Macroeconomic Effects of Collateral Requirements and Financial Shocks

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Abstract

A common feature of economic downturns is the collapse in firms borrowing and tighter collateral conditions for the business sector. Once the economic crisis hit, it became increasingly difficult for firms to access credit when pledging collateral as banks attached much significance to its value. This article explores the implications of borrower's side collateral constraints in the business cycle. Entrepreneurs, who are collateral-constrained, are subject to speculative swings in collateral price and face shocks to the amount of revenues that a bank can recover after a default. The effect of an increase in collateral requirements is highly significant. It has a long-lasting impact on output and induces a strong phase of investment, which is amplified by a high credit supply. Our model assigns an important role for collateral in the shock decomposition, explaining almost half of the variance in business credit.

Keywords: Business Loan, Collateral, Financial Shocks.

JEL Codes: E32, E44, G21.

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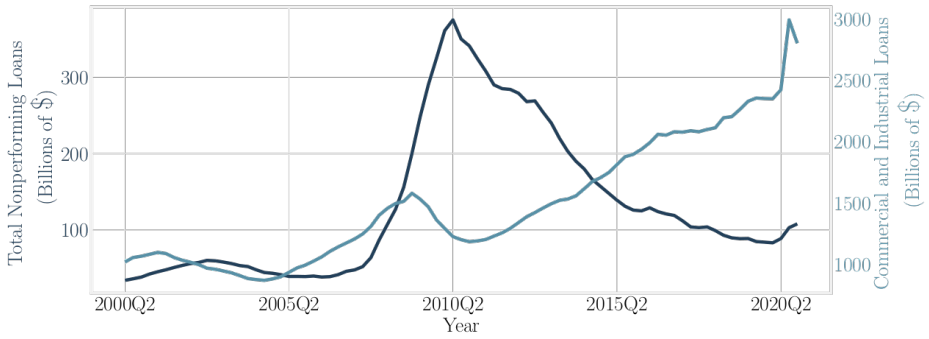
1. Introduction

The role of collateral in the financial system has attracted the attention of economists more than anytime. The lack of collateral during the global banking crisis of 2008 caused a considerable effect on banks' lending activities, which are dependent on not only interest rates, but also on collateral. The objective of banks' collateral policies is to control and avoid damages that may come from unsecured loans and improve bank stability. Banks may place stricter conditions using borrower-based measures, such as loan-to-value limits, ask for high-quality collateral, and lend less. The share of business loan supply as a percentage of gross domestic product (GDP) fell dramatically from 11% in 2009 to 8% in 2011 for the United States (see Figure 1). Throughout the same period, non-performing loans rose considerably, reaching their peak in 2010, while total business loans dropped continuously that same year, but rose again after 2011 (see Figure 1). We suspect that the movement of these three indicators is due to both low profitability in the business sector and the relaxation of credit policy prior to 2010 in the United States. In principle, strict collateral requirements would likely inhibit credit growth when economic crisis occur, which could explain changes in credit supply.[1] In this paper, we argue that the contraction and expansion of loans over the past decade are due to the change in collateral requirements and that the collateral channel exerts an influence on the economy.

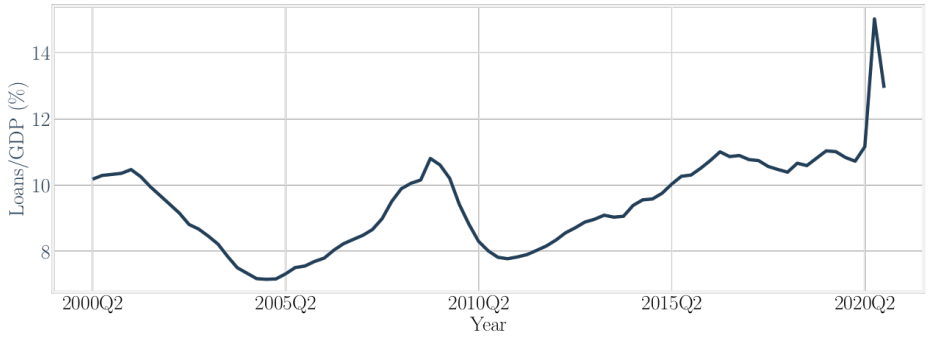
What is sorely lacking in the studies of collateral constraint frictions is empirical evidence on the role of collateral liquidation. Entrepreneurs typically offer assets as collateral to borrow from banks, whereas banks are subject to exogenous variations in the recovery value on delinquent debt. The contributions of our paper are twofold. The first is that it accounts for the collateral requirement shifter, interpreted as a shock to the amount of revenue that a bank can recover after a default. The impact of this shock materializes through both the collateral constraint and enforcement limitations channels. The second contribution is that it accounts for speculative swings in collateral price. Essentially, when the market price of capital differs from the capital's fundamental value. As clarified in later sections, the collateral requirement limit matters greatly for the emergence of speculative swings in collateral price. When an exogenous increase in collateral requirements occurs (relaxing the collateral constraint), demand for collateral increases, as does its price (capital demand curve shifting to the right). Subsequently, this raises collateral prices and induces firms to overborrow in future periods, thus placing upward pressure on investment and output. For a formal analysis of pecuniary externalities, see Bianchi and Mendoza (2018). As had long been the case, many macroeconomic models concerned with the effects of collateral constraint abstract from the interaction between speculative swings in price and collateral limits, and

[1] To clarify, a tight collateral requirement (low collateral requirement) reduces the credit supply, whereas a collateral relaxation (high collateral requirement) increases it. In either case, credit supply is driven by changes in collateral requirements.

assume that prices are uniquely based on fundamentals. Our paper aims to fill this gap in the literature.



A. Commercial and Industrial (CI) Loans and Non-performing Loans



B. Loans / GDP

FIGURE 1. Total Loans, Non-performing Loans, and Loan/GDP Ratio

Notes: Data from Federal Reserve Economic Data (FRED): Total nonperforming loans for commercial banks (USNP). Total stock of commercial and industrial loans issued by all commercial banks (BUSLOANSNSA) divided by gross domestic product (GDP). Quarterly data between 2000-2020

We focus on the role of collateral as a driver of aggregate fluctuations in the context of a dynamic stochastic general equilibrium (DSGE) model. In this particular context, entrepreneurs own specific collateral and are financially constrained. The two new ingredients are an exogenous collateral requirement shifter that affects firms' access to credit and influences bank recoveries in case of default, and eventually a speculative collateral price which emerges when the fundamental price differs from the market price of capital. Bernanke and Gertler (2000) introduced the non-fundamental component of capital price, and argued that both poor regulatory practices and imperfect rationality of investor explain the movement in the speculative price component. The model is estimated over the period 2000–2023 using both financial and real aggregate data.

First, we show a perceptible effect of collateral requirement limit on the economy. This has a long-lasting impact on output and induces a strong phase of investment, which is amplified by a high credit

supply. A positive collateral requirement shock implies a reduced default threshold; an increase in entrepreneurial net worth and collateral price. The bank lending channel propagates this shock through a rise in lending activity as capital increases in abundance, thus leading to an expansion in investment and output. This result contrasts with the well-known findings of Justiniano, Primiceri, and Tambalotti (2015), who showed that shocks to collateral have negligible effects on the macroeconomy. This contrast is plausible as, in their settings, the price feedback on collateral was curtailed and did not transmit sufficiently exogenous variations in the model. The model assigns an important role for collateral in the shock decomposition, explaining almost half of the variance in business credit. This is a logical implication as bank lending decisions primarily depend on the borrower's collateral, while investment-specific technical change (ISTC) plays no role in macroeconomic fluctuations, which is consistent with Schmitt-Grohe and Uribe (2012). The entrepreneur risk shock in the variance decomposition explains less of the variance of output, whereas it emerges as the most important shock in Christiano, Motto, and Rostagno (2014).

Second, a speculative price component implicitly emerged in the dynamic model when the collateral conditions were relaxed, largely due to high demand for collateral. The speculative swings in collateral price responded positively to an increase in loan-to-value limit and were largely persistent compared to the response of fundamental price. Third, collateral requirements were highly volatile during 2007–2009, reflecting the changes in collateral conditions on business loans. Finally, we provide empirical support for the aforementioned results. The Bayesian Vector Autoregressions evidence favors the model prediction and confirm a positive relationship between relaxed collateral terms and economic expansion and credit boom episodes. The slump in credit supply in the aftermath of 2019 and, similarly, during the global financial crisis of 2008, coincides with a large share of banks reporting stricter collateral limits.

A vast literature has introduced financial intermediation into mainstream macroeconomic models. Some major early contributions include Bernanke and Gertler (1989) and Bernanke, Gertler, and Gilchrist (1999). Other important research contributions on New Keynesian models with financial frictions include Gertler and Kiyotaki (2010), Gertler and Karadi (2011), Christiano, Motto, and Rostagno (2014), Del-Negro and Schorfheide (2013), Del-Negro, Giannoni, and Schorfheide (2015), Iacoviello (2015), Gertler, Kiyotaki, and Prestipino (2017), Guerrieri and Iacoviello (2017), and Akinci and Queralto (2022). Related studies that advanced this strand of literature include Brunnermeier and Sannikov (2014), who considered the effect of various policies on financial stability and the efficiency of financial regulations using a continuous time dynamic model; Jakab and Kumhof (2015), who studied the effect of financial shocks to illustrate the differences between intermediation of loanable funds and financing through money creation models of banking; Jermann and Quadrini (2012), who provided a more detailed discussion about the contribution of financial shocks to macroeconomic fluctuations; and He and Krishnamurthy (2019), who addressed

the effects of intermediary equity shocks in the presence of nonlinearity. Our main contribution is that our paper analyzes the impact of an exogenous collateral requirement shifter, which is crucial for the emergence of speculative swings in collateral price. Previous studies have restricted their analyses to financial shocks and disregarded the demand-induced collateral price volatility when credit supply is expanded. In contrast, we deem the presence of speculative swings and fundamental forces, reflected in asset prices important to uncovering the role of collateral requirement limit.

Moreover, this paper contributes to the existing literature on credit constraints. In particular, Justiniano, Primiceri, and Tambalotti (2015) studied the macroeconomic implications of leveraging cycles. Their model accounted for the housing sector and heterogeneity among households, with the authors finding that household leveraging and deleveraging have a small macroeconomic effect. Iacoviello and Pavan (2013) focused on how changes in credit conditions, with high downpayments, served to worsen the recession. In a different model with pecuniary externality, Bianchi and Mendoza (2018) found a significant effect of collateral constraint on the real economy. Geanakoplos and Zame (2007) claimed that market outcomes depend on collateral through many channels. In detail, the scarcity and amount of collateral that an agent must hold to secure their debt can imply restrictions on financing activities, and consequently, make collateral scarce in a way that induces significant social inefficiency. Other related works include Guerrieri and Lorenzoni (2017), who evaluated the channel by which the shock to the borrowing limit propagates to the economy in an incomplete market framework. For the propagation of credit supply shock in the Euro area, see Gerali et al. (2010). In a related study, Becard and Gauthier (2022) considered the impact of collateral shocks on the economy. Ferrante (2019) pursued a similar idea, but with the collateral value shock on the lender side instead, and this disturbance materializes through the bank incentive constraint. This paper's approach differs in two important ways. First, we assume that only entrepreneurs are financially constrained by banks' collateral requirements; therefore, bank lending is limited to business loans. Second, we assess the role of collateral by considering collateral price components: fundamental and speculative. The main advantage of accounting for the two components is that it allows us to understand the economic interaction between the relaxation of loan-to-value limit and the speculative price pressure.

This paper is also related to a growing literature investigating the implications of macro-prudential policies, e.g. Kim and Mehrotra (2018), Franz (2020), Aikman et al. (2021), Martin, Mendicino, and Van der Ghote (2021), Van der Ghote (2021), Drechsel and Kim (2022), and Ottonello, Perez, and Varraso (2022). Recent papers by Ottonello and Song (2022) and Mendicino et al. (2020) have considered the optimal collateral requirements, while Alpandaa and Zubairy (2017) studied the macroeconomic implications on household indebtedness. Moreover, Christensen et al. (2009), Lambertini, Mendicino, and Punzi (2013) and Rubio and Carrasco-Gallego (2014) focused on bank regulatory changes and explored the implication

of loan-to-value ratio as a macro-prudential tool. For macroprudential regulations in the US, multiple waves of major bank regulations were implemented, affecting how they evaluate and process collateral, as well as such macroprudential instruments as capital buffers and stress testing. It is worth our emphasizing that the key distinction between these papers and ours is that we exclusively focus on the implications of changes in collateral requirements in economies with financially-constrained entrepreneurs. Specifically, we embed an exogenous collateral disturbance into a medium-scale New Keynesian model, and focus on the interactions between collateral requirements limit and macroeconomic fluctuations. We assess the impact of collateral requirement limit on capital price and how it influences both the fundamental and speculative price.

Layout. Section 2 details the collateral collection process in the US, which motivated the inclusion of collateral constraint in the model. Section 3 presents the basic model. Section 4 examines the transmission mechanism of collateral shocks and its contribution to economic fluctuations, discusses the role of financial shocks, and estimates a Bayesian Vector Autoregressions model to assess the impacts of relaxed collateral requirement limit. Lastly, Section 5 draws relevant conclusions.

2. Motivating Evidence

This section documents the importance of collateral for business lending and economic activity, as well as the potential interaction between collateral liquidation and macroeconomic fluctuations, which supports the inclusion of collateral into macroeconomic models.

What Evidence Exists of Changes in Collateral Requirements? One way to motivate the inclusion of a collateral constraint into the medium-scale New Keynesian model is to explore the changes in collateral conditions.[2] We rely on the latest Small Business Lending Survey (SBLC). This is a nationally representative survey that collects information on small business lending practices. Here, we focus on the responses of surveyed banks about the tightening of collateral conditions in order to bolster our claim. In particular, in the response to the question, *In your opinion, how have your terms of C& I loans to US small businesses changed over the most recent calendar quarter?*, the respondent must specify the reason for change, of which collateral requirement is one. This information on collateral conditions has been collected on a quarterly basis since 2017.

[2]The terms "loan-to-value limit" and "collateral requirements" are used interchangeably throughout the paper.

TABLE 1. Collateral Requirements Amid the Recession

	Pre-Crisis				Recession		Post Crisis			
	2019Q1	2019Q2	2019Q3	2019Q4	2020Q1	2020Q2	2020Q3	2020Q4	2021Q1	2021Q2
Aggregate Data										
CI Loans/GDP	0.11	0.11	0.11	0.11	0.11	0.15	0.13	0.12	0.12	0.11
CI Loans	2.4%	0.0%	1.3%	0.3%	2.3%	22.1%	-4.9%	-5.9%	-2.5%	-2.4%
Survey Data										
Tightened considerably	0.0%	0.0%	0.0%	0.0%	0.9%	0.8%	2.5%	1.9%	0.9%	1.7%
Tightened somewhat	8.3%	10.3%	10.2%	12.5%	20.4%	26.3%	16.9%	17.1%	10.3%	6.9%
Total	8.3%	10.3%	10.2%	12.5%	21.2%	27.1%	19.5%	19.0%	11.2%	8.6%
N. Observations	133	126	128	120	113	118	118	105	107	116

CI Loans: Commercial and Industrial Loans, All Commercial Banks, Percent Change, Quarterly, Seasonally Adjusted.

CI Loans/GDP: Commercial and Industrial Loans, All Commercial Banks Billions of US Dollars, Seasonally Adjusted /Gross Domestic Product Billions of US Dollars, Seasonally Adjusted.

[c] Collateral requirement tightened considerably and tightened somewhat.

Table 1 details the tightness of the collateral requirements and the dynamics of business loans from aggregate and survey data. According to this data, 27% of surveyed banks reported a tightening of collateral conditions on business loans in the second quarter of 2020, signaling a decrease in loan-to-value ratio and limited access to credit amid the Covid-19 crisis. Business lending contraction is perhaps the most common feature of the post-crisis economy. However, the uncertainty and deterioration of borrower's conditions could lead to changes in collateral conditions, causing a sharp business credit crunch for significant period. This observation suggests that credit supply is indeed linked to changes in collateral requirements.

Additionally, credit supply to the business sector experienced a remarkable increase in the second quarter of 2020, despite banks having reported a tightening in collateral conditions. If we were to focus only on the collateral requirement tightness and the firm loan growth rate during the recession episode, we would be misled. Note that, over the course of the pandemic, the US government provided financial relief to firms in an effort to support those with depressed revenues and preserve small business resilience. Turning to the post-recession credit supply, a large decline in loan supply emerged, which could be linked to tightness in the collateral requirement indeed, banks reporting tighter collateral conditions were the highest. The contraction in credit supply to firms is in line with the observation in Figure 1, which depicts the fluctuation in credit supply between 2000 and 2020. The collateral requirements were tighter between 2007-2009, with evidence from the "Senior Loan Officer Opinion Survey on Bank Lending Practices" indicating that over 50% of banks reported an increase in collateral requirements for small firms during

the global financial crisis.[3] Furthermore, the global financial crisis and the Covid-19 pandemic differ substantially in terms of characteristics and origin, but they share the common feature of a drop in firms borrowing. Departing from Table 1, it is clear that the decline in business loan supply after the Covid-19 recession is associated with more banks reporting tighter collateral terms.

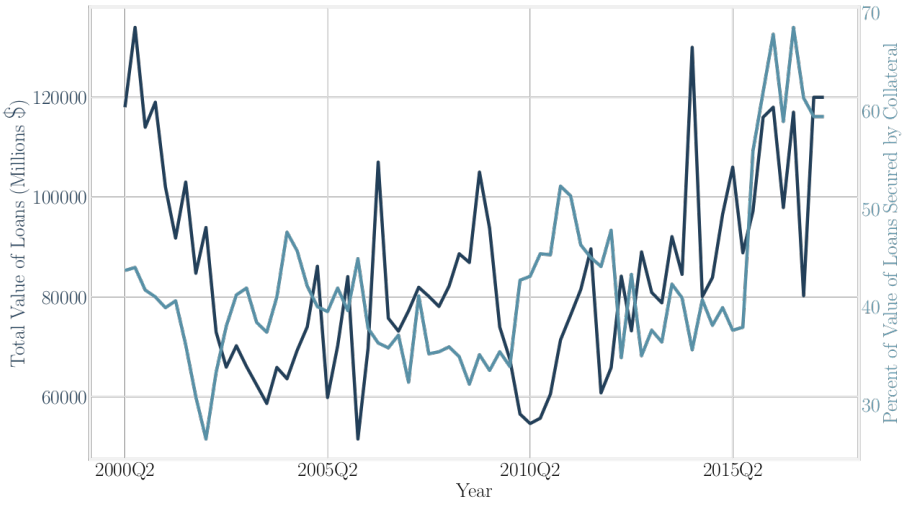


FIGURE 2. Loans with Pledged Collateral

Notes: This survey data is retrieved from FRED, Federal Reserve Bank of St. Louis. The "Survey of Terms of Business Lending" reports a quarterly collection of quantitative and qualitative information on bank lending to small businesses. Quarterly data between 2000-2017

Collateralized Business Loans. To argue that the collateral channel itself is economically important, we now evaluate the importance of collateralized debt. Figure 2 shows the percentage of loans secured by collateral and the volume of loans. The share of secured loans increased continuously to reach over 60% of total business loans in 2017. This tendency did not necessarily lead to a drop or rise in lending activity; however, one may assume that banks tend to prefer and approve loans that are secured by collateral as banks can recover the pledged asset in case of default. In principle, business assets can be leveraged to secure loans, but, with tighter collateral conditions especially during uncertain times credit supply to firms may be reduced. Furthermore, two events in 2008 and 2014 saw an uptick in loan supply, while the level of collateralized loans remained the same. Between 2009 and 2013, the percentage of loans approved by banks secured by collateral varied between 40% to 50%. These observed dynamics in business credit supply could partly be driven by the growth in the share of asset-based lending.

[3] In Appendix A, we report the evolution of the share of banks reporting an increase in collateral requirements for small firms.

Additionally, the inclusion of collateral constraints into the model seeks to capture the tightness in collateral conditions that essentially affect credit growth. An important contribution by Lian and Ma (2021) showed that cash flow-based debt represents 80% of total lending to US non-financial firms. We believe that allowing for two types of debt, namely asset- and cash-flow-based debt, could be a possible extension. In either collateral constraint form, the borrowers pledge capital K. The only difference is the absence of the price and return on capital, which implies that the interaction between the collateral requirement shock and the omitted aggregates will be reduced.

What would happen if we were to modify the assumption on asset-based lending? Consider a collateral constraint $\kappa_t K_{t+1} \geq (1 + R^e_{t-1})M_{t-1}$ instead of $(1 + R^k_t)\kappa_{t-1}Q^k_{t-1}K_{t-1} \geq (1 + R^e_{t-1})M_{t-1}$. Cash flow-based debt may also function as asset-based debt, with the only difference being the absence of capital price and return on capital. We would expect that equilibrium price and returns on capital would be less sensitive to changes in collateral requirements. The equilibrium interaction between these aggregates will be reduced. The focus of this study is not to analyze the form of the collateral constraint, but rather the macroeconomic implication of this constraint on the real economy. Indeed, we seek to asses collateral constraint on firms that essentially leverage their own assets to access external financing, meaning we need a collateral constraint that captures such important elements as collateral price, return on assets, and the pledged collateral.

As we have already shown, banks tend to tighten their collateral requirements during uncertain times, leading to a drop in credit supply. This raises questions as to the impact of collateral on firm lending, economic activity, and the relevance of this channel in quantitatively explaining the movement of macroeconomic aggregates. The main conclusions from the data, as described so far, constitute the motivation for writing this paper and focusing on the macroeconomic impact of collateral.

3. A Model with Collateral Constraint and Enforcement Limitation

Our model is based on Bernanke, Gertler, and Gilchrist (1999) and Christiano, Motto, and Rostagno (2014), augmented with an exogenous collateral constraint shock, and a shock to the amount of revenues that a bank can recover after a default. Not only did we add a risky collateral constraint, as in Becard and Gauthier (2022) and Drechsel (2023), but also a collateral shifter in the enforcement limitation of entrepreneurs, commonly referred to in the literature as the participation constraint of entrepreneurs. Furthermore, we introduced a non-fundamental component to collateral price that implicitly identifies the firm value. The goal of this study is to measure the importance of collateral requirement variations in the business cycle.

Entrepreneurs.

Each entrepreneur purchases capital good K_{t-1} at price Q^k_{t-1} using loans M_{t-1} obtained from banks and net worth N_{t-1}, such that $K_{t-1}Q^k_{t-1} = M_{t-1} + N_{t-1}$. After purchasing capital, entrepreneurs are hit by an idiosyncratic shock ω_t, which captures the idiosyncratic risk in businesses and describes the case when entrepreneurs are unable to pay their debt. Entrepreneurs go bankrupt when $\omega \leq \bar{\omega}_t$ as they are unable to pay the interest and principal. Letting risk shock $\sigma_{e,t}$ denote the standard deviation of $\log \omega$ and follows an exogenous process $\sigma_{e,t} = \rho_{\sigma_e}\sigma_{e,t-1} + \epsilon^{\sigma_e}_t$, the disturbance $\epsilon^{\sigma_e}_t$ has a mean of zero and a standard deviation of σ_{σ^e}; thus, the risk level increases when σ_e rises. We adopt the standard financial accelerator structure, as in Bernanke, Gertler, and Gilchrist (1999) and Christiano, Motto, and Rostagno (2004). For a given distribution of the idiosyncratic shock ω, there exists an endogenous equilibrium threshold $\bar{\omega}_t$, such that all entrepreneurs with $\omega < \bar{\omega}_t$ will optimally default.

After observing the idiosyncratic shock, the entrepreneur chooses the capital utilization rate u_t, and provides capital services $u_t\omega_t K_t$ at rental rate r^k_t. Formally, the entrepreneur enjoys the average gross nominal rate of return on capital: $(1 + R^k_{t+1}) = \frac{(1-\tau^k)\left[r^k_{t+1}u_{t+1} - \frac{a(u_{t+1})}{\chi_{t+1}}\right]P_{t+1} + (1-\delta)Q^k_{t+1} + \tau^k\delta Q^k_t}{Q^k_t}$, where τ^k is the capital tax, χ_t denotes the investment specific technical change shock, P_t represent the aggregate level price at time t, $a(.)$ is the adjustment cost function, and δ is the depreciation rate.

In each period, entrepreneurs must offer collateral to borrow from banks. They face a risky collateral constraint, defined as:

$$\boldsymbol{\kappa_{t-1}}(1 + R^k_t)Q^k_{t-1}K_{t-1}\bar{\omega}_t \geq (1 + R^e_{t-1})M_{t-1}. \tag{3.1}$$

The collateral limit κ, which can be interpreted as a loan-to-value ratio shifter, obeys an autoregressive process of order 1 with $\kappa_t = \rho_\kappa\kappa_{t-1} + \epsilon^\kappa_t$. This is equivalent to saying that the current collateral requirement depends on past realizations and a random disturbance ϵ^κ_t with a mean of zero and a standard deviation of σ^κ. The role of this disturbance is that a positive shock allows for entrepreneurs to pledge more collateral and relaxes the constraint, whereas a negative shock captures the tightness of the constraint. As clarified in Section 4.1, this disturbance explicitly affects the price of the pledged assets, the accumulation of capital, the rate of return on capital, the contractual interest rate, and the loan demand. Both entrepreneurs and banks know κ when the contract is signed.

Entrepreneurs maximize their earnings subject to the enforcement limitation:

$$[1 - F_t(\omega_{t+1})](1 + R^e_t)M_t + (1 - \mu)\int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega)(1 + R^k_{t+1})\boldsymbol{\kappa_t}Q^k_t K_t \geq (1 + R_{t+1})M_t. \tag{3.2}$$

where R^k_{t+1} is the rate of return on capital and R^e_{t+1} is the net interest rate paid by entrepreneurs on their debt M_t. $\bar{\omega}$ is the threshold level under which an entrepreneur declares bankruptcy and cannot pay the debt and R_{t+1} is the risk-free interest rate. The threshold value of the idiosyncratic risk in businesses $\bar{\omega}$ is

defined, such that if $\omega > \bar{\omega}$, the borrower retains the collateral and pays $(1 + R_t^e)M_t$. If $\omega < \bar{\omega}$, the bank will seize the collateral $\kappa_t(1 + R_{t+1}^k)Q_t^k K_t$ and pay a monitoring cost μ.[4]

The intuition behind this condition is as follows. The financial intermediary suffers no loss when providing loans to entrepreneurs. That is, in equilibrium, banks earn an expected return of $(1 + R_{t+1})M_t$ at the riskless interest rate R_{t+1}. Either the entrepreneur pays the debt $(1 + R_t^e)M_t$ with probability $[1 - F_t(\omega_{t+1})]$ and keeps the collateral, or declares bankruptcy with probability $\int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega)$ and the bank seizes the collateral $\kappa_t Q_t^k K_t$ net of monitoring cost μ that the bank pays to monitor defaulting entrepreneurs.

As per Bernanke, Gertler, and Gilchrist (1999), we adopt the assumption of the free entry and exit of entrepreneurs from the market. Financial wealth describes the case where only a fraction of entrepreneurs survive and accumulate their wealth in addition to that belonging to defaulting entrepreneurs. Thus, defaulting entrepreneurs exit the market. The net worth of surviving entrepreneurs is given by: $N_t = \gamma^e\big([1 - \kappa_{t-1}\Gamma_{t-1}(\bar{\omega}_t)](1 + R_t^k)Q_{t-1}^k K_{t-1}\big) + w$, where γ^e is the percentage of entrepreneurs who survive and w captures the transfer payment when new entrepreneurs enter the next period.

The assumption that the market price of capital differs from the capital's fundamental value is important to capture the idea that asset prices could reflect speculative swings and fundamental forces. This assumption on price volatility relates to Bernanke and Gertler (2000) and is important because price volatility is driven by fundamental and speculative forces, and our specification can distinguish between the two components. Therefore, if a speculative price component exists, then $Q_t^k \neq V_t/K_t$. This assumption implies that the relative price of capital is of the form

$$Q_t^k = \underbrace{\frac{V_t}{K_t}}_{\text{average capital price}} + \underbrace{\mathbf{s_t^*}}_{\text{speculative component}} \tag{3.3}$$

where V_t/K_t is the average capital price and s_t^* denotes the speculative component. The latter is seen as a price bubble that explains the fluctuations in capital market and macroeconomic aggregates. The model's implied-speculative component rationalizes the idea that entrepreneurs observe a speculative price pressure s_t^* whenever the value of Q^k differs from the average capital price. This idea aligns with recent studies measuring the impact of asset bubbles in a business cycle model, including Miao, Wang, and Xu (2015), who studied how an exogenous sentiment shock can affect the size of stock market bubbles and Bigio and Zilberman (2019), who generated an endogenous speculation in asset prices through belief heterogeneity, investor sentiments, and extreme market segmentation. The speculative component s_t^* is not an exogenous object, but rather an equilibrium outcome. Entrepreneurs can observe changes in Q_t^k and the optimality conditions for capital producers in our model determine the optimum supply for

[4] See Appendix F for detailed computations.

capital K_t and determine capital price Q_t^k. We characterize the aggregate firm value as in Schmitt-Grohe and Uribe (2012): $V_t = Y_t - W_t L_t - \frac{I_t}{\chi_t} + \beta \frac{\lambda_{z,t+1}}{\lambda_{z,t}} Q_{t+1}^k K_{t+1}$, where the aggregate firm value V_t equals the current dividend and presents the discounted value of future dividends.

Rest of the Model

Goods Production. The economy is populated by a continuum of firms and operates under monopolistic competition. Each firm has the final good stock, written as:

$$Y_t = \left[\int_0^1 Y_{j,t}^{\frac{1}{1+\nu_p}} dj \right]^{1+\nu_p}. \tag{3.4}$$

The final good is indexed by $j \in [0,1]$, and $1 \leq \nu_p < \infty$ is a shock that follows a standard AR(1) process $\nu_{p,t} = \rho_{\nu_p} \nu_{p,t-1} + \epsilon_t^{\nu_p}$, with $\epsilon_t^{\nu_p} \sim \mathcal{N}(0, \sigma_{\nu_p}^2)$. The shock to price markup is defined as: $\nu_{p,t} = \rho_{\nu_p} \nu_{p,t-1} + \epsilon_t^{\nu_p}$, where the shock captures changes in price markup, the disturbance $\epsilon_t^{\nu_p}$ has a mean of zero, and a standard deviation of σ_{ν_p}. A higher price markup ν_p implies that firms have market power and that the good is less substitutable. The monopolist produces the intermediate good j using two inputs, labor and capital, to produce a differentiated good according to the following intermediate good production function that relates outputs to inputs:

$$Y_{j,t} = \gamma_t (u_t K_{j,t})^{\alpha} (z_t L_{j,t})^{1-\alpha} - \Phi z_t^*, \tag{3.5}$$

where $L_{j,t}$ is the labor input, z_t is the exogenous productivity shock, and $K_{j,t}$ is the capital input proportional to u_t, the utilization rate of capital. The nonstationary productivity shock z_t, which is an important source of aggregate fluctuations, is assumed to have a growth rate of $\mu_{z,t}^* = z_t / z_{t-1}$. The technological trend $\mu_{z,t}^*$ is assumed to obey to the following law of motion: $\mu_{z,t}^* = \rho_{\mu_z^*} \mu_{z,t-1}^* + \epsilon_t^{\mu_z^*}$, where $\epsilon_t^{\mu_z^*}$ has a mean of zero and a standard deviation of $\sigma_{\mu_z^*}$. $\alpha \in (0,1)$ is the intermediate good elasticity that measures the responsiveness of $Y_{j,t}$ to changes in the utilization of capital and variations in labor. γ_t is a covariance stationary technology shock that evolves according to the following process: $\gamma_t = \rho_\gamma \gamma_{t-1} + \epsilon_t^{\gamma}$, where the innovation ϵ_t^{γ} has mean zero and the standard deviation σ_γ. Φ is a fixed cost proportional to $z_t^* = z_t \chi_t^{\alpha/(1-\alpha)}$, a process introduced to ensure we have a balanced growth path and χ_t represents investment specific technology change.

The nonstationary productivity shock z_t is assumed to have a growth rate of $\mu_{z,t}^* = z_t / z_{t-1}$. The technological trend $\mu_{z,t}^*$ adheres to an AR(1) process $\mu_{z,t}^* = \rho_{\mu_z^*} \mu_{z,t-1}^* + \epsilon_t^{\mu_z^*}$, where $\epsilon_t^{\mu_z^*}$ has a mean of zero and a standard deviation of $\sigma_{\mu_z^*}$. Following Justiniano, Primiceri, and Tambalotti (2011), we assumed nonstationary investment to have a growth rate of $\mu_{\chi,t} = \chi_t / \chi_{t-1}$. Investment-specific technical change shock also adheres to an AR(1) process $\mu_{\chi,t} = \rho_{\mu_\chi} \mu_{\chi,t-1} + \epsilon_t^{\mu_\chi}$, where $\epsilon_t^{\mu_\chi}$ represents an innovation to the growth rate of investment specific productivity, has a mean of zero, and a standard deviation of $\sigma_{\mu\chi}$.

We incorporated price stickiness into the model, drawn from the monopolistic power of the producer. The monopolist sets the price P_t of the good by adopting a variant of Calvo-type frictions. In any given period, the producer can reoptimize the price with probability $1-\zeta_p$; otherwise, it cannot reoptimize with probability ζ_p. Afterward, the price level à la Calvo is given by:

$$P_t = \left[(1-\zeta_p)(\tilde{P}_t)^{\frac{\nu_p}{1-\nu_p}} + \zeta_p(\tilde{\pi}P_{t-1})^{\frac{\nu_p}{1-\nu_p}}\right]^{\frac{1-\nu_p}{\nu_p}}. \tag{3.6}$$

If the producer cannot reoptimize, then it sets the price as $P_{j,t} = \tilde{\pi}_t P_{j,t-1}$, where $\tilde{\pi} = (\pi_{trg,t})^{i}(\pi_{t-1})^{1-i}$. Letting P_t denote the price of Y_t, the actual inflation is $\pi_{t-1} = P_{t-1}/P_{t-2}$, and $\pi_{trg,t}$ is the target inflation. $\tilde{P}_t$ is the price level that maximizes the expected discounted value of future profits. After setting the prices and considering the quantity demanded, the monopolist minimizes the production cost: $W_j L_{j,t} + \tilde{r}_t^k P_t u_t K_{j,t-1}$, subject to production function (3.5), to determine its demand for labor and capital inputs.

Labor Market. In this environment, labor services are provided by households to intermediate firms via labor contractors. The organization of the labor market adopted in the paper is similar to that presented in Christiano, Motto, and Rostagno (2014), which itself is a variant of Erceg, Henderson, and Levin (2000). Labor contractors combine differentiated labor inputs $L_{i,t}$ $i \in [0,1]$ that they convert into homogeneous labor L_t. We assumed labor services to have the Dixit-Stiglitz form:

$$L_t = \left[\int_0^1 (L_{i,t})^{\frac{1}{\nu_l}} di\right]^{\nu_l}$$

subject to $\int_0^1 W_{i,t}L_{i,t}di = W_t L_t$. Here, labor contractors determine the demand for labor type, where ν_l denotes the fixed markup, L_t is the quantity of homogenous labor, and $W_{i,t}$ is the wage rate. To define the aggregate wage W_t, we took the total labor supply expression $W_t L_t = \int_0^1 W_{i,t}L_{i,t}di$, then simplified it to obtain the expression for the aggregate wage: $W_t = [\int_0^1 W_{i,t}^{-(1+\nu_l)/\nu_l} di]^{-\nu_l/(1+\nu_l)}$. Then, to determine the optimal wage, we assumed there to be a monopoly union that represents all workers and sets their wages. The monopoly union faces Calvo-style frictions,

$$W_t = \left[(1-\zeta_l)(\tilde{W}_t)^{\frac{1}{1-\nu_l}} + \zeta_l(\tilde{\pi}_{l,t}(\mu_{z,t}^*)^{i_\mu}(\mu_{z,t}^*)^{1-i_\mu}W_{i-1})^{\frac{1}{1-\nu_l}}\right]^{1-\nu_l}. \tag{3.7}$$

With probability $1-\zeta_l$, the monopoly can reoptimize the wage. While with probability ζ_l, the monopoly cannot do so. If the monopoly cannot reoptimize, then it sets the wage according to: $W_{w,t} = (\mu_{z,t}^*)^{i_l}(\mu_z^*)^{1-i_l}\tilde{\pi}_{l,t}W_{w,t-1}$, where $\tilde{\pi}_{l,t} = (\pi_{t,t}^*)^{i_l}(\pi_{l,t-1}^*)^{1-i_l}$. The wage $W_{w,t}$ must be equal to the previous wage level at time $t-1$ adjusted by the inflation and growth rates.

Households. All households in this economy have identical preferences, which take the form:

$$\begin{aligned} &\text{maximize} \quad E_0 \Sigma_{t=0}^{\infty} \beta^t \left\{ \zeta_{c,t} (\log(C_t - bC_{t-1})) - \psi_l \frac{(L_t)^{1+\sigma_l}}{1+\sigma_l} \right\} \\ &\text{subject to} \quad (1+\tau^c) P_t C_t + T_t \leq (1-\tau^l) W_t L_t + (1+R_t) T_{t-1}, \end{aligned} \tag{3.8}$$

where $0 < \beta < 1$ is the discount factor, C_t is per capita consumption, T_t is the deposit, τ^c denotes the consumption tax, and b is the internal habit in consumption. The parameter σ_l is the curvature on disutility of labor, ψ_l is the disutility weight on labor, and $\zeta_{c,t}$ is the preference shock. The consumption preference evolves over time according to the law of motion: $\zeta_{c,t} = \rho_{\zeta_c} \zeta_{c,t-1} + \epsilon_t^{\zeta_c}$, where current consumption preferences are explained by past consumption preferences and disturbed by an error term $\epsilon_t^{\zeta_c}$, which has a mean of zero and a standard deviation of σ_{ζ^c}.

Capital Producers. In this environment, capital producers combine the previous capital with the investment goods to produce a new capital, which is then supplied to entrepreneurs. The representative capital producer operates the following technology:

$$K_t = (1-\delta) K_{t-1} + \left(1 - S\left(\zeta_{I,t}, \frac{I_t}{I_{t-1}}\right)\right) I_t, \tag{3.9}$$

where capital decays at the fixed rate $0 < \delta \leq 1$. According to this equation, the new capital depends on the existing capital and investment good I_t. As in Smets and Wouters (2007) and Christiano, Motto, and Rostagno (2014), the quantity of investment at period t is proportional to the adjustment cost function S, and $\zeta_{I,t}$ denotes the shock to the MEI, a disturbance to the process that transforms an investment good into productive capital, which is assumed to obey an AR(1) process $\zeta_{i,t} = \rho_{\zeta_i} \zeta_{i,t-1} + \epsilon_t^{\zeta_i}$, where $\rho_{\zeta_i} \in (0,1)$, the term $\epsilon_t^{\zeta_i}$ has a mean of zero and a standard deviation of σ_{ζ_i}.

Monetary Policy Rule. We assumed that monetary policy would obey a standard Taylor rule. The linearized form of monetary policy rule is given by:

$$R_t - R = \rho_p (R_{t-1} - R) + (1-\rho_p)[a_\pi (E_t \pi_{t+1} - \pi_{trg,t}) + a_{\Delta y} (Y_t - Y)] + \epsilon_{m,t}, \tag{3.10}$$

where ρ_p is a smoothing parameter, $\epsilon_{m,t}$ is the monetary policy shock, a_π is the policy weight on inflation, and $a_{\Delta y}$ is the policy weight on output growth. This rule states that changes in interest rates R_t depend on the deviation between a central bank's inflation target and expected inflation rate, as well as on the deviation between the output and its steady state.

Resource Constraints. We complete the setup of the model by specifying the main aggregate resource constraint, which can be written as:

$$Y_t = D_t + G_t + C_t + \frac{I_t}{\chi_t \mu_{\chi,t}} + a(u_t)\frac{K_t}{\chi_t \mu^*_{z,t}} + \theta\frac{1-\gamma^e}{\gamma^e}(N_{t+1} - w). \tag{3.11}$$

The first term on the right-hand side of the resource constraint equation $D_t^{w,e}$ represents banks' spending on monitoring entrepreneurs with $D_t = \kappa_{t-1}\mu G(\bar{\omega})(1+R_t^k)(Q_{t-1}^k K_t)/(\pi_t \mu^*_{z,t})$, while G_t denotes the government consumption, given by $G_t = z_t^* g_t$, where g_t is a shock that follows an AR(1) process $g_t = \rho_g g_{t-1} + \epsilon_t^g$, where the disturbance ϵ_t^g is an i.i.d process with a mean of zero and a standard deviation of σ_g. C_t is the aggregate consumption, the term $I_t/(\chi_t \mu_{\chi,t})$ defines the aggregate investment, and $a(u_t)K_t/(\chi_t \mu^*_{z,t})$ is the aggregate capital utilization costs of entrepreneurs.

Equilibruim and Solution. The dynamic competitive equilibrium is defined by a set of quantities and prices $\{\ \pi_t,\ \lambda_{1,t},\ r_t^k,\ I_t,\ u_t,\ \bar{\omega}_t,\ R_t^k,\ N_{t-1},\ \lambda_{z,t},\ C_t,\ \tilde{W}_t,\ L_t,\ K_t,\ R_t,\ P_t,\ W_t,\ F_{p,t},\ F_{w,t},\ Y_t,\ M_t,\ Q_t^k,\ \tilde{P}_t,\ s_t^*\ \}$, and subject to nine structural exogenous shocks $\{\ \nu_{p,t},\ \mu_{\chi,t},\ g_t,\ \mu^*_{z,t},\ \gamma_t,\ \sigma_{e,t},\ \zeta_{c,t},\ \zeta_{i,t},\ \kappa_t\ \}$. The model derivation and equilibrium are presented in the Appendix F and G. We detrended the model using a specific trend growth for each variable (Details are available in the Appendix H). Finally, we computed nonstochastic steady states, with the model steady states reported in Appendix I.

Formal Definition of Collateral Requirements. We defined the collateral requirement as an autoregressive process of order 1. This specification choice sought to capture the notion that the level of loan-to-value limit has an history dependence and an innovation term. The collateral constraint incorporated here is on the borrower's, not the lender's, side, meaning that we occasionally refer to it as "borrower's side collateral constraints". We can imagine a variety of appellations, such as "risky collateral constraints", that have been suggested by some readers, or simply "loan-to-value ratio". Considering the changes in collateral requirement as an exogenous borrower-based macroprudential policy is also an appropriate and valid interpretation of the collateral requirement shock. If this does not seem straightforward, consider a bank regulation as in Alpandaa and Zubairy (2017), where the importance of collateral requirements in addressing household indebtedness was effectively demonstrated. Prior research has also focused on the regulatory loan-to-value ratio. See, for example Lambertini, Mendicino, and Punzi (2013) and Rubio and Carrasco-Gallego (2014), who provided alternative definitions by assuming that loan-to-value ratio is countercyclically dependent on credit growth.

Role of Enforcement Limitation. One may ask why would it be optimal for the entrepreneur to default and lose everything, and why it would still be optimal to back up bad debt. In this setting, this is actually

far from the case. The collateral requirement shock propagates through the collateral constraint and enforcement limitations. Suppose κ_t is very small, and the bankruptcy risk is high, banks will therefore reduce credit supply, which will lessen investment. Entrepreneurs will then lose only a fraction, rather than all, of their collateral. The enforcement limitation guarantees that banks receive returns from supplying loans at risk-free interest rates R_t. That is, the value of expropriated collateral to recover the loss from bankruptcy would be equivalent to the amount that banks would receive from loan repayments in risk-free environment.

To save space, we have delegated the entire quantitative implementation to Appendix C. This Appendix presents the calibration and the estimation of the parameters of the models and reports the moments of the model and the data.

4. Results

This section first examines the transmission mechanism of collateral shocks and its contribution to economic fluctuations, and estimates a Bayesian Vector Autoregressions model to assess the impacts of relaxed collateral requirement limit.

4.1. The Effect of Collateral Shocks

This section examines the responses of the main macroeconomic aggregates to a collateral requirement shock. We measured the contribution of a loan-to-value limit through shock and historical decomposition analyses. We evaluated the model-implied collateral requirement limit, and finally estimated a BVAR model to assess the effects of collateral requirement.

Figure 3 reports the impact of a positive collateral requirement shock, which generates a rise in the entrepreneur's net worth and boosts external financing. This expansionary collateral requirement shock also implies that investment rises, but default threshold declines. The collateral constraint acts as a friction of firm financing, which directly affects the flow of capital to firms. The results are broadly consistent with the findings in Mendicino (2012) and Jensen et al. (2020). The expansion of credit supply to businesses benefits the collateral capacity of entrepreneurs and shifts the capital demand curve to the right. Moreover, collateral price Q^k rises faster and the speculative component is expanded even further due to the potential combination of lower risk levels and higher entrepreneur net worth. A large decrease in uncertainty serves to increase stock prices. This observation accords with Berger, Dew-Becker, and Giglio (2020), who found a negative relationship between volatility (changes in stock prices) and real activity, while uncertainty shocks are generally uncorrelated with realized volatility.

Notice that the adjustment observed in the speculative price component potentially relates to the

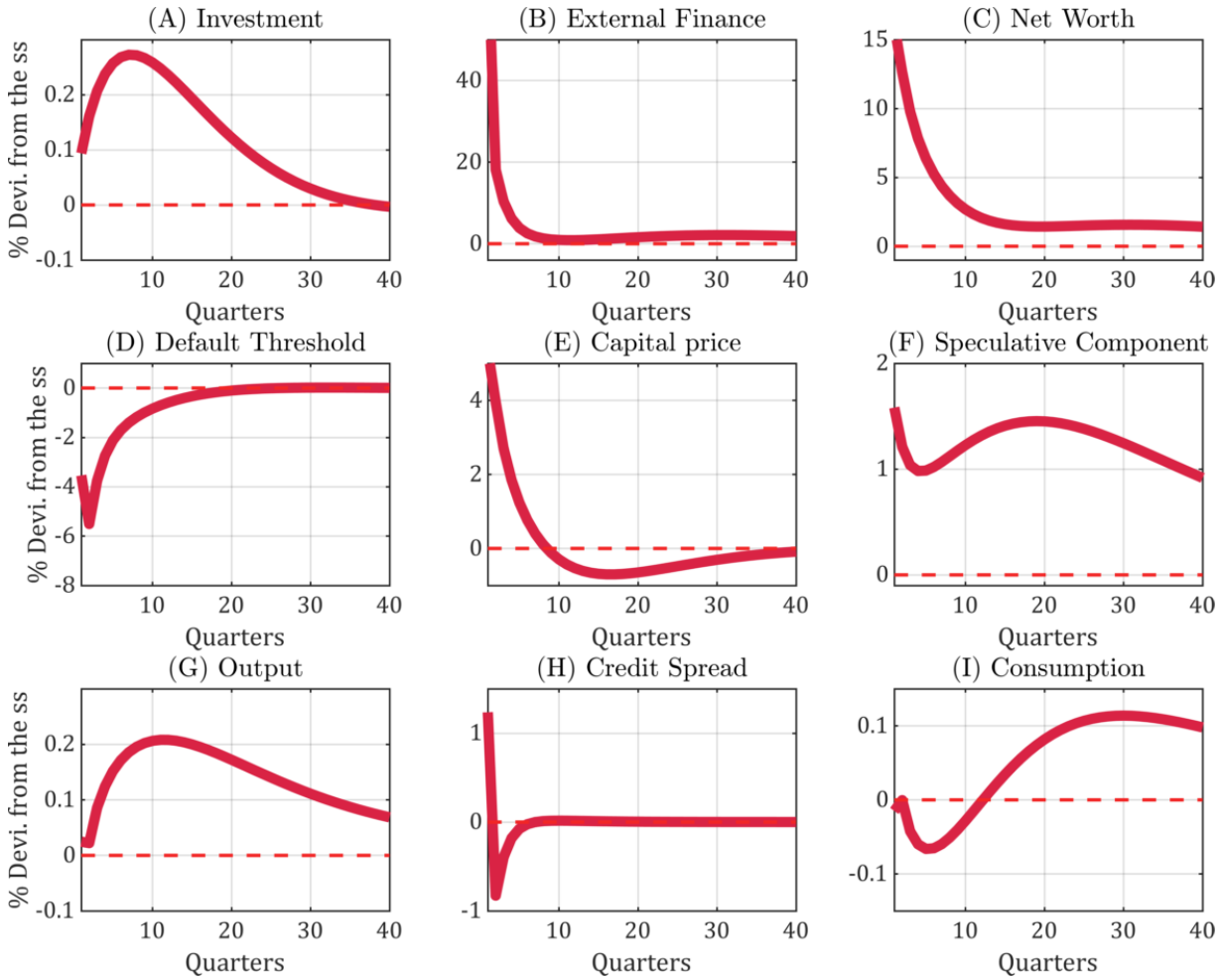


FIGURE 3. Impulse Responses to Collateral Requirement Shocks

Notes: Time horizons are quarterly. The figure shows the impulse responses of investment, external financing, and net worth to a positive collateral requirement shock (a 1% standard deviation). Parameters are set at posterior mean.

transition of credit spread back to the steady state. The relaxation in loan-to-value limits was initially associated with a large short-lived high premium in the entrepreneur interest rate over the risk-free interest rate, but the premium subdued rapidly after the 5th quarter. On the other hand, this shock induced a short-lived increase in interest rate and a temporary drop in consumption up to 5 quarters after the shock (the peak response). A decrease in interest rates occurred only after 10 quarters and coincided with a persistent increase in consumption. Many previous studies have emphasised the interaction between monetary policy and loan-to-value ratio, including Bachmann and Rüth (2020), who argued that the interaction between monetary policy and tightening of loan-to-value ratio shapes the response of collateral price, and Van der Ghote (2021), who studied the coordination between monetary policy and macroprudential instrument to contain the amplification of the capital price over the boom-bust cycle.

As previously discussed, this friction exerts a strong upward pressure on firm investment and thus raises the economic activity by over 0.2%. It also creates a high demand for capital and leads to further increases in collateral price. This mechanism was discussed in Görtz, Tsoukalas, and Zanetti (2022), where it was argued that the price feedback of Tobin Q can sever the financial channel and amplify the propagation of shocks. This result fits into the narrative of the credit boom episode. As banks relax the loan-to-value ratio when the likelihood of default among borrowers is relatively low, the induced credit expansion leads to an increase in firms investment and a rise in entrepreneurial earnings. This is consistent with Angelini, Neri, and Panetta (2014) who examined the key features of boom and bust cycles in credit supply, Liu, Wang, and Zha (2013), who showed how collateral constraint amplifies investment dynamics, and eventually Akinci and Queralto (2022), who presented the main triggers of a banking crisis through the lens of a macroeconomic framework that captures non-linear financial accelerator dynamics (see Appendix E for the detailed impulse responses of the estimated model to all shocks).

4.2. Measuring the Importance of Collateral: Shock and Historical Decomposition

We turn now to analyzing the economy model through the shock decomposition of the entire time horizon of 2000–2023 with all shocks included. We select the growth in macroeconomic variables, which are output, consumption, investment, external financing, worked hours, net worth, total factor productivity, government spending, and capital price.

Shock Decomposition. Panel A of Table (2) illustrates the contribution of all shocks to growth in nine macroeconomic aggregates over the entire 2000-2023 time horizon, while Panel B describes how the collateral requirement innovations explain the conditional variance of the selected economic aggregates over the time horizon: one quarter, 1 year, 2 years, and 5 years.

Regarding the price markup shocks, the estimation results show that markup disturbance explains 20% of the variance of output growth. This shock also explains a significant fraction of variation in the growth of consumption (11%), external financing (16%), and hours worked (70%). Put simply, changes in firm's market power appear to significantly affect real activity.

Additionally, we found the MEI to play a central role in driving fluctuations in the model. Over 51% of investment fluctuations could be attributed to this shock, and this disturbance also explained 20% of the unconditional variance of output growth. However, we fond that it accounted for 7% of the variance of hours worked growth and 3% of the variance of capital price growth. Both Greenwood, Hercowitz, and Krusell (1997) and Justiniano, Primiceri, and Tambalotti (2011) viewed the ISTC as one of the most important drivers of economic growth in business cycle frameworks. In contrast, our model suggests that the ISTC plays no role in the macroeconomic movement and accounts for only 1% of the variance of

TABLE 2. Variance Decomposition Predicted by the Model

	Y	*C*	*I*	*M*	*H*	*N*	*TFP*	*G*	*Q*
Panel A: unconditional variance decomposition									
Shocks [a]									
Price Markup	20	11	19	16	70	7	0	0	7
ISTC	1	0	0	0	0	0	0	0	0
Government Spending	1	0	0	0	0	0	0	100	0
Technology Trend	5	2	1	0	1	0	100	0	8
Stationary Technology	0	0	0	1	2	1	0	0	0
Entrepreneur Risk	5	0	12	25	0	57	0	0	77
Consumption Preference	42	84	2	2	14	3	0	0	0
MEI	21	1	51	8	7	9	0	0	3
Collateral requirement	6	0	14	47	6	21	0	0	5
Monetary Policy	1	0	0	0	0	1	0	0	0
Panel B: conditional variance decomposition - collateral requirements shock									
Horizon									
Q1	4	0	12	27	3	24	0	0	6
Q4	6	0	16	36	3	21	0	0	5
Q8	6	0	16	43	4	21	0	0	5
Q20	5	0	14	47	3	21	0	0	5

[a] Variance decomposition in percentage of the entire 2000–2023 time horizon. Parameters are set at posterior mean. Columns refer to the growth rate of output, consumption, investment, external financing, hours, net worth, total factor productivity, government spending, and capital price.

[b] Investment specific technical change.

[c] Marginal efficiency of investment.

output growth. The presence of three other financial shocks in this framework could explain why the effect of the ISTC vanishes. This is consistent with Schmitt-Grohe and Uribe (2012), who found minor effect of the ISTC on real variables.

According to the table, entrepreneur risk shock accounts for 57% of the variance of net worth growth and 77% of the variance of capital price growth, but explains only 17% of the variance of investment growth. It is useful to contrast these results with Christiano, Motto, and Rostagno (2014), whose main finding was that entrepreneur risk shock accounted for 60% of fluctuations in output growth. This is one important difference with the present paper, where entrepreneur risk shock accounted for only 5%. The estimation results clearly highlight that consumption preference substantially contributes to

the fluctuations of aggregate consumption. As discussed above, markup shocks explained 11% of the unconditional variance of consumption, in accordance with Iacoviello (2015). The model implies that consumption preference disturbance accounts for 42% of output growth and 14% of the variance of hours worked growth. Furthermore, we also found that government spending explained only 1% of the variance of output growth. Quantitatively, a shock to government spending appears to play a minor role in macroeconomic fluctuations.

Furthermore, the model includes two production disturbances: technology trend and stationary technology. These two shocks can modestly explain the movement in macroeconomic aggregates. This result stands in stark contrast to the existing literature, where these disturbances have tended to be assigned important roles. The estimated model technology trend explained only 5% of the variance of output growth and 2% of the variance of consumption. Meanwhile, stationary technology accounted for 2% of the unconditional variance hours worked growth and explained only 1% of net worth growth and external financing growth.

On the other hand, collateral requirements were an important source of aggregate fluctuations in the model. The shock accounted for 47% of fluctuations in external financing, 14% of the variance of investment, 5% of the variance of capital price, and 21% of the variance of net worth. It should be noted that the conditional variance decomposition over different horizons delivered similar results. Collateral requirement innovation explained almost half of the variance in external financing (47) within 5 years. This was technically expected since collateral constraint shocks predominantly affect the financial sector. These results are consistent with the findings of Jakab and Kumhof (2015) and Becard and Gauthier (2022). They also speak to the empirical evidence presented in Görtz, Tsoukalas, and Zanetti (2022), who emphasized the importance of the financial channel for business innovation activities.

Historical Decomposition. To deepen our understanding of the role of financial shocks, we used the model to quantify the extent to which they can drive economic fluctuations. As several economic shocks were incorporated into the model, we separated them into financial and real shocks. The latter include price markup, government spending, technology trend, stationary technology, and consumption preference, while the former include ISTC, entrepreneur risk, MEI, and collateral requirement. The example in the figure below illustrates how financial and real shocks shape macroeconomic fluctuations in the US economy by providing the historical decomposition of output growth.

Figure 4 displays the contribution of economic shocks to output growth, which appears to be driven by financial shocks, especially during the financial crisis and the recent pandemic. The model can thus be said to fairly capture the behavior of macroeconomic aggregates, attributing an important role for financial shocks. Fuentes-Albero (2019) demonstrated that output volatility was mainly explained by

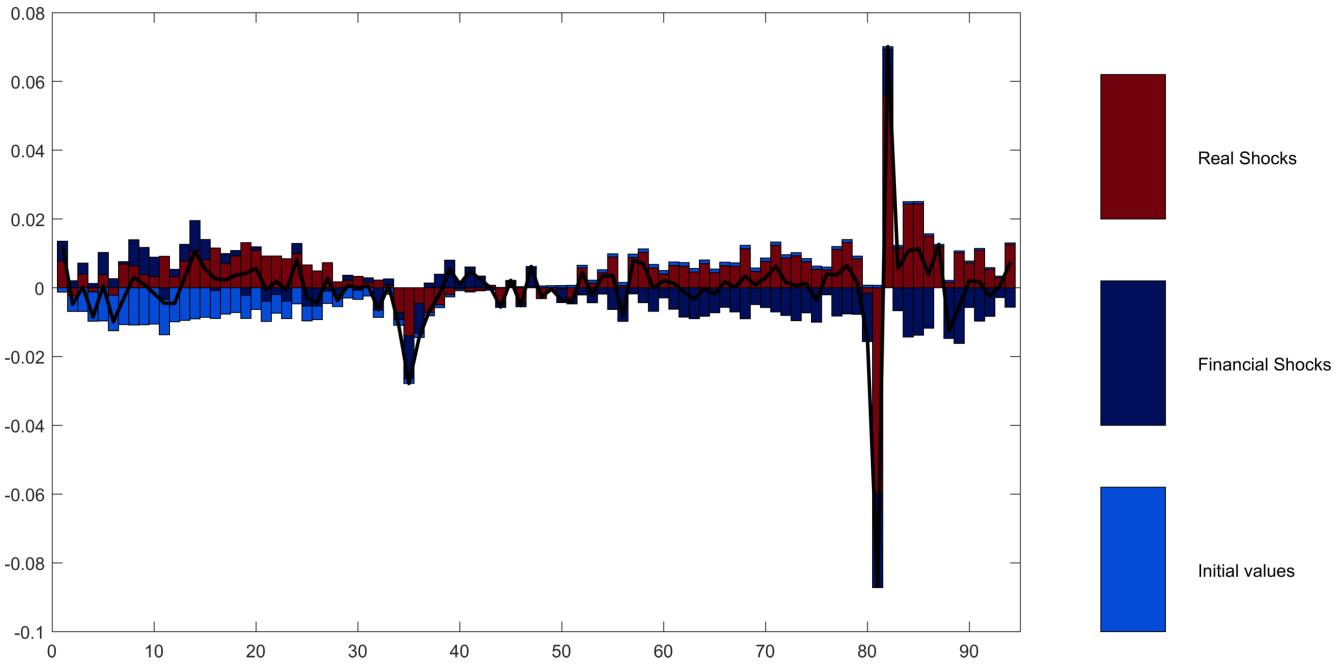


FIGURE 4. Contribution of Financial Shocks to Output Growth between 2000-2023

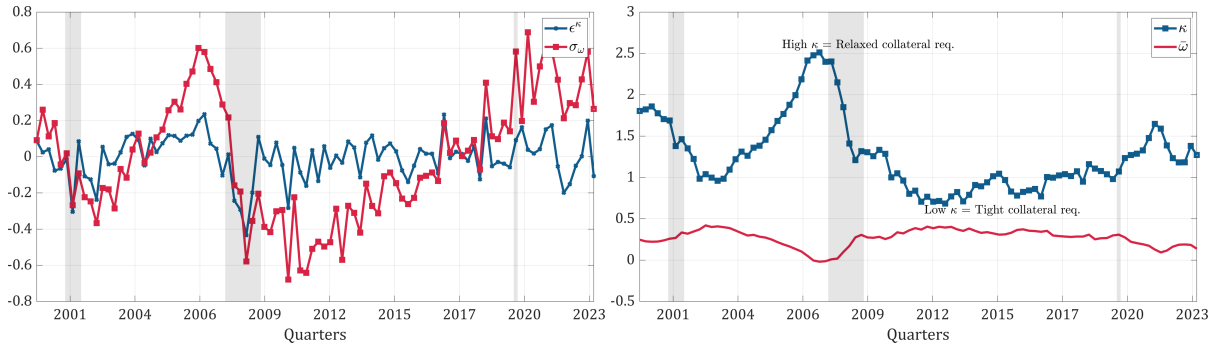
Notes: Shock decomposition of output and investment. The solid line represents the data and the colored bar represents the contribution of shocks in the aggregate movements. Real shocks include price markup, government spending, technology trend, stationary technology, and consumption preference. Financial shocks include ISTC, entrepreneur risk, MEI, and collateral requirement. Parameters are set at posterior mean.

financial factors in the 1970s, however, this association was weakened during the great moderation. This was due to the relaxation in credit conditions and the expansion of business loans. This idea was also emphasized by Jensen, Ravn, and Santoro (2018), who examined the association between high macroeconomic volatility and the relaxation of collateral constraints. This is also consistent with the recent work of Del-Negro et al. (2017), who found that a liquidity shock led to a general decline in funding for investment and output during the Lehman episode.

4.3. Collateral Requirement Process

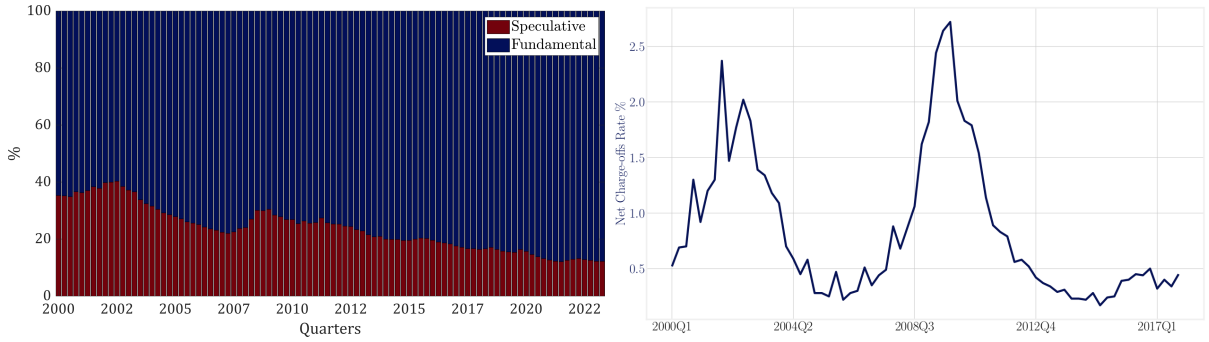
This section examines the model-implied collateral requirement over 2000–2023. The time variation of κ gives an intuition on how loan-to-value ratio has changed over time, and is interpreted as an indicator of changing in collateral conditions.

Panel A of Figure 5 portrays the exogenous innovations to collateral requirement and entrepreneur risk from the estimated model. Intuitively, the movement in ϵ^{κ} could be interpreted as an exogenous change in bank decisions by either tightening or easing collateral requirements. Innovation to ϵ_{σ_e} reflect exogenous shifts in risk level. Panel B shows the model-implied collateral requirement and default



(A) Smoothed shocks to loan-to-value limit and entrepreneur risk

(B) Simulated collateral requirements and endogenous default threshold



(C) Capital price

(D) CI net charge-offs rate

FIGURE 5. Collateral Requirements and Endogenous Capital Price: Fundamental Forces Versus Non-fundamental Swings

Notes: Panel A presents the model-implied smoothed innovation to collateral requirement and entrepreneur risk. Panel B describes the model-implied collateral requirements and default threshold. Panel C shows the model-implied average capital price (fundamental value) and speculative price (non-fundamental value) relative to the market price of capital. Model parameters are set at the posterior mean. Estimation sample: 2000Q1 and 2023Q4. Panel D shows the net charge-offs rate on commercial and industrial loans in the US. These data were retrieved from the FDIC. The data are quarterly between 2000Q1 and 2017Q4

threshold, while Panel C shows the speculative and fundamental price relative to the price of capital in percentage units predicted by the model. Given our model's features of an idiosyncratic risk in the business sector ω, a comparable real-world indicator of the entrepreneur probability of default would be the net charge-offs rate on business loans, as it would measure the percentage of loans that have been charged off (see Panel D).

Several conclusions can be drawn from these observed patterns. First, throughout the Great Recession, there were boom-bust cycles for collateral requirement limit κ capturing tightness in collateral conditions when the collateral requirement was low. Low loan-to-value limit essentially lowers credit

supply and affects a firm's capacity to invest, and thus influence total output. The model-implied collateral requirement reached its peak in early 2008. This sharp increase before the financial crisis was followed by a rapid decline at its onset. The model provides suggestive evidence for the volatility of collateral requirements during 2007–2009, and illustrates the idea that banks suddenly tighten their collateral requirement during periods of uncertainty.

Second, the default threshold of businesses idiosyncratic risk substantially increased between 2008-2009, meaning that more entrepreneurs fell below this threshold and declared bankruptcy. Additionally, in the middle of the global financial crisis, speculative price component reached a high level relative to capital price, whereas the fundamental value shrank substantially relative to Q^k. At the same time, banks were less willing to lend against collateral as the crisis drive down the fundamental value of business assets. We also observed that the speculative price component tended to be high during the IT bubble of 2000. Generally, banks often cut loan-to value limit during uncertain times, which further worsens the financial condition for the business sector. The model thus adequately generated a high level of risk around the global financial crisis, in line with the pattern observed in net charge-offs rate.

It would also appear that, over the entire period, the portion attributed to speculative swings in collateral price was reduced compared to the fundamental component. Between 2010-2019, the post-Great Recession period was associated with less volatile and tight loan-to-value limits. This stabilization coexisted with a decline in price speculation as well as an increase in fundamental value relative to capital price Q^k. The reverse reoccurred in 2021-2022, when the loan-to-value limit was relaxed, resulting in both a slight increase in the speculative price component and a decrease in the average capital price. This divergence in the patterns of the two capital price components could have potentially stemmed from the shift in borrower financial conditions and business economic prospects.

4.4. Bayesian Vector Autoregressions Evidence

To validate the dynamic interaction between collateral requirements and economic activity, we used the measure of collateral limit tightness from the Senior Loan Officer Opinion Survey on Bank Lending Practices as a proxy of collateral loosening or tightening, defined as follows:

$$\mathcal{X}_t = \textit{collateral Requirements tightness measure}_t$$

The measure represents the net percentage of domestic banks increasing collateral requirements for small firms between 1990-2023. A high measure results in a tightening of the loan-to-value limit, and vice versa. The identification assumption of the adverse shock to the collateral requirement rationalizes the idea of credit market liberalization. Moreover, our findings are potentially comparable to several papers that have identified the effects of financial shocks, including, Furlanetto, Ravazzolo, and Sarferaz

(2019), who provided a detailed discussion of the identification of credit shocks that capture changes in the condition of access to credit and the relaxations in collateral constraints. Alternatively, Caldara et al. (2016) used the excess bond premium as an empirical proxy of financial shock to disentangle the interaction between uncertainty and financial shocks during recessions.

We considered a standard reduced form VAR model with the following representation :

$$y_t = C + \Sigma_{j=1}^{p} B_j y_{t-1} + u_t$$

where C is a $n \times 1$ vector constant, and $B_j, ..., B_p$ are the $n \times n$ VAR matrix coefficients, y_t is the $n \times 1$ vector of endogenous variables, u_t is the vector of exogenous shocks with $u_t \sim \mathcal{N}(0, \Sigma)$, and Σ is the $n \times n$ variance covariance matrix. We adopted the hierarchical Bayesian procedure proposed by Giannone, Lenza, and Primiceri (2015), with a standard Minnesota Normal Inverse-Wishart prior and an optimal prior tightness selection. This approach accorded with the recent study of Ferreira, Miranda-Agrippino, and Ricco (2023), who generalized the Bayesian quasi-maximum likelihood approach to a local projection estimator. We estimated the Bayesian Vector Autoregressions model with 5 lags over 1999Q1-2023Q1. The vector of variables in the specification are: collateral requirement tightness measure $\mathcal{X}$, capital price Q, consumption C, external financing M, output Y, government spending G, hours H, investment I, net worth N, and total factor productivity TFP.

Figure 6 reports the impulse response function to collateral requirement innovation. The negative disturbance was interpreted as a relaxation in collateral conditions, as it reflected the decrease in the number of banks reporting a tightening of collateral terms. This disturbance led to an increase in entrepreneurial profits and triggered a favorable economic conditions for firms. Output reacted positively to collateral condition relaxation starting from the second quarter and also generated a credit boom episode. A sudden relaxation of collateral requirements boosted entrepreneurs investment and increased capital price still further. This result is similar to the general equilibrium implied impulses response functions discussed previously, and supports Iacoviello (2015)'s findings on how the relaxed borrowing capacity of entrepreneurs can increase investment levels and raise output. A relaxation of the collateral condition that increases the price of collateral leads to large and persistent effects on external financing. The expanded access of credit for businesses dampens hours worked and consumption in the short run, which is primarily related to the leveraging process of consumers.

In short, the credit supply crunch following the 2007 crisis and the global pandemic was partly driven by banks imposing stricter conditions on collateral. There is sound evidence to show that these restrictions worsened firms' access to external financing. As stated previously, a large share of banks reported a tightening of collateral limits during the economic crisis. This tendency explains the decline in credit supply to firms. Firms often leverage their own assets to secure debt when increases in uncertainty lead banks to become more prudent about the type and quality of pledged collateral. Banks may prefer

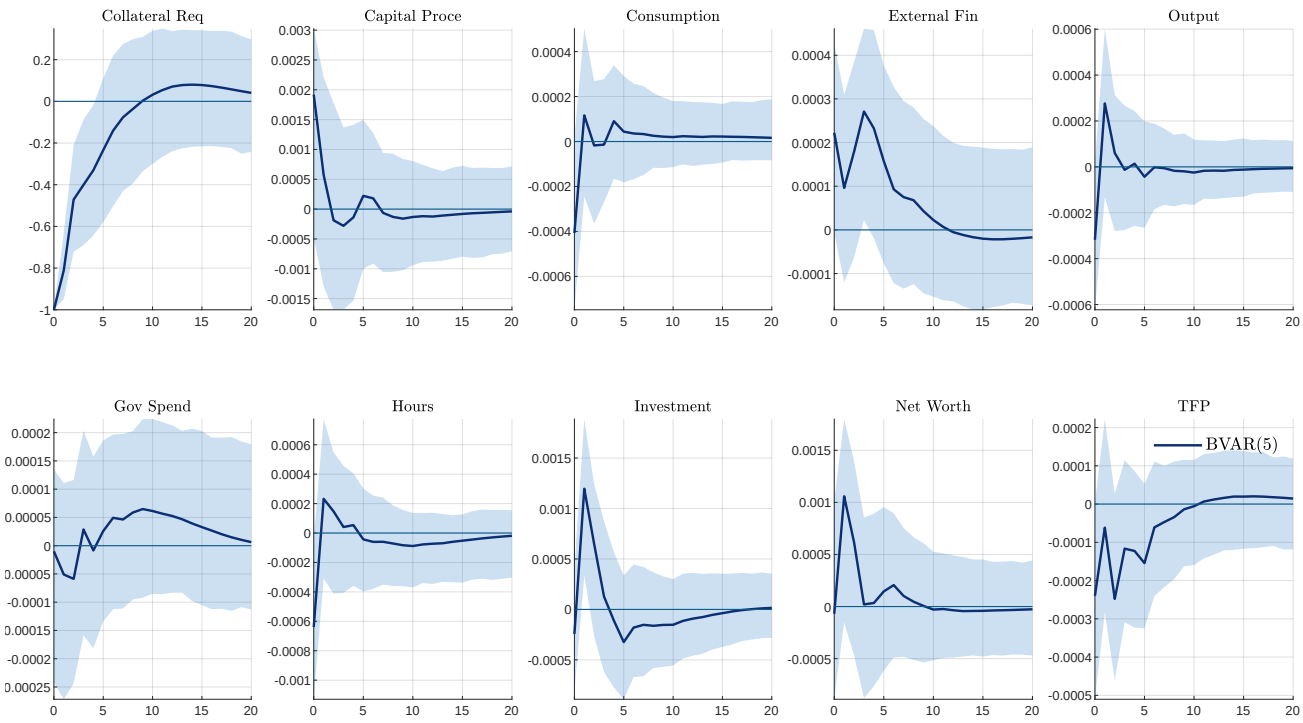


FIGURE 6. Impulse Response Function to a Negative Shock to $\mathcal{C}_t$

Notes: Estimation sample: 1999Q1-2023Q1. The blue area represents the 90 % posterior convergence bands.

high-quality collateral and more liquid assets. When the former grow scarce, banks can still grant loans by raising the downpayments that creditors should pay at the beginning of the loan contract. An adverse shock to loan-to-value limit reflects the inability to efficiently channel capital to firms, as well as a demand-induced scarcity that places strain on the available capital to invest. It is also the case that, when the price of collateral soars and default probability starts to increase, a cyclical correction may arise, which serves to shrink credit supply. This perfectly illustrates what happens during boom and bust episodes. Furthermore, the borrower-based macroprudential instrument and the ways in which banks evaluate and process collateral are of enormous importance in supporting credit to businesses, and stimulating investment and economic activity.

5. Concluding Remarks

In this paper, we estimated a DSGE model with collateral-constrained firms to analyze the impact of changes in collateral requirements and quantify their contribution. We have shown that a collateral

requirement shock significantly contributes to the shock decomposition analysis and explains a large component of credit supply. Relaxation of loan-to-value limit has a significant positive impact on business credit, causes a rise in collateral demand, and increases collateral price further. This leads to a general increase in investment and output. As a consequence of relaxed collateral conditions, speculative swings in collateral price increase along with the fundamental price. This analysis shows that incorporating the speculative price component is important to understanding the implication of borrower-based measures in collateral prices. From a policy perspective view, such an analysis highlights the importance of adopting policies that support collateral markets and can mitigate the consequences of financial system disruptions.

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Online Appendix

Appendix A. Additional Empirical Facts

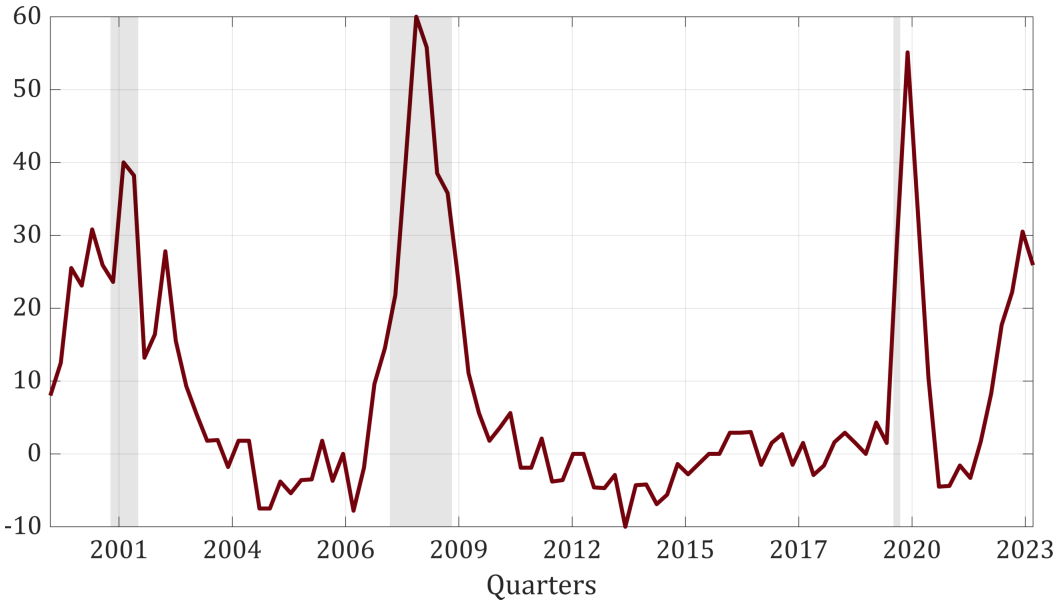


FIGURE A1. Net Percentage of Domestic Banks Increasing Collateral Requirements for Small Firms

Notes: Source: Board of Governors of the Federal Reserve System's Senior Loan Officer Opinion Survey on Bank Lending Practices, SUBLPDCISTQNQ, from Q4-1990 to Q3-2023

The Collateral Collection Process in the US. US banks typically proceed to the collection of collateral after clearly documenting a delinquent loan. Based on the Federal Deposit Insurance Corporation (FDIC) rules, banks should classify all loans that involve some risks by degree of non-payment risk. These risky loans must be classified into three categories: substandard, doubtful, and loss. This paper focuses on loans classified as loss and considered uncollectible debt. Even if the loan is classified as a loss, banks are able to recover all or part of the amount so long as the loan is secured by collateral. The estimated credit loss of a bank is given by the following equation:

$$\text{net charge-offs} = \text{gross charge-offs} - \text{subsequent recoveries of delinquent debt}$$

Gross charge-offs are the total amount of uncollectible debt, the amount of unpaid loans, and interest. The bank may have some recovery value on delinquent debt, which is determined by the value of collateral minus the cost of selling. The value of net charge-offs can be either positive or negative, reflecting the case that banks can make a loss or gain from collateral liquidation. According to the Federal Deposit

Insurance Act, banks should remove a delinquent debt from the books and the loan should be charged off when the loss occurs within the delinquency time frame adopted by the FDIC. Once the collateral dependent loan is classified as a loss, banks can take legal action and proceed to sell this collateral to repay the loan, such that the value of collateral minus the cost of selling it is sufficient for recovery.

The Evolution of Collateral Liquidation and Economic Growth. Entrepreneurs with financial needs may offer collateral to borrow from banks, which they would lose if they were to default on their loans. Table A1 depicts the annual average growth rates of gross domestic product, of commercial and industrial loans, uncollectible (CI) debt, and CI loan recoveries.

TABLE A1. Annual Average Growth Rates of Output, Commercial, and Industrial Loan Charge-offs, Commercial and Industrial Loan Recoveries, and Commercial and Industrial Loans.

Years	**Δ % GDP**	**Δ % CI Loans Charge-offs**	**Δ % CI Loans Recoveries**	**Δ % CI Loans**
1984-1988	7.24	8.62	5.41	0.72
1989-1998	5.53	2.84	0.29	0.98
1999-2008	4.94	6.74	2.61	1.31
2009-2018	3.71	-1.53	1.64	0.95

The growth rate of uncollectible debt between 1999-2008 of 6.7% reflects the height of the financial crisis. Around the same time, the level of recoveries on the delinquent debt reached a 2.6% average growth rate, while that of business lending dropped from 1.31% to 0.95% following the Lehman episode. The high level of uncollectible debt could potentially lead to tighter lending conditions for firms, put a strain on the capacity of firms to borrow and invest, and subsequently translate slowed economic growth.

TABLE A2. Collateral Requirements Amid the Recession

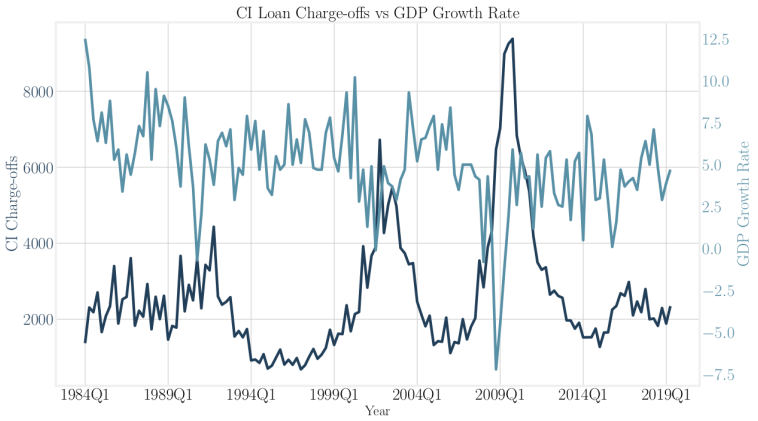
	Pre-Crisis				Recession			Post Crisis		
	2019Q1	2019Q2	2019Q3	2019Q4	2020Q1	2020Q2	2020Q3	2020Q4	2021Q1	2021Q2
Aggregate Data										
CI Loans/GDP	0.11	0.11	0.11	0.11	0.11	0.15	0.13	0.12	0.12	0.11
CI Loans	2.4%	0.0%	1.3%	0.3%	2.3%	22.1%	-4.9%	-5.9%	-2.5%	-2.4%
Survey Data										
Eased somewhat	5.3 %	0.8 %	1.6 %	0 %	0.9 %	1.7 %	0 %	0 %	1.9 %	2.6
Eased considerably	0.8 %	0 %	0 %	0 %	0 %	0 %	0 %	0 %	0 %	0
N. Observations	133	126	128	120	113	118	118	105	107	116

[a] Commercial and Industrial Loans, All Commercial Banks, Percent Change, Quarterly, Seasonally Adjusted.

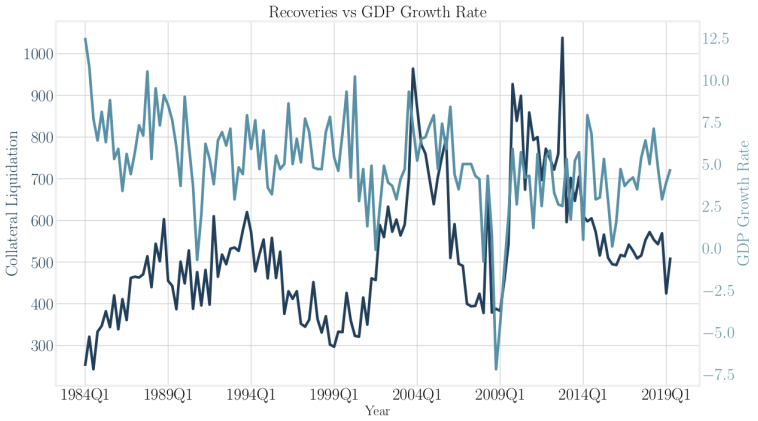
[b] Commercial and Industrial Loans, All Commercial Banks Billions of U.S. Dollars, Seasonally Adjusted /Gross Domestic Product Billions of U.S. Dollars, Seasonally Adjusted.

[c] Collateral requirement eased considerably and eased somewhat.

Panel A of Figure A2 shows that the level of uncollectible debt reached its peak between 2008 and 2009. Around the same time, the level of seized collateral also reached a relatively high level, with two-year lags of collateral liquidation, as shown in Panel B of Figure A2. One interpretation of the lagged level of liquidated collateral is that banks did not proceed to the liquidation of collateral immediately due to collateral having suddenly become illiquid and hard to sell. In fact, if banks foresee a high risk originating from some segments of the economy, such as the high level of loan charge-offs, then banks would be concerned about the liquidity of the collateral and thus scramble for better collateral.



A. Uncollectible Commercial and Industrial (CI) Loans vs. GDP Growth Rate



B. Liquidation of Collateral Commercial and Industrial (CI) Loans vs. GDP Growth Rate

FIGURE A2. Default, Collateral Liquidation and GDP Growth

Notes: Evolution of uncollectible commercial and industrial loans, liquidation of collateral for uncollectible commercial and industrial loans, and gross domestic product in the US. This data was retrieved from the FDIC's financial data and the Bureau of Economic Analysis. Quarterly data between 1984-2019.

Appendix B. Data Description

In order to evaluate the model and apply it to data, we use six US macroeconomic time series that cover the period 2000Q1-2023Q1. To make links between the macroeconomic data and the model, we add to the model the following equations:

$$gdpobs_t = \frac{(C_t + \frac{I_t}{\mu_{\chi,t}} + g_t)}{(C_{t-1} + \frac{I_{t-1}}{\mu_{\chi,t-1}} + g_{t-1})} \frac{\mu^*_{z,t}}{\mu^*_z} - 1 \qquad \text{change in output per capita}$$

$$consumptionobs_t = \frac{C_t}{C_{t-1}} \frac{\mu^*_{z,t}}{\mu^*_z} - 1 \qquad \text{change in consumption per capita}$$

$$investobs_t = \frac{I_t}{I_{t-1}} \frac{\mu^*_{z,t}}{\mu^*_z} - 1 \qquad \text{change in investment per capita}$$

$$hours_obs_t = \frac{L_t}{L} - 1 \qquad \text{change in hours worked per capita}$$

$$externobs_t = \frac{Q^k_t * K_t - N_t}{Q^k_{t-1} * K_{t-1} - N_{t-1}} \frac{\mu^*_{z,t}}{\mu^*_z} - 1 \qquad \text{change in external financing per capita}$$

$$netwobs_t = N_t / N_{t-1} - 1 \qquad \text{change in business net worth per capita}$$

$$capriceobs_t = s^*_t / s^*_{t-1} - 1 \qquad \text{change in capital price}$$

$$govobs_t = \frac{G_t}{G_{t-1}} - 1 \qquad \text{change in government spending per capita}$$

$$tfpobs_t = \frac{\mu^*_{z,t}}{\mu^*_{z,ss}}, \qquad \text{change in TFP}$$

Data are mainly obtained from the Federal Reserve Bank of St. Louis. Below, we describe the steps employed to construct the macroeconomic data used for estimation purposes and the data used to construct some plots.

TABLE A3. Data Sources

	Data Sources
Variables	**Description**
Gross Domestic Product	GDP, Gross Domestic Product, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/01/1947-01/01/2023)
Price Deflator for Gross Domestic Product	GDPDEF, Gross Domestic Product Implicit Price Deflator, Index 2012=100, Quarterly, Seasonally Adjusted, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/01/1947-01/01/2023)
Consumption	PCND, Personal Consumption Expenditures: Nondurable Goods, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/01/1947-01/01/2023)
	PCESV, Personal Consumption Expenditures: Services, Billions of Dollars, Quarterly, Seasonally Adjusted, Annual Rate, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/01/1947-01/01/2023)
Price Deflator for Personal Consumption Expenditures: Nondurable Goods	[DNDGRD3Q086SBEA], Personal Consumption Expenditures: Nondurable Goods (Implicit Price Deflator), Index 2012=100, Quarterly, Seasonally Adjusted, retrieved from FRED, Federal Reserve Bank of St. (01/01/1947-01/01/2023)
Price Deflator for Personal Consumption Expenditures: Services	[DSERRD3Q086SBEA], Personal Consumption Expenditures: Services Implicit Price Deflator, Index 2012=100, Quarterly, Seasonally Adjusted, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/04/1948-01/01/2023)
Investment	GPDI, Gross Private Domestic Investment, Billions of Dollars, Seasonally Adjusted Annual Rate, Quarterly, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/01/1945-01/01/2023)
	[PCDG], Personal Consumption Expenditures: Durable Goods, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/01/1945-01/01/2023)
Price Deflator for Gross Private Domestic Investment	[A006RD3Q086SBEA], Gross Private Domestic Investment Implicit Price Deflator, Index 2012=100, Quarterly, Seasonally Adjusted, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/01/1945-01/01/2023)
Price Deflator for Personal Consumption Expenditures: Durable Goods	[DDURRD3Q086SBEA], Personal Consumption Expenditures: Durable Goods Implicit Price Deflator, Index 2012=100, Quarterly, Seasonally Adjusted, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/01/1945-01/01/2023)
Hours Worked	HOANBS, Nonfarm Business Sector: Hours of All Persons, Index 2012=100, Seasonally Adjusted, Quarterly, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/01/1948-01/01/2023)
Capital Stock	[NNBTASQ027S] Nonfinancial Noncorporate Business; Total Assets, Level (DISCONTINUED), Millions of Dollars, Not Seasonally Adjusted, Quarterly, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/01/1948-01/01/2023)

	[TABSNNCB] Nonfinancial Corporate Business; Total Assets, Level, Billions of Dollars, Not Seasonally Adjusted, Quarterly, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/01/1948-01/01/2023)
Population	CNP16OV, Civilian Noninstitutional Population, Thousands of Persons, Monthly, Not Seasonally Adjusted, [CNP16OV 20180202], retrieved from FRED, Federal Reserve Bank of St. Louis. (01/01/1947-01/01/2023) Transformed Quarterly Data CNP16OV.
Volume of Business Loans, Survey of Terms of Business Lending	EVANQ, Total Value of Loans for All Commercial and Industry Loans, All Commercial Banks (DISCONTINUED), Millions of Dollars, Quarterly, Not Seasonally Adjusted, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/04/1997-01/04/2017)
Percent of Collateralized Business Loans, Survey of Terms of Business Lending	ESANQ, Percent of Value of Loans Secured by Collateral for All Commercial and Industry Loans, All Commercial Banks (DISCONTINUED), Percent, Quarterly, Not Seasonally Adjusted, retrieved from FRED, Federal Reserve Bank of St. Louis. (04/01/1997-04/01/2017)
Fed Funds Rate	FEDFUNDS, Effective Federal Funds Rate, Percent, Not Seasonally Adjusted, Monthly, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/07/1954-01/06/2023) (Transformed into Quarterly Data)
Relative Price Investment	A006RD3Q086SBEA, Gross Private Domestic Investment (Implicit Price Deflator), Index 2012=100, Seasonally Adjusted, Quarterly, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/04/1948-01/01/2023)
Wages	COMPNFB, Nonfarm Business Sector: Compensation per Hour, Index 2012=100, Seasonally Adjusted, Quarterly, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/04/1948-01/01/2023)
External Financing	TNWMVBSNNCB, Nonfinancial Corporate Business; Net Worth, Level, Billions of Dollars, Not Seasonally Adjusted, Quarterly, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/04/1948-01/01/2023)
Government Spending	GCE, Government Consumption Expenditures and Gross Investment, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/04/1948-01/01/2023)
Net Worth	TNWMVBSNNCB, Nonfinancial Corporate Business; Net Worth, Level, Billions of Dollars, Quarterly, Not Seasonally Adjusted, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/04/1948-01/01/2023)
Collateral requirement tightness measure	SUBLPDCISTQNQ, Net Percentage of Domestic Banks Increasing Collateral Requirements for Small Firms, Quarterly, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/04/1990-01/01/2023)

	Data Construction
Variables	**Description**
Series for Change in Output per Capita	We take the Gross Domestic Product [GDP] and divide it by the Implicit Price Deflator [GDPDEF_20180628] and by Population [CNP16OV_20180202]. Then, we take the log difference and remove the mean to match the series with the model equation.

Series for Change in Consumption per Capita	We take Personal Consumption Expenditures: Nondurable Goods [PCND] and divide it by the Implicit Price Deflator [DNDGRD3Q086SBEA]. We take personal Consumption Expenditures: Services [PCES], we divide it by the Implicit Price Deflator [DSERRD3Q086SBEA], and we sum these two series and divide by Population [CNP16OV_20180202]. Then, to combine the series with the model equation, we take the log difference and subtract the mean.
Series for Change in Investment per Capita	We take the Gross Private Domestic Investment [GDPI] and divide it by the Implicit Price Deflator [A006RD3Q086SBEA]. We take Personal Consumption Expenditures: Durable Goods [PCDG] and divide it by the Implicit Price Deflator [DDURRD3Q086SBEA]. We sum these two series and divide by Population [CNP16OV_20180202]. Then, to combine the series with the model equation, we take the log difference and subtract the mean.
Series for Change in Hours Worked per Capita	we take the index of Worked Hours in the Nonfarm Business Sector [HOANBS] and divide it by Population [CNP16OV_20180202]. Then, we take the log difference to combine with the model equation.
Series for Change in External Financing per Capita	we take the sum of Total Assets in Nonfinancial Noncorporate Business [NNBTASQ027S] and Nonfinancial Corporate Business [TABSNNCB], then we subtract the Net Worth of Nonfinancial Corporate Business and divide the result by the Implicit Price Deflator [GDPDEF_20180628] and by Population [CNP16OV_20180202]. Then, we take the log difference and subtract the mean to combine with the model equation.
Series for Change in Business Net Worth per Capita	we take the sum of Net Worth of Nonfinancial Corporate Business [TNWMVBSNNCB] and divide it by the Price Deflator [GDPDEF] and by Population [CNP16OV_20180202]. Then, to combine these series with the model equation, we take the log difference.
Series for Change in Capital Price	we take the Russell 3000 Price Index [RU3000PR] and we transform it to quarterly data. Then, we take the log difference to combine it with the model equation.
Change in Government Spending per Capita	We take the Government Consumption Expenditures and Gross Investment [GCE] and divide it by the Price Deflator [GDPDEF] and by Population [CNP16OV_20180202]. Then, we take the log difference and subtract the mean to combine it with the model equation.
Change in TFP	we multiply [HOANBS] by [COMPNFB] to obtain the aggregate labor factor L, and we compute the aggregate capital factor $K = [NNBTASQ027S] + [TABSNNCB]$. Then, we use the following formula to compute the total factor productivity $TFP = \frac{GDP}{K^{(1-\alpha)}L^{\alpha}}$ where $\alpha = 0.64$. Then we take the log difference to match it with the model equation.

Appendix C. Quantitative Implementation

This section first presents the calibrated parameters of the model, then estimates the unknown parameters and shocks using Bayesian methods. Finally, we report the moments of the model and the data, and compare the model predictions with the data.

C.1. Econometric Estimation of Parameters

The next step involves estimating the model through the following three stages: (i) calibrating economic parameters, (ii) constructing empirical data to combine it with the model equation, (iii) estimating the remaining economic and shock parameters using the Bayesian methods and quarterly US data over the period 2000–2023.

Calibrated Parameters. The baseline parameters are reported in Table A4.

TABLE A4. Calibrated Parameters

Par.	Description	Value	Target & Source
β	Discount rate	0.9966	R
σ^L	Curvature on disutility of labor	1	Parametrization
b	Habit persistence parameter	0.63	Iacoviello (2015)
ν^l	Steady state markup suppliers of labor	1.05	Justiniano, Primiceri, and Tambalotti (2011)
μ_z	Growth rate of the economy	1.00	Christiano, Motto, and Rostagno (2014)
δ	Depreciation rate on capital	0.025	Standard
α	Power on capital in production function	0.40	Standard
ν^p	Steady state markup, intermediate good firms	1.20	Becard and Gauthier (2022)
Φ	Fixed cost, intermediate goods	0.07	Christiano, Motto, and Rostagno (2014)
γ^e	Percent of entrepreneurs who survive	0.97	Christiano, Motto, and Rostagno (2014)
μ	Fraction of realized profits lost in bankruptcy	0.94	Christiano, Motto, and Rostagno (2004)
$Var(\log \omega)$	Variance of log of idiosyncratic productivity	0.24	Becard and Gauthier (2022)
w	Transfer from households	0.009	Christiano, Motto, and Rostagno (2014)
Θ	Fraction of net worth of entrepreneurs exiting the economy	0.10	Christiano, Motto, and Rostagno (2014)
τ^c	Tax rate on consumption	0.05	Parametrization
τ^k	Tax rate on capital income	0.32	Christiano, Motto, and Rostagno (2014)
τ^l	Tax rate on labor income	0.24	Christiano, Motto, and Rostagno (2014)
χ	Trend rate of ISTC	1.0035	Greenwood, Hercowitz, and Krusell (1997)
x	Growth rate of monetary base	3.71/400	π
g	Share of government consumption	0.2	G/Y

First, we set the discount factor β to 0.99, which yielded an annual policy rate R of 5.18%. We fixed the elasticity of labor supply σ^L and the consumption habit b to 1 and 0.63, respectively. We fixed the wage markup ν^l to 1.05. Following the literature, the depreciation rate on capital δ was set to 0.025. We set the price markup ν^p, the growth rate of the economy μ_z, and the power on capital in production function α to 1.2, 1.0036, and 0.4, respectively, as per the value ranges in the business cycle literature. We set the fraction of net worth of entrepreneurs exiting the economy Θ to 0.1. We took the value of the standard deviations of the idiosyncratic productivity shock 0.24, that is $\sigma = \sqrt{0.24}$, to solve for ω using the entrepreneur optimality condition. In steady states, the businesses bankruptcy rate is then computed as follows $\int_0^{\bar{\omega}} \omega dF(\omega) = 100 \times (\varphi(((log(\bar{\omega}) + \sigma^2/2)/\sigma) - \sigma)) = 5\%$, where φ refers to the cumulative distribution function of the standard normal distribution.

The percentage of entrepreneurs who survive γ^e was set at 0.9762, and the transfer received by entrepreneurs w fixed at 0.009. The parameter μ was chosen to target the value 1.566, that is, close to the data's ratio of external financing. Finally, the growth rate of monetary base x was set to 3.71/400, implying an inflation rate of 1%. We set the tax rate on consumption τ^c to 0.05. We also set tax rates on capital τ^k and on labor τ^l to 0.32 and 0.24, respectively. The trend rate of investment-specific technological change χ was set to 1.00035, and finally, we set $g = 0.2$ to match government spending.

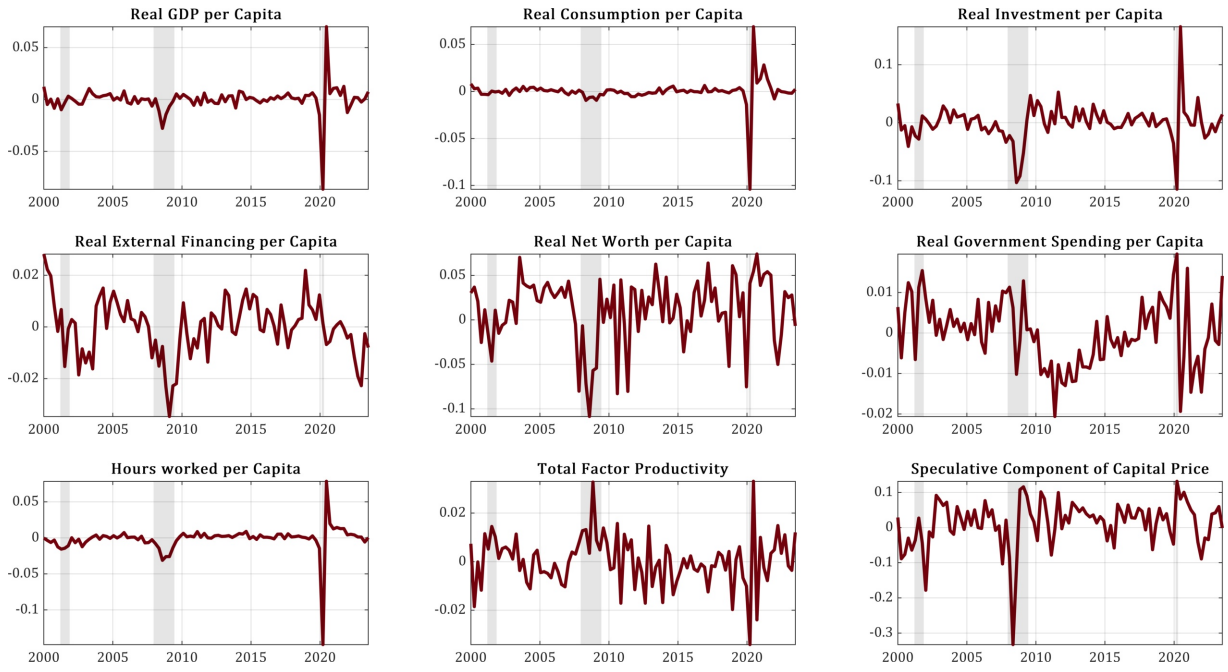


FIGURE A3. US Data

Notes: Quarterly data retrieved from Federal Reserve Bank of St. Louis Database (FRED).

Estimated Parameters. We now describe the procedure for estimating the model parameters. For this, we used quarterly US data, as shown in Figure A3. Data source, transformation, and measurement equations are extensively discussed in the Appendix B. We used nine observables: growth in consumption, growth in output, growth in investment, growth in business net worth, growth in external financing, growth in hours worked, growth in total factor productivity, growth in government spending, and growth in speculative component of capital price.

It is worth noting that our estimation procedure used the Wilshire 5000 price index as a proxy of the non-fundamental fluctuations in capital price s^*. The movement in capital market stock price conveys the speculative behavior in capital price during good and bad times.

Following a Bayesian approach, we chose prior distributions of selected parameters for estimation. Generally speaking, we relied on existing empirical literature where priors were common for cerain parameters. For autocorrelation parameters, we imposed Beta distribution with prior means varying between 0.5-0.7, and standard deviations ranging between 0.1-0.2. Regarding the standard deviation of shocks, we chose agnostic priors with Inverse Gamma distributions, with means of 0.0023 and 0.1167. We imposed a Normal distribution for the utilization costs, investment adjustment costs, the policy weight on inflation, and the output gap. The prior distribution of the remaining parameters was assumed to obey Beta distributions. The choice of priors plays a crucial role since it can distort the construction of posterior densities. Indeed, our adopted strategy did not eliminate the risk of weak identification of parameters.

We used the Metropolis–Hasting algorithm with 10, 000 draws per chain to estimate their posterior distributions over the period 2000–2023 (see Appendix D). The estimation results are reported in Table A5.

Our estimates showed substantial consistency regarding the values reported in related studies. The estimate of the Calvo price index parameter of $\zeta^p = 0.79$ was consistent with Guerrieri and Iacoviello (2017) and Justiniano, Primiceri, and Tambalotti (2011). For monitoring cost u^e the corresponding estimate was 0.87, that is, very close to the value reported in Christiano, Motto, and Rostagno (2014). Regarding the parameters of the Taylor rule, the monetary policy smoothing parameter had a mean posterior of $\rho_p = 0.90$, in line with the value of 0.85 reported in Becard and Gauthier (2022). However, the estimates of the weight on inflation in the Taylor rule corresponded to $\alpha^\pi = 1.02$, i.e., slightly lower than that documented in Drechsel (2023) and Becard and Gauthier (2022). The estimate of the investment adjustment cost parameter had a mean posterior of $S = 3.23$, in accordance with those reported in Liu, Wang, and Zha (2013) and Jensen, Ravn, and Santoro (2018). The estimated consumption habit parameter equaled 0.85, consistent with the estimates in Guerrieri and Iacoviello (2017). The curvature utilization cost had a posterior mean of $\sigma^a = 2.64$, which was relatively high compared to Candian and Dmitriev (2020)'s

TABLE A5. Prior and Posterior Distribution of Estimated Parameters

		Prior Distribution			Posterior Distribution [a]			
		Density	Mean	Std. Dev.	Mean	Median	Mode	Std. Dev.
Shock Parameters								
ϵ_{ν_p}	St. dev. Price Markup Shock	Inv. G.	0.0023	0.0033	0.0553	0.0547	0.0566	0.0013
ϵ_{μ^Υ}	St. dev. IST Shock	Inv. G.	0.0023	0.0033	0.0115	0.0116	0.0119	0.0008
ϵ_g	St. dev. Gov. Spend. Shock	Inv. G.	0.0023	0.0033	0.0085	0.0085	0.0081	0.0004
$\epsilon_{\mu_z^*}$	St. dev. Tech. Trend Shock	Inv. G.	0.0023	0.0033	0.0114	0.0108	0.0108	0.0009
ϵ_γ	St. dev. Stat. Tech. Shock	Inv. G.	0.0023	0.0033	0.0095	0.0092	0.0095	0.0006
ϵ_{σ^e}	St. dev. Entre. Risk Shock	Inv. G.	0.0825	0.1167	0.3365	0.3353	0.3287	0.0116
ϵ_{ζ^c}	St. dev. Cons. Pref. Shock	Inv. G.	0.0023	0.0033	0.1183	0.118	0.1187	0.0019
ϵ_{ζ^i}	St. dev. MEI Shock	Inv. G.	0.0023	0.0033	0.041	0.0408	0.0401	0.0013
ϵ_{κ^k}	St. dev. Coll. Requir. Shock	Inv. G.	0.0023	0.0033	0.116	0.1155	0.1145	0.0023
ϵ_m	St. dev. Mon. Pol. Shock	Inv. G.	0.0023	0.0033	0.5189	0.5191	0.5191	0.0009
ρ_{ν_p}	Autoc. Price Markup Shock	Beta	0.5	0.2	0.9919	0.9917	0.994	0.003
ρ_{μ^Υ}	Autoc. IST Shock	Beta	0.5	0.2	0.9552	0.9575	0.9637	0.0217
ρ_g	Autoc. Gov. Spend. Shock	Beta	0.5	0.2	0.9719	0.9716	0.9839	0.0091
$\rho_{\mu_z^*}$	Autoc. Tech. Trend Shock	Beta	0.5	0.2	0.0584	0.0446	0.0281	0.0411
ρ_γ	Autoc. Stat. Tech. Shock	Beta	0.5	0.2	0.9074	0.9105	0.9109	0.0151
ρ_{σ^e}	Autoc. Entre. Risk Shock	Beta	0.7	0.2	0.6722	0.6635	0.6773	0.0233
ρ_{ζ^c}	Autoc. Cons. Pref. Shock	Beta	0.5	0.1	0.8047	0.807	0.8099	0.0179
ρ_{ζ^i}	Autoc. MEI Shock	Beta	0.5	0.2	0.7704	0.7689	0.7665	0.0102
ρ_{κ^k}	Autoc. Coll. Requir. Shock	Beta	0.5	0.2	0.9933	0.9939	0.9932	0.002
Macroeconomic Parameters								
ζ^l	Calvo Wage Stickiness	Beta	0.75	0.1	0.9593	0.9535	0.9518	0.0135
b	Habit Parameter	Beta	0.5	0.1	0.8565	0.8587	0.8486	0.0137
ι^μ	Wage Indexing Weight on Technology Growth	Beta	0.5	0.15	0.4841	0.475	0.4547	0.0687
ι^w	Wage Indexing Weight on Inflation Target	Beta	0.5	0.15	0.3412	0.3516	0.3746	0.0327
μ^e	Monitoring Cost	Beta	0.275	0.15	0.8787	0.8917	0.8709	0.043
σ^a	Utilization Cost Curvature	Normal	1	1	2.6408	2.5802	2.8474	0.2427
S	Investment Adjustment Cost Curvature	Normal	5	3	3.2304	3.2368	3.214	0.0582
ζ^p	Calvo Price Stickiness	Beta	0.5	0.1	0.7907	0.793	0.8029	0.0101
ι^p	Price Indexing Weight on Inflation Target	Beta	0.5	0.15	0.7698	0.7889	0.7415	0.0508
α_π	Policy Weight on Inflation	Normal	1.5	0.25	1.021	1.0016	1.012	0.0845
$\alpha_{\Delta y}$	Policy Weight on Output Growth	Normal	0.25	0.1	0.0317	0.012	0.0129	0.0414
ρ_p	Policy Smoothing Parameter	Beta	0.75	0.1	0.9004	0.9009	0.8999	0.0048

[a] The table reports the results of the Bayesian estimation. Posterior statistics were constructed using 10, 000 draws per chain.

corresponding estimate of 1.38.

The autocorrelation of marginal efficiency of investment shock was similar to the value reported in

Justiniano, Primiceri, and Tambalotti (2011). The autocorrelation parameter of the collateral requirement shock was estimated to be relatively high, with a value of $\rho_\kappa = 0.99$, as in Jermann and Quadrini (2012) and Becard and Gauthier (2022). The value of the shock persistence of the stationary technology shock equaled $\rho_\gamma = 0.90$, in line with the value reported in Jermann and Quadrini (2012). In contrast, the history dependence of technology trend was shown to be weak. This was also the case for the entrepreneur risk, which exhibited a low persistence with $\rho_\sigma^e = 0.67$. We also documented the standard deviation of economic shocks. Entrepreneur risk and monetary policy were the most volatile disturbances, while government spending was the least. It should be evident that the model is highly stylized, thus making it difficult to discern between endogenous and exogenous sources of persistence, and its data may produce inaccurate parameter estimates.

C.2. Model Fit

To assess whether the model matched the data, we first compared the volatility, correlation, and autocorrelation in the model and the data, considering the key aggregates used in the estimation. Once done, we compared the model implied moments with the data.

Panel A of Table A6 reports the model implied volatility and the data. The model shows a high volatility of all aggregates with respect to the data. It is worth noting that investment, hours worked, net worth, and external financing were the most volatile aggregates. Overall, the model performed poorly in matching their analogues in the data. In terms of model implied correlation between output growth and the rest of macroeconomic aggregates, as recorded in Panel B of Table A6, the model performed modestly in terms of fitting the data. In the model, all economic aggregates were positively correlated to output growth, except for the speculative component of capital price, which negatively correlated to output growth. The model suggests that government spending is positively correlated with output, but this correlation is relatively weak and differs from the data inference (a negative correlation). On the other hand, the model predicted a negative relationship between output and speculative component, which stood at odds to the observed facts, since the data suggested a weak positive correlation.

Turning to Panel C of A6, we report the first order autocorrelation for each variable. In the data, a positive serial correlation was observed among all aggregates except for consumption, output, TFP, and hours, which had positive influence on themselves over time. However, the model predicted that net worth and government spending would negatively influence themselves in the 1^{st} lag. Finally, Panel D of Table A6 displays key model moments with equivalent data, showing that most of these moments were consistent with data properties.

This begs the question, why can the model not match certain moments? Generally speaking, it is not possible to effectively match all non-targeted moments. Logically, the data used in this analysis exhibited

TABLE A6. Data and Model Moments

	Y	C	I	M	H	N	TFP	G	s^*
Panel A: Standard Deviation									
Model	2.22	2.42	6.24	4.09	26.84	14.27	0.67	0.39	0.00
Data:	1.31	1.39	3.13	1.09	1.91	3.75	1.05	0.82	6.66
Panel B: Correl. with Δ Output									
Model	1.00	0.76	0.77	0.24	0.28	0.13	0.18	0.08	-0.19
Data	1.00	0.93	0.84	0.05	0.91	0.3	0.41	-0.24	0.07
Panel C: First Order Autocorrel.									
Model	0.72	0.69	0.77	0.75	0.99	-0.12	0.06	-0.01	0.00
Data	-0.22	-0.22	0.15	0.61	-0.11	0.26	-0.13	0.37	0.38

Panel D: Model versus Data

	Model	Data
Investment-Output Ratio $\left(\frac{I}{Y}\right)$	0.222	0.246
Consumption-Output Ratio $\left(\frac{C}{Y}\right)$	0.562	0.599
External Financing-Output Ratio $\left(\frac{K-N}{Y}\right)$	1.566	1.731
Gov. Spending-Output Ratio $\left(\frac{G}{Y}\right)$	0.2	0.189
Cost of External Finance (R^e)	6.21	5.115
Rate of Return on Capital (R^k)	10.51	10.726

[a] The columns Y, C, I, M, H, N, TFP, G, and Q refer to the growth rates of output, consumption, investment, external financing, net worth, total factor productivity, government spending, and speculative component of capital price.

a high degree of skewness, especially around the 2008 financial crisis, the market bubble of 2000, and the Covid-19 pandemic. This model is relevant for studying the effects of financial shocks and simplifies economic interactions to the extent to which moments observed in the data cannot be perfectly matched to the moments generated by the model. It is also difficult to handle the accuracy of model parameters, given that the selection of the prior standard deviation parameters can affect the posterior estimates and may imply that model moments are not perfectly matched with data moments. For further reference, see related papers by Lombardi and Nicoletti (2012) on prior elicitation when estimating dynamic models, and Gorodnichenko and Ng (2010) on the best approach to produce precise estimates.

Appendix D. Prior Posterior Densities

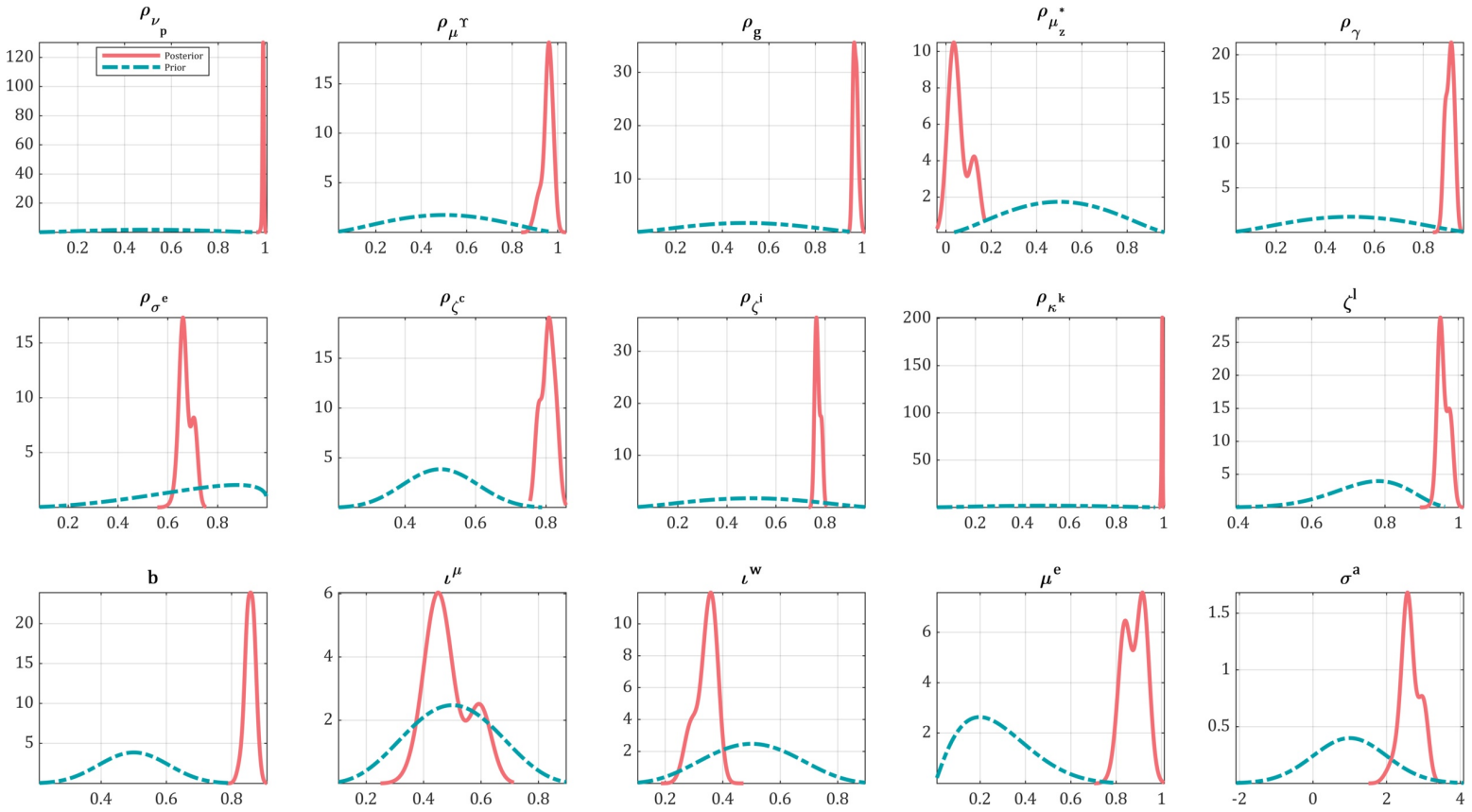


FIGURE A4. Prior and Posterior

Notes The posterior distribution is constructed over the period 2000–2023 using the Metropolis–Hasting algorithm with 10, 000 draws per chain needed to achieve convergence.

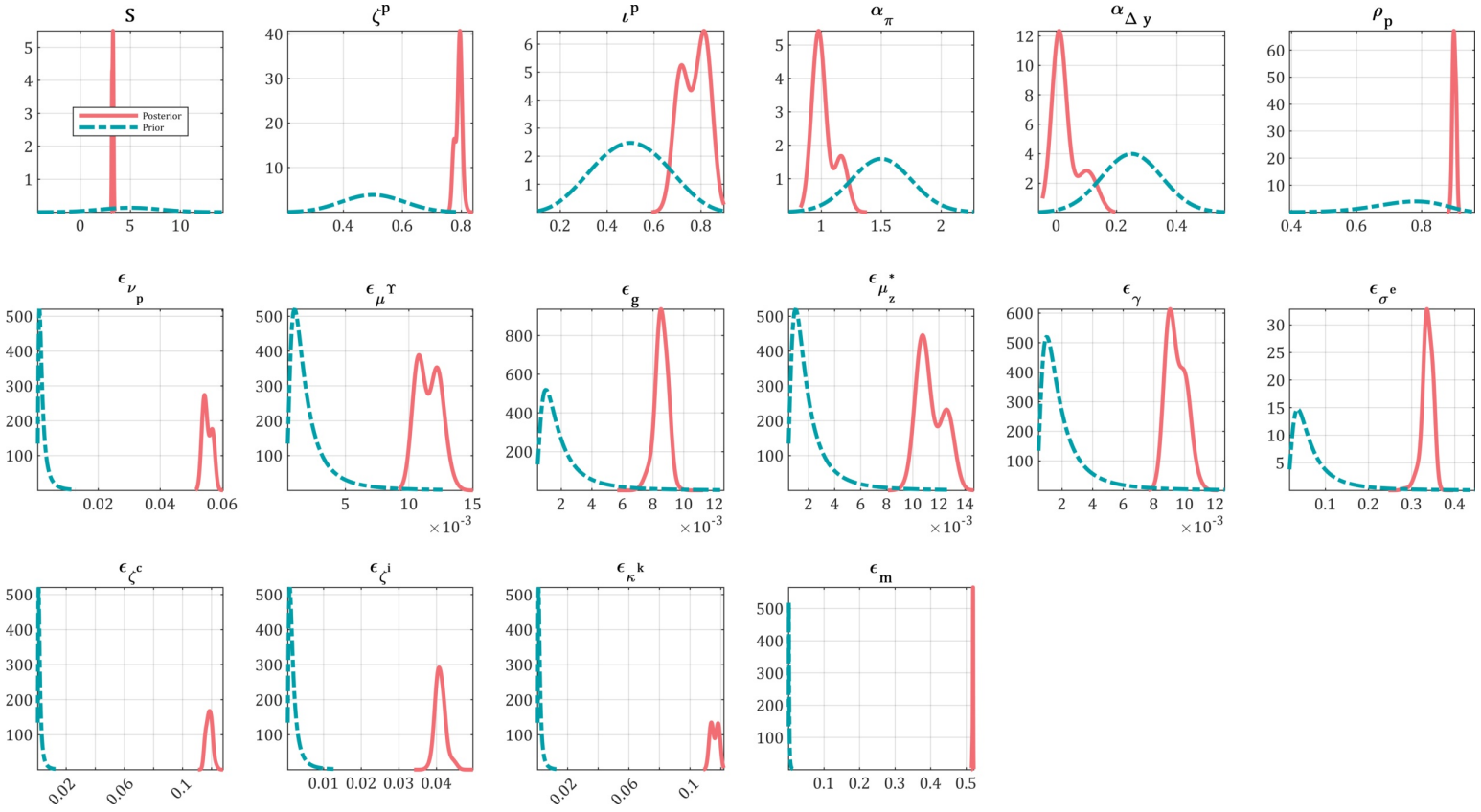


FIGURE A5. Prior and Posterior

Notes: The posterior distribution is constructed over the period 2000–2023 using the Metropolis–Hasting algorithm with 10, 000 draws per chain needed to achieve convergence.

Appendix E. Model Impulse Responses

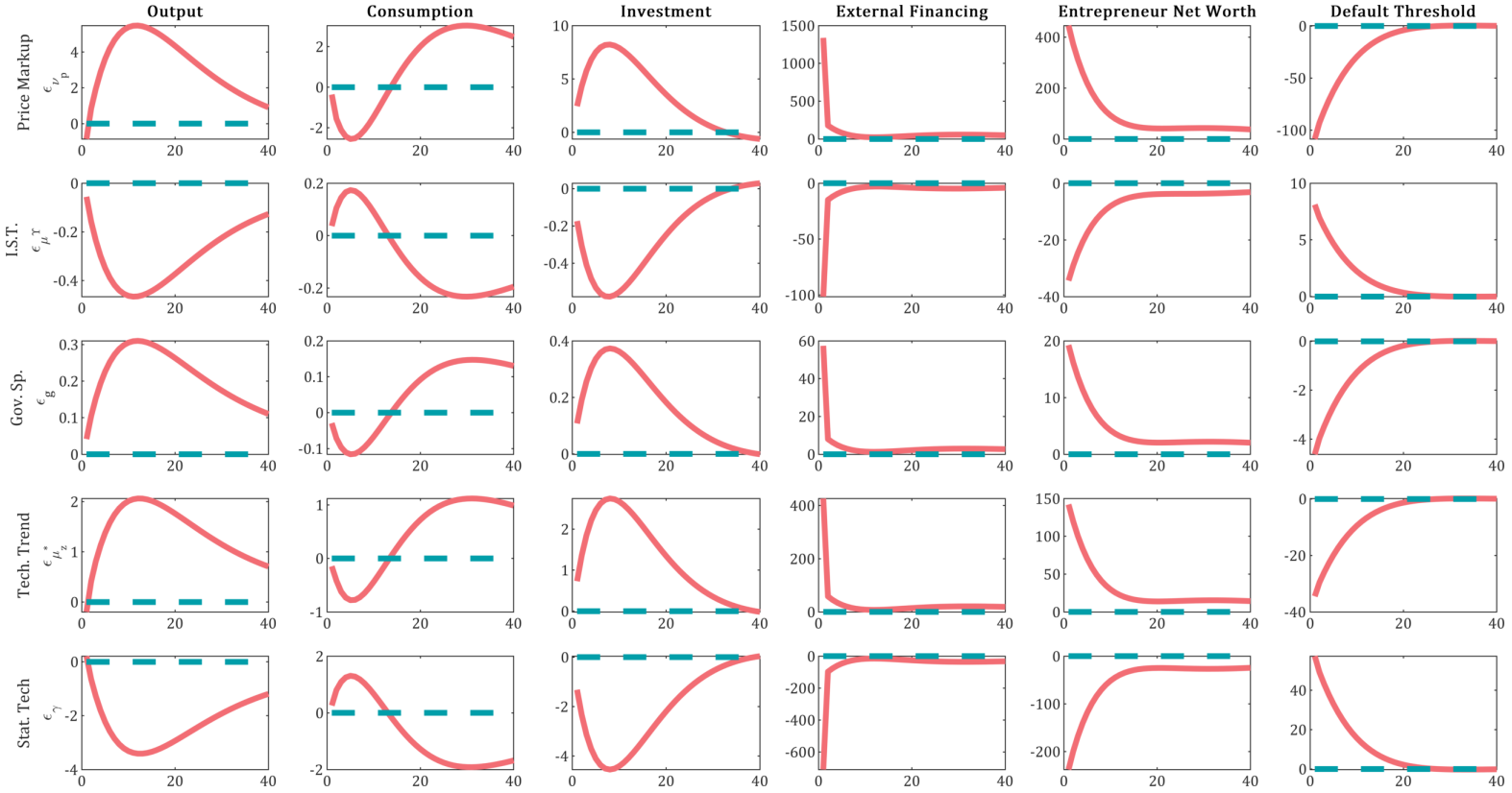


FIGURE A6. Impulse Responses of the Estimated Model

Notes: Time units are quarterly. The simulation shows the responses of output, consumption, investment, external financing, net worth, and default threshold to price markup, investment-specific technical change, government spending, technology trend, stationary technology, entrepreneur risk, consumption preference, marginal efficiency of investment, and collateral requirement shocks.

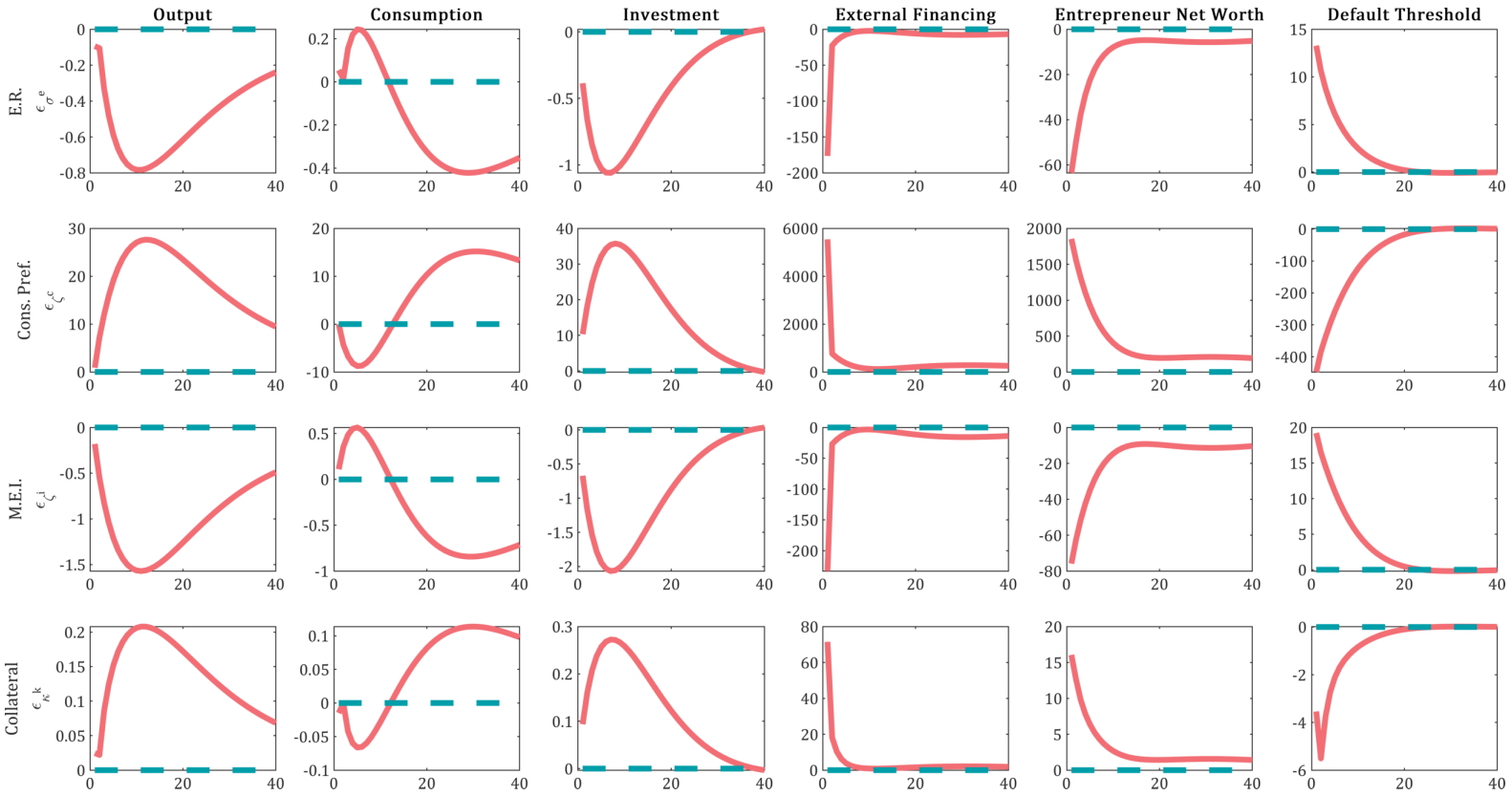


FIGURE A7. Impulse Responses of the Estimated Model

Notes: Time units are quarterly. The simulation shows the responses of output, consumption, investment, external financing, net worth, and default threshold to price markup, investment-specific technical change, government spending, technology trend, stationary technology, entrepreneur risk, consumption preference, marginal efficiency of investment, and collateral requirement shocks.

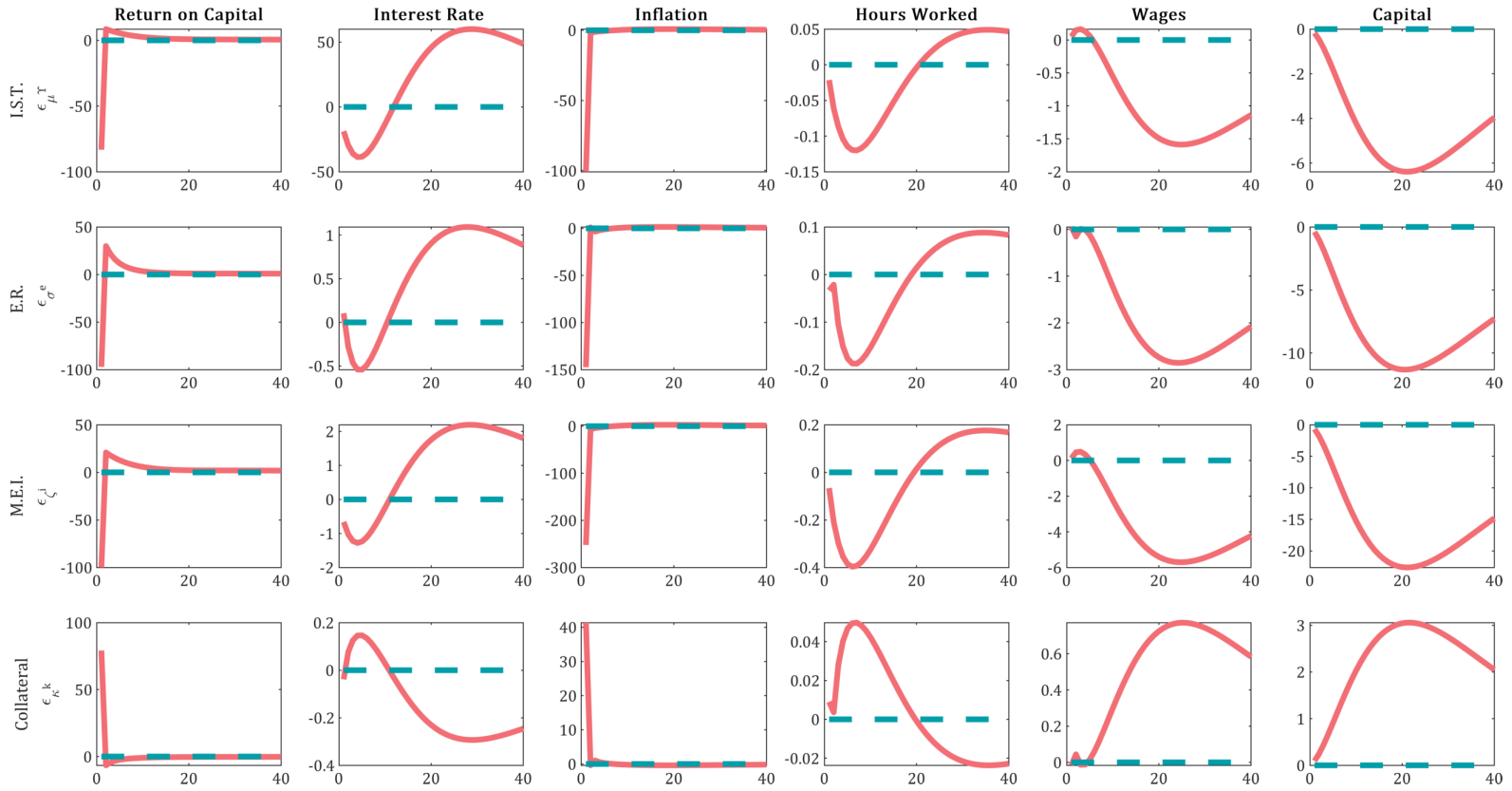


FIGURE A8. Impulse Responses of the Estimated Model

Notes: Time units are quarterly. The simulation shows the responses of return on capital, interest rate, inflation, hours worked, wages, and capital to investment-specific technical change, entrepreneur risk, marginal efficiency of investment, and collateral requirement shocks.

Appendix F. Model Computations

The model developed in this paper is based on Christiano, Motto, and Rostagno (2014), and augmented with collateral constraint to study the impact of collateral requirement variations on banking decisions and the macroeconomy. Furthermore, we add two important components to collateral price that implicitly identify the firm value. By characterizing the aggregate firm value as in Schmitt-Grohe and Uribe (2012), we can provide insight into how the bank lending channel propagates a positive collateral shock. We start by considering some general assumptions in this framework. First, in this economy, time is discrete, and there is a continuum of agents who live infinitely. They are rational, forward-looking, and maximize their profits. Second, the baseline model accounts for nominal rigidities in prices and wages as the price adjustment is less frequent, and where multiple agents maximize their utility and profits. We also assume that good retailers operate under monopolistic competition.

The main agents in this framework are households, bankers, entrepreneurs, firms, capital producers, labor contractors, and the government. In each period t, firms maximize their profits by converting an intermediate good to a final good, setting prices and wages under nominal inertia. Capital producers provide capital to entrepreneurs and adjust smoothly the cost of investment. Entrepreneurs maximize profits by offering capital services and borrowing from banks. Households maximize their lifetime utility by providing labor services, consuming the final good, and making deposits. Banks intermediate the flow of deposits and loans, and the government sets taxes and nominal interest rates. In equilibrium, all households, firms, and banks behave optimally, and all markets clear.

This appendix describes the computation of the equilibrium of a discrete time dynamic model using Lagrangian methods or substituting out the constraint. In section F, we sort all first-order conditions and definition terms. In section H, we detrend the model by using specific trend growth for each variable. In section I, we find the non-stochastic steady state.

F.1. Final Good Producers

The economy is populated by a continuum of firms and operates under monopolistic competition. Each firm has the final good stock, which writes:

$$Y_t = \left[\int_0^1 Y_{j,t}^{\frac{1}{1+\nu_p}} dj \right]^{1+\nu_p}$$

Final good producers purchase the good and resell it to consumers. Their objective is to maximize their profits.

$$P_t Y_t - \int_0^1 P_{j,t} Y_{j,t} dj$$

$$P_t\left[\int_0^1 Y_{j,t}^{\frac{1}{1+\nu_p}}dj\right]^{1+\nu_p} - \int_0^1 P_{j,t}Y_{j,t}dj.$$

The first-order condition with respect to $Y_{j,t}$ is given by

$$\begin{aligned}
(\partial Y_{j,t}): \qquad & P_t(1+\nu_p)\left[\int_0^1 Y_{j,t}^{\frac{1}{1+\nu_p}}dj\right]^{1+\nu_p-1}\frac{1}{1+\nu_p}Y_{j,t}^{(\frac{1}{1+\nu_p})-1} - P_{j,t} = 0 \\
& \left[\int_0^1 Y_{j,t}^{\frac{1}{1+\nu_p}}dj\right]^{\nu_p} Y_{j,t}^{(\frac{-\nu_p}{1+\nu_p})} = \frac{P_{j,t}}{P_t} \\
& \left[\int_0^1 Y_{j,t}^{\frac{1}{1+\nu_p}}dj\right]^{-(1+\nu_p)} Y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{\frac{1+\nu_p}{\nu_p}} \\
& Y_t^{-1}Y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\frac{1+\nu_p}{\nu_p}}.
\end{aligned}$$

From the first-order condition, $Y_{j,t}$ is given by

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\frac{1+\nu_p}{\nu_p}} Y_t.$$

To define the aggregate price of the final good P_t, we use an expression for the output, which is equal to price times quantities $P_tY_t = \int_0^1 P_{j,t}Y_{j,t}dj$; then, we simplify to obtain the expression for the aggregate price of the final good:

$$\begin{aligned}
P_tY_t &= \int_0^1 P_{j,t}\left(\frac{P_{j,t}}{P_t}\right)^{-\frac{1+\nu_p}{\nu_p}} Y_t dj \\
P_t &= \left[\int_0^1 P_{j,t}^{-\frac{1+\nu_p}{\nu_p}} dj\right]^{-\frac{\nu_p}{1+\nu_p}}.
\end{aligned}$$

F.2. Intermediate Good Producers

The problem of the intermediate good producer is to minimize the cost,

$$W_jL_{j,t} + r_t^k u_t K_{j,t-1},$$

subject to the production function,

(A1) $$Y_{j,t} = \gamma_t(u_tK_{j,t})^{\alpha}(z_tL_{j,t})^{1-\alpha} - \Phi z_t^*.$$

I can write the intermediate good producer problem in Lagrangian form as:

$$\mathcal{L} = \qquad (W_jL_{j,t} + r_t^k u_t K_{j,t}) + \lambda_{1,t}(Y_{j,t} - \gamma_t(u_tK_{j,t})^{\alpha}(z_tL_{j,t})^{1-\alpha} + \Phi z_t^*),$$

where λ_1 is the Lagrange multiplier associated with the production function. The first-order conditions with respect to labor and capital yield:

$$
\begin{aligned}
\frac{\partial \mathcal{L}_t}{\partial L_{j,t}}: \quad & W_t - (1-\alpha)\gamma_t (u_t K_{j,t})^{\alpha} (z_t L_{j,t})^{-\alpha} \Big) \lambda_{1,t} = 0 \\
& W_t - (1-\alpha)\gamma_t \left(\frac{u_t K_{j,t}}{z_t L_{j,t}} \right)^{\alpha} \lambda_{1,t} = 0 \\
& \lambda_{1,t} = \frac{W_t}{(1-\alpha)\gamma_t (u_t K_{j,t})^{\alpha} (z_t L_{j,t})^{-\alpha}} \\
\frac{\partial \mathcal{L}_t}{\partial u_t K_{j,t}}: \quad & r_t^k - \alpha \gamma_t \left(\frac{z_t L_{j,t}}{u_t K_{j,t}} \right)^{1-\alpha} \lambda_{1,t} = 0 \\
& \lambda_{1,t} = \frac{r_t^k}{\alpha \gamma_t (z_t L_{j,t})^{1-\alpha} (u_t K_{j,t})^{\alpha-1}}.
\end{aligned}
\tag{A2}
$$

We can eliminate the Lagrange multiplier using these two conditions, which imply the following:

$$
\begin{aligned}
\frac{W_t}{(1-\alpha)\gamma_t (u_t K_{j,t})^{\alpha} (z_t L_{j,t})^{-\alpha}} &= \frac{r_t^k}{\alpha \gamma_t (z_t L_{j,t})^{1-\alpha} (u_t K_{j,t})^{\alpha-1}} \\
\frac{W_t}{(1-\alpha)\gamma_t} \left(\frac{(z_t L_{j,t})}{(u_t K_{j,t})} \right)^{\alpha} &= \frac{r_t^k}{\alpha \gamma_t} \left(\frac{(u_t K_{j,t})}{(z_t L_{j,t})} \right)^{(1-\alpha)} \\
\frac{u_t K_{j,t}}{z_t L_{j,t}} &= \frac{\alpha}{1-\alpha} \frac{W_t}{r_t^k}.
\end{aligned}
$$

Then, if we integrate both $K_t = \left[\int_0^1 K_{j,t} dj \right]$ and $L_t = \left[\int_0^1 L_{j,t} dj \right]$, we obtain :

$$
\frac{u_t K_t}{z_t L_t} = \frac{\alpha}{1-\alpha} \frac{W_t}{r_t^k}.
$$

We further simplify this expression with $u_t K_t = K_t$ and $L_t = z_t L_t$.

$$
\frac{K_t}{L_t} = \left(\frac{\alpha}{1-\alpha} \right) \frac{W_t}{r_t^k}
$$

Then, the total cost function $W_j L_{j,t} + r_t^k u_t K_{j,t}$ is simply rewritten as:

$$
\begin{aligned}
& W_t L_t + r_t^k K_t \\
& W_t + r_t^k \frac{K_t}{L_t}.
\end{aligned}
$$

To define the marginal cost,[5] we divide the total cost by the productivity function $Y_t = \gamma_t (K_t)^{\alpha} (L_{,t})^{1-\alpha}$ and use the expression for $\frac{K_t}{L_t} = \left(\frac{\alpha}{1-\alpha}\right)\frac{W_t}{r_t^k}$.

$$
\begin{aligned}
\text{Marginal cost } = \lambda_{t,1} &= \frac{W_t + r_t^k \frac{K_t}{L_t}}{\gamma_t (K_t)^{\alpha} (L_{,t})^{1-\alpha}} \\
\lambda_{t,1} &= \frac{W_t + r_t^k \frac{K_t}{L_t}}{\gamma_t \left(\frac{K_t}{L_t}\right)^{\alpha}} \\
\lambda_{t,1} &= \frac{W_t + r_t^k \left(\frac{\alpha}{1-\alpha}\frac{W_t}{r_t^k}\right)}{\gamma_t \left(\frac{\alpha}{1-\alpha}\frac{W_t}{r_t^k}\right)^{\alpha}} \\
\lambda_{t,1} &= \frac{W_t + \left(\frac{\alpha}{1-\alpha} W_t\right)}{\gamma_t \left(\frac{\alpha}{1-\alpha}\frac{W_t}{r_t^k}\right)^{\alpha}} \\
\lambda_{t,1} &= \frac{1}{\gamma_t}\left[W_t + \left(\frac{\alpha}{1-\alpha} W_t\right)\right]\left[\frac{\alpha}{1-\alpha}\frac{W_t}{r_t^k}\right]^{-\alpha} \\
\lambda_{t,1} &= \frac{1}{\gamma_t}\left[W_t + \left(\frac{\alpha}{1-\alpha} W_t\right)\right]\left[\frac{1-\alpha}{\alpha}\frac{r_t^k}{W_t}\right]^{\alpha} \\
\lambda_{t,1} &= \frac{1}{\gamma_t}\left[W_t + \left(\frac{\alpha}{1-\alpha} W_t\right)\right]\left(\frac{1-\alpha}{\alpha}\right)^{\alpha} (r_t^k)^{\alpha} (W_t)^{-\alpha} \\
\lambda_{t,1} &= \frac{1}{\gamma_t}\left[W_t\left(1 + \frac{\alpha}{1-\alpha}\right)\right]\left(\frac{1-\alpha}{\alpha}\right)^{\alpha} (r_t^k)^{\alpha} (W_t)^{-\alpha} \\
\lambda_{t,1} &= \frac{1}{\gamma_t}\left[W_t\left(\frac{1}{1-\alpha}\right)\right]\left(\frac{1-\alpha}{\alpha}\right)^{\alpha} (r_t^k)^{\alpha} (W_t)^{-\alpha} \\
\lambda_{t,1} &= \frac{1}{\gamma_t}\left(\frac{1}{1-\alpha}\right)\left(\frac{1-\alpha}{\alpha}\right)^{\alpha} (r_t^k)^{\alpha} (W_t)^{(1-\alpha)} \\
\lambda_{t,1} &= \frac{1}{\gamma_t}\left(\frac{1}{1-\alpha}\right)\left(\frac{1}{\alpha}\right)^{\alpha} (1-\alpha)^{\alpha} (r_t^k)^{\alpha} (W_t)^{(1-\alpha)} \\
\lambda_{t,1} &= \frac{1}{\gamma_t}\left(\frac{1}{1-\alpha}\right)\left(\frac{1}{\alpha}\right)^{\alpha}\left(\frac{1}{(1-\alpha)}\right)^{-\alpha} (r_t^k)^{\alpha} (W_t)^{(1-\alpha)} \\
\lambda_{t,1} &= \left(\frac{1}{1-\alpha}\right)^{(1-\alpha)}\left(\frac{1}{\alpha}\right)^{\alpha} \frac{(r_t^k)^{\alpha} W^{(1-\alpha)}}{\gamma_t}. \qquad \text{(A3)}
\end{aligned}
$$

Considering a firm's pricing problem, we assume that prices are sticky, as in Erceg, Henderson, and Levin (2000). For those firms, the problem is to choose the price level that maximizes their profits:

$$
\begin{aligned}
\underset{P_{j,t}}{\text{maximize}} \quad & E\Sigma_{s=0}^{\infty} \beta^s \zeta^p [Y_{j,t}(P_{j,t}^* - \lambda_{1,j,t+s} P_{j,t+s})] \\
\text{subject to} \quad & Y_{j,t} = Y_t \left(\frac{P_{j,t}}{P_t}\right)^{\frac{\nu_p}{1-\nu_p}},
\end{aligned}
$$

[5] In equilibrium, marginal cost equals marginal utility $\lambda_{t,1}$.

we solve the firm problem by substituting $Y_{j,t}$ into the firm's profits function:

$$(\partial P^*_{j,t}): \qquad P^*_{j,t} = \nu_p \frac{E_t \Sigma_{k=0}^{\infty} \beta_k \zeta_k^p \left(\frac{P_{t+k}}{\tilde{\pi}_{t+k} P_t} \right) Y_{j,t+k} \lambda_{1,j,t+k}}{E_t \Sigma_{k=0}^{\infty} \beta_k \zeta_k^p Y_{j,t+k}}$$

where the price indexation $\tilde{\pi}_{t+k} = \pi_{trg,t}^{\iota^p} \pi_{t-1}^{1-\iota^p}$. To further simplify this condition, we divide by P_t and obtain:

$$\tilde{P}_t = \nu_p \frac{K_{p,t}}{F_{p,t}},$$

where $\tilde{P}_t = \frac{P^*_{j,t}}{P_t}$.

Then, the recursive form is written as:

$$K_{p,t} = \lambda_{1,j,t} \nu_{t,p} Y_t + \beta \zeta^p E_t \left[\left(\frac{\tilde{\pi_{t+1}}}{\pi_{t+1}} \right)^{\left(\frac{\nu_{t+1,p}}{1-\nu_{t+1,p}} \right)} K_{p,t+1} \right] \tag{A4}$$

$$F_{p,t} = \nu_{t,p} Y_t + \beta \zeta^p E_t \left[\left(\frac{\tilde{\pi_{t+1}}}{\pi_{t+1}} \right)^{\left(\frac{\nu_{t+1,p}}{1-\nu_{t+1,p}} \right)} F_{p,t+1} \right], \tag{A5}$$

with $\tilde{\pi_t} = \pi_{trg,t}^{\iota^p} \pi_{t-1}^{1-\iota^p}$ and $\pi_{t-1} = \frac{P_{t-1}}{P_{t-2}}$. We can now drop all indexes, since all firms have the same optimum, and given that $P_t = \left[\int_0^1 P_{j,t}^{\frac{1}{1+\nu_p}} dj \right]^{1+\nu_p}$, it follows that:

$$P_t = \left[(1-\zeta_p)(\tilde{P}_t)^{\frac{1}{1+\nu_p}} + \zeta_p (\tilde{\pi} P_{t-1})^{\frac{1}{1+\nu_p}} \right]^{1+\nu_p}.$$

We substitute with $\tilde{P}_t = \nu_p \frac{K_{p,t}}{F_{p,t}}$ such that $P_t = \left[(1-\zeta_p)(\nu_p \frac{K_{p,t}}{F_{p,t}})^{\frac{1}{1+\nu_p}} + \zeta_p (\tilde{\pi} P_{t-1})^{\frac{1}{1+\nu_p}} \right]^{1+\nu_p}$; then, we obtain:

$$P_t = \left[(1-\zeta_p) \left(\frac{1 - \zeta_p \left(\frac{\tilde{\pi_{t+1}}}{\pi_{t+1}} \right)^{\left(\frac{\nu_{t+1,p}}{1-\nu_{t+1,p}} \right)}}{1-\zeta_p} \right)^{\frac{1}{1+\nu_p}} + \zeta_p (\tilde{\pi} P_{t-1})^{\frac{1}{1+\nu_p}} \right]^{1+\nu_p} \tag{A6}$$

F.3. Labor Contractors

Labor services are provided by households to intermediate firms via labor contractors; these services are combined into homogeneous labor with:

$$L_t = \left[\int_0^1 L_{i,t}^{\frac{1}{1+\nu_l}} di \right]^{1+\nu_l}.$$

Labor contractors maximize their profits in a perfectly competitive market:

$$W_t L_t - \int_0^1 W_{i,t} L_{i,t} di$$

$$W_t \left[\int_0^1 L_{i,t}^{\frac{1}{1+\nu_l}} di \right]^{1+\nu_l} - \int_0^1 W_{i,t} L_{i,t} di.$$

The first-order condition with respect to labor $L_{i,t}$ can be written as:

$$
\begin{aligned}
(\partial L_{i,t}): \qquad & W_t(1+\nu_l)\left[\int_0^1 L_{i,t}^{\frac{1}{1+\nu_l}} di\right]^{1+\nu_l-1} \frac{1}{1+\nu_l} L_{i,t}^{(\frac{1}{1+\nu_l})-1} - W_{i,t} = 0 \\
& \left[\int_0^1 L_{i,t}^{\frac{1}{1+\nu_l}} di\right]^{\nu_l} L_{i,t}^{(\frac{-\nu_l}{1+\nu_l})} = \frac{W_{i,t}}{W_t} \\
& \left[\int_0^1 L_{i,t}^{\frac{1}{1+\nu_l}} di\right]^{-(1+\nu_l)} L_{i,t} = \left(\frac{W_{i,t}}{W_t}\right)^{\frac{1+\nu_l}{\nu_l}} \\
& L_t^{-1} L_{i,t} = \left(\frac{W_{i,t}}{W_t}\right)^{-\frac{1+\nu_l}{\nu_l}}.
\end{aligned}
$$

From the first-order condition, we obtain:

$$L_{i,t} = \left(\frac{W_{i,t}}{W_t}\right)^{-\frac{1+\nu_l}{\nu_l}} L_t.$$

To define the aggregate wage W_t, we take the total labour supply expression $W_t L_t = \int_0^1 W_{i,t} L_{i,t} di$, then simplify to obtain the expression for the aggregate wage:

$$
\begin{aligned}
W_t L_t &= \int_0^1 W_{i,t}\left(\frac{W_{i,t}}{W_t}\right)^{-\frac{1+\nu_l}{\nu_l}} L_t di \\
W_t &= \left[\int_0^1 W_{i,t}^{-\frac{1+\nu_l}{\nu_l}} di\right]^{-\frac{\nu_l}{1+\nu_l}}.
\end{aligned}
$$

The problem of wage setting is given as:

$$
\begin{aligned}
\underset{W_{i,t}}{\text{maximize}} \quad & E\Sigma_{s=0}^{\infty} \beta^s \zeta^{c,t+s} \zeta^l \left[-\Psi_L \int_0^1 \frac{L_{i,t+s}}{1+\sigma_L} di + \lambda_{z,t+s} W_{i,t} \Pi_{i,t+s}^w - L_{i,t+s} \right] \\
\text{subject to} \quad & L_{i,t+s} = \left(\frac{W_{i,t}\Pi_{i,t+s}^w}{W_{t+s}}\right)^{\frac{\nu_l}{1-\nu_l}} L_{t+s} \\
\text{where} \quad & \Pi_{i,t+s} = \Sigma_{k=1}^{s} (\mu_{z,t+k})^{\iota^\mu} L_t (\mu_z)^{1-\iota^\mu} \tilde{\pi}_{w,t+k} \\
\text{and} \quad & W_{t+s} = \pi_{w,t+s} \ldots \pi_{w,t+1} W_t
\end{aligned}
$$

$$
\frac{\partial \mathcal{L}_t}{\partial \tilde{W}_{i,t}} : W_{j,t} = \frac{\Psi_L E_t \Sigma_{s=0}^{\infty} \beta_s \zeta_{c,t+s} \zeta_{l,s} \nu_l \left[\left(\frac{\tilde{W} \Pi_{i,t+s}^w}{W_{t+s}} \right)^{\frac{\nu_l}{1-\nu_l}} L_{j,t+s} \right]^{1+\sigma_L}}{E_t \Sigma_{s=0}^{\infty} \beta_s \zeta_{c,t+s} \zeta_{l,s} L_{t+s} \left(\frac{\Pi_{i,t+s}^w}{\pi_{w,t+s} \ldots \pi_{w,t}} \right)^{\frac{\nu_l}{1-\nu_l}} \left(\frac{\tilde{W}_t}{W_t} \right)^{\frac{\nu_l}{1-\nu_l}} \lambda_{z,t+s} \Pi_{i,t+s}^w},
$$

where $\tilde{W}_t = W_{i,t}$. After some algebra, we obtain:

$$\left(\frac{\tilde{W}_t}{W_t}\right)^{\frac{1-\nu_l(1+\sigma_L)}{1-\nu_l}} \frac{W_t}{P_t} \frac{1}{\Psi_L} = \frac{K_{w,t}}{F_{w,t}}.$$

The recursive form is written as:

$$K_{w,t} = (L_t)^{1+\sigma_L} + \beta\zeta^l E_t\left[\left(\frac{\tilde{\pi}_{w,t+1}(\mu^*_{z,t+1})^{\iota_\mu}(\mu^*_z)^{1-\iota_\mu}}{\pi_{w,t+1}}\right)^{\frac{\nu_l}{1-\nu_l}(1+\sigma_L)} K_{w,t+1}\right] \tag{A7}$$

$$F_{w,t} = \frac{L_t}{\nu_l}P_t\lambda_z + \beta\zeta^l E_t\left(\tilde{\pi}_{w,t+1}(\mu^*_{z,t+1})^{\iota_\mu}(\mu^*_z)^{1-\iota_\mu}\right)^{\frac{1}{1-\nu_l}}\left(\frac{1}{\pi_{w,t+1}}\right)^{\frac{\nu_l}{1-\nu_l}}\frac{1}{\pi_{t+1}}F_{w,t+1}\Bigg]. \tag{A8}$$

Wages in this economy are sticky and are of the form:

$$W_t = \left[(1-\zeta_l)(\tilde{W}_t)^{\frac{1}{1+\nu_l}} + \zeta_l(\tilde{\pi}(\mu^*_{z,t})^{\iota_w}(\mu^*_z)^{1-\iota_w}W_{t-1})^{\frac{1}{1+\nu_l}}\right]^{1+\nu_l}, \tag{A9}$$

with $\tilde{\pi} = \pi^{\iota^w}_{trg,t}\pi^{1-\iota^w}_{t-1}$. Then, we substitute the last expression for $\left(\frac{\tilde{W}_t}{W_t}\right)^{\frac{1-\nu_l(1+\sigma_L)}{1-\nu_l}}$ $\frac{W_t}{P_t}\frac{1}{\Psi_L} = \frac{K_{w,t}}{F_{w,t}}$ and we obtain:

$$\frac{1}{\Psi_L}\left[\frac{1-\zeta^l\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}(\mu^*_{z,t})^{\iota_\mu}(\mu^*_z)^{1-\iota_\mu}\right)^{\frac{1}{1-\nu_l}}}{1-\zeta^l}\right]^{1-\nu_l(1+\sigma_L)}\frac{W_t}{P_t}F_{w,t} - K_{w,t} = 0.$$

F.4. Capital Producers

Capital producers accumulate capital according to the following equation:

$$K_t = (1-\delta)K_{t-1} + \left(1 - S\left(\zeta_{I,t}, \frac{I_t}{I_{t-1}}\right)\right)I_t. \tag{A10}$$

Capital producers maximize their profits:

$$\Pi_t = \left[Q^k_t\left[(1-\delta)K_{t-1} + \left(1 - S\left(\zeta_{I,t}, \frac{I_t}{I_{t-1}}\right)\right)I_t\right] - Q^k_t(1-\delta)K_{t-1} - \left(\frac{P_t}{\chi_t\mu}\right)\right].$$

The Lagrangian representation of the problem is given by:

$$\mathcal{L} = E_0\Sigma^\infty_{t=0}\beta\lambda_z\left[Q^k_t\left[(1-\delta)K_{t-1} + \left(1 - S\left(\zeta_{I,t}, \frac{I_t}{I_{t-1}}\right)\right)I_t\right] - Q^k_t(1-\delta)K_{t-1} - \left(\frac{P_t}{\chi_t\mu_\chi}\right)\right].$$

Given the capital producer profit function, the first-order condition with respect to investment is of the form:

$$\begin{aligned}\frac{\partial\mathcal{L}_t}{\partial I_t}: \quad & \lambda_{z,t}Q^k_t\left[1 - S\left(\zeta_{I,t}\frac{I_t}{I_{t-1}}\right) - \zeta_{I,t}\frac{I_t}{I_{t-1}}S'\left(\zeta_{I,t}\frac{I_t}{I_{t-1}}\right)\right] \\ & - \frac{\lambda_{z,t}P_t}{\chi_t\mu_{\chi,t}} + \beta\lambda_{z,t+1}Q^k_{t+1}\zeta_{I,t+1}\left(\frac{I_{t+1}}{I_t}\right)^2 S''\left(\zeta_{I,t+1}\frac{I_{t+1}}{I_t}\right) = 0.\end{aligned} \tag{A11}$$

F.5. Entrepreneurs

The rental income from a unit of capital is equal to the cost of capital utilization:

$$\frac{P_t}{\chi_t} a(u_t) K_t \omega_t = r_t^k P_t K_t \omega_t.$$

This takes the Lagrangian form:

$$\mathcal{L} = \frac{P_t}{\chi_t} a(u_t) K_t \omega_t - r_t^k P_t K_t \omega_t.$$

Given the capital utilization cost function, the first-order condition with respect to capital utilization is:

$$\begin{aligned} \frac{\partial \mathcal{L}_t}{\partial u_t K_t} : \qquad & \frac{P_t}{\chi_t} a'(u_t) \omega_t - r_t^k P_t \omega_t = 0 \\ & a'(u_t) = r_t^k \chi_t, \end{aligned} \tag{A12}$$

where $a'(u_t) = r_{ss}^k \exp\big(\sigma_a(u_t - 1)\big)$. Then, we substitute $a'(u_t)$ in the first-order condition, and it becomes:

$$r_{ss}^k \exp\big(\sigma_a(u_t - 1)\big) = r_t^k \chi_t,$$

or equivalently, with $r_t^k = r_t^k \chi_t$, we obtain $r_{ss}^k \exp\big(\sigma_a(u_t - 1)\big) = r_t^k$.
Entrepreneur revenues take the form:

$$\Big[(1 + R_{t+1}^k) Q_t^k\Big] \omega_t K_{t+1},$$

which embodies the revenues from capital, the cost associated with capital, and revenues from selling undepreciated capital to capital producers. This can be rewritten as:

$$\left[(1 - \tau^k)\big(u_{t+1} r_{t+1}^k - \frac{a(u_{t+1})}{\chi_{t+1}}\big) P_{t+1} + (1 - \delta) Q_{t+1}^k + \tau^k \delta Q_t^k\right] \omega_t K_{t+1}.$$

Equivalently,

$$\begin{aligned} (1 + R_{t+1}^k) Q_t^k = \quad & (1 - \tau^k)\left(u_{t+1} r_{t+1}^k - \frac{a(u_{t+1})}{\chi_{t+1}}\right) P_{t+1} + (1 - \delta) Q_{t+1}^k + \tau^k \delta Q_t^k \\ (1 + R_{t+1}^k) = \quad & \frac{(1 - \tau^k)\left(u_{t+1} r_{t+1}^k - \frac{a(u_{t+1})}{\chi_{t+1}}\right) P_{t+1} + (1 - \delta) Q_{t+1}^k + \tau^k \delta Q_t^k}{Q_t^k} \\ (1 + R_{t+1}^k) = \quad & \frac{(1 - \tau^k)\left(u_{t+1} r_{t+1}^k - \frac{a(u_{t+1})}{\chi_{t+1}}\right) P_{t+1} + (1 - \delta) Q_{t+1}^k}{Q_t^k} + \tau^k \delta. \end{aligned} \tag{A13}$$

Entrepreneur capital is equal to total loans and net worth:

$$Q_{t-1}^k K_{t-1} = M_{t-1} + N_{t-1}. \tag{A14}$$

The debt contract between an entrepreneur and a bank is signed before the realization of shocks. We define the threshold $\bar{\omega}_t$ below which entrepreneurs may default:

$$\bar{\omega}_t \kappa_{t-1}(1+R_t^k)Q_{t-1}^k K_{t-1} = (1+R_t^e)M_{t-1}.$$

The enforcement insurance condition is given by:

$$\begin{aligned}&[1-F_{t-1}(\bar{\omega}_t)](1+R_t^e)M_{t-1}\\&+(1-\mu_{t-1})\kappa_{t-1}\int_0^{\bar{\omega}_t}\omega dF_{t-1}(\omega)(1+R_t^k)Q_{t-1}^k K_{t-1} = (1+R_{t-1})M_{t-1}.\end{aligned}$$

Note that the equation of bank enforcement insurance (3.2) must hold with strict equality in every state of nature; it cannot be violated because that would mean that the bank would make profits or loss. The first term on the left-hand side of the equation corresponds to the returns from non-defaulting entrepreneurs. The second term corresponds to the returns from defaulting entrepreneurs whose collateral is seized by banks and net of monitoring cost μ. The right-hand side describes the return on loans given the interest rate demanded by banks R_{t+1}. Using the definitions:

$$\begin{aligned}Lev_t &= \frac{M_t}{N_t}\\ \Gamma_{t-1}(\bar{\omega}_t) &= \bar{\omega}_t[1-F_{t-1}(\bar{\omega}_t)] + G_{t-1}(\bar{\omega}_t)\\ G_{t-1}(\bar{\omega}_t) &= \int_0^{\bar{\omega}_t}\omega dF_{t-1}(\omega),\end{aligned}$$

the bank enforcement limitation can be rewritten as:

$$\begin{aligned}&[1-F_{t-1}(\bar{\omega}_t)]\bar{\omega}_t\kappa_{t-1}(1+R_t^k)Q_{t-1}^k K_{t-1}\\&+(1-\mu)\int_0^{\bar{\omega}_t}\omega dF_{t-1}(\omega)\kappa_{t-1}(1+R_t^k)Q_{t-1}^k K_{t-1} = (1+R_{t-1})M_{t-1}\\&\Bigg[[1-F_{t-1}(\bar{\omega}_t)]\bar{\omega}_t\\&+(1-\mu)\int_0^{\bar{\omega}_t}\omega dF_{t-1}(\omega)\Bigg]\kappa_{t-1}(1+R_t^k)Q_{t-1}^k K_{t-1} = (1+R_{t-1})M_{t-1}\\&\Bigg[[1-F_{t-1}(\bar{\omega}_t)]\bar{\omega}_t\\&+\int_0^{\bar{\omega}_t}\omega dF_{t-1}(\omega)-\mu\int_0^{\bar{\omega}_t}\omega dF_{t-1}(\omega)\Bigg]\kappa_{t-1}(1+R_t^k)Q_{t-1}^k K_{t-1} = (1+R_{t-1})M_{t-1}\\&\left[\Gamma_{t-1}(\bar{\omega}_t)-\mu G_{t-1}(\bar{\omega}_t)\right]\kappa_{t-1}(1+R_t^k)Q_{t-1}^k K_{t-1} = (1+R_{t-1})M_{t-1}\\&\left[\Gamma_{t-1}(\bar{\omega}_t)-\mu G_{t-1}(\bar{\omega}_t)\right]\kappa_{t-1}\frac{(1+R_t^k)}{(1+R_{t-1})} = \frac{M_{t-1}}{Q_{t-1}^k K_{t-1}}\\&\left[\Gamma_{t-1}(\bar{\omega}_t)-\mu G_{t-1}(\bar{\omega}_t)\right]\kappa_{t-1}\frac{(1+R_t^k)}{(1+R_{t-1})} = \frac{M_{t-1}}{M_{t-1}+N_{t-1}}\end{aligned}$$

$$\left[\Gamma_{t-1}(\bar{\omega}_t) - \mu G_{t-1}(\bar{\omega}_t)\right]\kappa_{t-1}\frac{(1+R_t^k)}{(1+R_{t-1})} = \frac{\frac{M_{t-1}}{N_{t-1}}}{\frac{M_{t-1}+N_{t-1}}{N_{t-1}}}$$

$$\left[\Gamma_{t-1}(\bar{\omega}_t) - \mu G_{t-1}(\bar{\omega}_t)\right]\kappa_{t-1}\frac{(1+R_t^k)}{(1+R_{t-1})} = \frac{\frac{M_{t-1}}{N_{t-1}}}{\frac{M_{t-1}}{N_{t-1}}+1}$$

$$\left[\Gamma_{t-1}(\bar{\omega}_t) - \mu G_{t-1}(\bar{\omega}_t)\right]\kappa_{t-1}\frac{(1+R_t^k)}{(1+R_{t-1})} = \frac{Lev_{t-1}}{Lev_{t-1}+1}$$

$$\left[\Gamma_{t-1}(\bar{\omega}_t) - \mu G_{t-1}(\bar{\omega}_t)\right]\kappa_{t-1}\frac{(1+R_t^k)}{(1+R_{t-1})}(1+Lev_{t-1}) = Lev_{t-1}$$

$$\kappa_{t-1}\left[\Gamma_{t-1}(\bar{\omega}_t) - \mu G_{t-1}(\bar{\omega}_t)\right]\frac{(1+R_t^k)}{(1+R_{t-1})}(1+Lev_{t-1}) = Lev_{t-1}.$$

The entrepreneur profits Π are given by:

$$\begin{aligned}
\Pi = & \int_{\bar{\omega}_t}^{\infty}\left[\omega_t(1+R_t^k)Q_{t-1}^k K_{t-1} - (1+R_t^e)M_{t-1}\right]dF_{t-1}\omega_t \\
= & \int_{\bar{\omega}_t}^{\infty}\left[\omega_t(1+R_t^k)Q_{t-1}^k K_{t-1} - \bar{\omega}_t\kappa_{t-1}(1+R_t^k)Q_{t-1}^k K_{t-1}\right]dF_{t-1}\omega_t \\
= & \int_{\bar{\omega}_t}^{\infty}\left[\left[\omega_t - \bar{\omega}_t\kappa_{t-1}\right]dF_{t-1}\right]\omega_t(1+R_t^k)Q_{t-1}^k K_{t-1} \\
= & \left[\int_{\bar{\omega}_t}^{\infty}\omega_t dF_{t-1} - \int_{\bar{\omega}_t}^{\infty}\bar{\omega}_t\kappa_{t-1}dF_{t-1}\right]\omega_t(1+R_t^k)Q_{t-1}^k K_{t-1} \\
= & \left[\int_{\bar{\omega}_t}^{\infty}\omega_t dF_{t-1}\omega_t - \bar{\omega}_t\kappa_{t-1}[1-F_{t-1}\bar{\omega}_t]\right](1+R_t^k)Q_{t-1}^k K_{t-1} \\
= & \left[[1-G_{t-1}(\bar{\omega}_t)] - \bar{\omega}_t\kappa_{t-1}[1-F_{t-1}\bar{\omega}_t]\right](1+R_t^k)Q_{t-1}^k K_{t-1} \\
= & \left[[1-G_{t-1}(\bar{\omega}_t)] - \kappa_{t-1}[\Gamma_{t-1}(\bar{\omega}_t) - G_{t-1}(\bar{\omega}_t)]\right](1+R_t^k)(N_{t-1}+M_{t-1}) \\
= & \left[[1-G_{t-1}(\bar{\omega}_t)] - \kappa_{t-1}[\Gamma_{t-1}(\bar{\omega}_t) - G_{t-1}(\bar{\omega}_t)]\right](1+R_t^k) \\
& (N_{t-1}+M_{t-1})\frac{N_{t-1}(1+R_{t-1})}{N_{t-1}(1+R_{t-1})} \\
= & \left[[1-G_{t-1}(\bar{\omega}_t)] - \kappa_{t-1}[\Gamma_{t-1}(\bar{\omega}_t) - G_{t-1}(\bar{\omega}_t)]\right] \\
& (1+R_t^k)\left(\frac{N_{t-1}+M_{t-1}}{N_{t-1}}\right)\frac{N_{t-1}(1+R_{t-1})}{(1+R_{t-1})} \\
= & \left[[1-G_{t-1}(\bar{\omega}_t)] - \kappa_{t-1}[\Gamma_{t-1}(\bar{\omega}_t) - G_{t-1}(\bar{\omega}_t)]\right] \\
& \frac{1+R_t^k}{1+R_{t-1}}\left[1+Lev_{t-1}\right]N_{t-1}(1+R_{t-1}).
\end{aligned}$$

Therefore, the entrepreneur's problem is to maximize their profits subject to the bank enforcement limitation defined above. The Lagrangian representation of the problem is:

$$\begin{aligned}\mathcal{L}_t = \quad & E_{t-1}\Bigg[\Bigg[\Big[1-G_{t-1}(\bar{\omega}_t)\Big]-\kappa_{t-1}\Big[\Gamma_{t-1}(\bar{\omega}_t)-G_{t-1}(\bar{\omega}_t)\Big]\Bigg]\frac{1+R_t^k}{1+R_{t-1}} \\ & \Bigg[1+Lev_{t-1}\Bigg](1+R_{t-1})N_{t-1} \\ & +\lambda_2\Bigg[\kappa_{t-1}\Big[\Gamma_{t-1}(\bar{\omega}_t)-\mu G_{t-1}(\bar{\omega}_t)\Big]\frac{(1+R_t^k)}{(1+R_{t-1})}(1+Lev_{t-1})-Lev_{t-1}\Bigg]\Bigg].\end{aligned}$$

The first-order condition yields:

$$\begin{aligned}\frac{\partial\mathcal{L}_t}{\partial\lambda_2}: \quad & \kappa_{t-1}\Big[\Gamma^{e'}_{t-1}(\bar{\omega}_t)-\mu G^{e'}_{t-1}(\bar{\omega}_t)\Big]\frac{(1+R_t^k)}{(1+R_{t-1})}(1+Lev_{t-1})-Lev_{t-1}=0 \\ \frac{\partial\mathcal{L}_t}{\partial Lev_{t-1}}: \quad & E_{t-1}\Bigg[\Big[1-G_{t-1}(\bar{\omega}_t)\Big]-\kappa_{t-1}\Big[\Gamma_{t-1}(\bar{\omega}_t)-G_{t-1}(\bar{\omega}_t)\Big]\Bigg] \\ & \frac{1+R_t^k}{1+R_{t-1}}(1+R_{t-1})N_{t-1} \\ & +E_{t-1}\Bigg[\lambda_2\Bigg(\kappa_{t-1}\Big[\Gamma_{t-1}(\bar{\omega}_t)-\mu G_{t-1}(\bar{\omega}_t)\Big]\frac{(1+R_t^k)}{(1+R_{t-1})}-1\Bigg)\Bigg]=0 \\ \frac{\partial\mathcal{L}_t}{\partial\bar{\omega}_t}: \quad & -E_{t-1}\Bigg[G^{e'}_{t-1}(\bar{\omega}_t)+\kappa_{t-1}\Big[\Gamma^{e'}_{t-1}(\bar{\omega}_t)-G^{e'}_{t-1}(\bar{\omega}_t)\Big]\Bigg] \\ & \frac{1+R_t^k}{1+R_{t-1}}\Bigg[1+Lev_{t-1}\Bigg](1+R_{t-1})N_{t-1} \\ & +E_{t-1}\Bigg[\lambda_2\Bigg(\kappa_{t-1}\Gamma^{e'}_{t-1}(\bar{\omega}_t)\frac{1+R_t^k}{1+R_{t-1}}(1+Lev_{t-1})\Bigg)\Bigg] \\ & -E_{t-1}\Bigg[\lambda_2\Bigg(\kappa_{t-1}\mu G^{e'}_{t-1}(\bar{\omega}_t)\frac{1+R_t^k}{1+R_{t-1}}(1+Lev_{t-1})\Bigg)\Bigg]=0 \\ & -E_{t-1}\Bigg[G^{e'}_{t-1}+\kappa_{t-1}\Big[\Gamma^{e'}_{t-1}(\bar{\omega}_t)-G^{e'}_{t-1}(\bar{\omega}_t)\Big]\Bigg]\frac{1+R_t^k}{1+R_{t-1}} \\ & \Bigg[1+Lev_{t-1}\Bigg](1+R_{t-1})N_{t-1} \\ & +E_{t-1}\Bigg[\lambda_2\Bigg(\kappa_{t-1}\Gamma^{e'}_{t-1}(\bar{\omega}_t)-\kappa_{t-1}\mu G^{e'}_{t-1}(\bar{\omega}_t)\Bigg)\frac{1+R_t^k}{1+R_{t-1}}(1+Lev_{t-1})\Bigg]=0 \\ \lambda_2 = \quad & \frac{\Big[G^{e'}_{t-1}(\bar{\omega}_t)+\kappa_{t-1}\Big[\Gamma^{e'}_{t-1}(\bar{\omega}_t)-G^{e'}_{t-1}(\bar{\omega}_t)\Big]\Big]\frac{1+R_t^k}{1+R_{t-1}}(1+Lev_{t-1})(1+R_{t-1})N_{t-1}}{\kappa_{t-1}\Big[\Gamma^{e'}_{t-1}(\bar{\omega}_t)-\mu G^{e'}_{t-1}(\bar{\omega}_t)\Big]\frac{1+R_t^k}{1+R_{t-1}}(1+Lev_{t-1})} \\ \lambda_2 = \quad & \frac{\Big[G^{e'}_{t-1}(\bar{\omega}_t)+\kappa_{t-1}\Big[\Gamma^{e'}_{t-1}(\bar{\omega}_t)-G^{e'}_{t-1}(\bar{\omega}_t)\Big]\Big]}{\kappa_{t-1}\Big[\Gamma^{e'}_{t-1}(\bar{\omega}_t)-\mu G^{e'}_{t-1}(\bar{\omega}_t)\Big]}(1+R_{t-1})N_{t-1}.\end{aligned}$$

Then, we substitute λ_2 into the first-order condition of leverage, and obtain:

$$\begin{aligned}
0 = \quad & E_{t-1}\Bigg[\Bigg[[1-G_{t-1}(\bar{\omega}_t)]-\kappa_{t-1}[\Gamma_{t-1}(\bar{\omega}_t)-G_{t-1}(\bar{\omega}_t)]\Bigg]\frac{1+R_t^k}{1+R_{t-1}}(1+R_{t-1})N_{t-1} \\
& + \frac{\left[G^{e'}_{t-1}(\bar{\omega}_t)+\kappa_{t-1}[\Gamma^{e'}_{t-1}(\bar{\omega}_t)-G^{e'}_{t-1}(\bar{\omega}_t)]\right]}{\kappa_{t-1}\left[\Gamma^{e'}_{t-1}(\bar{\omega}_t)-\mu G^{e'}_{t-1}(\bar{\omega}_t)\right]}(1+R_{t-1})N_{t-1} \\
& \left(\kappa_{t-1}\left[\Gamma_{t-1}(\bar{\omega}_t)-\mu G_{t-1}(\bar{\omega}_t)\right]\frac{(1+R_t^k)}{(1+R_{t-1})}-1\right)\Bigg].
\end{aligned}$$

This simplifies to:

$$\begin{aligned}
0 = \quad & E_{t-1}\Bigg[\Bigg[[1-G_{t-1}(\bar{\omega}_t)]-\kappa_{t-1}[\Gamma_{t-1}(\bar{\omega}_t)-G_{t-1}(\bar{\omega}_t)]\Bigg]\frac{1+R_t^k}{1+R_{t-1}} \\
& + \frac{\left[G^{e'}_{t-1}+\kappa_{t-1}[\Gamma^{e'}_{t-1}(\bar{\omega}_t)-G^{e'}_{t-1}(\bar{\omega}_t)]\right]}{\kappa_{t-1}\left[\Gamma^{e'}_{t-1}(\bar{\omega}_t)-\mu G^{e'}_{t-1}(\bar{\omega}_t)\right]} \\
& \left(\kappa_{t-1}\left[\Gamma_{t-1}(\bar{\omega}_t)-\mu G_{t-1}(\bar{\omega}_t)\right]\frac{(1+R_t^k)}{(1+R_{t-1})}-1\right)\Bigg]. \qquad \text{(A15)}
\end{aligned}$$

Given that $Q^k_{t-1}K_{t-1} = M_{t-1}+N_{t-1}$ and $Lev_t = \frac{M_t}{N_t}$, the bank enforcement limitation, defined above as $\kappa_{t-1}\left[\Gamma_{t-1}(\bar{\omega}_t)-\mu G_{t-1}(\bar{\omega}_t)\right]\frac{(1+R_t^k)}{(1+R_{t-1})} = \frac{Lev_{t-1}}{(1+Lev_{t-1})}$, can also be rewritten as:

$$\begin{aligned}
\kappa_{t-1}\left[\Gamma_{t-1}(\bar{\omega}_t)-\mu G_{t-1}(\bar{\omega}_t)\right]\frac{(1+R_t^k)}{(1+R_{t-1})} &= \frac{M_{t-1}}{(N_{t-1}+M_{t-1})} \\
\kappa_{t-1}\left[\Gamma_{t-1}(\bar{\omega}_t)-\mu G_{t-1}(\bar{\omega}_t)\right]\frac{(1+R_t^k)}{(1+R_{t-1})} &= \frac{Q^k_{t-1}K_{t-1}-N_{t-1}}{(Q^k_{t-1}K_{t-1})} \qquad \text{(A16)}
\end{aligned}$$

Entrepreneur net worth evolves according to:

$$N_t = \gamma^e V_t + w,$$

where V is entrepreneur equity and γ^e is the percentage of entrepreneurs who survive, obtain the aggregate profit, and receive w, which is a transfer payment when new entrepreneurs enter the next period.

Entrepreneur equity is of the form:

$$\begin{aligned}
V_t = \quad & (1+R^k_{t-1})Q^k_{t-1}K_{t-1}-(1+R_{t-1})(Q^k_{t-1}K_{t-1}-N_{t-1}) \\
& +\mu\kappa_{t-1}G_{t-1}(\bar{\omega}_t)(1+R^k_{t-1})Q^k_{t-1}K_{t-1} \\
= \quad & (1+R^k_{t-1})Q^k_{t-1}K_{t-1}-(1+R_{t-1})Q^k_{t-1}K_{t-1}+(1+R_{t-1})N_{t-1} \\
& +\mu\kappa_{t-1}G_{t-1}(\bar{\omega}_t)(1+R^k_{t-1})Q^k_{t-1}K_{t-1} \\
= \quad & Q^k_{t-1}K_{t-1}[(1+R^k_{t-1})-(1+R_{t-1})+\mu\kappa_{t-1}G_{t-1}(\bar{\omega}_t)(1+R^k_{t-1})] \\
& +(1+R_{t-1})N_{t-1}.
\end{aligned}$$

Then, we write the entrepreneur net worth as:

$$N_t = \gamma^e \Bigg(Q^k_{t-1} K_{t-1} [(1 + R^k_{t-1}) - (1 + R_{t-1}) + \mu \kappa_{t-1} G_{t-1}(\bar{\omega}_t)(1 + R^k_{t-1})] + (1 + R_{t-1}) N_{t-1}) \Bigg) + w_t. \tag{A17}$$

The aggregate firm value is given by:

$$V_t = Y_t - W_t L_t - \frac{I_t}{\chi_t} + \beta \frac{\lambda_{z,t+1}}{\lambda_{z,t}} Q^k_{t+1} K_{t+1},$$

which equals the current dividend and the present discounted value of future dividends.
The relative price of capital is given by:

$$Q^k_t = \frac{V_t}{K_t} + s^*_t,$$

where the price of capital includes two components: average capital price and a speculative component. The latter component can be interpreted as a price bubble that can explain the fluctuations in the capital market and macroeconomic aggregates. The relative price of capital can be rewritten as:

$$Q^k_t = \frac{Y_t - W_t L_t - \frac{I_t}{\chi_t}}{K_t} + \beta \frac{\lambda_{z,t+1}}{\lambda_{z,t}} Q^k_{t+1} \frac{K_{t+1}}{K_t} + s^*_t.$$

F.6. Households

The objective of a household is to maximize its utility subject to the budget constraints:

$$\begin{aligned} &\text{maximize} \quad E_0 \Sigma^{\infty}_{t=0} \beta^t \left\{ \zeta_{c,t} (\log(C_t - b C_{t-1}) - \psi_l \frac{(L_t)^{1+\sigma_l}}{1+\sigma_l} \right\} \\ &\text{subject to} \quad (1 + \tau^c) P_t C_t + T_t \leq (1 - \tau^l) W_t L_t + (1 + R_t) T_{t-1}. \end{aligned}$$

We solve the problem by the Lagrangian method. The household problem in the Lagrangian form is given by:

$$\begin{aligned} \mathcal{L} = \quad & E_0 \Sigma^{\infty}_{t=0} \beta \Bigg[\zeta_{c,t} (\log(C_t - b C_{t-1}) - \psi_l \frac{(L_t)^{1+\sigma_l}}{1+\sigma_l} \\ & + \lambda_{z,t} \Big((1 + \tau^c) P_t C_t + T_t - W_t L_t - (1 + R_t) T_{t-1} \Big) \Bigg]. \end{aligned}$$

The first-order condition with respect to consumption and deposit is of the form:

$$\frac{\partial \mathcal{L}_t}{\partial C_t}: \quad \lambda_{z,t}(1 + \tau_c) P_t - \frac{\zeta_{c,t}}{C_t - b C_{t-1}} + b \beta E_t \frac{\zeta_{c,t+1}}{C_{t+1} - b C_t} = 0 \tag{A18}$$

$$\frac{\partial \mathcal{L}_t}{\partial T_t}: \quad \lambda_{z,t} - \beta E_t \lambda_{z,t+1}(1 + R_{t+1}) = 0 \tag{A19}$$

$$\frac{\partial \mathcal{L}_t}{\partial L_t}: \quad -\psi_l (L_t)^{\sigma_l} - \lambda_{z,t} W_t = 0. \tag{A20}$$

F.7. Resource Constraint

$$Y_t = D_t + G_t + C_t + \frac{I_t}{\chi_t \mu_{\chi,t}} + \frac{a(u_t)K_t}{\chi_t} + \frac{\Theta(1-\gamma^e)V_t}{P_t},$$

where D_t is the entrepreneur monitoring cost,

$$D_t = \kappa_{t-1}\mu G(\bar{\omega})(1+R_t^k)\frac{Q_{t-1}^k K_t}{P_t},$$

and G_t denotes government consumption,

$$G_t = z_t^* g_t.$$

The resource constraint can be rewritten as:

$$Y_t = D_t + G_t + C_t + \frac{I_t}{\chi_t \mu_{\chi,t}} + \frac{a(u_t)K_t}{\chi_t} + \frac{\Theta(1-\gamma^e)}{\gamma^e}(N_t - w). \tag{A21}$$

F.8. Monetary Policy

The monetary authority sets the policy rate according to:

$$R_t - R = \rho_p(R_{t-1} - R) + (1-\rho_p)[a_\pi(E_t\pi_{t+1} - \pi_t) + a_{\Delta y}(Y_t - Y).] \tag{A22}$$

F.9. Exogenous Shocks

Price markup shock:

$$\nu_{p,t} = \rho_{\nu_p}\nu_{p,t-1} + \epsilon_t^{\nu_p}. \tag{A23}$$

Investment-specific technology shock:

$$\mu_{\chi,t} = \rho_{\mu_\chi}\mu_{\chi,t-1} + \epsilon_t^{\mu_\chi}. \tag{A24}$$

Government spending shock:

$$g_t = \rho_g g_{t-1} + \epsilon_t^g. \tag{A25}$$

Technology trend shock:

$$\mu_{z,t}^* = \rho_{\mu_z^*}\mu_{z,t-1}^* + \epsilon_t^{\mu_z^*}. \tag{A26}$$

Stationary technology shock:

$$\gamma_t = \rho_\gamma \gamma_{t-1} + \epsilon_t^\gamma. \tag{A27}$$

Entrepreneur risk shock:

(A28) $$\sigma_{e,t} = \rho_{\sigma_e}\sigma_{e,t-1} + \epsilon_t^{\sigma_e}.$$

Consumption preference shock:

(A29) $$\zeta_{c,t} = \rho_{\zeta_c}\zeta_{c,t-1} + \epsilon_t^{\zeta_c}.$$

Marginal efficiency of investment shock:

(A30) $$\zeta_{i,t} = \rho_{\zeta_i}\zeta_{i,t-1} + \epsilon_t^{\zeta_i}.$$

Collateral requirement shock:

(A31) $$\kappa_t = \rho_\kappa\kappa_{t-1} + \epsilon_t^{\kappa}.$$

Appendix G. Equilibrium Definition

Household's optimality conditions. The first order conditions with respect to C_t, T_t, L_t are as follows:

(A1) $$\lambda_{z,t}(1+\tau_c)P_t - \frac{\zeta_{c,t}}{C_t - bC_{t-1}} + b\beta E_t\frac{\zeta_{c,t+1}}{C_{t+1} - bC_t} = 0$$

(A2) $$\lambda_{z,t} - \beta E_t\lambda_{z,t+1}(1+R_{t+1}) = 0$$

(A3) $$-\psi_l(L_t)^{\sigma_l} - \lambda_{z,t}W_t = 0.$$

For each period, the representative household chooses how much to consume C_t, to save T_t, and decides on labor supply L_t. This choice maximizes the utility function using the budget constraint. The household takes P_t, R_{t+1}, W_t, $\lambda_{z,t}$ as given.

Labor contractors optimality conditions. Labor services are provided by households to intermediate firms via labor contractors. Labor contractors are introduced into the model to buy the differentiated labor from households and sell it to intermediate good producers. They set the wage rate W_t subject to calvo frictions, with probability $1 - \zeta_w$ contractors can optimize wages and with probability ζ_w they can keep the wage unchanged. The calvo wage auxilary variables $K_{w,t}$ and $F_{w,t}$, the aggregate wage index W_t, and

the first order condition with respect to the optimized wage $\tilde{W}_t$ are as follows:

$$K_{w,t} = (L_t)^{1+\sigma_L} + \beta\zeta^l E_t\left[\left(\frac{\tilde{\pi}_{w,t+1}(\mu^*_{z,t+1})^{\iota_\mu}(\mu^*_z)^{1-\iota_\mu}}{\pi_{w,t+1}}\right)^{\frac{\nu_l}{1-\nu_l}(1+\sigma_L)} K_{w,t+1}\right] \tag{A4}$$

$$F_{w,t} = \frac{L_t}{\nu_l}P_t\lambda_{z,t} + \beta\zeta^l E_t\left(\tilde{\pi}_{w,t+1}(\mu^*_{z,t+1})^{\iota_\mu}(\mu^*_z)^{1-\iota_\mu}\right)^{\frac{1}{1-\nu_l}}\left(\frac{1}{\pi_{w,t+1}}\right)^{\frac{\nu_l}{1-\nu_l}}\frac{1}{\pi_{t+1}}F_{w,t+1}\Bigg] \tag{A5}$$

$$W_t = \left[(1-\zeta_l)(\tilde{W}_t)^{\frac{1}{1+\nu_l}} + \zeta_l(\tilde{\pi}(\mu^*_{z,t})^{\iota_w}(\mu^*_z)^{1-\iota_w}W_{t-1})^{\frac{1}{1+\nu_l}}\right]^{1+\nu_l} \tag{A6}$$

$$\frac{K_{w,t}}{F_{w,t}} = \left(\frac{\tilde{W}_t}{W_t}\right)^{\frac{1-\nu_l(1+\sigma_L)}{1-\nu_l}}\frac{W_t}{P_t}\frac{1}{\Psi_L}. \tag{A7}$$

The labor contractor solves for $F_{w,t}$, $\tilde{W}_t$, W_t, $K_{w,t}$ and takes L_t, P_t, $\lambda_{z,t}$, $\pi_{w,t+1}$ as given.

Producers optimality conditions. The production function is given by:

$$Y_{j,t} = \gamma_t(u_tK_{j,t})^{\alpha}(z_tL_{j,t})^{1-\alpha} - \Phi z^*_t.$$

The producer optimality conditions with respect to capital and labor and the euler equation are derived as follows:

$$\begin{aligned}\lambda_1 &= \frac{W_t}{(1-\alpha)\gamma_t(u_tK_{j,t})^{\alpha}(z_tL_{j,t})^{-\alpha}}\\ \lambda_{t,1} &= \frac{r^k_t}{\alpha\gamma_t(z_tL_{j,t})^{1-\alpha}(u_tK_{j,t})^{\alpha-1}}\\ \frac{K_t}{L_t} &= \left(\frac{\alpha}{1-\alpha}\right)\frac{W_t}{r^k_t}.\end{aligned} \tag{A8}$$

The Lagrange multiplier λ_1 associated with the production function is derived as follows:

$$\lambda_{t,1} = \left(\frac{1}{1-\alpha}\right)^{(1-\alpha)}\left(\frac{1}{\alpha}\right)^{\alpha}\frac{(r^k_t)^{\alpha}(W_t)^{(1-\alpha)}}{\gamma_t}. \tag{A9}$$

The producer solves for K_t, L_t, $\lambda_{t,1}$, and takes W_t, r^k_t as given.

Calvo price setting. The calvo price auxilary variables $K_{p,t}$ and $F_{p,t}$, the aggregate price index P_t, and the first order condition with respect to the optimized price $\tilde{P}_t$ are as follows:

$$K_{p,t} = \lambda_{1,j,t}\nu_{t,p}Y_t + \beta\zeta^p E_t\left[\left(\frac{\tilde{\pi_{t+1}}}{\pi_{t+1}}\right)^{\left(\frac{\nu_{t+1,p}}{1-\nu_{t+1,p}}\right)} K_{p,t+1}\right] \tag{A10}$$

$$F_{p,t} = \nu_{t,p}Y_t + \beta\zeta^p E_t\left[\left(\frac{\tilde{\pi_{t+1}}}{\pi_{t+1}}\right)^{\left(\frac{\nu_{t+1,p}}{1-\nu_{t+1,p}}\right)} F_{p,t+1}\right] \tag{A11}$$

$$P_t = \left[(1-\zeta_p)(\tilde{P}_t)^{\frac{1}{1+\nu_p}} + \zeta_p(\tilde{\pi}P_{t-1})^{\frac{1}{1+\nu_p}}\right]^{1+\nu_p} \tag{A12}$$

$$\tilde{P}_t = \nu_p\frac{K_{p,t}}{F_{p,t}}. \tag{A13}$$

The producer solves for $K_{p,t}$, $F_{p,t}$, P_t, $\tilde{P}_t$ and takes $\lambda_{t,1}$, Y_t, π_{t+1} as given.

Capital producer optimality conditions. Capital producers accumulate capital according to the following expression:

$$K_t = (1-\delta)K_{t-1} + \left(1 - S\left(\zeta_{I,t}, \frac{I_t}{I_{t-1}}\right)\right)I_t. \tag{A14}$$

The first order condition of investment is as follows:

$$\begin{aligned}&\lambda_{z,t}Q_t^k\left[1 - S\left(\zeta_{I,t}\frac{I_t}{I_{t-1}}\right) - \zeta_{I,t}\frac{I_t}{I_{t-1}}S'\left(\zeta_{I,t}\frac{I_t}{I_{t-1}}\right)\right]\\ &- \frac{\lambda_{z,t}P_t}{\chi_t\mu_{\chi,t}} + \beta\lambda_{z,t+1}Q_{t+1}^k\zeta_{I,t+1}\left(\frac{I_{t+1}}{I_t}\right)^2 S''\left(\zeta_{I,t+1}\frac{I_{t+1}}{I_t}\right) = 0.\end{aligned} \tag{A15}$$

The capital producer solves for investment I_t and capital K_t, and takes $\lambda_{z,t}$, P_t, Y_t, W_t, L_t, χ_t, Q_t^k as given.

Entrepreneur's optimality conditions. The relative price of capital is given by:

$$Q_t^k = \frac{Y_t - W_tL_t - \frac{I_t}{\chi_t}}{K_t} + \beta\frac{\lambda_{z,t+1}}{\lambda_{z,t}}Q_{t+1}^k\frac{K_{t+1}}{K_t} + s_t^*.$$

Entrepreneurs set the utlization rate of capital u_t at a rental rate of r_t^k.

$$r_t^k = r_{ss}^k \exp\left(\sigma_a(u_t - 1)\right) \tag{A16}$$

$$(1 + R_{t+1}^k) = \frac{(1-\tau^k)\left(u_{t+1}\tilde{r}_{t+1}^k - \frac{a(u_{t+1})}{\chi_{t+1}}\right)P_{t+1} + (1-\delta)Q_{t+1}^k}{Q_t^k} + \tau^k\delta. \tag{A17}$$

Entrepreneurs solve for the utilization rate u_t and capital rental rate r_t^k and take R_{t+1}^k, χ_{t+1}, Q_{t+1}^k, P_{t+1} as given.

Each entrepreneur buys capital K_t at price Q_t^k from capital producers, selects the optimal debt contract by choosing the value of the firm $Q_t^kK_t$ and the loan M_t that maximizes the expected net worth N_t:

$$K_{t-1}Q_{t-1}^k = M_{t-1} + N_{t-1}.$$

The first order condition with respect to default threshold is derived as follows:

$$\begin{aligned}&E_{t-1}\Bigg[\Bigg[\left[1 - G_{t-1}(\bar{\omega}_t)\right] - \kappa_{t-1}\left[\Gamma_{t-1}(\bar{\omega}_t) - G_{t-1}(\bar{\omega}_t)\right]\Bigg]\frac{1+R_t^k}{1+R_{t-1}}\\ &+ \frac{\left[G_{t-1}^{e'} + \kappa_{t-1}\left[\Gamma_{t-1}^{e'}(\bar{\omega}_t) - G_{t-1}^{e'}(\bar{\omega}_t)\right]\right]}{\kappa_{t-1}\left[\Gamma_{t-1}^{e'}(\bar{\omega}_t) - \mu G_{t-1}^{e'}(\bar{\omega}_t)\right]}\\ &\left(\kappa_{t-1}\left[\Gamma_{t-1}(\bar{\omega}_t) - \mu G_{t-1}(\bar{\omega}_t)\right]\frac{(1+R_t^k)}{(1+R_{t-1})} - 1\right)\Bigg] = 0.\end{aligned} \tag{A18}$$

The bank enforcement limitation holds:

$$\kappa_{t-1}\left[\Gamma_{t-1}(\bar{\omega}_t) - \mu G_{t-1}(\bar{\omega}_t)\right]\frac{(1+R_t^k)}{(1+R_{t-1})} = \frac{Q_{t-1}^k K_{t-1} - N_{t-1}}{(Q_{t-1}^k K_{t-1})}. \tag{A19}$$

The collateral constraint holds:

$$\bar{\omega}_t \kappa_{t-1}(1+R_t^k)Q_{t-1}^k K_{t-1} = (1+R_t^e)M_{t-1}.$$

The entrepreneurs net worth evolves as follows:

$$\begin{aligned} N_t =& \gamma_t^e\Big(Q_{t-1}^k K_{t-1}[(1+R_{t-1}^k) - (1+R_{t-1}) + \mu\kappa_{t-1}G_{t-1}(\bar{\omega}_t)(1+R_{t-1}^k)] \\ &+ (1+R_{t-1})N_{t-1})\Big) + w. \end{aligned} \tag{A20}$$

The entrepreneur solves for R_t^k, ω, M_{t-1}, N_{t-1}, takes as given $F_t(\omega)$, R_{t-1}, $\Gamma_{t-1}(\bar{\omega}_t)$, $G_{t-1}(\bar{\omega}_t)$, K_{t-1}, and Q_{t-1}^k.

The Taylor rule is defined as:

$$R_t - R = \rho_p(R_{t-1} - R) + (1-\rho_p)[a_\pi(E_t\pi_{t+1} - \pi_{trg,t}) + a_{\Delta y}(Y_t - Y)].$$

The resource constraint is given by:

$$Y_t = D_t + G_t + C_t + \frac{I_t}{\chi_t \mu_{\chi,t}} + a(u_t)\frac{K_t}{\chi_t \mu_{z,t}^*} + \theta\frac{1-\gamma^e}{\gamma^e}(N_{t+1} - w).$$

The dynamic competitive equilibrium is defined by a set of quantities and prices $\{\ \pi_t, \lambda_{1,t}, r_t^k, I_t, u_t, \bar{\omega}_t, R_t^k, N_{t-1}, \lambda_{z,t}, C_t, \tilde{W}_t, L_t, K_t, R_t, P_t, W_t, F_{p,t}, F_{w,t}, Y_t, M_t, Q_t^k, \tilde{P}_t, s_t^*\ \}$.

Appendix H. Detrended Model

$$y_t = \gamma_t\left(\frac{u_t k_t}{\mu_z^* \chi}\right)^\alpha (l_t)^{1-\alpha} - \Phi \tag{A1}$$

$$r_t^k = \alpha\gamma_t\left(\frac{\chi\mu_{z,t}^* l_t}{u_t k_t}\right)\lambda_{1,t} \tag{A2}$$

$$\lambda_1 = \left(\frac{1}{1-\alpha}\right)^{(1-\alpha)}\left(\frac{1}{\alpha}\right)^\alpha \frac{(r_t^k)^\alpha w^{(1-\alpha)}}{\gamma_t} \tag{A3}$$

(A4) $$K_{p,t} = \lambda_{1,t}\nu_{t,p}y_t + \beta\zeta^p E_t\left[\left(\frac{\pi_{t+1}^{\sim}}{\pi_{t+1}}\right)^{\left(\frac{\nu_{t+1,p}}{1-\nu_{t+1,p}}\right)} K_{p,t+1}\right]$$

(A5) $$F_{p,t} = \nu_{t,p}y_t + \beta\zeta^p E_t\left[\left(\frac{\pi_{t+1}^{\sim}}{\pi_{t+1}}\right)^{\left(\frac{\nu_{t+1,p}}{1-\nu_{t+1,p}}\right)} F_{p,t+1}\right]$$

(A6) $$p_t = \left[(1-\zeta_p)\left(\frac{1-\zeta_p\left(\frac{\pi_{t+1}^{\sim}}{\pi_{t+1}}\right)^{\left(\frac{\nu_{t+1,p}}{1-\nu_{t+1,p}}\right)}}{1-\zeta_p}\right)^{\frac{1}{1+\nu_p}} + \zeta_p(\tilde{\pi}p_{t-1})^{\frac{1}{1+\nu_p}}\right]^{1+\nu_p}$$

(A7) $$K_{w,t} = (l_t)^{1+\sigma_L} + \beta\zeta^l E_t\left[\left(\frac{\tilde{\pi}_{w,t+1}(\mu_{z,t+1}^*)^{\iota_\mu}(\mu_z^*)^{1-\iota_\mu}}{\pi_{w,t+1}}\right)^{\frac{\nu_l}{1-\nu_l}(1+\sigma_L)} K_{w,t+1}\right]$$

(A8) $$F_{w,t} = \frac{l_t}{\nu_l}p_t\lambda_z + \beta\zeta^l E_t\left(\tilde{\pi}_{w,t+1}(\mu_{z,t+1}^*)^{\iota_\mu}(\mu_z^*)^{1-\iota_\mu}\right)^{\frac{1}{1-\nu_l}}\left(\frac{1}{\pi_{w,t+1}}\right)^{\frac{\nu_l}{1-\nu_l}}\frac{1}{\pi_{t+1}}F_{w,t+1}\Bigg]$$

(A9) $$w_t = \left[(1-\zeta_l)(\tilde{w}_t)^{\frac{1}{1+\nu_l}} + \zeta_l(\tilde{\pi}(\mu_{z,t}^*)^{\iota_w}(\mu_z^*)^{1-\iota_w}w_{t-1})^{\frac{1}{1+\nu_l}}\right]^{1+\nu_l}$$

(A10) $$k_t = \frac{(1-\delta)k_{t-1}}{\mu_{z,t}^*} + \left(1 - S\left(\zeta_{I,t}, \frac{i_t\mu_{z,t}^*\chi}{i_{t-1}}\right)\right)i_t$$

(A11) $$\begin{aligned}&\lambda_{z,t}q_t^k\left[1 - S\left(\frac{\zeta_{I,t}\mu_{z,t}^*\chi i_t}{i_{t-1}}\right) - \frac{\zeta_{I,t}i_t\mu_{z,t}^*\chi}{i_{t-1}}S'\left(\frac{\zeta_{I,t}i_t\mu_{z,t}^*\chi}{i_{t-1}}\right)\right]\\ &-\frac{\lambda_{z,t}}{\mu_{\chi,t}} + \frac{\beta\lambda_{z,t+1}q_{t+1}}{\mu_{z,t+1}^*\chi}\left(\frac{\zeta_{I,t+1}\mu_{z,t+1}^*\chi i_{t+1}}{i_t}\right)^2 S''\left(\frac{\zeta_{I,t+1}i_{t+1}}{i_t}\right) = 0\end{aligned}$$

(A12) $$m_{t-1} = \frac{q_{t-1}^k k_{t-1} - n_{t-1}}{\pi\mu_{z,t}^*}$$

(A13) $$a'(u_t) = r_t^k$$

(A14) $$(1+R_{t+1}^k) = \frac{(1-\tau^k)(u_{t+1}r_{t+1}^k - a(u_{t+1})) + (1-\delta)q_{t+1}^k}{\chi q_t^k}\pi_t + \tau^k\delta$$

$$0 = E_{t-1}\Bigg[\left(1-\kappa_{t-1}\Gamma_{t-1}(\bar{\omega}_t)\right)\frac{1+R^k_t}{1+R_{t-1}} + \frac{\kappa_{t-1}\Gamma^{e'}_{t-1}(\bar{\omega}_t)}{\kappa_{t-1}\Gamma^{e'}_{t-1}(\bar{\omega}_t)-\kappa_{t-1}\mu G^{e'}_{t-1}(\bar{\omega}_t)} \left(\kappa_{t-1}\left[\Gamma_{t-1}(\bar{\omega}_t)-(1-\mu)G_{t-1}(\bar{\omega}_t)\right]\frac{(1+R^k_t)}{(1+R_{t-1})}-1\right)\Bigg] \tag{A15}$$

$$\kappa_{t-1}\left[\Gamma_{t-1}(\bar{\omega}_t)-(1-\mu)G_{t-1}(\bar{\omega}_t)\right]\frac{(1+R^k_t)}{(1+R_{t-1})} = \frac{q^k_{t-1}k_{t-1}-n_{t-1}}{(q^k_{t-1}k_{t-1})} \tag{A16}$$

$$n_t = \frac{\gamma^e}{\pi\mu^*_z}\Big(q^k_{t-1}k_{t-1}[(1+R^k_{t-1})-(1+R_{t-1})+\mu G_{t-1}(\bar{\omega}_t)(1+R^k_{t-1})] + (1+R_{t-1})n_{t-1}\Big)+w \tag{A17}$$

$$\lambda_{z,t}(1+\tau_c)P_t - \frac{\zeta_{c,t}\mu^*_{z,t}}{\mu^*_{z,t}c_t - bc_{t-1}} + b\beta E_t\frac{\zeta_{c,t+1}\mu^*_{z,t+1}}{c_{t+1}-bc_t} = 0 \tag{A18}$$

$$\lambda_{z,t} + \beta E_t\frac{1}{\mu^*_{z,t+1}\pi_{t+1}}\lambda_{z,t+1}(1+R_{t+1}) = 0 \tag{A19}$$

$$y_t = d_t + g_t + c_t + \frac{i_t}{\mu_{\chi,t}} + \frac{a(u_t)k_t}{\chi\mu^*_{z,t}} + \frac{\Theta(1-\gamma^e)}{\gamma^e}(n_t - w) \tag{A20}$$

Appendix I. Steady State

In the steady state, $\pi = \tilde{\pi}$ and $\pi^* = \pi$. We can use the steady-state forms from equations (A6), (A4), and (A5) to derive:

$$\lambda_1 = \frac{1}{\nu_p}. \tag{A1}$$

Given (A11) and assuming $S = S'' = 0$, we obtain:

$$q = \frac{1}{\mu_\chi}. \tag{A2}$$

The steady-state form of R^k is:

$$R^k = ((1-\tau^k)r^k + 1 - \delta)\frac{\pi}{\chi} + \tau^k\delta - 1. \tag{A3}$$

From (A19), we find the steady-state form of R:

$$R = \frac{\pi \mu_z^*}{\beta} - 1. \tag{A4}$$

Using (A15) we can solve for $\bar{\omega}$.

Then, we use the detrended law of motion for net worth (A17) and the detrended enforcement limitation condition (A16) to solve numerically for k and n.

We then compute investment i:

$$we = \left[1 - \left(\frac{\frac{(1-\delta)}{\chi}}{\mu_z^*}\right)\right]k. \tag{A5}$$

Then, we can solve for w using:

$$\lambda_{1,t} = \left(\frac{1}{1-\alpha}\right)^{(1-\alpha)} \left(\frac{1}{\alpha}\right)^{\alpha} \frac{(r^k)^{\alpha} w^{(1-\alpha)}}{\gamma}. \tag{A6}$$

We solve for l using:

$$r^k = \alpha\gamma\left(\frac{\chi \mu_z^* l}{uk}\right)\lambda_{1,t}. \tag{A7}$$

We solve for y using:

$$y = \gamma\left(\frac{uk}{\mu_z^* \chi}\right)^{\alpha} (l)^{1-\alpha} - \Phi. \tag{A8}$$

We set a value for parameter g to obtain the government spending in the steady state,

$$G = g * y. \tag{A9}$$

We use the resource constraint to solve for consumption c:

$$y = d + G + c + \frac{i}{\mu_\chi} + \frac{a(u)k}{\chi \mu_z^*} + \frac{\Theta(1-\gamma^e)}{\gamma^e}(n - w). \tag{A10}$$

We solve for λ_z by using the first-order condition of consumption:

$$\lambda_z = \left(\frac{1}{\frac{(1+\tau_c)}{c}}\right)\left(\frac{\mu_z^* - b\beta}{\mu_z^* - b}\right) \tag{A11}$$

$$F_p = \frac{(y\lambda_z)}{(1 - \beta\zeta_p)} \tag{A12}$$

$$F_w = \frac{(\lambda_z l(1-\tau^l))}{((1-\beta\zeta^l)(\nu_l))} \tag{A13}$$

(A14) $$\psi^l = \frac{w\lambda_z}{L^{\sigma^l}}$$

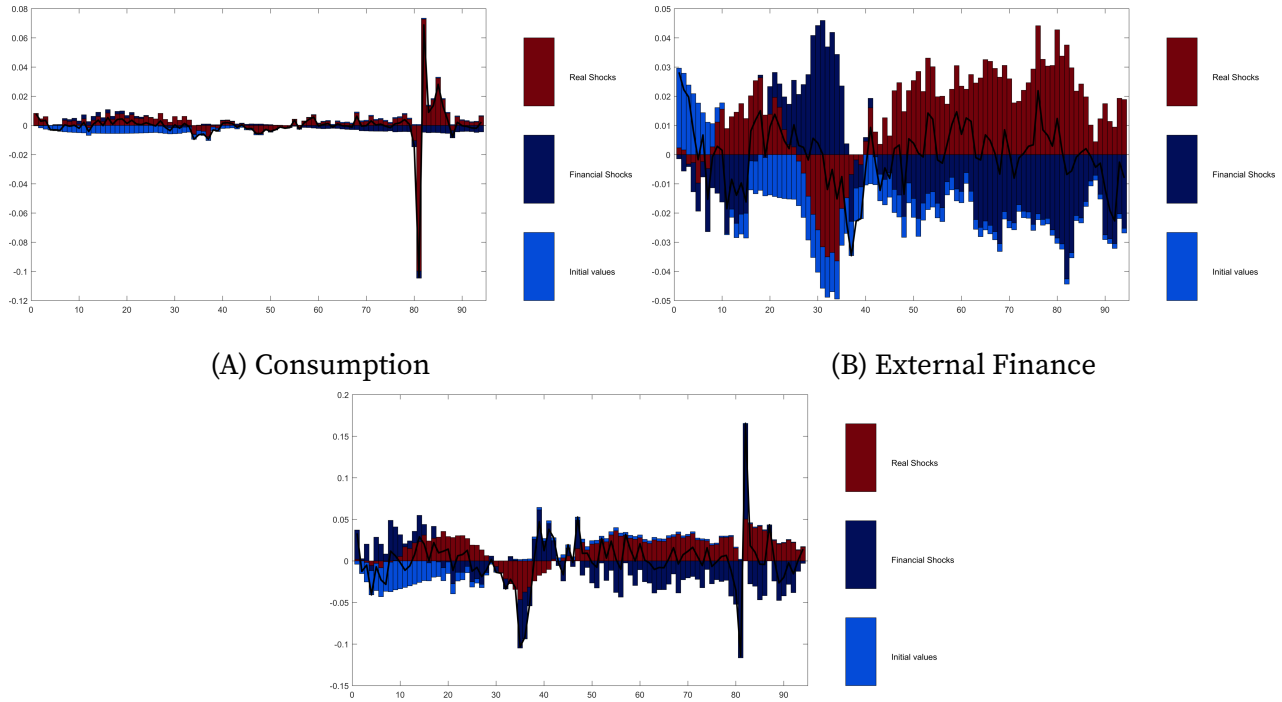
(A15) $$m = \frac{(qk - n)}{\pi\mu_z^*}.$$

We can also solve for D:

$$D = \kappa\mu G(\bar{\omega})(1 + R^k)\frac{QK}{\mu_z^*\pi}.$$

Appendix J. Shock Decomposition

Incorporating financial data into the model reduces considerably the role of productivity shocks in macroeconomic fluctuations. Financial shocks significantly shape macroeconomic fluctuations, especially during the economic downturn of 2008.



(A) Consumption

(B) External Finance

(C) Investment

FIGURE A9. Consumption, External Finance and Investment

Notes: Shock decomposition of consumption, external finance, and investment. The solid line represents the data and colored bar represents the contribution of shocks in the aggregate movements. Real shocks include price markup, government spending, technology trend, stationary technology, and consumption preference. Financial shocks include ISTC, entrepreneur risk, MEI, and collateral requirement. Parameters are set at posterior mean.

Figure A9 Panel A shows the relative contribution of both financial and real shocks to the volatility of consumption growth. This volatility is mainly explained by real shocks. Figure A9 Panel B illustrates the sensitivity of external financing to both financial and real shocks. Financial shocks appear to contribute largely to movements in external financing growth. A natural outcome of this analysis is that financial factors in macroeconomic models are non-negligible, at least under the present specification where ISTC is muted. In fact, financial shocks are important in explaining the behavior of macroeconomic variables and broadly predict macroeconomic fluctuations, especially around the period of financial crisis. Panel C of Figure A9 indicates that the effects of financial shocks on investment growth are relatively high

for the entire period. A key feature of the model is that financial shocks contribute substantially to the investment dynamics, especially around the global financial crisis of 2008 and the Covid-19 crisis.

Implications for Aggregates. A key assumption in this model allows entrepreneurs to pledge their assets as collateral, risking the loss of this collateral if they default. Under a collateral-constrained environment, entrepreneurs will use their assets to secure their debt. If they declare bankruptcy, then the bank can seize the pledged assets.

The overall conclusions that stand out from this analysis are the following. First, the main result is that the model assigns a minor role for the ISTC shock in the shock decomposition. This contradicts the existing literature, where the ISTC shock dominates. Suggesting that, when there is a shock that affects the transformation of investment goods into productive capital, such a shock will have a modest impact on the macroeconomy. We find that collateral disturbance accounts for 26% of the variance of net worth growth and 36% of the variance of capital price. This shock also accounts for 6% of the variance of output growth and 3% of the variance of consumption growth. However, even if the contribution of risk shock is less powerful in magnitude and accounts for only 1% of the variance of output growth, it explains a significant fraction of net worth growth (57%) and capital price growth (23%).The model of Christiano, Motto, and Rostagno (2014) suggests that the entrepreneur risk shock accounts for 60% of the fluctuations in the output growth and emerges as the most important shock.

Second, the magnitude of collateral shock is greater than the rest of shocks for external financing and capital price, suggesting that collateral shock plays an important role in the movement of financial aggregate. Accordingly, we find that the MEI shock has a significant effect on growth in output and investment. The contribution of ISTC shock appears to be small; nevertheless, financial shocks tend to dominate in the shock decomposition. This supports the finding of Jakab and Kumhof (2015) in which the contribution of collateral shock is higher in magnitude. Finally, financial shocks imply different degrees of impact on the macroeconomy. The effects of an entrepreneur risk shock and an ISTC shock are less marked than a MEI shock and a collateral requirement shock. These two shocks appear to be dominant in explaining the movement in output, investment, loan supply, and capital price.

Appendix K. Financial Shocks in the Model.

For comparison purposes, Figure A10 displays the response of aggregate output to different financial shocks. In principle, an increase in entrepreneur risk leads to a sharp decrease in output by more than −2% and adjust gradually to reach its steady state. Similarly, output declines in response to a positive shock to the MEI by more than −4%, confirming the important effect of this shock which is transmitted through the investment channel and represents a disturbance to the process that transforms investment into capital. Whereas, an investment technical change can depress output by close to −1.5%.

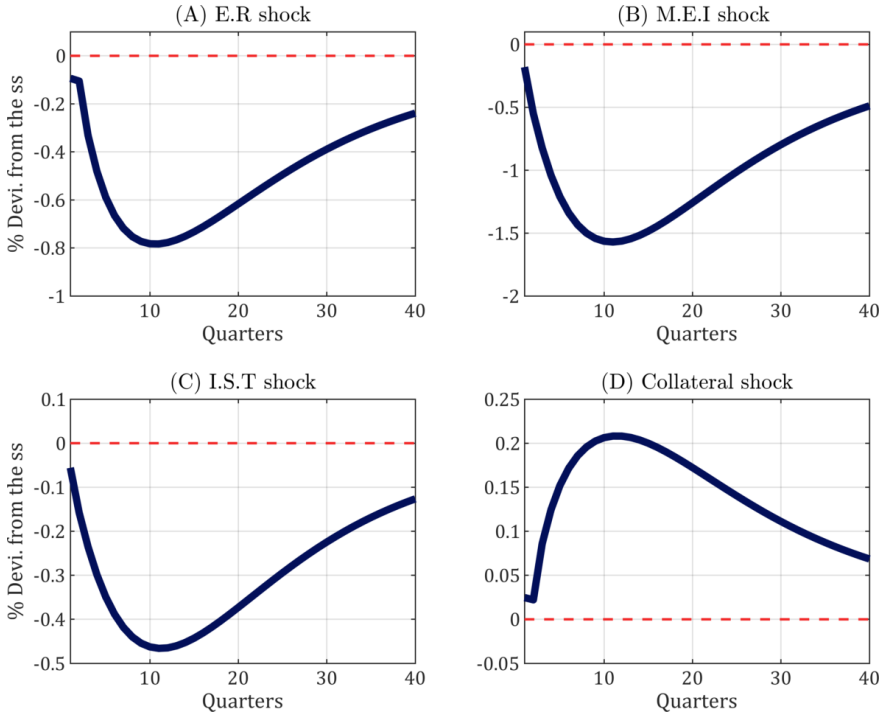


FIGURE A10. Real Activity Responses to Financial Shocks

Notes: Time horizon is quarter. The figure shows the impulse responses of output to a positive collateral requirement shock.

Figure A11 depicts the impulse responses of capital to different financial shocks. Panel A present the case when entrepreneur risk shock hits the economy, which could lead to less abundant capital with a decrease by −2.5%. The reason is that during uncertain times when default goes up, as liquidity erodes, these assets can become relatively scarce. As shown in Panel C, capital appears to respond negatively to ISTC shock, with an impact of −1.5%. Whereas, the MEI shock affects substantially the collateral capacity of entrepreneurs, implying a decrease by −5%, see Panel D. In contrast, Panel B show the response of capital to an increase in collateral requirements. This shock leads to an increase in capital stock; however, it will gradually recover and decrease to reach its steady state. The MEI shocks affect to a large degree

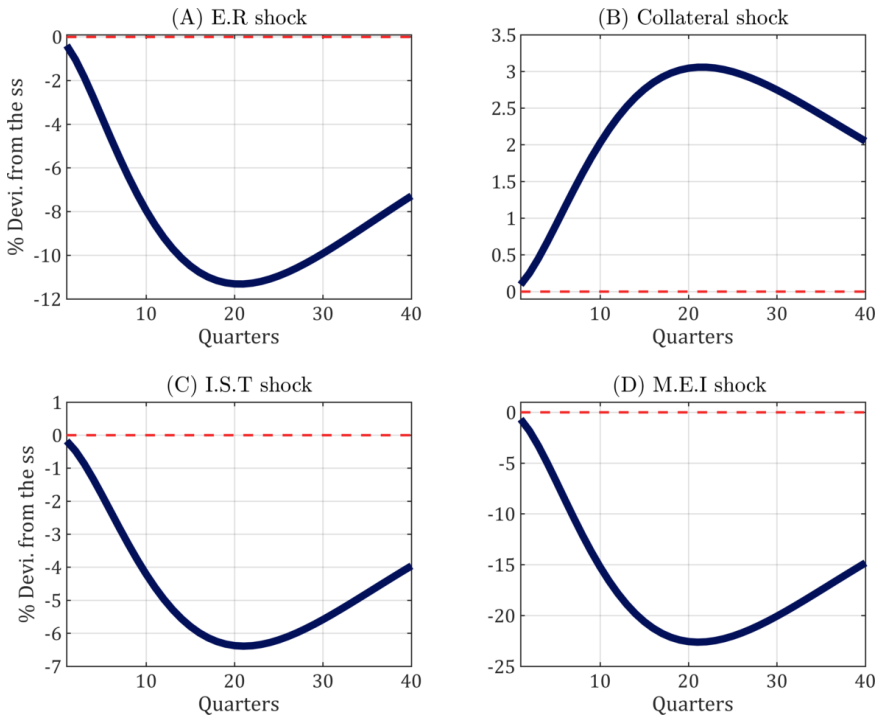


FIGURE A11. Impulse Responses of Capital to Financial Shocks

Notes: Time horizon is quarter. The figure shows the impulse responses of capital to the collateral requirement, entrepreneur risk, ISTC, and MEI shocks.

total capital and is relevant over the business cycle, as it dominates in terms of impact. In turn, shocks to the entrepreneur risk, ISTC, and MEI induce a scarcity of capital, while it becomes abundant when the loan-to value limit is relaxed.