship with aggregate arbitrage inventory depends on the relative holding-to-trading costs of arbitrageurs and hedgers. This spread-inventory relationship identifies two distinct arbitrage regimes: "Risk-On" and "Risk-Off." In the Risk-On regime, where arbitrageurs with high-risk appetites incur lower holding-to-trading costs, the spread is overcorrected by aggressive arbitrage. In the Risk-Off regime, where arbitrageurs with low-risk appetites face higher holding-to-trading costs, the spread is undercorrected by passive arbitrage. Competitive arbitrage may exceed socially efficient speeds, as arbitrageurs prioritize short-term inventory risk minimization at the expense of market liquidity. A redistribution policy targeting inventory neutrality can incentivize long-term liquidity provision, reducing price gaps and improving market efficiency.

Journal of Economic Literature Classification Numbers: D53, G11, G12

Keywords: Arbitrage, transaction cost, liquidity, market efficiency, excess volatility

*I am deeply grateful to my advisor, Nicolae Gârleanu, for his unwavering support during my time at Washington University in St. Louis. I also thank Darrell Duffie, Hong Liu, Andy Neuhierl, Costas Azariadis, Lasse Pedersen, Valentin Haddad, and Phil Dybvig for their insightful comments. Any remaining errors are my own.

†Department of Economics, Washington University in St. Louis, One Brookings Drive, St. Louis MO 63130. E-mail: won-chang.choi@wustl.edu

1. Introduction

The classical asset pricing theory imposes the no-arbitrage condition, under which assets with identical fundamentals must trade at the same price. This classical theory implicitly assumes that specialized traders such as arbitrageurs can respond instantaneously to trading opportunities. For instance, when asset prices temporarily deviate from their fundamental values due to non-fundamental reasons, arbitrage capital is assumed to flow immediately into markets, exploiting these opportunities and eliminating price deviations with infinite trading pressure. However, empirical evidence shows that arbitrage capital often responds inelastically to arbitrage opportunities and flows slowly into trades, leading to persistent violations of the Law of One Price.[1] To address this empirical challenge, I depart from the traditional assumption that arbitrageurs can deploy an unlimited amount of capital instantly to exploit price discrepancies without incurring price impacts. Instead, I consider "volume-based price effects" as limits to arbitrage, which prevent arbitrageurs from immediately adjusting their trading positions across markets.

I develop a dynamic general equilibrium model within a stylized arbitrage framework, where arbitrageurs with limited trading capacity intermediate the trading needs of hedgers across two segmented markets (e.g., Gromb and Vayanos, 2002; 2018). In this setting, hedgers aim to share liquidity risks stemming from idiosyncratic endowment shocks and demand liquidity (e.g., Lo, Mamaysky, and Wang, 2004; Gârleanu, 2009), while arbitrageurs supply liquidity by taking the opposite side of these trades. To simplify preferences, I assume that all traders have mean-variance preferences over the flow of their wealth net of the risk-free rate. Both arbitrageurs and hedgers face inventory-holding costs and convex trading costs, which constrain the liquidity provision of arbitrageurs and the liquidity demand of hedgers. Therefore, both groups engage in a tug-of-war when executing their order flows and adjusting their asset inventories.

The model is built on a two-way feedback mechanism that endogenizes volume-based price

[1] For example, in fixed-income and merger arbitrage, yield spreads between similar bonds and price discrepancies between target and acquirer firms can persist for extended periods (e.g., Mitchell, Pedersen, and Pulvino, 2007). Additionally, equity markets often experience gradual price adjustments (e.g., Harris and Gurel, 1986; Greenwood, 2005; Coval and Stafford, 2007). More recently, Bogousslavsky, Collin-Dufresne, and Sağlam (2021) have documented slow price reversals at high frequency following a significant trading glitch.

adjustments in a competitive arbitrage equilibrium. Specifically, I consider an economy with infinitely many price-taking arbitrageurs and hedgers, where both groups, subject to convex trading costs, gradually rebalance their positions by optimizing trading speeds and trajectories toward target allocations (e.g., Gârleanu and Pedersen, 2013; 2016). While individual arbitrageurs' order flows do not exert market power, the aggregate flow of all arbitrageurs' combined capital induces volume-based price adjustments during the market-clearing process. Consequently, aggregate arbitrage trades impact asset prices, which, in turn, influence the individual trading behaviors of arbitrageurs, creating a feedback loop that shapes further aggregate arbitrage activity. Therefore, in equilibrium, each arbitrageur needs to account for both their own price impacts and the illiquidity costs that arise from the aggregate trading of all arbitrageurs.

The model is designed to address the following questions: How effectively and quickly do competitive arbitrageurs allocate capital across markets when volume-based price impacts arise endogenously? How does the tug-of-war between arbitrageurs and hedgers shape arbitrage strategies and asset price dynamics in equilibrium? How does the value of competitive arbitrage depend on the trading costs and risk tolerance of arbitrageurs? Finally, can a redistribution policy between arbitrageurs and hedgers improve market efficiency? If so, how do such policy interventions influence the behavior of competitive arbitrageurs and the resulting asset price dynamics?

I find that the asset spread decomposes into price adjustments driven by two factors: the risk premiums that arbitrageurs require for holding risky inventories and sharing hedgers' liquidity risks across periods, and the flow of aggregate arbitrage trading volumes. The spread initially widens to compensate arbitrageurs for bearing inventory risks and managing liquidity risks over subsequent periods. Additionally, as each arbitrageur gradually executes trades to minimize trading costs, their slow-moving aggregate arbitrage capital induces further volume-based spread adjustments (e.g., Chordia, Roll, and Subrahmanyam, 2001). Consistent with empirical evidence (e.g., Chordia and Subrahmanyam, 2004), the equilibrium result establishes a positive relationship between the spread and liquidity risk, as risk-averse arbitrageurs demand compensation for both sharing hedgers' liquidity risks and covering the trading costs of liquidity provision. However, the spread may either increase or decrease with the level of aggregate arbitrage inventory, depending on the relative

holding-to-trading costs of arbitrageurs and hedgers.

This spread-inventory relationship defines two distinct arbitrage regimes: "Risk-On" and "Risk-Off." The Risk-On regime characterizes an arbitrage economy where arbitrageurs with high-risk tolerance incur lower holding-to-trading costs, whereas the Risk-Off regime describes an economy where arbitrageurs with low-risk tolerance face higher holding-to-trading costs.[2] In the Risk-On regime, the spread-inventory relationship is negative because equilibrium asset prices adjust to incentivize hedgers, who have higher holding-to-trading costs, to reallocate inventory, resulting in price movements unfavorable to arbitrageurs. This occurs as arbitrageurs in the Risk-On regime prioritize minimizing trading costs over inventory-holding risks. However, as empirically supported by traditional inventory-based models (e.g., Grossman and Miller, 1988), a positive spread-inventory relationship emerges under the Risk-Off regime. Here, the spread widens with increased aggregate arbitrage inventory to incentivize arbitrageurs with higher holding-to-trading costs to hold risky inventory. Unlike in the Risk-On regime, arbitrageurs in the Risk-Off regime focus on minimizing inventory-holding risks over trading costs.

The equilibrium arbitrage strategies differ significantly between these two regimes because of the distinct spread adjustments arising from the flow of aggregate arbitrage capital. Specifically, the trading speed of competitive arbitrageurs is endogenously adjusted—either accelerated or decelerated—depending on the relationship between the spread and aggregate arbitrage capital. In the Risk-On regime, as more capital flows into arbitrage trades, arbitrageurs anticipate that this influx will compress spreads and reduce expected returns on their positions. Consequently, they accelerate their rebalancing rate in competitive equilibrium and provide liquidity more aggressively by adjusting their positions against the aggregate inventory of other arbitrageurs. Conversely, in the Risk-Off regime, as cumulative capital flows into arbitrage trades increase, arbitrageurs expect widened spreads and higher expected returns. To capitalize on this spread adjustment, they decelerate their trading speed in competitive equilibrium and provide liquidity more passively by aligning their positions with the aggregate arbitrage inventory.

[2]The Risk-On and Risk-Off (RORO) paradigms refer to an investment framework where asset prices are driven by shifts in investors' risk tolerance, influencing their investment strategies.

The impulse response analysis of asset prices to liquidity shocks shows that asset prices persistently deviate from their fundamental value due to the volume-based price impacts of arbitrage capital. The patterns of spread convergence depend on the relative contributions of volume-based price impacts and inventory-holding risk premiums to the spread dynamics. In the Risk-On regime, where arbitrageurs prioritize minimizing price impact costs over inventory-holding risks, competitive arbitrageurs aggressively front-load their trading opportunities, quickly narrowing the pricing gap caused by liquidity shocks. However, this aggressive accumulation of arbitrage capital leads to an excess inventory phase, as arbitrageurs struggle to align their current positions with rapidly changing targets. To liquidate these positions at better prices, arbitrageurs demand compensation from hedgers, inducing a temporary inversion in the asset spread. Following the overshoot, the spread gradually converges to zero as arbitrageurs unwind their inventories. This oscillating convergence pattern aligns with findings in the slow-moving capital literature (e.g., Duffie, 2010; Bogousslavsky et al., 2021). However, the Risk-Off regime does not exhibit such spread inversion. Instead, the spread responds more strongly to liquidity shocks and converges monotonically, driven primarily by the inventory-holding risk premiums required by passive arbitrageurs with low risk tolerance. The analysis further shows that the half-life of the spread is longer in the Risk-Off regime than in the Risk-On regime.

The model generates flow-based return predictability when aggregate arbitrage trading flows impact asset prices (e.g., Lou, 2012). In the absence of volume-based price effects from arbitrage capital flows, asset returns align perfectly with fundamentals and follow a purely random IID process. However, when arbitrage capital flows influence prices, returns exhibit non-zero autocorrelation in response to a liquidity shock. Specifically, in the Risk-On regime, where volume-based price adjustments have a stronger influence on the asset spread, returns transition from short-term reversal to longer-term momentum. Short-term reversals arise as constrained arbitrageurs respond to the mean-reverting liquidity shock by demanding more compensation beyond the asset's fundamental value. Longer-term momentum emerges due to the gradual response of arbitrageurs to the shock. Yet, the Risk-Off regime exhibits no subsequent momentum, only reversal. This difference occurs because, in the Risk-Off regime, the asset spread is driven primarily by risk compensation

for arbitrageurs rather than by their volume-based price impacts.

I also examine how the presence of slow-moving arbitrage capital amplifies price volatility across segmented markets, beyond fluctuations driven by asset fundamentals. The results show that when competitive arbitrageurs bear a relatively larger share of trading costs or inventory-holding risks, price volatility is exacerbated by non-fundamental liquidity shocks. This amplification arises from the lead-and-lag price adjustments caused by the slow movement of arbitrage capital and the risk premiums demanded by risk-averse arbitrageurs for sharing hedgers' liquidity risks. A calibration exercise shows that liquidity shocks can amplify price volatility to more than twice the level of fluctuations driven by asset fundamentals (e.g., Koijen and Gabaix, 2021).

Beyond analyzing asset price dynamics, I also investigate market efficiency in an arbitrage economy with slow-moving capital. To rank the welfare of arbitrage economies, I adopt the approach of Dávila, Graves, and Parlatore (2024), focusing on equilibrium price and trading coefficients, which respectively measure the extent of Law-of-One-Price violations and the level of liquidity provision. As benchmarks, I consider two polar cases: a frictionless arbitrage economy and an economy without arbitrage. The findings reveal that in a normal Risk-On regime, where arbitrageurs have sufficiently high-risk tolerance, welfare criteria fall between these two benchmarks. Additionally, lower trading costs for arbitrageurs increase the value of arbitrage. However, in the Risk-Off regime, where arbitrageurs lack sufficient risk-bearing capacity to hold risky inventory, the welfare order breaks down. In this regime, as arbitrageurs prioritize minimizing short-term inventory-holding risks, the speed of competitive arbitrage exceeds the socially efficient rate. Paradoxically, lower trading costs for arbitrageurs can reduce the value of arbitrage by forcing hedgers to offer wider spreads to incentivize liquidity provision from less constrained but passive arbitrageurs. I further demonstrate that a redistribution policy, which subsidizes arbitrageurs' price impact costs without altering the total wealth flow between arbitrageurs and hedgers, can improve welfare. Specifically, such a policy should aim to minimize the price impact of arbitrage capital and incentivize arbitrageurs to provide long-term liquidity.

Related Literature. This paper contributes to the literature on transaction costs, which has

a substantial body of work on proportional and fixed trading costs.[3] While fixed and proportional trading costs remain relevant, recent research has increasingly focused on price impact costs due to their empirical significance for institutional traders.[4] Gârleanu and Pedersen (2013; 2016) establish the optimal trading strategy for a trader facing price impact costs, and Collin-Dufresne, Daniel, and Sağlam (2020) extend this dynamic portfolio choice framework to regime-shifting environments, where the trader seeks to minimize stochastic price impact costs.[5] I extend this partial equilibrium approach to a general equilibrium arbitrage framework, where each arbitrageur and hedger accounts for not only their negligible individual price impacts but also the price adjustments caused by aggregate arbitrage trading volume. In this general equilibrium setting, the trading speeds and target portfolios of arbitrageurs and hedgers are jointly determined alongside asset price dynamics. I show that arbitrage speeds may accelerate or decelerate as arbitrageurs decide whether to adjust their trading positions against or align with the aggregate arbitrage inventory, based on the relative importance of trading costs and inventory-holding costs for all arbitrageurs.

This paper also contributes to the literature on limits to arbitrage and intermediary asset pricing. Following Shleifer and Vishny (1997), extensive research has focused on the constraints faced by financial intermediaries or arbitrageurs. For instance, Gromb and Vayanos (2002; 2018) and Liu and Longstaff (2004) explore the collateral constraints imposed on arbitrageurs, while Xiong (2001), Kyle and Xiong (2001), Kondor (2009), and Kondor and Vayanos (2019) examine their funding constraints. Additionally, Brunnermeier and Pedersen (2009) and Gârleanu and Pedersen (2011) investigate margin constraints, and He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014) study the impact of equity constraints on financial intermediaries. These studies demonstrate how trading frictions endogenously limit the trading capacity of arbitrageurs, pre-

[3] A large body of literature examines the effect of trading costs on trading strategies; notable examples include Constantinides (1986), Davis and Norman (1990), Duffie and Sun (1990), Dumas and Luciano (1991), Shreve and Soner (1994), Liu and Lowenstein (2002), Liu (2005), Jang et al. (2007), among others. Additionally, another strand of literature focuses on the impact of trading costs on asset prices and liquidity; see, for instance, Amihud and Mendelson (1986), Grossman and Laroque (1990), Vayanos (1998), Lo et al. (2004), Garleanu (2009), and others.

[4] Gârleanu and Pedersen (2022) report that institutional investors account for approximately 50% of the US equity market in 2017, and Edelen, Evans, and Kadlec (2007) find that price-impact costs make up about 65% of the total trading costs for mutual funds, compared to just 17% for proportional transaction costs related to bid-ask spreads.

[5] The optimal execution literature has also examined trading strategies that minimize quantity-driven trading costs (e.g., Perold, 1988; Almgren and Chriss, 1999; Jarrow and Subramanian, 2001). For a comprehensive review of optimal execution, see Obizhaeva and Wang (2013).

venting them from fully exploiting trading opportunities. My contribution to this literature lies in introducing volume-based price impacts of intermediary capital and presenting a novel mechanism where these volume-based illiquidity costs render the competitive market endogenously incomplete.[6] Specifically, the model generates a financially stressed scenario under the Risk-Off paradigm, where arbitrageurs are more likely to excessively liquidate rather than provide liquidity.

Additionally, this paper contributes to the literature on slow-moving capital, which explains persistent illiquidity in asset markets.[7] For example, Duffie, Gârleanu, and Pedersen (2005; 2007) and Duffie and Strulovici (2012) develop search-based models to capture the slow price reversal phenomenon, while Bacchetta and van Wincoop (2010) address the forward discount puzzle through infrequent rebalancing. Chien et al. (2012) highlight excess volatility in economies with infrequent rebalancers, and Bogousslavsky (2016) further explores return autocorrelation within a similar framework. However, in the existing literature, the trading frequency of slow-moving traders is exogenously determined. My paper complements this literature by endogenizing the trading speeds of competitive arbitrageurs. Moreover, the arbitrage process in a Risk-Off economy offers a flow-based explanation for liquidity distress, as arbitrageurs collectively decelerate their trading due to their limited inventory risk-bearing capacity, consistent with the findings of Dow, Han, and Sangiorgi (2021).

This paper also contributes to the literature on market inelasticity and its relationship to investor behavior. Haddad, Huebner, and Loualiche (2024) show that the rise of passive investing reduces competitive market forces, leading to more inelastic markets. While their analysis focuses on the static effects of weakened competition, I develop a dynamic framework in which arbitrage economies are endogenously inelastic as slow-moving arbitrageurs shift between aggressive and passive strategies. This shift depends on the relative importance of inventory-holding costs and trading costs. Specifically, when trading costs dominate as the primary driver of inelasticity (i.e., Risk-On), competitive arbitrageurs adopt an acceleration strategy, which aligns with the behavior

[6] Tuckman and Vila (1992) were among the first to recognize holding costs as factors that limit arbitrage activity in a partial equilibrium framework. My work extends this idea by incorporating price impact costs into a general equilibrium framework, where volume-based price impacts and arbitrage opportunities are endogenously determined.

[7] For an extensive survey, see Duffie (2010).

of inelastic investors described in Haddad et al. (2024). Conversely, when inventory-holding costs become the main constraint (i.e., Risk-Off), arbitrageurs adopt a deceleration strategy, making markets more vulnerable to non-fundamental liquidity shocks.

Lastly, this paper relates to the welfare analysis of constrained arbitrage. Gromb and Vayanos (2002) show that competitive but constrained arbitrageurs generate social value by supplying liquidity to investors, while a social planner can enhance welfare by incentivizing arbitrageurs to take on risks closer to socially optimal levels. Dávila et al. (2024) build on this by showing that pricing gaps or price impacts are sufficient metrics for quantifying the total social value of closing arbitrage gaps. Extending this framework, I assess the value of arbitrage through pricing gaps and arbitrage trading volumes. My work explicitly characterizes the speed of arbitrage, highlighting how excessive rebalancing behaviors in competitive equilibrium—driven by short-term risk minimization—can be tempered to promote long-term liquidity provision. I demonstrate that a redistribution policy can mitigate Law-of-One-Price violations. Furthermore, the findings suggest a new channel for financial stability: effective policies in inelastic markets should aim for inventory neutrality, enabling arbitrageurs to reallocate capital efficiently without incurring volume-based price impacts, thereby restoring market efficiency.

The paper is organized as follows: Section 2 establishes the model. Section 3 formulates and solves the individual trading strategy of each trader. Section 4 analyzes competitive equilibrium under symmetric arbitrage. In Section 5, I extend the equilibrium analysis to include redistribution and market welfare. Finally, I conclude in Section 6.

2. Model

2.1 Assets

All traders can borrow and lend capital at a constant risk-free rate r. Two risky assets, indexed by $j = 1, 2$, are traded in segmented markets with zero net supply. Each asset j generates

identical cash flows D_t, which follow an Ornstein-Uhlenbeck process:

$$dD_t = \kappa_D \left(\bar{D} - D_t\right) dt + \sigma_D dZ_t, \tag{1}$$

where σ_D is the volatility of the dividend, κ_D is its mean-reversion rate, and $\bar{D}$ represents its long-term mean. The term $(Z_t)_{t\geq 0}$ is a standard Brownian motion, representing a fundamental or common cash flow shock. These risky assets can be considered a set of stocks or asset classes with closely related fundamentals. Without loss of generality, I assume that the asset fundamental is perfectly identical and denote it by F_t. The asset fundamental is defined as the present value of expected future cash flows, discounted at the risk-free rate:

$$F_t = E_t \left[\int_t^\infty e^{-r(s-t)} D_s ds\right] = \frac{\bar{D}}{r} + \frac{D_t - \bar{D}}{r + \kappa_D}. \tag{2}$$

Equation (2) implies that the asset attains a long-run stationary fundamental value of $\frac{\bar{D}}{r}$.

An arbitrage opportunity arises from a discrepancy in the prices of two assets with identical cash flows. Arbitrageurs seek profit opportunities derived solely from asset spreads:

$$\Delta_t = P_t^{(1)} - P_t^{(2)}. \tag{3}$$

2.2 Hedgers in Segmented Markets

In each segmented market, there are rational hedgers, indexed by $j \in \{1, 2\}$. The population measure of infinitely many hedgers is normalized to one in both markets. Each hedger in market j has identical mean-variance preferences over the net change in their wealth $dW_{net,s}^{(j)} \equiv dW_s^{(j)} - rW_s^{(j)} ds$ each period:[8]

$$U_t^{(j)} = \int_t^\infty e^{-\rho(s-t)} \left(E_s \left[dW_{net,s}^{(j)}\right] - \frac{\gamma}{2} Var_s \left[dW_{net,s}^{(j)}\right]\right) ds, \tag{4}$$

[8]This mean-variance preference, defined over the *flow* of wealth net of the risk-free rate, avoids the dynamic inconsistency issue raised by Basak and Chabakauri (2010), which arises when preferences are applied to the *stock* of wealth over time.

with the hedger's risk aversion $\gamma > 0$ and the subjective discount factor $\rho > 0$. Each hedger optimizes their portfolio $x_t^{(j)}$ to maximize this lifetime utility in (4).

Liquidity Shocks. Each hedger in segmented markets receives nontradable income through cumulative cash flows $Y_t^{(j)}$:

$$Y_t^{(j)} = \int_0^t \eta_s dZ_s^{(j)}, \tag{5}$$

where $(Z_t^{(j)})_{t\geq 0}$ is a sequence of liquidity shocks to each hedger in market j, and $(\eta_t)_{t\geq 0}$ represents their time-varying volatility. These shocks are constructed to alternate in sign and perfectly correlate with shifts in the asset fundamental, such that $Z_t^{(j)} \equiv (-1)^j Z_t$ for $j = 1, 2$. This implies that when hedgers in one market experience asset supply shocks, hedgers in the other market must experience asset demand shocks, ensuring no aggregate liquidity risk across markets. Additionally, since these liquidity risks are perfectly aligned with the asset fundamental risk, the fundamental risk becomes an essential component of asset demands for hedgers.

To preserve the stationarity of hedgers' liquidity demand in each market, I model the size of liquidity risks η_t as an Ornstein-Uhlenbeck process:

$$d\eta_t = -\kappa_\eta \eta_t dt + \sigma_\eta dZ_t^\eta, \tag{6}$$

where σ_η represents its volatility, κ_η is the mean-reversion rate, and $(Z_t^\eta)_{t\geq 0}$ represents liquidity risk shocks. This transient nature of liquidity risk imposes restrictions on arbitrage opportunities, ensuring that active arbitrage opportunities are short-lived and larger active trading opportunities tend to vanish more quickly (e.g., Tuckman and Vila, 1992).

Transaction Costs. Each hedger incurs the same trading costs. Specifically, trading costs incurred at time t from their trading activities are defined by

$$TC\left(\tau_t^{(j)}\right) = \frac{\lambda}{2}\left(\tau_t^{(j)}\right)^2, \tag{7}$$

where $\tau_t^{(j)} \equiv \frac{dx_t^{(j)}}{dt}$ is the trading intensity of hedger j, which represents how quickly they adjust their

portfolio.[9] The coefficient $\lambda \geq 0$ denotes a transaction cost parameter specific to hedgers.

2.3 Arbitrageurs

In both markets, there are rational arbitrageurs indexed by $a \in A$, with a unit measure of the population of arbitrageurs. They do not receive nontradable income and are not constrained by market segmentation. Their simultaneous presence across markets allows them to exploit superior trading opportunities, effectively positioning them as liquidity providers. Arbitrageurs have mean-variance preferences over the net change in their wealth $dW^{(a)}_{net,t} \equiv dW^{(a)}_s - rW^{(a)}_s ds$:

$$U^{(a)}_t = \int_t^\infty e^{-\rho(s-t)} \left(E_s \left[dW^{(a)}_{net,s} \right] - \frac{\gamma_a}{2} Var_s \left[dW^{(a)}_{net,s} \right] \right) ds \tag{8}$$

with the arbitrageur's risk aversion $\gamma_a > 0$ and the subjective discount factor $\rho > 0$. The arbitrageur optimizes their portfolio $\boldsymbol{x}^{(a)}_t \equiv \left(x^{(a)}_{1t}, x^{(a)}_{2t} \right)$ to maximize their lifetime utility (8).

Transaction Costs. Arbitrageurs also face covex transaction costs in executing trades as follows:

$$TC^{(a)} \left(\boldsymbol{\tau}^{(a)}_t \right) = \frac{\lambda_a}{2} \boldsymbol{\tau}^{(a)\prime}_t \boldsymbol{\tau}^{(a)}_t \tag{9}$$

where $\boldsymbol{\tau}^{(a)}_t \equiv \left(\frac{dx^{(a)}_{1t}}{dt}, \frac{dx^{(a)}_{2t}}{dt} \right)$ is the arbitrageur's trading intensity, which reflects the rate at which arbitrage capital of each individual arbitrageur is deployed. The parameter $\lambda_a \geq 0$ governs the cost of trading, effectively placing a limit on the arbitrage activity.[10]

2.4 Market Clearing in Segmented Markets

The aggregate trading activities of hedgers and arbitrageurs across segmented markets determine competitive market prices for assets. To distinguish the aggregate position of hedgers and arbitrageurs from individual traders' positions, I denote the aggregate demand of hedgers and ar-

[9] Throughout this paper, I assume that each hedger's trading policies: i) adapt to the filtration generated by their information; ii) satisfy absolute continuity; and iii) are square-integrable in the L^2 sense.

[10] Similarly to hedgers, each arbitrageur's trading policies are assumed to: i) adapt to the filtration generated by their information; ii) satisfy absolute continuity; and iii) are square-integrable in the L^2 sense.

bitrageurs as $X_t^{(j)} = \int_{i_j} x_t^{(j)}(i_j)di_j$ and $\boldsymbol{X}_t^{(a)} = \int_{i_a} \boldsymbol{x}_t^{(a)}(i_a)di_a$, respectively, with auxiliary indices i_j and i_a. The market clearing condition for each market $j = 1, 2$ is then written as:

$$X_t^{(j)} = -X_{jt}^{(a)}. \tag{10}$$

The market clearing condition (10) must hold at all times. This implies that if there is an excess demand for assets by hedgers in market j of $X_t^{(j)}$, arbitrageurs will step in and supply assets of $-X_{jt}^{(a)}$ to resolve the temporary excess demand.

3. Optimal Trading Strategies of Individual Traders

To formulate individual optimization problems within a Markovian framework, we first need to identify the state variables that govern each trader's investment opportunities. Specifically, I focus on the competitive equilibrium, where the liquidity demand of hedgers in one market is exactly offset by an equivalent but opposite demand in another market. Consequently, each arbitrageur maintains *symmetric* positions across segmented markets, represented by $x_{jt}^{(a)} = (-1)^j x_t^{(a)}$, where $x_t^{(a)} \geq 0$ denotes the size of their own capital. This symmetric market structure simplifies the state space by reducing its dimension, as the aggregate state of arbitrage requires information from only one of the two markets. Thus, I define the state vector as $\boldsymbol{\Omega}_t \equiv \left(X_t^{(a)}, \eta_t\right)$ where $X_t^{(a)}$ represents aggregate arbitrage inventory or arbitrage capital and η_t is the liquidity risk of hedgers.

I look for the competitive equilibrium where the price functions take the form:

$$P_t^{(j)} = F_t - \left(p_X^{(j)} X_t^{(a)} + p_\eta^{(j)} \eta_t\right), \tag{11}$$

where $p_X^{(j)} \equiv (-1)^j p_X$ and $p_\eta^{(j)} \equiv (-1)^j p_\eta$ are constant price coefficients determined in equilibrium. I further postulate the law of motions of aggregate inventories for arbitrageurs and hedgers:

$$\frac{dX_t^{(a)}}{dt} = q_\eta^{(a)} \eta_t - q_X^{(a)} X_t^{(a)} \quad \text{and} \quad \frac{dX_t^{(j)}}{dt} = q_\eta^{(j)} \eta_t - q_X X_t^{(j)}, \tag{12}$$

where $q_\eta^{(a)}$ and $q_X^{(a)}$ are trading coefficients of a representative arbitrageur, and $q_\eta^{(j)} \equiv (-1)^j q_\eta$ and

q_X are the trading coefficients of representative hedgers determined in equilibrium.

3.1 Investment Opportunities

Investment opportunities for hedgers and arbitrageurs in each market are defined by an instantaneous excess return on a zero-wealth portfolio, $dQ_t^{(j)} \equiv dP_t^{(j)} + D_t dt - rP_t^{(j)}dt$. Given the price function in (11), this excess return follows stochastic processes:

$$dQ_t^{(j)} = \left(\mu_\eta^{(j)}\eta_t + \mu_X^{(j)}X_t^{(a)}\right)dt + \boldsymbol{\sigma}^{(j)\prime}\boldsymbol{dZ_t^Q}, \tag{13}$$

where $\mu_\eta^{(j)} \equiv (r+\kappa_\eta)p_\eta^{(j)} - q_X^{(a)}p_X^{(j)}$ and $\mu_X^{(j)} \equiv \left(r + q_X^{(a)}\right)p_X^{(j)}$ are factor loadings for the investment factors: η_t and $X_t^{(a)}$. The return volatility vector is defined as $\boldsymbol{\sigma}^{(j)} \equiv \left[\sigma_F, -p_\eta^{(j)}\sigma_\eta\right]$, with the fundamental volatility $\sigma_F \equiv \frac{\sigma_D}{r+\kappa_D}$ and the vector of shocks to excess returns $\boldsymbol{dZ_t^Q} \equiv [dZ_t, dZ_t^\eta]$.

3.1. Hedger's Optimization

Each trader must account for their inventory value when formulating optimal trading strategies (e.g., Constantinides, 1986). Hence, an individual hedger in market j solves an optimization problem with the state space $\boldsymbol{\Omega}_t^{(j)} \equiv \left(x_t^{(j)}, \boldsymbol{\Omega}_t\right)$. The budget constraint of the hedger is:

$$dW_{net,t}^{(j)} = x_t^{(j)}dQ_t^{(j)} - \frac{\lambda}{2}\left(\tau_t^{(j)}\right)^2 dt + dY_t^{(j)}. \tag{14}$$

The hedger aims to maximize their expected utility (4), given the budget constraint (14), by selecting an admissible trading intensity $\tau_t^{(j)}$. By substituting the budget constraint (14) into the expected utility (4), the optimization problem can be re-expressed as:

$$\begin{aligned} V^{(j)}\left(x_t^{(j)}; \boldsymbol{\Omega}_t\right) = \max_{\left(\tau_s^{(j)}\right)_{s\geq t}} E_t \int_t^\infty e^{-\rho(s-t)} \Bigg[& \left(\mu_\eta^{(j)}\eta_s + \mu_X^{(j)}X_s^{(a)}\right)x_s^{(j)} \\ & - \frac{\gamma}{2}\Sigma^Q\left(x_s^{(j)} + (-1)^j\frac{\sigma_F}{\Sigma^Q}\eta_s\right)^2 - \frac{\gamma}{2}\frac{Var_t[dY_s^{(j)}]}{\Sigma^Q}p_\eta^2\sigma_\eta^2 - \frac{\lambda}{2}\left(\tau_s^{(j)}\right)^2\Bigg] ds, \end{aligned} \tag{15}$$

where I use the symmetric property of return volatilities $\Sigma^Q \equiv \boldsymbol{\sigma}^{(j)\prime}\boldsymbol{\sigma}^{(j)}$. The first term represents

the expected return from the current portfolio $x_t^{(j)}$, the second and third terms penalize risk originating from the modified portfolio and unhedgeable liquidity risks respectively, while the last term penalizes transaction costs.

For each hedger in market j, I conjecture their value functions in the quadratic form

$$V^{(j)}\left(x_t^{(j)};\mathbf{\Omega}_t\right)=\frac{1}{2}\mathbf{\Omega}_t^{(j)\prime}\boldsymbol{V}_{\mathbf{\Omega}\mathbf{\Omega}}{}^{(j)}\mathbf{\Omega}_t^{(j)}+v_0 \tag{16}$$

with a symmetric 3×3 matrix of value function coefficients $\boldsymbol{V}_{\mathbf{\Omega}\mathbf{\Omega}}{}^{(j)}$.

Proposition 1. ***(Hedger's Trading Strategy)*** *Each hedger in market j's value function (16) satisfies the HJB equation if $\left(v_{xx}^{(j)},v_{xX}^{(j)},v_{x\eta}^{(j)},v_{XX}^{(j)},v_{X\eta}^{(j)},v_{\eta\eta}^{(j)},v_0\right)$ satisfy a system of seven scalar equations for each hedger. The individual hedger's trading intensity is:*

$$\tau\left(x_t^{(j)};\mathbf{\Omega}_t\right)=M_{rate}\left(M_{aim}^{(j)}\left(\mathbf{\Omega}_t\right)-x_t^{(j)}\right) \tag{17}$$

with the optimal trading speed $M_{rate}=\sqrt{\frac{\gamma\Sigma^Q}{\lambda}+\frac{\rho^2}{4}}-\frac{\rho}{2}$ and aim portfolio

$$M_{aim}^{(j)}\left(\mathbf{\Omega}_t\right)=\frac{\mu_\eta^{(j)}\eta_t-(-1)^j\gamma\sigma_F\eta_t}{\gamma\Sigma^Q+\lambda M_{rate}\kappa_\eta}+\frac{\mu_X^{(j)}X_t^{(a)}}{\gamma\Sigma^Q+\lambda M_{rate}q_X^{(a)}}+v_{xX}^{(j)}\frac{q_\eta^{(a)}\eta_t}{\gamma\Sigma^Q+\lambda M_{rate}\kappa_\eta}. \tag{18}$$

The Feynman-Kac representation of the aim portfolio follows

$$M_{aim}^{(j)}\left(\mathbf{\Omega}_t\right)=K\int_t^\infty e^{-(s-t)(\rho+M_{rate})}E_t\left(Markowitz_s^{(j)}\right), \tag{19}$$

where $Markowitz_t^{(j)}$ represents their Markowitz portfolios defined by

$$Markowitz_t^{(j)}=\underbrace{\frac{\left(\mu_\eta^{(j)}+v_{xX}^{(j)}q_\eta^{(a)}-(-1)^j\gamma\sigma_F\right)\eta_t+\mu_X^{(j)}X_t^{(a)}}{\gamma\Sigma^Q}}_{\textit{liquidity portfolio}}. \tag{20}$$

Proof. See Appendix A. □

Proposition 1 describes an optimal trading strategy for each hedger in market j. As shown in

Garleanu and Pedersen (2016), each hedger with convex trading costs implements an aim trading strategy: continuously tracking an aim portfolio $M_{aim}^{(j)}(\mathbf{\Omega}_t)$ at a constant trading speed M_{rate}. Their target portfolios have exposures to the two investment factors $\mathbf{\Omega}_t$. In particular, each hedger forms a feedback portfolio, represented by the term in (18) proportional to $v_{xX}^{(j)}$. This feedback portfolio adjusts for anticipated price impacts from trading by *other* hedgers. The Feynman-Kac formula expresses their aim portfolios as a weighted average of their static Markowitz portfolios, $Markowitz_t^{(j)}$, over subsequent periods. As each hedger needs to hedge their liquidity risks, this Markowitz portfolio can be considered a liquidity portfolio.

3.2 Arbitrageur's Optimization

An arbitrageur's optimization problem is formulated based on their own state space $\mathbf{\Omega}_t^{(a)} \equiv \left(x_t^{(a)}, \mathbf{\Omega}_t\right)$. In the absence of an income stream, the arbitrageur's budget constraint becomes:

$$dW_{net,t}^{(a)} = \boldsymbol{x}_t^{(a)\prime} \boldsymbol{dQ_t} - \frac{\lambda_a}{2} \boldsymbol{\tau}_t^{(a)\prime} \boldsymbol{\tau}_t^{(a)} dt, \tag{21}$$

where $\boldsymbol{dQ_t} \equiv \left(dQ_t^{(1)}, dQ_t^{(2)}\right)$. The market symmetry simplifies the budget constraint to:

$$dW_{net,t}^{(a)} = x_t^{(a)} dQ_t^{\Delta-} - \lambda_a \left(\tau_t^{(a)}\right)^2 dt, \tag{22}$$

where $dQ_t^{\Delta-} \equiv dQ_t^{(2)} - dQ_t^{(1)} = r\Delta_t dt - d\Delta_t$. The budget constraint of arbitrageurs differs from that of hedgers due to their unique investment opportunities and the trading costs involved in executing arbitrage. Specifically, in symmetric markets, arbitrageurs can profit from shorting the price difference between markets 1 and 2 ($-d\Delta_t$) and earning a risk-free return on the cash held as collateral ($r\Delta_t dt$). Additionally, arbitrageurs incur transaction costs *twice* for changing one unit of their inventory due to the cross nature of arbitrage transactions.

The arbitrageur's optimization problem is to choose an admissible symmetric trading intensity $\tau_t^{(a)}$ to maximize the expected utility (8) subject to the budget constraint (22). By plugging the arbitrageur's budget constraint into their expected utility, the arbitrageur's optimization problem

can be restated as follows:

$$
\begin{aligned}
&V^{(a)}\left(x_t^{(a)};\mathbf{\Omega}_t\right) \\
&= \max_{\left(\tau_s^{(a)}\right)_{s\geq t}} E_t \int_t^{\infty} e^{-\rho(s-t)} \left[\left(\mu_\eta^{\Delta-}\eta_s + \mu_X^{\Delta-}X_s^{(a)}\right)x_s^{(a)} - \frac{\gamma_a}{2}\Sigma_\Delta^Q\left(x_s^{(a)}\right)^2 - \lambda_a\left(\tau_s^{(a)}\right)^2\right] ds,
\end{aligned}
\tag{23}
$$

where $\mu_\eta^{\Delta-} \equiv \mu_\eta^{(2)} - \mu_\eta^{(1)}$, $\mu_X^{\Delta-} \equiv \mu_X^{(2)} - \mu_X^{(1)}$, and $\Sigma_\Delta^Q \equiv \boldsymbol{\sigma}^{\boldsymbol{\Delta}-\prime}\boldsymbol{\sigma}^{\boldsymbol{\Delta}-} = 4p_\eta^2\sigma_\eta^2$ with $\boldsymbol{\sigma}^{\Delta-} \equiv \boldsymbol{\sigma}^{(2)} - \boldsymbol{\sigma}^{(1)} = [0, -2p_\eta\sigma_\eta]$. The first term represents expected returns on arbitrage portfolios, the second term is a penalty for arbitrage risks, and the third term is a penalty for trading costs. The trade-offs between trading benefits and penalties depend on arbitrage risk arising from unpredictable price changes due to uncertain hedgers' liquidity demands. Under *risky arbitrage* ($\sigma_\eta \neq 0$), an arbitrageur must weigh the return-risk trade-off and the return-cost trade-off. Conversely, an arbitrageur engaged in *riskless arbitrage* ($\sigma_\eta = 0$) only considers the trade-off between returns and transaction costs.

For each arbitrageur, I conjecture their value functions in the quadratic form

$$
V^{(a)}\left(x_t^{(a)};\mathbf{\Omega}_t\right) = \mathbf{\Omega}_t^{(a)\prime}\boldsymbol{V}_{\mathbf{\Omega}\mathbf{\Omega}}{}^{(a)}\mathbf{\Omega}_t^{(a)} + v_0^{(a)} \tag{24}
$$

with a symmetric 3×3 matrix for the value function coefficients $\boldsymbol{V}_{\mathbf{\Omega}\mathbf{\Omega}}{}^{(a)}$.

Proposition 2. ***(Arbitrageur's Trading Strategy)*** *Each arbitrageur's value function (24) satisfies the HJB equation if* $\left(v_{xx}^{(a)}, v_{xX}^{(a)}, v_{x\eta}^{(a)}, v_{XX}^{(a)}, v_{X\eta}^{(a)}, v_{\eta\eta}^{(a)}, v_0^{(a)}\right)$ *satisfy a system of seven scalar equations. The individual arbitrageur's trading intensity is*

$$
\tau\left(x_t^{(a)};\mathbf{\Omega}_t\right) = \begin{cases} M_{rate}^{(a)}\left(M_{aim}^{(a)}\left(\mathbf{\Omega}_t\right) - x_t^{(a)}\right), & \sigma_\eta > 0 \\ \dfrac{\Delta_t}{2\lambda_a}, & \sigma_\eta = 0 \end{cases} \tag{25}
$$

where the optimal trading speed $M_{rate}^{(a)} \equiv \sqrt{\frac{\gamma_a\Sigma_\Delta^Q}{2\lambda_a} + \frac{\rho^2}{4}} - \frac{\rho}{2}$ *and aim portfolio*

$$
M_{aim}^{(a)}(\mathbf{\Omega}_t) = \frac{\mu_\eta^{\Delta-}\eta_t}{\gamma_a\Sigma_\Delta^Q + 2\lambda_a M_{rate}^{(a)}\kappa_\eta} + \frac{\mu_X^{\Delta-}X_t^{(a)}}{\gamma_a\Sigma_\Delta^Q + 2\lambda_a M_{rate}^{(a)}q_X^{(a)}} + 2v_{xX}^{(a)}\frac{q_\eta^{(a)}\eta_t}{\gamma_a\Sigma_\Delta^Q + 2\lambda_a M_{rate}^{(a)}\kappa_\eta}. \tag{26}
$$

The Feynman-Kac representation of the aim portfolio follows

$$M_{aim}^{(a)}\left(\mathbf{\Omega}_t\right) = K^{(a)} \int_t^{\infty} e^{-(s-t)(\rho + M_{rate})} E_t \left(Markowitz_s^{(j)} \right), \tag{27}$$

where $Markowitz_t^{(a)}$ *represents their Markowitz portfolios defined by*

$$Markowitz_t^{(a)} = \frac{\left(\mu_\eta^{\Delta^-} + 2v_{xX}^{(a)} q_\eta^{(a)}\right)\eta_t + \mu_X^{\Delta^-} X_t^{(a)}}{\gamma_a \Sigma_\Delta^Q}. \tag{28}$$

Proof. See Appendix A. □

Proposition 2 illustrates that, similar to hedgers, each arbitrageur develops aim trading strategies for a given state of $\mathbf{\Omega}_t$. These strategies, however, diverge based on the existence of arbitrage risk. In the presence of such risk, risk-averse arbitrageurs adopt an aim trading strategy by aligning with an aim portfolio $M_{aim}^{(a)}(\mathbf{\Omega}_t)$ at a constant rebalancing rate $M_{rate}^{(a)}$. They also hold a feedback portfolio proportional to $v_{xX}^{(a)}$ to adjust for anticipated price impacts associated with the arbitrage activities of *other* arbitrageurs. The Feynman-Kac formula shows that, in the presence of arbitrage risk, each arbitrageur facing trading costs implements a *dynamic* spread trading strategy, holding a weighted average of all future Markowitz portfolios. This approach allows them to gradually harvest current and anticipated future mispricings rather than exploiting them immediately. In the absence of arbitrage risk, however, arbitrageurs adopt a *static* spread trading strategy, ignoring their current capital $x_t^{(a)}$ and maintaining only the current asset spread Δ_t, regardless of the trajectories of future spreads.

4. Arbitrage Equilibrium in Competitive Markets

This section examines the arbitrage economy within a competitive market framework. The equilibrium analysis assumes a symmetric competitive equilibrium, where individual arbitrageurs cannot influence prices directly but collectively contribute to volume-based price effects in aggregate. In this equilibrium, all traders—both hedgers and arbitrageurs—act as price takers, solve their respective optimization problems, and adopt identical optimal trading strategies within their

groups.

Definition. ***(Competitive Equilibrium)*** *Let* $\mathbf{\Omega}_t \equiv \left(X_t^{(a)}, \eta_t\right)$ *be an aggregate state vector. A symmetric competitive market equilibrium consists of stochastic processes: asset prices* $P_t^{(j)}$*, trading policies of hedgers* $\tau^{(j)}\left(x_t^{(j)}; \mathbf{\Omega}_t\right)$ *for* $j = 1, 2,$*, and trading policies of arbitrageurs* $\tau^{(a)}\left(x_t^{(a)}; \mathbf{\Omega}_t\right)$*, satisfying the following conditions: for each segmented market* $j = 1, 2$*,*

1. ***Individual optimization.*** *Each hedger and arbitrageur solve their optimization problems by taking the price* P_t *as given.*

2. ***Consistency.*** *Each trader must adopt an identical trading strategy within their group:*

$$x_t^{(j)} = X_t^{(j)} \qquad \textit{and} \qquad \frac{dX_t^{(j)}}{dt}\left(\mathbf{\Omega}_t\right) = \tau^{(j)}\left(x_t^{(j)}; \mathbf{\Omega}_t\right)\Big|_{x_t^{(j)} = X_t^{(j)}}, \tag{29}$$

$$x_t^{(a)} = X_t^{(a)} \qquad \textit{and} \qquad \frac{dX_t^{(a)}}{dt}\left(\mathbf{\Omega}_t\right) = \tau^{(a)}\left(x_t^{(a)}; \mathbf{\Omega}_t\right)\Big|_{x_t^{(a)} = X_t^{(a)}}. \tag{30}$$

3. ***Market clearing.*** *Each price* $P_t^{(j)}$ *adjusts to satisfy the market-clearing condition (10).*

4.1 Model Parameters

For numerical analysis, I select reference values for the model parameters. The absolute risk aversion for hedgers is set to $\gamma = 10^{-9}$, representing individuals managing a \$1 billion portfolio with a relative risk aversion coefficient of 1. In contrast, the absolute risk aversion coefficient for arbitrageurs, γ_a, is chosen within the range from 0 to 10×10^{-9}. This range implies that if the arbitrageur's relative risk aversion coefficient is 1, their managed wealth varies from \$0.1 billion ($\gamma_a = 10^{-8}$) to infinity ($\gamma_a = 0$). For the transaction cost parameters λ and λ_a, I consider a broad spectrum from 0 to 10×10^{-9} to capture different scenarios reported in the literature. For instance, Engle et al. (2008) estimate that trades equivalent to 1.59% of a stock's daily volume cause a 10 basis point price impact. Based on this estimate, Garleanu and Pedersen (2013) derive a cost parameter λ using the formula: $\$1 \times 0.001 = \frac{\lambda}{2} \times \$10^9 \times 0.0159$, which results in $\lambda \approx 1.3 \times 10^{-10}$. This assumes that investors' participation rates are proportional to their assets under management (AUM), specifically 1.59% for a \$1 billion portfolio. Additionally, Collin-Dufresne et al. (2020)

report similar estimates, ranging from 2×10^{-10} to 3×10^{-10} across different liquidity regimes.

Moreover, both the risk-free rate r and the traders' subjective discount rate ρ are fixed at 5%, aligning with historical averages of short-term U.S. Treasury bills. The dividend volatility σ_D is set at 0.25, and the mean-reversion speed κ_D is set to 1. The long-term dividend rate $\bar{D}$ is 0.05, resulting in a steady-state price level $\bar{P} = \$1$ and a dividend yield $\frac{\bar{D}}{\bar{P}} = 0.05$. This parameterization implies that the asset return premium normalizes to zero at the steady state, i.e., $\frac{\bar{D}}{\bar{P}} - r = 0$. For liquidity risks, the mean-reversion rate κ_η is set at 0.25 based on Campbell et al. (1993), indicating a half-life of 2.78 trading days, with its volatility σ_η set at 0.25. I consider these parameter values to capture the liquidity effects of trading costs and will adjust them to investigate various market conditions.

Parameter	γ	γ_a	$\{\lambda, \lambda_a\}$	r	ρ	κ_D	σ_D	$\bar{D}$	κ_η	σ_η
Value	10^{-9}	$[0, 10]\times10^{-9}$	$[0, 10]\times10^{-9}$	0.05	0.05	1	0.25	0.05	0.25	0.25

Table 1: Parameterization.

Assumption 1. $(\rho = r)$ *All traders have no trading incentive due to the rate differential* $(\rho - r)$.[11]

4.2 Benchmark under Unconstrained Arbitrage

Proposition 3. ***(Unconstrained Arbitrage)*** *For* $\lambda_a = 0$*, there exists a competitive equilibrium such that:*

1. *The law of one price holds across markets:*

$$P_t^{*(j)} = F_t \qquad and \qquad dQ_t^{*(j)} = \sigma_F dZ_t. \tag{31}$$

[11] By Assumption 1, I concentrate on transaction costs as the primary driver of intertemporal trading by arbitrageurs and hedgers, thus isolating the impact of rate differentials $(\rho - r)$. This allows for a cleaner examination of trading behaviors under transaction costs and their asset pricing implications.

2. The law of motion for aggregate arbitrage capital in each market follows

$$dX_t^{*(a)} = \begin{cases} q_X^{*(a)} \left(M_{aim,\eta}^{*(a)} \eta_t - X_t^{(a)} \right) dt, & \lambda > 0 \\ \dfrac{1}{\sigma_F} d\eta_t, & \lambda = 0 \end{cases} \tag{32}$$

where $q_X^{*(a)} \equiv \sqrt{\frac{\gamma\sigma_F^2}{\lambda} + \frac{r^2}{2}} - \frac{r}{2}$ *and* $M_{aim,\eta}^{*(a)} \equiv \frac{\gamma\sigma_F}{\gamma\sigma_F^2 + \lambda q_X^{*(a)} \kappa_\eta}$. *In particular,* $q_X^{*(a)} = M_{rate}\Big|_{(p_X, p_\eta)=\mathbf{0}}$.

Proof. See Appendix A. □

Proposition 3 establishes the frictionless benchmark under unconstrained arbitrage. Without limits to arbitrage, competitive arbitrageurs can immediately move capital without incurring any price impact and perfectly intermediate the trading needs of hedgers. Only the risk-free rate is required by arbitrageurs, without the need for additional compensation. Consequently, arbitrage capital flows caused by changing the two investment factors, η_t and $X_t^{(a)}$, do not generate any price effects ($p_\eta^* = p_X^* = 0$), and thus, the Law of One Price holds at all times ($\Delta_t = 0$). In this frictionless world, both the current and expected availability of arbitrage capital are irrelevant in predicting the behavior of asset prices, as asset prices are solely driven by their fundamental value.

In a frictionless environment, aggregate trading volume may still be slow-moving in equilibrium if hedgers with trading costs gradually demand liquidity. Only when these demanders face no trading frictions ($\lambda \downarrow 0$) does arbitrage trading volume perfectly align with the movements of liquidity risks, which follow a mean-reverting stochastic process with infinite variations. Additionally, in the absence of price adjustments due to aggregate capital flows, arbitrageurs are not required to further adjust their positions to account for aggregate illiquidity costs arising from the price impacts of other arbitrageurs. Consequently, there is no need for acceleration or deceleration strategies within the arbitrage group.

I will use this benchmark result as a reference point to evaluate the effectiveness of competitive arbitrage. As suggested by Dávila et al. (2024), pricing gaps and price impacts can sufficiently measure the social value of arbitrage. Specifically, I adopt both price and trading coefficients to assess market welfare in the arbitrage economy. The price coefficients $\boldsymbol{p} \equiv (p_\eta, |p_X|)$ measure price

efficiency by capturing deviations from the Law of One Price and the extent to which arbitrage corrects mispricing. Additionally, the trading coefficients $\boldsymbol{q} \equiv (q_X^{(a)}, q_\eta^{(a)})$ measure the benefits that arbitrageurs provide to liquidity demanders. These coefficients reflect how effectively arbitrageurs intermediate liquidity across segmented markets.

Definition. ***(Value of Arbitrage)*** *An economy under* $(\boldsymbol{p}, \boldsymbol{q})$ *is superior to an economy under* $(\boldsymbol{p}', \boldsymbol{q}')$ *if* $\boldsymbol{p}' \geq \boldsymbol{p}$ *and* $\boldsymbol{q} \geq \boldsymbol{q}'$.

The above definition allows us to rank the arbitrage equilibrium according to price discrepancy and trading volume. For instance, consider two economies under frictionless arbitrage: $(\boldsymbol{p}^*, \boldsymbol{q}^*)$ with $(\lambda, \lambda_a) = (c, 0)$ for a constant $c \in \mathbb{R}_{++}$ and $(\boldsymbol{p}^{**}, \boldsymbol{q}^{**})$ with $(\lambda, \lambda_a) = (0, 0)$. Although the the Law of One Price is feasible in both economies, according to our definition of the value of arbitrage, the economy under $(\boldsymbol{p}^{**}, \boldsymbol{q}^{**})$ is superior to the economy under $(\boldsymbol{p}^*, \boldsymbol{q}^*)$ because unconstrained hedgers benefit more from frictionless arbitrage in the form of their higher trading volume. This implies that the value of arbitrage can be higher in more liquid markets where hedgers are less constrained and more eager to share their liquidity risks. This is consistent with the prediction of Dávila et al. (2024) that for a given arbitrage gap, the value of arbitrage is higher in more liquid markets. However, this might not hold true if arbitrageurs face higher inventory-holding risks.

Proposition 4. ***(Equilibrium without Arbitrageurs)*** *If* $\lambda_a \uparrow \infty$, *the markets become autarkic with the following properties:*

1. $p_\eta^{aut} = \frac{\gamma \sigma_F}{r + \kappa_\eta}$, $p_X^{aut} = -\frac{\gamma\left(\sigma_F^2 + \left(\frac{\gamma \sigma_F}{r+\kappa_\eta}\right)\sigma_\eta^2\right)}{r}$, *and* $\boldsymbol{q}^{aut} = \boldsymbol{0}$.
2. $p_\eta^{aut} = \max p_\eta(\lambda_a)$ *and* $|p_X^{aut}| = \max |p_X(\lambda_a)|$.

Proof. See Appendix A. □

Proposition 4 shows that no trading occurs when arbitrageurs face infinite trading costs. As no liquidity is provided by arbitrageurs to hedgers, the hedgers become worse off. Additionally, without arbitrage, the price coefficients set an upper bound on their magnitude, resulting in the largest pricing gaps in response to changes in the investment factors $\boldsymbol{\Omega}_t$. This is intuitive because, without any quantity adjustment from liquidity providers, price adjustments must be largest. Clearly, an

economy without arbitrage is inferior to an economy with frictionless arbitrage according to our welfare order. Furthermore, Proposition 4 asserts that the economy without arbitrage is the most inferior due to both the largest price discrepancies and the lack of benefits for liquidity demanders. Consequently, arbitrage is expected to improve the welfare across segmented markets.

Propositions 3 and 4 suggest that there is room for the market to benefit from the participation of less constrained arbitrageurs, as they can provide more liquidity to hedgers and better minimize pricing errors relative to the asset's fundamental value.

4.3 Riskless Arbitrage

Before exploring the general case of risky arbitrage, I first examine a more tractable special case: riskless arbitrage, defined by the complete elimination of arbitrage risk ($\sigma_\eta \downarrow 0$). This scenario reflects situations where arbitrageurs can perfectly predict the trading needs of liquidity demanders and execute orders from hedgers without incurring inventory-holding risk. Nonetheless, even in the absence of inventory-holding risk, exploiting arbitrage opportunities is not cost-free.

Proposition 5. ***(Riskless Arbitrage)*** *For $\sigma_\eta = 0$, there exists a competitive equilibrium where the price coefficients are given by*

$$\bar{p}_X = -\lambda_a \bar{q}_X^{(a)} < 0, \tag{33}$$

$$p_\eta = \lambda_a \bar{q}_\eta^{(a)} > 0, \tag{34}$$

with the trading coefficients $\bar{q}_X^{(a)} = \sqrt{\frac{\gamma\sigma_F^2}{\lambda+\lambda_a} + \frac{r^2}{4}} - \frac{r}{2}$ and $\bar{q}_\eta^{(a)} = \frac{\gamma\sigma_F}{(\lambda+\lambda_a)(r+\kappa_\eta+\bar{q}_X^{(a)})}$.

1. *The asset spread is determined by*

$$\bar{\Delta}_t = \underbrace{2\lambda_a \times \left(\frac{d\bar{X}_t^{(a)}}{dt}\right)}_{\textit{volume-based price adjustment}}. \tag{35}$$

2. The law of motion for arbitrage capital follows

$$\frac{d\bar{X}_t^{(a)}}{dt} = \bar{q}_X^{(a)} \left(\bar{M}_{aim,\eta}^{(a)} \eta_t - X_t^{(a)} \right), \tag{36}$$

where $\bar{M}_{aim,\eta}^{(a)} \equiv \frac{\gamma \sigma_F}{\gamma \sigma_F^2 + (\lambda + \lambda_a) \kappa_\eta \bar{q}_X^{(a)}}$.

Proof. See Appendix A. □

Proposition 5 summarizes the competitive equilibrium under riskless arbitrage. Equation (33) shows that if competitive arbitrageurs face trading costs ($\lambda_a > 0$), then LOOP no longer holds across segmented markets because the aggregate state variables $X_t^{(a)}$ and η_t influence asset prices. First, the price coefficient $\bar{p}_X$ in (33) shows that equilibrium asset prices adjust unfavorably for arbitrageurs and benefit hedgers in response to changes in aggregate arbitrage inventory ($\frac{\partial \bar{P}_t^{(j)}}{\partial X_{jt}^{(a)}} = -\bar{p}_X > 0$). This occurs because, under riskless arbitrage with zero inventory-holding risk ($\Sigma_\Delta^Q = 0$), arbitrageurs always have a lower (unit) holding-to-trading cost ratio than hedgers. Consequently, equilibrium asset prices must incentivize hedgers, who face higher holding-to-trading cost ratios, to reallocate their inventory and participate in trades. This results in a negative correlation between the spread and aggregate arbitrage inventory ($\frac{\partial \bar{\Delta}_t}{\partial X_t^{(a)}} < 0$), which contradicts the positive spread-inventory relationship in traditional inventory models (e.g., Grossman and Miller, 1988). This result arises because, in the absence of inventory-holding risk, the spread reflects only the price impact costs incurred by arbitrageurs, who collectively exert downward pressure on prices when selling inflated assets and upward pressure when buying deflated assets across segmented markets.

The price coefficient $\bar{p}_\eta$ in (34) shows how asset prices respond to an increase in hedgers' liquidity risks in segmented markets. Hedgers must offer price concessions to arbitrageurs because constrained arbitrageurs require compensation for the costs of liquidity provision over time ($\frac{\partial \bar{P}_t^{(j)}}{\partial \eta_t^{(j)}} = -\bar{p}_\eta < 0$). Consequently, the equilibrium spread widens with higher liquidity risks across segmented markets ($\frac{\partial \bar{\Delta}_t}{\partial \eta_t} > 0$). Liquidity shocks in segmented markets cause reservation prices to diverge: increasing for hedgers with demand risks and decreasing for those with supply risks. Anticipating these shifts, arbitrageurs demand price concessions to cover expected trading costs, which further

widens the spread and incentivizes them to provide liquidity.

Proposition 5 also provides a closed-form expression for the asset spread $\bar{\Delta}_t$, which measures the violation of the law of one price. Equation (35) shows that in the absence of arbitrage risk, the asset spread is entirely driven by volume-based price adjustments, reflecting the marginal cost of providing liquidity across markets. For competitive arbitrageurs to engage in trading, the marginal benefit of holding an arbitrage position, $\bar{\Delta}_t$, must at least cover the marginal price impact cost of executing the trades required to establish that position, $2\lambda_a \times \left(\frac{d\bar{X}_t^{(a)}}{dt}\right)$. This result aligns with the classical argument by Demsetz (1968) that in a deterministic liquidity demand environment, a sufficiently competitive market for liquidity provision ensures that the spread size equals the marginal cost of providing immediacy.

Equation (36) describes the law of motion for arbitrage capital, which remains slow-moving despite each arbitrageur's static spread trading strategy (see Proposition 2). While arbitrageurs do not account for the history of their own capital when setting trading strategies, they must internalize the aggregate capital of all other arbitrageurs. Consequently, arbitrage capital gradually aligns with its target level, $\bar{M}^{(a)}_{aim,\eta}\eta_t$, at the rate of $\bar{q}^{(a)}_X$, similar to the frictionless benchmark case. However, the dynamics of capital under limits to arbitrage differ notably. First, as arbitrageurs face trading costs when accommodating hedgers' immediate liquidity needs, the instantaneous trading volume is shaped by constraints from both the demand and supply sides. Second, arbitrageurs adjust the rate of capital flows in tandem with asset price movements. By observing asset prices, they effectively identify the availability of arbitrage opportunities and competitive pressures, executing trades progressively over time. This endogenous adjustment of capital deployment, driven by price fluctuations, reveals a feedback mechanism between arbitrage capital flows and asset prices.

A. Value of Riskless Arbitrage

Proposition 6. ***(Value of Riskless Arbitrage)*** *For* $\{\lambda, \lambda_a\} \in (0, \infty)$ *and* $\sigma_\eta = 0$*, the following properties hold in equilibrium:*

1. $\boldsymbol{p}^{aut} > \bar{\boldsymbol{p}} > \boldsymbol{p}^*$ *and* $\boldsymbol{q}^* > \bar{\boldsymbol{q}} > \boldsymbol{q}^{aut}$.

2. The value of arbitrage increases for a smaller value of λ_a.

3. For small λ_a,

$$\bar{p}_X = -\lambda_a q_X^{*(a)} + o(\lambda_a), \tag{37}$$

$$\bar{p}_\eta = \lambda_a q_\eta^{*(a)} + o(\lambda_a), \tag{38}$$

$$\bar{q}_X^{(a)} = \left[1 - \left(\frac{\lambda_a}{\lambda}\right) C_X^q\right] q_X^{*(a)} + o(\lambda_a), \tag{39}$$

$$\bar{q}_\eta^{(a)} = \left[1 - \left(\frac{\lambda_a}{\lambda}\right) C_\eta^q\right] q_\eta^{*(a)} + o(\lambda_a), \tag{40}$$

where $C_X^q \equiv 1 - \frac{q_X^{*(a)}}{r+2q_X^{*(a)}}$, $C_\eta^q \equiv 1 - \frac{q_X^{*(a)}(r+q_X^{*(a)})}{(r+\kappa_\eta+q_X^{*(a)})(r+2q_X^{*(a)})}$, *and* $C_\eta^M \equiv \frac{q_X^{*(a)}\kappa_\eta}{(r+\kappa_\eta+q_X^{*(a)})(r+2q_X^{*(a)})}$.

Proof. See Appendix A. □

Proposition 6 characterizes the value of arbitrage under riskless arbitrage. Compared to the frictionless benchmark, the amount of liquidity supplied to hedgers decreases ($\boldsymbol{q}^* > \bar{\boldsymbol{q}}$), and the price impacts of the investment factors, as well as the resulting pricing gaps, become larger ($\bar{\boldsymbol{p}} > \boldsymbol{p}^*$) under riskless arbitrage. Despite limits to arbitrage, arbitrageurs still improve welfare across segmented markets compared to the autarkic case without their presence. Moreover, the value of arbitrage increases in more liquid markets where arbitrageurs are less constrained in providing liquidity. This is because, for a given risk-sharing capacity of hedgers, less constrained arbitrageurs can set a higher level of liquidity provision and execute trading at a faster rate. On the other hand, unlike the frictionless benchmark, easing the trading constraints for liquidity demanders (a lower λ) does not necessarily improve market welfare. This is because with arbitrageurs still facing constraints, larger spreads are required to compensate for the increased liquidity demand from less constrained hedgers.

Proposition 6 quantifies the change in market welfare from the frictionless benchmark associated with a small increase in the arbitrage cost parameter λ_a. First, the price impact coefficients are proportional to the equilibrium trading coefficients, $q_X^{*(a)}$ and $q_\eta^{*(a)}$, derived under frictionless arbitrage. Specifically, arbitrageurs face adverse price impacts when increasing their capital by

$\lambda_a q_X^{*(a)}$, while they require price concessions of $-\lambda_a q_\eta^{*(a)}$ from hedgers to provide liquidity. Second, the trading coefficients, $\bar{q}_\eta^{(a)}$ and $\bar{q}_X^{(a)}$, decrease approximately by $\frac{\lambda_a}{\lambda} q_X^{*(a)}$ and $\frac{\lambda_a}{\lambda} q_\eta^{*(a)}$, respectively, as arbitrageurs adjust their trading strategies in response to the net price impact costs relative to the price impacts from their counterparts. This suggests that arbitrage trading volume can decline not only because arbitrageurs reduce their trading speed to mitigate price impact costs but also because they lower their target liquidity provision.

B. Net Price Impact from Lead-and-Lag Capital Flows

Corollary 1. ***(Net Price Impact from Lead-and-Lag Capital Flows I)*** *Under riskless arbitrage, the price impacts of arbitrage capital flows across markets are neutralized such that*

$$\frac{\bar{p}_X}{\bar{q}_X^{(a)}} + \frac{\bar{p}_\eta}{\bar{q}_\eta^{(a)}} = 0 \tag{41}$$

with $\frac{\bar{p}_X}{\bar{q}_X^{(a)}} = \left[\frac{\partial \bar{P}_t^{(j)}}{\partial \bar{\tau}_{jt}^{(a)}}\right]_{d\bar{X}_t^{(a)}}$ *and* $\frac{\bar{p}_\eta}{\bar{q}_\eta^{(a)}} = \left[\frac{\partial \bar{P}_t^{(j)}}{\partial \bar{\tau}_t^{(j)}}\right]_{d\eta_t}$.

We have demonstrated that trading costs lead arbitrage capital to gradually align with the trading needs of hedgers over time, and the presence of slow-moving capital generates volume-based price effects in equilibrium. Corollary 1 shows that the price impacts of capital inflows and outflows into the arbitrage fund offset each other under riskless arbitrage. For example, when hedgers buy more risky assets in response to liquidity shocks, they allocate capital to arbitrageurs who facilitate these trades. Arbitrageurs use this capital inflow to acquire the assets, generating buying pressure with a price impact of $\frac{\bar{p}_\eta}{\bar{q}_\eta^{(a)}} = \lambda_a$. Conversely, when arbitrageurs unwind their positions, the resulting capital outflows create a price impact of $\frac{\bar{p}_X}{\bar{q}_X^{(a)}} = -\lambda_a$. Together, these capital inflows and outflows result in a net-zero price impact, as shown in (41). This occurs because arbitrageurs face symmetric price impact costs for both buying and selling across segmented markets and take no inventory-holding risk, eliminating the need for additional risk compensation during the lag period.

4.4 Risky Arbitrage

While arbitrage is often conceptualized as a risk-free profit opportunity, real-world arbitrage typically involves risk. For instance, Grossman and Miller (1988) highlight that order flows may not arrive simultaneously, exposing arbitrageurs to *non-simultaneous* price risks during inventory holding periods. Similarly, in this model, arbitrageurs face inventory-holding risk due to asynchronous order flows, as they cannot perfectly anticipate the intertemporal trading needs of hedgers.

Proposition 7. ***(Risky Arbitrage)*** *For $\lambda_a \in (0,\infty)$ and $\sigma_\eta > 0$, there exists a competitive equilibrium, with price coefficients given by*

$$p_X = -\frac{\lambda\lambda_a}{\gamma\Sigma^Q + (\gamma_a/2)\Sigma^Q_\Delta}\left(\frac{\gamma\Sigma^Q}{\lambda} - \frac{\gamma_a\Sigma^Q_\Delta}{2\lambda_a}\right) q_X^{(a)}, \tag{42}$$

$$p_\eta = \lambda_a q_\eta^{(a)} + \frac{\bar{\gamma}\sigma_F}{r+\kappa_\eta} > 0, \tag{43}$$

where $q_X^{(a)} = \sqrt{\frac{\gamma\Sigma^Q + \frac{\gamma_a}{2}\Sigma^Q_\Delta}{\lambda+\lambda_a} + \frac{r^2}{4}} - \frac{r}{2}$, $q_\eta^{(a)} = \frac{\gamma\sigma_F}{(\lambda+\lambda_a)(r+\kappa_\eta+q_X^{(a)})}$, *and* $\bar{\gamma} \equiv \frac{\frac{\gamma_a}{2}\Sigma^Q_\Delta}{(\lambda+\lambda_a)(r+\kappa_\eta)(r+q_X^{(a)})+\gamma\Sigma^Q+\frac{\gamma_a}{2}\Sigma^Q_\Delta}\gamma$.

1. *The asset spread is determined by*

$$\Delta_t = \underbrace{2\lambda_a \times \left(\frac{dX_t^{(a)}}{dt}\right)}_{\text{volume-based price adjustment}} + \frac{\gamma_a\Sigma^Q_\Delta}{r+q_X^{(a)}}X_t^{(a)} + \frac{1}{r+\kappa_\eta}\frac{\bar{\gamma}Cov_t[dQ_t, d(Y_t^{(2)} - Y_t^{(1)})]}{dt}. \tag{44}$$

2. *For $\lambda \in (0,\infty)$ and a small σ_η, there exists a threshold $\hat{\gamma}_a$ such that the economy is characterized as* ***Risk-On*** *for $\frac{1}{\gamma_a} > \frac{1}{\hat{\gamma}_a}$, where $M_{rate} > q_X^{(a)} > M_{rate}^{(a)}$. For $\frac{1}{\gamma_a} \leq \frac{1}{\hat{\gamma}_a}$, the economy is characterized as* ***Risk-Off****, where $M_{rate}^{(a)} \geq q_X^{(a)} \geq M_{rate}$.*

   - *In a* ***Risk-On*** *economy, $\frac{\partial\Delta_t}{\partial\eta_t} > 0$ and $\frac{\partial\Delta_t}{\partial X_t^{(a)}} < 0$.*
   - *In a* ***Risk-Off*** *economy, $\frac{\partial\Delta_t}{\partial\eta_t} > 0$ and $\frac{\partial\Delta_t}{\partial X_t^{(a)}} \geq 0$.*

3. The law of motion for arbitrage capital follows:

$$\frac{dX_t^{(a)}}{dt} = q_X^{(a)} \left(M_{aim,\eta}^{(a)} \eta_t - X_t^{(a)} \right), \tag{45}$$

$$\Leftrightarrow \quad \left. \frac{dx_t^{(a)}}{dt} \right|_{x_t^{(a)}=X_t^{(a)}} = M_{rate}^{(a)} \left(\left[\pi_a M_{aim,\eta}^{(a)} \eta_t + (1-\pi_a) X_t^{(a)} \right] - x_t^{(a)} \right) \Big|_{x_t^{(a)}=X_t^{(a)}}, \tag{46}$$

where $\pi_a \equiv \frac{q_X^{(a)}}{M_{rate}^{(a)}}$ *and* $M_{aim,\eta}^{(a)} \equiv \frac{\gamma \sigma_F}{\gamma \Sigma^Q + (\gamma_a/2)\Sigma_\Delta^Q + (\lambda+\lambda_a)\kappa_\eta q_X^{(a)}}$.

- *In a* ***Risk-On*** *economy,* $\pi_a > 1$.
- *In a* ***Risk-Off*** *economy,* $\pi_a \in [0, 1]$.

Proof. See Appendix A. □

Proposition 7 extends the competitive equilibrium model by incorporating both the risk and cost factors associated with arbitrageurs. Under risky arbitrage, the limited risk-bearing capacity of arbitrageurs introduces additional price effects, influencing the price impacts of the state variables, $X_t^{(a)}$ and η_t. First, the price coefficient p_X in (42) shows that, unlike the case of riskless arbitrage, the relative ratio of (unit) holding-to-trading costs between arbitrageurs and hedgers primarily determines the relationship between asset prices and aggregate arbitrage inventory. When arbitrageurs have lower holding-to-trading costs than hedgers ($\frac{\gamma_a \Sigma_\Delta^Q}{2\lambda_a} < \frac{\gamma \Sigma^Q}{\lambda}$), equilibrium asset prices move unfavorably against arbitrageurs and favorably toward hedgers, resembling the price behavior under riskless arbitrage where arbitrageurs have zero holding-to-trading costs. Conversely, when arbitrageurs have higher holding-to-trading costs than hedgers ($\frac{\gamma_a \Sigma_\Delta^Q}{2\lambda_a} > \frac{\gamma \Sigma^Q}{\lambda}$), equilibrium asset prices adjust to incentivize arbitrageurs to reallocate inventory, moving unfavorably against hedgers. By rearranging p_X in (42), the price coefficient can also be expressed as:

$$p_X = -\left(\frac{\lambda_a}{\lambda + \lambda_a} - \psi_a \right) (\lambda + \lambda_a)\, q_X^{(a)}, \tag{47}$$

where $\psi_a \equiv \frac{(\gamma_a/2)\Sigma_\Delta^Q}{\gamma \Sigma^Q + (\gamma_a/2)\Sigma_\Delta^Q}$ represents the share of inventory-holding risk borne by arbitrageurs relative to hedgers. Equation (47) shows that equilibrium asset prices respond differently to aggregate

arbitrage capital depending on the relative importance of trading costs and inventory-holding risk faced by competitive arbitrageurs. If arbitrageurs have relatively high risk appetites ($\psi_a < \frac{\lambda_a}{\lambda+\lambda_a}$), the relationship between asset prices and arbitrage capital reflects a cost-driven price effect. On the other hand, if arbitrageurs have relatively low risk appetites ($\psi_a > \frac{\lambda_a}{\lambda+\lambda_a}$), the risk-driven price effect becomes more pronounced.

The price coefficient p_η in (43) shows that equilibrium asset prices must adjust to compensate arbitrageurs not only for their trading costs related to target liquidity provision but also for their role in sharing hedgers' liquidity risks. Since arbitrageurs cannot perfectly hedge price risks due to non-simultaneous order flows, they bear the burden of sharing liquidity risks in equilibrium, requiring price concessions of $\frac{\bar{\gamma}\sigma_F}{r+\kappa_\eta}$. Here, $\bar{\gamma}$ represents the effective aggregate risk aversion coefficient, which increases with the share of inventory-holding risk borne by arbitrageurs ($\bar{\gamma} \propto \gamma_a \Sigma_\Delta^Q$). Greater inventory-holding risk for arbitrageurs necessitates larger price concessions from hedgers.

Proposition 7 also provides a closed-form expression for the asset spread. Under risky arbitrage, the spread is influenced by not only volume-based price adjustments, as in the case of riskless arbitrage, but also additional risk premiums demanded by risk-averse arbitrageurs due to the presence of arbitrage risk. First, Equation (44) shows that in response to an increase in η_t, the spread size monotonically increases to compensate for arbitrageurs' expected costs of target liquidity provision and the required risk premium for bearing liquidity risks offloaded by hedgers. This required risk premium is proportional to the instantaneous covariance between returns and hedgers' liquidity shocks, $\frac{Cov_t[dQ_t, d(Y_t^{(2)} - Y_t^{(1)})]}{dt}$. However, the spread does not exhibit a linear relationship with arbitrage inventory. The relationship depends on the relative size of trading costs and inventory-holding risk faced by arbitrageurs. Changes in $X_t^{(a)}$ influence the spread in opposing ways: as $X_t^{(a)}$ increases, arbitrageurs reduce their trading volume, $\tau_t^{(a)}$, which lowers the volume-based price adjustment. At the same time, arbitrageurs with limited risk-bearing capacity demand a higher risk premium for holding additional risky capital. Consequently, the spread-inventory relationship can be positive when arbitrageurs face relatively higher inventory-holding risk compared to trading costs (e.g., Grossman and Miller, 1988). However, the relationship turns out to be negative when arbitrageurs face relatively lower inventory-holding risk compared to trading costs.

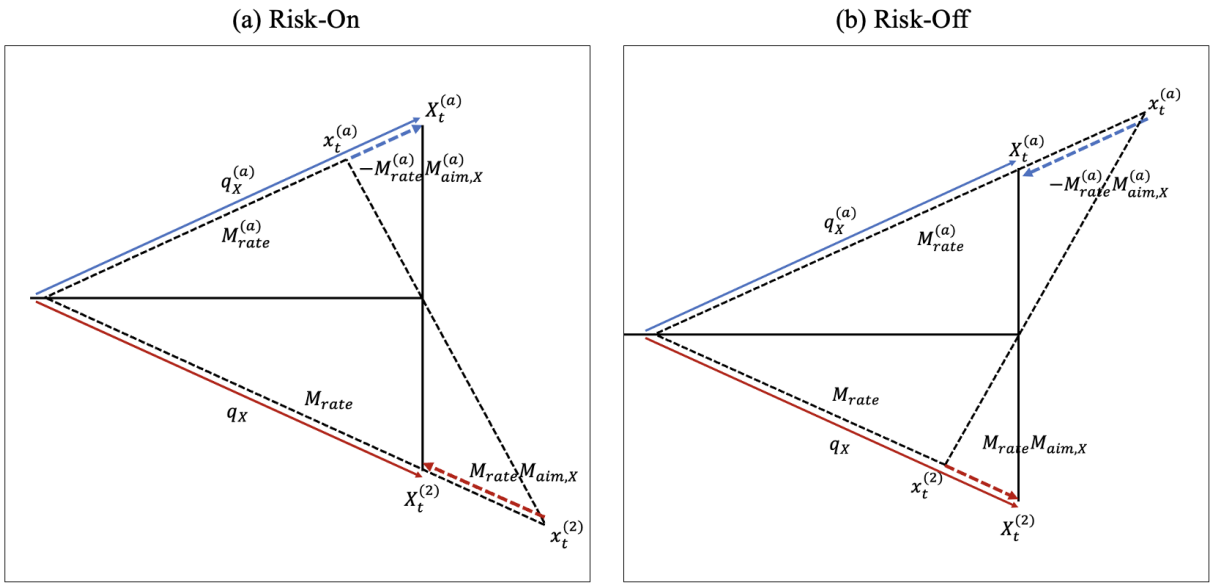


Figure 1: Endogenous Trading Speeds and Volume-based Price Effects

Proposition 7 shows that the arbitrageur's risk-bearing capacity (γ_a^{-1}) is a key determinant of the spread-inventory relationship, and divides the economy into two regimes: "Risk-On" and "Risk-Off." These regimes are differentiated by the cut-off value $\hat{\gamma}_a$, which is defined by the condition where the relative size of unit holding-to-trading costs is equal between arbitrageurs and hedgers ($\frac{\gamma\Sigma^Q(\hat{\gamma}_a)}{\lambda} = \frac{\hat{\gamma}_a\Sigma_\Delta^Q(\hat{\gamma}_a)}{2\lambda_a}$). In the Risk-On regime ($\gamma_a^{-1} > \hat{\gamma}_a^{-1}$), arbitrageurs have higher risk appetites, such that ($\frac{\gamma\Sigma^Q}{\lambda} > \frac{\gamma_a\Sigma_\Delta^Q}{2\lambda_a}$). Conversely, in the Risk-Off regime ($\gamma_a^{-1} < \hat{\gamma}_a^{-1}$), arbitrageurs have lower risk appetites, such that ($\frac{\gamma_a\Sigma_\Delta^Q}{2\lambda_a} > \frac{\gamma\Sigma^Q}{\lambda}$). Through a monotonic transformation, these conditions on relative holding-to-trading costs can also be expressed in terms of relative trading speeds between arbitrageurs and hedgers. As previously discussed, equilibrium price responses to arbitrage inventory differ across the two regimes.

In equilibrium, competitive arbitrageurs adjust their trading speeds endogenously in response to volume-based price adjustments, depending on the prevailing regime. To illustrate this endogenous adjustment, consider the optimal trading strategy at the state $\eta_t = 0$ and $X_t^{(a)} > 0$:

$$\tau_t^{(a)} = M_{rate}^{(a)}\left(M_{aim,X}^{(a)}X_t^{(a)} - x_t^{(a)}\right), \tag{48}$$

where $M^{(a)}_{aim,X} \equiv \frac{\overset{\Delta}{\mu}_X^-}{\gamma_a \Sigma^Q_\Delta + 2\lambda_a M^{(a)}_{rate} q^{(a)}_X}$. Equation (48) shows that as more capital has flowed into arbitrage trades, arbitrageurs in a Risk-On regime anticipate that this influx will compress spreads and reduce expected returns on their spread positions ($M^{(a)}_{aim,X} < 0$). Consequently, they aim to sell to avoid diminished returns, accelerating their liquidation speed. This acceleration strategy is optimal for arbitrageurs in this regime because their trading speed adjusts endogenously from the baseline speed $M^{(a)}_{rate}$, which excludes volume-based price effects, to the equilibrium trading speed $q^{(a)}_X = M^{(a)}_{rate} - M^{(a)}_{rate} M^{(a)}_{aim,X}$, which incorporates these effects. after considering the volume-based price effects. The term "Risk-On" reflects that arbitrageurs with high risk-bearing capacity are eager to transact aggressively with hedgers, collectively increasing their trading speeds ($q^{(a)}_X > M^{(a)}_{rate}$). For hedgers, the trading speed adjustment is the opposite of that of arbitrageurs. These endogenous speed adjustments for both types of traders in the Risk-On regime, driven by the volume-based price effects of arbitrage capital, are depicted in Figure 1-(a).

In contrast, under the Risk-Off paradigm, the volume-based price effects work in the opposite direction to those in the Risk-On regime. Equation (48) suggests that as cumulative capital flows into arbitrage trades increase, arbitrageurs expect widened spreads and higher expected returns ($M^{(a)}_{aim,X} > 0$). In response to these price adjustments, they aim to buy more at better prices. This anticipatory behavior decelerates their liquidation speed by $-M^{(a)}_{rate} M^{(a)}_{aim,X}$, as depicted in Figure 1-(b). The term "Risk-Off" reflects a more passive or conservative behavior among competitive arbitrageurs, where they collectively reduce their trading speeds and wait for more favorable terms of trade from hedgers ($q^{(a)}_X < M^{(a)}_{rate}$).

Proposition 7 extends the endogenous adjustment of arbitrageurs' trading strategies by examining how they dynamically adjust both their trading speeds and target positions for liquidity provision. The result shows that each arbitrageur adopts a flexible approach, shifting between aggressive and passive strategies based on the volume-based price adjustments in each regime. According to Equation (46), each arbitrageur accumulates capital $x^{(a)}_t$ by tracking a portfolio that combines two components: future liquidity provision, $M^{(a)}_{aim,\eta}\eta_t$, and the current aggregate inventory of other arbitrageurs, $X^{(a)}_t$, at the rate $M^{(a)}_{rate}$. The weights $(\pi_a, 1-\pi_a)$ assigned to these

components depend on the arbitrageur's trading speed $M_{rate}^{(a)}$ and the equilibrium trading speed $q_X^{(a)}$, which reflects the volume-based price effects of arbitrage capital.

In a Risk-On economy, the volume-based price effects of aggregate arbitrage capital lead arbitrageurs to adopt more aggressive trading strategies collectively than they would as individuals. Consequently, each arbitrageur places greater weight on liquidity provision, $M_{aim,\eta}^{(a)}\eta_t$, while adjusting their positions against the current aggregate arbitrage inventory $X_t^{(a)}$. Conversely, in a Risk-Off economy, where inventory-holding risk makes arbitrageurs more conservative or passive, they track a convex combination of these two components. This results in more conservative weights ($\pi_a \in [0,1]$), aligning their strategies more closely with the aggregate arbitrage inventory.

A. Value of Risky Arbitrage

Proposition 8. ***(Value of Risky Arbitrage)*** *For $\lambda_a \in (0,\infty)$ and $\sigma_\eta \neq 0$, the following properties hold in equilibrium:*

1. $q_X^{(a)} > \bar{q}_X^{(a)}$ *and* $q_\eta^{(a)} < \bar{q}_\eta^{(a)}$.
2. *Let $\hat{\gamma}_a$ be the threshold that differentiates the Risk-On and Risk-Off economies. There exists $Supp^{normal} \subset [0,\hat{\gamma}_a)$ such that*
   - *For $\gamma_a \in Supp^{normal}$, $\boldsymbol{p}^{aut} > \boldsymbol{p} > \boldsymbol{p}^*$ and $\boldsymbol{q}^* > \boldsymbol{q} > \boldsymbol{q}^{aut}$.*
3. *In a **Risk-Off** economy,*
   - $p_\eta^{aut} > p_\eta > p_\eta^*$ *and* $q_\eta^{*(a)} > q_\eta^{(a)} > q_\eta^{aut(a)}$.
   - $|p_X^{aut}| > |p_X| \geq |p_X^*|$ *and* $q_X^{(a)} > q_X^{*(a)} > q_X^{aut(a)}$.

Proposition 8 characterizes the value of arbitrage in two risky environments. Under risky arbitrage, competitive arbitrageurs become more myopic, lowering their target liquidity provision and increasing their rebalancing rates. By raising their trading speed, risk-averse arbitrageurs are able to minimize the duration of holding risky capital that deviates from its target. Consequently, risky arbitrage tends to be more short-term oriented, whereas riskless arbitrage aligns with a longer-term perspective on liquidity provision.

The second statement defines $Supp^{normal}$ as the subset of the arbitrageur's risk-bearing capacity that may fall within the Risk-Off economy.[12] We refer to this refined interval as the "*normal*' range of risk-bearing capacity in a Risk-On economy. In this normal Risk-On regime, where arbitrageurs are less concerned with inventory-holding risk, arbitrage activity improves market welfare by providing liquidity to hedgers and reducing mispricings relative to an economy without arbitrageurs. Moreover, market welfare is well-ordered, with the value of arbitrage bounded by the frictionless benchmark.

In contrast, the welfare rankings across the three economies break down in a Risk-Off economy, where arbitrageurs have relatively low risk-bearing capacity. While arbitrage intervention can still improve price efficiency by reducing pricing gaps, the welfare order observed in the normal Risk-On economy does not hold. This is because the equilibrium rebalancing rate exceeds the socially optimal rate of arbitrage ($q_X^{(a)} > q_X^{*(a)}$), and the target level of liquidity provision is reduced ($q_\eta^{(a)} < q_\eta^{*(a)}$). This excessive rebalancing and decreased liquidity provision lead to market inefficiencies, as arbitrageurs under the Risk-Off paradigm prioritize risk reduction over market stabilization.

Figure 2 illustrates the impact of transaction costs on the relationship between asset prices and arbitrage capital. In a normal Risk-On economy, arbitrageurs with relatively high inventory risk-bearing capacity require less compensation from hedgers to provide liquidity, especially when trading costs are lower. Consequently, the relationship between asset prices and arbitrage capital is positive, with this effect diminishing in more liquid markets. In contrast, this relationship changes in a Risk-Off economy. At point **A** in Figure 2, as Risk-Off arbitrageurs face lower transaction costs, the relationship between asset prices and arbitrage capital turns more negative. This shift indicates that hedgers must offer greater incentives to maintain liquidity provision, as Risk-Off arbitrageurs are more willing to offload their risky capital in more liquid markets. As a result, the value of arbitrage diminishes in these markets when the risk-bearing capacity of Risk-Off arbitrageurs is insufficient to sustain their trading activities.

[12]There exists some level of risk-bearing capacity in a Risk-On economy where the established welfare order does not hold.

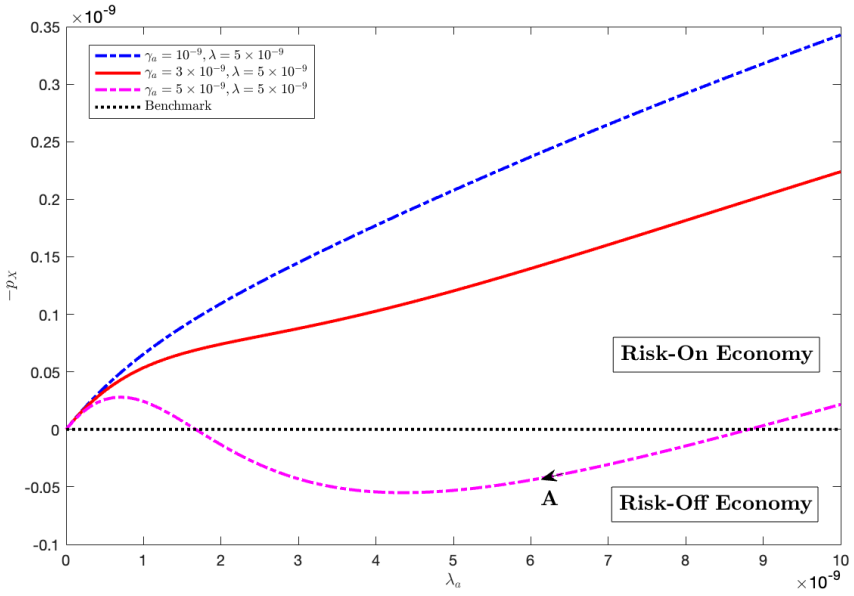


Figure 2: Effect of Trading Costs on the Relationship between Asset Prices and Arbitrage Capital

B. Net Price Impact from Lead-and-Lag Capital Flows

Corollary 2. ***(Net Price Impact from Lead-and-Lag Capital Flows II)*** *Under risky arbitrage, the price impacts of arbitrage capital flows across markets are neutralized such that*

$$\frac{p_X}{q_X^{(a)}} + \frac{p_\eta}{q_\eta^{(a)}} = (\lambda + \lambda_a)\psi_a + \frac{(\gamma_a/2)\Sigma_\Delta^Q}{(r+\kappa_\eta)(r+q_X^{(a)})} \tag{49}$$

with $\frac{p_X}{q_X^{(a)}} = \left[\frac{\partial P_t^{(j)}}{\partial \tau_{jt}^{(a)}}\right]_{dX_t^{(a)}}$ *and* $\frac{p_\eta}{q_\eta^{(a)}} = \left[\frac{\partial P_t^{(j)}}{\partial \tau_t^{(j)}}\right]_{d\eta_t}$.

Corollary 2 shows that, unlike in the riskless arbitrage case, the price impacts of capital inflows and outflows to the arbitrage fund do not fully offset under risky arbitrage. For example, when hedgers increase demand for assets due to liquidity shocks, capital flows into arbitrage funds. Arbitrageurs respond by purchasing assets, creating upward price pressure of $\frac{p_\eta}{q_\eta^{(a)}} = \lambda_a + \frac{(\gamma_a/2)\Sigma_\Delta^Q}{(r+\kappa_\eta)(r+q_X^{(a)})}$. Here, λ_a represents the unit price impact cost, while $\frac{(\gamma_a/2)\Sigma_\Delta^Q}{(r+\kappa_\eta)(r+q_X^{(a)})}$ reflects the price adjustment to compensate arbitrageurs for bearing hedgers' liquidity risks. On the other hand, when arbitrageurs liquidate positions and capital flows out, prices face downward pressure of $\frac{p_X}{q_X^{(a)}} = -\lambda_a + (\lambda + \lambda_a)\psi_a$.

The term $-\lambda_a$ reflects the unit selling cost, while $(\lambda + \lambda_a)\psi_a$ represents the price adjustment to induce arbitrageurs to hold risky capital. As a result, the overall price impact does not fully reverse the effect of buying pressures. Equation (49) shows that while direct price impact costs offset, the net aggregate price impact from lead-and-lag capital flows remains positive due to inventory-holding risk. This net effect grows with the arbitrageur's inventory-holding risk $(\gamma_a/2)\Sigma^Q_\Delta$.

4.5 Asset Price Dynamics

In this section, I examine the impulse responses of asset prices and trading positions, investigating how asset prices can persistently deviate from their fundamental values in both Risk-On and Risk-Off regimes. Specifically, I consider a scenario where hedgers and arbitrageurs, starting with no initial inventories, respond to liquidity shocks, following the experimental framework commonly used in the slow-moving capital literature (e.g., Duffie (2010)).

Proposition 9. ***(Persistent Law-Of-One-Price Violation)*** *Let $\hat{\mathbf{\Omega}}_t = (0, \eta_t)$ such that $\Delta(\hat{\mathbf{\Omega}}_t) \neq 0$. Then the following properties hold in equilibrium:*

1. *For $\lambda_a \in (0, \infty)$, the persistent violation of the law of one price occurs:*
   - *In a **Risk-On** economy, the spread exhibits a smaller initial response to liquidity shocks compared to a Risk-Off economy and inverts during the reversal phase.*
   - *In a **Risk-Off** economy, the spread shows a larger initial response to liquidity shocks than in a Risk-On economy and decreases monotonically during reversal.*
2. *The half-life of the asset spread is defined as $T^\Delta_{1/2} \equiv \left\{T > 0 | \frac{1}{2}\Delta_t = E_t[\Delta_{t+T}]\right\}$, and its approximated value is given by*

$$T^\Delta_{1/2} \approx \frac{1}{2}\left(\kappa_\eta + \frac{q^{(a)}_X}{C_{1/2}}\left[\frac{\lambda_a}{\lambda + \lambda_a} - \psi_a\right]\right)^{-1}. \tag{50}$$

where $C_{1/2} \equiv \frac{1}{\lambda+\lambda_a}\left(\lambda_a + \frac{(\gamma_a/2)\Sigma^Q_\Delta}{(r+\kappa_\eta)(r+q^{(a)}_X)}\right)$.

- *$T^{Risk\text{-}On\Delta}_{1/2} < T^{aut\Delta}_{1/2} < T^{Risk\text{-}Off\Delta}_{1/2}$ where $T^{aut\Delta}_{1/2} \approx \frac{1}{2}\kappa_\eta^{-1}$.*

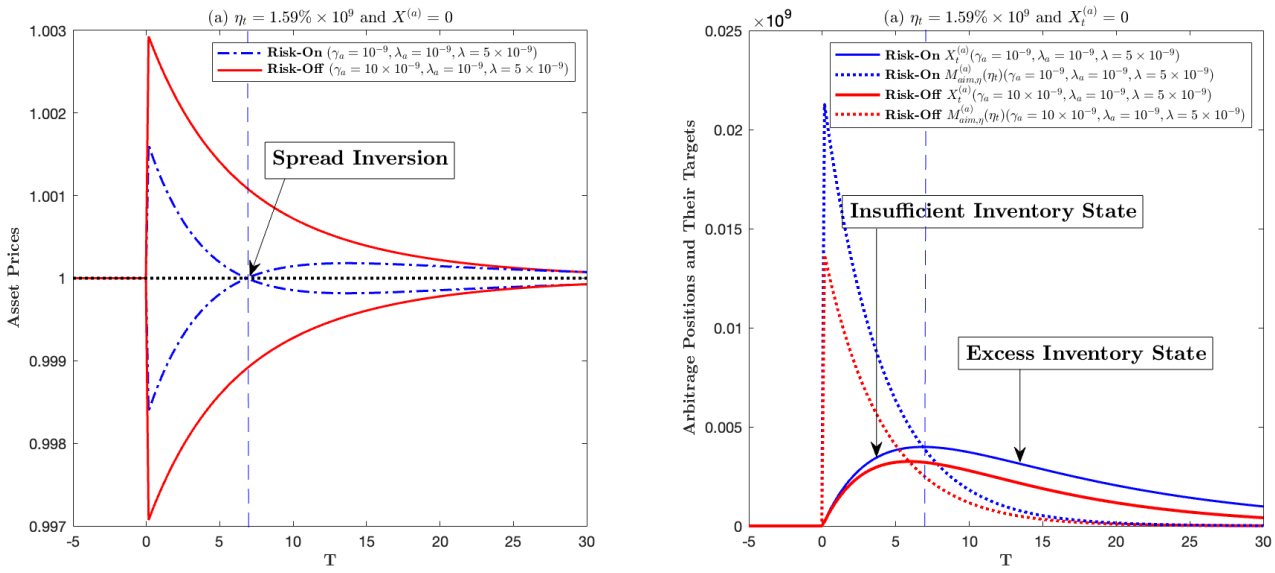


Figure 3: Impulse Responses of Asset Prices and Trading Positions

Proof. See Appendix B. □

Proposition 9 describes the persistent violation of the law of one price when risk-averse arbitrageurs are constrained by price impact costs. As a direct consequence of price impact costs, asset prices in segmented markets exhibit slow recovery following non-fundamental liquidity shocks. Although asset prices eventually converge to their fundamental value, the patterns of price discovery differ between the Risk-On and Risk-Off regimes. Figure 3-(a) illustrates the impulse responses of the asset spread following large liquidity shocks, designed to cause a 1.59% shift in hedgers' asset allocations from a $1 billion AUM, which gradually decay over time.

In the Risk-On regime, the asset spread initially widens to incentivize constrained arbitrageurs to increase their target liquidity provision. As arbitrage trading volume begins to align with its target level, the rise in aggregate arbitrage trading compresses arbitrage profits for each arbitrageur. In response, competitive arbitrageurs front-load their trading opportunities, quickly narrowing the pricing gaps caused by these shocks. However, this aggressive accumulation of arbitrage capital leads to an "excess inventory phase," as arbitrageurs struggle to align their current positions with their rapidly changing targets, as illustrated in Figure 3-(b). To liquidate these excess positions at

favorable terms, arbitrageurs demand compensation from hedgers, inducing a temporary inversion in the asset spread. This price overshooting allows arbitrageurs to sell at prices above the fundamental value in one market while buying below it in another, as shown in Figure 3-(a). Following the overshoot, arbitrageurs adopt a more cautious approach to unwinding their inventories, and the spread gradually converges to zero. This oscillating convergence pattern aligns with findings in the slow-moving capital literature (e.g., Duffie, 2010; Bogousslavsky et al., 2021).

In the Risk-Off regime, the asset spread initially widens, similar to the Risk-On regime, but responds more strongly to liquidity shocks to incentivize highly risk-averse arbitrageurs to provide liquidity. Due to the passive trading behavior of arbitrageurs under this regime, the rise in aggregate arbitrage trading further widens the spread. Consequently, asset prices decrease monotonically over time without any price overshooting, gradually converging to their fundamental value. Arbitrageurs undercorrect initial pricing gaps more than in the Risk-On regime, increasing the market's vulnerability to liquidity shocks. Their conservative approach to liquidity provision extends the half-life of the asset spread compared to that in the Risk-On regime.

4.6 Return Predictability

Proposition 10. ***(Return Autocorrelation)*** *The instantaneous return autocorrelation is given by*

$$\varrho_t^Q(T) \equiv \frac{1}{T}\frac{\frac{1}{dt}Cov_t(dQ_t^{(j)}, dQ_{t+T}^{(j)})}{\frac{1}{dt}Var_t(dQ_t^{(j)})} = B_1(T, \kappa_\eta) + B_2(T, q_X^{(a)}),$$

where $B_1(T, \kappa_\eta)$ *and* $B_2(T, q_X^{(a)})$ *are non-zero constants for* $\lambda_a > 0$ *and* $T > 0$.

- *In the* ***Risk-On*** *regime, there exists a threshold* $\hat{T}$ *such that* $\varrho_t^Q(T) \lesseqgtr 0$ *for* $T \lesseqgtr \hat{T}$.
- *In the* ***Risk-Off*** *regime,* $\varrho_t^Q(T) < 0$ *for all* $T > 0$.

Proof. See Appendix B. □

Proposition 10 shows that when arbitrageurs incur volume-based price impacts during capital

movements, asset returns deviate from fundamentals and do not follow a purely random IID process. Instead, they exhibit non-zero autocorrelation. In the Risk-On regime, where volume-based price adjustments play a stronger role in driving the asset spread, returns transition from short-term reversal to longer-term momentum. Short-term reversals occur as constrained arbitrageurs require compensation beyond the asset's fundamental value, with liquidity risks being mean-reverting. Longer-term momentum arises from the delayed portfolio adjustments of arbitrageurs, sustaining initial price trends over extended periods. In contrast, the Risk-Off regime exhibits no subsequent momentum, only reversal. This difference occurs because, in the Risk-Off regime, the spread is driven primarily by risk compensation for arbitrageurs with low-risk tolerance, rather than by volume-based price adjustments.

4.7 Excess Price Volatility

Proposition 11. ***(Excess Volatility)*** $P_t^{(j)} \xrightarrow{t\to\infty} \mathcal{N}\left(\frac{\bar{D}}{r}, \bar{\sigma}_P^2\right)$ *with the price volatility:*

$$\bar{\sigma}_P^2 = \bar{\sigma}_F^2 + \left[\left(\frac{\kappa_\eta}{\kappa_\eta + q_X^{(a)}}\right)\left(\frac{p_\eta}{q_\eta^{(a)}}\right)^2 + \left(\frac{q_X^{(a)}}{\kappa_\eta + q_X^{(a)}}\right)\left(\frac{p_X}{q_X^{(a)}} + \frac{p_\eta}{q_\eta^{(a)}}\right)^2\right]\left(\frac{q_\eta^{(a)}\sigma_\eta}{\sqrt{2\kappa_\eta}}\right)^2, \tag{51}$$

$$= \bar{\sigma}_F^2\left(1 + \frac{\kappa_D}{\kappa_\eta}\left[\frac{\kappa_\eta}{\kappa_\eta + q_X^{(a)}}\left(\frac{\left[\frac{\lambda_a}{\lambda+\lambda_a}\right](r+\kappa_\eta) + \psi_a q_X^{(a)}}{r + \kappa_\eta + q_X^{(a)}}\right)^2 + \frac{q_X^{(a)}}{\kappa_\eta + q_X^{(a)}}\psi_a^2\right]\left(\frac{\gamma\sigma_\eta}{r+\kappa_\eta}\right)^2\right), \tag{52}$$

where $\bar{\sigma}_F \equiv \frac{\sigma_F}{\sqrt{2\kappa_D}}$ *is the unconditional volatility of the asset's fundamental value.*

- $\lim_{\lambda_a \downarrow 0} \bar{\sigma}_P = \bar{\sigma}_F$.

Proof. See Appendix B. □

Proposition 11 explains how slow-moving arbitrage capital increases asset price volatility in segmented markets. In perfectly liquid markets where arbitrageurs face no trading costs, capital can flow instantaneously between markets and ensure that asset prices perfectly align with their fundamental value. However, when arbitrage capital becomes slow-moving due to trading costs, additional price variations are required in equilibrium to compensate arbitrageurs for providing

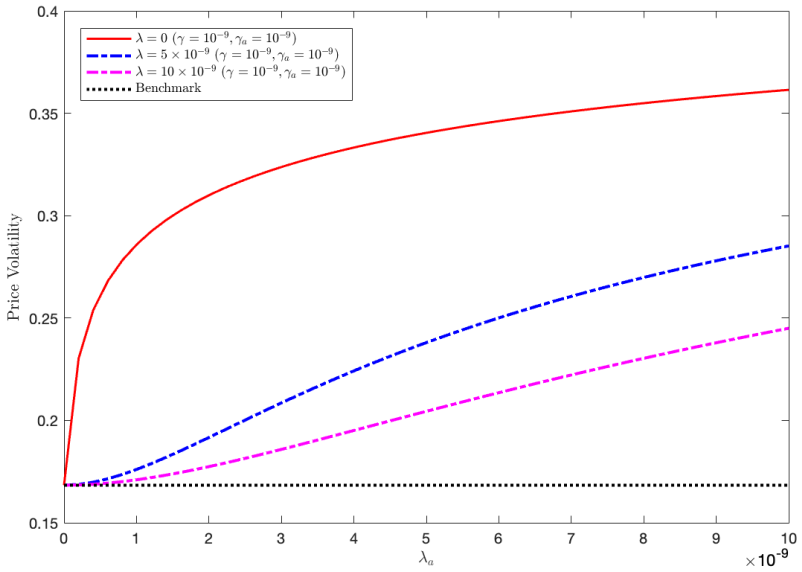


Figure 4: Excess Price Volatility and Transaction Costs

liquidity, as these provisions are both costly and risky.

The closed-form expression for price volatility in (51) shows that the asset price becomes excessively volatile due to variations in the risk premium and additional price adjustments caused by slow-moving arbitrage capital. In response to continuous arrivals of liquidity shocks, asset prices are initially driven by the risk premium required by arbitrageurs. This price adjustment is captured by the term $\frac{\kappa_\eta}{\kappa_\eta + q_X^{(a)}} \left(\frac{p_\eta}{q_\eta^{(a)}} \right)^2 \left(\frac{q_\eta^{(a)} \sigma_\eta}{\sqrt{2\kappa_\eta}} \right)^2$. Moreover, as arbitrage capital gradually adjusts to match its target allocation and moves in and out between arbitrageurs and hedgers, volume-based price adjustments further contribute to price volatility. This additional price adjustment is captured by the term $\frac{q_X^{(a)}}{\kappa_\eta + q_X^{(a)}} \left(\frac{p_X}{q_X^{(a)}} + \frac{p_\eta}{q_\eta^{(a)}} \right)^2 \left(\frac{q_\eta^{(a)} \sigma_\eta}{\sqrt{2\kappa_\eta}} \right)^2$.

The volatility expression (52) highlights that the excessive component of price volatility is primarily influenced by two key distributional variables: arbitrageurs' share of total trading costs $\frac{\lambda_a}{\lambda + \lambda_a}$ and their share of total inventory-holding risks ψ_a. An increase in either the arbitrageur's share of trading costs or their share of inventory-holding risks amplifies price volatility. Figure 4 illustrates how volatility depends on the cost parameters of both arbitrageurs and hedgers. When arbitrageurs face greater trading costs, the non-fundamental component of price volatility is amplified to com-

pensate for their greater cost burden of providing liquidity and for their greate risk burden of sharing liquidity risks from hedgers. Furthermore, lower trading costs for hedgers exacerbate price volatility, as arbitrageurs with unchanged trading capacity demand additional compensation to manage the increased order flow from these less constrained hedgers.

5. Redistribution Policy and Market Welfare

In this framework, hedgers in two segmented markets are willing to trade risky assets with each other. In the absence of limits to arbitrage, arbitrageurs can effectively uphold the law of one price and supply hedgers with a socially efficient amount of liquidity. However, when trading frictions are present, arbitrageurs intermediate the trading needs of hedgers in exchange for compensation, creating pricing gaps across segmented markets. These pricing gaps are essential for constrained arbitrageurs to participate and provide liquidity. Given this mechanism, would a social planner's intervention—subsidizing the costs associated with price impacts to compensate arbitrageurs—improve market welfare according to our criteria?

To examine the effect of redistribution on market welfare, we define the wealth redistribution processes for each hedger and arbitrageur as follows:

$$\tilde{W}_t^{(j)} \equiv \tilde{W}_0^{(j)} - \int_0^t \frac{\theta}{2} \left(\tau_s^{(a)}\right)^2 ds, \tag{53}$$

$$\tilde{W}_t^{(a)} \equiv \tilde{W}_0^{(a)} + \int_0^t \theta \left(\tau_s^{(a)}\right)^2 ds, \tag{54}$$

where $\theta \in [0, \lambda_a]$ is the parameter that controls the size of the subsidy associated with the price impact costs faced by arbitrageurs. Note that there is no net bias in wealth redistribution, as the sum of the wealth redistribution processes in (53) and (54) remains zero at all times. Under this redistribution policy, the new wealth processes for each hedger and arbitrageur are given by:

$$dW_{t,\theta}^{(j)} = rW_{t,\theta}^{(j)}dt + x_t^{(j)}dQ_t^{(j)} - \frac{\lambda + \theta}{2}\left(\tau_t^{(j)}\right)^2 dt + dY_t^{(j)}, \tag{55}$$

$$dW_{t,\theta}^{(a)} = rW_{t,\theta}^{(a)}dt + x_t^{(a)}dQ_t^{\Delta^-} - (\lambda_a - \theta)\left(\tau_t^{(a)}\right)^2 dt, \tag{56}$$

where $W_{t,\theta}^{(j)} \equiv W_t^{(j)} + \tilde{W}_t^{(j)}$ and $W_{t,\theta}^{(a)} \equiv W_t^{(a)} + \tilde{W}_t^{(a)}$.

From the new wealth processes and the mean-variance preferences of each arbitrageur and hedger, their maximization problems can be rewritten as:

$$\begin{aligned} V_\theta^{(j)}\left(x_t^{(j)}, \mathbf{\Omega}_t\right) = \max_{\left(\tau_{s,\theta}^{(j)}\right)_{s\geq t}} E_t \int_t^\infty e^{-r(s-t)} \Big[\left(\mu_{\eta,\theta}^{(j)} \eta_s + \mu_{X,\theta}^{(j)} X_s^{(a)}\right) x_s^{(j)} \\ -\frac{\gamma}{2}\left(\Sigma_\theta^Q \left(x_s^{(j)}\right)^2 + 2\sigma_F (-1)^j \eta_s x_s^{(j)} + (\eta_s)^2\right) - \frac{\lambda_\theta}{2}\left(\tau_{s,\theta}^{(j)}\right)^2 \Big] ds, \end{aligned} \tag{57}$$

$$\begin{aligned} V_\theta^{(a)}\left(x_t^{(a)}, \mathbf{\Omega}_t\right) = \max_{\left(\tau_{s,\theta}^{(a)}\right)_{s\geq t}} E_t \int_t^\infty e^{-r(s-t)} \Big[\left(\mu_{\eta,\theta}^{\Delta^-} \eta_s + \mu_{X,\theta}^{\Delta^-} X_s^{(a)}\right) x_s^{(a)} \\ -\frac{\gamma_a}{2}\Sigma_{\Delta,\theta}^Q \left(x_s^{(a)}\right)^2 - \lambda_{a,\theta}\left(\tau_{s,\theta}^{(a)}\right)^2 \Big] ds, \end{aligned} \tag{58}$$

where $\lambda_\theta \equiv \lambda + \theta$ and $\lambda_{a,\theta} \equiv \lambda_a - \theta$. By substituting the modified cost parameters, I use dynamic programming to compute a competitive equilibrium for the adjusted problem. Specifically, I aim to determine the competitive equilibrium under redistribution by conjecturing the price functions:

$$P_{t,\theta}^{(j)} = F_t - \left(p_{X,\theta}^{(j)} X_t^{(a)} + p_{\eta,\theta}^{(j)} \eta_t\right), \tag{59}$$

where $p_{X,\theta}^{(j)} \equiv (-1)^j p_{X,\theta}$ and $p_{\eta,\theta}^{(j)} \equiv (-1)^j p_{\eta,\theta}$ are price coefficients determined later in equilibrium. Additionally, the law of motion for arbitrage capital is given by

$$\frac{dX_{t,\theta}^{(a)}}{dt} = q_{\eta,\theta}^{(a)} \eta_t - q_{X,\theta}^{(a)} X_t^{(a)} \tag{60}$$

where $q_{\eta,\theta}^{(a)}$ and $q_{X,\theta}^{(a)}$ are trading coefficients that will be determined in equilibrium.

Proposition 12. ***(Competitive Equilibrium under Redistribution)*** *For $\theta \in (0, \lambda_a]$, there exists a competitive equilibrium such that:*

1. *At $\theta = \lambda_a$, $p_{X,\theta} = 0$, $p_{\eta,\theta} = 0$, $q_{X,\theta}^{(a)} = \bar{q}_X^{(a)}$, and $q_{\eta,\theta}^{(a)} = \bar{q}_\eta^{(a)}$.*

   - $\boldsymbol{p^\theta = p^*}$ *and* $\boldsymbol{q^\theta < q^*}$

2. ***(Riskless Arbitrage)*** *For $\theta \in (0, \lambda_a)$ and $\sigma_\eta = 0$, the price coefficients are determined by*

$$\bar{p}_{X,\theta} = \bar{p}_X + \theta \bar{q}^{(a)}_{X,\theta}, \tag{61}$$

$$\bar{p}_{\eta,\theta} = \bar{p}_\eta - \theta \bar{q}^{(a)}_{\eta,\theta}, \tag{62}$$

with the trading coefficients $\bar{q}^{(a)}_{X,\theta} = \bar{q}^{(a)}_X$ and $\bar{q}^{(a)}_{\eta,\theta} = \bar{q}^{(a)}_\eta$.

- $\bar{\boldsymbol{p}}^{\boldsymbol{\theta}} < \bar{\boldsymbol{p}}$ *and* $\bar{\boldsymbol{q}}^{\boldsymbol{\theta}} = \bar{\boldsymbol{q}}$

3. ***(Risky Arbitrage)*** *For $\theta \in (0, \lambda_a)$ and $\sigma_\eta > 0$, the price coefficients are determined by*

$$p_{X,\theta} = -(\lambda_a - \theta) q^{(a)}_{X,\theta} + \psi_{a,\theta}(\lambda + \lambda_a) q^{(a)}_{X,\theta}, \tag{63}$$

$$p_{\eta,\theta} = (\lambda_a - \theta) q^{(a)}_{\eta,\theta} + \frac{\bar{\gamma}_\theta \sigma_F}{r + \kappa_\eta}, \tag{64}$$

where $q^{(a)}_{X,\theta} = \sqrt{\frac{\gamma \Sigma^Q_\theta + (\gamma_a/2)\Sigma^Q_{\Delta,\theta}}{\lambda + \lambda_a} + \frac{r^2}{4}} - \frac{r}{2}$, $q^{(a)}_{\eta,\theta} = \frac{\gamma \sigma_F}{(\lambda+\lambda_a)(r+\kappa_\eta+q^{(a)}_{X,\theta})}$, $\psi_{a,\theta} \equiv \frac{(\gamma_a/2)\Sigma^Q_{\Delta,\theta}}{\gamma \Sigma^Q_\theta + (\gamma_a/2)\Sigma^Q_{\Delta,\theta}}$, and $\bar{\gamma}_\theta \equiv \frac{\frac{\gamma_a}{2}\Sigma^Q_{\Delta,\theta}}{(\lambda+\lambda_a)(r+\kappa_\eta)(r+q^{(a)}_{X,\theta}) + \gamma \Sigma^Q_\theta + \frac{\gamma_a}{2}\Sigma^Q_{\Delta,\theta}} \gamma$. The following properties hold in equilibrium:

- *$q^{(a)}_X > q^{(a)}_{X,\theta}$, $q^{(a)}_{\eta,\theta} > q^{(a)}_\eta$, and $p_\eta > p_{\eta,\theta}$.*
- *For a small σ_η,*
  - *For sufficiently small γ_a, $|p_X| > |p_{X,\theta}|$ and $\frac{\partial |p_{X,\theta}|}{\partial \theta} < 0$.*
  - *For sufficiently large γ_a, there exists $\hat{\theta} \in (0, \lambda_a)$ such that $p_{X,\hat{\theta}} = 0$.*

Proof. See Appendix A. □

Proposition 12 characterizes the competitive equilibrium under the redistribution policy. It shows that a social planner can achieve the Law of One Price by fully subsidizing the price impact costs incurred by arbitrageurs. While this intervention ensures that asset prices become efficient, it results in a welfare loss due to reduced trading volumes among hedgers. Since hedgers bear the full cost of subsidizing arbitrageurs' price impact costs, their trading activity for risk-sharing declines when the arbitrageurs' cost parameter rises.

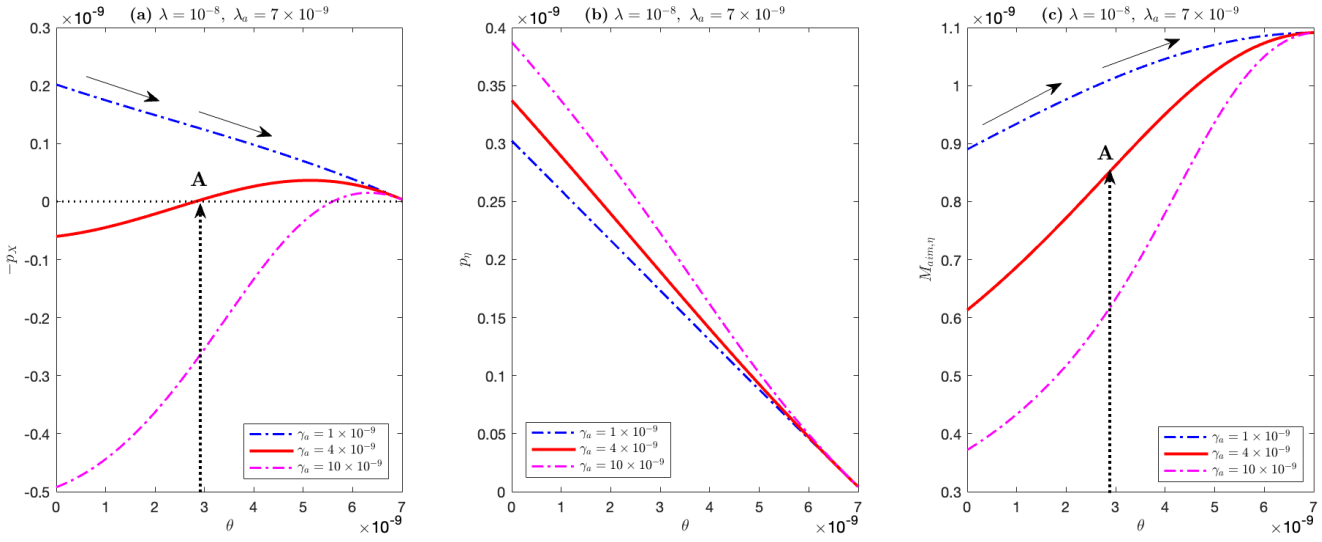


Figure 5: Redistribution Policy and Market Welfare

In the case of riskless arbitrage, a social planner can enhance market welfare by subsidizing arbitrage costs. Interestingly, despite redistributing costs and benefits between the two groups, the equilibrium trading volume remains unchanged before and after wealth redistribution, as the gains to arbitrageurs are exactly offset by the increased costs to hedgers. This redistribution policy reduces both the spread size and price impacts *without* altering aggregate trading volume. On the other hand, under risky arbitrage, the social planner can improve market welfare by increasing the target liquidity provision of arbitrageurs and decreasing the speed of competitive arbitrage that may have been excessive. For example, in a competitive equilibrium where arbitrageurs have relatively high risk-bearing capacity (e.g., Risk-On), arbitrageurs may have accelerated their trading excessively, aiming to reach their target asset allocation more quickly than others. By mitigating the volume-based price effects of arbitrage capital, a social planner can reduce the excessive trading speed of competitive arbitrageurs, and encourage them to adopt a longer-term perspective, ultimately increasing their liquidity provision.

Furthermore, Proposition 12 suggests that when arbitrageurs have low risk-bearing capacity (i.e., Risk-Off), an effective redistribution policy should aim for inventory neutrality so that asset prices remain stable despite changes in arbitrage capital. This inventory-neutral policy can prevent

highly risk-averse arbitrageurs from becoming more passive and seeking additional compensation due to fluctuations in arbitrage capital. In fact, it enables passive arbitrageurs to provide liquidity more actively. Figure 5-(a) depicts this inventory-neutral θ-policy. By eliminating the price impact of arbitrage capital, the policy increases the price efficiency and incentivizes arbitrageurs to adopt a long-term approach to liquidity provision, as depicted in Figures 5-(b) and 5-(c).

5. Conclusion

I investigate asset price dynamics and market efficiency within a general equilibrium arbitrage framework, where competitive arbitrageurs, facing convex trading costs and inventory-holding costs, intermediate the trading needs of hedgers in two segmented markets. The model captures volume-based price impacts of aggregate arbitrage trading volumes through a two-way feedback mechanism, where aggregate trading behavior influences price dynamics, and price dynamics, in turn, shape individual trading strategies. In equilibrium, the spread positively responds to liquidity risks but may either increase or decrease with aggregate arbitrage capital, depending on the relative size of inventory-holding costs and trading costs between arbitrageurs and hedgers. When arbitrageurs face relatively lower holding-to-trading costs, the spread negatively responds to aggregate arbitrage inventory, and each arbitrageur adopts an accelerated trading strategy. Conversely, when arbitrageurs face relatively higher holding-to-trading costs, the spread-inventory relationship becomes positive, and each arbitrageur adopts a decelerated trading strategy.

I investigate how non-fundamental liquidity shocks lead to persistent deviations in asset prices from their fundamental value and amplify price volatility through the volume-based price effects of slow-moving arbitrage capital. The analysis shows that the asset spread exhibits slow recovery following liquidity shocks, with the convergence pattern varying based on arbitrageurs' relative holding-to-trading costs. When arbitrageurs face relatively higher trading costs than inventory-holding costs, the spread is corrected by aggressive arbitrage and becomes inverted during the reversal phase. This spread inversion rationally compensates arbitrageurs who prioritize minimizing trading costs over inventory-holding costs. Furthermore, the analysis reveals that greater volume-

based price impacts of arbitrage capital amplify price volatility beyond fluctuations driven by asset fundamentals.

In addition to analyzing asset price dynamics, I investigate market efficiency in arbitrage economies by quantifying the extent of Law-of-One-Price violations and the level of liquidity provision. Two benchmark cases are considered: a frictionless arbitrage economy and an economy without arbitrage. The findings show that when arbitrageurs have sufficiently high-risk tolerance, welfare criteria lie between these two benchmarks. Furthermore, lower trading costs for arbitrageurs enhance the value of arbitrage. However, when arbitrageurs lack sufficient risk-bearing capacity to hold risky inventory, the welfare order breaks down. In this case, as arbitrageurs prioritize minimizing short-term inventory-holding risks, causing the speed of competitive arbitrage to exceed the socially efficient rate. Paradoxically, lower trading costs for arbitrageurs can diminish the value of arbitrage because hedgers are required to offer additional compensation to incentivize liquidity provision from less constrained but passive arbitrageurs. I further show that a redistribution policy subsidizing arbitrageurs' price impact costs, without altering the total wealth flow between arbitrageurs and hedgers, can improve welfare. Such a policy should aim to minimize the volume-based price impacts of arbitrage capital and encourage arbitrageurs to provide long-term liquidity.

References

[1] **Amihud, Y., and Mendelson, H.**, 1986. "Asset Pricing and the Bid-Ask Spread," Journal of Financial Economics, 17(2), 223-249.

[2] **Almgren, R., and Chriss, N.**, 2000. "Optimal Execution of Portfolio Transactions," Journal of Risk, 3, 5–39.

[3] **Bacchetta, P., and van Wincoop, E.**, 2010. "Infrequent Portfolio Rebalancing, Speculative Bubbles, and the Exchange Rate Disconnect Puzzle," American Economic Review, 100(1), 313-336.

[4] **Basak, S., and Chabakauri, G.**, 2010. "Dynamic Mean-Variance Asset Allocation," Review of Financial Studies, 23(8), 2970-3016.

[5] **Bogousslavsky, V.**, 2016. "Infrequent Rebalancing, Return Autocorrelation, and Seasonality," Journal of Finance, 71(6), 2967-3006.

[6] **Bogousslavsky, V., Collin-Dufresne, P., and Sağlam, M.**, 2021. "Slow-moving capital and execution costs: Evidence from a major trading glitch," Journal of Financial Economics, 139(3), 922-949.

[7] **Brunnermeier, M. K., and Pedersen, L. H.**, 2009. "Market Liquidity and Funding Liquidity," Review of Financial Studies, 22(6), 2201-2238.

[8] **Brunnermeier, M. K., and Sannikov, Y.**, 2014. "A Macroeconomic Model with a Financial Sector," American Economic Review, 104(2), 379-421.

[9] **Campbell, J. Y., Grossman, S. J., and Wang, J.**, 1993. "Trading Volume and Serial Correlation in Stock Returns," Quarterly Journal of Economics, 108(4), 905-939.

[10] **Chien, Y., Cole, H., and Lustig, H.**, 2012. "Is the Volatility of the Market Price of Risk due to Intermittent Portfolio Rebalancing?," American Economic Review, 102(6), 2859-2896.

[11] **Chordia, T., Roll, R., and Subrahmanyam, A.**, 2001. "Market Liquidity and Trading Activity," Journal of Finance, 56(2), pp. 501–530.

[12] **Chordia, T., and Subrahmanyam, A.**, 2004. "Order Imbalance and Individual Stock Returns," Journal of Financial Economics, 72, 485-518.

[13] **Collin-Dufresne, P., Daniel, K., and Sağlam, M.**, 2020. "Liquidity regimes and optimal dynamic asset allocation," Journal of Financial Economics, 136(2), 379-406.

[14] **Constantinides, G. M.**, 1986. "Capital Market Equilibrium with Transaction Costs," Journal of Political Economy, 94(4), 842-862.

[15] **Coval, J., and Stafford, E.**, 2007, "Asset fire sales (and purchases) in equity markets," Journal of Financial Economics, 86, 479–512.

[16] **Dávila, E., Graves, D., and Parlatore, C.**, 2024. "The Value of Arbitrage," Journal of Political Economy, 132(6).

[17] **Davis, M. H. A., and Norman, A. R.**, 1990. "Portfolio Selection with Transaction Costs," Mathematics of Operations Research, 15(4), 676-713.

[18] **Demsetz, H.**, 1968. "The Cost of Transacting," The Quarterly Journal of Economics, 82(1), 33-53.

[19] **Dow, J., Han, J., and Sangiorgi, F.**, 2021. "Hysteresis in price efficiency and the economics of slow moving capital," Review of Financial Studies, 34, 2857–2909.

[20] **Duffie, D.**, 2010. "Presidential Address: Asset Price Dynamics with Slow-Moving Capital," Journal of Finance, 65(4), 1237-1267.

[21] **Duffie, D., Garleanu, N., and Pedersen, L. H.**, 2005. "Over-the-Counter Markets," Econometrica, 73(6), 1815–1847.

[22] **Duffie, D., Garleanu, N., and Pedersen, L. H.**, 2007. "Valuation in Over-the-Counter Markets," Review of Financial Studies, 20(6), 1865–1900.

[23] **Duffie, D., and Strulovici, B.**, 2012. "Capital Mobility and Asset Pricing," Econometrica, 80(6), 2469–2509.

[24] **Duffie, D., and Sun, T.**, 1990. "Transaction Costs and Portfolio Choice in a Discrete-

Continuous-Time Setting," Journal of Economic Dynamics and Control, 14(1), 35-51.

[25] **Dumas, B., and Luciano, E.**, 1991. "An Exact Solution to a Dynamic Portfolio Choice Problem under Transaction Costs," The Journal of Finance, 46(2), 577-595.

[26] **Engle, R., Ferstenberg, R., and Russell, J.**, 2008. "Measuring and Modeling Execution Cost and Risk," Working paper, University of Chicago.

[27] **Gârleanu, N.**, 2009. "Portfolio Choice and Pricing in Illiquid Markets," Journal of Economic Theory, 144(2), 532-564.

[28] **Gârleanu, N., and Pedersen, L. H.**, 2011. "Margin-Based Asset Pricing and Deviations from the Law of One Price," Review of Financial Studies, 24(6), 1980-2022.

[29] **Gârleanu, N., and Pedersen, L. H.**, 2013. "Dynamic Trading with Predictable Returns and Transaction Costs," Journal of Finance, 68(6), 2309-2340.

[30] **Gârleanu, N., and Pedersen, L. H.**, 2016. "Dynamic Portfolio Choice with Frictions," Journal of Economic Theory, 165, 487-516.

[31] **Greenwood, R.**, 2005a, "Short and long term demand curves for stocks: Theory and evidence," Journal of Financial Economics, 75, 607–650.

[32] **Gromb, D., and Vayanos, D.**, 2002. "Equilibrium and welfare in markets with financially constrained arbitrageurs," Journal of Financial Economics, 66(2-3), 361-407.

[33] **Gromb, D., and Vayanos, D.**, 2018. "The Dynamics of Financially Constrained Arbitrage," Journal of Finance, 73(4), 1713-1750.

[34] **Grossman, S. J., and Laroque, G.**, 1990. "Asset Pricing and Optimal Portfolio Choice in the Presence of Illiquid Durable Consumption Goods," Econometrica, 58(1), 25-51.

[35] **Grossman, S. J., and M. H. Miller**, 1986. "Liquidity and Market Structure," Journal of Finance, 43(3), 617-633.

[36] **Haddad, V., Huebner, P., and Loualiche, E.**, 2024. "How Competitive is the Stock Market? Theory, Evidence from Portfolios, and Implications for the Rise of Passive Investing,"

American Economic Review, Forthcoming. Available as a working paper.

[37] **Harris, L., and Gurel, E.**, 1986, "Price and volume effects associated with changes in the S&P 500: New evidence for price pressures," Journal of Finance, 41, 815–830.

[38] **He, Z., and Krishnamurthy, A.**, 2013. "Intermediary Asset Pricing," American Economic Review, 103(2), 732-770.

[39] **Hendershott, T., and Seasholes, M. S.**, 2007. "Market Maker Inventories and Stock Prices," American Economic Review, 97(2), 210-214.

[40] **Koijen, R.S.J. and Gabaix, X.**, 2021. "In Search of the Origins of Financial Fluctuations: The Inelastic Markets Hypothesis," NBER Working Paper No. 28967.

[41] **Jang, B.-G., Koo, H. K., Liu, H., and Loewenstein, M.**, 2007. "Liquidity Premia and Transaction Costs," The Journal of Finance, 62(5), 2329-2366.

[42] **Jarrow, R. A., and Subramanian, A.**, 2001. "The Liquidity Discount," Mathematical Finance, 11(4), 447-474.

[43] **Kondor, P.**, 2009. "Risk in Dynamic Arbitrage: The Price Effects of Convergence Trading," Journal of Finance, 64(2), 638-658.

[44] **Kondor, P., and Vayanos, D.**, 2019. "Liquidity Risk and Arbitrage Capital," Journal of Finance, 74(6), 2695-2732.

[45] **Kyle, A. S., and W. Xiong**, 2001. "Contagion as a Wealth Effect," Journal of Finance, 56(4), 1401-1440.

[46] **Liu, H.**, 2005. "Optimal Consumption and Investment with Transaction Costs and Multiple Risky Assets," The Journal of Finance, 60(1), 219-246.

[47] **Liu, H., and Loewenstein, M.**, 2002. "Optimal Portfolio Selection with Transactions Costs and Finite Horizons," The Review of Financial Studies, 15(3), 805-835.

[48] **Liu, J., and Longstaff, F. A.**, 2004. "Losing Money on Arbitrage: Optimal Dynamic Portfolio Choice in Markets with Arbitrage Opportunities," The Review of Financial Studies,

17(3), 611-641.

[49] **Lo, A. W., H. Mamaysky, and J. Wang**, 2004. "Asset Prices and Trading Volume under Fixed Transactions Costs," Journal of Political Economy, 112(5), 1054-1090.

[50] **Lou, D.**, 2012. "A Flow-Based Explanation for Return Predictability," The Review of Financial Studies, 25(12), 3457–3489.

[51] **Mitchell, M., Pedersen, L. H., and Pulvino, T.**, 2007. "Slow Moving Capital," American Economic Review, 97(2), 215-220.

[52] **Obizhaeva, A. A., and Wang, J.**, 2013. "Optimal Trading Strategy and Supply/Demand Dynamics," Journal of Financial Markets, 16, 1–32.

[53] **Perold, A. F.**, 1988. "The Implementation Shortfall: Paper versus Reality," Journal of Portfolio Management, 14, 4–9.

[54] **Shleifer, A., and Vishny, R. W.**, 1997. "The Limits of Arbitrage," The Journal of Finance, 52(1), 35-55.

[55] **Shreve, S. E., and Soner, H. M.**, 1994. "Optimal Investment and Consumption with Transaction Costs," The Annals of Applied Probability, 4(3), 609-692.

[56] **Tuckman, B. and J. L. Vila**, 1992. "Arbitrage with Holding Costs: A Utility-Based Approach," Journal of Finance, 47(4), 1283-1302.

[57] **Vayanos, D.**, 1998. "Transaction Costs and Asset Prices: A Dynamic Equilibrium Model," Review of Financial Studies, 11(1), 1-58.

[58] **Xiong, W.**, 2001. "Convergence Trading with Wealth Effects: An Amplification Mechanism in Financial Markets," Journal of Financial Economics, 62(2), 247-292.

Appendix A

A.0 Competitive Equilibrium in the Benchmark

I first derive the benchmark value of each trader's value function with $\lambda_a = 0$. Hereafter, I will use distinct notation for variables in the benchmark case, denoting a variable x as x^*.

Assume there is no arbitrage limit, so $\lambda_a = 0$. In this case, $p^*_\eta = p^*_X = 0$, which will be verified later, and investment opportunities for hedgers follow an IID process:

$$P_t^{*(j)} = F_t. \tag{A.1}$$

First, let me derive the value function of arbitrageurs in the benchmark case. Since the law of one price holds, arbitrage opportunities are zero in every period; any such opportunities are immediately eliminated by arbitrageurs. In this frictionless equilibrium, the trading positions of arbitrageurs are determined by the market-clearing condition such that $x_t^{*(a)} = -(-1)^j x_t^{*(j)}$, and their lifetime utility is zero:

$$V^{*(a)} = 0. \tag{A.2}$$

Next, I derive the value function of hedgers in this case. Each hedger solves the following optimization problem:

$$V^{*(j)} = \min_{\tau_t^{*(j)}} E_t \int_t^\infty e^{-r(s-t)} \left(\frac{\gamma \sigma_F^2}{2} \left(x_s^{*(j)} + (-1)^j \frac{\eta_s}{\sigma_F} \right)^2 + \frac{\lambda}{2} \tau_s^{*(j)} \right) ds. \tag{A.3}$$

Note that when $\lambda = 0$, each hedger solves their portfolio problem pointwise and has the following frictionless hedging portfolio:

$$x_{0,t}^{*(j)} = -(-1)^j \frac{1}{\sigma_F} \eta_t. \tag{A.4}$$

For $\lambda > 0$, similar to the frictional case, we set HJB equations and maximize the trading

intensity such that

$$\tau_t^{*(j)} = \frac{1}{\lambda}\frac{\partial V^{*(j)}}{\partial x_t^{(j)}}. \tag{A.5}$$

Plugging (D.5) into the HJB for each hedger, we have the following equation:

$$rV^{*(j)} = -\frac{\gamma}{2}\left(\sigma_F^2(x^{(j)})^2 + 2\sigma_F(-1)^j\eta x^{(j)} + \eta^2\right) + \frac{1}{2\lambda}\left(\frac{\partial V^{*(j)}}{\partial x^{(j)}}\right)^2 + (-\kappa_\eta\eta)\frac{\partial V^{*(j)}}{\partial \eta} + \frac{1}{2}\frac{\partial^2 V^{*(j)}}{\partial \eta^2}\sigma_\eta^2. \tag{A.6}$$

To solve it, I conjecture the value function for each hedger in quadratic form as follows:

$$V_t^{*(j)} = -\frac{1}{2}v_{xx}^{*(j)}(x^{(j)})^2 + \frac{1}{2}v_{\eta\eta}^{*(j)}\eta^2 + v_{x\eta}^{*(j)}x^{(j)}\eta + v_0^{*(j)}, \tag{A.7}$$

and express the value function as:

$$\begin{aligned} rV^{*(j)} = &-\frac{\gamma}{2}\left(\sigma_F^2(x^{(j)})^2 + 2\sigma_F(-1)^j\eta x^{(j)} + \eta^2\right) + \frac{1}{2\lambda}\left((v_{xx}^{*(j)})^2(x^{(j)})^2 + (v_{x\eta}^{*(j)})^2\eta^2 - 2v_{xx}^{*(j)}v_{x\eta}^{*(j)}x^{(j)}\eta\right) \\ &+ (-\kappa_\eta)v_{x\eta}^{*(j)}x^{(j)}\eta + (-\kappa_\eta)v_{\eta\eta}^{*(j)}\eta^2 + \frac{1}{2}v_{\eta\eta}^{*(j)}\sigma_\eta^2. \end{aligned} \tag{A.8}$$

The right-hand side of (D.8) is quadratic in $x_t^{(j)}$ and η_t. Identifying the quadratic and constant terms yields seven scalar equations in $\left(v_{xx}^{*(j)}, v_{x\eta}^{*(j)}, v_{\eta\eta}^{*(j)}, v_0^{*(j)}\right)$ for each $j = 1, 2$:

$$0 = \frac{1}{\lambda}(v_{xx}^{*(j)})^2 + rv_{xx}^{*(j)} - \gamma\sigma_F^2, \tag{A.9}$$

$$0 = -rv_{x\eta}^{*(j)} - (-1)^j\gamma\sigma_F + (-\kappa_\eta)v_{x\eta}^{*(j)} - \frac{v_{xx}^{*(j)}}{\lambda}v_{x\eta}^{*(j)}, \tag{A.10}$$

$$0 = -rv_{\eta\eta}^{*(j)} - \gamma + (-2\kappa_\eta)v_{\eta\eta}^{*(j)} + \frac{\left(v_{x\eta}^{*(j)}\right)^2}{\lambda}, \tag{A.11}$$

$$0 = -rv_0^{*(j)} + \frac{1}{2}\sigma_\eta^2 v_{\eta\eta}^{*(j)}. \tag{A.12}$$

By solving the algebraic equations (D.7-10), we obtain

$$\frac{v_{xx}^{*(j)}}{\lambda}\left(\equiv M_{rate}^{*}\right)=\sqrt{\frac{\gamma\sigma_F^2}{\lambda}+\frac{\rho^2}{4}}-\frac{\rho}{2}, \tag{A.13}$$

$$\begin{aligned}\frac{v_{x\eta}^{*(j)}}{\lambda}&=-\frac{1}{\lambda}\frac{(-1)^j\gamma\sigma_F}{r+\kappa_\eta+\frac{v_{xx}^{*}}{\lambda}}=-\frac{v_{xx}^{*}}{\lambda}\frac{(-1)^j\gamma\sigma_F}{v_{xx}^{*}(r+\kappa_\eta)+\frac{(v_{xx}^{*})^2}{\lambda}}\\&=M_{rate}^{*}(-1)^j\underbrace{\left(-\frac{\gamma\sigma_F}{\gamma\sigma_F^2+\lambda M_{rate}^{*}\kappa_\eta}\right)}_{M_{aim,\eta}^{*}}=M_{rate}^{*}(-1)^jM_{aim,\eta}^{*},\end{aligned} \tag{A.14}$$

$$v_{\eta\eta}^{*(j)}\left(=v_{\eta\eta}^{*}\right)=\frac{1}{r+2\kappa_\eta}\left(-\gamma+\lambda\left(M_{rate}^{*}M_{aim,\eta}^{*}\right)^2\right), \tag{A.15}$$

$$v_0^{*(j)}\left(=v_0^{*}\right)=\frac{\sigma_\eta^2}{2r}v_{\eta\eta}^{*}. \tag{A.16}$$

The market-clearing condition yields $q_X^{*(a)}=M_{rate}^{*}$ and $q_\eta^{*(a)}=M_{rate}^{*}M_{aim,\eta}^{*}$.

A.1 Competitive Equilibrium under Constrained Arbitrage

Let $\mathbf{\Omega}_t\equiv\left(X_t^{(a)},\eta_t\right)$ be the aggregate state vector. We seek a competitive equilibrium where the price function in segmented markets takes the linear form:

$$P_t^{(j)}=F_t-\left(p_X^{(j)}X_t^{(a)}+p_\eta^{(j)}\eta_t\right) \tag{A.17}$$

where $p_X^{(j)}\equiv(-1)^jp_X$ and $p_\eta^{(j)}\equiv(-1)^jp_\eta$ are constant coefficients to be determined in equilibrium. We conjecture the law of motion for aggregate arbitrage capital:

$$\frac{dX_t^{(a)}}{dt}=q_\eta^{(a)}\eta_t-q_X^{(a)}X_t^{(a)} \tag{A.18}$$

where the coefficients $\left(q_X^{(a)},q_\eta^{(a)}\right)$ are to be determined in equilibrium.

Based on our conjectures, investment opportunities for each hedger are postulated in terms

of the aggregate state variables:

$$\begin{aligned} dQ_t^{(j)} &= dP_t^{(j)} + D_t dt - rP_t^{(j)} dt \\ &= dQ_t^F + (r+\kappa_\eta) p_\eta^{(j)} \eta_t dt - p_\eta^{(j)} \sigma_\eta dZ_t^\eta - p_X^{(j)} \left(\frac{dX_t^{(a)}}{dt} - rX_t^{(a)} \right) dt \\ &= \left(\mu_\eta^{(j)} \eta_t + \mu_X^{(j)} X_t^{(a)} \right) dt + \boldsymbol{\sigma}^{(j)\prime} \boldsymbol{dZ_t^Q}. \end{aligned} \tag{A.19}$$

where $\mu_\eta^{(j)} \equiv (r+\kappa_\eta) p_\eta^{(j)} - q_\eta^{(a)} p_X^{(j)}$ and $\mu_X^{(j)} \equiv (r+q_X^{(a)}) p_X^{(j)}$ are the expected returns' exposures to investment factors for hedgers. Additionally, $\boldsymbol{\sigma}^{(j)} \equiv \left[\sigma_F, -p_\eta^{(j)} \sigma_\eta \right]$ and $\boldsymbol{dZ_t^Q} \equiv [dZ_t, dZ_t^\eta]$ represent the return volatility vector and shock processes, respectively.

Using the definition of asset spread $\Delta_t \equiv P_t^{(1)} - P_t^{(2)}$, we describe the arbitrageur's investment opportunities in terms of the aggregate state variables:

$$\begin{aligned} dQ_t^{\Delta_-} &= r\Delta_t dt - d\Delta_t \\ &= \left(2(r+\kappa_\eta) p_\eta \eta_t dt - 2p_X \left(\frac{dX_t^{(a)}}{dt} - rX_t^{(a)} \right) dt \right) + \boldsymbol{\sigma}^{\Delta_-\prime} \boldsymbol{dZ_t^Q} \\ &= \left(\mu_\eta^{\Delta_-} \eta_t + \mu_X^{\Delta_-} X_t^{(a)} \right) dt + \boldsymbol{\sigma}^{\Delta_-\prime} \boldsymbol{dZ_t^Q}, \end{aligned} \tag{A.20}$$

where $\mu_\eta^{\Delta_-} \equiv \mu_\eta^{(2)} - \mu_\eta^{(1)}$ and $\mu_X^{\Delta_-} \equiv \mu_X^{(2)} - \mu_X^{(1)}$ are the expected returns' exposures to investment factors for arbitrageurs. $\boldsymbol{\sigma}^{\Delta_-} \equiv [0, 2p_\eta \sigma_\eta]$ is the return volatility vector.

We formulate the maximization problem for each hedger. The budget constraint for hedger j is given by

$$dW_t^{(j)} = rW_t^{(j)} dt + x_t^{(j)} dQ_{jt} - \frac{\lambda}{2} \left(\tau_t^{(j)} \right)^2 dt + dY_t^{(j)}.$$

The hedger, with mean-variance preferences, maximizes their lifetime utility, given by:

$$\begin{aligned} V^{(j)} \left(x_t^{(j)}, \boldsymbol{\Omega}_t \right) = \max_{\left(\tau_s^{(j)} \right)_{s \geq t}} E_t \int_t^\infty e^{-\rho(s-t)} \Bigg[& \left(\mu_\eta^{(j)} \eta_s + \mu_X^{(j)} X_s^{(a)} \right) x_s^{(j)} \\ & - \frac{\gamma}{2} \left(\Sigma^Q \left(x_s^{(j)} \right)^2 + 2\sigma_F (-1)^j \eta_s x_s^{(j)} + (\eta_s)^2 \right) - \frac{\lambda}{2} \left(\tau_s^{(j)} \right)^2 \Bigg] ds. \end{aligned}$$

where $\Sigma^Q \equiv \boldsymbol{\sigma}^{(j)\prime}\boldsymbol{\sigma}^{(j)}$.

Using dynamic programming, the Hamilton-Jacobi-Bellman (HJB) equation for hedgers is derived as:

$$\begin{aligned}\rho V^{(j)}\left(x^{(j)},\boldsymbol{\Omega}\right) = \max_{\tau^{(j)}}\Bigg\{&\left(\mu_\eta^{(j)}\eta+\mu_X^{(j)}X^{(a)}\right)x^{(j)}-\frac{\gamma}{2}\left(\Sigma^Q\left(x^{(j)}\right)^2+2\sigma_F(-1)^j\eta x^{(j)}+\eta^2\right)\\&-\frac{\lambda}{2}\left(\tau^{(j)}\right)^2+\frac{\partial V^{(j)}}{\partial x^{(j)}}\tau^{(j)}+\frac{\partial V^{(j)}}{\partial \eta}\left(-\kappa_\eta\eta\right)+\frac{1}{2}\frac{\partial^2 V^{(j)}}{\partial \eta^2}\sigma_\eta^2+\frac{\partial V^{(j)}}{\partial X^{(a)}}\frac{dX_t^{(a)}}{dt}\Bigg\}.\end{aligned} \tag{A.21}$$

Next, we formulate the arbitrageur's maximization problem, starting with their budget constraint:

$$dW_t^{(a)} = rW_t^{(a)}dt + \boldsymbol{x}_t^{(a)\prime}\boldsymbol{dQ_t} - \frac{\lambda_a}{2}\left(\boldsymbol{\tau}_t^{(a)}\right)'\left(\boldsymbol{\tau}_t^{(a)}\right)dt.$$

Using market symmetry, the budget constraint can be rewritten as:

$$dW_t^{(a)} = rW_t^{(a)}dt + x_t^{(a)}dQ_{\Delta t}^{-} - \lambda_a\left(\tau_t^{(a)}\right)^2 dt,$$

Thus, the arbitrageur's maximization problem is:

$$\begin{aligned}&V^{(a)}\left(x_t^{(a)},\boldsymbol{\Omega}_t\right)\\&=\max_{\left(\tau_s^{(a)}\right)_{s\geq t}} E_t\int_t^\infty e^{-\rho(s-t)}\left[\left(\mu_\eta^{\Delta^-}\eta_s+\mu_X^{\Delta^-}X_s^{(a)}\right)x_s^{(a)}-\frac{\gamma_a}{2}\Sigma_\Delta^Q\left(x_s^{(a)}\right)^2-\lambda_a\left(\tau_s^{(a)}\right)^2\right]ds.\end{aligned}$$

where $\Sigma_\Delta^Q \equiv \boldsymbol{\sigma}^{\Delta^-\prime}\boldsymbol{\sigma}^{\Delta^-}$. The HJB equation for the arbitrageur is

$$\begin{aligned}\rho V^{(a)}\left(x^{(a)},\boldsymbol{\Omega}\right) = \max_{\tau^{(a)}}\Bigg\{&\left(\mu_\eta^{\Delta^-}\eta+\mu_X^{\Delta^-}X^{(a)}\right)x^{(a)}-\frac{\gamma_a}{2}\Sigma_\Delta^Q\left(x^{(a)}\right)^2-\lambda_a\left(\tau^{(a)}\right)^2\\&+\frac{\partial V^{(a)}}{\partial x^{(a)}}\tau^{(a)}+\frac{\partial V^{(a)}}{\partial \eta}\left(-\kappa_\eta\eta\right)+\frac{1}{2}\frac{\partial^2 V^{(a)}}{\partial \eta^2}\sigma_\eta^2+\frac{\partial V^{(a)}}{\partial X^{(a)}}\frac{dX_t^{(a)}}{dt}\Bigg\}.\end{aligned} \tag{A.22}$$

Maximizing the right-hand side of (A.21) and (A.22) with respect to the trading intensity

yields

$$\tau^{(j)} = \frac{1}{\lambda}\frac{\partial V^{(j)}}{\partial x^{(j)}} \qquad \text{and} \qquad \tau^{(a)} = \frac{1}{2\lambda_a}\frac{\partial V^{(a)}}{\partial x^{(a)}}. \tag{A.23}$$

We conjecture the value function for hedgers in the quadratic form:

$$V^{(j)}\left(x^{(j)}, \mathbf{\Omega}\right) = \frac{1}{2}\mathbf{\Omega}_t^{(j)\prime}\boldsymbol{V}_{\mathbf{\Omega}\mathbf{\Omega}}{}^{(j)}\mathbf{\Omega}_t^{(j)} + v_0^{(j)} \tag{A.24}$$

where $\mathbf{\Omega}_t^{(j)} = \left[x^{(j)}, \mathbf{\Omega}\right]$ and $\boldsymbol{V}_{\mathbf{\Omega}\mathbf{\Omega}}{}^{(j)} = \begin{bmatrix} -v_{xx}^{(j)} & v_{xX}^{(j)} & v_{x\eta}^{(j)} \\ v_{Xx}^{(j)} & -v_{XX}^{(j)} & v_{X\eta}^{(j)} \\ v_{\eta x}^{(j)} & v_{\eta X}^{(j)} & v_{\eta\eta}^{(j)} \end{bmatrix}$ for $j = 1, 2$. Similarly, the arbitrageur's value function is conjectured as

$$V^{(a)}\left(x^{(a)}, \mathbf{\Omega}\right) = \mathbf{\Omega}_t^{(a)\prime}\boldsymbol{V}_{\mathbf{\Omega}\mathbf{\Omega}}{}^{(a)}\mathbf{\Omega}_t^{(a)} + v_0^{(a)} \tag{A.25}$$

where $\mathbf{\Omega}_t^{(a)} = \left[x^{(a)}, \mathbf{\Omega}\right]$ and $\boldsymbol{V}_{\mathbf{\Omega}\mathbf{\Omega}}{}^{(a)} = \begin{bmatrix} -v_{xx}^{(a)} & v_{xX}^{(a)} & v_{x\eta}^{(a)} \\ v_{Xx}^{(a)} & -v_{XX}^{(a)} & v_{X\eta}^{(a)} \\ v_{\eta x}^{(a)} & v_{\eta X}^{(a)} & v_{\eta\eta}^{(a)} \end{bmatrix}$.

Given these conjectured value functions, the optimal trading policies for each trader in (A.23) are:

$$\tau_t^{(j)} = \frac{v_{xx}^{(j)}}{\lambda}\left[\left(\frac{v_{x\eta}^{(j)}\eta_t + v_{xX}^{(j)}X_t^{(a)}}{v_{xx}^{(j)}}\right) - x_t^{(j)}\right], \tag{A.26}$$

$$\tau_t^{(a)} = \frac{v_{xx}^{(a)}}{\lambda_a}\left[\left(\frac{v_{x\eta}^{(a)}\eta_t + v_{xX}^{(a)}X_t^{(a)}}{v_{xx}^{(a)}}\right) - x_t^{(a)}\right]. \tag{A.27}$$

First, we identify the value function coefficients for each hedger. By substituting the optimal

trading intensity from (A.26) into the HJB equation (A.21), we express the HJB equation as:

$$\begin{aligned}\rho V^{(j)} = &\left(\left[(r+\kappa_\eta)p_\eta^{(j)} - q_\eta^{(a)}p_X^{(j)}\right]\eta + \left(r+q_X^{(a)}\right)p_X^{(j)}X^{(a)}\right)x^{(j)} - \frac{\gamma}{2}\left(\Sigma^Q(x^{(j)})^2 + 2\sigma_F(-1)^j\eta x^{(j)} + \eta^2\right)\\ &+ \frac{1}{2\lambda}\left((v_{xx}^{(j)})^2(x^{(j)})^2 + (v_{x\eta}^{(j)})^2\eta^2 + (v_{xX}^{(j)})^2(X^{(a)})^2 - 2v_{xx}^{(j)}v_{x\eta}^{(j)}x^{(j)}\eta - 2v_{xx}^{(j)}v_{xX}^{(j)}x^{(j)}X^{(a)}\right.\\ &\qquad\left. + 2v_{xX}^{(j)}v_{x\eta}^{(j)}X^{(a)}\eta\right) + (-\kappa_\eta)v_{x\eta}^{(j)}x^{(j)}\eta + (-\kappa_\eta)v_{\eta\eta}^{(j)}\eta^2\\ &+ (-\kappa_\eta)v_{X\eta}^{(j)}X^{(a)}\eta + \frac{1}{2}v_{\eta\eta}^{(j)}\sigma_\eta^2 + \left(-v_{XX}^{(j)}X^{(a)} + v_{xX}^{(j)}x^{(j)} + v_{X\eta}^{(j)}\eta\right)\left(q_\eta^{(a)}\eta - q_X^{(a)}X^{(a)}\right)\end{aligned} \tag{A.28}$$

The right-hand side of (A.28) is quadratic in $\mathbf{\Omega}_t^{(j)}$. Identifying the quadratic and constant terms yields seven scalar equations in $\left(v_{xx}^{(j)}, v_{xX}^{(j)}, v_{x\eta}^{(j)}, v_{XX}^{(j)}, v_{X\eta}^{(j)}, v_{\eta\eta}^{(j)}, v_0^{(j)}\right)$ for each hedger:

$$0 = \frac{1}{\lambda}(v_{xx}^{(j)})^2 + rv_{xx}^{(j)} - \gamma\Sigma^Q, \tag{A.29}$$

$$0 = -rv_{xX}^{(j)} + (-1)^j\left(r+q_X^{(a)}\right)p_X - \frac{v_{xx}^{(j)}}{\lambda}v_{xX}^{(j)} - q_X^{(a)}v_{xX}^{(j)}, \tag{A.30}$$

$$0 = -rv_{x\eta}^{(j)} + (-1)^j\left([(r+\kappa_\eta)p_\eta - q_\eta^{(a)}p_X] - \gamma\sigma_F\right) + (-\kappa_\eta)v_{x\eta}^{(j)} - \frac{v_{xx}^{(j)}}{\lambda}v_{x\eta}^{(j)} + q_\eta^{(a)}v_{xX}^{(j)}, \tag{A.31}$$

$$0 = rv_{XX}^{(j)} + \frac{(v_{xX}^{(j)})^2}{\lambda} + 2q_X^{(a)}v_{XX}^{(j)}, \tag{A.32}$$

$$0 = -rv_{X\eta}^{(j)} + (-\kappa_\eta)v_{X\eta}^{(j)} + \frac{v_{xX}^{(j)}}{\lambda}v_{x\eta}^{(j)} - q_\eta^{(a)}v_{XX}^{(j)} - q_X^{(a)}v_{X\eta}^{(j)}, \tag{A.33}$$

$$0 = -rv_{\eta\eta}^{(j)} - \gamma + (-2\kappa_\eta)v_{\eta\eta}^{(j)} + \frac{\left(v_{x\eta}^{(j)}\right)^2}{\lambda} + 2q_\eta^{(a)}v_{X\eta}^{(j)}, \tag{A.34}$$

$$0 = -rv_0^{(j)} + \frac{1}{2}\sigma_\eta^2 v_{\eta\eta}^{(j)}. \tag{A.35}$$

Since the system of the above nonlinear equations exhibits a triangular structure, the simul-

taneous equations can be solved recursively as follows:

$$v_{xx}^{(j)}/\lambda = \sqrt{\frac{\gamma\Sigma^Q}{\lambda} + \frac{r^2}{4}} - \frac{r}{2}, \tag{A.36}$$

$$v_{xX}^{(j)}/\lambda = (-1)^j \left(\frac{1}{\lambda} \frac{\left(r + q_X^{(a)}\right) p_X}{r + v_{xx}^{(j)}/\lambda + q_X^{(a)}} \right), \tag{A.37}$$

$$v_{x\eta}^{(j)}/\lambda = (-1)^j \frac{1}{\lambda} \frac{[(r+\kappa_\eta)p_\eta - q_\eta^{(a)} p_X] - \gamma\sigma_F}{r + v_{xx}^{(j)}/\lambda + \kappa_\eta} + \frac{v_{xX}^{(j)}}{\lambda} \frac{q_\eta^{(a)}}{r + v_{xx}^{(j)}/\lambda + \kappa_\eta}, \tag{A.38}$$

$$v_{XX}^{(j)} = -\frac{1}{r + 2q_X^{(a)}} \frac{(v_{xX}^{(j)})^2}{\lambda}, \tag{A.39}$$

$$v_{X\eta}^{(j)} = -\frac{q_\eta^{(a)} v_{XX}^{(j)} - (v_{x\eta}^{(j)}/\lambda) v_{xX}^{(j)}}{r + \kappa_\eta + q_X^{(a)}}, \tag{A.40}$$

$$v_{\eta\eta}^{(j)} = \frac{2q_\eta^{(a)} v_{X\eta}^{(j)} + (v_{x\eta}^{(j)})^2/\lambda - \gamma}{r + 2\kappa_\eta}, \tag{A.41}$$

$$v_0^{(j)} = \frac{\sigma_\eta^2}{2r} v_{\eta\eta}^{(j)}. \tag{A.42}$$

The optimal trading intensity for each hedger, as given in (A.26), can be expressed as:

$$\tau_t^{(j)} = M_{rate} \left[M_{aim}^{(j)}(\mathbf{\Omega}_t) - x_t^{(j)} \right], \tag{A.43}$$

where $M_{rate} \equiv \sqrt{\frac{\gamma\Sigma^Q}{\lambda} + \frac{\rho^2}{4}} - \frac{\rho}{2}$ is the optimal trading speed of hedgers and their aim portfolio takes the following form:

$$M_{aim}^{(j)}(\mathbf{\Omega}_t) = \frac{\mu_\eta^{(j)}\eta_t - (-1)^j \gamma\sigma_F \eta_t}{\gamma\Sigma^Q + \lambda M_{rate}\kappa_\eta} + \frac{\mu_X^{(j)} X_t^{(a)}}{\gamma\Sigma^Q + \lambda M_{rate} q_X^{(a)}} + v_{xX}^{(j)} \frac{q_\eta^{(a)}\eta_t}{\gamma\Sigma^Q + \lambda M_{rate}\kappa_\eta}. \tag{A.44}$$

By applying the Feynman-Kac formula to (A.30) and (A.31), we can express the aim portfolio of an individual hedger in market j as:

$$M_{aim}^{(j)}(\mathbf{\Omega}_t) = K \int_t^\infty e^{-(r+M_{rate})(s-t)} E_t \left(\frac{\left(\mu_\eta^{(j)} + v_{xX}^{(j)} q_\eta^{(a)} - (-1)^j \gamma\sigma_F\right)\eta_s + \mu_X^{(j)} X_s^{(a)}}{\gamma\Sigma^Q} \right) ds, \tag{A.45}$$

where $K \equiv \frac{\gamma\Sigma^Q}{M_{rate}}$.

Similarly, by substituting the arbitrageur's optimal trading policy from (A.27) into the HJB equation (A.22), we can rewrite their HJB equation as:

$$\begin{aligned} rV^{(a)} = {} & 2\left(\left[(r+\kappa_\eta)p_\eta - q_\eta^{(a)}p_X\right]\eta + \left(r+q_X^{(a)}\right)p_X X^{(a)}\right)x^{(a)} - \frac{\gamma_a}{2}\Sigma_\Delta^Q\left(x^{(a)}\right)^2 \\ & + \frac{1}{4\lambda_a}\Big(4(v_{xx}^{(a)})^2(x^{(a)})^2 + 4(v_{x\eta}^{(a)})^2\eta^2 + 4(v_{xX}^{(a)})^2(X^{(a)})^2 - 8v_{xx}^{(a)}v_{x\eta}^{(a)}x^{(a)}\eta - 8v_{xx}^{(a)}v_{xX}^{(a)}x^{(a)}X^{(a)} \\ & \qquad + 8v_{xX}^{(a)}v_{x\eta}^{(a)}X^{(a)}\eta\Big) + 2(-\kappa_\eta)v_{x\eta}^{(a)}x^{(a)}\eta + 2(-\kappa_\eta)v_{\eta\eta}^{(a)}\eta^2 \\ & + 2(-\kappa_\eta)v_{X\eta}^{(a)}X^{(a)}\eta + v_{\eta\eta}^{(a)}\sigma_\eta^2 + 2\left(-v_{XX}^{(a)}X^{(a)} + v_{xX}^{(a)}x^{(a)} + v_{X\eta}^{(a)}\eta\right)\left(q_\eta^{(a)}\eta - q_X^{(a)}X^{(a)}\right). \end{aligned} \tag{A.46}$$

The right-hand side of (A.46) is quadratic in $\mathbf{\Omega}_t^{(a)}$. By identifying the quadratic and constant terms, we derive seven scalar equations for each arbitrageur:

$$0 = \frac{1}{\lambda_a}(v_{xx}^{(a)})^2 + rv_{xx}^{(a)} - \frac{\gamma_a}{2}\Sigma_\Delta^Q, \tag{A.47}$$

$$0 = -2rv_{xX}^{(a)} + 2\left(r+q_X^{(a)}\right)p_X - 2\frac{v_{xx}^{(a)}}{\lambda_a}v_{xX}^{(a)} - 2q_X^{(a)}v_{xX}^{(a)}, \tag{A.48}$$

$$0 = -2rv_{x\eta}^{(a)} + 2\left[(r+\kappa_\eta)p_\eta - q_\eta^{(a)}p_X\right] + 2(-\kappa_\eta)v_{x\eta}^{(a)} - 2\frac{v_{xx}^{(a)}}{\lambda_a}v_{x\eta}^{(a)} + 2q_\eta^{(a)}v_{xX}^{(a)}, \tag{A.49}$$

$$0 = rv_{XX}^{(a)} + \frac{(v_{xX}^{(a)})^2}{\lambda_a} + 2q_X^{(a)}v_{XX}^{(a)}, \tag{A.50}$$

$$0 = -2rv_{X\eta}^{(a)} + 2(-\kappa_\eta)v_{X\eta}^{(a)} + 2\frac{v_{xX}^{(a)}}{\lambda_a}v_{x\eta}^{(a)} - 2q_\eta^{(a)}v_{XX}^{(a)} - 2q_X^{(a)}v_{X\eta}^{(a)}, \tag{A.51}$$

$$0 = -rv_{\eta\eta}^{(a)} + 2(-\kappa_\eta)v_{\eta\eta}^{(a)} + \frac{\left(v_{x\eta}^{(a)}\right)^2}{\lambda_a} + 2q_\eta^{(a)}v_{X\eta}^{(a)}, \tag{A.52}$$

$$0 = -rv_0^{(a)} + \sigma_\eta^2 v_{\eta\eta}^{(a)}. \tag{A.53}$$

Solving this triangular system of nonlinear equations, we obtain:

$$v_{xx}^{(a)}/\lambda_a = \sqrt{\frac{\gamma_a \Sigma_\Delta^Q}{2\lambda_a} + \frac{r^2}{4}} - \frac{r}{2}, \tag{A.54}$$

$$v_{xX}^{(a)}/\lambda_a = \frac{1}{\lambda_a} \frac{\left(r + q_X^{(a)}\right) p_X}{r + v_{xx}^{(a)}/\lambda_a + q_X^{(a)}}, \tag{A.55}$$

$$v_{x\eta}^{(a)}/\lambda_a = \frac{1}{\lambda_a} \frac{(r + \kappa_\eta)\, p_\eta - q_\eta^{(a)} p_X}{r + v_{xx}^{(a)}/\lambda_a + \kappa_\eta} + \frac{v_{xX}^{(a)}}{\lambda_a} \frac{q_\eta^{(a)}}{r + v_{xx}^{(a)}/\lambda_a + \kappa_\eta}, \tag{A.56}$$

$$v_{XX}^{(a)} = -\frac{1}{r + 2q_X^{(a)}} \frac{(v_{xX}^{(a)})^2}{\lambda_a}, \tag{A.57}$$

$$v_{X\eta}^{(a)} = -\frac{q_\eta^{(a)} v_{XX}^{(a)} - (v_{x\eta}^{(a)}/\lambda_a) v_{xX}^{(a)}}{r + \kappa_\eta + q_X^{(a)}}, \tag{A.58}$$

$$v_{\eta\eta}^{(a)} = \frac{2q_\eta^{(a)} v_{X\eta}^{(a)} + (v_{x\eta}^{(a)})^2/\lambda_a}{r + 2\kappa_\eta}, \tag{A.59}$$

$$v_0^{(a)} = \frac{\sigma_\eta^2}{r} v_{\eta\eta}^{(a)}. \tag{A.60}$$

The optimal trading intensity of each arbitrageur in (A.27) can be expressed as:

$$\tau_t^{(a)} = M_{rate}^{(a)} \left[M_{aim}^{(a)}(\mathbf{\Omega}_t) - x_t^{(a)} \right], \tag{A.61}$$

where $M_{rate}^{(a)} \equiv \sqrt{\frac{\gamma_a \Sigma_\Delta^Q}{2\lambda_a} + \frac{\rho^2}{4}} - \frac{\rho}{2}$ is the optimal trading speed of arbitrageurs, and their aim portfolio takes the following form:

$$M_{aim}^{(a)}(\mathbf{\Omega}_t) = \frac{\mu_\eta^{\Delta^-} \eta_t}{\gamma_a \Sigma_\Delta^Q + 2\lambda_a M_{rate}^{(a)} \kappa_\eta} + \frac{\mu_X^{\Delta^-} X_t^{(a)}}{\gamma_a \Sigma_\Delta^Q + 2\lambda_a M_{rate}^{(a)} q_X^{(a)}} + 2v_{xX}^{(a)} \frac{q_\eta^{(a)} \eta_t}{\gamma_a \Sigma_\Delta^Q + 2\lambda_a M_{rate}^{(a)} \kappa_\eta}. \tag{A.62}$$

Applying the Feynman-Kac formula to (A.48) and (A.49), the aim portfolio of each arbitrageur can be further expressed as:

$$M_{aim}^{(a)}(\mathbf{\Omega}_t) = K^{(a)} \int_t^\infty e^{-(r + M_{rate}^{(a)})(s-t)} E_t \left(\frac{\left(\mu_\eta^{\Delta^-} + 2v_{xX}^{(a)} q_\eta^{(a)}\right) \eta_s + \mu_X^{\Delta^-} X_s^{(a)}}{\gamma_a \Sigma_\Delta^Q} \right) ds, \tag{A.63}$$

where $K^{(a)} \equiv \frac{\gamma_a \Sigma_\Delta^Q}{M_{rate}^{(a)}}$.

In a symmetric competitive equilibrium, each arbitrageur's trading intensity $\tau_t^{(a)}$ must simultaneously solve the arbitrageur's optimization problem and satisfy the market-clearing condition. Since an individual arbitrageur's optimal trading intensity is proportional to their own capital level and its aggregate level, the trading intensity of all arbitrageurs can be aggregated by substituting the individual inventory level with the aggregate one. To solve for $q_X^{(a)}$ and $q_\eta^{(a)}$, we impose the consistency condition:

$$x_t^{(a)} = X_t^{(a)}, \tag{A.64}$$

and solve the following fixed-point problem:

$$\tau^{(a)}\left(= \frac{dx_t^{(a)}}{dt}\right) = \frac{dX_t^{(a)}}{dt}. \tag{A.65}$$

Using the consistency condition (A.64) and the expression for the optimal trading policy in (A.27), we express the representative arbitrageur's optimal trading intensity as

$$\tau_t^{(a)} = \frac{v_{x\eta}^{(a)}}{\lambda_a}\eta_t - \left(M_{rate}^{(a)} - \frac{v_{xX}^{(a)}}{\lambda_a}\right) X_t^{(a)}. \tag{A.66}$$

From (A.65) and (A.66), we obtain the fixed-point problem for $q_X^{(a)}$:

$$q_X^{(a)} = M_{rate}^{(a)} - \frac{v_{xX}^{(a)}}{\lambda_a}. \tag{A.67}$$

Solving for $q_X^{(a)}$, we find

$$\begin{aligned} q_X^{(a)} &= M_{rate}^{(a)} - \frac{1}{\lambda_a}\frac{\left(r + q_X^{(a)}\right) p_X}{r + M_{rate}^{(a)} + q_X^{(a)}}, \\ &= -\left(\frac{r}{2} + \frac{p_X}{2\lambda_a}\right) + \sqrt{\left(\frac{r}{2} + \frac{p_X}{2\lambda_a}\right)^2 + \frac{\gamma_a \Sigma_\Delta^Q - 2r p_X}{2\lambda_a}}. \end{aligned} \tag{A.68}$$

The coefficient $q_\eta^{(a)}$ solves the following fixed-point problem:

$$q_\eta^{(a)} = \frac{1}{\lambda_a} \frac{(r+\kappa_\eta)\, p_\eta - q_\eta^{(a)} p_X}{r + M_{rate}^{(a)} + \kappa_\eta} + \frac{v_{xX}^{(a)}}{\lambda_a} \frac{q_\eta^{(a)}}{r + M_{rate}^{(a)} + \kappa_\eta}. \tag{A.69}$$

Solving for $q_\eta^{(a)}$, we express $q_\eta^{(a)}$ as:

$$\begin{aligned} q_\eta^{(a)} &= \frac{1}{1 - \frac{1}{\lambda_a} \frac{v_{xX}^{(a)} - p_X}{r + M_{rate}^{(a)} + \kappa_\eta}} \left(\frac{1}{\lambda_a} \frac{(r+\kappa_\eta)\, p_\eta}{r + M_{rate}^{(a)} + \kappa_\eta} \right), \\ &= \frac{1}{1 + \frac{p_X}{\lambda_a} \frac{M_{rate}^{(a)}}{\left(r + M_{rate}^{(a)} + \kappa_\eta\right)\left(r + M_{rate}^{(a)} + q_X^{(a)}\right)}} \left(\frac{1}{\lambda_a} \frac{(r+\kappa_\eta)\, p_\eta}{r + M_{rate}^{(a)} + \kappa_\eta} \right). \end{aligned} \tag{A.70}$$

From the definition of the arbitrage portfolio $X_{jt}^{(a)} = (-1)^j X_t^{(a)}$, the equilibrium trading intensity of the arbitrageur in market j can be written as:

$$\frac{dX_{jt}^{(a)}}{dt} = q_\eta^{(a)} \eta_t^{(j)} - q_X^{(a)} X_{jt}^{(a)}. \tag{A.71}$$

where $\eta_t^{(j)} \equiv (-1)^j \eta_t$. Similarly, by imposing the aggregation condition $x_t^{(j)} = X_t^{(j)}$ and using the market-clearing condition $X_{jt}^{(a)} = -X_t^{(j)}$, the optimal trading policy for the representative hedger is written as:

$$\tau_t^{(j)} = \frac{v_{x\eta}}{\lambda} \eta_t^{(j)} - \left(M_{rate} + \frac{v_{xX}}{\lambda} \right) X_t^{(j)}. \tag{A.72}$$

where $v_{x\eta} \equiv (-1)^j v_{x\eta}^{(j)}$ and $v_{xX} \equiv (-1)^j v_{xX}^{(j)}$.

Using the expression for the value function coefficients, we also derive the law of motion for the aggregate inventory of hedgers:

$$\frac{dX_t^{(j)}}{dt} = q_\eta \eta_t^{(j)} - q_X X_t^{(j)}. \tag{A.73}$$

where the trading coefficients for the representative hedger are determined by

$$q_X = M_{rate} + \frac{1}{\lambda}\frac{(r+q_X)\,p_X}{r+M_{rate}+q_X} \tag{A.74}$$
$$= -\left(\frac{r}{2}-\frac{p_X}{2\lambda}\right)+\sqrt{\left(\frac{r}{2}-\frac{p_X}{2\lambda}\right)^2+\frac{\gamma\Sigma^Q+rp_X}{\lambda}},$$
$$q_\eta = \frac{1}{1-\frac{p_X}{\lambda}\frac{M_{rate}}{(r+M_{rate}+\kappa_\eta)\left(r+M_{rate}+q_X^{(a)}\right)}}\left(\frac{1}{\lambda}\frac{(r+\kappa_\eta)\,p_\eta-\gamma\sigma_F}{r+M_{rate}+\kappa_\eta}\right). \tag{A.75}$$

Since the market-clearing condition for each market must hold at every point in time, the equilibrium response of each hedger and arbitrageur to changes in $\mathbf{\Omega}_t$ satisfies the system of two algebraic equations:

$$q_X(p_X,p_\eta)-q_X^{(a)}(p_X,p_\eta)=0, \tag{A.76}$$
$$q_\eta(p_X,p_\eta)+q_\eta^{(a)}(p_X,p_\eta)=0. \tag{A.77}$$

If a set of prices exists that satisfies these conditions, the competitive equilibrium is established as defined by our model.

A.2 Proposition 4

Consider an autarkic economy where the trading cost for arbitrageurs approaches infinity, $\lambda_a \uparrow \infty$. In this scenario, transactions become *inactive* regardless of changes in the state variables $\mathbf{\Omega}_t$. This implies that the price must adjust to changes in the state variables such that the trading coefficients q_X in (A.74) and q_η in (A.75) become zero:

$$p_\eta^{aut} = \frac{\gamma\sigma_F}{r+\kappa_\eta}, \tag{A.78}$$
$$p_X^{aut} = \frac{\gamma\left(\sigma_F^2+p^{aut}\sigma_\eta^2\right)}{r}. \tag{A.79}$$

To show that $p_\eta^{aut} = \max p_\eta(\lambda_a)$ and $|p_X^{aut}| = \max|p_X(\lambda_a)|$, first choose a sufficiently large $\hat{\lambda}_a > 0$ and define the compact set $[0,\hat{\lambda}_a]$ for λ_a. Since the equilibrium price coefficients $|p_X(\lambda_a)|$

and $p_\eta(\lambda_a)$ are continuous on the compact interval, by the extreme value theorem, they must attain their maximum values within the interval. Thus, there exist point λ_a^* and λ_a^{**} within $[0, \hat{\lambda}_a]$ such that

$$p_\eta(\lambda_a^*) = \max_{\lambda_a \in [0,\hat{\lambda}_a]} p_\eta(\lambda_a) \qquad and \qquad |p_X(\lambda_a^{**})| = \max_{\lambda_a \in [0,\hat{\lambda}_a]} |p_X(\lambda_a)|.$$

As $\lambda_a \uparrow \infty$, the market reaches an autarkic state, and the price coefficients converge to the autarkic values p_η^{aut} and p_X^{aut}. By the continuity assumption, for any $\epsilon > 0$, there exists some $\hat{\lambda}_a$ such that for all $\lambda_a > \hat{\lambda}_a$:

$$|p_\eta(\lambda_a) - p_\eta^{aut}| < \epsilon \qquad and \qquad |p_X(\lambda_a) - p_X^{aut}| < \epsilon.$$

Given that p_η^{aut} and p_X^{aut} are the limiting values as $\lambda_a \uparrow \infty$, and that we have already identified a point within the compact set $[0, \hat{\lambda}_a]$ where the maximum of $p_\eta(\lambda_a)$ and $|p_X(\lambda_a)|$ are attainched, it follows that p_η^{aut} must be the overall maximum value of $p_\eta(\lambda_a)$ over the interval $[0, \infty)$. Similarly, $|p_X^{aut}|$ must be the overall maximum of $|p_X(\lambda_a)|$ over the interval $[0, \infty)$.

A.3 Proposition 5 and Proposition 6

We omit the derivation of the equilibrium trading coefficients and price coefficients for the case of riskless arbitrage, as they can be easily derived as a special case of the risky arbitrage scenario, which will be addressed later. By evaluating the solution obtained under risky arbitrage at $\sigma_\eta = 0$, we derive the following solutions for the trading and price coefficients:

$$\bar{q}_X^{(a)} = \sqrt{\frac{\gamma \sigma_F^2}{\lambda + \lambda_a} + \frac{r^2}{4}} - \frac{r}{2}, \tag{A.80}$$

$$\bar{q}_\eta^{(a)} = \frac{\gamma \sigma_F}{(\lambda + \lambda_a)\left(r + \kappa_\eta + \bar{q}_X^{(a)}\right)}, \tag{A.81}$$

$$\bar{p}_X = -\lambda_a \bar{q}_X^{(a)}, \tag{A.82}$$

$$\bar{p}_\eta = \lambda_a \bar{q}_\eta^{(a)}. \tag{A.83}$$

By the definition $\bar{M}_{aim,\eta} \equiv \frac{\bar{q}_\eta^{(a)}}{\bar{q}_X^{(a)}}$, we have

$$\bar{M}_{aim,\eta}^{(a)} = \frac{\gamma\sigma_F}{\gamma\sigma_F^2 + (\lambda + \lambda_a)\bar{q}_X^{(a)}}. \tag{A.84}$$

Recall that $q_X^{*(a)} = \sqrt{\frac{\gamma\sigma_F^2}{\lambda} + \frac{r^2}{4}} - \frac{r}{2}$ and $q_\eta^{*(a)} = \frac{\gamma\sigma_F}{\lambda\left(r+\kappa_\eta+q_X^{*(a)}\right)}$. It is straightforward that $\bar{\boldsymbol{q}} > \boldsymbol{q}^{aut}$ and $\boldsymbol{p} > \boldsymbol{p}^{\boldsymbol{*}}$. Since Proposition 4 establishes that $\boldsymbol{p}^{aut} > \bar{\boldsymbol{p}}$, it remains to demonstrate that $\boldsymbol{q}^{\boldsymbol{*}} > \bar{\boldsymbol{q}}$. A direct comparison between $\bar{q}_X^{(a)}$ in $(A.80)$ and $q_X^{*(a)}$ confirms that $q_X^{*(a)} > \bar{q}_X^{(a)}$.

Next, to prove $q_\eta^{*(a)} > \bar{q}_\eta^{(a)}$, consider the following inequality:

$$\begin{aligned}
q_\eta^{*(a)} > \bar{q}_\eta^{(a)} \;&\Leftrightarrow\; \frac{1}{\lambda\left(r+\kappa_\eta+q_X^{*(a)}\right)} > \frac{1}{(\lambda+\lambda_a)\left(r+\kappa_\eta+\bar{q}_X^{(a)}\right)},\\
&\Leftrightarrow\; \lambda_a\left(r+\kappa_\eta\right) + (\lambda+\lambda_a)\,\bar{q}_X^{(a)}(r+\bar{q}_X^{(a)})\left(\frac{r+q_X^{*(a)}}{r+\bar{q}_X^{(a)}}\right) > \lambda q_X^{*(a)}(r+q_X^{*(a)}),\\
&\Leftrightarrow\; \lambda_a\left(r+\kappa_\eta\right) + \gamma\sigma_F^2\left(\frac{q_X^{*(a)}-\bar{q}_X^{(a)}}{r+\bar{q}_X^{(a)}}\right) > 0.
\end{aligned}$$

For the second part, we show that $\bar{\boldsymbol{q}}$ decreases in λ_a. It is evident that $\bar{q}_X^{(a)}$ decreases as λ_a increases. Furthermore, since the function $\lambda_a\bar{q}_X^{(a)}$ in the denominator of $\bar{q}_\eta^{(a)}$ increases with λ_a, it follows that $\bar{q}_\eta^{(a)}$ also decreases as λ_a increases.

By taking a first-order Taylor expansion of the functions defined in (A.80–A.84) around $\lambda_a = 0$, we can derive the expressions for Equations (37–42) in Proposition 6.

A.4 Proposition 7

We derive the equilibrium expressions for the trading coefficients $\left(q_X^{(a)}, q_\eta^{(a)}\right)$ and the price coefficients (p_η, p_X). From the optimality condition for $q_X^{(a)}$ in (A.69) and q_X in (A.75), we obtain

the following equations:

$$\lambda_a \left(r + q_X^{(a)}\right) q_X^{(a)} = \frac{\gamma_a}{2}\Sigma_\Delta^Q - \left(r + q_X^{(a)}\right) p_X, \tag{A.85}$$

$$\lambda \left(r + q_X\right) q_X = \gamma \Sigma^Q + \left(r + q_X\right) p_X. \tag{A.86}$$

By applying the market-clearing conditions and solving for $q_X^{(a)}$, we find:

$$q_X^{(a)} = \sqrt{\frac{\gamma\Sigma^Q + (\gamma_a/2)\Sigma_\Delta^Q}{\lambda + \lambda_a} + \frac{r^2}{4}} - \frac{r}{2} \geq 0. \tag{A.87}$$

After extensive computation, we can also express the equilibrium price coefficient p_X by

$$\begin{aligned} p_X &= -\left(\frac{\lambda_a \gamma \Sigma^Q - \lambda(\gamma_a/2)\Sigma_\Delta^Q}{\gamma\Sigma^Q + (\gamma_a/2)\Sigma_\Delta^Q}\right) q_X^{(a)}, \\ &= -\lambda_a q_X^{(a)} + (\lambda + \lambda_a)\psi_a q_X^{(a)} = -\left[\lambda_a - (\lambda + \lambda_a)\psi_a\right] q_X^{(a)}, \end{aligned} \tag{A.88}$$

where $\psi_a \equiv \frac{\gamma_a \Sigma_\Delta^Q}{2\gamma\Sigma^Q + \gamma_a \Sigma_\Delta^Q}$.

Lemma 1. *For a small σ_η, p_η is an increasing function of γ_a.*

Define the function $f(\gamma_a) \equiv \lambda_a - (\lambda + \lambda_a)\psi_a$. The function $f(\gamma_a)$ is continuous and strictly decreasing in γ_a because ψ_a is an increasing and continuous function of γ_a by Lemma 1. Evaluating the endpoints, we have:

$$f(0) = \lambda_a > 0 \quad \text{and} \quad \lim_{\gamma_a \to \infty} f(\gamma_a) = -\lambda < 0.$$

By the Intermediate Value Theorem, there must be at least one value $\hat{\gamma}_a$ such that $f(\hat{\gamma}_a) = 0$. Because $f(\gamma_a)$ is strictly decreasing, this cutoff value $\hat{\gamma}_a$ is unique. Thus, $p_X = -f(\gamma_a) q_X^{(a)}$ changes sign exactly at this unique cutoff $\hat{\gamma}_a$. For $\gamma_a < \hat{\gamma}_a$, $p_X < 0$, and for $\gamma_a > \hat{\gamma}_a$, $p_X > 0$.

From the optimality conditions for $q_\eta^{(a)}$ in (A.70) and q_η in (A.75), we can express the price

coefficient p_η as

$$p_\eta = \frac{\gamma\sigma_F}{r+\kappa_\eta}\left(\frac{\Lambda_a}{\Lambda+\Lambda_a}\right), \tag{A.89}$$

where $\Lambda \equiv \lambda\left(r+M_{rate}+\kappa_\eta\right) - p_X \frac{M_{rate}}{r+M_{rate}+q_X^{(a)}}$ and $\Lambda_a \equiv \lambda_a(r+M_{rate}^{(a)}+\kappa_\eta) + p_X \frac{M_{rate}^{(a)}}{r+M_{rate}^{(a)}+q_X^{(a)}}$.

Substituting p_η in (A.89) into $q_\eta^{(a)}$ in (A.70), we obtain:

$$q_\eta^{(a)} = \frac{\gamma\sigma_F}{\Lambda+\Lambda_a}. \tag{A.90}$$

Then, we can rewrite the equilibrium price coefficient p_η in terms of $q_\eta^{(a)}$ as follows:

$$\begin{aligned}
p_\eta &= \frac{\gamma\sigma_F}{\Lambda+\Lambda_a}\left(\frac{\Lambda_a}{r+\kappa_\eta}\right), \\
&= q_\eta^{(a)} \frac{\lambda_a(r+M_{rate}^{(a)}+\kappa_\eta) + p_X \frac{M_{rate}^{(a)}}{r+M_{rate}^{(a)}+q_X^{(a)}}}{r+\kappa_\eta}, \\
&= \lambda_a q_\eta^{(a)} + q_\eta^{(a)} \frac{\lambda_a M_{rate}^{(a)} + p_X \frac{M_{rate}^{(a)}}{r+M_{rate}^{(a)}+q_X^{(a)}}}{r+\kappa_\eta}, \\
&= \lambda_a q_\eta^{(a)} + q_\eta^{(a)} \frac{\lambda_a M_{rate}^{(a)}(r+M_{rate}^{(a)}+q_X^{(a)}) + \left(-\lambda_a q_X^{(a)} + \psi_a(\lambda+\lambda_a) q_X^{(a)}\right) M_{rate}^{(a)}}{(r+\kappa_\eta)(r+M_{rate}^{(a)}+q_X^{(a)})}, \\
&= \lambda_a q_\eta^{(a)} + q_\eta^{(a)} \frac{(\gamma_a/2)\Sigma_\Delta^Q + \psi_a(\lambda+\lambda_a) q_X^{(a)} M_{rate}^{(a)}}{(r+\kappa_\eta)(\rho+M_{rate}^{(a)}+q_X^{(a)})}, \\
&= \lambda_a q_\eta^{(a)} + \psi_a(\lambda+\lambda_a) q_\eta^{(a)} \frac{\frac{\gamma\Sigma^Q + (\gamma_a/2)\Sigma_\Delta^Q}{\lambda+\lambda_a} + q_X^{(a)} M_{rate}^{(a)}}{(r+\kappa_\eta)(r+M_{rate}^{(a)}+q_X^{(a)})}, \\
&= \lambda_a q_\eta^{(a)} + \psi_a(\lambda+\lambda_a) q_\eta^{(a)} \left(\frac{q_X^{(a)}(r+q_X^{(a)}) + q_X^{(a)} M_{rate}^{(a)}}{(r+\kappa_\eta)(r+M_{rate}^{(a)}+q_X^{(a)})}\right), \\
&= \lambda_a q_\eta^{(a)} + \psi_a(\lambda+\lambda_a) q_\eta^{(a)} \frac{q_X^{(a)}}{r+\kappa_\eta}.
\end{aligned} \tag{A.91}$$

Now, we claim that

$$(\lambda+\lambda_a)\, q_X^{(a)} = \lambda M_{rate} + \lambda_a M_{rate}^{(a)} - p_X\left(\frac{M_{rate}}{r+M_{rate}+q_X^{(a)}} - \frac{M_{rate}^{(a)}}{r+M_{rate}^{(a)}+q_X^{(a)}}\right). \tag{A.92}$$

Starting from the left-hand side, we will show that this holds indeed as follows:

$$\begin{aligned}
(\lambda+\lambda_a)\, q_X^{(a)} &= \frac{(\lambda+\lambda_a)\, q_X^{(a)}(r+q_X^{(a)})}{r+q_X^{(a)}}, \\
&= \frac{\gamma\Sigma^Q + (\gamma_a/2)\Sigma_\Delta^Q}{r+q_X^{(a)}}, \\
&= \frac{\lambda M_{rate}\,(r+M_{rate}) + \lambda_a M_{rate}^{(a)}\left(r+M_{rate}^{(a)}\right)}{r+q_X^{(a)}}, \\
&= \lambda M_{rate} + \lambda_a M_{rate}^{(a)} + \lambda M_{rate}\frac{M_{rate}-q_X^{(a)}}{r+q_X^{(a)}} + \lambda_a M_{rate}^{(a)}\frac{M_{rate}^{(a)}-q_X^{(a)}}{r+q_X^{(a)}} \\
&= \lambda M_{rate} + \lambda_a M_{rate}^{(a)} - p_X\left(\frac{M_{rate}}{r+M_{rate}+q_X^{(a)}} - \frac{M_{rate}^{(a)}}{r+M_{rate}^{(a)}+q_X^{(a)}}\right).
\end{aligned} \tag{A.93}$$

Therefore, $q_\eta^{(a)}$ can be also written as :

$$q_\eta^{(a)} = \frac{\gamma\sigma_F}{(\lambda+\lambda_a)\left(r+\kappa_\eta+q_X^{(a)}\right)} \geq 0, \tag{A.94}$$

and $M_{aim,\eta}^{(a)}$ is expressed by

$$M_{aim,\eta}^{(a)} = \frac{\gamma\sigma_F}{\gamma\Sigma^Q + (\gamma_a/2)\Sigma_\Delta^Q + (\lambda+\lambda_a)\kappa_\eta q_X^{(a)}} \geq 0. \tag{A.95}$$

By using the expressions for p_X in (A.88) and p_η in (A.91), we can express $\frac{p_X}{q_X^{(a)}} + \frac{p_\eta}{q_\eta^{(a)}}$ as:

$$\frac{p_X}{q_X^{(a)}} + \frac{p_\eta}{q_\eta^{(a)}} = (\lambda+\lambda_a)\,\psi_a\left(\frac{r+\kappa_\eta+q_X^{(a)}}{r+\kappa_\eta}\right). \tag{A.96}$$

A.5 Proposition 8

Let $\lambda \in (0,\infty)$. For $\gamma_a \in [\hat{\gamma}_a, \infty)$, the economy falls into the Risk-On regime, and the following inequality holds:

$$\frac{\gamma\sigma_F^2}{\lambda} \leq \left(\frac{2\gamma_a}{\lambda_a} - \frac{\gamma}{\lambda}\right) p_\eta^2(\gamma_a)\sigma_\eta^2. \tag{A.97}$$

Since this inequality implies that:

$$\frac{\gamma\sigma_F^2}{\lambda} < \left(\frac{2\gamma_a}{\lambda_a} + \frac{\gamma}{\lambda_a}\right) p_\eta^2(\gamma_a)\sigma_\eta^2, \tag{A.98}$$

we have the equivalence:

$$\frac{\gamma\sigma_F^2}{\lambda} < \left(\frac{2\gamma_a}{\lambda_a} + \frac{\gamma}{\lambda_a}\right) p_\eta^2(\gamma_a)\sigma_\eta^2 \qquad \Leftrightarrow \qquad q_X^{*(a)} < q_X^{(a)}(\gamma_a), \tag{A.99}$$

for $\gamma_a \in [\hat{\gamma}_a, \infty)$. Additionally, since $q_X^{(a)} \geq q_X^{*(a)}$ for $\gamma_a \in [\hat{\gamma}_a, \infty)$, the following inequality also holds:

$$\frac{1}{\lambda\left(r + \kappa_\eta + q_X^{*(a)}\right)} > \frac{1}{(\lambda + \lambda_a)\left(r + \kappa_\eta + q_X^{(a)}\right)} \qquad \Leftrightarrow \qquad q_\eta^{*(a)} > q_\eta^{(a)}(\gamma_a). \tag{A.100}$$

Define $Supp^{normal} \equiv [0, \gamma_a']$ and choose γ_a' such that

$$q_X^{*(a)} = q_X^{(a)}(\gamma_a') \qquad \Leftrightarrow \qquad \frac{\gamma\sigma_F^2}{\lambda} = \left(\frac{2\gamma_a'}{\lambda_a} + \frac{\gamma}{\lambda_a}\right) p_\eta^2(\gamma_a')\sigma_\eta^2. \tag{A.101}$$

By rearranging, Equation (A.101) implies that:

$$\frac{\gamma\sigma_F^2}{\lambda} - \frac{2\gamma_a'}{\lambda_a} p_\eta^2(\gamma_a')\sigma_\eta^2 = \frac{\gamma}{\lambda_a} p_\eta^2(\gamma_a')\sigma_\eta^2. \tag{A.102}$$

Since $\frac{\gamma}{\lambda_a} p_\eta^2(\gamma_a)\sigma_\eta^2 > -\frac{\gamma}{\lambda} p_\eta^2(\gamma_a)\sigma_\eta^2$ holds at $\gamma_a = \gamma_a'$, it follows that:

$$\frac{\gamma\sigma_F^2}{\lambda} > \left(\frac{2\gamma_a'}{\lambda_a} - \frac{\gamma}{\lambda}\right) p_\eta^2(\gamma_a')\sigma_\eta^2. \tag{A.103}$$

Therefore, $\gamma_a' < \hat{\gamma}_a$ where $\hat{\gamma}_a$ is defined such that

$$\frac{\gamma\sigma_F^2}{\lambda} = \left(\frac{2\hat{\gamma}_a}{\lambda} - \frac{\gamma}{\lambda}\right) p_\eta^2(\hat{\gamma}_a)\sigma_\eta^2. \tag{A.104}$$

Since $\gamma_a' < \hat{\gamma}_a$, it follows that $Supp^{normal} \subset [0, \hat{\gamma}_a)$. Thus, for $\gamma_a \in Supp^{normal}$, $q_X^{*(a)} > q_X^{(a)}$ holds. To show that $q_\eta^{*(a)} > q_\eta^{(a)}$ for $\gamma_a \in Supp^{normal}$, consider the following inequalities:

$$\frac{1}{\lambda\left(r+\kappa_\eta+q_X^{*(a)}\right)} > \frac{1}{(\lambda+\lambda_a)\left(r+\kappa_\eta+q_X^{(a)}(\gamma_a)\right)},$$

which is equivalent to:

$$\lambda_a\left(r+\kappa_\eta\right)+(\lambda+\lambda_a)q_X^{(a)}(\gamma_a) > \lambda q_X^{*(a)},$$

further leading to:

$$\lambda_a\left(r+\kappa_\eta\right)\left(r+q_X^{*(a)}\right)+p_\eta^2(\gamma_a)\left(\frac{r+q_X^{*(a)}}{r+q_X^{(a)}(\gamma_a)}\right)+\gamma\sigma_F^2\left(\frac{q_X^{*(a)}-q_X^{(a)}(\gamma_a)}{r+q_X^{(a)}(\gamma_a)}\right) > 0.$$

This inequality confirms that $q_\eta^{*(a)} > q_\eta^{(a)}(\gamma_a)$ for $\gamma_a \in Supp^{normal}$.

A.6 Proposition 11

For each trader, define their wealth redistribution processes $d\tilde{W}_t^{(j)}$ and $d\tilde{W}^{(a)}$ as follows:

$$\tilde{W}_t^{(j)} \equiv \tilde{W}_0^{(j)} + \int_0^t \frac{\theta}{2}\left(\tau_s^{(a)}\right)^2 ds, \tag{A.105}$$

$$\tilde{W}_t^{(a)} \equiv \tilde{W}_0^{(a)} + \int_0^t \theta\left(\tau_s^{(a)}\right)^2 ds, \tag{A.106}$$

where $\theta \in [0, \lambda_a]$. The revised wealth processes of each hedger and arbitrageur become:

$$d\left(W_t^{(j)}+\tilde{W}_t^{(j)}\right) = rW_t^{(j)}dt + x_t^{(j)}dQ_t^{(j)} - \frac{\lambda+\theta}{2}\left(\tau_t^{(j)}\right)^2 dt + dY_t^{(j)}, \tag{A.107}$$

$$d\left(W_t^{(a)}+\tilde{W}_t^{(a)}\right) = rW_t^{(a)}dt + x_t^{(a)}dQ_t^{\Delta_-} - (\lambda_a-\theta)\left(\tau_t^{(a)}\right)^2 dt. \tag{A.108}$$

Based on (A.107) and (A.108) and the mean-variance preferences for each trader, the maxi-

mization problem for each trader can be rewritten as:

$$V_\theta^{(j)}\left(x_t^{(j)};\mathbf{\Omega}_t\right) = \max_{\left(\tau_{\theta,s}^{(j)}\right)_{s\geq t}} E_t\int_t^\infty e^{-r(s-t)}\left[\left(\mu_{\eta,\theta}^{(j)}\eta_s+\mu_{X,\theta}^{(j)}X_s^{(a)}\right)x_s^{(j)}\right.$$
$$\left.-\frac{\gamma}{2}\left(\Sigma_\theta^Q\left(x_s^{(j)}\right)^2+2\sigma_F(-1)^j\eta_s x_s^{(j)}+(\eta_s)^2\right)-\frac{\tilde{\lambda}}{2}\left(\tau_{\theta,s}^{(j)}\right)^2\right]ds,$$
$$V_\theta^{(a)}\left(x_t^{(a)};\mathbf{\Omega}_t\right) = \max_{\left(\tau_{\theta,s}^{(a)}\right)_{s\geq t}} E_t\int_t^\infty e^{-r(s-t)}\left[\left(\mu_{\eta,\theta}^{\Delta_-}\eta_s+\mu_{X,\theta}^{\Delta_-}X_s^{(a)}\right)x_s^{(a)}\right.$$
$$\left.-\frac{\gamma_a}{2}\Sigma_{\Delta,\theta}^Q\left(x_s^{(a)}\right)^2-\tilde{\lambda}_a\left(\tau_{\theta,s}^{(a)}\right)^2\right]ds,$$

where $\tilde{\lambda}\equiv\lambda+\theta$ and $\tilde{\lambda}_a\equiv\lambda_a-\theta$. By replacing the cost parameters with these modified ones, the solution method can be directly applied to the modified problem.

Case I ($\theta=\lambda_a$). Here, each arbitrageur executes their trading as if they face no constraints. Hence, $p_{X,\theta}=0$ and $p_{\eta,\theta}=0$ in equilibrium. Moreover, the equilibrium trading coefficients become $q_{X,\theta}^{(a)}=\sqrt{\frac{\gamma\sigma_F^2}{\lambda+\lambda_a}+\frac{r^2}{4}}-\frac{r}{2}$ and $q_{\eta,\theta}^{(a)}=\frac{\gamma\sigma_F}{(\lambda+\lambda_a)\left(r+\kappa_\eta+q_X^{(a)}\right)}$. The law of motion for arbitrage capital is given by $\frac{dX_{\theta,jt}^{(a)}}{dt}=-\frac{dX_{\theta,t}^{(j)}}{dt}$ with

$$\frac{dX_{\theta,t}^{(j)}}{dt}=M_{rate}^\theta\left((-1)^jM_{aim,\eta}^\theta\eta_t-X_t^{(j)}\right), \tag{A.109}$$

where $M_{rate}^\theta\equiv\sqrt{\frac{\gamma\sigma_F^2}{\lambda+\lambda_a}+\frac{r^2}{4}}-\frac{r}{2}$ and $M_{aim,\eta}^\theta\equiv-\frac{\gamma\sigma_F}{\gamma\sigma_F^2+(\lambda+\lambda_a)M_{rate}^\theta\kappa_\eta}$.

Case II ($\theta\in[0,\lambda_a)$). The trading coefficients in θ-equilibrium are characterized as:

$$q_{X,\theta}^{(a)}=\sqrt{\frac{\gamma\Sigma_\theta^Q+(\gamma_a/2)\,\Sigma_{\Delta,\theta}^Q}{\lambda+\lambda_a}+\frac{r^2}{4}}-\frac{r}{2}, \tag{A.110}$$
$$q_{\eta,\theta}^{(a)}=\frac{\gamma\sigma_F}{(\lambda+\lambda_a)\left(r+\kappa_\eta+q_{X,\theta}^{(a)}\right)}, \tag{A.111}$$

and the corresponding price coefficients are given by:

$$p_{X,\theta} = -\left(\lambda_a - \theta\right) q^{(a)}_{X,\theta} + (\lambda + \lambda_a)\psi_{a,\theta} q^{(a)}_{X,\theta}, \tag{A.112}$$

$$p_{\eta,\theta} = \left(\lambda_a - \theta\right) q^{(a)}_{\eta,\theta} + (\lambda + \lambda_a)\psi_{a,\theta} q^{(a)}_{\eta,\theta} \frac{q^{(a)}_{X,\theta}}{r + \kappa_\eta}. \tag{A.113}$$

Case II-(a) ($\sigma_\eta = 0$). In this case, the equilibrium trading coefficients simplify to:

$$\bar{q}^{(a)}_{X,\theta} = \sqrt{\frac{\gamma\sigma_F^2}{\lambda + \lambda_a} + \frac{r^2}{4}} - \frac{r}{2} = \bar{q}^{(a)}_X, \tag{A.114}$$

$$\bar{q}^{(a)}_{\eta,\theta} = \frac{\gamma\sigma_F}{(\lambda + \lambda_a)\left(r + \kappa_\eta + \bar{q}^{(a)}_{X,\theta}\right)} = \bar{q}^{(a)}_\eta, \tag{A.115}$$

with price coefficients:

$$\bar{p}_{X,\theta} = \bar{p}_X + \theta \bar{q}^{(a)}_X \in (\bar{p}_X, 0], \tag{A.116}$$

$$\bar{p}_{\eta,\theta} = \bar{p}_\eta - \theta \bar{q}^{(a)}_\eta \in [0, \bar{p}_\eta). \tag{A.117}$$

To compare the equilibrium behaviors before and after redistribution under small risky arbitrage, we first approximate the equilibrium coefficients $\left(p_X, p_\eta, q^{(a)}_X, q^{(a)}_\eta\right)$ around their known values $\left(\bar{p}_X, \bar{p}_\eta, \bar{q}^{(a)}_X, \bar{q}^{(a)}_\eta\right)$. Specifically, we seek a competitive equilibrium such that:

$$\begin{aligned}
p_X &= \bar{p}_X + \hat{p}_X \sigma_\eta^2 + o(\sigma_\eta^2), \\
p_\eta &= \bar{p}_\eta + \hat{p}_\eta \sigma_\eta^2 + o(\sigma_\eta^2), \\
q^{(a)}_X &= \bar{q}^{(a)}_X + \hat{q}^{(a)}_X \sigma_\eta^2 + o(\sigma_\eta^2), \\
q^{(a)}_\eta &= \bar{q}^{(a)}_X + \hat{q}^{(a)}_\eta \sigma_\eta^2 + o(\sigma_\eta^2).
\end{aligned}$$

where $\left(\hat{p}_X, \hat{p}_\eta, \hat{q}^{(a)}_X, \hat{q}^{(a)}_\eta\right)$ are coefficients to be determined.

Using the expressions for $q^{(a)}_X$, $q^{(a)}_\eta$, p_X and p_η, and applying a first-order Taylor expansion,

we obtain the following approximations:

$$
\begin{aligned}
q_X^{(a)} &= \bar{q}_X^{(a)} + \frac{\gamma + 2\gamma_a}{2(\lambda+\lambda_a)\sqrt{\frac{\gamma\sigma_F^2}{\lambda+\lambda_a} + \frac{r^2}{4}}}\bar{p}_\eta^2\sigma_\eta^2 + o(\sigma_\eta^2),\\
q_\eta^{(a)} &= \bar{q}_\eta^{(a)} - \frac{\gamma\sigma_F(\gamma + 2\gamma_a)}{2(\lambda+\lambda_a)^2\left(r + \kappa_\eta + \bar{q}_X^{(a)}\right)^2\sqrt{\frac{\gamma\sigma_F^2}{\lambda+\lambda_a} + \frac{r^2}{4}}}\bar{p}_\eta^2\sigma_\eta^2 + o(\sigma_\eta^2),\\
p_X &= -\lambda_a\left(\bar{q}_X^{(a)} + \frac{\gamma + 2\gamma_a}{2\sqrt{\frac{\gamma\sigma_F^2}{\lambda+\lambda_a} + \frac{r^2}{4}}}\bar{p}_\eta^2\sigma_\eta^2\right) + (\lambda+\lambda_a)\frac{2\gamma_a\bar{p}_\eta^2\sigma_\eta^2}{\gamma\sigma_F^2}\bar{q}_X^{(a)} + o(\sigma_\eta^2),\\
&= \bar{p}_X + \left((\lambda+\lambda_a)\frac{2\gamma_a}{\gamma\sigma_F^2}\bar{q}_X^{(a)} - \lambda_a\frac{\gamma + 2\gamma_a}{2(\lambda+\lambda_a)\sqrt{\frac{\gamma\sigma_F^2}{\lambda+\lambda_a} + \frac{r^2}{4}}}\right)\bar{p}_\eta^2\sigma_\eta^2 + o(\sigma_\eta^2),\\
p_\eta &= \bar{p}_\eta + \left(-\lambda_a\frac{\gamma\sigma_F(\gamma + 2\gamma_a)}{2(\lambda+\lambda_a)^2\left(r + \kappa_\eta + \bar{q}_X^{(a)}\right)^2\sqrt{\frac{\gamma\sigma_F^2}{\lambda+\lambda_a} + \frac{r^2}{4}}}\right.\\
&\quad \left. + \frac{2\gamma_a\bar{q}_X^{(a)} +}{\sigma_F(r+\kappa_\eta)\left(r + \kappa_\eta + \bar{q}_X^{(a)}\right)}\right)\bar{p}_\eta^2\sigma_\eta^2 + o(\sigma_\eta^2).
\end{aligned}
$$

Similarly, we analyze the θ-equilibrium where the price coefficients and trading coefficients are linear in σ_η^2 as follows:

$$
\begin{aligned}
p_X^\theta &= \bar{p}_X^\theta + \hat{p}_X^\theta\sigma_\eta^2 + o(\sigma_\eta^2),\\
p_\eta^\theta &= \bar{p}_\eta^\theta + \hat{p}_\eta^\theta\sigma_\eta^2 + o(\sigma_\eta^2),\\
q_{X,\theta}^{(a)} &= \bar{q}_X^{(a)} + \hat{q}_{X,\theta}^{(a)}\sigma_\eta^2 + o(\sigma_\eta^2),\\
q_{\eta,\theta}^{(a)} &= \bar{q}_{X,\theta}^{(a)} + \hat{q}_{\eta,\theta}^{(a)}\sigma_\eta^2 + o(\sigma_\eta^2).
\end{aligned}
$$

Using the solution forms for $q_{X,\theta}^{(a)}$, $q_{\eta,\theta}^{(a)}$, p_X^θ and p_η^θ, we derive their approximation solutions as

follows:

$$q_{X,\theta}^{(a)} = \bar{q}_{X,\theta}^{(a)} + \frac{\gamma + 2\gamma_a}{2(\lambda+\lambda_a)\sqrt{\frac{\gamma\sigma_F^2}{\lambda+\lambda_a} + \frac{r^2}{4}}}(\bar{p}_\eta^\theta)^2\sigma_\eta^2 + o(\sigma_\eta^2), \tag{A.118}$$

$$q_{\eta,\theta}^{(a)} = \bar{q}_{\eta,\theta}^{(a)} - \frac{\gamma\sigma_F(\gamma + 2\gamma_a)}{2(\lambda+\lambda_a)^2\left(r + \kappa_\eta + \bar{q}_X^{(a)}\right)^2\sqrt{\frac{\gamma\sigma_F^2}{\lambda+\lambda_a} + \frac{r^2}{4}}}(\bar{p}_\eta^\theta)^2\sigma_\eta^2 + o(\sigma_\eta^2), \tag{A.119}$$

$$\begin{aligned} p_{X,\theta} &= -(\lambda_a - \theta)\left(\bar{q}_{X,\theta}^{(a)} + \frac{\gamma + 2\gamma_a}{2(\lambda+\lambda_a)\sqrt{\frac{\gamma\sigma_F^2}{\lambda+\lambda_a} + \frac{r^2}{4}}}\bar{p}_{\eta,\theta}^2\sigma_\eta^2\right) + (\lambda+\lambda_a)\frac{2\gamma_a\bar{p}_{\eta,\theta}^2\sigma_\eta^2}{\gamma\sigma_F^2}\bar{q}_{X,\theta}^{(a)} + o(\sigma_\eta^2), \\ &= \bar{p}_{X,\theta} + \left((\lambda+\lambda_a)\frac{2\gamma_a}{\gamma\sigma_F^2}\bar{q}_{X,\theta}^{(a)} - (\lambda_a - \theta)\frac{\gamma + 2\gamma_a}{2(\lambda+\lambda_a)\sqrt{\frac{\gamma\sigma_F^2}{\lambda+\lambda_a} + \frac{r^2}{4}}}\right)\bar{p}_{\eta,\theta}^2\sigma_\eta^2 + o(\sigma_\eta^2), \end{aligned} \tag{A.120}$$

$$\begin{aligned} p_{\eta,\theta} = \bar{p}_{\eta,\theta} + \Bigg(&-(\lambda_a - \theta)\frac{\gamma\sigma_F(\gamma + 2\gamma_a)}{2(\lambda+\lambda_a)^2\left(r + \kappa_\eta + \bar{q}_X^{(a)}\right)^2\sqrt{\frac{\gamma\sigma_F^2}{\lambda+\lambda_a} + \frac{r^2}{4}}} \\ &+ \frac{2\gamma_a\bar{q}_X^{(a)}}{\sigma_F(r+\kappa_\eta)\left(r + \kappa_\eta + \bar{q}_X^{(a)}\right)}\Bigg)\bar{p}_{\eta,\theta}^2\sigma_\eta^2 + o(\sigma_\eta^2). \end{aligned} \tag{A.121}$$

Note that for any $\theta \in (0, \lambda_a)$, the following inequalities hold:

$$q_X^{(a)} > q_{X,\theta}^{(a)}, \quad q_{\eta,\theta}^{(a)} > q_\eta^{(a)}, \quad \textit{and} \quad p_\eta > p_\eta^\theta. \tag{A.122}$$

Since $p_X < 0$ and $p_{X,\theta} < 0$ for sufficiently small γ_a and σ_η, $|p_X| - |p_{X,\theta}| \approx \theta\bar{q}_X^{(a)} > 0$ for $\theta \in (0, \lambda_a)$. Additionally, $\frac{\partial|p_{X,\theta}|}{\partial\theta} \approx -\bar{q}_X^{(a)} < 0$ for sufficiently small γ_a and σ_η.

Now, taking the partial derivative of $p_{X,\theta}$ with respect to γ_a yields:

$$\frac{\partial p_{X,\theta}}{\partial\gamma_a} = \frac{2(\lambda+\lambda_a)\bar{q}_{X,\theta}^{(a)}}{\gamma\sigma_F^2} - (\lambda - \theta)\frac{2}{(\lambda+\lambda_a)(r + 2\bar{q}_{X,\theta}^{(a)})}. \tag{A.123}$$

The condition $\frac{\partial p_{X,\theta}}{\partial \gamma_a} > 0$ can be then rewritten as:

$$\begin{aligned} \frac{(\lambda+\lambda_a))\bar{q}^{(a)}_{X,\theta}(r+2\bar{q}^{(a)}_{X,\theta})}{\gamma\sigma_F^2} &> \frac{\lambda_a-\theta}{\lambda+\lambda_a}, \\ \frac{\gamma(\sigma_F^2+p^2_{\eta,\theta}\sigma_\eta^2)+2\gamma_a p^2_{\eta,\theta}\sigma_\eta^2+(\lambda+\lambda_a))\bar{q}^{(a)}_{X,\theta}\bar{q}^{(a)}_{X,\theta}}{\gamma\sigma_F^2} &> \frac{\lambda_a-\theta}{\lambda+\lambda_a}. \end{aligned} \tag{A.124}$$

Thus, p^θ_X is an increasing and continuous function of γ_a for any fixed $\theta \in (0, \lambda_a)$. Since $p_{X,\theta}$ is less than zero at $\gamma_a = 0$ and becomes positive for sufficiently large γ_a, the Intermediate Value Theorem guarantees the existence of a threshold $\hat{\gamma}_a > 0$ such that $p^\theta_X(\hat{\gamma}_a) = 0$ for some $\theta \in (0, \lambda_a)$.

Appendix B

B.1 Anticipated Dynamics of Asset Prices and Trading Positions

To investigate the dynamics of $X_t^{(a)}$ in response to liquidity shocks, I employ Theorem 8.1.5 of Arnold (1974). This theorem establishes the existence of a unique solution $\boldsymbol{\Omega_{t+T}}$ given by:

$$\boldsymbol{\Omega_{t+T}} = e^{\boldsymbol{\Phi} T}\boldsymbol{\Omega_t} + \int_t^{t+T} e^{\boldsymbol{\Phi}(t+T-s)}\boldsymbol{\sigma_\Omega} \boldsymbol{dZ_s}, \tag{B.1}$$

where $\boldsymbol{\Omega}_t$ represents the initial state vector, and $e^{\boldsymbol{\Phi} T}$ is specified as:

$$e^{\boldsymbol{\Phi} T} = \begin{bmatrix} e^{-q_X^{(a)} T} & -\frac{q_\eta^{(a)}}{\kappa_\eta - q_X^{(a)}}\left(e^{-\kappa_\eta T} - e^{-q_X^{(a)} T}\right) \\ 0 & e^{-\kappa_\eta T} \end{bmatrix},$$

provided that $q_X^{(a)} \neq \kappa_\eta$. Consequently, the stochastic processes $\left(X_{t+T}^{(a)}, \eta_{t+T}\right)$ are governed by:

$$\eta_{t+T} = e^{-\kappa_\eta T}\eta_t + \int_t^{t+T} e^{-\kappa_\eta (t+T-s)}\sigma_\eta dZ_s^\eta, \tag{B.2}$$

$$X_{t+T}^{(a)} = e^{-q_X^{(a)} T} X_t^{(a)} - \psi_\eta^X \left[\left(e^{-\kappa_\eta T} - e^{-q_X^{(a)} T}\right)\eta_t + \int_t^{t+T}\left(e^{-\kappa_\eta (t+T-s)} - e^{-q_X^{(a)}(t+T-s)}\right)\sigma_\eta dZ_s^\eta\right],$$

where $\psi_\eta^X = \frac{q_\eta^{(a)}}{\kappa_\eta - q_X^{(a)}}$.

Given the solution for the asset prices in equilibrium, I derive

$$P_{t+T}^{(j)} = P_t^{(j)} + \frac{1}{r+\kappa_D}(D_{t+T} - D_t) - \left[p_\eta^{(j)}(\eta_{t+T} - \eta_t) + p_X^{(j)}\left(X_{t+T}^{(a)} - X_t^{(a)}\right)\right] \tag{B.3}$$

By substituting the expressions for $(X_{t+T}^{(a)}, \eta_{t+T})$ from $(B.2)$ into (B.3), the asset price at time

$t+T$ $P_{t+T}^{(j)}$ can be expressed as a function of initial conditions $\mathbf{\Omega}_t$ and a series of shocks:

$$\begin{aligned}P_{t+T}^{(j)} = F_t - \left(p_\eta^{(j)}\eta_t + p_X^{(j)}X_t^{(a)}\right) - \frac{D_t - \bar{D}}{r+\kappa_D}g(T,\kappa_D) + \left[\left(p_\eta^{(j)} - \psi_\eta^X p_X^{(j)}\right)g(T,\kappa_\eta) + \psi_\eta^X p_X^{(j)} g(T,q_X^{(a)})\right]\eta_t \\ + p_X^{(j)} g(T,q_X^{(a)})X_t^{(a)} + \int_t^{t+T} e^{-\kappa_D(t+T-s)}\sigma_F dZ_s - p_\eta^{(j)}\int_t^{t+T} e^{-\kappa_\eta(t+T-s)}\sigma_\eta dZ_s^\eta \\ + \psi_\eta^X p_X^{(j)}\int_t^{t+T}\left(e^{-\kappa_\eta(t+T-s)} - e^{-q_X^{(a)}(t+T-s)}\right)\sigma_\eta dZ_s^\eta.\end{aligned} \tag{B.4}$$

where $g(T,\kappa_\eta) = 1 - e^{-\kappa_\eta T}$ and $g(T,q_X^{(a)}) = 1 - e^{-q_X^{(a)}T}$.

From (B.4), the conditional mean and the conditional variance are computed as follows:

$$\begin{aligned}E\left[P_{t+T}^{(j)}\right] &= F_t - \left(p_\eta^{(j)}\eta_t + p_X^{(j)}X_t^{(a)}\right) - \frac{D_t - \bar{D}}{r+\kappa_D}g(T,\kappa_D) \\ &+ \left[\left(p_\eta^{(j)} - \frac{q_\eta^{(a)}}{\kappa_\eta - q_X^{(a)}}p_X^{(j)}\right)g(T,\kappa_\eta) + \left(\frac{q_\eta^{(a)}}{\kappa_\eta - q_X^{(a)}}\right)p_X^{(j)}g(T,q_X^{(a)})\right]\eta_t + p_X^{(j)}g(T,q_X^{(a)})X_t^{(a)},\end{aligned} \tag{B.5}$$

$$\begin{aligned}Var_t\left[P_{t+T}^{(j)}\right] &= Var_t[P_{t+T}] = \frac{\sigma_F^2}{2\kappa_D}g(T,2\kappa_D) + \frac{p_\eta^2\sigma_\eta^2}{2\kappa_\eta}g(T,2\kappa_\eta) \\ &+ \left(\psi_\eta^X\right)^2 p_X^2\sigma_\eta^2\left[\frac{1}{2\kappa_\eta}g(T,2\kappa_\eta) + \frac{1}{2q_X^{(a)}}g(T,2q_X^{(a)}) - \frac{2}{\kappa + q_X^{(a)}}g(T,\kappa_\eta + q_X^{(a)})\right] \\ &- 2\psi_\eta^X p_X p_\eta\sigma_\eta^2\left[\frac{1}{2\kappa_\eta}g(T,2\kappa_\eta) - \frac{1}{\kappa_\eta + q_X^{(a)}}g(T,\kappa_\eta + q_X^{(a)})\right].\end{aligned} \tag{B.6}$$

B.2 Excess Price Volatility

By taking the limit as $T \to \infty$ in the expressions for the conditional mean and conditional variance, I derive the stationary distribution of asset prices:

$$E\left[P_t^{(j)}\right] = \frac{\bar{D}}{r}, \tag{B.7}$$

$$Var\left[P_t^{(j)}\right] = \frac{\sigma_F^2}{2\kappa_D} + \sigma_\eta^2\left[\frac{p_\eta^2}{2\kappa_\eta} + \left(\psi_\eta^X p_X\right)^2\left(\frac{1}{2\kappa_\eta} + \frac{1}{2q_X^{(a)}} - \frac{2}{\kappa_\eta + q_X^{(a)}}\right) - 2\psi_\eta^X p_X p_\eta\left(\frac{1}{2\kappa_\eta} - \frac{1}{\kappa_\eta + q_X^{(a)}}\right)\right]. \tag{B.8}$$

To establish the presence of excess volatility, the following inequality must hold:

$$\frac{p_\eta^2}{2\kappa_\eta} + \left(\psi_\eta^X p_X\right)^2 \left(\frac{1}{2\kappa_\eta} + \frac{1}{2q_X^{(a)}} - \frac{2}{\kappa_\eta + q_X^{(a)}}\right) - 2\psi_\eta^X p_X p_\eta \left(\frac{1}{2\kappa_\eta} - \frac{1}{\kappa_\eta + q_X^{(a)}}\right) \geq 0. \tag{B.9}$$

By manipulating the terms on the left-hand side of the inequality (B.8), I get:

$$\begin{aligned}
LHS =& \frac{p_\eta^2}{2\kappa_\eta} + (\psi_\eta^X p_X)^2 \frac{(\kappa_\eta - q_X^{(a)})^2}{2q_X^{(a)}\kappa_\eta(\kappa_\eta + q_X^{(a)})} + \psi_\eta^X p_X p_\eta \frac{\kappa_\eta - q_X^{(a)}}{\kappa_\eta(\kappa_\eta + q_X^{(a)})}, \\
=& \frac{p_\eta^2}{2\kappa_\eta} + \frac{1}{2q_X^{(a)}} \frac{(q_\eta^{(a)})^2 p_X^2}{\kappa_\eta(\kappa_\eta + q_X^{(a)})} + \frac{q_\eta^{(a)} p_X p_\eta}{\kappa_\eta(\kappa_\eta + q_X^{(a)})}, \\
=& \frac{\kappa_\eta q_X^{(a)} p_\eta^2 + \left(p_\eta q_X^{(a)} + p_X q_\eta^{(a)}\right)^2}{2\kappa_\eta q_X^{(a)}(\kappa_\eta + q_X^{(a)})}, \\
=& \left[\left(\frac{\kappa_\eta}{\kappa_\eta + q_X^{(a)}}\right)\left(\frac{p_\eta}{q_\eta^{(a)}}\right)^2 + \left(\frac{q_X^{(a)}}{\kappa_\eta + q_X^{(a)}}\right)\left(\frac{p_X}{q_X^{(a)}} + \frac{p_\eta}{q_\eta^{(a)}}\right)^2\right]\left(\frac{q_\eta^{(a)}}{\sqrt{2\kappa_\eta}}\right)^2, \qquad \text{(B.10)} \\
=& \left[\left(\frac{\kappa_\eta}{\kappa_\eta + q_X^{(a)}}\right)\left(\frac{\lambda_a + \psi_a(\lambda + \lambda_a)\frac{q_X^{(a)}}{r+\kappa_\eta}}{(\lambda + \lambda_a)\left(1 + \frac{q_X^{(a)}}{r+\kappa_\eta}\right)}\right)^2 + \left(\frac{q_X^{(a)}}{\kappa_\eta + q_X^{(a)}}\right)\psi_a^2\right]\left(\frac{\gamma\sigma_F}{r + \kappa_\eta}\right)^2 \frac{1}{2\kappa_\eta}. \qquad \text{(B.11)}
\end{aligned}$$

Hence, if $\lambda_a > 0$, there is always an excess volatility across markets.

B.3 Persistent Violations of the Law of One Price

Consider an economy beginning at $\hat{\mathbf{\Omega}}_t \equiv (0, \eta_t)$ with $\eta_t > 0$. Then the expected spread level at $t + T$ is given by

$$\begin{aligned}
E[\Delta_{t+T}|\hat{\mathbf{\Omega}}_t] &= 2p_\eta \eta_{t+T} + 2p_X X_{t+T}^{(a)} \qquad \text{(B.12)} \\
&= 2p_\eta e^{-\kappa_\eta T}\eta_t + p_X \frac{\partial E[X_{t+T}^{(a)}|\hat{\mathbf{\Omega}}_t]}{\partial \eta_t}\eta_t,
\end{aligned}$$

where the expected change in aggregate arbitrage position in response to η_t is

$$\frac{\partial E[X_{t+T}^{(a)}|\mathbf{\Omega}_t]}{\partial \eta_t} = -\frac{q_\eta^{(a)}}{\kappa_\eta - q_X^{(a)}}\left(e^{-\kappa_\eta T} - e^{-q_X^{(a)}T}\right) \geq 0. \tag{B.13}$$

The spread level at time t is given by

$$\Delta_t = 2p_\eta \eta_t > 0. \tag{B.14}$$

Suppose $\kappa_\eta > q_X^{(a)}$. Defining $T' > 0$ such that $e^{-\kappa_\eta T'} = 0$ and $e^{-q_X^{(a)} T'} > 0$, we can express the expected spread at $t + T'$ as

$$E[\Delta_{t+T'} | \hat{\mathbf{\Omega}}_t] = \frac{2p_X q_\eta^{(a)}}{\kappa_\eta - q_X^{(a)}} e^{-q_X^{(a)} \hat{T}} \eta_t. \tag{B.15}$$

The spread level is negative in a Risk-On regime ($p_X < 0$) and positive in a Risk-Off regime ($p_X > 0$). Now, suppose $q_X^{(a)} > \kappa_\eta$. Defining $T' > 0$ such that $e^{-q_X^{(a)} T'} = 0$ and $e^{-\kappa_\eta T'} > 0$, we can write the expected spread at $t + T'$ as

$$E[\Delta_{t+T'} | \hat{\mathbf{\Omega}}_t] = 2 \left(p_\eta + \frac{p_X q_\eta^{(a)}}{q_X^{(a)} - \kappa_\eta} \right) e^{-\kappa_\eta T'} \eta_t. \tag{B.16}$$

The spread becomes positive is positive in a Risk-Off regime ($p_X \geq 0$). To show the negative spread in a Risk-On regime ($p_X < 0$) for $q_X^{(a)} > \kappa_\eta$, we proceed by contradiction:

$$\begin{aligned} E[\Delta_{t+T'} | \hat{\mathbf{\Omega}}_t] > 0 \quad &\Leftrightarrow \quad p_\eta q_X^{(a)} + p_X q_\eta^{(a)} > p_\eta \kappa_\eta \\ &\Leftrightarrow \quad \frac{p_\eta q_X^{(a)} + p_X q_\eta^{(a)}}{p_\eta} > \kappa_\eta \\ &\Leftrightarrow \quad q_X^{(a)} \left(\frac{\psi_a(\lambda + \lambda_a) + \psi_a(\lambda + \lambda_a) \frac{q_X^{(a)}}{r + \kappa_\eta}}{\lambda_a + \psi_a(\lambda + \lambda_a) \frac{q_X^{(a)}}{r + \kappa_\eta}} \right) > \kappa_\eta. \end{aligned} \tag{B.17}$$

To be consistent with $q_X^{(a)} > \kappa_\eta$, it must be the case that:

$$\psi_a(\lambda + \lambda_a) \geq \lambda_a \quad \Leftrightarrow \quad p_X \geq 0, \tag{B.18}$$

which contradicts the assumption $p_X < 0$. Therefore, $E[\Delta_{t+T'} | \hat{\mathbf{\Omega}}_t] < 0$ for $p_X < 0$.

To compute the half-life of the asset spread, let us first approximate the expected value of

asset spreads at time $t+T$ as follows:

$$E_t[\Delta_{t+T}] \approx 2\left(p_\eta - \frac{p_X q_\eta^{(a)}}{\kappa_\eta - q_X^{(a)}}\right)(1-\kappa_\eta T)\eta_t + \frac{2p_X q_\eta^{(a)}}{\kappa_\eta - q_X^{(a)}}(1-q_X^{(a)}T)\eta_t. \tag{B.19}$$

The half-life is defined as the time T at which the spread reaches hlaf of its initial value:

$$\frac{1}{2}\Delta_t = E_t[\Delta_{t+T}]. \tag{B.20}$$

Equation (B.20) yields

$$\begin{aligned} p_\eta &= 2\left(p_\eta - \frac{p_X q_\eta^{(a)}}{\kappa_\eta - q_X^{(a)}}\right)(1-\kappa_\eta T) + \frac{2p_X q_\eta^{(a)}}{\kappa_\eta - q_X^{(a)}}(1-q_X^{(a)}T), \\ &= 2p_\eta - 2\kappa_\eta p_\eta T - \frac{2p_X q_\eta^{(a)}}{\kappa_\eta - q_X^{(a)}} + \frac{2p_X q_\eta^{(a)}\kappa_\eta T}{\kappa_\eta - q_X^{(a)}} + \frac{2p_X q_\eta^{(a)}}{\kappa_\eta - q_X^{(a)}} - \frac{2p_X q_\eta^{(a)} q_X^{(a)} T}{\kappa_\eta - q_X^{(a)}}, \\ &= 2p_\eta - 2\kappa_\eta p_\eta T + \frac{2p_X q_\eta^{(a)}}{\kappa_\eta - q_X^{(a)}}(\kappa_\eta - q_X^{(a)})T \end{aligned} \tag{B.21}$$

Solving for T, we obtain the expression

$$\begin{aligned} T_{1/2} &= \frac{p_\eta}{2\kappa_\eta p_\eta - 2p_X q_\eta^{(a)}}, \\ &= \frac{1}{2}\left(\kappa_\eta + q_X^{(a)}\frac{\lambda_a - \psi_a(\lambda+\lambda_a)}{\lambda_a + \frac{q_X^{(a)}}{\rho+\kappa_\eta}\psi_a(\lambda+\lambda_a)}\right)^{-1}. \end{aligned} \tag{B.22}$$

B.4 Return Autocorrelation

Consider the autocovariance between asset returns at two distinct points in time. Then the return autocovariance is computed as follows:

$$\begin{aligned} &Cov_t\left(dQ_t^{(j)}, dQ_{t+T}^{(j)}\right), \\ &= Cov_t\left(\sigma_F dZ_t - p_\eta^{(j)}\sigma_\eta dZ_t^\eta, \left(\mu_\eta^{(j)}\eta_{t+T} + \mu_X^{(j)}X_{t+T}^{(a)}\right)T + \sigma_F dZ_{t+T} - p_\eta^{(j)}\sigma_\eta dZ_{t+T}^\eta\right), \\ &= Cov_t\left(-p_\eta^{(j)}\sigma_\eta dZ_t^\eta, \left(\mu_\eta^{(j)}\eta_{t+T} + \mu_X^{(j)}X_{t+T}^{(a)}\right)T\right). \end{aligned} \tag{B.23}$$

To compute the impact of a one-time liquidity shock at t, we solve the following impulse-response system:

$$d\eta_t = -\kappa_\eta \eta_t dt, \tag{B.24}$$

$$dX_t^{(a)} = \left(q_\eta^{(a)} \eta_t - q_X^{(a)} X_t^{(a)}\right) dt, \tag{B.25}$$

with the initial condition $\eta_t = \sigma_\eta dZ_t^\eta$ and $X_t^{(a)} = 0$.

By substituting the solution to the system of ODEs into the previous covariance expression, we obtain:

$$\frac{1}{dt} Cov_t\left(dQ_t^{(j)}, dQ_{t+T}^{(j)}\right) = -p_\eta^{(j)} \sigma_\eta^2 \left[\left(\mu_\eta^{(j)} - \mu_X^{(j)} \psi_\eta^X\right) e^{-\kappa_\eta T} + \mu_X^{(j)} \psi_\eta^X e^{-q_X^{(a)} T}\right] T. \tag{B.26}$$

Next, using the definition of the spread return autocorrelation measure, we compute:

$$\varrho_t^Q(T) = \frac{1}{T} \frac{\frac{1}{dt} Cov_t(dQ_t^{(j)}, dQ_{t+T}^{(j)})}{\frac{1}{dt} Var_t(dQ_t^{(j)})} = B_1(T, \kappa_\eta) + B_2(T, q_X^{(a)}). \tag{B.27}$$

where $B_1(T, \kappa_\eta)$ and $B_2(T, q_X^{(a)})$ are non-zero constants for $\lambda_a > 0$ and $T > 0$ and defined as:

$$B_1(T, \kappa_\eta) = -\left(\frac{\mu_\eta^{(j)} - \mu_X^{(j)} \psi_\eta^X}{p_\eta}\right) e^{-\kappa_\eta T}, \tag{B.28}$$

$$B_2(T, \kappa_\eta) = -\frac{\mu_X^{(j)} \psi_\eta^X}{p_\eta^{(j)}} e^{-q_X^{(a)} T}. \tag{B.29}$$

Since $Sgn(-\frac{\mu_\eta^{(j)}}{p_\eta^{(j)}}) < 0$ at $T = 0$, it follows that $\varrho_t^Q(0) < 0$.

Suppose $\kappa_\eta > q_X^{(a)}$. Defining $\hat{T} > 0$ such that $e^{-\kappa_\eta \hat{T}} = 0$ and $e^{-q_X^{(a)} \hat{T}} > 0$, we can express the spread return autocorrelation at $\hat{T}$ as:

$$\varrho_t^Q(\hat{T}) = -\frac{\mu_X^{(j)} \psi_\eta^X}{p_\eta^{(j)}} e^{-q_X^{(a)} \hat{T}} = -\frac{\mu_X^{(j)} q_\eta^{(a)}}{p_\eta^{(j)} (\kappa_\eta - q_X^{(a)})} e^{-q_X^{(a)} \hat{T}}. \tag{B.30}$$

The autocorrelation value in (B.30) is positive in a Risk-On regime and negative in a Risk-Off

regime. Now, suppose $q_X^{(a)} > \kappa_\eta$. Defining $\hat{T}$ such that $e^{-q_X^{(a)}\hat{T}} = 0$ and $e^{-\kappa_\eta \hat{T}} > 0$, we can write the spread return autocorrelation at $\hat{T}$ as:

$$\begin{aligned}\varrho_t^Q(\hat{T}) &= -\frac{1}{p_\eta^{(j)}}\left(\mu_\eta^{(j)} - \mu_X^{(j)}\frac{q_\eta^{(a)}}{\kappa_\eta - q_X^{(a)}}\right)e^{-\kappa_\eta\hat{T}}, \\ &= \frac{r+\kappa_\eta}{p_\eta(q_X^{(a)}-\kappa_\eta)}\left(\kappa_\eta p_\eta - (q_X^{(a)}p_\eta + p_X q_\eta^{(a)})\right)e^{-\kappa_\eta\hat{T}}. \end{aligned} \tag{B.31}$$

First, we need to show that in the Risk-Off regime, $\varrho_t^Q(\hat{T}) < 0$. Note that

$$\begin{aligned}\varrho_t^Q(\hat{T}) < 0 \quad &\Leftrightarrow \quad p_\eta q_X^{(a)} + p_X q_\eta^{(a)} > p_\eta\kappa_\eta \\ &\Leftrightarrow \quad \frac{p_\eta q_X^{(a)} + p_X q_\eta^{(a)}}{p_\eta} > \kappa_\eta \\ &\Leftrightarrow \quad q_X^{(a)}\left(\frac{\psi_a(\lambda+\lambda_a) + \psi_a(\lambda+\lambda_a)\frac{q_X^{(a)}}{r+\kappa_\eta}}{\lambda_a + \psi_a(\lambda+\lambda_a)\frac{q_X^{(a)}}{r+\kappa_\eta}}\right) > \kappa_\eta. \end{aligned} \tag{B.32}$$

To be consistent with $q_X^{(a)} > \kappa_\eta$, it must be the case that:

$$\psi_a(\lambda+\lambda_a) \geq \lambda_a \quad \Leftrightarrow \quad p_X \geq 0, \tag{B.33}$$

which implies the Risk-Off regime. Second, to show that $\varrho_t^Q(\hat{T}) > 0$ in the Risk-On regime for $q_X^{(a)} > \kappa_\eta$, we proceed by contradiction:

$$\begin{aligned}\varrho_t^Q(\hat{T}) > 0 \quad &\Leftrightarrow \quad p_\eta q_X^{(a)} + p_X q_\eta^{(a)} < p_\eta\kappa_\eta \\ &\Leftrightarrow \quad \frac{p_\eta q_X^{(a)} + p_X q_\eta^{(a)}}{p_\eta} < \kappa_\eta \\ &\Leftrightarrow \quad q_X^{(a)}\left(\frac{\psi_a(\lambda+\lambda_a) + \psi_a(\lambda+\lambda_a)\frac{q_X^{(a)}}{r+\kappa_\eta}}{\lambda_a + \psi_a(\lambda+\lambda_a)\frac{q_X^{(a)}}{r+\kappa_\eta}}\right) < \kappa_\eta. \end{aligned} \tag{B.34}$$

Since $\lambda_a > \psi_a(\lambda+\lambda_a)$ in the Risk-On regime, the inequality (B.34) implies that $q_X^{(a)} < \kappa_\eta$, which contradicts the assumption $q_X^{(a)} > \kappa_\eta$. Therefore, $\varrho_t^Q(\hat{T}) > 0$ in the Risk-On regime.

Appendix C

Proof of Lemma 1.

Given the solution form of the price coefficient p_η, we aim to show that p_η is increasing in γ_a for small σ_η. To apply the implicit function theorem, consider the function:

$$F(p_\eta, \gamma_a) = p_\eta - \lambda_a q_\eta^{(a)} - \psi_a(\lambda + \lambda_a) q_\eta^{(a)} \frac{q_X^{(a)}}{r + \kappa_\eta}. \tag{C.1}$$

Then, the derivative of the function F with respect to p_η is given by

$$\begin{aligned}\frac{\partial F}{\partial p_\eta} &= 1 - \lambda_a \frac{\partial q_\eta^{(a)}}{\partial p_\eta} \\ &\quad - \left(\frac{\partial \psi_a}{\partial p_\eta}(\lambda + \lambda_a) q_\eta^{(a)} \frac{q_X^{(a)}}{r + \kappa_\eta} + \psi_a(\lambda + \lambda_a) \frac{\partial q_\eta^{(a)}}{\partial p_\eta} \frac{q_X^{(a)}}{r + \kappa_\eta} + \psi_a(\lambda + \lambda_a) q_\eta^{(a)} \frac{\partial q_X^{(a)}}{\partial p_\eta} \frac{1}{r + \kappa_\eta} \right).\end{aligned} \tag{C.2}$$

The partial derivatives are computed as follows:

$$\frac{\partial q_\eta^{(a)}}{\partial p_\eta} = -\frac{\gamma \sigma_F}{(\lambda + \lambda_a)(r + \kappa_\eta + q_X^{(a)})^2} \cdot \frac{\partial q_X^{(a)}}{\partial p_\eta}, \tag{C.3}$$

$$\frac{\partial q_X^{(a)}}{\partial p_\eta} = \frac{p_\eta \sigma_\eta^2 (\gamma + 2\gamma_a)}{(\lambda + \lambda_a)\left(q_X^{(a)} + \frac{r}{2}\right)}, \tag{C.4}$$

$$\frac{\partial \psi_a}{\partial p_\eta} = \frac{4\gamma_a p_\eta \sigma_\eta^2 \gamma \sigma_F^2}{(\gamma \sigma_F^2 + \gamma \sigma_\eta^2 p_\eta^2 + 2\gamma_a \sigma_\eta^2 p_\eta^2)^2}. \tag{C.5}$$

Plugging these expressions into Equation (C.2), we obtain:

$$\begin{aligned}\frac{\partial F}{\partial p_\eta} &= 1 + \frac{\lambda_a \gamma \sigma_F p_\eta \sigma_\eta^2 (\gamma + 2\gamma_a)}{(\lambda + \lambda_a)^2 (r + \kappa_\eta + q_X^{(a)})^2 \left(q_X^{(a)} + \frac{r}{2}\right)} - \frac{4\gamma_a p_\eta \sigma_\eta^2 \gamma \sigma_F^2 (\lambda + \lambda_a) q_\eta^{(a)} q_X^{(a)}}{(\gamma \sigma_F^2 + \gamma \sigma_\eta^2 p_\eta^2 + 2\gamma_a \sigma_\eta^2 p_\eta^2)^2 (r + \kappa_\eta)} \\ &\quad + \frac{\psi_a (\lambda + \lambda_a) \gamma \sigma_F p_\eta \sigma_\eta^2 (\gamma + 2\gamma_a) q_X^{(a)}}{(\lambda + \lambda_a)^2 (r + \kappa_\eta + q_X^{(a)})^2 \left(q_X^{(a)} + \frac{r}{2}\right)(r + \kappa_\eta)} \\ &\quad - \frac{\psi_a (\lambda + \lambda_a) \gamma \sigma_F p_\eta \sigma_\eta^2 (\gamma + 2\gamma_a)(r + \kappa_\eta + q_X^{(a)})}{(\lambda + \lambda_a)^2 (r + \kappa_\eta + q_X^{(a)})^2 \left(q_X^{(a)} + \frac{r}{2}\right)(r + \kappa_\eta)}.\end{aligned} \tag{C.6}$$

Rearranging the terms in the above expression, we derive the following:

$$\begin{aligned}\frac{\partial F}{\partial p_\eta} &= 1 + \frac{[\lambda_a - \psi_a(\lambda+\lambda_a)]\gamma\sigma_F p_\eta \sigma_\eta^2(\gamma+2\gamma_a)}{(\lambda+\lambda_a)^2(r+\kappa_\eta+q_X^{(a)})^2\left(q_X^{(a)}+\frac{r}{2}\right)} - \frac{4\gamma_a p_\eta \sigma_\eta^2 \gamma \sigma_F^2 \gamma \sigma_F}{(\lambda+\lambda_a)^2(r+q_X^{(a)})^2 q_X^{(a)}(r+\kappa_\eta+q_X^{(a)})(r+\kappa_\eta)}, \\ &= 1 + \frac{2\gamma\sigma_F p_\eta \sigma_\eta^2(\gamma+2\gamma_a)}{(\lambda+\lambda_a)(r+\kappa_\eta)(r+q_X^{(a)})(r+\kappa_\eta+q_X^{(a)})}\left(\frac{\left[\frac{\lambda_a}{\lambda+\lambda_a}-\psi_a\right](r+\kappa_\eta)(r+q_X^{(a)})}{(r+\kappa_\eta+q_X^{(a)})\left(r+2q_X^{(a)}\right)}\right. \\ &\qquad \left. - \frac{2\gamma_a}{\gamma+2\gamma_a}\frac{\gamma\sigma_F^2}{\gamma(\sigma_F^2+p_\eta^2\sigma_\eta^2)}\psi_a\right).\end{aligned} \tag{C.7}$$

Since the absolute value of the term in the brackets in (C.7) is no greater than 1, and σ_η^2 is small, the term multiplying σ_η^2 will also be small. Thus, we can conclude that $\frac{\partial F}{\partial p_\eta} > 0$.

Next, to compute $\frac{\partial F}{\partial \gamma_a}$, we differentiate each term in F that depends on γ_a:

$$\frac{\partial F}{\partial \gamma_a} = -\lambda_a \frac{\partial q_\eta^{(a)}}{\partial \gamma_a} - \left(\frac{\partial \psi_a}{\partial \gamma_a}(\lambda+\lambda_a)q_\eta^{(a)}\frac{q_X^{(a)}}{r+\kappa_\eta} + \psi_a(\lambda+\lambda_a)\frac{\partial q_\eta^{(a)}}{\partial \gamma_a}\frac{q_X^{(a)}}{r+\kappa_\eta} + \psi_a(\lambda+\lambda_a)q_\eta^{(a)}\frac{\partial q_X^{(a)}}{\partial \gamma_a}\frac{1}{r+\kappa_\eta}\right) \tag{C.8}$$

The partial derivatives are computed as follows:

$$\frac{\partial q_\eta^{(a)}}{\partial \gamma_a} = -\frac{\gamma\sigma_F}{(\lambda+\lambda_a)(r+\kappa_\eta+q_X^{(a)})^2}\cdot\frac{\partial q_X^{(a)}}{\partial \gamma_a}, \tag{C.9}$$

$$\frac{\partial q_X^{(a)}}{\partial \gamma_a} = \frac{p_\eta^2\sigma_\eta^2}{(\lambda+\lambda_a)\sqrt{\frac{\gamma(\sigma_F^2+p_\eta^2\sigma_\eta^2)+2\gamma_a p_\eta^2\sigma_\eta^2}{\lambda+\lambda_a}+\frac{r^2}{4}}} = \frac{p_\eta^2\sigma_\eta^2}{(\lambda+\lambda_a)\left(q_X^{(a)}+\frac{r}{2}\right)}, \tag{C.10}$$

$$\frac{\partial \psi_a}{\partial \gamma_a} = \frac{2p_\eta^2\sigma_\eta^2(\gamma\sigma_F^2+\gamma\sigma_\eta^2 p_\eta^2)}{(\gamma\sigma_F^2+\gamma\sigma_\eta^2 p_\eta^2+2\gamma_a\sigma_\eta^2 p_\eta^2)^2}. \tag{C.11}$$

Substituting these derivatives into Equation (C.8), we obtain:

$$\begin{aligned}\frac{\partial F}{\partial \gamma_a} = &\frac{\lambda_a \gamma \sigma_F p_\eta^2 \sigma_\eta^2}{(\lambda+\lambda_a)^2 (r+\kappa_\eta+q_X^{(a)})^2 \left(q_X^{(a)}+\frac{r}{2}\right)} - \frac{2\gamma p_\eta^2 \sigma_\eta^2 (\sigma_F^2+\sigma_\eta^2 p_\eta^2)(\lambda+\lambda_a) q_\eta^{(a)} q_X^{(a)}}{(\gamma\sigma_F^2+\gamma\sigma_\eta^2 p_\eta^2+2\gamma_a \sigma_\eta^2 p_\eta^2)^2 (r+\kappa_\eta)} \\ &+ \frac{\psi_a(\lambda+\lambda_a)\gamma\sigma_F p_\eta^2 \sigma_\eta^2 q_X^{(a)}}{(\lambda+\lambda_a)^2 (r+\kappa_\eta+q_X^{(a)})^2 \left(q_X^{(a)}+\frac{r}{2}\right)(r+\kappa_\eta)} \\ &- \frac{\psi_a(\lambda+\lambda_a)\gamma\sigma_F p_\eta^2 \sigma_\eta^2 (r+\kappa_\eta+q_X^{(a)})}{(\lambda+\lambda_a)^2 (r+\kappa_\eta+q_X^{(a)})^2 \left(q_X^{(a)}+\frac{r}{2}\right)(r+\kappa_\eta)}\end{aligned} \tag{C.12}$$

Rearranging terms, we derive the following expression:

$$\frac{\partial F}{\partial \gamma_a} = \frac{[\lambda_a - \psi_a(\lambda+\lambda_a)]\gamma\sigma_F p_\eta^2 \sigma_\eta^2}{(\lambda+\lambda_a)^2 (r+\kappa_\eta+q_X^{(a)})^2 \left(q_X^{(a)}+\frac{r}{2}\right)} - \frac{2\gamma p_\eta^2 \sigma_\eta^2 (\sigma_F^2+\sigma_\eta^2 p_\eta^2)\gamma\sigma_F}{(\lambda+\lambda_a)^2 (r+q_X^{(a)})^2 q_X^{(a)} (r+\kappa_\eta+q_X^{(a)})(r+\kappa_\eta)} \tag{C.13}$$

By manipulating terms, we derive condition equivalent to $\frac{\partial F}{\partial \gamma_a} > 0$ as follows:

$$\begin{aligned}&(r+\kappa_\eta)(r+q_X^{(a)}) \frac{\lambda_a - \psi_a(\lambda+\lambda_a)}{(r+\kappa_\eta+q_X^{(a)})\left(r+2q_X^{(a)}\right)} < \frac{\gamma(\sigma_F^2+\sigma_\eta^2 p_\eta^2)(\lambda+\lambda_a)}{\gamma(\sigma_F^2+\sigma_\eta^2 p_\eta^2)+2\gamma_a\sigma_\eta^2 p_\eta^2}, \\ &(r+\kappa_\eta)(r+q_X^{(a)}) \frac{\lambda_a - \psi_a(\lambda+\lambda_a)}{(r+\kappa_\eta+q_X^{(a)})\left(r+2q_X^{(a)}\right)} < \psi_a(\lambda+\lambda_a).\end{aligned} \tag{C.14}$$

We analyze the conditions under which these inequalities hold:

- **Risk-Off regime** ($\lambda_a \leq \psi_a(\lambda+\lambda_a)$): Under this condition, the inequality holds trivially, as the left-hand side is non-positive.
- **Risk-On regime** ($\lambda_a > \psi_a(\lambda+\lambda_a)$): Here, the inequality can be rearranged as:

$$\frac{(r+\kappa_\eta)(r+q_X^{(a)})}{(r+\kappa_\eta+q_X^{(a)})\left(r+2q_X^{(a)}\right)} < \frac{\psi_a(\lambda+\lambda_a)}{\lambda_a - \psi_a(\lambda+\lambda_a)}. \tag{C.15}$$

Since this inquality implies $\frac{\lambda_a}{2(\lambda+\lambda_a)} < \psi_a$, the right-hand side of (C.15) is greater than 1, ensuring that (C.15) holds in this regime as well. Therefore, for small σ_η, there exists a function p_η that is increasing in γ_a.