From Labor to Intermediates: Firm Growth, Input Substitution, and Monopsony

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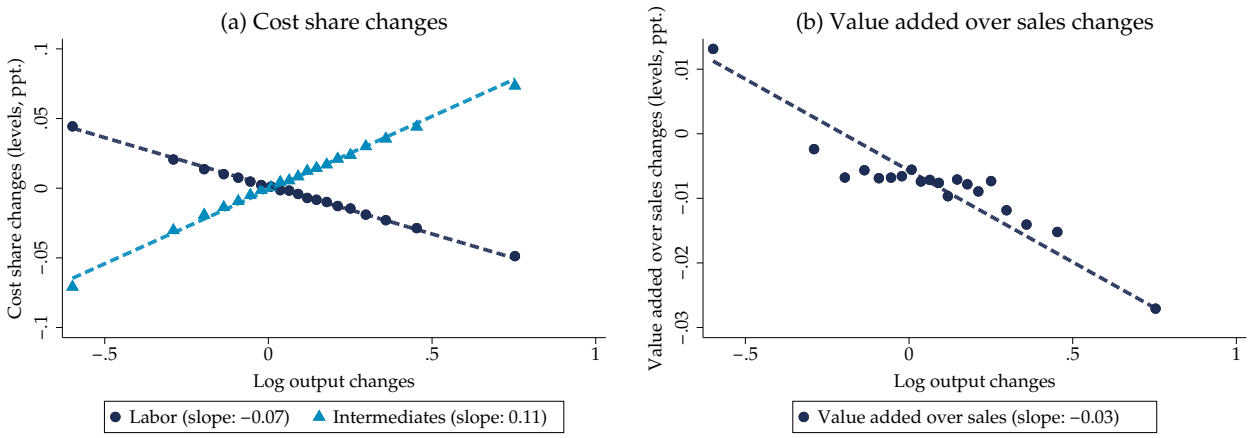
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Abstract

We document and dissect a new stylized fact about firm growth: the shift from labor to intermediate inputs. This shift plays out in terms of input quantities, cost and output shares, and output elasticities. We establish this fact using German firm-level data and replicate it in administrative firm data from 11 additional countries. We also document these patterns in micro-aggregated industry data for 20 European countries (and, with respect to industry cost shares, for the US). We rationalize this novel regularity within a parsimonious model featuring (i) an elasticity of substitution between intermediates and labor that exceeds unity, and (ii) an increasing shadow price of labor relative to intermediates, due to monopsony power over labor or labor adjustment costs. The shift from labor to intermediates accounts for one half to one third of the decline in the labor share in growing firms (the remainder is due to wage markdowns and markups) and rationalizes most of the labor share decline in growing industries.

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Figure 1: The shift from labor to intermediate inputs: within-firm 4-year change in cost shares and in and value added over sales ratios (levels) against output growth.



Notes: The figure reports binned scatter plots of the within-firm changes of labor and intermediate cost shares and value added over sales ratios in levels (i.e., percentage points) against log output growth (deflated sales). All panels depict 4-year differences and control for industry-year fixed effects. Panel (a) reports results for cost shares. Panel (b) reports results for value added over sales ratios. The data covers German manufacturing firms and the period of 1995 to 2017; further details on the datasets, samples, and empirical specifications are discussed in the main part of the paper.

1 Introduction

How do firms grow? Standard Cobb-Douglas production and competitive input markets predict that firms simply scale up inputs proportionately, with constant cost shares. Our paper uncovers and rationalizes clear departures from this benchmark. Figure 1 motivates the new stylized fact that motivates our analysis: as firms grow, they shift production inputs from labor to intermediate inputs, lowering their ratio of value added to sales. This shift from labor to intermediates holds in terms of cost shares (Figure 1 Panel (a)), but also for input quantities, output shares (i.e., the labor share), and output elasticities. Here, intermediate inputs consistent mainly of mainly materials, energy and XXX, while temporary agency labor or services play a smaller role given their small cost shares. To rationalize this new set of facts, we offer a *parsimonious* model of firm growth,[1] which relies on two features: (i) a labor-intermediates substitution elasticity above unity, and (ii) an increasing shadow price of labor relative to intermediates (most likely due to monopsony). Our paper connects those two features to draw their joint implications for firm growth.

We study firm growth in German micro data of manufacturing firms. The data include firm-specific output prices, allowing us to address the price biases in output elasticities derived from production function estimation (De Loecker et al. (2016), Bond et al. (2021), De Ridder et al. (2024)). We draw on OLS regressions as well as an IV strategy using export demand shocks as a firm growth shifter unrelated to factor-biased price or technological changes. We also establish the shift from labor to intermediates in nonparametric scatter plots of raw firm-level data and, as a complementary robustness check, we also infer output elasticities from cost shares.

Additionally, we confirm our main results in administrative firm-level data from 11 addi-

[1]For our thought experiment and in our empirical exercise, we think of firm growth as driven by factor-neutral factors such as TFP growth or product demand shifts. Our model formalizes this through cost minimization, taking scale as given, and our empirical analyses also includes an instrument for growth that relies on product demand shifts.

tional countries as well as in industry-level data for 20 European countries using harmonized data from the Competitiveness Research Network (CompNet) for manufacturing *and* non-manufacturing industries—and for a subset of outcomes also for US manufacturing industries. We estimate our regressions at various horizons, from 1- to 10-year changes, and across different size classes of firms. As we do not find evidence for non-homotheticities of the production function explaining our results, we rationalize our findings with a parsimonious production perspective on input substitutability as described below.

The shifts from labor to intermediates in quantities and the reduction in output elasticities of labor accounts for *about one half to one third* of the negative effect of firm growth on the firm-level labor share. (In log changes, the labor share is equal to the output elasticity minus markups and markdowns.) Hence, our framework provides a novel, technological explanation for the negative association between the labor share and firm growth, complementing existing approaches that focus on large firms' product or labor market power. Importantly, unlike the existing literature on cross-sectional firm *size* gradients and concentration (e.g., Autor et al. (2020) and De Loecker et al. (2020)), we focus on *firm growth* in panel data.[2]

A parsimonious model of firm growth can account for the entire set of findings. It rests on two features: substitutability between intermediates and labor, and an increasing shadow price of labor relative to intermediates. The combination of the two features is required to account for our findings—and hence the empirical findings support the model we propose, rejecting alternatives such as firm growth under Cobb-Douglas production (unit substitutability) and/or perfect input markets. In a nutshell, under an increasing relative shadow price of labor, as firms grow, firms lower their *relative* labor demand, compared to that for intermediate inputs, because labor becomes relatively expensive. If labor and intermediates are substitutes, this shift toward intermediates translates into a lower output elasticity of labor relative to intermediates, which lowers the labor cost share, and the labor share in output (holding constant returns to scale, markups, and wage markdowns).

As the first ingredient, substitution elasticities above unity imply that as the intermediate-labor input ratio increases, output elasticities shift from labor to intermediates—consistent with our firm growth facts. Quantitatively, our firm growth regressions identify substitution elasticities well above one, ranging from 1.8 to 2.7 (OLS) and from 3.8 to 4.2 (IV). Our paper situates these values in a systematic meta analysis of existing estimates.[3]

As the second ingredient, to account for *why* growing firms choose to shift their input mix from labor to capital, our model features an increasing shadow price of labor relative to intermediates. A natural source is a finitely elastic firm-specific labor supply curve, i.e., firms holding monopsony power over labor. (Alternatively, adjustment costs may play a role, at least in the short run.) In fact, for a given substitution elasticity and under the assumption of firm cost minimization, our input mix estimates identify the elasticity of the labor (shadow) price to the firm (i.e., the inverse labor supply

[2]We find much smaller (but qualitatively similar) cross-sectional differences in output elasticities by firm *size* than by firm *growth*, perhaps due to firm-specific permanent factors shaping input intensities and output elasticities.

[3]Existing estimates of the intermediate-labor substitution elasticity are difficult to compare as they depend on disparate identification strategies and production model assumptions. Most closely aligning with our approach, Huneeus et al. (2022) and Chan (2023) estimate substitution elasticities between labor and intermediates between 1.05 and 1.62 and between 1.6 and 9.6, respectively.

elasticity). We offer a range of implied elasticities across our specifications and wage measurement, and situate them in a meta-analysis of existing estimates using data from Sokolova and Sorensen (2021). Our results also imply that supply elasticities appear higher in the long run (i.e., labor markets are more competitive), for our IV estimates, and when we net out markdown shifts or use direct (average) wage estimates.

Our paper focuses on *firm*-level growth, and drawing aggregate implications would necessitate input-output network analysis or an open economy perspective. However, we confirm, to an attenuated degree, the firm-level patterns at the industry level, drawing on CompNet data for XXX countries and the United States. In fact, at this level, the negative association between output growth and labor shares is fully accounted for by labor output elasticities, with no role for markups or wage markdowns. Hence, reductions in the output elasticity of labor may act as a new, production-function-based factor in aggregate labor share declines.[4]

Additional related literature. Broadly, our study complements work showing that estimates of the substitution elasticity between production factors are inconsistent with a Cobb-Douglas production model (Chirinko et al., 2011, Raval, 2019).

Our paper also adds to existing studies that document factor substitution in response to firm-specific shocks and trends—where the existing literature has largely focused on capital-labor substitution. For example, Acemoglu and Restrepo (2019), Acemoglu and Restrepo (2020), Dauth et al. (2021), and Deng et al. (2023) study substitution of labor with robots. Lashkari et al. (2024) analyse how non-homotheticities in the production function cause firms to shift toward higher IT-capital intensities. Similarly, Hubmer and Restrepo (2021) use Compustat data and show that capital-output elasticities have increased in the largest Compustat firms in the most recent years, consistent with automation. Karabarbounis and Neiman (2014) discuss the role of declining capital prices in the global labor share decline. Dhyne et al. (2022) show that labor adjustments to demand shocks are weaker in the short- than in the long-run. Huneeus et al. (2022) estimate that labor and intermediates are substitutes and show that, as a result, firms with access to cheaper suppliers have lower labor shares.

More closely related to our paper, Castro-Vincenzi and Kleinman (2024) use *aggregate* data and study how rising material prices may lower labor shares if labor and materials are *complements*. We focus on firm-level mechanisms and firm growth, and find empirical evidence supporting that intermediates and labor are substitutes and that, as firms and industries grow, shadow prices of labor relative to intermediates increase. In parallel work, Chan et al. (2024) document that larger firms have (cross-sectionally) higher returns to scale due to higher intermediate input output elasticities and study the resulting implications for efficiency losses under financial frictions. Our two papers complement each other as we focus on shifts from labor to intermediates within firms due to firm growth, analyze implications for firm and industry labor shares, and provide a parsimonious micro-foundation for our findings based on substitution elasticities and imperfect input markets.

Finally, by focusing on input mix and production function dynamics accompanying firm growth, our study complements existing studies of firm growth using one-input (labor) models and measures

[4]With this final analysis, our paper generalizes and provides a micro-founded explanation for the aggregate time series results for Germany in Mertens (2022), who shows that output elasticities of labor and labor shares declined over the last decades. Similarly Elsby et al. (2013) argue that China's accession to the WTO, has contributed to aggregate labor share declines in many advanced countries by driving offshoring of labor-intensive tasks.

of firm size (e.g., Sterk et al., 2021), and we leave firm growth life cycles for future research.

Outline. The paper is organized as follows. Section 2 provides a formal model and derives predictions. Section 3 describes the firm-level data, sample, and production function estimation. Section 4 uses German firm-level data to empirically establish the shift from labor to intermediates. Section 5 interprets the results quantitatively, identifies parameters of interest, and discusses alternative explanations for our findings. Section 6 draws implications of our findings for firm-level labor shares. Section 7 transfers our analysis to the European industry level showing that our key results transfer to the industry level for many other countries and outside of manufacturing. Section 8 concludes.

2 Theory

This section provides a parsimonious framework for firm growth and its effects on factor intensities, output elasticities, and cost and output shares. Section 2.1 presents our production model, which we use in Section 2.2 to formulate testable predictions about firm growth.

2.1 Firm Optimization

Production function. We consider a constant returns to scale (CRS) CES production function of firm i in period t that transforms labor (L_{it}), intermediates (M_{it}), and capital (K_{it}) into output (Q_{it}):

$$Q_{it} = \Omega_{it}\Lambda_i^K K_{it}^{1-\kappa}\left(\Lambda_i^{LM}\alpha_i^L L_{it}^{\frac{\sigma-1}{\sigma}} + \Lambda_i^{LM}\alpha_i^M M_{it}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}\kappa} \tag{1}$$

Ω_{it} represents firm productivity. α_i^L, α_i^M, Λ_i^{LM}, and Λ_i^K are distribution parameters. Capital enters multiplicatively with an exponent $(1-\kappa)$. Labor and intermediates enter as a CES nest with substitution elasticity σ.

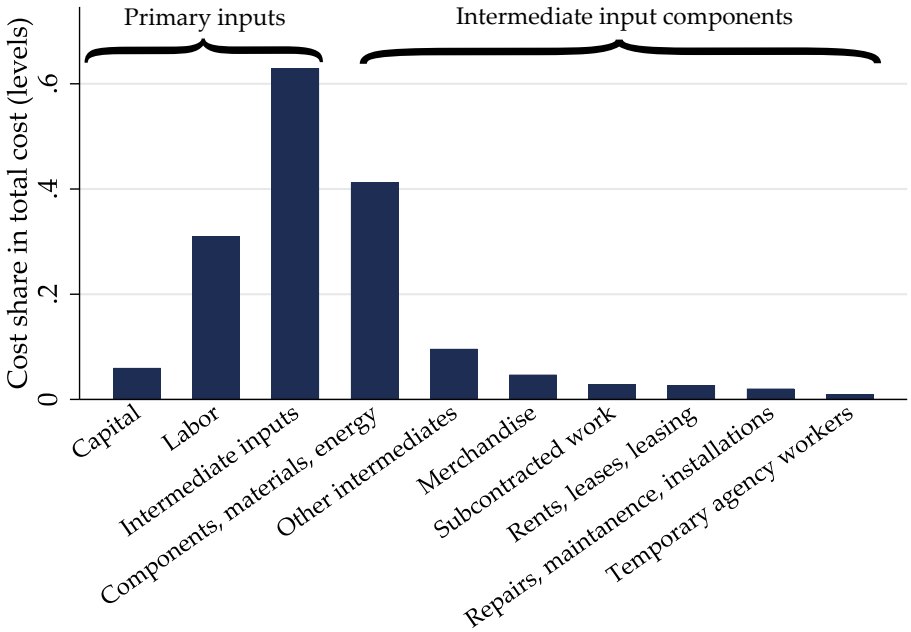
We choose this homothetic constant elasticity of substitution (CES) specification for its analytical simplicity, and as it can fully explain our main empirical finding of a shift from labor to intermediates in production through the substitution elasticity σ (Section 5.3 discusses non-homotheticities).

The specification foreshadows our main result, that the substitution from labor to intermediates is the primary pattern accompanying firm growth, with capital intensity shifts being comparatively unimportant. In fact, in the production function in Equation (1), the capital output elasticity is constant. In our empirical analysis, we will allow output elasticities (and cost shares) of all inputs to vary. As the level of the capital output elasticity is small (see below), its level changes in response to output growth are small as well, and one could indeed think of $(1-\kappa)$ as being approximately constant in our context.

Cost shares. Figure 2 details input cost shares for German manufacturing firms (data described in Section 3). The average capital cost share is just 6% (which equals $1-\kappa$ under perfect input markets).[5] The average capital output elasticity that we will later estimate is 0.12. The figure also decomposes the intermediate input cost share. Two thirds of intermediate inputs consist of materials, energy, and external product components (e.g., car tires). Of the remainder, half are classified as "other

[5] In our firm-level data, labor's average share of value added is 80%.

Figure 2: Cost shares of production inputs.



Notes: The figure reports average firm-level cost shares for capital, labor, intermediate inputs, and the components of intermediate inputs. Capital costs are approximated as 8% of the capital stock. Separate information for temporary agency worker cost shares is available from 1999, all other variables are available from 1995. The data covers German manufacturing firms and is described in Section 3.

intermediate inputs," which include services like transport, postage, insurance, and legal services. The other half comprises merchandise, subcontracted work, and rents. Temporary agency labor represents only about 1% of total costs.

Cost minimization. We rely on cost minimization at a given output level to study the input and production function dynamics accompanying firm growth. Cost minimization conveniently introduces firm size and growth as quasi-parameters and permit us to cast our reduced form empirical regression equations as structural equations. This section focuses on key equations. Appendix B.1 details all derivations.

We allow for imperfect product market competition and input market frictions, such as monopsony power and adjustment costs, creating wedges between the marginal costs of production inputs and their unit costs. This feature aligns with our empirical analysis that accommodates firm- and time-specific markups and input wedges. We will revisit these assumptions when formulating growth predictions in Section 2.2.

Cost minimization implies a FOC for each input, labor, capital, and intermediates, $X = \{L, K, M\}$:

$$P_{it}^X \underbrace{\left(1 + \frac{\partial P_{it}^X}{\partial X_{it}} \frac{X_{it}}{P_{it}^X} + \frac{\partial \chi^X}{\partial X_{it}}\right)}_{\gamma_{it}^X} = \lambda_{it} \frac{\partial Q_{it}}{\partial X_{it}}, \tag{2}$$

where P^X is price for input X (e.g., wage for labor L), χ^X is an adjustment cost function, λ_{it} is marginal cost, and γ_{it}^X is the input price wedge, such that $P_{it}^X \gamma_{it}^X$ is the input *shadow price*. We express the adjustment cost function in flexible ("quasi-static") terms (as in Bond et al., 2021) without

formulating specific timing assumption to highlight the key take-away from Equation (2): monopsony power and adjustment costs raise marginal factor cost beyond an input's price, P_{it}^X.[6]

Using the production function and cost minimization, we derive two key equations. First, we show how the substitution elasticity relates to (and hence can be identified by) the co-movement of relative output elasticity and relative input ratio changes:

$$\frac{\frac{\partial Q_{it}}{\partial L_{it}}}{\frac{\partial Q_{it}}{\partial M_{it}}} = \frac{\alpha_i^L}{\alpha_i^M}\left(\frac{L_{it}}{M_{it}}\right)^{\frac{-1}{\sigma}} \quad \Leftrightarrow \quad \frac{\theta_{it}^L}{\theta_{it}^M} = \frac{\alpha_i^L}{\alpha_i^M}\left(\frac{L_{it}}{M_{it}}\right)^{\frac{\sigma-1}{\sigma}} \quad \Rightarrow \quad \frac{\sigma-1}{\sigma} = \frac{\Delta\ln(\theta_{it}^L) - \Delta\ln(\theta_{it}^M)}{\Delta\ln(L_{it}) - \Delta\ln(M_{it})}, \tag{3}$$

where in the last step we have taken changes within firms. $\theta^X = \frac{\partial Q}{\partial X}\frac{X}{Q}$ is the output elasticity of input $X = \{L, K, M\}$.

Equation (3) is reminiscent of the substitution elasticity in Hicks (1932). The difference is that we connect changes in input quantities to changes in output elasticities rather than marginal products. If labor and intermediates are substitutes ($\sigma > 1$, consistent with our evidence below), the two equations imply that a decrease in the labor-intermediates ratio will decrease the ratio of the labor output elasticity to the intermediate output elasticity. With Cobb-Douglas ($\sigma = 1$), no such change would occur, and complements ($\sigma < 1$) would imply the opposite.

Second, inserting Equation (2) into Equation (3), we recover the implied shadow cost ratio change that would rationalize a given shift in input mix at the firm level for a given level of σ:

$$\Delta\ln\left(P_{it}^L\gamma_{it}^L\right) - \Delta\ln\left(P_{it}^M\gamma_{it}^M\right) = \frac{\Delta\ln(L_{it}) - \Delta\ln(M_{it})}{-\sigma}. \tag{4}$$

Hence, for any $\sigma > 0$, an increase in the shadow price of labor relative to intermediates lowers the labor to intermediate ratio, with σ guiding the quantitative elasticity level.

We will use Equations (3) and (4) in our empirical analysis to estimate substitution elasticities and the implied shadow price ratios that accompany the production function dynamics of firm growth.

Cost and output shares. Changing output elasticities have important implications. Appendix B.1 shows that using Equation (2) for all inputs pins down an input X's cost share as follows:

$$CS_{it}^X = \frac{P_{it}^X X_{it}}{P_{it}^X L_{it} + P_{it}^M M_{it} + P_{it}^K K_{it}} = \frac{\frac{\theta_{it}^X}{\gamma_{it}^X}}{\frac{\theta_{it}^L}{\gamma_{it}^L} + \frac{\theta_{it}^M}{\gamma_{it}^M} + \frac{\theta_{it}^K}{\gamma_{it}^K}}. \tag{5}$$

Similarly, reformulating Equation (2) defines an input's output share (e.g., the labor share in output) as a function of markups, input cost markdowns (i.e., γ_{it}^X, the wedge between the input's marginal cost and its price), and output elasticities (Appendix B.1):

$$OS_{it}^X = \frac{P_{it}^X X_{it}}{P_{it}Q_{it}} = \frac{\theta_{it}^X}{\mu_{it}\gamma_{it}^X}, \tag{6}$$

where $\mu_{it} = \frac{P_{it}}{\lambda_{it}}$ is the output price over (total) marginal cost markup. Thus, a decrease in an input's

[6] Under profit maximization, we could also write γ_{it}^X as the wedge between marginal revenue products and input costs.

output elasticity leads to a reduction in both the cost share and output share of an input.[7] For instance, as reflected in Equation (3), a shift in output elasticities due to a shift in the input mix (resulting from $\sigma \neq 1$, i.e., away from Cobb-Douglas) would pass through into cost and output shares.

2.2 Predictions for Firm Growth

To make predictions about firm growth, we require additional structure and model labor and intermediate supply as isoelastic, $P_{it}^X = a_{it}^X X_{it}^{\varepsilon^X}$ for $X = \{L, M\}$, where a_{it}^X is a baseline input price normalization and ε_{it}^X is the inverse input supply elasticity. For simplicity, we also assume that markups and capital shadow costs do not depend on firm scale and that γ_{it}^L and γ_{it}^M are fully determined by the above supply functions, which will be sufficient to make growth predictions consistent with our empirical results. Importantly, in our empirical analysis, we will relax these assumptions and allow for firm- and time-specific markups and wage markdowns (resulting from monopsony or adjustment costs).

Inserting the input demand equations into Equation (2) pins down labor and intermediate demand as functions of marginal products and parameters (see Appendix B.2):

$$X_{it} = \left(\frac{\lambda_{it} \alpha_i^X}{(1+\varepsilon^X) a_{it}^X} \right)^{\frac{1}{\varepsilon^X}} \left(\frac{\partial Q_{it}}{\partial X_{it}} \right)^{\frac{1}{\varepsilon^X}}, \quad \text{for} \quad X = \{L, M\}. \tag{7}$$

Inserting the production function and expressing the resulting equation in terms of the labor-intermediates ratio yields:

$$\frac{L_{it}}{M_{it}} = \varrho_{it} \lambda_{it}^{\frac{\sigma+\kappa-1}{\kappa}\left(\frac{1}{\sigma\varepsilon^L+1} - \frac{1}{\sigma\varepsilon^M+1}\right)} Q_{it}^{\left(\frac{1}{\sigma\varepsilon^L+1} - \frac{1}{\sigma\varepsilon^M+1}\right)}, \tag{8}$$

where ϱ_{it} (expression in Appendix B.2) captures effects that are unrelated to firm growth (i.e., parameters, baseline prices, and TFP). $\lambda_{it}^{\frac{\sigma+\kappa-1}{\kappa}\left(\frac{1}{\sigma\varepsilon^L+1} - \frac{1}{\sigma\varepsilon^M+1}\right)}$ captures the effect of marginal costs, which increase with quantities produced due to increasing supply curves. The key insight from Equation (8) is that the response of *input ratios* (and thus *output elasticities* and, in turn, cost and output shares) to an increase in output (i.e., firm growth) depends on $\frac{1}{\sigma\varepsilon^L+1} - \frac{1}{\sigma\varepsilon^M+1}$.

Growth predictions. Table 1 summarizes our predictions for various assumption about labor-intermediate substitution elasticities and relative input supply elasticities.[8] This section's discussion remains qualitative. Our quantitative interpretation is presented in Section 5, where we back out the implied values for σ and labor supply elasticities, $\epsilon^L = \frac{1}{\varepsilon^L}$, identified by our empirical estimates.

Potential cases. We use a CRS Cobb-Douglas production function with perfect markets as a benchmark to fix ideas (CES with $\sigma = 1$), and consider departures under input complementarity ($\sigma < 1$) and

[7] Note that we can express θ_{it}^X in terms of returns to scale ($RTS_{it} = \theta_{it}^L + \theta_{it}^M + \theta_{it}^K$) and the input output elasticity relative to other output elasticities, $\theta_{it}^X = \frac{\theta_{it}^X}{RTS_{it}} RTS_{it}$, which separates returns to scale from the relative technological importance of inputs vis-à-vis other production factors.

[8] We focus on the empirically relevant case of $\sigma + \kappa > 1$, where κ is approximately the sum of labor and intermediate output elasticities or, under constant returns to scale and perfect input markets, the sum of labor and intermediate cost shares. κ is therefore close to unity (see discussion above and our output elasticity estimates in Appendix Table A.1). Our estimates of σ are well above unity (see Table 4).

Table 1: Growth predictions for different substitution and supply elasticities (ceteris paribus).

	$\sigma < 1$	$\sigma = 1$	$\sigma > 1$	
L **less** elastic than M $(\varepsilon^L)^{-1} < (\varepsilon^M)^{-1}$	$\downarrow$	$\downarrow$	$\downarrow$	
	$\uparrow$	$=$	$\downarrow$	
	$\uparrow$	$=$	$\downarrow$	
	$\uparrow$	$=$	$\downarrow$	
L **as** elastic as M $(\varepsilon^L)^{-1} = (\varepsilon^M)^{-1}$	$=$	$=$	$=$	$\Delta \ln L_{it}/M_{it}$
	$=$	$=$	$=$	$\Delta \ln \theta^L_{it}/\theta^M_{it}$
	$=$	$=$	$=$	$\Delta \ln CS^L_{it}/CS^M_{it}$
	$=$	$=$	$=$	$\Delta \ln OS^L_{it}/OS^M_{it}$
L **more** elastic than M $(\varepsilon^L)^{-1} > (\varepsilon^M)^{-1}$	$\uparrow$	$\uparrow$	$\uparrow$	
	$\downarrow$	$=$	$\uparrow$	
	$\downarrow$	$=$	$\uparrow$	
	$\downarrow$	$=$	$\uparrow$	

Notes: Ceteris paribus means constant returns to scale and non-changing market imperfections with output growth ($\Delta \ln Q_{it}$). The shaded area denotes the region of the parameter space that is consistent with our empirical findings. These assumptions are relaxed in the empirical analysis.

substitutability ($\sigma > 1$). We differentiate three cases for firm-specific input supply elasticities: labor is more or less elastic than intermediates ($(\varepsilon^L)^{-1} > (\varepsilon^M)^{-1}$ and $(\varepsilon^L)^{-1} < (\varepsilon^M)^{-1}$), and the inputs are equally elastic ($(\varepsilon^L)^{-1} = (\varepsilon^M)^{-1}$).

The shaded, top-right area in Table 1 highlights the combination of parameters implied by our empirical evidence: substitutability and labor being less elastically supplied to the firm than intermediates.

Rejected by evidence: equally elastic supply. We first consider the benchmark of equal supply elasticities, which nests the competitive input prices case (the middle row of Table 1). In this case, input ratios, output elasticities, cost shares, and output shares are constant as firms grow for any value of the substitution elasticity.

Rejected by evidence: Cobb-Douglas and less elastic labor supply. Second, consider the case of an increasing relative shadow price of labor (the top row). With Cobb-Douglas ($\sigma = 1$), firms substitute from labor to intermediates but output elasticities, and thus cost and output shares, remain unchanged. That is, even though labor becomes relatively more expensive as firms grow, the quantity substitution from labor exactly offsets the price increase leaving labor cost and output shares constant.

Rejected by evidence: complements and less elastic labor supply. Now, consider that labor and intermediates are complements ($\sigma < 1$). In this case, a relative reduction in labor quantities increases the output elasticity of labor relative to intermediates. Intuitively, firms' reduce their labor-intermediate ratio less than one-to-one with the input price ratio increase, leading cost and output shares of labor to *increase*.

Case supported by evidence: substitutes and monopsony. Input substitutability ($\sigma > 1$) implies the opposite: the reduction in the labor-intermediate input quantity ratio translates into declines in output elasticity ratios and cost and output shares of labor. As labor becomes expensive, firms substitute away from it by more than one-to-one. This scenario aligns with our empirical evidence discussed below.

Rejected by evidence and implausible: more elastic labor supply. For completeness, we also

consider the case where intermediate inputs are less elastically supplied (or less flexible) than labor. In this scenario, the opposite sign for quantities emerges across the board, irrespective of σ.

The role of input wedges and markups. As noted, the previous predictions hold ceteris paribus with respect to input wedges (γ^L/γ^M) and markups. This simplified framework can fully explain our findings. However, in our empirical analysis, we will directly measure both markups and relative input wedges and explore their dynamics in response to firm growth in Section 6.

Markups tend to rise with firm growth, which dampens input responsiveness to growth relatively more for the more elastically supplied input (this can be seen in Equation (8) as $\lambda_{it} = \frac{P_{it}}{\mu_{it}}$). However, empirically, this attenuation is small and does not overturn the reduction in the labor-intermediate input ratio, because, empirically, markups only increase moderately as firms grow.

Regarding changing input wedges, we find they *amplify* the shift from labor to intermediates as the relative labor wedge increases with firm growth. One implication is that, in this case, even under Cobb-Douglas, we may observe declines in the labor share of costs and output with *constant* output elasticities. We formally discuss this case in Section 6, but note that we will have direct estimates of output elasticities in our baseline analysis using the production function methods outlined below.

3 Firm-level Data

We now describe the German firm-level data, the sample, and the estimation of output elasticities.

Production data. The firm-product-level panel data for Germany's manufacturing sector cover the period of 1995-2017. The data are collected and supplied by the German Statistical Offices.[9] The unit of observation are firms (not establishments or plants, although 90% of firms are single-plant firms). Firms are defined as legal units.[10] The variables include sales, employment, investment, intermediate input costs, wage bills, depreciation, and *product quantities and prices at a ten-digit product classification*.[11] The data cover 40% of firms with at least 20 employees and consist of a rotating panel that is drawn anew every 4-5 years. Labor is defined as the number of employees on September 30th. All other variables pertain to the full calendar year. We clean and prepare the data following Mertens (2022) and provide further details on data preparation, capital stock construction, variable definitions, and summary statistics in Appendix C.[12]

Supplementary trade data. To provide *causal* evidence on the relationship between output growth (i.e., a factor-neutral demand shifter) and our variables of interest, we merge bilateral trade flows from the United Nations Comtrade Database on the firm-product-year level to the German micro data (1995 to 2017). We translate product codes in both datasets into the PRODCOM2002 classification using official concordance tables following the code of Bräuer et al. (2023). As described below, we will apply the IV approach by Hummels et al. (2014) to instrument output changes with foreign export demand to study how output elasticities change in response to exogenous output changes (more precisely,

[9]Data source: RDC of the Federal Statistical Office and Statistical Offices of the Federal States, DOI: 10.21242/42131.2017.00.03.1.1.0, 10.21242/42221.2018.00.01.1.1.0, and 10.21242/42111.2018.00.01.1.1.0.

[10]The term "legal unit" refers to the smallest legally independent unit that keeps accounts for commercial or tax purposes.

[11]Examples of products are "Tin sheets and tapes, thicker than 0.2mm" or "Workwear: long trousers for men, cotton".

[12]The dataset has been used in various studies, e.g., Mertens (2020), 2022, 2023, Mertens and Müller (2022), Haelbig et al. (2023), Mertens et al. (2022), Bräuer et al. (2023), and Bighelli (2023).

output shifts in response to factor-neutral demand shifters).

Production function estimation. To allow for time-varying and firm-specific output elasticities, we rely on the following translog production function, where lower case letters denote logs:[13]

$$\begin{aligned} q_{it} = \beta_l l_{it} + \beta_k k_{it} + \beta_m m_{it} + \beta_{l2} l_{it}^2 + \beta_{k2} k_{it}^2 + \beta_{m2} m_{it}^2 + \\ \beta_{lk} l_{it} k_{it} + \beta_{lm} l_{it} m_{it} + \beta_{km} k_{it} m_{it} + \beta_{lkm} l_{it} k_{it} m_{it} + \omega_{it} + \nu_{it}, \end{aligned} \tag{9}$$

where ν_{it} is an i.i.d. error term. ω_{it} is logged total factor productivity. Labor enters Equation (9) in quantities, intermediates and capital enter as expenditures deflated by industry-year-specific deflators. *The production function is estimated for each industry separately* (NACE Rev. 1.1 two-digit), although we omit industry indices in Equation (9).

We detail our production function estimation approach in Appendix D. Most importantly, we apply a *correction for price biases (De Loecker et al., 2016, Bond et al., 2021) by constructing a firm-specific output price index from our firm-product-level price data* as in Eslava et al. (2004) and by additionally including an input price control function using information on firms' output prices and market shares. The latter follows the firm-level adaption of the control function method in De Loecker et al. (2016) by Mertens (2022).[14]

Using, the estimated coefficients from Equation (9) (varying across industries) and the information on input levels (varying across firms), we compute output elasticities for each firm-year observation as $\theta_{it}^X = \frac{\partial q}{\partial x}$ for input $X = \{L, M, K\}$.

Our production function estimation allows for imperfect product market competition, labor adjustment costs, imperfect input markets, and non-constant returns to scale. However, the control function approach relies on Hicks-neutrality in productivity to formulate a control function for productivity based on flexible production inputs (raw materials and energy expenditures, see Appendix D).

Alternative approach: cost shares. To complement our analysis, we measure output elasticities using cost shares, which avoids relying on Hicks-neutral productivity and does not require parametric assumptions. Under constant returns to scale and perfect input markets, cost shares equal output elasticities, $\theta_{it}^X = \frac{P^X X}{\sum_{X'} P^{X'} X'}$ (see Equation (5)). While these are typical assumptions in cross-sectional settings, for our within-firm analyses in changes, it is sufficient to assume non-changing returns to scale and input market imperfections (i.e., we considerably relax the conventional assumptions when using cost share appraoches). Reassuringly, our results are highly robust to both ways of measuring output elasticities.

Summary statistics and sample. Appendix Table A.1 provides summary statistics of our sample. Our sample sizes vary between 180,000 and 50,000 depending on the time horizon of the analysis (1-10

[13]While we ultimately find that our evidence can be rationalized by our simple CES production function from Section 2.1, it is useful to start with this general specification, which permits non-constant returns to scale and a direct estimation of more flexible output elasticities (e.g., with varying capital output elasticities). Additionally, we incorporate flexible non-parametric cost share estimates for output elasticities as an alternative specification. Later, we will find that the estimated output elasticity patterns can be well explained by a CES production function. Moreover, the production function estimation in the European firm and industry data in Section 7 draws on a translog production function as well.

[14]This approach relies on the positive correlation between output and input prices (e.g., due to high quality outputs requiring high quality inputs). To account for the dependence of input decisions on productivity, we utilize a control function approach similar to Wooldridge (2009) and proxy productivity with information on expenditures on raw materials and energy.

years).

4 Firm-level Evidence: Reduced Form Analysis

We now study firm growth and associated dynamics of input use, output elasticities, and cost and output shares in the micro data. This section presents our empirical results. Section 5 will interpret these reduced form moments structurally and argue how they identify the implied substitution elasticities and input supply elasticities.

4.1 OLS Regressions

Strategy. For each firm i in year t and firm-level outcome $\mathbb{O}_{it}$ (input quantities, cost shares, output shares, output elasticities, and, later, markups and markdowns), we estimate an OLS regressions in log differences across h years (i.e., $\Delta^h x_t = x_{t+h} - x_t$) of the form:

$$\Delta^h \ln(\mathbb{O}_{it}) = \beta_Q^h \Delta^h \ln Q_{it} + \upsilon_{jt} + \nu_{it}. \tag{10}$$

Q_{it} is deflated sales, and υ_{jt} captures 4-digit NACE rev. 1.1 industry (j) times year (t) fixed effects.[15] The difference specification accounts for unobserved constant firm characteristics.[16] ν_{it} is an error term. We cluster standard errors at the firm level. The results are firm-weighted (i.e., each firm has the same weight). We provide robustness checks by size (sales) quintiles below, finding similar results for all key outcomes.

The coefficient of interest is β_Q^h. It captures the percent effect (co-movement) of a one percent change in firms' output on firm-level outcome, $\mathbb{O}_{it}$. We examine time differences h ranging from one year to ten years.

Results. Table 2 presents OLS regression results for various outcomes (1- to 10-year changes across panels). As a companion exhibit, Figure 3 visualizes firm-level relationships from Table 2 using binned scatter plots and residualizing variables by industry-year fixed effects as in Equation (10)—generally revealing clearly linear relationships that support the linear regression specification. As OLS coefficients are precisely estimated, we focus our discussion on point estimates (standard errors are in the regression tables).

[15] We use the industry output price deflator provided by the statistical office. Using our own firm-specific output price index yields similar results.

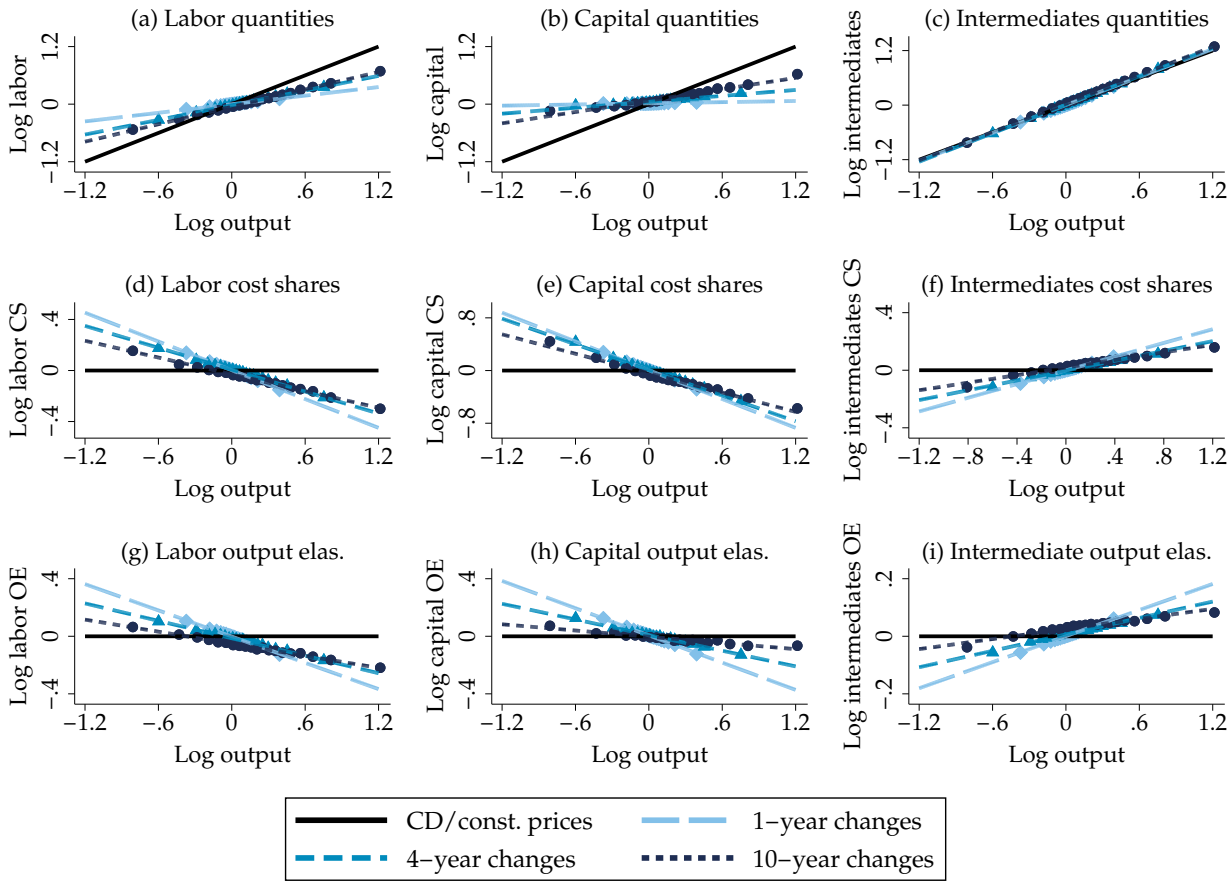
[16] In unreported robustness checks, we also ran specifications with firm effects to take out firm-specific trends. This specification leads to slightly higher coefficient estimates, including at longer horizons, which is plausibly consistent with that specification's remaining variation capturing more transitory fluctuations around the firm-specific trends.

Table 2: Firm-level adjustments in response to firm growth. OLS regressions.

	$\ln(L_{it})$	$\ln(K_{it})$	$\ln(M_{it})$	$\ln(CS^L_{it})$	$\ln(CS^K_{it})$	$\ln(CS^M_{it})$	$\ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\ln(\frac{\theta^K_{it}}{RTS_{it}})$	$\ln(\frac{\theta^M_{it}}{RTS_{it}})$	$\ln(RTS_{it})$
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output growth	0.298***	0.0405***	1.025***	-0.376***	-0.728***	0.238***	-0.304***	-0.315***	0.151***	0.0062***
	(0.0040)	(0.0022)	(0.0033)	(0.00393)	(0.0033)	(0.0022)	(0.004)	(0.0063)	(0.0015)	(0.0003)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950
R^2	0.215	0.043	0.741	0.343	0.556	0.340	0.230	0.137	0.325	0.053
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output growth	0.510***	0.206***	1.045***	-0.286***	-0.649***	0.171***	-0.202***	-0.181***	0.0951***	0.0167***
	(0.0064)	(0.0067)	(0.0045)	(0.0062)	(0.0071)	(0.0034)	(0.0060)	(0.0074)	(0.0025)	(0.0006)
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492
R^2	0.471	0.144	0.860	0.348	0.476	0.355	0.220	0.170	0.268	0.163
Panel C: 10-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output growth	0.606***	0.395***	1.041***	-0.219***	-0.490***	0.131***	-0.140***	-0.0720***	0.0582***	0.0278***
	(0.0066)	(0.0082)	(0.0043)	(0.0061)	(0.0083)	(0.0033)	(0.0053)	(0.0069)	(0.0024)	(0.0007)
Observations	49,915	49,915	49,915	49,915	49,915	49,915	49,915	49,915	49,915	49,915
N of firms	10,595	10,595	10,595	10,595	10,595	10,595	10,595	10,595	10,595	10,595
R^2	0.594	0.256	0.901	0.296	0.329	0.303	0.188	0.149	0.200	0.268

Notes: The table reports OLS regressions from estimating the specification in Equation (10). The dependent variables in Columns (1)-(10) are the logs of labor, capital, intermediates, labor cost shares, capital cost shares, intermediate cost shares, labor output elasticities over returns to scale, capital output elasticities over returns to scale, intermediate input output elasticities over returns to scale, and returns to scale, respectively. Panels A-C report on regressions of changes in those dependent variables on changes in log output for 1-, 4- , and 10-year differences. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm-level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Figure 3: Firm-level adjustments in response to firm growth (OLS, binned scatter plots).



Notes: The figure reports binned scatter plots from estimating the specification in Equation (10) with OLS for various differences against log output changes for the following dependent variables in log changes: labor, capital, intermediate quantities, output elasticities over returns to scale, and cost shares. It also includes the prediction from a Cobb-Douglas production framework with firms optimizing against constant input price (ratios). All panels report results that are residualized by industry-year fixed effects. German firm-level data.

Input quantities. Table 2 Columns (1)-(3) and Figure 3 Panels (a)-(c) report effects on input use. Intermediate inputs exhibit an about unit elasticity with output growth, while labor and capital increase in a much lower proportion. These relative slopes result in a declining ratio of labor (and capital) to intermediate inputs.[17] The small standard errors allow us to reject the hypothesis of proportionate input growth, which would be expected in a Cobb-Douglas model with constant or uniformly shifting (shadow) input prices.

Cost shares. One possible explanation for the shift from labor to intermediates is divergence of input prices, while production remains consistent with a Cobb-Douglas model. In this scenario, cost shares would remain stable, as firms adjust input quantities inversely proportionately to rising input prices. (Input wedges are not captured by monetary costs and hence cost shares—we explore their role in Section 6.)

[17]Unless one is willing to believe that intensive-margin responses exceed the extensive margin ones, even in the long run, our headcounts employment measure is unlikely to account for the flatness of the labor response. We also study employment adjustments in 11 countries with different labor market regimes (Tables 7 and 8) and find similar results. See also Footnote 22 for implications for substitution elasticities.

Table 2, Columns (4)-(6) and Figure 3, Panels (d)-(f) report effects on cost shares, i.e., input expenditures divided by total costs.[18] The data reveal a striking shift towards intermediate costs, away from labor (and capital), indicating that firms are increasingly outsourcing production. This is reflected in the declining ratio of value-added to total shipments shown in Figure 1

Complementing the cost share analysis in logs, we also run the specification in level (ppt.) changes (as in Figure 1), showing that intermediate cost shares absorb the decrease in the labor cost shares and, with a quantitatively much smaller role, the shift in capital cost shares (Appendix Tables A.2 and A.3). (We also study the labor income share in Section 6.)

Output elasticities. The prediction of constant input cost shares in a Cobb-Douglas model relies on the assumption of fixed input wedges and stable output elasticities. Indeed, Equation (5) shows that with constant input wedges and returns to scale, shifts in cost shares correspond directly to changes in output elasticities. To account for the possibility of varying input wedges, Table 2 Columns (7)-(9) and Figure 3 Panels (g)-(i) analyze output elasticities based on our production function estimates.

We find a strong negative relationship between output growth and labor output elasticities. Column (7) reveals that a 10 percent increase in output reduces labor output elasticities by 3 percent, with a coefficient of -0.30 (SE 0.004) at a one-year horizon. This shift away from labor is accompanied by a significant increase in reliance on intermediates, as shown in Column (9). Interestingly, capital output elasticities decline as well (Table 2 Column (8) and Figure 3 Panel (h)).

For additional clarity, Appendix Tables A.2 and A.3 replicate our findings in levels rather than logs for the dependent variables. The level changes in labor and capital output elasticities offset those in intermediate output elasticities, with labor and intermediates driving most of the variation. This result is consistent with the much smaller capital cost shares and output elasticities (Figure 2 and Table A.1).

Notably, we normalize the output elasticities in our regressions by the returns-to-scale parameter to account for any changes in scale, which is consistent with our constant returns-to-scale production function in Section 2. However, the results remain consistent without this adjustment (Appendix Tables A.2 and A.3). Column (10) also shows that returns to scale are relatively stable in the short term and only change slightly over longer horizons.

Effects across horizons. Comparing horizons across the panels highlights interesting dynamics: the coefficient for labor output elasticities drops to -0.20 and -0.14 for 4- and 10-year horizons, respectively. As we explore in Section 5, the evidence for horizon-dependency could be consistent with short-run labor adjustment costs that shape firms' input mix and production modes.

Heterogeneity: types of intermediates. The strong response of intermediates and substitution from labor may partially reflect that intermediate inputs also include intermediate services, unfinished product components, and similar items (see Figure 2). Appendix Table A.4 reports additional regressions for logged cost shares of all available sub-categories of intermediates as dependent variables. The effects are as expected. We find strong effects for product components and materials. Effects are negative for intermediate components that are arguably complementary to capital inputs, such as repair, maintenance, and installation services. We find the strongest effects for temporary agency

[18] We approximate capital costs as 8 percent of the capital stock; for logged specifications, this homogeneous multiplicative factor does not identify the coefficient.

workers, indicating that firms rely on those more flexible labor inputs when growing (see De Leon et al. (2024)), particularly in the short run. However, *quantitatively*, this response does not play an important role for the overall intermediate effect, because, on average, temporary agency workers account for only 1% of total costs (Figure 2).

Heterogeneity: Size heterogeneity. Appendix Table A.5 reproduces our regressions splitting firms into five size quintiles. Across all size groups, results align closely with our main results (Table 2).

Heterogeneity: Industries. In Appendix Figure A.2, we report key results by industry. Results are similar across industries and no clear pattern of heterogeneity emerges.

Additional analysis: cross-sectional results. Our paper focuses on within-firm changes. For completeness, we provide results for the cross-section in Appendix Figure A.1.[19]

4.2 Causal Effects: IV Strategy using Export Demand Shocks

We now use an instrumental variable strategy that draws on foreign product demand shocks as variation in firm growth that is plausibly unrelated to shifts in production function parameters (output elasticities) or input price shocks (conditional on industry-year fixed effects). This analysis aims to trace out the dynamics of firm growth corresponding to our structural equation in Section 2, where we focused on shifts in firm output while holding those confounding factors constant.

Across outcomes, IV coefficients are similar to the OLS counterparts. This result is consistent with our OLS regressions largely reflecting input-unbiased sources of growth such as shifters in product demand, TFP, or input prices across the board.

Strategy. We follow an established literature using trade-shocks as exogenous shifter (see Autor et al., 2016 for a review). In particular, we follow Hummels et al. (2014) and instrument changes in firms' output with changes in world export demand (excluding Germany). We first compute the total exports for each product, g, from a country group, n, to the world:

$$EX_{gt} = \sum_{c} ex_{gct}^{n \to world}, \tag{11}$$

where n contains Australia, Norway, Sweden, Singapore, New Zealand, Great Britain, Canada, Japan, and the US. c denotes the country. Our country selection follows the strategy in Dauth et al. (2014), with the exception that we additionally include the US (results are robust to excluding the US). There are two reasons for this selection. First, we choose other industrialized countries as their economies and thus specialization in types of export goods is more plausibly similar to Germany. Second, this selection of farther-away countries (excluding Germany's direct neighbors and members of its EUR currency union) helps mitigate potential endogeneity concerns arising from unobserved shocks that might be correlated between Germany and other nations.

[19]The cross-sectional analysis reflects qualitatively different forces such as permanent heterogeneity. Our empirical analysis confirms that large firms have lower labor cost shares and labor output elasticities but paints a less clear picture likely due to input price differences and/or other (permanent) heterogeneities (e.g., the α and Λ terms in Equation (1)). The effect for the labor output elasticity falls to -0.03, again precisely estimated. Scatter plots in Appendix Figure A.1 reveal a slightly concave pattern, consistent with large shorter-run elasticities not extending to cross-sectional variation in firm size, which is right-skewed.

Equation (11) constructs a product-level measure. To compute firm-specific export demand shocks, we calculate weighted averages of product-level trade flows, using the sales shares of products within firms' product portfolios as weights:

$$INS_{it} = \sum_{g} s_{git=0} \ln EX_{gt}. \tag{12}$$

s_{git} denotes the firm-specific sales weight of product g. To limit anticipatory effects, we fix the weights for each firm to its first year of observations.

Having constructed these firm-specific instruments, we instrument $\Delta \ln Q_{it}$ in Equation (10) with ΔINS_{it}, i.e., we use export demand shocks to instrument output changes.

Importantly, 80% of firms in our German manufacturing firm sample export in a given year. As we do not observe firm-product-specific export shares, our product sales share weights include domestic sales as well. Furthermore, we restrict our IV analysis to 1- and 4-year differences as the first stage is not sufficiently strong (a low F-statistic) for 10-year changes.

First stage. Table 3 is analogous to Table 2 and reports the corresponding IV results. Column (1) shows the first stage coefficient from regressing output growth on our instrument. We find a highly statistically significant positive association. The F-statistic is 102.6 for one-year changes and 48.57 for four-year changes. We visualize the first stage regressions in Figure 4 Panel (a).

Results: IV estimates. Table 3 Columns (2)-(11) reports the second stage IV results, where we instrument output growth with export demand shocks. Across outcomes, IV coefficients are similar to the OLS counterparts. For instance, the labor output elasticity coefficient is -0.327 (-0.242) at the one-year (four-year) horizon, nearly identical to the OLS results.

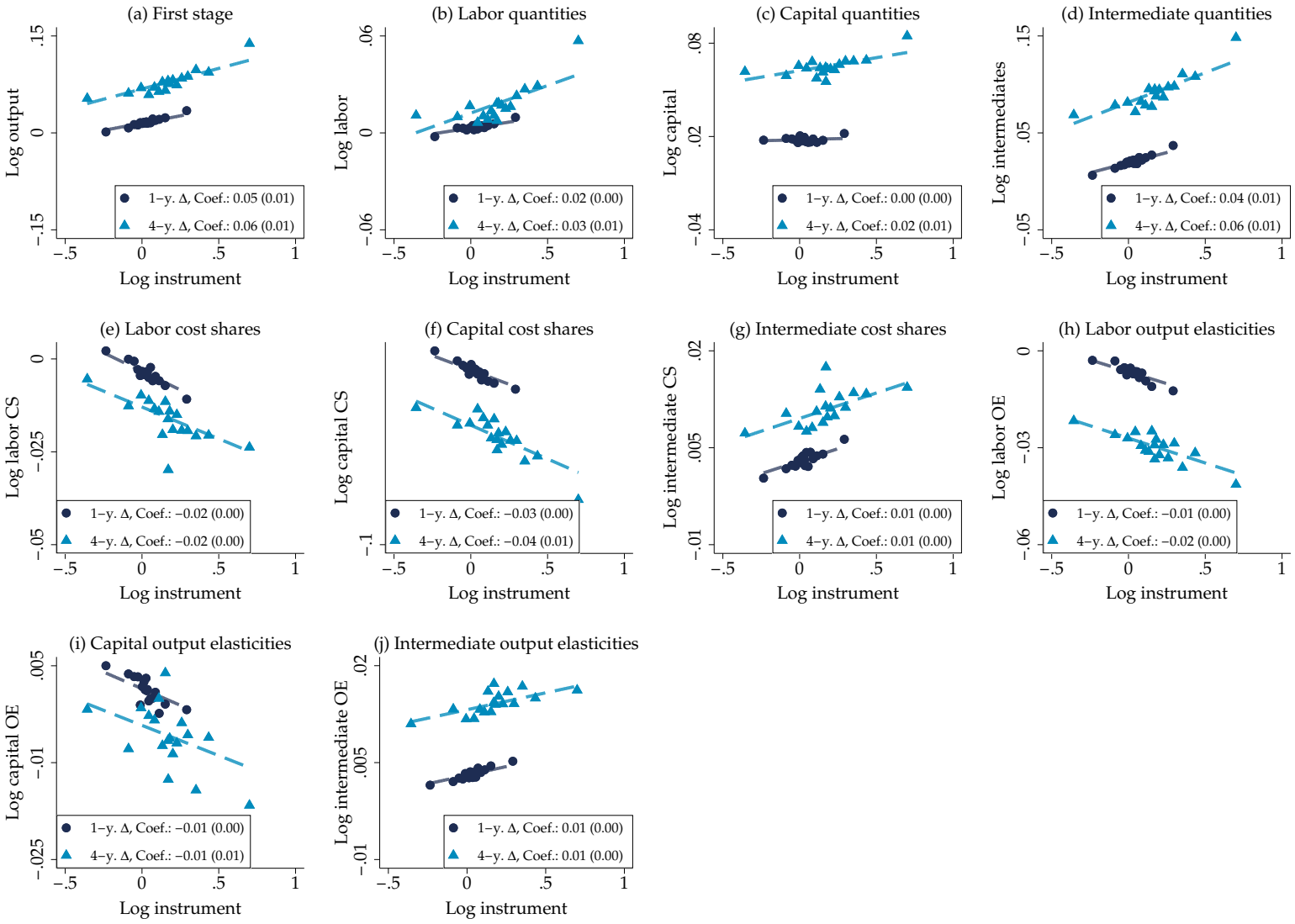
Results: Reduced form estimates. Figure 4 Panels (b)-(j) (associated regression tables available on request) report reduced form estimates (i.e., we directly regress the dependent variables from Table 3 on the instrument). Results are fully consistent with our IV regressions and similar for one and four-year changes. For instance, the coefficients for labor output elasticities (divided by returns to scale) are approximately -0.015 under both time horizons. The corresponding intermediate input output elasticity coefficients are both 0.005, and labor cost share coefficients are -0.02 and -0.017, for one- and your-year differences, respectively.

Table 3: Firm-level adjustments in response to firm growth. IV regressions.

	1st stage	$\ln(L_{it})$	$\ln(K_{it})$	$\ln(M_{it})$	$\ln(\frac{P^L_{it}L_{it}}{C_{it}})$	$\ln(\frac{P^K_{it}K_{it}}{C_{it}})$	$\ln(\frac{P^M_{it}M_{it}}{C_{it}})$	$\ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\ln(\frac{\theta^K_{it}}{RTS_{it}})$	$\ln(\frac{\theta^M_{it}}{RTS_{it}})$	$\ln(RTS_{it})$
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Export demand shock	0.0451***										
	(0.0045)										
Log output growth		0.324***	0.0612	0.929***	-0.444***	-0.684***	0.171***	-0.327***	-0.229**	0.119***	0.0082
		(0.0543)	(0.0461)	(0.0508)	(0.0521)	(0.0565)	(0.0292)	(0.0623)	(0.0980)	(0.0198)	(0.0053)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950
First-stage F-Statistic		102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6
R^2	0.205	0.214	0.042	0.736	0.336	0.554	0.321	0.229	0.132	0.314	0.052
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Export demand shock	0.0635***										
	(0.0091)										
Log output growth		0.536***	0.262***	0.963***	-0.269***	-0.570***	0.124***	-0.242***	-0.147	0.0823***	0.0222***
		(0.0737)	(0.0973)	(0.0542)	(0.0664)	(0.0969)	(0.0369)	(0.0706)	(0.0938)	(0.0268)	(0.0075)
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492
First-stage F-Statistic		48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57
R^2	0.183	0.470	0.139	0.855	0.347	0.471	0.339	0.217	0.169	0.266	0.156

Notes: The table reports IV regressions from estimating the specification in Equation (10) using foreign demand shocks as instruments (Equation (12)). Column (1) reports the first-stage regression results. The dependent variables in Columns (2)-(11) are the logs of labor, capital, intermediates, labor cost shares, capital cost shares, intermediate cost shares, labor output elasticities over returns to scale, capital output elasticities over returns to scale, intermediate input output elasticities over returns to scale, and returns to scale, respectively. All regressions of changes in those dependent variables on changes in log output for 1- and 4-year differences. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm-level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Figure 4: First stage and reduced form results, 1- and 4-year changes.



Notes: The figure reports binned scatter plots for firm-level first stage (Panel (a)) and reduced form regression (Panels (b)-(j)). The first stage regresses log output on the instrument, whereas the reduced form regressions do so for the logs of labor, capital, and intermediate quantities, labor, capital, and intemediate cost shares, and labor, capital, and intermediate output elasticities divided by returns to scale (Panels (b)-(j), respectively). Regressions are in one-year and four-year changes. All panels report results that are residualized by industry-year fixed effects. German firm-level data.

Table 4: Implied substitution elasticities, changes in input cost ratios, and firm-specific labor supply elasticities.

	OLS				IV			
	σ	$\Delta \ln\left(\frac{L_{it}}{M_{it}}\right)$	$\Delta \ln\left(\frac{P_{it}^L \gamma_{it}^L}{P_{it}^M \gamma_{it}^M}\right)$	$\epsilon^L = \frac{1}{\varepsilon^L}$	σ	$\Delta \ln\left(\frac{L_{it}}{M_{it}}\right)$	$\Delta \ln\left(\frac{P_{it}^L \gamma_{it}^L}{P_{it}^M \gamma_{it}^M}\right)$	$\epsilon^L = \frac{1}{\varepsilon^L}$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1-year diff.	2.673	-0.727	0.272	1.10	3.805	-0.605	0.159	2.04
4-year diff.	2.259	-0.535	0.236	2.16	4.158	-0.427	0.103	5.20
10-year diff.	1.837	-0.435	0.237	2.56				

Notes: The table reports substitution elasticities (Columns (1) and (5)), changes in input factor ratios (Columns (2) and (6)), changes in shadow factor price ratios (Columns (3) and (7)), and implied labor supply elasticities, assuming perfectly elastic intermediate input supply (Columns (4) and (8)), based on our OLS (Columns (1)-(4)) and IV (Columns (5)-(8)) regressions from Tables 2 and 3 that regress logged output elasticities over returns to scale and logged input factors on logged output in within-firm differences.

5 Quantitative and Structural Interpretation

We now interpret our reduced form results structurally and quantitatively through the lens of the production model presented in Section 2. Subsequently, we discuss alternative accounts.[20] Importantly, our estimates in this section allow for firm- and year-specific markups and input wedges.

Table 4 summarizes the identified parameters and the mapping from reduced-form empirical moments using the identification arguments from Section 2 (that we further detail below) for the substitution elasticity between labor and intermediates and the firm-specific labor supply elasticity. Figures 5 summarizes existing estimates of the two key parameters, along with the values our study implies. Throughout, we focus on our *firm growth* estimates.

5.1 Identification of Substitution Elasticity

Identification argument. Our identification of the substitution elasticity rests on Equation (3), which identifies the labor-intermediates substitution elasticity, σ, from the co-movement of output elasticities and input quantities:

$$\frac{\sigma - 1}{\sigma} = \frac{\Delta \ln(\theta_{it}^L) - \Delta \ln(\theta_{it}^M)}{\Delta \ln(L_{it}) - \Delta \ln(M_{it})} \quad \Rightarrow \quad \sigma = \frac{1}{1 - \frac{\Delta \ln(\theta_{it}^L) - \Delta \ln(\theta_{it}^M)}{\Delta \ln(L_{it}) - \Delta \ln(M_{it})}}. \tag{13}$$

To calculate σ, we use within-firm changes (Δ) *conditional on firm growth*, i.e., we insert the estimated OLS and IV coefficients from our firm growth regressions into Equation (13).[21] That is, for each right-hand side component of Equation (13), we plug in the corresponding regression coefficient for

[20] As before, we focus on within-firm changes, which nets out fixed, unobserved firm-specific factors, which are captured by α_i^L and α_i^M in our model (see Equation (3)). Large firms seem to have higher intermediate/labor ratios, with smaller differences in labor and intermediate input output elasticities (Appendix Figure A.1). As our focus is on firm growth, we omit a detailed discussion of how to rationalize those facts with permanent heterogeneity.

[21] Compared to the OLS estimates, the IV counterparts can be viewed as additionally purging the changes of potential confounders, such as input-biased shifters in prices or production function parameters, that may underlie some of the OLS variation in firm growth.

that outcome estimated in Section 4, separately by time horizon and for OLS or IV:

$$\widehat{\sigma} = \frac{1}{1 - \frac{\rho_{\Delta \ln(\theta^L),\Delta \ln(Y)} - \rho_{\Delta \ln(\theta^M),\Delta \ln(Y)}}{\rho_{\Delta \ln(L),\Delta \ln(Y)} - \rho_{\Delta \ln(M),\Delta \ln(Y)}}}, \tag{14}$$

where the ρ variables correspond to the regression coefficients from Equation (10), for various horizons and outcome variables. By relating reduced form regression coefficients to a structural equation, our method can be viewed as a continuous or a firm-growth application of the reform-based difference-in-differences strategies in the public finance literature (e.g., Curtis et al. (2021),Lobel (2024)). Hence, our difference-based estimates of changes in output elasticities and input factors in response to firm growth identify values for the substitution elasticity, σ. Given the precisely estimated coefficients, we focus on point estimates.

Implied parameter values. Table 4 Columns (1) and (4) report the implied values of σ based on our OLS and IV estimates. We find that labor and intermediates are indeed substitutes, with σ exceeding unity in all specifications. The OLS estimates range from 2.67 in the short run to 1.84 in the longer run. IV estimates of σ are around 4 for 1- and 4-year changes (we do not have IV estimates for 10-year changes).[22] These substitution elasticity estimates above one provide a coherent explanation for the decline in relative output elasticities following the reduction in the labor-intermediate quantity ratio.

The relatively high substitution elasticities may also reflect that intermediate inputs also include intermediate services, unfinished products, and similar items (see Figure 2).[23]

Meta-analysis of existing substitution elasticity estimates. Quantitatively, our firm growth regressions identify substitution elasticities well above one, ranging from 1.8 to 2.7 (OLS) and from 3.8 to 4.2 (IV). Figure 5 Panel (a) situates these values in a systematic meta analysis of existing estimates (we indicate our own estimates by "MS"). Existing estimates of the intermediate-labor substitution elasticity rely on disparate identification strategies and production model assumptions. We therefore differentiate between approaches that estimate substitution elasticities between intermediates and a capital-labor bundle (diamonds) and between intermediates and labor (triangles). The latter type of estimates are in line with our approach and typically higher (we report means of estimates in Figure 5 Panel (a); values in Chan (2023), for instance, range from 1.6 to 9.6). We also note that our estimates are derived from within-firm changes.

5.2 Identification of (Relative) Firm-Specific Labor Supply Elasticity

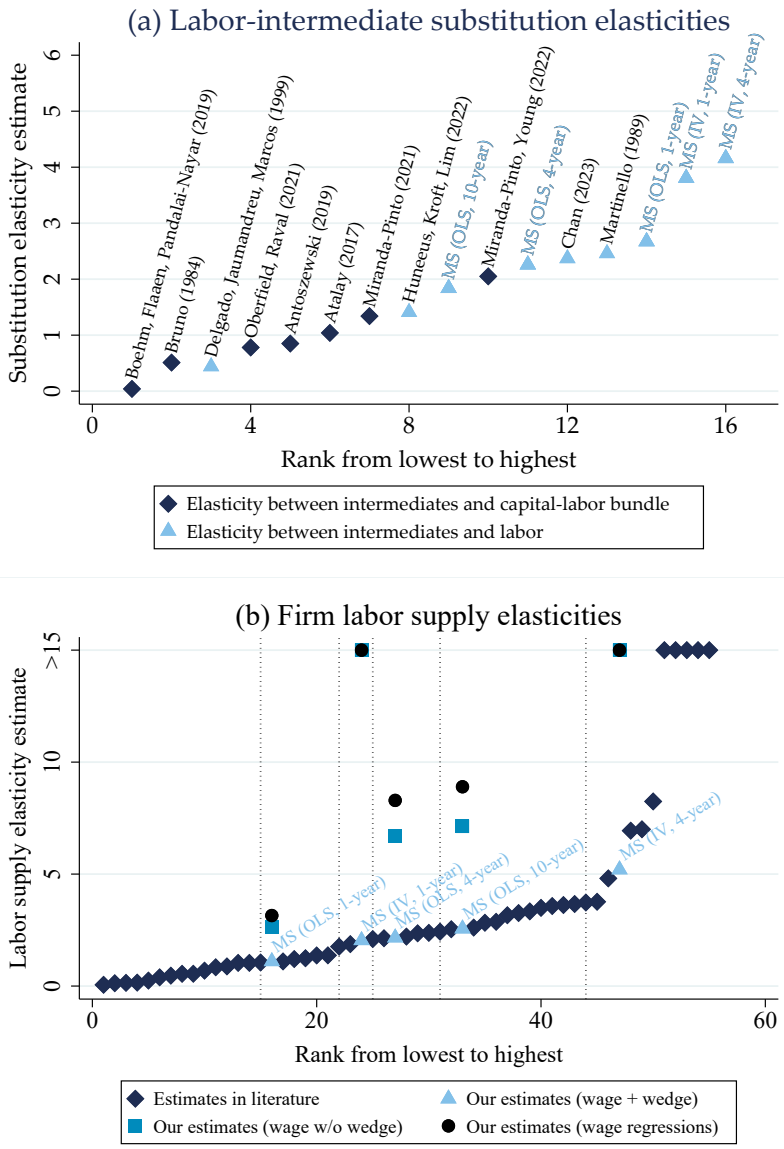
Why do firms change their input mix (and thus output elasticities) as they grow? Firms' cost minimization provides a natural answer, as it ties firms' optimal input mix to input prices. We now trace out the implied input price ratio, and translate it into an implied labor supply elasticity to the firm.

Step 1: identifying the implied input price ratio firm growth gradient. Using our estimates of σ

[22] We speculate that if we had available intensive-margin hours or capacity utilization measures rather than headcounts only, this would mildly amplify the labor responses to output growth, boosting the estimate of σ (see Equation 13. See also Footnote 17.

[23] As an aggregate perspective, consistent with labor and intermediates being substitutes, manufacturing intermediate to labor expenditure ratios increased while intermediate input to labor price ratios declined. See Appendix Figure A.3 for evidence on Germany and the US.

Figure 5: Meta-analysis: substitution and labor supply elasticities vs. literature.



Notes: The figure reports estimates of labor-intermediate substitution elasticities (Panel (a)) and labor supply elaticities (Panel (b)) from the literature and from our analysis (our own estimates are indicated by "MS" and in light blue). For substitution elasticity estimates, we exclude negative values, focus on samples that encompass the most firms, and report the mean value of estimates for a given paper as many studies report multiple separate values by industries, years, or with different methods. Data on labor supply elasticities come from the meta-study by Sokolova and Sorensen (2021). We exclude negative estimates. If available, we report medians of IV estimates. If no IV-approach was used, we report medians of all other estimates. (We prefer medians due to outliers in this statistic.) For a list of studies entering the data, we refer to Sokolova and Sorensen (2021).

and our firm growth regression coefficients, we can infer the implied (average) change in the shadow price ratio (specifically, the change that is caused by firm growth in our empirical specification). We insert our regression coefficients into Equation (4) to back out the effect of firm growth on within-firm

changes in the shadow price ratio:

$$\Delta \ln \left(\frac{P_{it}^L \gamma_{it}^L}{P_{it}^M \gamma_{it}^M} \right) = \frac{\Delta \ln(L_{it}) - \Delta \ln(M_{it})}{-\sigma} \tag{15}$$

$$\Rightarrow \widehat{\rho}_{\Delta \ln(\frac{P^L \gamma^L}{P^M \gamma^M}), \Delta \ln(Y)} = \frac{\rho_{\Delta \ln(L), \Delta \ln(Y)} - \rho_{\Delta \ln(M), \Delta \ln(Y)}}{-\widehat{\sigma}} \tag{16}$$

To the best of our knowledge, our paper is the first to combine estimates of output elasticities with a standard production model to measure unobserved input price variation.

We report results for $\widehat{\rho}_{\Delta \ln(\frac{P^L \gamma^L}{P^M \gamma^M}), \Delta \ln(Y)}$ in Table 4 Columns (3) and (6). OLS estimates indicate changes in the log price ratio of 0.27 in the short run and 0.23 in the long run. IV estimates are 0.16 and 0.10 for short- and longer-run changes, respectively. Our findings thus suggest that, as firms grow, the shadow price of labor increases relative to that of intermediate inputs. This increase is stronger in the short run than in the long run. As a result, firms substitute labor with intermediates and change their modes of production as reflected in their output elasticities.

Potential sources of the relative increase in labor prices. What forces cause relative labor costs to rise as firms grow? One natural explanation is that the firm-specific labor supply elasticity is low (compared to that of intermediate inputs), suggesting the presence of monopsony power in labor markets, which raises wages as firms hire more workers. The persistence of rising relative labor costs even over a long-term (10-year) horizon reinforces the monopsony-based explanation. However, the fact that relative labor costs are notably higher in the short run also points to short-run adjustment costs, which are plausibly more significant for labor than for intermediate inputs. Adjustment costs can thus be an important driver of (short-run) changes in wage ratios, and, through that, input quantity and output elasticity ratios.

Step 2: Identifying the firm-specific factor supply elasticities. To identify the firm-specific labor supply elasticity, we make two key assumptions. First, intermediate inputs are perfectly elastically supplied.[24] Second, we assume that the markdown, γ_{it}^L, is constant in firm growth (we still permit a baseline wedge)—an assumption we later relax by measuring γ_{it}^L and studying its firm growth gradient directly. Those two assumptions imply that the input shadow price gradient is solely due to wage increases, allowing us to infer the implied firm-specific labor supply elasticities from our estimated input price changes and the employment effects:

$$\widetilde{\epsilon}^L = \frac{\Delta \ln(L_{it})}{\Delta \ln \left(\frac{P_{it}^L \gamma_{it}^L}{P_{it}^M \gamma_{it}^M} \right)} = \frac{\Delta \ln(L_{it})}{\Delta \ln(P_{it}^L)}, \tag{17}$$

[24] We are not aware of comparable estimates for the firm-specific supply elasticities for intermediates. We assume them to be supplied elastically in our quantitative interpretation. Bilal and Lhuillier (2022) makes an analogous assumption for a labor-only model involving labor service purchases (such as temp work agencies), although our intermediates are largely made of goods rather than services. For temporary agency workers as one facet of outsourcing, Drenik et al. (2023) show that wage premia partially extend to outsourced labor. Huneeus et al. (2022) focus on intermediate input price variation from production networks as a driver of cross-sectional outsourcing differentials.

which we identify on the basis of the following ratio:

$$\widetilde{\epsilon}^{L} = \frac{\rho_{\Delta \ln(L),\Delta \ln(Y)}}{\widehat{\rho}_{\Delta \ln(\frac{P^L \gamma^L}{P^M \gamma^M}),\Delta \ln(Y)}}. \tag{18}$$

That is, we identify the labor supply elasticity by dividing the changes in labor quantities from Tables 2 and 3 by the changes in input price ratios from Table 4 as constructed above. We emphasize that this parameter, $\widetilde{\epsilon}^{L}$, is the *inverse* of the supply elasticity, ε^{L}, in Section 2. Moreover, the $\widehat{\rho}$ term in the denominator is *not* a regression coefficient but the inferred input shadow price ratio gradient backed out in Equation (15) above.

Results: implied firm-specific labor supply elasticities. We report implied labor supply elasticities, ϵ^{L}, in Figure 4 (triangles). The values range from 1.10 to 2.56 for our OLS- and from 2.04 to 5.20 for our IV-results. The higher long-run than short-run elasticities either reflect horizon-dependence of labor supply elasticities, or may reflect firm-side adjustment costs (non-constant γ^{L}_{it}). Nevertheless, even in the long run, supply elasticities remain relatively low. These estimates showcase the quantitative basis for the monopsony-driven incentive for firms to shift from labor to intermediate inputs.

Robustness: permitting markdowns and direct wage estimates. By using the shadow price of labor (the product of the wage and wage markdown), our initial estimates of the labor supply elasticity may be confounded by the markdown variation. We now calculate firm-specific labor supply elasticities with respect to wages, under two alternative assumptions. We report these alternative values vertically above our initial estimates in Figure 5 Panel (b).

First, we report results that subtract the measured markdown effect on $\gamma^{L}_{it}/\gamma^{M}_{it}$ that we estimate in Section 6 when studying the labor share implications (squares). As we find that markdowns *increase* as firms growth, this calculation pushes up the labor supply elasticities in our study. Intuitively, by eliminating the effect of increasing wage markdowns, a given shift in labor now corresponds with a smaller change in labor prices, implying higher supply elasticities.

Second, we also measure wages directly, using the average firm wage (wage bill per head) from the firm-level data and run regressions of wage changes on output growth as before (dots). Particularly for the OLS strategy, average wages increase in firm growth (consistent with monopsony), which, together with the labor quantity changes, yields a direct estimate of the wage change along the labor supply curve. Since those wages move less than the inferred shadow price ratios above, the implied firm-specific labor supply elasticities are again higher than our baseline measure. For IV, the wage effect point estimates are close to zero, these values imply large elasticities, but the wider confidence intervals for the wages accommodate also elasticities more consistent with the literature. Overall, we caveat that average wages are subject to composition bias and that we cannot merge matched employer-employee data or data on worker skills to our firm data to use cleaner wage measures. The additional wage regressions are reported in Appendix Table A.6.

Meta-analysis of existing parameter estimates. Figure 5 Panel (b) summarizes existing estimates of labor supply elasticities based on the meta-analysis of Sokolova and Sorensen (2021), along with the values our study implies, including the additional estimates from the robustness checks discussed above. (While estimates are ranked by size, for our own estimates, we provide their respective

robustness checks vertically stacked.) Overall, our estimates fall well into the range of existing estimates and highlight a lower short- than long-run labor supply elasticity. The figure also illustrates that supply elasticities are higher for our IV estimates, and when we net out markdown shifts or use direct (average) wage estimates. We note again that our estimates are based on within-firm firm growth regressions.

5.3 Alternative Mechanisms

Automation. Recent work has highlighted capital-based technological change as one factor that reduces the importance of labor to firms (Acemoglu and Restrepo, 2020, Hubmer and Restrepo, 2021). Such technological change could explain a decline in labor's output elasticity with firm growth. However, rather than an increase, we document a decline in capital output elasticities with firm growth. Instead, intermediate inputs *unrelated to capital services* gain in importance (as discussed above, maintenance and repair services decline, Appendix Table A.4). Therefore, while automation is an important process shaping changes in firms' production and aggregate labor market trends, it appears not to rationalize our findings in the context of idiosyncratic firm growth. Additionally, we reiterate that our causality runs from plausibly input-neutral output shifters to changes in output elasticities.

Biased technological change. A potential concern is that our output elasticities may be mismeasured due to the assumption of Hicks-neutrality. However, we do not believe that measurement error, particularly factor-biased technological shocks, can explain our findings.

First, our results for cost shares and output elasticities are quantitatively similar (see Tables 2 and 3). This similarity is reassuring because these metrics reflect firms' output elasticities under different assumptions. Our direct output elasticity estimates rely on Hicks-neutrality and a specific model of firm behavior, while allowing for varying input market imperfections and returns to scale. In contrast, changes in cost shares capture changes in output elasticities under non-changing input market imperfections and non-changing returns to scale, but accommodate non-Hicks-neutral productivity processes without estimating the underlying production function. In Appendix Table A.7, we compute substitution elasticities from cost share estimates, which suggest even stronger substitutability between labor and intermediates.

Second, we employ an instrumental variable approach, using foreign export demand shocks as instruments for firm growth. These shocks are plausibly largely independent of firm-specific factor-biased technological shocks that could influence output elasticities and overall output. Again, here, our causality runs from firm growth to changes in output elasticities rather than from changes in technology to changes in output.

Fixed costs and size dependence. Our data do not allow us to differentiate between fixed and flexible labor inputs. If firms' production involves fixed labor costs that scale up less than proportionately with flexible labor, labor output elasticities decline as firms grow. Effectively, fixed costs break the homotheticity of a production function (Dhyne et al., 2022, Savagar and Kariel, 2024).

We would expect fixed costs to be particularly relevant for small firms. However, when we split our firm sample into size quintiles (by year and industry), we find similar results across size (sales)

Table 5: Implied substitution elasticities, changes in input cost ratios, and firm-specific labor supply elasticities by size quintiles (OLS, 4-year changes).

	σ	$\Delta \ln \left(\frac{L_{it}}{M_{it}} \right)$	$\Delta \ln \left(\frac{P_{it}^L \gamma_{it}^L}{P_{it}^M \gamma_{it}^M} \right)$	$\widetilde{\epsilon}^L = 1/\varepsilon^L$
	(1)	(2)	(3)	(4)
Overall	2.259	-0.535	0.236	2.16
1st quintile	2.007	-0.546	0.272	1.86
2nd quintile	2.236	-0.550	0.246	1.97
3rd quintile	2.186	-0.540	0.247	1.99
4th quintile	2.327	-0.519	0.223	2.37
5th quintile	2.277	-0.526	0.231	2.29

Notes: The table replicates Table 4 by size quintile for OLS. Size quintiles are computed by year and industry. Size is measured by sales.

quintiles for our regression results (Appendix Table A.5). As a result, Table 5 shows that also our substitution and supply elaticities are similar across firms of different sizes (we rely on OLS as IV first stages become too weak in this sample split). Therefore, fixed labor costs that break the homotheticity of the production function are unlikely to explain the decline in relative labor quantities and output elasticities with firm growth. However, in Appendix Table A.5, we find that declines in capital output elasticities and increases in intermediate inputs are somewhat moderated at larger firms. This result is consistent with large initial capital fixed costs in firms' production.

Non-homothetic CES. Fixed costs are not the only factor that can break the homotheticity of a production function. Our paper proposes a homothetic CES production function that rationalizes changes in relative output elasticities through changes in relative input quantities (and prices). Alternatively, intermediate inputs might become more efficient at larger scale, such that firms' relative labor output elasticities decline as a *direct* result of firm growth. Recently, Lashkari et al. (2024) discussed such a mechanism in context of IT inputs becomming more efficient at larger scale. An adaptation of their framework to our CES model above and intermediates instead of IT can be described by the following non-homothetic production function:

$$Q_{it} = \Omega_{it} \Lambda_i^K K_{it}^{1-\kappa} \left(\Lambda_i^{LM} \alpha_i^L L_{it}^{\frac{\sigma-1}{\sigma}} + \Lambda_i^{LM} \alpha_i^M \left(\frac{M_{it}}{Q_{it}^{\eta}} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}\kappa}, \tag{19}$$

where $\eta > -1$ is a firm-size scaling parameter that captures the non-homoetheticity. If $\eta = 0$, Equation (19) collapses to Equation (1). Under this non-homothetic CES, the ratio of output elasticities is (derived in Appendix B.3):

$$\frac{\theta_{it}^L}{\theta_{it}^M} = \frac{\alpha_i^L}{\alpha_i^M} \left(\frac{L_{it}}{M_{it}} \right)^{\frac{\sigma-1}{\sigma}} Q_{it}^{\frac{-\eta}{\sigma}}. \tag{20}$$

Compared to the homothetic production function (Equation (3)), the ratio of output elasticities now features an additional *direct* effect of firm size on the output elasticity ratio: growing firms' labor output elasticity relative to their intermediate input output elasticity increases (decreases) if $\eta < 0$ ($\eta > 0$).

Equation (20) suggests an empirical horse race testing between the relevance of the non-

homotheticity versus the input factor ratio in regulating the observed changes in output elasticities. Specifically, we regress log changes in labor output elasticities on log changes in input factor ratios *and* on log changes in output. We report results from this exercise in Appendix Tables A.8 and A.9. We find that once we control for the input factor ratio, there is no statistically or economically significant negative effect of log output changes on (relative) log labor output elasticity changes. Therefore, this informal test suggests that a non-homotheticity in output is unlikely to be a main factor behind shifts in labor relative to intermediate output elasticities with firm growth. Instead, the changes in the input ratio and relative input prices appear to explain why growing firms move toward less labor- and more intermediate-intensive production modes, and also offer a natural dynamic explanation for the stronger short-run effects. In addition, we reiterate that across firm size groups, we find similar results (discussed above).

6 Implications for Firm-level Labor Shares

We now examine a key implication of firm growth on labor's output elasticity, shaped by factor substitution and monopsony: the decline in the labor share, driven by production function properties rather than direct market power. The distinction is essential: while monopsony plays a role in this mechanism by raising labor prices in growing firms, leading to substitution from labor to intermediates, the novel effect on the labor share that our framework uncovers works through output elasticities and hence production function properties rather through increasing markdowns.

6.1 Labor Share Decomposition and Identification

We adapt our framework from Section 2 to conceptualize this link and provide identification of the underlying additional drivers of the labor share.

The labor share. Taking logs of Equation (6) yields a statistical log decomposition of a firm's labor share in output into three terms: the output elasticity, the markup, and the markdown:

$$\ln(LS_{it}) = \ln(\theta^L_{it}) - \ln(\mu_{it}) - \ln(\gamma^L_{it}). \tag{21}$$

Identifying price markups and wage markdowns. To separate the three determinants of the labor share, we additionally construct measures of markups and markdowns from our production function estimation. Due to the translog structure of our production function, these variables are *firm- and time-specific*.

We derive output price markups using the production approach of Hall (1986) and De Loecker and Warzynski (2012). That is, assuming that intermediate inputs are a flexible input and that firms' take intermediate input prices as given, such that $\gamma^M_{it} = 1$, markups, μ_{it}, can be derived from the first order condition for intermediates within our framework (see Appendix B.4):

$$\mu_{it} = \frac{P_{it}}{\lambda_{it}} = \theta^M_{it} \frac{P_{it}Q_{it}}{P^M_{it}M_{it}}. \tag{22}$$

Following recent work (e.g., Dobbelaere and Mairesse, 2013, Yeh et al., 2022), we define an expression

for firms' wage markdowns, γ_{it}^L, by combining the first order conditions for intermediates and labor and again assuming that $\gamma_{it}^L = 1$ (see Appendix B.4):

$$\gamma_{it}^L = \frac{\theta_{it}^L}{\theta_{it}^M} \frac{P_{it}^M M_{it}}{P_{it}^L L_{it}}. \tag{23}$$

Importantly, we acknowledge that Equation (23) also captures any biased technological differences between firms that are not captured by output elasticities (but they will be captured by cost shares as discussed above).

Relaxing $\gamma_{it}^M = 1$: absolute vs. relative wedges. While we follow the conventional argument that $\gamma_{it}^M = 1$ is required for these two identification strategies, we note that γ_{it} is effectively a measure of *relative* input cost markdowns for labor vs. intermediate inputs. Assuming exogenous intermediate input prices and flexible intermediate inputs imposes $\gamma_{it}^M = 1$ and hence $\gamma_{it}^L = \frac{\gamma_{it}^L}{\gamma_{it}^M}$. In fact, the relative wedge exactly aligns with our theory as we focus on the reallocation between intermediates and labor.

Relaxing $\gamma_{it}^M = 1$: levels vs. changes. Moreover, for our specification in *changes*, we can relax the assumption that $\gamma_{it}^M = 1$; instead, it suffices to assume that within a firm, the intermediate wedge is constant ($\gamma_{it}^M = \gamma_i^M$). This is a considerably weaker assumption.

Total labor wedge. Finally, as a complement (and again assuming $\gamma_{it}^M = 1$), we also define and study the total labor wedge, which is the combined distortion from markup and labor market imperfections (this is sometimes used as a markup measure, under the assumption of perfect labor markets):

$$\mu_{it}\gamma_{it}^L = \theta_{it}^L \frac{P_{it}Q_{it}}{P_{it}^L L_{it}}. \tag{24}$$

6.2 Results

We now show and dissect the empirical results applying the decomposition and identification from above.

Strategy. To measure and decompose the effects of firm growth on firm-level labor shares, we estimate the regression model in Equation (10) for changes in log labor shares, log markups, log markdowns, and log labor wedges as outcome variables.

Throughout, we focus on the sales labor share because this is the relevant labor share definition that corresponds to the output elasticity of labor and our firm-level production function framework. However, we also include the labor share in value added ($\frac{P_{it}^L L_{it}}{VA_{it}}$) as a robustness check (value added, VA_{it}, is revenue minus intermediate input expenditures). Again, we present OLS and IV results and estimate regressions for 1-, 4-, and 10-year differences.

Results. Table 6 is organized as its counterparts, Tables 2 and 3, showing the OLS (Panels A-C) and IV (Panels D-E) effects of firm output growth on changes in labor shares, markups, markdowns, labor wedges, and value added labor shares by time horizon. We additionally add our previous estimates on labor output elasticities to facilitate the analysis. As indicated in Equation (21), the coefficients on

markups, wage markdowns, and output elasticities *add up* to that on the labor share. As before, we also visualize the variation underlying the regression estimates in Figure 6.

Declining labor shares. We find a strong negative effect of firm output on labor shares. OLS results for sales labor shares range from -0.581 to -0.303 for 1- to 10-year differences. Results for value added labor shares as well as IV estimates are similar. Again, as for output elasticities, effects somewhat shrink as we widen the time horizons.

Half to a third of the effect: declining output elasticity of labor. In the short and medium run, up to the 4-year differences, *half* of this negative effect of output growth on labor shares is driven by the declining output elasticity of labor. This result holds for OLS and IV regressions. For 10-year differences, the declining output elasticity still accounts for a third of the effect (as changing returns to scale become increasingly important over longer time horizons).

The remainder: conventional market power effects. The remaining half (short- to medium-run) or two thirds (long-run) of the within-firm effect of firm growth on labor share changes is explained by the fact that markups and wage markdowns increase in firm size. As we control for industry fixed effects, output growth is effectively reflecting an increase in firms' market shares, and one may expect a firm's market power to increase with its market share (as in Atkeson and Burstein, 2008 for product markets and Berger et al., 2022 for labor markets). Notably, over longer horizons, wage markdowns become much more important than product markups for explaining the decline in labor shares as firms grow.[25]

7 Evidence from other Countries and Aggregate Industry Implications

We now extend the analysis to other countries and sectors using firm and industry data. First, we replicate our baseline OLS regressions on administrative data (i.e., the equivalents to the German micro data) in six additional European countries. Subsequently, we study aggregate industry dynamics using a micro-aggregated dataset for 20 European countries and for the United States to show that our firm-level results carry over to industry aggregates (despite aggregation biases). Section 7.1 presents the data. Section 7.2 reproduces key results from our firm-level analysis for six other countries. Section 7.3 extends our firm-level regressions to the industry level and approximates industry substitution elasticities based on our firm-level equations. Section 7.4 discusses implications for industry labor shares.

7.1 International Industry Panel Data: CompNet Data

CompNet data. We use the 9th (most recent) vintage of the CompNet data (CompNet, 2023), which is a micro-aggregated database for 22 European countries.[26] The data are collected and provided by the Competitiveness Research Network (henceforth, CompNet). CompNet sources its data from representative administrative firm-level records located within European national statistical institutes and

[25] Also in the cross section, market power effects (particularly markdowns) are more important, consistent with Autor et al. (2020) and De Loecker et al. (2020) (see Figure A.4).

[26] We drop Malta due to insufficient observations for output elasticities and the UK, which did not provide industry-level data to CompNet.

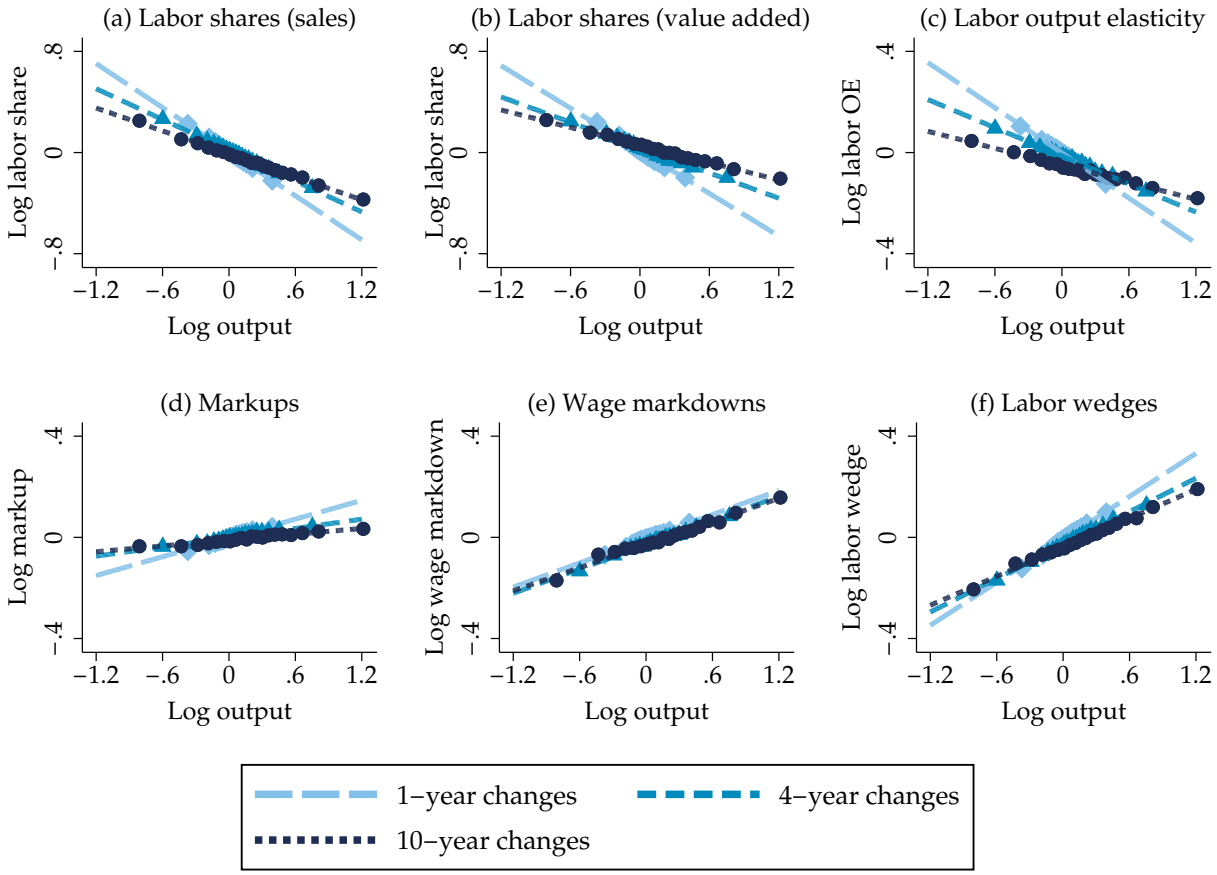
Table 6: Labor share, markup, markdown, and output elasticity changes in response to firm growth.

	$\ln(LS_{it})$	$\ln(\mu_{it})$	$\ln(\gamma_{it})$	$\ln(\mu_{it}\gamma_{it})$	$\ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\ln(\theta^L_{it})$	$\ln(\frac{P^L_{it}L_{it}}{VA_{it}})$
Panel A: OLS, 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output growth	-0.581***	0.124***	0.159***	0.283***	-0.304***	-0.298***	-0.561***
	(0.0040)	(0.0025)	(0.0048)	(0.0044)	(0.005)	(0.0046)	(0.0076)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950
R^2	0.534	0.096	0.115	0.191	0.230	0.215	0.190
Panel B: OLS, 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output growth	-0.406***	0.0608***	0.160***	0.220***	-0.202***	-0.185***	-0.334***
	(0.0061)	(0.0032)	(0.0065)	(0.0055)	(0.0060)	(0.0063)	(0.0081)
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492
R^2	0.461	0.126	0.193	0.239	0.220	0.201	0.188
Panel C: OLS, 10-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output growth	-0.303***	0.0383***	0.152***	0.191***	-0.140***	-0.112***	-0.231***
	(0.0062)	(0.0030)	(0.0060)	(0.0048)	(0.0053)	(0.0057)	(0.0068)
Observations	49,915	49,915	49,915	49,915	49,915	49,915	49,915
N of firms	10,595	10,595	10,595	10,595	10,595	10,595	10,595
R^2	0.378	0.125	0.213	0.253	0.188	0.161	0.178
Panel D: IV, 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output growth	-0.674***	0.186***	0.169**	0.355***	-0.327***	-0.318***	-0.842***
	(0.0521)	(0.0418)	(0.0759)	(0.0697)	(0.0623)	(0.0645)	(0.120)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950
First-stage F-Statistic	102.6	102.6	102.6	102.6	102.6	102.6	102.6
R^2	0.524	0.084	0.115	0.185	0.229	0.215	0.160
Panel E: IV, 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output growth	-0.443***	0.154***	0.0688	0.223***	-0.242***	-0.220***	-0.603***
	(0.0726)	(0.0444)	(0.0800)	(0.0713)	(0.0706)	(0.0742)	(0.119)
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492
First-stage F-Statistic	48.57	48.57	48.57	48.57	48.57	48.57	48.57
R^2	0.459	0.072	0.180	0.239	0.217	0.199	0.135

Notes: The table reports OLS and IV regressions estimating the specification in Equation (10). The dependent variables in Columns (1)-(7) are the logs of the labor share in sales, markups, wage markdowns, labor wedge, labor output elasticity divided by returns to scale, labor output elasticity, and labor share in value added, respectively. Panels A-C rely on OLS and regress changes in those dependent variables on changes in log output for 1-, 4- , and 10-year differences. Panels D-E rely on IV and regress changes in those dependent variables on changes in log output for 1-, and 4-year differences. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm-level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

central banks (akin to the US Census data). The CompNet team distributes harmonized data collection protocols (i.e., Stata codes) across the data providers and invests significant efforts in harmonizing the input data to maximize comparability across countries. The protocols compute micro-aggregated results. From these results, the CompNet team constructs the CompNet database. The data is aggre-

Figure 6: Labor share, markups, markdowns, and output elasticity changes in response to firm growth (OLS, binned scatter plots).



Notes: The figure reports binned scatter plots estimating the specification in Equation (10) with OLS for various differences. The graphs report binned scatter plots for changes in the logs of labor shares in sales and value added, labor output elasticities (not divided by returns to scale), markups, wage markdowns, and the labor wedge against log output changes. All panels report results that are residualized by industry-year fixed effects. German firm-level data.

gated at various levels. We use the country-industry-level data (NACE Rev. 2, two-digit industries), which is the most detailed aggregation level available.

The data contain, among other features, country-industry-level information on firms' sales, inputs, expenditures, markups, wage markdowns, and output elasticities. They cover 1999-2021, and we focus on manufacturing (NACE Rev. 2 two-digit industries 10-33) which has the best coverage.[27] Yet, we will also show key results for non-manufacturing industries.[28] Yearly coverage varies across countries as shown in Appendix Table A.10. We focus on the data containing firms with at least 20 employees as this is available for more countries and is consistent with our German micro data. To ensure representativeness, the data is weighted by firm population weights.[29] For further details on

[27]Manufacturing drives much of the decline in labor shares in most developed countries (Dao et al., 2019).

[28]This includes the NACE rev. 2 industries 41-43 (construction), 45-47 (wholesale/retail trade and repair of motor vehicles and motorcycles), 49-53 (transportation/storage), 55-56 (accommodation/food services), 58-63 (information and communication technology), 68 (real estate), 69-75 (professional/scientific/technical activities), and 77-82 (administrative/support service activities).

[29]Our results hold for the subset of countries for which we have data based on firms of all size classes.

the data, we refer to CompNet's User Guide (CompNet, 2023).

While there is no comparable data available for the US, we do provide a limited set of results for the US based on the NBER-CES Manufacturing Industry Database (which lacks output elasticity estimates) for 1958-2016.[30]

Production function estimation. The CompNet data provides information on industry-level of markups, wage markdowns, and output elasticities using various types of estimation approaches. We rely on measures derived from industry-specific translog specifications that are estimated using the two-step control function approach of Ackerberg et al. (2015) (in our German firm level-data we instead use a one-step approach similar to Wooldridge, 2009).[31] The log production function in the CompNet data is almost identical to Equation (9) with exception of excluding the triple interaction term. It is specified as follows and estimated in the micro firm-level panel data separately by country-industry cells (i.e., coefficients differ by industry-country):

$$q_{it} = \beta_l l_{it} + \beta_k k_{it} + \beta_m m_{it} + \beta_{l2} l_{it}^2 + \beta_{k2} k_{it}^2 + \beta_{m2} m_{it}^2 + \beta_{lk} l_{it} k_{it} + \beta_{lm} l_{it} m_{it} + \beta_{km} k_{it} m_{it} + \omega_{it} + \nu_{it}. \quad (25)$$

Lower case letters denote logs. As there is no firm-specific price information observed in the firm-level data underlying the CompNet data, output, q_{it}, is measured as deflated sales using industry-specific deflators. To account for firm-specific price variation within industries, the CompNet estimation routine additionally controls for firms' market shares when estimating the production function.[32] Labor is defined as number of employees. Intermediates and capital are defined as deflated intermediate input expenditures and deflated capital stocks (book values) using country-industry-year-specific deflators. Markups and wage markdowns are computed as described in Equations (22) and (23). Output elasticities are computed as in the German micro data: $\theta_{it}^X = \frac{\partial \ln(Q_{it})}{\partial \ln(X_{it})}$ for each input $X = \{L, K, M\}$.

Industry aggregation. Industry-level values are constructed as weighted means of the firm-level micro variables, using sales weights for output elasticities, labor cost weights for markdowns, and intermediate input cost weights for markups (i.e., the denominators of the markup and markdown expressions).[33] This weighted aggregation differs from our previous unweighted firm-level analysis and allows us to focus on representative industry-aggregates. Consistently, all other variables are computed as country-industry totals and their ratios.

Industry summary statistics and sample. To construct a harmonized sample, we only keep country-industry pairs for which we observe labor shares, output elasticities, cost shares, markups, and wage markdowns.[34] We provide summary statistics of key variables by country from the country-industry-

[30] While the NBER CES database currently ends in 2018, capital, which is required to construct cost shares, is missing after 2016.

[31] See Ackerberg et al. (2015) for details on this estimation strategy. In a nutshell, the estimator controls for unobserved productivity (ω_{it}) using a control function containing intermediate inputs, labor, and capital. The approach assumes a simple firm decision model where intermediate inputs and labor are flexible and intermediate input demand depends on capital, labor, and productivity. Under certain restrictions, one can invert the intermediate input demand function to approximate productivity as a function of intermediates, labor, and capital.

[32] This approach follows De Loecker et al. (2020). Under a Cournot model, market shares perfectly capture markup variation. Therefore, market shares can help absorbing some of the unobserved price variation.

[33] Results are robust to using sales weights for all variables, which is also available in the CompNet data (unreported).

[34] Some country-industry-year cells report missing values in the CompNet data because the number of firms in the underlying micro data was too small either for passing country-specific disclosure rules, or for estimating the production functions.

level data in Appendix Table A.10. The table also reports the yearly coverage for each country in CompNet.

Commissioning our own firm-level regressions in the CompNet micro data. To supplement the country-industry-level CompNet data, we collaborated with the CompNet team to incorporate additional firm-level regressions into their *10th* vintage data collection. These regressions replicate our firm-level analysis in 11 countries and extent our initial analysis beyond manufacturing, which we could not do with our German manufacturing micro data. The 10th vintage data includes a slightly different sample of countries and covers more recent years than the currently available 9th vintage country-industry data (we document yearly coverage below). Due to the time-intensive nature of running our codes, we, so far, only received results for a subset of data providers: France, Hungary, Poland, Portugal, Slovakia, Slovenia, Estonia, Latvia, The Netherlands, Romania, and Switzerland.[35]

7.2 Firm-level Results for 11 other European Countries

Tables 7 and 8 presents firm-level results (OLS) for the 11 countries using the CompNet infrastructure. Results are provided separately for manufacturing (Table 7) and all sectors (Table 8). Columns (1)–(9) show regression coefficients on log output growth based on Equation (10), estimated using OLS with 4-year changes (similar to Table 2). Column (10) presents the estimated elasticities of substitution between labor and intermediates, calculated from the coefficients on input quantities and output elasticities, as described in Equation (14). Column (11) provides a robustness check by using cost share coefficients instead of output elasticity coefficients, which is an important robustness check for the CompNet data analysis due to the lack of firm-specific price data when estimating production functions.[36] Lastly, Column (12) reports the percentage contribution of changes in firm-level labor output elasticities to declining labor shares with output growth.

Results. Our previous firm-level findings for Germany are confirmed across all 11 other countries: input quantities, cost shares, and output elasticities shift from labor toward intermediates, with substitution elasticities consistently exceeding unity. Quantitatively, most results align closely with our findings for Germany, although labor output elasticities decline even more sharply in most countries. This stronger decline also explains the larger implied substitution elasticities. As a result, with the exception of France, the contribution of changes in labor output elasticities to declines in firm-level labor shares with firm growth is higher than in the German data.

Manufacturing vs. all sectors. Results are similar in manufacturing (Table 7) and all-sector samples (Table 8).

Young and mature firms. Our German firm data lacks information on firms' registration years. However, in six of the eleven countries included in the firm-level CompNet data analysis, this information is available. In Appendix Table A.11, we replicate our analysis for young (no older than five years) and

[35] With further cooperation from data providers, we can extend our analysis to additional countries (possibly including Germany with non-manufacturing sectors) and provide these results upon request.

[36] We also provided also cost-share based estimates of substitution elasticities for the German micro data in Appendix Table A.7.

mature (older than five years) manufacturing sector firms. The results show no significant differences between the two groups, and similar findings hold when using firms from all sectors (not reported).

Table 7: Firm-level results for other countries: Coefficients on log output growth, substitution elasticities, and contirbution of labor output elasticity changes to labor share changes (manufacturing, 4-year changes, OLS).

	$\ln(L_{it})$	$\ln(M_{it})$	$\ln(CS^L_{it})$	$\ln(CS^M_{it})$	$\ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\ln(\frac{\theta^M_{it}}{RTS_{it}})$	$\ln(\theta^L_{it})$	$\ln(LS_{it})$	$\ln(\frac{P^L_{it}L_{it}}{VA_{it}})$	σ^{OE}	σ^{CS}	LS contrib. of θ^L_{it}
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
France (1995-21)	0.496***	1.027***	-0.0882***	0.313****	-0.131***	0.196***	-0.122***	-0.373***	-0.350***	2.60	4.01	33%
Obs.: 282,245	(0.004)	(0.004)	(0.003)	(0.004)	(0.002)	(0.002)	(0.002)	(0.003)	(0.003)			
Hungary (2003-22)	0.464***	1.057***	-0.307***	0.159***	-0.330***	0.121***	-0.343***	-0.409***	-0.340***	4.18	4.67	84%
Obs.: 35,902	(0.007)	(0.007)	(0.008)	(0.006)	(0.007)	(0.003)	(0.007)	(0.008)	(0.010)			
Poland (2002-22)	0.535***	1.058***	-0.262***	0.134***	-0.281***	0.0901***	-0.281***	-0.337***	-0.259***	3.44	4.12	83%
Obs.: 98,385	(0.004)	(0.003)	(0.004)	(0.003)	(0.004)	(0.001)	(0.004)	(0.004)	(0.004)			
Portugal (2004-22)	0.414***	1.115***	-0.347***	0.236***	-0.393***	0.198***	-0.403***	-0.469***	-0.352***	6.37	5.94	86%
Obs.: 33,478	(0.007)	(0.007)	(0.007)	(0.006)	(0.007)	(0.005)	(0.008)	(0.007)	(0.009)			
Slovakia (2000-23)	0.513***	1.019***	-0.266***	0.166***	-0.273***	0.102***	-0.264***	-0.412***	-0.381***	3.86	6.84	64%
Obs.: 9,605	(0.017)	(0.020)	(0.015)	(0.010)	(0.013)	(0.007)	(0.014)	(0.016)	(0.025)			
Slovenia (2002-23)	0.547***	1.091***	-0.285***	0.166***	-0.355***	0.135***	-0.351***	-0.360***	-0.236***	10.07	5.85	98%
Obs.: 7,452	(0.016)	(0.013)	(0.015)	(0.011)	(0.017)	(0.007)	(0.017)	(0.016)	(0.015)			
Estonia (2004-23)	0.547***	1.102***	-0.243***	0.151***	-0.393***	0.127***	-0.394***	-0.292***	-0.192***	15.86	3.45	135%
Obs.: 1,627	(0.031)	(0.024)	(0.029)	(0.014)	(0.033)	(0.01)	(0.034)	(0.031)	(0.034)			
Latvia (2005-21)	0.495***	0.940***	-0.239***	0.0272***	-0.258***	0.125***	-0.257***	-0.327***	-0.423***	7.18	2.49	79%
Obs.: 2,760	(0.023)	(0.022)	(0.028)	(0.0055)	(0.022)	(0.010)	(0.022)	(0.022)	(0.039)			
Netherlands (2007-22)	0.369***	1.116***	-0.303***	0.169***	-0.358***	0.143***	-0.363***	-0.353***	-0.185***	3.04	2.72	103%
Obs.: 30,210	(0.009)	(0.008)	(0.011)	(0.007)	(0.008)	(0.004)	(0.008)	(0.011)	(0.008)			
Romania (2005-23)	0.526***	1.031***	-0.269***	0.126***	-0.232***	0.0827***	-0.238***	-0.363***	-0.337***	2.65	4.59	66%
Obs.: 21,642	(0.0076)	(0.005)	(0.008)	(0.004)	(0.006)	(0.003)	(0.007)	(0.008)	(0.013)			
Switzerland (2009-22)	0.460***	1.055***	-0.303***	0.189***	-0.287***	0.124***	-0.283***	-0.434***	-0.365***	3.23	5.78	65%
Obs.: 11,875	(0.016)	(0.013)	(0.015)	(0.010)	(0.013)	(0.0057)	(0.014)	(0.016)	(0.017)			

Notes: The table reports OLS regressions from estimating the specification in Equation (10) for different countries by OLS (Columns (1)-(9)) using 4-year differences. The table reports coefficients on log output growth in Columns (1)-(9). The dependent variables in Columns (1)-(9) are the logs of labor and intermediate quantities, cost shares, and output elasticities divided by returns to scale, labor output elasticities, labor shares in sales, and labor shares in value added, respectively. All regressions are of changes in those dependent variables on changes in log output for 4-year differences. All regressions control for two-digit industry-year fixed effects. Standard errors are clustered at the firm-level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data. Columns (10) and (11) reports estimates of the substitution elasticity between labor and intermediates based on Equation (14) using output elasticity coefficients (Column (10)) and cost share coefficients (Column (11)). Column (12) reports the contribution of changes in labor output elasticities (Column (7)) to changes in labor shares (Column (8)). Firm-level data on manufacturing from CompNet data providers. Firm-level data from CompNet data providers.

Table 8: Firm-level results for other countries: Coefficients on log output growth, substitution elasticities, and contirbution of labor output elasticity changes to labor share changes (all sectors, 4-year changes, OLS).

	$\ln(L_{it})$	$\ln(M_{it})$	$\ln(CS_{it}^{L})$	$\ln(CS_{it}^{M})$	$\ln(\frac{\theta_{it}^{L}}{RTS_{it}})$	$\ln(\frac{\theta_{it}^{M}}{RTS_{it}})$	$\ln(\theta_{it}^{L})$	$\ln(LS_{it})$	$\ln(\frac{P_{it}^{L}L_{it}}{VA_{it}})$	σ^{OE}	σ^{CS}	LS contrib. of θ_{it}^{L}
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
France (1995-21)	0.496***	1.027***	-0.088***	0.313****	-0.131***	0.196***	-0.122***	-0.373***	-0.350***	2.60	4.01	33%
Obs.: 282,245	(0.004)	(0.004)	(0.003)	(0.004)	(0.002)	(0.002)	(0.002)	(0.003)	(0.003)			
Hungary (2003-22)	0.464***	1.057***	-0.307***	0.159***	-0.330***	0.121***	-0.343***	-0.409***	-0.340***	4.18	4.67	84%
Obs.: 35,902	(0.007)	(0.007)	(0.008)	(0.006)	(0.007)	(0.003)	(0.007)	(0.008)	(0.010)			
Poland (2002-22)	0.535***	1.058***	-0.262***	0.134***	-0.281***	0.090***	-0.281***	-0.337***	-0.259***	3.44	4.12	83%
Obs.: 98,385	(0.004)	(0.003)	(0.004)	(0.003)	(0.004)	(0.001)	(0.004)	(0.004)	(0.004)			
Portugal (2004-22)	0.414***	1.115***	-0.347***	0.236***	-0.393***	0.198***	-0.403***	-0.469***	-0.352***	6.37	5.94	86%
Obs.: 33,478	(0.007)	(0.007)	(0.007)	(0.006)	(0.007)	(0.005)	(0.008)	(0.007)	(0.009)			
Slovakia (2000-23)	0.513***	1.019***	-0.266***	0.166***	-0.273***	0.102***	-0.264***	-0.412***	-0.381***	3.86	6.84	64%
Obs.: 9,605	(0.017)	(0.020)	(0.015)	(0.010)	(0.013)	(0.007)	(0.014)	(0.016)	(0.025)			
Slovenia (2002-23)	0.547***	1.091***	-0.285***	0.166***	-0.355***	0.135***	-0.351***	-0.360***	-0.236***	10.07	5.85	98%
Obs.: 7,452	(0.016)	(0.013)	(0.015)	(0.011)	(0.017)	(0.007)	(0.017)	(0.016)	(0.015)			
Estonia (2004-23)	0.493***	1.079***	-0.261***	0.142***	-0.348***	0.151***	-0.344***	-0.323***	-0.219***	6.74	3.20	107%
Obs.: 4,020	(0.023)	(0.016)	(0.021)	(0.011)	(0.022)	(0.011)	(0.022)	(0.024)	(0.026)			
Latvia (2005-21)	0.529***	0.859***	-0.172***	0.0081**	-0.145***	0.074***	-0.132***	-0.320***	-0.361***	2.97	2.20	41%
Obs.: 13,405	(0.012)	(0.013)	(0.015)	(0.003)	(0.010)	(0.007)	(0.010)	(0.012)	(0.015)			
Netherlands (2007-22)	0.497***	1.089***	-0.230***	0.130***	-0.257***	0.107***	-0.249***	-0.270***	-0.144***	2.60	2.55	92%
Obs.: 142,659	(0.005)	(0.004)	(0.005)	(0.003)	(0.004)	(0.002)	(0.004)	(0.005)	(0.004)			
Romania (2005-23)	0.523***	1.012***	-0.266***	0.096***	-0.153***	0.044***	-0.163***	-0.350***	-0.328***	1.67	3.86	50%
Obs.: 58,441	(0.005)	(0.003)	(0.005)	(0.002)	(0.004)	(0.002)	(0.005)	(0.005)	(0.008)			
Switzerland (2009-22)	0.548***	1.058***	-0.232***	0.172***	-0.215***	0.119***	-0.220***	-0.347***	-0.277***	2.90	4.81	63%
Obs.: 28,572	(0.011)	(0.012)	(0.01)	(0.01)	(0.009)	(0.005)	(0.009)	(0.010)	(0.011)			

Notes: The table reports OLS regressions from estimating the specification in Equation (10) for different countries by OLS (Columns (1)-(9)) using 4-year differences. The table reports coefficients on log output growth in Columns (1)-(9). The dependent variables in Columns (1)-(9) are the logs of labor and intermediate quantities, cost shares, and output elasticities divided by returns to scale, labor output elasticities, labor shares in sales, and labor shares in value added, respectively. All regressions are of changes in those dependent variables on changes in log output for 4-year differences. All regressions control for two-digit industry-year fixed effects. Standard errors are clustered at the firm-level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data. Columns (10) and (11) reports estimates of the substitution elasticity between labor and intermediates based on Equation (14) using output elasticity coefficients (Column (10)) and cost share coefficients (Column (11)). Column (12) reports the contribution of changes in labor output elasticities (Column (7)) to changes in labor shares (Column (8)). Firm-level data on all sectors from CompNet data providers. Firm-level data from CompNet data providers.

7.3 Industry-level Dynamics in 20 European Countries and the United States

We now study the industry-level analog of our firm-level analysis. We expect divergence in effect sizes given aggregation biases, intra-industry intermediate input patterns now being absorbed, and because market- vs. firm-specific labor supply elasticities may differ (see, e.g., Berger et al. (2022)).

Industry growth regressions. We transfer our firm-level growth analyses to the industry level. We regress changes in log input quantities, log cost shares, and log output elasticities on log industry output growth in the aggregated *country-industry* data provided by CompNet. The horizons are 1-, 4-, and 8-year changes (10-year changes are not feasible with CompNet data), controlling for country-year and industry-year fixed effects as we pool country-industry pairs (denoted by c and j, respectively). (Results are similar when including country-industry fixed effects, but we omit them to approximate the firm-level specification.)

Results. Table 9 and Figure 7 present industry-level results akin to the firm-level results in Table 2 and Figure 3. Our analysis shows that the relationships observed in firm-level data also hold at the European industry level: as industries expand, labor-intermediate ratios, labor cost shares, and labor output elasticities decrease, while intermediate cost shares and output elasticities rise. The point estimates are precisely estimated and indicate that 10% higher industry output growth leads to a 0.5% reduction in the labor output elasticity.[37] The binned scatter plots illustrate clear monotonic, close to linear relationships between output growth and the outcome variables.

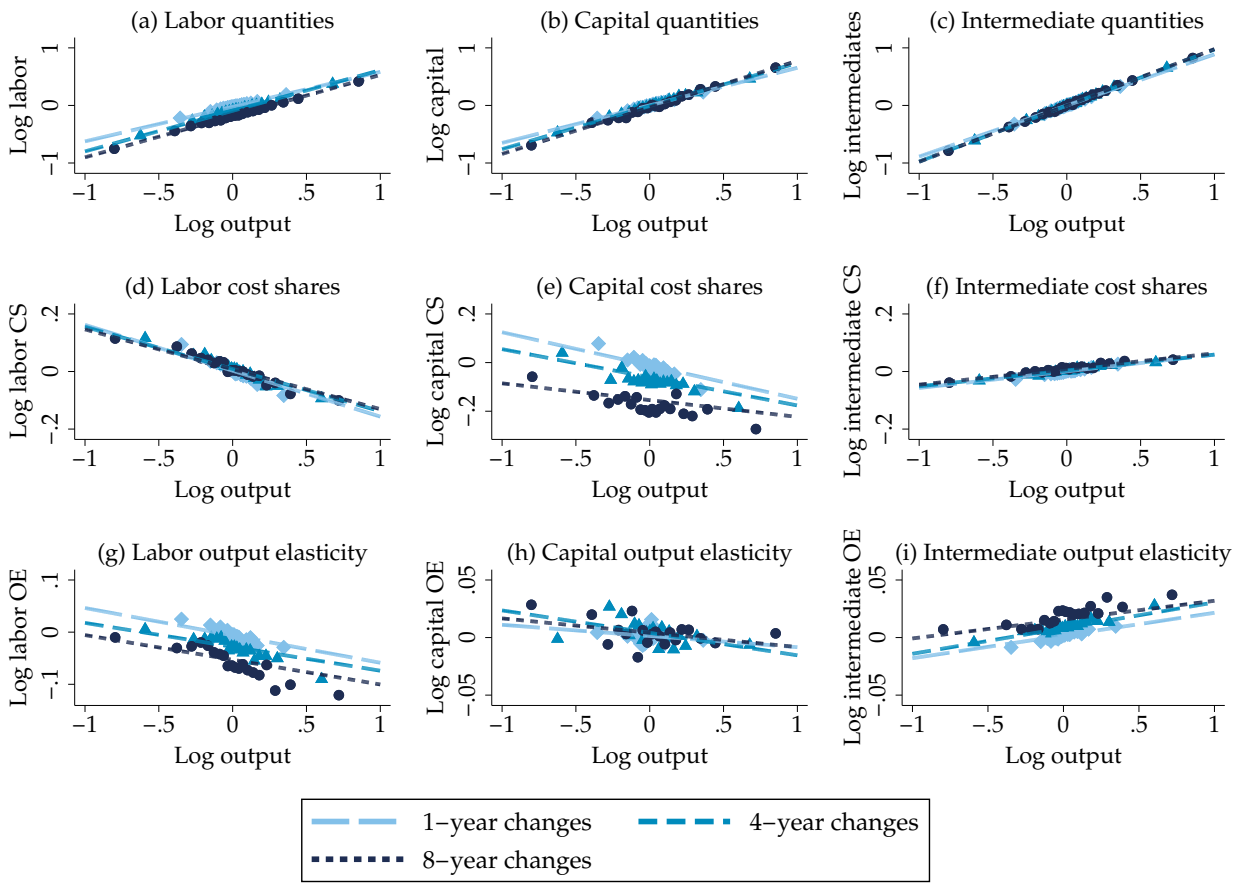
[37] Without taking logs, coefficients on industry-level labor, capital, and intermediate output elasticities divided by returns to scale are -0.0112, -0.00003, and 0.0148, respectively (4-year changes). Hence, the increase in intermediate input output elasticities is again largely rationalized by the decline in labor output elasticities.

Table 9: Industry-level adjustments in response to industry growth (Europe). OLS regressions.

	$\ln(L_{cjt})$	$\ln(K_{cjt})$	$\ln(M_{cjt})$	$\ln(CS^{L}_{cjt})$	$\ln(CS^{K}_{cjt})$	$\ln(CS^{M}_{cjt})$	$\ln(\frac{\theta^{L}_{cjt}}{RTS_{cjt}})$	$\ln(\frac{\theta^{K}_{cjt}}{RTS_{cjt}})$	$\ln(\frac{\theta^{M}_{cjt}}{RTS_{cjt}})$	$\ln(RTS_{cjt})$
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output growth	0.604***	0.652***	0.888***	-0.159***	-0.137***	0.058***	-0.053***	-0.001	0.020**	0.0016
	(0.038)	(0.048)	(0.036)	(0.020)	(0.028)	(0.012)	(0.013)	(0.016)	(0.008)	(0.0014)
Industry-year FE	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes
Country-year FE	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes
Observations	5,233	5,233	5,233	5,233	5,233	5,233	5,233	5,233	5,233	5,233
R^2	0.780	0.584	0.813	0.313	0.396	0.221	0.217	0.178	0.212	0.153
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output growth	0.705***	0.749***	0.974***	-0.147***	-0.116***	0.055***	-0.046***	-0.020	0.022**	0.0038
	(0.034)	(0.034)	(0.023)	(0.024)	(0.036)	(0.015)	(0.013)	(0.015)	(0.009)	(0.0023)
Industry-year FE	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes
Country-year FE	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes
Observations	4,207	4,207	4,207	4,207	4,207	4,207	4,207	4,207	4,207	4,207
R^2	0.880	0.746	0.907	0.399	0.545	0.279	0.252	0.229	0.243	0.164
Panel C: 8-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output growth	0.716***	0.809***	0.979***	-0.139***	-0.069*	0.054***	-0.047***	-0.012	0.016*	0.007**
	(0.026)	(0.043)	(0.016)	(0.022)	(0.038)	(0.001)	(0.016)	(0.019)	(0.008)	(0.004)
Industry-year FE	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes
Country-year FE	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes
Observations	2,904	2,904	2,904	2,904	2,904	2,904	2,904	2,904	2,904	2,904
R^2	0.911	0.815	0.946	0.451	0.581	0.322	0.298	0.278	0.241	0.183

Notes: The table reports OLS regressions from estimating industry versions of specification in Equation (10). The dependent variables in Columns (1)-(10) are the logs of country-industry-level total labor, capital, and intermediates, industry-level labor, capital, and intermediate cost shares, country-industry-level labor, capital, intermediate input output elasticities over returns to scale, and country-industry-level returns to scale, respectively. Panels A-C report on regressions of changes in those industry-level dependent variables on changes in industry-level log output for 1-, 4- , and 8-year differences. All regressions control for country-year and industry-year fixed effects and we pool country-industry pairs. Standard errors are clustered at the country-industry level. Significance: 0.01***, 0.05**, 0.1*. CompNet manufacturing data.

Figure 7: Industry-level adjustments in response to industry growth (OLS, binned scatter plots).



Notes: The figure reports binned scatter plots from estimating industry versions of the specification in Equation (10) with OLS for various differences. Panels (a)-(i) report on regressions of changes in the logs of country-industry-level labor, capital, and intermediate quantities, country-industry-level labor, capital, and intermediate cost shares, and country-industry-level labor, capital, intermediate output elasticities over returns to scale against country-industry-level log output changes, respectively. All panels control for country-year and industry-year fixed effects and we pool country-industry pairs. CompNet manufacturing data.

Country analysis and substitution elasticities. Table 10 Columns (1)-(3) present results for changes in industry-level labor-intermediate quantity, output elasticity, and cost share ratios in response to industry growth, based on country-specific *regression coefficients* from reproducing Table 9 by countries (4-year changes). As before, we also estimate substitution elasticities using Equation (14). Column (4) reports these substitution elasticity estimates based on the coefficients on output elasticities and input quantities (σ^{OE}) from our industry growth regressions. Column (5) uses cost share coefficients instead of output elasticity coefficients (σ^{CS}).

Results. In almost all countries, industry growth is associated with a shift from industry-level labor to intermediates in terms of quantities, cost shares, and output elasticities. Industry-level labor-intermediate substitution elasticities exceed unity in almost all countries. European-level values, derived from Table 9, and US values derived from running the same regressions with the NBER-CES Manufacturing Industry Database (1958-2016) are reported at the bottom of the table.[38] The

[38]For US cost shares, capital costs are approximated as 8% of the real capital stock as nominal values are not reported; labor and material expenditures are reported in nominal terms.

Table 10: Implied industry-level substitution elasticities and changes in input, output elasticity, and cost share ratios based on 4-year differences.

	$\Delta \ln\left(\frac{L_{cjt}}{M_{cjt}}\right)$	$\Delta \ln\left(\frac{\theta^L_{cjt}}{\theta^M_{cjt}}\right)$	$\Delta \ln\left(\frac{CS^L_{cjt}}{CS^M_{cjt}}\right)$	σ^{OE}	σ^{CS}
	(1)	(2)	(3)	(4)	(5)
Belgium (2000-2020)	-0.23	-0.10	-0.16	1.80	3.36
Croatia (2002-2021)	-0.70	-0.16	-0.64	1.31	12.71
Czech Republic (2005-2020)	-0.40	-0.01	-0.31	1.03	4.67
Denmark (2001-2020)	-0.13	-0.01	-0.09	1.12	3.13
Finland (1999-2020)	-0.09	-0.08	-0.13	5.72	-2.45
France (2004-2020)	-0.40	-0.05	-0.22	1.13	2.25
Germany (2001-2018)	-0.06	0.01	-0.05	0.85	6.59
Hungary (2003-2020)	-0.51	-0.09	-0.38	1.21	4.03
Italy (2006-2020)	-0.18	-0.09	-0.12	2.11	2.89
Latvia (2007-2019)	-0.43	-0.11	-0.25	1.33	2.46
Lithuania (2000-2020)	-0.15	-0.08	-0.11	2.04	3.59
Netherlands (2007-2019)	-0.11	0.03	-0.06	0.81	2.51
Poland (2002-2020)	-0.13	-0.01	-0.09	1.09	2.98
Portugal (2010-2020)	-0.68	0.02	-0.48	0.98	3.42
Romania (2005-2020)	-0.22	-0.03	-0.13	1.17	2.58
Slovakia (2000-2020)	-0.18	-0.09	-0.16	1.98	12.73
Slovenia (2002-2021)	-0.09	0.01	-0.13	0.92	-2.49
Spain (2008-2020)	-0.44	-0.23	-0.16	2.11	1.60
Sweden (2003-2020)	-0.03	-0.04	0.02	-5.77	0.64
Switzerland (2009-2020)	-0.31	-0.19	-0.24	2.63	4.58
Europe (1999-2021, manufac.)	-0.27	-0.07	-0.20	1.34	4.01
USA (1958-2016, manufac.)	-0.31		-0.23		3.88
Europe (1999-2021, non-manufac.)	-0.23	-0.08	-0.15	1.48	2.72

Notes: The table reports implied changes in labor-intermediate input ratios (Column (1)), output elasticity ratios (Column (2)), cost shares (Column (3)), and substitution elasticities based on output elasticity coefficients (Column (4)) and cost share coefficients (Column (5)) as estimated from country-specific versions of the country-industry-level regressions reported in Table 9 (4-year changes). All results are based on manufacturing industries, expect for the last row, which uses non-manufacturing industries. Industries are 2-digit NACE rev. 2 industries for Europe and 6-digit NAICS industries for the US. CompNet data and NBER-CES Manufacturing Industry Database.

US data lack output elasticity estimates, but changes in labor and intermediate quantities and cost shares are similar in Europe and the US. Also, substitution elasticities based on cost shares are similar (approximately 4 in both regions). Substitution elasticity estimates based on direct output elasticity estimates are 1.34 in Europe (manufacturing).

Manufacturing vs. non-manufacturing sectors. The last row extends the CompNet data analysis to non-manufacturing country-industry pairs as a robustness test. Substitution elasticities exceed unity and are comparable between manufacturing and non-manufacturing industries, indicating that our findings are consistent across different sectors of the economy. This generalization echoes the robustness check using non-manufacturing firms in firm-level micro data across several CompNet countries reported in Section 7.2.

Table 11: Industry labor share, market imperfection, and output elasticity changes in response to industry growth (Europe). OLS regressions.

	$\ln(LS_{cjt})$	$\ln(\mu_{cjt})$	$\ln(\gamma_{cjt})$	$\ln(\frac{\theta^L_{cjt}}{RTS_{cjt}})$	$\ln(\theta^L_{cjt})$	$\ln(\frac{P^L_{cjt}L_{cjt}}{VA_{cjt}})$
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)
Log output growth	-0.319***	-0.031***	0.004	-0.053***	-0.051***	-0.162***
	(0.041)	(0.009)	(0.013)	(0.013)	(0.014)	(0.029)
Observations	5,233	5,233	5,233	5,233	5,233	5,233
R^2	0.544	0.257	0.332	0.217	0.214	0.330
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)
Log output growth	-0.216***	-0.032***	0.006	-0.046***	-0.042***	-0.116***
	(0.029)	(0.007)	(0.015)	(0.013)	(0.014)	(0.023)
Observations	4,207	4,207	4,207	4,207	4,207	4,207
R^2	0.547	0.341	0.438	0.252	0.243	0.383
Panel C: 8-year diff.	(1)	(2)	(3)	(4)	(5)	(6)
Log output growth	-0.194***	-0.031***	0.026	-0.047***	-0.040**	-0.128***
	(0.024)	(0.009)	(0.021)	(0.0048)	(0.017)	(0.021)
Observations	2,904	2,904	2,904	2,904	2,904	2,904
R^2	0.516	0.400	0.412	0.298	0.282	0.406

Notes: The table reports OLS regressions estimating industry versions of the specification in Equation (10). The dependent variables in Columns (1)-(6) are the logs of country-industry-level labor shares in sales, markups, wage markdowns, labor output elasticities divided by returns to scale, labor output elasticities, and labor shares in value added, respectively. Panels A-C report on regressions of changes in those dependent variables on changes in country-industry-level log output for 1-, 4- , and 8-year differences. All regressions control for country-year and industry-year fixed effects. Standard errors are clustered at the country-industry-level. Significance: 0.01***, 0.05**, 0.1*. CompNet manufacturing data.

7.4 Industry Labor Share Implications

As with the firm-level analysis, we now study the industry labor share consequences of the decline of industry labor output elasticities with industry growth. We reiterate that our main focus is on firm-level growth, and drawing aggregate implications would necessitate input-output network analysis or an open economy perspective.

Labor shares and their drivers. Table 11 and Figure 8 provide European country-industry counterparts to the firm-level analyses shown in Table 2 and Figure 3. As before, we regress country-industry log changes in outcome variables on output growth, controlling for country-year and industry-year fixed effects. Due to aggregation bias, we do not expect that coefficients on output elasticities, markups, and wage markdowns sum to the coefficient on labor shares as they did in our firm-level analysis.[39]

Results. The results qualitatively align with our firm-level findings: as industries grow, industry labor shares decline, primarily due to a reduction in labor output elasticities. However, markups also *decline*, while changes in wage markdowns are statistically insignificant, which is both incompatible with a decline in labor shares. Therefore, increases in markups or wage markdowns cannot explain the negative relationship between labor shares and growth at the industry level. Instead, the industry analysis points to a substantial role for labor output elasticities. Quantitatively, the slope of the aggre-

[39] Aggregation biases may arise from compositional effects, weighting choices, Jensen's inequality, intra-industry trade, and shifts in labor demand and supply.

Figure 8: Labor share, market imperfection, and output elasticity changes in response to industry growth (OLS, binned scatter plots).



Notes: The figure reports binned scatter plots estimating industry versions of the specification in Equation (10) with OLS for various differences. The graphs relate changes in country-industry-level logs of labor shares in sales and value added, labor output elasticities (not divided by returns to scale), markups, and wage markdowns against country-industry-level log output changes. All regressions control for country-year and industry-year fixed effects. CompNet data.

gate output elasticity is about a quarter of the slope of the labor share in sales (4-year specification).[40] Notably, as with the firm-level analysis, the value-added labor share also declines, highlighting that the rise in intermediates mainly reflects a shift away from labor.[41]

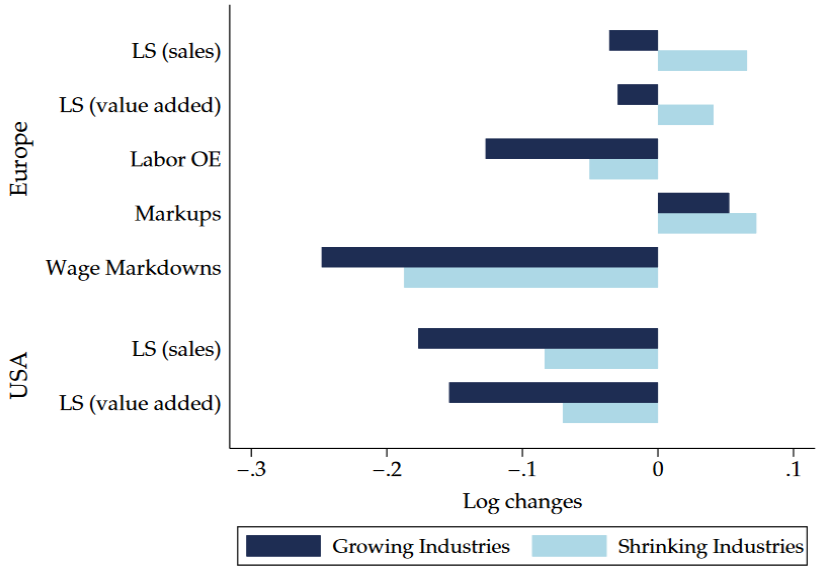
Simple check: industry splits. We close our paper with one more check at the industry level. We split our manufacturing data into growing and shrinking industries and study how labor shares, output elasticities, markups, and wage markdowns have changed between countries' first and last years in the CompNet data and for the US (NBER-CES data, where we focus on the period 1998–2016 to approximate the CompNet years and study only labor shares due to the lack of output elasticities).[42]

[40] In addition to classical aggregation biases, note that under competitive markets, the industry labor share equals the industry output elasticity of labor: $\sum_n s_{it} LS_{it} = \sum_n s_{it} \theta^L_{it}$, where s_{it} is the sales-weight. Under imperfect markets, the pass-through from changes in industry output elasticities of labor to changes in industry labor shares are additionally shaped by the distribution of firms' markups and wage markdowns. Nevertheless, qualitatively, it remains true that the partial effect of a change in the aggregate labor-output elasticity on the labor share, holding fixed market imperfections, is positive. Equally, the partial effect (ceteris paribus) of changes in industry markups and wage markdowns on industry labor shares is negative. Importantly, "holding fixed" refers here not only to the industry level, but also to the firm distribution.

[41] This observation aligns with previous research that identifies the offshoring of labor-intensive tasks as a major factor contributing to the declining labor share (e.g., Elsby et al., 2013).

[42] As yearly coverage varies between countries in CompNet, we first calculate log changes in each of these variables for each country-industry pair between the first and last year in the data, i.e., we pool differences for country-industry pairs of different lengths (see Table 10 for the yearly country coverage). Subsequently, we take averages of these changes across all country-industry-pairs.

Figure 9: Changes in labor shares, output elasticities, markups, and wage markdowns in growing and shrinking industries



Notes: The figure report average changes in the logs of labor shares, labor output elasticities, markups, and wage markdowns across country-industry pairs for growing and shrinking industries. Changes are first computed for each pair and then average across all pairs. Time spans differ across country-industry pairs and not all industries are available for all years within a country. European results are reported in the top five bars and based on CompNet data (1999-2021) and NACE Rev. 2 two-digit industries. US results are reported in the bottom two bars and based on the NBER-CES Manufacturing Industry Database (1998-2016) and NAICS 6-digit industries. There are 154 shrinking and 196 growing country-industry pairs in the CompNet data. For the US data, we observe 209 shrinking and 155 growing industries.

Figure 9 reports the result.

Examining the European results, a first striking result is that labor shares decreased within expanding industries, but remained stable in contracting industries. Expanding industries also experienced a much stronger decline in labor output elasticities. Markups and wage markdowns exhibit trends that should, in theory, *increase* labor shares in growing industries relative to shrinking ones—specifically, markups increase less and wage markdowns decrease more in expanding industries. Through the lens of our framework, changes in labor output elasticities are therefore the only factor that can account for the observation that labor shares declined only in growing but not in shrinking industries.

In the US, we see a marked decline in labor shares, particularly within expanding industries—consistent with the output elasticity channel (but we lack direct output elasticity measures). Additionally, the decline in labor shares within shrinking US industries may point to market power playing a more substantial role in the US (see De Loecker et al. (2020) and Autor et al. (2020) for that channel).

8 Conclusion

We have documented and dissected a new fact about firm growth: the declining importance of labor and the increasing importance of intermediate inputs in firms' production. As labor and intermediates function as substitutes, this shift in the input mix reduces (increases) the output elasticity of labor (intermediates). As a result, the labor (intermediates) share falls (increases) in growing firms. This

shift from labor to intermediates explains between one-third (10-year horizon) and one-half (1-/4-year horizon) of the decline in labor shares in growing firms and accounts for most of the decline in labor shares within growing industries. We establish these patterns using OLS and IV regression in rich German firm-level micro data, administrative firm-level data from 11 additional countries, as well as in micro-aggregated industry data for 20 European countries. We rationalize the facts in a production function framework characterized by (i) an elasticity of substitution between intermediates and labor that exceeds one, and (ii) an increasing shadow price of labor (e.g., due to monopsony or adjustment costs).

The findings also have broader implications. For instance, many current estimates of factor misallocation and productivity rely on Cobb-Douglas production functions, which assume constant output elasticities. Moreover, our results imply that, generally, any shocks that affect output growth will alter output elasticities, input mixes and cost shares, with large magnitudes for horizons of up to 10 years. Ignoring this regularity may confound analyses of firm-level effects from productivity shocks, trade, competition, or subsidies on a wide range of related outcomes in the short and medium term.

Our findings also show that not only the steady state distribution of firm size may be distorted by monopsony, but also firm growth. We also show how firms respond to those constraints on growth: with input substitution toward intermediates, i.e., outsourcing production, for which we estimate a high elasticity of substitution with labor. In our case, these intermediate inputs are supplied by other firms and are largely materials (rather than, e.g., services or temporary agency work). It thus appears that monopsony lengthens the supply chain as firms, or, more broadly, industries and the aggregate economy, grow.

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Appendix of:
From Labor to Intermediates:
Firm Growth, Input Substitution, and Monopsony

Matthias Mertens and Benjamin Schoefer

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A Appendix Tables and Figures

Table A.1: Summary statistics of the German manufacturing sample

	Mean (1)	**p25** (2)	**Median** (3)	**p75** (4)	**St.Dev.** (5)	**Obs.** (6)
Number of employees	366.63	53	109	272	2559.55	183,813
Real wage (1995 values)	34,162	25,972	33,823	41,485	11,492	183,813
Labor share (sales)	0.30	0.21	0.29	0.38	0.12	183,813
Labor share (value added)	0.80	0.63	0.76	0.88	2.89	183,813
Labor cost share	0.31	0.22	0.30	0.39	0.13	183,813
Capital cost share	0.06	0.03	0.05	0.07	0.04	183,813
Intermediates cost share	0.63	0.54	0.64	0.73	0.14	183,813
Materials, energy, ext. components cost share	0.41	0.29	0.41	0.53	0.17	183,813
Merchandise cost share	0.05	0.00	0.00	0.05	0.10	183,813
Subcontracted work by other companies cost share	0.03	0.00	0.00	0.03	0.06	183,813
Repairs, maintenance, installations cost share	0.02	0.01	0.03	0.04	0.02	183,813
Rents, leases, leasing cost share	0.03	0.01	0.02	0.04	0.03	183,813
Temporary agency worker cost share	0.01	0.00	0.00	0.01	0.03	164,410
Other intermediates cost share	0.10	0.05	0.08	0.12	0.06	183,813
Markup	1.09	0.97	1.05	1.17	0.20	183,813
Wage markdown	1.08	0.71	0.96	1.32	0.55	183,813
Output elasticity of labor	0.31	0.23	0.31	0.38	0.11	183,813
Output elasticity of capital	0.12	0.08	0.11	0.15	0.06	183,813
Output elasticity of intermediates	0.64	0.57	0.64	0.71	0.10	183,813
Returns to scale	1.06	0.98	1.05	1.13	0.12	183,813

Notes: This table presents summary statistics for selected variables from the German manufacturing sector firm-level data. Columns (1)-(5) show the mean, 25th percentile, median, 75th, and standard deviation, respectively. Column (6) reports the number of non-missing observations. German micro-data.

Table A.2: Firm-level adjustments in cost share and output elasticities to firm growth. Different specification: not logged, not dividing by RTS. OLS regressions.

	$\frac{w_{it}L_{it}}{C_{it}}$	$\frac{r_{it}K_{it}}{C_{it}}$	$\frac{z_{it}M_{it}}{C_{it}}$	$\frac{\theta^L_{it}}{RTS_{it}}$	$\frac{\theta^K_{it}}{RTS_{it}}$	$\frac{\theta^M_{it}}{RTS_{it}}$	$\ln(\theta^L_{it})$	$\ln(\theta^K_{it})$	$\ln(\theta^M_{it})$	θ^L_{it}	θ^K_{it}	θ^M_{it}	RTS_{it}
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Log output growth	-0.103***	-0.0380***	0.141***	-0.0710***	-0.0201***	0.0911***	-0.298***	-0.309***	0.157***	-0.0723***	-0.0203***	0.0991***	0.00647***
	(0.0011)	(0.0003)	(0.0011)	(0.0007)	(0.0003)	(0.0009)	(0.0046)	(0.0064)	(0.00129)	(0.0008)	(0.0004)	(0.0007)	(0.0004)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950
R^2	0.303	0.419	0.413	0.266	0.191	0.277	0.215	0.132	0.397	0.278	0.183	0.425	0.157
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Log output growth	-0.0692***	-0.0366***	0.106***	-0.0438***	-0.0136***	0.0574***	-0.185***	-0.164***	0.112***	-0.0420***	-0.0124***	0.0729***	0.0185***
	(0.00157)	(0.0006)	(0.00179)	(0.00108)	(0.000525)	(0.0014)	(0.0063)	(0.0077)	(0.0021)	(0.0014)	(0.0007)	(0.0013)	(0.0008)
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492
R^2	0.303	0.419	0.413	0.266	0.191	0.277	0.201	0.160	0.369	0.220	0.154	0.388	0.157
Panel C: 10-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Log output growth	-0.0535***	-0.0275***	0.0810***	-0.0305***	-0.0051***	0.0356***	-0.112***	-0.0441***	0.0860***	-0.0236***	-0.0018***	0.0559***	0.0304***
	(0.0016)	(0.0006)	(0.0018)	(0.0011)	(0.0005)	(0.0014)	(0.00569)	(0.0073)	(0.0021)	(0.00137)	(0.0007)	(0.0013)	(0.0008)
Observations	49,915	49,915	49,915	49,915	49,915	49,915	49,915	49,915	49,915	49,915	49,915	49,915	49,915
N of firms	10,595	10,595	10,595	10,595	10,595	10,595	10,595	10,595	10,595	10,595	10,595	10,595	10,595
R^2	0.259	0.318	0.347	0.222	0.155	0.201	0.161	0.141	0.322	0.174	0.142	0.334	0.262

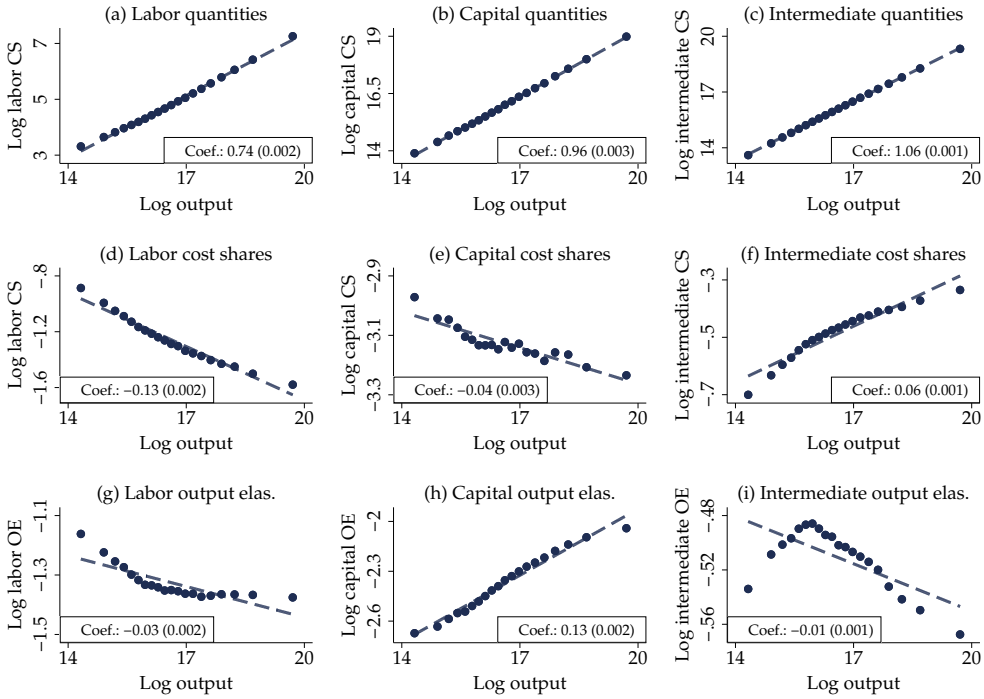
Notes: The table reports OLS regressions from estimating the specification in Equation (10). The dependent variables in Columns (1)-(13) are non-logged labor, capital, and intermediate input cost shares, and output elasticities divided by returns to scale, and logged and non-logged labor, capital, and intermediate input output elasticities no divided by returns to scale, respectively. Panel A-C report on regressions of changes in those dependent variables on changes in log output for 1-, 4- , and 10-year differences. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm-level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Table A.3: Firm-level adjustments in cost share and output elasticities to firm growth. Different specification: not logged, not dividing by RTS. IV regressions.

	1st stage	$\frac{w_{it}L_{it}}{C_{it}}$	$\frac{r_{it}K_{it}}{C_{it}}$	$\frac{z_{it}M_{it}}{C_{it}}$	$\frac{\theta^L_{it}}{RTS_{it}}$	$\frac{\theta^K_{it}}{RTS_{it}}$	$\frac{\theta^M_{it}}{RTS_{it}}$	$\ln(\theta^L_{it})$	$\ln(\theta^K_{it})$	$\ln(\theta^M_{it})$	θ^L_{it}	θ^K_{it}	θ^M_{it}	RTS_{it}
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Export demand shock	0.0451***													
	(0.0045)													
Log output growth		-0.0933***	-0.0283***	0.122***	-0.0626***	-0.0133***	0.0759***	-0.318***	-0.220**	0.127***	-0.0672***	-0.0135***	0.0914***	0.0107*
		(0.0144)	(0.0042)	(0.0153)	(0.0098)	(0.0039)	(0.0116)	(0.0645)	(0.0995)	(0.0170)	(0.0113)	(0.00455)	(0.0102)	(0.00575)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950
First-stage F-Statistic		102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6
R^2	0.205	0.294	0.427	0.400	0.309	0.208	0.329	0.215	0.127	0.387	0.277	0.171	0.424	0.049
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Export demand shock	0.0635***													
	(0.0091)													
Log output growth		-0.0555***	-0.0280***	0.0836***	-0.0465***	-0.0070	0.0536***	-0.220***	-0.125	0.105***	-0.0445***	-0.0005	0.0739***	0.0289***
		(0.0177)	(0.0062)	(0.0201)	(0.0127)	(0.0058)	(0.0157)	(0.0742)	(0.0973)	(0.0227)	(0.0153)	(0.0076)	(0.0143)	(0.00876)
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492
First-stage F-Statistic		48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57
R^2	0.297	0.403	0.402	0.266	0.175	0.276	0.183	0.199	0.158	0.368	0.220	0.121	0.388	0.139

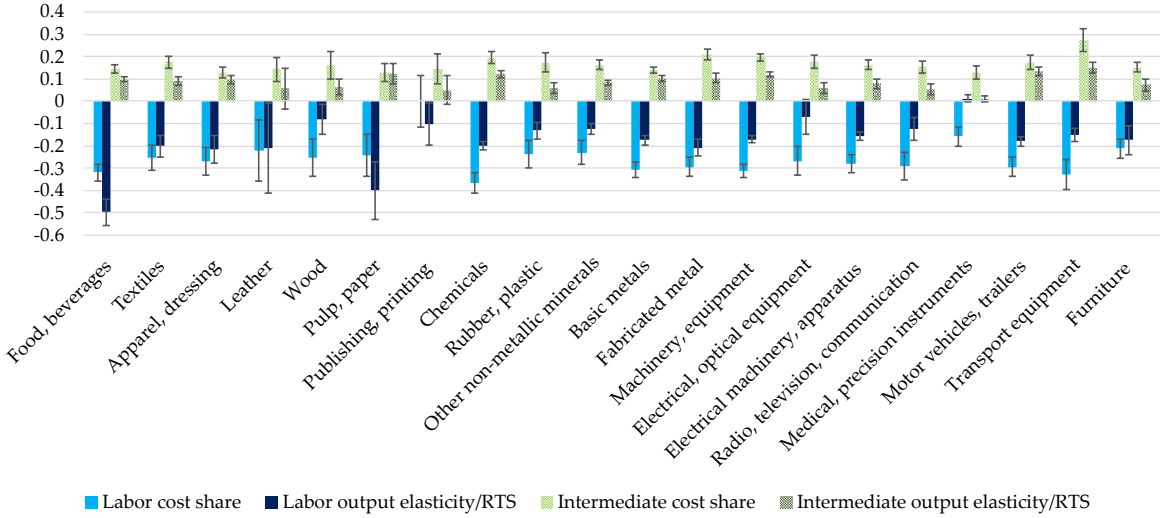
Notes: The table reports IV regressions from estimating the specification in Equation (10) using foreign demand shocks as instruments (Equation (12)). Column (1) reports the first-stage regression results. The dependent variables in Columns (2)-(14) are non-logged labor, capital, and intermediate input cost shares, and output elasticities divided by returns to scale, and logged and non-logged labor, capital, and intermediate input output elasticities no divided by returns to scale, respectively. Panel A-C report on regressions of changes in those dependent variables on changes in log output for 1-, 4- , and 10-year differences. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm-level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Figure A.1: The cross-sectional relationships: output elasticities and cost shares



Notes: The figure reports binned scatter plots estimating the specification in Equation (10) in levels with OLS. Panels (a)-(i) report on regressions of the logs of labor, capital, and intermediate quantities, labor, capital, intermediate cost shares, and labor, capital, and intermediate output elasticities over returns to scale on log output, respectively. All panels report results that are residualized by industry-year fixed effects. German firm-level data.

Figure A.2: Firm-level adjustments in labor and intermediate input output elasticities and cost shares separately by industry, 4-year changes, OLS



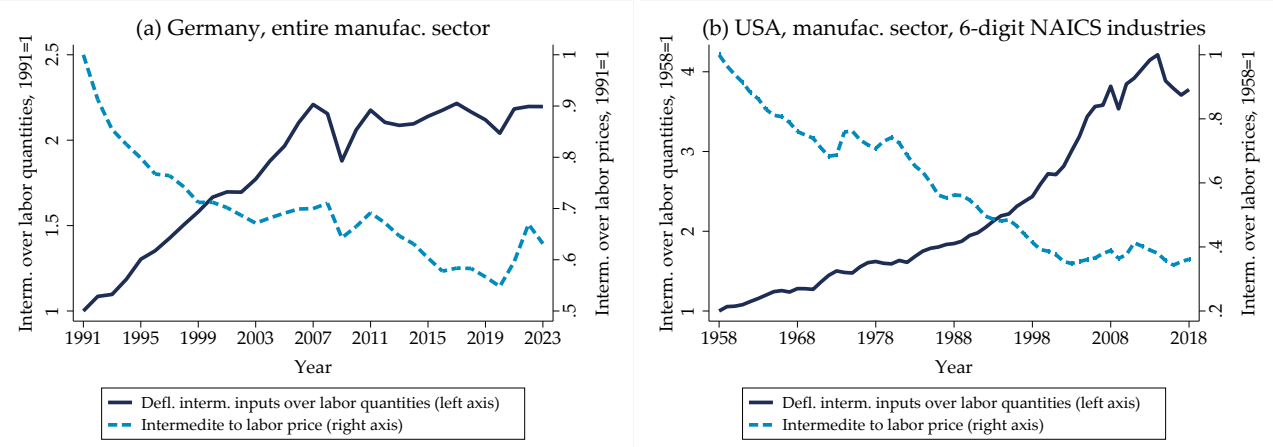
Notes: The figure reports OLS regressions from estimating Equation (10) in 4-year changes by two-digit NACE manufacturing industries. The dependent variables are logged labor and intermediate cost shares and logged labor and intermediate output elasticities divided by returns to scale. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm-level. The error bars indicate 95% confidence intervals. Results for 1- and 10-year changes are similar. German firm-level data.

Table A.4: Firm-level adjustments in intermediate cost shares by intermediate type (OLS).

	$\ln(E_{it}^{CS})$	$\ln(Merch_{it}^{CS})$	$\ln(Sub_{it}^{CS})$	$\ln(Rep_{it}^{CS})$	$\ln(Rent_{it}^{CS})$	$\ln(Temp_{it}^{CS})$	$\ln(Other_{it}^{CS})$
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output growth	0.297***	0.408***	0.388***	-0.104***	-0.417***	1.932***	-0.107***
	(0.001)	(0.029)	(0.022)	(0.011)	(0.011)	(0.0323)	(0.008)
Observations	183,807	83,975	96,062	176,845	175,391	86,927	183,813
N of firms	29,950	15,277	19,842	29,598	29,340	18,048	29,950
R^2	0.108	0.069	0.061	0.029	0.055	0.205	0.030
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output growth	0.187***	0.429***	0.218***	-0.113***	-0.281***	1.427***	-0.054***
	(0.001)	(0.044)	(0.035)	(0.014)	(0.018)	(0.0419)	(0.011)
Observations	70,933	36,495	35,962	68,859	67,827	32,512	70,936
N of firms	11,492	6,096	6,737	11,366	11,149	5,640	11,492
R^2	0.128	0.117	0.115	0.072	0.079	0.232	0.065
Panel C: 10-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output growth	0.137***	0.217***	0.148***	-0.072***	-0.218***	0.434***	0.011
	(0.008)	(0.051)	(0.038)	(0.015)	(0.021)	(0.0416)	(0.012)
Observations	48,950	21,102	22,376	46,996	46,143	19,064	48,953
N of firms	10,381	4,542	5,743	10,189	9,996	5,647	10,381
R^2	0.130	0.126	0.120	0.074	0.080	0.124	0.068

Notes: The table reports on OLS regressions from estimating the specification in Equation (10) for 1-year (Panel A), 4-year (Panel B), and 10-year differences. The dependent variables in Columns (1)-(7) are the logged values of raw materials, energy, and external components cost shares, merchandise cost shares, subcontracted works performed by other companies cost shares, repairs, maintenance, and installation cost shares, rents, leases, and leasing cost shares, temporary agency worker cost shares, and other intermediate inputs cost shares, respectively. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm-level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Figure A.3: Changes in aggregate intermediate to labor prices and quantities



Notes: The figures report changes in prices for labor (wages) and intermediate inputs (price index) and the ratio of deflated intermediate inputs to labor quantities for Germany (Panel (a)) and the US (Panel (b)). Values are normalized to unity in the first year. German data refers to the aggregate manufacturing sector. US data refers to averages across 6-digit NAICS industries. Data from the Federal Statistical Office of Germany and the NBER-CES Manufacturing Industry Database.

Table A.5: Firm-level adjustments in output elasticities in response to firm growth: effects by size quintiles (4-year changes).

	$\ln(L_{it})$	$\ln(K_{it})$	$\ln(M_{it})$	$\ln(CS^L_{it})$	$\ln(CS^K_{it})$	$\ln(CS^M_{it})$	$\ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\ln(\frac{\theta^K_{it}}{RTS_{it}})$	$\ln(\frac{\theta^M_{it}}{RTS_{it}})$	$\ln(\theta^{RTS}_{it})$
Panel A: 1st size quintile	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output growth	0.505***	0.259***	1.051***	-0.248***	-0.594***	0.183***	-0.179***	-0.183***	0.095***	0.024***
	(0.011)	(0.016)	(0.009)	(0.0111)	(0.0172)	(0.0072)	(0.010)	(0.017)	(0.005)	(0.001)
Observations	14,984	14,984	14,984	14,984	14,984	14,984	14,984	14,984	14,984	14,984
N of firms	4,408	4,408	4,408	4,408	4,408	4,408	4,408	4,408	4,408	4,408
R^2	0.558	0.290	0.862	0.426	0.492	0.421	0.337	0.255	0.351	0.357
Panel B: 2nd size quintile	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output growth	0.484***	0.217***	1.034***	-0.280***	-0.625***	0.174***	-0.206***	-0.175***	0.098***	0.018***
	(0.012)	(0.015)	(0.009)	(0.0106)	(0.0153)	(0.0062)	(0.011)	(0.014)	(0.004)	(0.001)
Observations	13,276	13,276	13,276	13,276	13,276	13,276	13,276	13,276	13,276	13,276
N of firms	4,508	4,508	4,508	4,508	4,508	4,508	4,508	4,508	4,508	4,508
R^2	0.540	0.268	0.872	0.456	0.505	0.457	0.383	0.295	0.384	0.325
Panel C: 3rd size quintile	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output growth	0.492***	0.208***	1.032***	-0.299***	-0.630***	0.169***	-0.200***	-0.164***	0.093***	0.017***
	(0.012)	(0.013)	(0.010)	(0.0131)	(0.0134)	(0.0071)	(0.011)	(0.013)	(0.005)	(0.001)
Observations	13,211	13,211	13,211	13,211	13,211	13,211	13,211	13,211	13,211	13,211
N of firms	4,265	4,265	4,265	4,265	4,265	4,265	4,265	4,265	4,265	4,265
R^2	0.546	0.295	0.877	0.461	0.559	0.463	0.353	0.355	0.394	0.312
Panel D: 4th size quintile	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output growth	0.528***	0.178***	1.047***	-0.295***	-0.687***	0.160***	-0.202***	-0.169***	0.094***	0.014***
	(0.015)	(0.012)	(0.009)	(0.0138)	(0.0129)	(0.0063)	(0.013)	(0.010)	(0.005)	(0.001)
Observations	13,276	13,276	13,276	13,276	13,276	13,276	13,276	13,276	13,276	13,276
N of firms	3,532	3,532	3,532	3,532	3,532	3,532	3,532	3,532	3,532	3,532
R^2	0.576	0.287	0.905	0.484	0.620	0.503	0.384	0.417	0.427	0.288
Panel E: 5th size quintile	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output growth	0.529***	0.150***	1.055***	-0.307***	-0.722***	0.162***	-0.196***	-0.206***	0.099***	0.008***
	(0.016)	(0.012)	(0.012)	(0.0165)	(0.0138)	(0.0085)	(0.015)	(0.021)	(0.006)	(0.002)
Observations	11,648	11,648	11,648	11,648	11,648	11,648	11,648	11,648	11,648	11,648
N of firms	2,115	2,115	2,115	2,115	2,115	2,115	2,115	2,115	2,115	2,115
R^2	0.596	0.292	0.910	0.501	0.686	0.537	0.363	0.359	0.456	0.310

Notes: The table reports on OLS regressions from estimating the specification in Equation (10) for 4-year differences. The dependent variables in Columns (1)-(10) are the logged values of labor, capital, and intermediates quantities, labor, capital, and intermediate cost shares, and labor capital and intermediate input output elasticities divided by returns to scale, respectively. Panels A-E report results for the 1st, 2nd, 3rd , 4th, and 5th quintile of the firm-level output distribution (computed within year and industry). All regressions control for industry-year fixed effects. Standard errors are clustered at the firm-level. Size is measured by sales. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Table A.6: Effect of firm growth on average wages. German micro data. (OLS)

	OLS	IV
	Real wage	Real wage
Panel A: 1-year changes	(1)	(2)
Log output growth	0.095***	-0.024
	(0.003)	(0.0562)
Observations	183,813	183,813
N of firms	29,950	29,950
First-stage F-Statistic		102.6
R^2	0.062	0.035
Panel B: 4-year changes	(1)	(2)
Log output growth	0.062***	0.031
	(0.003)	(0.046)
Observations	70,936	70,936
N of firms	29,950	29,950
First-stage F-Statistic		48.57
R^2	0.126	0.122
Panel C: 10-year changes	(1)	
Log output growth	0.068***	
	(0.003)	
Observations	48,953	
N of firms	10,381	
R^2	0.156	

Notes: The table reports OLS and IV regressions from estimating the specification in Equation (10) for 1-, 4- and 10-year differences. The dependent variable is the logged wage computed as the wage bill divided by the number of employees. Panels A-C report results for 1-, 4- and 10-year differences, respectively All regressions control for industry-year fixed effects. Standard errors are clustered at the firm-level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Table A.7: Implied substitution elasticities, changes in input cost ratios, and firm-specific labor supply elasticities: based on cost shares as output elasticity measures

	OLS			IV		
	σ	$\Delta \ln \left(\frac{w_{it}\gamma_{it}^{L}}{z_{it}\gamma_{it}^{M}} \right)$	ϵ_{it}^{L}	σ	$\Delta \ln \left(\frac{w_{it}\gamma_{it}^{L}}{z_{it}\gamma_{it}^{M}} \right)$	ϵ_{it}^{L}
	(1)	(2)	(3)	(4)	(5)	(6)
1-year diff.	6.434	0.113	2.64	2.28 - inf.	0.00 - 0.35	0.62 - inf.
4-year diff.	6.859	0.078	6.54	12.559	0.034	15.765
10-year diff.	5.118	0.085	7.13			

Notes: The table reports substitution elasticities (Columns (1) and (4)), changes in effective factor price ratios (Columns (2) and (5)), and implied labor supply elasticities assuming perfectly elastic intermediate input supply (Columns (3) and (6)) based on our OLS (Columns (1)-(3)) and IV (Columns (4)-(6)) regressions from Tables 2 and 3 of logged output elasticities over returns to scale and logged input factors on logged output in differences. As IV point estimates suggest a substitution elasticity of infinity for 1-year changes, we report intervals using 95% confidence intervals from all point estimates entering the computation for this specification. This yields much larger intervals than directly computing confidence intervals for substitution elasticites and other values.

Table A.8: Testing the role of non-homotheticity in the production function, OLS results

	$\ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\ln(\theta^L_{it})$	$\ln(\theta^L_{it})$	$\ln(\theta^L_{it})$	$\ln(\frac{\theta^L_{it}}{\theta^M_{it}})$	$\ln(\frac{\theta^L_{it}}{\theta^M_{it}})$	$\ln(\frac{\theta^L_{it}}{\theta^M_{it}})$
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Log output growth	-0.304***		0.006*	-0.298***		0.0338***	-0.455***		0.0206***
	(0.004)		(0.003)	(0.005)		(0.004)	(0.00549)		(0.00390)
Log labor-interm. input ratio		0.424***	0.427***		0.440***	0.456***		0.644***	0.655***
		(0.004)	(0.004)		(0.004)	(0.004)		(0.00373)	(0.00396)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950
R^2	0.230	0.466	0.466	0.215	0.471	0.472	0.276	0.618	0.618
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Log output growth	-0.202***		0.027***	-0.185***		0.061***	-0.297***		0.0512***
	(0.006)		(0.005)	(0.006)		(0.005)	(0.00785)		(0.00555)
Log labor-interm. input ratio		0.412***	0.427***		0.427***	0.461***		0.622***	0.651***
		(0.006)	(0.007)		(0.006)	(0.007)		(0.00656)	(0.00726)
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492
R^2	0.220	0.462	0.463	0.201	0.463	0.468	0.245	0.595	0.597
Panel C: 10-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Log output growth	-0.139***		0.048***	-0.111***		0.093***	-0.197***		0.0911***
	(0.005)		(0.004)	(0.006)		(0.005)	(0.00742)		(0.00522)
Log labor-interm. input ratio		0.402***	0.430***		0.415***	0.470***		0.610***	0.664***
		(0.006)	(0.007)		(0.006)	(0.007)		(0.00651)	(0.00729)
Observations	48,953	48,953	48,953	48,953	48,953	48,953	48,953	48,953	48,953
N of firms	10,381	10,381	10,381	10,381	10,381	10,381	10,381	10,381	10,381
R^2	0.189	0.492	0.497	0.163	0.485	0.503	0.199	0.612	0.623

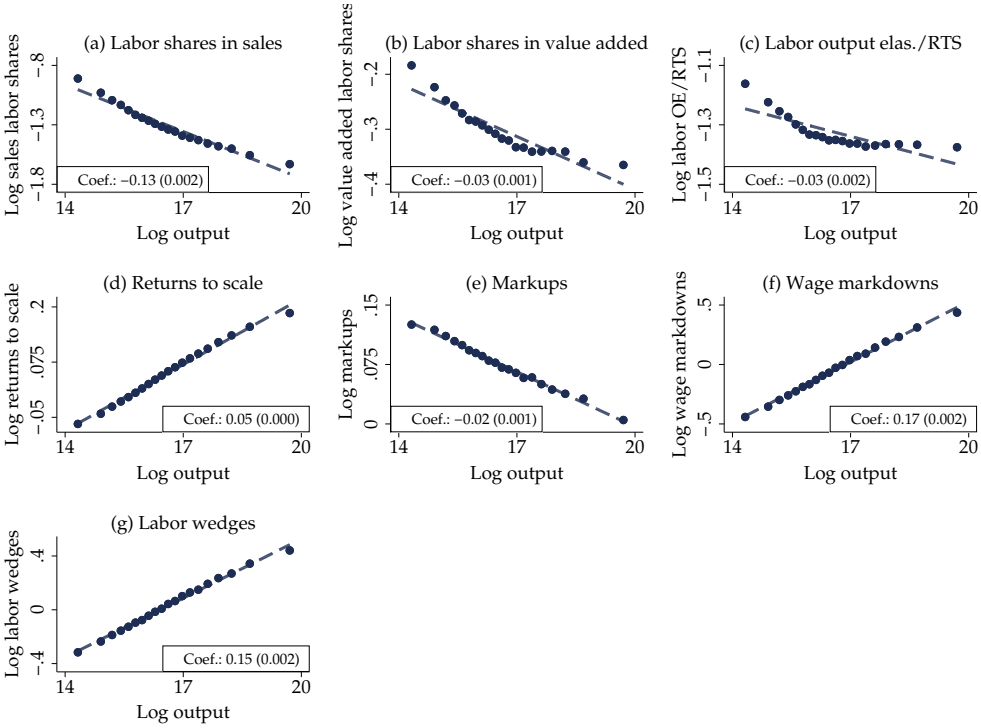
Notes: The table reports OLS regressions from estimating the specification in Equation (10) with additionally controlling for the log of the labor-intermediate input ratio. The dependent variable in Columns (1)-(3) is the log of the labor output elasticity divided by returns to scale. The dependent variable in Columns (4)-(6) is the log of the labor output elasticity. The dependent variable in Columns (7)-(9) is the log of the labor output elasticity divided by the intermediate input output elasticity. Panels A-C report on regressions of changes in those dependent variables on changes in log output and log labor-intermediate input ratios for 1-, 4- , and 10-year differences. Standard errors are clustered at the firm-level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Table A.9: Testing the role of non-homotheticity in the production function, IV results

	$\ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\ln(\theta^L_{it})$	$\ln(\theta^L_{it})$	$\ln(\theta^L_{it})$	$\ln(\frac{\theta^L_{it}}{\theta^M_{it}})$	$\ln(\frac{\theta^L_{it}}{\theta^M_{it}})$	$\ln(\frac{\theta^L_{it}}{\theta^M_{it}})$
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Log output growth	-0.327***		-0.100	-0.318***		-0.074	-0.445***		-0.0792
	(0.062)		(0.076)	(0.065)		(0.077)	(0.0753)		(0.0815)
Log labor-interm. input ratio		0.424***	0.374***		0.440***	0.403***		0.644***	0.605***
		(0.004)	(0.038)		(0.004)	(0.039)		(0.00373)	(0.0406)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813
First-stage F-value	102.6		90.01	102.6		90.01	102.6		90.01
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950
R^2	0.229	0.466	0.456	0.215	0.471	0.462	0.276	0.618	0.612
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Log output growth	-0.242***		-0.087	-0.220***		-0.049	-0.324***		-0.0772
	(0.071)		(0.075)	(0.074)		(0.078)	(0.0911)		(0.0857)
Log labor-interm. input ratio		0.412***	0.364***		0.427***	0.399***		0.622***	0.579***
		(0.006)	(0.042)		(0.006)	(0.044)		(0.00656)	(0.0482)
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492
First-stage F-value	48.57		39	48.57		39	48.57		39
R^2	0.217	0.462	0.445	0.199	0.463	0.452	0.244	0.595	0.583

Notes: The table reports IV regressions from estimating the specification in Equation (10) using foreign demand shocks as instruments (Equation (12)) and with additionally controlling for the log of the labor-intermediate input ratio. The dependent variable in Columns (1)-(3) is the log of the labor output elasticity divided by returns to scale. The dependent variable in Columns (4)-(6) is the log of the labor output elasticity. The dependent variable in Columns (7)-(9) is the log of the labor output elasticity divided by the intermediate input output elasticity. Panels A and B report on regressions of changes in those dependent variables on changes in log output and log labor-intermediate input ratios for 1-, and 4-year differences. Standard errors are clustered at the firm-level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Figure A.4: The cross-sectional relationships: labor shares, output elasticities, markups, markdowns, labor wedges, and returns to scale.



Notes: The figure reports binned scatter plots estimating the specification in Equation (10) in levels with OLS. Panels (a)-(g) report on regressions of the logs of labor shares in sales and value added, labor output elasticities divided by returns to scale, returns to scale, markups, wage markdowns, and labor wedges, respectively. All panels report results that are residualized by industry-year fixed effects. German firm-level data.

Table A.10: CompNet summary statistics, averages of industry averages (manufacturing).

Country	Lab. share (sales)	Lab. share (value added)	Lab. cost share	Lab. output elast.	Interm. cost share	Interm. output elast.	Markup	Wage Markdown
Belgium (2000-2020)	0.17	0.42	0.21	0.31	0.74	0.67	1.23	1.26
Croatia (2002-2021)	0.19	0.40	0.25	0.28	0.67	0.64	1.32	1.05
Czech Republic (2005-2020)	0.16	0.59	0.17	0.18	0.78	0.79	1.19	0.75
Denmark (2001-2020)	0.25	0.72	0.26	0.26	0.69	0.74	1.18	0.80
Finland (1999-2020)	0.21	0.69	0.22	0.21	0.75	0.77	1.15	0.84
France (2004-2020)	0.21	0.39	0.31	0.38	0.62	0.57	1.51	1.37
Germany (2001-2018)	0.22	0.74	0.21	0.36	0.69	0.77	1.13	1.53
Hungary (2003-2020)	0.17	0.61	0.17	0.19	0.77	0.77	1.12	0.86
Italy (2006-2020)	0.17	0.63	0.17	0.17	0.78	0.82	1.19	0.77
Latvia (2007-2019)	0.18	0.53	0.21	0.23	0.73	0.75	1.25	0.99
Lithuania (2000-2020)	0.19	0.66	0.20	0.20	0.74	0.76	1.15	1.14
Netherlands (2007-2019)	0.21	0.68	0.22	0.22	0.74	0.75	1.11	0.85
Poland (2002-2020)	0.16	0.54	0.17	0.17	0.78	0.78	1.18	0.79
Portugal (2010-2020)	0.19	0.62	0.20	0.25	0.74	0.74	1.12	1.09
Romania (2005-2020)	0.18	0.61	0.17	0.20	0.77	0.77	1.14	0.90
Slovakia (2000-2020)	0.16	0.50	0.18	0.19	0.76	0.76	1.14	1.09
Slovenia (2002-2021)	0.20	0.64	0.21	0.20	0.74	0.74	1.12	0.83
Spain (2008-2020)	0.17	0.64	0.19	0.21	0.79	0.77	1.17	0.96
Sweden (2003-2020)	0.22	0.38	0.32	0.41	0.62	0.56	1.31	1.52
Switzerland (2009-2020)	0.26	0.70	0.28	0.25	0.68	0.72	1.18	0.79

Notes: The table reports country-level averages of industry-level labor shares (in sales and value added), labor cost shares, labor output elasticities, intermediate input cost shares, intermediate input output elasticities, markups, and wage markdowns, i.e., we average across for each country across country-industry-level values. Country-industry-level aggregates for labor shares are computed from firms' total labor costs, total sales, and total value added. Country-industry cost shares are computed as total input expenditures divided by total costs. Country-industry-level averages for labor output elasticities are sales-weighted aggregates of firm-level values. Country-industry-level average markups are derived as intermediate input cost-weighted averages of firm-level values. Country-industry-level average wage markdowns are derived as labor cost-weighted averages of firm-level values. CompNet manufacturing data. Firms with at least 20 employees.

Table A.11: Firm-level results for other countries: Coefficients on log output growth, substitution elasticities, and contribution of labor output elasticity changes to labor share changes (4-year changes, OLS). Young and mature manufacturing firms.

	$\ln(L_{it})$	$\ln(M_{it})$	$\ln(CS^L_{it})$	$\ln(CS^M_{it})$	$\ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\ln(\frac{\theta^M_{it}}{RTS_{it}})$	$\ln(\theta^L_{it})$	$\ln(LS_{it})$	$\ln(\frac{P^L_{it}L_{it}}{VA_{it}})$	σ^{OE}	σ^{CS}	LS contrib. of θ^L_{it}
Panel A: young firms	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
France (1995-21)	0.477***	1.018***	-0.109***	0.227***	-0.131***	0.195***	-0.122***	-0.318***	-0.305***	2.52	2.64	38%
Obs.: 21,988	(0.008)	(0.010)	(0.007)	(0.008)	(0.004)	(0.006)	(0.005)	(0.007)	(0.008)			
Hungary (2003-22)	0.488***	1.052***	-0.249***	0.134***	-0.287***	0.116***	-0.299***	-0.330***	-0.274***	3.50	3.12	91%
Obs.: 3,966	(0.016)	(0.013)	(0.015)	(0.011)	(0.015)	(0.006)	(0.015)	(0.015)	(0.019)			
Slovakia (2000-23)	0.570***	1.014***	-0.255***	0.134***	-0.248***	0.0830***	-0.242***	-0.375***	-0.370***	3.93	8.07	65%
Obs.: 1,157	(0.031)	(0.031)	(0.031)	(0.021)	(0.028)	(0.013)	(0.029)	(0.031)	(0.051)			
Slovenia (2002-23)	0.590***	1.097***	-0.220***	0.140***	-0.279***	0.140***	-0.273***	-0.266***	-0.163***	5.76	3.45	103%
Obs.: 719	(0.036)	(0.026)	(0.032)	(0.023)	(0.038)	(0.017)	(0.038)	(0.033)	(0.029)			
Latvia (2005-21)	0.563***	0.973***	-0.224**	0.022	-0.205***	0.125***	-0.208***	-0.272***	-0.409***	5.13	2.49	76%
Obs.: 236	(0.053)	(0.053)	(0.090)	(0.0140)	(0.054)	(0.031)	(0.058)	(0.062)	(0.126)			
Netherlands (2007-22)	0.439***	1.107***	-0.251***	0.152***	-0.308***	0.126***	-0.313***	-0.293***	-0.141***	2.85	2.52	107%
Obs.: 1,774	(0.030)	(0.028)	(0.032)	(0.024)	(0.024)	(0.012)	(0.024)	(0.033)	(0.035)			
Panel B: mature firms	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
France (1995-21)	0.498***	1.029***	-0.0824***	0.330***	-0.131***	0.199***	-0.122***	-0.382***	-0.357***	2.64	4.48	32%
Obs.: 253,635	(0.004)	(0.004)	(0.003)	(0.004)	(0.002)	(0.002)	(0.002)	(0.003)	(0.004)			
Hungary (2003-22)	0.454***	1.060***	-0.326***	0.168***	-0.343***	0.124***	-0.356***	-0.434***	-0.354***	4.36	5.41	82%
Obs.: 31,916	(0.008)	(0.007)	(0.009)	(0.006)	(0.007)	(0.003)	(0.008)	(0.009)	(0.011)			
Slovakia (2000-23)	0.491***	1.018***	-0.272***	0.176***	-0.283***	0.108***	-0.274***	-0.428***	-0.391***	3.88	6.67	64%
Obs.: 8,399	(0.018)	(0.022)	(0.015)	(0.012)	(0.015)	(0.008)	(0.016)	(0.018)	(0.028)			
Slovenia (2002-23)	0.533***	1.097***	-0.306***	0.179***	-0.376***	0.137***	-0.371***	-0.387***	-0.244***	11.06	7.14	96%
Obs.: 6,656	(0.017)	(0.014)	(0.016)	(0.012)	(0.019)	(0.008)	(0.019)	(0.016)	(0.016)			
Latvia (2005-21)	0.479***	0.938***	-0.245***	0.0305***	-0.269***	0.127***	-0.267***	-0.338***	-0.424***	7.29	2.50	80%
Obs.: 2,482	(0.026)	(0.024)	(0.029)	(0.006)	(0.024)	(0.010)	(0.025)	(0.024)	(0.042)			
Netherlands (2007-22)	0.363***	1.117***	-0.308***	0.171***	-0.364***	0.145***	-0.369***	-0.359***	-0.188***	3.14	2.74	103%
Obs.: 28,411	(0.010)	(0.008)	(0.012)	(0.007)	(0.008)	(0.004)	(0.009)	(0.012)	(0.008)			

Notes: The table reports OLS regressions from estimating the specification in Equation (10) for different countries by OLS (Columns (1)-(9)) using 4-year differences. The table reports coefficients on log output growth in Columns (1)-(9). The dependent variables in Columns (1)-(9) are the logs of labor and intermediate quantities, cost shares, and output elasticities divided by returns to scale, labor output elasticities, labor shares in sales, and labor shares in value added, respectively. All regressions of changes in those dependent variables on changes in log output for 4-year differences. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm-level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data. Columns (10) and (11) reports estimates of the substitution elasticity between labor and intermediates based on Eq. (14) using output elasticity coefficients (Column (10)) and cost share coefficients (Column (11)). Column (12) reports the contribution of changes in labor output elasticities (Column (7)) to changes in labor shares (Column (8)). Panel A reports results for firms not older than five years. Panel B reports results for firms older than five years. Firm-level micro data from CompNet data providers for a subset of CompNet countries with information on registration years.

B Theoretical Appendix

B.1 Derivations for Section 2.1

The notation follows the main text. We can define the production function in general terms, $Q_{it}(K_{it}, L_{it}, M_{it}, \Omega_{it}) = Q_{it}(.)$ and write firms' cost minimization as a Lagrangian function:

$$\begin{aligned}\mathcal{L}_{it} = P_{it}^{L}(L_{it})L_{it} + \chi^{L}(L_{it})P_{it}^{L} + P_{it}^{M}(M_{it})M_{it} + \chi^{M}(M_{it})P_{it}^{M} \\ + P_{it}^{K}(K_{it})K_{it} + \chi^{K}(K_{it})P_{it}^{K} - \lambda_{it}(Q_{it} - Q_{it}(.)),\end{aligned} \tag{A.1}$$

where, for simplicity, we model input adjustment costs through convex functions $\chi^{X}(X_{it})$ with $X = \{L, M, K\}$ and write the optimization in a quasi-static way as in Bond et al. (2021). One interpretation of this setting is that each input is associated with a baseline quantity, $\bar{X_{it}}$, and that firms incur adjustment costs when choosing an input quantity that differs from $\bar{X_{it}}$. The first order conditions for each production input lead to Equation (2) from the main text:

$$\frac{\partial \mathcal{L}_{it}}{\partial X_{it}} = 0 \quad \Rightarrow \quad P_{it}^{X} \underbrace{\left(1 + \frac{\partial P_{it}^{X}}{\partial X_{it}} \frac{X_{it}}{P_{it}^{X}} + \frac{\partial \chi^{X}}{\partial L_{it}}\right)}_{\gamma_{it}^{X}} = \lambda_{it} \frac{\partial Q_{it}}{\partial X_{it}}. \tag{A.2}$$

Multiplying Equation (A.2) with $\frac{X_{it}}{Q_{it}}$ using the definition of the output elasticity, $\theta^{X} = \frac{\partial Q}{\partial X}\frac{X}{Q}$, and noting that $\lambda_{it} = \frac{P_{it}}{\mu_{it}}$ (price over markup) leads to:

$$X_{it}P_{it}^{X} = P_{it}Q_{it}\frac{\theta_{it}^{X}}{\mu_{it}\gamma_{it}^{X}}. \tag{A.3}$$

Rearranging this equation yields the share of input expenditures in sales as a function of output elasticities, markups, and input price markdowns,γ_{it}^{X}, which is also Equation (6) of the main text:

$$OS_{it}^{X} = \frac{P_{it}^{X}X_{it}}{P_{it}Q_{it}} = \frac{\theta_{it}^{X}}{\mu_{it}\gamma_{it}^{X}}. \tag{A.4}$$

Similarly, we can define Equation (A.3) for each input and recover expressions for input cost shares (which equals Equation (5) of the main text):

$$CS_{it}^{X} = \frac{P_{it}^{X}X_{it}}{P_{it}^{X}L_{it} + P_{it}^{M}M_{it} + P_{it}^{K}K_{it}} = \frac{\frac{\theta_{it}^{X}}{\gamma_{it}^{X}}}{\frac{\theta_{it}^{L}}{\gamma_{it}^{L}} + \frac{\theta_{it}^{M}}{\gamma_{it}^{M}} + \frac{\theta_{it}^{K}}{\gamma_{it}^{K}}}. \tag{A.5}$$

For the next derivations, we rely on the production function (Equation (1)). The partial derivatives with respect to labor and intermediate yield the first part of Equation (3):

$$\frac{\frac{\partial Q_{it}}{\partial L_{it}}}{\frac{\partial Q_{it}}{\partial M_{it}}} = \frac{\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}} K_{it}^{(1-\kappa)\frac{\sigma-1}{\sigma\kappa}} \Lambda_{i}^{K} \Lambda_{i}^{LM} \alpha_{i}^{L} \kappa L_{it}^{\frac{-1}{\sigma}} Q_{it}^{\frac{\sigma\kappa-\sigma+1}{\sigma\kappa}}}{\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}} K_{it}^{(1-\kappa)\frac{\sigma-1}{\sigma\kappa}} \Lambda_{i}^{K} \Lambda_{i}^{LM} \alpha_{i}^{M} \kappa M_{it}^{\frac{-1}{\sigma}} Q_{it}^{\frac{\sigma\kappa-\sigma+1}{\sigma\kappa}}} = \frac{\alpha_{i}^{L}}{\alpha_{i}^{M}} \left(\frac{L_{it}}{M_{it}}\right)^{\frac{-1}{\sigma_{it}}}. \tag{A.6}$$

Multiplying this expression with $\frac{L_{it}}{Q_{it}}/\frac{M_{it}}{Q_{it}}$ yields the second part of Equation (3):

$$\frac{\theta_{it}^L}{\theta_{it}^M} = \frac{\alpha_i^L}{\alpha_i^M}\left(\frac{L_{it}}{M_{it}}\right)^{\frac{\sigma-1}{\sigma}}. \tag{A.7}$$

Finally, taking logs and differences (the latter eliminates the firm fixed effect, $\frac{\alpha_i^L}{\alpha_i^M}$) yields the last part of Equation (3):

$$\frac{\sigma-1}{\sigma} = \frac{\Delta\ln(\theta_{it}^L) - \Delta\ln(\theta_{it}^M)}{\Delta\ln(L_{it}) - \Delta\ln(M_{it})}. \tag{A.8}$$

B.2 Derivations for Section 2.2

The notation follows the main text. To recover Equation (7) from the main text, we impose more structure on our model and define inverse input supply functions as $P_{it}^X = a_{it}^X X_{it}^{\varepsilon^X}$ for $X = \{L, M\}$ and abstract from adjustment costs for labor and intermediates. For simplicity, we also assume that markups and capital shadow costs do not depend on firm scale. Inserting the input supply functions into Equation (A.1) and dropping adjustment cost terms (except for capital) yields the following Lagrangian:

$$\mathcal{L}_{it} = a_{it}^L L_{it}^{1+\varepsilon^L} + a_{it}^M M_{it}^{1+\varepsilon^M} + P_{it}^K(K_{it})K_{it} + \chi^K(K_{it})P_{it}^K - \lambda_{it}(Q_{it} - Q_{it}(.)). \tag{A.9}$$

The first order condition for labor and intermediates yield Equation (7) from the main text:

$$\frac{\partial\mathcal{L}_{it}}{\partial L_{it}} = 0 \quad \Rightarrow \quad \left(1+\varepsilon^L\right) a_{it}^L L_{it}^{\varepsilon^L} = \lambda_{it}\frac{\partial Q_{it}}{\partial L_{it}} \quad \Rightarrow \quad L_{it} = \left(\frac{\lambda_{it}}{\left(1+\varepsilon^L\right) a_{it}^L}\frac{\partial Q_{it}}{\partial L_{it}}\right)^{\frac{1}{\varepsilon^L}} \tag{A.10}$$

$$\frac{\partial\mathcal{L}_{it}}{\partial M_{it}} = 0 \quad \Rightarrow \quad M_{it} = \left(\frac{\lambda_{it}}{\left(1+\varepsilon^M\right) a_{it}^M}\frac{\partial Q_{it}}{\partial M_{it}}\right)^{\frac{1}{\varepsilon^M}}. \tag{A.11}$$

The first order condition for capital is still Equation (A.2):

$$\frac{\partial\mathcal{L}_{it}}{\partial K_{it}} = 0 \quad \Rightarrow \quad P_{it}^K\gamma_{it}^K = \lambda_{it}\frac{\partial Q_{it}}{\partial K_{it}}. \tag{A.12}$$

Inserting the derivative of the production function with respect to labor (see Equation (A.6)) into Equation (A.10) yields:

$$L_{it} = \left(\frac{\lambda_{it}\Lambda_i^K\Lambda_i^{LM}\alpha_i^L\kappa\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}}}{\left(1+\varepsilon^L\right)a_{it}^L}K_{it}^{(1-\kappa)\frac{\sigma-1}{\sigma\kappa}}L_{it}^{\frac{-1}{\sigma}}Q_{it}^{\frac{\sigma\kappa-\sigma+1}{\sigma\kappa}}\right)^{\frac{1}{\varepsilon^L}} \tag{A.13}$$

$$= \left(\frac{\lambda_{it}\Lambda_i^K\Lambda_i^{LM}\alpha_i^L\kappa\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}}}{\left(1+\varepsilon^L\right)a_{it}^L}K_{it}^{(1-\kappa)\frac{\sigma-1}{\sigma\kappa}}Q_{it}^{\frac{\sigma\kappa-\sigma+1}{\sigma\kappa}}\right)^{\frac{\sigma}{(\sigma\varepsilon^L+1)}}. \tag{A.14}$$

Deriving the same expression for intermediates and combining it with Equation (A.14) yields:

$$\frac{L_{it}}{M_{it}} = \frac{\left(\frac{\lambda_{it}\Lambda_i^K\Lambda_i^{LM}\alpha_i^L\kappa\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}}}{(1+\varepsilon^L)a_{it}^L}\right)^{\frac{\sigma}{(\sigma\varepsilon^L+1)}}}{\left(\frac{\lambda_{it}\Lambda_i^K\Lambda_i^{LM}\alpha_i^M\kappa\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}}}{(1+\varepsilon^M)a_{it}^M}\right)^{\frac{\sigma}{(\sigma\varepsilon^M+1)}}} \frac{\left(K_{it}^{(1-\kappa)\frac{\sigma-1}{\sigma\kappa}}Q_{it}^{\frac{\sigma\kappa-\sigma+1}{\sigma\kappa}}\right)^{\frac{\sigma}{(\sigma\varepsilon^L+1)}}}{\left(K_{it}^{(1-\kappa)\frac{\sigma-1}{\sigma\kappa}}Q_{it}^{\frac{\sigma\kappa-\sigma+1}{\sigma\kappa}}\right)^{\frac{\sigma}{(\sigma\varepsilon^M+1)}}}. \tag{A.15}$$

The derivative of the production function with respect to capital is:

$$\frac{\partial Q_{it}}{\partial K_{it}} = (1-\kappa)\frac{Q_{it}}{K_{it}}. \tag{A.16}$$

Inserting this expression into Equation (A.12) allows us to write capital demand as:

$$K_{it} = \left(\frac{\lambda_{it}(1-\kappa)}{P_{it}^K\gamma_{it}^K}\right)Q_{it}. \tag{A.17}$$

Inserting this capital demand equation into Equation (A.15) recovers Equation (8) from the main text:

$$\frac{L_{it}}{M_{it}} = \frac{\left(\frac{\lambda_{it}\Lambda_i^K\Lambda_i^{LM}\alpha_i^L\kappa\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}}}{(1+\varepsilon^L)a_{it}^L}\left(\frac{\lambda_{it}(1-\kappa)}{P_{it}^K\gamma_{it}^K}\right)^{\frac{(1-\kappa)(\sigma-1)}{\sigma\kappa}}\right)^{\frac{\sigma}{(\sigma\varepsilon^L+1)}}}{\left(\frac{\lambda_{it}\Lambda_i^K\Lambda_i^{LM}\alpha_i^M\kappa\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}}}{(1+\varepsilon^M)a_{it}^M}\left(\frac{\lambda_{it}(1-\kappa)}{P_{it}^K\gamma_{it}^K}\right)^{\frac{(1-\kappa)(\sigma-1)}{\sigma\kappa}}\right)^{\frac{\sigma}{(\sigma\varepsilon^M+1)}}} \frac{\left(Q_{it}^{(1-\kappa)\frac{\sigma-1}{\sigma\kappa}}Q_{it}^{\frac{\sigma\kappa-\sigma+1}{\sigma\kappa}}\right)^{\frac{\sigma}{(\sigma\varepsilon^L+1)}}}{\left(Q_{it}^{(1-\kappa)\frac{\sigma-1}{\sigma\kappa}}Q_{it}^{\frac{\sigma\kappa-\sigma+1}{\sigma\kappa}}\right)^{\frac{\sigma}{(\sigma\varepsilon^M+1)}}} \tag{A.18}$$

$$= \varrho_{it}\lambda_{it}^{\frac{\sigma+\kappa-1}{\kappa}\left(\frac{1}{\sigma\varepsilon^L+1}-\frac{1}{\sigma\varepsilon^M+1}\right)}Q_{it}^{\frac{(1-\kappa)(\sigma-1)+\sigma\kappa-\sigma+1}{\sigma\kappa}\left(\frac{\sigma}{(\sigma\varepsilon^L+1)}-\frac{\sigma}{(\sigma\varepsilon^M+1)}\right)} \tag{A.19}$$

$$= \varrho_{it}\lambda_{it}^{\left(\frac{\sigma+\kappa-1}{\kappa(\sigma\varepsilon^L+1)}-\frac{\sigma+\kappa-1}{\kappa(\sigma\varepsilon^M+1)}\right)}Q_{it}^{\left(\frac{1}{\sigma\varepsilon^L+1}-\frac{1}{\sigma\varepsilon^M+1}\right)}, \tag{A.20}$$

where $\varrho_{it} = \frac{\left(\frac{\Lambda_i^K\Lambda_i^{LM}\alpha_i^L\kappa\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}}}{(1+\varepsilon^L)a_{it}^L}\left(\frac{(1-\kappa)}{P_{it}^K\gamma_{it}^K}\right)^{\frac{(1-\kappa)(\sigma-1)}{\sigma\kappa}}\right)^{\frac{\sigma}{(\sigma\varepsilon^L+1)}}}{\left(\frac{\Lambda_i^K\Lambda_i^{LM}\alpha_i^M\kappa\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}}}{(1+\varepsilon^M)a_{it}^M}\left(\frac{(1-\kappa)}{P_{it}^K\gamma_{it}^K}\right)^{\frac{(1-\kappa)(\sigma-1)}{\sigma\kappa}}\right)^{\frac{\sigma}{(\sigma\varepsilon^M+1)}}}$ is a function of parameters and markdowns only.

B.3 Non-homothetic CES Production Function: Derivations

Notation follows the main text. To derive Equation (20), we follow the same derivation steps as for Equations (A.6) and (A.7) in Appendix B.1. Specifically, the ratio of marginal products from the non-homothetic production function in Equation (19) from the main text writes:

$$\frac{\frac{\partial Q_{it}}{\partial L_{it}}}{\frac{\partial Q_{it}}{\partial M_{it}}} = \frac{\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}}K_{it}^{(1-\kappa)\frac{\sigma-1}{\sigma\kappa}}\Lambda_i^K\Lambda_i^{LM}\alpha_i^L\kappa L_{it}^{\frac{-1}{\sigma}}Q_{it}^{\frac{\sigma\kappa-\sigma+1}{\sigma\kappa}}}{\Omega_{it}^{\frac{\sigma-1}{\sigma\kappa}}K_{it}^{(1-\kappa)\frac{\sigma-1}{\sigma\kappa}}\Lambda_i^K\Lambda_i^{LM}\alpha_i^M\kappa\left(\frac{M_{it}}{Q_{it}^\eta}\right)^{\frac{-1}{\sigma}}Q_{it}^{\frac{\sigma\kappa-\sigma+1}{\sigma\kappa}}} = \frac{\alpha_i^L}{\alpha_i^M}\left(\frac{L_{it}}{M_{it}}\right)^{\frac{-1}{\sigma_{it}}}Q_{it}^{\frac{-\eta}{\sigma}}. \tag{A.21}$$

Multiplying this expression with $\frac{L_{it}}{Q_{it}}/\frac{M_{it}}{Q_{it}}$ yields Equation (20) of the main text:

$$\frac{\theta_{it}^L}{\theta_{it}^M} = \frac{\alpha_i^L}{\alpha_i^M}\left(\frac{L_{it}}{M_{it}}\right)^{\frac{\sigma-1}{\sigma}}Q_{it}^{\frac{-\eta}{\sigma}}. \tag{A.22}$$

B.4 Markups and Wage Markdowns

The notation follows the main text. We now show how we derive the equations for firms' markups and wage markdowns in terms of observables and output elasticities. This follows Hall (1986), De Loecker and Warzynski (2012), and Dobbelaere and Mairesse (2013). As discussed in the main text, we assume that intermediates are supplied perfectly elastically and that there are no adjustment costs (as standard in the literature for this derivation). *Strictly speaking, for our firm-level analysis in changes, a weaker assumption is sufficient: we can permit intermediate input market imperfections γ_{it}^M but those must be constant over time within a firm, i.e., $\gamma_{it}^M = \gamma_i^M$* (indeed, our results provide suggestive evidence for that property). This implies the following cost-minimization problem:

$$\mathcal{L}_{it} = P_{it}^L(L_{it})L_{it} + P^M M_{it} + P^K K_{it} - \lambda_{it}(Q_{it} - Q_{it}(.)), \tag{A.23}$$

where, for simplicity, we also abstract from capital market imperfections and $\lambda_{it} = \frac{P_{it}}{\mu_{it}}$. The first order conditions for labor and intermediates are:

$$P_{it}^M = \frac{P_{it}}{\mu_{it}} \frac{\partial Q_{it}}{\partial M_{it}} \tag{A.24}$$

$$P_{it}^L \gamma_{it}^L = \frac{P_{it}}{\mu_{it}} \frac{\partial Q_{it}}{\partial M_{it}}. \tag{A.25}$$

Rearranging Equation (A.24) yields an expression for the markup that we can identify in the data on the basis of the intermediate output share and output elasticity:

$$\mu_{it} = \frac{P_{it}}{MC_{it}} = \theta_{it}^M \frac{P_{it}Q_{it}}{P_{it}^M M_{it}}. \tag{A.26}$$

Combining Equations (A.25) and (A.24) recovers an expression for the wage markdown:

$$\gamma_{it}^L = \frac{\theta_{it}^L}{\theta_{it}^M} \frac{P_{it}^M M_{it}}{P_{it}^L L_{it}}. \tag{A.27}$$

Rearranging Equation (A.25) also yields the equation for the measure of combined distortions from markup and labor market imperfections:

$$\mu_{it}\gamma_{it}^L = \theta_{it}^L \frac{P_{it}Q_{it}}{P_{it}^L L_{it}}. \tag{A.28}$$

C Further Details on the German Firm-Product Level data

Table C.1: Variable definition in the German microdata.

Variable	Definition
L_{it}	Labor in headcounts.
P_{it}^{L}	Firm wage (firm average), defined as gross salary before taxes (including mandatory social costs) + "other social expenses" (including expenditures for company outings, advanced training, and similar costs) divided by the number of employees.
K_{it}	Capital derived by a perpetual inventory method as described below.
M_{it}	Deflated total intermediate input expenditures, defined as expenditures for raw materials, energy, intermediate services, goods for resale, renting, repairs, and contracted work conducted by other firms.
E_{it}	Deflated expenditures for raw, auxiliary, and operating materials and energy inputs (includes external product components). E_{it} is part of M_{it}.
$Merch_{it}$	Deflated expenditures for merchandise. $Merch_{it}$ is part of M_{it}.
Sub_{it}	Deflated expenditures for subcontracted work performed by other companies. Sub_{it} is part of M_{it}.
Rep_{it}	Deflated expenditures for repairs, maintenance, installation, and assembly. Rep_{it} is part of M_{it}.
$Temp_{it}$	Deflated expenditures for temporary agency workers. $Temp_{it}$ is part of M_{it}.
$Rent_{it}$	Deflated expenditures for rent, leases, leasing. Rep_{it} is part of M_{it}.
$Other_{it}$	Deflated expenditures for Other intermediate costs (insurance, postage, transport, etc.). $Other_{it}$ is part of M_{it}.
$z_{it}M_{it}$	Nominal values of total intermediate input expenditures.
$P_{it}Q_{it}$	Nominal total revenue, defined as total gross output, including, among others, sales from own products, sales from intermediate goods, revenue from offered services, and revenue from commissions/brokerage.
Q_{it}	Quasi-quantity measure of physical output, i.e., $P_{it}Q_{it}$ deflated by a firm-specific price index (denoted by PI_{it}).
PI_{it}	Firm-specific Törnqvist price index, derived as in Eslava et al., 2004. See Appendix D.1 for its construction.
P_{igt}	Price of a product g.
$share_{igt}$	Revenue share of a product g in total firm revenue.
ms_{it}	Weighted average of firms' product market shares in terms of revenues. The weights are the sales of each product in firms' total product market sales.
G_{it}	Headquarter location of the firm. 90% of firms in our sample are single-plant firms.
D_{it}	A four-digit industry indicator variable. The industry of each firm is defined as the industry in which the firm generates most of its sales.
Exp_{it}	Dummy-variable being one, if firms generate export market sales.
$NumP_{it}$	The number of products a firm produces.

Notes: The table list all variables and variable definitions used in the paper. Nominal values are deflated by a 2-digit industry-level deflator for intermediate inputs and which is supplied by the federal statistical office of Germany.

Data access. The data can be accessed at the Research Data Centres of the Federal Statistical Office of Germany and the Statistical Offices of the German Länder (states). Data request can be made at: https://www.forschungsdatenzentrum.de/en/request. The statistics we used are: "AFiD-Modul Produkte," "AFiD-Panel Industriebetriebe," "AFiD-Panel Industrieunternehmen," "Investitionserhebung im Bereich Verarbeitendes Gewerbe, Bergbau und Gewinnung von Steinen und Erden," "Panel der Kostenstrukturerhebung im Bereich Verarbeitendes Gewerbe, Bergbau und Gewinnung von Steinen und Erden." The data are combined by the statistical offices and provided as a merged dataset.

Variable definitions. Table C.1 presents an overview on the variable definitions of all variables used in this article. This includes variables used in other sections of the appendix.

Outlier cleaning. We exclude the top and bottom two percent outliers with respect to value-added over revenue and revenue over labor, capital, intermediate input expenditures, and labor costs. We replace quantity and price information with missing values for products displaying a price deviation from the average price in the top and bottom one percent. We also drop the sectors 16 (tobacco), 23 (mineral oil and coke), and 37 (recycling) as the observation count is insufficient to derive estimates of firms' production functions in these industries.

Capital stock estimation. As capital stocks are not directly observed, we calculate a time series of capital stocks for every firm using the perpetual inventory method of Bräuer et al. (2023):

$$K_{it} = K_{it-1}(1 - depr_{jt-1}) + I_{it-1}, \tag{C.1}$$

where K_{it}, $depr_{jt-1}$ and I_{it-1} denote firm i's capital stock, the depreciation rate of capital, and investment. Investment captures firms' total investment in buildings, equipment, machines, and other investment goods. Nominal values are deflated by a two-digit industry-level deflator supplied by the German Statistical Office.

We derive the industry- and year-specific depreciation rate from official information on the expected lifetime of capital goods (supplied by the statistical offices). To do so, we formulate the lifetime of a capital good LT as a function of its depreciation rate:

$$LT = depr \int_0^\infty (1 - depr)^t t dt. \tag{C.2}$$

Using partial integration gives:

$$LT = depr \left[\frac{(1 - depr)^t}{\ln(1 - depr)} t \right]_0^\infty - depr \int_0^\infty \frac{(1 - depr)^t}{\ln(1 - depr)} dt, \tag{C.3}$$

where the first term on the right-hand side equals zero because $0 < depr < 1$. Integrating the remaining expression yields:

$$LT = \frac{depr}{\ln(1 - depr) * \ln(1 - depr)}, \tag{C.4}$$

which we can numerically solve for $depr$. As the lifetime of capital goods is separately given for years and capital good types (buildings and equipment), we derive a depreciation rate for each year and capital good type separately. To derive a single industry-specific depreciation rate, we weight the depreciation rates for buildings and equipment respectively with the industry-level share of building capital in total capital and equipment capital in total capital (this information is supplied by the statistical offices). For the practical implementation, we assume that the depreciation rate of a firm's whole capital stock equals the depreciation rate of newly purchased capital.

The initial capital stock for the perpetual inventory method is derived from reported tax depreciation. We do not use the reported tax depreciation when calculating capital stock series as tax depreciation may vary due to state-induced tax incentives and might therefore not reliably reflect the

true amount of depreciated capital. Given that firms likely tend to report too high depreciation levels due to such tax incentives, our first capital values within a capital series are likely overestimated. However, over time, observed investment decisions gradually receive a larger weight in estimated capital stocks, mitigating the impact of the first capital stock. Given that we estimate very reasonable output elasticities (see Table A.1), we are confident that our capital variables reliably reflect firms' true capital stocks.[43]

Deriving a time-consistent industry classification. During our long time-series, the NACE classification of industries (and thus firms into industries) changed twice. Once in 2002 and once in 2008. Because our estimation of the production function requires a time-consistent industry classification at the firm level (as we allow for industry-specific production functions), it is crucial to recover a time-consistent NACE industry classification. Recovering such a time-consistent industry classification from official concordance tables is, however, problematic as they contain many ambiguous sector reclassifications. To address this issue, we follow the procedure described in Mertens (2022) and use information on firms' product mix to classify firms into NACE Rev 1.1 industries based on their main production activities. This procedure exploits the fact that the first four digits of the ten-digit GP product classification reported in the German data are identical to the NACE classification (i.e., they indicate the industry of the product). Applying this method demands a consistent reclassification of all products into the GP2002 scheme (which corresponds to the NACE Rev 1.1 scheme). Reclassifying products is, due to the granularity of the ten-digit classification, less ambiguous than reclassifying industries. In the few ambiguous cases, we can follow the firms' product mix over the reclassification periods and unambiguously reclassify most products (i.e., we observe what firms produce before and after reclassification years). Having constructed a time-consistent product-industry classification according to the GP2002 scheme, we attribute every firm to the NACE Rev 1.1 industry in which it generates most of its revenue. When comparing the classification with the one of the statistical offices for the years 2002-2008 (years in which industries are already reported in NACE Rev 1.1), Mertens (2022) finds that this two-digit and four-digit classification of firms into industries matches the classification of the statistical offices in 95% and 86% of all cases, respectively.

[43] As firms likely tend to overstate their capital depreciation, our capital stocks are likely a closer approximation of the true capital stock used in firms' production processes than capital measures based on book values.

D Production Function Estimation in the German Data

We follow Mertens (2022) in estimating the production function. This approach yields firm- and time-specific output elasticities by assuming the following translog production function (throughout, lower case letter denote logs):

$$q_{it} = \boldsymbol{\phi}'_{it}\boldsymbol{\beta} + \omega_{it} + \epsilon_{it}\,. \tag{D.1}$$

q_{it} denotes the log of produced quantities and $\boldsymbol{\phi}'_{it}$ captures the production inputs capital (k_{it}), labor (l_{it}), and intermediates (k_{it}) and its interactions. The industry-specific production function that we will estimate for each two-digit Nace rev. 1.1 industry is specified in logs as:

$$\begin{aligned} q_{it} =& \beta_l l_{it} + \beta_m m_{it} + \beta_k k_{it} + \beta_{ll} l_{it}^2 + \beta_{mm} m_{it}^2 + \beta_{kk} k_{it}^2 \\ &+ \beta_{lk} l_{it} k_{it} + \beta_{lm} l_{it} m_{it} + \beta_{km} k_{it} m_{it} + \beta_{lkm} l_{it} k_{it} m_{it} + \omega_{it} + \epsilon_{it}. \end{aligned} \tag{D.2}$$

The output elasticity of labor is

$$\frac{\partial q_{it}}{\partial l_{it}} = \beta_l + 2\beta_{ll} l_{it} + \beta_{lm} m_{it} + \beta_{lk} k_{it} + \beta_{lkm} k_{it} m_{it}\,. \tag{D.3}$$

ϵ_{it} is an i.i.d. error term and ω_{it} denotes Hicks-neutral productivity and follows a Markov process. ω_{it} is unobserved to the econometrician, yet firms know ω_{it} before making input decisions for flexible inputs (intermediates in our case). We assume that only firms' input decision for intermediates depends on productivity shocks. Labor and capital do not respond to contemporary productivity shocks. However, our results are similar when allowing labor to respond to productivity innovations. In fact, the CompNet routine for the production function estimation models labor as flexible and we find consistent results in both datasets.
There are three issues preventing us from estimating the production function in Equation (D.1) using OLS:

(1) We need to estimate a physical production model to recover the relevant output elasticities. Although we observe product quantities, quantities cannot be aggregated across the various products of multi-product firms. Relying on the standard practice to apply sector-specific output deflators does not solve this issue if output prices vary within industries.

(2) We do not observe firm-specific input prices for capital and intermediate inputs. If input prices are correlated with input decisions and output levels, an endogeneity issue arises.

(3) The fact that productivity is unobserved and that firms' flexible input decisions depend on productivity shocks, creates another endogeneity problem.

We now discuss how we solve these three identification problems.

D.1 Solving Challenge (1) by Deriving a Firm-specific Output Price Index

As we cannot aggregate output quantities across different products of a firm (a common problem), we follow Eslava et al. (2004) and construct a firm-specific price index from observed output prices. We use this price index to purge observed firm revenue from price variation by deflating firm revenues with this price index.[44] We construct firm-specific Törnqvist price indices for each firm's composite revenue from its various products in the following way:

$$PI_{it} = \prod_{g=1}^{n} \frac{p_{igt}}{p_{igt-1}}^{1/2(share_{igt}+share_{igt-1})} PI_{it-1}\,. \tag{D.4}$$

[44] This approach has also been applied in various other studies, such as Smeets and Warzynski (2013).

PI_{it} is the price index, p_{igt} is the firm-specific price of good, g, that we observe in the data, and $share_{igt}$ is the share of this good in total product market sales of firm i in period t. The growth of the index value is the product of the individual products' price growths, weighted with the average sales share of that product over the current and the last year. The first year available in the data is the base year (i.e., $PI_{it=1995} = 100$). If firms enter after 1995, we follow Eslava et al. (2004) and use an industry average of the computed firm price indices as a starting value. Similarly, we impute missing product price growth information in other cases with an average of product price changes within the same industry.[45] After deflating firm revenue with this price index, we have a quasi-quantity measure of output, for which, with a slight abuse of notation, we denote by q_{it}.[46]

D.2 Solving Challenge (2) by Accounting for Unobserved Input Price Variation

While the recent literature stresses the so-called "output-price bias" when estimating production functions, previous work has also highlighted that unobserved input prices introduce another identification problem. To control for input price variation across firms, we use a firm-level analog of De Loecker et al. (2016) and define a price-control function from firm-product-level output price information that we add to the production function in Equation (D.1):

$$q_{it} = \boldsymbol{\phi}'_{it}\boldsymbol{\beta} + B_{it}((pi_{it}, ms_{it}, G_{it}, D_{it}) \times \boldsymbol{\phi}^c_{it}) + \omega_{it} + \epsilon_{it}\,. \tag{D.5}$$

$B_{it}(.) = B_{it}((pi_{it}, ms_{it}, G_{it}, D_{it}) \times \boldsymbol{\phi}^c_{it})$ is the price control function consisting of our logged firm-specific output price index (pi_{it}), a logged sales-weighted average of firms' product market sales shares (ms_{it}), a headquarter location dummy (G_{it}) and a four-digit industry dummy (D_{it}). $\boldsymbol{\phi}^c_{it} = [1; \boldsymbol{\phi}_{it}]$, where $\boldsymbol{\phi}_{it}$ includes the production function input terms as specified in Equation (D.2). These are either in monetary terms and deflated by an industry-level deflator (capital and intermediates) or already reported in quantities (labor). The constant entering $\boldsymbol{\phi}^c_{it}$ highlights that elements of $B(.)$ enter the price control function linearly and interacted with $\boldsymbol{\phi}_{it}$ (a consequence of the translog production function). The idea behind the price-control function $B(.)$ is that output prices, product market shares, firm location, and firms' industry affiliation are informative about firms' input prices. Particularly, we assume that product prices and market shares contain information about product quality and that producing high-quality products requires expensive high-quality inputs. As De Loecker et al. (2016) discuss, this reasoning motivates the addition of a control function containing output price and market share information to the right-hand side of the production function to control for unobserved input price variation emerging from input quality differences across firms. We also include year, location, and four-digit industry dummies into $B(.)$ to further absorb the remaining differences in local and four-digit industry-specific input prices.

Conditional on elements in $B(.)$, we assume that there are no remaining input price differences across firms. Although restrictive, this assumption is more general than the ones employed in most other studies estimating production functions without having access to firm-specific price data and which implicitly assume that firms face identical input and output prices within industries.

A notable difference between the original approach of De Loecker et al. (2016) and our version is that they estimate product-level production functions, whereas we transfer their framework to the

[45] For roughly 30% of all product observations in the data, firms do not have to report quantities as the statistical office views them as not being meaningful.

[46] Note that, as discussed in Bond et al. (2021), using an output price index does not fully purge firm-specific price variation. There remains a base year difference in prices. Yet, using a firm-specific price index follows the usual practice of using price indices to deflate nominal values, we are thus following the best practice. Moreover, it is the only available approach when pooling multi- and single-product firms. Estimating the production function separately by single-plant firms requires other strong assumptions like perfect input divisibility of all inputs across all products. Finally, our results are also robust to using cost-share approaches to estimate the production function, which requires other assumptions (constant returns to scale, competitive input markets, and the absence of adjustment costs).

firm level. For that, we use firm-product-specific sales shares in firms' total product market sales to aggregate firm-product-level information to the firm-level. This implicitly assumes that i) such firm aggregates of product quality increase in firm aggregates of product prices and input quality, ii) firm-level input costs for inputs entering as deflated expenditures increase in firm-level input quality, and iii) product price elasticities are equal across the various products of a firm. These or even stricter assumptions are always implicitly invoked when estimating firm-level production functions.

Finally, note that even if some of the above assumptions do not hold, including the price control function is still preferable to omitting it. This is because the price control function can nevertheless absorb some of the unobserved price variations and does not require that input prices vary between firms with respect to all elements of $B_{it}(.)$. The estimation can regularly result in coefficients implying that there is no price variation at all. The attractiveness of a price control function lies in its agnostic view about the existence and degree of input price variation.

D.3 Solving Challenge (3) by Controlling for Unobserved Productivity

To address the dependence of firms' intermediate input decision on unobserved productivity, we follow Olley and Pakes (1996) and Levinsohn and Petrin (2003) and employ a control function approach. We base our control function on firms' consumption of energy and materials, which we denote by e_{it} and which are components of total intermediate inputs. Inverting the demand function for e_{it} defines an expression for productivity:

$$\omega_{it} \equiv g_{it}(.) = g_{it}(e_{it}, k_{it}, l_{it}, \mathbf{\Gamma}_{it}). \tag{D.6}$$

$\mathbf{\Gamma}_{it}$ captures state variables of the firm, that in addition to k_{it} and l_{it} affect firms demand for e_{it}. Ideally, $\mathbf{\Gamma}_{it}$ should include a wide set of variables affecting productivity and demand for e_{it}. We include dummy variables for export (EX_{it}) activities, the log of the number of products a firm produces ($NumP_{it}$), and the average wage a firm pays (P^L_{it}) into $\mathbf{\Gamma}_{it}$. The latter absorbs unobserved quality and price differences that shift input demand for e_{it}. Remember that productivity follows a first-order Markov process. Firms can shift this Markov process as described in Doraszelski and Jaumandreu (2013) and De Loecker (2013), giving rise to the following law of motion for productivity: $\omega_{it} = h_{it}(\omega_{it-1}, \mathbf{T}_{it-1}) + \xi_{it} = h_{it}(.) + \xi_{it}$, where ξ_{it} denotes the innovation in productivity and $\mathbf{T}_{it} = (EX_{it}, NumP_{it})$ reflects the fact that we allow for learning effects from export market participation and (dis)economies of scope through adding and dropping products to influence firm productivity.[47] Plugging Equation (D.6) and the law of motion for productivity into Equation (D.5) gives

$$q_{it} = \boldsymbol{\phi}'_{it}\boldsymbol{\beta} + B_{it}(.) + h_{it}(.) + \epsilon_{it} + \xi_{it}\,, \tag{D.7}$$

which constitutes the basis of our estimation.

D.4 Identifying Moments

We estimate Equation (D.7) separately by two-digit NACE Rev. 1.1 industries using a one-step estimator as in Wooldridge (2009).[48] Our estimator uses lagged values of flexible inputs (i.e., intermediates) as instruments for their contemporary values to address the dependence of firms' flexible input decisions

[47] Doraszelski and Jaumandreu (2013) also highlight the role of R&D investment in shifting firms' productivity process. We would also like to add this information to the productivity model but do not observe R&D expenditures for the early years in our data.

[48] We approximate $h_{it}(.)$ by a third-order polynomial in all of its elements, except for the variables in $\mathbf{\Gamma}_{it}$. Those we add linearly. $B_{it}(.)$ is approximated by a flexible polynomial where we interact the output price index with elements in $\boldsymbol{\phi}_{it}$ and add the vector of market shares, the output price index, and the location and industry dummies linearly. Interacting further elements of $B_{it}(.)$ with $\boldsymbol{\phi}_{it}$ creates too many parameters to be estimated. This implementation is similar to De Loecker et al. (2016).

on realizations of ξ_{it}. Similarly, we use lagged values of terms including firms' market share and output price index as instruments for their contemporary values as we consider these to be flexible variables.[49] We define identifying moments jointly on ϵ_{it} and ξ_{it}:

$$E[(\epsilon_{it} + \xi_{it})\mathbf{Y}_{it}] = 0. \tag{D.8}$$

$\mathbf{Y}_{it}$ includes lagged interactions of intermediate inputs with labor and capital, contemporary interactions of labor and capital, contemporary location and industry dummies, the lagged output price index, lagged market shares, lagged elements of $h_{it}(.)$, and lagged interactions of the output price index with production inputs. Formally this implies:

$$\mathbf{Y}'_{it} = (J_{it}(.), A_{it-1}(.), T_{it-1}(.), \Psi_{it}(.), \boldsymbol{\vartheta}_{it-1}) \ , \tag{D.9}$$

where we defined:

- $J_{it}(.) = (l_{it},\, k_{it},\, l_{it}^2,\, k_{it}^2,\, l_{it}k_{it},\, G_{it},\, D_{it})$,
- $A_{it}(.) = (m_{it},\, m_{it}^2,\, l_{it}m_{it},\, k_{it}m_{it},\, l_{it}k_{it}m_{it},\, ms_{it},\, \pi_{it})$,
- $T_{it}(.) = \big((l_{it},\, k_{it},\, l_{it}^2,\, k_{it}^2,\, l_{it}k_{it},\, m_{it},\, m_{it}^2,\, l_{it}m_{it},\, k_{it}m_{it},\, l_{it}k_{it}m_{it}) \times \pi_{it}\big)$,
- $\Psi_{it}(.) = \sum_{n=0}^{3} \sum_{w=0}^{3-b} \sum_{h=0}^{3-n-b} l_{it-1}^{n}\, k_{it-1}^{b}\, e_{it-1}^{h}$, and
- $\boldsymbol{\vartheta}_{it-1} = (Exp_{it-1},\, NumP_{it-1},\, P_{it-1}^{L})$,
- with P_{it}^{L} denoting the average wage a firm pays.

[49] These timing assumptions also address any simultaneity concerns with respect to the price variables entering the right-hand side of our estimation.

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