Working Paper Series Nº. 1-2024

Natural Rate of Interest, Monetary Policy and Shocks in Tunisia

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Central Bank of Tunisia
Sptember 2024

Abstarct

We provide an estimate of the natural rate of interest for Tunisia using the approach of Holston, Laubach and Williams (2017) based on a Neo-Wicksellian semi-structural model. The quarterly frequency estimates cover the period from 1989 to the end of 2023 and show a downward trend, especially from 2011 onwards. This decline is largely explained by the decline in the growth potential of the Tunisian economy. The estimation of the natural rate of interest, which served as a benchmark, and the calculation of its deviation from the observed real interest rate, allowed us to characterize the stance of monetary policy by subdividing the entire period into a phase designated as restrictive from 1989 to 2002 and another considered accommodative from 2003 to 2023. The results and the interest rate gap are consistent with the sign of the output-gap and the trend of inflation. The restrictive period was matched by a negative output-gap accompanied by a downward trend in inflation. The accommodative period was matched by a positive output-gap, reflecting the pressures of aggregate demand above potential for GDP, accompanied by an upward trend in inflation.

Keywords: Natural interest rate, potential growth, monetary policy stance, Kalman filter.

JEL classification : C32, E43, E52, O40.

The opinions expressed in this document are those of the authors and do not imply the Central Bank of Tunisia in any way.

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Natural Interest Rates and Monetary Policy in Tunisia

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February 2024

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Introduction

In the early 1990s, for both developed and emerging market central banks, the nominal interest rate became the preferred instrument used in the conduct of monetary policy. Known as **the key interest rate**, its variations now indicate the direction of this policy with the main objective of controlling inflation[1]. This instrument has gradually imposed itself on Central Banks to the detriment of the one advocated by Friedman's (1968) monetarist approach, which consists in controlling the evolution of the volume of the Monetary Base.

The setting of the nominal key interest rate is very often based on the "so-called" Taylor rule (1993) in which the concepts of: (1) the natural real interest rate, (2) the deviation of output from its potential and (3) the deviation of inflation from its target occupy an important place.

The major difficulty faced by all central bankers, who are directly inspired by Taylor's rule, lies in determining the unobservable variables, which in this case are the natural rate of interest and potential GDP.

More recently, especially after the 2008 Financial Crisis, the concept of natural interest rates has gained prominence in macroeconomic theory in relation with the continued decline in real interest rates. Nominal interest rates have approached their lower bound (equal to zero), which has drastically restricted the scope for monetary policy (Borio, 2021).

Despite the large amount of work devoted to estimating it, the natural rate of interest, which is unobservable and does not exist as such, has not been clearly defined. It is a theoretical construct, an abstract concept that has proven difficult to pin down, like the concepts of natural unemployment rate and potential output (Buncic et al, 2023). Thus, the concept of the natural rate of interest raises the question of both its definition and that of its measurement.

Despite the difficulties inherent in its definition and measurement, the concept of natural interest rates is very instructive, especially when one seeks to characterize the monetary policy of an economy in terms of whether it is accommodative or restrictive. It derives its importance from the fact that it provides an anchor for the conduct of monetary policy. It is based on the idea that there are real economic forces that drive the economy towards a long-term equilibrium with stable inflation and a zero output-gap. The role of monetary authorities should then be to steer monetary policy and market interest rates towards this equilibrium natural rate of interest (Constancio, 2016). It is in this perspective that we will try to measure the natural rate of interest for the Tunisian

[1] Central banks' mandates may have other juxtaposed objectives such as full employment and financial stability.

economy. The estimation of this rate will serve as a benchmark for monetary policy in Tunisia, thus allowing it to characterize its stance.

To this end, we will return to the plurality of definitions of the concept of natural interest rate and attempts to measure it before moving on to its estimation for Tunisia based on the approach that is a reference in this area, namely that of Laubach and Williams (LW, 2003, 2015), taken up and enriched by Holston, Laubach and Williams (HLW, 2016, 2017 and 2023).

The first section is an overview of the New Wicksilian literature[2] on the natural rate of interest and the methods used to estimate it empirically. The second section describes the HLW (2017) approach in detail. The third section is an application of the HLW model with Tunisian data and finally the fourth section exploits the results obtained to analyze potential growth in Tunisia and characterize the conduct of monetary policy based on the concept of natural and observed real interest rate differential.

I Definitions and Measures of the Natural Rate of Interest

The natural rate of interest has been defined in several ways. These various definitions have resulted in multiple empirical estimation methods. We first present the most common definitions and then the empirical estimation methods that have accompanied them.

I.1 The Natural Rate of Interest: An Abstract Concept That Is Difficult to Grasp

The natural rate of interest has its origins in distant works dating back to the economists Thornton (1760-1815) and especially those of Wicksell (1898, 1936). Wicksell defines the natural rate of interest as follows:

"There is a certain interest rate on loans that is neutral in relation to commodity prices and does not tend to increase or decrease them. It is necessarily the same rate of interest that would be determined by supply and demand if the money were not used and if all loans were made in the form of real capital goods. It is much the same to describe it as the present value of the natural rate of interest on capital. »

Largely based on this definition, several versions of the natural rate of interest have been proposed. The most commonly used are:

(a) **The natural rate of interest is a neutral interest rate.** It is often referred to in terms of output gaps and inflation. If it prevails in an economy, it would have no influence on inflation (inflation is equal to its desired level, $\pi = \pi^*$) and no influence on the output-gap,

[2] The neo-Keynesian macroeconomic literature that adopts Wicksell's definition of the natural rate of interest.

the observed real GDP would be equal to the potential GDP. This is clearly useful when thinking about monetary policy. If inflation is in line with the central bank's target and output is at its potential, the real interest rate that would achieve this dual objective would be defined as the natural rate of interest.

(b) **The natural rate of interest is in the real sphere, and its determinants are real factors**. It is a non-monetary theory of the interest rate. In the absence of monetary disruptions, it would be the one that equalized savings and investment. The natural rate of interest is pegged to the productive sector of the economy and is unrelated to inflation. It is determined by long-term structural factors such as demographics, technological progress and household preferences, i.e. factors that are expected to affect potential growth.

(c) **The natural rate of interest is a reference rate.** This definition, developed in the DSGE models of the New Keynesians, suggests that there is a natural short-run rate of interest, which can be perceived as a benchmark, a benchmark, for the non-equilibrium dynamics of an economy that is constantly impacted by stochastic shocks and operating with rigid prices and other types of rigidities. This is the prevailing rate in an economy without disruption and friction, with perfect price flexibility and instantaneous adjustment of markets (Woodford, 2003).

In the absence of a single, universally agreed definition of the natural rate of interest, empirical estimation attempts were timid before 2003 and more numerous since the publication of the work of Woodford (2003) and LW (2003).

After the 2008 global financial crisis, the persistence of ultra-low interest rates in most advanced economies increased interest in the question of whether the level of the natural interest rate has fallen permanently and why? thus raising the question of its measurement (cf., HLW (2017) and Borio (2021)). In the following section, we present the various empirical methods of estimating the natural rate of interest.

I.2 Measures of the natural rate of interest: Several definitions, several measures.

Empirical methods for estimating the natural rate of interest can be classified into three categories (cf. LW 2015 and Chervyakov-König 2017b): **(a)** univariate filtering methods, **(b)** dynamical and stochastic general equilibrium models (DSGE) and **(c)** multivariate semi-structural econometric models.

(a) Univariate filtering approaches are the simplest way to estimate the natural rate of interest. The main idea is that the observed real interest rate should converge to its long-run equilibrium trajectory – i.e. the natural interest rate (Weber et al, 2008). Methods such as the Hodrick-Prescott filter (Hodrick and Prescott, 1997) or the bandpass filter (Christiano and Fitzgerald, 1999) smooth out short-term variations while preserving the long-term trend component of the series. The same can be achieved by using Stock and Watson's (2007) Stochastic Volatility Model of Unobserved Components (UC-SV) where the variance of short-run shocks is assumed to follow a random course. Even though univariate filtering methods are relatively easy to implement, Weber et al (2008) and LW (2016) come to the same conclusion: the estimates can only be interpreted roughly, as they are highly dependent on the choice of the observed real interest rate and must be associated with different measures of expected inflation, leading to a multitude of point estimates. Univariate filtering methods are only useful for obtaining a first intuition about the data (Chervyakov and König, 2017b).

(b) The DSGE models of the New Keynesians, also called Neo-Wicksellian by Woodford, make it possible to calculate a short-term real interest rate that is referred to as a natural rate (cf. Woodford, 2003 and Gali, 2008). This rate is defined as the **marginal product of capital**, it is formulated as a pure real economy problem without any reference to monetary variables and inflation (Arena, 2020). It is expressed in terms of calibrated, observable and unobservable parameters, representing the shares of wages and capital income in output, the intertemporal discount rate of households, the marginal disutility of labour, and technological progress. It is a particular rate that would exist in an economy without disruption and friction, with perfect price flexibility and instantaneous adjustment of markets (Woodford, 2003). It plays a counter-factual role, vis-à-vis an economy, with rigidities and gradual adjustment of prices, where monetary shocks are introduced through the action of central banks on nominal interest rates according to the so-called Taylor rule.

(c) In contrast to the univariate approach, multivariate approaches are based on relatively simple semi-structural models to determine the natural rate of interest. They are based on structural relationships between several observable and unobservable variables, including the natural rate of interest. Unobservable variables are obtained using a statistical filter. These models can be enriched with a variety of additional ad-hoc specifications and assumptions. Laubach and Williams (LW 2003, 2015), pioneers in the field, establish links with the theoretical foundations of the natural rate of interest of the New Keynesians including a Phillips curve and an IS curve and, using the Kalman filter, arrive at a simultaneous estimation of the

natural real interest rate, potential output and trend growth. The Kalman filter allows an estimation of unobservable variables using actually observed series such as real GDP and inflation. Estimates of unobservable variables, also known as latent variables, are adjusted for the difference in expectations for inflation and real GDP (LW 2016, p. 7).

The approach of HLW (2016, 2017 and 2023), which has its origins in the work of LW (2003, 2015), has been widely used and has boosted work that has sought to measure the natural rate of interest, alongside of course alternative methods that also reflect the diversity of its definitions. Given its relevance and its status as a reference in this area, we have chosen it to measure the natural rate of interest in Tunisia.

In the following, we will discuss this approach in detail by presenting its theoretical foundations and empirical approach[3] before using it to estimate the natural rate of interest and potential GDP growth in Tunisia.

II. The Theoretical Foundations of HLW's Neo-Wicksellian Semi-Structural Model

HLW (2016) consider that their approach is based on two distinct schools of thought that converge on the definition of the natural rate of interest. These currents are (1) the Wicksellian current, which defines the natural rate of interest as the rate that is compatible with stable inflation and output at its potential level, and (2) the new Keynesian current, since Woodford 2003, which retains a similar definition but to which it adds a Phillips curve and an IS curve to describe the dynamics governing the output-gap as a function of the natural rate of interest.

In the following, we present the equations of the HLW (2017) model in its structural and reduced forms.

II.1 The basic equations of the HLW model in its structural form

The basic model of HLW (LW 2003 and HLW 2017) is a semi-structural framework that takes into account developments in inflation, output and interest rates. It includes an IS curve that relates the output-gap to interest rate differentials, a retrospective Phillips curve that relates inflation to the output-gap, and an equation that relates the natural rate to its determinants.

[3] The Federal Reserve Bank of New York dedicates a special page to Measuring the Natural Rate of Interest where it presents the results for the two models of LW (2003) and HLW (2017) and provides the series and R-codes used for the calculations as well as the basic articles.
https://www.newyorkfed.org/research/policy/rstar

The IS curve is of the form:

$$\widetilde{y_t} = E_t[\tilde{y}_{t+1}] - \rho^{-1}(i_t - E_t[\pi_{t+1}] - r_t^*) \qquad (1)$$

The Phillips curve that describes the dynamic relationship between inflation and the output-gap is of the form:

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa\widetilde{y_t} \qquad (2)$$

Where $\widetilde{\boldsymbol{y_t}}$ is the output-gap $(\tilde{y}_t = y_t - y_t*)$, which measures the percentage difference between the logarithm of real GDP y_t and the potential GDP y_t^* **,** $\boldsymbol{i_t}$ is the nominal interest rate, $\boldsymbol{\pi_t}$ the inflation rate measured by the increase in the consumer price index, and r_t^* is the natural rate of interest and the operator **E** expresses the anticipation of variables for future periods.

The parameters **ρ** and **κ**, determined from analytical developments in the framework of the DSGE models of the New Keynesians, depend on parameters relating to household preferences and technology, such as the intertemporal elasticity of substitution between domestic and foreign goods, distortions of the price adjustment mechanism, etc.

The IS curve and the Phillips curve include anticipated variables $E_t[\tilde{y}_{t+1}]$ and $E_t[\pi_{t+1}]$. Expectations are formulated by HLW as adaptive using the lagged values of the variables in question (see equations 5 and 6 in the reduced model below).

For **the natural rate of interest**, HLW (2017) start from an expression of the natural rate of interest **r*** similar to that obtained in the DSGE models with households that have a utility function with constant intertemporal elasticity of substitution for consumption. This equation is written as follows:

$$r_t* = \frac{1}{\sigma} g_c + \theta$$

where σ is the intertemporal elasticity of substitution of consumption, the growth rate of per capita consumption, and θ is the intertemporal preference rate. g_c

LW (2003, 2016) and HLW (2017, 2023) consider the medium-run natural interest rate to be a linear function of the potential GDP growth rate g_t and a z_t component that captures all determinants other than those related to potential GDP such as marginal propensity to save and population growth rate. They write the natural rate of interest in the following form:

$$r_t* = c\, g_t + z_t \qquad (3)$$

Finally, the model in its structural form consists of 3 equations and is written as follows:

Courbe IS : $$\widetilde{y_t} = E_t[\tilde{y}_{t+1}] - \rho^{-1}(i_t - E_t[\pi_{t+1}] - r_t^*) \quad (1)$$

Phillips curve: $$\pi_t = \beta E_t[\pi_{t+1}] + \kappa \widetilde{y_t} \quad (2)$$

Natural Interest Rate: $$r_t * = c\ g_t + z_t \quad (3)$$

II.2 Equations of the HLW model in its reduced form

For the purposes of estimating their model, HLW (2017) relaxes the restrictions imposed in the New Keynesian models at two levels:

(1) Using a reduced form of the model with little regard for the precision of the delays used for endogenous variables.

(2) By retaining short-term shocks that only affect the output-gap and inflation, unlike the natural rate of interest which does not suffer any shock because it is defined as a low-frequency concept and modelled as being influenced by factors that evolve relatively slowly.

The equations of the HLW (2017) model in its reduced form are as follows:

Output (PIB): $$y_t = \widetilde{y_t} + y_t * \quad (4)$$

Output-gap (IS): $$\widetilde{y_t} = a_{y,1}\tilde{y}_{t-1} + a_{y,2}\tilde{y}_{t-2} + \frac{\alpha_r}{2}\sum_{j=1}^{2}\left(r_{t-j} - r_{t-j}^*\right) + \epsilon_{\tilde{y},t} \quad (5)$$

Inflation (Phillips): $$\pi_t = \mathrm{b}_\pi \pi_{t-1} + (1 - \mathrm{b}_\pi)\pi_{t-2,4} + \mathrm{b}_y \tilde{y}_{t-1} + \epsilon_{\pi,t} \quad (6)$$

Natural interest rate: $$r_t * = c\ g_t + z_t \quad (7)$$

Potential GDP: $$y_t^* = y_{t-1}^* + g_{t-1} + \epsilon_{y^*,t} \quad (8)$$

Growth of y_t^*: $$g_t = g_{t-1} + \epsilon_{g,t} \quad (9)$$

Other determinants: $$z_t = z_{t-1} + \epsilon_{z,t} \quad (10)$$

Real Interest Rate: $$r_t = i_t - E_t[\pi_{t+1}] \quad (11)$$

Expected inflation: $$E_t[\pi_{t+1}] = (\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3})/4 \quad (12)$$

Interest Rate Spread: $$i_t - E_t[\pi_{t+1}] - r_t^* = r_t - r_t^* \quad (13)$$

Where is the natural logarithm of real GDP, is the trend or potential component of real GDP in log, is the output-gap (the difference between real GDP and potential GDP), is the short-term real interest rate, inflation as measured by changes in the consumer price index (CPI), is the moving average of the second, third and fourth CPI lags.$y_t y_t^* \widetilde{y_t} = 100\ (y_t - y_t *) r_t \pi_t \pi_{t-2,4}$

Stochastic terms capture transitory shocks that affect the output-gap and inflation, while permanent shocks arise from changes in that reflect persistent changes in the relationship between the short-term real interest rate and the output-gap, and follow a normal distribution with the standard deviations $\epsilon_{\tilde{y},t}, \epsilon_{\pi,t}, r_t^*, \epsilon_{y^*,t}, \epsilon_{g,t}, \epsilon_{z,t}, \sigma_{y*,t}, \sigma_{g,t}$ and $\sigma_{z,t}$, respectively.

The logarithm of potential GDP y_t^* is assumed to follow a random walk with drift g_t which, itself as well as z_t, follow a random process (equations 8, 9 and 10).

The natural rate of interest that appears in the second term on the right in equation (5) includes all shocks that affect inflation and the output-gap. The second term describes the difference between the real interest rate and the natural interest rate r_t^* (Equation 13)

The whole dynamic of stabilizing the output-gap and inflation in the model is done by this last equation. If the real interest rate is at its natural level, the output gap and inflation are stable from one period to the next.

The estimation of the HLW model (2017) is done in three stages. Each step corresponds to a space-state model. The equations used in the first and second steps are simplified versions of the final model. Variables are omitted in the first step, and parameters are restricted in the second step.

II.3 The three-stage estimation procedure

HLW (2017) use Stock and Watson's (1998) unbiased median estimate of the signal-to-noise ratio in two preliminary steps based on restricted versions of the complete model in its reduced form (Equations 4 to 13) to obtain estimates of signal-to-noise ratios:

$$\lambda_g = \frac{\sigma_g}{\sigma_{y*}} \text{ and.} \lambda_z = \frac{a_r \sigma_z}{\sigma_{\tilde{y}}}$$

These ratios are then kept fixed in step 3 of their procedure, which allows the estimation of the remaining parameters of the model.

HLW (2017) adopt a state-space formulation of the final model as well as for its intermediate versions, which are written as follows:

$$y_t = A'.x_t + H'\xi_t + \epsilon_t \quad (14)$$

$$\xi_t = F'\xi_{t-1} + \eta_t \quad (15)$$

In the measurement equation (14), y_t is the vector of observable endogenous variables (real GDP y_t and inflation rate π_t) at date t, x_t is the vector of exogenous variables and lagged endogenous variables and ξ_t is the vector of unobservable variables (potential GDP y_t^* , growth rate of potential

GDP g_t and the natural rate of interest r_t^*). The stochastic terms ϵ_t and $\eta_t$are assumed to be uncorrelated and follow a normal distribution with zero mean and covariance matrices R and Q, where R is assumed to be a diagonal matrix. The estimation method is based on a three-step sequential estimation procedure.

(a) In the **first step**, drawing on Kuttner (1994), HLW applies the Kalman filter to estimate potential GDP, without considering the difference between real and natural interest rates in equation (5), which becomes equation (5'), and assuming that the growth rate g of potential GDP is constant in equation (8):

$$\widetilde{y_t} = a_{y,1}\tilde{y}_{t-1} + a_{y,2}\tilde{y}_{t-2} + \epsilon_{\tilde{y},t} \quad (5')$$

$$\pi_t = \mathrm{b}_\pi\pi_{t-1} + (1 - \mathrm{b}_\pi)\pi_{t-2,4} + \mathrm{b}_\mathrm{y}\tilde{y}_{t-1} + \epsilon_{\pi,t} \quad (6)$$

$$y_t^* = y_{t-1}^* + g + \epsilon_{y^*,t} \quad (8)$$

The estimated parameters of the model make up the vector θ1:

$$\theta_1 = \left[a_{y,1}, a_{y,2}, b_\pi, b_y, g, \sigma_{\tilde{y}}, \sigma_\pi, \sigma_{y*}\right]$$

From the potential GDP estimated by equation (5') as the first difference, HLW calculates the exponential of the Andrews and Ploberger (1994) statistic which is used to detect structural breaks at an unknown date,[4] which allows us to estimate $\lambda_g = \frac{\sigma_g}{\sigma_{y*}}$.

(b) In the **second step**, we use the estimate of λ_g in the first step. In equation (5'') we reinclude the gap between the interest rate and the interest rate.

$$\tilde{y}_t = a_0 + a_{y,1}\tilde{y}_{t-1} + a_{y,2}\tilde{y}_{t-2} + \frac{\alpha_r}{2}(r_{t-1} + r_{t-2}) + a_g g_{t-1} + \epsilon_{\hat{y},t} \quad (5'')$$

$$\pi_t = \mathrm{b}_\pi\pi_{t-1} + (1 - \mathrm{b}_\pi)\pi_{t-2,4} + \mathrm{b}_\mathrm{y}\tilde{y}_{t-1} + \epsilon_{\pi,t} \quad (6)$$

$$y_t^* = y_{t-1}^* + g_{t-1} + \epsilon_{y^*,t} \quad (8)$$

$$g_t = g_{t-1} + \epsilon_{g,t} \quad (9)$$

$$z_t = z_{t-1} + \epsilon_{z,t} \quad (10)$$

[4] For more details on this method, see Perron (2005).

At this point, all the equations in the model, of which there are five, are estimated. The Wald test[5] is performed to check whether the intercept value has changed at the IS curve at an unknown date to obtain an estimate of $\lambda_z = a_r \frac{\sigma_z}{\sigma_{\tilde{y}}}$

The vector of the parameters to be estimated by the maximum likelihood in step 2 is:

$$\theta_2 = \left[a_{y,1}, a_{y,2}, a_r, a_0, a_g, b_\pi, b_y, g, \sigma_{\tilde{y}}, \sigma_\pi, \sigma_{y*}\right]$$

(c) In the **third and final step**, the values of λ_g and λ_z , which are estimated in the first and second steps, are used to estimate the remaining parameters of the model by the maximum likelihood method.

Throughout the estimate, the parameters and are constrained. At the IS curve, a negative value is imposed on the coefficient a_r, reflecting the negative relationship between output and the real interest rate. The coefficient b_y, is kept positive at the level of the Phillips curve to describe the inflationary effects of a positive output-gap and vice versa.

These restrictions on the signs of the coefficients constitute a minimum initialization of the model, which facilitates convergence during the estimation.

The vector of the parameters to be estimated by the maximum likelihood in step 3 is:

$$\theta_3 = \left[a_{y,1}, a_{y,2}, a_r, b_\pi, b_y, \sigma_{\tilde{y}}, \sigma_\pi, \sigma_{y*}\right]$$

At this stage is no longer constant and the natural rate of interest resumes its definition g

$$r_t * = c\ g_t + z_t \qquad (3)$$

Having presented in detail the semi-structural model of HLW and the empirical method for measuring the natural rate of interest and the potential growth rate, we will in the next section estimate the natural rate of interest for Tunisia by this method.

III. Estimation of the natural rate of interest for Tunisia

We will begin by defining the measurement variables that will be used for the estimation of the so-called latent unobservable variables (subsection 1). Then, we will present in subsection 2 how we took into account the COVID-19 effect in our calculations, again drawing on the approach of HLW (2023). Finally, we will present our results and the lessons we draw from them in terms of characterizing the conduct of monetary policy in Tunisia over the period 1989-2023 Q4.

III.1 Data and Variable Definitions

[5] Wald's statistic in exponential.

Estimates of r* using HLW are obtained from quarterly data covering the period, from 1988Q1 to 2023Q4. The variables used are real GDP, inflation rate, expected inflation, real interest rate, and stringency index to capture the COVID-19 effect. We detail in the following how we obtained each of these variables:

Real GDP is a chained series of the seasonally adjusted quarterly GDP published by the Nation Instiute of Statistics. The series dates to 1988Q1 and was calculated on several bases. Only the 1990s to date have been published by the NSI. The series from 1988 to 1990 was obtained on our own. The harmonization of the data published with different bases has been carried out in accordance with growth rates. The series is presented at constant 2015 prices.

Inflation is calculated from the monthly consumer price index published by the NSI. After calculating the average for each quarter, we calculate the annualized growth rate (Q/T-4) of that quarterly average expressed as a percentage.

Expected Inflation is calculated as the average of current inflation π_t and that of the last three quarters.

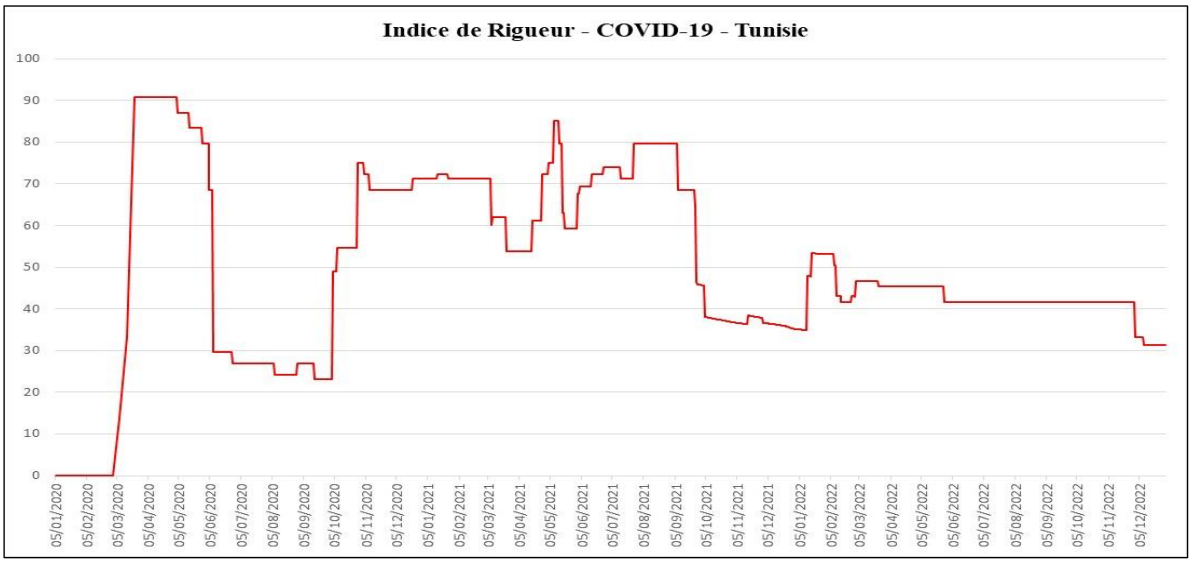
Expected inflation: $$E_t[\pi_{t+1}] = (\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3})/4 \quad (12)$$

The Observed Real Interest Rate (in percentage) r_t is obtained by subtracting the quarterly average of the money market rate i_t published by the Central Bank of Tunisia minus the quarterly average of expected inflation and $[\pi_{t+1}]$.

Real Interest Rate: $$r_t = i_t - E_t[\pi_{t+1}] \quad (11)$$

The COVID_19 Stringency Index, (published by OxCGRT - Oxford Coronavirus Government Response Tracker-), calculates the stringency of government policies to face the COVID-19 pandemic. It is a composite index of nine measures of a country's response to the pandemic. The nine measures are: school closures; workplace closures; cancellation of public events; restrictions on public gatherings; the shutdown of public transport; stay-at-home requirements; public information campaigns; restrictions on internal movement and control of international travel. https://ourworldindata.org/covid-stringency-index

Figure 1: The COVID-19 Stringency Index for Tunisia



III.2 Taking into account the COVID-19 effect

Like HLW (2023), we have taken into account the impact of the COVID-19 pandemic on the natural interest rate by including a double adjustment in the version of (HLW, 2017). **The first adjustment** is to introduce variability over time for random shocks that affect the IS curve and the Phillips curve, respectively. The variance of these shocks is no longer constant over time and is now modelled as being higher during the COVID period (years 2020, 21 and 22). $\sigma_{\tilde{y}}\ et\ \sigma_{\pi}$

To do this, we introduce parameters called Kappa (κ_t) that apply to the COVID years (κ2020, κ2021 and κ2022). These are scale variables that multiply random shocks $\sigma_{\tilde{y}}\ et\ \sigma_{\pi}$ by κt, where κt takes values equal to 1 before and after COVID and greater than or equal to 1 during the three years 2020, 21 and 22. The addition of these parameters increases the volatility of shocks affecting the IS and Phillips curves. The quantities κ2020, κ2021 and κ2022 are estimated by the maximum likelihood method, like all other parameters in the model, but constrained to be greater than or equal to 1.

The Second Adjustment is to introduce a persistent COVID supply shock on top of the demand and supply shocks already predicted in the model. It aims to capture the direct effects of COVID-19 on the output-gap by modifying potential GDP. This is to add a proxy for the direct effects of government restrictions and lockdowns put in place to deal with the pandemic. This proxy variable, named $(\boldsymbol{d_t})$**,** is approximated by the Stringency Index COVID_19 (Stringency Index) Published by OxCGRT (Oxford Coronavirus Government Response Tracker, cf, Hale et al, 2021).

This index calculates the stringency of government policies in response to the COVID-19 pandemic.

The stringency index adjusts potential GDP as follows:

$$y^*_{t,COVID} = y^*_t + \frac{\emptyset}{100}\, d_t$$

where y_t^* is the potential GDP, d_t is the COVID-19 stringency index, and $\emptyset$the parameter that allows the transmission of the impact of the stringency index to potential GDP. The parameter $\emptyset$ is estimated using the maximum likelihood method in the same way as all parameters including the scale variables κ2020, κ2021 and κ2022.

The output-gap adjusted by the COVID-19 stringency index replaces the initial output-gap in the space-state model measurement equations, the IS and Phillips equations. The equations of state (8, 9 and 10) of the latent (Unobservable) variables are not modified and keep the same expression. The vectors of the parameters to be estimated by the maximum likelihood method for steps 1, 2 and 3 with COVID-19 are written as follows, respectively:

$$\theta'_1 = \left[a_{y,1}, a_{y,2}, b_\pi, b_y, g, \sigma_{\tilde{y}}, \sigma_\pi, \sigma_{y*},\ \emptyset, \kappa_{2020Q2-Q4}, \kappa_{2021},\ \kappa_{2022}\right]$$

$$\theta'_2 = \left[a_{y,1}, a_{y,2}, a_r, a_0, a_g, b_\pi, b_y, \sigma_{\tilde{y}}, \sigma_\pi, \sigma_{y*},\ \emptyset, \kappa_{2020Q2-Q4}, \kappa_{2021}, \kappa_{2022}\right]$$

$$\theta'_3 = \left[a_{y,1}, a_{y,2}, a_r, b_\pi, b_y, \sigma_{\tilde{y}}, \sigma_\pi, \sigma_{y*},\ \emptyset,\ C, \kappa_{2020Q2-Q4}, \kappa_{2021}, \kappa_{2022}\right]$$

III.3 Results of the Empirical Estimates

In our approach, the Kalman filtering technique is used to estimate unobserved variables, including r_t*, gt, and z_t that vary over time. The Kalman filter partially adjusts the estimates of the unobserved components for the differences between the model's forecasts of inflation and real GDP and the real values of these variables until the latter statistically converge.

Estimated in the first stage, the value of λ_g for Tunisia is slightly higher than $\lambda_g$of the United States, Canada and the Eurozone. However, the value of λ_z (estimated in the second stage) is lower than that of the advanced countries, implying a quasi-stability in the evolution of the other factors influencing r^*.

The estimated parameters are mostly significant at 5%, the associated *t-statistics* show that they are estimated with some accuracy. The parameters introduced to reflect the covid effect on the potential level of output $y*$ are significant at 5%. The parameter $\phi = -0.073$ value shows a

negative impact of the Covid-19 shock. The parameters κ are relevant only for the year 2020. For 2021 and 2022, the calculations give κ2021 and κ2022 equal to 1.

The mean errors show that the unobservable variables $r*$,g and $y*$ are estimated with a precision comparable to that of the United States and the Eurozone.

Table 1 on the following page reports the results of the empirical estimate for Tunisia and includes the results of HLW (2023) for the United States, Canada and the Eurozone.

IV. Growth Potential, Natural Interest Rates and the Conduct of Monetary Policy

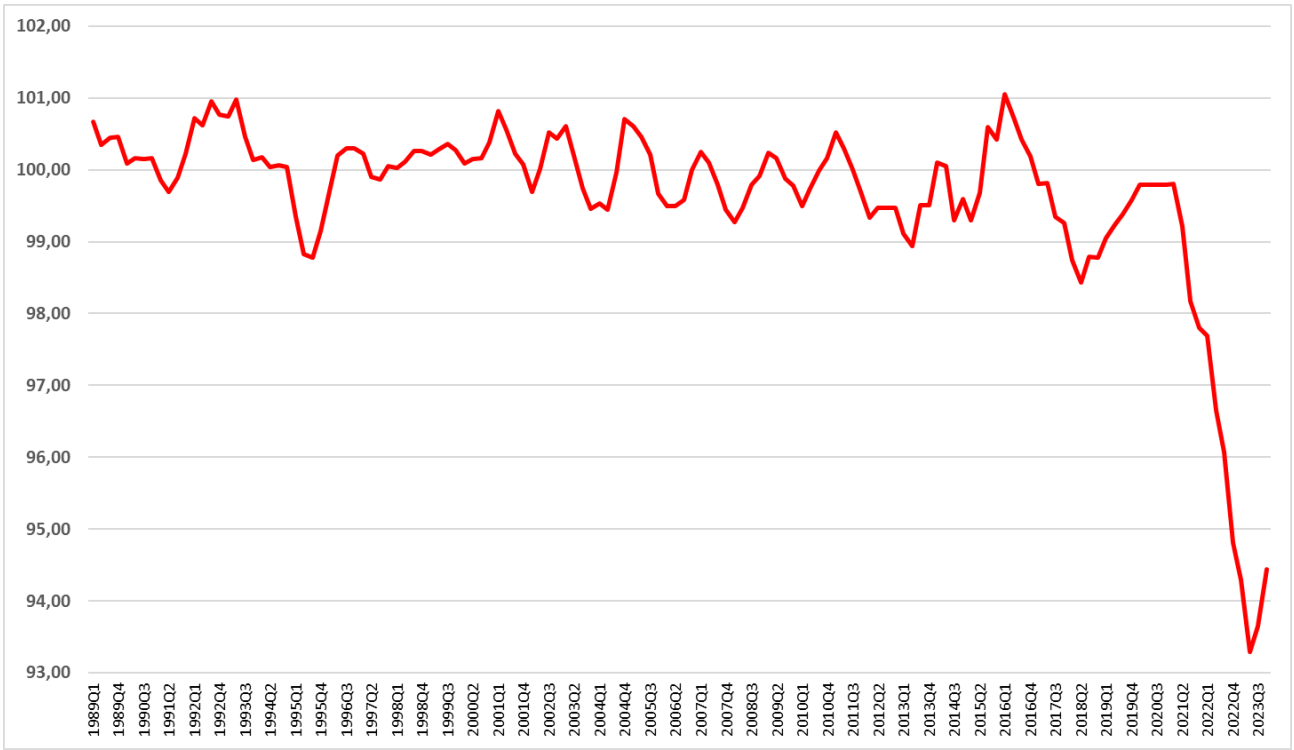
The results show a major impact of potential growth (g) on the natural interest rate and a negligible effect of other determinants (z) outside the COVID-19 period. Indeed, as shown in Chart 2 which presents the ratio (c g/r*), more than 99% of the evolution of the natural interest rate is explained by the growth of the potential of GDP and this for the entire period from 1989 to 2020 Q4, from 2021 Q3 the impact of GDP decreases to 94% reflecting the COVID-19 effect.

Parameters	Tunisia		USA	Canada	Euro Area
Estimation period	1988:Q1-2023:Q4	t-Student	1961:Q1-2023:Q4	1961:Q1-2023:Q4	1972:Q1-2023:Q3
λ_g	0,086		0,064	0,057	0,031
λ_z	0,002		0,018	0,011	0,029
a_{y1}	1,013	(-6,90)	1,428	1,338	1,639
a_{y2}	-0,139	(0,92)	-0,496	-0,389	-0,695
a_r	-0,106	(1,25)	-0,066	-0,077	-0,032
b_π	1,174	(19,50)	0,692	0,570	0,679
b_y	0,025	(0,47)	0,085	0,040	0,077
c	0,621	(1,02)	1,110	1,132	0,998
ϕ	-0,073	(6,14)	-0,114	-0,049	-0,133
$\kappa_{2020Q2-Q4}$	10,192	(1,96)	7,638	10,307	18,443
κ_{2021}	1	(1,93)	1,601	1,000	1,806
κ_{2022}	1	(0,92)	2,183	1,467	1,692
σ_{ygap}	0,678	(2,70)	0,435	0,563	0,312
σ_π	0,641	(15,14)	0,794	1,345	1,023
σ_{y*}	0,577	(2,19)	0,512	0,463	0,387

σ_g	0,050	0,132	0,106	0,048
σ_z	-0,009	0,118	0,082	0,284
σ_{r^*}	0,032	0,188	0,145	0,288
S.E. (Moy période)				
r*	1,921	1,214	1,578	4,011
g	0,644	0,404	0,402	0,237
y*	1,832	1,493	2,578	1,778
S.E. (obs finale)				
r*	1,194	1,602	1,703	5,980
g	0,946	0,596	0,576	0,340

Values in parentheses represent t-Students.

Figure 2: Contribution of the Potential GDP Growth Rate to the Natural Rate of Interest



In what follows, we will first analyse the evolution of the growth rate of potential GDP as the main determinant of the natural rate of interest. And then we'll make the connection between the natural rate of interest, on the one hand, and monetary policy and inflation, on the other.

IV.1 Growth Potential and Natural Interest Rate: A Continued Decline from 2011 onwards

Calculations of potential growth show a significant deterioration from 2011 onwards. The potential GDP growth rate has fallen from a relatively stable average of 4.5% over the entire period 1989-2010, to an extremely low level of 0.8% in 2023Q4.

The Tunisian economy has deteriorated sequentially, each time reaching a lower level where each level is the result of an exogenous shock negatively impacting the Tunisian economy.

The first shock coincided with the fall of the Ben Ali regime and the social and security disruptions that accompanied it, with potential growth (g) falling by 2 points to 2.64% between 2011 and 2015. The second level from 2016 to 2019 corresponds to the effects of the terrorist attacks in Bardo, Sousse and Tunis in 2015. Growth slowed to 2.09%. The COVID-19 crisis has exacerbated the situation, bringing potential growth down to 1.47% and finally the year 2023 shows an even lower growth potential of less than 1% in the order of 0.8%. This is alarming, as the Tunisian economy has not been able to recover its growth potential at any time. The weakness of private and public investment, the narrowness of the State's fiscal space and the absence of external financing make it difficult to envisage a recovery in the potential of the Tunisian economy in the short term. The following two charts show the growth rate of potential GDP and the growth rate of observed real GDP.

Figure 3: Potential GDP growth rate in %

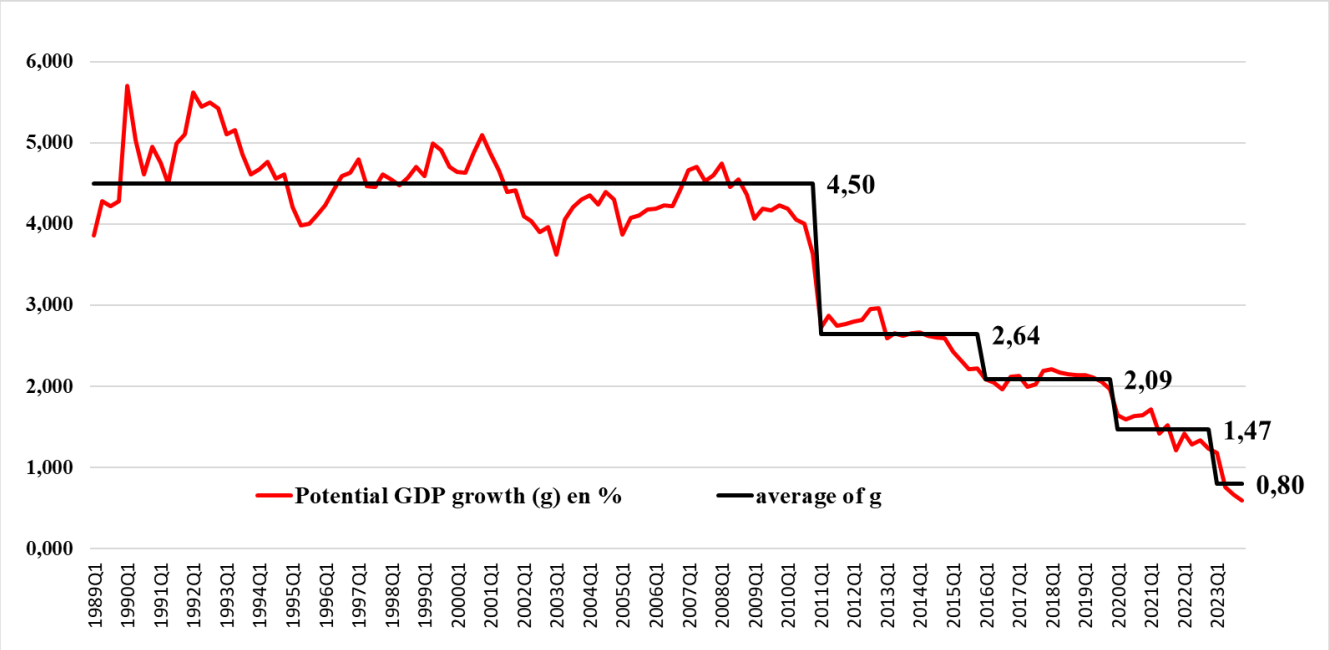
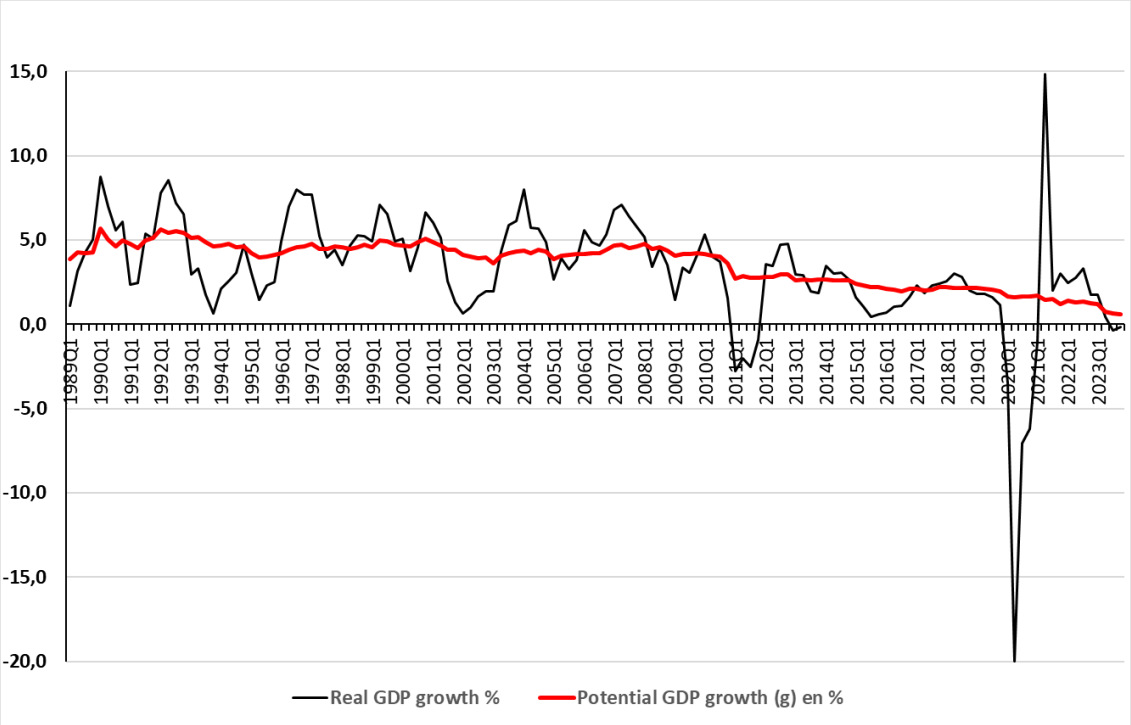


Chart 4: Growth Rates of Potential and Observed Real GDP

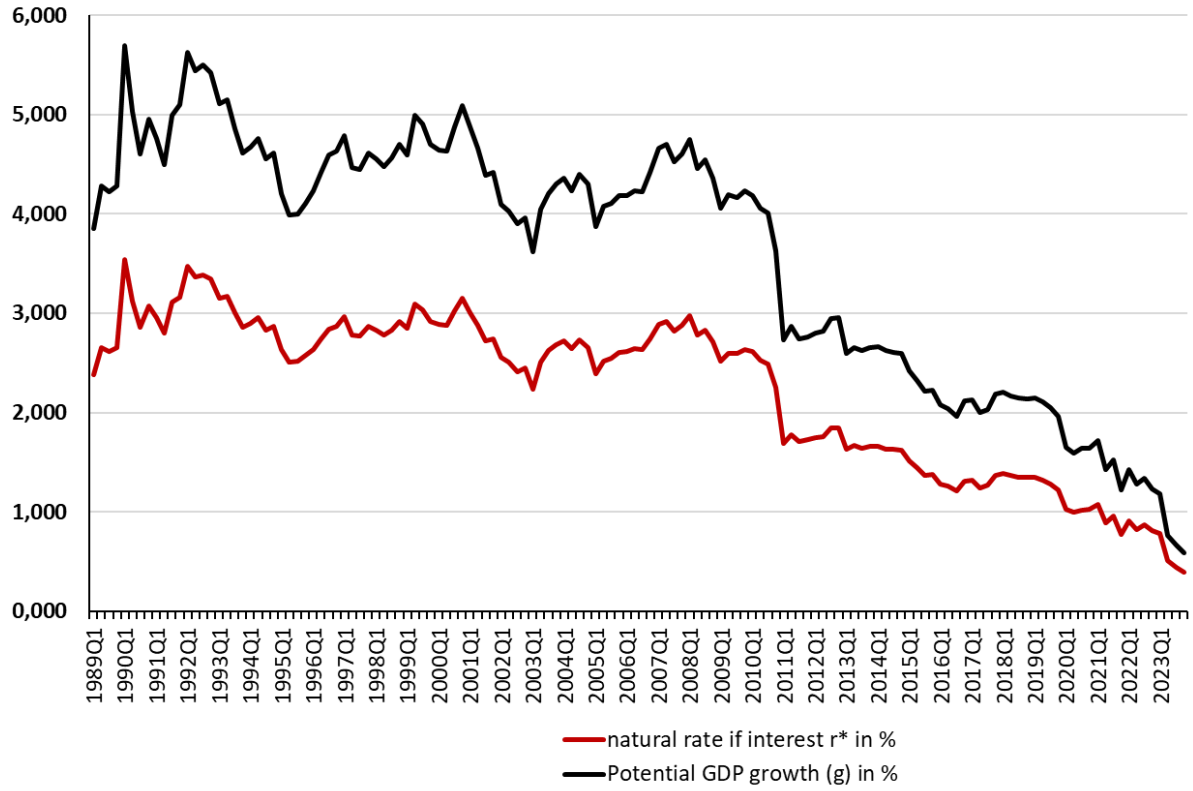


Strongly impacted by potential growth, the natural interest rate is at the same pace as that of potential GDP growth and shows a continuous decline, especially from 2011 onwards. The impact of other factors (z_t) on the natural rate of interest is small except at the end of the period. The estimate of parameter (c), which relates the growth rate of potential GDP to the natural rate of interest, is equal to 0.62 (c = 0.62), which normally gives a natural rate of interest less than gt.

$$r_t * = 0.62\ g_t + z_t$$

Chart 5 shows the natural rate of interest and the growth rate of potential GDP.

Figure 5: Growth rate of potential GDP and natural interest rate in %

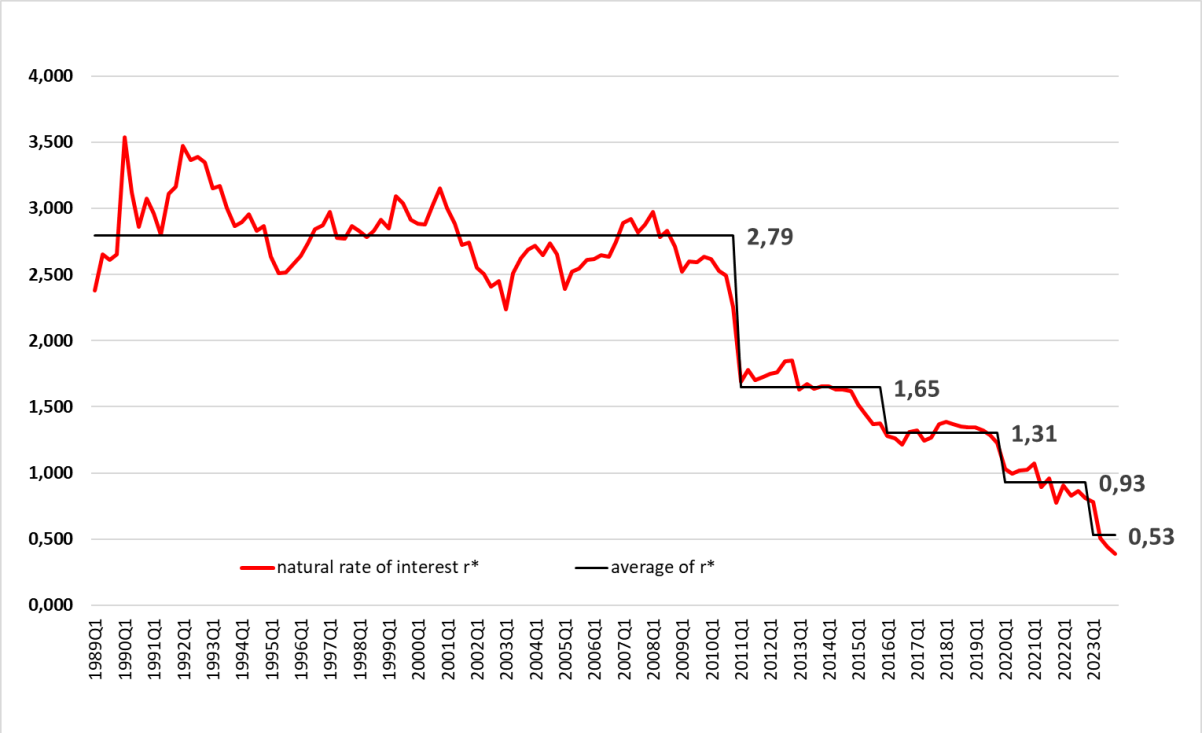


The exploitation of the results relating to the natural rate of interest is fruitful for the evaluation and characterization of the conduct of monetary policy in Tunisia.

IV.2 Natural Interest Rates and the Monetary Policy Approach

As noted above, the natural rate of interest is on a downward trend, especially after 2011, as shown in the following chart:

Graph 6: Natural interest rate in %

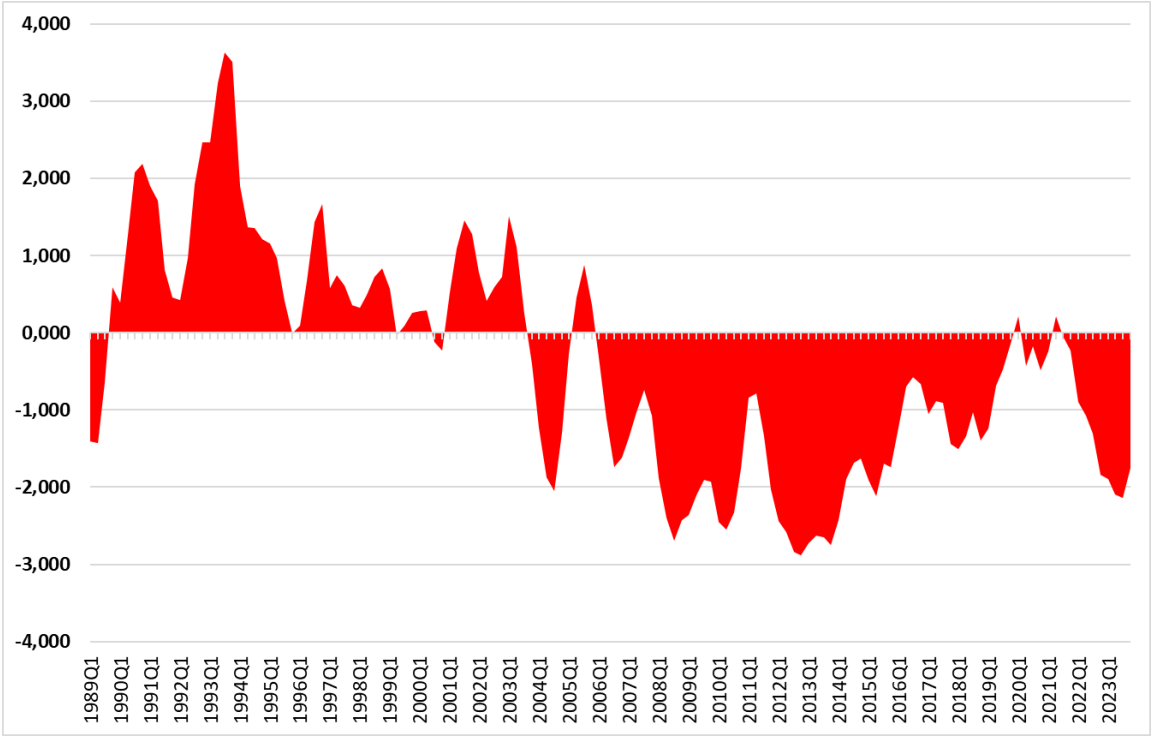


We will now use the estimate of the natural rate of interest to identify some observations on monetary policy and its conduct over the entire period. The difference between the observed real interest rate (r) and the natural rate of interest r*, which we will call the "interest rate gap" (r-r*), allows us to characterize the stance of monetary policy where r* would be the rate that would bring inflation back to its target and GDP to its potential level. Thus, the monetary policy stance will be considered as:

- Restrictive if: $r_t - r^* > 0$
- Neutral if: $r_t - r^* = 0$
- Accommodative if: $r_t - r^* < 0$

The results show that Tunisia's monetary policy was rather restrictive over the period 1990-2002 and accommodative from 2003 until the Covid-19 pandemic in 2020Q1. It was rather neutral between 2020Q2 and 2021Q3 and then became accommodative again from 2021Q4 to 2023Q4.

Chart 7: Interest Rate Gap (r-r*)

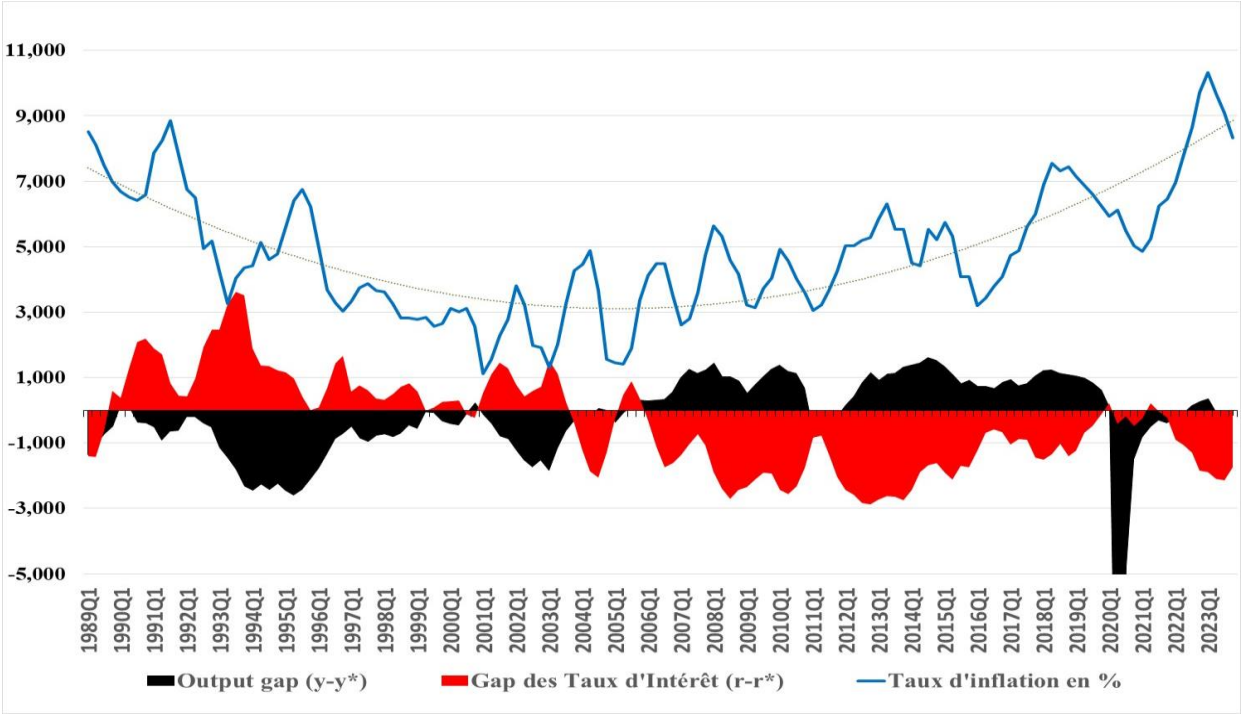


IV.3. Monetary Policy Stance, Output Gap and Inflation

Looking at the relationship between the output-gap and the monetary policy stance, we can see that when monetary policy was restrictive (1990-2002 and r-r*>0), the output-gap was negative, and the economy was below its potential (solid black section in Figure 8). When monetary policy was characterized as accommodative (2003-2023 and r-r*<0), the economy was above its potential.

On the other hand, when looking at the relationship between the inflation rate and the monetary policy stance, we can see that inflation followed a downward trend during the restrictive monetary policy phase (1990-2002) and an upward trend during the accommodative phase (2003-2023). The results are consistent with the evolution of inflation, which fell during the restrictive phase, from an average of 7% in the early 1990s to less than 2% in 2001, and which gradually increased during the accommodative phase, from 3% in 2003 to more than 9% in 2023.

Figure 8: Interest Rate Gap (r-r*), Output-gap (y-y*) and Inflation



During the 2011-2023 period, which saw growth potential declining continuously following a succession of major negative shocks (regime change in 2011, terrorist attacks in 2015 and COVID-19 in 2020), Tunisia lived beyond its potential due to a high budget deficit resulting in a growing debt ratio increasing the debt-to-GDP ratio from 40% in 2010 to 80% in 2023.

Forced to play its role as "lender of last resort", the Central Bank of Tunisia has been accommodating to the public treasury through open-market operations and the refinancing of banks that subscribed to treasury bond issues. The Central Bank of Tunisia, which has thus met the Treasury's financing needs, has at the same time pursued a policy of action on its key interest rate aimed at containing some of the pressure on domestic demand (positive output-gap) and the resulting inflationary slippages.

Finally, in the absence of macroeconomic stabilization and tight fiscal policy, the policy interest rate instrument alone could not limit inflationary pressures. It is in this sense that monetary policy appears to be accommodative during this period in light of the interest rate gap, which in a way reveals the limits of its use as the sole indicator of the stance of monetary policy, especially for an economy that is in a phase of fiscal dominance.

Conclusion

After an overview of the theoretical and empirical literature on the natural rate of interest, we estimated it for Tunisia using the Neo-Wicksellian semi-structural approach of Holston, Laubach and Williams considering the Covid-19 effect. The results show a continuous decline in the natural rate of interest r^*over the entire analysis period from 1989 to 2023.

The exercise involved in this work has been instructive in many ways. We started with the idea of estimating a natural rate of interest for the Tunisian economy with a view to providing a benchmark, from the real sphere, for monetary policy. This exercise allowed us to draw several lessons about potential GDP growth, the natural real interest rate, the conduct of monetary policy and the output-gap.

With regard to potential GDP growth, the main determinant of the natural rate of interest from the real sphere, we have seen a radical and alarming decline in the potential of the Tunisian economy after 2011. The economy has suffered three successive negative shocks, the first in 2011 with regime change, the second in 2015 with terrorist attacks, and the third in 2020 with the COVID-19 pandemic. At no time has the economy been able to recover and return to its previous potential.

About the natural rate of interest, the main focus of our work, the same downward trend emerges and reflects the impact of the weakening of GDP growth potential. The difference between the observed real interest rate and the natural interest rate, acting as a benchmark, allowed us to characterize the stance of Tunisian monetary policy as restrictive between 1990 and 2003 and accommodative from 2003 to the end of 2023.

The results also show a coherence between the characterization of monetary policy and the inflationary pressures of aggregate demand as measured by the output-gap. Indeed, the restrictive phase coincided with a negative output-gap and vice versa for the so-called accommodative phase, which reveals a positive output-gap and significant pressure from aggregate demand on prices. Monetary policy, through the instrument of the key interest rate, could not on its own curb inflationary pressures, especially when the Tunisian economy found itself in a situation of fiscal dominance.

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