Multilateral Comovement in a New Keynesian World: A Little Trade Goes a Long Way*

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Abstract

International linkages and nominal rigidities result in a global system of Phillips curves tying each country's inflation to a trade-weighted average of real activity abroad. Moreover, disturbances propagate across countries directly through pairwise trade and indirectly through knock-on network effects. We estimate that absent trade, the average cross-country correlation of output growth falls by 90 percent, while that of inflation, and of inflation and output growth, fall by 1/2. We show analytically that knock-on effects and nominal rigidities are key features underlying these findings. These features also help resolve the Backus-Kehoe-Kydland puzzle, producing lower cross-country correlations in consumption than output.

Keywords: International Comovement, Multilateral Trade, New Keynesian Phillips Curve
JEL Codes: E31, E32, F41, F44

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1 Introduction

Inflation and output growth exhibit substantial comovement between countries. Remarkably, the correlation between inflation and output growth at business cycle frequencies, a hallmark moment for the New Keynesian framework, is at times as pronounced *across* countries as it is *within* countries. However, given that large economies, including the U.S. or the European Union (E.U.), purchase goods and services mostly from themselves, with shares of domestic purchases exceeding 80 percent of total purchases, the quantitative study of inflation dynamics has largely proceeded in the context of closed-economy models. Is it then the case that international trade linkages have only a small role in explaining comovement across countries and that global shocks are the main drivers of common variations in output growth or inflation? In this paper, we develop a structural framework featuring multiple large open economies with nominal rigidities that suggests otherwise.

The structural framework we develop brings together nominal rigidities in the tradition of New Keynesian models (e.g., Justiniano, Primiceri, and Tambalotti (2010); Herbst and Schorfheide (2015); Bianchi and Melosi (2017)) with an emerging literature on dynamic models featuring multiple regions (e.g., Devereux, Gente, and Yu (2023); Huo, Levchenko, and Pandalai-Nayar (2023); Kleinman, Liu, and Redding (2023)). In particular, the network structure of trade gives foreign demand shocks knock-on effects that enhance cross-country comovement relative to previous work in international real business cycles (IRBC) (e.g., Kose and Yi (2006); Corsetti, Dedola, and Leduc (2008); Bai and Ríos-Rull (2015); Uribe and Schmitt-Grohé (2017)) and New Keynesian open economy (NKOE) models (e.g., Corsetti and Pesenti (2001); Devereux and Engel (2002)).

Network effects in our framework arise by way of both *backward* trade linkages, summarizing a country's import network, and *forward* trade linkages, summarizing its export network. Consequently, despite relatively low observed trade shares across countries, country-specific shocks generate sizeable cross-country comovement in output growth, in inflation, and between output growth and inflation.[1] In a large class of existing NKOE models, these network effects are mitigated by perfect pass-through from exchange rates to local prices, leading to substantially smaller spillover effects than we find (Justiniano and Preston (2010)). In contrast, consistent with empirical findings in Devereux and Yetman (2010), our model features imperfect exchange rate pass-through to local prices, which amplifies the knock-on effects from multilateral trade in the propagation of shocks across countries. Moreover, these features deliver greater cross-country comovement in output than consumption, thus helping to resolve the Backus-Kehoe-Kydland puzzle intrinsic to the IRBC literature (Backus, Kehoe, and Kydland (1992)). Lower cross-country comovement in consumption arises here because price stickiness emphasizes the role of demand forces in driving economic fluctuations.

International linkages between countries subject to nominal rigidities imply important depar-

[1] In that sense, our analysis generalizes the seminal contributions of Clarida, Gali, and Gertler (2002) and Galí and Monacelli (2005) to a setting with multiple large countries with network implications from trade for global comovement across countries.

tures from the conventional closed-economy New Keynesian Phillips curve (NKPC). First, whereas marginal costs are only a function of domestic input costs in a closed economy, they now also depend on foreign input costs and exchange rates of all trading partner countries. Second, these input costs, and notably wages, in part reflect demand conditions abroad.

The first departure from the closed-economy NKPC implies a global system of Phillips curves explicitly connected by weights reflecting multilateral trade relationships. Inflation in one country is thus connected to variations in output growth, consumption, and exchange rates in every other country. The coefficients in this system of Phillips curves provide a structural interpretation for the weights used in empirical work relating domestic inflation to foreign real activity (e.g., Borio and Filardo (2007); Eickmeier and Pijnenburg (2013); Bianchi and Civelli (2015)).

The second departure from standard New Keynesian mechanisms links input costs to foreign demand conditions. Because labor supply is imperfectly elastic within countries, multilateral trade implies that positive demand shocks in one country induce higher output and, thus, higher input costs across multiple countries. In equilibrium, foreign disturbances affect output growth not only through pairwise trade between countries but also indirectly through knock-on effects arising from the network structure of trade. For instance, as Europe increases domestic production to meet higher demand in the U.S., there is a rise in wages and other input costs in Europe, which raises European demand for foreign goods, prompting higher imports from Japan and so on.

Our modeling approach follows Devereux and Yetman (2010) in introducing nominal rigidities as a wedge between the prices of traded goods and local final goods. Global supply chains then matter for local inflation and output even if exchange rates were determined in frictionless financial markets. Thus, international comovement arises in potentially two ways. First, it may follow from the endogenous propagation of idiosyncratic country shocks through economic linkages, as explored in Imbs (2006) or Boeckelmann, Imbs, and Pauwels (2024). Second, cross-country comovement may be driven more directly by global factors, as discussed in Mumtaz, Simonelli, and Surico (2011). This paper incorporates these two mechanisms within a single structural framework summarized by a parsimonious set of equations. In the absence of trade, the model collapses to the canonical closed-economy New Keynesian model country by country. More generally, however, New Keynesian forces interact with the effects of foreign input costs and exchange rates on marginal costs and of trade flows on local demand.

We use our structural model to quantify the role of both global factors and international linkages for cross-country comovement in output growth, in inflation, and between output growth and inflation. Our findings derive from a full-information Bayesian estimation of almost all structural parameters (e.g., Justiniano and Primiceri (2008); Schmitt-Grohé and Uribe (2012)). This approach allows us to estimate the cross-country effects of shocks while accounting for the network effects of trade and other general equilibrium channels. Our data cover five world regions—the U.S. China, the E.U., Japan, and Canada. We allow for heterogeneous structural parameters, such as in the degree of nominal rigidities across countries.

Our estimation reveals that despite trade shares generally being less than 20 percent of national purchases, multilateral trade plays a key role in explaining international comovement in output growth, in inflation, as well as between output growth and inflation. Absent trade, the average cross-country correlation in output growth falls by 90 percent, that in inflation by 58 percent, and that between output growth and inflation by 44 percent.[2] We show analytically that nominal rigidities and demand shocks are central to these comovement findings. In contrast, because domestic productivity shocks move both consumption and output at home, the role of trade is mitigated where these shocks are concerned. Finally, we show that these New Keynesian features help resolve the Backus-Kehoe-Kydland puzzle and produce cross-country output correlations that exceed those of consumption. In this setting, an increase in demand idiosyncratic to any one country raises production across multiple other countries.

Estimating a New Keynesian model with multiple large countries poses two key computational challenges to dynamic models with a spatial dimension. First, there are 2.5 times as many parameters and about double the number of equations relative to Smets and Wouters (2007).[3] Second, steady state calculations are more involved than typical closed-economy New Keynesian models, requiring an iterative procedure that can take up over half the time needed to compute the likelihood for a given set of parameters. We use state-of-the-art adaptive sequential Monte Carlo algorithms (Cai, Del Negro, Herbst, Matlin, Sarfati, and Schorfheide (2021)) to overcome these challenges.

This paper is organized as follows. Section 2 reviews some motivating data. Section 3 presents the economic environment. Section 4 highlights key equilibrium conditions that describe the interplay between sticky prices and multilateral trade. Section 5 discusses the model solution in linearized form. Section 6 explores the mechanisms underlying global spillovers from country-specific shocks. Section 7 describes our quantitative findings and explores their robustness along different dimensions. Section 8 concludes. The online Appendix contains derivations of all results, a comprehensive description of the data and statistical methods, discussions of departures from our benchmark assumptions, and additional figures and tables complementing our main analysis.

2 GDP Growth and Inflation Across Countries

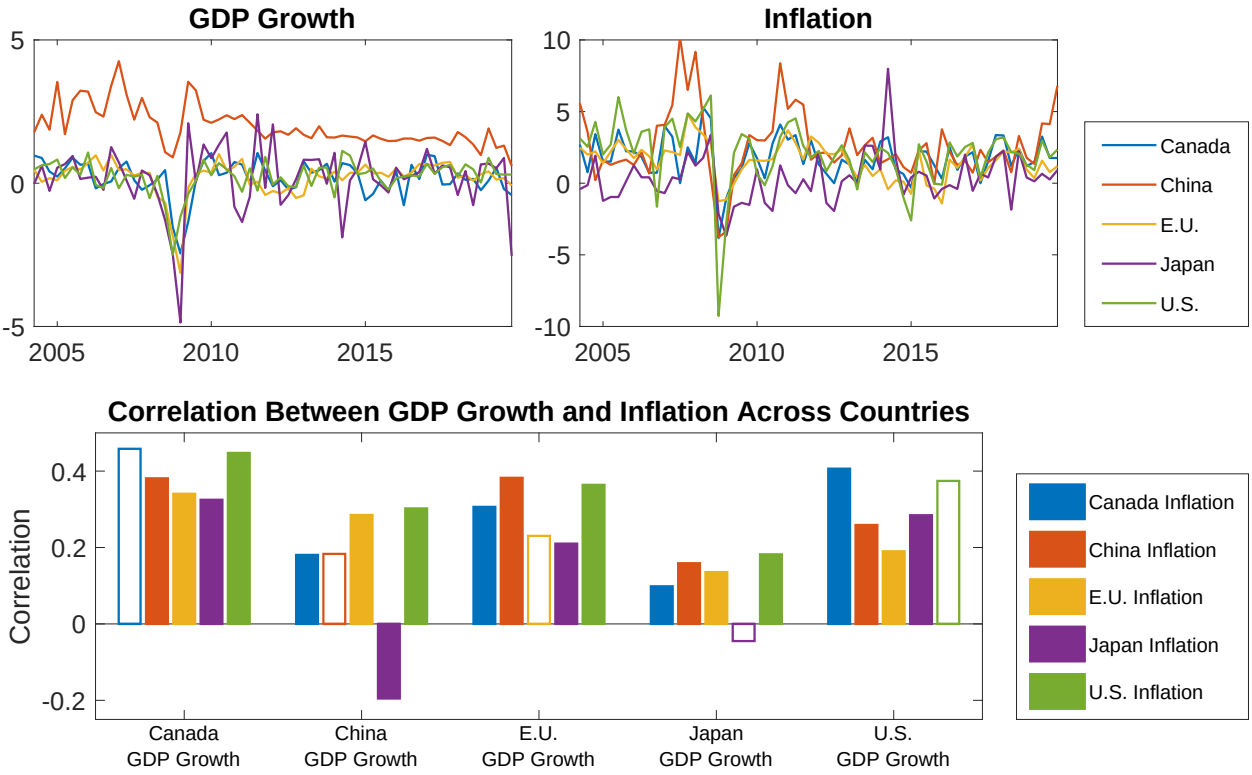
We begin by documenting some key stylized facts that motivate the paper using data from five world regions, the U.S. China, the E.U., Japan, and Canada, over the period 2004Q2 through 2019Q4. These regions cover roughly 62 percent of world GDP over that period.

The top left-hand panel of Figure 1 illustrates the behavior of real GDP growth in each region.

[2] Auer, Levchenko, and Sauré (2019) find that trade linkages can account for a substantial share of Producer Price Index (PPI) inflation across countries conditional on local cost shocks embodying variations in productivity or prices of primary factors. Here, we retain the cross-country comovement in inflation when wages are determined in equilibrium in a setting with nominal rigidities. This setting simultaneously gives rise to comovement in output and between output and inflation across countries.

[3] This particular curse of dimensionality is noted by Guerron-Quintana (2013) as a significant obstacle to the estimation of New Keynesian models with more than two countries.

Figure 1: Comovement in Real GDP and Inflation Within and Across Regions



Notes: In upper panels, GDP growth rates are quarter-over-quarter and deflated by actual inflation, while inflation rates are quarter-over-quarter and annualized. In the lower panel, each bar corresponds to the correlation between GDP growth in country i with inflation in country j. Filled bars correspond to cross-country correlations (i.e., $i \neq j$), and empty bars correspond to own-country correlations (i.e., $i = j$) without trade. Each group of bars corresponds to a country i's GDP growth, and each color corresponds to a country j's inflation.

While there are differences across regions, particularly with respect to the level of GDP growth in China, the panel shows positive comovement in real activity across countries. The mean pairwise correlation in GDP growth across countries is around 0.4, with Europe and Japan exhibiting the largest correlation in real activity at 0.6. The top right-hand panel of Figure 1 similarly shows that inflation rates tend to move together across countries, with a mean pairwise correlation of 0.45. The degree of comovement in inflation between a given pair can be as large as 0.8, as in the case of Canada and the U.S. Similar correlations have been documented in the literature for various sets of countries and sample periods.[4]

While both of these observations have been well documented for a wide range of countries and time periods, the bottom panel of Figure 1 shows a less known fact—inflation and output comove

[4]For real variables, Kose, Otrok, and Whiteman (2003) study a 60-country sample from 1960 through 1990, Stock and Watson (2005) focus on G7 countries between 1960 and 2002, Del Negro and Otrok (2008) consider 19 developed countries from 1970 through 2005, and Ferroni and Klaus (2015) analyze the Euro Area between 1998 and 2012. For inflation, Ciccarelli and Mojon (2010) analyze 22 OECD countries from 1971 through 2004 and Mumtaz and Surico (2012) study 13 countries from 1961 through 2004.

positively not only *within* countries but also *across* countries. Interestingly, the correlation between real activity and inflation may be as large or larger across countries as it is within countries. For instance, the correlation of E.U. real GDP growth with E.U. inflation is around 0.2 but well above 0.3 with U.S. inflation and almost 0.4 with Chinese inflation. Similarly, Canadian real GDP growth comoves as much with Canadian inflation as it does with U.S. inflation. Overall, mean cross-country and own-country correlations are of similar magnitude, at 0.25 and 0.24, respectively. The notion of a Phillips curve, therefore, is potentially as relevant across as within countries.[5]

Two mechanisms potentially underlie the comovement between countries depicted in Figure 1. One relates to global factors in productivity, policy, or other disturbances. Another is spillovers across countries arising from the endogenous propagation of country-specific shocks through international linkages. How does the interplay between global factors and the propagation of country-specific shocks shape the dynamics of output and inflation across countries? What do these interactions imply for the comovement shown in Figure 1? This paper presents a quantitative framework to address these questions.

3 Economic Environment

This section presents a New Keynesian world economy where multilateral trade allows disturbances abroad to spill over to domestic variables through network effects. There are N countries indexed by $n = 1, ..., N$. As in the prototypical New Keynesian closed-economy model, each country has three types of agents: firms that produce various types of outputs, households that consume final goods aggregated from these outputs, and a monetary authority that sets interest rates.

We model the interplay between sticky prices and multilateral trade by having two layers of intermediate inputs rather than the single layer of monopolistically competitive firms in the standard model. The first layer consists of different varieties that may be purchased from different countries. We refer to this first layer of intermediate varieties as *tradable goods*. The second layer consists of intermediate inputs that are subject to nominal rigidities, as in Calvo (1983), and assembled into final goods. We refer to this second layer of intermediate goods as *sticky-price goods*. Final goods can be used for consumption, as materials to produce tradable goods via roundabout production, or to pay for costs associated with trades in international financial markets.

We introduce trade in financial assets by allowing households to not only purchase and sell bonds domestically but also trade state-contingent assets internationally. Frictions in global financial markets determine how much countries can run trade deficits or surpluses, thereby influencing the behavior of exchange rates and the way that shocks spill over across countries. Securities traded in global financial markets are subject to these portfolio adjustment costs and exogenous disturbances that, in equilibrium, introduce deviations from uncovered interest parity (UIP).

[5] Borio and Filardo (2007) more formally include global variables in a reduced form Phillips curve and find a significant and increasingly important role for them in determining domestic inflation in their sample of 16 advanced economies and the Euro area between 1980 and 2005, consistent with the high cross-country correlations here.

Finally, a monetary authority in each country sets policy according to a country-specific Taylor rule. To the degree that some coordination exists between different central banks, we allow for common factors across countries' monetary policy shocks.

3.1 Households

The representative household in country n has preferences given by

$$\sum_{t=0}^{\infty} \beta^t \left\{ \frac{(C_{n,t}/A_{n,t})^{1-\sigma_n} - 1}{1-\sigma_n} - \omega_n \frac{L_{n,t}^{1+\eta_n}}{1+\eta_n} \right\}, \ 0 < \beta < 1, \ \sigma_n, \omega_n, \eta_n > 0, \tag{1}$$

where $L_{n,t}$ denotes labor supplied by the household, $C_{n,t}$ is a bundle of consumption goods, and $A_{n,t}$ summarizes country n's technical progress. As in Lubik and Schorfheide (2005), the presence of the term $A_{n,t}$ in equation (1) ensures that each country evolves along a balanced growth path in the long run, allowing hours worked, $L_{n,t}$, to remain stationary even if the intertemporal elasticity of substitution, $1/\sigma_n$, differs from 1.

Households in country n face the budget constraint

$$\begin{aligned} P_{n,t}C_{n,t} + \frac{B_{n,t}}{R_{n,t}} + E_t\left[\frac{V_{t+1}}{\mathcal{E}_{rn,t}} S_{n,t+1}\right] + \text{Portfolio Adjustment Costs} \\ = W_{n,t}L_{n,t} + \Pi_{n,t} + B_{n,t-1} + \frac{1}{\mathcal{E}_{rn,t}} S_{n,t}, \end{aligned} \tag{2}$$

where $P_{n,t}$ is the nominal price of final goods, $W_{n,t}$ denotes nominal wages, and $\Pi_{n,t}$ represents nominal firm profits. All prices in country n are expressed in units of the local currency. The term $B_{n,t}$ denotes the quantity of a bond traded domestically by households at price $1/R_{n,t}$ at date t that pays 1 unit of the local currency in the following period. As in the usual closed-economy household budget constraint, the first two terms on the left-hand side capture spending on consumption and bond purchases, respectively, while the first three terms on the right-hand side capture labor income, profits, and bond returns.

The budget constraint (2) differs from the usual closed-economy household budget constraint in that countries trade state-contingent securities internationally, $S_{n,t+1}$, subject to portfolio adjustment costs. The security is purchased at date t by country n at price V_{t+1}, expressed in the currency of a reference country r. It pays off 1 unit of the reference country currency, translated into local currency units by the exchange rate $\mathcal{E}_{rn,t}$ in the budget constraint (2).[6]

As in Schmitt-Grohé and Uribe (2003), trading securities internationally is subject to portfolio adjustment costs. Similar to Ohanian, Restrepo-Echavarria, and Wright (2018), these are borne

[6] Thus, if reference country r uses, say, the Euro and country n is the U.S., $\mathcal{E}_{rn,t}$ is expressed in €/\$. Moreover, $\frac{1}{\mathcal{E}_{rn,t}} S_{n,t}$ then represents a payoff in dollars from $S_{n,t}$ shares of the state-contingent security purchased at date $t-1$. Observe further that no-arbitrage in currency markets implies that $\mathcal{E}_{nn',t} = \mathcal{E}_{nn'',t}\mathcal{E}_{n''n',t}$.

separately for each state contingent security:

$$\text{Portfolio Adjustment Costs} = E_t\left[P_{n,t}A_{n,t}\Theta_n\left(\frac{V_{t+1}}{\mathcal{E}_{rn,t}P_{n,t}}\frac{S_{n,t+1}}{A_{n,t}}+\frac{1}{\tau}\xi_{n,t}\right)\right].$$

The function $\Theta_n(\cdot)$ reflects a cost incurred by country n when adjusting its financial position. We assume that $\Theta_n(0)=\Theta_n'(0)=0$ and $\Theta_n''(\cdot)=\tau>0$ so that portfolio adjustment costs increase at an increasing rate the larger country n's financial position is in absolute value. The curvature parameter τ captures the degree of financial frictions in global financial markets: $\tau\to 0$ corresponds to perfect capital markets, while $\tau\to\infty$ represents financial autarky. The function Θ_n varies with two objects. First, portfolio adjustment costs incurred by country n depend on $\frac{V_{t+1}S_{n,t+1}}{\mathcal{E}_{rn,t}P_{n,t}A_{n,t}}$, the real value of state-contingent securities purchased in global financial markets in units of local goods (i.e., the consumption bundle, $C_{n,t}$), normalized by $A_{n,t}$ to account for country n's level of technical progress. Second, the term $\frac{1}{\tau}\xi_{n,t}$ captures exogenous disturbances that affect country n's ability to trade in international financial markets.

Substantively, the financial market adjustment costs, Θ_n, and the shocks, $\xi_{n,t}$, allow the estimated model to match exchange rate movements. In particular, they imply deviations from the UIP condition that are an important feature of exchange rate data. Since exchange rates generally affect the behavior of domestic inflation in an international setting (see Clarida, Gali, and Gertler (2002) and Galí and Monacelli (2005)), UIP deviations are part of global inflation dynamics.

3.2 Firms

The production side consists of final goods and two layers of intermediate goods. As in the conventional New Keynesian framework, final goods are produced using intermediate inputs subject to nominal rigidities. We refer to this layer of intermediate goods as sticky-price goods. However, sticky-price goods are now produced using another layer of intermediate varieties that can be purchased from other countries, thus tying down local inflation to global supply chains.

Final Goods: We denote final goods in each country by $\mathcal{C}_{n,t}$. These goods are assembled by firms using sticky-price intermediate goods, $y_{n,t}(j)$, with the technology

$$\mathcal{C}_{n,t}=\left[\int_0^1 y_{n,t}(j)^{\frac{\varepsilon_{n,t}-1}{\varepsilon_{n,t}}}dj\right]^{\frac{\varepsilon_{n,t}}{\varepsilon_{n,t}-1}},\ \varepsilon_{n,t}>1.$$

We index the elasticity of substitution between intermediate varieties by t to account for potential time-varying markups, denoted by $\mu_t=\frac{\varepsilon_t}{\varepsilon_t-1}>1$. Firms that produce final goods are competitive and, given the constant-returns-to-scale technology, make zero profits. Given a price $P_{n,t}(j)$ for

variety j, cost minimization implies the demand function

$$y_{n,t}(j) = \left(\frac{P_{n,t}(j)}{P_{n,t}}\right)^{-\varepsilon_{n,t}} \mathcal{C}_{n,t}, \tag{3}$$

where $P_{n,t} = \left[\int_0^1 P_{n,t}(j)^{1-\varepsilon_{n,t}} dj\right]^{\frac{1}{1-\varepsilon_{n,t}}}$ such that $P_{n,t}\mathcal{C}_{n,t} = \int_0^1 P_{n,t}(j) y_{n,t}(j) dj$.

Final goods may be used by households either for consumption proper, $C_{n,t}$, or to pay for portfolio rebalancing costs, $A_{n,t}\Theta(.)$. Final goods may also be used as materials, $M_{n,t}$, by firms producing traded intermediate inputs. Thus, we have that

$$\mathcal{C}_{n,t} = C_{n,t} + M_{n,t} + E_t\left[A_{n,t}\Theta_n\left(\frac{V_{t+1}}{\mathcal{E}_{rn,t}P_{n,t}}\frac{S_{n,t+1}}{A_{n,t}} + \frac{1}{\tau}\xi_{n,t}\right)\right]. \tag{4}$$

As in Basu (1995), the fact that final goods can be used as materials implies a roundabout production structure that captures the feedback effects from input-output relationships.

Non-Tradable Sticky-Price Intermediate Goods: In country n, each variety j of sticky-price goods is produced using a composite input bundle, $Q_{n,t}(j)$, according to

$$y_{n,t}(j) = A_{n,t}Q_{n,t}(j). \tag{5}$$

This input bundle consists of different varieties (described below) that are traded across countries. Crucially, shocks affecting production costs abroad will then affect firm choices at home. We denote the price index associated with the composite bundle by $P^Q_{n,t}$ so that $P^Q_{n,t}Q_{n,t}(j) = MC_{n,t}y_{n,t}(j)$, where $MC_{n,t} = P^Q_{n,t}/A_{n,t}$ is the nominal marginal unit cost faced by sticky-price goods producers. Their real marginal cost is then given by

$$mc_{n,t} = \frac{P^Q_{n,t}}{P_{n,t}A_{n,t}}, \tag{6}$$

which reflects foreign production costs through the price index of tradable goods, $P^Q_{n,t}$.[7]

As in Calvo (1983), in each period a firm producing sticky-price goods in country n is unable to choose its price optimally with probability θ_n. It then sets its price using the indexation rule, $P_{n,t}(j) = (1+\pi_n)P_{n,t-1}(j)$, where π_n denotes country n's long-run or steady-state inflation rate.[8] With probability $1-\theta_n$, the firm is able to reset its price and optimally chooses a price, $P^*_{n,t}$, that

[7] This is in contrast to the model's closed-economy counterpart or even alternative open economy settings in which producers subject to sticky prices only used locally sourced inputs.

[8] The online Appendix considers a more general indexation rule that includes lagged inflation, denoted $1+\pi_{n,t-1} = P_{n,t-1}/P_{n,t-2}$. In that case, firms that are unable to choose their price set $P_{n,t}(j) = (1+\pi_{n,t-1})^{\varrho_n}(1+\pi_n)^{1-\varrho_n}P_{n,t-1}(j)$, $\varrho_n \in [0,1]$, where ϱ_n and $1-\varrho_n$ indicate the degree of indexation to lagged inflation and long-run inflation respectively.

maximizes its expected present discounted value of future profits,

$$E_t\left[\sum_{k=0}^{\infty}\beta^{t+k}\frac{\Lambda_{n,t+k}}{\Lambda_{n,t}}\theta^k\left(\frac{P^*_{n,t}(1+\pi_n)^k}{P_{n,t+k}}\right)^{-\varepsilon_{n,t}}\left(\frac{P^*_{n,t}(1+\pi_n)^k}{P_{n,t+k}}-mc_{n,t+k}\right)\mathcal{C}_{n,t+k}\right], \tag{7}$$

where $\beta\frac{\Lambda_{n,t+k}}{\Lambda_{n,t}}$ is the discount factor from t to $t+k$.

The implied first-order conditions indicate that under constant markups, $\mu=\frac{\varepsilon}{\varepsilon-1}>1$, the nominal price chosen by sticky-price producers at date t, $P^*_{n,t}$, is proportional to a weighted average of their expected future nominal marginal costs. Inflation, therefore, will tend to rise when those marginal costs are expected to increase. Importantly, because domestic marginal costs depend on conditions abroad, foreign shocks will affect domestic prices and inflation.

Globally Traded Intermediate Goods: The second layer of intermediate goods consists of tradables whose production follows for the most part the framework originally laid out in Eaton and Kortum (2002).[9]

Tradable goods come in varieties that differ in their production technology. In particular, varieties are indexed by a vector of country-specific productivity parameters, $\mathbf{z}=\{z_1,z_2,...,z_N\}$, where each element z_n reflects the productivity of firms producing that variety in country n. Formally, the production of tradable variety $\mathbf{z}$ in country n is given by

$$q_{n,t}(\mathbf{z})=z_{n,t}\ell_{n,t}(\mathbf{z})^{\gamma}M_{n,t}(\mathbf{z})^{1-\gamma},\ \gamma\in(0,1), \tag{8}$$

where $\ell_{n,t}(\mathbf{z})$ and $M_{n,t}(\mathbf{z})$ denote respectively local labor and materials used in the production of that variety.[10] As in Basu (1995), the roundabout production structure, whereby final goods also serve as materials, helps capture the role of input-output relationships in generating international comovement in real activity underscored by Caselli, Koren, Lisicky, and Tenreyro (2020), Huo et al. (2023) and Devereux et al. (2023).

Producers of sticky-price goods in country n can acquire any tradable variety $\mathbf{z}$ necessary to produce those goods from any country, including their own. From here on, we assume that sticky-price goods $j\in[0,1]$ are symmetric and abstract from the j index, $Q_{n,t}(j)=Q_{n,t}$. Thus, $\forall j\in[0,1]$, the demand for tradable variety $\mathbf{z}$ in country n is

$$Q_{n,t}(\mathbf{z})=\sum_{n'}Q_{nn',t}(\mathbf{z}),$$

[9] In its basic form, this framework provides microfoundations for home bias and trade elasticities arising from the dispersion of product-level productivity shocks and trade costs while yielding demand functions for goods produced abroad that can also be obtained using Armington preferences.

[10] We follow the convention adopted by Eaton and Kortum (2002) in indexing varieties by their productivity vectors, $\mathbf{z}$, rather than an arbitrary label $i\in[0,1]$. Observe that any differences in production and prices across varieties reflect only productivity differences. Therefore, denoting the output of variety i by $q_{n,t}(i)$, and letting $\mathbf{z}(i)$ represent its corresponding productivity vector, it follows that $q_{n,t}(i)=q_{n,t}(\mathbf{z}(i))$. Thus, one need only refer to $q_{n,t}(\mathbf{z})$.

where $Q_{nn',t}(\mathbf{z})$ denotes traded varieties produced by country n' and used by country n. These tradable varieties are then aggregated into a single composite tradable bundle according to a CES aggregator, $Q_{n,t} = \left[\int Q_{n,t}(\mathbf{z})^{\frac{\gamma-1}{\gamma}} d\Phi(\mathbf{z})\right]^{\frac{\gamma}{\gamma-1}}$, $\gamma > 1$, which is then used as an input by sticky-price firms in equation (5). The function Φ denotes the joint *c.d.f.* from which $\mathbf{z}$ is drawn.

As in Eaton and Kortum (2002), the elements of $\mathbf{z}$ are jointly drawn from a Fréchet distribution with shape parameter φ, $\Phi(\mathbf{z}) = \exp\left\{-\sum_n (z_n)^{-\varphi}\right\}$. This distributional assumption ensures analytically tractable aggregate import demand functions (discussed in detail below). The shape parameter, φ, governs how much a given variety $\mathbf{z}$ is likely to differ in productivity across countries and implies a trade elasticity of $1 + \varphi$. Low values of φ imply large cross-country variations in productivity. In contrast, as $\varphi \to \infty$, the level of productivity associated with tradable varieties becomes common across countries.

Sticky-price producers in country n acquire each variety $\mathbf{z}$ from the country that supplies it most cheaply given its production costs and bilateral trade costs. The technology in equation (8) implies nominal production costs in country n' of $W_{n',t}^{\gamma} P_{n',t}^{1-\gamma} / z_{n',t}$, where $W_{n't}$ is the nominal wage in n' and $P_{n',t}$ the price of final goods in that country, both in local currency units. The cost of acquiring tradable varieties from abroad also includes an allowance for shipping. In particular, goods imported by country n from country n' are subject to iceberg shipping costs such that only a fraction $1/\kappa_{nn'} \in (0, 1)$ of those goods make it to their destination. This implies that the price paid for variety $\mathbf{z}$ in country n, $P_{n,t}(\mathbf{z})$, satisfies

$$P_{n,t}(\mathbf{z}) = \min_{n'} \kappa_{nn'} \mathcal{E}_{nn',t} \frac{W_{n',t}^{\gamma} P_{n',t}^{1-\gamma}}{z_{n',t}}, \tag{9}$$

where $\mathcal{E}_{nn',t}$ converts prices quoted in the currency of country n' into that of country n. It also follows that $q_{n,t}(\mathbf{z}) = \sum_{n'} \kappa_{n'n} Q_{n'n,t}(\mathbf{z})$ since actual output of any tradable variety $\mathbf{z}$, $q_{n,t}(\mathbf{z})$, must account for losses in transit in meeting the demand for that variety. As we show below, this model structure allows us to match empirical trade shares across all country pairs.[11]

Observe in equation (9) that because of roundabout production, the price of tradable goods paid by country n depends not only on labor costs abroad, $W_{n',t}$, but also on final good prices abroad, $P_{n',t}$. Tradable goods prices thus inherit in part nominal rigidities in final goods prices.

3.3 Monetary Authority

As in Taylor (1993, 1999), monetary policy in country n follows a standard monetary policy rule,

$$R_{n,t} = R_{n,t-1}^{\rho_n} R_{n,t}^{*1-\rho_n} e^{\nu_{n,t}} \text{ where } R_{n,t}^{*} = R_n \left(\frac{1+\pi_{n,t}}{1+\pi_n}\right)^{\phi_{n,\pi}} \left(\frac{Y_{n,t}}{\overline{Y}_{n,t}}\right)^{\phi_{n,Y}}, \tag{10}$$

[11] This is in contrast to Devereux and Yetman (2010), with whom we share an emphasis on demand shocks, but who abstract from domestic production and do not explore the comovement patterns central to this paper.

where $R_{n,t}$ is the nominal interest rate with value R_n in steady state, $\overline{Y}_{n,t}$ denotes trend output,[12] and $\nu_{n,t}$ is an exogenous monetary policy shock.[13] With $\phi_{n,\pi} > 0$ and $\phi_{n,Y} > 0$, the monetary authority increases nominal interest rates in response to high inflation or output relative to their long-run paths. We allow the monetary policy rule to differ across countries to account for different responses to inflation or the output gap. Unlike closed-economy settings, we also allow the shock, $\nu_{n,t}$, to be correlated across countries to account for potential coordination in policy.

4 Equilibrium Conditions in a World with Trade and Sticky Prices

4.1 Some Key Equilibrium Relationships

We now highlight several key equilibrium conditions that describe the role of trade in an environment with nominal rigidities. Details of all derivations are provided in the online Appendix.

Aggregate Import Shares: The economic environment above yields tractable expressions that describe aggregate trade shares as functions of their underlying determinants. In the notation introduced above, $\kappa_{nn'}Q_{nn',t}$ represents real imports of variety $\mathbf{z}$ by country n. Letting $p_{n',t}(\mathbf{z}) = W_{n',t}^{\gamma}P_{n',t}^{1-\gamma}/z_{n',t}$ denote the price charged by country n' for variety $\mathbf{z}$ quoted in local currency, $\mathcal{E}_{nn',t}p_{n',t}(\mathbf{z})\kappa_{nn'}Q_{nn',t}$ is the value of country n's imports from n' in n's currency. Then,

$$X_{nn',t} = \int \mathcal{E}_{nn',t}p_{n',t}(\mathbf{z})\kappa_{nn'}Q_{nn',t}(\mathbf{z})d\Phi(\mathbf{z})$$

represents expenditures by country n on all traded intermediate varieties from country n'. It follows that $X_{n,t} = \sum_{n'} X_{nn',t}$ represents country n's total expenditures on varieties from abroad.

Under the maintained assumptions, each country purchases all tradable varieties. It acquires any one variety $\mathbf{z}$ from a single other country (frequently itself). The choice of whom to purchase a given variety from depends on the production cost of that variety in different countries as well as shipping costs. The assumption that $\mathbf{z}$ is Fréchet distributed ensures that the total value of trade between any country pair is a smooth function of foreign marginal production costs. In particular, the share of country n's expenditures on varieties from country n' at date t, $\varpi_{nn',t}$, is given by

$$\varpi_{nn',t} = \frac{X_{nn',t}}{X_{n,t}} = \frac{\left(\kappa_{nn'}\mathcal{E}_{nn',t}W_{n',t}^{\gamma}P_{n',t}^{1-\gamma}\right)^{-\varphi}}{\sum_{n''}\left(\kappa_{nn''}\mathcal{E}_{nn'',t}W_{n'',t}^{\gamma}P_{n'',t}^{1-\gamma}\right)^{-\varphi}}. \tag{11}$$

Thus, the share of country n's expenditures on imports from country n' reflects the costs of acquiring

[12] In our model, trend output is proportional to the stochastic trend A_t.

[13] Similar to Cúrdia, Del Negro, and Greenwald (2014) and Leeper, Traum, and Walker (2017), we abstract from the zero lower bound, keeping the system log-linear. However, in Section D of the online Appendix, we reestimate the model using data on shadow rates in place of nominal interest rates during periods with binding effective lower bounds for robustness.

varieties from n', including production costs, $W_{n',t}$ and $P_{n',t}$, as well as bilateral shipping costs, $\kappa_{n,n'}$, relative to all other countries (all expressed in n's currency).

We highlight three important features of the matrix of import shares, $\boldsymbol{\varpi}_t = \{\varpi_{nn',t}\}$. First, $\boldsymbol{\varpi}_t$ has the properties of stochastic matrix in the sense that it is a real square matrix whose rows add to one, a feature that will prove useful below. Second, the larger the trade elasticity, $1+\varphi$, the more import shares depend on relative wages and exchange rates. With large $1+\varphi$, varieties are produced with similar productivities in all countries. As a result, small shifts in wages or exchange rates lead to large changes in the number of varieties traded between country pairs. In contrast, $\lim_{\varphi\to 0} \varpi_{nn't} = 1/N \ \forall n, n'$. Finally, equation (11) implies that, conditional on wages and exchange rates, trade costs, $\kappa_{nn'}$, can be directly inferred from long-run import shares, $\varpi_{nn'}$. This feature makes it possible to condition the quantitative analysis only on those shares rather than trade costs.

Marginal Cost and the Price of the Input Bundle: Recall that the marginal cost faced by producers of sticky-price goods in country n depends on the price of their input bundle, $P^Q_{n,t}$, in equation (6). The assumption in Eaton and Kortum (2002) that $\mathbf{z}$ is drawn from a Fréchet distribution with shape parameter φ implies that the price of the input bundle is

$$P^Q_{n,t} = \Gamma\left(\Upsilon\right)^{\frac{1}{1-\gamma}} \left(\sum_{n'} \left(\kappa_{nn'} \mathcal{E}_{nn',t} W^{\gamma}_{n',t} P^{1-\gamma}_{n',t} \right)^{-\varphi} \right)^{-\frac{1}{\varphi}}, \tag{12}$$

where $\Gamma(\Upsilon)$ is the Gamma function evaluated at $\Upsilon = 1 + \frac{1-\gamma}{\varphi}$. Equation (12) illustrates how production costs, $W_{n't}$ and $P_{n't}$, trade costs, $\kappa_{nn'}$, and exchange rates, $\mathcal{E}_{nn',t}$, across different countries all factor into the price of the input bundle, $P^Q_{n,t}$, paid by country n firms. This mechanism is analogous to how these same factors determine import shares in equation (11). In that sense, equation (12) captures the role of *backward trade linkages*, or alternatively country n's import network, in determining domestic marginal production costs. Because these multilateral trade linkages directly impact the real profits of country n firms, they also affect their pricing decisions. Moreover, to the degree that final goods prices, $P_{n't}$, are sticky around the world, so are the marginal costs of Calvo producers in every country.

Aggregate Market Clearing for Traded Intermediate Goods: Given perfectly competitive markets, the total cost of producing tradable varieties in country n, i.e., its wage bill and material costs, $W_{n,t}L_{n,t} = W_{n,t}\int \ell_{n,t}(\mathbf{z})d\Phi(\mathbf{z})$ and $P_{n,t}M_{n,t} = P_{n,t}\int M_{n,t}(\mathbf{z})d\Phi(\mathbf{z})$, respectively, exhaust the value of varieties it produces and exports to foreign countries. In equilibrium, exports equate to the demand for country n's domestic product by economies abroad. Foreign demand for n's goods in turn amounts to expenditures abroad on country n's varieties, or $\mathcal{E}_{nn',t}X_{n',t}$ for trading partner

n' (in n's currency), weighted by their import share in country n's goods, $\varpi_{n'n,t}$. It follows that

$$W_{n,t}L_{n,t} + P_{n,t}M_{n,t} = \sum_{n'} \mathcal{E}_{nn',t}\varpi_{n'n,t}X_{n',t}. \tag{13}$$

Equation (13), therefore, highlights how *forward trade linkages*, or alternatively country n's export network, help determine the value of domestic output.

4.2 Steady State

As in much of the New Keynesian closed-economy literature, our objective is in part to estimate our multi-country framework approximated around its steady state and explore its quantitative implications. However, unlike much of the related literature, the steady state here possesses unique features that arise from international trade linkages. We discuss salient implications of these features here and refer the reader to the online Appendix for details.

Country-Specific Balanced Growth Paths: In departing from the closed-economy environment, we focus on a world balanced growth path that has two features. First, steady state growth rates for variables such as consumption, real wages, and inflation are country-specific. Second, the world steady-state growth path nevertheless features constant country shares in the world economy so that no country's share of the world economy grows or shrinks indefinitely.

For a given economy n, the derivation of country-specific balanced growth paths for real quantities and steady-state inflation follows that of closed-economy models. In particular, real quantities grow at rate $g_n = E[g_{n,t}]$, which denotes the long-run mean of technical progress, $g_{n,t} = \frac{A_{n,t}}{A_{n,t-1}}$. In addition, prices increase at the rate $1 + \pi_n$, which also determines the steady state interest rate through the Fisher equation, $1 + \pi_n = \frac{\beta R_n}{g_n}$.

The existence of a global balanced growth path where no country eventually dominates or becomes irrelevant in the world economy is shown in the online Appendix. This requires, among other conditions, that the real exchange rate for each country n relative to the reference country, $\mathcal{E}_{rn,t}\frac{P_{n,t}}{P_{r,t}}$, grow in the long run at a rate that reflects the difference in technical progress between country n and the reference country.[14]

Trade Balances: While the model features transition dynamics whereby different countries can run trade deficits or surpluses, the form of portfolio adjustment costs does not ultimately pin down countries' long-run net asset positions. In other words, analogous to the static framework of Eaton

[14]The derivations of these paths and conditions that must hold for global balanced growth open up applications beyond nominal rigidities, for example with respect to the study of long-run global change. See Müller, Stock, and Watson (2020) for a statistical analysis of the joint long-run evolution of GDP per capita for 113 countries over 118 years from 1900 to 2017. The assumption that varieties $\mathbf{z}$ are drawn from a Fréchet distribution in much of the trade literature is not innocuous. In some instances, country-specific balanced growth paths within a stable world economy, where country shares are constant in the long run, will not be feasible given constant trade costs, $1/\kappa_{nn'} \in (0, 1)$. See Section B of the online Appendix.

and Kortum (2002), the environment does not provide a theory of countries' steady-state trade deficits or surpluses. As such, we adopt a benchmark defined by balanced trade in steady state.

However, even over periods as long as our sample (16 years), a country's net asset position can differ from zero on average. We address this observation following the approach first proposed by Dekle, Eaton, and Kortum (2007) in a static environment but now applied to the steady state of a dynamic setting. Specifically, we first write a variant of the economic environment whose steady state can be matched to observed average regional trade imbalances over our entire sample, 2004Q2–2019Q4. This can be done by allowing for exogenous frictions in international asset markets even in the long-run, that is $\xi_n \neq 0$. Then, for the set of parameters implied by a world with long-run trade imbalances, we calculate counterfactual steady-state allocations and prices corresponding to zero trade imbalances. More specifically, for the implied set of parameters, we compute counterfactual steady-state allocations consistent with $\xi_n = 0 \ \forall n$, in which case real net asset positions are zero for all countries, $S_n = 0 \ \forall n$. Analogously to Dekle et al. (2007), the resulting counterfactual world steady-state allocations without imbalances can then be used to produce a linearized model for estimation and analysis using standard linear rational expectations toolkits.

5 The Linearized Global New Keynesian Model

This section describes the solution to our model economy, casting some of its equations in terms of expressions familiar from the New Keynesian literature and international trade. To obtain conventional New Keynesian expressions as special cases, we log-linearize the deflated-detrended equilibrium equations around their steady state. Thus, all variables in the expressions below refer to their appropriately deflated-detrended counterparts.[15] We use a 'hat' over variables to denote log-deviations from steady-state (i.e., $\widehat{x}_{n,t} = \log(x_{n,t}) - \log(x_n)$), use dx to denote level deviations from steady state, (i.e., $dx_{n,t} = x_{n,t} - x_n$), and express all equations in vector form, stacking country-specific variables into vectors $\widehat{\boldsymbol{x}}_t = (\widehat{x}_{1,t}, ..., \widehat{x}_{N,t})'$.

To develop intuition, we discuss key economic forces at work in our setting using a simplified version of our world economy. In this version, we set the materials share of gross output, $1 - \gamma$, to zero. The domestic implications of foreign production costs can then be written in terms of labor costs only (i.e., wages). We further assume that the demand faced by sticky-price firms is subsidized at rate $1 + \tau_n$, set to the inverse of the markup to free our discussion from steady-state distortions arising from monopolistic pricing.[16] Thus, the long-run real marginal cost of sticky price goods is 1 in all countries (i.e., $mc_n = 1 \ \forall n$). In the quantitative analysis, however, we allow for both pricing distortions, $mc_{n,t} \neq 1$, and roundabout production, $1 - \gamma \neq 0$. We let $\mathcal{E}_{n,t} \equiv \mathcal{E}_{rn,t}$ for notational convenience and, without loss of generality, take country 1 to be the reference country,

[15] See Section A.3 of the online Appendix for details of the model normalization with respect to country-specific inflation, $1 + \pi_n$, and technical progress, g_n.

[16] Specifically, firms' profits are then given by $\frac{\Pi_{n,t}(j)}{P_{n,t}} = \left(\frac{P_{n,t}(j)}{P_{n,t}}\right)^{-\varepsilon_{n,t}} \mathcal{C}_{n,t} \left((1+\tau_n)\frac{P_{n,t}(j)}{P_{n,t}} - mc_{n,t}\right)$, where the subsidy to sticky-price firms is financed by lump sum taxes, $T_{n,t}$.

(i.e., $r = 1$). Under no arbitrage in currency markets, $\mathcal{E}_{1,t} = \mathcal{E}_{r,t} = \mathcal{E}_{rr,t} = 1$. Similarly, we define the trend-adjusted real exchange rate, which converts country n final goods into units of country 1 final goods, as $e_{n,t} = \mathcal{E}_{n,t}\frac{A_{n,t}P_{n,t}}{A_{1,t}P_{1,t}}$. Finally, while we allow for country-specific preference and technology parameters in the estimation, σ_n, η_n, ω_n, θ_n, etc., we set these parameters to be the same in this section to simplify notation.

The interaction between nominal rigidities, international trade, and international financial flows can be summarized by two main blocks of equations. The first block describes conventional New Keynesian mechanisms in all countries. The second block captures forces underlying international trade in goods and financial assets. Crucially, these two blocks are linked through the marginal cost of sticky price goods, a central element of the New Keynesian Phillips curve (NKPC), that now reflects export prices of trading partner countries and the fundamentals that drive them.

5.1 The New Keynesian Block

The New Keynesian block reflects intertemporal mechanisms at home along with a description of monetary policy and domestic labor supply in vector form:

$$\widehat{\boldsymbol{C}}_t = E_t\widehat{\boldsymbol{C}}_{t+1} - \frac{1}{\sigma}\left(\widehat{\boldsymbol{R}}_t - E_t\widehat{\boldsymbol{\pi}}_{t+1} - E_t\widehat{\boldsymbol{g}}_{t+1}\right), \tag{14}$$

$$\widehat{\boldsymbol{\pi}}_t = \frac{(1-\theta)(1-\beta\theta)}{\theta}(\widehat{\boldsymbol{mc}}_t + \widehat{\boldsymbol{\mu}}_t) + \beta(E_t[\widehat{\boldsymbol{\pi}}_{t+1}]), \tag{15}$$

$$\widehat{\boldsymbol{R}}_t = \rho\widehat{\boldsymbol{R}}_{t-1} + (1-\rho)(\phi_\pi\widehat{\boldsymbol{\pi}}_t + \phi_Y\widehat{\boldsymbol{Y}}_t) + \widehat{\boldsymbol{\nu}}_t, \tag{16}$$

$$\widehat{\boldsymbol{w}}_t = \eta\widehat{\boldsymbol{L}}_t + \sigma\widehat{\boldsymbol{C}}_t, \tag{17}$$

where we have stacked the equations for each country. Equations (14)-(16) replicate familiar expressions from the New Keynesian literature in vector form: equation (14) is the standard household Euler equation for each country, equation (15) describes a vectorized NKPC, and equation (16) describes a monetary policy rule for each country. Finally, equation (17) describes labor supply in all countries with its attendant income and substitution effects. The model block includes three exogenous shocks in every country: shocks to technical progress, $\widehat{\boldsymbol{g}}_t$, markup shocks, $\widehat{\boldsymbol{\mu}}_t$, and monetary policy shocks, $\widehat{\boldsymbol{\nu}}_t$.

In the absence of trade, the New Keynesian block reduces to a set of conventional closed-economy three-equation New Keynesian models (e.g., An and Schorfheide (2007); Galí (2008)), thus yielding isolated New Keynesian economies (i.e., autarky). In particular, absent trade, the real marginal cost of sticky-price goods, $\widehat{\boldsymbol{mc}}_t$, is determined by payments to domestic inputs, in this case wages, $\widehat{\boldsymbol{w}}_t = \eta\widehat{\boldsymbol{L}}_t + \sigma\widehat{\boldsymbol{C}}_t$. Since, absent trade, linearized output, $\widehat{\boldsymbol{Y}}_t$, equates to consumption, $\widehat{\boldsymbol{C}}_t$, the marginal cost faced by domestic firms ultimately reflects a domestic output gap, $\widehat{\boldsymbol{mc}}_t = (\eta+\sigma)\widehat{\boldsymbol{Y}}_t$.

When countries trade, the marginal cost of sticky-price goods no longer equates to payments for domestic inputs. Instead, marginal cost now reflects global and country-specific forces operating through adjustments in countries' trade balances and net asset positions along with global market

clearing. These international forces are described in the second block of equations.

5.2 The International Block

The second block of equations relates to relationships that arise under international trade. These include i) a set of equations describing the marginal cost of producing sticky-price goods in each country as a function of cost conditions abroad, ii) a set of equations describing market clearing conditions for traded goods, iii) a set of equations describing the balance of payments in each country, and iv) a set of equations describing the evolution of countries' financial assets.

i) Marginal Cost of Producing Sticky-Price Goods:

Unlike a closed economy where marginal costs are determined only by the price of domestic inputs, marginal costs depend on costs in all trading partner countries. In particular, equation (12) implies

$$\widehat{\boldsymbol{e}}_t + \widehat{\boldsymbol{mc}}_t = \boldsymbol{\varpi}(\widehat{\boldsymbol{e}}_t + \widehat{\boldsymbol{w}}_t), \tag{18}$$

where $\widehat{\boldsymbol{e}}_t$, $\widehat{\boldsymbol{mc}}_t$, and $\widehat{\boldsymbol{w}}_t$ are vectors of exchange rates, marginal costs, and wages, respectively, in log-linearized form. The left-hand side of equation (18) is the marginal cost of producing sticky-price goods for each country, expressed in real units of the reference country's goods. This is determined by wages in each country (given $\gamma = 1$), expressed in the reference country's goods, weighted by the matrix of import shares, $\boldsymbol{\varpi} = \{\varpi_{nn'}\}$. The influence that country n' has on country n increases in proportion to how much n imports from n', $\varpi_{nn'}$. As such, $\varpi_{nn'}$ captures the role of backward trade linkages in determining marginal cost.

ii) Market Clearing for Traded Goods:

There are N market clearing conditions for traded goods, one for each country. These can be expressed in vector form in a way that describes the makeup of the real trade balance. From equation (13),

$$\underbrace{\widehat{\boldsymbol{Y}}_t - \widehat{\boldsymbol{C}}_t}_{\text{real trade balance}} = \underbrace{(\boldsymbol{\psi}^T - \boldsymbol{I})\widehat{\boldsymbol{C}}_t}_{\substack{\text{relative demand} \\ \text{in export markets}}} - \underbrace{(1+\varphi)}_{\substack{\text{price elasticity} \\ \text{of trade}}} \underbrace{\big(\boldsymbol{I} - \boldsymbol{\psi}^T\boldsymbol{\varpi}\big)(\widehat{\boldsymbol{e}}_t + \widehat{\boldsymbol{w}}_t)}_{\substack{\text{relative price} \\ \text{at destination}}}, \tag{19}$$

where $\boldsymbol{\psi} = \{\psi_{n'n}\}$ denotes the matrix of (steady state) export shares from country n to n'.[17]

The left-hand side of the equation, $\widehat{\boldsymbol{Y}}_t - \widehat{\boldsymbol{C}}_t$, captures real quantities produced and consumed, defining the real trade balance. Deviations in countries' real detrended GDP from their steady state are $\widehat{\boldsymbol{Y}}_t = \widehat{\boldsymbol{L}}_t$. The real trade balance depends on the relative demand in export markets, the first term on the right-hand side of equation (19). It moves towards a surplus when the consumption of

[17] Formally, $\psi_{nn'} = \frac{\varpi_{nn'} e_n mc_n \mathcal{C}_n}{\sum_{n''} \varpi_{n''n'} e_{n''} mc_{n''} \mathcal{C}_{n''}}$.

home goods abroad, $\boldsymbol{\psi}^T\widehat{\boldsymbol{C}}_t$, increases. Here, each foreign country's consumption of home-produced goods is weighted by its share of exports from the home country. Conversely, the real trade balance moves towards a deficit when domestic consumption, $\boldsymbol{I}\widehat{\boldsymbol{C}}_t$, increases.

The real trade balance also depends on the relative price of exports in foreign markets, the second term on the right-hand side of equation (19). This term compares the price of exports in each country (in real units of the reference country), $\widehat{\boldsymbol{e}}_t+\widehat{\boldsymbol{w}}_t$, with the corresponding prices of goods produced in other countries weighted by $\boldsymbol{\psi}^T\boldsymbol{\varpi}$. As such, the term $(\boldsymbol{I}-\boldsymbol{\psi}^T\boldsymbol{\varpi})(\widehat{\boldsymbol{e}}_t+\widehat{\boldsymbol{w}}_t)$ represents the substitution from countries with higher costs to those with lower costs.

Importantly, this substitution means that the trade balance in country n depends on country n' not only through economic conditions in country n', but also through conditions in other countries that trade with n'. The import matrix, $\boldsymbol{\varpi}$, indicates that in response to an increase in wages in a country n'', country n' will substitute toward country n (and thereby increase country n's trade balance according to how much n exports to n'). Country n' reacts more strongly to real wages in places from which it imports heavily, substituting away from these other countries and towards country n proportionally to its import shares with n''. Concretely, suppose that real wages in China increase, $\widehat{e}_{n'',t}+\widehat{w}_{n'',t}>0$. All else equal, Japan reacts by substituting toward U.S. exports and away from China in proportion to its import share from China (captured in $\boldsymbol{\varpi}$). This shift in trade in turn increases the U.S. trade balance in proportion to its exports towards Japan (captured in $\boldsymbol{\psi}^T$). Equation (19) thus emphasizes the role of both backward trade linkages, $\boldsymbol{\varpi}$, and forward linkages, $\boldsymbol{\psi}^T$, in driving the effects of foreign prices on the trade balance.[18]

The price elasticity of trade, $1+\varphi$, governs the scale of real trade adjustments to relative export prices. Larger values of $1+\varphi$ imply less dispersion in the productivity of any given variety across countries. In that case, firms are more sensitive to a given relative price change and substitute inputs more readily across countries resulting in larger trade effects.

iii) Balance of Payments:

Rearranging countries' household budget constraints (2), linearized around a steady state with no monopolistic trade distortions, yields the following expression relating the trade balance and the terms of trade to financial flows:

$$\underbrace{\widehat{\boldsymbol{Y}}_t-\widehat{\boldsymbol{C}}_t}_{\text{real trade balance}}+\underbrace{(\boldsymbol{I}-\boldsymbol{\varpi})(\widehat{\boldsymbol{e}}_t+\widehat{\boldsymbol{w}}_t)}_{\substack{\text{terms of trade}\\ \times\text{ s.s. import shares}}}=\underbrace{Vg_1(1+\pi_1)E_t\left[d\boldsymbol{s}_{t+1}/(\boldsymbol{e}\times\boldsymbol{C})\right]-d\boldsymbol{s}_t/(\boldsymbol{e}\times\boldsymbol{C})}_{\text{financial flows}}, \tag{20}$$

[18] These knock-on effects stand in contrast to much of the open economy New Keynesian literature modeling either two large countries or individual small open economies. Guerron-Quintana (2013) and Aggarwal, Auclert, Rognlie, and Straub (2022) go a step further by considering multiple small open economies, but Guerron-Quintana (2013) acknowledges that such models abstract from the multiple layers of interactions across countries that are central in an environment with multiple large countries and highlighted herein.

where '/' and '×' in this case denote respectively element-wise (Hadamard) division and product operations. The left-hand side of the equation is a vector of trade balances for each country in local units. It can be decomposed into a real trade balance of quantities produced and consumed, $\widehat{\boldsymbol{Y}}_t - \widehat{\boldsymbol{C}}_t$, and a measure of the terms of trade for each country, scaled by import shares, $(\boldsymbol{I} - \boldsymbol{\varpi})(\widehat{\boldsymbol{e}}_t + \widehat{\boldsymbol{w}}_t)$.[19] The terms of trade captures the wedge between production costs (which are a function of local factor prices) and consumption costs (which are a function of foreign factor prices and exchange rates). As emphasized by Kehoe and Ruhl (2008), while terms of trade fluctuations do not affect real GDP, they do affect welfare. Importantly, the equation highlights that the terms of trade also affect the domestic value of the trade balance that needs to be covered by financial flows.

The right-hand side captures financial flows normalized by consumption. Keeping track of financial flows is necessary when international financial markets are subject to frictions. These frictions imply that adjustments to countries' net asset positions are costly and thus distort consumption allocations across time and states of nature. To the extent that portfolio rebalancing costs affect financial flows, the balance-of-payments equation implies that they also affect the real trade balance and terms of trade. When portfolio adjustment costs are infinitely high, trade balance holds period by period, creating a tight link between consumption, output, and exchange rates.

iv) Financial Asset Allocations:

Finally, around a steady state with balanced trade, the Euler equation governing the evolution of countries' financial assets satisfies

$$\underbrace{\mathbf{1}\widehat{C}_{1,t} - \widehat{\boldsymbol{C}}_t}_{\text{relative consumption}} = \frac{1}{\sigma} \underbrace{\widehat{\boldsymbol{e}}_t}_{\substack{\text{real} \\ \text{exchange rate}}} + \underbrace{\beta \frac{\tau}{\sigma} \overbrace{(d\boldsymbol{S}_t/\boldsymbol{e} - \mathbf{1}dS_{1,t})}^{\substack{\text{cumulative financial} \\ \text{asset position}}} + \overbrace{d\boldsymbol{\Xi}_t - d\boldsymbol{\Xi}_{1,t}}^{\substack{\text{cumulative portfolio} \\ \text{adjustment shocks}}}}_{\text{asset pricing wedge}}, \tag{21}$$

$$\text{where } d\boldsymbol{S}_t = \sum_{k=-\infty}^{t} d\boldsymbol{s}_k;\ d\boldsymbol{\Xi}_t = \frac{1}{\sigma} \sum_{k=-\infty}^{t-1} d\boldsymbol{\xi}_k,$$

and $\mathbf{1}$ denotes a $N \times 1$ vector of ones. Absent frictions in global financial markets, defined by $\tau = 0$ and $d\boldsymbol{\xi}_{t-k} = 0\ \forall k$, equation (21) reduces to the expression first highlighted by Backus and Smith (1993). In that case, the (log) ratio of consumption between any two countries, $\mathbf{1}\widehat{C}_{1,t} - \widehat{\boldsymbol{C}}_t$, reflects variations in their real exchange rate, $(1/\sigma)\widehat{\boldsymbol{e}}_t$, (i.e., the relative price of consumption between the two countries). Put simply, a depreciation of the domestic currency goes hand in hand with higher consumption of home goods (as consumption goods abroad become more expensive).

However, Backus and Smith (1993) also note that the data do not generally support a clear connection between countries' relative consumption and their real exchange rate. This 'consumption-

[19] The terms of trade is the ratio of a country's export prices to import prices (in common units, here the reference country's final goods). Thus country n's linearized terms of trade are given by $\widehat{e}_{n,t} + \widehat{w}_{n,t} - \frac{1}{1-\varpi_{nn}} \sum_{n' \neq n} \varpi_{nn'} (\widehat{e}_{n',t} + \widehat{w}_{n',t}) = \frac{1}{1-\varpi_{nn}} \left(\widehat{e}_{n,t} + \widehat{w}_{n,t} - \sum_{n'} \varpi_{nn'} (\widehat{e}_{n',t} + \widehat{w}_{n',t}) \right)$, or in matrix form, $\left(diag(\boldsymbol{I} - \boldsymbol{\varpi})^{-1} \right) (\boldsymbol{I} - \boldsymbol{\varpi})(\widehat{\boldsymbol{e}}_{n,t} + \widehat{\boldsymbol{w}}_{n,t})$.

exchange rate disconnect puzzle' suggests frictions in international financial markets that operate along two dimensions in equation (21):[20] one relates to exogenous disturbances affecting each country's trading, $d\boldsymbol{\Xi}_t - d\boldsymbol{\Xi}_{1,t}$, and the other relates to endogenous portfolio adjustments (relative to the reference country), $d\boldsymbol{S}_t/\boldsymbol{e} - \mathbf{1}dS_{1,t}$, where $d\boldsymbol{S}_t$ is a vector representing the history of countries' net asset positions (in linearized form). This history matters because financial frictions affect consumption growth through the Euler equation, and thus have a permanent effect on consumption levels. For example, following a one-time financial disturbance that lowers consumption in country n relative to the reference country, country n households will from then on keep consumption smooth at a level lower than they otherwise would have absent the shock.[21]

6 Foreign Spillovers to the Domestic Economy

We now combine the two blocks of equations to derive the implications for cross-country correlations in output and inflation. First, we show that our model implies a set of interconnected Phillips curves. Next, we present a special case of our model whose analytical solution makes the role of knock-on network effects for cross-country comovement transparent.

6.1 The Global Phillips Curve

Implicit in the two blocks of equations above is a system of interrelated Phillips curves. This system defines a *global Phillips curve* for vectors of variables pertaining to all countries, highlighting how domestic inflation in each country is connected to real activity and exchange rates of trading partners. The global Phillips curve thus provides a useful summary of how trade connects nominal and real variables across countries, similar to how the conventional Phillips curve summarizes the trade-off between output and inflation in a closed economy.

In the NKPC (15) for each country, the marginal cost of producing sticky-price goods depends on multilateral trade considerations. It follows from the labor supply equation (17) in the New Keynesian block and the expression for marginal cost (18) in the international block that

$$\widehat{\boldsymbol{mc}}_t = \boldsymbol{\varpi}(\eta\widehat{\boldsymbol{Y}}_t + \sigma\widehat{\boldsymbol{C}}_t) + (\boldsymbol{\varpi} - \boldsymbol{I})\widehat{\boldsymbol{e}}_t.$$

In other words, in equation (18), the marginal cost at home depends not only on domestic production costs (domestic wages in this case), but also on the real exchange rate and production costs of imported goods (wages in trading partner countries). These in turn reflect real activity embodied in the determination of labor supply abroad, $\widehat{\boldsymbol{w}}_t = \eta\widehat{\boldsymbol{Y}}_t + \sigma\widehat{\boldsymbol{C}}_t$.

[20] See Kollmann (1995), Benigno and Thoenissen (2008), and Corsetti, Dedola, and Leduc (2008), for examples of papers that specifically address the Backus and Smith (1993) puzzle through financial frictions.

[21] An implication of this history dependence is that the model can imply very persistent real exchange rate fluctuations in response to transitory financial or monetary shocks, in line with the 'purchasing power parity puzzle' documented in Rogoff (1996).

The global Phillips curve is then given, in vector form, by

$$\widehat{\boldsymbol{\pi}}_t = \kappa \left[\boldsymbol{\varpi}(\eta \widehat{\boldsymbol{Y}}_t + \sigma \widehat{\boldsymbol{C}}_t) + (\boldsymbol{\varpi} - \boldsymbol{I})\widehat{\boldsymbol{e}}_t + \widehat{\boldsymbol{\mu}}_t \right] + \beta(E_t[\widehat{\boldsymbol{\pi}}_{t+1}]), \tag{22}$$

where $\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}$ is the coefficient of inflation on the output gap in the conventional closed economy model. In this context, however, the coefficient, κ, applies to an import-share weighted average of domestic and foreign production costs, $\boldsymbol{\varpi}(\eta \widehat{\boldsymbol{Y}}_t + \sigma \widehat{\boldsymbol{C}}_t)$, and also applies to variations in real exchange rates, $(\boldsymbol{\varpi} - \boldsymbol{I})\widehat{\boldsymbol{e}}_t$. Put another way, inflation in country n no longer moves one-for-one with country n's own output gap (conditional on expected inflation), but also depends on real activity and exchange rates in trading partner countries. Specifically, the comovement between inflation in country n and real activity and exchange rates in country n' is proportional to country n's import share from country n', $\varpi_{nn'}$.[22]

The global Phillips curve (22) imposes theoretical structure on the weights that different foreign countries have in determing domestic inflation. The choice of weights is a problem that Borio and Filardo (2007) and Bianchi and Civelli (2015), among others, had to contend with in studying the global determinants of domestic inflation. The global Phillips curve implies that conditional on international output and consumption data, import-weighted exchange rates are appropriate.

To quantify the role of foreign spillovers for domestic inflation, one approach is to estimate equation (22) using direct observations or proxies of the variables. However, in equilibrium, variations in exchange rates and output are ultimately determined by global or country-specific shocks operating through adjustments in countries' trade balances and asset positions as well as global market clearing. Simply put, the variables on the right-hand side of (22) are endogenous. Real exchange rates, in particular, deviate from UIP according to

$$\begin{aligned} E_t\widehat{\boldsymbol{e}}_{t+1} - \widehat{\boldsymbol{e}}_t = &- \widehat{\boldsymbol{R}}_t + E_t[\widehat{\boldsymbol{\pi}}_{t+1} + \widehat{\boldsymbol{g}}_{t+1}] + \mathbf{1}\left(\widehat{R}_{1,t} - E_t[\widehat{\pi}_{1,t+1} + \widehat{g}_{1,t+1}]\right) \\ &- \beta\tau E_t\left(\frac{1}{e} d\boldsymbol{s}_{t+1} - \mathbf{1} ds_{1,t+1}\right) - (d\boldsymbol{\xi}_t - \mathbf{1} d\xi_{1,t}), \end{aligned} \tag{23}$$

where $\beta\tau E_t\left(\frac{1}{e} d\boldsymbol{s}_{t+1} - \mathbf{1} ds_{1,t+1}\right)$ are costs arising from endogenous portfolio adjustments and $(d\boldsymbol{\xi}_t - \mathbf{1} d\xi_{1,t})$ are exogenous disturbances that affect countries' trading in global financial markets. Therefore, in a generalized open economy setting, financial frictions create an additional challenge to directly estimating New Keynesian Phillips curves. Moreover, in equilibrium, spillovers from output on the right-hand side of (22) are driven in part by network effects involving export shares

[22] The global Phillips curve as written in equation (22) depends not only on output in all countries, but also on expenditures (here, consumption) and exchange rates. Therefore, it differs from more compact expressions developed in frameworks with complete markets such as Galí and Monacelli (2005) and Clarida et al. (2002), where equilibrium trade relationships and financial market equilibrium conditions may be used to substitute for consumption and exchange rates. Importantly, our multi-country setup allows the expression to highlight the role of heterogeneous trade linkages. Another difference is that our expression holds for total headline inflation, rather than just inflation in domestically produced goods. This difference stems from our assumption that local sticky price producers use goods produced by workers in all countries rather than local labor.

that do not directly appear in that equation, and that we illustrate explicitly in the next section. These endogeneity issues add to existing complications in using limited-information approaches to estimate closed-economy NKPCs, including ambiguous interpretations of the output gap (e.g., Galí and Gertler (1999); Neiss and Nelson (2005)) or measurements of inflation expectations (e.g., Roberts (1995); Sbordone (2005)). Mavroeidis, Plagborg-Møller, and Stock (2014) highlight and summarize how these complications translate into a broad range of estimates in the literature. Given these challenges, our quantitative analysis in Section 7 takes a full-information Bayesian approach, utilizing the complete set of cross-equation restrictions from our model for identification.

6.2 Spillovers from Idiosyncratic Demand Shocks: An Analytical Example

We now provide further intuition on the nature of cross-country spillovers through an analytical example. As in closed-economy New Keynesian models, this example highlights the importance of demand shocks with nominal rigidities. To obtain closed-form solutions, we make two key assumptions. First, all shocks are *i.i.d.* across countries and over time. Therefore, all comovement in this example arises from idiosyncratic country-specific shocks through endogenous spillovers rather than exogenous global shocks. Second, following Clarida et al. (2002), we replace the monetary policy rule (10) with a simple forward-looking inflation-centric rule in every country:

$$\widehat{\boldsymbol{R}}_t = \phi E_t \widehat{\boldsymbol{\pi}}_{t+1} + \sigma \boldsymbol{\nu}_t.$$

For transparency, we let all elements of the monetary shocks, $\boldsymbol{\nu}_t$, have unit variances. However, for the discussion in this section, this is without loss of generality.

These assumptions imply a solution for consumption of the form

$$\widehat{\boldsymbol{C}}_t = -\boldsymbol{\nu}_t.$$

That is, $\widehat{C}_{n,t}$ depends only on its own-country monetary policy shocks. Since these shocks are independent across countries in this example, consumption is also uncorrelated across countries, $E[\widehat{\boldsymbol{C}}_t \widehat{\boldsymbol{C}}_t^T] = \boldsymbol{I}$. As expected, lower nominal rates, $\nu_{n,t} < 0$, in a given country n raises its consumption relative to trend. In what follows, we delve into the mechanisms at work by focusing on the limit case with no frictions in global financial markets, $\tau = 0$ and $d\boldsymbol{\xi}_t = 0 \ \forall t$. Note that because of real trade frictions, exchange rate variations imply that consumption may differ across countries even with frictionless financial markets (recall (21)).

International Comovement from Demand Shocks: Under the above assumptions, analytical expressions for output and inflation provide intuition for how the model reproduces the positive correlations described in Section 2.

The solution for output is, in vector form,

$$\widehat{\boldsymbol{Y}}_t = -\mathcal{A}_{YY}^{-1}\boldsymbol{\psi}^T\boldsymbol{\nu}_t, \text{ where } \mathcal{A}_{YY}^{-1} = (1-b)(\boldsymbol{I} - b\boldsymbol{\psi}^T\boldsymbol{\varpi})^{-1}, \tag{24}$$

where $b = \frac{(1+\varphi)\eta}{1+(1+\varphi)\eta}$. Absent trade, $\boldsymbol{\varpi} = \boldsymbol{\psi}^T = \boldsymbol{I}$ and equation (24) reduces to $\widehat{\boldsymbol{Y}} = -\boldsymbol{\nu}_t = \widehat{\boldsymbol{C}}_t$, so that each country's production adjusts to meet its own local demand. With trade, the export matrix $\boldsymbol{\psi}^T$ captures the first layer of trade spillovers: when an expansionary monetary shock, $\nu_{n',t} < 0$, increases demand in country n', output in country n increases in proportion to its exports to country n'. The matrix $\mathcal{A}_{YY}^{-1}$ summarizes subsequent rounds of trade in the form of a Leontief inverse capturing the effects of backward linkages, $\boldsymbol{\varpi}$, and forward linkages, $\boldsymbol{\psi}^T$. The strength of these effects depends on model elasticities with respect to trade, $1+\varphi$, and labor, $1/\eta$. We discuss these successive rounds of trade below after deriving additional results on the model solution.

The solution for inflation also reflects spillovers from foreign demand shocks,

$$\widehat{\boldsymbol{\pi}}_t = \kappa\eta\boldsymbol{\varpi}\widehat{\boldsymbol{Y}}_t - \kappa\sigma\boldsymbol{\nu}_t + \kappa\boldsymbol{\mu}_t, \tag{25}$$

which is the global Phillips curve (22) in this case. As before, domestic inflation comoves with output abroad to the extent that countries are connected through the import matrix $\boldsymbol{\varpi}$. However, consumption increases in country n' are also offset by a depreciating exchange rate, as implied by equation (21) when $\tau = 0$.[23] The solutions for output, equation (24), and inflation, equation (25), imply the following proposition.

Proposition 1. *For any matrix $\boldsymbol{M}$, let $\boldsymbol{M} \geq 0$ if and only if $m_{ij} \geq 0\ \forall ij$. Then, under the maintained assumptions, $E[\widehat{\boldsymbol{Y}}_t\widehat{\boldsymbol{Y}}_t^T] \geq 0$, $E[\widehat{\boldsymbol{\pi}}_t\widehat{\boldsymbol{\pi}}_t^T] \geq 0$ and $E[\widehat{\boldsymbol{\pi}}_t\widehat{\boldsymbol{Y}}_t^T] \geq 0$.*

Proof. See Appendix A.

Proposition 1 states that in the presence of nominal rigidities, demand shocks imply positive comovement in output across countries, positive comovement in inflation across countries, and positive comovement between inflation and output across countries, as in Figure 1. The global comovement properties highlighted in Proposition 1 derives from both direct and indirect knock-on effects through trade.

Consider the effects on country n of an expansionary monetary shock in country n', $\nu_{n',t} < 0$, that raises consumption in n', $\widehat{C}_{n',t} = -\nu_{n',t} > 0$. From equation (24) and the Neumann series expression for $\mathcal{A}_{YY}^{-1}$ in Proposition 1, we can summarize the effects of this increase in demand on

[23] The net effect reduces to the term $-\kappa\sigma\boldsymbol{\nu}_t$, as the depreciating exchange rate increases inflation in country n in response to an expansionary monetary shock. In a closed economy, (25) becomes the familiar expression, $\widehat{\boldsymbol{\pi}}_t = \kappa(\eta+\sigma)\widehat{\boldsymbol{Y}}_t + \kappa\boldsymbol{\mu}_t$, with $\widehat{\boldsymbol{Y}}_t = \widehat{\boldsymbol{C}}_t = -\boldsymbol{\nu}_t$ as inflation now only depends on domestic conditions.

output growth as

$$\widehat{\boldsymbol{Y}}_t = \underbrace{-(1-b)\boldsymbol{\psi}^T\boldsymbol{\nu}_t}_{\text{direct effects}} - \underbrace{(1-b)\left(b\boldsymbol{\psi}^T\boldsymbol{\varpi} + b^2(\boldsymbol{\psi}^T\boldsymbol{\varpi})^2 + b^3(\boldsymbol{\psi}^T\boldsymbol{\varpi})^3 + ...\right)\boldsymbol{\nu}_t}_{\text{indirect effects}}, \tag{26}$$

which illustrates both direct and indirect spillovers. These direct and indirect effects then extend to inflation via equation (25).

Since consumption in country n' includes foreign goods, there is a *direct* positive demand spillover onto its trading partners, including country n. The effect on country n is greater the more country n exports to country n', as reflected in the matrix of export shares, $\boldsymbol{\psi}^T$. More generally, production in each country immediately increases in proportion to its exports to country n' via forward linkages, $-(1-b)\boldsymbol{\psi}^T\boldsymbol{\nu}_t > 0$.

However, additional *indirect* or knock-on effects also arise through repeated rounds of trade created by import or export substitution, thus involving both backward and forward linkages, the terms $-(1-b)b^m(\boldsymbol{\psi}^T\boldsymbol{\varpi})^m\boldsymbol{\nu}_t,\ m > 0$, on the right-hand-side of (26). Specifically, with the rise in the demand for goods in country n' (arising from its expansionary shock) comes an increase in labor demand and thus wages in country n'. Therefore, another country, say n'', that imports from country n' will substitute away from n' towards other countries including n, more so if a large fraction of imports in country n'' originates from n' in the import matrix, $\boldsymbol{\varpi}$. This effect induces a further increase in the demand for goods produced by country n, the extent of which will be larger the more n exports to n'' in the export matrix, $\boldsymbol{\psi}^T$. Finally, the propagation of the initial stimulative shock in country n' continues further afield as higher export activity and production in country n raises its wages so that its trading partners, country n''' say, redirect some of their imports from n back towards n'. This redirection induces higher labor demand and wages in country n' thus generating new demand for foreign goods from n' and creating a new round of trade, $-(1-b)b^{m+1}(\boldsymbol{\psi}^T\boldsymbol{\varpi})^{m+1}\boldsymbol{\nu}_t > 0$, and so on.

The strength of these spillovers depends on the trade and labor elasticities, $1+\varphi$ and $1/\eta$, respectively, through $b = \frac{(1+\varphi)\eta}{1+(1+\varphi)\eta}$ in the solution for output growth and inflation. The greater the trade elasticity, $1+\varphi$, the more sensitive countries are to changes in export prices and the more easily they substitute across trading partners. The more inelastic labor supply, or the smaller $1/\eta$, the more wages increase in country n' as the demand for its goods rises, thus magnifying the extent of trade substitutions across countries.

Finally, besides the positive cross-country comovement in real activity or inflation implied by nominal rigidities and trade, these features also mean that inflation *at home* comoves positively with real activity *abroad*, captured in part by the global Phillips curve (22). In this setting, higher employment at home and abroad imposes a higher disutility of labor and higher wages everywhere. Therefore, higher employment in any given country potentially induces higher inflation in all of its trading partners.

Comparison with Previous New Keynesian Open Economy Models: The knock-on effects in our model stand in contrast to the limited propagation of shocks between countries typical of previous work in the NKOE literature (e.g., Justiniano and Preston (2010)). A key difference that helps us resolve this standing puzzle lies in the way nominal rigidities affect the prices of traded goods and translate to local inflation. In the canonical NKOE model, stickiness affects traded good prices at the origin and exchange rate movements pass through entirely unimpeded to final prices at the destination. In contrast, in this paper, sticky price producers use imported goods as inputs so that the exchange rate pass-through to local prices is incomplete, consistent with the evidence in Devereux and Yetman (2010).

In Section F of the online Appendix, we present a model variant that nests both approaches. In a calibrated version of this more general model, the positive cross-country correlations described in Proposition 1 continue hold, with comovement amplified by multiple rounds of knock-on effects as in our baseline model. Moreover, we show analytically that these knock-on effects disappear only in the polar case with perfect exchange rate pass-through, contrary to empirical evidence. In this polar case, there is only a single round of direct effects of demand shocks on local output, constraining endogenous comovement to the small trade shares observed in the data.

The Backus-Kehoe-Kydland Puzzle: We end this section with two observations. First, Backus, Kehoe, and Kydland (1992) underscore that IRBC models generally imply cross-country consumption correlations that are higher than those of output, an implication that runs counter to the data. In contrast, since $E[\widehat{C}_{n,t}\widehat{C}_{n',t}] = 0$ while $E[\widehat{Y}_{n,t}\widehat{Y}_{n',t}] > 0$ for any country pair (n, n') in our setting, cross-country output correlations always exceed their consumption counterpart as in the data. Here, an idiosyncratic increase in consumption in any country raises production in all other countries. In fact, because of multilateral trade, an increase in demand in country n not only induces higher production in all of its trading partners, n', but also in all of n''s trading partners, n'', and so on. Therefore, uncorrelated demand shocks, while generating uncorrelated consumption movements across countries in this case, nevertheless produce correlated output fluctuations.

The second observation is that *i.i.d.* shocks to the growth rate of a country's TFP do not spill over to other countries in this example. Put another way, TFP shocks are absent from the key equations underlying Proposition 1. Contrary to demand shocks, local TFP shocks raise not only the supply of goods at home but also the demand for these goods at home via higher domestic wages. This mechanism, therefore, mitigates the role of international trade with respect to TFP shocks, and eliminates it entirely when these shocks are *i.i.d*, in which case TFP shocks generate cross-country comovement in neither consumption nor output.

7 Quantitative Findings

To study the quantitative implications of our model, we calibrate the steady-state values of productivity growth, inflation, employment, and the wage bill to their averages over the period 2004Q2 through 2019Q4, as well as those of the trade matrix over the same period. We also calibrate the material share and then estimate the remaining parameters using Bayesian methods.

7.1 Shock Structure

Our model includes four shocks: TFP growth shocks, $g_{n,t}$, markup shocks, $\mu_{n,t}$, monetary policy shocks, $\nu_{n,t}$, and shocks to portfolio costs, $\xi_{n,t}$. The first three shocks are standard in the New Keynesian literature while the last shock ensures that we match the behavior of real exchange rates, as discussed in Section 6, through the modified UIP condition, (23).

To allow for common shocks across countries, we consider a factor structure for the first three shocks.[24] In particular, for a generic shock $\mathsf{x}_{n,t}$, $\mathsf{x} \in \{g, \mu, \nu\}$:

$$\mathsf{x}_{n,t} = \lambda_{\mathsf{x},n}\mathsf{x}_t^G + \tilde{\mathsf{x}}_{n,t}, \tag{27}$$

$$\mathsf{x}_{n,t}^G = \rho_{\mathsf{x}}^G\mathsf{x}_{t-1}^G + \sigma_{\mathsf{x}}^G\epsilon_t^G, \tag{28}$$

$$\tilde{\mathsf{x}}_{n,t} = \tilde{\rho}_{\mathsf{x},n}\tilde{\mathsf{x}}_{n,t-1} + \tilde{\sigma}_{\mathsf{x},n}\tilde{\epsilon}_{n,t}. \tag{29}$$

Thus, each shock consists of a global factor, x_t^G, and an idiosyncratic country component, $\tilde{\mathsf{x}}_{n,t}$, both of which follow AR(1) processes. Since $\lambda_{\mathsf{x},n}$ and σ_{x}^G cannot be separately identified, we normalize $\sigma_{\mathsf{x}}^G = 1/100$. The sign of $\lambda_{\mathsf{x},n}$ is identified through its prior.[25]

With this factor structure, comovement across countries arises from either common global shocks or spillovers from country-specific shocks through trade. Note that trade also affects the propagation of global shocks.

7.2 Data and Calibrated Parameters

In the quantitative application, we rely on data for the U.S., China, the E.U., Japan, and Canada over the period 2004Q2 through 2019Q4. The first four countries capture the world's largest economies, while Canada is included as a major trading partner of the U.S.[26] The sample period allows us to have a balanced sample and captures most of the period following China's integration in the World Trade Organization. Details of the data sources and construction as well as estimation are in Section D of the online Appendix.

[24] Shocks to portfolio costs only matter to the degree that they are country specific. Global shocks in international financial markets cancel out in equation (21) unless they impact each country differently.

[25] Baumeister and Hamilton (2015) and Matthes and Schwartzman (2021) similarly use a prior for identification, but do so in the context of shock identification in vector autoregressions. Unlike those applications, the sign of $\lambda_{\mathsf{x},n}$ here is a normalization without economic content, making the identification scheme innocuous.

[26] Observe that Proposition 1 does not hinge on the number of countries. One can also verify that as $b \to 1$, all cross-country pairwise correlations in output approach 1 irrespective of N.

Table 1: Calibrated Parameters

Panel A: Steady States

		Canada	China	E.U.	Japan	U.S.
$400\,(g_n - 1)$	Productivity Growth	0.76	7.93	0.80	0.70	0.92
$400\pi_n$	Inflation	1.76	2.68	1.55	0.31	2.05
L_n	Employment	0.03	1.70	0.22	0.12	0.26
$\mathcal{E}_n W_n L_n$	Wage Bill	1.56	7.83	12.04	5.14	16.39

Panel B: Import Trade Matrix (ϖ)

	Canada	China	E.U.	Japan	U.S.
Canada	0.674	0.047	0.036	0.014	0.230
China	0.009	0.775	0.074	0.086	0.057
E.U.	0.007	0.105	0.779	0.029	0.080
Japan	0.006	0.082	0.031	0.843	0.039
U.S.	0.043	0.055	0.040	0.019	0.842

Notes: In Panel A, productivity growth and inflation are annualized and calibrated based on quarterly averages of per capita GDP growth and inflation, respectively, over the 2004Q2 to 2019Q4 sample period. Total employment, L_n, is in trillions of hours, calibrated to the product of employment and average hours, using annual data from the Penn World Table over the 2004 to 2019 sample period. The wage bill, $\mathcal{E}_n W_n L_n$, is calibrated to average GDP in trillions of current U.S. dollars, using annual data from the World Bank and OECD over the 2004 to 2019 sample period. In Panel B, import shares correspond to average Import Partner Shares over the 2004Q2 through 2019Q4 sample period, using data from World Integrated Trade Solution compiled by the World Bank. Diagonal elements are calibrated to match the share of domestically produced output. Off-diagonal elements are proportional to import shares from respective countries, scaled so that each row sums to one.

The calibrated parameters are summarized in Table 1. The steady state parameters in Panel A reflect time series means for each country over the sample period. To calibrate the matrix of import shares in Panel B, we set its diagonal elements to the share of domestic goods in consumption for each country. The off-diagonal elements, $\varpi_{nn'}$, are proportional to import shares of country n from country n' for every country pair (n, n'), scaled so that each row sums to one (since the model does not capture the entire world economy). The import trade matrix implies that most countries purchase goods mostly from themselves. Finally, we calibrate the share of value added, $1 - \gamma$, to 0.5, consistent with the share of materials in gross output across countries.

7.3 Estimation

We estimate the remaining parameters of the model using Bayesian methods. The estimation relies on observed data on per capita GDP growth, inflation, interest rates, and exchange rates in the measurement equations for each country. GDP growth, inflation, and interest rates are standard observables in the estimation of closed-economy New Keynesian models (e.g., An and Schorfheide (2007); Aruoba, Cuba-Borda, and Schorfheide (2018)). We keep to this conventional approach for the choice of observables but take into account data from multiple countries simultaneously. In addition, we include exchange rates as observables to account for their role in determining trade

Table 2: Priors

Parameter		Distribution	Mean	Std Dev
Global Parameters				
$100(\beta^{-1}-1)$	Discount Rate	Gamma	0.20	0.10
φ	Fréchet Parameter (Trade Elasticity Minus One)	Gamma	5.50	2.00
τ	Financial Frictions	Gamma	5.00	3.00
$\rho^G_{\mathrm{x},n}$	Global Shock Persistence	Beta	0.50	0.20
Country-Specific Parameters				
σ_n	Inverse Intertemporal Elasticity of Substitution	Gamma	2.00	0.50
η_n	Inverse Labor Supply Elasticity	Gamma	2.00	0.50
θ_n	Calvo Price Stickiness	Beta	0.70	0.10
ρ_n	Taylor Rule: Persistence	Beta	0.50	0.20
$r_{y,n}$	Taylor Rule: Output Coefficient	Gamma	0.50	0.25
$r_{\pi,n}$	Taylor Rule: Inflation Coefficient	Gamma	1.50	0.25
$\tilde{\rho}_{\mathrm{x},n}$	Idiosyncratic Shock Persistence	Beta	0.50	0.20
$100\tilde{\sigma}_{\mathrm{x},n}$	Idiosyncratic Shock Standard Deviation	Inverse Gamma	0.70	4.00
$\lambda_{\mathrm{x},n}$	Factor Loading	Normal	0.20	0.50

patterns and, as suggested by the global Phillips curve (22), variations in cross-country inflation.

Model parameter priors are summarized in Table 2. The prior for the price elasticity of trade, $1+\varphi$, covers the range of estimates in the literature.[27] We use a relatively flat prior for the degree of adjustment costs in global financial markets, τ. Priors for the factor loadings are chosen so that global factors account for approximately half of the variation of each shock at the prior mode. As mentioned earlier, these priors also help normalize the sign of the factor loadings by placing a small probability that factor loadings for a given shock are negative for a majority of the shock realizations. The remaining priors are comparable to the existing literature (e.g., Lubik and Schorfheide (2005); Adolfson, Laséen, Lindé, and Villani (2007); An and Schorfheide (2007)).

Accounting for international linkages within a multi-country sticky price framework raises two computational challenges relative to existing work. First, with five countries, there are a total of 90 parameters to be estimated, with 63 equations characterizing the equilibrium even though the model we lay out is in some sense already the smallest version of an environment with nominal rigidities and multilateral trade that one would reasonably consider for quantitative purposes.[28] Second, the calculation of the steady state, described in Section 4.2, is relatively time-consuming. Recall that in applying the approach developed by Dekle et al. (2007) to the steady state of our dynamic setting, we first solve for a variant of the model whose steady state matches observed average trade

[27] Microeconomic estimates of trade elasticities center around 4 but are as large as 17. See Footnote 44 in Caliendo and Parro (2015) as well as section 4.2 in Head and Mayer (2014) for comprehensive summaries of estimates. Macroeconomic estimates, however, tend to center around 2.

[28] In comparison, the medium-scale New Keynesian model in Smets and Wouters (2007) has 36 parameters and 32 equations.

Table 3: Posterior Means for Selected Parameters

Parameter		Mean Estimate				
Global Parameters						
φ	Fréchet Parameter (Trade Elasticity Minus One)	0.48				
τ	Financial Frictions	8.8×10^{-4}				
Country-Specific Parameters		Canada	China	E.U.	Japan	U.S.
σ_n	Inverse Intertemporal Elasticity of Substitution	3.08	2.10	2.72	2.43	3.40
η_n	Inverse Labor Supply Elasticity	2.71	3.12	3.41	2.91	2.83
θ_n	Calvo Price Stickiness	0.85	0.76	0.86	0.88	0.76

imbalances over our entire sample, then use the implied parameters to calculate counterfactual steady-state allocations and prices corresponding to zero trade imbalances. The latter step uses an iterative procedure that takes up to half the time needed to compute the model's likelihood for a given set of parameters. This step, therefore, potentially doubles the time taken to compute the likelihood relative to the typical steady-state calculations in prototypical New Keynesian models.

To address these computational challenges, we estimate the model using the adaptive sequential Monte Carlo (SMC) algorithm described in Cai et al. (2021). Relative to standard Markov Chain Monte Carlo methods, SMC is more robust to irregular posteriors and improves computational efficiency by allowing for parallelization.[29] The SMC algorithm constructs a sequence of intermediate *bridge distributions* which begin with the prior and end with the posterior. In some cases, Cai et al. (2021)'s adaptive approach to constructing the bridge distributions is key to obtaining accurate results. The procedure allows us to begin with particle draws from the prior and gradually modify those particles until they approximate the posterior.[30] Details of the algorithm can be found in Section D of the online Appendix.

Table 3 presents posterior means for selected parameters of interest. We plot the prior and posterior distributions for all model parameters in Section E of the online Appendix. Overall, posterior estimates align with values used in both the New Keynesian and trade literatures.

We find a price elasticity of trade, $1+\varphi$, of 1.48, consistent with estimates typically found in the international macroeconomics literature for the price elasticity of trade (e.g., Reinert and Roland-Holst (1992); Blonigen and Wilson (1999); Drozd, Kolbin, and Nosal (2021)). The aggregate data thus favor a lower elasticity than is typically obtained in the trade literature.

The posterior mean for τ of 8.8×10^{-4} implies low endogenous portfolio costs. In particular, the model at its posterior mean implies that UIP deviations in equation (23) between the U.S. and

[29] These advantages have increasingly led to the use of SMC methods to estimate DSGE models (e.g., Creal (2007); Herbst and Schorfheide (2014)). In our case, the estimation takes 17 hours on an Intel 1.8 GHz workstation with 16 cores using MATLAB.

[30] At each stage, the distribution is represented by a set of particles where each particle consists of a parameter value and a weight. With a large number of particles, the weighted average of particle values converges to expectations of the bridge distribution. To generate particles from the stage n distribution, we modify the stage $n-1$ particle values and weights appropriately. See Herbst and Schorfheide (2015) for a textbook treatment on SMC methods.

Canada, China, the E.U., and Japan are primarily driven by (differences in) exogenous financial shocks, $d\boldsymbol{\xi}_t - \mathbf{1}d\xi_{1,t}$, with standard deviations that are, respectively, 4.1, 0.3, 5.3, and 6.6 times as large as that of the endogenous portfolio costs, $\beta\tau E_t\left(d\boldsymbol{s}/\boldsymbol{e}_{t+1} - \mathbf{1}ds_{1,t+1}\right)$. The estimate is far in the left tail of our prior, with a *c.d.f.* of less than 0.001, suggesting that the data heavily favor such a small value.

The rest of Table 3 reports the posterior means for key structural parameters that are also present in closed-economy New Keynesian models. While these exhibit heterogeneity across countries, the estimates lie in the range estimated in closed-economy settings. The estimates of the inverse intertemporal elasticity of substitution, σ_n, lie between 2.10 and 3.40, the estimates of the inverse labor supply elasticity, η_n, lie between 2.71 and 3.41, and the Calvo price setting parameter, θ_n, has a posterior mean of between 0.76 and 0.88.[31] Section D of the online Appendix additionally reports impulse responses of foreign variables to a U.S. monetary policy shock in our estimated model. These are also broadly consistent with existing empirical estimates.

7.4 Multilateral Trade and Comovement Between Countries

We now consider a series of counterfactual exercises that highlight key quantitative implications of New Keynesian economies with multilateral trade linkages. We use the Kalman smoother to extract the shocks implied by our model under the posterior mean, then feed the same shocks into versions of the model with alternative assumptions on trade, price stickiness, and other model mechanisms.

Cross-Country Correlations: Table 4, Panel A, explores the role of multilateral trade in explaining global cross-country comovement. Our estimated model indicates that absent trade, cross-country correlations in output growth, inflation, and between output growth and inflation, all decline substantially. This finding is shared broadly across all countries and results primarily from knock-on network effects. It also arises despite the presence of global factors.

As reported in line A3 of Table 4, the average correlation in GDP growth across countries is 0.39 in the data but falls drastically, by about 90 percent, to 0.04 without trade. Moreover, the absence of multilateral trade reduces the average cross-country comovement in inflation by 58 percent, from 0.45 in the data to 0.19 without trade. Finally, the comovement between output growth and inflation across countries falls by 44 percent, from 0.25 in the data to 0.14 without trade. Multilateral trade, therefore, plays a key role in determining the global comovement properties of both real and nominal variables. The effect of trade on the average (pairwise) comovement of

[31] Estimating New Keynesian models, Smets and Wouters (2007) and Herbst and Schorfheide (2015) report posterior means for σ_n of 1.38 and 2.83, respectively, for the U.S.; Iiboshi, Shintani, and Ueda (2018) report a posterior mean of 1.55 for Japan. Across a broader range of methodologies, Havránek (2015) reports a mean estimate of between 0.3 and 0.4 for $1/\sigma_n$. For η_n, Smets and Wouters (2007) and Justiniano et al. (2010) find estimates of 1.83 and 3.79, respectively, for the U.S.; Iiboshi et al. (2018) find a posterior mean of 3.92 for Japan. For θ_n, Justiniano et al. (2010) report a median estimate of 0.84 for the U.S.; Sugo, Ueda, et al. (2007) find a mean estimate of 0.88 for θ_n using data from Japan; Galí, Gertler, and Lopez-Salido (2001) reports estimates of between 0.67 and 0.92 for Europe.

Table 4: Mean Cross-Country Correlations Under Different Counterfactual Models

Panel A: Role of Trade

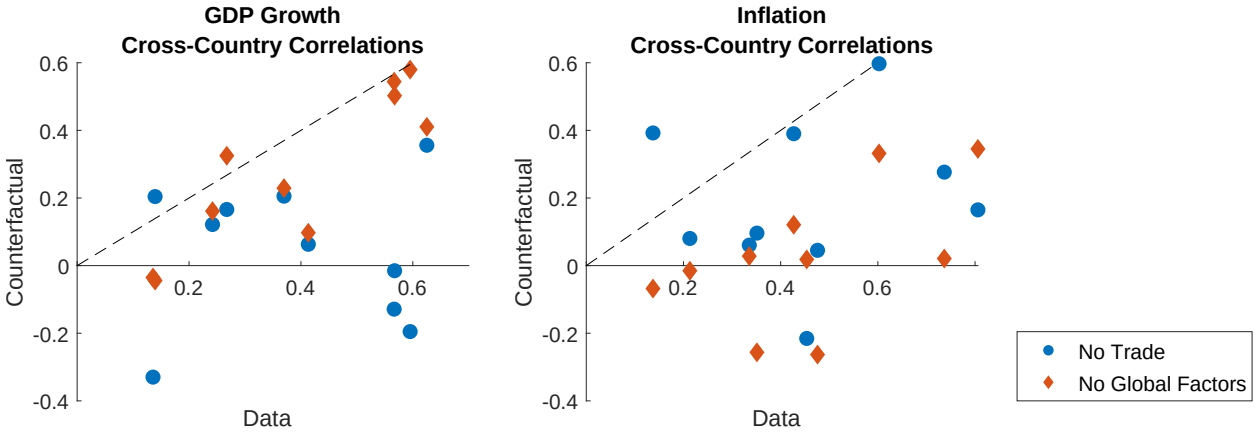
	Mean Cross-Country Correlation			
Model	GDP Growth	Inflation	GDP Growth–Inflation	Consumption Growth
A1. Data	0.39	0.45	0.25	0.18
A2. Baseline	0.39	0.45	0.25	0.05
A3. No Trade	0.04	0.19	0.14	0.04
A4. Small Trade Elasticity	0.07	0.03	0.02	0.12

Panel B: Attendant Mechanisms

	Mean Cross-Country Correlation			
Model	GDP Growth	Inflation	GDP Growth–Inflation	Consumption Growth
B1. Flexible Prices, Financial Autarky	0.19	0.32	−0.25	0.31
B2. No Roundabout Production	0.28	0.34	0.20	0.07
B3. Weaker Exchange Rate Pass-through	0.34	0.41	0.27	0.05

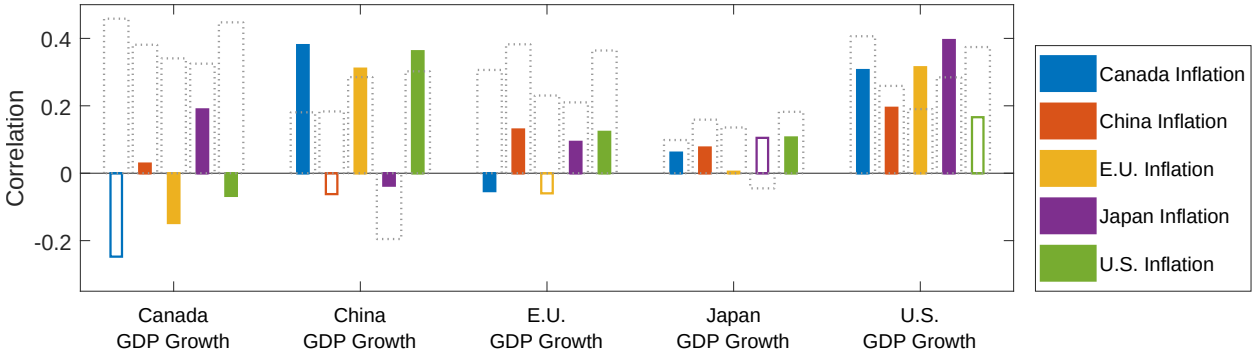
Notes: Panel A summarizes the role of trade for cross-country correlations and Panel B considers other mechanisms in the model. Line A1 presents correlations in the data; line A2 presents correlations in the full model with parameters set at the posterior mean; line A3 corresponds to the model without trade; lines A4 corresponds to the model with a small trade elasticity, $1 + \varphi = 1.072$, corresponding to a Fréchet parameter, φ, that is 0.15 times its posterior mean under the baseline estimation; line B1 corresponds to the model with low price rigidity, with θ_n at 0.01 times its posterior mean for each country n and high portfolio costs, $\tau = 10^6$; line B2 corresponds to the model without roundabout production, $\gamma = 1$; line B3 corresponds to a model with weaker exchange rate pass-through, where exchange rate pass-through is dampened by $\lambda = 0.76$.

Figure 2: Cross-Country Correlations in Data and Counterfactuals



Notes: Each marker indicates the correlation between a country pair's GDP growth (left panel) or inflation (right panel). Horizontal axis indicates correlation in the data; vertical axis indicates correlation in a counterfactual. Blue circles correspond to the counterfactual without trade; red diamonds correspond to the counterfactual without global factors.

Figure 3: Counterfactual Correlation Between GDP Growth and Inflation Without Trade



Notes: Each bar corresponds to correlation between GDP growth in country i with inflation in country j. Filled bars correspond to cross-country correlations (i.e., $i \neq j$) without trade, empty bars correspond to own-country correlations (i.e., $i = j$) without trade, gray dotted bars correspond to correlations in the data. Each group of bars corresponds to a country i's GDP growth and each color corresponds to a country j's inflation.

inflation across countries is less pronounced than for output because markup shocks, including their global factor, have a direct effect on inflation independently of trade shares.

The left panel of Figure 2 further highlights the role of trade and global factors in driving GDP growth cross-country correlations. The 45° degree line corresponds to a perfect alignment of model and data. In the counterfactual without trade (blue dots), GDP growth correlations decline notably from the 45° line in almost all cross-country pairs, with four pairs even yielding negative correlations. In contrast, the markers for the counterfactual without global factors (red diamonds) continue to lie close to the 45° line, indicating that shutting down global factors has little effect on any of these cross-country correlations. For example, the correlation in output growth between the U.S. and the E.U. is 0.57 in the data, falls to -0.13 without trade, but remains as high as 0.54 with trade but without global factors.

While average cross-country correlation in inflation is similarly driven to a large degree by trade, global factors play an important role as well. As indicated in the right panel of Figure 2, trade and global factors have varying relative importance depending on the specific country pair. For instance, the correlation in inflation between Canada and the U.S., two major trading partners, falls from 0.81 in the data all the way down to 0.16 without trade. Absent global factors but with trade, this correlation falls much less to 0.35. In contrast, global factors appear more important in driving the correlation in inflation between Canada and Japan, countries that are not as tightly connected by trade. The correlation observed in the data, 0.43, remains high at 0.39 without trade, but falls to 0.12 without global factors. As indicated in the global Phillips curve (22), while inflation inherits the comovement properties of output, it is also driven in each country by inflationary supply shocks specific to that country (i.e., markup shocks).

The pairwise comovement between GDP growth and inflation also falls substantially across countries absent multilateral trade. This finding is evident in Figure 3, which compares the pairwise

correlations between inflation and output across countries in the model without multilateral trade linkages (but with global factors) to the observed correlations reported in Figure 1.

Taken together, these findings underscore the notion that multilateral trade plays a fundamental role in driving international comovement despite countries trading mostly with themselves in Table 1, i.e., a little trade goes a long way. The intuition follows mainly from two features of multilateral trade and nominal rigidities. First, while countries trade mostly with themselves, the multilateral nature of trade and its implied network of backward and forward international linkages mean that the comovement in output between countries is substantially enhanced by knock-on network effects reflected in the Leontief inverse, the terms $b^m(\boldsymbol{\psi}^T\boldsymbol{\varpi})^m,\ m > 0$, in equation (26), capturing the different rounds of trade generated by disturbances to the economic environment. Second, the global Phillips curve described by equation (22) makes clear that every country's inflation rate potentially comoves with real activity in every other country in a way that is proportional to their trade shares, reflected here in the elements of $\boldsymbol{\varpi}$. This relationship in turn implies that the cross-country comovement between output and inflation depends in large part on the comovement in output between countries.

In line A4 of Table 4, we illustrate the quantitative importance of knock-on network effects by reducing the price elasticity of trade to $1+\varphi = 1.072$, corresponding to a Fréchet parameter φ that is 0.15 times its benchmark value, thereby reducing the indirect effects from trade. Our analytical example in Section 6.2 provided intuition for how smaller elasticities of trade, $1+\varphi$, diminished the role of indirect knock-on effects. In particular, knock-on effects vanish in equation (26) when $b = 0$, or alternatively $1 + \varphi = 0$. However, the parameterization of the Fréchet distribution imposes that $1 + \varphi > 1$, with the general model becoming non-invertible near $1 + \varphi = 1$. Thus, some indirect effects remain under the parameterization in line A4 of Table 4. Nevertheless, line A4 already points to large declines in global comovement related to output growth, inflation, and output growth and inflation.

Finally, the last column of line A2 in Table 4 quantitatively confirms the intuition in the analytical example above about the importance of nominal rigidities and multilateral trade for the Backus-Kehoe-Kydland puzzle. Backus et al. (1992) observe that IRBC models commonly imply that consumption growth is more correlated across countries than output growth even though the opposite is true in the data. For our sample of countries, the corresponding average pairwise correlations are 0.39 for output growth and 0.18 for consumption growth. The baseline model matches observed GDP growth cross-country correlations by construction and yields a lower positive consumption growth cross-country correlation, 0.05. The special case underlying Proposition 1 made it clear that even with uncorrelated consumption across countries, direct demand spillovers amplified through knock-on effects propagating through multilateral trade led to positive cross-country comovement in output growth. This intuition carries over to our quantitative findings.

7.5 Attendant Mechanisms

In this final section, we explore the role of attendant mechanisms working with nominal rigidities and multilateral trade, some highlighted in previous work. As in the previous section, we do so by studying counterfactual economies where key model mechanisms are either shut down or mitigated. Findings from these exercises are summarized in Panel B of Table 4.

Demand Shocks and Sticky Prices vs. Supply Shocks and Flexible Prices: We first compare our findings to those obtained under a neoclassical benchmark defined by flexible prices and financial autarky. This version of the model is in the vein of recent work by Huo et al. (2023) or Boeckelmann et al. (2024) who focus on the role of sectoral linkages and trade in explaining the comovement in real activity across multiple countries. Thus, line B1 of Table 4 sets the Calvo price stickiness parameter, θ, to 1 percent of the posterior mean (for each country) in the estimated model, and increases the financial frictions parameter, τ, to 10^6.

This combination of parameters results in notably lower comovement in real activity than in the data, with an average cross-country correlation in output growth that is about 1/2 its observed value, 0.19 instead of 0.39. In addition, cross-country inflation comovement declines by 30 percent and, counterfactually, comovement between inflation and output growth across countries becomes negative. In addition, the Backus-Kehoe-Kydland puzzle reemerges, with cross-country consumption correlations that exceed those of output.

Two key differences relative to our baseline estimation explain these results. First, as prices become more flexible, demand shocks cannot spread as easily across countries. As a result, demand shocks in individual countries lead to higher inflation and a stronger interest rate response under the maintained monetary policy rules. These higher rates dampen the effects of the initial shocks. Second, under financial autarky, countries no longer run trade surpluses or deficits in response to shocks. As countries trade mostly with themselves, trade patterns then remain largely unaffected by shocks and result in lower comovement across countries.

Roundabout Production: As in Auer et al. (2019), Afrouzi and Bhattarai (2023), and Devereux et al. (2023), our results are partly driven by the feedback effects from input-output relationships stemming from roundabout production. This feature by itself generates cross-country comovement in that foreign shocks not only affect domestic consumption but also feed back into the domestic production of traded goods. In addition, because of roundabout production, the input cost of traded goods inherit nominal rigidities from sticky materials prices.

To gauge the importance of these mechanisms, we consider a counterfactual in which we set the share of materials in gross output, $1-\gamma$, to zero. The production of tradables then depends only on local labor. Line B2 in Table 4 shows that removing roundabout production from our model reduces GDP growth cross-country comovement by 30 percent, inflation comovement by 25 percent, and GDP growth-inflation comovement by 20 percent. While these effects confirm the importance

of allowing for roundabout production in our estimation, they are nevertheless substantially smaller than the overall effects of trade captured in line A3.

The Effects of Exchange Rate Pass-Through: Similar to Devereux and Yetman (2010), our production structure featuring nominal rigidities in the price of final goods implies less than perfect pass-through of exchange rates to final goods prices, consistent with the observation that variations in exchange rates are not perfectly reflected in domestic inflation. Nakamura and Zerom (2010), Burstein and Gopinath (2014), and others emphasize that exchange rate fluctuations do not pass through perfectly to imported goods prices either. This is in contrast to our benchmark model where in equation (18) for instance, import weighted exchange rate movements, $\boldsymbol{\varpi}\widehat{\boldsymbol{e}}_t$, are reflected one for one in the price of the imported bundle, $\widehat{\boldsymbol{e}}_t + \widehat{\boldsymbol{mc}}_t$.

We explore the extent to which this matters for our results by dampening the pass-through of exchanges rates in equation (18) by a scalar, λ, such that

$$\widehat{\boldsymbol{e}}_t + \widehat{\boldsymbol{mc}}_t = \boldsymbol{\varpi}(\widehat{\lambda \boldsymbol{e}}_t + \widehat{\boldsymbol{w}}_t).$$

We also apply this adjustment to equation (19) to dampen the pass-through of exchanges rates to the relative price of goods in destination countries.

Line B3 in Table 4 sets $\lambda = 0.76$.[32] We find that a weaker exchange rate pass-through generally lowers cross-country comovement by around 10 percent for output growth and inflation. Dampening the pass-through has a relatively small effect in our setting because cross-country comovement arises mostly from the transmission of foreign input costs, namely wages and the price of materials, to import prices rather than from exchange rate variations. Lowering λ further to 0.5 results in cross-country output growth comovement that is about 1/3 lower compared to 90 percent lower absent trade altogether.

8 Concluding Remarks

There is substantial comovement across countries in output growth, in inflation, and between output growth and inflation despite many countries purchasing goods and services largely from themselves. This paper shows that a model with nominal rigidities and multilateral trade reconciles these facts. Sizeable comovement arise even with low trade shares because domestic variables respond to foreign disturbances through both direct effects and knock-on effects from the network structure of trade.

International linkages in a world with nominal rigidities are associated with important changes to the standard closed-economy New Keynesian Phillips curve. These changes imply a global system of Phillips curves explicitly connected by multilateral trade relationships, thus relating inflation in each country to variations in output growth, consumption, and exchange rates in all of its trading

[32] Estimates of exchange rate pass-through to import prices are relatively wide, with 0.76 corresponding to the unconditional estimate in Gopinath, Boz, Casas, Díez, Gourinchas, and Plagborg-Møller (2020).

partners. Moreover, in equilibrium, variations in output growth, consumption, and exchange rates in turn reflect the effects of foreign disturbances.

Using Bayesian methods to quantify the role of global factors and international linkages, we find that country-specific shocks alone explain almost 90 percent of the observed average pairwise cross-country correlation in output growth. These idiosyncratic shocks also explain around 1/2 the cross-country comovement in inflation, as well as between output and inflation. In addition, our framework helps resolve the Backus-Kehoe-Kydland puzzle and produces cross-country correlations in output that exceed those of consumption. We show analytically that nominal rigidities and demand shocks are central to these comovement patterns.

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Appendix A: Proof of Proposition 1

From equation (24), the solution for output in every country is in vector form,

$$\widehat{\boldsymbol{Y}}_t = -\mathcal{A}_{YY}^{-1}\boldsymbol{\psi}^T\boldsymbol{\nu}_t, \text{ where } \mathcal{A}_{YY}^{-1} = (1-b)(\boldsymbol{I} - b\boldsymbol{\psi}^T\boldsymbol{\varpi})^{-1},$$

with implied variance-covariance matrix given by,

$$E\left[\widehat{\boldsymbol{Y}}_t\widehat{\boldsymbol{Y}}_t^T\right] = \mathcal{A}_{YY}^{-1}\boldsymbol{\psi}^T\boldsymbol{\psi}\left(\mathcal{A}_{YY}^{-1}\right)^T.$$

Since $\boldsymbol{\psi}^T \geq 0$, it immediately follows that $E\left[\widehat{\boldsymbol{Y}}_t\widehat{\boldsymbol{Y}}_t^T\right] \geq 0$ whenever $\mathcal{A}_{YY}^{-1} \geq 0$.

To show that $\mathcal{A}_{YY}^{-1} \geq 0$, recall that $\mathcal{A}_{YY} = a(\boldsymbol{I} - b\boldsymbol{\psi}^T\boldsymbol{\varpi})$ where $b = \frac{(1+\varphi)\eta}{1+(1+\varphi)\eta}$ and $a = \frac{1}{1-b}$. Because the matrices of import and export shares, $\boldsymbol{\varpi}$ and $\boldsymbol{\psi}^T$ respectively, are stochastic matrices, so is their product, $\boldsymbol{\psi}^T\boldsymbol{\varpi}$. It follows that $\rho(\boldsymbol{\psi}^T\boldsymbol{\varpi}) = 1$ where $\rho(\boldsymbol{\psi}^T\boldsymbol{\varpi}) = \max_{\Lambda(\boldsymbol{\psi}^T\boldsymbol{\varpi})}\{|\lambda|\}$ such that $\Lambda(\boldsymbol{\psi}^T\boldsymbol{\varpi}) = \{\lambda|\lambda \text{ is an eigenvalue of } \boldsymbol{\psi}^T\boldsymbol{\varpi}\}$, i.e., the spectral radius of $\boldsymbol{\psi}^T\boldsymbol{\varpi}$ is 1. In addition, since $b < 1$, the spectral radius of $b\boldsymbol{\psi}^T\boldsymbol{\varpi}$ is strictly less than 1, $\rho(b\boldsymbol{\psi}^T\boldsymbol{\varpi}) < 1$. We can then write $\mathcal{A}_{YY}^{-1}$ as

$$\mathcal{A}_{YY}^{-1} = \left[a(\boldsymbol{I} - b\boldsymbol{\psi}^T\boldsymbol{\varpi})\right]^{-1} = \frac{1}{a}\sum_{m=0}^{\infty} b^m(\boldsymbol{\psi}^T\boldsymbol{\varpi})^m.$$

Given that $\boldsymbol{\psi}^T\boldsymbol{\varpi} \geq 0$, it follows that $\mathcal{A}_{YY}^{-1} \geq 0$ and therefore $E\left[\widehat{\boldsymbol{Y}}_t\widehat{\boldsymbol{Y}}_t^T\right] \geq 0$.

The solution for inflation above is given by

$$\begin{aligned}\widehat{\boldsymbol{\pi}}_t &= \kappa\eta\boldsymbol{\varpi}\widehat{\boldsymbol{Y}}_t - \kappa\sigma\boldsymbol{\nu}_t + \kappa\boldsymbol{\mu}_t \\ &= -\kappa\left(\eta\boldsymbol{\varpi}\mathcal{A}_{YY}^{-1}\boldsymbol{\psi}^T + \sigma\right)\boldsymbol{\nu}_t + \kappa\boldsymbol{\mu}_t.\end{aligned}$$

Therefore, we can express the variance-covariance matrix for inflation, $\widehat{\boldsymbol{\pi}}$ as

$$E\left[\widehat{\boldsymbol{\pi}}_t\widehat{\boldsymbol{\pi}}_t^T\right] = \kappa^2\left[\eta^2\boldsymbol{\varpi}\mathcal{A}_{YY}^{-1}\boldsymbol{\psi}^T\boldsymbol{\psi}\mathcal{A}_{YY}^{-1}\boldsymbol{\varpi}^T + \eta\sigma\left(\boldsymbol{\varpi}\mathcal{A}_{YY}^{-1}\boldsymbol{\psi}^T + \boldsymbol{\psi}(\mathcal{A}_{YY}^{-1})^T\boldsymbol{\varpi}^T\right) + \sigma^2\boldsymbol{I}\right].$$

Since $\boldsymbol{\varpi}\mathcal{A}_{YY}^{-1}\boldsymbol{\psi}^T = \frac{1}{a}\sum_{m=0}^{\infty} b^m(\boldsymbol{\varpi}\boldsymbol{\psi}^T)^{m+1}$ and $\boldsymbol{\varpi}\boldsymbol{\psi}^T \geq 0$, we have that $E\left[\widehat{\boldsymbol{\pi}}_t\widehat{\boldsymbol{\pi}}_t^T\right] \geq 0$.

Finally, we have

$$E\left[\widehat{\boldsymbol{\pi}}_t\widehat{\boldsymbol{Y}}_t^T\right] = \kappa^2\left[\eta\boldsymbol{\varpi}E\left[\widehat{\boldsymbol{Y}}_t\widehat{\boldsymbol{Y}}_t^T\right] + \sigma\boldsymbol{\psi}\left(\mathcal{A}_{YY}^{-1}\right)^T\right] \geq 0$$

since $\boldsymbol{\varpi}E\left[\widehat{\boldsymbol{Y}}_t\widehat{\boldsymbol{Y}}_t^T\right] = \frac{1}{a}\sum_{m=0}^{\infty} b^m\boldsymbol{\varpi}(\boldsymbol{\psi}^T\boldsymbol{\varpi})^m \geq 0$ and $\boldsymbol{\psi}\left(\mathcal{A}_{YY}^{-1}\right)^T = \frac{1}{a}\sum_{m=0}^{\infty} b^m\left[(\boldsymbol{\psi}^T\boldsymbol{\varpi})^m\boldsymbol{\psi}^T\right]^T \geq 0$.