Optimal Carbon Taxation with Concerns for Redistribution*

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Abstract

We provide a general formula for optimal carbon taxes in a second best world where governments may have concerns for redistribution, but only have access to nonlinear taxes on labor income. Our formula requires adding to standard estimates of the social cost of carbon an extra term that takes into account its potentially adverse consequences on earnings inequality. This extra term depends on a few sufficient statistics, including elasticities of relative wages with respect to changes in carbon emissions across quantiles of the income distribution. Combining a model of the US economy with detailed Census data, we provide the first estimates of these elasticities and explore their implications for carbon taxation.

*This version: October 2024. This is EXTREMELY preliminary and incomplete. Please do not circulate.

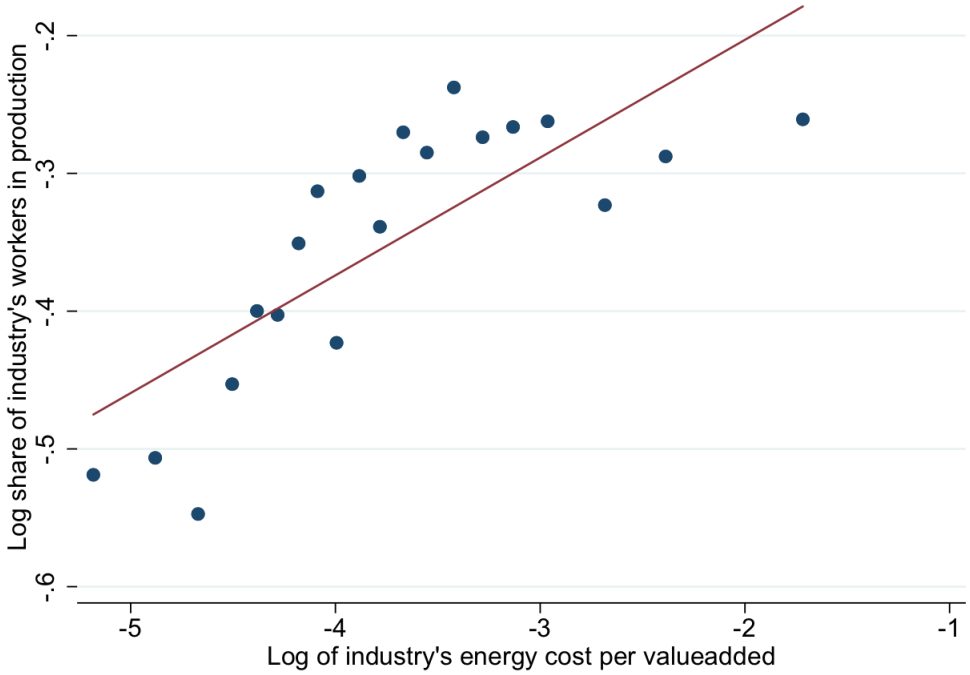
1 Introduction

We provide a general formula for optimal carbon taxes in a second best world where governments may have concerns for redistribution, but only have access to nonlinear taxes on labor income. Our formula requires adding to standard estimates of the social cost of carbon an extra term that takes into account its potentially adverse consequences on earnings inequality. This extra term depends on a few sufficient statistics, including elasticities of relative wages with respect to changes in carbon emissions across quantiles of the income distribution. Combining a model of the US economy with detailed Census data, we provide the first estimates of these elasticities and explore their implications for carbon taxation.

2 Carbon Emissions and Redistribution: Motivating Facts

A lot has been written about carbon taxation and redistribution via the expenditure side, i.e. the fact that the consumption baskets of poor and rich people may differ in its carbon content. In this paper, we focus instead on the incidence of carbon taxation on the income side, i.e. the fact that the demand for skilled vs. unskilled workers (from different region) may also be affected by carbon taxation.

2.1 Skill- and Carbon-Intensity: Between Industries



2.2 Skill- and Carbon-Intensity: Within Industries

Within a given sector, we expect a similar relationship.

3 Optimal Taxation with Concerns for Redistribution

3.1 Environment

We model the US economy as comprising many local labor markets, indexed by $m \in \mathcal{M}$, and multiple labor groups within each market, indexed by $l \in \mathcal{L}$. In our empirical analysis, each market m corresponds to a separate commuting zone and each labor group l corresponds either to college or non-college graduates.Within each market and each group, there is a continuum of workers with heterogeneous productivity $\theta > 0$. We let G_{ml} denote the cumulative distribution of productivity among workers of group l in market m. The associated density is g_{ml}. If the wage per efficiency unit for a labor group in a market is w_{ml}, then workers with productivity θ receive θw_{ml} per hours worked. Throughout our analysis, we normalize the total measure of workers in the economy to one and denote by π_{ml} the fixed share of the population in a given market m and labor group l.

Preferences. All workers derive utility from consuming an aggregate final good, disutility from supplying work hours, and disutility from experiencing carbon emissions. In any market m and for any labor group l, a worker of type θ has utility

$$U_{ml}(\theta) = D_{ml}(\theta) - \frac{\epsilon}{1+\epsilon} L_{ml}(\theta)^{\frac{1+\epsilon}{\epsilon}} - \chi_{ml}(\theta) C, \tag{1}$$

where $D_{ml}(\theta)$ denotes her consumption of the aggregate final good; $L_{ml}(\theta)$ denotes her labor supply, with $\epsilon > 0$ the labor supply elasticity; and C denotes total carbon emissions, which are taken as given by all workers and have marginal utility cost $\chi_{ml}(\theta) > 0$.

Technology. The aggregate final good is produced by a representative firm using intermediate goods from different sectors, indexed by $k \in \mathcal{K}$, that are produced by different firms, indexed by $n \in \mathcal{N}_k$. The production function of the representative firm is

$$Y = \prod_{k \in \mathcal{K}} (Y_k)^{\beta_k}, \tag{2}$$

$$Y_k = \Big[\sum_{n \in \mathcal{N}_k} \alpha_n^{1/\sigma} (y_n)^{(\sigma-1)/\sigma} \Big]^{\sigma/(\sigma-1)}, \tag{3}$$

where y_n denotes the output of firm n used in the production of the aggregate final good, $\sigma > 0$ is the elasticity of substitution between goods produced by different firms within the same sector; and $\alpha_n \geq 0$ is an exogenous technological parameter. The aggregate final good can be used either for consumption or investment in physical capital.

Intermediate good producers belong to one of three broad sector categories: mining ($\mathcal{K}_M$), which extracts fossil $f \in \mathcal{F} \equiv \{\text{coal, oil, gas}\}$; energy ($\mathcal{K}_E$), which processes fossil fuel and includes oil refining, gas utilities, and electric utilities; and production ($\mathcal{K}_P$), which consumes fossil fuels and includes manufacturing, agriculture, and services. By definition, $\mathcal{K} = \mathcal{K}_M \cup \mathcal{K}_E \cup \mathcal{K}_P$. Each intermediate good producer n is located in a single local labor market, with $\mathcal{N}_{mk}$ the set of producers from sector k located in market m.[1] By definition, $\mathcal{N}_k = \cup_{m \in \mathcal{M}} \mathcal{N}_{mk}$. Without risk of confusion, we also denote $\mathcal{N}_m \equiv \cup_{k \in \mathcal{K}} \mathcal{N}_{mk}$ the set of intermediate producers in market m and $\mathcal{N} \equiv \cup_{m \in \mathcal{M}} \mathcal{N}_m$ the set of all producers. Intermediate good producers combine physical capital, local labor from different groups, and intermediate goods from different sectors. Gross output of a firm n from market m that operates in a sector k is equal to

$$q_n = \varphi_n (K_n)^{\beta_{Kn}} (L_n)^{\beta_{Ln}} (Q_n)^{\beta_{Qn}}, \tag{4}$$

$$Q_n = \Big[\prod_{j \in \mathcal{K}_E \cup \{\text{coal}\}} (Q_{jn})^{\beta_{jk}} \Big]^{(\beta_{En}/\beta_{Ek})} \Big[\prod_{j \in \mathcal{K}_M \cup \mathcal{K}_P \setminus \{\text{coal}\}} (Q_{jn})^{\beta_{jk}} \Big]^{(1-\beta_{En})/(1-\beta_{Ek})} \tag{5}$$

$$Q_{jn} = \Big[\sum_{r \in \mathcal{N}_j} \alpha_r^{1/\sigma} q_{rn}^{(\sigma-1)/\sigma} \Big]^{\sigma/(\sigma-1)}, \tag{6}$$

$$L_n = \Big[\sum_{l \in \mathcal{L}} \delta_{ln}^{1/\eta} L_{ln}^{(\eta-1)/\eta} \Big]^{\eta/(\eta-1)}, \tag{7}$$

where K_n is firm n's demand for physical capital; L_{ln} is its employment of workers of type l from market m; q_{rn} is its demand for firm r's output; $\eta > 0$ is the elasticity of substitution between different labor groups; and $\varphi_n, \beta_{fn}, \beta_{Ln}, \beta_{Kn}, \beta_{Qn}, \delta_{ln}, \beta_{jk}, \beta_{En}, \beta_{Ek} \geq 0$ are exogenous technology parameters, with $\beta_{Kn} + \beta_{Ln} + \beta_{Qn} \leq 1$, $\sum_{l \in \mathcal{L}} \delta_{ln} = 1$, $\beta_{Ek} = \sum_{j \in \mathcal{K}_E \cup \{\text{coal}\}} \beta_{jk}$, and $\sum_{j \in \mathcal{K}} \beta_{jk} = 1$. For each firm n from market m that operates in a production sector $k \in \mathcal{K}_P$, technology is the same, except that they do not demand extracted fossil fuels, i.e. equations (4)-(6) hold with the additional restriction $\beta_{fn} = 0$ for all $f \in \mathcal{F}$.

A few remarks are in order. First, the firm-level shifters $\{\alpha_n\}$ appearing in (6) are the same as those appearing in (3). This a standard restriction that derives from the fact that we will only be able to observe a firm's total sales in the data. For the same reason, we

[1] For expositional purposes, we do not draw a distinction between firms and plants in our model. In practice, a given firm may operate multiple plants in different markets. In such circumstances, one should interpret the production function described below as the production function of a plant, with the total output of the firm equal to sum of the output of its plants across all markets.

assume that input-output linkages are sector- rather than firm-specific, i.e., β_{jk} may only vary with the sectors j and k in which the selling and buying firms operate. In contrast, we let firms be heterogeneous in terms of their productivity, as reflected in φ_n below, as well as in terms of their cost shares on capital, labor, intermediate goods, and energy, as reflected in β_{Kn}, β_{Ln}, β_{Qn}, and β_{En}, since such information IS available for firms in the oil refining and manufacturing sectors. Finally, we allow the possibility that $\beta_{Kn}+\beta_{Ln}+\beta_{Qn}<1$. This will be relevant for firms in the mining sector for which natural resources of fossil fuel are treated as a fixed factor of production whose levels implicitly affect firms' productivity and supply elasticities.

Taxes. There are two types of taxes: carbon taxes and labor income taxes. Carbon taxes create a wedge between the price received by firms that extract and sell fossil fuels and the price paid by firms that buy and burn fossil fuels. We let carbon taxes t_m vary by the market m in which firms burning fossil fuel are located. For any firm $n \in \mathcal{N}_f$ that extracts fossil fuel f and any firm $r \in \mathcal{N}_m$ that is located in market m, the price p_{nr} paid by firm r is equal to the price p_n received by the seller n plus the adjustment due to the carbon tax,

$$p_{nr} = p_n + \varepsilon_f t_m,$$

where ε_f denotes carbon emissions per unit of fossil f. For future reference, we let $C_m \equiv \sum_{f\in\mathcal{F}}\sum_{n\in\mathcal{N}_f}\sum_{r\in\mathcal{N}_m}\varepsilon_f q_{nr}$ denote the total carbon emissions associated firms from market m. Income taxes $T(x)$ are may vary non-linearly with earnings, x. Earnings include both wages and total payments to fixed factors in the mining sector Π, which we assume are uniformly distributed across the population. In the observed equilibrium, we assume that income taxes satisfy

$$T(x) = x - kx^{1-\phi}$$

with $\phi \in [0,1]$ and k such that government's budget is balanced. We do not assume, however, that income taxes have to satisfy this restriction away from the initial equilibrium; as a matter of fact, we have allowed variations of the income tax schedule that do not in order to derive our optimal tax formula.

Market Structure . All markets are perfectly competitive. Good markets are perfectly integrated. We use the aggregate final good as our numeraire. Labor markets are segmented, with w_{ml} the wage per efficiency units of labor group l in market m.

Competitive Equilibrium. In a competitive equilibrium, for any $m \in \mathcal{M}$, $l \in \mathcal{L}$, and $\theta > 0$, workers maximize their utility, as described in (1), subject to their budget constraint,

$$D_{ml}(\theta) = \theta w_{ml} L(\theta) + \Pi - T(\theta w_{ml} L(\theta) + \Pi); \tag{8}$$

the representative final good producer maximizes its profits

$$Y - \sum_{n \in \mathcal{N}} p_n y_n;$$

for any $m \in \mathcal{M}$ and $n \in \mathcal{N}_m$, intermediate good producers maximize their profits

$$p_n q_n - K_n - \sum_{l \in \mathcal{L}} w_{ml} L_{ln} - \sum_{r \in \mathcal{N}} p_{rn} q_{rn}$$

subject to technological constraints described in (4)-(6); for any $m \in \mathcal{M}$, $n \in \mathcal{N}_m$, and $r \in \mathcal{N}$, prices satisfy the non-arbitrage condition,

$$p_{rn} = \begin{cases} p_r + \varepsilon_f t_m & \text{if } r \in \mathcal{N}_f \text{ for } f \in \mathcal{F}, \\ p_r & \text{otherwise,} \end{cases}$$

good and labor markets clear,

$$\begin{aligned} \sum_{m,l} \int D_{ml}(\theta) dG_{ml}(\theta) + \sum_{n \in \mathcal{N}} K_n &= Y \\ \sum_{r \in \mathcal{N}} q_{nr} + y_n &= q_n \text{ for all } n, \\ \sum_{n \in \mathcal{N}_m} L_{ln} &= \int L(\theta) dG_{ml}(\theta) \text{ for all } m \text{ and } l; \end{aligned} \tag{9}$$

and the government's budget is balanced,

$$\int T(\theta w_{ml} L(\theta) + \Pi) dG_{ml}(\theta) + \sum_{m \in \mathcal{M}} t_m C_m = 0. \tag{10}$$

3.2 Carbon Taxes and Wage Inequality

Given carbon taxes $\boldsymbol{t} \equiv \{t_m\}$ and an income tax schedule T, the competitive equilibrium induces a schedule of utilities $\{U_{ml}(\theta)\}$. This, in turn, induces a cumulative distribution over utilities, summarized by the utility levels $\bar{U} \equiv \{\bar{U}(z)\}$ associated with each quantile $z \in [0,1]$. For any worker at the quantile $z \in [0,1]$ of both the utility and wage distribution,

let $\bar{w}(z)$, $\bar{L}(z)$, and $\bar{x}(z)$ denote the common wage per hour, hours worked, and earnings. Wage inequality around each quantile is summarized by $\omega(z) = d\ln\bar{w}(z)/dz$.

Now consider a small change in the carbon taxes $\delta\boldsymbol{t} \equiv \{\delta t_m\}$ accompanied by a variation of the income tax schedule δT that is designed to keep utility levels fixed at all quantiles of the income distribution, $d\bar{U}(z) = 0$ for all z. The goal of this subsection is to compute the log-change in relative wages $\delta\ln\omega(z)$ associated with such a variation.

Let $\boldsymbol{w}_m \equiv \{w_{ml}\}_{l\in\mathcal{L}}$ denote the vector of factor prices in market m; and let $\boldsymbol{w} \equiv \{\boldsymbol{w}_m\}_{m\in\mathcal{M}}$ denote the entire vector of factor prices. Without loss of generality, we choose units of account for all labor groups $l \in \mathcal{L}$ and labor $m \in \mathcal{M}$ such that $w_{ml} = 1$ prior to the tax variation.

We start with a series of observations linking the vector of factor prices $\boldsymbol{w}$ to the wages $\bar{w}(z)$ paid across different quantiles. Since any worker of group l in market m is paid θw_{ml} per hour worked, the fraction of workers of that group that receive a wage less than w is $G_{ml}(w/w_{ml})$. Thus, the wage $\bar{w}(z)$ associated with each quantile z is equal to the solution $\hat{w}(z,\boldsymbol{w})$ to the following equation,

$$\sum_{m\in\mathcal{M}}\sum_{l\in\mathcal{L}} \pi_{ml} G_{ml}(w/w_{ml}) = z \text{ for all } z. \tag{11}$$

Differentiating (11) with respect to z, we then obtain

$$\hat{\omega}(z,\boldsymbol{w}) \equiv \partial\ln\hat{w}(z,\boldsymbol{w})/\partial z = [\sum_{m\in\mathcal{M}}\sum_{l\in\mathcal{L}} \pi_{ml}\frac{\hat{w}(z,\boldsymbol{w})}{w_{ml}} g_{ml}(\hat{w}(z,\boldsymbol{w})/w_{ml})]^{-1} \text{ for all } z. \tag{12}$$

Along any variation of the carbon price and income tax, including the variation of interest such that $d\bar{U}(z) = 0$ for all z, the local change in relative wages $\delta\ln\omega(z)$ must be related to the changes in factor prices $\{\delta\ln w_{ml}\}$ through

$$\delta\ln\omega(z) = \sum_{m\in\mathcal{M}}\sum_{l\in\mathcal{L}} \frac{\partial\ln\hat{\omega}(z,\boldsymbol{w})}{\partial\ln w_{ml}}\delta\ln w_{ml}, \tag{13}$$

where the vector of partial derivatives $\{\partial\ln\hat{\omega}(z,\boldsymbol{w})/\partial\ln w_{ml}\}$ can be computed by differentiating (11) and (12) with respect to each factor price w_{ml} and using the normalization $w_{ml} = 1$ prior to the tax variation,

$$\frac{\partial\ln\hat{\omega}(z,\boldsymbol{w})}{\partial\ln w_{ml}} = \pi_{ml}(z)[\varepsilon_{ml}(z) - \sum_{m'\in\mathcal{M}}\sum_{l'\in\mathcal{L}} \pi_{m'l'}(z)\varepsilon_{m'l'}(z)]$$

with $\pi_{ml}(z)$ the share of the population in quantile z of the wage distribution that is of labor group l and market m and $\varepsilon_{ml}(z)$ the elasticity of the productivity density for that same

group market and quantile,

$$\pi_{ml}(z) \equiv \frac{\pi_{ml} g_{ml}(\bar{w}(z))}{\sum_{m' \in \mathcal{M}} \sum_{l' \in \mathcal{L}} \pi_{m'l'} f_{m'l'}(\bar{w}(z))},$$
$$\varepsilon_{ml}(z) \equiv \frac{\bar{w}(z) g'_{ml}(\bar{w}(z))}{g_{ml}(\bar{w}(z))}.$$

The next step of our analysis consists in solving for the changes in factor prices $\{\delta \ln w_{ml}\}$ along the variation of interest. We then express all labor market clearing conditions as demand equals supply relative to that factor.

Consider first the demand side. For any firm $n \in \mathcal{N}_k$, let β_{jn} denote the share of total cost on intermediates that firm n spends on intermediates from sector j,

$$\beta_{jn} \equiv \begin{cases} \frac{\beta_{jk}\beta_{En}}{\beta_{Ek}} & \text{if } j \in \mathcal{K}_E \cup \{\text{coal}\}, \\ \frac{\beta_{jk}(1-\beta_{En})}{(1-\beta_{Ek})} & \text{if } j \in \mathcal{K}_M \cup \mathcal{K}_P \setminus \{\text{coal}\}; \end{cases}$$

let m_{rn} denote the share of firm n's total cost spent on a firm $r \in \mathcal{N}_j$,

$$m_{rn} \equiv \beta_{Qn}\beta_{jn} \frac{\alpha_r (p_{rn})^{1-\sigma}}{(P_{jn})^{1-\sigma}},$$

with $P_{jn} \equiv [\sum_{r \in \mathcal{N}_j} \alpha_r (p_{rn})^{1-\sigma}]^{1/(1-\sigma)}$; let $M \equiv \{(p_r/p_{rn}) m_{rn}\}$ denote the firm-level domestic input-output matrix (evaluated at the seller prices $\{p_n\}$); and let $B \equiv \{b_{rn}\}$ denote the associated Leontief inverse, i.e. $B = [1 - M]^{-1}$. Combining profit-maximization by firms with utility maximization by households and good market clearing, we can solve for the relative demand $RD_{ml}(\boldsymbol{t}, \boldsymbol{w}) \equiv L_{ml}/L_{m_0 l_0}$ for workers of group l in market m as a function of the vector of carbon taxes t and the vector of wages per efficiency units $\boldsymbol{w}$,

$$RD_{ml}(\boldsymbol{t}, \boldsymbol{w}) = \frac{w_{ml}^{-\eta}}{w_{m_0 l_0}^{-\eta}} \frac{\sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}_{mk}} \delta_{l n} \tilde{w}_n^{\eta - 1}(\boldsymbol{w}_m) \beta_{Ln} \left[\sum_{j \in \mathcal{K}, r \in \mathcal{N}_j} \tilde{b}_{nr}(\boldsymbol{t}, \boldsymbol{w}) \beta_j \alpha_r \tilde{p}_r(\boldsymbol{t}, \boldsymbol{w})^{1-\sigma} \tilde{P}_j(\boldsymbol{t}, \boldsymbol{w})^{\sigma - 1} \right]}{\sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}_{m_0 k}} \delta_{l_0 n} \tilde{w}_n^{\eta - 1}(\boldsymbol{w}_{m_0}) \beta_{Ln} \left[\sum_{j \in \mathcal{K}, r \in \mathcal{N}_j} \tilde{b}_{nr}(\boldsymbol{t}, \boldsymbol{w}) \beta_j \alpha_r \tilde{p}_r(\boldsymbol{t}, \boldsymbol{w})^{1-\sigma} \tilde{P}_j(\boldsymbol{t}, \boldsymbol{w})^{\sigma - 1} \right]}, \tag{14}$$

with

$$\begin{aligned}
\tilde{w}_n(\boldsymbol{w}_m) &\equiv [\sum_{l\in\mathcal{L}} \delta_{ln} w_{ml}^{1-\eta}]^{1/(1-\eta)},\\
\tilde{P}_j(\boldsymbol{t},\boldsymbol{w}) &\equiv [\sum_{n\in\mathcal{N}_j} \alpha_n \tilde{p}_n^{1-\sigma}(\boldsymbol{t},\boldsymbol{w})]^{1/(1-\sigma)},\\
\tilde{P}_{jn}(\boldsymbol{t},\boldsymbol{w}) &\equiv [\sum_{r\in\mathcal{N}_j} \alpha_r \tilde{p}_{rn}^{1-\sigma}(\boldsymbol{t},\boldsymbol{w})]^{1/(1-\sigma)}\\
\tilde{p}_n(\boldsymbol{t},\boldsymbol{w}) &\equiv \exp\{[\ln\varphi_n + \beta_{Ln}\ln\tilde{w}_n(\boldsymbol{w}_m) + \beta_{Qn}\tilde{p}_{Qn}(\boldsymbol{t},\boldsymbol{w})]\},\\
\tilde{p}_{Qn}(\boldsymbol{t},\boldsymbol{w}) &\equiv \left(\prod_{j\in\mathcal{K}} \beta_{jn}^{-\beta_{jn}}\right) \exp\{[\sum_{j\in\mathcal{K}} \beta_{jn}\ln\tilde{P}_{jn}(\boldsymbol{t},\boldsymbol{w})]\},\\
\tilde{p}_{rn}(\boldsymbol{t},\boldsymbol{w}) &\equiv \begin{cases} \tilde{p}_r(\boldsymbol{t},\boldsymbol{w}) + \varepsilon_f t_m & \text{if } r\in\mathcal{N}_f \text{ and } n\in\mathcal{N}_m,\\ \tilde{p}_r(\boldsymbol{t},\boldsymbol{w}) & \text{otherwise,}\end{cases}
\end{aligned}$$

and $\{\tilde{b}_{nr}(\boldsymbol{t},\boldsymbol{w})\}$ are the coefficient of the Leontief inverse associated with $\tilde{M}(\boldsymbol{t},\boldsymbol{w}) \equiv \{\tilde{m}_{nr}(\boldsymbol{t},\boldsymbol{w})\}$ such that

$$\tilde{m}_{nr}(\boldsymbol{t},\boldsymbol{w}) \equiv \beta_{Qr}\beta_{jr} \frac{\tilde{p}_n(\boldsymbol{t},\boldsymbol{w})}{\tilde{p}_{nr}(\boldsymbol{t},\boldsymbol{w})} \frac{\alpha_n[\tilde{p}_{nr}(\boldsymbol{t},\boldsymbol{w})]^{1-\sigma}}{[\tilde{P}_{jr}(\boldsymbol{t},\boldsymbol{w})]^{1-\sigma}} \text{ for } n\in\mathcal{N}_j \text{ and } r\in\mathcal{N}_k.$$

Next consider the labor supply side. At any given quantile z, the amount $\bar{L}(z)$ of hours supplied is equal to the solution to

$$\max_L U \text{ subject to } D = \bar{w}(z)L + \Pi - T(\bar{w}(z)L + \Pi).$$

Summing up across workers of different types θ within a given labor group l and market m, we can then express the total efficiency units of labor L_{ml} supplied by these workers as

$$L_{ml} = \pi_{ml} \times \int \theta \bar{L}(\bar{w}^{-1}(\theta w_{ml})) g_{ml}(\theta) d\theta \tag{15}$$

Changing variable to integrate across quantiles $z = \bar{w}^{-1}(\theta w_{ml})$, we can then express relative labor supply $RS_{ml}(\boldsymbol{w},\{\bar{L}(z)\}) \equiv L_{ml}/L_{m_0 l_0}$ as a function of the distribution of labor supply and the vector of wages per efficiency units $\boldsymbol{w}$,

$$RS_{ml}(\boldsymbol{w},\{\bar{L}(z)\}) = \frac{\pi_{ml}}{\pi_{m_0 l_0}} \frac{\int_0^1 \hat{\omega}(z,\boldsymbol{w})(\hat{w}(z,\boldsymbol{w})/w_{ml})^2 \bar{L}(z) g_{ml}(\hat{w}(z,\boldsymbol{w})/w_{ml}) dz}{\int_0^1 \hat{\omega}(z,\boldsymbol{w})(\hat{w}(z,\boldsymbol{w}))^2 \bar{L}(z) g_{m_0 l_0}(\hat{w}(z,\boldsymbol{w})) dz}. \tag{16}$$

In equilibrium, relative demand for each factor must be equal to its relative supply

$$RD_{ml}(\boldsymbol{t},\boldsymbol{w}) = RS_{ml}(\boldsymbol{w},\{\bar{L}(z)\}) \text{ for all } (m,l) \neq (m_0,l_0) \text{ and } l.$$

Taking log, totally differentiating, and using again the normalization $w_{ml} = 1$ prior to the tax variation, we therefore get

$$\begin{aligned}\sum_{m' \in \mathcal{M}} \frac{\partial \ln RD_{ml}(\boldsymbol{t},\boldsymbol{w})}{\partial t_{m'}} \delta t_{m'} + \sum_{m' \in \mathcal{M}} \sum_{l' \in \mathcal{L}} \frac{\partial \ln RD_{ml}(t_C,\boldsymbol{w})}{\partial \ln w_{m'l'}} \delta \ln w_{m'l'} \\ = \sum_{m' \in \mathcal{M}} \sum_{l' \in \mathcal{L}} \frac{\partial \ln RS_{ml}(\boldsymbol{w},\{\bar{L}(z)\})}{\partial \ln w_{m'l'}} \delta \ln w_{m'l'} + \int_0^1 \delta \ln \bar{L}(z) [d\Lambda_{ml}(z) - d\Lambda_{m_0 l_0}(z)]\end{aligned}$$

with $\Lambda_{ml}(z)$ the share of earnings of workers of type l in market m that are received by workers in quantiles below z,

$$\Lambda_{ml}(z) = \frac{\int_0^z \bar{w}(z') \bar{L}(z') \pi_{ml}(z') \bar{w}'(z') dz'}{\int_0^1 \bar{w}(z') \bar{L}(z') \pi_{ml}(z') \bar{w}'(z') dz'}.$$

To conclude our analysis, we use the fact that, as established in the robots paper, changes in labor supply $\delta \ln \bar{L}(z)$ and relative wages $\delta \ln \omega(z)$ are related through

$$\delta \ln \bar{L}(z) = -\frac{\epsilon}{\epsilon + 1} \delta \ln \omega(z).$$

Substituting in the previous expression, we obtain

$$\begin{aligned}\sum_{m' \in \mathcal{M}} \frac{\partial \ln RD_{ml}(\boldsymbol{t},\boldsymbol{w})}{\partial t_{m'}} \delta t_{m'} + \sum_{m' \in \mathcal{M}} \sum_{l' \in \mathcal{L}} \frac{\partial \ln RD_{ml}(t_C,\boldsymbol{w})}{\partial \ln w_{m'l'}} \delta \ln w_{m'l'} \\ = \sum_{m' \in \mathcal{M}} \sum_{l' \in \mathcal{L}} \frac{\partial \ln RS_{ml}(\boldsymbol{w},\{\bar{L}(z)\})}{\partial \ln w_{m'l'}} \delta \ln w_{m'l'} - \int_0^1 \frac{\epsilon}{\epsilon + 1} \delta \ln \omega(z) [d\Lambda_{ml}(z) - d\Lambda_{m_0 l_0}(z)].\end{aligned} \tag{17}$$

The partial derivatives of RD_{ml} and RS_{ml} with respect to t_C and $\boldsymbol{w}$ can be directly computed from (14) and (16), respectively. We report those in Appendix A. The changes in relative wages $\delta \ln \omega(z)$ is itself a known linear function of the changes in relative factor prices $\{\delta \ln w_{ml}\}$ given by (13). Thus, (17) provides a system of $M \times L - 1$ linear equations in $\{\delta \ln w_{ml}\}$. To solve for $\{\delta \ln w_{ml}\}$ as a function of $\delta \boldsymbol{t} \equiv \{\delta t_{m'}\}$, we combine the previous system with our choice of numeraire, which implies

$$\delta \ln P = 0 = \sum_{j \in \mathcal{K}} \beta_j \sum_{m' \in \mathcal{M}} \frac{\delta \ln \tilde{P}_j(\boldsymbol{t},\boldsymbol{w})}{\partial t_{m'}} \delta t_{m'} + \sum_{m' \in \mathcal{M}} \sum_{l' \in \mathcal{L}} \sum_{l' \in \mathcal{L}} \frac{\partial \ln \tilde{P}_j(\boldsymbol{t},\boldsymbol{w})}{\partial \ln w_{m'l'}} \delta \ln w_{m'l'}.$$

Once the factor prices have been solved for, we can substitute in (13) to obtain $\delta \ln \omega(z)$, as also described in Appendix A, and plug into our optimal tax formula, i.e. (25) below.

3.3 Carbon Taxes and Carbon Emissions

We now compute the change in total carbon emission $\delta \boldsymbol{C} \equiv \{\delta C_m\}$ associated with the same variation. The total level of demand of each good is proportional to total spending on the aggregate final good, Y. From the market clearing condition (9), it is equal to the sum of final consumption and investment

$$Y = D + K,$$

with $D \equiv \sum_{m,l} \int D_{ml}(\theta) dG_{ml}(\theta)$ and $K \equiv \sum_{n \in \mathcal{N}} K_n$. Using the workers' budget constraints (8) and the government's budget constraint, this can be rearranged as

$$Y = \sum_{m \in \mathcal{M}} \sum_{l \in \mathcal{L}} w_{ml} L_{ml} + \sum_{m \in \mathcal{M}} t_m C_m + K,$$

with the total efficiency units of labor supplied by each labor group in each market such that

$$L_{ml} = \pi_{ml} \times \int_0^1 \hat{\omega}(z, \boldsymbol{w}) (\hat{w}(z, \boldsymbol{w}) / w_{ml})^2 \bar{L}(z) g_{ml}(\hat{w}(z, \boldsymbol{w}) / w_{ml}) dz \equiv L^s_{ml}(\boldsymbol{w}, \{\bar{L}(z)\}); \quad (18)$$

the carbon emissions associated with burning fossil fuels in different locations m, $C_m \equiv \sum_{f \in \mathcal{F}} \sum_{n \in \mathcal{N}_f} \sum_{r \in \mathcal{N}_m} \varepsilon_f q_{nr}$, such that

$$C_m = \sum_{f \in \mathcal{F}} \sum_{n \in \mathcal{N}_f} \sum_{r \in \mathcal{N}_m} \varepsilon_f \left[\beta_{Qr} \beta_{jr} \frac{\alpha_n (\tilde{p}_{nr}(\boldsymbol{t}, \boldsymbol{w}))^{-\sigma}}{(\tilde{P}_{jr}(\boldsymbol{t}, \boldsymbol{w}))^{1-\sigma}} \right] \left[\sum_{j \in \mathcal{K}, v \in \mathcal{N}_j} \tilde{b}_{rv}(\boldsymbol{t}, \boldsymbol{w}) \beta_j \alpha_v \tilde{p}_v(\boldsymbol{t}, \boldsymbol{w})^{1-\sigma} \tilde{P}_j(\boldsymbol{t}, \boldsymbol{w})^{\sigma-1} \right] Y; \quad (19)$$

and investment $K \equiv \sum_{n \in \mathcal{N}} K_n$ such that

$$K = \sum_{n \in \mathcal{N}} \beta_{Kn} \left[\sum_{j \in \mathcal{K}, r \in \mathcal{N}_j} \tilde{b}_{nr}(\boldsymbol{t}, \boldsymbol{w}) \beta_j \alpha_r \tilde{p}_r(\boldsymbol{t}, \boldsymbol{w})^{1-\sigma} \tilde{P}_j(\boldsymbol{t}, \boldsymbol{w})^{\sigma-1} \right] Y. \quad (20)$$

Along the variation of interest, we therefore have

$$\delta \ln Y = \sum_{m \in \mathcal{M}} \sum_{l \in \mathcal{L}} \frac{w_{ml} L_{ml}}{Y} [\delta \ln w_{ml} + \delta \ln L^s_{ml}(\boldsymbol{w}, \{\bar{L}(z)\})] + \sum_{m \in \mathcal{M}} \frac{C_m [\delta t_m + t_m \delta \ln C_m]}{Y} + \frac{K}{Y} \delta \ln K \quad (21)$$

For the same reason as in Section 3.2, the changes in labor supply are given by

$$\delta \ln L^s_{ml}(\boldsymbol{w}, \{\bar{L}(z)\}) = \sum_{m' \in \mathcal{M}} \sum_{l' \in \mathcal{L}} \frac{\partial \ln L^s_{ml}(\boldsymbol{w}, \{\bar{L}(z)\})}{\partial \ln w_{m'l'}} \delta \ln w_{m'l'} - \int_0^1 \frac{\epsilon}{\epsilon+1} \delta \ln \omega(z) d\Lambda_{ml}(z). \quad (22)$$

Given changes in factor prices $\{\delta \ln w_{ml}\}$, we therefore compute $\delta \ln L^s_{ml}(\boldsymbol{w}, \{\bar{L}(z)\})$ using the partial derivative with respect to factor prices given by 18 and the changes in relative wages given by 13. Next, consider the changes in carbon emission and capital. Differentiating (19) and (20), we obtain

$$\delta \ln C_m = \sum_{f \in \mathcal{F}} \sum_{n \in \mathcal{N}_f} \sum_{r \in \mathcal{N}_m} \left(\frac{\varepsilon_f q_{nr}}{C_m}\right) \left\{ \delta \ln \left[\frac{(\tilde{p}_{nr}(\boldsymbol{t}, \boldsymbol{w}))^{-\sigma}}{(\tilde{P}_{jr}(\boldsymbol{t}, \boldsymbol{w}))^{1-\sigma}} \right] + \delta \ln \left[\sum_{j \in \mathcal{K}, v \in \mathcal{N}_j} \tilde{b}_{rv}(\boldsymbol{t}, \boldsymbol{w}) \beta_j \alpha_v \tilde{p}_v(\boldsymbol{t}, \boldsymbol{w})^{1-\sigma} \tilde{P}_j(\boldsymbol{t}, \boldsymbol{w})^{\sigma-1} \right] + \right. \quad (23)$$

$$\delta \ln K = \sum_n \left(\frac{K_n}{K}\right) \left\{ \delta \ln \left[\sum_{j \in \mathcal{K}, r \in \mathcal{N}_j} \tilde{b}_{nr}(\boldsymbol{t}, \boldsymbol{w}) \beta_j \alpha_r \tilde{p}_r(\boldsymbol{t}, \boldsymbol{w})^{1-\sigma} \tilde{P}_j(\boldsymbol{t}, \boldsymbol{w})^{\sigma-1} \right] + \delta \ln Y \right\} \quad (24)$$

Given changes in carbon taxes $\delta \boldsymbol{t}$ and the associated changes in factor prices $\{\delta \ln w_{ml}\}$, we can use (23) and (24) to solve for $\delta \ln C_m$ and $\delta \ln K$ as a function of $\delta \ln Y$. Substituting into (21), we can then solve for $\delta \ln Y$. The change in carbon emissions $\delta \ln C_m$ associated with burning fossil fuels in different locations is then obtained by plugging $\delta \ln Y$ into (23). Given initial carbon emissions C_m, we get δC_m required to implement (25).

3.4 Optimal Carbon Taxes

Following the same approach as in Costinot and Werning (2023) in an environment with externalities from carbon emissions, one can show that the optimal carbon price satisfies

$$t_m = \text{Pigouvian Tax} + \int \tau(z) \bar{x}(z) \frac{\epsilon}{\epsilon+1} \frac{\delta \ln \omega(z)}{\delta C_m} dz, \quad (25)$$

where $\tau(z)$ is the marginal income tax at quantile z; $\delta \ln \omega(z)$ is the local change in relative wages computed in Section (3.2); and δC_m is the change in carbon emission from market m computed computed in Section (3.3).

4 Optimal Carbon Taxation Revisited

Our plan is to use US micro-level data in order to calibrate the model from Section 3 and evaluate $\int \tau(z) \bar{x}(z) \frac{\epsilon}{\epsilon+1} \frac{\delta \ln \omega(z)}{\delta C_m} dz$, i.e. the extent to which carbon taxation should be affected

by concerns for redistribution.

5 Concluding Remarks

We provide a general formula for optimal carbon taxes in a second best world where governments may have concerns for redistribution, but only have access to nonlinear taxes on labor income. Our formula requires adding to standard estimates of the social cost of carbon an extra term that takes into account its potentially adverse consequences on earnings inequality. This extra term depends on a few sufficient statistics, including elasticities of relative wages with respect to changes in carbon emissions across quantiles of the income distribution. Combining a model of the US economy with detailed Census data, we provide the first estimates of these elasticities and explore their implications for carbon taxation.

References

Costinot, Arnaud and Ivan Werning, "Robots, Trade, and Luddism: A Sufficient Statistics to Optimal Technology Regulation," *Review of Economic Studies*, 2023, *90* (5), 2261–2291.

A Cross-Price Elasticities

A.1 Relative Factor Demand

In Section (3.2), we have established that relative factor demand is equal to

$$RD_{ml}(p_C,\boldsymbol{w}) = w_{ml}^{-\eta}\frac{\sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}_{mk}}\delta_{ln}\tilde{w}_n^{\eta-1}(\boldsymbol{w}_m)\beta_{Ln}e_n(p_C,\boldsymbol{w})}{\sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}_{m_0k}}\delta_{l_0n}\tilde{w}_n^{\eta-1}(\boldsymbol{w}_{m_0})\beta_{Ln}e_n(p_C,\boldsymbol{w})},$$

with

$$\begin{aligned}
\tilde{w}_n(\boldsymbol{w}_m) &\equiv [\sum_{l\in\mathcal{L}}\delta_{ln}w_{ml}^{1-\eta}]^{1/(1-\eta)},\\
\tilde{P}_k(p_C,\boldsymbol{w}) &\equiv [\sum_{n\in\mathcal{N}_k}\alpha_n\tilde{p}_n^{1-\sigma}(p_C,\boldsymbol{w})]^{1-\sigma},\\
\tilde{p}_n(p_C,\boldsymbol{w}) &\equiv \exp\{[\ln\phi_n+\beta_{Ln}\ln\tilde{w}_n(\boldsymbol{w}_m)+\beta_{Kn}\ln r_m+\beta_{En}\ln\tilde{p}_{En}(p_C)+\beta_{Mn}\tilde{p}_{Mn}(p_C,\boldsymbol{w})]\},\\
\tilde{p}_{Mn}(p_C,\boldsymbol{w}) &\equiv \left(\prod\theta_{nk}^{-\theta_{nk}}\right)\exp\{[\sum_k\theta_{nk}\ln\tilde{P}_k(p_C,\boldsymbol{w})]\},\\
\tilde{p}_{En}(p_C) &\equiv [\sum_{s\in\mathcal{S}}\zeta_{sn}(p_{sm}+\varepsilon_{sn}p_C)^{1-\kappa}]^{1/(1-\kappa)},
\end{aligned}$$

and

$$\tilde{e}_n(p_C,\boldsymbol{w}) = \alpha_n\tilde{p}_n(p_C,\boldsymbol{w})^{1-\sigma}\tilde{P}_{k(n)}(p_C,\boldsymbol{w})^{\sigma-1}\left[\tilde{P}_{k(n)}(p_C,\boldsymbol{w})^{1-\gamma}I+\sum_{n'}\theta_{n'k}\beta_{Mn'}\tilde{e}_{n'}(p_C,\boldsymbol{w})\right].$$

The goal of this appendix is to express the semi-elasticity $\partial\ln RD_{ml}(p_C,\boldsymbol{w})/\partial p_C$ as well as the elasticities $\{\partial\ln RD_{ml}(p_C,\boldsymbol{w})/\partial\ln w_{m'l'}\}$ as a function of the three structural elasticities η, σ and γ and initial shares. Using Shepard's Lemma, we get

$$\begin{aligned}
\frac{\partial\ln RD_{ml}(p_C,\boldsymbol{w})}{\partial p_C} =& (1-\sigma)\sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}_{mk},\mathcal{N}_{m_0k}}(s_{mln}-s_{m_0l_0n})(x_{Cn}-\bar{x}_{Ck})\\
&+(1-\gamma)\sum_{k\in\mathcal{K}}(\bar{s}_{mlk}-\bar{s}_{m_0l_0k})\bar{x}_{Ck}
\end{aligned}\tag{A.1}$$

where s_{mln} denotes the share of workers of type l in market m that are employed by firm n, $\bar{s}_{mlk}$ denotes the share of these workers that are employed in sector k, x_{Cn} denotes carbon emission per dollar of output, d_n denotes the share of expenditure on goods from sector k that goes to firm n, and $\bar{x}_{Ck}$ denotes the average carbon intensity among firms from sector k, weighted by their expenditure

shares within that sector,

$$
\begin{aligned}
s_{mln} &\equiv \frac{L_{mln}}{\sum_{n' \in \mathcal{N}_m} L_{mln'}} \text{ for all } l \in \mathcal{L}, m \in \mathcal{M} \text{ and } n \in \mathcal{N}_m, \\
x_{Cn} &\equiv \frac{\sum_{s \in \mathcal{S}} \varepsilon_{sn} E_{sn}}{\sum_{l \in \mathcal{L}} w_{ml} L_{mln} + r_m K_{mn} + \sum_{s' \in \mathcal{S}} (p_{s'm} + \varepsilon_{s'n} p_C) E_{s'n}} \text{ for all } n \in \mathcal{N}_m, \\
d_n &\equiv \frac{p_n q_n}{\sum_{n' \in \mathcal{N}_k} p_{n'} q_{n'}} \text{ for all } n \in \mathcal{N}_k, \\
\bar{s}_{mlk} &\equiv \frac{\sum_{n \in \mathcal{N}_{mk}} L_{mln}}{\sum_{n \in \mathcal{N}_m} L_{mln}} \text{ for all } l \in \mathcal{L}, m \in \mathcal{M} \text{ and } k \in \mathcal{K}, \\
\bar{x}_{Ck} &\equiv \sum_{n \in \mathcal{N}_k} d_{nk} x_{Cn} \text{ for all } k \in \mathcal{K}.
\end{aligned}
$$

The wage elasticities can be computed in a similar manner,

$$
\begin{aligned}
\frac{\partial \ln RD_{ml}(p_C, \boldsymbol{w})}{\partial \ln w_{m'l'}} = &-\eta \mathbb{1}_{\{ml = m'l'\}} + (\eta - 1) \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}_{mk}, \mathcal{N}_{m_0 k}} (s_{mln} - s_{m_0 l_0 n}) x^L_{m'l'n} \\
&+ (1 - \sigma) \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}_{mk}, \mathcal{N}_{m_0 k}} (s_{mln} - s_{m_0 l_0 n})(x_{m'l'n} - \bar{x}_{m'l'k}) \\
&+ (1 - \gamma) \sum_{k \in \mathcal{K}} (\bar{s}_{mlk} - \bar{s}_{m_0 l_0 k}) \bar{x}_{m'l'k}. \qquad \text{(A.2)}
\end{aligned}
$$

where $x^L_{m'l'n}$ denotes the share of firm n's labor cost devoted to group l' from market m'; $x_{m'l'n}$ denotes spending on labor group l' from market m' per dollar of output; and $\bar{x}_{m'l'k}$ denotes the average of $x_{m'l'n}$ among firms from sector k, weighted by their expenditure shares within that sector,

$$
\begin{aligned}
x^L_{m'l'n} &\equiv \frac{w_{m'l'n} L_{m'l'n}}{\sum_{l \in \mathcal{L}} w_{mln} L_{mln}} \text{ for all } n \in \mathcal{N}_m, \\
x_{m'l'n} &\equiv \frac{w_{m'l'n} L_{m'l'n}}{\sum_{l \in \mathcal{L}} w_{mln} L_{mln} + r_m K_{mn} + \sum_{s' \in \mathcal{S}} (p_{s'm} + \varepsilon_{s'} p_C) E_{s'n}} \text{ for all } n \in \mathcal{N}_m, \\
\bar{x}_{m'l'k} &\equiv \sum_{n \in \mathcal{N}_k} d_{nk} x_{m'l'n} \text{ for all } k \in \mathcal{K}.
\end{aligned}
$$

Note that since firms in market m only hire labor from that market, we have $x_{m'l'n} = 0$ if $m' \neq m$.

A.2 Relative Factor Supply

In Section (3.2), we have established that relative factor supply is equal to

$$
RS_{ml}(\boldsymbol{w}, \{\bar{L}(z)\}) = \frac{\pi_{ml}}{\pi_{m_0 l_0}} \frac{\int_0^1 \hat{\omega}(z, \boldsymbol{w}) (\hat{w}(z, \boldsymbol{w}) / w_{ml})^2 \bar{L}(z) f_{ml}(\hat{w}(z, \boldsymbol{w}) / w_{ml}) dz}{\int_0^1 \hat{\omega}(z, \boldsymbol{w}) (\hat{w}(z, \boldsymbol{w}))^2 \bar{L}(z) f_{m_0 l_0}(\hat{w}(z, \boldsymbol{w})) dz},
$$

with

$$z = \sum_{m\in\mathcal{M}}\sum_{l\in\mathcal{L}} \pi_{ml} F_{ml}(\hat{w}(z,\boldsymbol{w})/w_{ml}),$$
$$\hat{\omega}(z,\boldsymbol{w}) = \frac{1}{\sum_{m\in\mathcal{M}}\sum_{l\in\mathcal{L}} \pi_{ml} \frac{\hat{w}(z,\boldsymbol{w})}{w_{ml}} f_{ml}(\hat{w}(z,\boldsymbol{w})/w_{ml})}.$$

Taking log and differentiating, we obtain

$$\begin{aligned}\frac{\partial \ln RS_{ml}(\boldsymbol{w},\{\bar{L}(z)\})}{\partial \ln w_{m'l'}} = &\int_0^1 [\frac{\partial \ln \hat{\omega}(z,\boldsymbol{w})}{\partial \ln w_{m'l'}} + (2+\varepsilon_{ml}(z))(\pi_{m'l'}(z) - \mathbb{1}_{\{ml=m'l'\}})] d\Lambda_{ml}(z) \\ &- \int_0^1 [\frac{\partial \ln \hat{\omega}(z,\boldsymbol{w})}{\partial \ln w_{m'l'}} + (2+\varepsilon_{m_0l_0}(z))\pi_{m'l'}(z)] d\Lambda_{m_0l_0}(z)\end{aligned} \tag{A.3}$$

with

$$\begin{aligned}\frac{\partial \ln \hat{\omega}(z,\boldsymbol{w})}{\partial \ln w_{m'l'}} &= \pi_{m'l'}(z)[\varepsilon_{m'l'}(z) - \sum_{m''\in\mathcal{M}}\sum_{l''\in\mathcal{L}} \pi_{m''l''}(z)\varepsilon_{m''l''}(z)], \\ \pi_{ml}(z) &= \frac{\pi_{ml} f_{ml}(\bar{w}(z))}{\sum_{m'\in\mathcal{M}}\sum_{l'\in\mathcal{L}} \pi_{m'l'} f_{m'l'}(\bar{w}(z))}, \\ \varepsilon_{ml}(z) &= \frac{\bar{w}(z) f'_{ml}(\bar{w}(z))}{f_{ml}(\bar{w}(z))}, \\ \Lambda_{ml}(z) &= \frac{\int_0^z \bar{w}(z')\bar{L}(z')\pi_{ml}(z')dz'}{\int_0^1 \bar{w}(z')\bar{L}(z')\pi_{ml}(z')dz'}.\end{aligned}$$

A.3 Putting it Together

To summarize, the change in factor prices $\{\delta \ln w_{m'l'}\}$ associated with the change in carbon prices δp_c that is the solution to

$$\begin{aligned}&\frac{\partial \ln RD_{ml}(p_C,\boldsymbol{w})}{\partial p_C}\delta p_c + \sum_{m'\in\mathcal{M}}\sum_{l'\in\mathcal{L}} \frac{\partial \ln RD_{ml}(p_C,\boldsymbol{w})}{\partial \ln w_{m'l'}}\delta \ln w_{m'l'} \\ &\quad = \sum_{m'\in\mathcal{M}}\sum_{l'\in\mathcal{L}} \frac{\partial \ln RS_{ml}(\boldsymbol{w},\{\bar{L}(z)\})}{\partial \ln w_{m'l'}}\delta \ln w_{m'l'} \\ &\qquad - \int_0^1 \frac{\epsilon}{\epsilon+1}[\sum_{m'\in\mathcal{M}}\sum_{l'\in\mathcal{L}} \frac{\partial \ln \hat{\omega}(z,\boldsymbol{w})}{\partial \ln w_{m'l'}}\delta \ln w_{m'l'}][d\Lambda_{ml}(z) - d\Lambda_{m_0l_0}(z)] \text{ for all } ml \neq m_0l_0\end{aligned}$$

where the factor demand and supply elasticities are given by (A.1)-(A.3).

B Data

B.1 Industry Classifications

The paper defines oil and gas as a single pooled energy type for defining industries, though distinguishes oil from gas in measuring emissions intensity. Specifically, the input-output links like θ_{nk} and $d_{nn'}$ treat oil and gas as a single energy type, while carbon intensity measures like x_{Cn} treat them

separately. The rationale for this approach is that the US input-output table defines a single industry, "oil and gas," and NAICS industry codes also have a single detailed 6-digit code for "oil and gas production," 21111. In part this is because many wells produce both oil and gas. Because oil and gas are mining rather than manufacturing industries, the paper does not conduct establishment-level analysis of these oil and gas extraction. While it is possible to separate oil from gas in the US input-output table by assuming that use of oil and gas in each industry is proportional to the national share of MMBTU produced from oil versus gas (e.g., **LYUBICH ET AL AEAPP**), this assumption is accurate on average but not for each individual industry, so we do not impose it here.

Our construction of the US input-output table from make and use tables follows **LYUBICH ET AL AEAPP**. For calculating materials shares θ_{nk}, we define one set of 381 industries observed as inputs and outputs. Obtaining this classification requires dealing with four specific sets of unusual industries. First are industries that are never used as an intermediate input to other industries, only as an input to final demand. These industries have no associated industry (NAICS) code in the BEA's IO-NAICS concordance. We exclude these industries from calculation of θ_{nk} since they have zero materials input shares anyways. This group includes customs duties, which used as an input to imports which is a final demand column (4200ID); owner-occupied housing, which is listed as an input to final demand but no other industries (531HSO); and federal defense spending, which is an input to final demand but no other industries (S00500)

The second group of industries which require special consideration are those with zero output but that are listed as input rows in the input-output table. This includes state and local government educational services (GSLGE, which is distinct from the "elementary and secondary schools industry", which we keep); state and local hospitals and health services (GSLGH); and state and local other services (GSLGO). We exclude these.

The third group of special industries are those where governments provide services that the private sector also provides. For these, we recode the government industries to the same industry as the private industries. So federal electric utilities (S00101) and state and local electric utilities (S00202) both become part of 221100 (electricity generation and distribution); and state and local passenger transit (S00201) becomes part of "passenger and ground transportation" 485000).

The fourth group of special industries includes 'other government services.' This includes "other federal government" (S00102); "other state and local government enterprises" (S00203", and "federal general government-nondefense" (S00600). These don't link to any industry (NAICS) code and we exclude them from calculation of materials shares θ_{nk}.

We then aggregate industries in the input-output table so that each 6-digit 2012 NAICS industry maps to a unique industry in this paper's final classification. In addition to combining private and public industry codes for electricity (S00101, S00202, 221100) and transportation (S002021 and 485000), as discussed above, we also combine construction (23) into a single industry, and we combine tenant-occupied housing into other real estate (531HST and 531ORE).

B.2 Input-Output Tables

We work with input-output tables from the US Bureau of Economic Analysis (BEA), following their guidelines (https://www.bea.gov/sites/default/files/methodologies/industry_primer.pdf). We use data from 2012, a benchmark year. We use input-output tables after redefinitions. We work with values in producer prices.

We use three files from the input output tables–the make, use, and import tables. The make table describes the commodities that each industry produces. The make table has industries on rows and commodities on columns. The use table describes commodities that each industry uses. The use

table has has commodities on rows and industries on columns. The import table shows the value of imports of each commodity that an industry uses. The BEA creates the import table by imposing the proportionality assumption that each industry uses imports of a commodity in the same proportion as the ratio of imports to domestic supply of the commodity

Gross output appears in both the make and use tables. The right-most column of the make table shows gross output, which equals the sum of sales to firms as intermediate goods and to final demand. The bottom row of the use table also shows gross output, which equals the sum of expenditures on intermediate goods and value added. The right-most column of the make table and the bottom row of the use table show the same measure of gross output.

We deal carefully with imports. The use table describes total use of each commodity, regardless of whether the commodity is imported or domestically produced. Thus, each intermediate and final good flow described in the use table represents the sum of domestic and foreign production of that good. Gross output (the right-most column of the use table), however, represents only domestic supply of a commodity. Because most of the use table describes the sum of domestic and foreign production, but gross output represents only domestic supply, the use table must subtract imports. For this reason, imports appear with a negative in final demand. For the roughly 15 percent of commodities with substantial net imports, the use table therefore describes final demand as negative.

To avoid negative final demand, we work with a domestic use table. Specifically, we subtract the import matrix from the use table. The resulting domestic use table describes the domestic supply of commodities (in each row) to each industry (in each column). Final demand in the domestic use table includes only domestically supplied final demand. Gross output in the domestic use table is the same as in the original use table.