Monetary Policy and Capital Flows: Dissecting the Transmission Mechanism*

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Abstract

We examine the transmission channels of monetary policy in an open economy. Through an analytical decomposition, we establish five main results: (i) expenditure switching plays a prominent role in driving the output response; (ii) capital mobility may amplify or dampen the effects of monetary policy; (iii) an exchange rate depreciation stemming from foreign UIP shocks or monetary contraction abroad can be contractionary (iv) exchange rate depreciations stemming from domestic UIP shocks or domestic monetary expansion are always expansionary; (v) capital mobility may amplify or dampen the effects of monetary policy.

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1 Introduction

How do capital mobility and exchange rates influence the transmission of monetary policy in an open economy? To study this questions, we study a canonical open economy model with nominal rigidities and provide a decomposition of the transmission channels of monetary policy.

Our results are as follows. First, the direct effects are composed of an intertemporal substitution channel and an expenditure switching channel. A depreciation of the exchange rate reduces the relative price of non-tradables, leading households to substitute consumption toward domestically produced goods. Second, monetary policy has dynamic effects, which may amplify or attenuate the effects of monetary policy. When the elasticity of substitution over time is larger than the elasticity across goods, a monetary policy expansion leads to consumption rising over output, which implies that capital inflows feed into more current aggregate demand—yet they lower aggregate demand going forward, considering the increase in debt. These results revert when the elasticity of substitution over time is lower than the elasticity across goods. Finally, when the elasticities are equal, an economy with a closed capital account and an economy with an open capital account respond in the same way. These results also have implications for the role of heterogeneity. Specifically, when an open economy amplifies the effect of monetary policy, it also follows that introducing a fraction of non-Ricardian hand-to-mouth agents reduces the aggregate responses to monetary policy.[1]

We then provide an analogous decomposition of the transmission channels of capital controls. Like an increase in the nominal interest rate, a tax on capital inflows generates a contraction in consumption through an intertemporal substitution effect. Moreover, the resulting decrease in capital inflows implies that the exchange rate appreciates in the future. Given a nominal interest rate, the exchange rate appreciates today, generating a contraction in demand for non-tradables through expenditure switching effects. Through the aggregate income channel, this leads to a further contraction in aggregate demand and output.

A calibration of the model underscores that monetary policy and capital controls operate through very different channels. While the intertemporal substitution channel accounts for about 95% of the effects of capital controls, expenditure switching accounts

[1] Kaplan, Moll and Violante (2018) show that in a simple TANK closed economy model, a higher share of hand-to-mouth agents increases the relative importance of general equilibrium effects, but the aggregate response to monetary policy shocks remains the same. In our open economy model, this result holds only in the case in which the elasticity of substitution over time is equal to the elasticity of substitution across goods.

for about one-third of the effects of monetary policy.

Related Literature Kaplan, Moll and Violante (2018) and Auclert (2019) provide a decomposition of the effects of monetary policy in a closed economy model and evaluate the differences in transmission channels between representative agent and heterogeneous agent models. One contribution of our paper is to develop an open economy decomposition and uncover the central role of capital flows and expenditure switching resulting from exchange rate fluctuations. In parallel and independent work, Auclert, Rognlie, Souchier and Straub (2021) also provide a related decomposition in an open economy setting. One difference is that they consider the response to a shock to the world interest rate whereas we consider a shock to the domestic nominal interest rate. Moreover, they consider a policy that keeps the domestic real rate constant, muting intertemporal substitution effects.

2 Model

We consider a small open economy with nominal rigidities. There is an infinite horizon and two types of goods: tradables and non-tradables. In this section, we describe the decisions of households and firms and the general equilibrium.

2.1 Households

There is a continuum of identical households of measure one. Households' preferences

$$\sum_{t=0}^{\infty} \beta^t \left[U(c_t) - v(h_t) \right], \tag{1}$$

where $\mathbb{E}_t$ denotes the time t expectation operator, $\beta \in (0,1)$ is the discount factor and z_t represents a discount factor shock. The utility function over consumption $u(\cdot)$ is strictly increasing and concave, and $v(\cdot)$ denotes an increasing and convex disutility function of labor. We assume that these functions are isoelastic of the form

$$U(c_t) = \frac{c_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}, \quad v(h_t) = \frac{h_t^{1+\phi}}{1+\phi}, \tag{2}$$

where σ is the elasticity of intertemporal substiution and ϕ is the inverse of the Frisch elasticity. The consumption good c_t is a composite of tradable consumption c_t^T and non-

tradable consumption c_t^N, according to a constant elasticity of substitution aggregator:

$$c_t = \left[\omega(c_t^T)^{1-\frac{1}{\gamma}} + (1-\omega)(c_t^N)^{1-\frac{1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}}, \quad \text{where } \omega \in (0,1).$$

The elasticity of substitution between tradable and non-tradable consumption is γ. For convenience, we use $u(c^T, c^N)$ to denote the utility as a function of the two consumption goods.

In each period t, households supply h_t units of labor and are endowed with y_t^T units of tradable goods. We assume that y_t^T is stochastic and follows a first-order Markov process. Households receive a wage rate, W_t, collect profits, ϕ_t^N, all expressed in terms of domestic currency, which serves as the numeraire, and receive government transfers T_t. Households trade two types of one-period non-state-contingent bonds in credit markets: a real bond b_{t+1}^*, which pays a gross return R_t^* units of tradables, and a nominal bond b_{t+1}, which pays R_t in units of domestic currency. The domestic government controls the nominal rate R_t. Both bonds are potentially subject to a tax/subsidy τ_t. When $\tau_t > 0$, households face a tax on debt issuance and a subsidy on savings. Conversely, when $\tau_t < 0$, households face a subsidy on debt issuance and taxes on savings.

The budget constraint of the representative household is therefore given by

$$P_t^N c_t^N + P_t^T c_t^T + \frac{1}{1+\tau_t}\left[\frac{b_{t+1}}{R_t} + P_t^T \frac{b_{t+1}^*}{R_t^*}\right] = \phi_t^N + W_t h_t + P_t^T (y_t^T + T_t) + b_t + P_t^T b_t^\star, \quad (3)$$

where P_t^N and P_t^T denote respectively the price of non-tradables and tradables (in terms of the domestic currency). The left-hand side represents total expenditures in tradable and non-tradable goods and purchases of bonds while the right-hand side represents total income, including the returns from bond holdings.

Optimality conditions. The households' problem consists of choosing sequences of $\{c_t^N, c_t^T, h_t, b_{t+1}, b_{t+1}^*\}$ to maximize the expected present discounted value of utility (1), subject to (3) and taking as given the sequence of tradable endowments $\{y_t^T\}$, profits $\{\phi_t^N\}$, transfers $\{T_t\}$, and prices $\{W_t, P_t^N, P_t^T, R_t, R_t^*\}$.

The first-order conditions for consumption and labor yield

$$\frac{W_t}{P_t^N} = \frac{v'(h_t)}{u_N(c_t^T, c_t^N)} \tag{4}$$

$$\frac{P_t^N}{P_t^T} = \frac{1-\omega}{\omega}\left(\frac{c_t^T}{c_t^N}\right)^{\frac{1}{\gamma}}, \tag{5}$$

where u_N denotes the marginal utility of non-tradable consumption in period t. Condition (4) is the labor supply optimality condition equating the marginal rate of substitution between leisure and non-tradable consumption with the wage rate in terms of non-tradables. Condition (5) equates the marginal rate of substitution between tradables and non-tradables to the relative price.

The first-order conditions for the nominal and real bond holdings yield

$$u_T(c_t^T, c_t^N) = \beta R_t^*(1+\tau_t)u_T(c_{t+1}^T, c_{t+1}^N) \tag{6}$$

$$\frac{u_T(c_t^T, c_t^N)}{P_t^T} = \beta R_t(1+\tau_t)\frac{u_T(c_{t+1}^T, c_{t+1}^N)}{P_{t+1}^T}. \tag{7}$$

where u_T denotes the marginal utility of tradable consumption. Households equate the marginal benefit from saving in nominal or real bonds to the marginal costs of cutting tradable consumption today to buy the bonds.

2.2 Firms

The non-tradable good is produced by a continuum of firms in a perfectly competitive market. Each firm produces a non-tradable good according to a production technology given by $y_t^N = n_t^\alpha$ and perceive profits given by

$$\phi_t^N = P_t^N n_t^\alpha - W_t n_t. \tag{8}$$

We assume that prices are perfectly rigid, $P_t^N = \bar{P}^N$, and that firms produce goods to satisfy demand. That is, labor demand in equilibrium is given by $n = (c^N)^{1/\alpha}$. We note that results can be extended to allow for partially sticky prices or optimal price setting under monopolistic competition.[2]

[2]We have conducted simulations with one-period-in-advance price setting with similar results.

2.3 Government

The government sets a nominal interest rate R_t and a tax on all forms of bond issuances τ_t. As is common in the literature, this tax can be interpreted as a capital control or as a macroprudential policy (see e.g., Bianchi, 2011; Schmitt-Grohé and Uribe, 2016; and Fornaro and Romei, 2019). The tax is assumed to be rebated lump-sum to households, an assumption that is without loss of generality given that Ricardian equivalence holds.[3] That is, the government budget constraint is

$$T_t = -\frac{\tau_t}{1+\tau_t}\left[\frac{b_{t+1}}{P_t^T R_t} + \frac{b_{t+1}^*}{R_t^*}\right]. \tag{9}$$

2.4 Prices, UIP and Exchange Rates

We assume that the law of one price holds for the tradable good, that is, $P_t^T = e_t P_t^{T*}$, where e is the nominal exchange rate defined as the price of the foreign currency in terms of the domestic currency, and P^{T*} is the price of the tradable good denominated in foreign currency.

Using the Euler equations for international bond (6) and domestic bond (7), we can equate the marginal benefits from buying the real and nominal bond. Together with the law of one price, this implies that the nominal exchange rate must satisfy the risk-adjusted uncovered interest parity condition:

$$R_t^* = R_t \frac{e_t}{e_{t+1}} \frac{P_t^{T,*}}{P_{t+1}^{T,*}}, \tag{10}$$

Condition (10) is a standard condition that relates the the foreign real interest rate and the domestic nominal interest rate to the expected depreciation of the domestic currency.

[3] We abstract from other the so-called unconventional fiscal policies that can relax the zero lower bound (see e.g. Correia, Farhi, Nicolini and Teles, 2013). We also abstract from differential taxes on domestic and foreign currency bonds. As examined in Acharya and Bengui (2018), differential taxes on bonds across currencies can also help relax the zero lower bound. As long as there are some limitations on the use of these policies (either political or economic), the first best cannot be implemented and our key results would remain.

2.5 Competitive Equilibrium

Market clearing for labor requires that the units of labor supplied by households equal the aggregate labor demand by firms:

$$h_t = n_t. \tag{11}$$

Market clearing for the non-tradable good requires that output be equal to non-tradable consumption:

$$y_t^N = c_t^N. \tag{12}$$

We assume that the bond denominated in domestic currency is traded only domestically. We make this assumption to abstract from portfolio problems and from the possibility of inflating away external debt.[4] Market clearing therefore implies

$$b_{t+1} = 0. \tag{13}$$

Combining the budget constraints of households, firms, and the government, as well as market clearing conditions, we arrive at the resource constraint for tradables, or the balance of payment condition:

$$c_t^T - y_t^T = b_t^* - \frac{b_{t+1}^*}{R_t^*}, \tag{14}$$

which says that the trade balance must be financed with net bond issuances.

An equilibrium, given government policies, is defined as follows.

Definition 1. Given an initial condition b_0^*, exogenous process $\{R_t^*, y_t^T\}_{t=0}^{\infty}$, a rigid price $\bar{P}^N$, and government policies $\{R_t, \tau_t\}_{t=0}^{\infty}$, an equilibrium is a stochastic sequence of prices $\{e_t, P_t^{T*}, W_t\}$ and allocations $\{c_t^T, c_t^N, b_{t+1}^*, n_t, h_t\}_{t=0}^{\infty}$ such that

(i) households optimize, and hence the following conditions hold: (4), (5), (6), (7);

(ii) firms choose hours to meet demand, $h_t^{\alpha} = c_t^N$;

(iii) labor market clears (11) and the domestic currency bond is in zero net supply (13);

(iv) the government budget constraint (9) is satisfied;

(v) the law of one price holds: $P_t^T = e_t P_t^{T*}$.

[4] See Fanelli (2017) for an interesting study of optimal monetary policy with nominal external debt and incomplete markets. In his model, the government can commit to future policies and uses monetary policy to improve risk-sharing in addition to the standard objectives.

3 Transmission of Monetary and Macroprudential Policy

Following Kaplan, Moll and Violante (2018) and Auclert (2019), our approach consists of first studying how an individual household responds in partial equilibrium to these policies, then analyzing how these polices play out in general equilibrium by disentangling the direct effects of policies on households decisions from the indirect effects emerging from general equilibrium. For analytical convenience, we consider a version of the model starting with zero net foreign asset position and without uncertainty. We then show how the decomposition can be applied to a stochastic version of the model, which we solve numerically.

3.1 Partial Equilibrium Response

Our approach requires us to first characterize the households policies as a function of an arbitrary sequence of prices and government policies. We place no restrictions on prices and policies other than the condition that the return of the domestic bond in real terms equals the foreign bond.[5]

Households' policies. We can express consumption of tradables and non-tradables and foreign bond holdings as a function of the sequence of income levels, relative prices, interest rates and taxes. That is,

$$c_0^N = \mathcal{C}^N\left(\left\{\bar{P}^N/(P^{T*}e_t), R_t, \tau_t, Y_t\right\}_{t\geq 0}\right), \tag{15a}$$

$$c_0^T = \mathcal{C}^T\left(\left\{\bar{P}^N/(P^{T*}e_t), R_t, \tau_t, Y_t\right\}_{t\geq 0}\right), \tag{15b}$$

$$b_1^* = \mathcal{B}^*\left(\left\{\bar{P}^N/(P^{T*}e_t), R_t, \tau_t, Y_t\right\}_{t\geq 0}\right), \tag{15c}$$

where Y_t is defined value of the income in period t in units of the composite consumption, or the "real income". That is,

$$\mathcal{P}_t Y_t \equiv y_t^T + T_t + \frac{W_t h_t + \phi_t^N}{P_t^T}, \tag{16}$$

[5]Otherwise, the deviation from arbitrage would imply that households' policies would be unbounded.

and $\mathcal{P}_t$ denotes the ideal price index (i.e., the minimum expenditure, denominated in units of tradables, required to buy one unit of the composite good c_t):

$$\mathcal{P}_t = \left[\omega^\gamma + (1-\omega)^\gamma \left(\frac{\bar{P}^N}{e_t P_t^{T*}}\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}. \tag{17}$$

In addition, for future reference, the share of expenditures in tradables is denoted by

$$\tilde{\omega}_t \equiv P_t^T c_t^T / (P_t^T c_t^T + P^N c_t^N). \tag{18}$$

We have the following lemma characterizing the solutions of the household problem in closed form.

Lemma 1. *For any sequence of nominal exchange rates $\{e_t\}$, taxes $\{\tau_t\}$, and households' income $\{Y_t\}$, we have that savings decision, tradable consumption, and non-tradable consumption at date 0 are given by*

$$c_0^T = \omega^\gamma \left(\mathcal{P}_0\right)^{\gamma-1} \mu_0 \sum_{t=0}^{\infty} Q_{t|0} \mathcal{P}_t Y_t. \tag{19a}$$

$$c_0^N = \frac{P^{T*} e_0}{\bar{P}^N}\left[1 - \omega^\gamma \left(\mathcal{P}_0\right)^{\gamma-1}\right] \mu_0 \sum_{t=0}^{\infty} Q_{t|0} \mathcal{P}_t Y_t, \tag{19b}$$

$$b_1^* = R_0^*(1+\tau_0)\left[\mathcal{P}_0 Y_0 - \mu_0 \sum_{t=0}^{\infty} Q_{t|0} \mathcal{P}_t Y_t\right], \tag{19c}$$

where $Q_{t|0} \equiv \prod_{s=0}^{t-1}[R_s^*(1+\tau_s)]^{-1}$ and $\mu_0 = 1/\sum_{t=0}^{\infty} \beta^{t\sigma}\left[Q_{t|0}\frac{\mathcal{P}_0}{\mathcal{P}_t}\right]^{\sigma-1}$ is the marginal propensity to consume.

Proof. In Appendix A.1 □

Because preferences are additively separable over time, we can use two-stage budgetting. Specifically, we split the household problem into choosing the optimal expenditures on the composite consumption over time and then deciding how expenditures at each point in time are allocated between tradables and non-tradables.[6] In particular, households spend at $t = 0$ a fraction μ_0 of the annuity value of lifetime income and split it between

[6]Notice that by taking as given the household income Y as exogenously given for the household, we are, in effect, taking as given a number of hours worked. Thanks to the separability of consumption and leisure in the utility function, the solutions from Lemma 1 apply to any choice of hours. When we consider the general equilibrium response, we consider the optimally supplied amount of hours.

tradables and non-tradables, with weights given by $\omega^{\gamma}\left(\mathcal{P}_0\right)^{\gamma-1}$ and $1-\omega^{\gamma}\left(\mathcal{P}_0\right)^{\gamma-1}$, respectively. In turn, the marginal propensity to consume, μ_0, depends on the price of the period 0 composite consumption relative to all future prices and the intertemporal elasticity of substitution. Likewise, the share of expenditures between tradables and non-tradables in period 0 depends on the relative price of tradables and non-tradables in period 0 and the elasticity of substitution across the two goods.

3.2 Transmission Channels of Monetary Policy

In this section, we propose an analytical decomposition of the effects of monetary policy. As we will see, there are two critical elements of the transmission channel not present in closed economy studies (Auclert, 2019 and Kaplan et al., 2018). First, monetary policy affects the exchange rate between the domestic and foreign currencies. Through the risk-adjusted UIP condition (10), a temporary decline in the nominal interest rate leads to a depreciation of the nominal exchange rate and critically shifts the relative price between tradables and non-tradables. Second, monetary policy has dynamic effects. By affecting capital flows, monetary policy shifts aggregate demand over time.

We consider a one-period decline in the nominal interest rate at $t=0$, which reverts to the steady state value at $t=1$. We assume that $\beta R^*=1$, so that there is no consumption tilting after the shock. Moreover, we restrict attention to a policy that keeps the exchange rate at a level is consistent with the first-best allocation for $t\geq 1$. Given these assumptions, we have a stationary equilibrium for $t>1$ with a constant exchange rate, employment, and non-tradable consumption. Notice, however, that *the stationary level for the exchange rate and allocations do not revert to the initial steady state values*. Indeed, monetary policy will have dynamic effects because the new steady state values depend on how the economy reacts to the temporary shock at $t=0$.[7]

In Lemma 1, we derived the individually optimal policies for consumption and savings for arbitrary prices and government policies. We can now use the fact that the non-tradable goods market clears in general equilibrium to obtain a decomposition of the channels by which monetary policy affects the economy. The results are displayed in the proposition below.

Proposition 1 (Monetary policy decomposition)**.** *Consider a change in the nominal interest*

[7]We note that our approach can be extended to allow for persistent shocks and alternative assumption about future monetary policy.

rate R_0. To a first-order, the effects on output are given by

$$\frac{dy_0^N}{y_0^N} = \underbrace{-\beta\sigma(1-\tilde{\omega})\frac{dR_0}{R_0}}_{\textit{Intertemporal Subst.}} + \underbrace{\tilde{\omega}\gamma\frac{de_0}{e_0}}_{\textit{Expend. Switching}} \underbrace{-(1-\beta)(1-\tilde{\omega})\left[(1-\kappa)\sigma+\kappa\gamma\right]\frac{dR_0}{R_0}}_{\textit{Aggregate Income}} \tag{20}$$

and the effects on capital flows are given by

$$\frac{db_1^*}{\mathcal{P}c} = \underbrace{\sigma(1-\tilde{\omega})\frac{dR_0}{R_0}}_{\textit{Intertemporal Subst.}} \underbrace{-\left[(1-\tilde{\omega})\sigma+\tilde{\omega}\gamma\right](1-\tilde{\omega})\frac{dR_0}{R_0}}_{\textit{Aggregate Income}} \tag{21}$$

where $de_0/e_0 = -\left[1-(1-\beta)(1-\kappa)\left(1-\sigma\gamma^{-1}\right)\right]dR_0/R_0$, $\kappa \equiv \tilde{\omega}\left[1+\frac{\alpha(1-\tilde{\omega})(\gamma-\sigma)}{\alpha\sigma+(1-\alpha+\phi)\gamma\sigma}\right]^{-1}$. *We also have that* $\kappa \in (0,1)$, *and the effects on* y_t^N *for* $t>0$ *and* c_t^T *are presented in Appendix A.2*

Proof. In Appendix A.2. □

Equation (20) decomposes the effects of a change in the interest rate on non-tradable output in three channels: intertemporal substitution, expenditure switching and aggregate income.

Consider first the intertemporal substitution channel, a channel that is common with closed economy studies. To fix ideas, consider a reduction in the nominal interest rate, which reduces the relative price of consuming in period 0 relative to future periods. The intertemporal substitution channel denotes the shift in consumption from the future to the present that originates from the change in the relative intertemporal price of consumption. To a first order, the optimal households' response in terms of non-tradable consumption is given by the discounted change in the real interest rate, times the IES. Notice that a 1% reduction in the nominal interest rate gives rise to a reduction in the interest rate in terms of the composite consumption (i.e., the real interest rate) of $(1-\tilde{\omega})\%$ where $\tilde{\omega}$ is defined in (18).[8]

The second channel is expenditure switching. Through (10), the decrease in the interest rate generate a proportional depreciation of the exchange rate.[9] Given a sticky price for non-tradables in domestic currency and the price for tradables in foreign currency, the depreciation reduces today's relative price of non-tradables and leads households to switch consumption toward non-tradables. The expenditure switching channel represents the

[8] See Lemma A.1 in the appendix for this result.

[9] As the proposition states, the relative change in the exchange rate relative to the interest rate depend on the relative values of γ and σ. We expand on this in Section 3.2.1.

increase in non-tradable consumption that results from the reduction in the relative price for non-tradables, keeping lifetime constant in units of the composite consumption. For a 1% depreciation of the nominal exchange rate, the magnitude of this effect is given by $\tilde{\omega}$ times the elasticity of substitution between tradables and non-tradables. Notice that the more open the economy is, as measured by $\tilde{\omega}$, the larger is the expenditure switching channel (and the smaller is the intertemporal substitution channel).[10]

These two channels, expenditure switching and intertemporal substitution, concern the direct partial equilibrium response of households. This partial equilibrium response assumes that production of non-tradable goods is not affected by the monetary expansion. However, in general equilibrium, as demand increases, firms respond by producing more. The increased production raises households' labor income and profits and generates second-round effects on demand. By a classic Keynesian cross logic, higher demand feeds into higher output. Notice that the marginal propensity to consume that matters is the one associated with non-tradables. In particular, given relative prices, a one unit increase in lifetime income leads to a $(1-\beta)(1-\tilde{\omega})$ increase in c^N. The total effect is given by the marginal propensity to consume non-tradables times an average of the elasticities of substitution across sectors and time weighted by κ.

3.2.1 Capital Flows and Dynamic Effects

In this section, we analyze how monetary policy has dynamic effects by affecting capital flows, a feature that contrasts with the canonical closed economies where a purely transitory monetary policy shock does not alter future output and prices. We tackle two interrelated questions. First, how does monetary policy affect the sign of the change in capital flows. Second, how does an open capital account affect the output response to a monetary policy shock.

Effect on capital flows. The second item in Proposition 1 characterizes the effects of monetary policy on capital flows. A monetary expansion has two opposite effects on capital flows. For a given level of aggregate income, a decline in the domestic nominal interest rate lowers the price of consuming today versus tomorrow, leading households to substitute toward current consumption. Given aggregate income, this reduces aggregate savings and leads to an increase in capital inflows. On the other hand, the temporary

[10]There is of course an old tradition in international macro describe the expenditure switching and expenditure tilting effect (also called expenditure changing or in our case intertemporal substitution). The contribution here is a formal decomposition, which also includes general equilibrium effects.

increase in income, resulting from expenditure switching and intertemporal substitution, leads to higher savings, and therefore capital outflows.

The net effects of a decrease in the nominal interest rate on aggregate savings depend crucially on the relative importance of the two elasticities. If the elasticity across time σ is higher than the elasticity across sectors γ, the intertemporal substitution channel dominates the aggregate income one and the increase in consumption exceeds the increase in income. As a result, we have an increase in capital inflows, and savings decrease. On the other hand, if the elasticity across time is lower than the elasticity across sectors, we have that the aggregate income channel dominates the intertemporal substitution one and the increase in income exceeds the increase in consumption, resulting in capital outflows. In the knife-edge case in which the elasticities are equal ($\sigma = \gamma$), a monetary expansion does not induce movement in the country's net foreign asset position, as aggregate income and current consumption increase by the same amount.[11]

These results are summarized in the following corollary.

Corollary 1. *Consider a reduction in R. If $\sigma < \gamma$, aggregate savings increase, and if $\sigma > \gamma$, aggregate savings decrease. Finally, if $\sigma = \gamma$, savings remain constant.*

Proof. In Appendix A.3. □

Amplification or attenuation? Do capital flows amplify or attenuate the effects of monetary policy? To examine this, we analyze the effects of monetary policy when we shut down the capital account and compare the response with our baseline model with an open capital account. As it turns out, the result depends on the relative elasticities of substitution across time and sectors. When $\sigma > \gamma$, households decrease savings in response to a monetary policy expansion, as characterized in Corollary 1. This implies that households shift aggregate demand from the future to the present. In this context, closing the capital account contracts aggregate demand and raises output. Thus, capital flows amplify the effects of monetary policy (i.e., output increases by more than if the capital account were closed). On the other hand, when $\sigma < \gamma$, households increase savings, and shutting down the capital account boosts aggregate demand. Thus, capital flows attenuate the effects of monetary policy. Finally, when $\sigma = \gamma$, the response of monetary policy is independent of whether the capital account is open or closed.

[11] An alternative intuition for how the relative elasticities matter for the response of capital flows to monetary policy can be obtained by noting that given the preference structure, $\sigma > \gamma$ if and only if $u_{TN} > 0$. That is, given that a monetary policy expansion increases c^N, a positive cross partial derivative implies that households value more tradables and therefore borrow more. The role of these two elasticities in driving the current account response to a monetary policy shock was first highlighted by Lane (2001).

The following proposition summarizes these results.

Proposition 2. *Consider a monetary expansion under a closed capital account and an open capital account. We have that*

i) If $\sigma > \gamma$, output increases by more with an open capital account

ii) If $\sigma < \gamma$, output increases by more with a closed capital account

iii) If $\sigma = \gamma$, the output response is the same under open and closed capital account.

Proof. In Appendix A.4. □

3.3 Transmission Channel of Macroprudential Policy

We now turn to describing the transmission channels of capital controls. We keep the nominal interest rate constant and consider a one-period change in the tax on debt at $t = 0$ (which reverts to the steady state value at $t = 1$). [12]

Analogously to Proposition 1, the next proposition presents the channels by which capital controls affect the economy.

Proposition 3 (Capital controls decomposition)**.** *Consider a change in the tax on debt τ_0. To a first order, the effects on output are given by*

$$\frac{dy_0^N}{y_0^N} = \underbrace{-\beta\sigma d\tau_0}_{\text{Intertemporal subst.}} + \underbrace{\tilde{\omega}\gamma\frac{de_0}{e_0}}_{\text{Expend. Switching}} \underbrace{-(1-\beta)(1-\kappa)\sigma d\tau_0}_{\text{Aggregate Income}}$$

and the effects on capital flows are given by

$$\frac{db_1^*}{\mathcal{P}c} = \underbrace{\sigma d\tau_0}_{\text{Intertemporal Subst.}} \underbrace{-(1-\tilde{\omega})\sigma d\tau_0}_{\text{Aggregate Income}}$$

where $\frac{de_0}{e_0} = -(1-\beta)\frac{1-\kappa}{1-\tilde{\omega}}\sigma\gamma^{-1}d\tau_0$.

Proof. In Appendix A.5. □

[12] As we proceeded above, we assume that for $t \geq 1$, the central bank implements a monetary policy consistent with the first-best allocation.

A tax on debt increases the return on savings and leads households to substitute consumption intertemporally toward the future. The intertemporal substitution channel is again proportional to $\beta\sigma$, but now the change in the real interest rate is given by $d\tau_0$. So the total effect is given by $\beta\sigma d\tau_0$, implying that for changes of the same magnitude in the nominal interest rate and the tax rate, we have a larger intertemporal substitution effect with the latter.

As foreign savings increase, the future exchange rate appreciates to close the output gap. Through the UIP condition, this implies that the current exchange rate also appreciates, generating an expenditure switching effect away from non-tradable consumption. The form of the expenditure switching channel is isomorphic to the one in (20). However, as we will see, it is smaller than the one for monetary policy because the exchange rate responds only indirectly through the future changes in the exchange rate.

Finally, we have the aggregate income channel. As output contracts in response to the reduction in the demand for consumption, income falls, and this further reduces consumption.

3.4 Quantitative Inspection of Transmission Channels

We now solve the stochastic version of our model economy non-linearly and simulate the impulse response to a monetary policy shock and a capital control shock.[13] We solve the model non-linearly using a time iteration algorithm.

The time period is one-quarter, and data are calibrated using United Kingdom data between 1980 and 2019.[14] The labor supply elasticity is set to one-third, as in Gali and Monacelli (2005) and α is set one.

We set β to 0.99 to target the historical average net foreign asset position (NFA) as a share of GDP of -17.4 % in the UK. The world interest rate is set to a deterministic value of 1.4%, which implies $\beta R^* < 1$ to have a well-defined ergodic distribution of the net foreign asset position. Finally, we use $\gamma = 1$ as a baseline value but consider also $\gamma = 0.5$ and $\gamma = 2$. The weight on tradable consumption in the CES function ω is calibrated to match a

[13] See Appendix B for the numerical solution method used to decompose the effects of monetary policy and capital controls shocks on output.

[14] We focus on the United Kingdom because an as an example of an advanced small open economy. We note that the problem of the zero lower bound has indeed been more pervasive for advanced economies although a side effect of the recent increase in central bank credibility in emerging markets appears to be the increase in vulnerability to liquidity traps, as can be seen from the recent experiences of countries such as Chile and Peru (see Matthew Bristow "Paul Krugman Says the Liquidity Trap Has Spread to Emerging Markets" Bloomberg May 12, 2020).

24% share of tradable output in the total value of production observed in the data over the period 1980-2019, implying that $\omega = 0.247$. The resulting values are presented in Table 1.

Table 1: Calibration

Description	Parameter Value	Source/Target
Intertemporal elasticity	$\sigma = 1$	Standard value
Technology	$\alpha = 1$	Gali and Monacelli (2005)
Frisch elasticity parameter	$\phi = 3$	Gali and Monacelli (2005)
Weight on tradables in CES	$\omega = 0.24$	Share of tradable output = 24%
Discount factor	$\beta = 0.99$	Standard value
Intratemporal elasticity	$\gamma \in \{0.5, 1, 2\}$	

The overall response in output and decomposition are displayed in Table 2. We illustrate the results for different values of the intra and inter-temporal elasticities of substitution, keeping the rest of the parameters constant. We highlight that the results are very close to those that one would obtain from directly applying Proposition 1, suggesting that the approximation remains accurate in the presence of uncertainty and higher-order terms.

Table 2 shows that monetary policy and capital controls operate quantitatively through entirely different channels. While capital controls work almost entirely through intertemporal substitution, expenditure switching plays a key role for monetary policy. In the case of unitary elasticities, expenditure switching accounts for about one-third of the overall effects of monetary policy. As the elasticity of substitution across sectors γ increases, demand becomes more sensitive to changes in the nominal exchange rate, leading to a larger increase in output following a monetary expansion. When $\gamma = 2$, expenditure switching accounts for about one-half of the overall effects. Meanwhile, the aggregate income accounts for at most 5% of the overall effects of monetary policy, a finding that is common with the closed economy representative agent model. Moreover, the modest contribution of the aggregate income channel also extends to the effects of capital controls.

The table also shows that for values of $\gamma = 1$, the effects on output of a 1 percentage point cut in the nominal interest rate and a 1 percentage point decrease in the tax on debt are about the same. While this suggests that these policies may be substitutes in terms of the output response, they work through distinct channels, as we showed, and they also have different implications for capital flows. In the next section, we leverage this analysis and take a normative perspective on how policy makers should jointly use these policies.

Table 2: Transmission channels of monetary policy and capital controls

	$\sigma = 0.5$			$\sigma = 1$		
	$\gamma = 0.5$	$\gamma = 1$	$\gamma = 2$	$\gamma = 0.5$	$\gamma = 1$	$\gamma = 2$
Monetary Policy						
Change in y^N (pp)	0.50	0.62	0.91	0.89	1.00	1.27
Intertemporal subst.	0.37	0.36	0.34	0.74	0.71	0.69
Expend. switching	0.11	0.24	0.53	0.12	0.25	0.53
Aggregate income	0.02	0.02	0.04	0.03	0.04	0.05
Capital controls						
Change in y^N (pp)	0.50	0.50	0.50	1.01	1.01	1.01
Intertemporal subst.	0.48	0.48	0.47	0.97	0.96	0.96
Expend. switching	0.00	0.00	0.01	0.01	0.01	0.01
Aggregate income	0.02	0.02	0.02	0.03	0.04	0.04

Note: The monetary policy shock we consider is a 1 percentage point decrease in the nominal interest rate for one quarter (annualized). The capital controls shock is a 1 percentage point decrease in the tax on debt for one quarter (annualized). All responses are reported in annualized terms.

3.5 Extensions with Hand-to-Mouth Households

Our decomposition is conducted in the context of a canonical open economy model. However, the decomposition can be applied in richer frameworks to further investigate quantitative insights.

Our straightforward extension is to allow for heterogeneity. In particular, we extend the analysis to the case where a fraction of households are non-ricardian hand-to-mouth households. To be more concrete, we suppose that there is a fraction Λ of households that do not have access to capital markets, i.e. there are *hand-to-mouth* households who consume their entire endowment of tradables, that is $c_t^{h,T} = y_t^T$. The overall response of output and decomposition in an economy where the proportion of hand-to-mouth households is 30% (as in Kaplan et al., 2018) are displayed in Table 3

Table 3 shows that when it is the case introducing a fraction of hand-to-mouth agents does not affect the aggregate responses to monetary policy. When an open economy amplifies the effect of monetary policy, however, introducing a fraction of hand-to-mouth agents then reduces the aggregate responses to monetary policy.

Proposition 2 also helps implications for the importance of departing from a represen-

Table 3: Transmission channels of monetary policy

	$\sigma = 0.5$			$\sigma = 1$		
	$\gamma = 0.5$	$\gamma = 1$	$\gamma = 2$	$\gamma = 0.5$	$\gamma = 1$	$\gamma = 2$
RANK						
Change in y^N (pp)	0.50	0.62	0.91	0.89	1.00	1.27
Intertemporal subst.	0.37	0.36	0.34	0.74	0.71	0.69
Expend. switching	0.11	0.24	0.53	0.12	0.25	0.53
Aggregate income	0.02	0.02	0.04	0.03	0.04	0.05
TANK						
Change in y^N (pp)	0.50	0.72	1.18	0.78	1.00	1.45
Intertemporal subst.	0.27	0.26	0.25	0.53	0.51	0.49
Expend. switching	0.11	0.25	0.54	0.11	0.25	0.55
Aggregate income	0.12	0.21	0.39	0.14	0.24	0.41

Note: The monetary policy shock we consider is a 1 percentage point decrease in the nominal interest rate for one quarter (annualized). All responses are reported in annualized terms.

tative agent environment. In the context of a closed economy, this increases the aggregate income channel but mitigates the intertemporal substitution channel, while the aggregate response remains the same (Kaplan et al., 2018). In our open economy, however, this offsetting of partial equilibrium and general equilibrium effects emphasized by Werning (2015) does not hold unless $\sigma = \gamma$. That is, if $\sigma < \gamma$ ($\sigma > \gamma$), the response of output to a monetary expansion is increasing (decreasing) in the share of agents that are hand-to-mouth.[15] On the other hand, expenditure switching effects are almost unaffected by Λ.

4 UIP shocks and Contractionary Depreciations

This section extends the model and assumes that bond holdings provide liquidity services that enter the utility function. Households' preferences are now described by

$$\sum_{t=0}^{\infty} \beta^t \left[U(c_t) - v(h_t) + \zeta_t^* \Theta(b_{t+1}^*) + \zeta_t \Phi\left(\frac{b_{t+1}}{e_t}\right) \right], \tag{22}$$

[15] A related result by Auclert et al. (2021) provides an irrelevance result on idiosyncratic risk in open economy, analogous to Werning's results in closed economy.

where U and v are given by (2) and

$$\Theta(b^*_{t+1}) = \left[1 + \exp\left(-b^*_{t+1}\right)\right]^{-1}$$
$$\Phi\left(\frac{b_{t+1}}{e_t}\right) = \left[1 + \exp\left(-\frac{b_{t+1}}{e_t}\right)\right]^{-1}$$

where $\zeta_t \geq 0$ and $\zeta^*_t \geq 0$ are preference parameters that introduce a wedge in the standard interest parity condition (10), which become,[16]

$$R_t \frac{e_t}{e_{t+1}} = \left[1 + \frac{\zeta^*_t \Theta'(b^*_{t+1}) - \zeta_t \frac{e_{t+1}}{e_t} \Phi'(b_{t+1})}{\beta u_T(c^T_{t+1}, c^N_{t+1}) + \zeta_t \frac{e_{t+1}}{e_t} \Phi'(b_{t+1})}\right] R^*_t \tag{24}$$

we interpret ζ_t and ζ^*_t as shocks to the interest parity condition (UIP shocks). We compare the transmission channels of UIP shocks to monetary policy shocks and shocks to the foreign interest rate. The results are presented in Table 4.

The first lesson from Table 4 is that and are foreign UIP shocks are not isomorphic to monetary policy shocks, and depreciations originating from foreign UIP shocks can be contractionary. To understand why, consider a positive foreign UIP shock (ζ^*_0 increases). Then, for given interest rates R_0 and R^*_0 and for given future exchange rates e_{t+1} for $t \geq 1$, the effective return of foreign currency bonds (which includes the liquidity benefits of holding foreign bonds) increases. The exchange rate therefore depreciates. Moreover, from (23), households increase savings, as they perceive a higher benefit from saving in foreign currency bonds. The UIP shock can therefore have a contractionary effect (as shown in Table 4 for $\sigma < \gamma$), which is not present with the monetary policy shock. In fact, Table 4 shows that foreign UIP shocks are isomorphic to shocks to the foreign interest rate,

This result highlights the role of the source of exchange rate depreciation in determining its impact on domestic consumption and output.

Another takeaway from Table 4 is that, in contrast to foreign UIP shocks, shocks to domestic UIP shocks are always expansionary. This is because, for given R_0, R^*_0 and given future exchange rates e_{t+1}, a negative domestic UIP shock (ζ_t falls) leads to a fall in the return of domestic currency bonds relative to the foreign currency ones, which in turn

[16] The Euler equation for foreign bonds is given

$$\frac{u_T(c^T_t, c^N_t)}{R^*_t} = \beta u_T(c^T_{t+1}, c^N_{t+1}) + \zeta^*_t \Theta'(b^*_{t+1}) \tag{23}$$

These conditions equate the marginal cost of cutting consumption today to buy foreign bonds to the marginal benefit from saving in foreign bonds. Recall also that domestic currency bonds are in zero net supply.

implies that the exchange rate must depreciate to prevent arbitrage. However, contrary to foreign UIP shocks, the marginal benefits from savings decrease. The increase in current consumption stimulates further production.

Table 4: Transmission channels of interest rate shocks and UIP shocks

	$\sigma = 1$		
	$\gamma = 0.5$	$\gamma = 1$	$\gamma = 2$
Shock on R_0			
Change in y^N (pp)	1.01	1.24	1.39
Intertemporal subst.	0.67	0.81	0.90
Expend. switching	0.30	0.35	0.37
Aggregate income	0.05	0.08	0.12
Change in e_0 (pp)	1.50	1.41	1.34
Shock on ζ_0			
Change in y^N (pp)	0.62	0.81	0.95
Intertemporal subst.	0.41	0.53	0.62
Expend. switching	0.18	0.23	0.25
Aggregate income	0.03	0.05	0.08
Change in e_0 (pp)	0.91	0.92	0.91
Shock on ζ_0^*			
Change in y^N (pp)	0.14	-0.04	-0.17
Intertemporal subst.	0.32	0.20	0.12
Expend. switching	-0.18	-0.24	-0.27
Aggregate income	0.01	0.00	-0.01
Change in e_0 (pp)	-0.91	-0.95	-0.97
Shock on R_0^*			
Change in y^N (pp)	0.16	-0.03	-0.17
Intertemporal subst.	0.32	0.19	0.09
Expend. switching	-0.17	-0.22	-0.25
Aggregate income	0.00	0.00	-0.01
Change in e_0 (pp)	-0.85	-0.89	-0.92

Note: The interest rate shocks we consider are a 1 percentage point decrease in the nominal interest rates for one quarter (annualized) and the UIP shocks are decreases in ζ_0 and ζ_0^* by 0.05. All responses are reported in annualized terms.

5 Conclusion

In this paper, we provide an analytical decomposition of the transmission channels of monetary and macroprudential policies in an open economy. Our decomposition reveals that the expenditure-switching channel accounts for about one-quarter of the overall effects of monetary policy. Macroprudential policy instead operates primarily through intertemporal substitution. We also show that whether exchange rate depreciations are expansionary or contractionary depends on the source of the shock. In particular, we show that an exchange rate depreciation stemming from foreign UIP shocks or monetary contraction abroad can be contractionary, while exchange rate depreciations stemming from domestic UIP shocks or domestic monetary expansion are always expansionary.

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APPENDIX

A Proofs for Section 3

A.1 Proof of Lemma 1 (Household problem)

Proof. We split the optimization problem of the households into an intra and an inter-temporal problem. The inter-temporal problem determines the consumption-savings and labor supply decision of the households, while the intra-temporal problem determines the allocation of consumption expenditures among tradable goods and non-tradable goods.

The intra-temporal problem of the households consists of choosing (c_t^N, c_t^T) to maximize the consumption bundle $c = \left[\omega(c^T)^{\frac{\gamma-1}{\gamma}} + (1-\omega)(c^N)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}}$ for any given expenditure level $\mathcal{Z}_t$

$$c_t^T + \frac{\bar{P}^N}{P_t^{T*} e_t} c_t^N = \mathcal{Z}_t \tag{A.1}$$

Maximizing c subject to (A.1) yields the following optimality conditions

$$c_t^T = \omega^\gamma (\mathcal{P}_t)^{\gamma-1} \mathcal{Z}_t \tag{A.2}$$

$$\frac{\bar{P}^N}{P_t^{T*} e_t} c_t^N = \left[1 - \omega^\gamma (\mathcal{P}_t)^{\gamma-1}\right] \mathcal{Z}_t \tag{A.3}$$

where $\mathcal{P}$ is defined as the price index that equates total consumption expenditures $\mathcal{Z}_t$ to $\mathcal{P}_t c_t$, which is given by

$$\mathcal{P}_t = \left[\omega^\gamma + (1-\omega)^\gamma \left(\frac{P^N}{P_t^{T*} e_t}\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}. \tag{A.4}$$

Let us consider now the inter-temporal problem of the households. Recall that Y_t is defined as:

$$\mathcal{P}_t Y_t \equiv y_t^T + T_t + (W_t h_t + \phi_t^N)/P_t^T$$

Using this definition and the budget constraint (3) then becomes

$$\mathcal{P}_t c_t + \frac{1}{1+\tau_t}\left[\frac{b_{t+1}}{R_t} + P_t^T \frac{b_{t+1}^*}{R_t^*}\right] = \mathcal{P}_t Y_t \tag{A.5}$$

Let's define the discount rate as $Q_{t|0} \equiv \prod_{s=0}^{t-1} \left(R_s^*(1+\tau_s)\right)^{-1}$. Integrating forward (A.5) and the standard terminal conditions, the inter-temporal problem of the households can

be formulated as follow

$$\max_{\{c_t,h_t\}} \sum_{t=0}^{\infty} \beta^t \left[\frac{(c_t)^{1-\frac{1}{\sigma}}}{1-\sigma^{-1}} - \frac{(h_t)^{1+\phi}}{1+\phi} \right]$$

subject to

$$\sum_{t=0}^{\infty} Q_{t|0} \mathcal{P}_t c_t = \sum_{t=0}^{\infty} Q_{t|0} \mathcal{P}_t Y_t \tag{A.6}$$

The first-order condition of that problem yields the following Euler equation:

$$c_t = \left[\beta R_t^* (1+\tau_t) \frac{\mathcal{P}_t}{\mathcal{P}_{t+1}} \right]^{-\sigma} c_{t+1} \tag{A.7}$$

Plugging (A.7) into the inter-temporal budget constraint (A.6), we obtain that consumption of the composite good in period 0 is given by

$$c_0 = \mu_0 \sum_{t=0}^{\infty} \left(Q_{t|0} \frac{\mathcal{P}_t}{\mathcal{P}_0} \right) Y_t, \tag{A.8}$$

where

$$\mu_0 \equiv \left[\sum_{t=0}^{\infty} (\beta^\sigma)^t q_{t|0}^{1-\sigma} \right]^{-1}$$

is the marginal propensity to consume and

$$q_{t|0} \equiv Q_{t|0} \mathcal{P}_t / \mathcal{P}_0 \tag{A.9}$$

is the effective real interest rate on consumption between period 0 and period t.

We derive the optimal savings decision of the household as the difference between the net income and consumption expenditures. The optimal savings in period 0 is given by

$$b_1^* = R_0^* (1+\tau_0) \left[\mathcal{P}_0 Y_0 - \mu_0 \sum_{t=0}^{\infty} Q_{t|0} \mathcal{P}_t Y_t \right]. \tag{A.10}$$

Finally, we substitute (A.8) into (A.1) and the demand equations (A.2) and (A.3) to obtain households policies for tradable consumption and non-tradable expenditures at date 0

$$c_0^T = \omega^\gamma (\mathcal{P}_0)^{\gamma-1} \mu_0 \sum_{t=0}^{\infty} Q_{t|0} \mathcal{P}_t Y_t, \tag{A.11a}$$

$$\frac{\bar{P}^N}{P^{T*} e_0} c_0^N = \left[1 - \omega^\gamma (\mathcal{P}_0)^{\gamma-1} \right] \mu_0 \sum_{t=0}^{\infty} Q_{t|0} \mathcal{P}_t Y_t. \tag{A.11b}$$

□

A.2 Proof of Proposition 1 (Decomposition of monetary policy)

Preliminaries. We decompose the overall effect of a temporary decline in R at time $t = 0$ which reverts to the steady state value at $t = 1$. Moreover, we assume that for $t \geq 1$, the economy features a level for the exchange rate that is consistent with the first-best allocation. Because we restrict attention to the case in which $\beta R^* = 1$ so that tradable consumption remains constant in the steady state, it is straightforward to see that $R = R^*$ in the steady state. Note that $\tau_t = 0$ which implies that $T_t = 0$.

The proof covers both the decomposition of the overall effect of a monetary expansion on the demand for nontradables c_0^N and the decomposition of the effect a monetary expansion on the demand for tradables c_0^T.

Recall that by definition, the consumption functions and the function for foreign bond holdings are given by:

$$
\begin{aligned}
c_0^N &= \mathcal{C}^N\left(\left\{\bar{P}^N/(P^{T*}e_t), R_t, \tau_t, Y_t\right\}_{t\geq 0}\right), \\
c_0^T &= \mathcal{C}^T\left(\left\{\bar{P}^N/(P^{T*}e_t), R_t, \tau_t, Y_t\right\}_{t\geq 0}\right), \\
b_1^* &= \mathcal{B}^*\left(\left\{\bar{P}^N/(P^{T*}e_t), R_t, \tau_t, Y_t\right\}_{t\geq 0}\right),
\end{aligned}
$$

where recall that Y_t is defined as in (16).

Using the results from Proposition 1 and (A.9), we can express the households' policies (A.10), (A.11a) and (A.11b) as:

$$
c_0^N = \frac{P^{T*}e_0}{\bar{P}^N}\left[1-\omega^\gamma(\mathcal{P}_0)^{\gamma-1}\right]\left[\sum_{t=0}^{\infty}\beta^{t\sigma}q_{t|0}^{1-\sigma}\right]^{-1}\sum_{t=0}^{\infty}q_{t|0}\mathcal{P}_0Y_t \tag{A.13a}
$$

$$
c_0^T = \omega^\gamma(\mathcal{P}_0)^{\gamma-1}\left[\sum_{t=0}^{\infty}\beta^{t\sigma}q_{t|0}^{1-\sigma}\right]^{-1}\sum_{t=0}^{\infty}q_{t|0}\mathcal{P}_0Y_t \tag{A.13b}
$$

$$
\frac{b_1^*}{R_0^*} = \mathcal{P}_0Y_0 - \left[\sum_{t=0}^{\infty}\beta^{t\sigma}q_{t|0}^{1-\sigma}\right]^{-1}\sum_{t=0}^{\infty}q_{t|0}\mathcal{P}_0Y_t \tag{A.13c}
$$

The overall effects of a decline in the nominal interest rate on initial consumption of

non-tradables c_0^N can thus be decomposed as follow:

$$dc_0^N = \underbrace{\sum_{t=0}^{\infty} \frac{\partial \mathcal{C}^N}{\partial e_t} de_t}_{\text{Expend. switching}} + \underbrace{\sum_{t=0}^{\infty} \frac{\partial \mathcal{C}^N}{\partial q_{t|0}} dq_{t|0}}_{\text{Intertemporal subst.}} + \underbrace{\sum_{t=0}^{\infty} \frac{\partial \mathcal{C}^N}{\partial Y_t} dY_t}_{\text{GE effects}} \tag{A.14}$$

This decomposition also applies to the overall effects of a decline in the nominal interest rate on tradable consumption c_0^T and for foreign bond holdings b_1^*.

It will be useful to derive the the effects of an exchange depreciation on the consumer price index. As established in Lemma A.1, when the exchange rate depreciates by 1%, the price index falls by $(1 - \tilde{\omega})$%.

Lemma A.1. *The impact of an exchange rate depreciation on the consumer price index is given by*

$$\frac{d\mathcal{P}_t}{\mathcal{P}_t} = -(1 - \tilde{\omega}_t) \frac{de_t}{e_t} \tag{A.15}$$

Proof. Differentiating the consumer price index (A.4) with respect to e_t yields

$$\begin{aligned} \frac{d\mathcal{P}_t}{de_t} &= (1-\omega)^\gamma \left(\frac{\bar{P}^N}{P_t^{T*} e_t} \right)^{1-\gamma} \frac{1}{e_t} \left[\omega^\gamma + (1-\omega)^\gamma \left(\frac{\bar{P}^N}{P_t^{T*} e_t} \right)^{1-\gamma} \right]^{\frac{\gamma}{1-\gamma}} \\ &= -(1-\omega)^\gamma \left[\frac{P_t^{T*} e_t}{\bar{P}^N} \mathcal{P}_t \right]^{\gamma-1} \frac{\mathcal{P}_t}{e_t} \end{aligned} \tag{A.16}$$

Next, use the optimality that describes the household's demand for nontradable goods and substitute (A.2) to obtain

$$\begin{aligned} c_t^N &= \left[\frac{1-\tilde{\omega}}{\omega} \frac{P^{T*} e_t}{\bar{P}^N} \right]^\gamma c_t^T \\ &= \frac{P_t^{T*} e_t}{\bar{P}^N} (1-\omega)^\gamma \left[\frac{P_t^{T*} e_t}{\bar{P}^N} \mathcal{P}_t \right]^{\gamma-1} \mathcal{P}_t c_t \end{aligned} \tag{A.17}$$

where we substitute $\mathcal{Z}_t = \mathcal{P}_t c_t$ in (A.2). Using the definition of the expenditure share in tradables (18), we have that $\frac{\bar{P}^N}{P_t^{T*} e_t} c_t^N = (1 - \tilde{\omega}_t) \mathcal{P}_t c_t$. Thus, (A.17) implies that

$$1 - \tilde{\omega}_t = (1-\omega)^\gamma \left[\frac{P_t^{T*} e_t}{\bar{P}^N} \mathcal{P}_t \right]^{\gamma-1} \tag{A.18}$$

Finally, substitute (A.18) into (A.16) to get

$$\frac{d\mathcal{P}_t}{\mathcal{P}_t} = -(1 - \tilde{\omega}_t) \frac{de_t}{e_t}$$

□

We now proceed by calculating the different terms in the decomposition.

Expenditure switching. We start with the expenditure switching channel. Using equation (A.15) in Lemma A.1, the derivatives of the policy functions, (A.13b), (A.13a) and (A.13c), with respect to e_t (starting from the stationary equilibrium, with $b_1^* = 0$) are

$$\sum_{t=0}^{\infty} \frac{\partial \log \mathcal{C}^N}{\partial e_t} de_t = \tilde{\omega}\gamma \; d\log e_0 \tag{A.19a}$$

$$\sum_{t=0}^{\infty} \frac{\partial \log \mathcal{C}^T}{\partial e_t} de_t = -(1-\tilde{\omega})\gamma \; d\log e_0 \tag{A.19b}$$

$$\sum_{t=0}^{\infty} \frac{\partial \mathcal{B}^*}{\partial e_t} de_t = 0 \tag{A.19c}$$

Intertemporal substitution. Consider next the derivative of the policy functions with respect to $q_{t|0}$. Letting $\iota_0 \equiv \frac{P^{T*}e_0}{\bar{P}^N}(1-\tilde{\omega}_0)$, we have

$$\frac{\partial \mathcal{C}^N}{\partial q_{t|0}} = -(1-\sigma)\beta^{\sigma t} q_{t|0}^{-\sigma} \left[\sum_{t=0}^{\infty} \beta^{t\sigma} q_{t|0}^{1-\sigma}\right]^{-2} \iota_0 \sum_{t=0}^{\infty} q_{t|0}\mathcal{P}_0 Y_t + \iota_0 \left[\sum_{t=0}^{\infty} \beta^{t\sigma} q_{t|0}^{1-\sigma}\right]^{-1} \mathcal{P}_0 Y_t$$

Using that $q_{0|0} = 1$ and that $q_{t|0}$ is constant for all $t > 0$, given that the economy reaches the stationary equilibrium at $t = 1$.

$$\begin{aligned}\sum_{t=0}^{\infty} \frac{\partial \mathcal{C}^N}{\partial q_{t|0}} dq_{t|0} &= (\sigma-1)\sum_{t=1}^{\infty} \beta^{\sigma t} q_{t|0}^{1-\sigma} \frac{dq_{t|0}}{q_{t|0}} \cdot \mu_0^2 \iota_0 \sum_{t=0}^{\infty} q_{t|0}\mathcal{P}_0 Y_t + \mu_0 \iota_0 \sum_{t=1}^{\infty} \mathcal{P}_0 Y_t \; dq_{t|0} \\ &= \left[(\sigma-1)(1-\mu_0)\mu_0\iota_0 \sum_{t=0}^{\infty} q_{t|0}\mathcal{P}_0 Y_t + \mu_0\iota_0 \sum_{t=1}^{\infty} q_{t|0}\mathcal{P}_0 Y_t\right] \frac{dq_{t|0}}{q_{t|0}}\end{aligned} \tag{A.20}$$

Moreover, note that

$$\begin{aligned}\sum_{t=1}^{\infty} q_{t|0}\mathcal{P}_0 Y_t &= \sum_{t=0}^{\infty} q_{t|0}\mathcal{P}_0 Y_t - \mathcal{P}_0 c_0 + \mathcal{P}_0(c_0 - Y_0) \\ &= \sum_{t=0}^{\infty} q_{t|0}\mathcal{P}_0 Y_t - \mathcal{P}_0 \; \mu_0 \sum_{t=0}^{\infty} q_{t|0} Y_t + \mathcal{P}_0(c_0 - Y_0) \\ &= (1-\mu_0)\sum_{t=0}^{\infty} q_{t|0}\mathcal{P}_0 Y_t - \frac{b_1^*}{R_0^*}\end{aligned} \tag{A.21}$$

Plugging (A.21) into (A.20), we get

$$\sum_{t=0}^{\infty} \frac{\partial \mathcal{C}^N}{\partial q_{t|0}} dq_{t|0} = \left[\sigma(1-\mu_0)\mu_0\iota_0 \sum_{t=0}^{\infty} q_{t|0}\mathcal{P}_0 Y_t - \mu_0\iota_0 \frac{b_1^*}{R_0^*} \right] \frac{dq_{t|0}}{q_{t|0}}$$
$$= \left[\sigma(1-\mu_0)c_0^T - \mu_0\iota_0 \frac{b_1^*}{R_0^*} \right] \frac{dq_{t|0}}{q_{t|0}} \tag{A.22}$$

Differentiating $q_{t|0} = Q_{t|0}\mathcal{P}_t/\mathcal{P}_0$, we obtain that the change in the relative price induced by the temporary decline in the nominal interest rate yields $d\log q_{0|0} = 0$ and

$$d\log q_{t|0} = -(1-\tilde{\omega})d\log R_0 \quad \text{for } t \geq 1 \tag{A.23}$$

Finally substituting (A.23) into (A.22) and starting from the stationary equilibrium, with $b_1^* = 0$, we arrive to the expression in the proposition.

$$\sum_{t=0}^{\infty} \frac{\partial \log \mathcal{C}^N}{\partial q_{t|0}} dq_{t|0} = -\beta\sigma(1-\tilde{\omega})d\log R_0 \tag{A.24}$$

Proceeding similar for tradable consumption (with $\iota_0 = \tilde{\omega}_0$), we have

$$\sum_{t=0}^{\infty} \frac{\partial \log \mathcal{C}^T}{\partial q_{t|0}} dq_{t|0} = -\beta\sigma(1-\tilde{\omega})d\log R_0 \tag{A.25}$$

Next, we turn to deriving the effects on foreign bond holdings. We have

$$\sum_{t=0}^{\infty} \frac{\partial \mathcal{B}^*}{\partial q_{t|0}} dq_{t|0} = -R_0^* \left[(\sigma-1)(1-\mu_0)\mu_0 \sum_{t=0}^{\infty} q_{t|0}\mathcal{P}_0 Y_t + \mu_0 \sum_{t=1}^{\infty} q_{t|0}\mathcal{P}_0 Y_t \right] \frac{dq_{t|0}}{q_{t|0}}$$
$$= -R_0^* \left[\sigma(1-\mu_0)\mathcal{P}_0 c_0 - \mu_0 \frac{b_1^*}{R_0^*} \right] \frac{dq_{t|0}}{q_{t|0}} \tag{A.26}$$

where the second equality uses (A.21) and (A.7). We then substitute (A.23) into (A.26), and starting from the stationary equilibrium, with $b_1^* = 0$, we arrive to the expression in the proposition

$$\sum_{t=0}^{\infty} \frac{\partial \mathcal{B}^*}{\partial q_{t|0}} dq_{t|0} = \mathcal{P}c \cdot \sigma(1-\tilde{\omega}) \frac{dR_0}{R_0} \tag{A.27}$$

Effects through income. Using the definition of Y_t and totally differentiating with respect to e_t and y_t^N, we obtain

$$dY_t = \frac{\partial Y_t}{\partial e_t} de_t + \frac{\partial Y_t}{\partial y_t^N} dy_t^N \tag{A.28}$$

The total effect of the change in income on non-tradable consumption is thus given by

$$\sum_{t=0}^{\infty} \frac{\partial \mathcal{C}^N}{\partial Y_t} dY_t = \underbrace{\sum_{t=0}^{\infty} \frac{\partial \mathcal{C}^N}{\partial Y_t} \left[\frac{\partial Y_t}{\partial e_t} de_t \right]}_{\text{valuation effects}} + \underbrace{\sum_{t=0}^{\infty} \frac{\partial \mathcal{C}^N}{\partial Y_t} \left[\frac{\partial Y_t}{\partial y_t^N} dy_t^N \right]}_{\text{aggregate income channel}} \tag{A.29}$$

where

$$\frac{\partial \mathcal{C}^N}{\partial Y_t} = \frac{P^{T*} e_0}{\bar{P}^N} \left[1 - \omega^\gamma (\mathcal{P}_0)^{\gamma-1} \right] \mu_0 q_{t|0} \mathcal{P}_0 \tag{A.30}$$

We also use (A.28) to split the general equilibrium through income on tradable consumption and foreign bond holdings into the valuation effects and the aggregate income channel where

$$\frac{\partial \mathcal{C}^T}{\partial Y_t} = \omega^\gamma (\mathcal{P}_0)^{\gamma-1} \mu_0 q_{t|0} \mathcal{P}_0,$$

$$\frac{\partial \mathcal{B}^*}{\partial Y_0} = R_0^* (1 - \mu_0) \cdot \mathcal{P}_0 \quad \text{and} \quad \frac{\partial \mathcal{B}^*}{\partial Y_t} = -R_0^* \mu_0 \cdot q_{t|0} \mathcal{P}_0 \quad \text{for } t \geq 1 \tag{A.31}$$

Valuation effects. From (16), we can see that an increase in the nominal exchange rate reduces the value of income measured in units of the CPI. In particular,

$$\begin{aligned} \frac{\partial Y_t}{\partial e_t} &= -\frac{1}{\mathcal{P}_t} \frac{W_t h_t + \phi^N}{P^{T*} e_t} \frac{1}{e_t} - \frac{1}{\mathcal{P}_t^2} \frac{\partial \mathcal{P}_t}{\partial e_t} Y_t \\ &= -\frac{1}{\mathcal{P}_t} \frac{\bar{P}^N y_t^N}{P_t^T} \frac{1}{e_t} + (1 - \tilde{\omega}_t) \frac{1}{e_t} Y_t \\ &= -(1 - \tilde{\omega}_t)(c_t - Y_t) \frac{1}{e_t} \end{aligned} \tag{A.32}$$

where the last equality combine (A.3), (A.18) and the market clearing condition for the non-tradable good $c_t^N = y_t^N$ to get $\bar{P}^N c_t^N / P_t^T = (1 - \tilde{\omega}_t) \mathcal{P}_t c_t$. From (A.30) and (A.32), we have

$$\begin{aligned} \sum_{t=0}^{\infty} \frac{\partial \mathcal{C}^N}{\partial Y_t} \left[\frac{\partial Y_t}{\partial e_t} de_t \right] &= \iota_0 \mu_0 \sum_{t=0}^{\infty} Q_{t|0} \mathcal{P}_t (Y_t - c_t)(1 - \tilde{\omega}_t) \frac{de_t}{e_t} \\ &= \iota_0 \mu_0 \mathcal{P}_0 (Y_0 - c_0) \left[(1 - \tilde{\omega}_0) \frac{de_0}{e_0} - (1 - \tilde{\omega}_1) \frac{de_1}{e_1} \right] \\ &= \iota_0 \mu_0 \frac{b_1^*}{R_0^*} \left[(1 - \tilde{\omega}_0) \frac{de_0}{e_0} - (1 - \tilde{\omega}_1) \frac{de_1}{e_1} \right] \end{aligned} \tag{A.33}$$

where the second equality uses $\sum_{t=1}^{\infty} Q_{t|0} \mathcal{P}_t (Y_t - c_t) = \mathcal{P}_0 (c_0 - Y_0)$ from (A.6) and $e_t = e_1$ for $t \geq 1$. The last equality uses the flow budget constraint, that is $P_0 (c_0 - Y_0) = b_1^* / R_0^*$.

Hence, starting from the stationary equilibrium, with $b_1^* = 0$, we arrive to

$$\sum_{t=0}^{\infty} \frac{\partial \log \mathcal{C}^N}{\partial Y_t} \left[\frac{\partial Y_t}{\partial e_t} de_t \right] = 0 \tag{A.34}$$

Proceeding similar for tradable consumption $\mathcal{C}^T$ (with $\iota_0 = \tilde{\omega}_0$), we have that starting from the stationary equilibrium, with $b_1^* = 0$

$$\sum_{t=0}^{\infty} \frac{\partial \log \mathcal{C}^T}{\partial Y_t} \left[\frac{\partial Y_t}{\partial e_t} de_t \right] = 0 \tag{A.35}$$

For foreign bond holdings, we use again $\sum_{t=1}^{\infty} Q_{t|0} \mathcal{P}_t (Y_t - c_t) = \mathcal{P}_0 (c_0 - Y_0)$ from (A.6) and $e_t = e_1$ for $t \geq 1$, to obtain

$$\begin{aligned}
\sum_{t=0}^{\infty} \frac{\partial \mathcal{B}^*}{\partial Y_t} \left[\frac{\partial Y_t}{\partial e_t} de_t \right] &= R_0^* (1 - \mu_0) \mathcal{P}_0 \frac{\partial Y_0}{\partial e_0} de_0 - R_0^* \mu_0 \sum_{t=1}^{\infty} Q_{t|0} \mathcal{P}_t \frac{\partial Y_t}{\partial e_t} de_t \\
&= R_0^* (1 - \mu_0) \mathcal{P}_0 (Y_0 - c_0)(1 - \tilde{\omega}_0) \frac{de_0}{e_0} - R_0^* \mu_0 \sum_{t=1}^{\infty} Q_{t|0} \mathcal{P}_t (Y_t - c_t)(1 - \tilde{\omega}_t) \frac{de_t}{e_t} \\
&= b_1^* \left[(1 - \mu_0)(1 - \tilde{\omega}_0) \frac{de_0}{e_0} + \mu_0 (1 - \tilde{\omega}_1) \frac{de_1}{e_1} \right]
\end{aligned} \tag{A.36}$$

Thus, starting from the stationary equilibrium, with $b_1^* = 0$, we arrive to

$$\sum_{t=0}^{\infty} \frac{\partial \mathcal{B}^*}{\partial Y_t} \left[\frac{\partial Y_t}{\partial e_t} de_t \right] = 0 \tag{A.37}$$

Aggregate income. We now deriving the change in real labor income, dy_0^N induced by the temporary decline in nominal interest rate. In equilibrium $y^N = c^N$, and from (A.2) and using Lemma A.1 we have

$$y_t^N = \frac{P_t^{T*}}{\bar{P}^N} e_t \left[1 - \omega^{\gamma} (\mathcal{P}_t)^{\gamma - 1} \right] \mathcal{P}_t c_t \quad \Rightarrow \quad \frac{dy_t^N}{y_t^N} = \gamma \tilde{\omega} \frac{de_t}{e_t} + \frac{dc_t}{c_t} \tag{A.38}$$

Given that we reach a stationary equilibrium at $t = 1$, we have $e_{t+1} = e_t$ and $c_{t+1}^T = c_1^T$ for all $t \geq 1$. In addition the stationary level of tradable consumption is $c_1^T = y^T + (1 - \beta) b_1^*$. Thus, the next period level of consumption can be expressed as $c_{t+1} = \omega^{-\gamma} \mathcal{P}_{t+1}^{-\gamma} [y^T + (1 - \beta) b_1^*]$ and replacing this in (A.7) yields

$$c_t = \left[\beta R^* (1 + \tau_t) \frac{\mathcal{P}_t}{\mathcal{P}_{t+1}} \right]^{-\sigma} \omega^{-\gamma} \mathcal{P}_{t+1}^{-\gamma} \left[y^T + (1 - \beta) b_1^* \right] \tag{A.39}$$

Differentiating this equation and using (A.39) we obtain

$$\frac{dc_t}{c_t} = -\sigma d\tau_t - \sigma(1-\tilde{\omega})\frac{dR_t}{R_t} + \gamma(1-\tilde{\omega})\frac{de_{t+1}}{e_{t+1}} + (1-\beta)\frac{db_1^*}{c^T} \tag{A.40}$$

Recall that absent capital controls $d\tau_t = 0$ for all t. We then plug (A.40) into (A.38) to get

$$\frac{dy_t^N}{y_t^N} = -\left[(1-\tilde{\omega})\sigma + \tilde{\omega}\gamma\right]\frac{dR_t}{R_t} + \gamma\frac{de_1}{e_1} + (1-\beta)\frac{db_1^*}{c^T} \tag{A.41}$$

Use (A.41) to determine the effects of changes in y^N on foreign bond holdings

$$\begin{aligned}\sum_{t=0}^{\infty}\frac{\partial \mathcal{B}^*}{\partial Y_t}\left[\frac{\partial Y_t}{\partial y_t^N}dy_t^N\right] &= (1-\mu_0)R_0^*(1-\tilde{\omega}_0)\mathcal{P}_0 c_0\frac{dy_0^N}{y_0^N} - \mu_0 R_0^*\sum_{t=1}^{\infty}Q_{t|0}\mathcal{P}_t c_t(1-\tilde{\omega}_t)\frac{dy_t^N}{y_t^N}\\ &= \beta R^*(1-\tilde{\omega})\mathcal{P}c\left[\frac{dy_0^N}{y^N} - \frac{dy_1^N}{y^N}\right]\\ &= -(1-\tilde{\omega})\mathcal{P}c\ \left[(1-\tilde{\omega})\sigma + \tilde{\omega}\gamma\right]d\log R_0 \end{aligned} \tag{A.42}$$

The second equality evaluates the first one at the stationary equilibrium. We then combine (A.27), (A.37) and (A.42) and arrive to

$$db_1^* = (1-\tilde{\omega})(\sigma-\gamma)c^T d\log R_0 \tag{A.43}$$

Next, we determine de_1. Because e_1 is consistent with the first-best allocation e_1 satisfies (**??**). We define $\kappa \equiv \tilde{\omega}\left[1 + \frac{\alpha(1-\tilde{\omega})(\gamma-\sigma)}{\alpha\sigma+(1-\alpha+\phi)\gamma\sigma}\right]^{-1}$. It is straightforward to see that $\kappa \in [0,1]$. The derivative of e_1 evaluated at the stationary equilibrium is given by

$$d\log e_1 = -\frac{1-\kappa}{1-\tilde{\omega}}\frac{1}{\gamma}(1-\beta)\frac{db_1^*}{c^T} \tag{A.44}$$

$$= (1-\kappa)(1-\beta)(\gamma-\sigma)\frac{1}{\gamma}d\log R_0 \tag{A.45}$$

where the second equality uses (A.43) to substitute for db_1^*. We substitute (A.43) and (A.45) into to get the change in output induced by a change in R_0,

$$\frac{dy_t^N}{y_t^N} = -\left[(1-\tilde{\omega})\sigma + \tilde{\omega}\gamma\right]\frac{dR_t}{R_t} + (1-\beta)(\kappa-\tilde{\omega})(\sigma-\gamma)\frac{dR_0}{R_0} \tag{A.46}$$

The aggregate income effect on nontradable consumption is given by

$$\sum_{t=0}^{\infty}\frac{\partial \mathcal{C}^N}{\partial Y_t}\left[\frac{\partial Y_t}{\partial y_t^N}dy_t^N\right] = \mu_0\iota_0\sum_{t=0}^{\infty}Q_{t|0}\mathcal{P}_t c_t(1-\tilde{\omega}_t)\frac{dy_t^N}{y^N}$$

Starting from the stationary equilibrium and plugging equation (A.46) into this equation, we have that

$$\begin{aligned}\sum_{t=0}^{\infty} \frac{\partial \log \mathcal{C}^N}{\partial Y_t}\left[\frac{\partial Y_t}{\partial y_t^N} dy_t^N\right] &= (1-\beta)(1-\tilde{\omega})\frac{dy_0^N}{y^N} + (1-\beta)(1-\tilde{\omega})\sum_{t=1}^{\infty}\beta^t \frac{dy_t^N}{y^N} \\ &= -(1-\beta)(1-\tilde{\omega})\left[(1-\tilde{\omega})\sigma + \tilde{\omega}\gamma + (\kappa-\tilde{\omega})(\gamma-\sigma)\right] d\log R_0 \\ &= -(1-\beta)(1-\tilde{\omega})\left[(1-\kappa)\sigma + \kappa\gamma\right] d\log R_0 \end{aligned} \tag{A.47}$$

Proceeding similar for $\mathcal{C}^T$, we use (A.43) and (A.45) to get

$$\sum_{t=0}^{\infty} \frac{\partial \log \mathcal{C}^T}{\partial Y_t}\left[\frac{\partial Y_t}{\partial y_t^N} dy_t^N\right] = -(1-\beta)(1-\tilde{\omega})\left[(1-\kappa)\sigma + \kappa\gamma\right] d\log R_0 \tag{A.48}$$

Conclusion of Proof of Proposition 1 We combine (A.19a), (A.24), (A.34) and (A.47) to obtain the decomposition of the overall effect of a temporary change in the nominal interest rate R_0 on non-tradable consumption into three channels

$$\frac{dc_0^N}{c_0^N} = \underbrace{\tilde{\omega}\gamma\frac{de_0}{e_0}}_{\text{Expend. switching}} \underbrace{-\beta\sigma\,(1-\tilde{\omega})\frac{dR_0}{R_0}}_{\text{Intertemporal subst.}} \underbrace{-(1-\beta)\left[(1-\kappa)\sigma+\kappa\gamma\right](1-\tilde{\omega})\frac{dR_0}{R_0}}_{\text{aggregate income channel}} \tag{A.49}$$

Because of market clearing, this also corresponds to the decomposition of the effect of a temporary decrease in the nominal interest rate on output as in proposition 1. We use the UIP condition and (A.45) to obtain the change in the nominal exchange rate

$$\frac{de_0}{e_0} = -\frac{dR_0}{dR_0} + (1-\kappa)(1-\beta)(\gamma-\sigma)\frac{1}{\gamma}\frac{dR_0}{dR_0} \tag{A.50}$$

and the overall effects on y_t^N for $t>0$ follows from (A.46) with $dR_t = 0$

$$\frac{dy_t^N}{y_t^N} = (1-\beta)(\kappa-\tilde{\omega})(\sigma-\gamma)\frac{dR_0}{R_0}, \quad \text{for } t>0$$

Similarly, we combine (A.19b), (A.25), (A.35) and (A.48) to obtain the decomposition of the overall effect of a temporary change in the nominal interest rate R_0 on tradable consumption

$$\frac{dc_0^T}{c_0^T} = \underbrace{-(1-\tilde{\omega})\gamma\frac{de_0}{e_0}}_{\text{Expend. switching}} \underbrace{-\beta\sigma\,(1-\tilde{\omega})\frac{dR_0}{R_0}}_{\text{Intertemporal subst.}} \underbrace{-(1-\beta)\left[(1-\kappa)\sigma+\kappa\gamma\right](1-\tilde{\omega})\frac{dR_0}{R_0}}_{\text{aggregate income channel}}$$

Finally, from (A.27), (A.37) and (A.42) it follows that the overall effect of a temporary

change in the nominal interest rate on savings can be decomposed as

$$\frac{db_1^*}{\mathcal{P}c} = \underbrace{\sigma\,(1-\tilde{\omega})\frac{dR_0}{R_0}}_{\text{Intertemporal subst.}} - \underbrace{[(1-\tilde{\omega})\sigma+\tilde{\omega}\gamma]\,(1-\tilde{\omega})\frac{dR_0}{R_0}}_{\text{aggregate income channel}}$$

A.3 Proof of Corollary 1

Proof. The proof of the corollary follows directly from equation (21) in Proposition 1. From equation (21),

$$\begin{aligned}\frac{db_1^*}{\mathcal{P}c} &= \sigma\,(1-\tilde{\omega})\frac{dR_0}{R_0} - [\sigma+\tilde{\omega}(\gamma-\sigma)]\,(1-\tilde{\omega})\frac{dR_0}{R_0}\\ &= \tilde{\omega}(1-\tilde{\omega})(\sigma-\gamma)\frac{dR_0}{R_0}\end{aligned} \tag{A.51}$$

It then follows that

$$\text{sign}\left(\frac{db_1^*}{dR_0}\right) = \text{sign}\,(\sigma-\gamma)$$

The response of aggregate savings to a monetary expansion thus follows. □

A.4 Proof of Proposition 2

Proof. Because next period foreign bond holdings is the only state variable for future income and exchange rates, under closed capital capital $db_1^* = 0$, a temporary monetary expansion has no effect on future income and exchange rate, $dy_t^N = 0$ and $de_t = 0$ for $t \geq 1$. We then turn to calculating each component of the decomposition of the effects of a change in R_0 on nontradable consumption (A.14) under closed capital account. Differentiating (A.13b) with respect to e_0 and using (A.15), for given $q_{t|0}$ and Y_t, the expenditure switching effect is given by

$$\begin{aligned}\frac{\partial \log \mathcal{C}^N}{\partial e_0}de_0 &= \tilde{\omega}\gamma\; d\log e_0\\ &= -\tilde{\omega}\gamma\; d\log R_0\end{aligned} \tag{A.52}$$

noting that from the UIP condition $d\log e_0 = -d\log R_0 + d\log e_1$. It can easily be shown that the expression for intertemporal substitution effect, the direct effect from changes in $q_{t|0}$, is not affected by the assumption that the capital account is closed. We again arrive to (A.24), that is

$$\sum_{t=0}^{\infty}\frac{\partial \log \mathcal{C}^N}{\partial q_{t|0}}dq_{t|0} = -\beta\sigma(1-\tilde{\omega})d\log R_0 \tag{A.53}$$

It is straightforward to see from (A.32) that the valuation effects are once again nil,

$$\frac{\partial \log \mathcal{C}^N}{\partial Y_0}\left[\frac{\partial Y_0}{\partial e_0}de_0\right] = 0 \tag{A.54}$$

Next, we turn to the aggregate income effects. First, we have

$$\frac{\partial \mathcal{C}^N}{\partial Y_0} = \frac{P^{T*}e_0}{\bar{P}^N}\left[1-\omega^\gamma(\mathcal{P}_0)^{\gamma-1}\right]\mu_0 \tag{A.55}$$

Then, use (A.41) with $db_1^* = 0$ and $de_1 = 0$ to obtain

$$\frac{dy_0^N}{y_0^N} = -\left[(1-\tilde{\omega})\sigma + \tilde{\omega}\gamma\right]\frac{dR_0}{R_0} \tag{A.56}$$

Equations (A.55) and (A.56) in turn imply, starting from the stationary equilibrium, that

$$\frac{\partial \log \mathcal{C}^N}{\partial Y_0}\left[\frac{\partial Y_0}{\partial y_0^N}dy_0^N\right] = -(1-\beta)\left[(1-\tilde{\omega})\sigma + \tilde{\omega}\gamma\right]\frac{dR_0}{R_0} \tag{A.57}$$

Finally, combine (A.52), (A.53), (A.54) and (A.57) to get the decomposition of the overall effect of a temporary change in R_0 on c^N under closed capital account

$$\frac{dc_0^N}{c_0^N} = \underbrace{\tilde{\omega}\gamma\frac{dR_0}{R_0}}_{\text{Expend. switching}} \underbrace{-\beta\sigma\,(1-\tilde{\omega})\frac{dR_0}{R_0}}_{\text{Intertemporal subst.}} \underbrace{-(1-\beta)\left[(1-\tilde{\omega})\sigma+\tilde{\omega}\gamma\right](1-\tilde{\omega})\frac{dR_0}{R_0}}_{\text{aggregate income channel}} \tag{A.58}$$

Because of market clearing, this also corresponds to the decomposition of the effect of a temporary decrease in the nominal interest rate on output under closed capital account.

Next, subtract (A.58) from (A.49) and use (A.50) to obtain the difference between the effect of a change in R_0 on output under open capital account, $d\log y_0^{N,o}$ and its effect on output under closed capital account $d\log y_0^{N,c}$. We have

$$\begin{aligned} d\log y_0^{N,o} - d\log y_0^{N,c} &= \tilde{\omega}(1-\beta)(1-\kappa)\,(\gamma-\sigma)\,\frac{dR_0}{R_0} - (1-\beta)(1-\tilde{\omega})\,(\kappa-\tilde{\omega})\,(\gamma-\sigma)\,\frac{dR_0}{R_0} \\ &= \tilde{\omega}(2-\tilde{\omega}-\kappa)(1-\beta)\,(\gamma-\sigma)\,\frac{dR_0}{R_0} \end{aligned}$$

Note that $2-\tilde{\omega}-\kappa = (1-\tilde{\omega})+(1-\kappa) > 0$. Thus, if $\gamma > \sigma$, the effect of a monetary expandion $dR_0 < 0$ on output is larger with a closed capital account. Conversely, if $\gamma < \sigma$, the effect of a monetary expandion is larger with a open capital account. The effect is the same with open and closed capital account if $\gamma = \sigma$. □

A.5 Proof of Proposition 3 (Decomposition of capital controls)

We decompose the overall effect of a temporary increase in a capital control τ at time $t = 0$ which reverts to the steady state value at $t = 1$ under passive monetary policy response (i.e., R remains constant). We assume that for $t \geq 1$ the economy features a level for the exchange rate that is consistent with the first-best allocation.

Proof. We follow the same procedure as in section A.2. The expression of the households' policy for non-tradable consumption and foreign bond holdings are given by (A.13b) and (A.13c) where $q_{t|0} = (\mathcal{P}_t/\mathcal{P}_0)\prod_{s=0}^{t-1}(R_s^*(1+\tau_s))^{-1}$. Totally differentiate these function, we obtain the following decomposition of the overall effects of an increase in τ_0 on non-tradable consumption

$$dc_0^N = \underbrace{\sum_{t=0}^{\infty}\frac{\partial \mathcal{C}^N}{\partial e_t}de_t}_{\text{Expend. switching}} + \underbrace{\sum_{t=0}^{\infty}\frac{\partial \mathcal{C}^N}{\partial q_{t|0}}dq_{t|0}}_{\text{Intertemporal subst.}} + \underbrace{\sum_{t=0}^{\infty}\frac{\partial \mathcal{C}^N}{\partial Y_t}dY_t}_{\text{GE effects}}$$

$$db_1^* = \underbrace{\sum_{t=0}^{\infty}\frac{\partial \mathcal{B}^*}{\partial e_t}de_t}_{\text{Expend. switching}} + \underbrace{\sum_{t=0}^{\infty}\frac{\partial \mathcal{B}^*}{\partial q_{t|0}}dq_{t|0}}_{\text{Intertemporal subst.}} + \underbrace{\sum_{t=0}^{\infty}\frac{\partial \mathcal{B}^*}{\partial Y_t}dY_t}_{\text{GE effects}}$$

Expenditure switching. start by characterizing the expenditure switching channel. Using equation (A.15) in Lemma A.1, the derivative of (A.13b) and (A.13c) with respect to e_t, starting from the stationary equilibrium, are

$$\sum_{t=0}^{\infty}\frac{\partial \log \mathcal{C}^N}{\partial e_t}de_t = [1-\tilde{\omega}(1-\gamma)]\, d\log e_0 \tag{A.59}$$

$$\sum_{t=0}^{\infty}\frac{\partial \log \mathcal{B}^*}{\partial e_t}de_t = 0 \tag{A.60}$$

Intertemporal substitution. First, notice that the change in $q_{t|0}$ induced by the decline in τ_0 is given by $d\log q_{0|0} = 0$ and for $t \geq 1$

$$\begin{aligned} d\log q_{t|0} &= -d\tau_0 - (1-\tilde{\omega})d\log R_0, \\ &= -d\tau_0, \end{aligned} \tag{A.61}$$

Substituting (A.61) into (A.22) starting from the stationary equilibrium, with $b_1^* = 0$, we arrive to the expression in the proposition

$$\sum_{t=0}^{\infty}\frac{\partial \log \mathcal{C}^N}{\partial q_{t|0}}dq_{t|0} = -\sigma\beta d\tau_0 \tag{A.62}$$

Substituting (A.61) into (A.26) starting from the stationary equilibrium, with $b_1^* = 0$, we arrive to the expression in the proposition

$$\sum_{t=0}^{\infty} \frac{\partial \log \mathcal{B}^*}{\partial q_{t|0}} dq_{t|0} = \sigma \beta d\tau_0 \tag{A.63}$$

Effects through income. Using the definition of Y_t, with $T_t \neq 0$, and totally differentiating with respect to e_t, T_t and y_t^N, we have that

$$dY_t = \frac{\partial Y_t}{\partial e_t} de_t + \frac{\partial Y_t}{\partial T_t} dT_t + \frac{\partial Y_t}{\partial y_t^N} dy_t^N$$

The total effect of the change in Y_t on non-tradable consumption is thus given by

$$\sum_{t=0}^{\infty} \frac{\partial \mathcal{C}^N}{\partial Y_t} dY_t = \underbrace{\sum_{t=0}^{\infty} \frac{\partial \mathcal{C}^N}{\partial Y_t} \left[\frac{\partial Y_t}{\partial e_t} de_t \right]}_{\text{valuation effects}} + \underbrace{\sum_{t=0}^{\infty} \frac{\partial \mathcal{C}^N}{\partial Y_t} \left[\frac{\partial Y_t}{\partial T_t} dT_t \right]}_{\text{fiscal channel}} + \underbrace{\sum_{t=0}^{\infty} \frac{\partial \mathcal{C}^N}{\partial Y_t} \left[\frac{\partial Y_t}{\partial y_t^N} dy_t^N \right]}_{\text{aggregate income channel}}$$

where $\partial \mathcal{C}^N / \partial Y_t$ is given by (A.30). Similarly, we decompose the total effect of a change in Y_t on foreign bond holdings where $\partial \mathcal{B}^* / \partial Y_t$ is given by (A.31). Consider first the valuation effects. Using directly (A.33) and (A.36), and starting at a stationary equilibrium, we have

$$\sum_{t=0}^{\infty} \frac{\partial \log \mathcal{C}^N}{\partial Y_t} \left[\frac{\partial Y_t}{\partial e_t} de_t \right] = 0 \quad \text{and} \quad \sum_{t=0}^{\infty} \frac{\partial \log \mathcal{B}^*}{\partial Y_t} \left[\frac{\partial Y_t}{\partial e_t} de_t \right] = 0 \tag{A.64}$$

Differentiating the government budget, $T_t = -\frac{\tau_t}{1+\tau_t} \frac{b_{t+1}^*}{R_t^*}$, leads to $dT_0 = -\frac{b_1^*}{R^*} d\tau_0$ and $dT_t = 0$ for $t \geq 1$. Thus, starting at the stationary equilibrium with $b_1^* = 0$, $dT_0 = 0$ and we have

$$\sum_{t=0}^{\infty} \frac{\partial \log \mathcal{C}^N}{\partial T_t} \left[\frac{\partial Y_t}{\partial T_t} dT_t \right] = 0 \quad \text{and} \quad \sum_{t=0}^{\infty} \frac{\partial \log \mathcal{B}^*}{\partial T_t} \left[\frac{\partial Y_t}{\partial T_t} dT_t \right] = 0 \tag{A.65}$$

Finally, for the aggregate income channel, substitute (A.40) into into (A.38), along with $dR_t = 0$ for all t as the nominal interest R remain constant, to get

$$dy_t^N = \left[-\sigma d\tau_t + \gamma \frac{de_1}{e_1} + (1-\beta) \frac{db_1^*}{c^T} \right] y_t^N \tag{A.66}$$

We use (A.66) to evaluate the effects of changes in y^N on foreign bond holdings starting from the stationary equilibrium. We have

$$\sum_{t=0}^{\infty} \frac{\partial \mathcal{B}^*}{\partial Y_t} \left[\frac{\partial Y_t}{\partial y_t^N} dy_t^N \right] = \beta R^* (1-\tilde{\omega}) \mathcal{P} c \left[\frac{dy_0^N}{y^N} - \frac{dy_1^N}{y^N} \right]$$
$$= -(1-\tilde{\omega}) \mathcal{P} c \, \sigma d\tau_0 \tag{A.67}$$

Combining (A.60), (A.63)-(A.65) and (A.67), we arrive to the decomposition of a change in the tax on debt τ_0 on capital flows as in Proposition 3

$$\frac{db_1^*}{\mathcal{P}c} = \underbrace{\sigma d\tau_0}_{\text{Intertemporal Subst.}} - \underbrace{(1-\tilde{\omega})\sigma d\tau_0}_{\text{Aggregate Income}}$$

This in turn implies that the overall effect of capital controls on foreign bond holdings is given by

$$db_1^* = \sigma c^T d\tau_0 \tag{A.68}$$

We then substitute (A.68) into (A.44) to obtain

$$d \log e_1 = -\frac{1-\kappa}{1-\tilde{\omega}} (1-\beta) \sigma \frac{1}{\gamma} d\tau_0 \tag{A.69}$$

Next, we plug (A.68) and (A.69) into (A.66) and use (A.30), starting from the stationary, to obtain the effects of changes in y^N on non-tradable consumption. Formally, we have

$$\sum_{t=0}^{\infty} \frac{\partial \log \mathcal{C}^N}{\partial Y_t} \left[\frac{\partial Y_{t|0}}{\partial y_t^N} dy_t^N \right] = (1-\beta)(1-\tilde{\omega}) \frac{dy_0^N}{y^N} + (1-\beta)(1-\tilde{\omega}) \sum_{t=1}^{\infty} \beta^t \frac{dy_t^N}{y^N}$$
$$= (1-\beta) \left[-\sigma(1-\tilde{\omega}) - (1-\kappa)\sigma + \sigma(1-\tilde{\omega}) \right] d\tau_0$$
$$= -(1-\beta)(1-\kappa)\sigma d\tau_0 \tag{A.70}$$

We finally combine (A.59), (A.62)-(A.65) and (A.70) to get the decomposition of the overall effect of capital controls on non-tradable consumption

$$d \log c_0^N = \underbrace{\tilde{\omega}\gamma \, d \log e_0}_{\text{Expend. switching}} - \underbrace{\beta\sigma d\tau_0}_{\text{Intertemporal subst.}} - \underbrace{(1-\beta)(1-\kappa)\sigma d\tau_0}_{\text{aggregate income channel}}$$

Because of market clearing, this also corresponds to the decomposition of the effect of capital controls on output. □

B Numerical Solution Method for Transmission Channels

The computation of the transmission channels of monetary policy requires solving for functions $\{\mathcal{E}(b,s), \mathcal{B}(b,s), \mathcal{C}^T(b,s), \mathcal{Y}^N(b,s), \mathcal{R}(b,s)\}$ such that:

$$\mathcal{C}^T(b,s) + \frac{\mathcal{B}(b,s)}{R} = y^T + b \tag{A.71}$$

$$\left[\mathcal{Y}^N(b,s)\right]^{\phi} \left[u_N\left(\mathcal{C}^T(b,s), \mathcal{Y}^N(b,s)\right)\right]^{-1} = 1 \tag{A.72}$$

$$u_T\left(\mathcal{C}^T(b,s), \mathcal{Y}^N(b,s)\right) = \beta R \mathbb{E}_{s'|s}\left[u_T\left(\mathcal{C}^T(\mathcal{B}(b,s),s'), \mathcal{Y}^N(\mathcal{B}(b,s),s)\right)\right] \tag{A.73}$$

$$\mathcal{E}(b,s) = \bar{P}^N \frac{\omega}{1-\omega}\left[\frac{\mathcal{Y}^N(b,s)}{\mathcal{C}^T(b,s)}\right]^{\frac{1}{\gamma}}, \tag{A.74}$$

$$\mathcal{R}(b,s) = R^*\left\{\mathbb{E}_{s'|s}\left[\Lambda(\mathcal{B}(b,s),s')\frac{\mathcal{E}(b,s)}{\mathcal{E}(\mathcal{B}(b,s),s')}\right]\right\}^{-1} \tag{A.75}$$

The steps for the algorithm are the following:

1. Generate grids $G_b = \{b_1, b_2, ..., b_M\}$ for the bond position and $G_s = \{s_1, s_2, ..., s_N\}$ for the shock state space, and choose an interpolation scheme for evaluating the functions outside the grid of bonds. The piecewise linear approximation is used to interpolate the functions and the grid for bonds contains 500 points.

2. **Solve for the policy functions.** Guess policy functions $\mathcal{E}, \mathcal{B}, \mathcal{C}^T$, $\mathcal{Y}^N$, $\mathcal{R}$ at time H, $\forall b \in G_b$ and $\forall s \in G_s$. Set $i = 1$. Solve for the values of $\mathcal{E}^N_{h-i}(b,s)$, $\mathcal{B}_{h-i}(b,s)$, $\mathcal{C}^T_{h-i}(b,s)$, $\mathcal{Y}^N_{h-i}(b,s)$, $\mathcal{R}^N_{h-i}(b,s)$ at time $h-i$ using (A.71)-(A.75) and $\mathcal{B}_{h-i+1}(b,s)$, $\mathcal{C}^T_{h-i+1}(b,s)$, $\mathcal{Y}^N_{h-i+1}(b,s)$, $\forall b \in G_b$ and $\forall s \in G_s$. using (A.71)-(A.73) and a root finding algorithm solve for $\mathcal{B}^T_{h-i}(b,s)$, $\mathcal{Y}^N_{h-i}(b,s)$ and $\mathcal{C}^T_{h-i}(b,s)$. Solve for $\mathcal{E}_{h-i}(b,s)$ and $\mathcal{R}_{h-i}(b,s)$ using (A.74) and (A.75). The competitive equilibrium is found if $\|\sup_{B,s} x_{h-i}(b,s) - x_{h-i+1}(b,s)\| < \epsilon$ for $x \in \{\mathcal{B},, \mathcal{C}^T, \mathcal{Y}^N\}$. Otherwise, set $x_{h-i}(b,s) = x_{h-i+1}(b,s)$, $i \approx i+1$ and solve A.71)-(A.75) again (until convergence).

3. **Decomposition of monetary policy shocks.** For a given state (b,s), we denote by $\bar{y}^N(b,s), \bar{c}^T(b,s), \bar{R}(b,s), \bar{e}(b,s), \bar{b}_1(b,s)$, the value of the corresponding variable before the monetary policy shock. Consider a 1% increase in R, i.e. $R = 1.01 \cdot \bar{R}(b,s)$

   (a) *Overall effect.* Using a root finding algorithm solve for $\{e, c^T, b_1\}$ using

$$c^T + \frac{b_1}{R^*} = y^T + b \tag{A.76}$$

$$e = \frac{R^*}{R}\left[\mathbb{E}_{s'|s}\frac{\Lambda(b_1,s')}{\mathcal{E}(b_1,s')}\right]^{-1} \tag{A.77}$$

$$u_T(c^T, y^N) = \beta R^* \mathbb{E}_{s'|s}\left[u_T\left(\mathcal{C}^T(b_1,s'), \mathcal{Y}^N(b_1,s')\right)\right] \tag{A.78}$$

then determine output using

$$y^N = \left[\frac{1-\omega}{\omega}\frac{e}{\bar{P}^N}\right]^{\gamma} c^T \tag{A.79}$$

(b) *Partial Equilibrium effect.* Using a root finding algorithm to solve for $\hat{b}_1$ using

$$u_T(\hat{c}^T, \hat{c}^N) = \beta R^* \mathbb{E}_{s'|s}\left[u_T\left(\hat{c}^T(\hat{b}_1, s'), \hat{c}^N(\hat{b}_1, s')\right)\right] \tag{A.80}$$

It is important to notice here that we need individual consumption holding constant the path for output. To determine $\hat{c}^T, \hat{c}^N$ and $\hat{c}^T(\hat{b}_1, s'), \hat{c}^N(\hat{b}_1, s')$, note first that for any $\hat{b}_1$ the following quantities can be determined

$$\hat{e} = \frac{R^*}{R}\left[\mathbb{E}_{s'|s}\frac{\Lambda(\hat{b}_1, s')}{\mathcal{E}(\hat{b}_1, s')}\right]^{-1}$$

$$\hat{\mathcal{P}} = \left[\omega^{\gamma} + (1-\omega)^{\gamma}\left(\frac{\bar{P}^N}{\hat{e}}\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$$

$$\hat{\mathcal{P}}\hat{c} = \left[y^T + \frac{\bar{P}^N}{\hat{e}}\bar{y}^N(b,s)\right] + b - \frac{\hat{b}_1}{R^*}$$

$$\hat{c}^T = \left(\omega\hat{\mathcal{P}}\right)^{\gamma}\hat{c} \tag{A.81}$$

$$\hat{c}^N = (1-\omega)^{\gamma}\left(\frac{\hat{e}}{\bar{P}^N}\hat{\mathcal{P}}\right)^{\gamma}\hat{c} \tag{A.82}$$

we proceed similar to determine $c^T(b_1, s')$ and $c^N(b_1, s')$ where we hold output invariant of households decisions, that is next period output in next period budget constraint is $\mathcal{Y}^N(\mathcal{B}(b, s'), s')$. Then, after solving for b_1 using (A.80), $\hat{c}^N$ is determined by (A.82).

i. *Intertemporal substitution.* c^N_{IS} is determined using

$$c^N_{IS} = (1-\omega)^{\gamma}\left(\frac{\bar{e}}{\bar{P}^N}\bar{\mathcal{P}}\right)^{\gamma}\hat{c}, \tag{A.83}$$

$$\text{with } \bar{\mathcal{P}} = \left[\omega^{\gamma} + (1-\omega)^{\gamma}\left(\frac{\bar{P}^N}{\bar{e}}\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$$

ii. *Expenditure switching.* c^N_{ES} is determined as a residual, $c^N_{ES} = \hat{c} - c^N_{IS}$

(c) *Aggregate Income effect.* Given c^N and $\hat{c}$ we have $c^N_y = c^N - \hat{c}^N$.

We proceed similar for the transmission channels of capital controls.