Bitcoin Mining for Carbon Emission Reduction

Working draft; (Comments welcomed)

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Abstract

While Bitcoin mining consumes a huge amount of electricity, does it necessarily translate into increased carbon emission? Using an analytical model featuring endogenous renewable energy adoption decisions, we show that with appropriate electricity price policies, the high electricity demand from Bitcoin mining may actually subsidize the capacity building of renewable energy plants and thus *lower* total carbon emission. A key intuition is that unlike other electricity uses, Bitcoin mining intensity can be elastically dialed up or down without disrupting operations, and thus can replace fuel-based electricity generation as an effective shock absorber for the volatile supply of renewable electricity generation. To corroborate this seemingly counterintuitive result at first sight, we are working on quantifying the potential reductions in carbon emission from introducing Bitcoin mining by calibrating the models with empirical data.

Keywords: bitcoin, blockchain, carbon emission, mining, renewable energy, sustainability

1 Introduction

Bitcoin mining (or more generally the operations of proof-of-work blockchains) has long been criticized for its high electricity consumption. Media headlines often depict Bitcoin mining as being estimated to consume as much electricity as a middle-size country (e.g., Norway). Some studies even further extrapolate these estimates and equate electricity consumption to carbon emission and environmental impact. For example, several years ago Mora, Rollins, Taladay, Kantar, Chock, Shimada and Franklin (2018) even made a bold claim that Bitcoin mining can single-handedly push global warming up by 2°C.

Although it is indisputable that Bitcoin mining indeed consumes a huge amount of electricity, we point out that it requires more careful studies to draw a further conclusion of whether the high energy consumption necessarily drives more carbon emission or climate damage. Indeed, if policymakers can design appropriate incentives so that Bitcoin mining is induced to mainly use electricity generated from renewable sources, or if the rise of Bitcoin mining further incentivizes the capacity building of renewable energy plants, the high electricity demand from Bitcoin mining may actually subsidize renewable energy adoption, and thus (maybe counter-intuitively to some people) actually *lower* carbon emission and help fight against global warming.

This paper develops an analytical model to formalize the above reasoning. To reiterate, the models demonstrate that the increasing electricity consumption caused by Bitcoin mining may NOT necessarily lead to increasing carbon emission. In contrary, with proper demand response programs, Bitcoin mining may actually help reduce carbon emission.

Several key characteristics of both renewable energy and Bitcoin mining help realize the above-mentioned possibility. First, with respect to renewable energy, electricity generation from renewable sources features (1) high fixed cost (to initially build up capacity), (2) low or even zero variable cost to produce electricity generation (once capacity is already built

in place), and (3) imperfect correlation between renewable energy production (supply) and electricity demand. To see these features, note that with respect to (1) and (2), electricity generation from solar/hydro/wind all feature high initial cost to purchase panels/build dams/install turbines, but once in place these renewable sources can also generate electricity almost for free; With respect to (3), electricity supply from solar/hydro/wind goes down during the night/dry season/low winds and may vary with volatile short-term weather changes, yet such supply shocks are unlikely to be perfectly correlated with households or industry electricity demands.

Given that electricity has to be consumed as soon as it is produced,[1] characteristic (3) above (imperfect supply-demand correlation) imposes a serious friction against renewable energy adoption: Fuel energy, whose supply is more controllable, has to be in place to synchronize electricity demand and supply, despite significant carbon footprints. On the other hand, the high fixed cost involved in building renewable energy capacity makes it difficult commercially, especially given that much electricity demand has to be met by fuel energies for the reason just mentioned.

All the pain points enumerated above can almost be perfectly resolved by Bitcoin mining, thanks to the latter's two salient characteristics: (4) Bitcoin mining consumes a lot of electricity, and more importantly (5) Bitcoin mining is memoryless (as it only involves brute-force hash operations), meaning that it can be turned off without compromising continued mining operations once turned on again – indeed, Bitcoin mining can endure disruptions at extremely high frequencies (e.g., about 10^{-15} second for the Antminer S19 ASIC mining rig).

Characteristic (4) ensures that Bitcoin mining creates sizable demand for electricity, which helps prospective renewable energy providers overcome the barrier of high fixed cost, especially smooth over time periods during which other household/industry electricity de-

[1] Although energy storage solutions like batteries/pumps do exist and are developing, their capacities today are still limited to negligible levels compared to total electricity generation.

mands are low. On the other hand, during time periods of peak household/industry electricity demands, characteristic (5) ensures that Bitcoin mining can be temporarily shut down to prioritize other difficult-to-disrupt electricity needs. The shutdown can also be implemented voluntarily given well-designed demand response programs, which changes the electricity prices charged to Bitcoin miners depending on the demand/supply imbalance in the grid (indeed, such practices has already been adopted by ERCOT – the electrical grid in the state of Texas). In short, Bitcoin mining's large electricity demand subsidizes renewable capacity building, its flexible electricity demand substitutes fuel energy for demand-supply synchronization, and their unique combination helps Bitcoin mining relieve reliance on fuel energy and reduce carbon emission.

It is also worth noting that the combination of both large and flexible electricity demand (point (4) and (5) above) is quite unique to Bitcoin mining: Other large electricity consumers (e.g., electrolysis for aluminum production, data centers, or increasingly AI training for large models) cannot be easily turned off and resume later without incurring large disruptions or lengthy "warm-up" periods. Therefore Bitcoin mining enjoys a unique advantage in reducing carbon emission and safeguarding toward a sustainable future.

We develop a series of models with increasing complexity (and increasing closeness to reality). For best illustration of intuition, our most basic model takes a social planner's perspective and ask whether introducing Bitcoin mining (with appropriate central planning) helps reduce carbon emission. The basic model closely follows the intuition described earlier and feature the following model ingredients: (1) stochastic energy demand (from non-Bitcoin mining related household/industry needs); (2) stochastic renewable energy supply (also dependent on the renewable capacity in place); (3) costly but controllable electricity supply from fuel energy; (4) high fixed cost for renewable energy capacity building; (5) high (low) carbon emission from fuel (renewable) sources for electricity generation; and for the first model (6) centrally-planned electricity demand from Bitcoin mining. Analysis of the ba-

sic model suggests that the intuition discussed earlier robustly holds under fairly general conditions.

From the basic model, we also carry out extensions that take a mechanism design perspective and ask to what extent the socially-optimal outcome can be sustain in equilibrium. Within this scope, we conduct the following analyses. First, we endogenize the central planning of the Bitcoin mining intensity, and instead design optimal demand response programs to induce the desirable level of Bitcoin mining assuming miners' profit-maximizing incentives. Second, we allow the grid to behave as a profit-maximizing entity rather than a benevolent social planner, and investigate the extent to which the equilibrium outcome coincides with the socially optimal one. To the extent that discrepancies exist, we will look for appropriate regulations to internalize externalities. We also explore different market structures (as a grid often has large market powers in a region, but may complete with each other given that Bitcoin mining is a global business).

Finally, we bring our model to data, and quantify the magnitude of potential carbon reductions from Bitcoin mining by calibrating all model parameters to actual market level data.

Related literature This paper relates to large literature on bitcoin mining. On the theory side, Biais, Bisiere, Bouvard and Casamatta (2019) model bitcoin mining as a coordination game and presents the possibility of multiple equilibrium. Cong, He and Li (2021) studies the dynamics of mining pool formation and show that the rise of mining pools may lead to significant electricity consumption (though it does not talk about the source of electricity and thus not the implications on renewables). Prat and Walter (2021) propose a model that uses the exchange rate of Bitcoin against the US dollar to predict the computing power of Bitcoin's network. Arnosti and Weinberg (2022) shows that bitcoin mining has the tendency toward significant centralization given even small initial differences in variable costs. Chatzigiannis,

Baldimtsi, Griva and Li (2022) develops the optimal rule and a robo-advisor for miners to allocate computing power across mining pools, based on the insight in Cong, He and Li (2021). Bertucci, Bertucci, Lasry and Lions (2024) models bitcoin mining as a mean-field game.

On the empirical side, the most related paper to ours is Halaburda and Yermack (2023), who study the operations and financial valuations of 13 cryptocurrency mining companies listed on the NASDAQ stock exchange and have facilities in North America. Their empirical finding largely supports the feasibility of the idea presented here: Specifically, they find that miners using Texas wind power are offline more than other miners, in a more erratic pattern. Their model shows that miners using sustainable energy may be more profitable than those using conventional sources, despite the shutdowns, as they benefit from extremely low prices when there is oversupply of sustainable energy (e.g., strong winds). The model also shows that it may be beneficial for the electric utility to offer miners compensation for curtailment of their activity when there is undersupply of energy (e.g. lack of wind), which we also observe in our sample. This compensation further increases profits of the miners. They find a negative and significant beta between crypto mining stocks and an index of electric utilities, suggesting that ownership of a crypto mining company might provide a useful channel for risk management in the electric power industry. Several industry reports also touches on the themes discussed in this paper. For example, Ibañez and Freier (2023) contain numerous verbal arguments on why Bitcoin mining may be environmentally friendly that echos the several key aspects of the current paper, although they do not distill an end-to-end theory. In a Bitcoin Policy Institute report, Margot Paez and Troy Cross document several data patterns observed consistent with bitcoin mining's flexible electricity demand.

TODO: complete the lit review.

2 Model

This section formalizes the key intuitions above in a series of analytical models. We lay out the key modeling ingredients as follows:

(1) stochastic energy demand: we assume the quantity of demand for electricity from (non-Bitcoin mining related) household/industry needs within a representative period to follow

$$D = D(\tilde{D}, P), \tag{1}$$

where $\tilde{D}$ is a positive random variable, P is the (unit) electricity price charged to households/industries, and $D(\cdot, \cdot)$ is increasing in $\tilde{D}$ and non-increasing in P.

For simplicity, in Section 2.1 we will start with $D(\tilde{D}, P) = \tilde{D}$. Other simplifying examples may have $D = \tilde{D} - \lambda P$ where $\lambda > 0$ is the slope of a linear demand function.

(2) stochastic renewable energy supply: for a given level of the renewable capacity k in place (which is determined by costly initial investment ex ante), the quantity of supply of renewable energy within a representative period is given by

$$S = S(\tilde{S}, k), \tag{2}$$

where $\tilde{S}$ is a positive random variable with support in $[0, 1]$ and $S(\cdot, \cdot)$ is increasing in both $\tilde{S}$ and k. For simplicity, in Section 2.1 we will assume $S = \tilde{S}k$. The variable cost of generating electricity from renewable energy is assumed to be zero.

(3) costly but controllable electricity supply from fuel energy: we assume that grid has perfect control over the quantity of supply of fuel-based energy within a representative

period. Assuming this quantity is Q, then the (variable) cost incurred would be

$$c \cdot Q, \tag{3}$$

where $c > 0$ captures the (unit) cost of fuels.

(4) high fixed cost for renewable energy capacity building: We assume that it costs the grid

$$C \cdot k \tag{4}$$

to build up the capacity of renewable energy production.

(5) high (low) carbon emission from fuel (renewable) sources for electricity generation: we assume that the emission from any level of renewable energy production is 0, while the emission from Q fuel-based energy production is

$$\theta \cdot Q. \tag{5}$$

Notice that the total emission is proportional to the variable cost incurred by the grid, and hence we will often equate the objective of emission reduction with variable cost reduction.

2.1 Central planner

Our goal is to allow either the grid or regulators to initiate demand response programs that dynamically adjusts the price charged to bitcoin miners and thus indirectly influences the intensity of Bitcoin mining. To help gain intuition of our result, we can start with the limit case and entertain an extreme version of demand response programs, in which the electricity prices to bitcoin miners can be arbitrarily set by regulators and thus to effectively control

the quantity of electricity demand from Bitcoin mining. We call this limiting version of the model a "centrally planned" one. Later section will show that the centrally planned model may actually be much less restrictive than what it may first appear.

In the centrally planned model, we stipulate the quantity of electricity demand d from bitcoin mining as a function of household/industry electricity demand D — the realization of $\tilde{D}$ (recall that we set the slope of the demand function λ to be 0 in this case), the renewable supply factor S — realization of $\tilde{S}$, the cost of fuel c, and existing renewable capacity k.

With the regulator's ability to control the quantity of electricity demand from bitcoin mining, the grid aims to minimize its expected total (initial and variable) cost of fulfilling electricity demand from household/industry electricity needs. Formally, the grid's problem is given by:

$$\min_{k} \left\{ C \cdot k + c \cdot \mathbb{E}[\max\{\tilde{D} + d(\tilde{D}, \tilde{S}) - \tilde{S} \cdot k, 0\}] \right\}. \tag{6}$$

We are interested in comparing this scenario with the one in which bitcoin mining is banned (that is, $d = 0$). Our first result is summarized in the following proposition.

Proposition 1 (The impact of introducing bitcoin mining)**.** *Compared to a world in which bitcoin mining is not allowed, introducing (centrally-planned) bitcoin mining leads to*

(1) more renewable capacity building (i.e., a higher k);

(2) lower total cost to the grid (i.e., a lower (6));

(3) lower total emission (i.e., a lower $\theta \cdot \mathbb{E}[\max\{\tilde{D} + d(\tilde{D}, \tilde{S}) - \tilde{S} \cdot k, 0\}]$).

The proof of Proposition 1 can be visualized in Figure 1. Bullet point (2) within Proposition 1 should also be immediately, in that restricting the optimal $d(\cdot, \cdot)$ to 0 certainly cannot help with the grid's objective.

Note that in Figure 1, the upper-left panel illustrates, for a given level of renewable capacity k and a given realization of household/industry demand $\tilde{D}$ and renewable factor

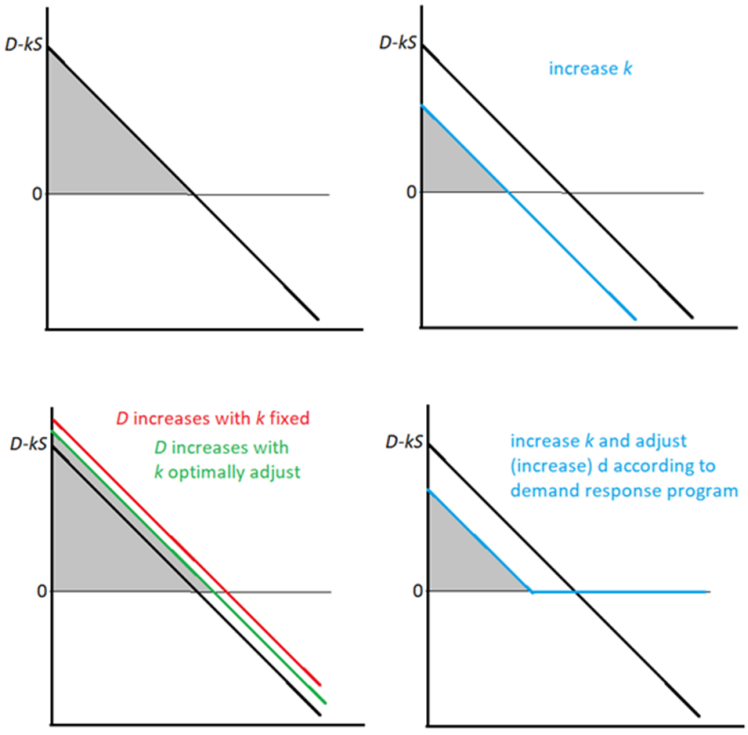


Figure 1: Visualization of the centrally-planned problem

$\tilde{S}$, the grey area illustrates (in proportion) the grid's total variable cost and total emission in the baseline case in which no bitcoin mining is introduced. On the upper-right corner, we see that a higher level of renewable capacity k can reduce the grid's total variable cost and total emission (the grey area), but such savings will be counteracted by a higher initial cost of renewable capacity building (not illustrated in the graphs). In order for the grid to increase its optimal level, one may seek an increase in extra electricity demand.

In the lower-left panel, we illustrate a naive approach. In this graph, the quantity of demand increases (from black to red), and the optimal capacity adjusts accordingly (increases, as the green line illustrates). However, (by Le Chatelier's principle) the change increase in k is not large enough to offset the increase in D nor the grey area, and total emission will increases if we just naively add more electricity demand.

The lower-right panel demonstrates how we can add electricity demand smartly. We

only add extra electricity demand (from Bitcoin mining) when household/industry demand exceeds renewable supply, and otherwise cap extra electricity demand from Bitcoin mining to only absorb unconsumed renewable supply (in electricity grid parlance, we "curtail" the electricity demand from bitcoin miners when the load from household/industry exceeds the supply from renewable sources). By adding extra electricity demand (from Bitcoin mining) when household/industry demand exceeds renewable supply, we incentivize the grid to invest in renewable capacity to reduce the probability of such situation happens.

By capping extra electricity demand from Bitcoin mining to only absorb unconsumed renewable supply when household/industry demand does not meet renewable supply, we do not increase emission in this situation. It turns out the lower chance of having emission at all may trump the extra emission when this does happen, and thus introducing bitcoin mining (with proper control of its intensity) would lower total emission.

2.1.1 Example

Assume that $\tilde{D} \sim \text{Uniform}[0, \bar{D}]$ and $\tilde{S} \sim \text{Uniform}[0, \bar{S}]$. Then (6) becomes

$$\min_k \left\{ C \cdot k + c \cdot \frac{1}{\bar{D}\bar{S}} \int_0^{\bar{D}} \int_0^{\bar{S}} \max\{D + d(D,S) - S \cdot k, 0\} dS dD \right\} \tag{7}$$

Without bitcoin mining Without bitcoin mining, we get

$$\min_k \left\{ C \cdot k + c \cdot \frac{1}{\bar{D}\bar{S}} \int_0^{\bar{D}} \int_0^{\bar{S}} \max\{D - S \cdot k, 0\} dS dD \right\}.$$

If $k\bar{S} \geq \bar{D}$, then the objective becomes

$$\min_k \left\{ C \cdot k + c \cdot \frac{1}{\bar{D}\bar{S}} \int_0^{\bar{D}} \int_0^{\frac{D}{k}} (D - S \cdot k) dS dD \right\} \tag{8}$$

$$= \min_k \left\{ C \cdot k + c \cdot \frac{1}{\bar{D}\bar{S}} \int_0^{\bar{D}} \frac{D^2}{2k} dD \right\} \tag{9}$$

$$= \min_k \left\{ C \cdot k + c \cdot \frac{1}{\bar{S}} \frac{\bar{D}^2}{6k} \right\} \geq 2\sqrt{\frac{\bar{D}^2 C c}{6\bar{S}}}. \tag{10}$$

with equality when $k = \sqrt{\frac{\bar{D}^2 c}{6\bar{S}C}}$.

If $k\bar{S} < \bar{D}$, then the objective becomes

$$\min_k \left\{ C \cdot k + c \cdot \frac{1}{\bar{D}\bar{S}} \int_0^{\bar{S}} \int_{kS}^{\bar{D}} (D - S \cdot k)\, dD dS \right\} \tag{11}$$

$$= \min_k \left\{ C \cdot k + c \cdot \frac{1}{\bar{D}\bar{S}} \int_0^{\bar{S}} \left(\frac{1}{2}\bar{D}^2 - \frac{1}{2}(kS)^2 - S \cdot k\bar{D} + (kS)^2 \right) dS \right\} \tag{12}$$

$$= \min_k \left\{ C \cdot k + c \cdot \frac{1}{\bar{D}\bar{S}} \left(\frac{1}{2}\bar{D}^2\bar{S} - \frac{1}{2}\bar{S}^2 \cdot k\bar{D} + \frac{1}{6}k^2\bar{S}^3 \right) \right\} \tag{13}$$

$$= \min_k \left\{ C \cdot k + c \cdot \frac{1}{\bar{D}} \left(\frac{1}{2}\bar{D}^2 - \frac{1}{2}\bar{S} \cdot k\bar{D} + \frac{1}{6}k^2\bar{S}^2 \right) \right\} \tag{14}$$

$$\geq -\frac{3\bar{D}}{2c\bar{S}^2} \left(\frac{1}{2}\bar{S}c - C \right)^2 + \frac{1}{2}c\bar{D}. \tag{15}$$

with equality when $k = \frac{3\bar{D}}{c\bar{S}^2}(\frac{1}{2}\bar{S}c - C)$.

Therefore, we have

$$\begin{cases} \text{If } C > \frac{1}{2}\bar{S}c, & k^* = 0, & \text{total cost} = \frac{1}{2}c\bar{D} \\ & & \text{variable cost} = \frac{1}{2}c\bar{D} \\ \text{If } \frac{1}{6}\bar{S}c < C \leq \frac{1}{2}\bar{S}c, & k^* = \frac{3\bar{D}}{c\bar{S}^2}(\frac{1}{2}\bar{S}c - C), & \text{total cost} = -\frac{3\bar{D}}{2c\bar{S}^2}\left(\frac{1}{2}\bar{S}c - C\right)^2 + \frac{1}{2}c\bar{D} \\ & & \text{variable cost} = \frac{1}{8}c\bar{D} + \frac{3\bar{D}}{2c\bar{S}^2}C^2 \\ \text{If } C \leq \frac{c\bar{S}}{6}, & k^* = \sqrt{\frac{\bar{D}^2 c}{6\bar{S}C}}, & \text{total cost} = 2\sqrt{\frac{\bar{D}^2 C c}{6\bar{S}}} \\ & & \text{variable cost} = \sqrt{\frac{\bar{D}^2 C c}{6\bar{S}}}. \end{cases} \tag{16}$$

With bitcoin mining With bitcoin mining, we show that there exists some stipulated quantity of electricity demand from bitcoin mining $d(D, S \cdot k)$ that leads to lower emission when the grid's optimal renewable capacity building.

Suppose we specify $d(D,S) = \begin{cases} S\cdot k - D, & \text{if } S\cdot k - D \geq 0 \\ d, & \text{if } S\cdot k - D < 0 \end{cases}$ where $d > 0$, that is, we let bitcoin mining consume a positive amount of electricity when household/industry needs exceed renewable supply, and absorb extra supply from renewable when otherwise. The d effectively serves as penalty to under building of renewable capacity. Then the grid's objective becomes

$$\min_k \left\{ C\cdot k + c\cdot \frac{1}{\bar{D}\bar{S}} \int_0^{\bar{D}} \int_{0\leq S\leq \bar{S}, S\cdot k<D} (D + d - S\cdot k) dS dD \right\} \tag{17}$$

If $k\bar{S} \geq \bar{D}$, then the objective becomes

$$\begin{aligned}
& \min_k \left\{ C\cdot k + c\cdot \frac{1}{\bar{D}\bar{S}} \int_0^{\bar{D}} \int_0^{\frac{D}{k}} (D + d - S\cdot k) dS dD \right\} && (18)\\
= & \min_k \left\{ C\cdot k + c\cdot \frac{1}{\bar{D}\bar{S}} \int_0^{\bar{D}} \left(\frac{1}{2}\frac{D^2}{k} + d\frac{D}{k} \right) dD \right\} && (19)\\
= & \min_k \left\{ C\cdot k + c\cdot \frac{1}{\bar{D}\bar{S}} \left(\frac{1}{6}\frac{\bar{D}^3}{k} + d\frac{\bar{D}^2}{2k} \right) \right\} && (20)\\
\geq & 2\sqrt{\frac{Cc}{\bar{S}} \left(\frac{\bar{D}^2}{6} + d\frac{\bar{D}}{2} \right)}. && (21)
\end{aligned}$$

with equality when $k = \sqrt{\frac{c}{\bar{S}} \left(\frac{\bar{D}^2}{6C} + d\frac{\bar{D}}{2C} \right)}$.

If $k\bar{S} < \bar{D}$, then the objective becomes

$$\begin{aligned}
& \min_k \left\{ C\cdot k + c\cdot \frac{1}{\bar{D}\bar{S}} \int_0^{\bar{S}} \int_{kS}^{\bar{D}} (D + d - S\cdot k) dD dS \right\} && (22)\\
= & \min_k \left\{ C\cdot k + c\cdot \frac{1}{\bar{D}\bar{S}} \int_0^{\bar{S}} \left(\frac{1}{2}(\bar{D}^2 - (kS)^2) + (d - kS)(\bar{D} - kS) \right) dS \right\} && (23)\\
= & \min_k \left\{ C\cdot k + c\cdot \frac{1}{\bar{D}\bar{S}} \left((\frac{1}{2}\bar{D}^2 + d\bar{D})\bar{S} - \frac{1}{2}(d + \bar{D})k\bar{S}^2 + \frac{1}{6}k^2\bar{S}^3 \right) \right\} && (24)\\
= & \min_k \left\{ C\cdot k + c\cdot \frac{1}{\bar{D}} \left((\frac{1}{2}\bar{D}^2 + d\bar{D}) - \frac{1}{2}(d + \bar{D})k\bar{S} + \frac{1}{6}k^2\bar{S}^2 \right) \right\} && (25)
\end{aligned}$$

$$\geq \quad c(\frac{1}{2}\bar{D}+d)-\frac{3}{2}\frac{\bar{D}}{c\bar{S}^2}\left(C-\frac{c}{\bar{D}}\frac{1}{2}(d+\bar{D})\bar{S}\right)^2. \tag{26}$$

with equality when $k=\frac{3}{2}(d+\bar{D})\frac{1}{\bar{S}}-3C\frac{\bar{D}}{c\bar{S}^2}$.

Therefore, we have

$$\begin{cases} \text{If } C>\frac{d+\bar{D}}{2\bar{D}}c\bar{S}, & k^*=0, & \text{total cost}=c\left(\frac{1}{2}\bar{D}+d\right) \\ & & \text{variable cost}=c\left(\frac{1}{2}\bar{D}+d\right) \\ \text{If } \frac{3d+\bar{D}}{6\bar{D}}c\bar{S}<C\leq\frac{d+\bar{D}}{2\bar{D}}c\bar{S}, & k^*=\frac{3}{2}\frac{d+\bar{D}}{\bar{S}}-3C\frac{\bar{D}}{c\bar{S}^2}, & \text{total cost}=c(\frac{1}{2}\bar{D}+d)-\frac{3}{2}\frac{\bar{D}}{c\bar{S}^2}\left(C-\frac{c}{\bar{D}}\frac{1}{2}(d+\bar{D})\bar{S}\right)^2 \\ & & \text{variable cost}=\frac{c}{8}\bar{D}+\frac{1}{4}cd+\frac{3}{2}\frac{\bar{D}}{c\bar{S}^2}C^2-\frac{3}{8}\frac{c}{\bar{D}}d^2 \\ \text{If } C\leq\frac{3d+\bar{D}}{6\bar{D}}c\bar{S}, & k^*=\sqrt{\frac{c}{\bar{S}}\left(\frac{\bar{D}^2}{6C}+d\frac{\bar{D}}{2C}\right)}, & \text{total cost}=2\sqrt{\frac{Cc}{\bar{S}}\left(\frac{\bar{D}^2}{6}+d\frac{\bar{D}}{2}\right)} \\ & & \text{variable cost}=\sqrt{\frac{Cc}{\bar{S}}\left(\frac{\bar{D}^2}{6}+d\frac{\bar{D}}{2}\right)}. \end{cases} \tag{27}$$

Therefore, we have that

- In case $\frac{c\bar{S}}{6}<C\leq\frac{3d+\bar{D}}{6\bar{D}}c\bar{S}$, mining has lower variable cost if $\sqrt{\frac{Cc}{\bar{S}}\left(\frac{\bar{D}^2}{6}+d\frac{\bar{D}}{2}\right)}<\frac{1}{8}c\bar{D}+\frac{3\bar{D}}{2c\bar{S}^2}C^2$ which holds when

$$d<\frac{\bar{D}\left(c\bar{S}-6C\right)^2\left(3c^2\bar{S}^2+4cC\bar{S}+12C^2\right)}{96c^3C\bar{S}^3}.$$

- In case $\frac{1}{2}c\bar{S}<C\leq\frac{d+\bar{D}}{2\bar{D}}c\bar{S}$, mining has lower variable cost if $\frac{c}{8}\bar{D}+\frac{1}{4}cd+\frac{3}{2}\frac{\bar{D}}{c\bar{S}^2}C^2-\frac{3}{8}\frac{c}{\bar{D}}d^2<\frac{1}{2}c\bar{D}$ which holds when

$$d>\frac{1}{3}\bar{D}\left(1+\frac{2\sqrt{9C^2-2c^2\bar{S}^2}}{c\bar{S}}\right)$$

or

$$d<\frac{1}{3}\bar{D}\left(1-\frac{2\sqrt{9C^2-2c^2\bar{S}^2}}{c\bar{S}}\right).$$

In sum, we prove that there exists a (centrally-planned) quantity of electricity demand from bitcoin mining that reduces total emission under the grid's optimal renewable capacity building decision. This result demonstrate how Bitcoin mining can actually contribute to sustainability objectives with the right policies.

TO DO: derive the optimal (centrally-planned) quantity of electricity demand from bitcoin mining. Sharper characterizations beyond existence proof, taking into account the joint distribution of $\tilde{D}$ and $\tilde{S}$

TO DO: Consider moving this section after Section 2.2.

2.2 Profit-maximizing grid

In Section 2.1 and we demonstrate how a cost-minimizing grid may optimally increase renewable capacity building in face of extra electricity demand from bitcoin miners and a stipulated bitcoin mining electricity demand pattern (either through direct control or indirect influence through demand response programs). Careful readers may notice that in Section 2.1 the emission reduction bitcoin mining quantity cannot be chosen by the grid itself, as its total cost minimization quantity would be to set bitcoin mining to zero. This may give the impression that the grid has to suffer in order to facilitate carbon emission reduction.

The crux of the above observation is that in previous models the grid is not directly compensated by miners. Instead miners either absorb extra renewable supply (which does not affect the grid's cost) or add additional demand for fuel energy (which increases the grid's cost). The situation stands a chance to change, however, if the grid can be directly compensated by miners. In this section, we expand the previous models to directly consider the electricity cost charged to households/industries and miners and to explore to what extent we can make emission reduction electricity production incentive compatible for the grid. This result can be interpreted as evaluating to what extent bitcoin miners can subsidize

renewable energy production and lower total emission.

Setup We assume that the grid charges a fixed price $p > c$ to households/industries. Since p is higher than the variable cost of electricity generation (c in case of fuel-based electricity or 0 in case of renewable-based electricity), the grid will always optimally satisfy all households/industry electricity demand.

On the other hand, to bitcoin miners the grid charges a price of $P(\tilde{D}, \tilde{S} \cdot k)$. In other words, the grid dynamically adjusts the price charged to bitcoin miners in real time. This assumption makes sense in practice because (1) Bitcoin mining is a memoryless process, so that it can easily be turned on or off and could thus be responsive to price changes; (2) almost all Bitcoin mining today is conducted at large mining farms, and they do actively negotiate electricity price deals with the grid, so that the grid can work on complicated pricing schedules with miners in advance.

For a given pricing strategy to miners, and the miners' electricity demand function $d(P, \xi)$, where ξ captures all non-price factors that influences the demand function, the objective function of the grid is now given by

$$\max_k \left\{ \mathbb{E}\left[p\tilde{D} + P(\tilde{D}, \tilde{S}k) \cdot d\left(P(\tilde{D}, \tilde{S}k), \xi \right) - c \cdot \max\{ \tilde{D} + d\left(P(\tilde{D}, \tilde{S}k), \xi \right) - \tilde{S}k, 0 \} - Ck \right] \right\} \tag{28}$$

It is easily verified that without revenue terms and assuming perfectly controllable $d(\cdot, \cdot)$, the profit-maximization objective is a duality of the cost-minimization problem studied in Section 2.1.

Furthermore, it is reasonable to argue that the revenue terms indeed do not affect the grid's decisions. This is because on the one hand, the revenue from households/industries $p\tilde{D}$ is completely independent of the grid's decisions, while the the revenue from bitcoin miners $P(\tilde{D}, \tilde{S}k) \cdot d\left(P(\tilde{D}, \tilde{S}k), \xi \right)$ can be interpreted as miners' operating costs, which under a first-

order approximation of perfect competition between miners would equal to mining revenues, which may also be argued to be independent of the grid's decisions (mining revenue within a given period equals the product of (1) number of blocks produced within the period, which is hard-coded in Bitcoin's protocol given its on average 10 min block time; (2) block reward which consists of fixed reward and exogenous transaction fee revenues; (3) Bitcoin price which is arguably exogenous to grid's decisions – we will discuss more about this later). Therefore, as a first approximation one may interpret our findings in Section 2.1 as a reasonable approximation of the reality already.

The next section will show that the insight we discuss in the above section also applies to settings in which bitcoin mining is not perfectly competitive.

2.2.1 Characterizing the miner demand function

It is our intention to assume a general miner demand function $d(\cdot, \cdot)$. Here we conduct a simply analysis to illustrate how $d(\cdot, \cdot)$ may look like, with the caveat that there are more nuances that we abstract away.

As we have already seen in the discussion above, if the mining industry is perfectly competitive, then the results in Section 2.1 captures the reality. Now we instead follow the literature (e.g., Cong, He and Li (2021); Arnosti and Weinberg (2022)) to consider an oligopolistic mining industry. Specifically, assume n symmetric miners. Then each miner i's objective is

$$\max_{d_i} R\frac{d_i}{d_i + \sum_{j\neq i} d_j} - d_i P \tag{29}$$

where d_i is miner i's computing power allocation, R is the total dollar-based block reward within a representative period, and P is the electricity price.

By first order condition, in equilibrium we have

$$R\frac{\sum_{j\neq i} d_j}{(d_i + \sum_{j\neq i} d_j)^2} = P.$$

Summing over all i-s we have that

$$R\frac{(n-1)\sum_j d_j}{(\sum_j d_j)^2} = nP,$$

and thus $\sum_j d_j = R\frac{(n-1)}{nP}$. In other words, we have that

$$d = \frac{(n-1)}{n}\frac{R}{P} \tag{30}$$

Then again we show that with oligopolistic miners, we still have Pd to be exogenous and independent of the grid's decisions.

TODO: further extension to asymmetric oligopoly.

Demand response program Equation (30) gives how total bitcoin mining demand responds to the charged electricity price P as well as the bitcoin market (captured by R) and the mining industry conditions (captured by n). Equation (30) is a convenient outcome for demand response programs, in that one can reverse-engineer the real-time price of electricity charged to bitcoin miners to produce any given pattern of the bitcoin mining demand. More precisely, one can just set $P = \frac{(n-1)}{n}\frac{R}{d}$.

TO DO: compare the total profit to the grid under emission reduction mining demands and the profit with no mining.

TO DO: Consider moving this section before Section 2.1, that is, state the general problem here and then do the analysis in 2.1.

2.3 Grid Competition

We have so far considered one single monopolistic grid. While this assumption is likely true in the local market, this assumption certainly is not accurate from a global perspective, which is relevant since bitcoin mining is a global business. This section extends the analysis to study an oligopoly market.

TO BE COMPLETED

TO DO: consider one grid or multiple grids versus an outside supply of electricity (that does not offer demand responses).

3 Discussions and empirical evaluations

Admittedly, the analytical results we have derived in this paper may be counterintuitive at first sight. Therefore, it is helpful to calibrate the model with real parameters to demonstrate the quantitative magnitude of our results. For this calibration exercise, we first take note all the exogenous parameters within the model:

- the unit price of electricity charged to households/industry p, in units of \$/kwh. We take it to be \$0.15/kwh (see https://www.energybot.com/electricity-rates/).

- the quantity of demand of electricity from households/industry $\tilde{D}$, in units of kwh, characterized by mean, standard deviation, or higher moments. According to the EIA (see https://www.eia.gov/todayinenergy/detail.php?id=12711), a representative region typically has a total electricity consumption of 57 - 155 GW per hour. We thus assume it follows a uniform distribution within 0 and 200 GW. Note that the exact level the quantity of demand is not important, as we can always "scale" the economy and interpret other quantities (for example, renewable capacities) relative to non-mining electricity demand.

- the random utilization rate of renewal energy $\tilde{S}$, as a percentage number, characterized by mean, standard deviation, or higher moments. In electricity parlance, this ratio is known as the capacity factor, which is the ratio of actual electrical energy output over a given period of time to the theoretical maximum electrical energy output over that period. Capacity factors varies across different renewable energy sources, ranging from over 90% for nuclear, 32.2% to 34.7% for wind farms in the U.S. (see www.eia.gov/electricity/monthly/), 23% to 45% for hydroelectric dams (see en.wikipedia.org/wiki/Capacity_factor), and 12.0% to 29.1% for photovoltaic (solar) power stations (see en.wikipedia.org/wiki/Capacity_factor). There is also significant inter-temporal variations. For calibration we set $\tilde{S}$ to be a uniform distribution between 0% and 50% (the exact level is not that important for our qualitative arguments, as we can always adjust the capacity building K accordingly).

- the unit cost of electricity generated from fuel energy c, in units of \$/kwh. This number ranges from 3-15 cents per kwh.

- the number of bitcoin miners (mining pools) n. We can pick it to be 30.

- the dollar value of bitcoin mining reward R, in units of \$. Assuming a 10-minute average block time, 6.25 BTC per block, and a \$60,000 bitcoin price, then R would be about $6 \times 6.25 \times 60,000 = \$2,250,000$.

However, it appears none of the numbers really matter that much for qualitative confirmation. What really matters is the correlation between $\tilde{D}$ and $\tilde{S}$.

TO DO: find data to calibrate the above model parameters and then calculate the optimal endogenous variables: k, d, and P.

TO DO: beyond simple calibration, look for accurate data for an accurate quantification.

4 Conclusion

While bitcoin mining has long been argued to be environmentally unfriendly, we point out this conventional wisdom is not necessarily correct. In a simple model, we demonstrate how combining the unique features of bitcoin mining and well-designed demand response program can actually lower total emission. We plan to go beyond the existence proof and finish up deriving the optimal policy under general conditions soon.

Our results have multiple policy implications:

- Banning bitcoin mining, like some jurisdictions have chosen to do (e.g., China) may actually be counter-effective and missing important incentives for renewable adoption;
- To incentivize renewable buildup, a properly-designed demand response program is paramount; offering bitcoin miners the market rate of electricity would not have the desirable effect;
- the emission-reducing pricing to miners may not be in line with the profit-maximizing policy chosen by a monopolistic grid – therefore regulatory intervention may be desirable;
- in order for the incentive scheme to work, we should allow adequate amount of electricity assumption by bitcoin miners as compared to households/industries.

References

Arnosti, Nick, and S Matthew Weinberg. 2022. "Bitcoin: A natural oligopoly." *Management Science*, 68(7): 4755–4771. 5, 17

Bertucci, Charles, Louis Bertucci, Jean-Michel Lasry, and Pierre-Louis Lions. 2024. "A Mean Field Game Approach to Bitcoin Mining." *SIAM Journal on Financial Mathematics*, 15(3): 960–987. 6

Biais, Bruno, Christophe Bisiere, Matthieu Bouvard, and Catherine Casamatta. 2019. "The blockchain folk theorem." *Review of Financial Studies*, 32(5): 1662–1715. 5

Chatzigiannis, Panagiotis, Foteini Baldimtsi, Igor Griva, and Jiasun Li. 2022. "Diversification across mining pools: Optimal mining strategies under pow." *Journal of Cybersecurity*, 8(1): tyab027. 5

Cong, Lin William, Zhiguo He, and Jiasun Li. 2021. "Decentralized mining in centralized pools." *Review of Financial Studies*, 34(3): 1191–1235. 5, 6, 17

Halaburda, Hanna, and David Yermack. 2023. "Bitcoin mining meets Wall Street: A study of publicly traded crypto mining companies." National Bureau of Economic Research. 6

Ibañez, Juan Ignacio, and Alexander Freier. 2023. "Bitcoin's carbon footprint revisited: Proof of Work mining for renewable energy expansion." *Challenges*, 14(3): 35. 6

Mora, Camilo, Randi L Rollins, Katie Taladay, Michael B Kantar, Mason K Chock, Mio Shimada, and Erik C Franklin. 2018. "Bitcoin emissions alone could push global warming above 2 C." *Nature Climate Change*, 8(11): 931–933. 2

Prat, Julien, and Benjamin Walter. 2021. "An equilibrium model of the market for bitcoin mining." *Journal of Political Economy*, 129(8): 2415–2452. 5