# Faraway, So Close:
# Business Cycle Effect of Long-Run Ambiguity

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**Abstract**

*This paper explores forward-looking ambiguity (Knightian uncertainty) in a model with homogeneous workers and credit-constrained heterogeneous entrepreneurs. Agents are ambiguity-averse, using a worst-case criterion to form expectations about future productivity. We compare our economy with one that lacks uncertainty and find that ambiguity: (i) lowers the productivity threshold for market entry, (ii) reduces the equilibrium interest rate, and (iii) shifts expenditures between workers and entrepreneurs. These results stem from persistent expectation-realization mismatches. While ambiguity does not affect stability, it alters the convergence rate to the steady state and helps explain key macroeconomic comovements.*

**Keywords**: ambiguity, collateral constraints, heterogeneous agents, transition dynamics.
**JEL codes**: E22, D81, D84, G14.

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*Count not that far that can be had,*
*Though sunset lie between –*
*Nor that adjacent, that beside,*
*Is further than the sun.*
Emily Dickinson.

# 1 Introduction

This paper proposes a business cycle theory in which exogenous ambiguity shocks drive movements in macroeconomic variables. Agents are ambiguity-averse in the sense that they dislike uncertainty and unquantifiable knowledge about future events. As a result, they make decisions based on worst-case probabilities derived from a spectrum of possible outcomes. We explore how ambiguity interacts with borrowing constraints in a credit-constrained economy, and how such effects are amplified by heterogeneity. Specifically, we integrate the concept of ambiguity as described by Epstein and Schneider (2007), Ilut and Schneider (2014), and Bidder and Dew-Becker (2016) into the analytical framework proposed by Itskhoki and Moll (2019).

We consider an economy populated by two types of infinite-lived agents, workers and entrepreneurs. Workers, who are are homogeneous, consume, save by holding bonds, and decide how much labor to supply. Entrepreneurs, who are heterogeneous in terms of assets and productivity, face borrowing constraints that limit the amount of capital they can use. Depending on the realization of a random productivity shock, they decide whether to engage in production (being *active*) or exit the market (being *inactive*). They also consume and accumulate wealth. Due to heterogeneity and borrowing constraints, entrepreneurs make positive profits.

Both workers and entrepreneurs face *Knightian uncertainty* and are ambiguity-averse. We assume that they employ a maxmin expected utility criterion (Gilboa and Schmeidler, 1989). Ambiguity pertains to the future levels of total factor productivity, although its present value is known with certainty. The interaction between certain knowledge of the current total factor productivity and forward-looking ambiguity on its future realizations drives the economy away from the steady state that would be achieved under perfect foresight. By comparing our *ambiguous* economy to the one with perfect foresight we observe the following. To hedge against the worst-case scenario, workers increase their savings, generating an excess capital supply. This leads to a lower long-run equilibrium interest rate. Interestingly, this form of ambiguity makes workers better off while entrepreneurs are worse off. Lastly, we observe that greater heterogeneity implies more significant and persistent effects of ambiguity on macroeconomic variables. Importantly, these results do not change in essence even when agents need to solve a signal extraction problem to estimate the worst-case scenario.

To sum up, the paper provides two main contributions. First, it investigates the role of forward-looking ambiguity in business cycle fluctuations (i.e., it proposes a theory on how future ambiguity influences current saving decisions and interacts with back-propagation effects). An ambiguity shock triggers procyclical movements in the interest rate, aggregate wealth, and workers' consumption. Second, it sheds light on the difference between short-term fluctuations driven by resolving ambiguity shocks and long-term trajectories shaped by persistent ambiguity, steering the economy away from the perfect-foresight steady state.

**Related literature** We build upon the models presented in Buera and Moll (2015) and Itskhoki and Moll (2019). Buera and Moll (2015) examines the aggregate implications of a credit crunch under various specifications of heterogeneity, highlighting the critical role of heterogeneity in understanding the sources of business cycles. Itskhoki and Moll (2019) sets the same model in continuous time to study optimal development policies and dynamic taxation. Aside from incorporating uncertainty, our economy shares the same fundamentals as the one analyzed in Itskhoki and Moll (2019) in its closed-economy version, which can serve as a benchmark. A similar setup is also found in Banerjee and Moll (2010), which introduces the concept of distinct types of capital misallocation – intensive and extensive – distinguishing the quantity of capital from the quality of its usage and attributing persistent inefficiencies to binding financial constraints and heterogeneity.

Modeling heterogeneity among entrepreneurs as a productivity shock with a Pareto distribution aligns with recent literature – see, e.g., Gabaix et al. (2016), Itskhoki and Moll (2019), and Achdou et al. (2022) – and empirical observations (Jones, 2015; Jones and Kim, 2018), which describe the right tail of the wealth distribution as following a Pareto distribution. This approach captures the extreme variability in productivity among entrepreneurs, highlighting that only those with sufficiently high productivity levels engage in production at the aggregate level.

We model ambiguity following Epstein and Schneider (2003), Epstein and Schneider (2007), and Ilut and Schneider (2014). Epstein and Schneider (2007) analyzes an economy where agents' confidence evolves as they acquire new information. Similarly, Ilut and Schneider (2014) examines a New Keynesian business cycle model that incorporates confidence shocks, demonstrating how variations in ambiguity can generate business cycles. Unlike these studies, we do not rely on nominal rigidities. We explore the interplay between ambiguity and heterogeneity, setting the economy within a continuous-time framework and specifically focus on forward-looking ambiguity. This forward-looking approach is similar to that of Bidder and Dew-Becker (2016), where long-run ambiguity influences investors' asset pricing decisions, and Bianchi and Melosi (2016),

where agents are subject to belief waves.

In a small open economy with credit constraints, entrepreneurial heterogeneity, and *hand-to-mouth* workers, Carbonari and Maurici (2023) shows that *present* ambiguity has both short- and long-run implications. Similar to what happens in our economy, agents' hedging strategies are at the root of short-run fluctuations. However, the underlying mechanism is entirely different, as in our model, workers can accumulate assets, and ambiguity results in higher steady-state consumption levels for them.

Other recent models with ambiguity that have examined business cycle phenomena are Nimark (2014), Altug et al. (2020), and Michelacci and Paciello (2024). Nimark (2014) presents a business cycle model with higher-order beliefs and considers the impact of signals observed after unusual events that increase uncertainty and disagreement among agents. Using a smooth ambiguity model (Klibanoff et al., 2005, 2009), Altug et al. (2020) studies the business cycle implications of ambiguity aversion in a model in a standard neoclassical model with investment irreversibility. Michelacci and Paciello (2024) proposes a New Keynesian model with ambiguity-averse agents and associate their worst-case beliefs to the credibility of the monetary authority actions.

Classic references on ambiguity, robustness and model misspecification are Bewley (2002), Hansen and Sargent (2011), and Hansen and Sargent (2022). Chen and Epstein (2002), Epstein and Ji (2013), Lin and Riedel (2021), and Hansen and Miao (2022) explore ambiguity in continuous-time settings within the context of asset pricing.

**Outline** The paper is structured as follows. In Section 2, we lay out the structure of the economy and derive the optimal choices, both individual and aggregate, focusing on consumption and investment decisions. In Section 3, we characterize the competitive equilibrium under forward-looking ambiguity. In Section 4, we study the business cycle dynamics. Section 5 concludes. Proofs and derivations are in the Online Appendix.

# 2 Model

Consider an infinite-horizon closed economy operating in continuous time. As in Itskhoki and Moll (2019), there are two types of agents, each with unit mass. On one side, identical *workers* consume ($c$) and work during each period. They supply a variable quantity of labor hours ($\ell$) and save a fraction of their income by holding an asset ($b$). On the other side, heterogeneous *entrepreneurs*, differing in their productivity ($z$) and asset holdings ($a$), run the production process. Entrepreneurs hire workers at a competitive wage $w$ and rent capital at rate $r$. Firms combine labor ($n$) and capital ($k$) to produce a final good, which serves as the numeraire and is traded in a competitive market. In what follows, let $t^+ = \lim_{\tau \to t^+} \tau$ indicate the *next period* in a continuous

setting, meaning that $t^+ \mid t$ should be read as the next-period-ahead forecast. For notational convenience, we will omit time subscripts unless they are necessary.

**Ambiguity** Workers and entrepreneurs are ambiguity-averse. We assume the existence of market-wide information regarding the set of perceived distributions $(\mathcal{P}_{t^+|t})$ of *instantaneous* future TFP, based on the information available at time $t$.[1] Hence, ambiguity is represented by all possible elements $P \in \mathcal{P}_{t^+|t}$. Ambiguity influences choices through expectations about the aggregate total factor productivity (TFP). We model ambiguity aversion by considering a utility function that is the minimum of the expected utilities over the collection $\mathcal{P}_{t^+|t}$.

We denote the *true* TFP series as $\{A_\tau\}_{\tau \geq t}$. As in Ilut and Schneider (2014), the stochastic process governing the TFP is unknown to the agents, who face ambiguity regarding its future realizations.

For any $t$ and any $P \in \mathcal{P}_{t^+|t}$, we define the *perceived* TFP as the random variable $A^P_{t^+|t}$ defined on a probability space $(\Omega^P, \mathscr{F}^P, P)$. For a given $P$, the $\sigma$-algebra $\mathscr{F}^P$ conveys the information available at time $t$. Let $\mathcal{A}_t$ be the set of all points that have non-zero probability under at least one $P \in \mathcal{P}_{t^+|t}$,

$$\mathcal{A}_t = \bigcup_{P \in \mathcal{P}_{t^+|t}} \operatorname{supp}(P) \subseteq \mathbb{R},$$

and $\underline{A}_t \in \mathbb{R}$ be

$$\underline{A}_t = \inf \mathcal{A}_t.$$

We assume that, in any period, all the possible outcomes are bounded from below by $\underline{A}_t$,

$$P\left(A^P_{t^+|t} \geq \underline{A}_t\right) = 1,$$

for all $P \in \mathcal{P}_{t^+|t}$, with $\underline{A}_t > 0$.[2] We also assume that the Dirac-delta distribution centered at $\underline{A}_t$ is an element of the set of perceived distributions,[3]

$$\mathcal{D}(\underline{A}_t) \in \mathcal{P}_{t^+|t}.$$

This last assumption simplifies the analysis by restricting the focus on $\mathcal{D}(\underline{A}_t)$ within $\mathcal{P}_{t^+|t}$. Specifically, for the Dirac-delta distribution, the upper and lower bounds of the expected TFP coincide at $\underline{A}_t$, whereas for all other elements in $\mathcal{P}_{t^+|t}$, the upper bound must be strictly higher than $\underline{A}_t$.[4] Consequently, ambiguity-averse agents will exclusively

[1] This assumption can be seen as a limiting case of De Long et al. (1990).

[2] This is the continuous-time adaptation of Ilut and Schneider (2014), denoting the "*indicator of the quality of intangible information available at date t about TFP at t + 1*".

[3] Since our notation is the same as Itskhoki and Moll (2019), we denote the Dirac-delta distribution with $\mathcal{D}$ to avoid confusion with the entrepreneurs' discount rate, $\delta$, which will be defined below.

[4] This ensures that, by convexity, for all $P \in \mathcal{P}_{t^+|t} \backslash \mathcal{D}(\underline{A}_t)$ we have $\underline{A}_t < \mathbb{E}_P\left[A^P_{t^+|t}\right]$.

focus on $\mathcal{D}(\underline{A}_t)$, meaning the Dirac-delta distribution represents the worst-case scenario.

Workers and entrepreneurs behave as if they are optimizing based on a worst-case distribution, which is different from actual data-generating process.

For the sake of exposition, we start by setting each element of the sequence $\{A_\tau\}_{\tau \geq t}$ equal to $A$, and each perceived TFP lower bound $\{\underline{A}_\tau\}_{\tau \geq t}$ constant at $\underline{A}$, where $A > \underline{A}$. In Section 4, the lower bound is treated as time-varying to analyze the business cycle.

**Workers** The representative worker's preferences are given by

$$\min_{P \in \mathcal{P}_{t^+|t}} \mathbb{E}_P \int_t^\infty e^{-\rho(\tau - t)} u(c_\tau, \ell_\tau) \, \mathrm{d}\tau, \tag{1}$$

where $\rho$ is the workers' discount rate, and

$$u(c, \ell) = \ln c - \psi \frac{\ell^{1+\phi}}{1+\phi},$$

with $\psi, \phi > 0$. The worker is subject to the flow budget constraint

$$c + \dot{b} = rb + w\ell. \tag{2}$$

The resulting optimality conditions are

$$-\frac{u_\ell}{u_c} = \psi c \ell^\phi = w, \tag{3}$$

$$\frac{\dot{u}_c}{u_c} = \frac{\dot{c}}{c} = r - \rho, \tag{4}$$

along with the proper transversality condition.

**Entrepreneurs** Assuming a logarithmic instantaneous utility function, an entrepreneur maximizes

$$\min_{P \in \mathcal{P}_{t^+|t}} \mathbb{E}_P \int_t^\infty e^{-\delta(\tau - t)} \ln c_\tau^e \, \mathrm{d}\tau, \tag{5}$$

where $\delta$ is the entrepreneurs' discount rate, subject to the flow budget constraint

$$c^e + \dot{a} = \pi(a, z) + ra. \tag{6}$$

Given continuous time and logarithmic preferences, the optimal individual consumption for each entrepreneur is $c^e = \delta a$. Equation (6) characterizes the dynamics of entrepreneurial wealth, leading to a time-dependent distribution of assets, denoted by $\mathcal{G}_t(a)$.

We assume that idiosyncratic productivity follows a Pareto distribution with a shape parameter $\eta > 1$.[5] Productivity shocks are i.i.d. over time. The distribution function $\mathcal{F}(z)$ is supported on the interval $[1, \infty)$, and its density function is given by $\eta z^{-\eta-1}$.

Financial markets are incomplete, in that entrepreneurs face a collateral constraint given by

$$k \leq \lambda a, \tag{7}$$

with $\lambda \geq 1$.[6]

Upon making production decisions, $A$ is known, and the entrepreneur's production function is

$$A(zk)^{\alpha} n^{1-\alpha}, \tag{8}$$

with $\alpha \in (0, 1)$, while profits are

$$A(zk)^{\alpha} n^{1-\alpha} - wn - rk.$$

Profit maximization gives

$$k(a, z) = \lambda a \, \mathbb{I}(z \geq \underline{z}), \tag{9}$$

$$n(a, z) = \left[\frac{1-\alpha}{w} A\right]^{\frac{1}{\alpha}} zk(a, z), \tag{10}$$

$$\pi(a, z) = \alpha \left[\frac{1-\alpha}{w}\right]^{\frac{1-\alpha}{\alpha}} A^{\frac{1}{\alpha}} zk(a, z) - rk(a, z), \tag{11}$$

where $\underline{z}$ is the minimum level of idiosyncratic productivity to enter the market. Let $\zeta \equiv \frac{r}{\alpha}\left[\frac{w}{1-\alpha}\right]^{\frac{1-\alpha}{\alpha}} A^{-\frac{1}{\alpha}}$,[7] then

$$\underline{z} = \max\{\zeta, 1\}. \tag{12}$$

Condition (12) states that the cutoff level of $z$ is the maximum between the zero-profit productivity, $\zeta$, and the lower bound of the distribution $\mathcal{F}(z)$, 1. For the sake of simplicity, we restrict our attention to the parameter space (and initial conditions) such that $\underline{z} = \zeta > 1$.

[5]The shape parameter $\eta$ determines the tail heaviness of the distribution. A higher $\eta$ results in a thinner tail, indicating that extreme values are less common. As highlighted above, the motivation for this assumption is empirical, since the right tail of the wealth distribution in many countries is Pareto-shaped (Jones and Kim, 2018). Additionally, assuming a Pareto distribution simplifies the derivation of aggregate properties of our model economy.

[6]This is equivalent to imposing a debt limit on entrepreneurs. An entrepreneur can leverage debt, $d$, backed by a fraction $\vartheta$ of the acquired capital, $k$, which acts as collateral (i.e., $d \leq \vartheta k$). $\vartheta$ can be interpreted as the loan-to-value ratio (LTV). Entrepreneur's net worth is then given by $a \equiv k - d$, and the debt constraint becomes (7), where $\lambda \equiv (1-\vartheta)^{-1}$. Moreover, as $\lim_{\vartheta \to 1^-} \lambda = \infty$ it signals the efficient functioning of capital markets.

[7]Derived by setting (11) to zero and solving for $z$.

**Aggregation** The evolution of $\mathcal{G}_t(a)$ over time is relatively simple, as (6) can be expressed as a linear function of $a$. Consequently, it becomes feasible to monitor the evolution of aggregate entrepreneurial wealth. Let $x$ represent the total assets of entrepreneurs, given by $\int a \, \mathrm{d}i$, so $\int c^e \, \mathrm{d}i = \delta x$.

The aggregate levels of capital and labor demand can be expressed as

$$\kappa = \lambda \underline{z}^{-\eta} x, \tag{13}$$

$$\ell = \lambda \frac{\eta}{\eta - 1} \left[ \frac{1-\alpha}{w} A \right]^{\frac{1}{\alpha}} \underline{z}^{1-\eta} x. \tag{14}$$

Factor shares are given by

$$w\ell = (1-\alpha) y, \tag{15}$$

$$r\kappa = \frac{\alpha(\eta - 1)}{\eta} y, \tag{16}$$

with aggregate profits equal to

$$\frac{\alpha}{\eta} y.$$

The aggregate production is

$$y = A \Theta x^{\frac{\alpha}{\eta}} \kappa^{\frac{\alpha(\eta-1)}{\eta}} \ell^{1-\alpha}, \tag{17}$$

with $\Theta \equiv \left( \frac{\eta}{\eta - 1} \right)^{\alpha} \lambda^{\frac{\alpha}{\eta}}$, while the law of motion of the entrepreneurs' wealth is

$$\dot{x} = \frac{\alpha}{\eta} y + (r - \delta) x. \tag{18}$$

Finally, the feasibility constraint can be written as

$$\dot{\kappa} = y - \delta x - c. \tag{19}$$

# 3 Equilibrium under ambiguity

At each time $t$, given the initial aggregate levels for entrepreneurial wealth $x_0$ and workers' savings $b_0$, actual TFP $A$, and the worst-case distribution $\mathcal{D}(\underline{A})$, a competitive equilibrium consists of a sequence of aggregate variables $\{x, b, \kappa, y, c, \ell, \underline{z}\}_{\tau \geq t}$, wage $\{w\}_{\tau \geq t}$, and interest rate $\{r\}_{\tau \geq t}$ such that: (i) workers maximize (1) subject to (2), taking prices as given, (ii) entrepreneurs maximize (5) subject to (6), taking prices as given, (iii) market clearing conditions hold, (iv) entrepreneurial wealth evolves according to (18), and (v) aggregate wealth evolves according to (19). A steady-state equilibrium is a competitive equilibrium in which real variables are time-invariant.

**Ambiguous steady state** In this model, the ambiguity pertains to the future realization of the aggregate TFP. Consider now the following thought experiment. Based on the agents' expectation that the *true* level of TFP is $\underline{A}$, we can compute a worst-case steady state equilibrium, which we refer to as *ambiguous*. Since $A > \underline{A}$, the economy never actually reaches this allocation. However, due to pessimistic expectations, agents believe the opposite and make their decisions accordingly. Let $\widehat{s}$ denote the steady-state value for any variable $s$. *If* the true realization of the TFP were $\underline{A}$, we would have

$$\widehat{r} = \rho, \tag{20}$$

$$\widehat{c} = \left(1 - \frac{\delta}{\delta - \rho}\frac{\alpha}{\eta}\right)\widehat{y}, \tag{21}$$

$$\widehat{x} = \frac{1}{\delta - \rho}\frac{\alpha}{\eta}\widehat{y}, \tag{22}$$

$$\widehat{\kappa} = \frac{\alpha}{\rho}\frac{\eta - 1}{\eta}\widehat{y}, \tag{23}$$

$$\widehat{b} = \left(\alpha - \frac{\delta}{\delta - \rho}\frac{\alpha}{\eta}\right)\frac{\widehat{y}}{\rho}, \tag{24}$$

$$\widehat{\ell} = \left[\frac{1-\alpha}{\psi}\left(1 - \frac{\delta}{\delta - \rho}\frac{\alpha}{\eta}\right)^{-1}\right]^{\frac{1}{1+\phi}}, \tag{25}$$

$$\widehat{w} = (1-\alpha)\frac{\widehat{y}}{\widehat{\ell}}, \tag{26}$$

$$\widehat{y} = \underline{A}\Theta\widehat{x}^{\frac{\alpha}{\eta}}\widehat{\kappa}^{\frac{\alpha(\eta-1)}{\eta}}\widehat{\ell}^{1-\alpha}. \tag{27}$$

Notice that (21) gives the necessary condition $(\delta - \rho)\eta > \delta\alpha$. Moreover, to have $\widehat{b} > 0$, from (24), we must have $(\delta - \rho)\eta > \delta$. Lastly, combining (22) and (23) with (13), we get a necessary condition for $\widehat{\underline{z}} > 1$, that is, $\rho\lambda > (\delta - \rho)(\eta - 1)$.

The system (20)-(27) represents the steady state of an economy conditional on $\mathcal{D}(\underline{A})$, with $A > \underline{A}$ (the lowest lower bound among all distributions within $\mathcal{P}_{t^+|t}$). This instance, however, never occurs, since $A > \underline{A}$ by assumption. This implies that, under ambiguity, the economy never reaches such a long-run equilibrium. Nonetheless, as we will show, these conditions are essential to characterize the behavior of our economy. Indeed, agents systematically underestimate the actual value of the TFP, and the equilibrium dynamics arises from the interplay between *ex ante* pessimism and actual realizations. It turns out that the ambiguous steady state, achieved under the worst-case belief, is the point around which to linearize the behavior of the economy. This is because agents behave *as if* the economy converges there in the long run (Ilut and Schneider, 2014).[8]

It is convenient to express all endogenous variables in (4), (18), and (19) as functions of $c$, $x$, and $\kappa$. Equating (3) and (15) at the expected steady state, ambiguous aggregate

[8]In fact, the system (20)-(27) represents the terminal conditions in both economies, where $\underline{A}$ denotes either the actual TFP or its ambiguous expectation.

production can be rewritten as

$$\widehat{y} = \underline{A}^{\frac{\omega}{\alpha}} \Omega \widehat{x}^{\frac{\omega}{\eta}} \widehat{\kappa}^{\frac{\omega(\eta-1)}{\eta}} \widehat{c}^{1-\frac{\omega}{\alpha}}, \tag{28}$$

where $\omega \equiv \alpha \frac{1+\phi}{\alpha+\phi}$ and $\Omega \equiv \left[\frac{1-\alpha}{\psi}\right]^{\frac{1-\alpha}{\alpha+\phi}} \Theta^{\frac{\omega}{\alpha}}$. Similarly, $\widehat{r}$ is obtained via (16).

The dynamics of the economy, linearized around the ambiguous steady state, is described by the following system

$$\begin{bmatrix} \dot{c} \\ \dot{x} \\ \dot{\kappa} \end{bmatrix} = \mathbf{J} \begin{bmatrix} c - \widehat{c} \\ x - \widehat{x} \\ \kappa - \widehat{\kappa} \end{bmatrix}, \tag{29}$$

where the $\mathbf{J}$ is the Jacobian matrix and is computed in the Online Appendix.

**Lemma 1**

*The worst-case expectation $\underline{A}$ affects the dynamics of the economy only through its effects on $\widehat{c}$, $\widehat{x}$, and $\widehat{\kappa}$, that is*

$$\frac{\partial \mathbf{J}}{\partial \underline{A}} = \mathbf{0}.$$

Using Lemma 1, we can state the following.

**Proposition 1**

*The steady-state equilibrium exists and is (locally) stable. Furthermore, the existence and local stability of the steady-state equilibrium remain invariant to changes in the worst-case expectation parameter, $\underline{A}$.*

To ensure stability, $\mathbf{J}$ must have one eigenvalue with a positive real part, associated with the forward-looking variable, $c$, and two eigenvalues with negative real parts, associated with the predetermined variables, $x$ and $\kappa$. We restrict our analysis to the parameter space that satisfies these conditions. Therefore, from the stable branch of (29), consumption becomes[9]

$$c = \widehat{c} - \frac{Q_x(x - \widehat{x}) + Q_\kappa(\kappa - \widehat{\kappa})}{Q_c}. \tag{30}$$

Notice that (30) depends on worst-case expectation $\underline{A}$ at the steady state, through (28). Moreover, since $\widehat{c}$ is increasing in $\underline{A}$, a decrease in ambiguity (i.e., a higher $\underline{A}$) implies a higher workers' consumption in the ambiguous steady state.

By combining Lemma 1 with (30), we can express the effect of the worst-case expectation $\underline{A}$ on the rate of convergence of the economy towards the ambiguous steady state and claim the following.

[9] $Q_c$, $Q_x$, and $Q_\kappa$ are the entries of the left-eigenvector associated to the eigenvalue with positive real part. See the Online Appendix for the derivation.

**Proposition 2**

*More pessimistic expectations (i.e., a lower $\underline{A}$) lead to an acceleration in workers' consumption and a reduced accumulation of entrepreneurial wealth, resulting in a decline in aggregate capital, i.e.,*

$$\frac{\partial}{\partial \underline{A}} \begin{bmatrix} \dot{c} \\ \dot{x} \\ \dot{\kappa} \end{bmatrix} = -\mathbf{J} \frac{\partial}{\partial \underline{A}} \begin{bmatrix} -\frac{Q_x}{Q_c}\widehat{x} - \frac{Q_\kappa}{Q_c}\widehat{\kappa} \\ \widehat{x} \\ \widehat{\kappa} \end{bmatrix}.$$

Proposition 2 states that changes in the expected dynamics of consumption, entrepreneurial wealth, and aggregate capital, as influenced by varying pessimistic expectations, only depend on the ambiguous steady state. Although $\widehat{x}$ and $\widehat{\kappa}$ do not actually occur, they play a crucial role in determining how the convergence rate is expected to change in response to variations in $\underline{A}$.

The tension between long-term pessimistic expectations and $A > \underline{A}$ produces two countervailing effects. In the short run, it leads to a reduction in workers' consumption as agents increase their savings to prepare for future expected downturns – i.e., ambiguity induces higher savings due to precautionary motive like in Aiyagari (1994). In the long run, the economy converges to a steady state where the stream of workers' excessive savings enables them to achieve a level of consumption higher than what they would attain in a scenario without uncertainty about the realization of TFP. To illustrate this point, in Figure 1 we compare the equilibrium dynamics of our economy (blue), in which the actual TFP stands at $A$ while the perceived future TFP is $\underline{A}$, with that of an economy with the same fundamentals but without ambiguity (black), as in Itskhoki and Moll (2019).[10]

Along the convergence towards the steady state, compared to the case with no uncertainty, precautionary motive *initially* leads to a higher labor supply, as workers are willing to work more and accumulate more bonds – i.e., workers consume less (1e) to bolster savings (1b), offering higher labor hours at lower wages, as shown in Figure 1g and Figure 1h. Subsequently, workers' savings fuel aggregate wealth (1c) and drive higher wages, thus contracting labor demand (1h) and reducing entrepreneurial wealth (1a), which in turn lead to a lower interest rate (1d) and a lower productivity cutoff (1f).[11] Accordingly, aggregate production goes up (1i). Regarding consumption (1e), we observe that once the level of savings becomes sufficiently high, it exceeds the level achieved in an economy without uncertainty.[12] The over-accumulation of assets,

[10] We use the parameters from the calibration in Section 4. Here, we set the variances to zero.

[11] The fact that $r \neq \rho$ at the steady state is similar to Moll et al. (2022). However, in their case, the presence of dissipation shocks, which leave agents with zero assets and only their labor income, causes $r > \rho$.

[12] In Carbonari and Maurici (2023), where workers are *hand-to-mouth*, the effect of ambiguity on workers' consumption is reversed, resulting in a lower steady-state level compared to the case without uncertainty.

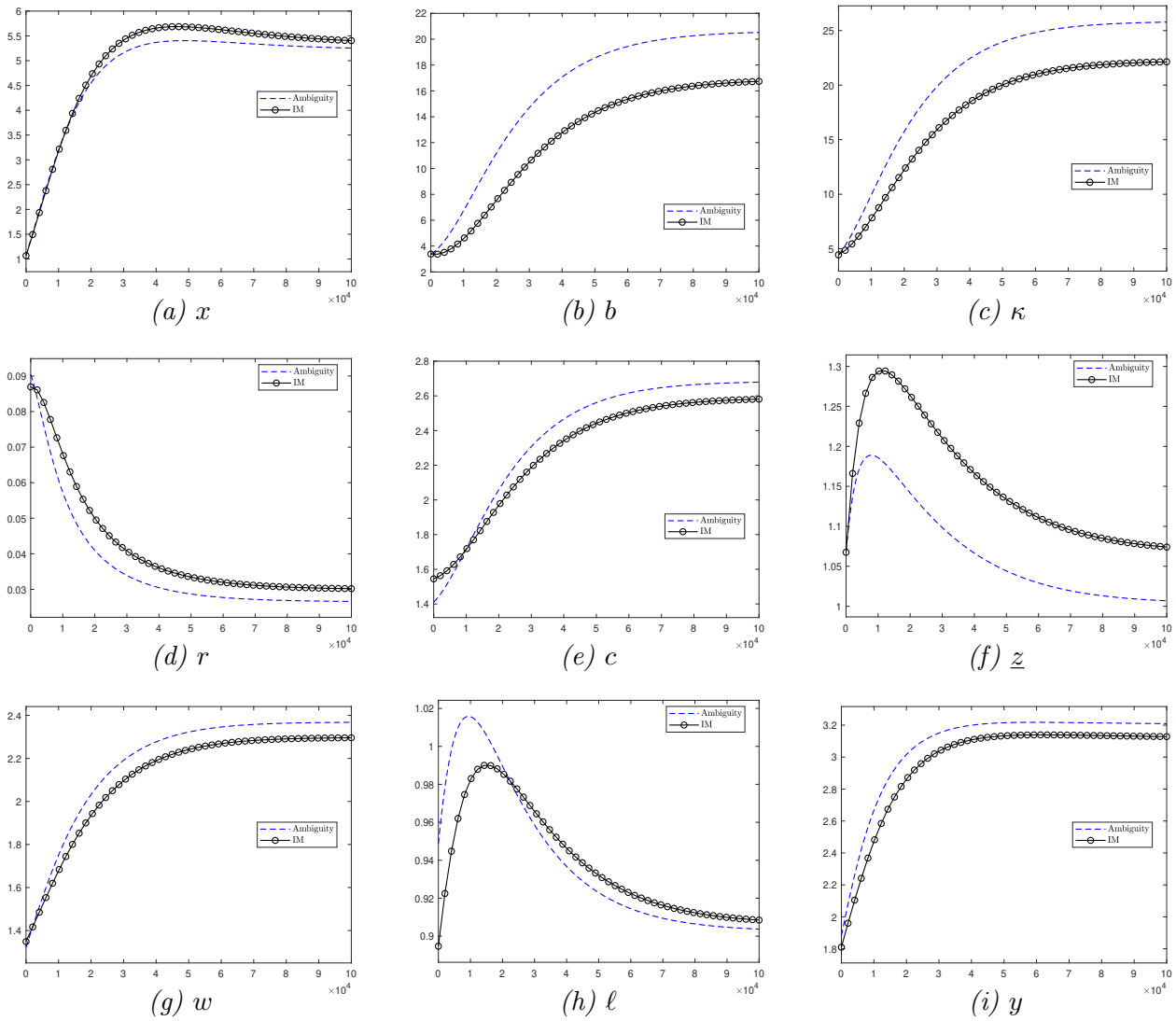



*Figure 1: Dynamics of the ambiguous economy (blue) and the Itskhoki and Moll (2019) economy (black).*

compared to the case with no ambiguity, also endures in the long run, characterized by a persistent mismatch between expectations and realizations. In the Online Appendix, we show how the interplay between ambiguous expectations and realizations can be captured by constructing vector fields induced by ambiguity.

An important feature of the model is that agents systematically underestimate the effective realization of the TFP. As in Carbonari and Maurici (2023), a key assumption is that a Dirac-delta distribution is an element of the set of perceived distributions. The Dirac-delta allows to reduce a stochastic optimization problem, in which the distribution used by agents to weigh the future is a choice variable, into a deterministic worst-case

optimization. This eliminates randomness because the distribution is fixed and consists of a single point.

As stated above, the Dirac delta distribution centered at the ambiguity lower bound eliminates any randomness. While this assumption significantly simplifies calculations, it may appear somewhat stringent. However, relaxing it would only alter the magnitude of our results, making them less extreme, but not their fundamental nature. This is because, by construction, for any element of $\mathcal{P}_{t^+|t}$, the ambiguous TFP is expected to be *at most* the true TFP, which can be thought of as a Dirac-delta centered at $A$, $\mathcal{D}(A)$. Consequently, for any less-extreme minimizing distribution, the results can always be expressed as a convex combination of the perfect-foresight case and the lower bound Dirac-delta.[13] When recovering the ambiguity-implied cycles, we also account for the possibility that $\underline{A}_t = A_{t^+}$, which represents correct forecasting.

**Signal extraction** In this paragraph, we assess the robustness of our results by considering the case where agents, who receive dispersed information about the fundamentals as in Hansen (2007), Angeletos and La'O (2013), and Huo et al. (2024), solve a signal extraction problem to estimate the true distribution of the TFP centered in the worst-case scenario.[14]

We adopt the standard benchmark in which agents aim at minimizing the mean squared error (MSE) of their next-period-ahead forecasts. It is convenient to define the worst-case scenario as a random variable $\xi \sim \mathcal{N}(\ln \underline{A}, \sigma_\xi)$, with agents receiving a signal $S$ in the form of

$$S = \xi + \chi, \tag{31}$$

where $\chi \sim \mathcal{N}(\ln A, \sigma_\chi)$ and $\xi$ and $\chi$ are independent. The signal (31) describes a situation in which both the ambiguous and the true TFP are signalled together.[15] Since agents are pessimistic, they are interested in identifying the effective realization of the worst-case scenario within the signal they receive, i.e., they aim at filtering out the true TFP component, $\chi$.

With Gaussian shocks, we can rely on signal-extraction functions of the form

$$h_0 + hS. \tag{32}$$

Agents minimize the MSE with respect to $\xi$ by appropriately choosing $h_0$ and $h$.

**Proposition 3**

*With Gaussian shocks, the signal-extraction problem implies that:*

[13] For a similar argument, see Battigalli et al. (2016).

[14] Auclert et al. (2020) also considers informational frictions as noisy information about shocks, applying standard Bayesian updating.

[15] Our results do not change if we assign a smaller weight to $\chi$, the true TFP component.

*1. the optimal parameters are given by*

$$h_0^* = -\frac{\sigma_\xi^2}{\sigma_\xi^2 + \sigma_\chi^2}\mathbb{E}[S] + \ln \underline{A}, \tag{33}$$

$$h^* = \frac{\sigma_\xi^2}{\sigma_\xi^2 + \sigma_\chi^2}, \tag{34}$$

*where* $\mathbb{E}[S] = \ln \underline{A} + \ln A$. *Thus*

$$\widehat{\xi}(S) = \frac{\sigma_\xi^2}{\sigma_\xi^2 + \sigma_\chi^2}(S - \mathbb{E}[S]) + \ln \underline{A}; \tag{35}$$

*2. the extracted signal is distributed as* $\widehat{\xi}(S) \sim \mathcal{N}\left(\ln \underline{A}, \frac{\sigma_\xi^2}{\sqrt{\sigma_\xi^2+\sigma_\chi^2}}\right)$.

Proposition 3 shows that the fundamental structure of ambiguous expectations remains unchanged. In particular, we have $\mathbb{E}\left[\widehat{\xi}(S)\right] = \ln \underline{A}$. This result supports the robustness of the findings derived above in the absence of the signal extraction problem. When the values of $\underline{A}$ and $A$ remain fixed over time, the dynamics of an economy in which agents address a signal extraction problem will oscillate around those of an economy characterized by ambiguity. Additionally, the distribution of the extracted signal is always less volatile than $\xi$. Therefore, as $\sigma_\chi \to \infty$, $\widehat{\xi}(S) \xrightarrow{d} \mathcal{D}(\ln \underline{A})$.

# 4 Business cycle

This section is divided into two parts. In the first part, we examine the impulse-response functions (IRFs) following a temporary ambiguity shock. By employing IRFs, we can trace the temporal progression of these effects, providing insights into how different macroeconomic variables respond and adjust over time following such a shock. In the second part, we calibrate our model to the U.S. economy using quarterly data from 1960Q1 to 2023Q3 and analyze the properties of the implied cycles.

**Impulse-response Functions (IRFs)** We now simulate the dynamics generated by an ambiguity shock, assuming that in the long run ambiguity disappears. Hence, the IRFs describe the short-run dynamics. We assume that the economy is at its non-ambiguous steady state, and, at time $t$, there is an ambiguity shock ($-10\%$).[16] $\underline{A}$ falls and recovers over time according to

$$\dot{\underline{A}} = \rho_{\underline{A}}(A - \underline{A}).$$

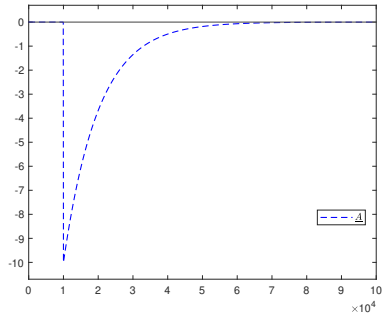
[16] A $-10\%$ shock corresponds to $0.9 \approx M = 0.93$, defined in the following paragraph.

To guarantee stability, we require $\rho_{\underline{A}} > 0$, which we set equal to 0.1.[17]

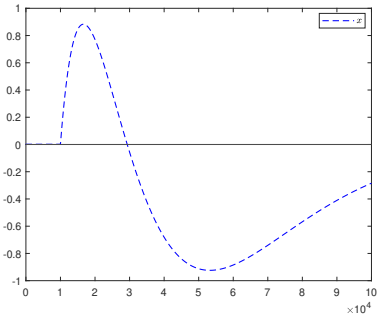
The negative shock on $\underline{A}$ that hits the economy is shown in Figure 2a while the remaining panels in Figure 2 reports the IRFs. Higher ambiguity causes an immediate drop in workers' consumption, accompanied by an increase in hours worked (2f and 2i) and asset holdings (2c). Excess labor supply reduces wages (2h) while aggregate output sharply increases (2j), thereby boosting interest rate (2e) and entrepreneurs' profits and wealth (2b). Over time, the ambiguity shock vanishes and the expectation converges towards the true value of the TFP. After the initial increase driven by workers' savings (2c) and entrepreneurial wealth (2b), aggregate capital begins to decline (2d), mirroring the reduction in workers' savings. This leads to a reduction of the interest rate (2e) and the productivity cutoff (2g), both driven by the excess supply of capital during the transition dynamics. Workers' consumption increases due to higher levels of past savings and reduced necessity for precautionary savings (2f). Lastly, aggregate production, after the initial increase, rapidly returns to its long-run level, reflecting the pattern of hours worked.

In the Online Appendix, we show the sensitivity of IRFs with respect to heterogeneity by letting $\eta$ vary.

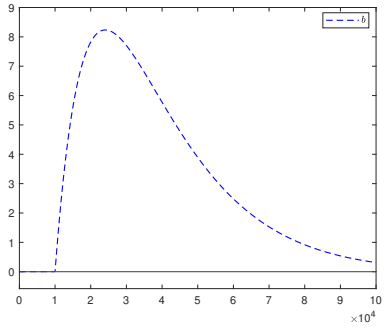
[17]Setting $\rho_{\underline{A}} = 0.1$ implies an exponential recovery of $e^{-\rho_{\underline{A}}} \approx 0.9$, which is standard in discrete-time models.



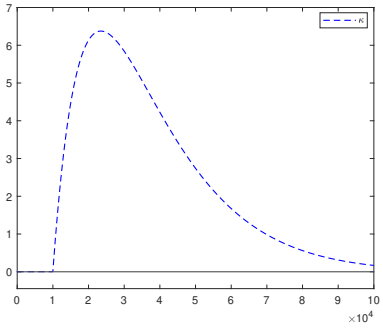

*(a)* $\underline{A}$



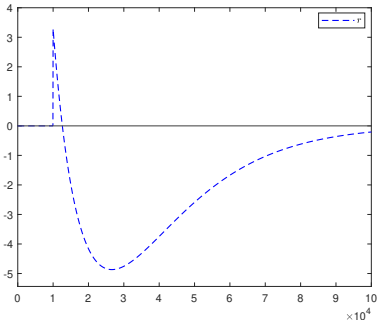

*(b)* $x$



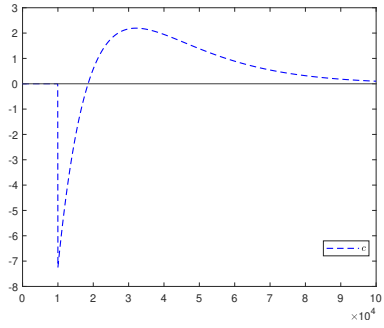

*(c)* $b$



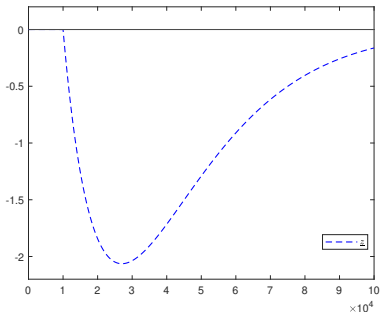

*(d)* $\kappa$



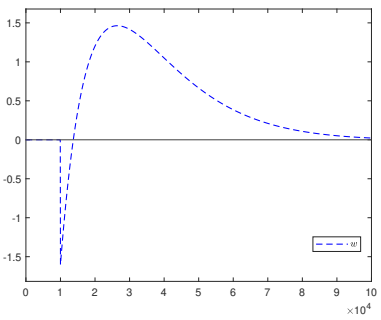

*(e)* $r$



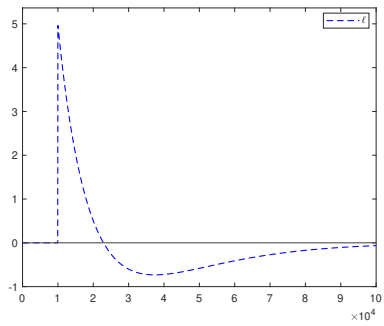

*(f)* $c$



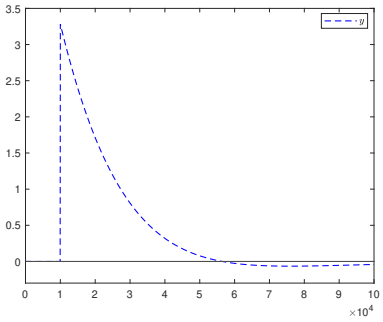

*(g)* $\underline{z}$




*(h)* $w$




*(i)* $\ell$




*(j)* $y$

*Figure 2: Responses to an ambiguity shock.*

**Calibration and cycles** In line with Altug et al. (2020), our parameter calibration aims to match two sets of cycle moments – correlations and standard deviations – of some filtered variables (i.e., total consumption, GDP, investments, and hours worked). For both simulated and actual data, we apply the HP-filter to their logarithms,[18] as in Hodrick and Prescott (1997), with a penalty parameter of 1600, consistent with standard practice for quarterly data.

| Parameter | Value | Description | Source |
|---|---|---|---|
| $\rho$ | 0.03 | workers' discount rate | standard value |
| $\delta$ | 0.1 | entrepreneurs' discount rate | standard value |
| $\alpha$ | 0.33 | capital share | standard value |
| $\lambda$ | 5 | financial constraint | Gatt (2024) |
| $\psi$ | 1 | marginal disutility of work | Itskhoki and Moll (2019) |
| $\phi$ | 1.22 | inverse Frisch elasticity | Chetty et al. (2011) |
| $\eta$ | 2.79 | inverse skill-heterogeneity | calibrated |
| $m$ | 0.18 | $\underline{A}_t/A_{t^+}$, lower bound | calibrated |
| $M$ | 0.93 | $\underline{A}_t/A_{t^+}$, unconstrained mean | calibrated |
| $\sigma$ | 0.045 | $\underline{A}_t/A_{t^+}$, unconstrained std. deviation | calibrated |
| $\sigma_A$ | 0.01 | $\ln A_{t^+}$, noise std. deviation | calibrated |

*Table 1: Parameter values for calibration.*

We model time-varying ambiguity, i.e., $\underline{A}_t = \underline{A}_t(A_{t^+}, m_t, M_t, \sigma_t)$, and allow for the possibility that $\underline{A}_t = A_{t^+}$. When computing the cycles, we describe ambiguity as a rescaled truncated normal distribution, defined by

$$\underline{A}_t = A_{t^+} \times \begin{cases} m_t & \text{if} \quad \mathcal{N}(M_t, \sigma_t) \leq m_t, \\ \mathcal{N}(M_t, \sigma_t) & \text{if} \quad m_t < \mathcal{N}(M_t, \sigma_t) \leq 1, \\ 1 & \text{if} \quad \mathcal{N}(M_t, \sigma_t) > 1. \end{cases}$$

Then $m_t \in (0, 1]$ denotes the maximum level of ambiguity, while $M_t$ and $\sigma_t^2$ are, respectively, the mean and variance parameters of a normal distribution.[19]

[18] In simulating the model, investments are periodically negative. To avoid taking the logarithm of a negative number we apply the following procedure. For each combination of parameters to calibrate, we evaluate the minimum of investments over time, $\underline{\dot{\kappa}}$, and define a new proxy for investments as $\dot{\kappa} + 1.01 \times |\underline{\dot{\kappa}}|$, which is strictly positive.

[19] This characterization of ambiguity can accommodate exogenous learning processes, as in Benhabib et al. (2015).

For calibration purposes, we assume that the logarithm of TFP follows the process

$$\ln A_{t^+} = \rho_A \ln A_t + \varepsilon_{t^+},$$

where $\varepsilon_{t^+} \sim \mathcal{N}(0, \sigma_A)$, while $\rho_A \in (-1, 1)$, governing the speed of convergence, is set to 0.9.[20]

Table 1 presents the calibrated parameters.[21] We also compute the model-implied moments for the economy described by Itskhoki and Moll (2019), which serves as a reference case with no ambiguity.

Panel A of Table 2 shows the implied standard deviations. As in the data, the volatility of consumption is less than that of aggregate production. However, both models predict that market consumption is too smooth, with standard deviations that are approximately 60% of the actual value. Regardless of the role of aggregate uncertainty, the standard deviation of output is roughly 70% of the observed value, while the volatility of overall investments exceeds that of output, reflecting a well-documented business cycle fact. Interestingly, ambiguity aversion appears to play a significant role in explaining investment volatility. Indeed, while our model perfectly reproduces this moment, the standard deviation for the model without uncertainty is approximately 40% higher than the observed value. Lastly, regarding the volatility of hours worked, although our model performs slightly better than the one with no uncertainty, the result is not satisfactory, with variability barely exceeding 30% of its observed value.

As for the remaining moments, Panel B of Table 2 shows that our model outperforms the one without ambiguity, effectively mirroring the correlations between consumption, production, and investment. It also provides reasonable results for the joint movements of hours worked and consumption, as well as hours worked and production. Notably, regardless of the role of ambiguity, the model-implied correlations between hours worked and investment are too high compared to the data.

To identify the usual medium-term business cycle we apply a band-pass filter, as described in Baxter and King (1999), with frequency bands ranging from 1.5 to 8 periods. Figure 3 plots the ambiguity-implied cycles, with ambiguity read on the left axis and the other variables on the right axis, allowing for effective comparison despite different scales. Ambiguity serves as an amplification shock for both output and workers' savings, as shown in Figure 3a and Figure 3e, although it exhibits a low correlation with the latter. Conversely, ambiguity displays much greater volatility compared to

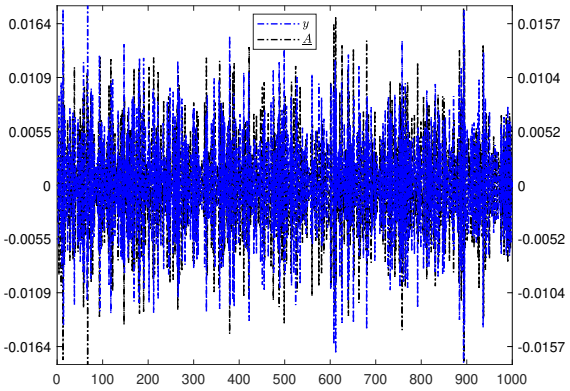
[20]To recover the cycles implied by ambiguity, we employ discretization to transform the theoretical differential equations into difference equations. We set $\mathrm{d}\tau = 1/100$ to represent instantaneous time changes. Subsequently, we average every 100 simulated observations.

[21]We perform 120,000 iterations, discarding the initial 10% to get rid of the impact of initial conditions. This process is executed for each parameter combination, with each parameter drawn from a grid of 8 elements.

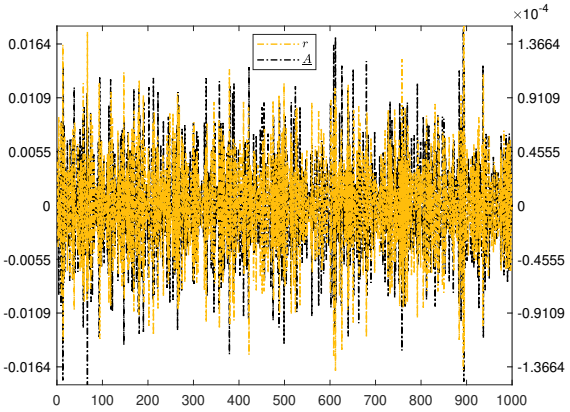
| Panel A. Standard deviations | | | | |
|---|---|---|---|---|
| | *Consumption* | *Production* | *Investment* | *Hours Worked* |
| Data, 1960Q1-2023Q3 | 1.13 | 1.52 | 6.40 | 0.93 |
| Model with ambiguity | 0.66 | 1.10 | 6.22 | 0.29 |
| Model without ambiguity | 0.67 | 1.00 | 8.93 | 0.15 |
| **Panel B. Correlations** | | | | |
| Data, 1960Q1-2023Q3 | *Consumption* | *Production* | *Investment* | *Hours Worked* |
| *Consumption* | · | · | · | · |
| *Production* | 80.79 | · | · | · |
| *Investment* | 56.44 | 88.05 | · | · |
| *Hours Worked* | 59.50 | 75.51 | 76.21 | · |
| Model with ambiguity | | | | |
| *Consumption* | · | · | · | · |
| *Production* | 85.04 | · | · | · |
| *Investment* | 54.88 | 90.57 | · | · |
| *Hours Worked* | 43.02 | 84.07 | 98.93 | · |
| Model without ambiguity | | | | |
| *Consumption* | · | · | · | · |
| *Production* | 99.95 | · | · | · |
| *Investment* | 99.05 | 99.41 | · | · |
| *Hours Worked* | 99.50 | 99.76 | 99.81 | · |

*Table 2: Actual vs Simulated data*

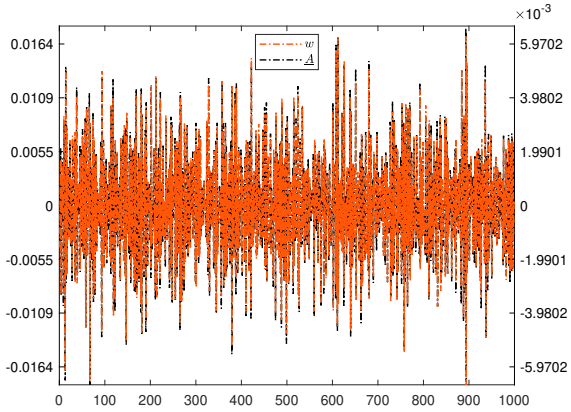
the interest rate, as illustrated in Figure 3b. Finally, Figure 3c and Figure 3d show a one-to-one effect of ambiguity on wages and entrepreneurial wealth.



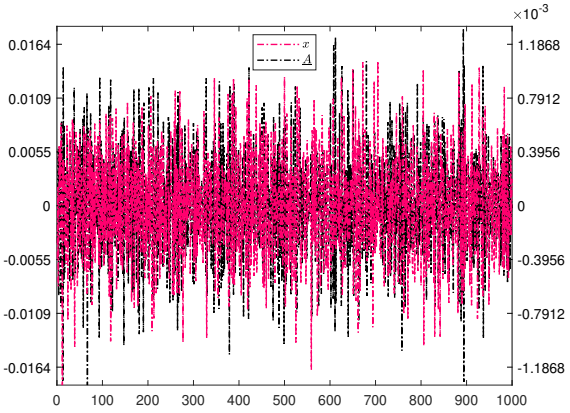

*(a) Ambiguity and production.*



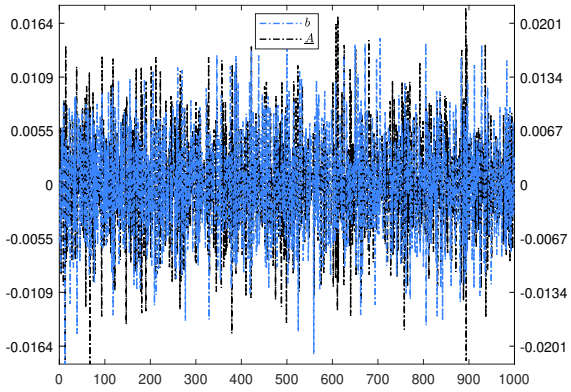

*(b) Ambiguity and interest rate.*




*(c) Ambiguity and wages.*




*(d) Ambiguity and entrepreneurial wealth.*




*(e) Ambiguity and workers' savings.*

*Figure 3: Business cycle.*

# 5 Conclusion

In this paper, we study a credit-constrained economy with heterogeneous entrepreneurs, as in Itskhoki and Moll (2019), where agents face Knightian uncertainty about the future

realization of the aggregate TFP. We show that, similar to Ilut and Schneider (2014), incorporating worst-case scenarios into long-run expectations drives the equilibrium away from its non-ambiguous counterpart. Furthermore, we prove that the (local) stability properties of the ambiguous equilibrium are not affected by ambiguity.

Comparing our economy with one having the same fundamentals but without ambiguity, we observe that, due to precautionary motives, workers accumulate more assets. This offsets the reduction in entrepreneurial wealth, lowers interest rates, and reduces the productivity cutoff. Workers consume less in the early stages but end up consuming more at the ambiguity-implied steady state. Additionally, according to our calibration, we find an increase in aggregate production.

We examine the back-propagation mechanisms of ambiguity shocks. By characterizing short- and long-term dynamics, we observe how a temporary ambiguity shock impacts the accumulation processes of entrepreneurs and workers.

Employing discretization techniques and time-varying ambiguity modeling, we outline the cyclical patterns driven by ambiguity. To show the implications of ambiguity across key economic variables, we calibrate the model to match the U.S. economy. Overall, introducing ambiguity significantly improves the ability of the model to reproduce actual moments.

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