The Effects of Reserves Accumulation on Sovereign Default Under Terms of Trade Shocks *

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Abstract

How important are terms of trade shocks for reserves accumulation? What are the benefits of holding international reserves for emerging economies when they face terms of trade shocks and their sovereign debt is denominated in foreign currency? To quantify these benefits, we develop a model of endogenous sovereign default with international reserves accumulation and exogenous terms of trade. Negative terms of trade shocks increase debt service costs while simultaneously raising the value of international reserves holding, leading to lower default risk. Calibrated to Mexico for the period 1997-2019, we find that half of the reserves predicted by the model are explained by the terms of trade channel. Moreover, with no international reserves' accumulation, default risk is higher, consumption falls more when the economy faced an adverse terms of trade shock, welfare is reduced, and the volatility of consumption raises regardless of whether the terms of trade are stochastic or deterministic.

Keywords: Sovereign Default, International Reserves, Terms of Trade
JEL Classification: E32, F31, F34, F41 , H63

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1 Introduction

Emerging economies are exposed to large and frequent terms of trade fluctuations[1] . First, they heavily depend on export revenues, often derived from a small number of primary commodities, and on imported inputs for production[2]. Second, due to a historical inability to issue public external debt in their own currency[3], they rely extensively on foreign-currency borrowing. When negative terms of trade shocks occur, export revenues decline, import costs rise, and the burden of repayment in foreign-currency debt increases. This last factor is critical, as it can give rise to sovereign risk. However, since the 1990s, emerging economies significantly increased their international reserve holdings[4][5]. Unlike debt, the value of international reserves can improve during adverse terms of trade shocks, providing a buffer that mitigates their risk exposure.

Motivated by stylized facts observed in emerging economies, our contribution to the literature is twofold: first, we document significant negative interaction effects between terms of trade (and international reserves) on sovereign default risk; second, we quantify the role of terms of trade shocks in explaining international reserves accumulation, and the benefits of holding reserves when debt is denominated in foreign currency. Using data from Mexico for the period 1997-2019, our model shows that higher terms of trade and reserves are linked to lower sovereign default risk, as observed in the data, that the terms of trade channel explains half of the international reserves predicted by the model, and that holding reserves reduces default risk, improves welfare, and decreases consumption volatility.

We begin by presenting three stylized facts regarding the relationship between sovereign risk, international reserves, debt, and terms of trade in emerging economies. First, we

[1]Terms of Trade are defined as the relative price of exports to imports

[2]For example, see Eaton and Kortum (2001).

[3]Despite the recent rise in the share of local currency debt documented by Du and Schreger (2022) and Ottonello and Perez (2019), emerging economies are still highly dependent on foreign currency borrowing. From a panel of 18 emerging economies between 2004 and 2020, about 1/3 of total public debt was external, of which 75% was denominated in foreign currency. See Arslanalp and Tsuda (2014).

[4]This fact is considered puzzling since emerging economies pay high spreads on debt issuance while receiving low returns on international reserves. Rodrik (2006) estimated the average cost of holding international reserves and issuing debt to be 1% of GDP. Favorable commodity prices, self-insurance against external shocks or export-led growth motives are often explanations of this rapid increased among emerging economies.

[5]Despite the rise of reserve accumulation being a worldwide phenomenon, it was more pronounced among emerging economies, as shown in Figure 5 in Appendix B. China considerably increased its reserves accumulation over the period, however, even if we exclude this country, emerging economies have increased steadily its reserves holding. See Figure 6 from Appendix B which shows the reserves accumulation for the countries in the sample described in the empirical evidence with and without China.

document that countries with lower public external debt and higher reserves exhibit lower sovereign default risk. Second, terms of trade shocks are positively correlated with output and inversely related to sovereign default risk. Finally, terms of trade tend to be more volatile than output. Additionally, a panel estimation for a sample of 16 emerging economies reveals significant negative interaction effects between terms of trade and reserves on sovereign default risk, consistent with the unconditional correlations.

To address these facts and quantify the benefits of reserve accumulation, we develop a dynamic stochastic model of endogenous sovereign default risk, building on the foundational work of Eaton and Gersovitz (1981) and subsequent quantitative models by Aguiar and Gopinath (2006) and Arellano (2008). Our model incorporates terms of trade shocks and allows for the accumulation of international reserves. In this framework, households consume both imported and non-tradable goods while exporting a fixed endowment. The benevolent sovereign issues long-term, non-contingent bonds to international creditors and accumulates a one-period risk-free asset (international reserves) both denominated in units of the imported good. The economy faces two sources of uncertainty: a stochastic endowment of the non-tradable good, and an external shock to the relative price of export and import goods, i.e. the terms of trade.

In the model, a negative terms of trade shock produces three key effects. First, it reduces the value of the exportable endowment while increasing the relative price of imported consumption goods. Second, it raises the burden of debt repayment, as the debt is denominated in units of imported goods, which become relatively more expensive. Finally, it increases the value of international reserves, which are also denominated in imported goods. Since these two financial assets are affected in opposite ways by adverse terms of trade shocks, holding international reserves serves as a buffer against the rising debt burden, particularly in the presence of rollover risk associated with long-maturity debt.

Using a model calibrated to Mexico, as is common in the sovereign default and reserves literature, we quantify the extent to which terms of trade fluctuations influence reserves accumulation. To do so, we compare the level of reserves in the benchmark economy with a version of the model where terms of trade are deterministic, examining the impact of removing this channel. Our findings show that when terms of trade are deterministic, reserves accumulation falls by half, and sovereign default risk increases by 15%.

From a normative perspective, we aim to quantify the value of holding international reserves across several dimensions. First, we assess the significance of reserve accumu-

lation by evaluating the model's short- and long-run implications in the absence of an optimal reserves strategy. In the short-run analysis, we introduce a negative terms of trade shock and compare the variables' responses with and without international reserves. In the benchmark calibration, a 3.6% decline in terms of trade (equivalent to the median drop observed in the data) results in a rise in the debt-to-GDP ratio of about 3.3 p.p. and an increase in sovereign spreads by 10 basis points compared to pre-shock levels, leading to reductions in both output and consumption. Due to lower export revenues, the reserves-to-GDP ratio declines by 1.5 p.p.. This combination of rising debt burden and falling reserves increases the risk of default. In the no-reserves model, the economy is even more vulnerable to the same shock, as sovereign spreads rise further relative to the benchmark, pushing consumption down more sharply.

By comparing the long-term outcomes of the benchmark model with an economy where the sovereign does not have access to international reserves, we find the economy becomes more vulnerable to terms of trade shocks. In the model without reserves accumulation, the probability of default increases by 10% relative to the benchmark calibration. This may seem counterintuitive in light of Bulow and Rogoff (1989), as their reputational model suggests that without additional punishments, no debt can be sustained in equilibrium when a small country holds foreign assets indexed to the same variables as the reputation contract. In that framework, reserves could be used during financial autarky, increasing the continuation value of default and thus raising the default probability.

However, in our model, both the value of default and repayment increase with international reserves [6]. The value of default rises due to the additional utility of holding reserves during financial autarky, while the value of repayment increases because reserves help smooth consumption in response to terms of trade fluctuations, particularly with long-term debt. The increase in default probability highlights that the benefits of holding reserves during repayment outweigh the benefits in a state of default[7].

Secondly, we evaluate the welfare benefits of holding international reserves by calculating the compensation households would need to be indifferent between having reserves and not. The results show that without reserves accumulation, welfare declines by 0.02% units of consumption. In addition to boosting welfare, holding reserves reduces consumption volatility. Indeed, we find that without reserves, consumption volatility increases, regardless of whether the terms of trade are stochastic or deterministic.

[6]The model includes a disutility cost of default.
[7]See Bianchi, Hatchondo, and Martinez (2018)

The rest of the paper continues as follows: Section 2 describes the state of the literature, in Section 3 we present the empirical evidence and motivation of the paper. Section 4 describes the model. Section 5 discusses the quantitative results of the model as well as a welfare analysis. In Section 6 we perform sensitivity analysis to test the predictions of the model. Finally, the paper concludes with Section 7 discussing the final remarks.

2 Related Literature

This paper builds on the foundational work of Eaton and Gersovitz (1981) in the sovereign default literature, extending the field through quantitative methods as developed by Aguiar and Gopinath (2006) and Arellano (2008), among others.[8] The recent rise in international reserves, particularly among emerging economies, has prompted the incorporation of reserves accumulation into quantitative models of sovereign default, providing insight into the joint determinants of reserves, debt, and default decisions. Alfaro and Kanczuk (2009) find that, in the context of one-period, non-contingent debt, these models struggle to explain the high levels of international reserves observed in the data, as reserves serve as a relatively poor hedge against income fluctuations.

Other studies offer alternative explanations for higher levels of reserves. For instance, Tavares (2023) examines how international reserves help lenders improve recovery rates post-default and act as a buffer during sudden stops. Meanwhile, Hernandez (2018) explores the role of reserves in crisis prevention. In a two-entity framework, Samano (2022) finds that a more patient monetary authority relative to the fiscal authority can sustain higher levels of reserves. Additionally, the growing literature on long-term debt issuance, as seen in Bianchi et al. (2018) and Bianchi and Sosa-Padilla (2023), highlights that the maturity mismatch between reserves and debt allows for the transfer of resources between repayment and default states, and within repayment states in which debt is more costly, creating a demand for reserves to hedge against rollover risk. Our model builds on this framework by incorporating long-term debt and rollover risk, focusing specifically on quantifying the benefits of holding reserves as a hedge against terms of trade shocks, particularly when rollover risk is present.

This paper is also related to the literature on precautionary motives for reserve accumulation in response to external shocks. Empirically, Aizenman and Lee (2007) provide evidence supporting the precautionary motive for holding international reserves

[8] For a comprehensive survey on sovereign default and quantitative solutions, see Aguiar and Amador (2014) and Aguiar, Chatterjee, Cole, and Stangebye (2016).

among developing economies as a form of self-insurance against sudden stops. Theoretical studies on precautionary reserve holding include Caballero and Panageas (2008) and Jeanne and Rancière (2011), which characterize sudden stop events as exogenous. Meanwhile, Calvo, Izquierdo, and Loo-Kung (2013), Hur and Kondo (2016), and Arce, Bengui, and Bianchi (2023) investigate how reserve accumulation impacts the likelihood of sudden stops and rollover crises. More recently, Jeanne and Sandri (2020) explores optimal reserve management in a financially closed economy, focusing on precautionary motives in response to export income shocks, although it does not address default directly. Our analysis contributes to this literature by measuring the benefits of holding reserves for precautionary reasons in the face of external shocks, specifically terms of trade fluctuations, and examining how this influences default incentives.

We also contribute to the literature on the effects of terms of trade fluctuations and commodity prices on emerging economies,[9] with a particular focus on their implications for sovereign default. Cuadra and Sapriza (2006) analyze how the level and volatility of terms of trade impact default incentives in emerging economies through a model of sovereign default with production. Popov and Wiczer (2014) examine the penalties associated with default in relation to terms of trade. In a two-country, two-goods setting, Gu (2021) explores the interaction between income, trade, terms of trade, and foreign-currency default risk, noting that terms of trade are endogenously determined by default risk. In contrast, our study treats terms of trade as exogenous to better understand how foreign factors influence default decisions in small open economies. The paper most closely related to ours is Hamann, Mendez-Vizcaino, Mendoza, and Restrepo-Echavarria (2023), which investigates the impact of oil price fluctuations on default risk among oil-exporting emerging economies with endogenous extraction. The key distinction is that while Hamann et al. (2023) treats reserves as a commodity good, we consider international reserves as a financial asset.

Lastly, this paper is closely related to the research on sovereign wealth funds (SWFs)[10][11]. SWFs are traditionally funded by commodity export revenues or foreign reserves and are designed to stabilize export revenues, save for future generations, or manage foreign reserves[12]. One branch of the SWF literature addresses concerns related to investment

[9] See Mendoza (1995), Mendoza (1997), Uribe (1997), Kose (2002), Ben Zeev, Pappa, and Vicondoa (2017), Fernández, Schmitt-Grohé, and Uribe (2017), Drechsel and Tenreyro (n.d.) for extensive discussions on how terms of trade and commodity price fluctuations drive business cycles in emerging economies.

[10] Defined by the International Forum of Sovereign Wealth Funds (IFSWF) as funds, typically owned and managed by governments (including central banks and various levels of government), that invest in diverse assets.

[11] For a comprehensive literature review, see Alhashel (2015).

[12] See Aizenman and Glick (2007), Beck and Fidora (2008), Butt, Shivdasani, Stendevad, and Wyman

strategies, transparency, and potential risks to the global economy, as discussed in Beck and Fidora (2008) and Butt et al. (2008). Another branch focuses on the reasons behind their establishment, including the relationship between SWF creation and the accumulation of foreign exchange reserves, particularly in emerging economies where excess reserves are linked to primary commodities, as explored by Aizenman (2008) and Beck and Fidora (2008). Our paper contributes to this literature by demonstrating how optimal reserve accumulation in our model is influenced by terms of trade shocks and export revenues, thus linking our findings to the concept of a quasi-SWF or a source of SWF funding.

To the best of our knowledge, this paper is the first to quantify the benefits of holding international reserves for mitigating terms of trade shocks, specifically in the context where sovereign debt is denominated in foreign currency.

3 Empirical Evidence

In this section, we describe key stylized facts about the dynamics of output, sovereign spreads, terms of trade, debt, and international reserves in emerging economies from 1997 to 2019. Our sample includes countries classified as emerging markets and developing economies by the IMF's World Economic Outlook and as low-income countries by the World Bank[13]. The 1997-2019 timeframe is chosen due to limited data availability on sovereign spreads for most countries before 1997. We will first outline the data[14] and then discuss our findings.

3.1 Data

We gathered data on output, sovereign spreads, terms of trade, debt, and international reserves at annual frequency for a sample of 16 emerging economies used in the literature[15]. Output data is represented by Gross Domestic Product in local currency at

(2008), and Urban (2011).

[13]Lower-middle income countries: Egypt, Ghana, Indonesia, Morocco, Nigeria, and the Philippines. Upper-middle income countries: Argentina, Brazil, China, Colombia, Dominican Republic, Ecuador, Mexico, Peru, Thailand, and Türkiye.

[14]For a detailed description of data sources and variables, see Appendix A.

[15]Sample of 16 Emerging Economies: Argentina, Brazil, China, Colombia, Dominican Republic, Ecuador, Egypt, Ghana, Indonesia, Mexico, Morocco, Nigeria, Peru, Philippines, Thailand, and Türkiye. The sample was selected based on data availability. For a detailed summary of the main statistics by country and the sample average, see Appendix B.

constant prices, sourced from the World Bank.

To measure sovereign risk, we use the EMBI+ (Emerging Market Bond Index Plus) constructed by JP Morgan, which is publicly available through the Global Financial Monitor from the World Bank. The EMBI+ serves as a benchmark index for assessing the total return performance of international government and corporate bonds issued by emerging market countries that meet specific liquidity and structural criteria[16]. While this proxy effectively captures sovereign risk, its use limits the number of countries and years included in the sample. However, it is advantageous because it is publicly available, unlike some other time series data.

In this paper, we define terms of trade as the relative value of a unit of exports (P^X) to a unit of imports (P^M), represented as $\frac{P^X}{P^M}$. Specifically, we use the net barter terms of trade, which is the percentage ratio of export unit value indexes to import unit value indexes, with the base year set to 2000, as provided by the World Bank.

Public external debt, expressed as a percentage of GDP, includes public and publicly guaranteed long-term external obligations. This category encompasses debts of the national government, public corporations, state-owned enterprises, development banks, other mixed enterprises, political subdivisions (or their agencies), autonomous public bodies, and private debt obligations guaranteed by public entities.

International reserves as a percentage of GDP are calculated by dividing the international reserves (excluding gold) by nominal GDP, both measured in current US dollars. These data series were obtained from the World Bank.

3.2 Stylized Facts

From the data, we gathered the following results regarding emerging economies:

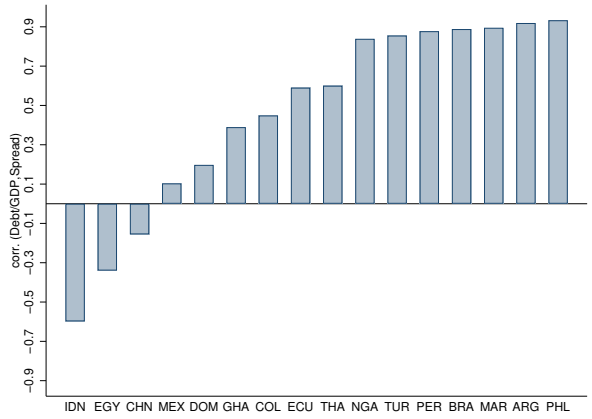
1. Lower public external debt and higher international reserves are correlated with lower sovereign default risk. Panel (a) of Figure 1 illustrates a positive unconditional correlation between public external debt and sovereign spreads for most emerging economies in our sample. Higher debt levels may elevate the perceived risk of repayment to foreign creditors. Conversely, Panel (b) shows a negative unconditional correlation between international reserves and sovereign spreads, observed in nearly all countries in the sample. Accumulating international reserves can serve as a buffer during periods of distress, mitigating the burden of debt repayment. However, reserves may also have

[16] Alternative proxies for sovereign default risk include credit default swaps, investment bank credit ratings, and other indicators such as the Institutional Investor Index.

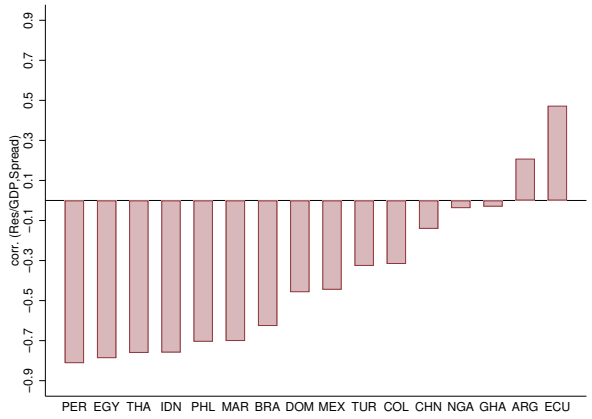
a counterintuitive effect by enhancing the value of financial autarky, potentially making default more attractive. This mechanism will be further examined quantitatively in our model. Both of these relationships are preserved when looking at the between-country correlation, as shown in Figures 10 and 11 in Appendix B.

Figure 1: Effect of External Debt and Reserves on sovereign spreads

(a) Debt-to-GDP ratio and Spreads



(b) Reserves-to-GDP ratio and Spreads

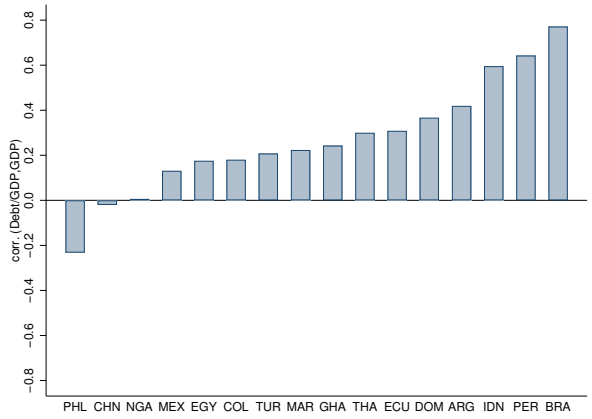


Note: The left panel shows the average relation between external debt as a percentage of GDP and sovereign spreads for the countries in the sample while the right panel depicts the average relation between international reserves as a percentage of GDP and sovereign spreads, both span over the period 1997-2019. Spreads correspond to EMBI+ series from JP Morgan publicly available at the World Bank. Debt-to-GDP ratio corresponds to the public external debt as a percentage of GDP from the World Bank. The reserves-to-GDP ratio stands for International Reserves excluding gold as a percentage of GDP from the World Bank.

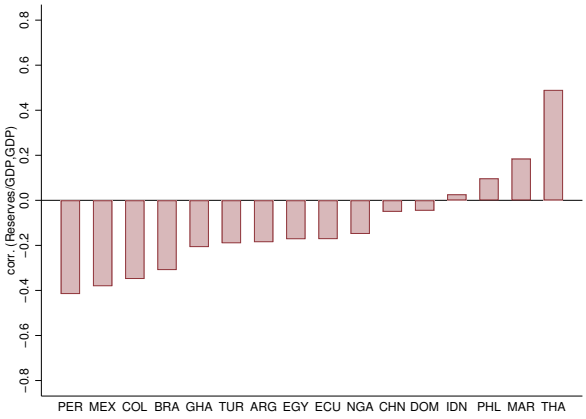
2. Higher terms of trade are associated with higher output and lower sovereign default risk. As highlighted by Mendoza (1995), large and frequent fluctuations in terms of trade are significant drivers of business cycles. The procyclical relationship between terms of trade and output, as highlighted in the left panel of Figure 2, reflects the heavy reliance on imported goods and commodity exports typical of many developing countries. Furthermore, the instability of their currencies often leads to external debt being denominated in foreign currency and adverse terms of trade can amplify the burden of this foreign-currency debt, potentially increasing the perceived risk of default. This is illustrated in the right panel of Figure 2, which shows a negative correlation between terms of trade and sovereign spreads for most countries in the sample.

Figure 2: Effect of External Debt and Reserves on sovereign spreads

(a) Terms of Trade and GDP



(b) Terms of Trade and Spreads

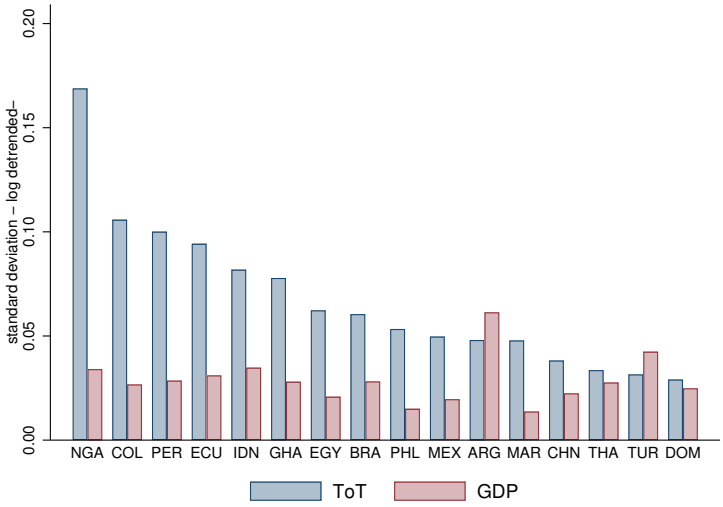


Note: The left panel shows the average unconditional correlation between terms of trade and GDP for the countries in the sample while the right panel depicts the average unconditional correlation between terms of trade and sovereign spreads for 1997-2019. The GDP corresponds to the Gross Domestic Product at constant prices from the World Bank. Terms of trade and GDP were lagged and detrended with the HP filter and smoothing parameter 100. Sovereign spreads correspond to EMBI+ series from JP Morgan publicly available at the World Bank. Terms of trade stand for the net barter terms of trade from the World Bank.

3. Terms of trade are more volatile than output. Figure 3 displays the volatility of terms of trade and GDP for each country in our sample, after logging and detrending both time series using the HP filter with a smoothing parameter of 100. Despite cross-country heterogeneity, all economies in the sample, except Argentina and Türkiye, exhibit greater volatility in their terms of trade compared to their output.

Despite the heterogeneity across countries, the higher volatility of terms of trade relative to the volatility of GDP seems to be distinctive to emerging economies. Indeed, if we replicate the analysis to a sample of advanced economies, we would find the opposite effect as depicted in Figure 9 in Appendix B.

Figure 3: Terms of Trade and Output volatility



Note: The graph depicts the volatility of GDP and terms of trade measured as the standard deviation from the cyclical component over 1997-2019 for the emerging economies in the sample. Both time series were logged and detrended using the HP filter with a smoothing parameter set to 100. GDP corresponds to the Gross Domestic Product at constant prices from the World Bank. Terms of trade stand for the net barter terms of trade from the World Bank.

Conditional correlations: panel regression: From the stylized facts we have gathered that the correlation between terms of trade and EMBI+ is negative for most of the countries in the sample, though this could be driven by confounding factors. Hence, to provide more robustness to the relationship between terms of trade and default risk we conduct a panel regression that includes other control variables often used in the literature on sovereign default. We proceed to estimate a panel fixed-effect regression as follows:

$$spr_{i,t} = \alpha + \beta\, log(tot)_{i,t-1} + \phi' X_{i,t-1} + \gamma_i + \delta_t + \epsilon_{i,t} \tag{1}$$
$$t = 1, ...T_i \quad i = 1, ..., N$$

where spr_{it} is the sovereign spread given by the EMBI+ in basic points, $log(tot)_{i,t-1}$ is the natural logarithm of the terms of trade and $X_{i,t-1}$ is a set of country controls which includes: GDP growth, the natural logarithm of the real effective exchange rate, the natural logarithm of public external debt-to-GDP ratio and the international reserves-to-GDP ratio. Finally, in the estimation, we include time-invariant country-fixed effects

(γ_i) and year-fixed effects ($\epsilon_{i,t}$). To overcome potential contemporaneous endogeneity, all explanatory variables are lagged one period, and as common practice in the literature, we exclude all the years involving default episodes following Catão and Mano (2017) [17]. Table 1 summarizes the estimation results of the empirical strategy for different model specifications.

Table 1: Effects of Terms of Trade on Sovereign Risk

(Dep. Variable: EMBI+ (b.p.))	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
$ln(tot)$	-0.056***	-0.055***	-0.053***	-0.027*	-0.039**	-0.048**
	(0.014)	(0.017)	(0.018)	(0.016)	(0.017)	(0.019)
$growth$		-0.178***	-0.176***	-0.160***	-0.117***	-0.045
		(0.043)	(0.040)	(0.041)	(0.029)	(0.028)
$ln(rer)$			-0.032*	-0.008	-0.015	-0.041***
			(0.017)	(0.018)	(0.012)	(0.010)
$ln(debt/gdp)$				0.016**	0.012	0.008
				(0.006)	(0.007)	(0.007)
$ln(res/gdp)$					-0.022***	-0.021***
					(0.005)	(0.007)
$constant$	0.316***	0.321***	0.481***	0.254*	0.286**	0.426***
	(0.066)	(0.079)	(0.150)	(0.151)	(0.135)	(0.126)
Obs	254	233	233	233	233	233
Countries	16	16	16	16	16	16
R^2	0.22	0.28	0.33	0.40	0.51	0.70
Country FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	No	No	No	No	No	Yes

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Note: All explanatory variables are lagged one period to control for contemporaneous endogeneity. Robust standard errors are reported in parenthesis at country-level clusters.

Column (1) reveals a significant negative correlation between terms of trade and default risk: a 10% improvement in the terms of trade leads to a 56 basis point reduction in the country's sovereign spread. Columns (2) to (5) introduce additional regressors commonly used in the literature, such as real GDP growth, the cyclical component of the real effective exchange rate, the debt-to-GDP ratio, and the international reserves-to-GDP ratio. Even after accounting for these controls and including year-fixed effects, Column (6) confirms the significant negative relationship between terms of trade and default risk.

The intuition behind this result is clear: an improvement in terms of trade reduces default risk by increasing export revenues and alleviating the debt burden. Additionally,

[17] See Appendix A for a detailed description each country's default years.

the coefficient for the real effective exchange rate is negative, statistically significant, and of similar magnitude to that of the terms of trade. If export prices equaled consumer goods prices and import prices matched the prices of goods consumed by the rest of the world, the terms of trade and real exchange rate would be equivalent. However, the difference in magnitude suggests a discrepancy in the composition of these goods, which still exerts a significant impact on sovereign spreads.

For international reserves, the correlation with sovereign spreads is both negative and significant, suggesting that higher reserves help reduce default risk by serving as a buffer against foreign liabilities. Specifically, a 10% increase in international reserves corresponds to a decrease in sovereign spreads of approximately 21 basis points.

Lastly, the coefficient estimates for terms of trade, international reserves, and public external debt maintain the same directional relationships with sovereign spreads as observed in the unconditional correlations presented in Facts 1 and 2. However, these results do not imply causality. In the next section, we will introduce a model consistent with the empirical findings, allowing us to explore the role of international reserves as a buffer against terms of trade fluctuations and their impact on default risk.

4 The Model

We consider a small open economy populated by a representative household, a benevolent sovereign, and foreign lenders. The household consumes a composite of an imported good and a non-tradable good (endowment), exports a fixed endowment, and receives a lump-sum transfer from the sovereign[18]. In addition to the stochastic process, which governs the non-tradable good endowment, the economy is subject to shocks to the relative price of exported to imported goods $\frac{p^x}{p^m}$, i.e terms of trade, which follows a Markov process. The sovereign engages in international borrowing by issuing long-period, non-contingent bonds and accumulates international reserves to provide resources to the household for consumption smoothing purposes. Debt contracts are non-enforceable, hence, the sovereign can choose to default on its debt at any point. In states of default, the sovereign is temporarily excluded from international credit markets for an exogenous length of time. Regardless of its credit status, the sovereign can accumulate international reserves.

[18]The implicit assumption behind this characterizations is that while both exportable and non-tradable goods are produced, there is no way to shift labor from one sector to another.

4.1 Household

The economy is populated by a representative household that maximizes her lifetime utility over consumption:

$$\max_{c_t} \ \mathbb{E}_0\left[\sum_{t=0}^{\infty} \beta^t u(c_t)\right] \tag{2}$$

where c_t denotes consumption at time t. The household discounts the future with the factor $\beta \in (0,1)$ while $\mathbb{E}_0$ stands for the expectation operator at time 0. The function $u(\cdot)$ represents the instantaneous utility of the household, which is increasing in consumption.

Let c_t be a composite of the consumption of an imported good c_t^m and a non-tradable good c_t^n with constant elasticity of substitution (CES):

$$c_t = [\omega(c_t^m)^{-\mu} + (1-\omega)(c_t^n)^{-\mu}]^{-\frac{1}{\mu}} \tag{3}$$

where $\omega > 0$ and $\mu > -1$.

In every period, the household receives an endowment of non-tradable goods which follows a first-order Markov process given by:

$$\ln(y_{t+1}^n) = \rho_y \ln(y_t^n) + \epsilon_{y,t+1} \tag{4}$$

with $\mid \rho_y \mid < 1$, $\epsilon_{y,t+1} \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma_y^2)$.

The exogenous process of y^n will capture domestic factors that affect the small open economy, independent of the terms of trade. In addition, in each period the household is endowed with an exportable good y_t^x, which we assume to be constant. Finally, the household receives a lump-sum transfer provided by the sovereign, all denominated in units of the imported good, considered the numeraire in this economy.

The household's budget constraint is given by:

$$c_t^m + p_t^n\, c_t^n = p_t^* y_t^x + \ p_t^n\, y_t^n + T_t \tag{5}$$

where p_t^n is the price of the non-tradable good relative to the imported good, p_t^* is the relative price of the exported endowment y_t^X in terms of the imported good ($\frac{p^x}{p^m}$, the terms of trade). In this paper, we assume the terms of trade are exogenous[19] and they

[19] We consider a small economy relative to international markets.

will follow a first-order Markov process:

$$\ln(p^*_{t+1}) = \rho_{p^*} \ln(p^*_t) + \varepsilon_{p^*,t+1} \tag{6}$$

with $\mid \rho_{p^*} \mid < 1$, $\varepsilon_{p^*,t+1} \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma^2_{p^*})$.

Finally, we assume that the household does not have access to credit markets and the only form of consumption smoothing is done operated external debt issuance and reserves accumulation carried out by the sovereign. Given the normalization, equation (5) is written in terms of the price of the imported good. The household maximizes (2) subject to (5) taking the lump sum transfer of the sovereign, the exportable good endowment, the non-tradable good endowment, and the terms of trade as given, following optimality condition:

$$\frac{c'_n}{c'_m} = p^n_t \tag{7}$$

which given the CES functional form, determines the following equation:

$$p^n_t = \frac{1-\omega}{\omega}\left(\frac{c^m_t}{c^n_t}\right)^{1+\mu} \tag{8}$$

Meanwhile, the trade balance condition can be expressed as:

$$p^*_t y^x_t - c^m_t = -T_t = TB_t \tag{9}$$

The left-hand side corresponds to the current account of the economy while the middle term stands for the negative of the capital account, the negative value of the lump-sum transfer given by the sovereign.

4.2 Sovereign

The sovereign is benevolent and its objective is to maximize the household's utility. The government issues long-period, non-contingent bonds to international creditors and accumulates international reserves to smooth household consumption. We will assume that debt and international reserves are traded in international markets and denominated in units of imported goods, which can be interpreted as debt and reserves denominated in foreign currency. This assumption maps to the fact that, although there has been a surge in local currency external debt issuance, most of the external debt in

emerging economies is denominated in foreign currency[20]. As a result, both assets will be subject to terms of trade shocks. Due to the incomplete nature of debt contracts, in every period the sovereign can decide to repay its debt or to default. If default is optimally chosen, the sovereign will be excluded from financial markets for an exogenous length of time. Independently of the credit status, the sovereign can accumulate international reserves.

The sovereign transfers a lump sum to households to smooth consumption, which results in the net position of its financial operations through bonds and reserves. Following Arellano and Ramanarayanan (2012) and Hatchondo and Martinez (2009), we will assume that a bond that is issued at time t promises an infinite stream of coupons that decay at an exogenous and constant rate $\delta \in (0,1]$. When $\delta = 1$, the maturity of the bond is one-period whereas if $\delta = 0$ it constitutes a consol. Hence, debt issuance can be defined as:

$$i_t = b_{t+1} - (1-\delta)b_t \tag{10}$$

where $b_t \in \mathbb{R}^+$ is the outstanding stock of debt due at time t and $b_{t+1} \in \mathbb{R}^+$ is the stock of debt at the beginning of $t+1$. Then, the sovereign's budget constraint in units of imported goods under repayment is determined by:

$$T_t = q_t(b_{t+1}, a_{t+1}, s_t)[b_{t+1} - (1-\delta)b_t] - \delta b_t + a_t - q_a\, a_{t+1} \tag{11}$$

where q_t stands for the price of bonds at time t which is a function of the stock of debt b_{t+1} and international reserves $a_{t+1} \in \mathbb{R}^+$ at period $t+1$ and the current state of the economy $s_t = \{y_t^n, p_t^*\}$. At a price q_t, the sovereign issues new debt equivalent to the amount $b_{t+1} - (1-\delta)b_t$, pays the decaying amount of outstanding debt δb_t, holds the reserves carried over from the previous period a_t and accumulates new reserves a_{t+1} at a price q_a. Bonds, reserves, the price of bonds, and the price of reserves are all denominated in units of the imported goods set to be the numeraire in this economy. If the sovereign chooses to default, its debt holdings become zero while it can still accumulate international reserves. The budget constraint under default can be expressed as:

$$T_t = a_t - q_a\, a_{t+1} \tag{12}$$

Sovereign's objective is to maximize the present discounted utility of the households

[20] See Arslanalp and Tsuda (2014)

described in equation (2), subject to (5) when (12) holds.

4.3 Foreign Lenders

Foreign lenders are deep-pocketed and provide debt and reserve assets to the sovereign. We assume lenders are risk-averse and in every period they price the bonds with their stochastic discount factor (SDF) $m_{t,t+1}$. To characterize the SDF many papers follow a reduced form approach such as Arellano (2008), Arellano and Ramanarayanan (2012) and Bianchi et al. (2018) in which they characterize a functional form of a kernel that depends on the stochastic income process of the small open economy. Although not critical for our mechanism, this feature allows us to improve the fit of the level of sovereign spreads observed in the data. We will follow an analogous approach by defining the SDF as a function of the domestic factors affecting the economy, in this paper given by the shock to the non-tradable good endowment:

$$m_{t,t+1} = exp\Big\{ -r^* - \kappa\epsilon_{y,t+1} - 0.5\kappa^2\sigma_y^2 \Big\} \tag{13}$$

In this case, $r^* \in [0,1]$ is the international interest rate and $\epsilon_{y,t+1}$ and σ_y^2 corresponds to the error and variance of the shock to the non-tradable endowment. The parameter $\kappa \geq 0$ corresponds to the market price of risk and it provides a measures of the size of the risk premium. Arellano (2008) uses a simpler functional form for the kernel which only depends on the shock of the income process and in which the analogous parameter to κ is a constant parameter. While Arellano and Ramanarayanan (2012) adds on the variance of the shock effect, the parameter κ is a time-variant function of the income process. In Bianchi et al. (2018) κ is defined as a two-stage Markov process. In this paper, we opt for a time-invariant constant since the feature improves the model fit without introducing an additional state variable.

The price of sovereign bonds is defined as:

$$q_t = \mathbb{E}_t \Big[m_{t,t+1}(1 - D_{t+1})[\delta + (1-\delta)q_{t+1}] \Big] \tag{14}$$

where $m_{t,t+1}$ is the SDF of the foreign lender, D_{t+1} is the default set -which will be defined in the next section, and q_{t+1} is the price of the bond in period $t+1$.

International reserves are considered to be a practically safe asset, hence investors price them with:

$$q_a = \mathbb{E}_{s'|s}\Big[exp\{-r^*\}\Big] \tag{15}$$

where r^* is the risk-free rate. Given the riskless nature of the asset, foreign lenders are risk-neutral toward international reserves and the SDF does not appear in their pricing function.

Timing: At the beginning of each period, the timing of actions is described as follows:

1. Shocks $s_t = \{y_t^n, p_t^*\}$ are realized, and along with the b_t and a_t, the state of the economy is determined
2. Given the optimality conditions of the household and the state of the economy, the sovereign chooses to default or not on the debt, $D_t = \{0, 1\}$, and transfer the net procedings to the household.
    a) If default is chosen ($D_t = 1$), the sovereign is excluded from international credit markets for an exogenous length of time, but still it chooses the optimal level of reserves a_{t+1} given the state of the economy.
    b) If repayment is chosen ($D_t = 0$), the sovereign decides optimal quantities of debt b_{t+1} and reserves a_{t+1} at prices $q(b_{t+1}, a_{t+1}, s_t)$.
3. Exports and Imports of goods take place.
4. The Household consumes the composite of non-tradable and imported goods.

4.4 Government Recursive Formulation

In this section, we will describe the recursive formulation of the sovereign's problem. In detail, the sovereign maximizes the utility of households and, to smooth consumption, borrows long-term, non-contingent bonds from international markets and accumulates international reserves. Given the non-contingent nature of the bonds, the sovereign can choose to default at any time. If we consolidate the budget constraints of the economy we obtain the following balance of payments equation under repayment:

$$c^m = p^*\ y^x + q(b', a', s)[b' - (1-\delta)b] - \delta b + a - q_a a' \tag{16}$$

and under default,

$$c^m = \ p^*\, y^x + a - q_a a' \tag{17}$$

and the non-tradable good market clearing condition:

$$c^n = y^n \tag{18}$$

Let $V(b, a, s)$ be the value function of the sovereign defined by:

$$V(b, a, s) = \max \left\{ V^R(b, a, s), V^D(a, s) \right\} \tag{19}$$

where $V^R(b, a, s)$ and $V^D(a, s)$ correspond to the value of repaying and defaulting, respectively, as a function of the level of debt, reserves, and the state of the economy $s = \{y^n, p^*\}$. The government's optimal default policy can be characterized by:

$$D(b, a, s) = \begin{cases} 1 & \text{if } V^R(b, a, s)) < V^D(a, s) \\ 0 & \text{otherwise} \end{cases}$$

which implies that it will choose to default whenever the value of default exceeds the value of repayment. Conversely, the sovereign will choose to repay the debt when the value of repayment exceeds the value of default. In detail, the value of repayment can be described as:

$$V^R(b, a, s) = \max_{b' \geq 0, a' \geq 0} \left\{ U(c(c^m, c^n)) + \beta \mathbb{E} \left[V(b', a', s') \mid s \right] \right\} \tag{20}$$

subject to (16) and (18). Under repayment, the sovereign chooses debt b' and reserves a' to maximize the household's utility.

Under default the problem is defined by:

$$V^D(a, s) = \max_{a' \geq 0} \left\{ U(c(c^m, c^n)) - \phi(y^n) + \beta \mathbb{E} \left[\theta V(0, a', s') + (1 - \theta) V^D(a', s') \mid s \right] \right\} \tag{21}$$

subject to (17) and (18) . If the sovereign chooses to default, it will suffer a disutility cost $\phi(y^n) \in \mathbb{R}$ which is a function of the non-tradable endowment and can be interpreted as a reputational cost on output. Next period, it can regain access to financial markets with an exogenous probability θ and re-enter with zero debt, otherwise it will remain in financial autarky.

Now, we can define the recursive equilibrium of this small open economy.

Definition 4.1. Recursive equilibrium. A Markov perfect equilibrium for this small open

economy for a state $s = \{y^n, p^*\}$ consists of:

- a set of the sovereign's optimal policies for its value function V, the value function of repayment V^R and default V^D;
- a set of rules for default D, bonds b' and international reserves a'^D, a'^R;
- a set of policies for household's consumption of imported goods (c_D^m, c_R^m) and non-tradable goods (c_D^n, c_R^n) in default and repayment;
- a bond price function $q(b', a', s)$;
  such that:
  a. Given the bond price $q(b', a', s)$, the optimal policies from the household $(c_D^m, c_R^m, c_D^n, c_R^n)$, the government's value function V, V^R and V^D and its policy functions D, b' and a'^D, a'^R solve (19) , (20) and (21),
  b. The equilibrium bond price $q(b', a', s)$ is determined by equation (14).

4.4.1 Model Properties

Proposition 4.1. The value of repayment is non-increasing in b, the value of default is independent of b and the default set grows in b. Given $s = \{y^n, p^*\}$ and a, for $b^0 \leq b^1$, $V^R(b^1, a, s) \leq V^R(b^0, a, s)$.

Proof. See Appendix E ■

This result has been proven by Eaton and Gersovitz (1981), Arellano (2008) and Chatterjee and Eyigungor (2012) and it is originated in the fact that since the value of default is independent of the level of debt b and the value of repayment is decreasing in b, it follows that the default threshold increases with b. Hence, if default is chosen for a certain level of debt b, for higher state of debt default would also be preferred.

Proposition 4.2. The value of repayment and default are non-decreasing in a.

Given $s = \{y^n, p^*\}$ and b, for $a^0 \leq a^1$, $V^R(b, a^0, s) \leq V^R(b, a^1, s)$ and $V^D(a^0, s) \leq V^D(a^1, s)$.

Proof. See Appendix E ■

This proposition arises from the fact that both the value of repayment and default are increasing in the level of reserves, which implies that while reserves can be useful under repayment to lessen the debt burden they also make autarky more attractive. In their

paper, Bianchi and Sosa-Padilla (2023) argue that despite both values being increasing in the level of reserves, the sovereign as a net debtor tends to run trade surpluses under repayment which implies a lower tradable consumption and a higher marginal utility, making the value of repayment more positively affected by reserves holdings [21].

Proposition 4.3. A lower realization of the terms of trade decreases household consumption (and welfare) if the Sovereign does not engage in asset purchases.

For any given a, b, and y^n, for all $p^{*0} < p^{*1}$ where $s^0 = \{y^n, p^{*0}\}$ and $s^1 = \{y^n, p^{*1}\}$, then $u(c^0) < u(c^1)$ where $c^0 = c(b, a, s^0) < c^1 = c(b, a, s^1)$ if $T = 0$.

Proof. See Appendix E ■

This is a similar result to the one discussed in Cuadra and Sapriza (2006). A negative shock to the terms of trade has two effects on the domestic economy. Firstly, it reduces the value of the fixed exportable endowment, thereby dampening the income of the household. Secondly, the shock changes the relative price of consumption goods, making imported goods relatively more expensive and reducing demand for them. Given the the functional form of consumption, both goods are normal and a reduction in income induces a fall in the consumption of the household and welfare.

Moreover, even if there is a financial intervention from the sovereign, the household's utility is increasing in the terms of trade, which implies that both the value of repayment and default are also increasing in the terms of trade. However, since the sign of the transfer depends on the optimal decision of bonds and reserves, *ex ante* we cannot determine whether one effect compensates the other.

Discussion of model assumptions:

Before solving the model, we wish to discuss the model's assumptions.

Firstly, we assumed all external debt issued by the sovereign was denominated in foreign currency. As described in the empirical section, emerging economies have struggled to issue debt in international markets in their own currency [22]. Although in recent years some emerging economies were successful in placing local currency debt in international markets, debt denomination is still predominantly tilted towards foreign currency (See Du and Schreger (2022) and Ottonello and Perez (2019), for example). In such cases, the effects of the terms of trade shocks on debt, international reserves, and default decisions become more relevant under the presence of foreign currency debt. For numerical

[21] In addition to this direct channel, in their model the introduction of nominal rigidities amplifies this effect.

[22] Eichengreen, Hausmann, and Panizza (2007)

reasons, excluding local currency debt in the model is less computationally demanding.

Secondly, we assumed the terms of trade are exogenous to the small open economy. It is reasonable to believe that since the economy is small relative to the international markets, they are not sizeable enough to set the price of their exported goods relative to their imports. Such an assumption has been made in previous work, such as Cuadra and Sapriza (2006) and Popov and Wiczer (2014). Conversely, in papers such as in Gu (2021), terms of trade are determined endogenously. In the context of this paper, determining the terms of trade exogenously for the small open economy arises from the interest in studying how external shocks, particularly those that affect the relative value of debt and reserves, shape and influence default incentives.

Next, to simplify the analysis, we defined the exportable good as a fixed endowment. An alternative and more comprehensive approach would be to endogenize the production decision of the exportable good to changes in the terms of trade. More competitive export prices might shift resources towards producing exportable goods to the detriment of non-tradable production.

In this paper, we only consider the need for imports as final consumption goods rather than inputs for production which could amplify the effects of terms of trade shocks on default decisions and potentially, the role of reserves accumulations. However, the consumption of imported goods already allows us to capture how these shocks affect household welfare and default risk. Considering the dependence on imported inputs could raise interesting considerations regarding how changes in the production structure of a small open economy could impact default risk. Nonetheless, this is beyond the scope of this paper.

Lastly, we wish to address the connection between the nature of our model and the notion of Sovereign Wealth Funds (SWFs). Since there is a correspondence between the price of commodities and the terms of trade, it could be argued that the mechanism underlying the reserves accumulation resembles a SWF determined by export revenues. In this paper, the importance of accumulating reserves is derived from the changes in the relative price of exports to imports (which affect the value of export revenues, imports, debt, and reserves) as opposed to the price of a single good or commodity. Additionally, the sovereign saves in international reserves which are a riskless type of asset. This investment strategy goes against the nature of SWF, since the aim is to maximize financial returns for the benefit of long-term public policies (Bernstein, Lerner, and Schoar (2013)).

5 Quantitative Analysis

In this section, we will discuss the calibration and functional forms used to quantitatively solve the model. We use Mexico to calibrate the model since it is an emerging economy frequently used as a benchmark in the literature of sovereign default and reserve accumulation. We will present the model fit by comparing the moments of the data with the model counterparts. Additionally, we will explore the effects of terms of trade shocks and study the role of reserve accumulation under such fluctuations. Firstly, Section 5.1 will briefly outline the solution algorithm; in Section 5.2 we will describe the calibration and functional forms used to numerically solve the model; in Section 5.3 we will show the simulation results under the benchmark calibration; Section 5.4 will showcase the long-run moments under no terms of trade variability; the Impulse Response analysis to terms of trade shock are presented in Section 5.5, followed by the long-run implications of not holding reserves in Section 5.6. The welfare implications of reserve accumulation are addressed in Section 5.7, including considerations regarding the effects on consumption volatility.

5.1 Algorithm

The problem is solved in its recursive form using value function iteration. Following Hatchondo and Martinez (2009), we take the limit of the finite horizon problem by approximating the value and bond price functions of the first period with a large enough number of periods such that the maximum difference between the functions in the first and second period is no larger than 10^{-3}. To solve for the optimal portfolio, for every state of the economy, we search over a grid of debt and reserves. The value functions of the sovereign under default and repayment (V^D and V^R, respectively) and the bond price (q) are approximated using linear interpolation over y^N and cubic spline interpolation over debt and reserves positions. We use 40 grid points for debt and reserves over the range [0,0.5], and 20 grid points for income and terms of trade realizations. Expectations are calculated using 50 quadrature points for the income and the terms of trade processes.

5.2 Functional Forms and Calibration

In this section, we will describe the functional forms and calibration used for the quantitative solution of the benchmark model set to match the features of the Mexican econ-

omy[23].

5.2.1 Functional Forms

I assume the per-period utility function of the household is of the form:

$$u(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma} \tag{22}$$

where $\sigma \in \mathbb{R}^{++} \setminus \{1\}$ is the coefficient of risk aversion and $u(c_t) = log\,(c_t)$ when $\sigma = 1$.

As mentioned before, consumption is defined as a composite of the consumption of imported and non-tradable goods, as follows:

$$c_t = [\omega(c_t^m)^{-\mu} + (1-\omega)(c_t^n)^{-\mu}]^{-\frac{1}{\mu}} \tag{23}$$

where $\omega > 0$ and $\mu > -1$.

Finally, the disutility cost of default follows the functional form used by (Bianchi & Sosa-Padilla, 2023):

$$\phi(y_t^n) = \phi_0 + \phi_1 ln(y_t^n) \tag{24}$$

in this paper, as a function of the endowment of the non-tradable production y_t^n with $\phi_0, \phi_1 \in \mathbb{R}^+$. As previously mentioned in the literature [24], having two parameters enables us to improve the fit of the spreads observed in the data. In addition, an interesting feature of this functional form is that it reduces the incentives of default in good times for positive values of ϕ_1. Indeed, if the punishment of defaulting proportional to output is negative enough, it would overcompensate the default during bad times and act as a subsidy. Finally, another attractive feature of this functional form is that it only depends on one exogenous state variable (y_t^n) and not on endogenous states (bonds or reserves). The departure in our specification is that in this paper the functional form depends on the endowment of the non-tradable good y_t^n as opposed to the endowment of tradable goods as in Bianchi and Sosa-Padilla (2023). Nonetheless, both specifications try to capture the effect of domestic factors that influence the small open economy.

[23] Figure 12 in Appendix B shows the dynamics of the main variables in the model for Mexico for the period under study.

[24] See Chatterjee and Eyigungor (2012).

5.2.2 Calibration

As previously mentioned, we will calibrate the benchmark model to Mexico for the period 1997-2019. A period in the model is set to be a year. Table 2 describes the set of parameters. The upper panel shows parameters taken from the literature or the data whereas the parameter values in the bottom panel were chosen through the simulated method of moments.

Parameter Values: The relative risk aversion σ is set to 2, a standard value in the literature. The risk-free interest rate is set to $r^* = 0.04$, standard in quantitative business cycles and sovereign default models at annual frequency.

For the probability of re-entering financial markets after default, we set $\theta = 0.11$ corresponding to a 9-year financial exclusion as Mexico faced during the default episode between 1982 and 1990. The share of importable goods in consumption ω was set at 0.29 to match Mexico's tradable share in GDP. For the computation, we obtained data for the non-tradable sectors from national accounts data for Mexico for 1993-2021, considering under this category: electricity, water, and gas supply through pipelines to the final consumer; construction, and tertiary activities. The exportable endowment y^x was normalized to 1.

For the stochastic processes, we set the persistence of output ρ_y to 0.66 following Samano (2022), and the persistence of the terms of trade ρ_{p^*} to 0.8, according to the literature. The standard deviation σ_y and σ_{p^*} were chosen to match the output and the terms of trade volatility, respectively. Finally, the coupon decaying rate of debt was set to $\delta = 0.2845$, as used in Bianchi et al. (2018) for the case of Mexico.

Targeted Moments: The last six parameters, the discount factor β, the disutility cost parameters ϕ_0 and ϕ_1, the risk premium parameter κ and the volatility of the non-tradable endowment σ_y and the terms of trade σ_{p^*} were chosen through the simulated method of moments to replicate features of the Mexican economy: the public external debt-to-GDP ratio in foreign currency, the default probability, the average EMBI+ spreads, the volatility of EMBI+ spreads, the volatility of output and the volatility of the terms of trade, respectively.

To match the sovereign spreads to the data, we compute in the model the debt-to-maturity yield under no default, that is, the interest rate the lenders will receive if they keep the bond until it fully matures. The yield should satisfy:

$$q_t = \sum_{j=1}^{\infty} \delta(1-\delta)^{j-1} e^{-jr} \tag{25}$$

This expression implies that the sovereign spread is determined by[25]:

$$S_t = log\left(1 - \delta + \frac{\delta}{q_t}\right) - r^* \tag{26}$$

Table 2: Calibration Values 1997-2019

Description		Value	Source/Target
Parameters from Literature and Data			
Relative risk aversion	σ	2	Literature
Risk Free Interest Rate	r^*	0.04	Literature
Prob. of access to financial markets	θ	0.11	Mexico's 9 years in default (1982-1990)
Share of tradables	ω	0.29	Mexico's Tradable/GDP 1993-2021
Exportable endowment	y^X	1	Normalization
Persistence of non-tradable output	ρ_y	0.66	(Samano, 2022)
Persistence of terms of trade process	ρ_{p^*}	0.8	Literature
Debt decaying rate	δ	0.2845	(Bianchi et al., 2018)
Parameters from Matching Moments			
Discount factor	β	0.955	Mexico's Debt/GDP
Disutility Cost of Default	ϕ_0	0.0664	Default Probability 2%
Disutility Cost of Default	ϕ_1	5.1	Mexico's EMBI+ 277 b.p.
Lender SDF Parameter	κ	0.5	Mexico's EMBI+ Volatility
Volatility of non-tradable output	σ_{ε_y}	0.015	Volatility of GDP
Volatility of terms of trade shocks	$\sigma_{\varepsilon_{p^*}}$	0.03	Volatility of ToT

To match the public debt, we used External Debt in the International Debt Statistics from the World Bank, which includes only public and publicly guaranteed debt that comprises long-term external obligations of public debtors [26]. Since the debt is denominated in local and foreign currency (while in our model debt is solely denominated in foreign currency), we obtained the share of foreign-currency public external debt from Arslanalp and Tsuda (2014). Over 2004-2020, 55% Mexico's public external debt was de-

[25]Where $S_t \equiv r_t - r^*$

[26]Including the national government, Public Corporations, State Owned Enterprises, Development Banks and Other Mixed Enterprises, political subdivisions, autonomous public bodies, and external obligations of private debtors that are guaranteed for repayment by a public entity.

nominated in foreign currency, leading to a stock of 16.4 % of GDP. To capture the mean and volatility of sovereign spread, we set to match Mexico's EMBI+ between 1997 to 2019 which averaged 277 b.p. and its standard deviation was 122 b.p. In the data, Mexico's output volatility was 2.0%, and the terms of trade volatility was 5.0%. Lastly, the probability of default is 2.0%, consistent with the two default episodes Mexico experienced over the past 100 years.

5.3 Simulation Moments

To test the model's fit, we compare moments in the data for Mexico with the model's counterparts over the period under study. Table 3 reports moments from the data and from the model simulations under the benchmark calibration as described in Section 5.2.2[27]. To do so, we simulated a time series of 300,000,000 observations and eliminated the first 200,000 observations to reduce the influence of initial conditions; we then computed the moments as the mean of the series[28].

[27] Table 13 in Appendix C depicts the data moments for the countries in the sample mentioned in Section 3.

[28] To calculate the long-run averages, default episodes were excluded.

Table 3: Simulation Statistics Mexico 1997-2019

	Data	Model
Targeted Moments		
$\mu(b/y)$ (%)	16.4	16.5
$\mu(spr)$ (%)	2.8	2.2
$\sigma(spr)$ (%)	1.2	1.9
$\sigma(y)$ (%)	2.0	2.0
$\sigma(tot)$ (%)	5.0	4.9
default prob (%)	2.0	2.0
Non-Targeted Moments		
$\mu(a/y)$ (%)	10.2	4.1
$\rho(c,y)$	0.88	0.69
$\rho(tot,spr)$	-0.38	-0.13
$\rho(tot,y)$	0.23	0.74
$\rho(spr,a/y)$	-0.45	-0.16
$\rho(b/y,spr)$	0.10	0.14
$\rho(tb/y,tot)$	0.43	0.55

Note: Simulate a time series of 300,000,000 observations and eliminate the first 200,000 observations to eliminate issues related to initial conditions. Moments are computed as the mean of the series under repayment. In the data, real output and consumption were logged and detrended with a HP filter with a smoothing parameter of 100.

The model successfully replicates most of the targeted moments from the data. As is common with sovereign default models, it struggles to simultaneously capture both the mean and volatility of sovereign spreads due to the rare occurrence of default events in theory. However, the deviations from the data are not substantial. Moreover, the model performs well in capturing several non-targeted moments. Notably, it explains approximately 40% of the international reserves observed during the period under study. While this rationalized level of reserves is lower than those documented by Bianchi et al. (2018), Bianchi and Sosa-Padilla (2023), and Samano (2022), these studies target a higher debt-to-GDP ratio, as they account for total external debt, whereas this paper focuses on external debt denominated in foreign currency.

In terms of the model's ability to capture the stylized facts outlined in Section 3.2, it accurately reflects Facts 1-3. Specifically, the model predicts both the direction and magnitude of the relationship between sovereign spreads and public external debt, as well as

the negative correlation with international reserves. The simulation results also show a negative correlation between terms of trade and spreads, and a positive correlation with output, consistent with the second stylized fact. Finally, the model accounts for the third fact by showing that the volatility of terms of trade exceeds that of output, in line with the method of matching moments.

5.4 Model under no terms of trade variability

In this section, we will study whether terms of trade matter to account for reserves accumulation by comparing the level of international reserves in the benchmark model to the level achieved in an economy with deterministic terms of trade. To do so, we set σ_{p^*} close to zero to make it numerically deterministic and re-calibrate the model to match the same targets. Table 4 shows the long-run moments for the benchmark calibration (Column (1)) and for the deterministic terms of trade (Column (2). The second column shows a model of deterministic terms of trade (No Δp^*)[29].

Table 4: Long-run moments with no terms of trade variability

	Benchmark	No Δp^*
Targeted Moments		
$\mu(b/y)$ (%)	16.5	16.4
$\mu(spr)$ (%)	2.2	2.4
$\sigma(spr)$ (%)	1.9	2.0
$\sigma(y)$ (%)	2.0	1.3
$\sigma(tot)$ (%)	4.9	0.0
default prob (%)	2.0	2.3
Non-Targeted Moments		
$\mu(a/y)$ (%)	4.1	2.2
$\sigma(a/y)$ (p.p.)	9.3	1.8

Note: Simulate a period of 300,000,000 observations and eliminate the first 200,000 observations to eliminate issues related to initial conditions. Moments are computed as the mean of the series under repayment. In the data, real output and consumption were logged and detrended with a HP filter with a smoothing parameter 100.

[29]The re-calibrated parameters are as follows: the parameters associated with the cost of defaulting are $\phi_0 = 0.0585$ and $\phi_1 = 4.972$, the lender's risk aversion is $\kappa = 0.5$, the discount factor is $\beta = 0.955$, $\sigma(\varepsilon_y) = 0.015$ and $\sigma(\varepsilon_{p^*}) = 0.00001$. All other parameter values are the same as in the benchmark calibration.

When terms of trade remain stable, the long-run reserves-to-GDP ratio falls to 2.2%, representing a 53% decline compared to the benchmark. Without terms of trade uncertainty, the model prompts lower reserves accumulation, as there is less need to buffer against shocks to export revenues, import costs, and foreign currency debt service. The incentive to default is increased, with the default probability raising 15% relative to the benchmark model. This result underscores the importance of holding international reserves as a safeguard even in the absence of terms of trade fluctuations. In Section 6 we further explore this by calculating long-run moments under one-period debt, as rollover risk associated with longer debt maturities amplifies the need for reserve accumulation.

5.5 Impulse Response Functions to terms of trade shocks

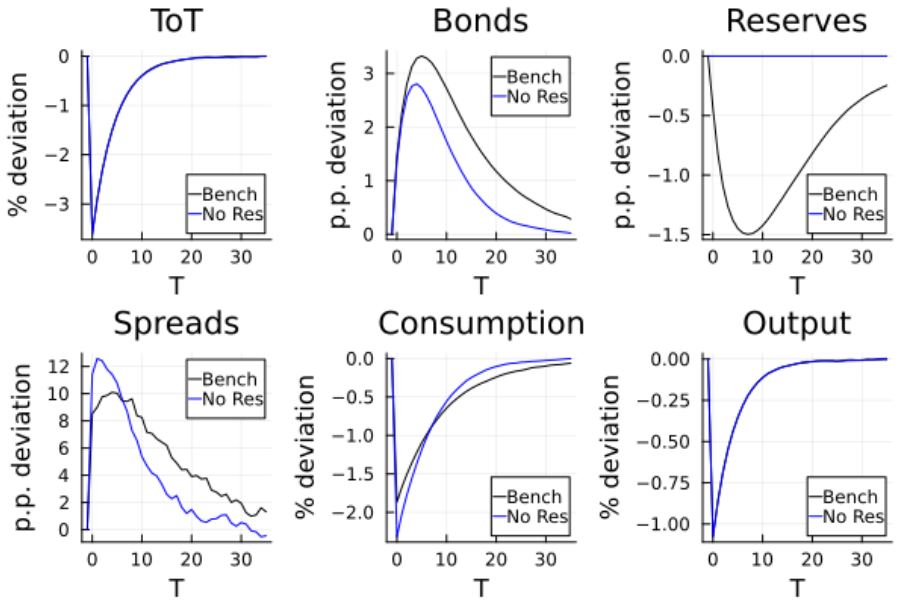
In this section, we study how the outcome variables respond in the short-un to shocks to the terms of trade and assess whether the responses differ if the sovereign does not accumulate reserves.

We construct impulse response functions (IRFs) following Koop, Pesaran, and Potter (1996) and in particular, the implementation to sovereign default models as in Arellano, Bai, and Mihalache (2020) and Centorrino, Li, and Mihalache (2024). We simulate 500,000 simulations of 200 periods. For the first 149 periods, the stochastic process follows the Markov Chain so that the cross-sectional distribution converges to the model's ergodic distribution. In period 150 of every simulation, we reduce the terms of trade realizations to replicate the median drop of terms of trade of 3.6% observed in the data for Mexico. From period 151 onward the terms of trade will continue with their underlying process. Then, we average each variable across all simulations[30] and plot the functions. We compare the IRFs after a terms of trade decline under: (i) the benchmark calibration and, (ii) a model with no reserve accumulation, described in the following section.

Figure 4 shows the responses of terms of trade, external debt-to-GDP, reserves-to-GDP, the percentage points change in the share of simulations in default, consumption, and output when the economy faces a decrease in the terms of trade of 3.6% for the benchmark model (the solid black line) and an economy in which the sovereign cannot accumulate international reserves subject to the same shock (the solid blue line).

[30] We average the simulations including the periods under default.

Figure 4: Impulse Response Function: *shock to* p^*



Notes: Impulse response functions (IRFs) for 500,000 simulations of 200 periods. Terms of trade, Consumption, and Output are shown as % deviations relative to the pre-shock values. Bond, Reserves, and Spreads in percentage points change relative to the pre-shock values. The economy faces a decrease in the terms of trade by 3.6% for the benchmark model and an economy in which the sovereign cannot accumulate international reserves.

Under the benchmark model, a drop in the terms of trade leads to an increase in debt-to-GDP of 3.3 p.p. and a decrease in reserves-to-GDP of 1.5 p.p. relative to the pre-shock values. A decline in the terms of trade makes debt repayment more expensive in terms of imported goods. Instead, this drop in the export income reduces the resources to asset saving, which contracts the reserves-to-GDP ratio. Meanwhile, the sovereign spreads increased due to the rise in the debt burden.

Both consumption and output fall due to the negative shock on exported income and the subsequent increase in the debt burden and decrease in asset saving. The responses under a model of no reserves accumulation are also shown in Figure 4 (solid blue line). As in the benchmark model, the simulated economies suffer from a drop in terms of trade of 3.6% relative to the median process. Under no reserve accumulation, the volume of debt-to-GDP increases to 2.8 p.p., a milder reaction than the benchmark model. Consumption and output respond similarly to the benchmark model, although the fall in consumption is 21% bigger under no reserves. When there is no reserves accumulation, sovereign spreads are much more reactive to terms of trade decline since the economy is more exposed to shocks when reserves cannot be used for hedging purposes.

5.6 Model under no reserves accumulation

To further quantify the value of holding international reserves, we focus on the long-run implications of not being able to accumulate reserves. We solved the model without the possibility of choosing the level of reserves (holding reserves numerically close to zero), re-calibrated the parameters to match the targeted moments, and re-computed the long-run moments as in the benchmark. Table 5 summarizes the results.

Table 5: Optimal Reserves vs. No Reserves Accumulation

	Data	Benchmark	No Reserves
$\mu(b/y)$ (%)	16.4	16.5	16.4
$\mu(spr)$ (%)	2.8	2.2	2.4
$\sigma(spr)$ (%)	1.2	1.9	2.1
$\sigma(y)(\%)$	2.0	2.0	2.0
$\sigma(tot)(\%)$	5.0	4.9	4.9
default prob (%)	2.0	2.0	2.2
$\mu(a/b)(\%)$	10.2	4.1	0.0
$\rho(tb/y, tot)$	0.43	0.55	0.41

Note: The benchmark model corresponds to the benchmark calibration described in section 5.2.2. The No Reserves model is solved by re-calibrating the parameters to match the targeted moments there is no option to accumulate international reserves.

The default probability under no reserves accumulation rises to 2.2%, translating into higher sovereign spreads. This result might seem counter-intuitive under the light of Bulow and Rogoff (1989) since having reserves increases the value of default, making autarky more attractive. In their reputational model, absent any direct punishments, no debt can be sustained in equilibrium while the small country holds foreign assets indexed to the same variables as the reputation contract (in our model, in addition to the market exclusion, the small open economy suffers a disutility cost of default). Holding reserves increases the value of default since they can be used during financial autarky, which could give rise to default risk. However, the value of repayment is also non-decreasing in international reserves. Reserves, whose value also fluctuates with the terms of trade, allow the sovereign to smooth consumption against terms of trade fluctuations, particularly in times in which low debt prices make it difficult to roll over debt. With this exercise, we confirm that the benefits of holding reserves in a state

of repayment dominate the ones in a state of default. In a context in which debt is denominated in foreign currency and the economy suffers from terms of trade shocks, having international reserves reduces the need to exercise the costly option of default to bear these shocks.

5.7 Welfare Analysis

Next, we conduct a welfare analysis to better understand the role of reserves under terms of trade shocks when debt is denominated in foreign currency. To do so, we compute the welfare losses from not accumulating international reserves as the equivalent variation in consumption relative to the baseline model, the benchmark. The alternative scenario corresponds to the model without reserve accumulation. The value function of the benchmark and the alternative model is described in equations (27):

$$\begin{aligned} V^B(\omega) &= \mathbb{E}\left\{ \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma} \left(((1+\lambda^*)c_t(\omega_A))^{1-\sigma} - 1 \right) \right\} \\ V^A(\omega_A) &= \mathbb{E}\left\{ \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma} \left[\left(c_t(\omega_A)^{1-\sigma} - 1 \right) \right]^{1-\sigma} \right\} \end{aligned} \tag{27}$$

$$\lambda^* = \left(\frac{(1-\sigma)(1-\beta)V^B(\omega)+1}{(1-\sigma)(1-\beta)V^A(\omega_A)+1} \right)^{\frac{1}{1-\sigma}} - 1 \tag{28}$$

where:

$$V^J(\omega) = \sum_{t=1}^{T} \frac{1}{N} \sum_{n=1}^{N} \beta^{t-1} u(c_t) \tag{29}$$

Table 6 shows the results of the welfare exercise that compares the alternative to the baseline model. For the alternative model, we preserved the parameters of the benchmark calibration to capture the transition effect of one regime versus the other. The exogenous state variables were determined stochastically by the underlining process and the endogenous state variables are set to zero.

The results indicate that in the absence of reserves accumulation the household suffers from a welfare loss of 0.02% (in units of consumption) due to the lack of resources to face the non-tradable good endowment and the terms of trade uncertainty.

Table 6: Welfare Analysis

Alternative Model	Baseline Model	(%)
No Reserves	Benchmark	0.02%

Note: The initial conditions for the exogenous state variables are determined by their underlining processes. The initial conditions for bonds and reserves were set to zero.

5.8 Consumption Volatility

The welfare analysis measures the consumption's long-run costs of not holding international reserves. Another way of capturing the value of having reserves to mitigate terms of trade shocks is to study how the volatility of consumption changes depending on whether or not the terms of trade fluctuate. We begin by calculating the standard deviation of consumption for (a) the benchmark, (b) the no reserves accumulation, (c) the no terms of trade variability, and (d) the no terms of trade variability with no reserves accumulation. Since models differ in their mean consumption, we calculate the coefficient of variability as the ratio of the standard deviation and mean of the consumption net of labor disutility to be able to compare how the volatility of consumption changes depending on whether terms of trade vary and whether the sovereign can accumulate reserves. For models (c) and (d) we re-calibrated the model parameters to match the targeted moments[31] and set σ_{p^*} close to zero to make it numerically deterministic.

Table 7: Volatility of Consumption

Models	$\sigma(C)/\mu(C)$	Change
(a) Bechmark	3.73	+0.32
(b) No Reserves	4.05	
(c) No Δp^*	0.68	+0.07
(d) No Δp^* + No Reserves	0.75	

Notes: For cases (c) and (d) we set σ_{p^*} close to zero to make it numerically deterministic and solved the model with the benchmark calibration. We consider the coefficient of variation as the ratio between the standard deviation and mean of consumption since models might differ in its mean.

Table 7 shows that when the sovereign does not accumulate international reserves,

[31] The matching moment parameters for model (c) are: $\beta = 0.955$, $\kappa = 0.5$, $\phi_0 = 0.0585$, and $\phi_1 = 4.972$. For model (d) the parameters are as follows: $\beta = 0.955$, $\kappa = 0.5$, $\phi_0 = 0.04$, and $\phi_1 = 3.190$.

the coefficient of variability increases by 0.32 units of consumption whereas the increase is much smaller when there are no terms of trade variability (+0.07). This result further highlights the value of holding reserves since, in addition to welfare improvements, consumption is less volatile when the economy faces terms of trade variability. In this case, international reserves are relatively more beneficial with terms of terms volatility.

6 Sensitivity

In this section, we conduct different sensitivity exercises to test the predictive power of the model. In each exercise, we modify the corresponding parameter while re-calibrating the rest to match the targeted moments. Table 8 summarizes the exercises' results.

Table 8: Sensitivity Analysis

	Benchmark	Lower ρ_{p^*}	No risk aversion	$\delta = 1$
Targeted Moments				
$\mu(b/y)$ (%)	16.5	18.4	17.4	17.9
$\mu(sp)$ (%)	2.2	2.3	2.4	1.8
$\sigma(sp)$ (%)	1.9	2.1	2.0	3.3
$\sigma(y)$ (%)	2.0	2.0	2.0	2.0
$\sigma(tot)$ (%)	4.9	4.9	4.9	5.0
default prob.(%)	2.0	2.1	2.3	1.6
Non-Targeted Moments				
$\mu(a/y)$ (%)	4.1	5.6	3.7	1.4
$\rho(tb/y, tot)$	0.43	0.55	0.54	0.49

Notes: Lower volatility of terms of trade calibration: $\beta = 0.955$, $\kappa = 0.5$, $\phi_0 = 0.0585$, and $\phi_1 = 4.972$. No risk aversion calibration: $\beta = 0.955$; $\kappa = 0.0$; $\phi_0 = 0.07$; $\phi_1 = 5.5$. 1-period bond calibration: $\beta = 0.951$; $\kappa = 0.6$, $\delta = 1.0$, $\phi_0 = 0.08$, $\phi_1 = 5.5$. The rest of the parameters are assumed to be the same as in the benchmark calibration.

Persistence of p^*: We show the model's responsiveness to changes in the persistence of the terms of trade process. In this exercise, we decreased the parameter in half to 0.4 and set $\beta = 0.955$, $\kappa = 0.5$, $\phi_0 = 0.0585$, and $\phi_1 = 4.972$. In the long-run, the model's solution predicts slightly higher levels of foreign debt and default risk. Since terms of trade shocks could be interpreted as an income shock (income on exports),

households tend to save a larger fraction of a temporary income shock and a smaller one when the shock is permanent. This helps explain why a less permanent shock on terms of trade is associated with higher international reserves. In terms of the non-targeted moments, it is noteworthy how the correlation of the trade balance as a percentage of GDP increases from 0.43 in the benchmark calibration to 0.55. This is in line with the Obstfeld-Svensson-Razin effect[32] which states that the effects of terms of trade depend on the persistence of its process and in particular that the Harberger-Laursen-Metzler effect[33] might be reduced or reverted when the persistence of the process is high.

No risk aversion: This exercise aims to assess the relevance of risk aversion to accounting for international reserves accumulation in the context of this model. To do this, we set $\kappa = 0$ so that $m_{t,t+1} = \exp\{-r^*\}$ in every period and $\beta = 0.955$, $\kappa = 0.0$, $\phi_0 = 0.07$, and $\phi_1 = 5.5$ to matched the levels of sovereign debt and spreads as in the benchmark calibration. Table 8 shows that when foreign lenders are risk neutral ($\kappa = 0$), the level of international reserves falls by 10% relative to the benchmark. This result indicates that, despite reducing the level of reserves, the lack of risk aversion is not the main driving force of reserves accumulation decisions. For instance, in Bianchi et al. (2018), the level of reserves halves when the lenders are risk-neutral. In their paper, the SDF of the foreign lenders is characterized as a two-state Markov process to capture the spikes in sovereign spreads' volatility, which are exogenous to the local conditions. When you remove this shock, the need for holding reserves in the presence of rollover risk reduces significantly. In this paper, the SDF is entirely dependent on the state of the local economy, hence, the need for international reserves absent the lender's risk aversion is even stronger than in Bianchi et al. (2018).

One-period bonds: To assess the importance of the rollover risk in the decision of reserves accumulation, we solve the model with one-period bonds ($\delta = 1$) and recalibrate the parameters corresponding to the method of matching moments[34]. Table 8 shows that, indeed, the rollover risk explains a large portion of the reserves accumulated in equilibrium as they reduce considerably to 1.4%. However, the fluctuations in terms of trade still create a non-negligible demand for holding reserves under one-period debt despite the difference in debt maturity. Although reserves provide insurance against rollover risk, they do not solely serve this purpose when the economy is subject to terms of trade fluctuations which affect the value of debt, imports, and export revenues. In

[32] See Obstfeld (1982) and Svensson and Razin (1983).

[33] A deterioration of the terms of trade reduces real income which causes a decrease in savings and deterioration of the current account and the trade balance. See Harberger (1950) and Laursen and Metzler (1950).

[34] The parameters are as follows: $\beta = 0.951$; $\kappa = 0.6$; $\phi_0 = 0.08$; $\phi_1 = 5.5$.

such a case, international reserves accumulation adds to the insurance properties of rollover risk.

7 Conclusion

We contribute to the literature by quantifying the benefits of holding international reserves in emerging economies, particularly when sovereign debt is denominated in foreign currency and the economy faces terms of trade shocks.

To this end, we develop a quantitative model of endogenous sovereign default with international reserves accumulation, where negative terms of trade shocks increase debt servicing costs while simultaneously raising the value of reserves. This opposing dynamic creates a hedging mechanism that can reduce default incentives. Using a model calibrated to Mexico from 1997 to 2019, we first explore the extent to which terms of trade fluctuations influence reserves accumulation, finding that without these fluctuations, reserves would be halved.

In subsequent exercises, we quantify the value of holding reserves across various dimensions. The results indicate that in the long run, the absence of reserves accumulation leads to higher default risk relative to the benchmark model. In the short run, negative terms of trade shocks lead to sharper increases in sovereign spreads and deeper drops in consumption when reserves are not accumulated. From a welfare perspective, the gains from holding reserves amount to approximately 0.02% in terms of consumption, while the absence of reserves increases consumption volatility, irrespective of the terms of trade volatility.

These quantitative exercises display the value of holding international reserves for emerging economies, especially when facing terms-of-trade shocks, which significantly shape their reserve accumulation decisions. Understanding how reserves influence these countries' responsiveness to shocks, default risks, and welfare is key for furthering the discussions on optimal reserve accumulation strategies.

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Uribe, M. (1997). Exchange-Rate-Based Inflation Stabilization: The Initial Real Effects of Credible Plans. *Journal of Monetary Economics*, *39*(2), 197-221. Retrieved from https://www.sciencedirect.com/science/article/pii/S0304393297000184 doi: https://doi.org/10.1016/S0304-3932(97)00018-4

A Data Sources

- **Sovereign Spread:**
  EMBI+ (Emerging Markets Bond Index) developed by JP Morgan and obtained from the Global Economic Monitor of World Bank. Annual frequency.

- **Gross Domestic Product:**
  Gross domestic product in constant local currency from World Bank. GDP is the sum of gross value added by all resident producers in the economy plus any product taxes and minus any subsidies not included in the value of the products. It is calculated without making deductions for the depreciation of fabricated assets or for the depletion and degradation of natural resources.

- **Real Exchange Rate:**
  Real Effective Exchange Rate (CPI-based) 2007 Index=100, annual data. from (Darvas, 2012). We made use of the annual dataset of 66 countries since it provided the longest time series. The dataset can be found: https://www.bruegel.org/2012/03/real-effective-exchange-rates-\for-178-countries-a-new-database/

- **Terms of Trade**: Net barter terms of trade index is calculated as the percentage ratio of the export unit value indexes to the import unit value indexes, measured relative to the base year 2000. Unit value indexes are based on data reported by countries that demonstrate consistency under UNCTAD quality controls, supplemented by UNCTAD's estimates using the previous year's trade values at the Standard International Trade Classification three-digit level as weights. Obtained from World Bank.

- **Government debt as a % GDP :**
  Public External debt stocks as a percentage of GDP in International Debt Statistics from World Bank. Public and publicly guaranteed debt comprises long-term external obligations of public debtors, including the national government, Public Corporations, State Owned Enterprises, Development Banks and Other Mixed Enterprises, political subdivisions (or an agency of either), autonomous public bodies, and external obligations of private debtors that are guaranteed for repayment by a public entity. Data are in current U.S. dollars.

- **Reserves as a % GDP:**
  Total Reserves in current US dollars (Indicator FI.RES.TOTL.CD) divided by Gross

Domestic Product in current uS$ (Indicator NY.GDP.MKTP.CD) in World Development Indicators from World Bank. Total reserves comprise holdings of monetary gold, special drawing rights, reserves of IMF members held by the IMF, and holdings of foreign exchange under the control of monetary authorities. The gold component of these reserves is valued at year-end (December 31) London prices. Data are in current U.S. dollars.

- **Default Episodes:**
  Sovereign External Default and Restructuring 1800-2012 from Carmen Reinhart's Database at the Behavioral Finance and Financial Stability of Harvard Business School. The episodes do not consider defaults on World War I debt to the United States and the United Kingdom but include post-1975 defaults on Official External Creditors. The default episodes considered for each country are the following: Argentina (1982-1993, 2001-2016), Brazil (1983-1994), Dominican Republic (1982-1994, 2005), Ecuador (1982-1995, 1999-2000, 2008-2009), Egypt (1984), Ghana (1987), Indonesia (1999-2000, 2002), Mexico (1982-1990), Morocco (1983, 1986-1990), Nigeria (1987-1994, 2004-2005), Philippines (1983-1992), Peru (1980, 1983-1997), Turkey (1982).

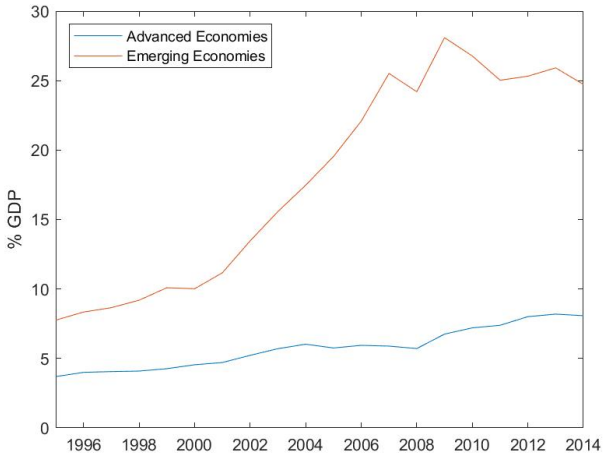
- **Consumption:**
  Final consumption expenditure (formerly total consumption) is the sum of household final consumption expenditure (formerly private consumption) and general government final consumption expenditure (formerly general government consumption). Data are in constant local currency from the World Bank.

- **Trade Balance % of GDP:**
  External balance on goods and services (% of GDP) External balance on goods and services (formerly resource balance) equals exports of goods and services minus imports of goods and services (previously non-factor services) from the World Bank.

- **Share of Mexico's Tradable Sector:**
  Time series obtained from national accounts data for Mexico for 1993-2021 from Banco de Mexico. For the non-tradable sectors, we considered under this category: electricity, water, and gas supply through pipelines to the final consumer; construction, and tertiary activities.

B Empirical Evidence

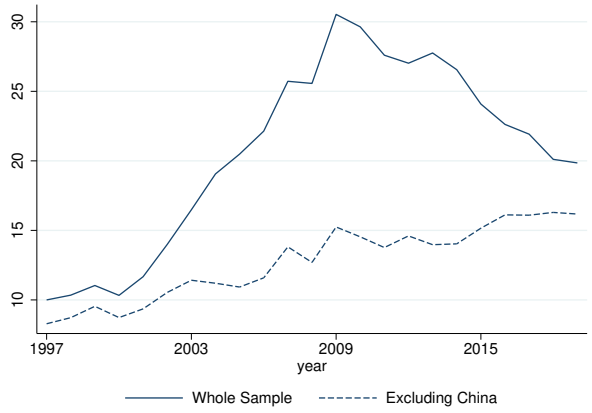
Figure 5: Aggregate International Reserves



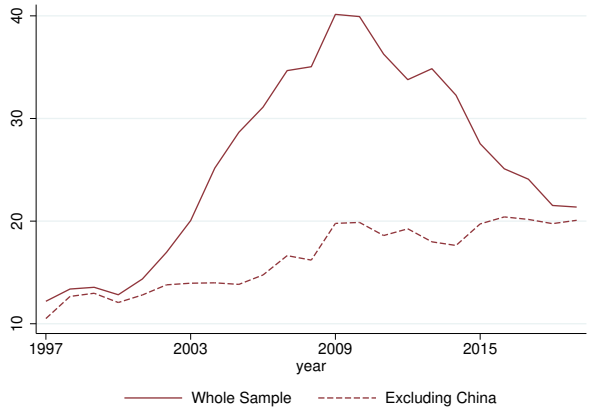
Source: Evolution of the weighted average of international reserves -excluding gold- as a percentage of GDP of advanced and emerging economies between 1995 to 2014, reported by the Currency Composition of Official Exchange Reserves (COFER) from IMF from 149 reporting countries consisting in the monetary authorities of members countries of the IMF, non-IMF members and other foreign reserves holding entities.

Figure 6: Aggregate International Reserves in Emerging Economies

(a) Relative GDP Weighted Average

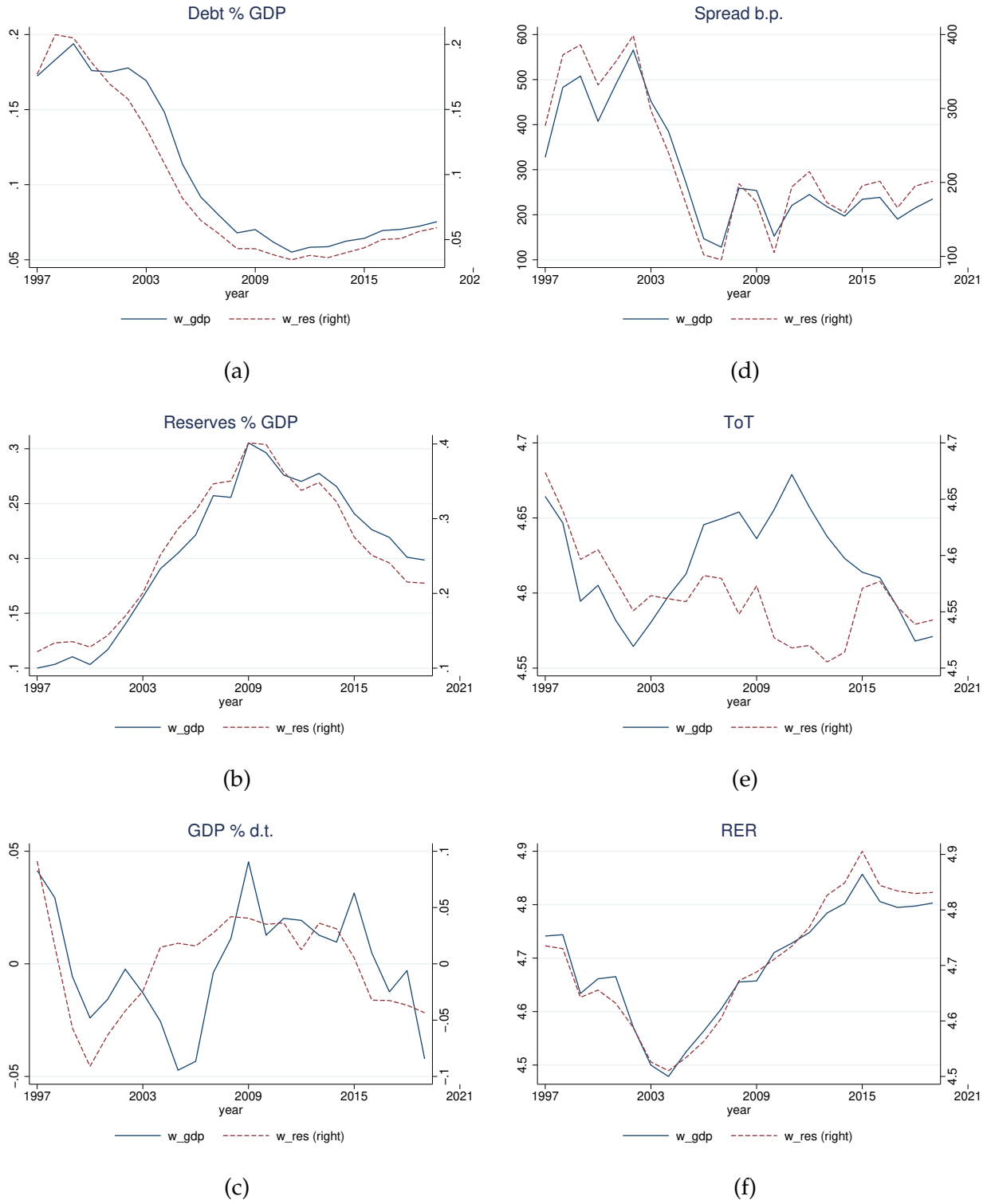


(b) Relative Reserves Weighted Average



Note: The two graphs depict the average international reserves accumulation for a sample of 16 emerging economies over the period 1997 to 2019. The left panel shows the average international reserves by using as weight the relative GDP for the whole sample (solid blue line) and the sample excluding China (dotted blue line). The right panel shows the average international reserves by using as weight the relative international reserves for the whole sample (solid red line) and the sample excluding China (dotted red line).

Figure 7: Dynamics of main variables for the countries in the sample - Weighted Average



Note: Blue solid lines correspond to the weighted average using relative GDP as weight. Red dotted lines used the relative international reserves as weights to compute the average for the sample.

Table 9: Summary Statistics for Emerging Economies

Country	$\sigma(ToT)$	$\sigma(gdp)$	$\sigma(spread)$	$\mu(spread)$	$\mu(debt/gdp)$	$\mu(res/gdp)$
Argentina	0.05	0.06	16.56	14.41	29.28	9.54
Brazil	0.06	0.03	3.27	4.64	9.94	11.77
China	0.04	0.02	0.49	1.33	3.94	29.99
Colombia	0.11	0.03	1.78	3.25	18.42	11.30
Dominican	0.03	0.02	2.41	4.79	20.31	5.64
Ecuador	0.09	0.03	6.49	10.96	35.02	3.69
Egypt	0.06	0.02	1.69	3.17	24.10	14.42
Ghana	0.08	0.03	1.77	5.76	45.83	10.36
Indonesia	0.08	0.03	1.19	2.29	26.15	13.22
Mexico	0.05	0.02	1.22	2.77	16.35	10.17
Morocco	0.05	0.01	1.77	3.34	29.97	21.00
Nigeria	0.17	0.03	4.84	6.88	14.58	11.21
Peru	0.10	0.03	1.81	3.05	21.77	24.02
Philippines	0.05	0.01	1.61	2.87	25.03	19.53
Thailand	0.03	0.03	1.33	1.70	9.98	34.47
Türkiye	0.03	0.04	1.82	3.97	15.70	10.04

Note: GDP at constant prices in local currency. Terms of trade corresponds to the unit value of exports with respect to the unit value of imports. Spread corresponds to the EMBI+ index (Emerging Markets Bond Index from JP Morgan). International reserves as a percentage of GDP. GDP and terms of trade are log and HP detrended with an HP filter and smoothing parameter 100. International reserves detrended with an HP filter and smoothing parameter 100. Sample of 16 Emerging Economies: Argentina, Brazil, China, Colombia, Dominican Republic, Ecuador, Egypt, Ghana, Indonesia, Mexico, Morocco, Nigeria, Peru, Philippines, Thailand, Turkey.

Table 10: Terms of Trade Correlations with Main Variables

	$\rho(tot, gdp)$	$\rho(tot, spread)$	$\rho(tot, \frac{res}{gdp})$
Argentina	0.42	-0.19	-0.17
Brazil	0.77	-0.31	-0.03
China	-0.02	-0.05	-0.15
Colombia	0.18	-0.35	-0.52
Dominican	0.37	-0.05	0.13
Ecuador	0.31	-0.17	0.02
Egypt	0.18	-0.17	0.25
Ghana	0.24	-0.21	0.17
Indonesia	0.59	0.03	-0.46
Mexico	0.13	-0.38	-0.07
Morocco	0.22	0.19	-0.33
Nigeria	0.01	-0.15	0.34
Peru	0.64	-0.42	0.21
Philippines	-0.23	0.09	-0.07
Thailand	0.29	0.49	0.07
Türkiye	0.21	-0.19	-0.12

Note: The international reserves as a percentage of GDP, the log of the GDP and the log of the terms of trade were detrended by using the HP filter with smoothing parameter 100 and are shown as a percentage deviation from the trend while spreads are shown as a percentage.

Table 11: Average foreign currency share of external debt

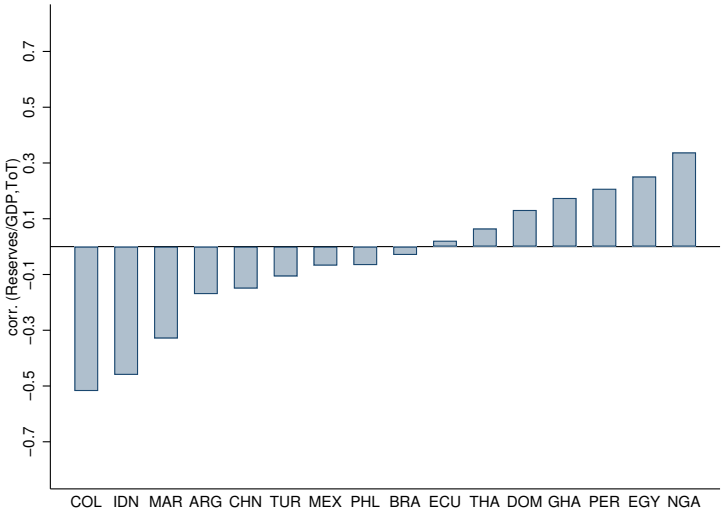
Country	FC Share	Period
Argentina	94%	2004-2020
Brazil	45%	2005-2020
China	79%	2004-2020
Colombia	81%	2009-2020
Egypt	89%	2004-2020
Indonesia	80%	2004-2020
Mexico	55%	2004-2020
Peru	79%	2004-2020
Philippines	91%	2004-2020
Thailand	32%	2004-2020
Türkiye	73%	2004-2020
Average	72%	2004-2020

Source: Authors' based on data by Arslanalp and Tsuda (2014)

Table 12: Country average for exports and imports 1997-2019

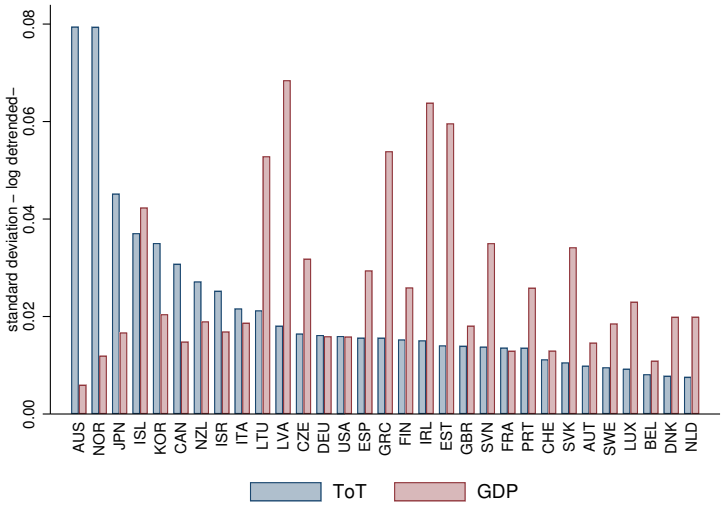
	$\mu(X/GDP)$	$\mu(M/GDP)$
Argentina	17.0	14.6
Brazil	13.3	12.6
China	24.6	21.2
Colombia	16.5	20.0
Dominican Rep	28.4	34.7
Ecuador	25.7	27.1
Egypt	20.1	26.6
Ghana	34.2	47.5
Indonesia	28.8	25.6
Mexico	29.3	30.8
Morocco	31.2	40.0
Nigeria	22.0	16.0
Peru	23.1	22.0
Philippines	34.3	37.8
Thailand	64.7	58.0
Türkiye	24.0	26.0

Figure 8: Correlation Res/GDP and ToT



Note: Reserves to GDP excluding gold and net barter terms of trade index for the period 1997-2019. The series of terms of trade was logged and detrended with the HP filter and smoothing parameter set to 100.

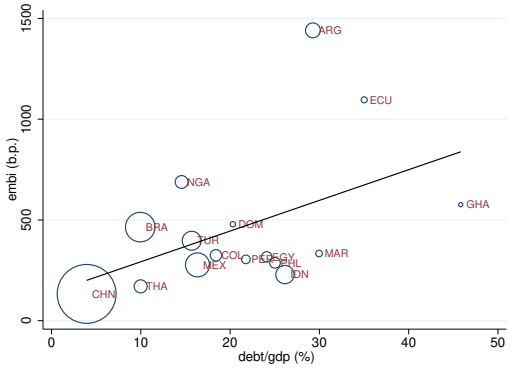
Figure 9: Terms of Trade and Output volatility for Advanced Economies



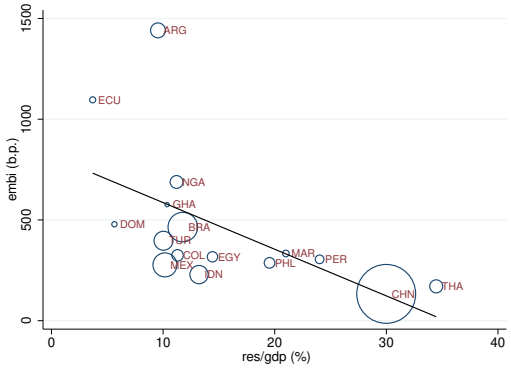
Note: Volatility of GDP and terms of trade as the standard deviation from the cyclical component (time series logged and detrended with HP filter and parameter 100) over 1997-2019 for a set of advanced economies. The countries in the sample are: Australia, Austria, Belgium, Canada, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Iceland, Ireland, Israel, Italy, Japan, Korea, Latvia, Lithuania, Luxembourg, Netherlands, New Zealand, Norway, Portugal, Slovak Republic, Slovenia, Spain, Sweden, Switzerland, United Kingdom and United States.

Figure 10: Between Country Effect of External Debt and Reserves on Sovereign Spreads

(a) Debt-to-GDP ratio and spreads

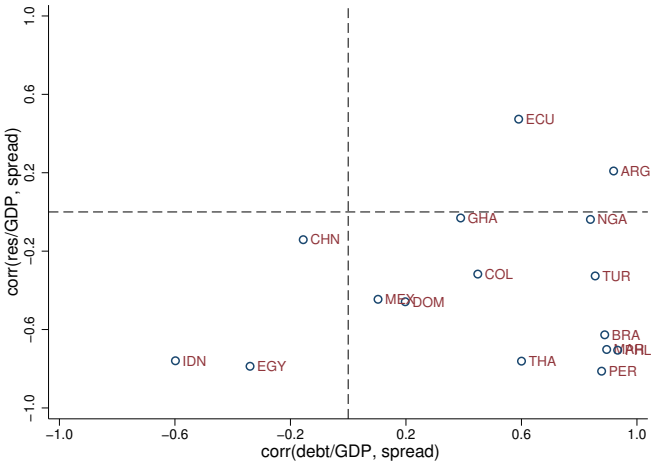


(b) Reserves-to-GDP ratio and spreads



Note: Data corresponds to public external debt-to-GDP ratio (in %), international reserves-to-GDP ratio excluding gold (in %) and EMBI+ (in bps). Both panels show the average for the period 1997-2019. The size of the circles corresponds to the country's relative GDP on the sample.

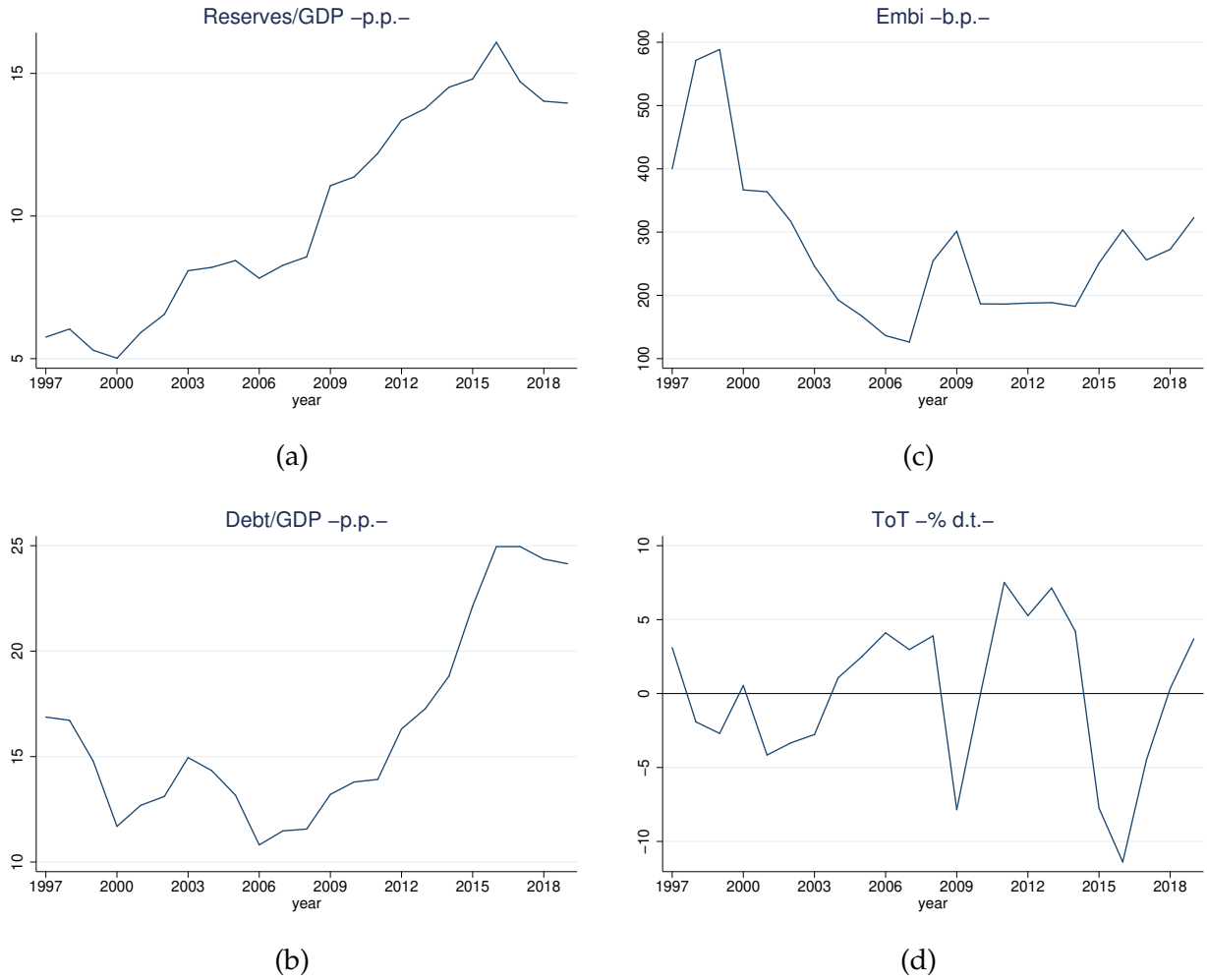
Figure 11: Between Country $Corr(Debt, EMBI+)$ and $Corr(Reserves, EMBI+)$



Note: Data corresponds to public external debt-to-GDP ratio (in %), international reserves-to-GDP ratio excluding gold (in %) and EMBI+ (in bps). Both panels show the average for the period 1997-2019.

C Quantitative Results

Figure 12: Evolution of Main Variables for Mexico in 1997-2019



(a) (c)

(b) (d)

Table 13: Sample Weighted Average Statistics

	Weight	
	Relative GDP	Relative Reserves
$\mu(b/y)$ (%)	10.7	9.0
$\mu(a/y)$ (%)	20.6	25.7
$\mu(spr)$ (p.p.)	297	227
$\sigma(tot)$ (%)	5.1	4.6
$\sigma(y)$ (%)	2.8	2.5
$\sigma(spr)$ (p.p.)	200	139
$\rho(tot, spr)$	-0.13	-0.07
$\rho(tot, y)$	0.20	0.12
$\rho(spr, a/y)$	-0.15	-0.20

D Sovereign Wealth Funds

As mentioned in Section 2, this paper is related to the literature on Sovereign Wealth Funds. The creation of these funds to stabilize export revenues, save for future generations, or manage foreign is associated with commodity prices, terms of trade, and the accumulation of international reserves as a source of financing. The following table summarizes the Sovereign Wealth Funds created by the countries in our sample, including the year of inception and its sources of financing.

Table 14: Sovereign Wealth Funds for Countries in Sample

Country	Fund	Inception	Origin
China	SAFE Investment Company	1997	Non-commodity
	National Social Security Fund (NSSF)	2000	Non-commodity
	China-Africa Development Fund (CADF)	2007	Non-commodity
	China Investment Corporation (CIC)	2007	Non-commodity
Colombia	Fondo de Ahorro y Estabilización Petrolera (FAEP)	1995	Oil Gas
	Fideicomiso Fondo de Ahorro y Estabilizacion (FAE)	2011	Oil Gas
Egypt	The Sovereign Fund of Egypt (SFE)	2018	Non-commodity
Ghana	Ghana Petroleum Funds (GPFs)	2011	Oil Gas
	Ghana Infrastructure Investment Fund (GIIF)	2015	Non-commodity
Indonesia	Indonesia Investment Authority	2021	Non-commodity
Mexico	Fondo Mexicano del Petróleo para la Estabilización y el Desarrollo (FMPED)	2013	Oil Gas
Nigeria	Nigeria Sovereign Investment Authority (NSIA)	2011	Oil Gas
	Bayelsa Development and Investment Corporation (BDIC)	2012	Non-commodity
Peru	Fiscal Stabilization Fund (FEF)	1999	Non-commodity
Phillipines	Maharlika Investment Fund	2023	Non-commodity
Türkiye	Turkey Wealth Fund	2016	Non-commodity

Note: Authors' elaboration based on Global SWF and the International Forum of Sovereign Wealth Funds (IFSWF).

Table 14 shows that 10 out of the 16 countries in our sample have created a Sovereign Wealth Fund at one point. We classified the origin of the funds to finance these SWF as non-commodity-based or oil and gas-based, to provide an idea of how relevant the price of commodities are for these economies regarding their SFWs. Only 5 out of the 16 SFWs created among the countries in our sample are commodity-based (oil & gas).

E Proofs

Proposition E.1. The value of repayment is non-increasing in b, the value of default is independent of b and the default set grows in b.

Given $s = \{y^n, p^*\}$ and a, for $b^0 \leq b^1$, $V^R(b^1, a, s) \leq V^R(b^0, a, s)$. The value of repayment is non-increasing in b, the value of default is independent of b and the default set grows in b.

Proof:

$$\begin{aligned} \frac{\partial V^R(b,a,s)}{\partial b} &= -\frac{\partial u}{\partial c}\frac{\partial c}{\partial c^m}\Big[\delta + (1-\delta)\, q\Big] < 0 \\ \frac{\partial V^D(a,s)}{\partial b} &= 0 \end{aligned} \tag{30}$$

Hence, if the default threshold is:

$$\phi(b,a,s) = V^R(b,a,s) - V^D(a,s) \tag{31}$$

then:

$$\begin{aligned} \frac{\partial \phi(b,a,s)}{\partial b} &= \frac{\partial V^R(b,a,s)}{\partial b} - \frac{\partial V^D(a,s)}{\partial b} \\ \frac{\partial \phi(b,a,s)}{\partial b} &= \frac{\partial V^R(b,a,s)}{\partial b} < 0 \end{aligned} \tag{32}$$

Proposition E.2. The value of repayment and default are non-decreasing in a.

Given $s = \{y^n, p^*\}$ and b, for $a^0 \leq a^1$, $V^R(b, a^0, s) \leq V^R(b, a^1, s)$ and $V^D(a^0, s) \leq V^D(a^1, s)$, i.e. the value of repayment and default are non-decreasing in reserves.

Proof:

$$\begin{aligned} \frac{\partial V^R(b,a,s)}{\partial a} &= \frac{\partial u}{\partial c}\frac{\partial c}{\partial c^{m,r}} > 0 \\ \frac{\partial V^D(a,s)}{\partial a} &= \frac{\partial u}{\partial c}\frac{\partial c}{\partial c^{m,d}} > 0 \end{aligned}$$

where:

$$\begin{aligned} c^{m,r} &= p^* y^x + q[b' - (1-\delta)b] - \delta b + a - q_a a' \\ c^{m,d} &= p^* y^x + a - q_a a' \end{aligned} \tag{33}$$

Proposition E.3. A lower realization of the terms of trade decreases household consumption (and welfare) if the Sovereign does not engage in asset purchases.

For any given a, b, and y^n, for all $p^{*0} < p^{*1}$ where $s^0 = \{y^n, p^{*0}\}$ and $s^1 = \{y^n, p^{*1}\}$, then $u(c^0) < u(c^1)$ where $c^0 = c(b, a, s^0) < c^1 = c(b, a, s^1)$ if $T = 0$.

Proof:

$$\frac{\partial u(c)}{\partial p^*} = \frac{\partial u}{\partial c}\frac{\partial c}{\partial c^m}\, y^x > 0$$