The Natural Rate of Interest in Small-Open Economies: Asymmetries and Fragmentation*

Work in Progress — Please Do Not Circulate

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Abstract

This paper develops a structural model to study the trend real interest rate in small-open economies, $\tilde{R}^*$. Impediments to global capital mobility can drive a wedge between the global trend real interest rate, R^*, and $\tilde{R}^*$, necessitating country-specific analyses. Our specific focus is to quantify the evolution of $\tilde{R}^*$ in the UK, a small-open international financial centre, relative to R^* in the rest of the world. The model captures five potential drivers of $\tilde{R}^*$: productivity growth, population growth, longevity, risk premia, and fragmentation of global capital markets. Against the backdrop of a decline in global neutral real interest rate R^* of around 3pp in the past half century, the model suggests a more muted decline—of around 2.5pp—in the UK. However, looking ahead, increased geo-economic fragmentation poses significant upside risks to UK equilibrium rates, of nearly 0.6pp.

Key Words: Capital mobility; Demographics; Long-run interest rates; Macroeconomic trends; Structural change.

JEL Codes: E21, E43, E13, J11.

*The views expressed here are solely those of the authors and so cannot be taken to represent those of the Bank of England.

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1 Introduction

Long-run prospects for interest rates are an ever-present topic in international policy debates (e.g., International Monetary Fund, 2023; Benigno, Hofmann, Barrau, and Sandri, 2024). Analysing how interest rates are likely to evolve requires an assessment of the underlying forces driving secular movements in interest rates. To date, the majority of such work has focused on the global component of trend real interest rates, 'Global R^*' (Cesa-Bianchi, Harrison, and Sajedi, 2023). However, against a backdrop of heightened concern around geo-economic fragmentation, debate has intensified around whether asymmetries in economic outlooks will impact differences in interest-rate prospects across countries. There is also increasing focus on whether the path of economic integration itself may contribute to these differences.

In this paper, we develop a structural life-cycle model to study the drivers of trend real interest rates in small-open economies. Within the model, Global R^*—which is determined by the factors that drive the global supply and demand for capital—acts as an anchor for domestic interest rates. However, international financial-market frictions create a wedge between Global R^* and the trend real interest rate in small-open economies, $\tilde{R}^*$. The more imperfect is global capital mobility, the less sensitive a country's trend real interest rate $\tilde{R}^*$ is to global factors. As a consequence, cross-country asymmetries in secular trends can generate different paths for interest rates across countries. And fragmentation, in the form of disintegration of global capital markets, will widen the wedge between Global R^* and country-specific $\tilde{R}^*$ in small-open economies.

Relative to existing work assessing trend real interest rates in small-open economies, our model is unique in its focus on a *range* a potential drivers—rather than a single one in isolation. We focus on five potential drivers: productivity growth, population growth, longevity, risk premia, and fragmentation of global capital markets. The model has two regions—the Home small-open economy and the rest of the world—each populated by overlapping generations of finitely-lived households that face age- and region-specific mortality rates. In the small-open economy, households save using domestic productive capital (with time-varying relative price), an exogenous stock of bonds issued by the domestic fiscal authority, and foreign assets. We allow for both a premium between the rate of return on domestic capital and the return on the domestic government bond (i.e., the real interest rate, $\tilde{R}^*$), and a premium between the rate of return on foreign assets (i.e., Global R^*) and the return on the domestic bond. We back out the latter empirically, exploiting differences between empirical measures of R^* for the US and

the UK, as well as data on the UK net foreign asset position. Conditional on the (exogenous) trends in each of the drivers, the equilibrium response of the small-open economy real interest rate in the model represents a simulated path for $\tilde{R}^*$.

We apply the model to study trend real interest rates in the UK over the period 1960-2020, treating the US as the primary driver of Global R^*. With perfect global capital mobility, Global R^* and UK-specific $\tilde{R}^*$ are equal, determined by the global supply and demand for capital. In this case, the trend real interest rate exhibits a gradual decline of nearly 3 percentage points from 1960 to 2020. In line with analyses of Global R^* (Cesa-Bianchi et al., 2023), the decline is predominantly driven by falling productivity growth and increased longevity globally.

Against this global downward trend, our analysis suggests the UK experienced a more moderate decrease in its equilibrium rate, $\tilde{R}^*$, of roughly 2.5pp, when accounting for realistic frictions to global capital mobility. While the fall in Global R^* continues to act as a significant anchor for UK equilibrium rates, impediments to capital mobility allow asymmetries in underlying drivers to play a role in generating a wedge between $\tilde{R}^*$ and Global R^*. In particular, the global productivity slowdown and demographic forces pushing the global risk-free rate lower appear to have been less pronounced for the UK. This asymmetry contributed to the UK's more muted decline in equilibrium rates over the last six decades.

We then turn our attention to the future outlook. First, we consider how cross-country asymmetries in key UK factors—such as population growth, longevity, productivity growth, and risk premia—relative to the US, could weigh on the future trajectory of $\tilde{R}^*$, using projected trends for these variables. Second, we assess how potential changes in the degree of global economic integration could further impact the equilibrium real interest rate going forward. Shifts in the level of cross-border capital mobility and trade linkages could alter the underlying drivers of $\tilde{R}^*$, leading to divergences from past patterns.

By analyzing both the country-specific asymmetries and the implications of evolving global integration, we aim to gain a more comprehensive understanding of how $\tilde{R}^*$ may evolve in the years ahead, building upon the insights from our previous decomposition analysis.

Even without assuming any projected divergence in the drivers, a slight movement towards less integrated capital markets poses an upward risk to $\tilde{R}^*$ estimates. In other words, the potential effect of reduced openness leads to a non-negligible wedge between Global R^* and $\tilde{R}^*$ of up to around 0.6pp, compared to a scenario with pure small-open economy assumptions.

Literature Review. Our work is most closely related to papers investigating the drivers of real interest rates in small-open economies. Compared to existing work using structural models to do this, our model extends on this by considering a range of potential drivers of wedges between global and country-specific real interest rates within a single framework. Lisack, Sajedi, and Thwaites (2021) and Carvalho, Ferrero, Mazin, and Nechio (2023), for example, focus solely on the role of demographic factors, while Kuncl and Matveev (2023) additionally account for productivity and government debt.

Other work providing country-specific estimates of equilibrium real interest rates differ from us in relying on closed-economy frameworks, and so are unable to investigate the potential impact of fragmentation. Semi-structural models estimate a high-frequency concept of equilibrium real interest rates: the real interest rate that stabilizes output at potential and inflation at target (e.g., Laubach and Williams, 2003; Holston, Laubach, and Williams, 2017; Harrison, Lloyd, Page, and von dem Berge, 2024). Our approach focuses on longer-term trends, abstracting from shorter-term deviations of real interest rates from $\tilde{R}^*$ and hence the effects of shocks on domestic equilibrium rates over shorter horizons. Empirical studies into country-specific trend real interest rates (e.g., Ferreira and Shousha, 2023; Davis, Fuenzalida, Huetsch, Mills, and Taylor, 2024) offer complementary evidence, but lack the structural decomposition of drivers that our model provides.

Analysis of global equilibrium real interest rates are a key input for our small-open economy analysis. Most closely related to our theoretical setup is the structural model for Global R^* by Cesa-Bianchi et al. (2023) which, like us, accounts for a range of factors that could drive trend real interest rates. Our estimates of Global R^*, and so UK $\tilde{R}^*$ when capital is perfectly mobile across borders, is similar to that from Cesa-Bianchi et al. (2023)—who, in turn, find similar global trends to Del Negro, Giannone, Giannoni, and Tambalotti (2019) and Kiley (2020).

Finally, our work contributes to a nascent literature studying the links between geo-economic fragmentation and interest rates (e.g., International Monetary Fund, 2023). Our modelling setup allows us to consider this within a structural framework, and make progress answering the question: how might fragmentation impact natural rates of interest across countries?

Outline. The remainder of the paper proceeds as follows. Section 2 lays out the structural model. Section 3 describes the empirical measures of global and UK-specific trends in the model and details the calibration. Section 4 presents the results of model simulations to analyze the drivers of UK $\tilde{R}^*$, relative to Global R^*, over the period 1960-2020. Section 5 reports

forward-looking experiments analysing the prospects for long-run interest rates in the UK vs. the rest of the world. Section 6 concludes.

2 The Model

The model is a neoclassical overlapping generations (OLG) model. A continuum of identical households are born each period, with an exogenous population growth rate, and face an exogenous survival probability each period, up to a maximum of J periods. Over their lifetime, households supply labour and capital to a monopolistic firm producing a single good, and pay taxes or receive transfers from the government.

This section describes each agent in the model, as well as the aggregation and market-clearing conditions that define the equilibrium.

2.1 Households

Each period, a continuum of mass N_t of households is 'born'. The growth rate of consecutive cohorts, $(1 + n_t) \equiv N_t/N_{t-1}$, is exogenous. A representative household born in period t faces an exogenous unconditional probability of surviving until age j, denoted by $\Pi_{t,j}$, with $\Pi_{t,1} = 1$ and $\Pi_{t,J+1} = 0$, where J is the maximum age.

Each household inelastically supplies ρ_j effective units of labour, for wage w_{t+j-1}.[1] After some fixed age, J^R, the household will 'retire', such that $\rho_j = 0$ for $j \geq J^R$.

Households can save using a single asset, $a_{t,j}$, which will pay a net real interest rate, r_{t+j-1}, in the next period. They choose their stream of consumption and asset holdings to maximise their expected lifetime utility, given by:

$$U_t = \sum_{j=1}^{J} \beta_j \Pi_{t,j} \log(c_{t,j})$$

subject to a budget constraint in each period:

$$c_{t,j} = \rho_j w_{t+j-1} + (1 + r_{t+j-2})a_{t,j-1} - a_{t,j} + \varpi_{t,j} \quad \text{for } j = 1, \ldots, J$$

where β_j is the utility weight on consumption at age j, which will be used to calibrate the life-cycle profile of wealth. Households are born with no assets, $a_{t,0} = 0$, and $a_{t,J}$ will be taken

[1] Note that $t + j - 1$ is the period in which the generation born in period t is aged j.

as given to match positive wealth holdings of the oldest households in the data.[2] $\varpi_{t,j}$ denotes a stream of non-labour income, which households take as exogenous.[3]

This problem yields the familiar Euler equations:

$$1 = \frac{\beta_{j+1}}{\beta_j} \frac{\Pi_{t,j+1}}{\Pi_{t,j}} \left(\frac{c_{t,j+1}}{c_{t,j}} \right)^{-1} (1 + r_{t+j-1}) \quad \text{for } j = 1, \ldots, J-1$$

Notice that $\Pi_{t,j+1}/\Pi_{t,j}$ is the probability of surviving until age $j+1$ conditional of having survived until age j.

2.2 Firms

Retailer. A monopolistic retailer buys an intermediate good and sells it as a final good with a net mark-up, μ, over its marginal cost. Normalising the price of the final good to 1, for consistency with the household's budget constraint above, this means the relative price of the intermediate good is given by $1/(1+\mu)$.

Letting Y_t denote total output, the aggregate profit of the retailer is given by:

$$\mathcal{P}_t = Y_t \frac{\mu}{1+\mu}$$

This profit is distributed equally across working-age cohorts:

$$\mathcal{P}_t = \sum_{j=1}^{J^R-1} p_{j,t} = (J^R - 1) p_t \quad \Longrightarrow \quad p_t = \frac{1}{J^R - 1} \mathcal{P}_t$$

Intermediate-Good Producer. The intermediate good is produced using a constant elasticity of substitution (CES) aggregation of capital and labour:

$$Y_t = \left(\alpha K_{t-1}^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(E_t L_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

where L_t is total effective units of labour and K_{t-1} is the stock of capital at the end of period $t-1$, used for production in period t, and E_t is a labour-augmenting technological process with an exogenous net growth rate e_t.

[2] To have $a_{t,J} > 0$, it should either be taken as given by the households or appear directly in the utility function, implying utility gained from deliberate bequests at the end of life.

[3] This will be defined fully below, and will comprise of taxes and transfers from the government, the monopolistic profits of the firm, and bequests.

The intermediate-good producer's profit maximisation can be written as:

$$\max_{K_{t-1},\, L_t} \quad \frac{1}{1+\mu} Y_t - \left(r_t^k p_t^k K_{t-1} + w_t L_t \right)$$

where p_t^k is the relative price of capital, r_t^k is the real rental rate that the firm pays for capital, and w_t is the wage per effective units of labour. This yields the usual first-order conditions:

$$\begin{aligned} r_t^k =& \frac{1}{1+\mu} \alpha \frac{1}{p_t^k} \left(\frac{Y_t}{K_{t-1}} \right)^{\frac{1}{\sigma}} \\ w_t =& \frac{1}{1+\mu} (1-\alpha) E_t \left(\frac{Y_t}{L_t} \right)^{\frac{1}{\sigma}} \end{aligned}$$

2.3 Government

The government collects lump-sum taxes, $\mathcal{T}_t$, from working-age cohorts, and gives lump-sum 'social security', $\mathcal{S}_t$, to retired cohorts. Both taxes and transfers are assumed to be uniform across these cohorts:

$$\begin{aligned} \mathcal{T}_t &= \sum_{j=1}^{J^R-1} \tau_{j,t} = (J^R - 1)\tau_t \implies \tau_t = \frac{1}{J^R - 1} \mathcal{T}_t \\ \mathcal{S}_t &= \sum_{j=J^R}^{J} s_{j,t} = (J - J^R) s_t \implies s_t = \frac{1}{J - J^R} \mathcal{S}_t \end{aligned}$$

In each period, the government has a stock of debt, G_{t-1}, on which it pays interest, r_{t-1}. Hence the government budget constraint is given by:

$$G_t = (1 + r_{t-1}) G_{t-1} + \mathcal{S}_t - \mathcal{T}_t$$

In the baseline exercise, the debt-to-GDP and spending-to-GDP ratios will be held fixed and lump-sum taxes will adjust to meet the budget constraint.

2.4 Financial Intermediary

At the end of each period, a financial intermediary takes the aggregate assets of the households, promising a net return of r_t per unit in the next period. They use part of the assets to buy the stock of government debt, which provides this safe rate of return.

The remaining assets are turned into capital goods, and stored for production in the next period. The financial intermediary earns the rental rate, net of depreciation, $(1 + r_{t+1}^k - \delta)$, on

this capital, as well as any capital gains from changes in the relative price of capital, p^k_{t+1}/p^k_t

We assume that there is an exogenous spread, φ_t, between the return on capital and the risk-free rate paid out to households such that:

$$1 + r_t = \left(1 + r^k_{t+1} - \delta\right) \frac{p^k_{t+1}}{p^k_t} - \varphi_t$$

This spread is treated as a resource cost to the economy, which can also be viewed as an additional cost to the financial intermediary, such as the cost of storing capital.

2.5 Aggregation and Market Clearing

To define the market clearing conditions, it is necessary to aggregate household-level variables across all J cohorts alive in each period.

Note that $(t-j+1)$ is the birth-year of the household aged j in period t, and $\Pi_{t-j+1,j} N_{t-j+1}$ is the mass of households aged j still alive in period t. Hence, for any household-level variable, $x_{t,j}$, the aggregate quantity in period t is given by:

$$X_t = \sum_{j=1}^{J} \Pi_{t-j+1,j} N_{t-j+1} x_{t-j+1,j}$$

Labour Markets. Total effective labour supply is given by:

$$L_t = \sum_{j=1}^{J} \Pi_{t-j+1,j} N_{t-j+1} \rho_j$$

Goods Markets. To ensure that goods markets clear, it remains to define the non-labour income of the household, $\varpi_{t,j}$, such that all resources are correctly distributed:

$$\sum_{j=1}^{J} \Pi_{t-j+1,j} N_{t-j+1} \varpi_{t-j+1,j} = \mathcal{P}_t + \mathcal{B}_t + \mathcal{S}_t - \mathcal{T}_t$$

For the working-age cohorts, this income comprises of the monopolistic profit of the retailer and the bequests, minus the taxes paid to the government. For retired cohorts, it comprises of

the social security transfer received from the government. In other words:

$$\varpi_{t-j+1,j} = \frac{p_t + b_t - \tau_t}{\Pi_{t-j+1,j} N_{t-j+1}} \qquad \text{for } j = 1, ..., J^R - 1$$
$$\varpi_{t-j+1,j} = \frac{s_t}{\Pi_{t-j+1,j} N_{t-j+1}} \qquad \text{for } j = J^R, ..., J$$

Bequests. Households that die at the end of each period leave their assets, along with the return, as bequests. Aggregate bequests can be defined as:

$$\mathcal{B}_t = (1 + r_{t-1}) \sum_{j=1}^{J} \left(1 - \frac{\Pi_{t-j,j+1}}{\Pi_{t-j,j}}\right) \Pi_{t-j,j} N_{t-j} a_{t-j,j}$$

Asset Markets in an Open Economy. Taking the UK as a small-open economy, we consider three arrangements for international asset markets: at one extreme, a case with fully flexible capital; at the other extreme, a case with no cross-border capital flows (i.e., a closed economy); and an intermediate case with some limits to free capital mobility.

In the first case, with fully flexible capital, the real interest rate of the small-open economy will be exogenously determined by the foreign ('global') rate—e.g., for the UK, this would correspond to the real interest rate path of the rest of the world, for which we use the US as our benchmark. This foreign interest rate, in turn, influences the capital-output ratio in the domestic economy through the first-order conditions of firms. However, there is no guarantee that the wealth of domestic households will adjust to match the level of capital demanded by domestic firms. Differences between economies mean that the interest rate that equilibrates asset markets in the US may not necessarily do so in the domestic economy.

Differences between domestic household wealth and the firms' capital will be filled by capital inflows, as foreign capital compensates the dearth of domestic wealth, or capital outflows, as domestic households send their excess wealth abroad. Formally, the asset-market clearing condition, rather than imposing that domestic household wealth equals domestic capital, instead defines the Net Foreign Asset, NFA_t, position of the economy:

$$\sum_{j=1}^{J} \Pi_{t-j+1,j} N_{t-j+1} a_{t-j+1,j} = NFA_t + G_t + p_t^k K_t$$

The aggregate assets of the household given to the financial intermediary are either sent abroad, used to buy government debt, or turned into capital.

This free-capital mobility case can be thought of as a case in which the price in international

capital markets is fully inelastic for the small-open economy, being set by foreign interest rates. Here, all of the adjustment in the domestic capital markets comes through the quantity of capital flows.

At the opposite extreme, the closed-economy case is one in which the quantity of capital flows is fixed, at zero, and hence all of the adjustment in domestic capital markets comes through the domestic real interest rate. Asset-market clearing is therefore:

$$\sum_{j=1}^{J} \Pi_{t-j+1,j} N_{t-j+1} a_{t-j+1,j} = G_t + p_t^k K_t$$

Considered in this way, then, there can also be an intermediate case in which both prices and quantities are allowed to adjust. This can be introduced by defining the domestic real interest as a function of foreign real interest rates, r_t^f, and NFA_t:

$$r_t = r_t^f - \phi \frac{NFA_t}{Y_t} \tag{1}$$

The parameter ϕ governs the elasticity of price, r_t, to changes in quantities, NFA. When $\phi = 0$, this implies $r_t = r_t^f$, and hence this is the small open economy case. As $\phi \to \infty$, any small change in NFA will imply an infinite deviation of the domestic interest rate from the foreign interest rate, so that in equilibrium $NFA_t = 0$, and hence this is the same as the closed-economy case. For an intermediate $\phi > 0$, some of the adjustment in domestic capital markets will come from domestic prices, r_t, and some from capital-flow quantities, NFA_t. This specification can be interpreted as a case with frictions in global capital markets, which limit the flow of capital across borders. On the one hand capital can flow between countries, meaning that foreign interest rates will play a role in the domestic economy, but on the other hand there are some frictions limiting their quantity so that there cannot be full arbitrage between capital in different countries, meaning that domestic factors will also continue to be important.

3 Calibration

The section describes our calibration strategy, distinguishing aggregate and age-specific parameters. In general, we set age-specific parameters to match life-cycle profiles, calculated from the Wealth and Assets Survey (WAS) for the UK and the Survey of Consumer Finances (SCF) for the US,[4] and calibrate aggregate parameters by matching macroeconomic data tar-

[4]The US will be used as the benchmark representing the 'Rest of the World' in our analysis.

gets.

Because the model will be used to simulate demographic trends from 1950 onwards, the parameter calibration will mechanically pin down a pre-1950 steady state that will be the initial point of the simulations. However, since there is not sufficient data before 1950 to convincingly calibrate to this period, instead, as will be made clear below, some of the macroeconomic targets will be matched during the dynamic simulation.

3.1 Aggregate Targets

Aggregate parameters include those related to production—the net mark-up μ, the capital share α, the elasticity of substitution between capital and labour σ, and the depreciation rate of capital δ— and well as government debt-to-GDP and spending-to-GDP ratios.

The elasticity of substitution between capital and labour in the CES production function, σ, is set to 0.7, in line with Sajedi and Thwaites (2016) and the references therein. Capital depreciation, δ, is set to 3.4% annual for US and 3.7% for UK, which is the average depreciation rate in those countries between 1970-79 from Penn World Tables (PWT) data.[5]

The parameters α and μ are jointly used to match the capital-output ratio and the labour-income share. As with the depreciation rates, PWT provides data on these two variables from 1950 to 2019. Since these two quantities, particularly the capital-output ratio, exhibit a trend over time as a result the demographic trends within the model, targeting their average values over this entire period is not particularly useful. Instead, similar to Lisack et al. (2021), our calibration will target their average values in 1970-79.[6].

The initial interest-rate level is set at 5% (annualised), which is in line with the Holston et al. (2023) estimates for the US.[7]

3.2 Life-cycle Profiles

The key age-specific parameters are the discount factors, β_j, and the effective units of labour, ρ_j. It is also necessary to set the maximum age, J, and the retirement age, J^R.

[5]While using the average over the entire sample period yields similar results, we selected this specific timeframe to maintain consistency with our calibration of both the capital-to-output ratio and labour-income share.

[6]In particular, the capital-output ratio is set to 3.9 for US and 4.3 for UK, while the labour-income share is set to 63.2% and 55.1%, respectively.

[7]Holston et al. (2023) estimate their model starting in 1961. The initial rate aligns with the 1961-69 average. However, this starting point is revised upward when additional factors, beyond ageing, are incorporated into the model.

The WAS provides detailed information on the wealth, assets, and income of UK households. To obtain profiles consistent with patterns observed for the US (e.g., Lisack et al., 2021), survey data from 2011 to 2020 is averaged.[8].

For the US, the calibration of profiles focuses on the most recent pre-COVID data available in the SCF—specifically the 2019 wave, which serves as a representative year.

To ensure that there is a sufficient sample within each age group, households in the survey are separated into 5-year age groups, according to the age of the 'reference person', and the focus will be on households aged between 20-89. In terms of the model, this means that each model period corresponds to 5 years, and that households enter the model at age 20-24, and live to a maximum age of 85-89, so that $J = 14$.

Following Lisack et al. (2021), the two key life-cycle profiles that will be targeted are the life-cycle profile of labour income and of net wealth, constructing real measures of net employment income and net financial wealth, respectively. The life-cycle profiles are recovered as the average of these variables within each age group.[9] These profiles are shown in Figure 1, in dark blue for UK and light blue for US.

Focusing on the UK, first, for net wealth shown on the left-hand panel, it is important to notice that the pattern observed in Lisack et al. (2021) for the US also appears in the data for the UK. Specifically, households accumulate net wealth over their working life as predicted by the life-cycle savings theory, reaching a peak at around 70-74 years old, just above retirement age. However, the decumulation of wealth after this point, and throughout retirement, is only partial. In line with the findings of Lisack et al. (2021), this data suggests that agents never reduce their wealth below the levels attained by around age 50-54.[10] To embed this profile of wealth holdings into the model, the age-specific discount factors, β_j, will be set to match this shape.[11]

Second, the right-hand panel also shows a hump-shaped profile of UK labour income similar to the US. On average, labour income begins to drop gradually from age 55-59, before reaching close to zero from age 75-79. While the pattern in the data is driven by a number of factors, including productivity, hours worked, labour effort, and possibly even compositional

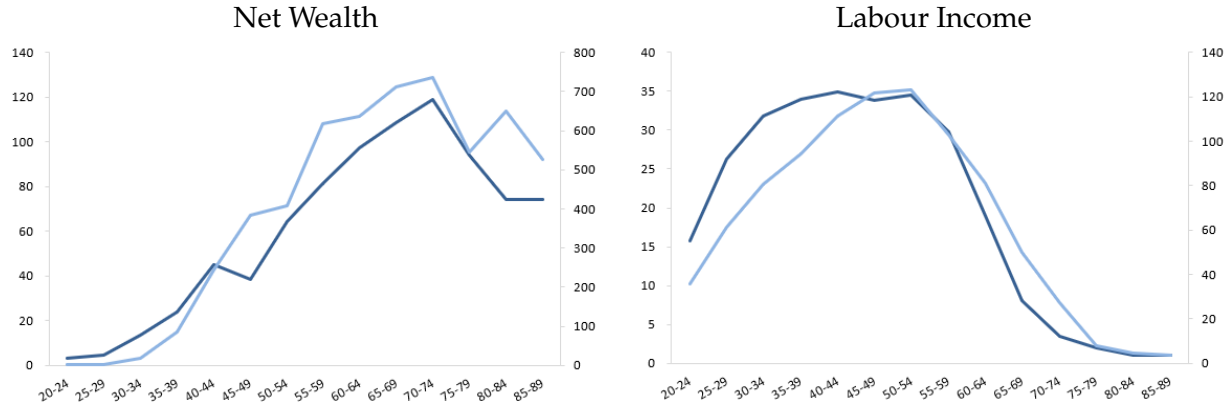
[8] In general, households tend to accumulate net wealth throughout their working life, as predicted by the life-cycle savings theory, and begin decumulating wealth after retirement, although this reversal is often partial. Labour income typically follows a hump-shaped pattern, declining after retirement.

[9] These are in fact weighted averages using the survey weights which translate the survey sample to the population so that it is representative along several characteristics, including age.

[10] Since WAS data is not disaggregated after age 80, we use same average calculated for the last two age groups for UK.

[11] Note that this is a shortcut to match this life-cycle profile using preference parameters; modelling the deeper reasons for this behaviour is outside the scope of this paper.

Figure 1: Estimated Life-cycle Profiles in the UK (dark, left) and US (light, right)



Thousands GBP and USD, respectively. Sources: WAS (2011-2020), and SCF (2019)

effects between high- and low-skilled workers, in the model all of these factors are captured by the age-specific effective units of labour, ρ_j. Since the model has no endogenous labour supply decision, and no heterogeneity within cohorts, this provides a simple and straightforward way of embedding the observed shape of labour income over the life-cycle. The downside of this approach is that this paper will be silent about how ageing will impact endogenous labour-supply decisions. Since labour income declines considerably after age 65, in the model $J^R = 10$, implying all households retire at age 65-69 with certainty.

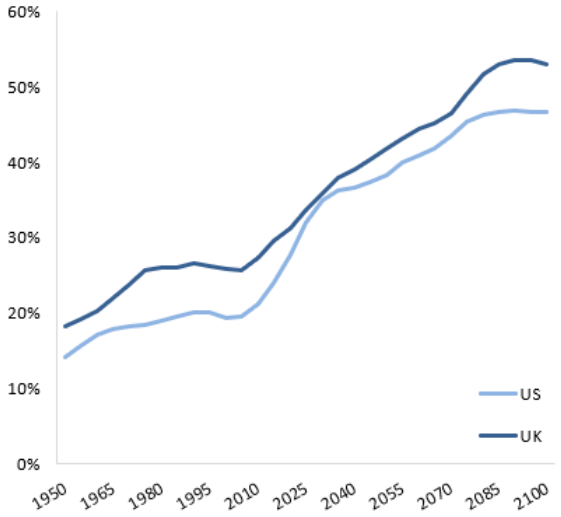
3.3 Demographics

The UN Population Statistics provides population sizes in 5-year age groups at 5-year intervals from 1950 to 2023, along with projections through to 2100. In the model, two exogenous demographic drivers—cohort growth rates and survival probabilities—are calibrated to match the observed age distribution of the population. Cohort growth rates serve as the model's population growth driver, while survival probabilities are used to calibrate longevity as a distinct factor. The resulting age distribution is summarized by the Old-Age Dependency Ratio (OADR), shown in Figure 2.

In principle, since this data begins in 1950, the dynamics of the model begin when the first cohort that will still be alive in 1950 enters the model, since their lifetime optimisation problem will not be at the initial steady state. In other words, the variables can move away from steady state as early as 1885. In the results shown below, the early dynamics are discarded and instead the figures will cover the period for which the demographics data is observed.

The OADR is consistently higher in the UK than in the US, indicating that the UK has a

Figure 2: Old-Age Dependency Ratio (65+/20-64 years-olds)



Source: UN Population Statistics, projections based on median fertility scenario

larger proportion of elderly individuals relative to its working-age population. In the absence of other drivers, this suggests that, under purely demographic pressures, we would expect capital outflows from the UK, ultimately leading to the accumulation of net foreign assets. However, that is not consistent with what we observe in the data.

3.4 Productivity Growth

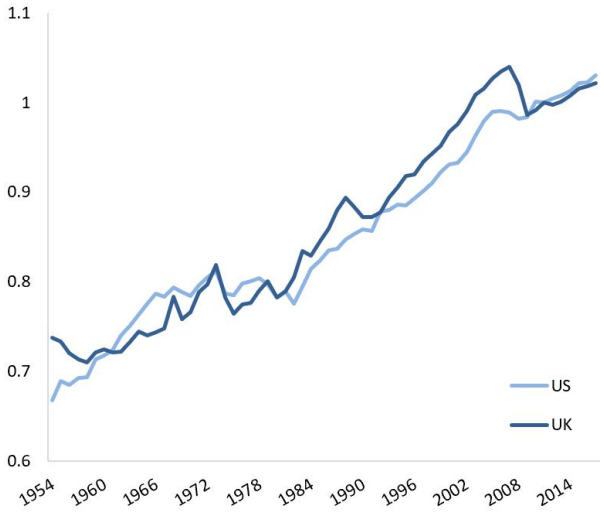
We calibrate the exogenous productivity process by using the growth rate of labour-augmenting technology as estimated by Ziesemer (2023). This annual dataset spans the period 1951-2020. Technological change is derived from a CES production function, employing data from the PWT.

Figure 3 plots the productivity profiles for the UK and US. It demonstrates that, after comparatively fast growth up to 2008, UK labour productivity declined considerably after the Global Financial Crisis—much more so than in the US—and has largely stagnated since then.

3.5 Risk Premia

To calibrate risk premia, we identify observable proxies for the convenience yield in financial markets. Following the methodology developed by Krishnamurthy and Vissing-Jorgensen (2012), we measure the convenience yield in the US and UK as the difference between the yield on BAA corporate bonds and the yield on 10-year Treasury bonds. This convenience

Figure 3: Productivity



Source: Ziesemer (2023). Labour-augmenting technical change data (CES estimation, 0.7 elasticity).

yield can be further decomposed into two key components: a safety premium, captured by the spread between BAA and AAA bonds, and a liquidity premium, represented by the spread between AAA bonds and 10-year Treasuries.

As shown in Figure 4, the risk premium—measured as the difference between the return on capital and the risk-free rate—spiked significantly during the GFC. Since then, the UK wedge has remained for the most part larger than that of the US.

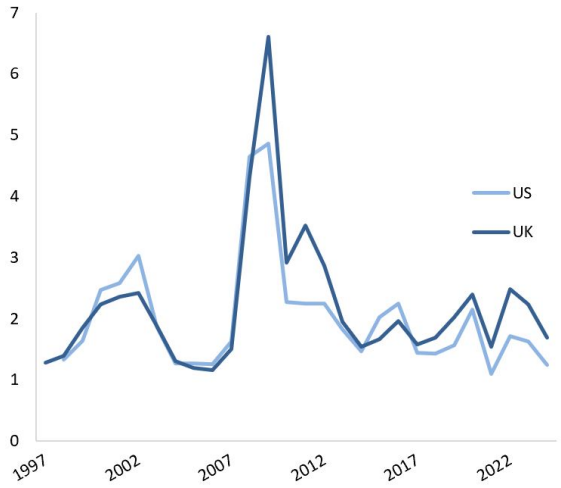
3.6 Open-Economy Wedge

Finally, we must calibrate the elasticity of the small-open economy real interest rate with respect to changes in quantities of net foreign assets, ϕ. As equation (1) captures, in the limiting case of perfect capital mobility, $\phi = 0$ such that the price in international capital markets is perfectly inelastic. In this case, the UK trend real interest rate $\tilde{R}^*$ will align with Global R^*.

Away from this limiting case, where $\phi \neq 0$, capital-flow frictions will generate a wedge between $\tilde{R}^*$ and R^*. The calibration of ϕ is therefore crucial to understanding how different driving forces affect $\tilde{R}^*$ in the UK. However, pinning it down is empirically challenging.

Our baseline calibration for ϕ is static. We attain it by taking equation (1) to the data, projecting long-term real rates in the UK relative to the US on the UK net-foreign asset to GDP

Figure 4: Return to capital and risk-free rate wedge



Source: Spread in p.p., computed as the difference between the yield on BAA bonds and 10-year Treasuries.

ratio. That is, we estimate the following regression:

$$\log\left(\frac{R_t^{UK,10y}}{R_t^{US,10y}}\right) = \phi_1 + \phi_2\left(\frac{NFA_t}{GDP_t}\right) + \varepsilon_t \tag{2}$$

where $R_t^{UK,10y}$ and $R_t^{US,10y}$ are yields on 10-year government bonds for UK and US, respectively, taken from FRED and NFA/GDP is the ratio of UK's net international investment position to GDP from the UK Office of National Statistics.[12] we additionally add dummy-variable controls for the periods post Global Financial Crisis, $\mathbb{1}_t^{GFC}$, and post Covid-19, $\mathbb{1}_t^{Covid}$.

Table 1 presents the results. Once translated to annual frequency, our estimates imply that an increase of 1pp in the net-foreign asset to GDP ratio is associated with a reduction in the interest-rate spread of around 0.58pp.

We complement this static calibration with a time-varying measure of ϕ. We attain this by comparing external empirical measures of $R*$ for the US and the UK, and backing out the implied ϕ 'residually'. The idea is as follows: with perfect capital mobility, these two measures should be identical, but in the data, they are not—see, for example, the UK and US measures of R* from Del Negro et al. (2019). One can then use data on the UK NFA position, along with the UK and US R^* measures from Del Negro et al. (2019), to estimate a time-varying measure of ϕ that satisfies equation (1) hold with equality.

[12] All the series are at quarterly frequency, not seasonally adjusted.

Table 1: Estimated Association Between Relative Long-Term Risk-Free Rates and Net Foreign Assets, Regression (2)

	$\log\left(R^{UK,10y}/R^{US,10y}\right)$
NFA/GDP ($\hat{\phi}_2$)	-0.121*
	(2.05)
$\mathbb{1}^{GFC}$	-0.329***
	(-4.93)
$\mathbb{1}^{Covid}$	-0.580***
	(-6.15)
Constant ($\hat{\phi}_1$)	0.127***
	(7.85)
Observations	186
R^2	0.347

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

4 Evolution of UK $\tilde{R}^*$, 1960-2020

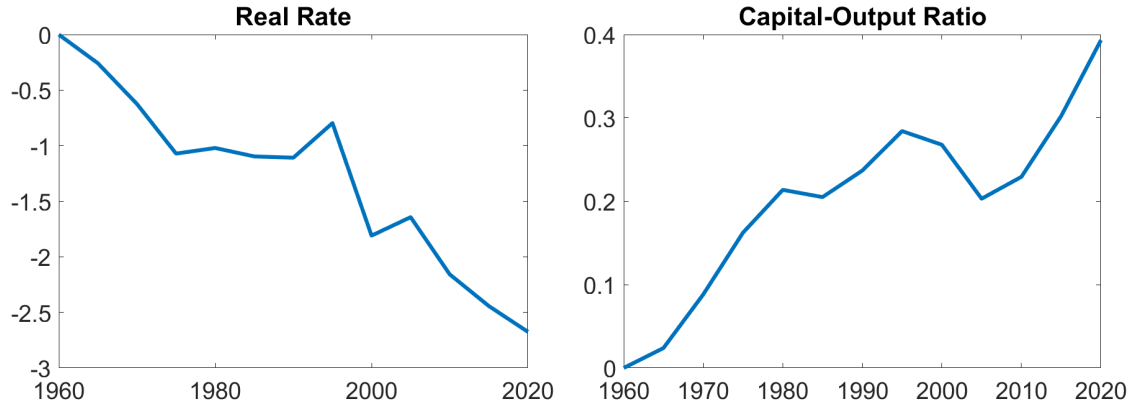
Building on this calibration, we now discuss the results from perfect-foresight deterministic simulations of the model. We set the model's initial steady state in 1950, but our analysis begins in 1960 to allow for comparison with other estimates. The exogenous variables—demographics, longevity, productivity growth, and risk premia—evolve from the 1950 initial steady state to a final steady state in 2020, following observed data over this period. In this section, we discuss the effect of those drivers on the (long-run) real interest rate in the UK over the period 1960-2020 under different configurations of capital-account openness.

4.1 Global R^* and $\tilde{R}^*$ with Perfect Capital Mobility

With perfect capital mobility, $\phi = 0$, the UK trend real rate $\tilde{R}^*$ corresponds with Global R^*. The global supply and demand for capital, driven in our simulations by US factors, determine $\tilde{R}^*$. Figure 5 shows the dynamics of the real interest rate and the capital-output ratio from the model simulations in this case.

When focusing solely on population growth and longevity, the capital-output ratio rises throughout the simulations, resembling the rise in the US OADR seen in Figure 2. This follows the intuition laid out from the start of this paper. From the perspective of a given household, ageing implies that they expect to spend more time in retirement, and so they need to accumulate more wealth while earning labour income in order to smooth their consumption. From the aggregate perspective, the age distribution is skewing increasingly towards the oldest cohorts, and so, given the shape of net wealth over the lifecycle, there is increasing weight put on the

Figure 5: Effects of Population Growth, Longevity, Productivity Growth, and Risk Premia on Global R^*: Change Relative to 1960



Note: Both panels present changes relative to 1960. Left panel presents change in percentage points, right panel presents change in fraction.

highest wealth households. Both of these factors raise total wealth, and hence capital, in the economy.

The converse of the rising capital-output ratio is a declining real interest rate. Due to diminishing returns to capital, a larger capital stock leads to a lower marginal return on capital, which, in the frictionless version of this model, corresponds to the real interest rate.

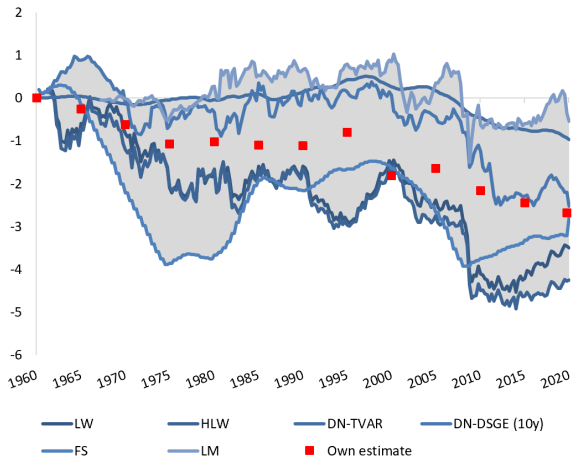
Moreover, slower trend US productivity growth contributes further to the reduction in the natural rate. Lower expected returns on investment have decreased the demand for capital.

Another determinant of the downward pressure on R^* is the increase in the risk premium, given a relatively stable return on capital. Evidence suggests that a rise in risk premia has significantly contributed to the decline in safe rates of return, both in the United States (Del Negro et al. (2017)) and globally (Del Negro et al. (2019)).

Overall, the real interest rate in the US declines gradually by nearly 3pp over the 60-year period.

Comparison With Alternative Estimates. Figure 6 compares our estimates of changes in US R^* from 1960 with alternative measures. Most strikingly, our near-3pp decline in R^* over the six decades to 2022 lies broadly in the centre of available estimates. Like others, we find that R^* declined from 1960 to around 1990, before increasingly slightly in the 1990s. Since then, R^* has declined further—driven in large part by ageing and productivity slowdown.

Figure 6: US selected R^* estimates



Note: Models included are Laubach-Williams (LW), Holston-Labauch-Williams (HLW), Del Negro et al (DN) TVAR and DSGE, Ferreira-Shousha (FS), and Lubik-Matthes (LM). All series are in p.p. differences w.r.t. 1960, except for those with a later initial value (1961 for LW and HLW, and 1967 for LM).

4.2 UK $\tilde{R}^*$ with Imperfect Capital Mobility

With imperfect capital mobility, $\phi \neq 0$, a wedge opens between the UK trend real rate $\tilde{R}^*$ and Global R^*.

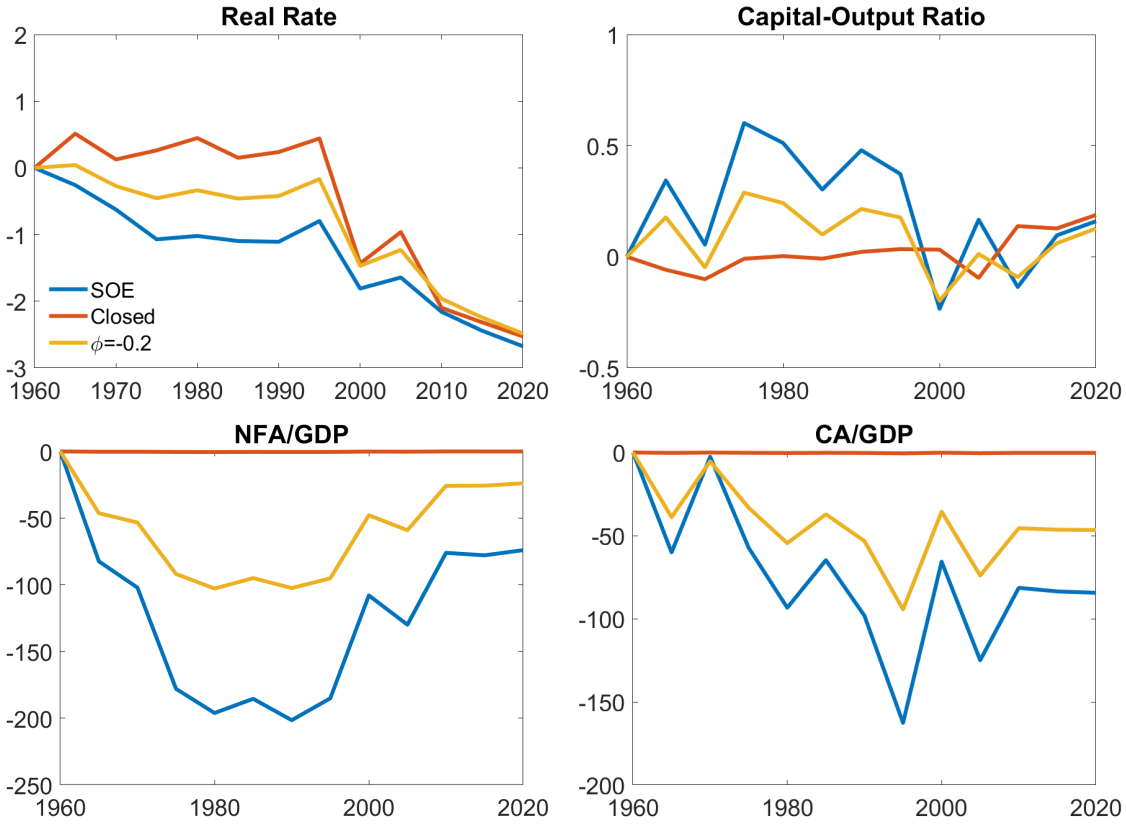
As shown in Figure 7, we explored three illustrative cases to assess the implications of varying degrees of global capital mobility. The blue line represents the small-open economy case ($\phi = 0$), corresponding to the US case depicted in Figure 5. The red line represents the UK as a closed economy ($\phi = -100$), and the yellow line illustrates an intermediate case with $\phi = -0.2$.

Against the backdrop of a roughly 3pp decline in the global neutral real interest rate R^*, over the past half-century, the model suggests a decrease of about 2.5pp for the UK when imperfect capital mobility is taken into account. This result aligns a capital-to-output ratio that does not increase substantially.

Differences in the trajectory of the real interest rate translate into a deterioration of the UK's net foreign asset (NFA) position. In terms of the general pattern, if demographic drivers alone were at play—given that the UK has an older population than the US (which we use as a proxy for the Rest of the World)—the UK would be expected to accumulate net foreign assets as foreign capital flows out. However, consistent with observed data, the UK instead accumulates net foreign liabilities, leading to a deterioration in the current account as well.

Accordingly, we need to investigate the specific factors contributing to such differences.

Figure 7: Effects of Population Growth, Longevity, Productivity Growth, and Risk Premia on UK $\tilde{R}^*$ for Varying Degrees of Global Capital Mobility: Change Relative to 1960



Note: All panels present changes relative to 1960. Top-left panel presents change in percentage points, while other panels presents change in fraction.

Figure 8 presents the decomposition of UK $\tilde{R}^*$ drivers for the estimated path under the previously depicted intermediate case of imperfect capital mobility (i.e., $\phi = -0.2$).

The key differences between countries seem to be related to the relative strength of the factors driving the downward trend in the natural rate of interest. Global R^* played a major role, but in the presence of impediments to capital mobility, asymmetries between some of the underlying drivers help explain the cross-country differences.

In particular, productivity and demographic forces pushing the risk-free rate downwards were less strong for the UK case in comparison to the rest of the world. However, risk premium trends, especially over the last 20 years of the sample, played a similar role.

Comparison With Alternative Estimates. In order to compare our UK-specific $\tilde{R}^*$ estimates, we depict the estimated path for the three capital mobility scenarios described previously, but

Figure 8: UK R^* decomposition
(p.p. changes w.r.t. 1960)



Note: estimated R^* corresponds to $\phi = -0.2$ case. 'Total' refers to UK real interest rate estimated by the model with all the drivers, the remaining refer to changes w.r.t. 1960 UK real interest rate estimated changing one driver at a time.

with respect to 1970 as the starting point in Figure 9[13]. We find that, consistent with findings from previous studies for UK and EA, $\tilde{R}^*$ declined around 2pp from 1970 to 2020, with most of the decrease happening in the last 20 years.

5 Potential Drivers of UK $\tilde{R}^*$ in the Future

Having taken stock of the model-implied decomposition of $\tilde{R}^*$ in the previous section, we now look ahead and ask: how might $\tilde{R}^*$ evolve in the future? First, we consider how cross-country asymmetries in UK population growth, longevity, productivity growth and risk premia, *vis-à-vis* the US, could weigh on $\tilde{R}^*$ in coming years, using projected trends. Second, we ask how changes in global integration could impact $\tilde{R}^*$.

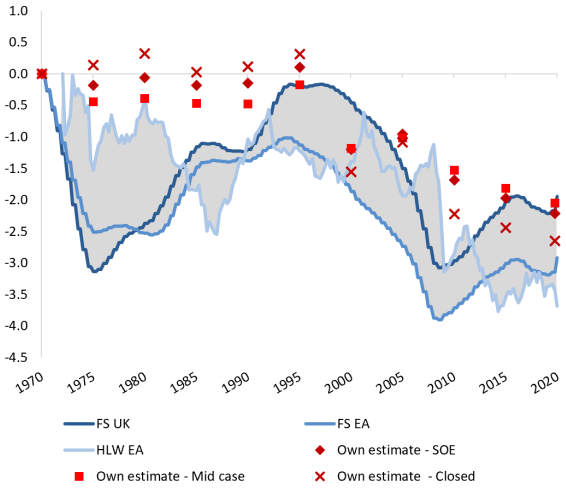
5.1 Key Drivers and Their Relative Trends, 2020-2050

In what follows, we assess how projected trends in economic drivers could impact trend real interest rates. We focus on cross-country asymmetries in four of the main drivers captured by our model: population growth, longevity, productivity growth and risk premia.

For population growth and longevity, the projections come from UN median fertility scenarios. As Figure 2 shows, these projections suggest that the UK will age faster than the US in the coming decades. Longevity, therefore, will push UK $\tilde{R}^*$ down relative to Global R^*.

[13]This allows us to introduce and compare the estimates from Holston et al. (2023) for Europe, in addition to the downside estimates for the UK and EA provided by Ferreira and Shousha (2023).

Figure 9: UK selected R^* estimates

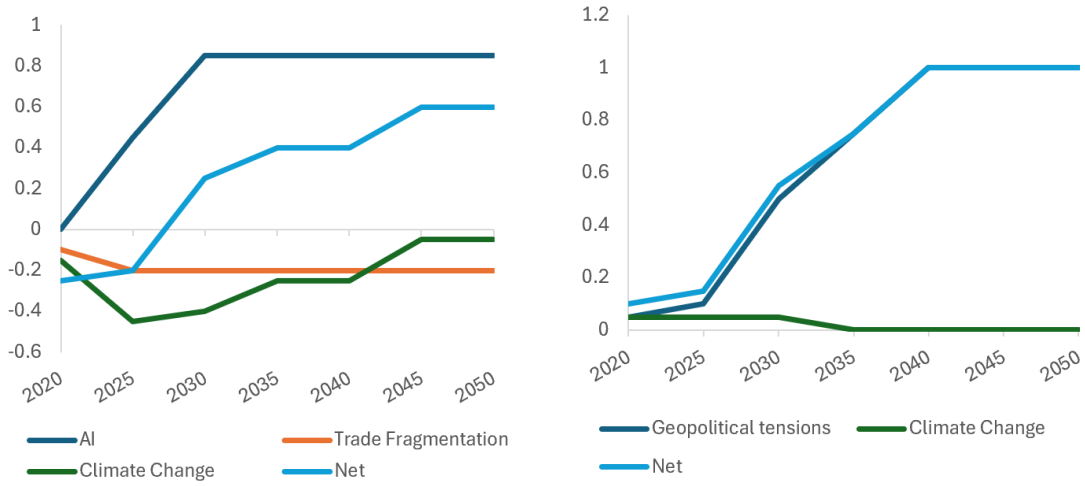


Source: models included are Ferreira-Shousha (FS), and Holston-Labauch-Williams (HLW) . Estimates refer to 'UK' or 'EA'. Own estimates in p.p. differences w.r.t. 1970.

Projections for productivity growth combine the net effects of three factors: AI adoption, trade fragmentation, and climate change. We construct the contribution from AI adoption using data on medium-term earnings per share expectations from S&P500 constituents, sourced from IBES. We use this to estimate the marginal increase in productivity growth consistent with observed and anticipated AI-related investments in the tech sector. Alone, this implies a 0.45pp increase in productivity in 2025, increasing to 0.85pp as AI adoption broadens across sectors.

While AI adoption positively impacts productivity growth, trade fragmentation and climate change are projected to exert negative pressure. For climate change, our projection relies on findings from the IMF (2024) and the Network for Greening the Financial System (NGFS). While climate change and related policies may initially dampen productivity growth, net-zero policies are projected to boost long-term productivity through lower energy costs, green innovation, and reduced climate-related risks. For consistency with debt-to-GDP effects, we derive productivity growth rates in our central and upside scenarios from the NGFS 'Net Zero 2050' and 'Delayed Transition' pathways, respectively. This scenario implies a peak productivity decline in 2025. To incorporate the impact of trade fragmentation, we account for the observed decline in 2020 due to global supply-chain disruptions from the Ukraine conflict, projecting an expected acceleration of up to -0.2pp going forward motivated by the risk of geopolitical tensions.

Figure 10: Productivity Growth and Risk Premia 2050 Scenarios



Note: Left and right panel are 5 year percentage points changes.

Figure 10 shows the net effects of these three factors on UK productivity growth. Combined, the three factors push up on UK productivity growth, placing upside pressure on UK equilibrium rates over time.

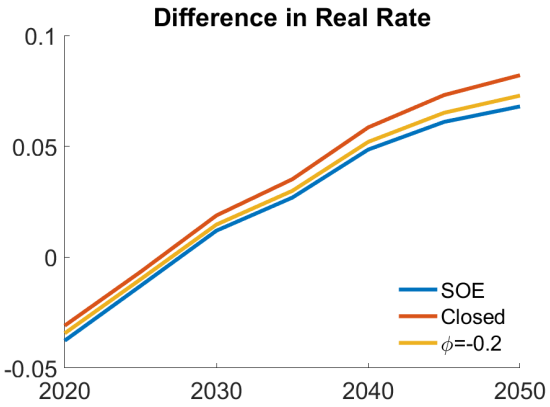
As shown in Figure 11, accounting for productivity asymmetries implies downward pressure on both global Global R^* and UK-specific $\tilde{R}^*$ through 2030, driven by trade fragmentation and the implications of climate change. However, from that point onward, as AI adoption expands across sectors, productivity growth exerts upward pressure on natural interest rates across all deglobalisation scenarios relative to 2020. The more disintegrated the capital markets, the greater this upward pressure, reaching up to around 0.1 percentage points by 2050 in the extreme case of total fragmentation (i.e., under a closed economy scenario).

Finally, we construct projections for risk premia. Although the trend in average equity risk premia across countries has been relatively stable since 2015,[14] other measures of risk, such as economic policy uncertainty and geopolitical risk indices, have surged in the aftermath of COVID-19 and recent conflicts in Ukraine and the Middle East. Therefore, our projections incorporates a gradual increase in risk premia, reaching 1pp per 5 years from 2040 onwards. We additionally account for climate-related risk premia, with our scenario assuming a cumulative 15bp increase over the 2020–2030 period. The net effect of these two factors is shown in Figure 10.

As shown in Figure 12, an increase of 1pp every 5 years through 2050, driven by a pes-

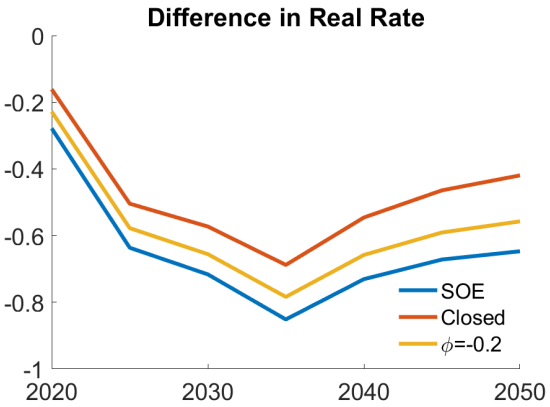
[14] Based on Jorda et al. (2020)'s macro-financial database.

Figure 11: Effect of Productivity - Prospects



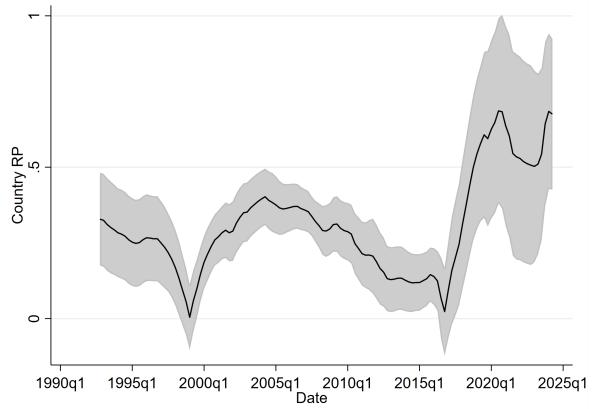
Note: panel is in percentage points, difference calculated w.r.t. estimate without productivity changes. Productivity prospects incorporated into estimations are the total (net) effect from AI developments, trade fragmentation effects and climate change impacts).

Figure 12: Effect of Risk Premia - Prospects



Note: panel is in percentage points, difference calculated w.r.t. estimate without risk premia changes. Risk Premia prospects incorporated into estimations are the total (net) effect from Covid and geopolitical tensions, and climate change impact.

Figure 13: Rolling window $|\hat{\phi}|$



Note: Country RP ϕ in absolute value, estimated with 15 years rolling windows in black, 95% confidence intervals in shaded grey area.

simistic outlook on geopolitical tensions, implies a steady and similar downward pressure on $\tilde{R}^*$. However, this trend diminishes with increasing fragmentation, potentially reducing the downward pressure by as much as up to 0.4 percentage points.

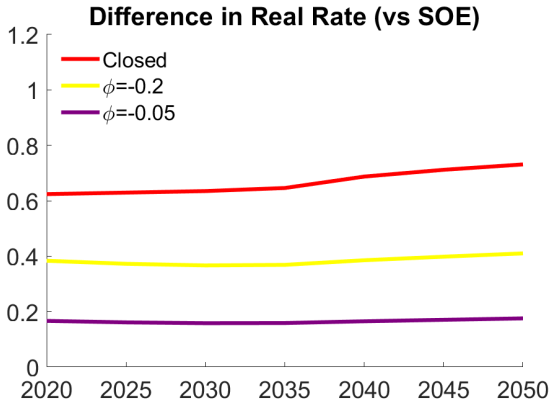
5.2 Effects of Fragmentation

Without assuming any projected changes in the economic drivers of natural rates, even slight changes in global capital mobility may impact the wedge between Global R^* and country-specific rates in small-open economies, $\tilde{R}^*$. As discussed throughout this paper, this factor is captured in the calibration of ϕ in equation (1). While our baseline calibration of ϕ is informed by a static estimation of regression (2), we can assess the extent to which the degree of global capital mobility has changed over time by estimating the same regression in a time-varying manner.

Figure 13 plots the implied estimates of ϕ over time from a 15-year rolling-window estimation of regression (2). They are plotted in absolute terms such that an increase in $|\phi|$ is associated with diminished global capital mobility or increased fragmentation. The estimates indicate that estimates of ϕ were broadly in line with the full-sample estimate of 0.12 (reported in Table 1) over the majority of the sample. However, in recent years, implied $|\phi|$ estimates have increased markedly—indicative of heightened geo-economic fragmentation impacting global capital mobility. Compared to the full-sample average, the estimates for the most recent period increase almost five-fold in magnitude, to near 0.6.

Figure 14 shows how estimates of UK $\tilde{R}^*$ may change with different values of ϕ. Assuming

Figure 14: Effects of Geo-economic Fragmentation on Wedge Between UK $\tilde{R}^*$ and Global R^*



Note: panel is in percentage point difference w.r.t. SOE estimation.

that the UK had a ϕ near zero before 2020, an increase in absolute value, for example to $\phi = 0.05$, would already imply an estimated $\tilde{R}^*_{2050}$ that is around 0.2pp higher compared to the Global R^* estimate. In the most extreme case, implied by Figure 13, the wedge between Global R^* and UK $\tilde{R}^*$ rises to over 0.6pp.

6 Conclusion

This paper has explored the behaviour and determinants of trend real interest rates in small-open economies $\tilde{R}^*$, focusing on the case of the UK over the period 1960-2020. Model simulations indicate that imperfections in global capital markets imply a muted decline in UK equilibrium rates than their global counterparts over the last 60 years—with UK $\tilde{R}^*$ declining by around 2.5pp relative to the near-3pp decline in Global R^*. The decline in productivity growth growth and demographic factors, which weighed most heavily on Global R^*, have been less pronounced in the UK. Such asymmetric trends explain the more limited reduction in the UK's equilibrium rate over the last six decades.

Looking ahead, asymmetries could continue to generate a wedge between equilibrium rates in the UK and globally.

Even without such asymmetries, any acceleration of geo-economic fragmentation poses an upside risk to estimates of $\tilde{R}^*$. Reduced openness could increase equilibrium rates in the UK by 0.6pp.

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