Market Dynamics of Risk-On and Risk-Off Incentives and their Effect on Asset Prices

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Abstract

Mutual fund asset managers' contracts create a dynamic, time-varying objective: initially, there is a strong drive to outperform the index. However, as performance improves, the focus shifts from outperforming to maintaining performance. Predictability arises from both the predictable changes in incentive contracts and the predictability of performance uncertainty. Uncertainty peaks when asset managers' performance converges to the benchmark performance but diminishes as performance diverges away from the benchmark. The equilibrium, expressed in closed form, reveals that these two predictable channels lead to momentum for both index and non-index assets when outperformance is more likely but otherwise results in a reversal.

1 Introduction

Institutional investors hold a significant proportion of publicly traded stocks and account for an even larger share of daily trading volume. Consequently, their actions play a crucial role in determining stock prices, and understanding the incentives behind these actions is essential for comprehending price dynamics. In March 2005, the U.S. Securities and Exchange Commission (SEC) implemented a rule requiring mutual funds to disclose the compensation structure of their portfolio managers in the Statement of Additional Information. These

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disclosures indicate that nearly all mutual fund managers' contracts include a performance-based bonus tied to the funds' performance relative to their benchmark index.

This paper develops a dynamic equilibrium model with benchmarking incentives that incorporates a critical element the literature has for the most part overlooked: the incentives to outperform the benchmark (risk-on) are powerful when asset managers underperform in a bid to get the bonus. However, as performance exceeds the benchmark, the incentives to outperform the benchmark become less relevant, and instead, asset managers' incentives shift to maintaining their performance (risk-off) to avoid losing the bonus.

Our main contribution is to show that momentum and reversal arise endogenously in equilibrium with a complete information environment and perfectly rational asset managers who face risk-on, risk-off incentives. The paper defines *momentum* as the tendency for expected returns (and future returns) to increase when current returns are rising. Conversely, it defines *reversal* as the tendency for expected returns (and future returns) to decrease when current returns are falling.

Given their time-varying incentives, asset managers' payoffs are convex when they underperform the benchmark but become concave as their returns exceed the benchmark. This results in an S-shaped objective, where the inflexion (or reference) point is the benchmark. Despite this added complexity, we characterize equilibrium prices in closed form.

The S-shaped objective function and its pricing implications are novel in asset management. It was originally designed to explain financial decisions of households that appeared at odds with the predictions of standard utility models. Friedman and Savage (1948) demonstrate that for an agent to both purchase insurance and a lottery ticket, the agent's utility must shift from concave at certain wealth levels to convex at others. Levy (1969) demonstrates that an objective function that only depends on the first three moments of the wealth's distribution leads to an inverted S-shape objective, concave below a reference and convex above it. Kahneman and Tversky (1979) show that an S-shape objective arises when risk-aversion is different towards losses and gains –so that the reference point is current wealth. The key distinction between these models and ours is that in our model the benchmark, and not the value of the assets under management, is the reference point. This difference significantly enhances the tractability of the equilibrium characterization.

Generally, pricing predictability is surprising because conventional wisdom suggests that if market participants expect prices to rise, they will quickly buy the asset to capture the

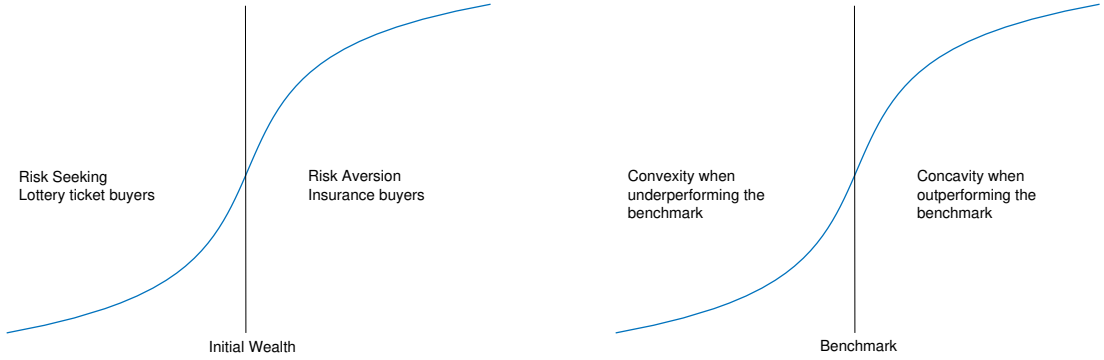


Figure 1. The figure on the left shows that an agent prefers a fair gamble (such as purchasing a lottery ticket) when their utility is convex with respect to absolute wealth. Conversely, when their utility is concave with respect to wealth, the agent prefers to pay a premium to avoid a fair gamble. The figure on the right demonstrates a similar intuition for asset managers. They are incentivized to gamble when their wealth is below the benchmark and they are underperforming. In contrast, they are incentivized to purchase insurance and maintain their wealth when it is above the benchmark.

return, causing an immediate price increase that neutralizes the expected return. Similarly, if they expect prices to fall, they will sell the asset immediately to avoid losses, causing an immediate price drop that neutralizes the expected decline in return. Consequently, the main explanations for momentum and reversal involve either an asymmetric information environment leading to partial price adjustments or irrational investor behavior. In these scenarios, momentum and reversal occur as participants gradually adjust their positions based on their observations of the market.

The return predictability analyzed in this paper is fully rational and stems directly from the no-arbitrage conditions, where all available information is fully reflected in prices at any given time. Predictability persists in this environment because it is not optimal for asset managers to trade on it. In this setting, asset managers respond immediately to new information, but their incentives may cause them to either overreact or underreact. These reactions shape return predictability patterns through equilibrium, resulting in patterns that emerge endogenously from the time-varying incentives faced by asset managers.

There are two equilibrium predictable channels of asset mangers' performance. The first channel involves *benchmarking incentives*: the asset manager's performance predicts

their benchmarking incentives. When performance is low, the incentives are strong, whereas when performance is high, the incentives are weak. The second equilibrium channel is the *uncertainty of performance*: When performance diverges from the benchmark, performance is more ceratin. Conversly, when performance converges to the benchmark, it becomes less certain whether the asset manager will perform. These two predictable channels lead to an asymmetric price reaction to news during periods of underperformance and overperformance.

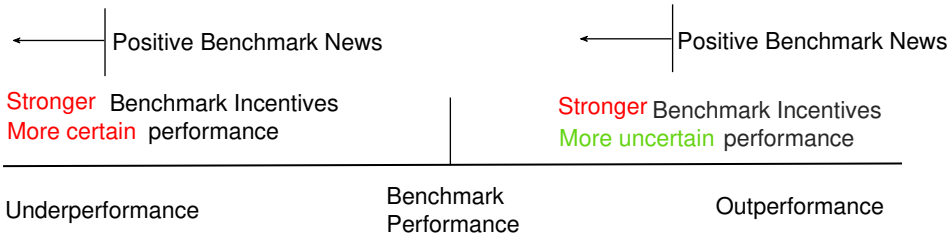


Figure 2. This figure illustrates the asymmetric reaction to news during periods of underperformance and overperformance. The left side of the figure represents scenarios where asset managers underperform, while the right side cases of overperformance. In both cases, positive benchmark news negatively impacts the performance of asset managers. For those outperforming, it reduces their overperformance, while for those underperforming, it exacerbates their underperformance. Lower performance leads to stronger benchmarking incentives. In contrast, the uncertainty of performance increases for overperforming asset managers but decreases for those underperforming.

Stronger benchmark incentives lead to a reversal in benchmark returns. In contrast, increased performance uncertainty leads to momentum in benchmark returns, while a decreased uncertainty causes a reversal. When asset managers underperform, the two predictability channels align, resulting in an eventual reversal in benchmark returns. In contrast, when asset managers outperform the benchmark, the two channels work in opposite directions. Due to the stronger predictability in performance uncertainty, the overall effect leads to eventual momentum. The logic is as follows.

The benchmark incentives prompt asset managers to hold a benchmark hedging position to avoid falling behind it. This hedging creates demand to hold the benchmark, unrelated to risk and return, leading to higher prices and lower returns compared to an otherwise equivalent asset. An unpredictable positive shock to the benchmark raises its price initially

due to the extra demand but subsequently lowers future prices due to decreased expected returns, ultimately inducing a reversal.

Performance uncertainty causes the benchmark to overreact to news. As asset managers' performance diverges from the benchmark, performance becomes more certain, and the overreaction diminishes. In contrast, as asset managers' performance converges with the benchmark, performance becomes less certain, and the overreaction becomes more pronounced. This channel have opposing effects on prices depending on whether asset managers outperform or underperform the benchmark.

When asset managers outperform the benchmark, an unpredictable positive shock to the benchmark initially raises its price due to an overreaction to news. This shock also increases future prices as it induces an even greater overreaction in the future due to heightened uncertainty, boosting expected returns and ultimately creating momentum.

Conversely, when asset managers underperform the benchmark, an unpredictable positive shock to the benchmark initially raises its price due to an overreaction to news. However, this shock decreases future prices because it induces a smaller overreaction in the future due to the reduced uncertainty, dropping expected returns and ultimately creating reversal.

The remainder of the paper is organized as follows. Section 2 summarizes the related literature; Section 3 sets up the economy with the time-varying incentives contract; Section 4 solves for the equilibrium in closed-form; Section 5 analyzes the assets' expected returns; Section 6 discusses momentum and reversal; Section 7 discusses additional implications, and Section 8 concludes.

2 Related Literature

This paper fits into the asset pricing literature investigating stock returns predictable patterns, and in particular, the drivers of momentum and reversal in stock returns. Jegadeesh and Titman (1993) first documented momentum and reversal in the cross section of the stock market, and Chan, Jegadeesh, and Lakonishok (1996) have thoroughly investigated the phenomenon. They showed that a portfolio that buys past winners and sells past losers will generate a significant positive return over a short-term window of one year or less, and that the strategy's performance partially reverts for longer horizons.

There are several competing theories, falling into a few broad categories, which explain

momentum. One of the most prevalent explanations of momentum is behavioural. In this category, Barberis, Shleifer, and Vishny (1998) studies an agent that due to behavioural biases incorrectly forms beliefs about stock earnings. Daniel, Hirshleifer, and Subrahmanyam (1998) study investors that overreact to confirming evidence and underreact to refuting evidence about their prior signal. Hong and Stein (1999) study a two agents model where agents have different restrictions on the way they process information. Barberis and Shleifer (2003) study an economy with two investors. One of these investors groups assets into styles and gradually moves funds from the worst performing style to the best performing style, while the second investor does not act on the predictable pattern. Lastly, Hillert, Jacobs, and Müller (2014) find that news coverage exacerbate momentum because investors' biases are stronger.

The most prevalent rational explanation is based on an information economy. In Holden and Subrahmanyam (2002), the setup induces a sequential information acquisition that feeds into prices. Allen, Morris, and Shin (2006) assumes that agents, instead of forecasting prices, forecast future forecasts. Shin (2006) endogenizes firms' disclosure decision jointly with prices. Banerjee, Kaniel, and Kremer (2009) investigate a heterogenous beliefs economy, where agents agree to disagree about their beliefs. Biais, Bossaerts, and Spatt (2010) investigates an overlapping generation setup with informed and uninformed traders. Cespa and Vives (2012) compares consensus opinion and prices and show that specific patterns of information generates momentum and reversal. Albuquerque and Miao (2014) shows that information about future earnings that is independent from current earning induces momentum and reversal. Ottaviani and Sørensen (2015) investigate a heterogenous beliefs economy where traders exhibit wealth constraints. Andrei and Cujean (2017) show that short-term momentum can be generated by an economy where private information flows at an increasing rate. Cujean and Hasler (2017) investigate an economy with heterogenous agents where some agents continuously update their beliefs while others do so intermittently.

Another strand of literature connects momentum to firms' specific attributes, such as growth options, dividend growth rate, revenues, and costs. Berk, Green, and Naik (1999), Johnson (2002), and Sagi and Seasholes (2007), do so in a partial equilibrium setup. Relatedly, Liu and Zhang (2008) identifies the loading on the growth rate of industrial production as the source for momentum, and Liu and Zhang (2014) study a partial equilibrium production based setup to study momentum.

More recent studies connect the asset management industry to momentum and reversal. This strand of literature is the most relevant to this paper. Dasgupta, Prat, and Verardo (2011) assume that investors observe asset managers' actions and form beliefs about their ability to perform. Asset managers internalize the reputational cost of underperformance, which causes a herding behavior that positively predicts short-term returns. Vayanos and Woolley (2013) suggest that a slow-moving flow between active and passive funds due to active funds' performance is the source of momentum and reversal.

Our paper is the first to show that a predictable pattern in asset managers' risk preferences generates momentum and reversal and that the predictable changes to asset managers' risk preferences arise due to the time-varying nature of their incentive contracts. In particular, the incentives to outperform the benchmark are stronger for asset managers who underperform. However, as performance improves, the incentives to outperform the benchmark become less relevant, and instead, asset managers' incentives shift to maintain their performance in a bid not to lose the bonus.

Furthermore, our article contributes to the expanding body of literature examining the impact of professional asset management on asset prices. In particular, our research aligns with a strand of this literature that investigates the asset pricing implications resulting from benchmarking incentives.

The paper most closely related to ours is Sotes-Paladino and Zapatero (2019). Their setup introduces a kink to the performance base fees of the asset manager, which changes the sensitivity of the objective to the relative (to the benchmark) wealth. Similar to our setup, the kink represents a shift in the performance sensitivity between the end-of-period underperformance and outperformance regions. While the kink introduces a convexity to the asset manager's objective, the slope at the outperformance region could be concave, which aligns with our setup. They find that the asset manager overinvests in assets outside the benchmark and underinvests in the benchmark, which aligns with our predictions when asset managers are likely to outperform (the risk-off region). The main difference from our paper is that they investigate the asset manager's behavior, given prices, in a partial equilibrium setup, while our setup focuses on the pricing implications in an equilibrium setup.

By and large, the asset management literature investigates a setup where asset managers' objective is always convex and does not become concave when outperformance is likely in a bid to maintain performance. The idea to embed the benchmark into the asset

manager's objective started by Brennan (1993) and Gómez and Zapatero (2003). Cuoco and Kaniel (2011) were the first to study the equilibrium implications of benchmarking incentives through performance-based fees. Later, Basak and Pavlova (2013) and Basak and Pavlova (2016) introduce a reduced form approach to incorporate the benchmark, which allows for much tractability. Buffa and Hodor (2023) extended their setup to investigate the pricing implications of heterogenous benchmarking incentives and Hodor and Zapatero (2023) to investigate the pricing implications of heterogenous horizons.

Unlike the literature, we investigate the pricing implications when the asset manager's benchmarking incentives are time-varying. The asset manager exhibits incentives to outperform the benchmark when underperformance is likely (similar to the literature), but these incentives weaken, and eventually flips when outperformance is more likely, in a bid to maintain performance (instead of outperforming the benchmark).

Lastly, our paper relates to Friedman and Savage (1948) early work. They investigated a case whereby a wealthy individual is willing to pay an insurance premium to avoid a fair gamble with an equal chance of winning or losing a dollar, while a poor individual is willing to purchase a lottery ticket to participate in such a fair gamble. They introduced the utility characteristics that jointly induce these two behaviors in a rational expected utility maximizing framework. Rationalizing these behaviors implies that when the individual wealth is low, the agent's marginal utility increases with wealth, and the utility is convex. In contrast, when individual wealth is high, the agent's marginal utility decreases with wealth, and utility becomes concave, eventually leading to an S-shape utility function: a convex segment followed by a concave segment.

We incorporate Friedman and Savage (1948) main insights into an asset management framework with relative wealth concerns. In particular, when wealth is expected to fall below the index, asset managers behave similarly to poor individuals and face risk-on incentives (purchasing lottery tickets): convex incentives that increase their marginal utility of wealth the further the wealth falls below the index. In contrast, when wealth is expected to surpass the index, asset managers behave similarly to wealthy individuals and face risk-off incentives (purchasing insurance): concave incentives that reduce their marginal utility of wealth the further the wealth increases above the index.

3 The Economy

This section lays out a simple and tractable model to study the joint equilibrium effect of risk-on and risk-off incentives of asset managers. We consider a simple and tractable standard pure-exchange finite horizon economy in which time t is continuous and goes from zero to T. The economy is populated by two types of investors: passive investors, $\mathcal{P}$, and asset managers, $\mathcal{A}$. While the economy could be generalized and accommodate multiple assets and different specifications of the index against which the asset managers' performance is evaluated, critical aspects of the mechanism can be analyzed with three assets. Accordingly, uncertainty is driven by a (3)-dimensional Brownian motion $\mathbf{Z} = (Z_1, Z_2, Z_3)'$.

3.1 Investment Opportunities

The are 3 risky assets and a single riskless bond in the economy. We set the bond as numeraire and normalize its interest rate to zero. The risky asset is denoted by $S_{kt}, k = 1, 2, 3$, and represents a claim on the dividends D_{kT} arriving at time T. We assume each risky asset is in unit supply and posit that it follows

$$dS_{kt} = S_{kt}\left(\mu_{kt}^S dt + \boldsymbol{\sigma}_{kt}^{S\prime} d\boldsymbol{Z}_t\right). \tag{1}$$

Prices (S_{kt}), the (instantaneous) vector of expected returns $\boldsymbol{\mu}_t^S \equiv (\mu_{1t}^S, \mu_{2t}^S, \mu_{3t}^S)'$, and the (instantaneous) volatility matrix $\boldsymbol{\Sigma}_t^S \equiv \left(\boldsymbol{\sigma}_{1t}^S, \boldsymbol{\sigma}_{2t}^S, \boldsymbol{\sigma}_{3t}^S\right)'$ are endogenous and determined in equilibrium. The bold symbols represent vectors and the $\prime$ represents the transpose throughout the analysis.

The dividends (D_{kT}) are determined by the dynamics of the processes

$$dD_{kt} = D_{kt}\left(\mu_k dt + \boldsymbol{\sigma}_k^{D\prime} d\boldsymbol{Z}_t\right), \quad D_{k0} > 0, \tag{2}$$

which we refer to as dividend news. The parameter μ_k and the vector $\boldsymbol{\sigma}_k^D$ are constants for the first two dividends. We follow Menzly, Santos, and Veronesi (2004), Basak and Pavlova (2013), and Buffa and Hodor (2023) and leave the remaining dividend news process unspecified, and, instead, we specify an aggregate process as soon follows. The procedure substantially improves the tractability of the model and provides closed-form precise equi-

librium characterization.

We assume that stocks' fundamentals are independent, implying that the vector $\boldsymbol{\sigma}_k^D$ (2) has a positive kth element equaling $\sigma_k > 0$, while the remaining entries are zero. This assumption provides clear interpretation to the type of shocks that affect prices. To simplify the analysis, we assume that the distributional properties of news are the same, $\sigma_1 = \sigma_2 = \sigma_3$ and $\mu_1 = \mu_2 = \mu_3$.

3.2 The Index

For the baseline analysis we assume that the first asset is the index.

$$S_t^I \equiv S_{1t}. \tag{3}$$

The asset manager's performance is evaluated against the performance of this index and the passive investor invests its entire wealth in it. The index does not include the remaining assets in the economy because we are interested in analyzing the asset managers' holdings within and outside its mandate.

For consistency of the analysis, we refer to I_t as the *index news*, which in the baseline economic setup equals the news about asset 1,

$$I_t \equiv D_{1t}, \quad \boldsymbol{\sigma}^I \equiv \boldsymbol{\sigma}_1^D. \tag{4}$$

3.3 Investors

The passive investors ($\mathcal{P}$) are endowed with $(1-\lambda)$ units of the index at time 0

$$W_0^{\mathcal{P}} = (1-\lambda)\, S_0^I \tag{5}$$

and remain passive until time T. The asset managers ($\mathcal{A}$) are endowed with the remaining (λ) units of the index and holds the entire supply of the other assets,

$$W_0^{\mathcal{A}} = \lambda S_0^I + S_{20} + S_{30} \equiv S_0^{\mathcal{A}} \tag{6}$$

The asset managers' initial endowment identifies the active section of the asset market, which is a claim on the total asset market dividends minus the passive index holdings' dividends,

$$V_T \equiv I_{1T} + D_{2T} + D_{3T} - (1 - \lambda) I_{1T}, \tag{7}$$

where we assume that this terminal value is determined by the process

$$dV_t = V_t (\eta dt + \boldsymbol{\nu}' d\boldsymbol{Z}_t), \quad V_0 > 0. \tag{8}$$

The parameter η is positive and the vector $\boldsymbol{\nu} \equiv (\nu_1, \nu_2, \nu_3)'$ has positive entries. We assume that $(\nu_1, \nu_2, \nu_3) = (\lambda\sigma_1/3, \sigma_2/3, \sigma_3/3)$ to mimic the aggregate active dividends and following the standard in the literature. We refer to V_t as the *market news*.

3.4 Risk-On and Risk-Off Incentives

Empirical evidence indicates that individuals sometimes choose to pay an insurance premium to avoid a fair gamble, while at other times they prefer to take the gamble. At first glance, these behaviors appear inconsistent with the expected utility maximization theory, which would require a rational agent to pay for insurance in some situations and buy a lottery ticket in others.

Friedman and Savage (1948) addressed this puzzle by introducing utility characteristics that explain these two behaviors within a rational, expected utility-maximizing framework. They demonstrated that rationalizing these behaviors means that, at certain wealth levels, an agent's marginal utility increases with wealth, while at other wealth levels, the marginal utility decreases.

The empirical evidence on asset managers and their benchmarking incentives aligns with the behaviors of individual agents as described by Friedman and Savage (1948). Specifically, when wealth is expected to fall below the index, asset managers act like individuals willing to take a fair gamble, facing risk-on incentives. These convex incentives increase their marginal utility of wealth as it falls further below the index. Conversely, when wealth is expected to exceed the index, asset managers behave like individuals willing to purchase insurance to avoid a fair gamble, facing risk-off incentives. These concave incentives reduce their marginal utility of wealth as it rises above the index.

Crucially, the arguments and insights of Friedman and Savage (1948) apply to asset managers' risk-on and risk-off incentives because the key factor for prices is the marginal utility of wealth. Changes in the index impact prices solely through their effect on the marginal utility of wealth.

Therefore, Friedman and Savage (1948)'s arguments in this context suggest that asset managers' marginal utility of wealth *increases* with the index level when their wealth is below the index, ($U_{WS^I} > 0$), indicating a stronger desire to increase wealth when they are underperforming. Conversely, when wealth is above the index, their marginal utility of wealth *decreases*, ($U_{WS^I} < 0$), indicating a weaker desire to increase wealth when they are outperforming the benchmark. Any changes in the index level that do not affect the marginal utility of wealth have no implications for prices. Figure 3 illustrates this argument.

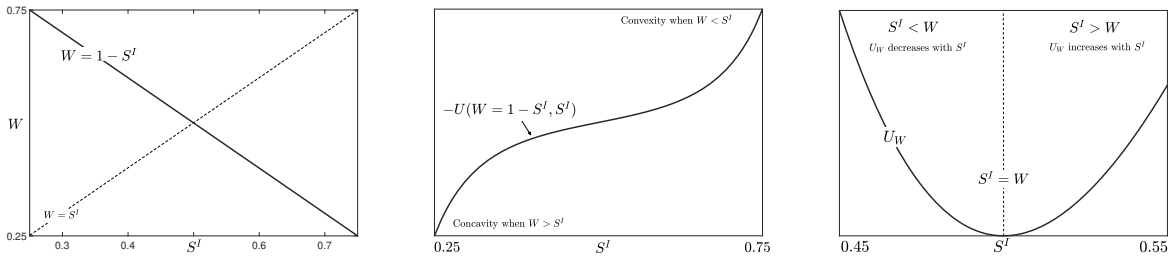


Figure 3. The left figure plots the $S^I - W$ plane. At the top left triangle, the asset managers have concave incentives because $W > S^I$, while at the bottom right triangle, the asset managers have convex incentives because $W < S^I$, where the solid linear line represents $W = -S^I + 1$. As we trace that line from the top left corner to the bottom right corner, incentives shift from concave to convex incentives. Indeed, the middle figure shows that the cross derivative U_{WS^I} is concave initially and becomes convex as the line crosses the dashed line. The cross derivative (U_{WS^I}) in the direction of the solid line (in the left figure) can be captured by the rate of change in $U_S(W, S^I)$ in the direction $(-1, 0)$, leading to a negative cross derivative: $-U_{WS^I}(W, S^I)$. Therefore, we include a minus sign in front of U to capture U_{WS^I}. The right figure shows what the cross derivative captures. When $W < S^I$ (right side of the figure), the marginal utility of wealth increases when S^I increases, indicating the asset manager desire to outperform. In contrast, when $W > S^I$ (left side of the figure), the marginal utility of wealth decreases when S^I increases, indicating the asset manager desire to maintain performance. The function U represents the asset managers' objective (11) with $\alpha_1 = 0.5$ and $\alpha_2 = 4.5$.

While many potential objective functions may satisfy the requirements for risk-on and

risk-off incentives. The goal is to introduce an objective that switches between risk-on and risk-off incentives based on the state of asset managers' wealth relative to the index without losing tractability.

To do so, we start with a variation of Basak and Pavlova (2013)s' asset manager objective function, and introduce the *risk-on objective*

$$-E\left[\frac{\left(S_T^I\right)^{1-\gamma_1}\left(W_T^{\mathcal{A}}\right)^{1-\gamma_2}}{\left(\gamma_2-1\right)\left(1-\gamma_1\right)}\right], \quad \gamma_2>1>\gamma_1>0. \tag{9}$$

Similar to the original objective function of Basak and Pavlova (2013), the risk-on objective (9) is always convex, $U_{WS^I}=\left(S_T^I\right)^{-\gamma_1}\left(W_T^{\mathcal{A}}\right)^{-\gamma_2}>0$. The parameter of risk aversion in this case is γ_2.

If we alternate the wealth and the index in the risk-on objective (9) and remove the minus sign, we attain a *risk-off objective*,

$$E\left[\frac{\left(W_T^{\mathcal{A}}\right)^{1-\gamma_1}\left(S_T^I\right)^{1-\gamma_2}}{\left(\gamma_2-1\right)\left(1-\gamma_1\right)}\right], \quad \gamma_2>1>\gamma_1>0, \tag{10}$$

which is always concave, $U_{WS^I}=-\left(S_T^I\right)^{-\gamma_2}\left(W_T^{\mathcal{A}}\right)^{-\gamma_1}<0$. The parameter of risk aversion in this case is γ_1.

Perhaps more importantly, by combining the risk-on (9) and risk-off (10) incentives we attain an objective function that switches between risk-on and risk-off depending on whether the wealth is below or above the index.

$$U\left(W^{\mathcal{A}},S^I\right)\equiv E\left[\mathbf{1}_{\text{Off}}\frac{\left(S_T^I\right)^{1-\gamma_2}\left(W_T^{\mathcal{A}}\right)^{1-\gamma_1}}{\left(\gamma_2-1\right)\left(1-\gamma_1\right)}-\mathbf{1}_{\text{On}}\frac{\left(S_T^I\right)^{1-\gamma_1}\left(W_T^{\mathcal{A}}\right)^{1-\gamma_2}}{\left(\gamma_2-1\right)\left(1-\gamma_1\right)}\right], \quad \gamma_2>1>\gamma_1>0. \tag{11}$$

The indicators $\mathbf{1}_{\text{Off}},\mathbf{1}_{\text{On}}$ in (11) allow to separate the asset pricing implications of each incentive individually and their joint effect. A fully risk-on asset manager has only the second component in (11), $\mathbf{1}_{\text{Off}}=0,\mathbf{1}_{\text{On}}=1$, while a fully risk-off asset manager has only the second component in (11), $\mathbf{1}_{\text{Off}}=1,\mathbf{1}_{\text{On}}=0$. When taken together, $\mathbf{1}_{\text{Off}}=1,\mathbf{1}_{\text{On}}=1$, the risk-off component dominates when $S^I<W^{\mathcal{A}}$, while the risk-on component dominates when $S^I>W^{\mathcal{A}}$, and the asset manager has both risk-on and risk-off incentives, in line with Friedman and Savage (1948) main insights. The parameter of risk aversion in this case is

endogenous and time-varying.

Besides the risk-on and risk-off incentives, the asset manager objective function (11) satisfies other useful characteristics: it increases and concave in wealth ($U_W > 0, U_{WW} < 0$), and satisfies the inada conditions stating that the marginal utility of wealth (U_W) takes the value of ∞ as wealth approaches zero, and the value 0 as wealth approaches infinity. These conditions are at the core of expected utility maximization.

There are two variations to the original Basak and Pavlova (2013) objective function. First, we introduce a risk aversion parameter larger than one (γ_2) instead of the typical log utility of wealth, and second, the objective is a power function of the index (γ_1) instead of being linear. The are two reasons for these variations.

The symmetrical structure of the asset manager's objective function (11) is the primary characteristic, inducing the shift between risk-on and risk-off incentives. If we were to follow Basak and Pavlova (2013) log utility of wealth for the risk-on incentives, the symmetrical structure would require a log of the index for the risk-off incentives. This outcome would, in turn, violates the requirement for the increasing-in-wealth objective function ($U_W > 0$) because the index log becomes negative when the index level falls below one, and U_W would eventually turn negative. Similarly, if we were to follow Basak and Pavlova (2013)s' linearity in the index for risk-on incentives, the symmetrical structure would require linearity in wealth for the risk-off incentives. This outcome would, in turn, violates the requirement for a decreasing marginal utility in wealth, $U_{WW} < 0$. Due to these two reasons, we introduce the two variations to the original asset manager objective function, as laid out by Basak and Pavlova (2013).

Starting with their initial endowments, the asset managers dynamically choose a portfolio $\boldsymbol{\pi}_t^{\mathcal{A}}$, which represents the fraction of wealth invested in each of the risky assets. The wealth process of the asset managers, therefore, follows the dynamics

$$\frac{dW_t^{\mathcal{A}}}{W_t^{\mathcal{A}}} = \boldsymbol{\pi}_t^{\mathcal{A}\prime} \left(\boldsymbol{\mu}_{St} dt + \boldsymbol{\Sigma}_t^S d\boldsymbol{Z}_t \right), \quad t \leq T. \tag{12}$$

4 The Equilibrium

We define the equilibrium in a standard way: equilibrium prices and portfolio holdings are such that (i) the asset managers choose their optimal portfolio for given prices, and (ii)

stocks, the bond, and consumption-good markets clear.

Asset managers' risk-on and risk-off objectives each is a constant relative risk aversion type on their own (11). However, when combined, the relative risk aversion becomes a weighted average of the two risk aversion coefficients (γ_1, γ_2). These weights fluctuate over time, depending on the probability of the asset managers to outperform.

Asset managers' risk-on and risk-off objectives are of a constant relative risk aversion type individually. However, when taken together, the relative risk aversion of the asset managers' objective (11) is a weighted average of the two risk aversion coefficients (γ_1, γ_2), with time-varying weights that depend on the likelihood of the asset managers to outperform or underperform. We denote the relative risk aversion by $RA(W, S^I)$. By applying the definition of relative risk aversion to the asset managers' objective function, we find that:

$$RA(W, S^I) = \gamma_1 \underbrace{\left(1 - \frac{1}{1 + \frac{(1-\gamma_1)}{(\gamma_2 - 1)} \left(W/S^I\right)^{(\gamma_2 - \gamma_1)}}\right)}_{\equiv 1-w} + \gamma_2 \underbrace{\left(\frac{1}{1 + \frac{(1-\gamma_1)}{(\gamma_2 - 1)} \left(W/S^I\right)^{(\gamma_2 - \gamma_1)}}\right)}_{\equiv w}, \quad (13)$$

where $w \in (0, 1)$ since $0 < \gamma_1 < 1 < \gamma_2$. The formulation shows that risk aversion, $RA(W, S^I)$, is endogenous, time-varying, and dependent on the ratio of the asset managers' wealth to the index. When the asset managers' wealth is low relative to the index, they are in the risk-on region since $W < S^I$. In this case, the risk aversion converges to the risk-aversion coefficient in the risk-on region γ_2 since the weight on γ_1 approaches zero. As their wealth increases relative to the index, the weight on γ_1 rises, and the weight on γ_2 falls, resulting in an overall decreasing relative risk aversion objective function. The risk aversion decreases as asset managers becomes wealthier.

Since market clearing requires asset managers to hold the entire active supply, the index price must equal its dividends at maturity, and in equilibrium, $RA(W, S^I) = RA(V, I)$. Interestingly, the state variable (V/I) that determines the variations in the risk aversion parameter also dictates how asset managers hedge against shifts between the risk-off and the risk-on regions, ultimately determining the equilibrium quantities.

Before we start the equilibrium analysis we first define the asset managers' *risk-on hedging*

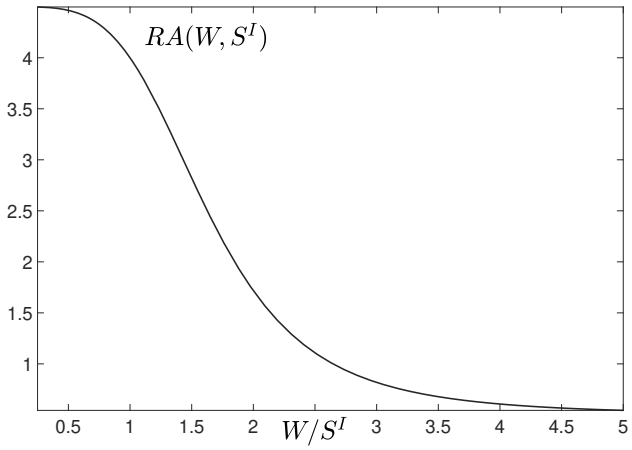


Figure 4. This figure plots the asset managers' risk aversion as a function of W/S^I. It converges to γ_2, the risk aversion coefficient of the risk-on region when $W/S^I \downarrow 0$, and to γ_1, the risk aversion coefficient of the risk-off region when $W/S^I \uparrow \infty$, resulting in a decreasing relative risk aversion utility.

demand, $\mathcal{H}_{it}$. In the joint incentives case,

$$\mathcal{H}_{it} \equiv \frac{1}{1 + \frac{1-\gamma_1}{\gamma_2-1} \left(V_t/I_t\right)^{\gamma_2-\gamma_1} \mathcal{E}_{it}}, \quad i = 1, 2, 3, 4, \tag{14}$$

where i refer to four different determinstic functions of time, $\mathcal{E}_{1t}$, $\mathcal{E}_{2t}$, $\mathcal{E}_{3t}$, $\mathcal{E}_{4t}$, which are given in (A.3), (A.6), (A.9), (A.12), respectively. We refer to $\mathcal{H}_{it}$ as the risk-on hedging demand because the closer it is to one, the more correlated the equilibrium quantities of the joint incentives case are with those of the purely risk-on incentives case. The deterministic functions $\mathcal{E}_{it}$ determines the speeds at which the hedging demand changes with the wealth-index ratio, $\partial\mathcal{H}_i/\partial(V/I)$. Thus, we refer to $\mathcal{H}_{it}$ for $i = 1, 2, 3, 4$ as the risk-on hedging demand. The same ratio of V/I governs the risk-on hedging demand ($\mathcal{H}_{it}$) and also the equilibrium risk aversion ($RA(V, I)$), which in turn determines all the equilibrium quantities. In fact, it is the only state variable that determines equilibrium.

We begin by analyzing the partial equilibrium effect of how shocks impact asset managers' wealth, given fixed prices. This approach allows us to identify the different shock propagation channels for risk-on and risk-off incentives, both individually and jointly. To facilitate this,

let us denote the (equilibrium) discount factor by ξ_t and its dynamics by

$$d\xi_t = -\xi_t \left(\boldsymbol{\theta}_t' d\boldsymbol{Z}_t \right). \tag{15}$$

By definition of the discount factor, the market price of risk vector process $\boldsymbol{\theta}_t$ can be represented as

$$\boldsymbol{\theta}_t = \left(\boldsymbol{\Sigma}_t^S \right)^{-1} \boldsymbol{\mu}_t^S, \tag{16}$$

which in a single asset economy simplifies to the ratio of an asset's (excess) expected return and its standard deviation.

Lemma 1 (**Risk Exposure**). *The asset managers' risk exposure due to risk-on incentives is given by*

$$\boldsymbol{\Sigma}_t^{S\prime} \boldsymbol{\pi}_t^{\mathcal{A}} = \boldsymbol{\theta}_t - (\gamma_2 - 1)\, \boldsymbol{\nu} + (1 - \gamma_1)\, \boldsymbol{\sigma}^I, \tag{17}$$

while the risk exposure due to risk-off incentives is given by

$$\boldsymbol{\Sigma}_t^{S\prime} \boldsymbol{\pi}_t^{\mathcal{A}} = \boldsymbol{\theta}_t + (1 - \gamma_1)\, \boldsymbol{\nu} - (\gamma_2 - 1)\, \boldsymbol{\sigma}^I. \tag{18}$$

In the presence of both risk-on and risk-off incentives the risk exposure is given by

$$\boldsymbol{\Sigma}^{S\prime} \boldsymbol{\pi}_t^{\mathcal{A}} = \boldsymbol{\theta}_t + (1 - \gamma_1)\, \boldsymbol{\nu} - (\gamma_2 - 1)\, \boldsymbol{\sigma}^I + (\gamma_2 - \gamma_1)\, \mathcal{H}_{2t} \left(\boldsymbol{\sigma}^I - \boldsymbol{\nu} \right). \tag{19}$$

The function $\mathcal{H}_{2t}$ is defined in (14).

There are three shock propagation channels. The first is the myopic mean-variance channel, where asset managers seek a risk exposure proportional to the assets' risk-return tradeoff, correlating their wealth with the market price of risk, θ_t. Lemma 1 demonstrates that this channel remains unaffected regardless of the type of incentives. The exact exposure will be determined in equilibrium once the market price of risk is identified.

The second channel is a risk-aversion adjustment channel, and Lemma 1 indicates that it has different implications for risk-on and risk-off incentives. For risk-on incentives (17), asset managers desire more wealth in bad economic states than in good economic states since the risk aversion coefficient is greater than one $\gamma_2 > 1$. The logic reverses for risk-off

incentives. For risk-off incentives (18), asset managers desire more wealth in good economic states than in bad economic states since the risk aversion coefficient is less than one $\gamma_1 < 1$. Overall, the risk-aversion adjustment identifies that outperforming fund managers (risk-off incentives) hedge against positive market news, while underperforming fund managers (risk-on incentives) hedge against negative market news.

The third channel is benchmark hedging. Similar to risk aversion adjustment channel, it has opposing implications for risk-on and risk-off incentives. When asset managers trail the index and exhibit risk-on incentives (17), they align their wealth with the index to protect it from further decline below the index. In contrast, when asset managers outperform the index and exhibit risk-off incentives (18), they stop protecting their portfolio from falling behind the benchmark. To maintain performance, they leverage assets with favorable risk-return tradeoffs.

The joint effect (risk-on and risk-off) is a time-varying weighted average of the two individual effects, with the weights controlled by the risk-on hedging demand, $\mathcal{H}_{2t}$. As asset managers' wealth decreases relative to the index ($\mathcal{H}_{2t}$ increases), they sell a portion of their risk-off portfolios and buy a portion of their risk-on portfolios. The portion increases as their wealth falls further below the index, and $\mathcal{H}_{2t}$ approaches one.

We proceed by uncovering the closed-form expressions for the equilibrium quantities, beginning with the market prices of risk.

Proposition 1 (**Market Prices of Risk**). *The market prices of risk in the presence of risk-on incentives is given by*

$$\boldsymbol{\theta}_t = -\left(1-\gamma_1\right)\boldsymbol{\sigma}^I + \gamma_2\boldsymbol{\nu}, \tag{20}$$

while in the presence of risk-off incentives, it is given by

$$\boldsymbol{\theta}_t = \left(\gamma_2-1\right)\boldsymbol{\sigma}^I + \gamma_1\boldsymbol{\nu}. \tag{21}$$

In the presence of both risk-on and risk-off incentives the market price of risk is given by

$$\boldsymbol{\theta}_t = \left(\gamma_2-1\right)\boldsymbol{\sigma}^I + \gamma_1\boldsymbol{\nu} + \left(\gamma_2-\gamma_1\right)\left(\boldsymbol{\nu}-\boldsymbol{\sigma}^I\right)\left(\mathcal{H}_{1t}\right). \tag{22}$$

The function $\mathcal{H}_{1t}$ is defined in (14).

The market prices of risk with pure risk-on incentives (20) align with findings documented in the literature over the past decade: the market price of risk decreases with benchmarking motives. Asset managers seek to buy the index to hedge against unexpected increases in the benchmark, creating additional demand for index assets. Equilibrium is restored by making these assets less attractive from a risk-return tradeoff perspective, thereby reducing their market prices of risk.

The logic reverses for pure risk-off incentives (21). Risk-off asset managers aim to maintain performance and use the funds in their benchmark hedging portfolio to invest in assets with attractive risk-return tradeoffs. This introduces an extra supply for the index. Equilibrium is restored by enhancing the risk-return tradeoff of the index, leading to a higher market price of risk.

The joint incentives case (22) is a time-varying weighted average of the two individual effects, with $\mathcal{H}_{1t}$ determining the weights. The greater the outperformance, the closer the market price of risk aligns with the risk-off incentive case (21). Conversely, the more significant the underperformance, the closer it aligns with the risk-on incentive case (20).

Consequently, the market price of risk is *cyclical* with asset managers' performance: as their relative performance improves, market prices of risk increase. Previous literature has demonstrated that benchmarking reduces the market price of risk for benchmarked assets. This paper refines that statement by asserting that both the benchmarking motives and underperforming the benchmark reduce the market price of risk for benchmaked assets. However, when asset managers outperform, this prediction reverses.

We proceed by characterizing the risk exposures.

Proposition 2 (**Volatility**). *The return volatilities in the presence of either pure risk-on incentives or pure risk-off incentives are given by*

$$\boldsymbol{\sigma}_{1t}^{S} = \boldsymbol{\sigma}_{1t}^{D}, \qquad \boldsymbol{\sigma}_{2t}^{S} = \boldsymbol{\sigma}_{2t}^{D}. \tag{23}$$

The return risk exposures in the presence of joint risk-on and risk-off incentives are given by

$$\boldsymbol{\sigma}_{1t}^{S} = \boldsymbol{\sigma}_{1}^{D} + (\gamma_2 - \gamma_1)\left(\boldsymbol{\sigma}^{I} - \boldsymbol{\nu}\right)\left(\mathcal{H}_{3t} - \mathcal{H}_{1t}\right), \tag{24}$$

$$\boldsymbol{\sigma}_{2t}^{S} = \boldsymbol{\sigma}_{2}^{D} - (\gamma_2 - \gamma_1)\left(\boldsymbol{\sigma}^{I} - \boldsymbol{\nu}\right)\left(\mathcal{H}_{1t} - \mathcal{H}_{4t}\right). \tag{25}$$

where $\mathcal{H}_{1t}$, $\mathcal{H}_{3t}$, $\mathcal{H}_{4t}$ are given in (14). Positive index news increases the index price, while positive non-index news decreases the index price:

$$\boldsymbol{\sigma}^S_{1t}(1) > 0,\ \boldsymbol{\sigma}^S_{1t}(2) < 0,\ \boldsymbol{\sigma}^S_{1t}(3) < 0. \tag{26}$$

In contrast, positive index news decreases the non-index price, while positive non-index news increases the non-index price:

$$\boldsymbol{\sigma}^S_{2t}(1) < 0,\ \boldsymbol{\sigma}^S_{2t}(2) > 0,\ \boldsymbol{\sigma}^S_{2t}(3) > 0. \tag{27}$$

In a traditional endowment economy without benchmark incentives, prices neither overreact nor underreact to news. In such environments, price immediately adjusts to reflect the arrival of new information. Since optimal portfolios remain unchanged by the arrival of news, the equilibrium does not change the exposure of prices to future news.

Proposition 2 reveals that pure incentives cause asset managers to respond to news differently than market participants in a traditional setup. However, similar to a traditional endowment economy, optimal portfolios (and risk exposures) remain unchanged by the arrival of news, meaning that asset managers' reactions to news are consistent across all states. Therefore, equilibrium does not alter the exposure of prices to future news. In contrast, in previous pure risk-on models, such as those by Basak and Pavlova (2013), and Buffa and Hodor (2023), asset managers' portfolios change with the arrival of news, indicating that equilibrium changes how the index price will react to future news. These earlier models suggest that an index stock overreacts to index news. In the pure risk-on (and risk-off) setups of this paper, this behavior does not occur.

The joint incentives case introduces both overreaction and underreaction to news. The index price overreacts to index news and underreacts to non-index news. Conversely, the non-index price overreacts to non-index news and underreacts to index news. This behavior is driven by asset managers' demand to hold the benchmark, which now depends on the likelihood of their eventual performance.

More explicitly, when index news arrives, the index price increases, making asset managers more likely to fall behind the benchmark and underperform. Anticipating this, asset managers aim to buy more of the benchmark as positive news arrives, increasing the demand for the index. Consequently, equilibrium causes the index price to overreact to index news.

To purchase more of the index, asset managers must sell other assets they hold, leading non-index assets to underreact to index news.

When non-index news arrives, the non-index price increases, boosting the overall market relative to the index. In this scenario, asset managers are more likely to outperform. Anticipating this, asset managers buy less of (or sell) the benchmark as positive non-index news arrives, decreasing the demand for the index. Consequently, equilibrium causes the index price to underreact to non-index news.

5 Expected Returns

In the presence of joint incentives, equilibrium forces are time-varying and may work in different directions, depending on the likelihood of asset managers' performance. This section demonstrates that expected returns summarize these varying forces and determine which effect prevails.

Given the representation of the market price of risk (16), we decompose expected returns into the market price of risk multiplied by the quantity of the associated risk,

$$\mu_{it}^S = \boldsymbol{\theta}_t' \boldsymbol{\sigma}_{it}^S, \quad i = 1, 2. \tag{28}$$

To illustrate the intuitions clearly, we rewrite the vector of market prices of risk, (22), as a sum of the *index news exposure* ($\boldsymbol{\sigma}^I$) and the *market news exposure* ($\boldsymbol{\nu}$),

$$\boldsymbol{\theta}_t = \left[(\gamma_2 - 1) - (\gamma_2 - \gamma_1)\left(\mathcal{H}_{1t}\right)\right] \boldsymbol{\sigma}^I + \left[\gamma_1 + (\gamma_2 - \gamma_1)\left(\mathcal{H}_{1t}\right)\right] \boldsymbol{\nu}. \tag{29}$$

The characterization shows that the market price of risk may react positively or negatively to index news, depending on whether asset managers outperform or underperform. The market price of risk decreases with positive index news when asset managers are more likely to underperform and the risk-on hedging $\mathcal{H}_{1t}$ exceeds the threshold (54). As asset managers' underperformance increases, risk-on incentives become more pronounced than risk-off incentives, leading asset managers to invest more in the benchmark. Equilibrium then depresses the market price of risk to make the index asset less attractive and more expensive, ultimately restoring balance.

Conversely, the market price of risk increases with positive index news when asset man-

agers are more likely to outperform, and $\mathcal{H}_{1t}$ is below the threshold (54). In this scenario, asset managers care less about the benchmark and offload some of their benchmark hedging portfolio. To restore equilibrium, the market price of risk for the index rises, making it cheaper and reducing the selling pressure.

The market price of risk (29) further indicates that positive market news consistently increases the market price of risk, accurately reflecting the asset market's risk-return tradeoff.

Similar to the market price of risk, we rewrite the vector of return exposures as a sum of index news exposure ($\boldsymbol{\sigma}^I$) and market news exposure ($\boldsymbol{\nu}$),

$$\boldsymbol{\sigma}_{1t}^S = \boldsymbol{\sigma}_1^D + (\gamma_2 - \gamma_1)(\mathcal{H}_{3t} - \mathcal{H}_{1t})\boldsymbol{\sigma}^I - (\gamma_2 - \gamma_1)(\mathcal{H}_{3t} - \mathcal{H}_{1t})\boldsymbol{\nu}, \tag{30}$$

$$\boldsymbol{\sigma}_{2t}^S = \boldsymbol{\sigma}_2^D - (\gamma_2 - \gamma_1)(\mathcal{H}_{1t} - \mathcal{H}_{4t})\boldsymbol{\sigma}^I + (\gamma_2 - \gamma_1)(\mathcal{H}_{1t} - \mathcal{H}_{4t})\boldsymbol{\nu}. \tag{31}$$

The representation reveals that the index price *overreacts to index news*, since

$$(\gamma_2 - \gamma_1)(\mathcal{H}_{3t} - \mathcal{H}_{1t}) > 0. \tag{32}$$

Positive index news increases the index price, suggesting that asset managers are more likely to underperform. Anticipating a higher likelihood of underperformance, asset managers buy more into the benchmark as good news arrive, indicating that the demand for the index rises with positive index news. Equilbrium is restored by reducing the expected return of the index asset.

The index price *underreacts to market news* for similar reasons. Positive market news lowers the index price relative to the non-index price, suggesting that asset managers are less likely to underperform. Anticipating a lower likelihood of underperformance, asset managers buy less of the benchmark asset as good market news arrives, indicating that the demand for the index decreases with positive market news. Equilbrium is restored by decreasing the expected return of the index asset.

The non-index price mirrors the index price. Positive index news lowers the non-index price as asset managers sell this asset to buy the index. Similarly, positive market news raises the non-index price because asset managers sell the benchmark and purchase this asset when they are more likely to outperform. Equilbrium is restored by the counter price movement.

The expected returns can be divided into two distinct components: a time-varying sign

component that alters the expected return sign between outperformance and underperformance, and a constant sign component that remains either positive or negative. Asset managers' performance dictates whether index news increases or decreases the market price of risk, but it never shifts the overreaction to underreaction, as evident from the return exposures. As a result, the time-varying sign component arises from the impact of index news on the market price of risk, whereas non-index news is part of the constant sign component.

The expected returns (28), which are given by the cross product between the market price of risk (29) and the return exposures (30) and (31), reveal that there are six propagation channels affecting it some belongs to the time-varying sign component and others to the constant component: (i) pure index news ($\sigma^I \times \sigma^I$) has a time-varying sign, whereas (ii) pure market news ($\nu \times \nu$) has a constant sign. Further, since market news co-moves with the index news, there are two additional cross terms: (iii) cross index and market news ($\sigma^I \times \nu$) has a time-varying sign, while (iv) cross market and index news ($\nu \times \sigma^I$) has a constant sign. Lastly, since the index asset belong to the index and the market we have two extra cross terms: (v) cross asset and index news ($\sigma^I \times \sigma_1^D$) that has a time-varying sign; and (vi) cross asset and market news ($\nu \times \sigma_1^D$) that has a constant sign. In sum, there are three time-varying sign components and three constant sign components for both assets.

News about the first asset are also news about the benchmark since the first asset is the benchmark. This suggests that the fifth channel aligns with the pure-index channel and the sixth channel aligns with the cross-index and market channel. Additionally, the non-index asset is not part of the benchmark, meaning the cross-asset and index news channel does not exist for the non-index asset. Consequently, we find that the index expected return is given by

$$\begin{aligned}
\mu_{1t}^S = &(\gamma_2 - \gamma_1)(\mathcal{H}_{3t} - \mathcal{H}_{1t})\left[(\gamma_2 - 1) - (\gamma_2 - \gamma_1)(\mathcal{H}_{1t})\right] \|\boldsymbol{\sigma}^I\|^2 \\
&- (\gamma_2 - \gamma_1)(\mathcal{H}_{3t} - \mathcal{H}_{1t})\left[\gamma_1 + (\gamma_2 - \gamma_1)(\mathcal{H}_{1t})\right] \|\boldsymbol{\nu}\|^2 \\
&- (\gamma_2 - \gamma_1)(\mathcal{H}_{3t} - \mathcal{H}_{1t})\left[(\gamma_2 - 1) - (\gamma_2 - \gamma_1)(\mathcal{H}_{1t})\right] \boldsymbol{\sigma}^{I'} \boldsymbol{\nu} \\
&+ (\gamma_2 - \gamma_1)(\mathcal{H}_{3t} - \mathcal{H}_{1t})\left[\gamma_1 + (\gamma_2 - \gamma_1)(\mathcal{H}_{1t})\right] \boldsymbol{\nu}' \boldsymbol{\sigma}^I \\
&+ \left[(\gamma_2 - 1) - (\gamma_2 - \gamma_1)(\mathcal{H}_{1t})\right] \boldsymbol{\sigma}^{I'} \boldsymbol{\sigma}_1^D \\
&+ \left[\gamma_1 + (\gamma_2 - \gamma_1)(\mathcal{H}_{1t})\right] \boldsymbol{\nu}' \boldsymbol{\sigma}_1^D, \qquad (33)
\end{aligned}$$

and the non-index expected return is given by

$$\begin{aligned}\mu_{2t}^{S} = &-(\gamma_2-\gamma_1)(\mathcal{H}_{1t}-\mathcal{H}_{4t})\left[(\gamma_2-1)-(\gamma_2-\gamma_1)(\mathcal{H}_{1t})\right]\|\boldsymbol{\sigma}^{I}\|^2\\ &+(\gamma_2-\gamma_1)(\mathcal{H}_{1t}-\mathcal{H}_{4t})\left[\gamma_1+(\gamma_2-\gamma_1)(\mathcal{H}_{1t})\right]\|\boldsymbol{\nu}\|^2\\ &+(\gamma_2-\gamma_1)(\mathcal{H}_{1t}-\mathcal{H}_{4t})\left[(\gamma_2-1)-(\gamma_2-\gamma_1)(\mathcal{H}_{1t})\right]\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\\ &-(\gamma_2-\gamma_1)(\mathcal{H}_{1t}-\mathcal{H}_{4t})\left[\gamma_1+(\gamma_2-\gamma_1)(\mathcal{H}_{1t})\right]\boldsymbol{\nu}'\boldsymbol{\sigma}^{I}\\ &+\left[\gamma_1+(\gamma_2-\gamma_1)(\mathcal{H}_{1t})\right]\boldsymbol{\nu}'\boldsymbol{\sigma}_2^{D}. \end{aligned} \tag{34}$$

The pure index news ($\|\sigma^I\|^2$) is a time-varying sign channel. Expected returns are negative during underperformance and positive during overperformance due to this channel. The index price consistently overreacts to index news, reflecting the higher likelihood of underperformance associated with such news. However, index news changes the sign of the market price of this risk. When asset managers underperform, the market price of risk becomes negative, reflecting the increased demand for hedging against lagging behind the index. Conversly, when asset managers outperform, the market price of risk becomes positive, reflecting the demand to maintain performance and offload the hedging instrument.

Pure market news ($\|\nu\|^2$) operates as a constant sign channel, resulting in consistently negative expected returns. The market price of risk consistently increases in response to positive market news to accurately reflect the assets' risk-return tradeoffs. However, the index price underreacts to positive market news because asset managers are more likely to outperform and reduce their benchmark loading. These two opposiing effects combine to produce an overall negative expected returns.

Cross-index and market news ($\sigma^I \times \nu$) function as a time-varying sign channel. Expected returns are positive during underperformance and negaitve during overperformance due to this channel. The index price consistently underreacts to market news, reflecting the lower likelihood of underperformance associated with such news. However, index news changes the sign of the market price of this risk from negative during underperformance to positive during overperformance. These two negative effects combine to produce an overall positive effect on expected returns during underperformance and a negative effect during outperformance.

Cross market and index news ($\nu \times \sigma^I$) function as a constant sign channel, resutling in consistently positive expected returns. The market price of risk consistently increases in response to positive market news to accurately reflect the assets' risk-return tradeoffs.

Similarly, the index price overreacts to positive index news because asset managers are more likely to underperform and increase their benchmark loading. These two positive effects combine to produce an overall positive expected returns.

Cross index and asset news ($\sigma^I \times \sigma_1^D$) operates much like pure index news ($\|\sigma^I\|^2$) beacause the first asset is the index. Expected returns are negative during periods of underperformance and positive during periods of overperformance through this channel. Similarly, cross market and asset news ($\nu \times \sigma_1^D$) operates much like cross-market and index news ($\nu \times \sigma^I$) beacause the first asset is the index. Expected returns are consistenly positive through this channel. However, in both cases the impact is smaller since the first asset accurately reflects news without factoring in the benchmark.

The logic reverses for the non-index asset in the first four channels. When the expected return of the non-index asset rises, the expected return of the index asset falls. This occurs because when the index price overreacts to index news, the non-index price underreacts to the same news, and vice versa. Cross-market and asset news ($\nu \times \sigma_2^D$) is similar for both index and non-index assets, as they react to news in the same manner without considering the benchmark. Lastly, ($\sigma^I \times \sigma_2^D$) does not exist because the second asset is not part of the benchmark. The following proposition summarizes our results.

Proposition 3 (**Expected Return**). *The expected returns for the index and non-index assets are given by (33) and (34), respectively. The expected return of the index in the case of extreme outperformance is strictly higher than in the case of extreme underperformance* ($\mu_{1t}^S \mid_{\mathcal{H}_{it}\downarrow 0} - \mu_{1t}^S \mid_{\mathcal{H}_{it}\uparrow 1} > 0$). *In contrast, the expected return of the non-index asset in extreme outperformance is strictly lower than in the case of extreme underperformance* ($\mu_{2t}^S \mid_{\mathcal{H}_{it}\downarrow 0} - \mu_{2t}^S \mid_{\mathcal{H}_{it}\uparrow 1} < 0$). *Furthermore, the index expected return*

- *decreases in outperformance if and if* $\underline{\gamma}_2 \leq \gamma_2$,
- *increases in underperformance if and if* $\bar{\gamma}_2 \geq \gamma_2$.

Conversly, the non-index expected return

- *increases in outperformance if and only if* $\underline{\underline{\gamma}}_2 \leq \gamma_2$,
- *decreases in underperformance if and only if* $\bar{\bar{\gamma}}_2 \geq \gamma_2$,

where $\underline{\gamma}_2$, $\bar{\gamma}_2$, $\underline{\underline{\gamma}}_2$, *and* $\bar{\bar{\gamma}}_2$ *are given in (A.65), (A.67), (A.70), and (A.71), respectively.*

Table 1 illustrates the six different channels for the index and non-index assets. The sum of the channels determines the expected return and whether it increases or decreases following the arrival of news.

Table 1. Expected Returns

This table illustrates how impact of six distinct channels on index and non-index expected returns: (i) pure index news ($\sigma^I \times \sigma^I$); (ii) pure market news ($\nu \times \nu$); (iii) cross index and market news ($\sigma^I \times \nu$); (iv) cross market and index news ($\nu \times \sigma^I$); (v) cross index and asset news ($\sigma^I \times \sigma_i^D$); (vi) cross market and asset news ($\nu \times \sigma_i^D$) affect the index and non-index expected returns. Each row represents a distinct channel. The θ_t column indicates whether index and market news results in a positive ($+$) or negative ($-$) market price of risk, for both underperformance (left table) and outperformance (right table). The σ_{it}^S column indicates the overreaction ($+$) and underreaction ($-$) to index and market news. The third column, μ_{it}^S, shows whether each channels results in a negative ($\downarrow$) or positive ($\uparrow$) expected return. Underperformance in the left table is indicated when the risk-on hedging demand ($\mathcal{H}_{1t}$) exceeds the threshold (54), while outperformance in the right table when the risk-on hedging demand ($\mathcal{H}_{1t}$) falls below the threshold.

		Underperformance					Outperformance				
		$\boldsymbol{\theta}_t$		$\boldsymbol{\sigma}_{1t}^S$		μ_{1t}^S	$\boldsymbol{\theta}_t$		$\boldsymbol{\sigma}_{1t}^S$		μ_{1t}^S
Index & Index:	$(\sigma^I \times \sigma^I)$	$-$	$\times$	$+$	$\Rightarrow$	$\downarrow$	$+$	$\times$	$+$	$\Rightarrow$	$\uparrow$
Market & Market:	$(\nu \times \nu)$	$+$	$\times$	$-$	$\Rightarrow$	$\downarrow$	$+$	$\times$	$-$	$\Rightarrow$	$\downarrow$
Index & Market:	$(\sigma^I \times \nu)$	$-$	$\times$	$-$	$\Rightarrow$	$\uparrow$	$+$	$\times$	$-$	$\Rightarrow$	$\downarrow$
Market & Index:	$(\nu \times \sigma^I)$	$+$	$\times$	$+$	$\Rightarrow$	$\uparrow$	$+$	$\times$	$+$	$\Rightarrow$	$\uparrow$
Index & Asset:	$(\sigma^I \times \sigma_1^D)$	$-$	$\times$	$+$	$\Rightarrow$	$\downarrow$	$+$	$\times$	$+$	$\Rightarrow$	$\uparrow$
Market & Asset:	$(\nu \times \sigma_1^D)$	$+$	$\times$	$+$	$\Rightarrow$	$\uparrow$	$+$	$\times$	$+$	$\Rightarrow$	$\uparrow$
		$\boldsymbol{\theta}_t$		$\boldsymbol{\sigma}_{2t}^S$		μ_{2t}^S	$\boldsymbol{\theta}_t$		$\boldsymbol{\sigma}_{2t}^S$		μ_{2t}^S
Index & Index:	$(\sigma^I \times \sigma^I)$	$-$	$\times$	$-$	$\Rightarrow$	$\uparrow$	$+$	$\times$	$-$	$\Rightarrow$	$\downarrow$
Market & Market:	$(\nu \times \nu)$	$+$	$\times$	$+$	$\Rightarrow$	$\uparrow$	$+$	$\times$	$+$	$\Rightarrow$	$\uparrow$
Index & Market:	$(\sigma^I \times \nu)$	$-$	$\times$	$+$	$\Rightarrow$	$\downarrow$	$+$	$\times$	$+$	$\Rightarrow$	$\uparrow$
Market & Index:	$(\nu \times \sigma^I)$	$+$	$\times$	$-$	$\Rightarrow$	$\downarrow$	$+$	$\times$	$-$	$\Rightarrow$	$\downarrow$
Market & Asset:	$(\nu \times \sigma_2^D)$	$+$	$\times$	$+$	$\Rightarrow$	$\uparrow$	$+$	$\times$	$+$	$\Rightarrow$	$\uparrow$

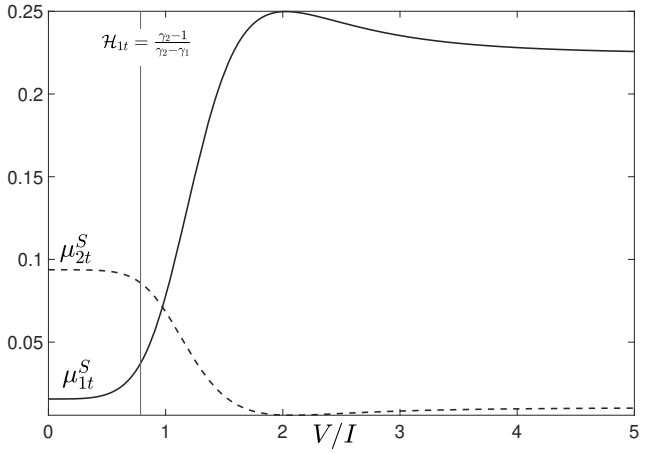


Figure 5. This figure illustrates the expected returns of index and non-index assets as a function of (V/I), the state variable that determines the likelihood of performance. A higher (V/I) suggests a greater chance of outperformance, leading to a higher expected return for the index asset and a lower expected return for the non-index asset. Conversely, a lower (V/I) indicates a higher likelihood of underperformance, resulting in a lower expected return for the index asset and a higher expected return for the non-index asset. The figure also demonstrates that the maximum expected return for the index asset and the minimum expected return for the non-index asset are both interior. Finally, the horizontal line marks the threshold between underperformance (left) and outperformance (right). The parameters are $\gamma_2 = 4.5$, $\gamma_1 = 0.5$, $\mu_1 = \mu_2 = 0.05$, $\sigma_1 = \sigma_2 = 0.25$, $\lambda = 0.5$, $V_t = 5$, $V_0 = 2.5$, $I_0 = 1$, $T = 5$, $t = 2$..

6 Time-Series Predictability

This section examines how an asset's past returns can predict its current return. Positive autocorrelation is known as *momentum*, while negative autocorrelation is referred to as *reversal*.

Both the quantity of risk and the market price of risk are crucial for identifying the existence and direction of predictability. Overreactions and underreactions introduce predictability by causing incorrect adjustments in asset prices to new information. The shifts in the index news market price of risk between underperformance and outperformance determine the direction of predictability, resulting in either momentum or reversal. These two effects are absent in a typical endowment economy that lacks overreaction and underreaction.

Before we delve into the predictability channels, let us first establish the predictability measure. Let's begin by defining the expected return dynamics for both the index and non-index assets as follows

$$d\mu_{1t}^{S} = \alpha_{1t}^{\mu} dt + \boldsymbol{b}_{1t}^{\mu'} d\boldsymbol{Z}_t, \qquad d\mu_{2t}^{S} = \alpha_{2t}^{\mu} dt + \boldsymbol{b}_{2t}^{\mu'} d\boldsymbol{Z}_t. \tag{35}$$

The covariance between the expected returns in time t and $t+\Delta$ (for $\Delta > 0$) is given by

$$\begin{aligned} \mathbb{C}\mathrm{ov}_t\left(\frac{dS_{kt}}{S_{kt}}, \frac{dS_{kt+\Delta}}{S_{kt+\Delta}}\right) =& \mathbb{C}\mathrm{ov}_t\left(\left(\mu_{kt}^{S} dt + \boldsymbol{\sigma}_{kt}^{S\prime} d\boldsymbol{Z}_t\right), \left(\mu_{kt+\Delta}^{S} d\left(t+\Delta\right) + \boldsymbol{\sigma}_{kt+\Delta}^{S\prime} d\boldsymbol{Z}_{t+\Delta}\right)\right) \\ =& \mathbb{C}\mathrm{ov}_t\left(\boldsymbol{\sigma}_{kt}^{S\prime} d\boldsymbol{Z}_t, \mu_{kt+\Delta}^{S} d\left(t+\Delta\right)\right) = \boldsymbol{\sigma}_{kt}^{S\prime} \mathbb{E}_t\left[\left(d\boldsymbol{Z}_t\right) \mu_{kt+\Delta}^{S}\right] d\left(t+\Delta\right). \end{aligned} \tag{36}$$

This covariance indicates that the predictability of asset k depends on how today's news ($d\boldsymbol{Z}_t$) influences the asset's expected return in $t+\Delta$. It reflects the response of the expected return $\mu_{kt+\Delta}^{S}$ to the news $d\boldsymbol{Z}_t$.

To obtain a closed-form characterization, we examine predictability as Δ approaches 0. Plugging the dynamics of μ_{kt}^{S} and taking the covariance leads to

$$\mathbb{C}\mathrm{ov}_t\left(\frac{dS_{kt}}{S_{kt}}, \frac{dS_{kt+\Delta\downarrow 0}}{S_{kt+\Delta\downarrow 0}}\right) = \boldsymbol{\sigma}_{kt}^{S\prime} \boldsymbol{b}_{kt}^{\mu} dt, \tag{37}$$

where $\boldsymbol{b}_{kt}^{\mu}$ is the exposure of asset k expected return to news (35). The following definition characterizes momentum and reversal for Δ approaches 0.

Definition 1 (**Momentum - Reversal**). *Asset k exhibits* momentum *if both the asset return and its expected return move in the same direction,*

$$\boldsymbol{\sigma}_{kt}^{S\prime} \boldsymbol{b}_{kt}^{\mu} > 0, \tag{38}$$

and exhibits reversal *if the asset return and the expected return move in opposite directions,*

$$\boldsymbol{\sigma}_{kt}^{S\prime} \boldsymbol{b}_{kt}^{\mu} < 0, \tag{39}$$

where $k = 1, 2$ and identifies the index and non-index assets, respectively.

The definition is straightforward. We identify momentum when returns rise (fall) and the equilibrium expectation is that they will continue to rise (fall). Conversely, we identify

a reversal when returns rise (fall) but the equilibrium expectation is that they will fall (rise) in the future.

More specifically, predictability stems from predictable changes in the expected return, (μ_{it}^S, due to its sensitivity to shocks, $\boldsymbol{b}_{kt}^{\mu}$). There are two predictable channels that emerge from the no arbitrage condition, $\boldsymbol{\theta}_t' \boldsymbol{\sigma}_{it}^S = \mu_{it}^S$. The first channel is through predictable change in the market prices of risk, $\boldsymbol{\theta}_t$, and the second is through the predictable change in the assets' risk exposures, $\boldsymbol{\sigma}_{it}^S$.

First, let's examine the first channel. Economically, the first channel arises from the predictable *changes in the incentives contract*. Since the incentives contract varies over time, news predictably alters the incentives to benchmark, which in turn changes asset managers' optimal exposures. This adjustment propagates through market clearing to affect the discount factor.

$$\partial \boldsymbol{\theta}_t / \partial \boldsymbol{\sigma}^I = (\gamma_2 - 1) - (\gamma_2 - \gamma_1)(\mathcal{H}_{1t}), \qquad \partial \boldsymbol{\theta}_t / \partial \boldsymbol{\nu} = \gamma_1 + (\gamma_2 - \gamma_1)(\mathcal{H}_{1t}). \tag{40}$$

The variation in these exposures resulting from changes in the state variable (V/I) that determines the liklihood of performance is then provided by

$$d\left(\partial \boldsymbol{\theta}_t / \partial \boldsymbol{\sigma}^I\right) = -d\left(\partial \boldsymbol{\theta}_t / \partial \boldsymbol{\nu}\right) = (\gamma_2 - \gamma_1)^2 \mathcal{H}_{1t}(1 - \mathcal{H}_{1t})\, d(V_t/I_t) / (V_t/I_t), \tag{41}$$

where the derivative of $\mathcal{H}_{1t}$ with respect to V/I is given in (A.17).

When positive index news arrives ($\partial \boldsymbol{\sigma}^I$), the likelihood of underperformance increases ($dV/I < 0$). Asset managers respond by hedging against further declines, increasing their allocation to the benchmark. As a result, the expected return of the benchmark decreases $d\left(\partial \boldsymbol{\theta}_t / \partial \boldsymbol{\sigma}^I\right) < 0$. Conversly, when positive market news arrives ($\partial \boldsymbol{\nu}$), the likelihood of underperformance decreases ($dV/I > 0$). Asset managers respond by reducing their allocation to the benchmark. Consequently, the expected return of the benchmark increases $d(\partial \boldsymbol{\theta}_t / \partial \boldsymbol{\sigma}^{\nu}) > 0$. This predictability channel remains consistent in both outperformance and underperformance.

The *change in the incentives contract* channel causes asset price reversals during periods of both underperformance and outperformance. The index asset is positively exposed to index news ($\partial \boldsymbol{\sigma}_{1t}^S / \partial \boldsymbol{\sigma}^I > 0$), meaning that positive index news boosts the asset's return. However, the predictabile *change in the incentives contract* implies that the expected return

of the index asset decreases, leading to a reversal. Figure 6 illustrates the idea. More concisely,

$$(V/I \downarrow) \quad \Rightarrow \quad \begin{array}{lll} d\left(\partial \boldsymbol{\theta}_t / \partial \boldsymbol{\sigma}^I\right) < 0 & \Rightarrow & \mu_{1t}^S \downarrow \\ \partial \boldsymbol{\sigma}_{1t}^S / \partial \boldsymbol{\sigma}^I > 0 & \Rightarrow & S_{1t} \uparrow \end{array} \Rightarrow \quad \text{Reversal.} \tag{42}$$

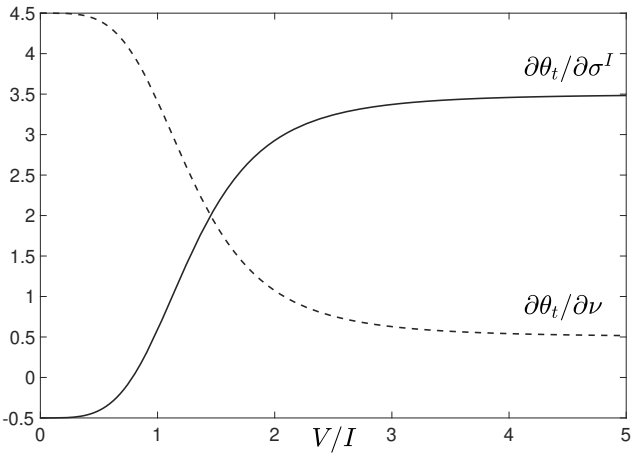


Figure 6. This figure illustrates how discount factor exposures (market prices of risk) change with the arrival of news, as a function of V/I, the state variable governing performance likelihood. A low market price of risk means assets correlated with this risk are highly valued, while a high market price of risk means these assets are less valued. The solid line shows that as performance worsens and asset managers' incentives to benchmark increase, assets correlated with benchmark news become more valuable. Conversely, the dashed line indicates that as performance deteriorates, assets correlated with market news become less valuable. This channel predicts a reversal in asset returns: returns (and prices) increase following index news (V/I decreases), but expected returns decrease due to this channel. A similar reversal channel applies due to market news. The parameters are as in Figure 5.

Next we examine the second channel, which arises from changes in asset exposures due to the arrival of news ($d\boldsymbol{\sigma}_{it}^S$). Economically, the second channel stems from the predictable change in the *uncertainty of performance*: when performance diverges from the benchmark, performance is more ceratin. Conversly, when performance converges to the benchmark, it becomes less certain. Heightened uncertainty means that shocks have a greater impact on asset managers' wealth, which, through market clearing, influences assets' risk exposures. A heightened uncertainty implies that shocks have a bigger impact on asset managers' wealth,

which, through market clearing, affects assets' exposures to risk. By taking the derivative of assets' risk exposures with respect to index and market news, we obtain

$$\partial \boldsymbol{\sigma}_{1t}^{S}/\partial \boldsymbol{\sigma}^{I} = 1 + (\gamma_2 - \gamma_1)(\mathcal{H}_{3t} - \mathcal{H}_{1t}), \qquad \partial \boldsymbol{\sigma}_{1t}^{S}/\partial \boldsymbol{\nu} = -(\gamma_2 - \gamma_1)(\mathcal{H}_{3t} - \mathcal{H}_{1t}), \quad (43)$$

$$\partial \boldsymbol{\sigma}_{2t}^{S}/\partial \boldsymbol{\sigma}^{I} = -(\gamma_2 - \gamma_1)(\mathcal{H}_{1t} - \mathcal{H}_{4t}), \qquad \partial \boldsymbol{\sigma}_{2t}^{S}/\partial \boldsymbol{\nu} = (\gamma_2 - \gamma_1)(\mathcal{H}_{1t} - \mathcal{H}_{4t}). \quad (44)$$

The variation in these exposures resulting from changes in the state variable (V/I) that determines the liklihood of performance is then provided by

$$d\left(\partial \boldsymbol{\sigma}_{1t}^{S}/\partial \boldsymbol{\sigma}^{I}\right) = -d\left(\partial \boldsymbol{\sigma}_{1t}^{S}/\partial \boldsymbol{\nu}\right) = -(\gamma_2 - \gamma_1)^2 (\mathcal{H}_{3t} - \mathcal{H}_{1t})(1 - \mathcal{H}_{1t} - \mathcal{H}_{3t})\, d(V_t/I_t)/(V_t/I_t), \quad (45)$$

$$d\left(\partial \boldsymbol{\sigma}_{2t}^{S}/\partial \boldsymbol{\sigma}^{I}\right) = -d\left(\partial \boldsymbol{\sigma}_{2t}^{S}/\partial \boldsymbol{\nu}\right) = (\gamma_2 - \gamma_1)^2 (\mathcal{H}_{1t} - \mathcal{H}_{4t})(1 - \mathcal{H}_{1t} - \mathcal{H}_{4t})\, d(V_t/I_t)/(V_t/I_t), \quad (46)$$

where the derivative of $\mathcal{H}_{it} - \mathcal{H}_{jt}$ with respect to V/I is given in (A.18) for $i = 1, 3$ and $j = 4, 1$.

Let us first discuss outperformance. When positive index news arrives $(\partial \boldsymbol{\sigma}^{I})$, it introduces more uncertainty about performance because asset managers' performance tend to align with the benchmark. This heightened uncertainty leads to greater asset volatility, making the index asset riskier, ($|\partial \boldsymbol{\sigma}_{1t}^{S}/\partial \boldsymbol{\sigma}^{I}| \uparrow$ and $|\partial \boldsymbol{\sigma}_{1t}^{S}/\partial \boldsymbol{\nu}| \uparrow$). Consequently, it leads to a higher expected return.

Analytically, (45) shows that the risk exposure to index news increases when performance converges to the benchmark, $d\left(\partial \boldsymbol{\sigma}_{1t}^{S}/\partial \boldsymbol{\sigma}^{I}\right) > 0$ because in outperformance $1 - \mathcal{H}_{1t} - \mathcal{H}_{3t} > 0$. Similarly, the risk exposure to market news becomes more negative when performance converges to the benchmark $d\left(\partial \boldsymbol{\sigma}_{1t}^{S}/\partial \boldsymbol{\nu}\right) < 0$.

When asset managers outperform the benchmark, the *uncertainty of performance* channel creates price *momentum*. This happens because the return of the index asset rises after positive index news, and also it expected return due to the heightened uncertainty of performance. Figure 7 illustrates the idea.

Next, we discuss underperformance. When positive index news arrives $(\partial \boldsymbol{\sigma}^{I})$, it introduces less uncertainty about performance because asset managers' performance diverges away from the benchmark. This reduced uncertainty leads to lower asset volatility, making the index asset less risky, ($|\partial \boldsymbol{\sigma}_{1t}^{S}/\partial \boldsymbol{\sigma}^{I}| \downarrow$ and $|\partial \boldsymbol{\sigma}_{1t}^{S}/\partial \boldsymbol{\nu}| \downarrow$). Consequently, it leads to a lower expected return.

Analytically, (45) shows that the risk exposure to index news decreases when performance

diverges away from the benchmark, $d\left(\partial \boldsymbol{\sigma}_{1t}^{S}/\partial \boldsymbol{\sigma}^{I}\right) < 0$ because in underperformance $1 - \mathcal{H}_{1t} - \mathcal{H}_{3t} < 0$. Similarly, the risk exposure to market news becomes less negative when performance converges to the benchmark $d\left(\partial \boldsymbol{\sigma}_{1t}^{S}/\partial \boldsymbol{\nu}\right) > 0$.

When asset managers underperform the benchmark, the *uncertainty of performance* channel leads to price *reversal*. This occurs because, although the return of the index asset rises after positive index news, its expected return decreases due to the reduced uncertainty of performance. Figure 7 illustrates the idea.

These two predictable channels, *change in the incentives contract* and *uncertainty of performance*, lead to an asymmetric price reaction to news during periods of underperformance and overperformance.

$$\text{Underperformance: } (V/I \downarrow) \quad \Rightarrow \quad \begin{array}{lll} d(\partial \boldsymbol{\sigma}_{1t}^{S}/\partial \boldsymbol{\sigma}^{I}) < 0 & \Rightarrow & \mu_{1t}^{S} \downarrow \\ \partial \boldsymbol{\sigma}_{1t}^{S}/\partial \boldsymbol{\sigma}^{I} > 0 & \Rightarrow & S_{1t} \uparrow \end{array} \Rightarrow \text{Reversal}, \tag{47}$$

$$\text{Outperformance: } (V/I \downarrow) \quad \Rightarrow \quad \begin{array}{lll} d\left(\partial \boldsymbol{\sigma}_{1t}^{S}/\partial \boldsymbol{\sigma}^{I}\right) > 0 & \Rightarrow & \mu_{1t}^{S} \uparrow \\ \partial \boldsymbol{\sigma}_{1t}^{S}/\partial \boldsymbol{\sigma}^{I} > 0 & \Rightarrow & S_{1t} \uparrow \end{array} \Rightarrow \text{Momentum}. \tag{48}$$

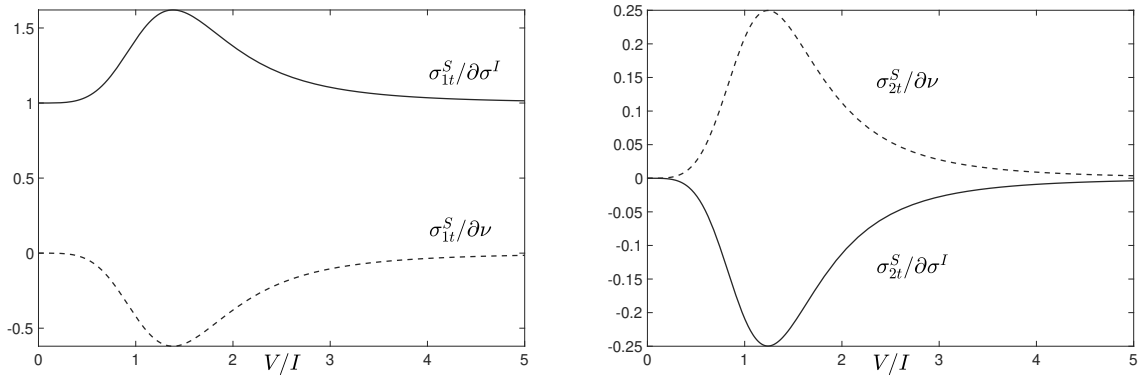


Figure 7. This figure illustrates return exposures for both index and non-index assets fluctuate with the arrival of news, based on the state variable V/I that governs the performance likelihood. The left panel demonstrates that index asset exposure to shocks peaks during periods of high performance uncertainy, and deminishes as the likelihood of either outperformance or underperformance increases. Similarly, the right panel depicts a comparable effects on the non-index asset. The parameters are as in Figure 5.

The two predictable channels lead to price reversal when asset managers underperform.

However, when they outperform, these channels produce conflicting predictions: the *change in the incentives contract* suggests reversal, while the *uncertainty of performance* indicates momentum. In the following proposition, we identify a parametric restriction on the risk aversion parameter γ_2. This ensures that the *uncertainty of performance* is at least as significant as the *change in the incentives contract*, resulting in momentum during outperformance and reversal during underperformance. Specifically, when the risk-on risk-aversion parameter falls within the specified range, $\underline{\gamma}_2 \leq \gamma_2 \leq \bar{\gamma}_2$, prices show overall momentum during outperformance periods and reversal in underperformance periods.

The same two mechanisms apply similarly to the non-index asset, though with a slightly different parametric restriction on the risk-on risk-aversion parameter. When asset managers underperform, the reversal channel is more dominant. Conversely, when outperformance is more likely, the momentum channel is stronger under the parametric restriction $\underline{\underline{\gamma}}_2 \leq \gamma_2 \leq \bar{\bar{\gamma}}_2$. The following proposition summarizes our main findings.

Proposition 4 (**Momentum and Reversal**). *The index asset exhibits momentum when asset managers outperform and reversal if they underperform, if and only if*

$$\underline{\gamma}_2 < \gamma_2 < \bar{\gamma}_2. \tag{49}$$

Similarly, the non-index asset exhibits momentum when asset managers outperform and reversal if they underperform, if and only if

$$\underline{\underline{\gamma}}_2 < \gamma_2 < \bar{\bar{\gamma}}_2. \tag{50}$$

The risk-on risk-aversion parameter thresholds $\underline{\gamma}_2$, $\bar{\gamma}_2$, $\underline{\underline{\gamma}}_2$, and $\bar{\bar{\gamma}}_2$ depend on the risk-off risk aversion parameter, γ_1, and the active management share λ. For a given γ_1, there always exist an active management share λ such that (49) holds. The same is true for the non-index asset under the parametric restriction in (A.85). The thresholds are given in (A.65), (A.67), (A.70), and (A.71), respectively.

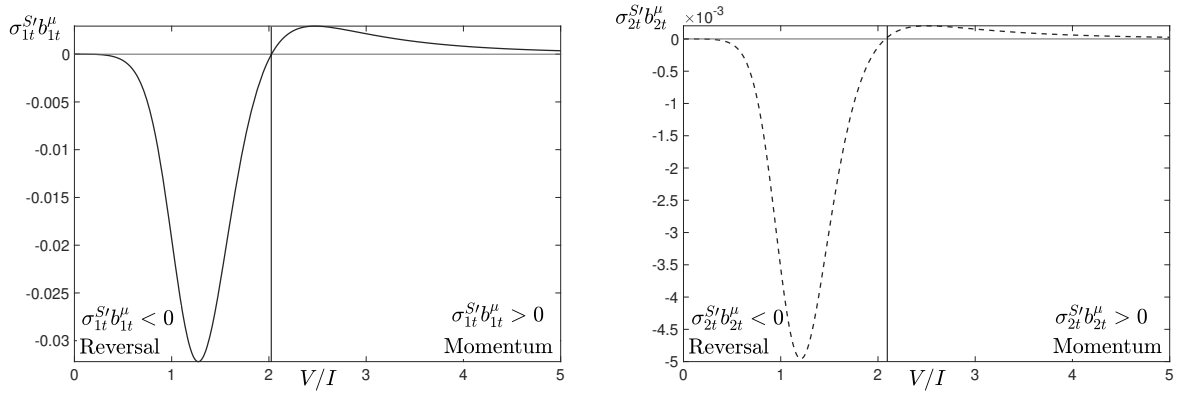


Figure 8. This figure illustrates the cross product between asset return exposure and expected return exposure. A negative measure indicates reversal, while a positive measure indicates momentum. The two panels demonstrate that both index and non-index assets exhibit reversal during underperformance and momentum during outperformance. The parameters are as in Figure 8.

7 Further Discussion

Proposition 5 (**Discount Factor**). *The discount factor in the presence of risk-on incentives is given by*

$$\xi_{0,t} = (I_t/I_0)^{1-\gamma_1} (V_t/V_0)^{-\gamma_2} \mathcal{E}_{1t}^{On}/\mathcal{E}_{10}^{On}, \tag{51}$$

while it is given by

$$\xi_{0,t} = (I_t/I_0)^{-(\gamma_2-1)} (V_t/V_0)^{-\gamma_1} \mathcal{E}_{1t}^{Off}/\mathcal{E}_{10}^{Off} \tag{52}$$

in presence of risk-off incentives. In the presence of both risk-on and risk-off incentives, the discount factor is given by

$$\xi_{0,t} = (I_t/I_0)^{1-\gamma_1} (V_t/V_0)^{-\gamma_2} \mathcal{E}_{1t}^{On}/\mathcal{E}_{10}^{On} \frac{1}{\mathcal{H}_{1t}/\mathcal{H}_{10}}, \tag{53}$$

where $\mathcal{E}_{1t}^{Off}$ and $\mathcal{E}_{1t}^{On}$ are determinstic functions of time given in (A.1) and (A.2), respectively, and $\mathcal{H}_{1t}$ is given in (14). The discount factor exhibits the following characteristics:

- *It decreases following market news:* $\partial \xi_t / \partial V_t < 0$.
- *It decreases following index news in the risk-off incentives case:* $\partial \xi_t / \partial I_t < 0$.
- *It increases following index news in the risk-on incentives case:* $\partial \xi_t / \partial I_t > 0$.
- *It has a U-shape with index news in the joint incentives case. It decreases with the index news* ($\partial \xi_t / \partial I_t < 0$) *if and only if the risk-on hedging demand is below a threshold,*

$$\mathcal{H}_{1t} < \frac{\gamma_2 - 1}{\gamma_2 - \gamma_1} < 1, \tag{54}$$

and increases with the index news ($\partial \xi_t / \partial I_t > 0$) *when* $\mathcal{H}_{1t}$ *passes the threshold.*

Regardless of the asset managers' incentives, Proposition 5 reveals that the state price density is inversely related to the market news, V_t — a feature that is similar to a traditional asset pricing model: an asset that pays off in bad states get a high value.

The risk-on incentives imply that assets that pay off when the index is high get a high value because asset managers with risk-on incentives are concerned about underperforming the index and are willing to pay high prices for assets that pay in these states. This feature is typical to an asset pricing model with benchmarking concerns, such as Basak and Pavlova (2013), Buffa and Hodor (2023), Hodor and Zapatero (2023), among others.

The logic flips with risk-off incentives: assets that pay off when the index is high get a low value. With risk-off incentives, asset managers are outperforming the index and do not desire to hold assets that pay off when the benchmark is high. Instead of hedging unexpected increases in the benchmark, outperforming asset managers hedge unexpected falls by taking a short position on the benchmark and investing the proceeds in other assets — a feature unique to risk-off incentives.

When asset managers face both risk-on and risk-off incentives, the joint incentives discount factor fluctuates between risk-on and risk-off pricing environments. When the benchmark news (I) is low relative to the market news (V), the risk-on hedging demand ($\mathcal{H}_{1t}$) is small, and the asset prices correlate with the purely risk-off prices. In these states, asset managers are more likely to outperform, and prices adjust to accommodate their risk-off portfolios. As the benchmark news (I) increases, the risk-on hedging demand ($\mathcal{H}_{1t}$) becomes more pronounced, and prices adjust to accommodate the heightened likelihood that asset managers will underperform. Eventually, the risk-on hedging demand ($\mathcal{H}_{1t}$) cross the threshold in (54) and becomes so pronounced that prices correlate with the purely risk-on prices.

Figure 9 illustrates this logic.

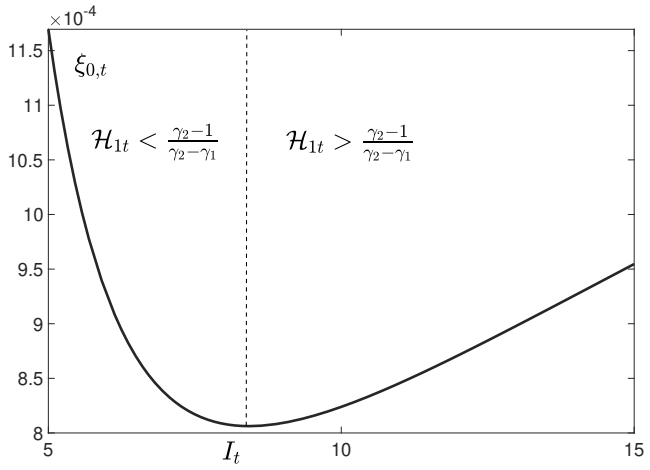


Figure 9. This figure plots the discount factor as a function of index news in the joint incentives case. The discount factor sets prices similar to the risk-off environment when the risk-on hedging demand is below a threshold. In contrast, the discount factor sets prices similar to the risk-on environment when the risk-on hedging demand crosses that threshold. The parameters are as in Figure 5.

Proposition 6 (**The Index Price**). *The index and non-index price-dividend ratios in the presence of risk-on incentives are given by*

$$S_t^I/I_t = \frac{\mathcal{E}_{3t}^{On}}{\mathcal{E}_{1t}^{On}}, \qquad S_{2t}/D_{2t} = \frac{\mathcal{E}_{4t}^{On}}{\mathcal{E}_{1t}^{On}}, \tag{55}$$

while in the presence of risk-off incentives, they are given by

$$S_t^I/I_t = \frac{\mathcal{E}_{3t}^{Off}}{\mathcal{E}_{1t}^{Off}}, \qquad S_{2t}/D_{2t} = \frac{\mathcal{E}_{4t}^{Off}}{\mathcal{E}_{1t}^{Off}}. \tag{56}$$

The index and non-index prices in the presence of both risk-on and risk-off incentives are given by

$$S_t^I/I_t = \frac{\mathcal{E}_{3t}^{On}}{\mathcal{E}_{1t}^{On}}\left[\frac{\mathcal{H}_{1t}}{\mathcal{H}_{3t}}\right], \qquad S_{2t}/D_{2t} = \frac{\mathcal{E}_{4t}^{On}}{\mathcal{E}_{1t}^{On}}\left[\frac{\mathcal{H}_{1t}}{\mathcal{H}_{4t}}\right]. \tag{57}$$

The comparison between the index and non-index price-dividend ratios reveals that the index price-dividend ratio

- *is higher than the non-index one with pure risk-on incentives.*
- *is lower than the non-index one with pure risk-off incentives, if the difference in risk-on and risk-off risk aversion parameters is larger than one:* $\gamma_2 - \gamma_1 \geq 1$.

In the joint incentives case, if $\gamma_2 - \gamma_1 \geq 1$*, the index price-dividend ratio is higher than the non-index one if and only if the ratio of the market news to the index news is below a deterministic threshold,*

$$\frac{V_t}{I_t} \leq \left[\frac{\left(\mathcal{E}_{3t}^{On} - \mathcal{E}_{4t}^{On}\right)}{\left(\mathcal{E}_{4t}^{Off} - \mathcal{E}_{3t}^{Off}\right)} \frac{(\gamma_2 - 1)}{(1 - \gamma_1)} \right]^{\frac{1}{\gamma_2 - \gamma_1}}. \tag{58}$$

Further, in the joint incentives case, the index price-dividend ratio S_t^I / I_t

- *decreases following news about the market:* $\partial \left(S_t^I / I_t\right) / \partial V_t < 0$,
- *increases following news about the index:* $\partial \left(S_t^I / I_t\right) / \partial I_t > 0$.

The results flip for the non-index price-dividend ratio, S_{2t}/D_{2t}*; it*

- *increases following news about the market:* $\partial \left(S_{2t}/D_{2t}\right) / \partial V_t > 0$,
- *decreases following news about the index:* $\partial \left(S_{2t}/D_{2t}\right) / \partial I_t < 0$,

where $\mathcal{H}_{1t}$*,* $\mathcal{H}_{3t}$*, and* $\mathcal{H}_{4t}$ *are given in (14), and* $\mathcal{E}_{1t}^{Off}$*,* $\mathcal{E}_{1t}^{On}$*,* $\mathcal{E}_{3t}^{Off}$*,* $\mathcal{E}_{3t}^{On}$*,* $\mathcal{E}_{4t}^{Off}$*,* $\mathcal{E}_{4t}^{On}$ *are given in (A.1), (A.2), (A.7), (A.8), (A.10), and (A.11), respectively.*

In pure risk-on incentives, asset managers desire to post higher returns when the benchmark is high than when it is low. These risk-on incentives induce the asset managers to buy the benchmark to hedge against unexpected increases in the benchmark, which, in turn, creates extra demand for the benchmark. Equilibrium is restored by increasing the prices of assets that belong to the benchmark to reduce their attractiveness, in line with the documented findings in the asset management literature.

In pure risk-off incentives, asset managers desire to post higher returns when the benchmark is low than when it is high. These risk-off incentives induce the asset managers to sell

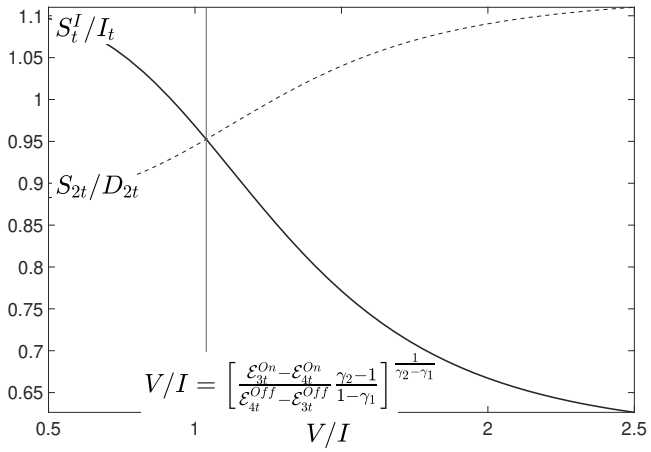


Figure 10. This figure plots the index price-dividend and non-index price-dividend ratios in the joint incentives case as a function of (V/I), the state variable governing the likelihood of performance. The figure shows that the index price dividend ratio falls with market-to-index news (V/I) while the non-index price dividend ratio increases. The vertical line identifies the threshold in (58). To the left of this threshold, the joint incentives asset managers are leaning towards risk-on incentives, and the index price-dividend ratio is more elevated than the non-index one, while to the right of the threshold, the joint incentives asset managers are leaning towards risk-off incentives and the index price-dividend ratio is depressed relative to the non-index one. The parameters are as in Figure 5.

short their benchmark to hedge against unexpected decreases in the benchmark, creating extra supply for the benchmark. Equilibrium is restored by decreasing the prices of assets that belong to the benchmark to increase their attractiveness.

In the joint incentives case, asset managers are more likely to fall behind the benchmark when the index news increases relative to the market news. As the benchmark level increases and the underperformance becomes more likely, the asset manager becomes more concerned about the benchmark. This concern translates to hedging against unexpected increases in the benchmark, which asset managers achieve by purchasing the benchmark assets. Equilibrium increases the benchmark asset prices to counter the extra demand for these assets.

In contrast, asset managers are more likely to outperform the benchmark when the index news decreases relative to the market news. As the benchmark level decreases and the outperformance becomes less likely, the asset manager becomes more concerned about the

benchmark. This concern translates to hedging against unexpected falls in the benchmark, which asset managers achieve by selling short the benchmark assets. Equilibrium decreases the benchmark asset prices to counter the extra supply for these assets.

8 Conclusion

The performance-based component of mutual fund asset managers' compensation encourages them to take on more risk when they are likely to underperform, in an effort to enhance their performance. When managers are likely to outperform and secure their bonus, either partially or fully, they are incentivized to switch from a risk-on strategy to a risk-off strategy to sustain their performance levels.

This paper incorporates time-varying incentives into a dynamic continuous-time asset pricing model and derives the equilibrium in a closed form. The likelihood that asset managers will eventually outperform is a key factor in determining equilibrium prices and the predictability of returns in particular. This likelihood is governed by a single state variable that captures the ratio of market news to index news.

When asset managers are at risk of underperforming, they allocate part of their wealth to hedge against falling further below the benchmark, effectively using the benchmark as a hedging tool. As the likelihood of underperformance decreases and performance becomes more likely, their optimal portfolio (and risk exposures) change significantly. To sustain strong relative performance, asset managers then invest in assets with favorable risk-return tradeoffs rather than simply buying the benchmark for hedging.

The paper's primary contribution demonstrates that the time-varying incentives contract leads to reversals during periods of underperformance and generates momentum during periofs of outperformance, affecting both index and non-index assets. To generate similar predictions, previous literature relied on either a slow information environment, where prices adjust gradually to new information, or on the assumption that investors are not fully rational.

Return predictability stems from two identifiable channels. The first channel involves predictably changes in the incentives contract: when new information arrives, it alters asset managers' incentives to benchmark, prompting them to adjust their optimal exposure. This adjustment, through market clearing, then influences returns. The second channel arises from performance uncertainty. When the chances of outperformance and underperformance

are equal, there's greater uncertainty about asset managers' performance. This uncertainty heightens their sensitivity to news, affecting returns through equilibrium. As the likelihood of either outperformance or underperformance increases, performance uncertainty decreases, and sensitivity to news diminishes.

A Proofs

Before we begin with the formal proofs, we solve the following expected values.

$$\mathcal{E}_{1t}^{\text{Off}} = (\mathbf{1}_{\text{Off}})\,\mathcal{E}_{1t}\,(1-\gamma_2,-\gamma_1) \equiv \frac{E_t\left[(I_T)^{1-\gamma_2}\,(V_T)^{-\gamma_1}\right]}{(I_t)^{1-\gamma_2}\,(V_t)^{-\gamma_1}} \tag{A.1}$$
$$= e^{\left((-\gamma_1)(\eta)-(\gamma_2-1)(\mu_1)+(\sigma_1\nu_1)(\gamma_1)(\gamma_2-1)+\frac{1}{2}(\sigma_1)^2(\gamma_2)(\gamma_2-1)+\frac{1}{2}\|\boldsymbol{\nu}\|^2(\gamma_1)(1+\gamma_1)\right)(T-t)},$$

$$\mathcal{E}_{1t}^{\text{On}} = (\mathbf{1}_{\text{On}})\,\mathcal{E}_{1t}\,(1-\gamma_1,-\gamma_2) \equiv \frac{E_t\left[(I_T)^{1-\gamma_1}\,(V_T)^{-\gamma_2}\right]}{(I_t)^{1-\gamma_1}\,(V_t)^{-\gamma_2}} \tag{A.2}$$
$$= e^{\left((-\gamma_2)(\eta)+(1-\gamma_1)(\mu_1)-(\sigma_1\nu_1)(\gamma_2)(1-\gamma_1)-\frac{1}{2}(\sigma_1)^2(\gamma_1)(1-\gamma_1)+\frac{1}{2}\|\boldsymbol{\nu}\|^2(\gamma_2)(1+\gamma_2)\right)(T-t)},$$

$$\frac{\mathcal{E}_{1t}^{\text{Off}}}{\mathcal{E}_{1t}^{\text{On}}} \equiv \mathcal{E}_{1t} = e^{\left((\gamma_2-\gamma_1)\left(\eta-\mu_1+\sigma_1\nu_1+\frac{1}{2}(\sigma_1)^2(\gamma_1+\gamma_2-1)-\frac{1}{2}\|\boldsymbol{\nu}\|^2(\gamma_1+\gamma_2+1)\right)\right)(T-t)}, \tag{A.3}$$

$$\mathcal{E}_{2t}^{\text{Off}} = (\mathbf{1}_{\text{Off}})\,\mathcal{E}_{2t}\,(1-\gamma_2,1-\gamma_1) \equiv \frac{E_t\left[(I_T)^{1-\gamma_2}\,(V_T)^{1-\gamma_1}\right]}{(I_t)^{1-\gamma_2}\,(V_t)^{1-\gamma_1}} \tag{A.4}$$
$$= e^{\left((1-\gamma_1)(\eta)-(\gamma_2-1)(\mu_1)-(\sigma_1\nu_1)(1-\gamma_1)(\gamma_2-1)+\frac{1}{2}(\sigma_1)^2(\gamma_2)(\gamma_2-1)+\frac{1}{2}\|\boldsymbol{\nu}\|^2(-\gamma_1)(1-\gamma_1)\right)(T-t)},$$

$$\mathcal{E}_{2t}^{\text{On}} = (\mathbf{1}_{\text{On}})\,\mathcal{E}_{2t}\,(1-\gamma_1,1-\gamma_2) \equiv \frac{E_t\left[(I_T)^{1-\gamma_1}\,(V_T)^{1-\gamma_2}\right]}{(I_t)^{1-\gamma_1}\,(V_t)^{1-\gamma_2}} \tag{A.5}$$
$$= e^{\left((1-\gamma_2)(\eta)+(1-\gamma_1)(\mu_1)-(\sigma_1\nu_1)(\gamma_2-1)(1-\gamma_1)-\frac{1}{2}(\sigma_1)^2(\gamma_1)(1-\gamma_1)+\frac{1}{2}\|\boldsymbol{\nu}\|^2(\gamma_2)(\gamma_2-1)\right)(T-t)},$$

$$\frac{\mathcal{E}_{2t}^{\text{Off}}}{\mathcal{E}_{2t}^{\text{On}}} \equiv \mathcal{E}_{2t} = e^{\left((\gamma_2-\gamma_1)\left(\eta-\mu_1+\frac{1}{2}\left[(\sigma_1)^2-\|\boldsymbol{\nu}\|^2\right](\gamma_1+\gamma_2-1)\right)\right)(T-t)}, \tag{A.6}$$

$$\mathcal{E}_{3t}^{\text{Off}} = (\mathbf{1}_{\text{Off}})\,\mathcal{E}_{3t}\,(2-\gamma_2,-\gamma_1) \equiv \frac{E_t\left[(I_T)^{2-\gamma_2}\,(V_T)^{-\gamma_1}\right]}{(I_t)^{2-\gamma_2}\,(V_t)^{-\gamma_1}} \tag{A.7}$$
$$= e^{\left(-\gamma_1\eta+(2-\gamma_2)(\mu_1)-(\sigma_1\nu_1)\gamma_1(2-\gamma_2)+\frac{1}{2}(\sigma_1)^2(\gamma_2-1)(\gamma_2-2)+\frac{1}{2}\|\boldsymbol{\nu}\|^2\gamma_1(1+\gamma_1)\right)(T-t)},$$

$$\mathcal{E}_{3t}^{\text{On}} = (\mathbf{1}_{\text{On}})\,\mathcal{E}_{3t}\,(2-\gamma_1,-\gamma_2) \equiv \frac{E_t\left[(I_T)^{2-\gamma_1}\,(V_T)^{-\gamma_2}\right]}{(I_t)^{2-\gamma_1}\,(V_t)^{-\gamma_2}} \tag{A.8}$$
$$= e^{\left(-\gamma_2\eta+(2-\gamma_1)(\mu_1)-(\sigma_1\nu_1)\gamma_2(2-\gamma_1)+\frac{1}{2}(\sigma_1)^2(\gamma_1-1)(\gamma_1-2)+\frac{1}{2}\|\boldsymbol{\nu}\|^2\gamma_2(1+\gamma_2)\right)(T-t)},$$

$$\frac{\mathcal{E}_{3t}^{\text{Off}}}{\mathcal{E}_{3t}^{\text{On}}} \equiv \mathcal{E}_{3t} = e^{(\gamma_2-\gamma_1)\left(\eta-\mu_1+2(\sigma_1\nu_1)+\frac{1}{2}(\sigma_1)^2(\gamma_2+\gamma_1-3)-\frac{1}{2}\|\boldsymbol{\nu}\|^2(\gamma_2+\gamma_1+1)\right)(T-t)}, \tag{A.9}$$

$$\mathcal{E}_{4t}^{\text{Off}} = (\mathbf{1}_{\text{Off}})\,\mathcal{E}_{4t}\,(1-\gamma_2,-\gamma_1) \equiv \frac{E_t\left[(I_T)^{1-\gamma_2}\,(V_T)^{-\gamma_1}\,D_{2T}\right]}{(I_t)^{1-\gamma_2}\,(V_t)^{-\gamma_1}\,D_{2t}}$$
$$= e^{\left[-(\gamma_2-1)\mu_1-\gamma_1\eta+\mu_2+(\sigma_1\nu_1)(\gamma_1)(\gamma_2-1)-(\sigma_2\nu_2)(\gamma_1)+\frac{1}{2}(\sigma_1)^2(\gamma_2-1)(\gamma_2)+\frac{1}{2}\|\boldsymbol{\nu}\|^2\gamma_1(1+\gamma_1)\right](T-t)}, \tag{A.10}$$

$$\mathcal{E}_{4t}^{\text{On}} = (\mathbf{1}_{\text{On}})\,\mathcal{E}_{4t}\,(1-\gamma_1,-\gamma_2) \equiv \frac{E_t\left[(I_T)^{1-\gamma_1}\,(V_T)^{-\gamma_2}\,D_{2T}\right]}{(I_t)^{1-\gamma_1}\,(V_t)^{-\gamma_2}\,D_{2t}}$$
$$= e^{\left[-(\gamma_1-1)\mu_1-\gamma_2\eta+\mu_2+(\sigma_1\nu_1)(\gamma_2)(\gamma_1-1)-(\sigma_2\nu_2)(\gamma_2)+\frac{1}{2}(\sigma_1)^2(\gamma_1-1)(\gamma_1)+\frac{1}{2}\|\boldsymbol{\nu}\|^2\gamma_2(1+\gamma_2)\right](T-t)}, \tag{A.11}$$

$$\frac{\mathcal{E}_{4t}^{\text{Off}}}{\mathcal{E}_{4t}^{\text{On}}} \equiv \mathcal{E}_{4t} = e^{(\gamma_2-\gamma_1)\left[-\mu_1+\eta+(\sigma_1\nu_1)+(\sigma_2\nu_2)+\frac{1}{2}(\sigma_1)^2(\gamma_2+\gamma_1-1)-\frac{1}{2}\|\boldsymbol{\nu}\|^2(\gamma_2+\gamma_1+1)\right](T-t)}. \tag{A.12}$$

We reference to these expected values throughout the proofs. Note that $\mathcal{E}_{it}$, $i=1,2,3,4$ is not well defined in the risk-off incentive case, since $\mathcal{E}_{it}^{\text{On}}=0$. In pure risk-on incentives $\mathcal{E}_{it}=0$, $i=1,2,3,4$ since $\mathcal{E}_{it}^{\text{Off}}=0$. By plugging the definition of $\mathcal{E}_{it}$ into $\mathcal{H}_{it}$ we find that

$$\mathcal{H}_{it} \equiv \frac{1}{1+\frac{1-\gamma_1}{\gamma_2-1}\,(V_t/I_t)^{\gamma_2-\gamma_1}\,\mathcal{E}_{it}} = \frac{\mathcal{E}_{it}^{\text{On}}}{\mathcal{E}_{it}^{\text{On}}+\frac{1-\gamma_1}{\gamma_2-1}\,(V_t/I_t)^{\gamma_2-\gamma_1}\,\mathcal{E}_{it}^{\text{Off}}}, \tag{A.13}$$

implying that $\mathcal{H}_{it}=0$ in the pure risk-off incentives and $\mathcal{H}_{it}=1$ in the pure risk-on

incentives. Notice that

$$\mathcal{H}_{3t} > \mathcal{H}_{1t} \iff \mathcal{E}_{1t} > \mathcal{E}_{3t} \iff e^{(\gamma_2-\gamma_1)\left(-\sigma_1\nu_1+\sigma_1^2\right)(T-t)} > 1, \tag{A.14}$$

$$\mathcal{H}_{1t} > \mathcal{H}_{4t} \iff \mathcal{E}_{4t} > \mathcal{E}_{1t} \iff e^{(\gamma_2-\gamma_1)(\sigma_2\nu_2)(T-t)} > 1. \tag{A.15}$$

The last inequalities are true because $\nu_1 = \lambda\sigma_1/3$, which implies that $\nu_1 < \sigma_1$ and because $\sigma_2 > 0$ and $\nu_2 > 0$. The definition of the risk-on hedging demand (14) and the ensuing analysis implies that in extreme underperformance the ratio of market news to index news approaches zero, $V/I \downarrow 0$, while in extreme outperformance the ratio approaches infinity, $V/I \downarrow \infty$, implying that

$$\lim_{V/I\to 0} \mathcal{H}_{it} - \mathcal{H}_{jt} \to 0, \qquad \lim_{V/I\to\infty} \mathcal{H}_{it} - \mathcal{H}_{jt} \to 0. \tag{A.16}$$

The derivative of the risk-on hedging demand with respect to V/I equals

$$\frac{d\mathcal{H}_{it}}{d\frac{V}{I}} = -\left(\gamma_2 - \gamma_1\right)\mathcal{H}_{it}\left(1 - \mathcal{H}_{it}\right)\left(V_t/I_t\right)^{-1} < 0, \tag{A.17}$$

and the derivative of $\mathcal{H}_{it} - \mathcal{H}_{jt}$ with respect to V/I is given by

$$\frac{d\mathcal{H}_{it}}{d\frac{V}{I}} - \frac{d\mathcal{H}_{jt}}{d\frac{V}{I}} = -\left(\gamma_2 - \gamma_1\right)\left(V_t/I_t\right)^{-1}\left(\mathcal{H}_{it} - \mathcal{H}_{jt}\right)\left(1 - \mathcal{H}_{it} - \mathcal{H}_{jt}\right), \tag{A.18}$$

which approaches zero at extreme outperformance and underperformance due to (A.16). Next, we find that $\mathcal{H}_{3t} - \mathcal{H}_{1t}$ converges to zero as fast as $\mathcal{H}_{1t}\left(1 - \mathcal{H}_{1t}\right)$ converges to zero, both when $\mathcal{H}_{it} \to 0$ and $\mathcal{H}_{it} \to 1$, and the limits are given by

$$\lim_{\mathcal{H}_{it}\to 0} = \lim_{V/I\to\infty} \frac{\mathcal{H}_{jt} - \mathcal{H}_{1t}}{\mathcal{H}_{1t}\left(1 - \mathcal{H}_{1t}\right)} = \frac{\mathcal{E}_{1t} - \mathcal{E}_{jt}}{\mathcal{E}_{jt}} > 0, \quad j = 3, 4, \tag{A.19}$$

$$\lim_{\mathcal{H}_{it}\to 1} = \lim_{V/I\to 0} \frac{\mathcal{H}_{jt} - \mathcal{H}_{1t}}{\mathcal{H}_{1t}\left(1 - \mathcal{H}_{1t}\right)} = \frac{\mathcal{E}_{1t} - \mathcal{E}_{jt}}{\mathcal{E}_{1t}} > 0. \tag{A.20}$$

Furthermore,

$$\lim_{V/I\to 0}\frac{\mathcal{H}_{1t}\left(1-\mathcal{H}_{1t}\right)}{V/I}=\lim_{V/I\to\infty}\frac{\mathcal{H}_{1t}\left(1-\mathcal{H}_{1t}\right)}{V/I}=0, \tag{A.21}$$

$$\lim_{V/I\to 0}\frac{\mathcal{H}_{jt}-\mathcal{H}_{1t}}{V/I}=\lim_{V/I\to\infty}\frac{\mathcal{H}_{jt}-\mathcal{H}_{1t}}{V/I}=0,\quad j=3,4. \tag{A.22}$$

Lastly,

$$\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}=\frac{\sigma_1^2}{N}\left(N-\lambda\right)>0, \tag{A.23}$$

$$\|\boldsymbol{\nu}\|^2-\boldsymbol{\nu}'\boldsymbol{\sigma}^I=\frac{\sigma_1^2}{N^2}\left(1-\lambda\right)\left(\left(N-1\right)-\lambda\right), \tag{A.24}$$

implying that $\|\boldsymbol{\nu}\|^2-\boldsymbol{\nu}'\boldsymbol{\sigma}^I$ approached zero as $\lambda\to 1$, and that

$$\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}>\|\boldsymbol{\nu}\|^2-\boldsymbol{\nu}'\boldsymbol{\sigma}^I. \tag{A.25}$$

Proof of Proposition 5 (Discount Factor). The security market is dynamically complete. As such, there exists a unique state price density process, ξ, and the no arbitrage relations

$$\xi_t S_{kt}=\mathbb{E}_t\left[\xi_T D_{kT}\right],\quad t\in[0,T], k=1,2,3, \tag{A.26}$$

is always satisfied. In our setup, we set $r=0$ for $t\leq T$, and thus, the state price density evolves according to

$$d\xi_t=-\xi_t\left(\boldsymbol{\theta}_t' d\boldsymbol{Z}_t\right),\quad t\leq T. \tag{A.27}$$

The vector process $\boldsymbol{\theta}_t$ is the cash-flow news market risk prices. Restating the dynamic budget constraint as

$$\xi_t W_t^{\mathcal{A}}=\mathbb{E}_t\left[\xi_T W_T^{\mathcal{A}}\right],\quad t\in[0,T], \tag{A.28}$$

and maximizing the asset manager's objective function (11), subject to (A.28) at time $t=0$,

we obtain the first order condition

$$y^{\mathcal{A}}\xi_T = \mathbf{1}_{\text{Off}}\frac{(I_T)^{1-\gamma_2}\left(W_T^{\mathcal{A}}\right)^{-\gamma_1}}{(\gamma_2-1)} + \mathbf{1}_{\text{On}}\frac{(I_T)^{1-\gamma_1}\left(W_T^{\mathcal{A}}\right)^{-\gamma_2}}{(1-\gamma_1)}, \tag{A.29}$$

where $y^{\mathcal{A}}$ is the Lagrange multiplier, and due to no-arbitrage condition at time T, $S_T^I = I_T$.

To clear markets, the asset managers must hold the entire active supply S_t^A, which at time T equals V_T, leading to

$$y^{\mathcal{A}}\xi_T = \mathbf{1}_{\text{Off}}\frac{(I_T)^{1-\gamma_2}(V_T)^{-\gamma_1}}{(\gamma_2-1)} + \mathbf{1}_{\text{On}}\frac{(I_T)^{1-\gamma_1}(V_T)^{-\gamma_2}}{(1-\gamma_1)}. \tag{A.30}$$

Since ξ_t is a martingale, we have

$$\xi_{0,t} = \frac{\mathbb{E}_t\left[\mathbf{1}_{\text{Off}}\frac{(I_T)^{1-\gamma_2}(V_T)^{-\gamma_1}}{(\gamma_2-1)} + \mathbf{1}_{\text{On}}\frac{(I_T)^{1-\gamma_1}(V_T)^{-\gamma_2}}{(1-\gamma_1)}\right]}{\mathbb{E}\left[\mathbf{1}_{\text{Off}}\frac{(I_T)^{1-\gamma_2}(V_T)^{-\gamma_1}}{(\gamma_2-1)} + \mathbf{1}_{\text{On}}\frac{(I_T)^{1-\gamma_1}(V_T)^{-\gamma_2}}{(1-\gamma_1)}\right]}. \tag{A.31}$$

By plugging (A.1) and (A.2) into (A.31) we find that

$$\xi_{0,t} = \frac{\frac{\mathbf{1}_{\text{Off}}}{\gamma_2-1}(I_t)^{1-\gamma_2}(V_t)^{-\gamma_1}\mathcal{E}_{1t}^{\text{Off}} + \frac{\mathbf{1}_{\text{On}}}{1-\gamma_1}(I_t)^{1-\gamma_1}(V_t)^{-\gamma_2}\mathcal{E}_{1t}^{\text{On}}}{\frac{\mathbf{1}_{\text{Off}}}{\gamma_2-1}(I_0)^{1-\gamma_2}(V_0)^{-\gamma_1}\mathcal{E}_{10}^{\text{Off}} + \frac{\mathbf{1}_{\text{On}}}{1-\gamma_1}(I_0)^{1-\gamma_1}(V_0)^{-\gamma_2}\mathcal{E}_{10}^{\text{On}}}. \tag{A.32}$$

To find the Lagrange multiplier, we plug the equilibrium discount factor (A.30) into the dynamic budget constraint (A.28) evaluated at time $t=0$, apply the market clearing condition so that the asset managers must hold the entire active supply and find that

$$\xi_0 S_0^A = \mathbb{E}\left[\frac{1}{y^{\mathcal{A}}}\left(\mathbf{1}_{\text{Off}}\frac{(I_T)^{1-\gamma_2}(V_T)^{1-\gamma_1}}{(\gamma_2-1)} + \mathbf{1}_{\text{On}}\frac{(I_T)^{1-\gamma_1}(V_T)^{1-\gamma_2}}{(1-\gamma_1)}\right)\right], \tag{A.33}$$

which eventually leads to

$$\begin{aligned} y^{\mathcal{A}} &= \frac{1}{\xi_0 S_0^A}\mathbb{E}\left[\left(\mathbf{1}_{\text{Off}}\frac{(I_T)^{1-\gamma_2}(V_T)^{1-\gamma_1}}{(\gamma_2-1)} + \mathbf{1}_{\text{On}}\frac{(I_T)^{1-\gamma_1}(V_T)^{1-\gamma_2}}{(1-\gamma_1)}\right)\right] \\ &= \frac{1}{\xi_0 S_0^A}\left[\frac{\mathbf{1}_{\text{Off}}}{\gamma_2-1}(I_0)^{1-\gamma_2}(V_0)^{1-\gamma_1}\mathcal{E}_{20}(1-\gamma_2,1-\gamma_1) + \frac{\mathbf{1}_{\text{On}}}{1-\gamma_1}(I_0)^{1-\gamma_1}(V_0)^{1-\gamma_2}\mathcal{E}_{20}(1-\gamma_1,1-\gamma_2)\right], \end{aligned} \tag{A.34}$$

where the second equality is due to (A.4) and (A.5) evaluated at time $t = 0$.

We conclude the proof with the discount factor derivatives. It is straightforward to show that $\xi_{0,t}$ is decreasing in V_t and increasing in I_t in the risk-on incentives case, (51), and $\xi_{0,t}$ is decreasing in V_t and I_t in the risk-off incentives case, (52). The derivative of the discount factor with respect to I_t in the joint incentives case equals

$$\frac{\partial \xi_{0,t}}{\partial I_t} = (1-\gamma_1)\,\frac{1}{I_t}\xi_{0,t} - \xi_{0,t}\frac{1}{\mathcal{H}_{1t}}\frac{\partial \mathcal{H}_{1t}}{\partial I_t}, \tag{A.35}$$

where the derivative of the risk-on hedging demand with respect to I_t equals

$$\frac{\partial \mathcal{H}_{1t}}{\partial I_t} = (\mathcal{H}_{1t})^2 \left(\frac{1-\gamma_1}{\gamma_2-1}\,(\gamma_2-\gamma_1)\,(I_t)^{\gamma_1-\gamma_2-1}\,(V_t)^{\gamma_2-\gamma_1}\,\mathcal{E}_{1t}\right) > 0. \tag{A.36}$$

By plugging the hedging demand derivative (A.36) into the discount factor derivative (A.35), and by removing strictly positive terms and rearranging, we find that $\partial \xi_{0,t}/\partial I_t < 0$ if and only if

$$(1-\gamma_1) - (\gamma_2-\gamma_1)\,(-\mathcal{H}_{1t}+1) < 0, \tag{A.37}$$

which leads to our desired result in (54). Similarly, the derivative of the discount factor with respect to V_t in the joint incentives case equals

$$\frac{\partial \xi_{0,t}}{\partial V_t} = (-\gamma_2)\,\frac{1}{V_t}\xi_t - \xi_t\frac{1}{\mathcal{H}_{1t}}\frac{\partial \mathcal{H}_{1t}}{\partial V_t}, \tag{A.38}$$

where the derivative of the risk-on hedging demand with respect to V_t equals

$$\frac{\mathcal{H}_{it}}{\partial V_t} = -(\mathcal{H}_{it})^2 \left(\frac{1-\gamma_1}{\gamma_2-1}\,(\gamma_2-\gamma_1)\,(I_t)^{\gamma_1-\gamma_2}\,(V_t)^{\gamma_2-\gamma_1-1}\,\mathcal{E}_{1t}\right) < 0, \quad i = 1,2,3,4. \tag{A.39}$$

By plugging the hedging demand derivative (A.39) into the discount factor derivative (A.38) and rearranging, we find that $\partial \xi_{0,t}/\partial V_t < 0$ if and only if

$$(\gamma_2-\gamma_1)\,\mathcal{H}_{1t} + \gamma_1 > 0, \tag{A.40}$$

which is always true because $\gamma_2 > \gamma_1$ and $\mathcal{H}_{1t} > 0$.

□

Proof of Lemma 1 (Risk Exposures). We utilize the no-arbitrage condition (A.28) at time t, the market clearing condition, and write

$$\xi_t W_t^{\mathcal{A}} = \mathbb{E}_t\left[\xi_T V_T\right] = \frac{1}{y^{\mathcal{A}}}\mathbb{E}_t\left[\left(\mathbf{1}_{\text{Off}}\frac{(I_T)^{1-\gamma_2}(V_T)^{1-\gamma_1}}{(\gamma_2-1)} + \mathbf{1}_{\text{On}}\frac{(I_T)^{1-\gamma_1}(V_T)^{1-\gamma_2}}{(1-\gamma_1)}\right)\right]. \tag{A.41}$$

By plugging (A.4) and (A.5) we obtain

$$\xi_t W_t^{\mathcal{A}} = \frac{1}{y^{\mathcal{A}}}\left[\frac{\mathbf{1}_{\text{Off}}}{\gamma_2-1}(I_t)^{1-\gamma_2}(V_t)^{1-\gamma_1}\mathcal{E}_{2t}^{\text{Off}} + \frac{\mathbf{1}_{\text{On}}}{1-\gamma_1}(I_t)^{1-\gamma_1}(V_t)^{1-\gamma_2}\mathcal{E}_{2t}^{\text{On}}\right] \tag{A.42}$$

Applying Itô's Lemma on both sides of the above equation leads to the desired result.

$$\boldsymbol{\pi}_t^{\mathcal{A}\prime}\boldsymbol{\Sigma}^S = \boldsymbol{\theta}_t + (1-\gamma_1)\boldsymbol{\nu} - (\gamma_2-1)\boldsymbol{\sigma}_1^D + (\gamma_2-\gamma_1)\mathcal{H}_{2t}^{\text{On}}\left(\boldsymbol{\sigma}_1^D - \boldsymbol{\nu}\right) \tag{A.43}$$

□

Proof of Proposition 1 (Market Prices of Risk). Applying Itô's Lemma on both sides of (A.32) leads to the desired result.

□

Proof of Proposition 6 (Stock Prices). We start from the no arbitrage condition (A.26), plug $\xi_{0,T}$ (A.32), and find that the deflated index price satisfies

$$S_t^I\xi_{0,t} = \frac{\mathbb{E}_t\left[\left(\mathbf{1}_{\text{Off}}\frac{(I_T)^{1-\gamma_2}(V_T)^{-\gamma_1}}{(\gamma_2-1)} + \mathbf{1}_{\text{On}}\frac{(I_T)^{1-\gamma_1}(V_T)^{-\gamma_2}}{(1-\gamma_1)}\right)I_T\right]}{\mathbb{E}\left[\mathbf{1}_{\text{Off}}\frac{(I_T)^{1-\gamma_2}(V_T)^{-\gamma_1}}{(\gamma_2-1)} + \mathbf{1}_{\text{On}}\frac{(I_T)^{1-\gamma_1}(V_T)^{-\gamma_2}}{(1-\gamma_1)}\right]}. \tag{A.44}$$

By plugging the expected values from (A.7) and (A.8), $\xi_{0,t}$ (A.32), and rearranging we obtain the desired result. Next, we characterize the non-index price. Similar to the index price, we find that the deflated non-index price satisfies

$$S_{2t}\xi_{0,t} = \frac{\mathbb{E}_t\left[\left(\mathbf{1}_{\text{Off}}\frac{(I_T)^{1-\gamma_2}(V_T)^{-\gamma_1}D_{2T}}{(\gamma_2-1)} + \mathbf{1}_{\text{On}}\frac{(I_T)^{1-\gamma_1}(V_T)^{-\gamma_2}D_{2T}}{(1-\gamma_1)}\right)\right]}{\mathbb{E}\left[\mathbf{1}_{\text{Off}}\frac{(I_T)^{1-\gamma_2}(V_T)^{-\gamma_1}}{(\gamma_2-1)} + \mathbf{1}_{\text{On}}\frac{(I_T)^{1-\gamma_1}(V_T)^{-\gamma_2}}{(1-\gamma_1)}\right]}. \tag{A.45}$$

By plugging the expected values from (A.10) and (A.11) and $\xi_{0,t}$ (A.32), and rearranging we obtain the desired result. We conclude the proof by taking the derivative of the index and non-index price dividend ratios, and find that

$$\partial\left(S_t^I/I_t\right)/\partial V_t = \frac{\mathcal{E}_{3t}^{\mathrm{On}}}{\mathcal{E}_{1t}^{\mathrm{On}}\left(\mathcal{H}_{3t}\right)^2}\left[\left(\partial\mathcal{H}_{1t}/\partial V_t\right)\mathcal{H}_{3t} - \left(\partial\mathcal{H}_{3t}/\partial V_t\right)\mathcal{H}_{1t}\right] \tag{A.46}$$

$$\partial\left(S_{2t}/D_{2t}\right)/\partial V_t = \frac{\mathcal{E}_{4t}^{\mathrm{On}}}{\mathcal{E}_{1t}^{\mathrm{On}}\left(\mathcal{H}_{4t}\right)^2}\left[\left(\partial\mathcal{H}_{1t}/\partial V_t\right)\mathcal{H}_{4t} - \left(\partial\mathcal{H}_{4t}/\partial V_t\right)\mathcal{H}_{1t}\right]. \tag{A.47}$$

By plugging the derivative of $\mathcal{H}_{it}$, $i = 1, 3, 4$, (14) with respect to V_t, into the price-dividend ratios (A.46) and (A.47), and by removing strictly positive terms and rearranging, we find that

$$\partial\left(S_t^I/I_t\right)/\partial V_t < 0 \iff \mathcal{E}_{3t} - \mathcal{E}_{1t} < 0, \tag{A.48}$$

$$\partial\left(S_{2t}/D_{2t}\right)/\partial V_t > 0 \iff \mathcal{E}_{4t} - \mathcal{E}_{1t} > 0. \tag{A.49}$$

The right hand side inequalities are true due to (A.14) and (A.15), respectively, implying that the left hand side inequality are also true, which is our desire result. In similar fashion, we obtain

$$\partial\left(S_t^I/I_t\right)/\partial I_t > 0 \iff \mathcal{E}_{3t} - \mathcal{E}_{1t} < 0, \tag{A.50}$$

$$\partial\left(S_{2t}/D_{2t}\right)/\partial I_t < 0 \iff \mathcal{E}_{4t} - \mathcal{E}_{1t} > 0. \tag{A.51}$$

The right hand side is always satisfied due to (A.14) and (A.15), respectively, implying that the left hand side inequality are also true, which is our desire result.

We continue with the comparison of the index price-dividend ratio and the non-index price-dividend ratio. Dividing the index price-dividend ratio by the non-index price-dividend ratio (55) in the pure risk-on incentives, we obtain

$$\begin{aligned}\frac{S_t^I/I_t}{S_{2t}/D_{2t}} = \frac{\mathcal{E}_{3t}^{\mathrm{On}}}{\mathcal{E}_{4t}^{\mathrm{On}}} &= e^{\left(\mu_1-\mu_2-(\sigma_1\nu_1)\gamma_2+(\sigma_2\nu_2)\gamma_2+(\sigma_1)^2(1-\gamma_1)\right)(T-t)} \\ &= e^{\left(-\left(\lambda\sigma^2/3\right)\gamma_2+\left(\sigma^2/3\right)\gamma_2+(\sigma)^2(1-\gamma_1)\right)(T-t)} > 1.\end{aligned} \tag{A.52}$$

We obtain the second equality by plugging the definitions of $\mathcal{E}_{3t}^{\mathrm{On}}$ (A.8) and $\mathcal{E}_{4t}^{\mathrm{On}}$ (A.11) and cancelling similar terms, and the last equality by using the assumptions stating that $\mu_1 = \mu_2$,

$\sigma_1 = \sigma_2$, and $\nu_1 = \lambda\sigma/3$, $\nu_2 = \sigma/3$. In similar fashion, dividing the index price-dividend ratio by the non-index price-dividend ratio (56) in the pure risk-off incentives, we obtain

$$\begin{aligned}\frac{S_t^I/I_t}{S_{2t}/D_{2t}} = \frac{\mathcal{E}_{3t}^{\text{Off}}}{\mathcal{E}_{4t}^{\text{Off}}} =& e^{\left(\mu_1-\mu_2-(\sigma_1\nu_1)\gamma_1+(\sigma_2\nu_2)\gamma_1-(\sigma_1)^2(\gamma_2-1)\right)(T-t)} \\ =& e^{\left(-\left(\lambda\sigma^2/3\right)\gamma_1+\left(\sigma^2/3\right)\gamma_1-(\sigma)^2(\gamma_2-1)\right)(T-t)} < 1. \end{aligned} \tag{A.53}$$

We obtain the second equality by plugging the definitions of $\mathcal{E}_{3t}^{\text{Off}}$ (A.7) and $\mathcal{E}_{4t}^{\text{Off}}$ (A.10) and cancelling similar terms, and the last equality by using the assumptions stating that $\mu_1 = \mu_2$, $\sigma_1 = \sigma_2$, and $\nu_1 = \lambda\sigma/3$, $\nu_2 = \sigma/3$. The inequality is true because the exponent simplifies to $\sigma^2\left(\frac{\gamma_1}{3}(1-\lambda)+1-\gamma_2\right) < 0$, which is negative because we assume that $\gamma_2 - \gamma_1 > 1$.

Lastly, dividing the index price-dividend ratio by the non-index price-dividend ratio (57) in the joint incentives case, we obtain

$$\frac{S_t^I/I_t}{S_{2t}/D_{2t}} = \frac{\frac{\mathcal{E}_{3t}^{\text{On}}}{\mathcal{H}_{3t}}}{\frac{\mathcal{E}_{4t}^{\text{On}}}{\mathcal{H}_{4t}}} = \frac{\left(\mathcal{E}_{3t}^{\text{On}} + \frac{1-\gamma_1}{\gamma_2-1}\left(V_t/I_t\right)^{\gamma_2-\gamma_1}\mathcal{E}_{3t}^{\text{Off}}\right)}{\left(\mathcal{E}_{4t}^{\text{On}} + \frac{1-\gamma_1}{\gamma_2-1}\left(V_t/I_t\right)^{\gamma_2-\gamma_1}\mathcal{E}_{4t}^{\text{Off}}\right)}, \tag{A.54}$$

where the last equality is obtained by plugging the definitions of $\mathcal{E}_{3t}^{\text{On}}$ (A.8) and $\mathcal{E}_{4t}^{\text{On}}$ (A.11) and the definitions of $\mathcal{H}_{3t}$ and $\mathcal{H}_{4t}$, (14), and cancelling similar terms. Thus, we find that $\frac{S_t^I/I_t}{S_{2t}/D_{2t}} > 1$ if and only if V/I is below the deterministic threshold in (58). The threshold is well defined because $\mathcal{E}_{3t}^{\text{On}} > \mathcal{E}_{4t}^{\text{On}}$ and $\mathcal{E}_{4t}^{\text{Off}} > \mathcal{E}_{3t}^{\text{Off}}$ since $\gamma_2 - \gamma_1 > 1$. □

Proof of Proposition 2 (Risk Exposures). We start with the joint incentives case. By applying Itô's Lemma on both sides of the deflated index price (A.44), given (A.7) and (A.8), we obtain

$$\boldsymbol{\sigma}_{1t}^S = \boldsymbol{\theta}_t + (2-\gamma_2)\,\boldsymbol{\sigma}^I - \gamma_1\boldsymbol{\nu} + (\gamma_2-\gamma_1)\left(\mathcal{H}_{3t}\right)\left(\boldsymbol{\sigma}^I - \boldsymbol{\nu}\right). \tag{A.55}$$

And, plugging θ_t (22) and rearranging leads to the desired result.

$$\boldsymbol{\sigma}_{1t}^S = \boldsymbol{\sigma}_1^D + (\gamma_2-\gamma_1)\left(\boldsymbol{\sigma}^I - \boldsymbol{\nu}\right)\left(\mathcal{H}_{3t} - \mathcal{H}_{1t}\right). \tag{A.56}$$

Note that $\mathcal{H}_{3t} > \mathcal{H}_{1t}$ since $\mathcal{E}_{1t} > \mathcal{E}_{3t}$, as (A.14) shows. Therefore, the first entry is always positive, $\boldsymbol{\sigma}_{1t}^S(1) > 0$ since $\nu_1 = \lambda\sigma/3 < \sigma$, while the second and third entries are always

negative because $\boldsymbol{\sigma}^I \equiv \boldsymbol{\sigma}_1^D = 0$ for these two entries. In similar fashion, by applying Itô's Lemma on both sides of the deflated non-index price (A.45), given (A.7) and (A.8), we obtain

$$\boldsymbol{\sigma}_{2t}^S = \boldsymbol{\theta}_t + \boldsymbol{\sigma}_2^D - (\gamma_2 - 1)\,\boldsymbol{\sigma}^I - \gamma_1 \boldsymbol{\nu} + (\gamma_2 - \gamma_1)\left(\boldsymbol{\sigma}^I - \boldsymbol{\nu}\right)(\mathcal{H}_{4t})\,. \tag{A.57}$$

And, plugging θ_t (22) and rearranging leads to the desired result. Note that $\mathcal{H}_{4t} < \mathcal{H}_{1t}$ since $\mathcal{E}_{1t} < \mathcal{E}_{4t}$, as (A.15) shows. Therefore, the first entry is always negative, $\boldsymbol{\sigma}_{2t}^S(1) < 0$, because $\nu_1 = \lambda\sigma/3 < \sigma$ and $\boldsymbol{\sigma}_2^D(1) = 0$, while the second and third entries are always positive because $\boldsymbol{\sigma}^I = 0$ and $\boldsymbol{\nu} > 0$ for the second and third entries, and $\boldsymbol{\sigma}_2^D(2) = \sigma > 0$.

$$\boldsymbol{\sigma}_{2t}^S = \boldsymbol{\sigma}_2^D + (\gamma_2 - \gamma_1)\left(\boldsymbol{\nu} - \boldsymbol{\sigma}^I\right)(\mathcal{H}_{1t} - \mathcal{H}_{4t})\,. \tag{A.58}$$

The pure incentives cases are two knife-edge cases of the joint incentives case. In the pure risk-off incentives ($\mathbf{1}_{\text{Off}} = 1, \mathbf{1}_{\text{On}} = 0$), the risk-on hedging demand equals zero, $\mathcal{H}_{it} = 0$, which leads to the simplifications in (23). Similarly, in the pure risk-on incentives ($\mathbf{1}_{\text{Off}} = 0, \mathbf{1}_{\text{On}} = 1$), the risk-on hedging demand equals one, $\mathcal{H}_{it} = 1$, which leads to the same simplifications. □

Proof of Proposition 3 (Expected Return). The index and non-index expected returns are given by (33) and (34), respectively, and obtained by taking the inner product between the market price of risk (29) and the return exposures (30) and (31). The six channels are identified by the six combinations of inner products between the two components of the market price of risk (σ^I, ν) and the three components of the asset exposures (30) and (31) (σ_i^D, σ^I, ν). By multiplying the characterizations for θ_t and $\boldsymbol{\sigma}_{1t}^S$, we obtain our desired result in (33) and (34)

Next, we show that the expected return in extreme undeperformance is strictly lower than the expected return in extreme outperformance. First, notice that the term,

$$(\gamma_2 - \gamma_1)\,(\mathcal{H}_{3t} - \mathcal{H}_{1t})\,, \tag{A.59}$$

in the first four channels is strictly positive (A.14) and approaches zero in extreme outperformance and underperformance (A.16). Therefore, in extreme outperformance the index

asset expected return becomes

$$\mu_{1t}^{S}\mid_{\mathcal{H}_{it}\downarrow 0}=(\gamma_2-1)\,\boldsymbol{\sigma}^{I'}\boldsymbol{\sigma}_1^{D}+\gamma_1\boldsymbol{\nu}'\boldsymbol{\sigma}_1^{D}, \tag{A.60}$$

while in extreme underperformance it becomes

$$\mu_{1t}^{S}\mid_{\mathcal{H}_{it}\uparrow 1}=-(1-\gamma_1)\,\boldsymbol{\sigma}^{I'}\boldsymbol{\sigma}_1^{D}+\gamma_2\boldsymbol{\nu}'\boldsymbol{\sigma}_1^{D}.$$

The difference between extreme outperformance and underperformance is strictly positive,

$$\mu_{1t}^{S}\mid_{\mathcal{H}_{it}\downarrow 0}-\mu_{1t}^{S}\mid_{\mathcal{H}_{it}\uparrow 1}=(\gamma_2-\gamma_1)\left(\boldsymbol{\sigma}^{I'}\boldsymbol{\sigma}_1^{D}-\boldsymbol{\nu}'\boldsymbol{\sigma}_1^{D}\right)>0, \tag{A.61}$$

because $\gamma_2-\gamma_1>0$, and (A.23). In similar fashion, the difference between extreme outperformance and underperformance for the non-index asset is

$$\mu_{2t}^{S}\mid_{\mathcal{H}_{it}\downarrow 0}-\mu_{2t}^{S}\mid_{\mathcal{H}_{it}\uparrow 1}=-(\gamma_2-\gamma_1)\,\boldsymbol{\nu}'\boldsymbol{\sigma}_2^{D}<0. \tag{A.62}$$

Next, we establish that the index expected return decreases in outperformance and increases in underperformance. We achieve it by showing the derivative of the expected return with respect to V/I is negative as $V/I\to\infty$ ($\mathcal{H}_{it}\to 0$) and positive as $V/I\to 0$ ($\mathcal{H}_{it}\to\infty$). We derive the derivative, $d\mu_{1t}^{S}/d(V/I)$, by plugging the derivatives of $\mathcal{H}_{it}$ and $\mathcal{H}_{it}-\mathcal{H}_{jt}$, given in (A.17) and (A.18), respectively, and grouping similar terms, which leads to

$$\begin{aligned}
d\mu_{1t}^{S}/d(V/I)=&\,(V_t/I_t)^{-1}(\gamma_2-\gamma_1)^2\,\mathcal{H}_{1t}(1-\mathcal{H}_{1t})\times\Big\{\\
&-\frac{(\mathcal{H}_{3t}-\mathcal{H}_{1t})}{\mathcal{H}_{1t}(1-\mathcal{H}_{1t})}(1-\mathcal{H}_{1t}-\mathcal{H}_{3t})\left[(\gamma_2-1)-(\gamma_2-\gamma_1)(\mathcal{H}_{1t})\right]\left(\|\boldsymbol{\sigma}^{I}\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)\\
&+\frac{(\mathcal{H}_{3t}-\mathcal{H}_{1t})}{\mathcal{H}_{1t}(1-\mathcal{H}_{1t})}(1-\mathcal{H}_{1t}-\mathcal{H}_{3t})\left[\gamma_1+(\gamma_2-\gamma_1)(\mathcal{H}_{1t})\right]\left(\|\boldsymbol{\nu}\|^2-\boldsymbol{\nu}'\boldsymbol{\sigma}^{I}\right)\\
&+(\gamma_2-\gamma_1)(\mathcal{H}_{3t}-\mathcal{H}_{1t})\,\|\boldsymbol{\sigma}^{I}-\boldsymbol{\nu}\|^2+\left(\|\boldsymbol{\sigma}^{I}\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)\Big\}.
\end{aligned} \tag{A.63}$$

The function multiplying the curly brackets is always positive and approaches zero as $V/I\to 0$ and $V/I\to\infty$ due to (A.21) and (A.22). Therefore, to show that the derivative is negative as $V/I\to\infty$, we establish that the function inside the curly brackets is negative as $V/I\to\infty$. Accordingly, the limit $V/I\to\infty$ of the function inside the curly brackets equals

$$-\frac{\mathcal{E}_{1t}-\mathcal{E}_{3t}}{\mathcal{E}_{3t}}(\gamma_2-1)\left(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)+\frac{\mathcal{E}_{1t}-\mathcal{E}_{3t}}{\mathcal{E}_{3t}}\gamma_1\left(\|\boldsymbol{\nu}\|^2-\boldsymbol{\nu}'\boldsymbol{\sigma}^I\right)+\left(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right) \tag{A.64}$$

because the limit of $\frac{(\mathcal{H}_{3t}-\mathcal{H}_{1t})}{\mathcal{H}_{1t}(1-\mathcal{H}_{1t})}$ is a strictly positive constant (A.20). Simplifying the term, we find that the derivative is negative if and only if

$$\underline{\gamma}_2 \equiv \frac{\mathcal{E}_{1t}}{\mathcal{E}_{1t}-\mathcal{E}_{3t}}+\gamma_1\frac{\left(\|\boldsymbol{\nu}\|^2-\boldsymbol{\nu}'\boldsymbol{\sigma}^I\right)}{\left(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)}<\gamma_2. \tag{A.65}$$

Similarly, the limit $V/I\to 0$ of the function inside the curly brackets equals

$$-\frac{\mathcal{E}_{1t}-\mathcal{E}_{3t}}{\mathcal{E}_{1t}}(1-\gamma_1)\left(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)-\frac{\mathcal{E}_{1t}-\mathcal{E}_{3t}}{\mathcal{E}_{1t}}\gamma_2\left(\|\boldsymbol{\nu}\|^2-\boldsymbol{\nu}'\boldsymbol{\sigma}^I\right)+\left(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right). \tag{A.66}$$

Simplifying the term, we find that the derivative is positive if and only if

$$\gamma_2<\frac{\left(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)}{\left(\|\boldsymbol{\nu}\|^2-\boldsymbol{\nu}'\boldsymbol{\sigma}^I\right)}\left(\frac{\mathcal{E}_{3t}}{\mathcal{E}_{1t}-\mathcal{E}_{3t}}+\gamma_1\right)\equiv\bar{\gamma}_2. \tag{A.67}$$

Similar to $d\mu^S_{1t}/d(V/I)$, we plug the derivatives of $\mathcal{H}_{it}$ and $\mathcal{H}_{it}-\mathcal{H}_{jt}$, given in (A.17) and (A.18), respectively, and group similar terms, leading to

$$\begin{aligned}
d\mu^S_{2t}/d(V/I) = {} & (V_t/I_t)^{-1}(\gamma_2-\gamma_1)^2\mathcal{H}_{1t}(1-\mathcal{H}_{1t})\times\Bigg\{ \\
& \frac{(\mathcal{H}_{1t}-\mathcal{H}_{4t})}{\mathcal{H}_{1t}(1-\mathcal{H}_{1t})}(1-\mathcal{H}_{1t}-\mathcal{H}_{4t})\left[(\gamma_2-1)-(\gamma_2-\gamma_1)(\mathcal{H}_{1t})\right]\left(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right) \\
& -\frac{(\mathcal{H}_{1t}-\mathcal{H}_{4t})}{\mathcal{H}_{1t}(1-\mathcal{H}_{1t})}(1-\mathcal{H}_{1t}-\mathcal{H}_{4t})\left[\gamma_1+(\gamma_2-\gamma_1)(\mathcal{H}_{1t})\right]\left(\|\boldsymbol{\nu}\|^2-\boldsymbol{\nu}'\boldsymbol{\sigma}^I\right) \\
& -(\gamma_2-\gamma_1)(\mathcal{H}_{1t}-\mathcal{H}_{4t})\|\boldsymbol{\sigma}^I-\boldsymbol{\nu}\|^2-\boldsymbol{\nu}'\boldsymbol{\sigma}^D_2\Bigg\}.
\end{aligned} \tag{A.68}$$

The function multiplying the curly brackets is always positive and approaches zero as $V/I\to 0$ and $V/I\to\infty$ due to (A.21) and (A.22). Therefore, to show that the derivative is positive as $V/I\to\infty$, we establish that the function inside the curly brackets is positive as $V/I\to\infty$.

Accordingly, the limit $V/I \to \infty$ of the function inside the curly brackets equals

$$\frac{\mathcal{E}_{4t} - \mathcal{E}_{1t}}{\mathcal{E}_{4t}} (\gamma_2 - 1) \left(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'} \boldsymbol{\nu} \right) - \frac{\mathcal{E}_{4t} - \mathcal{E}_{1t}}{\mathcal{E}_{4t}} \gamma_1 \left(\|\boldsymbol{\nu}\|^2 - \boldsymbol{\nu}' \boldsymbol{\sigma}^I \right) - \boldsymbol{\nu}' \boldsymbol{\sigma}_2^D, \tag{A.69}$$

which is positive if and only if

$$\gamma_2 > 1 + \frac{1}{(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'} \boldsymbol{\nu})} \left[\gamma_1 \left(\|\boldsymbol{\nu}\|^2 - \boldsymbol{\nu}' \boldsymbol{\sigma}^I \right) + \frac{\mathcal{E}_{4t}}{\mathcal{E}_{4t} - \mathcal{E}_{1t}} \boldsymbol{\nu}' \boldsymbol{\sigma}_2^D \right] \equiv \underline{\gamma}_2. \tag{A.70}$$

Similarly, the limit $V/I \to 0$ of the function inside the curly brackets is negative if and only if

$$\gamma_2 < \frac{1}{(\|\boldsymbol{\nu}\|^2 - \boldsymbol{\nu}' \boldsymbol{\sigma}^I)} \left[\frac{\mathcal{E}_{1t}}{\mathcal{E}_{4t} - \mathcal{E}_{1t}} \boldsymbol{\nu}' \boldsymbol{\sigma}_2^D - (1 - \gamma_1) \left(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'} \boldsymbol{\nu} \right) \right] \equiv \bar{\bar{\gamma}}_2 \tag{A.71}$$

□

Proof of Proposition 4 (Momentum and Reversal). By taking Itó's Lemma of the index asset expected return (μ_{1t}^S), given in (33), and matching the $d\boldsymbol{Z}_t$ terms with (35) we find that

$$\begin{aligned}
\boldsymbol{b}_{1t}^{\mu} = & (\gamma_2 - \gamma_1)^2 \left[(\mathcal{H}_{3t} - \mathcal{H}_{1t}) (1 - \mathcal{H}_{1t} - \mathcal{H}_{3t}) \right] \left[(\gamma_2 - 1) - (\gamma_2 - \gamma_1) (\mathcal{H}_{1t}) \right] \left(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'} \boldsymbol{\nu} \right) (\boldsymbol{\sigma}^I - \boldsymbol{\nu}) \\
& - (\gamma_2 - \gamma_1)^2 \left[(\mathcal{H}_{3t} - \mathcal{H}_{1t}) (1 - \mathcal{H}_{1t} - \mathcal{H}_{3t}) \right] \left[\gamma_1 + (\gamma_2 - \gamma_1) (\mathcal{H}_{1t}) \right] \left(\|\boldsymbol{\nu}\|^2 - \boldsymbol{\nu}' \boldsymbol{\sigma}^I \right) (\boldsymbol{\sigma}^I - \boldsymbol{\nu}) \\
& - (\gamma_2 - \gamma_1)^3 (\mathcal{H}_{3t} - \mathcal{H}_{1t}) \left[\mathcal{H}_{1t} (1 - \mathcal{H}_{1t}) \right] \|\boldsymbol{\sigma}^I - \boldsymbol{\nu}\|^2 (\boldsymbol{\sigma}^I - \boldsymbol{\nu}) \\
& - \left[(\gamma_2 - \gamma_1)^2 \mathcal{H}_{1t} (1 - \mathcal{H}_{1t}) \right] \left(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'} \boldsymbol{\nu} \right) (\boldsymbol{\sigma}^I - \boldsymbol{\nu}) .
\end{aligned} \tag{A.72}$$

By taking the inner product of $\boldsymbol{b}_{1t}^{\mu}$ and $\boldsymbol{\sigma}_{1t}^{S}$, we find that

$$
\begin{aligned}
\boldsymbol{\sigma}_{1t}^{S'}\boldsymbol{b}_{1t}^{\mu} = & \left(\gamma_2-\gamma_1\right)^2\mathcal{H}_{1t}\left(1-\mathcal{H}_{1t}\right)\times\Bigg\{ \\
& \left(\gamma_2-\gamma_1\right)\frac{\left(\mathcal{H}_{3t}-\mathcal{H}_{1t}\right)^2}{\mathcal{H}_{1t}\left(1-\mathcal{H}_{1t}\right)}\left(1-\mathcal{H}_{1t}-\mathcal{H}_{3t}\right)\left[\left(\gamma_2-1\right)-\left(\gamma_2-\gamma_1\right)\left(\mathcal{H}_{1t}\right)\right]\left(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)\|\boldsymbol{\sigma}^I-\boldsymbol{\nu}\|^2 \\
& -\left(\gamma_2-\gamma_1\right)\frac{\left(\mathcal{H}_{3t}-\mathcal{H}_{1t}\right)^2}{\mathcal{H}_{1t}\left(1-\mathcal{H}_{1t}\right)}\left(1-\mathcal{H}_{1t}-\mathcal{H}_{3t}\right)\left[\gamma_1+\left(\gamma_2-\gamma_1\right)\left(\mathcal{H}_{1t}\right)\right]\left(\|\boldsymbol{\nu}\|^2-\boldsymbol{\nu}'\boldsymbol{\sigma}^I\right)\|\boldsymbol{\sigma}^I-\boldsymbol{\nu}\|^2 \\
& -\left(\gamma_2-\gamma_1\right)^2\left(\mathcal{H}_{3t}-\mathcal{H}_{1t}\right)^2\|\boldsymbol{\sigma}^I-\boldsymbol{\nu}\|^2\|\boldsymbol{\sigma}^I-\boldsymbol{\nu}\|^2 \\
& -\left(\gamma_2-\gamma_1\right)\left(\mathcal{H}_{3t}-\mathcal{H}_{1t}\right)\left(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)\|\boldsymbol{\sigma}^I-\boldsymbol{\nu}\|^2 \\
& \frac{\left(\mathcal{H}_{3t}-\mathcal{H}_{1t}\right)}{\mathcal{H}_{1t}\left(1-\mathcal{H}_{1t}\right)}\left(1-\mathcal{H}_{1t}-\mathcal{H}_{3t}\right)\left[\left(\gamma_2-1\right)-\left(\gamma_2-\gamma_1\right)\left(\mathcal{H}_{1t}\right)\right]\left(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)^2 \\
& -\frac{\left(\mathcal{H}_{3t}-\mathcal{H}_{1t}\right)}{\mathcal{H}_{1t}\left(1-\mathcal{H}_{1t}\right)}\left(1-\mathcal{H}_{1t}-\mathcal{H}_{3t}\right)\left[\gamma_1+\left(\gamma_2-\gamma_1\right)\left(\mathcal{H}_{1t}\right)\right]\left(\|\boldsymbol{\nu}\|^2-\boldsymbol{\nu}'\boldsymbol{\sigma}^I\right)\left(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right) \\
& -\left(\gamma_2-\gamma_1\right)\left(\mathcal{H}_{3t}-\mathcal{H}_{1t}\right)\|\boldsymbol{\sigma}^I-\boldsymbol{\nu}\|^2\left(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right) \\
& -\left(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)^2\Bigg\}. \qquad \text{(A.73)}
\end{aligned}
$$

The function multiplying the curly brackets is always positive and approaches zero as $V/I \to 0$ and $V/I \to \infty$ due to (A.21) and (A.22). Therefore, to show that the momentum condition is satisfied as $V/I \to \infty$, we establish that the function inside the curly brackets is positive as $V/I \to \infty$ ($\mathcal{H}_{it} \to 0$). Accordingly, the limit $V/I \to \infty$ ($\mathcal{H}_{it} \to 0$) of the function inside the curly brackets is positive if and only if

$$\gamma_2 > \underline{\gamma}_2. \qquad \text{(A.74)}$$

Similarly, the reversal condition is satisfied as $V/I \to 0$ ($\mathcal{H}_{it} \to 1$), if and only if

$$\gamma_2 < \bar{\gamma}_2. \qquad \text{(A.75)}$$

We conclude by showing that there always exist an active share λ such that

$$\underline{\gamma}_2 < \bar{\gamma}_2. \qquad \text{(A.76)}$$

First, comparing the factors multiplying γ_1 in (A.65) and (A.67), it is clear that

$$\gamma_1 \frac{\left(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)}{\left(\|\boldsymbol{\nu}\|^2 - \boldsymbol{\nu}'\boldsymbol{\sigma}^I\right)} > \gamma_1 \frac{\left(\|\boldsymbol{\nu}\|^2 - \boldsymbol{\nu}'\boldsymbol{\sigma}^I\right)}{\left(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)},$$

due to (A.25). Second, comparing the remaining factors in (A.65) and (A.67), it is clear that

$$\frac{\left(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)}{\left(\|\boldsymbol{\nu}\|^2 - \boldsymbol{\nu}'\boldsymbol{\sigma}^I\right)} \frac{\mathcal{E}_{3t}}{\mathcal{E}_{1t} - \mathcal{E}_{3t}} > \frac{\mathcal{E}_{1t}}{\mathcal{E}_{1t} - \mathcal{E}_{3t}} \tag{A.77}$$

because the denominator shrinks to zero as the active increases, (A.24). Therefore, there is always exist a γ_2 such that index asset exhibits momentum and reversal.

We continue with the non-index asset. By taking Itó's Lemma of the non-index asset expected return (μ_{2t}^S), given in (34), and matching the $d\boldsymbol{Z}_t$ terms with (35) we find that

$$\begin{aligned}
\boldsymbol{b}_{2t}^{\mu} = &-(\gamma_2-\gamma_1)^2(\mathcal{H}_{1t}-\mathcal{H}_{4t})(1-\mathcal{H}_{1t}-\mathcal{H}_{4t})\left[(\gamma_2-1)-(\gamma_2-\gamma_1)(\mathcal{H}_{1t})\right]\left(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\nu}'\boldsymbol{\sigma}^I\right)(\boldsymbol{\sigma}^I-\boldsymbol{\nu}) \\
&+(\gamma_2-\gamma_1)^2(\mathcal{H}_{1t}-\mathcal{H}_{4t})(1-\mathcal{H}_{1t}-\mathcal{H}_{4t})\left[\gamma_1+(\gamma_2-\gamma_1)(\mathcal{H}_{1t})\right]\left(\|\boldsymbol{\nu}\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)(\boldsymbol{\sigma}^I-\boldsymbol{\nu}) \\
&+(\gamma_2-\gamma_1)^3(\mathcal{H}_{1t}-\mathcal{H}_{4t})\mathcal{H}_{1t}(1-\mathcal{H}_{1t})\|\boldsymbol{\sigma}^I-\boldsymbol{\nu}\|^2(\boldsymbol{\sigma}^I-\boldsymbol{\nu}) \\
&+(\gamma_2-\gamma_1)^2(\mathcal{H}_{1t}(1-\mathcal{H}_{1t}))\boldsymbol{\nu}'\boldsymbol{\sigma}_2^D(\boldsymbol{\sigma}^I-\boldsymbol{\nu}).
\end{aligned} \tag{A.78}$$

By taking the inner product of $\boldsymbol{b}_{2t}^{\mu}$ and $\boldsymbol{\sigma}_{2t}^S$, we find that

$$\begin{aligned}
\boldsymbol{\sigma}_{2t}^{S'}\boldsymbol{b}_{2t}^{\mu} = &(\gamma_2-\gamma_1)^2\mathcal{H}_{1t}(1-\mathcal{H}_{1t})\times\Big\{ \\
&(\gamma_2-\gamma_1)\frac{(\mathcal{H}_{1t}-\mathcal{H}_{4t})^2}{\mathcal{H}_{1t}(1-\mathcal{H}_{1t})}(1-\mathcal{H}_{1t}-\mathcal{H}_{4t})\left[(\gamma_2-1)-(\gamma_2-\gamma_1)(\mathcal{H}_{1t})\right]\left(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\nu}'\boldsymbol{\sigma}^I\right)\|\boldsymbol{\sigma}^I-\boldsymbol{\nu}\|^2 \\
&-(\gamma_2-\gamma_1)\frac{(\mathcal{H}_{1t}-\mathcal{H}_{4t})^2}{\mathcal{H}_{1t}(1-\mathcal{H}_{1t})}(1-\mathcal{H}_{1t}-\mathcal{H}_{4t})\left[\gamma_1+(\gamma_2-\gamma_1)(\mathcal{H}_{1t})\right]\left(\|\boldsymbol{\nu}\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)\|\boldsymbol{\sigma}^I-\boldsymbol{\nu}\|^2 \\
&-(\gamma_2-\gamma_1)(\mathcal{H}_{1t}-\mathcal{H}_{4t})\boldsymbol{\nu}'\boldsymbol{\sigma}_2^D\|\boldsymbol{\sigma}^I-\boldsymbol{\nu}\|^2 \\
&-(\gamma_2-\gamma_1)^2(\mathcal{H}_{1t}-\mathcal{H}_{4t})^2\|\boldsymbol{\sigma}^I-\boldsymbol{\nu}\|^2\|\boldsymbol{\sigma}^I-\boldsymbol{\nu}\|^2 \\
&-\frac{(\mathcal{H}_{1t}-\mathcal{H}_{4t})}{\mathcal{H}_{1t}(1-\mathcal{H}_{1t})}(1-\mathcal{H}_{1t}-\mathcal{H}_{4t})\left[(\gamma_2-1)-(\gamma_2-\gamma_1)(\mathcal{H}_{1t})\right]\left(\|\boldsymbol{\sigma}^I\|^2-\boldsymbol{\nu}'\boldsymbol{\sigma}^I\right)\boldsymbol{\sigma}_2^{D\prime}(\boldsymbol{\sigma}^I-\boldsymbol{\nu}) \\
&+\frac{(\mathcal{H}_{1t}-\mathcal{H}_{4t})}{\mathcal{H}_{1t}(1-\mathcal{H}_{1t})}(1-\mathcal{H}_{1t}-\mathcal{H}_{4t})\left[\gamma_1+(\gamma_2-\gamma_1)(\mathcal{H}_{1t})\right]\left(\|\boldsymbol{\nu}\|^2-\boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)\boldsymbol{\sigma}_2^{D\prime}(\boldsymbol{\sigma}^I-\boldsymbol{\nu}) \\
&+\left(\boldsymbol{\nu}'\boldsymbol{\sigma}_2^D\right)\boldsymbol{\sigma}_2^{D\prime}(\boldsymbol{\sigma}^I-\boldsymbol{\nu})+(\gamma_2-\gamma_1)(\mathcal{H}_{1t}-\mathcal{H}_{4t})\|\boldsymbol{\sigma}^I-\boldsymbol{\nu}\|^2\boldsymbol{\sigma}_2^{D\prime}(\boldsymbol{\sigma}^I-\boldsymbol{\nu})\Big\}.
\end{aligned} \tag{A.79}$$

The function multiplying the curly brackets is always positive and approaches zero as

$V/I \to 0$ and $V/I \to \infty$ due to (A.21) and (A.22). Therefore, to show that the reversal condition is satisfied as $V/I \to \infty$, we establish that the function inside the curly brackets is negative as $V/I \to \infty$ ($\mathcal{H}_{it} \to 0$). Accordingly, the limit $V/I \to \infty$ ($\mathcal{H}_{it} \to 0$) of the function inside the curly brackets is positive if and only if

$$\gamma_2 > \underset{=2}{\gamma}. \tag{A.80}$$

Similarly, the reversal condition is satisfied as $V/I \to 0$ ($\mathcal{H}_{it} \to 1$), if and only if

$$\gamma_2 < \bar{\bar{\gamma}}_2 \tag{A.81}$$

We conclude the proof by showing that there always exist an active share λ such that

$$\underset{=2}{\gamma} < \bar{\bar{\gamma}}_2. \tag{A.82}$$

First, comparing the factors multiplying γ_1 in (A.70) and (A.71), it is clear that

$$\gamma_1 \frac{\left(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)}{\left(\|\boldsymbol{\nu}\|^2 - \boldsymbol{\nu}'\boldsymbol{\sigma}^I\right)} > \gamma_1 \frac{\left(\|\boldsymbol{\nu}\|^2 - \boldsymbol{\nu}'\boldsymbol{\sigma}^I\right)}{\left(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)}, \tag{A.83}$$

due to (A.25). Second, comparing the remaining factors in (A.70) and (A.71), it is clear that

$$1 + \frac{1}{\left(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)}\left[\frac{\mathcal{E}_{4t}}{\mathcal{E}_{4t} - \mathcal{E}_{1t}}\boldsymbol{\nu}'\boldsymbol{\sigma}_2^D\right] < \frac{1}{\left(\|\boldsymbol{\nu}\|^2 - \boldsymbol{\nu}'\boldsymbol{\sigma}^I\right)}\left[\frac{\mathcal{E}_{1t}}{\mathcal{E}_{4t} - \mathcal{E}_{1t}}\boldsymbol{\nu}'\boldsymbol{\sigma}_2^D - \left(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)\right] \tag{A.84}$$

is true if

$$\frac{\mathcal{E}_{1t}}{\mathcal{E}_{4t} - \mathcal{E}_{1t}} \geq \frac{\left(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'}\boldsymbol{\nu}\right)}{\boldsymbol{\nu}'\boldsymbol{\sigma}_2^D}. \tag{A.85}$$

The denominator in the right hand side of (A.84) shrinks to zero as the active increases due to (A.24). Therefore, there is always exist a γ_2 such that the non-index asset exhibits momentum in outperformance and reversal in underperformance. □

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