Limited Liability in an Input-Output Economy

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Abstract

Trade credit is crucial in business-to-business transactions but poses challenges due to potential defaults. This study examines the distortionary effects of trade credit on market efficiency and resource allocations using a multi-sector, input-output economy model with endogenous defaults. Defaults arise in the model due to a limited liability constraint, which limits enforceable debt repayments. I show that the limited liability constraint induces persistent economic distortions by over-ordering intermediate inputs and excessive credit demand. The resultant misallocation reduces total factor productivity, final consumption and welfare. Importantly, this resource misallocation persists even when the deadweight loss from default is zero. Using sectoral input-output and firm-level data for the U.S.A., I estimate that trade credit distortion results in a welfare loss of around 2.07% of aggregate consumption.

Keywords: Limited liability, Input-output economy, Resource misallocation, Trade credit, Default risk, Defaultable contracts

I Introduction

Trade credit is ubiquitous in business-to-business transactions, with buying firms often purchasing intermediate inputs on credit from selling firms. However, such credit transactions potentially introduce risk into the system since borrowing firms could default on their repayment commitments if there is limited enforcement of debt contracts. While limited enforcement of contracts could occur for many reasons, one typical reason is limited liability constraints, which limit the enforceable debt repayment amount. Suppliers typically respond to this default risk by raising prices to include a risk premium.[1][2] The key question is whether the market's

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[1] As highlighted by Klapper et al (2012), suppliers frequently offer discounts for early payment, especially to small and non-investment-grade buyers, to mitigate the risk of default. In contrast, larger, rated buyers may have bargaining power; they often exercise it by demanding longer 'trial' credit periods before settling their payments.

[2] To illustrate, consider a typical contract like '2/10 Net30,' which implies an annual interest rate of 44% due to the risk of default. In such contracts, customers are granted a 30-day window to make full payment after receiving the goods, but they can enjoy a 2% discount if they settle the payment within the first ten days.

price response effectively balances the excessive credit demand due to limited liability. If it does, market efficiency prevails; otherwise, it results in resource misallocations and efficiency losses, which impact aggregate consumption.

To delve deeper into this issue, this paper integrates trade credit into a static, general equilibrium, multi-sector, input-output economy. Firms place orders for intermediate inputs from their suppliers on credit. However, without any additional frictions in the economy, limited liability in debt contracts introduces a potential distortion since firms can default if their realized productivity shocks render production plans unprofitable. This distortion induces borrowing firms to over-borrow while simultaneously inducing lending firms to build a risk premium into the prices they charge for their goods. [3]

The paper demonstrates that the buyer's over-borrowing effect *always* dominates the supplier's risk premium effect as long as debt defaults do not lead to a *total* destruction of the defaulting firm's output. Intuitively, as long as suppliers receive at least some fraction of a defaulting firm's output, the market risk premium only partially covers the default risk. As a result, buying firms consistently over-order intermediate inputs and demand excessive credit.

Figure 1 summarizes the distorting effects of limited liability on the economy. Over-borrowing leads to excessive defaults and over-ordering of intermediate inputs. Firms need a higher realized productivity shock to pay their debts, causing a higher default rate; combined with a positive deadweight loss of default, it results in higher output loss and lower sectoral total factor productivity(TFP). The over-ordering of intermediate inputs results in a misallocation of resources towards intermediate inputs, leaving fewer resources for producing consumption goods. As a result, TFP and welfare both fall. [4] The quantitative impact of both channels depends on the input-output structure and the extent of the deadweight loss of default. However, the resource misallocation effect of the second channel occurs even when there is no deadweight loss of default.

The over-borrowing effect is a distinctive feature of an input-output economy and is intricately connected to the interdependencies among sectors and default risk. To my knowledge, this mechanism is unique to this paper and a new introduction to the literature.

To gauge the extent of the limited liability distortion within the economy, I introduce a metric called "limited liability influence." This metric offers insights into how limited liability affects each sector by considering its ripple effects throughout the input-output economy. Upstream sectors, primarily responsible for providing intermediate inputs, experience the most significant impact from limited liability. In these sectors, buyer over-ordering results in elevated prices and increased production relative to an efficient economy without distortion. I analytically establish that the welfare loss in the distorted economy relative to the

[3] This paper investigates the distortions caused by limited liability in an input-output economy. While I acknowledge the beneficial role of limited liability in fostering investment and innovation, the paper aims to analyze the distortions it can create and understand the mechanisms through which limited liability leads to inefficiencies in this context. I recognize that the broader impact of limited liability on investment and creative destruction is an important topic, but it falls outside the scope of this study. By concentrating on these distortions, I aim to provide insights and potential interventions to address them.

[4] If suppliers do not receive any production leftovers after defaults, they charge the highest risk premium for any given default rate as they only receive payment when firms do not default. In this extreme case, over-borrowing and higher prices offset each other, resulting in no distortion.

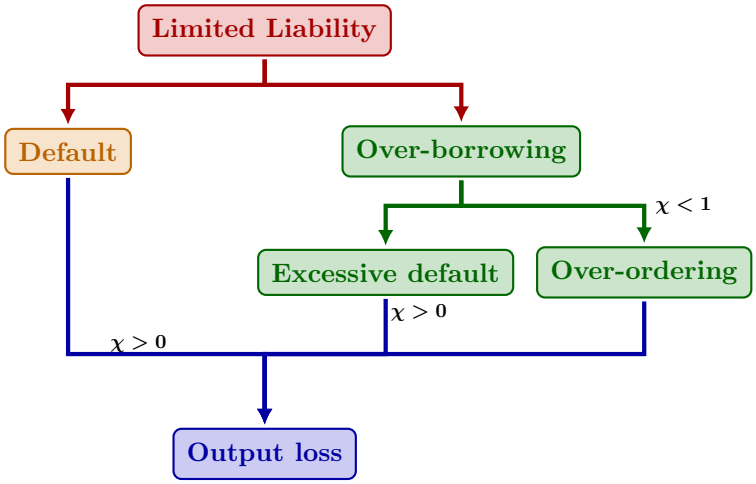


Fig. 1: Limited Liability Distortions. Over-borrowing leads to higher default rates and over-ordering of intermediate inputs. Positive deadweight loss of default, $\chi > 0$, decreases the sectoral TFP by wasting defaulting firms' output. Over-ordering leads to misallocation as long as the defaulting firms' output is not entirely wasted, $\chi < 1$.

efficient economy primarily depends on the influence of the limited liability constraint in downstream sectors engaged in producing consumer goods.

To examine the empirical implications of the model, I extend the model to include sectoral shocks and labour as a factor of production. I then use sectoral input-output and firm-level data from the U.S.A. for 1998-2021 to calibrate and estimate the model's parameters. The empirical analysis reveals that the consumption loss attributable to limited liability stands at 2.07%, with more than two-thirds of that loss attributable to the over-ordering channel.

I.A Literature Review

My research is situated within three strands of literature. The first literature is on limited liability. Previous research has extensively examined how limited liability for shareholders can impact firm investments. Some studies, like Brander and Lewis (1986), Perla et al (2022), Qin and Sun (2023), and Jacques et al (2022), have suggested that limited liability might lead to over-investment. This over-investment can increase output and welfare gains in a partial equilibrium setting. However, my research takes a different approach. In a general equilibrium input-output economy, I investigate how limited liability can lead to over-borrowing, causing resource misallocation and resulting in welfare losses.

This paper employs a contract structure with a one-period debt contract akin to Brander and Lewis (1986). On the other hand, I differentiate my research from studies by Aguiar et al (2019), who delve into equilibrium defaults and debt maturities, and Albuquerque and Hopenhayn (2004), who focus on optimal contracts under limited liability. Unlike these studies, my work does not primarily seek to establish optimal contracts within the limited liability framework but aims to investigate how limited liability contributes to misallocation.

Studies such as Townsend (1979), Carlstrom and Fuerst (1997), Williamson (1987), and Bernanke et al (1999) focused on limited enforcement in private information setting where part of defaulting firms output was wasted as the monitoring cost. In contrast to these papers, I assume symmetric information. However, lenders and borrowers cannot sign a contract contingent on the lender's realized productivity shocks and part of the defaulting firm output might be wasted as a deadweight loss of default.

My work is also closely related to the literature on the role of trade credit as a financial propagation mechanism, underlining the significant volume at which trade credit operates. [5] Empirical studies by Boissay and Gropp (2013), Jacobson and Von Schedvin (2015), and Raddatz (2010) demonstrate how trade credit helps transmit shocks from financially constrained customers to their suppliers.

While many papers (see Gatti et al (2010), Luo (2020), Kiyotaki and Moore (1997), Bigio (2023), and Boissay (2006)) primarily explore the impact of trade credit failures on liquidity shocks and the contraction of creditor balance sheets, it is essential to recognize that this is not the sole financial contagion mechanism. In my model, firms exhibit perfect customer risk pooling and implement optimal risk premiums on trade credit. However, limited liability leads firms to over-order and over-default despite their risk management efforts.

Empirical research on trade credit encounters significant hurdles arising from limited data availability and the intricacies of enforcing contracts. Weak enforcement mechanisms can obscure the connection between contract structures and the characteristics of suppliers and customers. Notably, Giannetti et al (2011) raises the possibility that this weak enforcement may be why higher penalties fail to incentivize timely repayments significantly, and firms with extended payment periods frequently miss their due dates.

Summers and Wilson (2002) empirically finds that riskier firms rely more on trade credit, resulting in elevated accounts payable relative to their total assets. However, empirical evidence suggests that suppliers do not substantially vary the price of trade credit (Petersen and Rajan (1997), Wilner (1997), and Ng et al (1999)). Furthermore, Wilner (2000) argues that the inability of trade creditors to differentiate prices for different customers leads to a scenario where risky firms prefer trade credit over credit market loans. Remarkably, both Giannetti et al (2011) and Summers and Wilson (2002) provide empirical support for the notion that firms actively engaged in trade credit perceive it as a cost-effective external funding option. My research further elucidates these empirical findings by delving into why suppliers do not charge a higher risk premium on trade credit, ultimately leading to over-borrowing, particularly among riskier firms, resulting in an increased reliance on trade credit.

Finally, my work is relevant to the literature exploring production networks and the transmission of financial shocks throughout the economy. In this literature, the multi-sectoral framework was introduced by early studies like Long Jr and Plosser (1983), Horvath (1998), and Horvath (2000) to examine how micro-sectoral shocks can generate macroeconomic fluctuations. I employ a similar framework to focus on

[5] The aggregate annual volume of domestic and international trade credit comes to over 40 percent of world GDP, or US$35 trillion, most involving trade within countries. In a developed economy like France, total inter-firm trade credit exceeds half of the country's GDP. (source: Worldbank) Trade credit is vital in the U.K. corporate sector, particularly for smaller companies, where it accounts for a significant portion of daily transactions. On average, U.K. firms maintain a trade credit-to-debt ratio of 43%. U.S.A. firms have a trade credit-to-debt ratio of 34%, indicating the substantial role of trade credit in financing and managing cash flow(Cunat (2007) and Wilson and Summers (2002)).

the transmission of distortions through the input-output economy, particularly emphasizing the impact of limited liability. Similar to Carvalho (2014), Baqaee and Farhi (2019), and Foerster et al (2011), I take an exogenous multi-sectoral input-output network. [6]

When the deadweight loss of default is nonexistent, my work aligns with the TFP decomposition found in Baqaee and Farhi (2020), emphasizing the first-order effects of rebated distortions in inefficient economies. In contrast, when defaulting firms' output is fully wasted, my model's predictions correspond to the findings in Liu (2019), primarily focusing on the first-order effects of fully wasted distortions.

Numerous studies, including those by Bigio et al (2016), Bigio and La'o (2020), Liu (2019), Luo (2020), and Altinoglu (2021), have explored various aspects of trade credit and financial distortions in input-output economies. Notably, studies like Altinoglu (2021), Luo (2020), and Bigio et al (2016) have used collateral or cash-in-advance requirements to forestall defaults. In such models, firms avoid defaulting, but external adverse financial shocks tighten credit constraints, resulting in reduced output. [7] In contrast, my research adopts a distinctive approach by spotlighting endogenously driven financial frictions that engender trade wedges between sectors. It uniquely incorporates endogenous default probabilities into the input-output network, marking a novel contribution to the existing literature.

My research bridges these three strands of literature by investigating how limited liability in business-to-business contracts affects the economy's dynamics, specifically focusing on the agency problem within an input-output framework.

I.B Layout

This article is organized as follows. Section II introduces the general framework. Section III explores the equilibrium solution. Section IV explains two types of distortions caused by the limited liability and their amplification through the network. Section V presents simple and tractable examples to see how input-output structure affects the magnitude of distortion. Section VI presents the concept of "limited liability influence" as a metric to gauge the distortions experienced by sectors resulting from limited liability. Section VII introduces optimal budget-balance tax policy to deal with this friction and restore efficient allocation. Section VIII explores two potential extensions to the benchmark model: the inclusion of labour and the incorporation of sectoral shocks. Section IX investigates sectors' exposure to limited liability distortion and the corresponding consumption loss within the U.S.A. economy. Section X briefly discusses potential avenues for future research. Section XI concludes. The Appendix includes proofs for the results pertaining to the general economy, as well as a comprehensive coverage of an extended model featuring cash-in-advance details and associated tax policies.

Notation Guideline: To ensure clarity and coherence in presenting various concepts and variables, I provide a comprehensive notation guide that will be consistently followed throughout the paper. Adhering to

[6] In contrast to my model, some papers try to explain the endogenous input-output network formation(Oberfield (2018)).
[7] These models have been critiqued for their realism, as firms face bankruptcy risks and credit spreads(Anderson and Cesa-Bianchi (2020), Campbell and Taksler (2003) and Campbell et al (2001)).

these conventions will facilitate a better understanding of the paper's content. Sectors will be distinguished by subscripts i and j, and f will represent firms. Sectoral variables will employ capital letters, for example, P_i and Y_i. Variables related to individual firms will utilize lowercase letters, such as p_f and y_f. Variables in an efficient economy will be denoted with a superscript e. Vectors, which will be in N by 1 dimensions, will be indicated using letters with no subscripts, e.g., x. Matrices, following N by N dimensions, will be represented by bold letters with no subscripts, like $\mathbf{A}$. The symbol $\circ$ will signify element-wise multiplication. The symbol $\otimes$ will denote the Khatri-Rao product (column-wise Kronecker product).

II General Framework

The economy is *static* and comprises of a representative household and N production sectors, indexed by $i \in \mathscr{N} = \{1, ..., N\}$. Each sector consists of a unit mass of firms indexed by $f \in [0, 1]$ operating in a perfectly competitive market, accompanied by a zero-profit sector aggregator tasked with transferring the output of these firms into a uniform sectoral good that households can consume as final goods or other firms use as intermediate inputs. Firms are heterogeneous in their sectors and the realized idiosyncratic productivity shocks. Firms buy *all* the intermediate inputs from sector aggregators on credit before realizing idiosyncratic productivity shock. These trade-credit contracts cannot be contingent upon idiosyncratic shocks, and firms retain the option to default due to limited liability constraints. In the event of default, a portion of the firm's output becomes lost as deadweight loss, with the remaining output going to suppliers (the creditors). The representative household, in turn, consumes sectoral goods utilizing the rebated profits.

It is beneficial to divide the period into two stages to clarify the sequence of events. In Stage I, idiosyncratic shocks have yet to be realized, while in Stage II, idiosyncratic shocks have occurred.

II.A Sectors' Technology

The sector aggregator, functioning as a wholesale retailer, gathers all goods produced within a specific sector to create the sectoral output. This process is demonstrated by Equation 1, where sectoral aggregator j purchases y_{fj} units from Firm f in Sector j at the price of $\hat{p}_{fj}$. The unit mass of firms competes in a perfectly competitive market within the sector.

$$Y_j \equiv \int_0^1 y_{fj} df. \tag{1}$$

The Sectoral aggregator has the option to sell its sectoral good either to the representative household as the final good C_j at the consumer price of P_j, or to Firm f in Sector i as an intermediate input, x_{fij}, at the price of p_{fij}. [8]

The market clearing condition for Commodity j is described as:

[8] Unlike the representative consumers, firms have the option to default, so p_{fij} is a function of Sector j's attributes and default risk of Firm f in Sector i. Therefore, P_j and p_{fij} do not necessarily equal at the equilibrium.

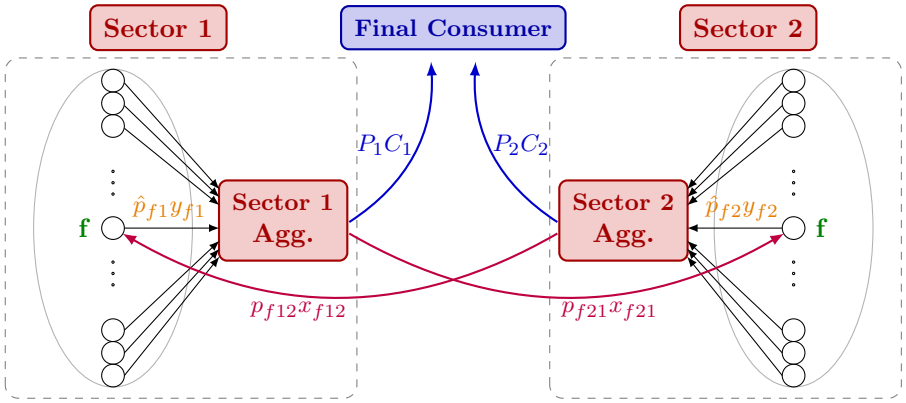


Fig. 2: Simplified Two-Sector Roundabout Economy Structure. Sector aggregators gather all products within a given sector and sell them to final consumers or firms. In Sector 1, all firms, including Firm f, buy intermediate inputs using credit from Sector Aggregator 2. Similarly, in Sector 2, every firm, including Firm f, obtains intermediate inputs on credit from Sector Aggregator 1. Firms in a sector can only sell their outputs to their own sector aggregator.

$$Y_j = C_j + \sum_{i=1}^{N} \int_0^1 x_{fij}\, df. \tag{2}$$

Figure 2 depicts a simplified two-sector model, each sector comprising a unit mass of firms. In Sector 1, we focus on Firm f purchasing intermediate inputs from the Sector 2 aggregator. Similarly, in Sector 2, every firm, including Firm f, obtains intermediate inputs on credit from the Sector 1 aggregator. Sector aggregators are central in combining firms' output within their respective sectors.

Firms exclusively trade with sector aggregators rather than directly with each other, simplifying the model. This implies that individual demand shocks at the firm level are not considered, resulting in uniform risk exposure for all firms within each sector. In other words, any changes in x_{fij} equally impact the demand of all firms in Sector j. [9] I acknowledge that in real-world scenarios, firms engage in trade with one another and encounter diverse levels of demand uncertainty and varying degrees of counterparties' default risk. However, my model simplifies these complexities to emphasize the essential financial friction mechanisms. [10]

II.B Sectors' Problem

The sector aggregator optimization problem is as follows:

$$\Pi_j = \underset{\left\{P_i, C_i, \left\{\{x_{fji}, p_{fji}\}_{f\in[0,1]}\right\}_{j=1}^{N}, \{\hat{p}_{fj}\}_{f\in[0,1]}\right\}}{\text{Max}} \left(P_i\, C_i + \sum_{j=1}^{N} \int_0^1 p_{fji}\, x_{fji}\, df \right) - \int_0^1 \hat{p}_{fj} y_{fj}\, df,$$

subject to the feasibility constraint:

[9] Another perspective on this is to consider that the creation of x_{fij} within Sector j is distributed proportionally across all firms in the sector based on their output proportions. For instance, if Tesla orders 5% of all batteries from the Battery sector, each battery producer allocates 5% of their output to fulfill Tesla's order. By assuming uniform risk exposure for all firms within a sector, we avoid factoring in individual demand shocks at the firm level. This approach lets us concentrate on the overall impacts of financial frictions. This assumption also simplifies the model's strategic interaction and ensures perfect diversification for firms in the supplier sector.

[10] Similar to the assumption of risky firms as customers and risk-free sector aggregators as suppliers, Lee and Rhee (2011) assumes a 5% probability of default for the customer, a zero probability for the supplier, and includes a 10% exogenous risk premium on the customer's side.

$$Y_i \geq C_i + \sum_{j=1}^{N} x_{ji}.$$

Sector Aggregator j has a constant-return aggregator of firms' outputs and operates as a zero-profit entity(i.e. $\Pi_j = 0$).

II.C Firms' Technology

To focus on the core idea, I am excluding factors of production. Firms will only purchase intermediate inputs from sector aggregators and produce output. We can later introduce labour and capital markets for similar outcomes. [11] Firm f in Sector i produces as follows:

$$y_{fi} \equiv A_f \, x_{fi}^{\alpha_i} \quad , \alpha_i \in (0,1),$$

where A_f represents the Hicks-neutral idiosyncratic productivity shock, drawn from a mean-one distribution with a continuous cumulative distribution function of G, bounded from below by zero.

x_{fi} is the aggregate intermediate input used by this firm, and α_i is the sector-specific degree of returns to scale. It is common in the literature to assume decreasing returns to scale (DRS) with respect to the intermediate inputs.[12]

The intermediate input aggregator for Firm f in Sector i follows

$$x_{fi} \equiv \left(\sum_{j=1}^{N} \omega_{ij} \, x_{fij}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}} \quad \text{for } i \in \{1, ..., N\} \ , \sum_{j=1}^{N} \omega_{ij} = 1,$$

where ω_{ij} is the share parameter of input j in the aggregate intermediate inputs used by firms in Sector i. x_{fij} represents the quantity of intermediate input j purchased by Firm f in Sector i. The equation implies that the intermediate input aggregator is a CES function of intermediate inputs, where the elasticity of substitution is given by γ.

x_{fij} is a *specific* input for Firm i, meaning that it can *only* be used by this firm in the production process. The literature often models contracts for intermediate inputs with two types of inputs: specific and generic (Cunat, 2007). Specifically, Kermani and Ma (2023) argues that asset specificity is crucial for investment response to uncertainty. In fact, the investment response to uncertainty is absent if assets are fully generic. Additionally, Bigio (2023) utilizes the presumption of input specificity to illustrate how delayed payments within the supply chain can result in reduced TFP. The conversion of Y_j into a specific input, x_{fij}, incurs no cost for Sector Aggregator j, although this process is irreversible.

[11] I will demonstrate how the model mechanisms are magnified with factors of production later.

[12] Horvath (1998), Bigio et al (2016), and Herskovic et al (2016) assume DRS for the production function with capital or labour as a factor of production. Moreover, Boissay (2006), Besley et al (2020), and Gatti et al (2010) assume DRS to investigate the financial propagation mechanism.

As intermediate inputs are specific, suppliers require time to adjust their production line for the orders. Therefore, firms must pre-order their inputs in Stage I. In this stage, sectors' outputs, prices, and the fraction of defaulting firms are equilibrium variables known to all parties. Hence, sector aggregators (suppliers) and firms (customers) have all the information necessary to sign the contract. Firms place input orders in Stage I and proceed to deliver, produce output, and make payments (in full or partially) for these input orders in Stage II. Payment occurs after delivery, effectively providing credit from sector aggregators to firms.

The share of Sector j in providing credit for Firm f in Sector i is denoted by $\tilde{\Delta}_{fij}$,

$$\tilde{\Delta}_{fij} \equiv \frac{p_{fij}\, x_{fij}}{\sum_{j=1}^{N} p_{fij}\, x_{fij}} = \frac{\text{Expenditure on Commodity } j}{\text{Total expenditure on inputs}}.$$

The term $\tilde{\Delta}_{fij}$ represents the fraction of Firm f's expenditures on Commodity j relative to its total expenditure on intermediate inputs. Since all expenditures are externally financed, this fraction also represents the share of Sector j in Firm f's total debt. In a Cobb-Douglas intermediate input aggregator(i.e. $\gamma = 1$), the credit share $\tilde{\Delta}_{fij}$ is equal to the share parameter of Commodity j in the intermediate input aggregator, denoted by ω_{ij}. In Figure 2, $\tilde{\Delta}_{f12} = \tilde{\Delta}_{f21} = 1$. In general $\sum_j \tilde{\Delta}_{fij} = 1$ by definition.

II.D Contract Structure

The contract structure is similar to Brander and Lewis (1986). [13] In Stage I, Firm f in Sector i orders a quantity of x_{fij} in advance from the Sector j's aggregator at a fixed promised price of p_{fij}. The firm has the option to default on its commitment. Sector aggregators, the creditors, commit not to renegotiate in the event of default to prevent the customer from misreporting their default status and bargaining for a lower price. Based on the Law of Large Numbers, the Sector j's aggregator can predict the fraction of firms in Sector i that will go bankrupt with certainty.

In Stage II, if Firm f in Sector i does not default, Sector j's aggregator receives a fixed payment of $p_{fij}\, x_{fij}$ for the input. This payment remains constant irrespective of the realization of productivity shocks, and the firm owner retains the associated profit.

If a firm's profit turns negative, default occurs. In that case, the bankruptcy court takes control of the firm, produces, and a fraction $\chi \in [0, 1]$ of the defaulting firm's output is wasted as the deadweight loss of default. In such a case, the sector j aggregator holds a claim over the residual income proportionate to $\tilde{\Delta}_{fij}$. Notably, no debt seniority leads to the sector j aggregator receiving $\tilde{\Delta}_{fij}\,(1-\chi)\hat{p}_{fi}\, y_{fi}$.

II.E Firms' Problem

Firms in a given sector must determine the optimal level of intermediate inputs to maximize the expected firm value,V_{fi}, in Stage I before realizing their individual productivity shocks. As firms operate under limited

[13] In Brander and Lewis (1986), the borrower and the creditor establish a debt contract in Stage I that cannot be contingent on the realized productivity shocks in Stage II. If the borrower's realized profit falls short of meeting the debt obligation, the borrower defaults and walks away, while the creditor receives the defaulting firm's output.

liability, their owners have the option to default if profits turn negative due to the realization of a low productivity shock. Equation 3 encapsulates the maximization of expected profit over all possible realizations of productivity shocks by selecting the optimal basket of inputs at given prices.

$$V_{fi} = \max_{\{x_{fij}\}_{j=1}^{N}} \mathop{\mathbb{E}}_{A_f \sim G} \left(\max\left\{0, \Pi_{fi}(A_f)\right\}\right), \tag{3}$$

s.t.

$$\Pi_{fi}(A_f) \equiv \hat{p}_{fi}\, A_f\, x_{fi}^{\alpha_i} - \sum_{j=1}^{N} p_{fij}\, x_{fij}, \tag{4}$$

$$x_{fi} \equiv \left(\sum_{j=1}^{N} \omega_{ij}\, x_{fij}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}. \tag{5}$$

The first constraint in Equation 4 characterizes the firm's profit. The initial term signifies revenue, influenced by realized productivity shocks, output price, and aggregate intermediate input utilization. The second term represents input costs, which remain constant irrespective of productivity shocks. The second constraint in Equation 5 describes the firm's aggregate intermediate input technology.

Firms make two decisions: choosing the optimal set of intermediate inputs and making the default decision. Firm f defaults if and only if its profit becomes negative, $\Pi_{fi}(A_f, x_{fi}) < 0$.

F.O.C of firm's problem implies that optimal x_{fij} should satisfy

$$\alpha_i\, \hat{p}_{fi} \left(\int_0^{\infty} A_f\, \mathscr{I}(A_f) dG\right) x_{fi}^{\alpha_i - 1}\, \frac{\mathrm{d}x_{fi}}{\mathrm{d}x_{fij}} \leq p_{fij} \left(\int_0^{\infty} \mathscr{I}(A_f) dG\right), \tag{6}$$

where $\mathscr{I}$ is an indicator function. It equals one when Firm f does not default at the realized productivity of A_f and zero otherwise. Equation 6 holds with equality for every Commodity j that $\omega_{ij} > 0$ in Equation 5.

II.F Households

In this economy, the representative household aims to maximize its aggregate consumption by optimizing its basket of consumption goods. This can be formulated as a standard consumption problem, where the CES function in Equation 8 represents aggregate consumption. Here, σ represents the elasticity of substitution between sector-produced goods, and ω_{ci} is the share parameter of Commodity i in the aggregate consumption basket. The household's optimization problem is shown in Equations 7 to 9.

$$\max_{\{C_i\}_{i=0}^{N}} C, \tag{7}$$

s.t.

$$C \equiv \left(\sum_{i=1}^{N} \omega_{ci} \, C_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \quad \sum_{i=1}^{N} \omega_{ci} = 1, \tag{8}$$

$$\sum_{i=1}^{N} P_i \, C_i = \sum_{i=1}^{N} \int_0^1 V_{fi} \, df. \tag{9}$$

The household faces a budget constraint, as shown in Equation 9. The two sides of the budget constraint coincide with nominal GDP, using respectively the final expenditure and income approaches. For simplicity, nominal GDP is normalized to one.

III Equilibrium Characterization

In this section, I apply equilibrium conditions to characterize the solution for this economy. Within sectors, a unit mass of price-taker firms operates in perfect competition, and a zero-profit sector aggregator with a constant-return aggregator, as described in Equation 1, combines firms' outputs. Hence, $\hat{p}_{fi} = P_i$ for all firms in Sector i.

Prior to the realization of individual productivity shocks, firms within a given sector are identical ex-ante in terms of their access to supplier contracts, technology, and demand.[14] Therefore, they exhibit uniform behaviour and encounter identical prices, leading to $x_{fij} = x_{ij}$ and $p_{fij} = p_{ij}$.

In the second stage, after productivity shocks are realized, firms begin to differentiate based on their individual productivity levels. Each firm makes a distinct decision regarding whether or not to default, depending on their realized productivity.

Applying the mentioned equilibrium conditions, Equation 4 can be rewritten as:

$$\Pi_{fi}(A_f) = P_i \, A_f \, x_i^{\alpha_i} - \sum_{j=1}^{N} p_{ij} \, x_{ij}. \tag{10}$$

Equation 10 shows that price-taker firms within a sector choose identical levels of intermediate input and accumulate equal debt in equilibrium. As a result, the only source of heterogeneity in firm profit within the sector is A_f, which strictly increases profit.

A firm in Sector i defaults when its productivity, A_f, falls below the profitability threshold, A^*i (i.e., $\Pi_{f,i}(A_i^*) = 0$). As depicted in Equation 10, firm revenue is positively linked to productivity shocks, while firms within the same sector share the same cost structure. A firm will default if and only if its productivity

[14] The only source of heterogeneity for firms is their sectors in Stage I.

falls below the threshold, resulting in negative profit. This condition is succinctly captured as:

$$\Pi_{fi}(A_f) < 0 \Longleftrightarrow A_f < A_i^*.$$

Therefore, the indicator function, $\mathscr{I}$, has a monotone property, and it is one if and only if $A_i^* < A_f$. Firms' problem in Equation 3 to Equation 5 can be restated as follow:

$$\max_{\{x_{ij}\}_{j=1}^N} \int_{A_i^*}^{\infty} P_i\, A_f\, x_i^{\alpha_i}\, dG - (1 - G\,(A_i^*)) \sum_{j=1}^{N} p_{ij}\, x_{ij},$$

s.t.

$$x_i \equiv \left(\sum_{j=1}^{N} \omega_{ij}\, x_{ij}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}},$$

where $G\,(A_i^*)$ is the fraction of defaulting firms in Sector i. [15]

The F.O.C of the optimal set of intermediate inputs, as depicted in Equation 6, can be reformulated as:

$$\alpha_i\, P_i\, x_i^{\alpha_i - 1} \frac{\mathrm{d}x_{fi}}{\mathrm{d}x_{fij}} \int_{A_i^*}^{\infty} A_f\, dG = (1 - G(A_i^*))\, p_{ij} \quad \forall\, j \;\; s.t. \;\; \omega_{ij} > 0.$$

III.A Default Threshold, A^*

To determine the default threshold A^*, two sets of conditions are essential: First, the F.O.C of the firm's problem with respect to all the intermediate inputs shown as:

$$\alpha_i\, P_i\, x_i^{\alpha_i} \frac{\mathrm{d}x_{fi}}{\mathrm{d}x_{fij}} \int_{A_i^*}^{\infty} A_f\, dG = (1 - G(A_i^*)) \sum_{j=1}^{N} p_{ij} x_{ij},$$

derived from the F.O.C of the optimal set of intermediate inputs.[16] Secondly, the firm's profit at the default threshold productivity, A^*, must be zero to signify the breakeven point. This condition is represented as:

$$\Pi(A_i^*) = 0 \Longleftrightarrow \underbrace{P_i\, A_i^* x_i^{\alpha_i}}_{\substack{\text{Revenue of} \\ \text{the threshold firm}}} = \underbrace{\sum_{j=1}^{N} p_{ij}\, x_{ij}}_{\text{Total debt}},$$

[15] This version is more familiar to readers of Bernanke et al (1999) and Williamson (1987), representing the unconditional expected profit and expected cost.

[16] Firm's solution to find the optimal set of $\{x_{ij}\}_{j=1}^N$ is

$$\frac{\mathrm{d}x_i}{\mathrm{d}x_{ij}}\, P_i\, \alpha_i\, x_i^{\alpha_i - 1} \int_{A_i^*}^{\infty} A_f dG = \left(1 - G\left(A_i^*\right)\right)\, p_{ij}.$$

By multiplying both side with x_{ij}, we drive

$$\frac{\mathrm{d}x_i}{\mathrm{d}x_{ij}}\, x_{ij}\, \alpha_i\, P_i\, x_i^{\alpha_i - 1} \int_{A_i^*}^{\infty} A_f dG = \left(1 - G\left(A_i^*\right)\right)\, p_{ij}\, x_{ij}.$$

By taking the sum over all intermediate inputs, we have

originated from the threshold productivity definition. Combining these two conditions leads to the following condition

$$A_i^* = \alpha_i E(A|A \geq A_i^*). \tag{11}$$

Equation 11 provides insights into the threshold productivity and the fraction of firms that default within a sector, regardless of the network structure. The threshold productivity, A_i^*, is determined only based on α_i and G. It captures the point at which firms in the sector cannot meet their debt obligations and default. By imposing weak assumptions, A_i^* exists and is unique. [17]

The threshold productivity value, A_i^*, escalates as α_i increases. Intuitively, α_i encapsulates three pivotal factors. First and foremost, it signifies the inverse of the markup, depicting the extent to which firms can cover debt obligations through profits.

The markup denotes the revenue-to-cost ratio. In Sector i, all firms procure the same intermediate input level, yet their revenues differ due to varying productivities. For realized productivity A_f, the markup can be expressed as:

$$markup_{fi}(A_f) = \frac{Revenue}{Cost} = \frac{P_i x_i^{\alpha_i} A_f}{P_i x_i^{\alpha_i} A^* i} = \frac{A_f}{A_i^*}.$$

Additionally, Equation 11 can be rewritten as:

$$\frac{1}{\alpha_i} = E\left(\frac{A}{A_i^*}|A \geq A_i^*\right), \tag{12}$$

where the right-hand side represents the expected markup conditional on no default. Without idiosyncratic uncertainty, firms in Sector i set the markup to $\frac{1}{\alpha_i}$. With the introduction of idiosyncratic uncertainty, they adjust the expected markup accordingly. A higher markup (lower α_i) bolsters firms against financial stress, increasing their capacity to fulfill debt obligations. Equation 12 elucidates that regardless of how many firms default, the expected markup for non-defaulting firms remains unchanged. This clarifies why prices play no role in Equation 11. Firms align final prices with average costs, so increased input costs are transmitted to customers through elevated output prices, leaving the default threshold unchanged.

Secondly, a higher α_i implies that firms borrow more inputs for each output unit, elevating debt levels. Consequently, the likelihood of default rises with higher debt burdens. Consistent with existing literature,

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$$\left(\sum_j \frac{\mathrm{d}x_i}{\mathrm{d}x_{ij}} x_{ij}\right) \alpha_i \, P_i \, x_i^{\alpha_i - 1} \int_{A_i^*}^{\infty} A_f dG = \left(1 - G\left(A_i^*\right)\right) \left(\sum_j p_{ij} \, x_{ij}\right).$$

Using the fact that x_i is CRS, $\sum_j \frac{\mathrm{d}x_i}{\mathrm{d}x_{ij}} x_{ij} = x_i$,

$$\alpha_i \cdot \overbrace{P_i \, x_i^{\alpha_i} \int_{A_i^*}^{\infty} A_f \, dG}^{\text{expected sale}} = \overbrace{(1 - G(A_i^*)) \cdot \sum_{j=1}^{N} p_{ij} \, x_{ij}}^{\text{expected input cost}}.$$

[17] See Online Appendix for the proof.

the second explanation becomes particularly relevant later, as higher debt-to-sales ratios amplify firm risk (see Bernanke et al (1999), Gatti et al (2010), and Kiyotaki and Moore (1997)).

Lastly, α_i reflects the sector's reliance on the production network. A higher α_i signifies heightened dependence on inputs for final output production.

Higher α amplifies trade credit demand, default probability, and lowers profit. This model prediction aligns with empirical evidence presented in Deloof and Jegers (1999) and Petersen and Rajan (1997).

III.B Recovery Rate, δ

Definition 1. *The recovery rate, δ, for a defaulting firm is*

$$\delta \equiv \frac{Recovered\ Amount}{Principal\ Debt}.$$

The recovery rate indicates the portion of unpaid debt creditors can recover if the borrower defaults. The recovery rate for defaulting firms in Sector i may vary between individual firms. However, during the contract signing in Stage I, creditors and borrowing firms in Sector i only consider the sector-level recovery rate, δ_i, because they lack additional information about idiosyncratic productivity shocks.[18] Sector Aggregator j is guaranteed a recovery value of $\delta_i\, p_{ij}\, x_{ij}$ on average from defaulting firms in Sector i

The recovery rate for Sector i

$$\delta_i = (1-\chi)\int_0^{A_i^*} \frac{P_i\, y_{fi}(A)}{\sum_{j=1}^{N} p_{ij}\, x_{ij}} dG'(A),$$

where the productivity distribution conditional on default, $G'(.)$, is $\frac{G(.)}{G(A_i^*)}$.

We are taking integral over productivity shocks below the threshold that firms in Sector i default. Inside the integral is the ratio of revenue over the total debt. This ratio equals 1 for the threshold productivity A_i^*. It is smaller than one for productivity below this threshold, so $\delta_i < 1$.

Utilizing the fact that the revenue for a firm in Sector i with A_i^* equals its input cost (or total debt), we arrive at:[19]

$$\delta_i = (1-\chi)\frac{E(A|A<A_i^*)}{A_i^*}. \tag{13}$$

This equation demonstrates that δ_i solely depends on χ, α_i, and G. In Stage I, when contracts are being signed, all agents know δ_i.

[18] Fims' sector is the only state variable in Stage I

[19]
$$\delta_i = (1-\chi)\int_0^{A_i^*} \frac{P_i\, A_f\, x_i^{\alpha_i}}{P_i\, A_i^*\, x_i^{\alpha_i}}\, dG'$$
$$= (1-\chi)\frac{\int_0^{A_i^*} A_f\, dG'}{A_i^*} = (1-\chi)\frac{E(A|A<A_i^*)}{A_i^*}$$

In Section III.C, I will delve into how sectors optimize their expected profit by determining bilateral trade quantities and prices.

III.C Bilateral Sectoral Prices

The sector optimization problem can be reformulated as follows:

$$\underset{\{P_i, C_i, \{x_{ji}, p_{ji}\}_{j=1}^N\}}{\text{Max}} P_i\, C_i + \sum_{j=1}^{N} p_{ji} \left(\int_{A_j^*}^{\infty} x_{ji}\, dG(A) + \int_0^{A_j^*} \delta_j\, x_{ji}\, dG(A) \right) - P_i\, Y_i,$$

subject to the feasibility constraint:

$$Y_i \geq C_i + \sum_{j=1}^{N} x_{ji}.$$

The sector aggregators function as zero-profit entities with CRS technology. A crucial aspect of their optimization involves bilateral sectoral prices, where prices are determined to equalize the expected revenue from selling one unit of output to final consumers and one unit of intermediate inputs to firms. This equilibrium condition is illustrated as follows:

$$P_i = \int_{A_j^*}^{\infty} p_{ji} dG(A) + \delta_j \int_0^{A_j^*} p_{ji} dG(A) \ , \ \forall j \in \{1, ..., N\}.$$

By rearranging it, we arrive at Equation 14, which expresses the price of bilateral trade as a function of sector price, recovery rate, and the fraction of defaulting firms. This equation demonstrates that the trading price between sectors depends on the supplier sector's marginal cost and the customer sector's default risk,$G(A_j^*)$, and recovery rate,δ_j, in the event of default.

$$p_{ji} = P_i \frac{1}{\left(1 - G\left(A_j^*\right)\right) + \delta_j G\left(A_j^*\right)} \ , \ \forall j \in \{1, ..., N\}. \tag{14}$$

$1 - G(A_j^*)$ represents the proportion of non-defaulting firms in sector j who pay their debt in full. $G(A_j^*)$ is the fraction of defaulting firms who pay δ_j of the their debt. The term $\frac{1}{(1-G(A_j^*))+\delta_j G(A_j^*)}$ in Equation 14 signifies the risk premium for firms in Sector j. An increase in the fraction of defaulting firms in the customer sector or a decrease in the recovery rate leads to a higher risk premium, resulting in a higher trading price.

III.D Equilibrium Definition

Definition 2. *An equilibrium is an allocation of* $\{C_i\}_{i=1}^N$ *and* $\left\{\{x_{ij}\}_{i=1}^N\right\}_{j=1}^N$ *and a sequence of* $\left\{P_i, A_i^*, \delta_i, \{p_{ij}\}_{j=1}^N\right\}_{i=1}^N$ *such that:*

(i) given $\left\{P_i, \{p_{ij}\}_{j=1}^N\right\}_{i=1}^N$, $\{\{x_{ij}\}_{i=1}^N\}_{j=1}^N$ *and* $\{A_i^*\}_{i=1}^N$ *satisfy Firms' problem in Equations 3 to 5;*

(ii) given $\left\{A_i^*, \delta_i, \{p_{ij}\}_{j=1}^N\right\}_{i=1}^N$, $\{\{x_{ij}\}_{i=1}^N\}_{j=1}^N$, $\{C_i\}_{i=1}^N$, *and* $\{P_i\}_{i=1}^N$ *satisfy Sectors' problem in Equations 2 and 14;*

(iii) given $\{P_i\}_{i=1}^N$, $\{C_i\}_{i=1}^N$ *satisfy Households' problem in Equations 7 to 9;*

(iv) $\{\delta_i\}_{i=1}^N$ *satisfy Equation 13.*

III.E Efficient Economy

Before delving into the limited liability distortion within the input-output economy, it is essential to establish the efficient allocation as our reference point. This frictionless environment allows us to present the social planner's allocation as efficient.

To ensure comparability with the benchmark model, the social planner allocates intermediate inputs before realizing idiosyncratic shocks. Even though there is no default risk, individual uncertainty remains. The constrained social planner's objective is to maximize aggregate consumption by allocating sectoral goods to final consumers and firms in various sectors. This problem can be summarized as follows:

$$\max_{\left\{C_i, \left\{\{x_{fij}\}_{f\in[0,1]}\right\}_{j=1}^N\right\}_{i=0}^N} \left(\sum_{i=1}^N \omega_{ci}\, C_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}},$$

s.t.

$$\int_0^1 y_{fi} df = C_i + \sum_{i=1}^N \int_0^1 x_{fji}\, df \quad \text{for } i \in \{1, ..., N\},$$

$$y_{fi} \equiv A_f\, x_{fi}^{\alpha_i} \quad , \alpha_i \in (0,1),$$

$$x_{fi} \equiv \left(\sum_{j=1}^N \omega_{ij}\, x_{fij}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}} \quad \text{for } i \in \{1, ..., N\} \ , \sum_{j=1}^N \omega_{ij} = 1,$$

where ω_{ci} is the share parameter of Commodity i in the aggregate consumption and $\sum_{i=1}^N \omega_{ci} = 1$.

Definition 3. *An efficient allocation is a sequence of* $\left\{C_i^e, \left\{\left\{x_{fij}^e\right\}_{f\in[0,1]}\right\}_{j=1}^N\right\}_{i=0}^N$ *that solves the social planner problem described in Section III.E.*

The social planner allocates intermediate inputs without prior knowledge of idiosyncratic shocks. Therefore, firms within the same sector are treated uniformly in an efficient equilibrium, and identical intermediate inputs are allocated to them, meaning $x_{fij}^e = x_{ij}^e$.

The F.O.C.s for the social planner problem are:

$$\frac{\omega_{ci}}{\omega_{cj}}\left(\frac{C_i^e}{C_j^e}\right)^{\frac{-1}{\sigma}} = \frac{P_i^e}{P_j^e} \qquad \forall\, i,j \text{ that } \omega_{ci}>0,\ \omega_{cj}>0,$$

$$\frac{\omega_{ij}}{\omega_{ik}}\left(\frac{x_{ij}^e}{x_{ik}^e}\right)^{\frac{-1}{\gamma}} = \frac{P_j^e}{P_k^e} \qquad \forall\, i,j,k \text{ that } \omega_{ij}>0,\ \omega_{ik}>0,$$

where $\{P_i^e\}_{i=1}^N$ represents the sequence of shadow prices. The first set of F.O.C.s depicts the M.R.S. between any two final goods, and the second set shows the M.R.T.S. between any two intermediate inputs within the production process.

I define A_i^{*e}, the efficient break-even point, using the shadow prices as follows:

$$\Pi_{fi}(A_i^{*e}) = 0 \iff P_i^e\ A_i^{*e}\ x_i^{e\alpha_i} = \sum_j P_j^e\ x_{ij}^e.$$

While profit lacks meaning in a social planner allocation, the concept of the efficient break-even point becomes valuable when discussing limited liability distortion.

The upcoming section examines the economic impacts of limited liability friction through two distinct channels: the misallocation of intermediate inputs and the deadweight loss from defaults. Notably, the former is specific to the input-output economy, while the latter is exacerbated within this framework.

IV Channels of Distortions

Before showing how limited liability appears as a distortion and propagates through the network, I need to define three key terms.

Definition 4. *Sectoral gross output, $\lambda_i \equiv P_i Y_i$, is the total value of sectoral goods sold to the final consumer as the final good and to firms as intermediate inputs.*

The sectoral gross output become Domar weights by setting the nominal GDP, PC, to one.

Definition 5. *The aggregate gross output, ϵ, is*

$$\epsilon = \sum_{i=1}^{N} \lambda_i.$$

The aggregate gross output represents the dollar value of total production in an economy. When setting nominal GDP to one, ϵ equals one in a horizontal economy without intermediate inputs. $\epsilon - 1$ signifies the total sale of intermediate inputs in a general economy.[20]

[20] ϵ is also referred to as the network multiplier in the literature.

Definition 6. *Sectoral TFP, $E(A|A^*, \chi)$, is*

$$E(A|A^*, \chi) = (1-\chi)\int_{A^*}^{\infty} A\, dG + \int_{0}^{A^*} A\, dG.$$

It represents the effective productivity of a sector while accounting for the deadweight loss associated with defaulting firms.

In an efficient economy, the sectoral TFP equals one, as the idiosyncratic productivity shocks follow a distribution with a mean of one. Additionally, when limited liability exists with no deadweight loss of default ($\chi = 0$), the sectoral TFP remains at one.

Definition 7. *Leverage ratio, LR, is*

$$\frac{total\ debt}{EBITDA},$$

where EBITDA is earnings before interest, taxes, depreciation, and amortization.

IV.A Understanding the Over-ordering Mechanism

Due to limited liability, firm owners are encouraged to over-borrow trade credit and over-demand intermediate inputs from suppliers. If the realized productivity shock exceeds the default threshold, they gain a higher profit from increased output due to higher inputs; otherwise, they get zero. This creates a moral hazard issue, as outlined in Lemma 1.

Lemma 1. *In a market with limited liability, the market value of intermediate inputs to the market value of sectoral output is different compared to an efficient economy.*

$$\frac{\frac{\sum_{j=1}^{N} P_j\, x_{ij}}{\lambda_i}}{\frac{\sum_{j=1}^{N} P_j^e\, x_{ij}^e}{\lambda_i^e}} = \Gamma_i,$$

where $\Gamma_i = \left(1 - G\left(A_i^*\right) + \delta_i\, G(A_i^*)\right) \frac{E(A|A>A_i^*)}{E(A|\chi, A_i^*)}$ is the over-ordering coefficient.[21]

Lemma 1 illustrates how limited liability in an economy alters the ratio of the market value of resources a sector uses to the market value of its outputs compared to an efficient economy. This disparity is quantified as the over-ordering coefficient. The term $\sum_{j=1}^{N} P_j\, x_{ij}$ shows the market value of Sector i inputs to generate sectoral output with a market value of λ_i. The moral hazard distorts the ratio of resources used to produce the sectoral goods, and by deviating from the optimal ratio, we incur welfare loss. Proposition 1 shows that this ratio is *always* bigger than one if the defaulting firms' output is not *fully* wasted.[22]

Proposition 1. *In the model outlined in Section II, the over-ordering coefficient, Γ_i, is defined as follows:*

[21] See Online Appendix for the proof.
[22] See Online Appendix for the proof.

$$\underbrace{(1-G(A_i^*)+G(A_i^*)\delta_i)}_{\textit{Inverse of risk premium}}\underbrace{\left(\frac{E(A|A>A_i^*)}{E(A|A_i^*,\chi)}\right)}_{\textit{Moral hazard problem}}. \tag{15}$$

If $\chi < 1$ and idiosyncratic productivity shocks have a mean-one distribution and follow the CDF G with lower support of $\underline{A}$, when $\alpha_j > \underline{A}$ (i.e., when $G(A^*) > 0$), Γ exceeds one.

$$\Gamma_i = 1 + (1-\alpha_i)\underbrace{(1-\chi)\frac{E(A|A<A_i^*)}{A_i^*}G(A_i^*)}_{\delta_i}\frac{E(A|A>A_i^*)}{E(A|\chi,A_i^*)} > 1.$$

The first term in Γ_i,

$$(1-G(A_i^*))+G(A_i^*)\ \delta_i,$$

is the inverse of the risk premium and is always smaller than one. This term is the market response to the moral hazard problem. By charging a higher price to cover the risk premium, the market tried to disincentive firm owners to *over-order*, but it is never strong enough to fully offset the moral hazard problem. Intuitively, as long as suppliers receive at least some fraction of a defaulting firm's output (i.e. $\delta_i > 0$), the market risk premium only partially covers the default risk. As suppliers raise prices in proportion to the associated risk, the sector's demand for inputs and credit decreases. Only when $\chi = 1$ and the recovery rate, δ_i, is zero, suppliers charge the complete default risk to non-defaulting firms, effectively neutralizing the moral hazard problem.

In contrast, the second term in Γ_i,

$$\frac{E(A|A>A_i^*)}{E(A|\chi,A_i^*)},$$

is always bigger than one. This term reflects the standard moral hazard problem leading to demand for more inputs and credit. This term represents firms' expected productivity in Sector i under the condition of not defaulting relative to the sectoral TFP, which is always bigger than one. This fraction indicates that the firm owners care about the expected productivity under the condition of not defaulting rather than the expected productivity when they are demanding intermediate inputs on credit. Over-ordering is the first channel in which limited liability affects the economy. In the extreme case of $\chi = 1$, these two terms in Γ_i cancel each other out, rendering this channel inactive.

The recovery rate, δ, is a form of insurance, offering creditors a safety net in the default scenario. This assurance, however, introduces moral hazard, a problem similar to what was observed by Krugman (1998) during the Asian currency crisis of the late 1990s. In this crisis, financial intermediaries with liabilities perceived as implicitly government-backed faced severe moral hazard issues due to their lack of regulation. In a more recent context, Crouzet and Tourre (2021) argues that interventions in corporate credit markets during the 2020 recession, involving the purchase of corporate debt with subsidies, incentivized excessive debt accumulation beyond the tax shield, thereby extending the motive for debt issuance. In all these examples,

some form of unpaid insurance weakens the market's response to default risk and leads to over-accumulating debt.

Upon revisiting Lemma 1, it becomes evident that in an economy with limited liability, the market value of sectoral output to sectoral input is lower than in an efficient economy. This indicates that the misallocation resulting from the over-ordering mechanism diminishes effective sectoral productivity. In simpler terms, it signifies that sectors are less effective at generating revenue from a given dollar of input resources.

A higher output elasticity of intermediate inputs, α, leads to increased firm dependence on trade credit per dollar of output and a simultaneous rise in default risk. This heightened risk does not significantly increase the risk premium charged by suppliers due to $\delta > 0$. This observation aligns with empirical evidence, indicating that suppliers maintain relatively stable trade credit prices, and risky firms often prefer trade credit and perceive it as a cost-effective financing option, even though the implied interest rates are high (see Ng et al (1999), Summers and Wilson (2002), and Giannetti et al (2011)).

Example: one-sector roundabout economy Consider a simplified one-sector roundabout economy, similar to the multi-sector economy in Figure 2, but with just a single sector. In this economy, firms and final consumers purchase intermediate inputs and consumption goods from the sector aggregator. The economy's production function is described as:

$$Y = A\, x^{\alpha}.$$

The feasibility constraint for consumption is given by $C = Y - x$. In the social planner solution, $x^e = (\alpha\, E(A))^{\frac{1}{1-\alpha}}$ of the sector's output is dedicated to intermediate inputs, with the remaining portion directed towards final consumption. However, due to the presence of limited liability, the market employs a different allocation strategy, assigning $x = \Gamma^{\frac{1}{1-\alpha}}\, x^e$ of the sector's output to intermediate inputs, thus leaving a smaller share available for final consumption, $(1 - \alpha\,\Gamma)\,(\alpha\,\Gamma)^{\frac{\alpha}{1-\alpha}}\, A^{\frac{1}{1-\alpha}}$. This phenomenon is at the heart of what we term the "over-ordering mechanism" and is distinct from the misallocation of factors of production. The over-ordering mechanism is a unique characteristic of input-output economies, emerging when decisions are made regarding allocating sectoral output to intermediate input production.

Figure 3 illustrates how sectoral output and consumption respond to variations in Γ. When $\Gamma < 1$, the economy produces less than what the social planner would ideally allocate and dedicates a larger portion to consumption. Conversely, when $\Gamma > 1$, the economy produces more than the social planner's allocation but allocates a greater share of output to intermediate inputs, consequently reducing the amount available for consumption. Limited liability is a constraint that pushes Γ above one.

Much like the observations in Jones (2011) and Bigio and La'o (2020), Figure 3 indicates that minor deviations from $\Gamma = 1$ have negligible effects on TFP and consumption loss. Conversely, a slight adjustment in Γ when it deviates significantly from one yields a marked improvement in aggregate consumption.

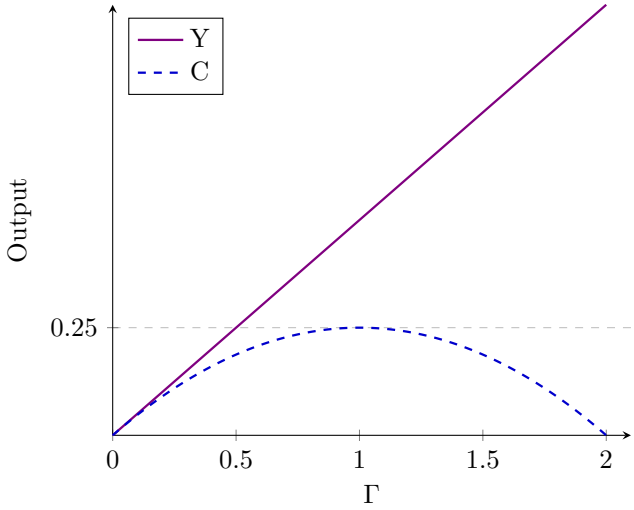


Fig. 3: One-Sector Roundabout Economy: $E(A) = 1$, $\chi = 0$, and $\alpha = 0.5$

While Jones (2011) underscores the influence of factor misallocation on aggregate TFP and how production networks can magnify these effects, it is crucial to note that factor misallocation can manifest even in a horizontal economy solely dedicated to producing consumption goods. Here, I posit that the "over-ordering mechanism" is a pertinent example of the misallocation of sectoral goods, which endures even in the absence of factors of production to be reallocated.

In the business cycle accounting framework as introduced by Chari et al (2007), the efficiency wedge can be expressed as $\Gamma^{\frac{\alpha}{1-\alpha}} \frac{1-\alpha\,\Gamma}{1-\alpha}$. This formulation underscores the impact of over-ordering on the aggregate TFP.

IV.B Deadweight Loss of Default, χ

Default can be costly, resulting in a waste of a firm's output during the bankruptcy process. The deadweight loss from default reduces sectoral TFP and output. This section focuses on understanding how over-borrowing induced by limited liability exacerbates this deadweight loss.

The leverage ratio is a key indicator of a firm's capacity to meet its debt obligations. A higher leverage ratio is associated with a higher chance of default. Corollary 1 shows that firms accumulate debts more than in an efficient economy, and sectors experience a higher leverage ratio.

Corollary 1. *The sectoral leverage ratio is higher than the efficient economy in a market with limited liability.*

$$\frac{LR}{LR^e} = \frac{E(A|A > A^*)}{E(A|\chi, A^*)} > 1. \tag{16}$$

Corollary 1 emphasizes that limited liability encourages borrowing firms to over-borrow credit from suppliers, thus raising the break-even threshold and the likelihood of default. [23] Brander and Lewis (1986) initially uncovered this critical insight in a partial equilibrium model. Their central argument is that limited

[23] It is important to note that the over-accumulation of corporate debt has been discussed in the literature. As pointed out by Sørensen (2017), the tax bias favouring debt financing under the corporate income tax has contributed to corporate debt ratios surpassing socially optimal levels. The socially optimal debt-asset ratio is estimated to be 4-5 percentage points lower than the current observed debt levels. My approach offers a different perspective on the issue of over-accumulation of corporate debt.

liability incentivizes firms to concentrate on states with higher marginal profits, given zero returns in default states, resulting in a more aggressive stance and excessive debt accumulation.

The second term of Γ also appears on the RHS of Equation 16, signifying that over-borrowing is a consequence of the moral hazard problem encountered by firm owners when ordering intermediate inputs. Furthermore, the repercussions are notable if the deadweight loss associated with default is positive (represented by $\chi > 0$), it leads to a lower sectoral TFP and sectoral output.

Lemma 2. *The break-even productivity threshold is higher than the efficient economy in a market with limited liability.*

$$\frac{A_i^*}{A_i^{*e}} = E(A|A > A_i^*).$$

Moreover, the sectoral TFP is less than the efficient economy in a market with limited liability by $-\chi \left(\int_{A_i^{*e}}^{A_i^*} A\, dG\right)$.

Lemma 2 summarizes limited liability's impact on excessive defaults and sectoral TFP. In an efficient economy with no default, sectoral TFP is one. In a market with limited liability and without over-borrowing driven by moral hazard, firms with productivity below A_i^{*e} default, reducing sectoral TFP by $\chi \int_0^{A_i^{*e}} A\, dG$. With over-borrowing, more firms default, and sectoral TFP decreases by $\chi \int_0^{A_i^*} A\, dG$. The difference between these two represents the reduction in sectoral TFP due to excessive default.

Additionally, Lemma 3 shows that the over-ordering coefficient Γ is strictly decreasing to the deadweight loss of default. As deadweight loss rises, defaulting firms' output after wasting a bigger fraction of χ reduces.[24] Consequently, creditors demand higher risk premiums for a given default rate to cover losses by collecting more from non-defaulting firms. This increased risk premium reduces borrowing firms' tendency to overextend trade credit and over-demand intermediate inputs, resulting in a lower Γ. However, higher deadweight loss from defaults also causes a more significant reduction in sectoral TFP and increased consumption loss.

Lemma 3. *Over-ordering distortion,* Γ*, In the model described in Section II, with mean-one idiosyncratic productivity shocks, the over-ordering coefficient of* Γ *is strictly decreasing in* χ*, as expressed by:*

$$\frac{\mathrm{d}\Gamma_i}{\mathrm{d}\chi} = \frac{(1-\alpha_i)\,(1-E(A|A_i^*,\chi))}{\alpha_i\, E(A|A_i^*,\chi)} < 0 \quad \forall\ i.$$

Limited liability drives over-borrowing among firms, resulting in two significant consequences:

1. Over-ordering of intermediate inputs causes the misallocation of sectoral goods towards intermediate input production and a reduction in resources for final goods.
2. A higher default rate, when combined with a positive deadweight loss of default, results in greater output loss.

The next section discusses how the limited liability distortion propagates in the economy.

[24] See the proof of Lemma 3 in Online Appendix

IV.C Propagation through Network

Deadweight loss of default and over-ordering happen for individual firms, and the created distortion propagates and magnifies through the network. Deadweight loss of default leads to lower sectoral TFP. A decline in sectoral TFP carries several negative consequences. It escalates output costs and final prices, adversely affecting consumers. Additionally, it diminishes the demand for intermediate inputs, impacting suppliers. This creates a chain reaction: affected customers decrease demand, and harmed suppliers respond by reducing orders from their own suppliers. This cascading effect causes the initial reduction in sectoral TFP to propagate across the network, leading to a more substantial output loss. Baqaee and Farhi (2020) has already shown the first-order effect of a sectoral TFP change on the real GDP.

The more intriguing case is the misallocation effect of limited liability, even without the deadweight loss of default. Over-ordering intermediate inputs leads to overproduction in some sectors and excessive resource usage, leaving other sectors under-resourced.

To examine the impact of over-ordering on the economy, we can derive the following equation from the feasibility constraint of each sectoral good:

$$\lambda_i = \beta_i + \sum_{j=1}^{N} \psi_{ji} \Gamma_j \lambda_j, \tag{17}$$

where β_i is defined as

$$\beta_i = \left(\frac{\omega_{ic}^{\sigma} p_i^{1-\sigma}}{\sum_{k=1}^{N} \omega_{kc}^{\sigma} p_k^{1-\sigma}} \right),$$

and ψ_{ji} as

$$\psi_{ji} = \alpha_j \left(\frac{\omega_{ji}^{\gamma} p_i^{1-\gamma}}{\sum_{k=1}^{N} \omega_{jk}^{\gamma} p_k^{1-\gamma}} \right).$$

Here, β_i is Sector i sale to final consumers, and ψ_{ji} is the share of Sector j's total sale spent on Commodity i as an intermediate input. [25] It depends on the elasticity of output to the aggregate intermediate input, α_j, the role of Commodity i in producing aggregate intermediate inputs, ω_{ji}, relative price of Commodity i compared to other suppliers of firms in Sector j, and the elasticity of substitution between different inputs, γ. $\boldsymbol{\beta}$ and $\boldsymbol{\psi}$ are equilibrium vector and matrix.

Overall, Equation 17 provides a framework for understanding the interdependence of sales shares among different sectors, considering final demand, input-output linkages, limited liability friction, and productivity distribution.

Over-ordering at the individual firm level triggers an excessive demand for intermediate inputs. This aggregated demand ripples through the economy, leading to over-ordering across sectors. Sector i on the LHS of Equation 17 signifies an upstream sector, supplying part of its output as intermediate inputs to other sectors. The over-ordering of its customers creates an excess demand for the inputs produced by Sector i.

[25] Since nominal GDP is normalized to one, β_i is also the share of sectoral good i in consumption expenditure

Consequently, Sector i must procure more aggressively from its suppliers to fulfill the increased demand for its product. This sets off a chain reaction: Sector i's suppliers also experience heightened demand, and this process echoes through the input-output linkages. The magnification of these effects occurs through this iterative cycle, ultimately affecting the economy.

Rewriting Equation 17 in a matrix format can provide a clearer illustration of this iterative cycle:

$$\boldsymbol{\lambda} = \left(I - \boldsymbol{\psi}^T diag(\Gamma)\right)^{-1} \boldsymbol{\beta},$$

where $\left(I - \boldsymbol{\psi}^T diag(\Gamma)\right)^{-1}$ is the Leontief inverse matrix. The Leontief inverse matrix measures sectors' direct and indirect exposure to the final consumption. Part of sectoral outputs consumed directly by final consumers is $\boldsymbol{\beta}$. $\left(\boldsymbol{\psi}^T diag(\Gamma)\right) \boldsymbol{\beta}$ represents the sectoral outputs employed by other sectors, directly contributing to the production of final goods in a single step. Similarly, Part of sectoral outputs turns into the final consumption goods by n steps is $\left(\boldsymbol{\psi}^T diag(\Gamma)\right)^n \boldsymbol{\beta}$. The summation of all of those terms forms the total sectoral sale.

Writing Equation 17 for the efficient economy where $\Gamma = 1$ shows that

$$\lambda_i^e = \beta_i^e + \sum_{j=1}^{N} \psi_{ji}^e \, \lambda_j^e.$$

Comparing this equation with Equation 17 is not straightforward as $\boldsymbol{\beta}$ and $\boldsymbol{\psi}$ are equilibrium variables. However, comparing λ and λ^e reveals the extent of a sector's vulnerability to limited liability friction. Customers' over-ordering inflates sectors in the benchmark model compared to the efficient economy. This discrepancy signifies the distortion caused by limited liability and underscores the consequential enlargement of sectors due to customer over-ordering.

The rest of Section IV.C explores an economy with sectors sharing similar output elasticity of intermediate inputs, leading to an invariant over-ordering coefficient across sectors, enabling analytical insights. The investigation focuses on illuminating the ripple effect of the over-ordering mechanism in an input-output economy.

Proposition 2. *For the economy with the limited liability described in Section II, if all sectors exhibit similar output elasticity of intermediate inputs, α,*

- $\epsilon = \frac{1}{1-\alpha\,\Gamma}$,
- $\dfrac{\mathrm{d}\log\,\epsilon}{\mathrm{d}\log\,\Gamma_i} = \alpha\,\Gamma\,\lambda_i$,
- $\dfrac{\mathrm{d}(\epsilon-1)}{\mathrm{d}\log\,\Gamma_i} = \dfrac{\alpha\,\Gamma}{1-\alpha\,\Gamma}\,\lambda_i$.

Proposition 2 illustrates that the aggregate gross output increase due to the over-ordering mechanism when the output elasticity of intermediate inputs, α, is invariant across sectors. $\epsilon - 1$ represents the market value of the intermediate input required to produce one unit of nominal GDP. Over-ordering of intermediate inputs places demand pressure on the upstream sectors, primarily responsible for producing these inputs in

the economy—this increased demand for inputs drives up their prices, signalling scarcity. Consequently, more resources are allocated to producing intermediate inputs in the upstream sectors, leaving fewer resources for downstream sectors to manufacture final goods.

Proposition 2 also indicates that the aggregate gross output elasticity of the over-ordering coefficient is determined by the market value of that sector's intermediate input, $\alpha\,\Gamma\,\lambda_i$. $\alpha\,\Gamma$ is the portion of the sector's sales spent on intermediate inputs, while $1-\alpha\,\Gamma$ accounts for the value-added generated during the production process.

Proposition 2 shows how over-ordering leads to the over-production of intermediate inputs without quantifying welfare loss. Corollary 2, with an additional assumption on the input-output structure, provides analytical results on welfare loss. Before delving into Corollary 2, I will define 'Upstreamness centrality' as a measure of the economy's verticality and formally outline the additional assumption used in the corollary.

Definition 8. *Upstreamness centrality, U, quantifies the extent to which sectoral output is distanced from the production of final consumption goods.*

In the context of this measurement, a downstream sector that exclusively directs its products to final consumers is attributed a value of one. This value escalates for sectors located progressively upstream in the production chain. The increasing values of this centrality metric, reduced by one, delineate the count of times a sector's output is utilized as intermediary inputs across multiple stages before culminating in the hands of final consumers. To elucidate, take the instance of a sector dedicated solely to furnishing downstream sectors. In such a scenario, its Upstreamness centrality, U, would be two, indicative of its output's involvement as an intermediary input once before ultimately reaching final consumers.

Definition 9. *Symmetric K-suppliers network is defined as a production network with the following characteristics:*

- *For any Sector i,* $\omega_{ic} = \frac{1}{N}$*;*
- *For any Sector i,* $\alpha_i = \alpha$*;*
- *Each firm has K suppliers;*
- *Each sector aggregator provides intermediate inputs for firms in K sectors;*
- *For any Sector i and j, if* ω_{ij} *is non-zero, it is uniformly assigned as* $\omega_{ij} = \frac{1}{K}$*.*

While acknowledging that real-world networks deviate significantly from symmetry, the concept of a Symmetric K-Suppliers Network is a valuable simplification. This abstraction aids us in comprehending the fundamental nature of distortion and its propagation within a network. It underscores that the outcomes do not depend on network granularity, as discussed in Acemoglu et al (2012) and Gabaix (2011). Moreover, the Symmetric K-Suppliers Network concept is remarkably versatile, encompassing a broad spectrum of networks. This range spans from a symmetric ring economy, where $K = 1$, to a symmetric complete economy,

realized when $K = N - 1$.[26] In this network classification, all sectors contribute equally to final goods and intermediate inputs. Outputs, prices, and U are invariant across the sectors.

Corollary 2 encapsulates the outcomes for this particular network archetype. Through the lens of this abstraction, we gain valuable insights into economic dynamics despite its departure from the intricate realities of real-world networks. [27]

Corollary 2. *For the economy with the limited liability described in Section II with a symmetric K-suppliers network,*

$$\lambda = \frac{1}{N(1-\alpha\,\Gamma)}\,,$$

$$\frac{\mathrm{d}\log\,\lambda}{\mathrm{d}\log\,\Gamma} = \frac{\alpha\Gamma}{1-\alpha\Gamma}\,,$$

$$U = \epsilon = \frac{1}{1-\alpha\Gamma}\,,$$

$$P_i = E(A|A^*,\chi)^{\frac{-1}{1-\alpha}}\,\frac{1}{N(1-\alpha\,\Gamma)}\,(\alpha\ \Gamma)^{\frac{-\alpha}{1-\alpha}}\ K^{\frac{\alpha}{1-\alpha}}\,,$$

$$C = \underbrace{E\,(A|A^*,\chi)^{\frac{1}{1-\alpha}}}_{\substack{\textit{reduced by}\\ \textit{deadweight loss of default}}} \cdot \underbrace{(1-\alpha\,\Gamma)\,(\alpha\ \Gamma)^{\frac{\alpha}{1-\alpha}}}_{\substack{\textit{reduced by}\\ \textit{over-ordering}}} \cdot K^{\frac{-\alpha}{1-\alpha}},$$

$$\frac{\mathrm{d}\log\,C}{\mathrm{d}\log\,\Gamma} = \frac{\alpha^2(1-\Gamma)}{(1-\alpha)(1-\alpha\Gamma)} < 0\,,\ \textit{and}$$

$$\frac{\mathrm{d}\log\,C}{\mathrm{d}\log\,\chi} = \left(\frac{E(A|A^*,\chi)-1}{E(A|A^*,\chi)}\right)\left(1+\frac{\alpha\,(1-\Gamma)}{(1-\alpha\,\Gamma)}\right),$$

where $\dfrac{\mathrm{d}\log\,C}{\mathrm{d}\log\,\chi} < 0$ *iff* $\Gamma < \frac{1+\alpha}{2\alpha}$ *and* $\chi > 0$.

This corollary extends the previous proposition by reaffirming that sectoral sales increase due to the over-ordering mechanism. Upstreamness centrality equals the aggregate gross output, ϵ, in the symmetric K-supplier network. This result shows that as $\alpha\,\Gamma$ increases, firms demand more intermediate inputs and buy more intermediate inputs on credit. Hence, producers are involved more in producing intermediate inputs. Thus, their distance from final consumers increases, and upstreamness centrality increases.

Corollary 2 succinctly illustrates the impact of limited liability on prices and real GDP, denoted as C. The interplay is evident— defaults under limited liability and the positive deadweight loss of default result in decreased sectoral TFP, as evidenced by the first term in P_i.

Limited liability impacts prices through another avenue: the over-ordering distortion. In a symmetric K-suppliers network, all sectors contribute equally to final consumption and intermediate inputs. As $\alpha\,\Gamma$ rises, demand for intermediate inputs surges, leaving fewer sectoral outputs for final consumers. Simultaneously, elevated demand raises prices, diminishing consumers' purchasing power, given their normalized nominal income.

[26] One-sector roundabout economy also belongs to this network structure.
[27] See the proof of Corollary 2 in Online Appendix

Increasing the over-ordering distortion negatively impacts aggregate consumption, contingent on Γ magnitude. To underscore the consumption loss solely attributed to misallocation(i.e. $\chi = 0$), consider $\alpha = 0.5$ and $\Gamma = 1.1$, resulting in $\epsilon = 2.22$—approximating the U.S.A.'s aggregate gross output from 1997 to 2021. Notably, the consumption loss under these conditions amounts to 1% of real GDP. The model is highly non-linear. For an economy with $\alpha = 0.8$ and $\Gamma = 1.1$, the consumption loss expands significantly to 12%. A higher α implies an elevated debt-to-GDP ratio, and the magnitude of the problem intensifies for any given Γ, inflicting a more substantial blow to aggregate consumption.[28]

A change in χ yields two opposing impacts on aggregate consumption. Firstly, the higher deadweight loss of default diminishes sectoral TFP, reducing aggregate consumption. Conversely, it elevates the risk premium and market responsiveness to the moral hazard issue, curbing over-ordering distortion. When $\Gamma < \frac{1+\alpha}{2\alpha}$, the dominance of the first effect prevails over the second. Consequently, an increase in χ culminates in reduced aggregate consumption.

IV.D Limited Liability in the Input-Output Literature

Before we explore how limited liability distortions operate in concrete examples in Section V, it is crucial to highlight the significance of Proposition 1 and its central finding: $\Gamma > 1$. In the existing literature, one approach to addressing limited liability friction involves mandating cash-in-advance payments to suppliers or factors of production like labour. For instance, firms may be required to make an upfront payment of a fraction θ of intermediate input costs, with the ability to borrow up to their collateral at the risk-free interest rate, denoted as r (as seen in Bigio et al (2016) and Altinoglu (2021)). In such models, it can be shown that the implied Γ is $\frac{1}{1+\theta r} < 1$. Consequently, firms tend to under-order intermediate inputs, and sectors operate below their optimal size due to insufficient production of intermediate inputs in the economy. These models introduce a higher cost for intermediate inputs(i.e., $1 + \theta r$) akin to the risk premium in this study. However, they do not encompass the moral hazard problem, which sets them apart from this paper.

In Appendix A, I extend the model to include a required cash-in-advance component and demonstrate that if the lower support of G is strictly positive, for θ exceeding a certain threshold, the fraction of defaulting firms becomes zero, and Γ becomes $\frac{1}{1+\theta r} < 1$, aligning with the results of Bigio et al (2016) and Altinoglu (2021). However, for θ values below a certain threshold, $\Gamma > 1$, affirming Proposition 1. Notably, a zero default rate may not align with real-world observations, making the benchmark model with a positive default rate a more apt explanation of how limited liability genuinely induces misallocation in an economy.

Liu (2019) assumes an exogenous bilateral tax on trade of intermediate leading to $\Gamma = \frac{1}{1+\text{tax rate}} < 1$. So Liu (2019) reaches the conclusion that the upstream sectors are under-demanded and below the optimal size. Thus, it suggests subsidies for upstream sectors and taxes for the downstream sectors to reduce the misallocations.

[28] Γ depends on χ, α, and G. Using a fixed Γ illustrates the extent to which the over-ordering mechanism can lead to consumption loss. In Appendix F, Figure F4 demonstrates the consumption loss when Γ responds endogenously to changes in α given fixed values of χ and G.

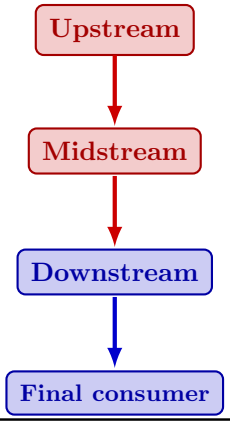
Proposition 1 highlights that the market response cannot entirely counterbalance the moral hazard incentive arising from the limited liability friction alone. Introducing additional frictions, such as cash-in-advance constraints or upfront payments, effectively complements the market's response to moral hazard, potentially mitigating or completely nullifying the over-ordering effect. Consequently, a model encompassing limited liability and cash-in-advance constraints can yield a more efficient allocation than one considering only a single friction. The existing literature has predominantly concentrated on cash-in-advance constraints (or modelling financial frictions with working capital requirements), potentially overestimating their economic impact.

V Illustrative Examples

This section will present two simplified production networks depicted in Figure 4. These economies consist of three sectors: Upstream (U), Midstream (M), and Downstream (D).

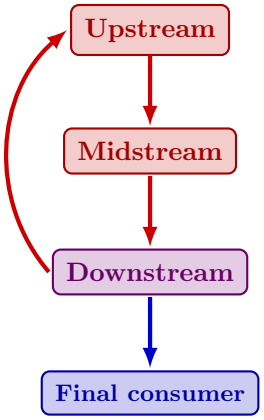
Figure 4a represents an extreme case of a vertical economy, where the production flow occurs strictly sequentially from Upstream to Midstream and finally to Downstream. Therefore, $\alpha_U = 0$, and firms in the upstream sectors have no limited liability problem.

Figure 4b illustrates the general case where the Midstream sector purchases intermediate inputs from the Upstream sector and sells its output to the Downstream sector. The Downstream sector sells to the Upstream and the final consumer. This configuration involves all sectors providing credit to one another.



Sector	D	M	U
λ	1	$\alpha\,\Gamma$	$\alpha^2\,\Gamma^2$
Upstreamness	1	2	3

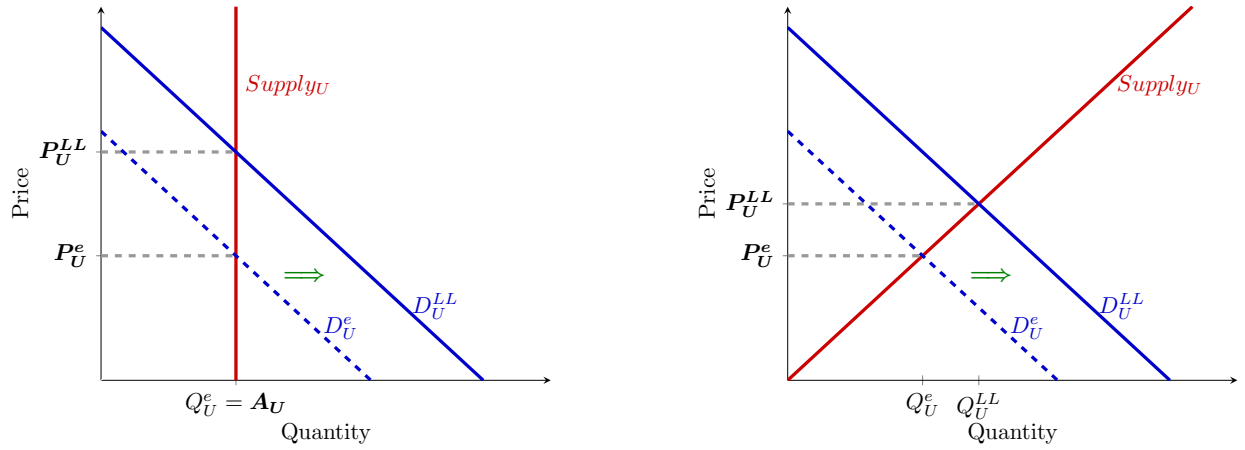
(a) Vertical Economy: $\epsilon = 1 + \alpha\,\Gamma + \alpha^2\,\Gamma^2$



Sector	D	M	U
λ	$\frac{1}{1-\alpha^3\,\Gamma^3}$	$\frac{\alpha\,\Gamma}{1-\alpha^3\,\Gamma^3}$	$\frac{\alpha^2\,\Gamma^2}{1-\alpha^3\,\Gamma^3}$
Upstreamness	$\frac{2\alpha^3\,\Gamma^3}{1-\alpha^3\,\Gamma^3} + 1$	$\frac{2\alpha^3\,\Gamma^3}{1-\alpha^3\,\Gamma^3} + 2$	$\frac{2\alpha^3\,\Gamma^3}{1-\alpha^3\,\Gamma^3} + 3$

(b) Three-Sector Roundabout Economy: $\epsilon = \frac{1}{1-\alpha\Gamma}$

Fig. 4: Two Simple Economies. The red links represent the flow of intermediate inputs, while the blue links signify the flow of final goods.



(a) Vertical Economy

(b) Three-Sector Roundabout Economy

Fig. 5: Upstream Sectoral Output Market with Fixed Factors of Production under Different Network Structures and $\chi = 0$. Limited liability prompts a rightward shift in demand. The supply curve's slope is influenced by the network structure and factors of production flexibility, dictating how demand pressure translates into adjustments in quantity and price. Importantly, in the benchmark model, the absence of a factor of production makes the network structure the sole determinant of variations in the supply slope. Over-ordering of intermediate inputs shifts demand from D_U^e to D_U^{LL}, resulting in an increase in prices from P_U^e to P_U^{LL}. The quantity and price response hinges on the Upstream sector's capacity expansion. In a vertical economy, where the Upstream sector is constrained to producing a unit of output, prices surge significantly while quantity remains unchanged. Conversely, in the three-sector roundabout economy, the Upstream sector can augment its output in response to heightened demand, purchasing more intermediate inputs. Limited liability-induced higher demand generates more moderate price increases and substantial sectoral output growth. Positive deadweight loss of default (e.g. $\chi > 0$) reduces sectoral TFP and shifts the supply curve to the left.

These simplified production networks serve as illustrative examples to emphasize how the economy's structure influences the extent of the propagation due to the limited liability. In these networks, each sector has only one creditor or supplier, making the elasticity of substitutions (γ and σ) less relevant. As a result, our attention can be fully directed towards analyzing the distortions arising from limited liability and the critical role of network structure in both propagating shocks and magnifying the distortions.

In the vertical economy, where a single allocation path exists, misallocation is not possible. Over-ordering of intermediate inputs under limited liability only results in increased prices, with no impact on quantities or final output (see Fig. 5a). Limited liability, while pushing demand to the right, merely elevates prices without affecting the quantity of intermediate inputs. Aggregate consumption in the vertical economy is determined as $E(A|A_D^*, \chi)E(A|A_M^*, \chi)^{\alpha}E(A|A_U^*, \chi)^{\alpha^2}$, which remains independent of Γ.[29]

Contrastingly, in a roundabout 3-sector economy, there is an infinite way to allocate the sectoral output of the Downstream sector into the final goods and intermediate inputs, so there is endless possible allocation and room for misallocation. Fig. 5b illustrates that under limited liability, the Upstream sector responds to higher demand by using more intermediate inputs produced by the Downstream sector, leading to increased output and prices, deviating from efficiency. Higher demand from the Upstream sector translates to higher

[29]Since $\alpha_U = 0$, $E(A|A_U^*, \chi) = E(A) = 1$. For $\chi = 0$, aggregate consumption becomes one.

demand for intermediate inputs from the Downstream sector, driving up prices of Downstream sectoral output and reducing consumption for the final consumer, who has a nominal income of one.

A flexible factor of production amplifies the Upstream sector's response to demand pressure, allowing for greater increases in sectoral outputs compared to scenarios with no factors of production.

The aggregate consumption in the three-sector general economy depicted in Figure 4b is as follows:

$$\underbrace{\left(1-(\alpha\,\Gamma)^3\right)(\alpha\,\Gamma)^{\frac{3\alpha^3}{1-\alpha^3}}}_{\text{Effected by over-ordering}}\underbrace{\left(E(A|A_D^*,\chi)E(A|A_M^*,\chi)^{\alpha}E(A|A_U^*,\chi)^{\alpha^2}\right)^{\frac{1}{1-\alpha^3}}}_{\text{Effected by the deadweight loss of default}}. \tag{18}$$

The first term of aggregate consumption, $\left(1-(\alpha\,\Gamma)^3\right)(\alpha\,\Gamma)^{\frac{3\alpha^3}{1-\alpha^3}}$, elucidates the impact of over-ordering mechanism on aggregate consumption dynamics. This mechanism plays a pivotal role by boosting the production of intermediate inputs. On the one hand, it amplifies output through $(\alpha\,\Gamma)^{\frac{3\alpha^3}{1-\alpha^3}}$, yet on the other, it reduces the availability of final consumption goods by $\left(1-(\alpha\,\Gamma)^3\right)$. [30] This intricate interplay underscores the nuanced effects of over-ordering distortion on the overall balance between intermediate inputs and final consumption goods within the economic framework. [31]

This comparison between the two economies underscores the significance of network structure in the propagation of distortions, elucidating how it can shape the outcomes of economic dynamics.

What is particularly intriguing is that the over-ordering mechanism is absent in two extreme scenarios: a horizontal economy where sectors do not engage in business-to-business trade and a vertical economy where over-ordering does not result in consumption loss. Over-ordering mechanism truly comes into play when the production network falls somewhere between these two extremes, mirroring the complexity of real-world economic systems.

Deadweight loss of default impacts sectoral TFP and subsequently increases prices across the economy. In the general three-sector model, the presence of network loops magnifies the impact on final consumption. As a result, the elasticity of consumption to χ is $\frac{1}{1-\alpha^3}$ times bigger than that of the vertical economy. This comparison underscores how network structure significantly influences the reach and strength of default deadweight loss effects on an economy's consumption dynamics. Similar to the over-ordering mechanism, the

[30] If $\Gamma > 1$,

$$\frac{\mathrm{d}\log\,C}{\mathrm{d}\log\,\Gamma}=\frac{3\alpha^3}{1-3\alpha^3}-\frac{3\Gamma^3\,\alpha^3}{1-3\Gamma^3\,\alpha^3}<0$$

In this equation, the first term of the elasticity signifies the increase in output resulting from over-ordering intermediate inputs. Meanwhile, the second term underscores the allocation of resources more towards intermediate inputs at the expense of final consumption goods.
The negative value of this elasticity indicates that as Γ increases beyond 1, consumption experiences a decrease. This dual effect—higher output due to over-ordering but a shift towards intermediate inputs—ultimately reduces consumption.

[31] Equation 18 provides a revealing insight: as the number of sectors, denoted by N, goes from 3 to infinity, the network's behaviour closely resembles that of a vertical economy, rendering over-ordering distortion inconsequential. With increasing N, the sale share of the most upstream sector, which acquires goods from downstream sectors, tends towards zero. Consequently, over-ordering ceases to influence the demand of the downstream sector, and as a result, it has no discernible impact on the final price and consumption. It's worth noting that if N goes to infinity, the average upstreamness for sectors approaches infinity—a counterintuitive outcome. For context, the upstream centrality for the U.S.A. economy is measured at 2.09 with a maximum of 4.65(Petrochemicals) (see Antràs et al (2012)). If we calibrate the U.S.A. economy with a ring economy similar to Figure 4b, $N = 4$ would be the best choice.

impact of the deadweight loss of default is more pronounced when the network structure diverges from these two extreme cases.[32]

For a straightforward simulation, I assume a uniform distribution, $A \sim U(0.5, 1.5)$. When $\alpha < 0.5$, defaults are absent, and the economy resembles a frictionless, perfectly competitive market. However, when $\alpha > 0.5$, financial frictions disrupt efficient resource allocation in the economy.

Figure 6 illustrates the economy's response to variations in α in the three-sector roundabout economy shown in Figure 4b. For this exercise, χ is set to zero, making the over-ordering mechanism the sole source of misallocation in the model.

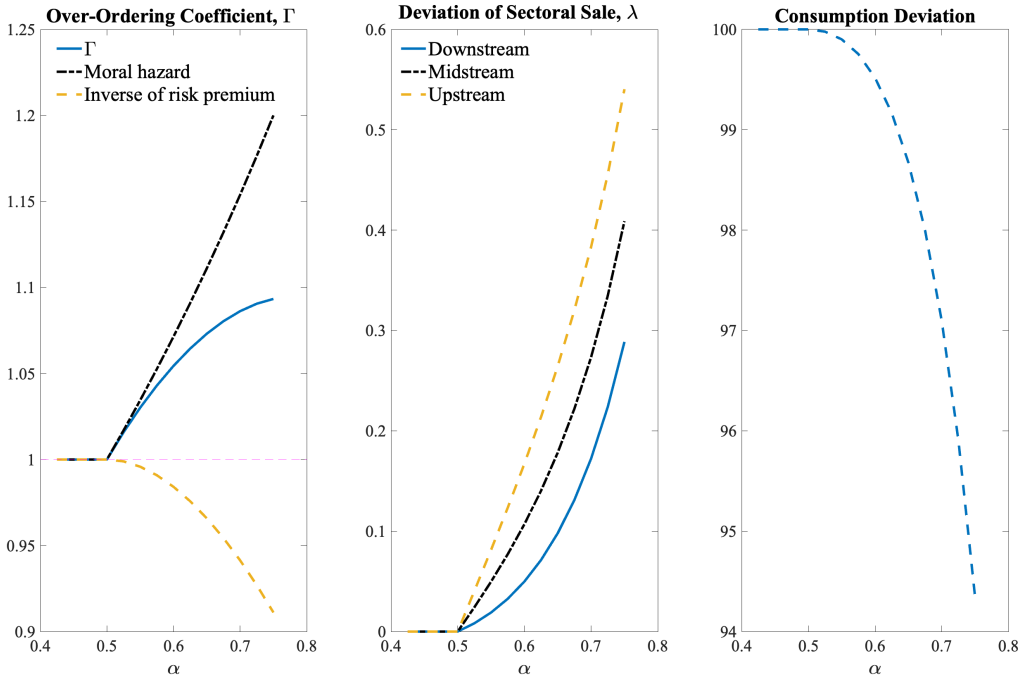


Fig. 6: Three-Sector Roundabout Economy: $\chi = 0$, $A \sim U(0.5 - 1.5)$

The first panel of the figure offers insights into the mechanics of the over-ordering phenomenon, influenced by Equation 15. In this panel, we observe the interplay of two contrasting forces: as α increases, firms accumulate more debt, leading to a higher default rate and exacerbating the moral hazard problem, quantified as $\frac{E(A|A>A^*)}{E(A|A^*,\chi)}$. Simultaneously, the market addresses this challenge by imposing higher risk premiums. However, these risk premiums do not entirely offset the issue, resulting in Γ surpassing a value of one. This panel

[32] In the roundabout economy, the elasticity of consumption concerning χ is $\frac{1}{1-\alpha^3}$ times larger than that of the vertical economy. In a horizontal economy, aggregate consumption is represented by $E(A|A^*, \chi)$ in a symmetric network where $\omega_{ic} = \frac{1}{3}$ for all $i = 1, 2, 3$. Comparing this with $\left(E(A|A^*_D, \chi)E(A|A^*_M, \chi)^{\alpha}E(A|A^*_U, \chi)^{\alpha^2}\right)$ in a vertical economy, we find that consumption loss is smaller in the horizontal economy than the vertical economy.

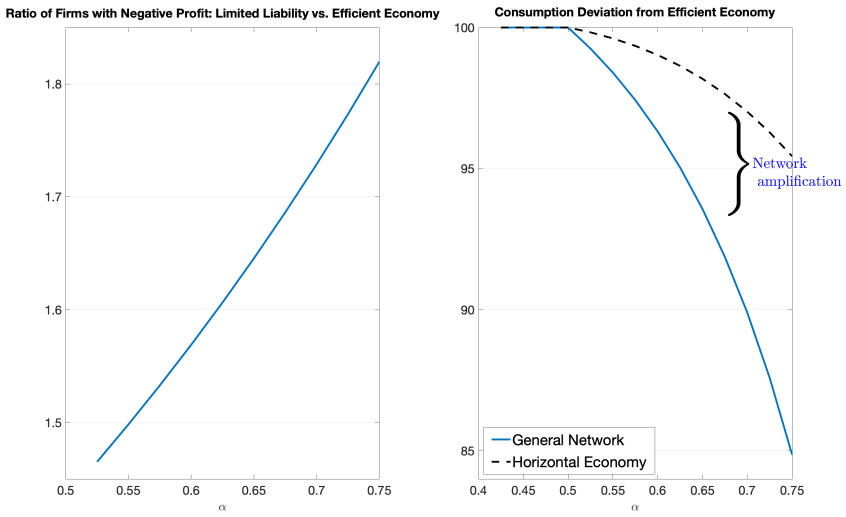


Fig. 7: Three-Sector Roundabout Economy with $\Gamma = 1$ and $A \sim U(0.5 - 1.5)$

effectively illustrates these dynamic forces and their collective impact, providing insight into the outcome of this conflict. [33]

The second panel of Figure 6 shows the heterogeneous response of sectoral sale sizes, influenced by Equation 17, within an input-output economy. The over-ordering distortions of customers, as per Equation 17, presents its most significant challenges to upstream sectors, which predominantly supply intermediate inputs to other sectors. Consequently, the impact on the upstream sector is the most pronounced, leading to a significant rise in its size.

The third panel of Figure 6 delves into the aggregate consumption loss stemming from the over-ordering distortion compared to an efficient economy. This panel vividly illustrates the highly non-linear nature of welfare loss. When $\alpha \leq 0.5$, firms can fully meet their debt obligations, generating no distortion. However, as α increases, firms increasingly rely on credit for input purchases, making it challenging for lower-productivity firms to repay their debts. Consequently, the default rate rises, and the over-ordering distortion exerts influence.

The first panel also applies to the vertical economy in Figure 4a. However, in the vertical economy, over-ordering leads to price increases but doesn't introduce any misallocation, resulting in zero consumption loss.

The deadweight loss of default, denoted as χ, has a twofold impact. It influences sectoral TFP by causing output loss and affects Γ, decreasing to χ. By maintaining $\Gamma = 1$, the over-ordering mechanism is mitigated. Figure 7 demonstrates how the deadweight loss of default distorts the economy exclusively through output loss. [34]

[33] Since $\chi = 1, E(A|A^*, \chi) = 1$, and $E(A|A > A^*)$ increases linearly as G follows a uniform distribution. Thus, the part of Γ comes from the moral hazard respond to α linearly.

[34] Because Γ is linked to χ with $\frac{d\Gamma}{d\chi} < 0$, it is not feasible to decompose consumption loss into two distinct channels. Thus, Consumption loss $<$ Consumption $\text{loss}_{\Gamma=1}$ + Consumption $\text{loss}_{\chi=0}$. Figure 6 explores Consumption $\text{loss}_{\chi=0}$, and Figure 7 explores Consumption $\text{loss}_{\Gamma=1}$.

In the left panel, we observe the excessive default rate, $\frac{G(A^*)}{G(A^{*e})}$, within a three-sector roundabout economy. As α increases, limited liability becomes a more significant challenge. Firms accumulate more debt and default more frequently, leading to a substantial increase in firms facing negative profits compared to an efficient economy. In an efficient economy, defaults do not occur. However, observing how over-borrowing pushes the break-even point higher, leading to increased default rates, is still valuable. Lemma 2 further highlights that $\frac{A^*}{A^{*e}}$ is not constant. Higher values of α elevate this ratio, increasing the excessive default rate.

The right panel compares the consumption loss caused by the deadweight loss of default in the general roundabout three-sector model and the horizontal economy, where sectors exclusively produce final goods for households.[35] This panel highlights the network's role in propagating shocks. Notably, consumption loss in the horizontal economy is significantly smaller than in the roundabout economy. This discrepancy arises because firms in the roundabout economy rely on intermediate inputs, and the lower TFP of suppliers increases input costs and overall production costs, ultimately leading to lower aggregate consumption.

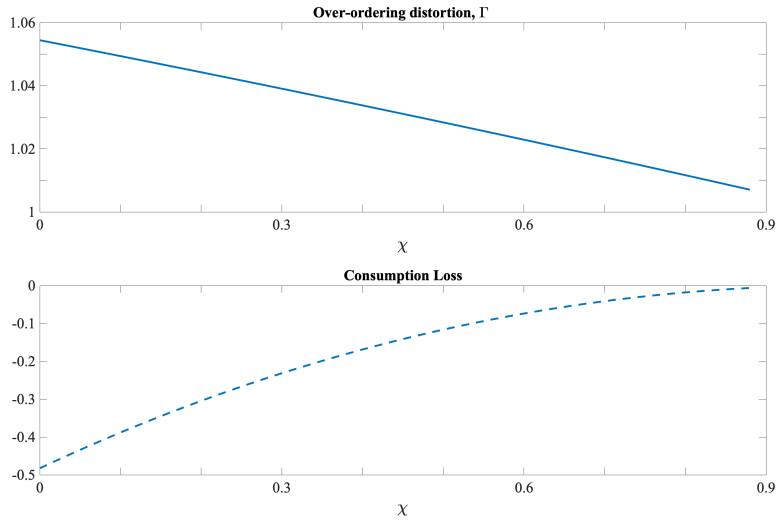


Fig. 8: Percentage Point Consumption Loss Attributable Only to Over-Ordering in a Three-Sector Roundabout Economy Compared to an Efficient Economy with $\Gamma = 1$, $A \sim U(0.5 - 1.5)$, and $\alpha = 0.6$

Figure 8 visualizes the interaction between the over-ordering mechanism and the deadweight loss of default highlighted in Lemma 3 within the context of a three-sector roundabout economy. As a larger portion of defaulting firms' output is wasted by higher χ, creditors respond by imposing higher risk premiums to compensate for potential losses. This, in turn, results in increased borrowing costs and a reduced demand for intermediate inputs, thereby weakening the over-ordering mechanism. The upper panel of Figure 8 illustrates that the over-ordering coefficient is most pronounced when $\chi = 0$, leading to the highest level of consumption loss.

[35] Since there is no business-to-business trade in the horizontal economy, I assumed similar default rates in both economies to focus on the role of the economy's structure in propagating distortion.

VI Limited Liability Influence

Limited liability distorts the economy in two channels:

1. **Over-ordering Mechanism:** This distortion leads to sectoral sizes exceeding the optimal levels observed in an efficient economy with $\Gamma = 1$. As a result, it drives up production costs and prices while concurrently reducing aggregate consumption.
2. **Deadweight Loss of Default:** Over-borrowing elevates the default rate, which results in higher sectoral output losses when combined with a positive deadweight loss of default. The input-output economy exacerbates the output loss of defaulting firms through network ripple effects.

Section VI introduces the concept of "limited liability influence," which measures how much limited liability affects a specific sector. It is important to note that Γ quantifies how severely a sector over-demands intermediate inputs but does not gauge the degree to which a sector faces over-demand due to the over-ordering tendencies of its customers.

Given that distortion is most pronounced in demand for upstream sectors, we anticipate a positive correlation between limited liability distortion and Upstreamness centrality. However, it is important to note that Upstreamness centrality alone cannot precisely pinpoint the extent of limited liability distortions. For instance, consider two sectors of the same size facing demands from another sector. In one case, the demand originates from a downstream sector, while in the other, it comes from an upstream sector. Despite experiencing the same limited liability distortion, the latter sector exhibits higher Upstreamness centrality because it connects to a sector with greater upstreamness. Thus, a specific and dedicated measure is required to accurately assess the influence of limited liability on sectors.

Quantifying the over-ordering channel poses a challenge. It is complex to directly measure this distortion, both for comparing sectors within an economy and for making cross-country comparisons. Additionally, the deadweight loss of defaults impacts sectoral TFP. However, the extent of this distortion is highly contingent on the network structure. Section V has already demonstrated how a similar χ could lead to different levels of consumption loss in horizontal, vertical, and roundabout three-sector economies. This is where the concept of limited liability influence proves invaluable. It provides a metric that elucidates how sectors are affected by the limited liability distortion. A critical feature of any measurement is its feasibility for computation using the available data, and this centrality measurement satisfies it.

Definition 10. ***Limited Liability Influence(LLI):*** *For Sector i, the distortion caused by limited liability is* $\Phi_i \equiv \log(\frac{p_i}{p_i^c})$

When a sector's customers are distorted by over-ordering, it encounters greater demand and subsequently raises its prices. Likewise, higher deadweight loss of defaults reduces sectoral TFP and production capacity, resulting in elevated prices. In both situations, rising prices are captured by LLI.

The sector least affected by these distortions has an LLI value of zero. As a sector's customers become more distorted by over-ordering, it experiences increased demand and raises its prices, so Φ becomes positive.

Γ_i shows how much Sector i distorts its suppliers while Φ_i indicates how much Sector i is distorted by output loss of its defaulting firms and other sectors via over-ordering.

Lemma 4. *The first-order approximation of* Φ *for the economy described in Section II is as follows:*

$$\Phi = \boldsymbol{\kappa}_2 \kappa_1 + \boldsymbol{\kappa}_2 \log \frac{\lambda}{\lambda^e}\,, \tag{19}$$

where

$$\log \frac{\lambda}{\lambda^e} = (\mathbf{I} - \boldsymbol{\Delta} - \boldsymbol{\kappa}_3\boldsymbol{\kappa}_2)^{-1}\boldsymbol{\kappa}_3\, \boldsymbol{\kappa}_2\, \kappa_1 + (\mathbf{I} - \boldsymbol{\Delta} - \boldsymbol{\kappa}_3\boldsymbol{\kappa}_2)^{-1}\boldsymbol{\Delta} \log \Gamma\,,$$

$$\kappa_1 = \left(-\frac{\alpha}{1-\alpha} \circ \hat{A}^* - \hat{z} + \frac{\alpha}{1-\alpha} \circ \hat{RP}\right)\,,$$

$$\boldsymbol{\kappa}_2 = (\frac{1}{1-\alpha} \circ \mathbf{I} - \frac{\alpha}{1-\alpha} \circ \tilde{\boldsymbol{\Delta}})^{-1}\,,$$

$$\boldsymbol{\kappa}_3 = \left(\big(\xi(1-\sigma) + (1-\xi)(1-\gamma)\big) \circ \mathbf{I} - \xi(1-\sigma) \circ (\Lambda' \otimes \mathbf{1}_{N\times N}) - (1-\gamma)\boldsymbol{\Delta}\tilde{\boldsymbol{\Delta}}\right).$$

If $\sigma = 1$ *or only one sector produces the final consumption good and* $\gamma = 1$ *or firms in a sector only have one supplier,* $\kappa_3 = 0$ *and* $\log \frac{\lambda}{\lambda^e} = (I - \boldsymbol{\Delta})^{-1}\boldsymbol{\Delta} \log \Gamma$*, where* $\boldsymbol{\Delta}$ *is the matrix with* $\frac{y_{ji}}{Y_i}$ *in entry* (i, j)*,* $\tilde{\boldsymbol{\Delta}}$ *is the matrix with* $\frac{x_{ij}p_j}{\sum_j x_{ij}p_j}$ *in entry* (i, j)*, so* $\sum_j \tilde{\Delta}_{ij} = 1$*,* ξ *is the vector with* $\xi_i = 1 - \sum_j \Delta_{ji} = \frac{C_i}{Y_i}$ *in entry* i*,* Λ *is the vector with* $\Lambda_i = \frac{P_i C_i}{PC}$ *in entry* i*,* $\hat{A}^*$ *is the vector with* $\hat{A}^*_i = \log(\frac{A^*_i}{\alpha_i})$ *in entry* i*,* $\hat{z}$ *is the vector with the sectoral TFP of Sector* i *in entry* i*, and* $\hat{RP}$ *is the vector with* $\hat{RP}_i = -\log(1 - G(A^*_i) + G(A^*_i)\delta_i)$ *in entry* i*.*

Lemma 4 provides a method to determine the first-order approximation of LLI across sectors. By accessing key parameters such as the sectoral default rate, distribution of idiosyncratic productivity shocks, G, and data from the input-output table, we can calculate the distortion factor Φ. Lemma 4 empowers us to perform cross-country or cross-sector analyses, offering valuable insights into the impact of limited liability across various contexts.

To further streamline this calculation, we can introduce additional constraints on the network. For example, by imposing the Cobb-Douglas assumption for both aggregate intermediate input and aggregate consumption, we can simplify the determination of Φ even more.

Corollary 3 shows the first-order approximation of economic consumption loss due to the limited liability. Since $\sum_{i=1}^{N} \Lambda_i = 1$, the consumption loss is the weighted average of LLI. Resource misallocation manifests as overproduction of intermediate inputs and underproduction of final consumption goods. In situations where default rates are consistent across all sectors, the over-ordering mechanism impacts upstream sectors more significantly than downstream sectors, leading to higher LLI for upstream sectors than downstream ones. However, Corollary 3 emphasizes a crucial insight: Sectors contribute to the overall welfare loss proportional to their share in the household expenditure on final goods. This means downstream sectors can lead to

significant consumption loss even if they have a low LLI, as long as they represent a notable share of the final expenditure.

Corollary 3. *The first-order effect of limited liability on consumption loss is*

$$\log(\frac{C}{C^e}) = -\Lambda'\Phi,$$

where Λ *is the vector with* $\Lambda_i = \frac{P_i C_i}{PC}$ *in entry* i.

Consider the vertical economy illustrated in Figure 4a. $-\Phi_D$ represents the consumption loss. When $\chi = 0$, the downstream sector has a zero LLI, leading to no consumption loss. [36] Over-ordering exclusively causes price hikes and does not result in consumption loss because there is no potential for misallocation, as the only possible allocation involves selling the final output of each sector to the next sector.

As χ becomes positive, sectoral TFP decreases, causing prices to rise. Consequently, Φ_D increases, and overall consumption decreases from one to $\left(E(A|A_D^*,\chi)E(A|A_M^*,\chi)^{\alpha}E(A|A_U^*,\chi)^{\alpha^2}\right)$.

Given the absence of factors of production, any consumption loss primarily stems from the reduction in aggregate TFP. As a result, the ratio $\frac{C}{C^e}$ indicates the efficiency wedge in a representative agent model.

Corollary 4. *If* $\chi = 0$*, the sectoral sale elasticity of* Γ *is as follows:*

$$\frac{\mathrm{d}\log\lambda}{\mathrm{d}\log\Gamma} = (I - \mathbf{\Delta})^{-1}\left(-\kappa_3\,\kappa_2 \circ \frac{\alpha}{1-\alpha} + \mathbf{\Delta}\right).$$

Corollary 4 provides insight into how sectoral sales respond to limited liability distortion. In scenarios with no default deadweight loss, over-ordering becomes the sole channel of distortion influencing sectoral sales, acting through the parameter Γ. This corollary has practical implications for empirical applications, as all the necessary data to calculate the sectoral sales elasticity of Γ can be sourced from input-output tables.

Subsequently, in Section VII, we explore optimal intervention policies to mitigate the distortions from limited liability.

VII Reclaiming Efficiency: The Role of Subsidies, Taxes, and Market Interventions

This section discusses essential policy measures to restore efficient allocation in an economy with limited liability, all while maintaining a balanced budget. The key policies are as follows:

Production Subsidies: These subsidies are provided to non-defaulting firms. The idea is to encourage firms to keep operating and producing, even when their realized productivity shocks are below the threshold. These subsidies act as a financial safety net for firms, ensuring they can continue operations without a high default risk.

[36] In the vertical economy with $\chi = 0$, $\Phi_M = \log(\Gamma)$ and $\Phi_U = 2\log(\Gamma)$, but their shares out of household expenditure is zero, so they do not affect aggregate consumption

Profit Taxes: Firms in each sector are subject to profit taxes. This means they must pay a portion of their profits to the government. It does not distort firms' incentives and ensures the government budget remains balanced.

While a detailed explanation can be found in Appendix B, a fundamental result is that economic efficiency, where there is *no* default, can be achieved with the correct subsidy and tax rates.

Indeed, firm default and exiting the market are often viewed as the core mechanisms of creative destruction in a market economy. This process ensures long-term growth and innovation by allowing less efficient or outdated firms to make way for newer, more competitive ones. The concept of an optimal exit rate becomes an intricate consideration in a dynamic model and is beyond the scope of a static model. However, One area of interest is developing intervention policies that eliminate distortions caused by over-borrowing while maintaining default due to limited liability without having its over-borrowing impact.

To address the issues of over-ordering and excessive default resulting from over-borrowing, policymakers can consider imposing a tax on intermediate inputs to counter the moral hazard incentives for buyers. The tax rate should be set at the buyers' over-ordering coefficient to nullify their motivation to over-order.

Analyzing the components of Γ in Proposition 1 demonstrates that a tax on trade credit, precisely equal to Γ, can effectively complement the market response and mitigate the incentive for firm owners to over-order intermediate inputs. Given $\Gamma \geq 1$, the government generates tax income and may provide household rebates. This tax on the bilateral sectoral trade is linked to suppliers' risk.

Policy intervention in the models with no default and cash-in-advance constraints that make trade credit more expensive leads to $\Gamma < 1$, signifying that firms under-order intermediate inputs. Consequently, the government must provide subsidies for intermediate inputs while imposing taxes on consumption goods to address this under-ordering phenomenon. In Appendix A.A, I present a detailed alternative tax scheme for scenarios where enforcing a strict credit policy is viable. In such situations, the government mandates a substantial upfront payment for trading intermediate inputs to ensure that firms do not default. Assuming that firm owners possess adequate collateral to borrow the upfront payment, the problem closely resembles the dynamics outlined in Liu (2019), Bigio et al (2016), and Altinoglu (2021). [37]

VIII Extensions

VIII.A Model with a Factor of Production

Thus far, the model has omitted production factors such as labour and capital. Incorporating factors of production, particularly labour, into the model exacerbates the impact of limited liability distortion.

[37] However, this scheme encounters a significant limitation – the real-world context typically includes a positive default rate. This implies that either the cash-in-advance constraint is not sufficiently stringent or not all firm owners possess enough collateral to provide the necessary upfront payment. In either case, some firms are still incentivized to default, and the moral hazard problem persists. Consequently, the intervention policies outlined in this section offer a more universal and resilient approach, applicable in scenarios where collateral constraints or strict credit policies pose challenges.

Firstly, over-ordering intermediate inputs results in the overproduction of these inputs, primarily concentrated in upstream sectors, and the underproduction of consumption goods, mainly affecting downstream sectors. Consequently, sectors compete fiercely for resources, significantly reallocating resources towards intermediate input production. The introduction of factors of production amplifies the role of resources in the production process, intensifying the magnitude of distortion caused by resource misallocation. Sectors now face a dual impact: the over-demand for intermediate inputs, which inflates production costs and end prices, and the over-demand for labour by upstream sectors, driving up wages and, subsequently, production costs for consumption goods in downstream sectors.

Secondly, the increased role of resources in production diminishes firm profits and markups, elevating the likelihood of default and pushing the default threshold higher. Consequently, the excess default distortion becomes more pronounced under these circumstances.

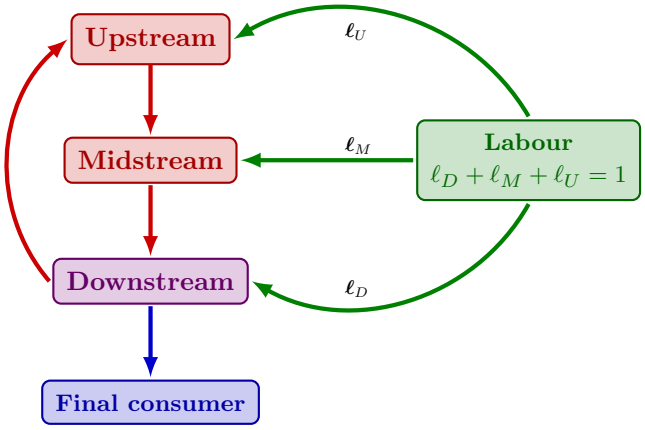


Fig. 9: General Three-Sector Roundabout Economy with Inelastic Labor Supply of 1

Figure 9 depicts the three-sector roundabout economy shown in Figure 4b, now including a labour market with an inelastic labour supply set at unity. In this context, over-ordering intermediate inputs within the economy compels the upstream sector (U) to produce at levels exceeding the optimal quantities found in an efficient economy, resulting in $\ell_U > \ell_U^e$. Additionally, U demands intermediate inputs from the downstream sector (D) in quantities greater than the optimal levels. Consequently, a smaller portion of D's output is allocated to consumption. Furthermore, U has less labour, $\ell_D < \ell_D^e$, available for production, further contributing to the contraction of consumption on two fronts.

The vertical economy in Figure 4a was not distorted by over-ordering because there was only one possible allocation, and only prices in midstream and upstream sectors went up. Now, higher prices in those sectors lead to more demand for labour and leave less labour for the downstream sector. So, the over-ordering causes consumption loss now in the vertical economy.

In Appendix C, I present a detailed model that includes labour and explores the impact of limited liability. I find a significant trend: introducing limited liability leads to an increase in the labour income share

and a decrease in the profit share within nominal GDP. This shift is driven by over-ordering, which forces sectors to acquire additional resources at any cost to meet increased demand. As a result, excess demand for intermediate inputs pushes wages higher, resulting in a greater labour income share.

Lemma 6 in Appendix C further elucidates that the wage response to limited liability distortion is contingent upon several critical factors, including the role of labour in the production process, the extent of distortion within sectoral sales, and their respective shares within the total labour demand.

VIII.B Model with Sectoral Shocks

Up to this point, our model has solely considered idiosyncratic productivity shocks. This section briefly overviews how the model responds to sectoral and aggregate shocks, building upon our understanding of how financial frictions, over-ordering, deadweight loss of default, and network amplification interact.[38]

Trade credit agreements are short-term contracts that typically settle within 30 to 90 days after delivering intermediate inputs. It is reasonable to assume that firms and sector aggregators are aware of realized sectoral and aggregate shocks before entering these contracts. Moreover, these trade credit agreements are flexible, allowing suppliers or customers to adjust contract terms based on the specific sectoral and aggregate shocks they anticipate. Additionally, contingency contracts can be executed to cover all potential realizations of the aggregate state variable, a vector of N by 1 of sectoral shocks.[39] In both scenarios, all relevant contract parameters, such as prices, quantities, default rates within each sector, and recovery rates, become contingent on the aggregate state variable. This eliminates aggregate-level uncertainty, leaving only the question of which firm within a sector may default.

A positive sectoral shock increases sectoral output and boosts firm productivity with the same input levels. The complementarity between sectors results in higher firm revenue, where output growth outpaces price reductions, increasing profits. This profit increase reduces default risks, creating a financial acceleration effect. Enhanced sectoral output can be attributed to the positive shock and reduced output loss due to lower default rates. As a result, TFP response to sectoral shocks exceeds one, the extent influenced by the default rate and deadweight loss, χ. Economies with higher default rates or more substantial deadweight loss experience more significant sectoral and aggregate TFP fluctuations.

While our study sheds light on this financial acceleration mechanism, its full potential in explaining business cycle fluctuations is a subject for future research beyond the scope of this study.

IX Empirical Results

This section investigates sectors' exposure to limited liability distortion and the corresponding consumption loss within the U.S.A. economy. To achieve a closer alignment between the model and real-world conditions,

[38] Appendix D provides an in-depth setup of the model with sectoral and aggregate shocks.
[39] The aggregate state space includes every possible realization of sectoral and aggregate shocks.

parameters are calibrated and estimated for the extended model, which has sectoral shocks and labour as a factor of production.[40]

I employ two primary datasets: the U.S.A. input-output use tables from 1997 to 2021 and the Annual Fundamental Compustate dataset covering 1998 to 2022. Our empirical analysis focuses on 65 sectors within the U.S.A. input-output data, where I have combined the "Housing" and "Other real estate" sectors.

The U.S.A. input-output tables offer invaluable sector-specific information for key variables, including β, α, $\mathbf{\Delta}$, $\tilde{\mathbf{\Delta}}$, ζ, and Λ. These parameters are crucial for our analysis.

Additionally, I consider two elasticity parameters: γ, representing the elasticity of substitution for intermediate inputs, and σ, characterizing the elasticity of substitution for aggregate consumption goods. In line with prior research by Baqaee and Farhi (2019), Baqaee and Farhi (2020), and Atalay (2017), $\gamma = 0.1$ and $\sigma = 0.9$ are adopted for the benchmark sectoral input-output model.

Furthermore, I estimate G using the Annual Fundamental Compustate dataset. In Appendix E, I comprehensively explain the data sources, the calibration and estimation process, and robustness tests to assess the sensitivity of our results to the calibration procedure.

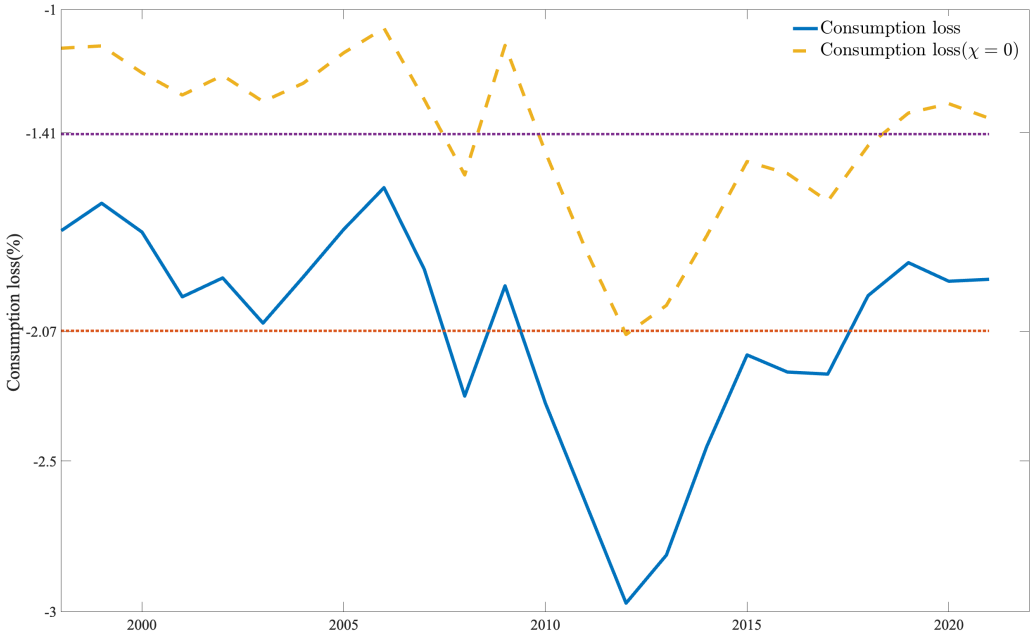


Fig. 10: Time Series of Consumption Loss and Average Over 1998-2021 ($\chi = 0.15$)

Figure 10 displays the time series of consumption loss resulting from limited liability in the U.S.A. economy spanning 1998 to 2021, with an average loss of 2.07% of total consumption.

[40] I explained the extended model with labour and sectoral shocks and Appendix C and D in detail.

Furthermore, this welfare loss can be dissected into two components: welfare loss stemming from over-ordering and welfare loss due to the deadweight loss of default. The dashed line in Figure 10 illustrates the contribution of the over-ordering mechanism to the overall welfare loss, with an average of 1.41% of total consumption attributed to this channel.

However, it is important to interpret these time-series findings cautiously. The model's static nature means that these results should be viewed skeptically. Notably, during the Great Recession of 2008-2009, the limited liability distortion was close to the average consumption loss. This could be attributed to factors such as quantitative easing policies, efforts to control the default rate, or the limitations of the first-order approximation in severe economic downturns, such as the 2008-2009 crisis or the COVID-19 pandemic in 2020.

It is worth noting that analyzing the specific contribution of limited liability distortion to business cycles falls outside the scope of this paper. Such an investigation would require a quantitative model and could be a subject for future research.

Table 1 offers a comprehensive overview of the ten most distorted and least distorted sectors within the U.S.A. economy. It is evident from the second column that there exists a substantial variation in LLI among these sectors. LLI levels range from 0.08 to 5.81, highlighting the significant disparities in how limited liability distortions impact sectors.

Panel A: The most distorted sectors			
Sector	$\Phi(\%)$	$\zeta(\%)$	U
Warehousing and storage	5.81	5.11	2.83
Primary metals	5.48	1.34	3.16
Wholesale trade	5.39	0.04	3.38
Other transportation and support activities	5.13	10.59	3.14
Pipeline transportation	4.71	0	3.81
Rail transportation	4.56	26.04	2.72
Oil and gas extraction	4.30	5.043	2.81
Mining, except oil and gas	4.21	10.57	2.91
Truck transportation	4.18	34.25	2.35
Management of companies and enterprises	4.11	0.14	2.92
Panel B: The least distorted sectors			
Sector	$\Phi(\%)$	$\zeta(\%)$	U
Apparel and leather and allied products	0.08	94.5	1.08
Motor vehicle and parts dealers	0.94	100	1
Publishing industries, except internet (includes software)	1.03	85.3	1.23
Amusements, gambling, and recreation industries	1.10	96.34	1.07
Computer and electronic products	1.21	64.66	1.63
Ambulatory health care services	1.39	96.86	1.032
Funds, trusts, and other financial vehicles	1.46	90.68	1.145
Construction	1.52	88.43	1.199
Miscellaneous manufacturing	1.55	79.14	1.27
Food and beverage and tobacco products	1.59	73.37	1.39

Table 1: Top Ten Most Distorted and Least Distorted Sectors in the U.S.A.

The third column of Table 1 shows the percentage of sectoral output allocated to final consumption, ζ. Notably, the most distorted sectors tend to be upstream sectors. Moreover, the fourth column represents the average Upstreamness centrality of sectors from 1998 to 2021. A centrality of one means the sector primarily sells output to final consumers, while higher centrality indicates a more significant role in the supply chain.

Table 1 highlights that LLI is closely correlated with ζ and Upstreamness centrality. However, it is essential to note that these factors cannot fully replace LLI and capture all the effects of limited liability on the economy.

The distortion observed in upstream sectors, which provide intermediate inputs for other sectors, primarily arises from over-ordering by their customers. Consequently, these sectors experience substantial price increases compared to an efficient economy without distortions. In contrast, downstream sectors, primarily providing final goods, experience minimal impact from limited liability distortions. The deadweight loss of default also reduces sectoral TFP, leading to reduced output. This, in turn, drives up sectoral output prices and LLI. Notably, the second mechanism is not dependent on a sector's position in the supply chain, reducing the correlation between U and ζ with LLI.

Corollary 3 underscores the significance of sectoral distortions in influencing economic consumption loss. The extent of this loss is closely tied to household expenditure allocation toward goods from those sectors. In the U.S.A. economy, the expenditure on final goods allocated to upstream sectors is notably low, resulting in a relatively minor influence on overall consumption loss. For instance, sectors like "Warehousing and storage" and "Primary metals" account for a mere 0.028% and 0.024% of total expenditure on final goods. Consequently, their effect on consumption loss is primarily indirect, as they contribute to the increased prices of intermediate inputs for downstream sectors.

Conversely, downstream sectors with lower distortions command a substantial share of expenditure on final goods. For example, "Construction" and "Ambulatory health care services" represent significant proportions, accounting for 8.5% and 5.7% of final consumption, respectively.

The concentration of expenditure on final goods within a limited number of downstream sectors, coupled with their relatively low LLI, highlights an important consideration. Using a representative roundabout economy with only one sector tends to overestimate the impact of limited liability distortion on the overall economy. If sectors exhibited more similarity in U and ζ, resulting in closer LLI values, their influence on aggregate consumption would be more evenly distributed. In such a situation, choosing the representative model offers a more nuanced approximation, avoiding excessive simplification.

The deadweight loss of default impacts TFP and leads to price increases across all sectors. As most sectors maintain relatively similar default rates, this mechanism uniformly affects prices, resulting in little change in relative sectoral sales due to the deadweight loss of default. The primary driver of sectoral sales deviation from an efficient economy, $\log\left(\frac{\lambda}{\lambda^e}\right)$, is the over-ordering mechanism, which primarily affects sectors positioned upstream in the supply chain, causing them to experience higher-than-optimal demand levels.

The connection between $\log\left(\frac{\lambda}{\lambda^e}\right)$, U, and LLI is remarkably strong, exhibiting a consistently high positive correlation. Between 1998 and 2021, the average cross-sectional correlation between sectoral sales deviation and LLI stood at 0.892, while the correlation between sectoral sales deviation and U was notably higher at 0.975.

The heightened correlation between U and $\log\left(\frac{\lambda}{\lambda^e}\right)$ compared to $\log\left(\frac{\lambda}{\lambda^e}\right)$ and LLI can be attributed to the closely intertwined relationship between the over-ordering mechanism and a sector's position in the supply chain. This connection results in a robust correlation between U and $\log\left(\frac{\lambda}{\lambda^e}\right)$. LLI, which encompasses both channels simultaneously, and the relatively limited explanatory power of the deadweight loss of default in understanding $\log\left(\frac{\lambda}{\lambda^e}\right)$, show a lower correlation with $\log\left(\frac{\lambda}{\lambda^e}\right)$ compared to U.

A sensitivity analysis presented in Table E4 in Appendix E further reinforces these observations. As the parameter χ increases from 0.15 to 0.3, the influence of deadweight loss of default grows, causing the correlation between the deviation of sectoral sales and LLI to decrease to 0.713. In contrast, the correlation with U only declines slightly to 0.965. Therefore, Upstreamness centrality emerges as a superior indicator for tracking the deviation of sectoral sales from the efficient economy despite not fully encompassing all the effects of limited liability.[41]

Table E5 in the appendix provides insights into the results' robustness concerning substitution elasticity. Variations in the elasticity of substitution among final goods do not substantially affect consumption loss because all downstream sectors over-order intermediate inputs, resulting in higher intermediate input prices. Limited liability primarily functions as a cost-push inflation mechanism, increasing the cost of intermediate inputs. Hence, all downstream sectors raise prices for the final consumers, making the degree of substitution between them relatively inconsequential.

Conversely, the elasticity of substitution between different sectoral goods used in producing the aggregate intermediate input, x_{fit}, becomes a significant factor. A higher value of γ indicates that upstream sectors cannot significantly raise their prices, as can be done with lower γ values. Increased elasticity fosters greater competition, resulting in less pronounced price increases for intermediate inputs due to over-ordering. Consequently, this leads to more modest increases in the prices of final goods. Since over-ordering influences the demand for intermediate inputs through Γ, γ plays a pivotal role, while σ has a comparatively muted effect.

X Future Research

The limited liability distortion is not exclusive to trade credit and business transactions. It is a broader issue linked to default risk. Depending on market dynamics, it can result in excessive or inadequate borrowing, causing resource misallocation. This distortion can extend to consumer-to-business transactions. Households purchase final goods with either cash or credit, and creditors penalize defaults on credit. If the risk premium in credit pricing is imbalanced, it can lead to suboptimal credit demand, causing misallocation and consumption loss. For example, a representative agent model incorporating two types of goods, cash and credit, can be used

[41] For a more detailed visual representation of these correlations over time, please refer to Figure E3 in Appendix E.

to model default risk friction in consumer-to-business transactions. This model can explore the monetary policy and business cycle implications of default risk on household debt within a cash-in-advance framework.

Another avenue for future research involves exploring the interplay between limited liability and other financial frictions, such as more restricted access to credit for S.M.E.s. Inevitable financial frictions may result in under-borrowing by firms. Failing to account for the moral hazard of limited liability can lead models to overestimate distortions. This paper establishes a benchmark model that can serve as a foundation for integrating additional financial frictions and evaluating their impact on the overall economy.

XI Conclusion

Trade credit plays a pivotal role in business-to-business transactions, where firms (customers) have the option to default on their commitments due to limited enforceability. This inherent limitation creates a classic moral hazard problem, leading customers to over-order and demand more credit than is optimal. Suppliers respond by charging higher prices to counterbalance this, effectively incorporating a risk premium into their transactions. My analysis demonstrates that the former force consistently prevails over the latter in a static general equilibrium multi-sectoral model with endogenous default decisions.

This imbalance leads to two distinct mechanisms. Firstly, firm owners accumulate more debt in this economic environment, increasing the probability of default and resulting in output wastage due to the deadweight loss of default. Secondly, limited liability causes an over-ordering of intermediate inputs, pushing upstream sectors to become bigger in sales than an efficient economy with no limited liability. This resource misallocation towards intermediate input production rather than consumer goods leads to consumption losses, even when the deadweight loss of default is absent. The extent of these distortions depends on the economic structure and inter-sectoral connections.

I introduce the Limited Liability Influence (LLI), a metric that encapsulates the impact of limited liability on sectors. LLI serves as a comprehensive measure, summarizing all the input-output structure's impact on how limited liability affects the economy. A higher LLI indicates that a sector experiences greater over-demand from its business customers or reduced sectoral TFP due to the deadweight loss of default, causing price increases in that sector. Additionally, I quantify consumption losses due to this friction at a first-order approximation. While downstream sectors may be less directly affected by limited liability, they profoundly impact the overall consumption loss.

A policy intervention involving lump-sum production subsidies to firms shifts the default threshold to the left, effectively reducing the default rate to zero. This can be budget-balanced through profit taxation.

Furthermore, all distortions are exacerbated by incorporating production factors into the model. The over-ordering of intermediate inputs leads to resource allocation favouring upstream sectors and depriving downstream sectors of factors of production like labour, magnifying the consumption loss. In alignment with the existing literature, by introducing sectoral shocks into the model, a positive sectoral shock reduces the

default rate within that sector and triggers a stronger sectoral TFP response. This implies that economies with higher financial frictions experience more pronounced sectoral TFP fluctuations when subjected to sectoral shocks.

The empirical findings from the U.S.A. economy show that limited liability distortion significantly affects sectors, particularly upstream ones, due to over-ordering. The average consumption loss is 2.07%, with 70% attributed to the over-ordering mechanism. The deviation of sectoral sales, Upstreamness centrality, and LLI are highly correlated, with Upstreamness centrality being a better predictor of sectoral sales deviation. The greater elasticity of substitution among sectoral goods in intermediate input production fosters competition in upstream sectors, resulting in more moderate price increases due to over-ordering.

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Appendix A Model with Cash-in-Advance Constraints

This section extends the model to incorporate cash-in-advance constraints, a common assumption in addressing limited liability within financial friction models (as observed in Altinoglu (2021) and Bigio et al (2016)).

In this extended framework, aggregate suppliers demand an upfront payment, denoted as θ_{fi}, to process orders from Firm f in Sector i.

Additionally, a third-party creditor, such as a bank, operates as a risk-neutral, zero-profit entity. Firm owners have the option to borrow the upfront cost from the bank at the risk-free rate of r, provided they put up collateral of equal value. For simplicity, we assume that all firm owners possess sufficient assets to use as collateral. In the event of a default, the bank can readily convert the collateral into credit at no cost.

We can rewrite the firm's problem in Equations 3 to 5 as follow:

$$\max_{\{x_{fij}\}_{j=1}^{N}} \mathop{\mathbb{E}}_{A_f \sim G} \left(\max\{-\theta_{fi}(1+r) \sum_{j=1}^{N} p_{fij}\, x_{fij}, \Pi_{fi}(A_f, x_{fi})\} \right) \tag{A.1}$$

s.t.

$$\Pi_{fi}(A_f, x_{fi}) \equiv \hat{p}_{fi}\, A_f\, x_{fi}^{\alpha_i} - (1-\theta_{fi}) \sum_{j=1}^{N} p_{fij}\, x_{fij} - \theta_{fi}(1+r) \sum_{j=1}^{N} p_{ij}\, x_{ij} \tag{A.2}$$

$$x_{fi} \equiv \left(\sum_{j=1}^{N} \omega_{ij}\, x_{fij}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}} \tag{A.3}$$

Similar to Section II, we apply the equilibrium conditions to simplify the firms' problem. Given that firm heterogeneity is solely determined by their respective sectors in the context of ordering intermediate inputs, we can simplify θ_{fi} and denote the upfront payment as θ_i.

$$\max_{\{x_{ij}\}_{j=1}^{N}} \int_{A_i^*}^{\infty} P_i\, A_f\, x_i^{\alpha_i}\, dG - (1 - G(A_i^*))(1-\theta_i) \sum_{j=1}^{N} p_{ij}\, x_{ij} - \theta_i(1+r) \sum_{j=1}^{N} p_{ij}\, x_{ij} \tag{A.4}$$

s.t.

$$x_i \equiv \left(\sum_{j=1}^{N} \omega_{ij}\, y_{ij}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}$$

To determine the default threshold, we need to find A_i^* at which the firm owner is indifferent between defaulting and not defaulting. This can be expressed as:

$$P_i\, A_i^*\, x_i^{\alpha_i} - (1-\theta_i) \sum_{j=1}^{N} p_{ij}\, x_{ij} - \theta_i(1+r) \sum_{j=1}^{N} p_{ij}\, x_{ij} = -\theta_i(1+r) \sum_{j=1}^{N} p_{ij}\, x_{fij}$$

$$P_i\, A_i^*\, x_i^{\alpha_i} = (1-\theta_i) \sum_{j=1}^{N} p_{ij}\, x_{ij} \tag{A.5}$$

Here, it is essential to note that the upfront payment for intermediate inputs is a sunk cost and does not influence the default decision. When deciding whether to default, firm owners solely consider their revenue and variable costs.

The first-order conditions with respect to $\{x_{ij}\}_{j=1}^{N}$ are

$$\alpha_i\, x^{\alpha_i - 1}\, \frac{\mathrm{d}x_i}{\mathrm{d}x_{ij}}\, P_i \int_{A_i^*}^{\infty} A_f\, dG = \left(\left(1 - G\left(A_i^*\right)\right)\left(1-\theta_i\right) + (1+r)\theta_i\right) p_{ij}$$

Utilizing the fact that the aggregate intermediate input follows a CRS technology, as indicated in Equation 5, we can rewrite it as:

$$\alpha_i\, x^{\alpha_i}\, P_i \int_{A_i^*}^{\infty} A_f\, dG = \left(\left(1 - G\left(A_i^*\right)\right)\left(1-\theta_i\right) + (1+r)\theta_i\right) \sum_{j=1}^{N} p_{ij}\, x_{ij} \tag{A.6}$$

By combining Equations A.5 and A.6, we can determine the default threshold.

$$\alpha_i E(A|A > A_i^*) = \left(1 + \frac{\theta_i(1+r)}{(1-G(A_i^*))(1-\theta_i)}\right) A_i^*$$

$$A_i^* = \left(1 - \frac{\theta_i(1+r)}{(1-G(A_i^*))(1-\theta_i) + \theta_i(1+r)}\right) \alpha_i E(A|A > A_i^*)$$

If $G(.)$ is supported from below with $\underline{A}$, to set the default rate to zero, $\theta \geq \theta^*$ where θ^* should satisfies

$$\underline{A} = \frac{1-\theta^*}{(1-\theta^*) + \theta^*(1+r)}\, \alpha_i$$

by rearranging the equation, we have

$$\theta^* = \frac{1 - \frac{\underline{A}}{\alpha_i}}{1 - \frac{\underline{A}}{\alpha_i} r} \tag{A.7}$$

$$\frac{\mathrm{d}\theta^*}{\mathrm{d}\underline{A}} = \frac{\frac{r-1}{\alpha_i}}{(1 - \frac{\underline{A}}{\alpha_i} r)^2} < 0 \tag{A.8}$$

Equation A.8 demonstrates that a possibility of lower productivity shocks leads to an increase in the cash-in-advance requirement. As the lower bound of productivity shocks, $\underline{A}$, approaches zero, θ^* grows and eventually converges to one.

If $\theta > \theta^*$, then $\Gamma = \frac{1}{1+\theta\, r} < 1$. This implies that modeling limited liability using cash-in-advance constraints, as demonstrated in Altinoglu (2021) and Bigio et al (2016), leads to the under-ordering of intermediate inputs. This outcome stands in stark contrast to the benchmark model's misallocation pattern.

$$\frac{\mathrm{d}\theta_i^*}{\mathrm{d}\alpha_i} = \frac{(1-r)\frac{A}{\alpha_i^2}}{(1-\frac{A}{\alpha_i}r)^2} > 0 \tag{A.9}$$

Equation A.9 illustrates that an increase in α_i corresponds to a higher cash-in-advance requirement. A higher α_i implies a greater leverage ratio and an increased likelihood of default. Consequently, creditors demand a higher upfront payment as a safeguard.

In this extended framework, the recovery rate for Sector i is modified as follows:

$$\delta_i = \theta_i + (1-\theta_i)(1-\chi)\int_0^{A_i^*} \frac{P_i\, y_{fi}(A)}{(1-\theta_i)\sum_{j=1}^{N} p_{ij}\, x_{ij}} dG^{'}(A)$$

where the productivity distribution conditional on default, $G^{'}(.)$, is $\frac{G(.)}{G(A_i^*)}$. Using Equation 11, we have

$$\delta_i = \theta_i + (1-\theta_i)(1-\chi)\frac{E(A|A<A_i^*)}{A_i^*} \tag{A.10}$$

Lemma 5. *In the economy detailed in Appendix A, Γ_i can be expressed as*

$$\Gamma = \frac{E(A|A>A_i^*)}{E(A|A_i^*,\chi)} \frac{(1-G(A_i^*)+G(A_i^*)\,\delta_i(A_i^*))}{1+\theta(\frac{r+G(A_i^*)}{1-G(A_i^*)})}$$

Γ is monotonically decreasing with respect to θ for any non-negative value of r.

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\theta} < 0$$

Appendix G provides a proof of Lemma 5. As a result of Lemma 5, one can introduce a parameter, θ^{**}, such that $\Gamma = 1$:

$$\theta^{**} = \frac{\frac{E(A|A>A_i^*)}{E(A|A_i^*,\chi)}\,(1-G(A_i^*)+(1-\chi)\frac{E(A|A<A_i^*)}{A_i^*}) - 1}{\left(\frac{r+G(A_i^*)}{1-G(A_i^*)} + \left((1-\chi)\frac{E(A|A<A_i^*)}{A_i^*} - G(A_i^*)\right)\frac{E(A|A>A_i^*)}{E(A|A_i^*,\chi)}\right)}$$

It is important to note that θ^{**} depends on the parameters r, α, G, and χ. The monotonous behaviour of Γ with respect to θ implies that $\theta^{**} < \theta^*$. When θ falls within this range, the probability of default is positive, and $\Gamma < 1$. When θ is less than θ^{**}, Γ exceeds 1, indicating the onset of over-ordering.

A.A Intervention Policies with Cash-in-Advance Constraints

Cash-in-advance constraints can effectively lower the default threshold, thereby increasing the default cost for firm owners. In Appendix A, I elaborate on the extended model with cash-in-advance constraints. In this setup, aggregate suppliers require firms in Sector i to pay a fraction of θ_i from the order upfront. Firm owners can borrow this credit at the intra-period risk-free interest rate of r from a third-party creditor, a zero-profit

entity like a bank, provided they offer collateral equal in value to the credit requested. Importantly, in the event of default, the bank can readily convert the collateral into credit without incurring any cost.

Within this extended model, if $\theta_i \geq \theta_i^*$, where $\theta^* = \frac{1-\frac{A}{\alpha_i}}{1-\frac{A}{\alpha_i}r}$, defaults never occur.[42] However, firms face higher costs when purchasing intermediate inputs, leading to $\Gamma_i = \frac{1}{1+\theta_i\, r}$. In this scenario, firms under-order intermediate inputs, rendering the model similar to Bigio et al (2016), Altinoglu (2021), and Liu (2019).

By implementing a credit policy such that $\theta \geq \theta_i^*$, defaults and the associated deadweight loss can be avoided. Consequently, the primary concern would then be resource misallocation stemming from under-ordering.

When $\theta \geq \theta_i^*$, the economy experiences underproduction of intermediate inputs and overproduction of final consumption due to $\Gamma_i < 1$. Firms' reduced use of intermediate inputs ultimately leads to lower overall economic production.

We can introduce two policy measures to rectify this misallocation: a tax on final consumption goods and subsidies for firms' intermediate input costs. Implementing a tax on final consumption goods decreases the demand for these goods. Subsidies on intermediate inputs increase the demand for these inputs by lowering their effective cost for firms. As a result, final consumers observe a price of $(1+\tau)P_i$ for sectoral good i, while firms in Sector i perceive a price of $(1-S_i)p_{ij}$ when purchasing sectoral good j from the sector aggregator. The tax rate τ is carefully adjusted to ensure budget balance for these government interventions. Proposition 3 succinctly summarizes this remedy.[43]

Proposition 3. *In an economy with limited liability, if the credit policy sets the required cash-in-advanced payments for firms in Sector i equal to $\theta_i = \theta_i^*$, and subsidies on each dollar of intermediate input costs are established at $\frac{\theta_i^* r}{1+\theta_i^* r}$, the resulting allocation is similar to an efficient economy.*

Additionally, to maintain this equilibrium, the tax rate on the final consumption good should remain constant and be equal to

$$\tau = \sum_i \alpha_i \lambda_i \frac{\theta_i^* r}{1+\theta_i^* r}$$

If all sectors have a similar α, the tax rate τ can be expressed as $\tau = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\theta^* r}{1+\theta^* r}\right)$.[44] $\frac{\alpha}{1-\alpha}$ is the ratio of intermediate input costs to the value-added in sectors. Higher α increases the importance of intermediate input in the production process and increases the share of input costs to nominal GDP. To fund subsidies for this volume of intermediate inputs, the tax rate on nominal GDP must increase accordingly. Furthermore, θ^* increases with α. A higher α corresponds to a higher debt accumulation for each dollar of revenue, indicating a higher leverage ratio. Consequently, suppliers demand a larger upfront payment to accommodate this increased financial risk.

The proposed tax scheme is straightforward: all downstream sectors are subject to the same tax rate,τ, while all sectors receive subsidies proportional to $1+\theta_i^*\, r$.

[42] See Appendix A for details
[43] See the proof in Appendix G
[44] $\tau = \alpha\, \frac{\theta^* r}{1+\theta^* r}\, \epsilon$ and $\epsilon = \frac{1}{1-\alpha}$

The tax policy introduced here is aligned with the recommendations in Liu (2019), which suggests subsidizing trade with upstream sectors while imposing taxes on downstream sectors. This approach is rooted in the argument that the bulk of economic friction accumulates within the upstream sectors, making them the most distorted overall. To rectify this allocation, subsidies are provided to upstream sectors, while downstream sectors face taxation.

Recognizing that this tax policy can only lead to an efficient allocation if specific conditions are met is crucial. First, the government must successfully implement cash-in-advance constraints characterized by θ^*. Second, firm owners must possess sufficient collateral to secure upfront payments from third-party creditors. The failure of any of these conditions causes the economy to deviate from the efficient economy.

Trade credit plays a pivotal role in various sectors, with many firms relying on credit for a significant portion of their input purchases. This suggests that in practice, θ may often be much lower than θ^*. Moreover, collateral constraints represent a significant source of financial friction, contributing to misallocation in the economy (Bernanke et al (1999), Gertler and Kiyotaki (2010)). The second assumption of having enough collateral for all firm owners may not be realistic in the real world. Lastly, real-world conditions typically involve non-zero default rates, suggesting that at least one of the conditions required for the first-best solution may not hold in practice.

Appendix B Detailed Policy Mechanisms and Impact

To re-establish efficient allocation in an economy with limited liability while ensuring a balanced budget, the government implements the following policy measures:

- **Production Subsidies:** Firms operating in Sector i receive subsidies, denoted as S_i, contingent on not defaulting. These lump-sum subsidies encourage firm owners to refrain from defaulting.
- **Profit Taxes:** Firms in each sector face a profit tax, denoted as τ_i. This tax is levied on the profits firms generate within Sector i. It helps generate government revenue to make sure the government budget remains balanced.

The firm's optimization problem in Sector i is as follows:

$$\max_{\{x_{fij}\}_{j=1}^{N}} \mathbb{E}_{A_f \sim G} \left(\max\{0, (1-\tau_i)\,\Pi_{fi}\,(A_f)\} \right),$$

s.t.

$$\Pi_{fi}\,(A_f) \equiv \hat{p}_{fi}\,A_f\,x_{fi}^{\alpha_i} - \sum_{j=1}^{N} p_{fij}\,x_{fij} + S_i,$$

$$x_{fi} \equiv \left(\sum_{j=1}^{N} \omega_{ij}\,x_{fij}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}.$$

S_i represents the lump-sum subsidies provided to firms in Sector i that do not default. These subsidies serve as an incentive to support production activities and mitigate the impact of policy-induced taxes on the sector's economic performance.

After applying the equilibrium conditions, we can rewrite the problem as follows:

$$\max_{\{x_{ij}\}_{j=1}^{N}} (1-\tau_i)\left(\int_{A_i^*}^{\infty} P_i\, A_f\, x_i^{\alpha_i} + S_i\, dG - (1-G\left(A_i^*\right))\sum_{j=1}^{N} p_{ij}\, x_{ij}\right),$$

s.t.

$$x_i \equiv \left(\sum_{j=1}^{N} \omega_{ij}\, y_{ij}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}.$$

At the break-even productivity, the following condition holds:

$$(1-\tau_i)\, P_i\, A_i^*\, x_i^{\alpha_i} + (1-\tau_i) S_i = (1-\tau_i)\left(\sum_{j=1}^{N} p_{ij}\, x_{ij}\right).$$

This equation elucidates that the tax on profits does not influence the default decision; it is solely the production subsidies that can impact this critical determination. The F.O.C.s of firm's problem with respect to $\{x_{ij}\}$ is expressed as:

$$(1-\tau_i)\,\alpha_i\,\frac{\mathrm{d}x_i}{\mathrm{d}x_{ij}}\, P_i\, x_i^{\alpha_i-1} E\left(A|A>A_i^*\right) = (1-\tau_i)\, p_{ij} \quad \forall\ \ i,j \text{ such that } \omega_{ij}>0.$$

This equation reinforces that the tax on profits does not affect the decision regarding the optimal set of intermediate inputs. The primary objective of the profit tax is to maintain a balanced budget. In Appendix F, the impact of profit tax and production subsidies on firm profit and the default threshold is depicted by Figure F5 for further clarity.

By utilizing the optimality conditions of firms, we can derive a condition similar to Equation 11:

$$\alpha_i\, E(A|A>A_i^*) = A_i^* + \frac{S_i}{P_i\, x_i^{\alpha_i}}.$$

Setting the default rate to zero eliminates limited liability distortions. This can be accomplished by setting $A_i^* = \underline{A}$, where $\underline{A}$ represents the lower bound of G.

To ensure that $A_i^* = \underline{A}$, the necessary subsidy S_i should be $\lambda_i(\alpha_i - \underline{A})$ when $\underline{A} < \alpha_i$.[45] To have a balanced budget at the sectoral level, we need to have $\tau_i = \frac{\alpha_i - \underline{A}}{1-\alpha_i}$. A^* depends on α_i. Higher values of α_i increase the default threshold, necessitating higher subsidies to mitigate this threshold back to the lowest realization of idiosyncratic productivity shocks, $\underline{A}$. In an efficient economy, sectoral TFP is one, and the input costs for firms in Sector i are $\alpha_i\lambda_i$. However, the revenue of a firm with the lowest productivity is $\underline{A}\lambda_i$. Consequently,

[45] If $\underline{A} > \alpha_i$, firms will not default without any intervention.

the production subsidies serve as a safeguard, ensuring that even firms with the lowest productivity levels can meet their financial obligations. Proposition 4 summarizes this result.

Proposition 4. *For the economy with limited liability described in Section II if A^* exists and is unique and, for each Sector i, non-defaulting firms receive lump-sum subsidies S_i and pay taxes on profits with a tax rate τ_i such that:*

- $S_i = \lambda_i(\alpha_i - \underline{A})$;
- $\tau_i = \frac{\alpha_i - \underline{A}}{1-\alpha_i}$.

Then, the economy can achieve efficient allocation.

Proposition 4 shows that we can attain an optimal allocation without resorting to strict credit policies. We can still achieve economic efficiency by avoiding the imposition of cash-in-advance constraints, which might lead to collateral constraints for some firms and introduce a different form of financial distortion.

Appendix C Model with Factor of Production

The model structure remains largely consistent with the benchmark model outlined in Section II, with some notable modifications to incorporate labour as a factor of production. These changes include:

1. labour Supply: In this extended model, the representative household supplies labour inelastically. As a result, the household's budget constraint is adjusted to incorporate labour income:

$$P_i C_i = \sum_{i=1}^{N} \int_{A_i^*}^{\infty} \Pi_i(A_f)\, dG(A) + w, \tag{C.1}$$

   where w represents the wage per unit of labour.

2. Production Function: The firm's production function is modified to account for the inclusion of labour:

$$y_{fi} = A_f\, \ell_{fi}^{\beta_i}\, x_{fi}^{\alpha_i} \quad , \quad \alpha_i + \beta_i < 1.$$

   Here, the Cobb-Douglas assumption is introduced for simplicity and to facilitate algebraic manipulations while maintaining the core concept of the factor of production - labour. The production function continues to exhibit DRS.

3. Labour Decision: Firms now make labour-related decisions after determining whether to default on their commitments. Importantly, firm owners are not permitted to default on their obligations related to labour compensation.

4. labour Market Clearing: To ensure labour market equilibrium, the total labour supplied by all firms in all sectors must equal one:

$$\sum_{i=1}^{N} \int \ell_{fi} df = 1. \tag{C.2}$$

These adjustments allow for a more comprehensive exploration of the model's implications when labour is integrated into the production process.

The firms' problem can be formulated using Equations C.3 to C.5, providing insight into their decision-making process:

$$\max_{\{x_{fij}\}_{j=1}^{N}} \mathop{\mathbb{E}}_{A_f \sim G} \left(\max\{0, \Pi_{fi}(A_f)\}\right), \tag{C.3}$$

s.t.

$$\Pi_{fi}(A_f) \equiv \max_{\ell_{fi}} \hat{p}_{fi}\, A_f\, \ell_{fi}^{\beta_i}\, x_{fi}^{\alpha_i} - \sum_{j=1}^{N} p_{fij}\, x_{fij} - w\, \ell_{fi}, \tag{C.4}$$

$$x_{fi} \equiv \left(\sum_{j=1}^{N} \omega_{ij}\, x_{fij}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}. \tag{C.5}$$

Equation C.4 shows that firms choose the optimal labour given by previously ordered intermediate inputs and realized productivity shocks.

For the defaulting firms, the bankruptcy court chooses the labour to maximize the profit of those firms. The optimal labour choices for those firms are as follows:

$$\max_{\ell_{fi}} \hat{p}_{fi}\, (1-\chi)\, A_f\, \ell_{fi}^{\beta_i}\, x_{fi}^{\alpha_i} - \sum_{j=1}^{N} p_{fij}\, x_{fij} - w\, \ell_{fi}.$$

F.O.C of firm's problem implies that optimal x_{fij} and ℓ_{fi} should satisfy:

$$\alpha_i\, \hat{p}_{fi} \left(\int_0^{\infty} A_f\, \mathscr{I}(A_f) dG \right) \ell_{fi}^{\beta_i}\, x_{fi}^{\alpha_i - 1}\, \frac{\mathrm{d}x_{fi}}{\mathrm{d}x_{fij}} \leq p_{fij} \left(\int_0^{\infty} \mathscr{I}(A_f) dG \right) \ \forall\, j,$$

where $\mathscr{I}$ is an indicator function that equals one when Firm f does not default at the realized productivity A_f, and zero otherwise. Another condition is given by:

$$\beta_i\, \hat{p}_{fi}\ (1 - (\mathscr{I}(A_f) - 1)\, \chi)\ A_f\, \ell_{fi}^{\beta_i - 1}\, x_{fi}^{\alpha_i} < w.$$

Labour demand after deciding whether to default is as follows:

$$\ell_{fi} = \left(\frac{\beta_i\, \hat{p}_{fi}\ (1 - (\mathscr{I}(A_f) - 1)\chi)\ A_f\, x_{fi}^{\alpha_i}}{w} \right)^{\frac{1}{1-\beta_i}}. \tag{C.6}$$

By applying the equilibrium conditions outlined in Section II (i.e., $\hat{p}_{fi} = P_i$, $p_{fij} = p_{ij}$, and $x_{fi} = x_i$) and utilizing the optimal labour choice, we can derive the firm's profit as follows:

$$\Pi_{fi}(A_f) = (1-\beta_i)\, P_i^{\frac{1}{1-\beta_i}}\, A_f^{\frac{1}{1-\beta_i}}\, x_i^{\frac{\alpha_i}{1-\beta_i}}\, w^{\frac{-\beta_i}{1-\beta_i}} - \sum_j p_{ij} x_{ij}. \tag{C.7}$$

Equation C.7 provides several insights into firm profitability. Firstly, it illustrates that a share of β_i of the firm's revenue is allocated to labour costs. Secondly, the term $\frac{1}{1-\beta_i}$ represents the neoclassical multiplier, which amplifies the profit sensitivity to the firm's decisions and the realized productivity level. This equation highlights that Π_{fi} is contingent on sector-specific variables and idiosyncratic productivity shocks. Consequently, the partial derivative $\dfrac{\mathrm{d}\Pi_{fi}}{\mathrm{d}A_f} > 0$ implies that there exists a default threshold at which $\Pi_{fi} > 0$, occurring only if $A_f > A_i^*$. This threshold is crucial in determining default outcomes for firms.

The default threshold, as derived from solving the firms' problem represented by Equations C.3 to C.5, is determined by the following equation:

$$A_i^* = \Big(\frac{\alpha_i}{1-\beta_i} E(A^{\frac{1}{1-\beta_i}} | A > A_i^*)\Big)^{1-\beta_i}.$$

Incorporating labour into the model introduces the neoclassical multiplier, which impacts the magnitude of welfare loss attributable to the deadweight loss of default. Additionally, the markup shifts from $\frac{1}{\alpha_i}$ to $\frac{1-\beta_i}{\alpha_i}$. This change contributes to a higher probability of default and elevates the default threshold.

The recovery rate in the model with labour is as follows:

$$\begin{aligned}
\delta_i &= (1-\chi) \int_0^{A_i^*} \frac{P_i\, y_{fi}(A) - w\, \ell_{fi}(A)}{\sum_{j=1}^N p_{ij}\, x_{ij}} dG'(A) \\
&= (1-\chi)(1-\beta_i) \int_0^{A_i^*} \frac{P_i\, y_{fi}(A)}{\sum_{j=1}^N p_{ij}\, x_{ij}} dG'(A) \\
&= (1-\chi) \frac{E(A^{\frac{1}{1-\beta_i}} | A < A_i^*)}{A_i^{*\,\frac{1}{1-\beta_i}}}.
\end{aligned} \tag{C.8}$$

The ratio of the market value of intermediate inputs to the market value of sectoral output is given by:

$$\begin{aligned}
\frac{\sum_j P_j\, x_{ij}}{P_i\, Y_i} &= \left(\frac{\alpha_i\, P_i^{\frac{1}{1-\beta_i}}\, E(A^{\frac{1}{1-\beta_i}} | A > A_i^*)\, x_i^{\frac{1}{1-\beta_i}}\, w^{\frac{-\beta_i}{1-\beta_i}}}{P_i^{\frac{1}{1-\beta_i}}\, E(A^{\frac{1}{1-\beta_i}} | A_i^*, \chi)\, x_i^{\frac{1}{1-\beta_i}}\, w^{\frac{-\beta_i}{1-\beta_i}}} \right) (1 - G(A_i^*) + G(A_i^*)\delta_i) \\
&= \left(\frac{\alpha_i\, E(A^{\frac{1}{1-\beta_i}} | A > A_i^*)}{E(A^{\frac{1}{1-\beta_i}} | A_i^*, \chi)} \right) (1 - G(A_i^*) + G(A_i^*)\delta_i)\,.
\end{aligned}$$

$$\frac{\frac{\sum_j P_j\, x_{ij}}{P_i\, Y_i}}{\frac{\sum_j P_j^e\, x_{ij}^e}{P_i^e\, Y_i^e}} = \Gamma_i = \left(\frac{E(A^{\frac{1}{1-\beta_i}} | A > A_i^*)}{E(A^{\frac{1}{1-\beta_i}} | A_i^*, \chi)} \right) (1 - G(A_i^*) + G(A_i^*)\delta_i)$$

This equation illustrates that the economy still experiences an over-ordering distortion even in the presence of labour.

Definition 11. *An equilibrium is an allocation of* $\{C_i\}_{i=1}^N$, $\{\{\ell_{fi}\}_{f\in[0,1]}\}_{i=1}^N$, $\{\{x_{ij}\}_{i=1}^N\}_{j=1}^N$, *a sequence of* $\{P_i, A_i^*, \delta_i, \{p_{ij}\}_{j=1}^N\}_{i=1}^N$, *and* w *such that:*

(i) given $\{P_i, \{p_{ij}\}_{j=1}^N\}_{i=1}^N$ *and* w , $\{\{x_{ij}\}_{i=1}^N\}_{j=1}^N$, $\{\{\ell_{fi}\}_{f\in[0,1]}\}_{i=1}^N$, *and* $\{A_i^*\}_{i=1}^N$ *satisfy Firms' problem in Equations C.3 to C.5 and C.6;*

(ii) Given $\{P_i, A_i^*, \delta_i, \{p_{ij}\}_{j=1}^N\}_{i=1}^N$, $\{\{x_{ij}\}_{i=1}^N\}_{j=1}^N$ *and* $\{C_i\}_{i=1}^N$ *satisfy Sectors' problem in Equations 2 to 14;*

(iii) Given $\{P_i\}_{i=1}^N$ *and* w, $\{C_i\}_{i=1}^N$ *satisfy Households' problem in Equations 7, 8 and C.1;*

(iv) $\{\delta_i\}_{i=1}^N$ *satisfy Equation C.8;*

(v) Market clearing condition in Equation C.2 holds.

I drive Corollary 5 to measure limited liability influence in a model that includes labour as a factor of production.

Corollary 5. *The first-order approximation of* Φ *for the economy described in Appendix C is*

$$\Phi = \boldsymbol{\kappa}_2 \kappa_1 + \boldsymbol{\kappa}_2\, \boldsymbol{\kappa}_w\, \log \frac{\lambda}{\lambda^e}\,, \tag{C.9}$$

where

$$\log \frac{\lambda}{\lambda^e} = (I - \boldsymbol{\Delta} - \boldsymbol{\kappa}_3\boldsymbol{\kappa}_2\boldsymbol{\kappa}_w)^{-1}\boldsymbol{\kappa}_3\boldsymbol{\kappa}_2\, \kappa_1 + (I - \boldsymbol{\Delta} - \boldsymbol{\kappa}_3\, \boldsymbol{\kappa}_2\, \boldsymbol{\kappa}_w)^{-1}\boldsymbol{\Delta}\hat{\Gamma},$$

$$\kappa_1 = (-\frac{\alpha}{(1-\beta)(1-\alpha-\beta)} \circ \hat{A}^* - \hat{z} + \frac{\alpha}{1-\alpha-\beta} \circ \hat{RP}),$$

$$\boldsymbol{\kappa}_2 = (\frac{1}{1-\alpha-\beta} \circ I - \frac{\alpha}{1-\alpha-\beta} \circ \tilde{\boldsymbol{\Delta}})^{-1},$$

$$\boldsymbol{\kappa}_3 = \left(\big(\xi(1-\sigma) + (1-\xi)(1-\gamma)\big) \circ \mathbf{I} - \xi(1-\sigma) \circ (\Lambda' \otimes \mathbf{1}_{N\times N}) - (1-\gamma)\boldsymbol{\Delta}\tilde{\boldsymbol{\Delta}} \right),$$

$$\boldsymbol{\kappa}_w = I + \frac{\beta}{1-\alpha-\beta} \circ \ell' \otimes \mathbf{1},$$

where $\boldsymbol{\Delta}$ *is the matrix with* $\frac{y_{ji}}{Y_i}$ *in entry* (i,j), $\tilde{\boldsymbol{\Delta}}$ *is the matrix with* $\frac{x_{ij}p_j}{\sum_j x_{ij}p_j}$ *in entry* (i,j), *so* $\sum_j \tilde{\Delta}_{ij} = 1$, ξ *is the vector with* $\xi_i = 1 - \sum_j \Delta_{ji} = \frac{C_i}{Y_i}$ *in entry* i, Λ *is the vector with* $\Lambda_i = \frac{P_i C_i}{PC}$ *in entry* i, $\hat{A}^*$ *is the vector with* $\hat{A}_i^* = \log(\frac{E(A^{\frac{1}{1-\beta_i}} | A > A_i^*)}{E(A^{\frac{1}{1-\beta_i}})})$ *in entry* i, $\hat{z}$ *is a vector representing the ratio of sectoral TFP for Sector* i *in the benchmark economy relative to an efficient economy. The* i*-th entry of this vector is defined*

as $\log(\frac{E(A^{\frac{1}{1-\beta_i}}|A_i^*,\chi)}{E(A^{\frac{1}{1-\beta_i}})})$, $\hat{RP}$ *is the vector with* $\hat{RP}_i = -\log(1-G(A_i^*)+G(A_i^*)\delta_i)$ *in entry* i, *and* ℓ *is the vector with* ℓ_i *in entry* i.

Lemma 6. *In the context of the economy described in Appendix C, the first-order effect of limited liability on the labour share of nominal GDP* ($\hat{w}$) *is given by:*

$$\sum_{i=1}^{N} \ell_i \log(\frac{\lambda_i}{\lambda_i^e}) = \hat{w}.$$

Lemma 6 illuminates the dynamic relationship between limited liability and the labour share of nominal GDP. The introduction of limited liability has a tangible impact on this economic indicator.

The introduction of limited liability tends to amplify sector sales through over-ordering distribution, bolsters overall demand and magnifies the aggregate gross output. Consequently, labour's share of income,w, is expected to expand within the model framework in response to limited liability.

Furthermore, the magnitude of this response hinges on several crucial factors. Firstly, the degree of distortion within sector sales is a key determinant. As sales distortion intensifies, sectors experience heightened demand, prompting them to offer higher wages at the expense of reduced profitability to meet the surge in demand. This dynamic pushes the labour income share within nominal GDP upwards while diminishing the profit share. Notably, the most severely distorted sectors often found upstream sectors significantly influence wages and augment the overall labour income share.

Secondly, sectors that employ a larger labour force, represented by higher ℓ, assume greater importance. Even minor distortions in these sectors can trigger significant disruptions in the labour market, necessitating the hiring of a substantial number of labourers to cope with the heightened demand from limited liability.

Appendix D Model with Sectoral Shocks

Now that we have gained insights into how financial frictions, over-ordering, and the deadweight loss of default interact with network amplification, examining the model's behaviour in the presence of sectoral and aggregate shocks is valuable. In this section, we will explore the effects of sectoral shocks on the model's default threshold and sectoral TFP.

To incorporate sectoral shocks into the model, we introduce a sequence of mean zero i.i.d sectoral productivity shocks denoted as $\mathscr{A}_{N\times 1} = (A_1, ..., A_N)'$, drawn from an arbitrary distribution F. Sectors and firms observe these shocks in Stage I before signing the contracts. Thus, there is no aggregate uncertainty when they sign the contracts. If $\mathscr{A}$ realize, firms in Sector i can pre-order and receive $x_{ij}(\mathscr{A})$ from Sector j, and they make payments of $p_{ij}(\mathscr{A})x_{ij}(\mathscr{A})$ if they avoid default. Furthermore, Sector j receives $\delta_i(\mathscr{A})$ of the principal from defaulting firms in Sector i, contingent on the specific value of $\mathscr{A}$. This setup allows sectors and firms to hedge perfectly against the impact of sectoral shocks in the model, so similar to the benchmark, idiosyncratic risk is the only risk that parties should deal with.

Incorporating sectoral shocks allows us to examine their impact on the model, especially on the default threshold and sectoral TFP. This analysis deepens our understanding of how financial frictions, network amplification, and sectoral shocks interact, providing a more comprehensive view of the model's dynamics and implications.

In the model with sectoral shocks, the production function becomes

$$y_{fi} = (A_f + A_i)x_{fi}^{\alpha_i} \quad , \alpha_i \in (0,1), \tag{D.1}$$

where A_i is the entry i of vector $\mathscr{A}$.

The sectoral shock acts as a shifter, altering the idiosyncratic productivity either to the right or left. Suppose we introduce a mean-one sectoral shock as a multiplication factor to the idiosyncratic shock. In that case, the model generates a counterfactual result where sectoral shocks have no effect on the default threshold or the fraction of defaulting firms. However, a positive productivity shock in a sector should decrease the likelihood of default and result in fewer defaulting firms.

To address this inconsistency, it is more appropriate to incorporate an additive mean zero sectoral shock. With this adjustment, a positive sectoral shock increases the overall productivity of the sector, making default less likely and reducing the fraction of defaulting firms. This aligns with our understanding of how sectoral shocks should impact default behaviour.

$$V_{fi}(\mathscr{A}) = \max_{\{x_{fij}\}_{j=1}^{N}} \mathop{\mathbb{E}}_{A_f \sim G} \left(\max\{0, \Pi_{fi}(\mathscr{A}, A_f\}\right), \tag{D.2}$$

s.t.

$$\Pi_{fi}(\mathscr{A}, A_f) \equiv \hat{p}_{fi}\,(A_f + A_i)\,x_{fi}^{\alpha_i} - \sum_{j=1}^{N} p_{fij}\,x_{fij}, \tag{D.3}$$

$$x_{fi} \equiv \left(\sum_{j=1}^{N} \omega_{ij}\, x_{fij}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}. \tag{D.4}$$

For a given set of sectoral shocks denoted as $\mathscr{A}$, firms aim to maximize their value by determining the optimal combination of intermediate inputs, $\{x_{fij}\}_{j=1}^{N}$, and deciding whether to default. A firm denoted as f defaults if and only if its profit becomes negative, represented as $\Pi_{fi}(\mathscr{A}, A_f) < 0$.

The F.O.C of the firm's problem implies that the optimal choice of $x_{fij}(\mathscr{A})$ must satisfy the following inequality:

$$\alpha_i\,\hat{p}_{fi}\left(\int_0^{\infty}(A_f + A_i)\,\mathscr{I}(\mathscr{A}, A_f)dG\right)\,x_{fi}^{\alpha_i - 1}\,\frac{\mathrm{d}x_{fi}}{\mathrm{d}x_{fij}} \leq p_{fij}\,\left(\int_0^{\infty}\,\mathscr{I}(\mathscr{A}, A_f)dG\right), \tag{D.5}$$

where $\mathscr{I}(\mathscr{A}, A_f)$ is the indicator function with the value of one when the firm does not default at the aggregate state of $\mathscr{A}$ and realized idiosyncratic productivity shock of A_f, otherwise, it is zero. At Stage I, when firms are ordering intermediate inputs, firms within the same sector are similar. They all face similar sectoral shocks and have similar customers. Consequently, we can apply the equilibrium conditions discussed in Section II. This leads to the following expression:

$$\Pi_{fi}(\mathscr{A}, A_f) = P_i(\mathscr{A})\,(A_f + A_i)\,x_i^{\alpha_i}(\mathscr{A}) - \sum_{j=1}^{N} p_{ij}(\mathscr{A})\,x_{ij}(\mathscr{A}). \tag{D.6}$$

Given sectoral shocks, $\dfrac{\mathrm{d}\Pi_{fi}}{\mathrm{d}A_f} > 0$, meaning that Firm f in Sector i does not default if and only if $A_f > A_i^*$, where $\Pi_{fi}(\mathscr{A}, A_i^*) = 0$. This represents the critical threshold for default decisions.

Using firms' optimiality conditions, Equation 11 is modified as follows:

$$A_i^* = \alpha_i E(A_f | A_f > A_i^*) - (1-\alpha_i)A_i. \tag{D.7}$$

Definition 12. *For a given $\mathscr{A}$, an equilibrium is an allocation of $\{C_i\}_{i=1}^N$ and $\{\{x_{ij}\}_{i=1}^N\}_{j=1}^N$ and a sequence of $\{P_i, A_i^*, \delta_i, \{p_{ij}\}_{j=1}^N\}_{i=1}^N$ such that:*

(i) Given $\mathscr{A}$ and $\{P_i, \{p_{ij}\}_{j=1}^N\}_{i=1}^N$, $\{\{x_{ij}\}_{i=1}^N\}_{j=1}^N$ and $\{A_i^*\}_{i=1}^N$ satisfy Firms' problem in Equations D.2 to D.4;

(ii) Given $\mathscr{A}$ and $\{A_i^*, \delta_i, \{p_{ij}\}_{j=1}^N\}_{i=1}^N$, $\{\{x_{ij}\}_{i=1}^N\}_{j=1}^N$, $\{C_i\}_{i=1}^N$, and $\{P_i\}_{i=1}^N$ satisfy Sectors' problem in Equations 2 and 14;

(iii) Given $\mathscr{A}$ and $\{P_i\}_{i=1}^N$, $\{C_i\}_{i=1}^N$ satisfy Households' problem in Equations 7 to 9;

(iv) $\{\delta_i\}_{i=1}^N$ satisfy Equation 13.

Definition 12 demonstrates that the equilibrium allocation, prices, recovery rate, and default threshold become functions of the aggregate state variable. Because there is no *aggregate* uncertainty in purchasing intermediate inputs and production, these equilibrium variables are independent of other realizations of aggregate state.

The subsequent proposition demonstrates how the default threshold responds to sectoral shocks centred around zero. It establishes a sufficient condition for a negative response, shedding light on the relationship between sectoral shocks and the default threshold.

Assumption 1. *G has a hazard rate such that $h(A) < \frac{1}{A(1-\alpha_i)}$.*

Proposition 5. *For the economy described in Section II with sectoral shock A_i, the default threshold response to the sectoral shock around 0 is*

$$\frac{\partial A_i^*}{\partial A_i}\Big|_{A_i=0} = \frac{\alpha_i - 1}{1 + A_i^* h(A_i^*)(\alpha_i - 1)}, \tag{D.8}$$

and $\frac{\partial A_i^*}{\partial A_i}\Big|_{A_i=0} < 0$ *if Assumption 1 holds.*

Lemma 7. *For the economy described in Section II with sectoral shock* A_i*, the industry TFP response to the default threshold is*

$$\frac{\partial E(A|A_i^*,\chi)}{\partial A_i^*} = -g(A_i^*)A_i^*\chi. \tag{D.9}$$

Lemma 7 states that when there is a positive default cost, an increase in the default threshold leads to a reduction in sectoral TFP.

Corollary 6. *For the economy described in Section II with sectoral shock* A_i*, the industry TFP response to the sectoral shock around 0 is*

$$\frac{\partial E(A|A_i^*,\chi)}{\partial A_i}\Big|_{A_i=0} = \frac{(1-G(A_i^*))\chi}{\frac{1}{h(A_i^*)\,A_i^*(1-\alpha_i)}-1} + (1-\chi\,G(A_i^*)). \tag{D.10}$$

If Assumption 1 holds and $G(A) < h(A)A(1-\alpha_i)$ *holds,* $\frac{\partial E(A|A_i^*,\chi)}{\partial A_i}\Big|_{A_i=0} > 1$.

Corollary 6 describes the response of sectoral TFP to a sectoral shock, considering the presence of financial frictions and a positive default cost. This corollary demonstrates that the response of sectoral TFP to the shock is different from 1 due to the influence of financial frictions.

Specifically, when a sector experiences severe financial frictions characterized by a high default rate, the corollary suggests that the sector's response to productivity shocks may be stronger. In other words, significant financial frictions can amplify the impact of productivity shocks on sectoral TFP, leading to larger and more volatile fluctuations in productivity. [46]

Appendix E Empirical Analysis

E.A Data

In this section, I describe the datasets used for the empirical analysis of Limited Liability Influence (LLI) in an input-output economy with labour, as outlined in Appendix C.

I utilize two datasets to quantify LLI: the U.S.A. input-output use tables from 1997 to 2021 and the Annual Fundamental Compustate dataset from 1998 to 2022. The U.S.A. input-output data comprises 66 sectors, where I merged the "Housing" and "Other real estate" sectors into one, resulting in a total of 65 sectors for empirical analysis.

The Annual Fundamental Compustate dataset is extensive. Within this dataset, a wealth of detailed information is available to capture various financial and operational performance aspects. 120,411 observations have information regarding interest payments, total debt, and debt structure from 1998 to 2022. Table E1 summarizes the sector composition of these observations.

[46] As an example, Assumption 1 and $G(A_i^*) < h(A)A(1-\alpha_i)$ holds for $G \sim U[0.5, 1, 5]$ at $A_i^* < 1(\ G(A_i^*) < 0.5)$

Category / Sector	Share (%)
Mining	**10.95**
Mining, except oil and gas	4.93
Oil and gas extraction	4.69
Manufacturing	**42.39**
Chemical products	11.7
Computer and electronic products	9.26
Trade and Transportation	**7.2**
Other retail	2.09
Water transportation	1.09
Services	**31.5**
Housing and other real estate	4.53
Data processing, internet publishing, and other information services	4.01
Other	**7.96**
Food services and drinking places	1.56
Ambulatory health care service	1.44

Table E1: Distribution of Categories and Sectors in Compustate Data, 1998-2022

Within this dataset, I extract sector-specific risk premiums for each year through a multi-step process. Initially, I calculate the credit spread for each data point. Subsequently, I compute the median credit spread for sector-year combinations, but this calculation is executed only if a sector has more than four observations for a given year. Notably, 57 sectors consistently maintain this threshold of four or more observations yearly.

However, addressing instances where specific sector-level risk premium data is unavailable for particular years is essential. In such cases, I adopt a pragmatic approach by substituting missing data with the average risk premium for that specific year. To provide context, the sectors "Construction", "Wholesale trade," "Management of companies and enterprises," and "Educational services" had no observations for the entire study duration; thus, I applied the annual average as a replacement. Similarly, sectors "Social assistance", "Legal services," "Transit and ground passenger transportation," and "Warehousing and storage" had no observations in specific years. For those particular years, I utilized the annual average as a surrogate.

To determine the credit spread for an individual firm, I utilize two key variables: the total interest and related expenses and the firm's total debt. This information allows me to derive the interest rate paid by the firm, as expressed by the formula:

$$\text{interest rate}_{fit} = \frac{\text{total interest and related expenses}_{fit}}{\text{total debt}_{fit}}.$$

Recognizing that the risk-free rate varies based on debt maturity, I also construct a risk-free rate that aligns with the maturity of a firm's debt. To achieve this, I calculate the share of short-term debt relative to the total debt (SD_{fit}) for that specific firm in that particular year. I then employ U.S.A. Treasury bond yields with one-year maturity (i_1) and U.S.A. Treasury bond yields with five-year maturity (i_5) for long-term debt to determine the risk-free rate:

$$\text{i}_{fit} = SD_{fit}\, i_{1t} + (1 - SD_{fit})\, i_{5t}.$$

Consequently, i_{fit} represents the risk-free interest rate that Firm f in Sector i during the year t could secure for external funding if it faced no risk of default. The discrepancy between the actual interest rate paid (interest rate$_{fit}$) and the risk-free rate constitutes the credit spread, which I utilize as a proxy for the risk premium in my analysis.

Additionally, my study incorporates variables such as "Sale" and "Cost of goods and services" (COGS). COGS accounts for all costs directly attributed to production, including materials, labour, and overhead expenses. To compute the sale-to-COGS ratio, I considered observations with positive COGS values and trimmed the top one percent of sales data. This ratio is then calculated for a dataset comprising 202,558 observations from 1998 to 2022.

Descriptive statistics

Variable	mean	median	Std. Dev.	Skewness	P10	P90	P99
Sale	1882.16	164.03	5393.92	5.08	3.62	4404.546	30744
Cogs	1211.429	85.72	3684.80	5.78	2.56	2767	20067.63
Sale to Cogs	3.15	1.59	37.23	152.19	1.04	4.41	20.75

Table E2: Descriptive Statistics of Variables for G Estimation . 202,558 observations from 1998 to 2022. The data for the top 1% of sales and negative sales and Cogs has been trimmed. The sectors of "Federal Reserve banks, credit intermediation, and related activities", "Computer and electronic products," and "Chemical products" stand out with the highest share of observations, accounting for 10.4%, 8.7%, and 8.7% of the dataset, respectively

Table E2 presents the descriptive statistics for the variables sale, COGS, and the sale-to-COGS ratio. It is important to note that all these variables exhibit a right-skewed distribution, with a pronounced long tail on the right side, particularly evident in the case of the sale-to-COGS ratio. This skewness indicates that the data tends to have higher values concentrated toward the lower end of the scale, with a few extreme values contributing to the elongated right tail.

E.B Calibration and estimation

The input-output tables provide critical sector-specific information for key variables, encompassing β, α, $\mathbf{\Delta}$, $\tilde{\mathbf{\Delta}}$, ζ, and Λ. To accurately adjust $\mathbf{\Delta}$ to reflect a closed economy, I implement the methodology proposed by Antràs et al (2012). In this approach, $\frac{y_{ji}}{Y_i}$ is replaced by $\frac{y_{ji}}{Y_i - \text{export}_i + \text{import}_i}$, thereby accounting for export and import dynamics

The parameters γ and σ represent the elasticity of substitution for intermediate inputs and consumption goods, respectively. Baqaee and Farhi (2019), Baqaee and Farhi (2020), and Atalay (2017) employ $\gamma = 0.1$ and $\sigma = 0.9$ in their sectoral input-output models. In the robustness section, I assess the sensitivity of the results to these parameters, exploring how sector complementarity affects welfare losses.

Estimating default costs is a delicate process, susceptible to measurement methods and sample variations. Previous studies have offered diverse estimates. For example, Glover (2016) found default costs to be 45%

of firm value. In contrast, Bris et al (2006), analyzing mostly smaller nonpublic firms from 1995 to 2001, discovered that bankruptcy fees exceeded the entire estate in 68% of Chapter 7 cases. Korteweg (2010) estimated default costs to range from 15% to 30% of firm value, while Altman (1984) estimated it at 16.7%. Besley et al (2020), examining U.K. firm data, estimated that credit frictions led to an average annual output loss of around 28%. I set $\chi = 0.15$ as the deadweight cost of default in the benchmark model. I will also conduct robustness tests to assess the sensitivity of the results to this parameter.

To estimate G based on the model's assumptions, I utilize sales and COGS data from the Annual Fundamental Compustate dataset. According to the model, the firm's sale-to-COGS ratio variations within a sector are influenced by idiosyncratic productivity shocks, the share of labour in the production function (β), and the default threshold. Assuming that the sale data is reported before incurring the deadweight loss of default, we can infer the idiosyncratic productivity distribution from this ratio. I define additive sectoral and aggregate productivity shocks for each sector and year, following the model described in Section VIII.B. This leads to the equation:

$$\frac{Sale_{fit}}{Cogs_{fit}} = \left(\frac{A_f + A_{it} + A_t}{A^*_{it}}\right)^{\frac{1}{1-\beta_i}}.$$

Taking the expectation over all firms in a specific sector and year, we obtain:

$$E\left(\frac{Sale_{fit}}{Cogs_{fit}}\right)^{1-\beta_i} = \frac{E(A) + A_{it} + A_t}{A^*_{it}}.$$

This equation helps us derive the following relationship:

$$\left(\frac{Sale_{fit}}{Cogs_{fit}}\right)^{1-\beta_i} - E\left(\frac{Sale_{fit}}{Cogs_{fit}}\right)^{1-\beta_i} = \frac{A_f - E(A)}{A^*_{it}}.$$

To estimate G, I aim to fit a beta distribution with lower bound $\underline{A}$ and upper bound $\overline{A}$. The beta distribution is suitable for modelling bounded distributions. Given that the beta distribution is constrained between zero and one, I apply a linear transformation to fit the data.

If we express A_f as $A_f = \underline{A} + \overline{A}\, x$, where $x \sim \beta(SP_1, SP_2)$, we need to estimate the parameters $\{\underline{A}, \overline{A}, SP_1, SP_2\}$. Since $E(A) = 1$ in the model, we can relate one parameter to the others using the equation:

$$\underline{A} + \overline{A}\frac{SP_1}{SP_1 + SP_2} = 1.$$

This allows us to derive $\overline{A}$ when we know the other three parameters:

$$\left(\frac{Sale_{fit}}{Cogs_{fit}}\right)^{1-\beta_i} - E\left(\frac{Sale_{fit}}{Cogs_{fit}}\right)^{1-\beta_i} = \frac{A_f - E(A)}{A^*_{it}}.$$

For a given $\underline{A}$, we have:

$$\min \left(\left(\frac{Sale_{fit}}{Cogs_{fit}} \right)^{1-\beta_i} - E\left(\frac{Sale_{fit}}{Cogs_{fit}} \right)^{1-\beta_i} \right) = \frac{\underline{A} - E(A)}{A^*_{it}}.$$

Using the fact that $E(A) = 1$, we can derive A^*_{it} for all sectors and years given a specific $\underline{A}$. To ensure that the lowest observed derivative of the sale-to-COGS ratio from the mean corresponds to $\underline{A}$, there should be a minimum of 10 observations in Sector i in Year t.

To estimate $\underline{A}$, SP_1, and SP_2 for the model described in Appendix C, I fix $\underline{A}$ and derive A^*_{it} and the distributions of A_f. I fit a beta distribution using the maximum likelihood method to obtain SP_1 and SP_2. At the optimal $\underline{A}$, the model should generate default thresholds close to $\{\{A^*_i\}_{i=1}^N\}_{t=1998}^{2021}$.

To find the optimal $\underline{A}$, I fit RP_{it} to the model and derive $\{\{\tilde{A^*}_i\}_{i=1}^N\}_{t=1998}^{2021}$ from:

$$RP_{it} = \left(1 + G(\tilde{A^*}_{it}) + G(\tilde{A^*}_{it})\, \delta_{it}(\tilde{A^*}_{it}, G, \chi)\right)^{-1}.$$

Now, I use the estimated default thresholds, $\{\{\tilde{A^*}_i\}_{i=1}^N\}_{t=1998}^{2021}$, to estimate the required $\tilde{\alpha}_{it}$ that satisfies Equation E.1 for the estimated $\tilde{G}(\cdot)$ and $\{\{\tilde{A^*}_i\}_{i=1}^N\}_{t=1998}^{2021}$:

$$\tilde{A}^*_{it} = \left(\frac{\tilde{\alpha}_{it}}{1-\beta_{it}} E(A^{\frac{1}{1-\beta_i}} | A > \tilde{A}^*_{it})\right)^{1-\beta_i}. \tag{E.1}$$

Finally, I select $\underline{A}$ in a way that minimizes the cost function Q:

$$Q(\underline{A}) = \sum_{t=1998}^{2021} \sum_{i=1}^{N} (\alpha_{it} - \tilde{\alpha}_{it})^2.$$

Calibration			**Estimation**	
Parameter	Value	Source	Parameter	Value
α_{it}		US input-output table	$\underline{A}$	0.66
β_{it}		US input-output table	$\overline{A}$	2.91
ζ_{it}		US input-output table	SP_1	2.54
Λ_{it}		US input-output table	SP_2	19.26
$\mathbf{\Delta}_t$		US input-output table		
$\tilde{\mathbf{\Delta}}_t$		US input-output table		
RP_{it}		Compustat data set		
σ	0.9	Baqaee and Farhi (2019)		
γ	0.1	Baqaee and Farhi (2019)		

Table E3: Parameter Calibration and Estimation. Input-output data and Compustate data sets from 1998-2021 were used to estimate G.

The summary of the calibration and estimation process is presented in Table E3. Figure E1 displays the distribution of idiosyncratic shocks in the data alongside the distribution estimated using $\tilde{G}$. It is evident that

the estimated distribution effectively captures the appropriate skewness and the heavy-tailed characteristics of the data.

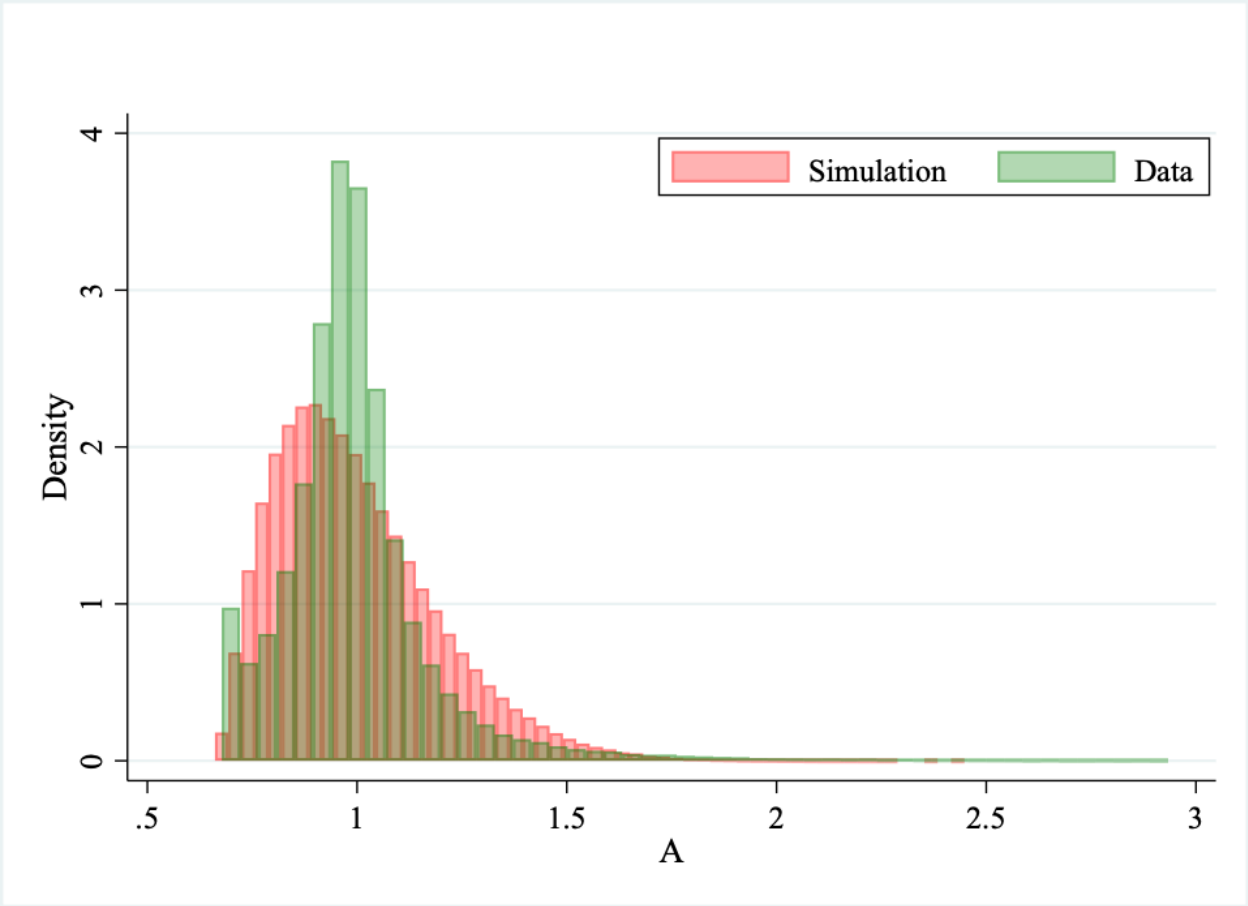


Fig. E1: The Idiosyncratic Productivity Shock Distribution in the Data and Simulation

E.C Empirical Results & Robustness tests

Now that we have all the parameters estimated or calibrated, we can compute LLI_{it}, which represents the first-order approximation of consumption loss, and analyze labour share changes for 1998-2021.

Figure E2 presents the time series of consumption loss alongside the risk premium. Notably, a strong negative correlation exists of -0.6964 (with a p-value of 0.00015) between the risk premium and consumption loss. A higher default rate exacerbates the moral hazard problem stemming from limited liability, leading to increased over-ordering and excessive default, distorting the economy to a greater extent. In 2012, the maximum consumption loss reached 2.97% of aggregate consumption, coinciding with the highest risk premium and default rate, reflecting the most pronounced distortion due to limited liability. Conversely, the minimum consumption loss occurred in 2006, at 1.59%, when the risk premium was low.

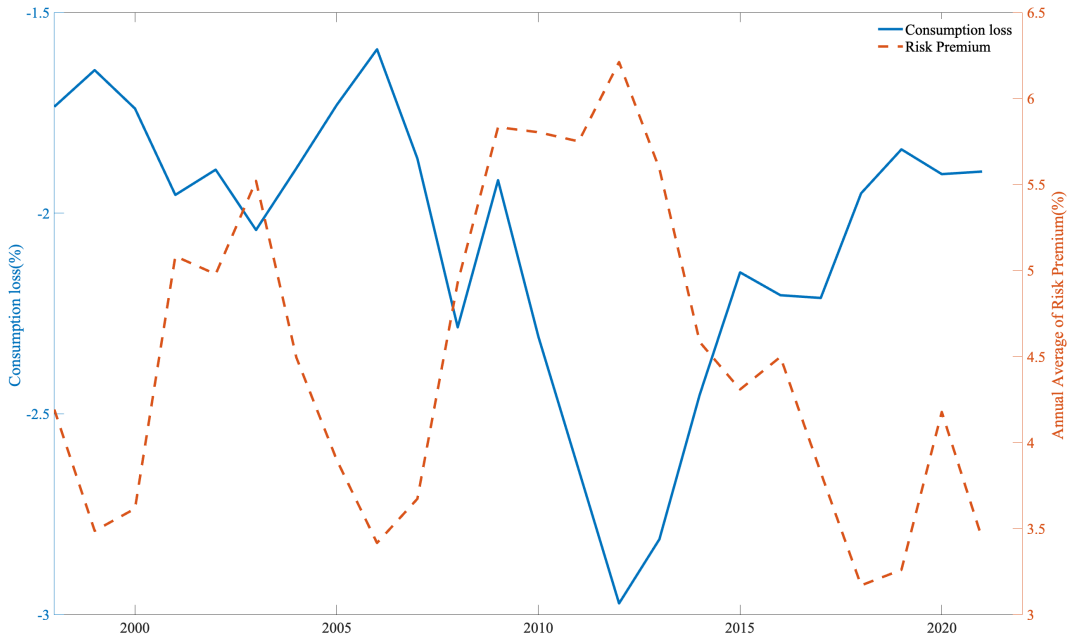


Fig. E2: Consumption Loss and Annual Average Risk Premium

The most distorting sectors have the highest Γ, causing a higher than socially optimal demand from upstream sectors. "Ambulatory health care services", "Educational services Publishing industries, except internet (includes software)", "Support activities for mining", and "Miscellaneous professional, scientific, and technical services" are among the sectors with the highest Γ.

The distortion caused by limited liability affects the economy through two main channels: over-ordering and the deadweight loss of default. The latter tends to diminish sectoral TFP, leading to an overall reduction in the production of intermediate inputs and final goods. This subsequently drives up prices across the board, be it for intermediate inputs or consumer goods. As a result, the deadweight loss of default does not substantially impact sectoral sales.

Conversely, sectoral sales deviations are primarily influenced by over-ordering, with upstream sectors experiencing the most pronounced effects. This dynamic occurs as resources are redirected towards upstream sectors to bolster the production of intermediate inputs for the economy, ultimately leading to increased sales in these sectors. Therefore, Upstreamness centrality, which solely considers a sector's position in the supply chain, is a more effective indicator for tracking sectoral sales deviations.

For instance, a uniform default rate across sectors may reduce real GDP but would not impact sectoral sales significantly, as relative prices remain unchanged. However, uniform changes in Γ across sectors disproportionately affect upstream sectors, leading to more substantial sales increases in upstream than downstream sectors.

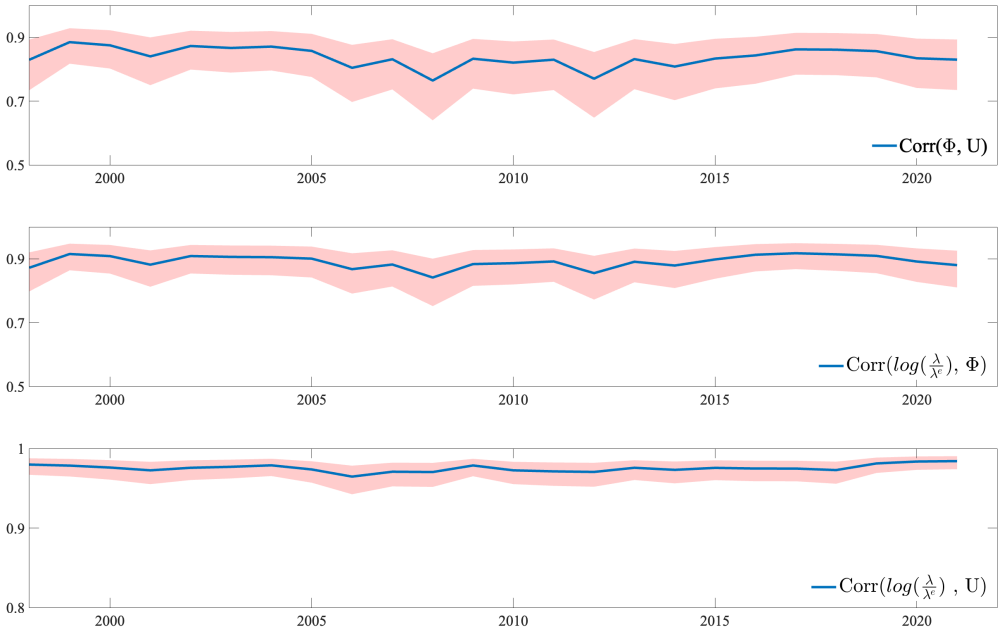


Fig. E3: Correlation Between log $\left(\frac{\lambda}{\lambda^e}\right)$, U, and Φ in the U.S.A. Economy (1998-2021) with 95% Confidence Interval.

Figure E3 visually depicts the interconnectedness of sectoral sales deviations from the efficient economy, LLI, and Upstreamness in the context of limited liability. It illustrates the correlation among sectoral sales (Domar weight) deviations from efficient sector sales, LLI, and Upstreamness centrality in the U.S.A. economy over time. In the first panel of Figure E3, the strong positive correlation between LLI and Upstreamness is evident, primarily due to their shared reliance on the over-ordering mechanism.

The second and third panels underscore how LLI and Upstreamness significantly correlate with sectoral sales deviations from the efficient economy, primarily driven by the over-ordering mechanism. Since a portion of LLI is shaped by the output loss caused by the deadweight loss of default, which has a limited impact on sectoral sales, Upstreamness centrality exhibits a stronger correlation with sectoral sales deviations than LLI.

Table E4 assesses the sensitivity of the results to changes in χ. As discussed earlier, an increase in χ leads to a decrease in the recovery rate and an increase in the risk premium, making the market response to the moral hazard problem resulting from limited liability more pronounced and causing a reduction in Γ (i.e., $\frac{\mathrm{d}\Gamma}{\mathrm{d}\chi} < 0$).

	$\hat{C}(\%)$	$Corr(\hat{C}, RP)$	$Corr(\Phi, U)$	$Corr(\Phi, \log(\frac{\lambda}{\lambda^e}))$	$Corr(\log(\frac{\lambda}{\lambda^e}), U)$
$\chi = 0$	-2.22	-0.66	0.855	0.912	0.974
$\chi = 0.15$	-2.07	-0.69	0.837	0.892	0.975
$\chi = 0.3$	-3.18	-0.93	0.61	0.713	0.965

Table E4: Robustness Test: Sensitivity of Results to Deadweight Loss of Default.

The market's response to the limited liability friction weakens in the first row of Table E4. Even though the excess default channel is turned off, the influence of the over-ordering channel strengthens, resulting in overall increased distortions in the economy.

The last row of Table E4 highlights that the recovery rate decreases for a substantial value of χ, and the risk premium increases at a given default rate. Here, the excess default channel becomes the dominant mechanism, weakening the over-ordering effect as the risk premium rises, bringing Γ closer to one.

Column 3 of Table E4 demonstrates that as χ increases and its role in LLI becomes more prominent, the risk premium and consumption loss become more closely correlated.

Furthermore, Column 4 of Table E4 illustrates that as the significance of χ in LLI grows, the correlation between upstreamness and LLI diminishes since upstreamness only has information regarding sector location in the supply chain and over-ordering channel.

A higher contribution of excess default to LLI results in lower sectoral TFP but does not significantly impact misallocation, especially if default rates are similar across sectors. Consequently, the predictive capacity of LLI for sectoral sales deviations diminishes, as indicated in Column 4 of the table.

Lastly, the last column of Table E4 underscores that regardless of the χ value, Upstreamness centrality consistently emerges as the superior predictor of sectoral sales deviations from the efficient economy.

Panel A: robustness test with respect to σ					
	$\hat{C}(\%)$	$Corr(\hat{C}, RP)$	$Corr(\Phi, U)$	$Corr(\Phi, \log(\frac{\lambda}{\lambda^e}))$	$Corr(\log(\frac{\lambda}{\lambda^e}), U)$
$\sigma = 0.7$	-2.06	-0.69	0.835	0.895	0.976
$\sigma = 0.9$	-2.07	-0.69	0.837	0.892	0.975
$\sigma = 1.1$	-2.07	-0.69	0.84	0.888	0.974
Panel B: robustness test with respect to γ					
$\gamma = 0.01$	-2.11	-0.677	0.837	0.895	0.973
$\gamma = 0.1$	-2.07	-0.69	0.837	0.892	0.975
$\gamma = 0.5$	-1.91	-0.7612	0.84	0.875	0.9834

Table E5: Robustness Test: Sensitivity of Results to Elasticity of Substitution.

Table E5 presents the results' robustness concerning the substitution elasticity. Given that the estimation focuses on the first-order impact of limited liability on consumption loss, the influence of elasticity values remains minimal.

As highlighted by Baqaee and Farhi (2019), it is worth noting that the significance of elasticity values becomes more pronounced when elasticities are very different from one and severe shock happens. Then, we need to move beyond the first-order approximation.

The variation in elasticity of substitution among final goods does not affect the consumption loss. This occurs because downstream sectors tend to over-order intermediate inputs and consequently face higher intermediate input prices. This phenomenon applies universally to all downstream sectors, rendering the degree of substitution between these sectors relatively insignificant. Therefore, limited liability effectively

acts as a cost-push inflation, increasing the cost of intermediate inputs. In this context, the substitutability between sectors has a limited impact.

In contrast, the elasticity of substitution between different sectoral goods used to produce the aggregate intermediate input denoted as x_{fit} becomes a significant factor. Higher values of γ imply that upstream sectors cannot increase their prices as extensively as they can with lower γ. Greater elasticity fosters higher competition, resulting in less substantial price increases for intermediate inputs due to over-ordering. Consequently, this leads to more moderate increases in the prices of final goods. As over-ordering influences the demand for intermediate inputs through Γ, γ plays a crucial role, whereas σ has a relatively muted effect.

Appendix F Figures and tables

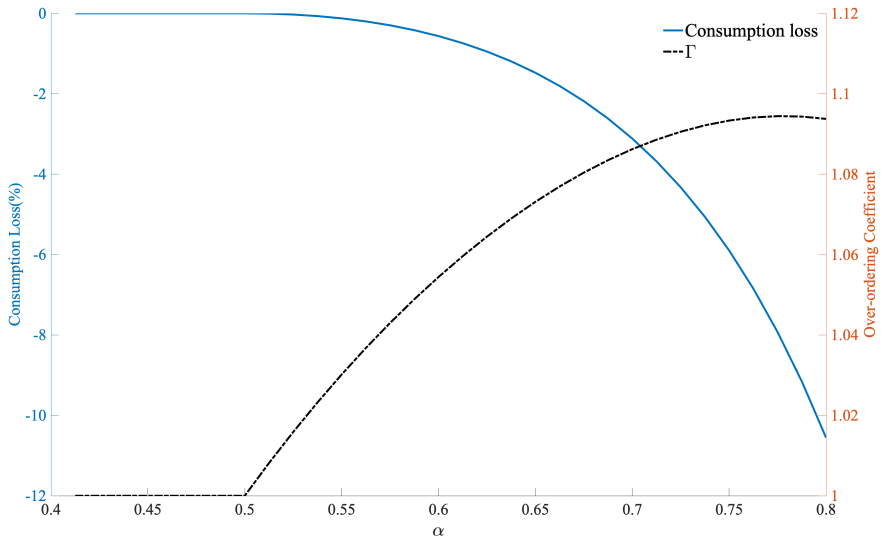


Fig. F4: Consumption Loss Due to the Over-ordering Mechanism in a Symmetric K-Suppliers Network: $A \sim U(0.5, 1.5)$, $\chi = 0$. As α changes, the over-ordering coefficient endogenously varies. An increase in the over-ordering coefficient or α increases the demand for intermediate inputs, increasing the importance of input trades. Consequently, the misallocation of intermediate inputs becomes costlier. For $\alpha < 0.5$, firms can repay their debts for any realization of idiosyncratic shocks. Thus, there is no default risk and over-ordering mechanism.

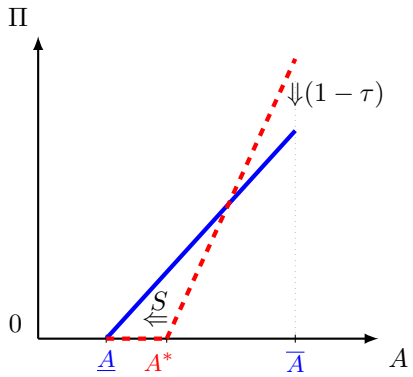


Fig. F5: Industrial Policy for Achieving Efficiency in Allocation.