New Trade Models, Same Old Emissions?

Robin Sogalla Joschka Wanner Yuta Watabe*

October 2024

Abstract

This paper investigates the role of firm heterogeneity in environmentally extended new trade models, contrasting Eaton-Kortum and Melitz models to Armington and Krugman models. We show that when emissions per sales are constant across firms – a standard assumption in the literature – all four models predict identical emission responses. However, when emissions per quantity are constant across firms, this equivalence breaks. We propose a generalized framework that nests both assumptions. Calibrating the model with multiple industries and estimating the key elasticity between emission intensity and productivity using German firm-level data, we find that firm heterogeneity considerably raises emissions from trade liberalization.

JEL-Codes: F11; F12; F18; Q56
Keywords: International trade; carbon emissions; firm heterogeneity; quantitative modeling

*Watabe: Institute of Developing Economies, JETRO. Contact: yuta_watabe@ide.go.jp; Sogalla: German Institute for Economic Research (DIW), Contact: rsogalla@diw.de; Wanner: University of Wuerzburg, Kiel Institute of the World Economy, CESifo. Contact: joschka.wanner@uni-wuerzburg.de.
Acknowledgments will be added.

1 Introduction

Are exporting firms cleaner than domestic producers? Does trade liberalization drive dirty producers out of business? And how does international trade alter market shares across firms with different emission intensities? The feature of firm heterogeneity in "New New Trade Theory"' and the availability of firm-level data have deeply influenced research on trade and the environment, focusing on new micro-level questions. Yet, the macro-level implications of these micro-level insights remain unexplored: Do new trade models with firm heterogeneity deliver different predictions for *aggregate* emissions?

In their seminal paper, **?** (henceforth ACR) show that a broad class of trade models with very different micro-foundations — including **?**-, **?**-, **?**-, and **?**-type models (henceforth Armington, Krugman, EK, and Melitz) — yield the same expression for gains from trade. Hence, even though firm heterogeneity offers a new source of gains from trade — the reallocation of production towards more productive firms — this new channel does not translate into larger overall gains. This ACR insight raises the question: Can heterogeneous firm-level responses to trade liberalization potentially change overall environmental outcomes or are they, in a similar vein to ACR, neutralized in an aggregate consideration?

Our first key contribution is to establish an ACR-type equivalence result for single-industry models: if emissions are proportional to the value of production and if this proportion is common across firms, the aggregate emission predictions of the Armington-, Krugman-, EK-, and Melitz-type models are identical. This equivalence condition applies to production emissions in the majority of new quantitative trade and environment models, including models based on Armington (e.g. **???**), Krugman (**?**), EK (e.g. **?????**), and Melitz (**???**).[1,2] The intuition behind this equivalence is straightforward: the EK- and Melitz-type models do take into account the crucial empirical insight of exporters producing less emission-intensively (see e.g. **?**, **?**, and **?**, as well as **?** and **?** for recent surveys). Trade liberalization induces a reallocation of production towards these cleaner firms, leading to an environmentally beneficial aggregate technique effect (i.e. an economy-wide reduction of emissions per unit of output). However, as another key fea-

[1] It also applies to the quantitative model in **?**, which is written in a general form that can be interpreted as either an Armington- or an EK-type model.

[2] All examples given here are multi-industry models. We will return to this distinction below but already prefigure that a broad equivalence result will continue to hold.

ture of exporters is that they are more productive (see, e.g. **?**), these firms produce *more*, resulting in an emission-increasing scale effect. In this class of models, these two effects exactly offset each other.[3]

Our second main contribution is to show that the emission equivalence across Armington-, Krugman-, EK-, and Melitz-type models breaks down when emissions are linked to produced quantities with fixed proportions (see **?**, for a prominent example). Instead, models with heterogeneous producers (EK, Melitz) may then lead to higher emissions in the case of trade liberalization. This is because productivity differences are still considered, and the resulting scale effect drives up emissions, while emission intensity differences are absent in this case. Hence, there is no counteracting technique effect. Trade liberalization may lead to a shift in production to high-productivity firms and, therefore, to larger quantities produced and higher emissions. In the special case of movement from an economy in autarky to equilibrium with trade, we show that all countries' emissions increase in the models with firm heterogeneity.

Our third contribution is to propose a generalized modeling strategy for emissions in trade and environment models along the lines of **?** (henceforth KR). In this case, emissions are linked to the quantities produced, but more productive producers are allowed to be cleaner. The productivity-emission intensity relationship is disciplined by an additional parameter: the productivity elasticity of emissions. We derive expressions for aggregate emissions in Armington, Krugman, EK, and Melitz models with KR-type emission intensities. Similar to the quantity-based approach, the different models do *not* yield the same emission outcomes. The EK and Melitz models imply a scale effect that drives emissions up and a counteracting technique effect due to reallocations between producers. The magnitude of the latter depends on the KR elasticity parameter. If the emission intensity advantage of highly productive firms becomes sufficiently strong, firm heterogeneity may lead to *lower* aggregate emissions in response to trade liberalization. In this setting, we can show that when emissions are elastic [inelastic], moving from autarky to a trade

[3] **?** denote the case in which reallocation across firms decreases emissions per output more strongly than it increases total output as the Pollution Reduction by Rationalization (PRR) Hypothesis. One way to think about our equivalence result is that the PRR Hypothesis is ruled because the net emission effect of rationalization (i.e. of reallocating market shares to more productive firms) is zero by construction in these models.

equilibrium results in lower [higher] national and global emissions.[4]

Our fourth contribution is to extend our analysis to a multi-industry setting. The distinction between different types of trade models is decisive for scale and technique effects *within* sectors. In multi-industry environments, reallocations *across* sectors induce an additional composition effect: Some countries specialize in emission-intensive industries, facing higher emissions, while others focus on clean industries, reducing their emissions. Conditional on equal trade outcomes, our (non-)equivalence results across emission modeling strategies carry over to multi-industry models. Yet, due to economies of scale, trade outcomes differ between perfectly competitive multi-sector models (Armington and EK) and multi-sector models with monopolistic competition (Krugman and Melitz). Hence, in the value emission case, Armington and EK models imply the same emission responses to trade, which differ from those of the Melitz and Krugman models only because of differences in trade responses. In the flexible emission specification, emission effects again vary across models, with the impact of firm heterogeneity depending on sectoral KR elasticities.

Our final contribution is quantifying our theoretical insights using the flexible emissions version of the multi-industry Armington, Krugman, EK, and Melitz models calibrated with global production and trade data. We estimate the key elasticity between emission intensity and firm productivity using German firm-level data. Simulating a global reduction in trade costs by 40%, we find that in the Armington and Krugman models (and equivalently, in EK and Melitz models with unit KR elasticities in all sectors), global emissions increase only mildly by 1.0 and 1.3 %, respectively, in response to sectoral production shifts across countries. Using the estimated elasticities in the EK and Melitz setups instead, the global emissions increase in response to trade liberalization rises substantially to 8.6 and 5.2%, respectively. These increases indicate that the emission-increasing scale effect induced by firm heterogeneity is stronger than the emission-decreasing technique effect.

Our work is related to three main strands of literature. First, we add to the literature on the interplay between international trade and the environment (recently surveyed e.g.

[4]The KR approach nests the value- and quantity-based approaches as special cases. If the KR elasticity equals one, emission outcomes correspond to the case in which emissions are proportional to values, and the corresponding equivalence results hold. If the KR elasticity equals zero, the emission outcomes equal the ones from the case in which emissions are proportional to quantities.

by **??**). Specifically, we contribute to studies focusing on the role of firm heterogeneity in the trade and environment context, which include both theory (e.g. **?????**) and empirics (e.g. **?????**). In this literature, we take a bird's-eye view of how incorporating (some) insights from the micro-level into a macro-level quantitative framework affects aggregate outcomes.

Second, we relate to the new quantitative trade literature (see **??**, for overviews) and, more specifically, environmental extensions of quantitative trade models (see e.g. **????**). We clarify how modeling choices in the trade component affect the environmental effects they predict.

Finally, we contribute to the longstanding tradition of decomposing emission changes into scale, composition, and technique effects (**?????**). We elucidate how different trade modeling strategies shape these effects and whether differences in decomposition translate into different aggregate outcomes.

The rest of the paper proceeds as follows. Section **??** shows a simple example that incorporates emissions into a Melitz-type trade model with heterogeneous firms. Section **??** establishes our general (non-)equivalence results under three emissions modeling choices, with a particular focus on the four canonical types of trade models. Section **??** extends the model to multiple industries and derives the corresponding analytical results. In Section **??**, we bring flexible versions of the models to the data and quantify the impact of firm heterogeneity on the emission effects of international trade. Section **??** concludes.

2 A primer on trade and emissions with firm heterogeneity

To gain an intuition for our equivalence result, we start with a single-industry Melitz model with two emission settings: endogenous abatement in the spirit of **?** and emissions linked to quantities in a fixed proportion.

Emissions proportional to value – In the model, firms have heterogeneous productivity

φ. The physical output from country i for market n is produced using labor l_{ni}:

$$q_{ni}(\varphi) = (1 - \xi_i(\varphi))\varphi l_{ni}(\varphi),$$

where ξ_i is the share of labor devoted to abatement. The emissions in the production process are given by:

$$z_{ni}(\varphi) = (1 - \xi_i(\varphi))^{1/\alpha_i} l_{ni}(\varphi),$$

where α_i is the elasticity of the pollution emissions intensity with respect to pollution abatement intensity. With this abatement function, emissions can be equivalently expressed as a second factor of production (**?**):

$$q_{ni}(\varphi) = \varphi(z_{ni}(\varphi))^{\alpha_i}(l_{ni}(\varphi))^{1-\alpha_i}.$$

Given a price for emissions, t_i, and wages, w_i, the firm-level emissions embodied in the trade flow from i to n are a constant share of the revenues, $x_{ni}(\varphi) = p_{ni}(\varphi)\tilde{q}_{ni}(\varphi)$, :

$$z_{ni}(\varphi) = \alpha_i \frac{x_{ni}(\varphi)}{\tilde{\sigma} t_i},$$

where $\tilde{q}_{ni}$ is the quantity of goods consumed in country n and $\tilde{\sigma}$ is the markup ratio, which is common and constant.[5] The aggregate emissions embodied in the trade flow from i to n are obtained by aggregating the emissions of each exporting firm.[6] Under standard assumptions on the relative magnitude of fixed costs, a unique productivity cutoff for every market, φ^*_{ni}, exists that determines the set of firms that export from i to n. Only firms with productivity greater than or equal to this cutoff export. The emissions embodied in the total exports from i to n are given by:

$$Z_{ni} = M_{ni} E[z_{ni}(\varphi)|\varphi \geq \varphi^*_{ni}] = M_{ni} \int_{\varphi_{ni*}} \frac{\alpha_i}{\tilde{\sigma} t_i} x_{ni}(\varphi) dG(\varphi \geq \varphi^*_{ni}) d\varphi = \frac{\alpha_i}{\tilde{\sigma} t_i} X_{ni} \qquad (1)$$

where $X_{ni} \equiv M_{ni} \int_{\varphi_{ni*}} x_{ni}(\varphi) dG(\varphi \geq \varphi^*_{ni}) d\varphi$ is the aggregate export value from country i

[5] There is a threshold value of t_i/w_i where all firms do not abate. We assume that t_i/w_i is high enough so all the firms abate emissions. The case where $t_i = 0$ is discussed in Section **??**.

[6] For simplification, we assume fixed costs of exporting are not associated with emissions.

to country n. M_{ni} is the mass of firms producing in i and selling to n. It is evident from (**??**) that the change in the aggregate bilateral export value is a sufficient statistic for the change in embodied emissions.

Emissions proportional to quantities – We now provide an alternative assumption regarding the generation of emissions. Specifically, we assume that emissions are proportional to the quantity produced:

$$z_{ni}(\varphi) = \mu_i q_{ni}(\varphi),$$

where we assume the following one-factor production function without abatement:

$$q_{ni}(\varphi) = \varphi l_{ni}.$$

With an analogous calculation as in the previous case, the emissions embodied in exports from i to n are given by:

$$Z_{ni} = M_{ni} E[z_{ni}(\varphi)|\varphi \geq \varphi^*_{ni}] = M_{ni} \int_{\varphi_{ni*}} \mu_i q_{ni}(\varphi) dG(\varphi \geq \varphi^*_{ni}) d\varphi = \mu_i Q_{ni} \tag{2}$$

where $Q_{ni} \equiv M_{ni} \int_{\varphi_{ni*}} q_{ni}(\varphi) dG(\varphi \geq \varphi^*_{ni}) d\varphi$ is the aggregate quantity of goods produced to export from i to n.[7] In this case, the change in the aggregate export *quantity* is a sufficient statistic for the change in embodied emissions.

There are two main differences in the treatment of the two cases. The first concerns how firms' productivity is related to their emission intensity. When emissions are proportional to the value, more productive firms use less labor and emit less per output unit.[8] In contrast, when emissions are proportional to quantity, more productive firms may use less labor but emit the same amount per output unit.

Trade liberalization changes the productivity composition of firms, which, in the value case, affects both the quantity and emissions per unit of output. However, in the case of quantity, it only changes the quantity produced. Regarding the decomposition of emission changes (**??**), trade in both cases induces an emission-increasing scale effect. It

[7] This quantity includes an iceberg component.

[8] Emission per quantity are inversely proportional to productivity ($z_{ni}/q_{ni} \propto 1/\varphi_{ni}$).

is counteracted by an emission-decreasing technique effect in the value case but the sole effect in the quantity case.[9]

The second difference lies in the applicability of the ACR results. In a scenario where emissions are proportional to the trade value, the model benefits from the ACR finding that changes in trade value coincide across a range of models. If emissions per value are common across different models (e.g., Armington, Krugman, EK, and Melitz) and the ACR result of equal changes in aggregate trade flow holds, the changes in aggregate emissions will be identical. However, in a scenario where emissions are proportional to quantity, even though the emission intensity is the same across models, the changes in trade quantity differ, breaking emission equivalence. In the next section, we formalize this intuition and present the analytical results for changes in aggregate emissions associated with trade shocks.

3 (Non-)Equivalence of emission effects

3.1 General model set-up

This section introduces a general model encompassing the Armington, Krugman, EK, and Melitz models with different emission mechanisms, including endogenous abatement as in **?**.[10] In this section, we consider a single-industry setting. We will show how this section's results can be embedded into a multi-sector setting in Section **??**.

The global economy comprises $i = 1, ..., N$ countries. Each country has a mass of consumers l_i, each supplying a unit of labor inelastically. There is a mass of varieties Ω, which is potentially endogenous.[11] A variety can be produced by multiple firms or a single firm.

Preferences – The preference of a representative consumer in country i is of the Dixit-Stiglitz form, maximizing utility from consuming a variety of goods. The associated price

[9]The exact decompositions for each case and each model are shown in Section **??**.

[10]We also discuss the case of energy usage being the source of emissions. See the Online Appendix for details.

[11]Following ACR, Ω may include either a continuum or a discrete number of goods.

index is:

$$P_i = \left(\int_{i \in \Omega_i} p_i(\omega)^{1-\sigma} d(\omega) \right)^{1/(\sigma-1)},$$

where Ω_i is the set of varieties available in country i and σ is the elasticity of substitution between varieties.

Production and emissions – If a firm in country i produces variety ω, it requires labor:

$$q_i(\omega) = f_{i,\omega}(l_i^p(\omega)),$$

where $l_i^p(\omega)$ is a labor input for production. We allow the production technology of variety ω to be country-specific. Emissions are associated with production and can be partially abated by labor input:

$$z_i(\omega) = g_{i,\omega}(q_i, l_i^z(\omega))$$

where $l_i^z(\omega)$ is the labor input for abatement of variety ω in country i. Again, we allow this abatement technology to be country-specific. We assume $g_{i,\omega}$ is increasing in q_i and nonincreasing in l_i^z.[12] Firms have to pay a carbon tax, t_i, per emission. The firm's cost minimization problem is:

$$\begin{aligned} C_{i,\omega}(w_i, t_i, \bar{q}) &= min_{l^p, l^z, z} w_i(l^p + l^z) + t_i z \\ \text{subject to } \bar{q} &= f_{i,\omega}(l^p) \\ z &= g_{i,\omega}(\bar{q}, l^z), \end{aligned}$$

where w_i and t_i are the wage and emission taxes in country i, respectively. The solution to this minimization problem is characterized by a variable cost function, $C_{i,\omega}(w_i, t_i, q)$, labor demand function, $l_{i,\omega}(w_i/t_i, q)$ and the emission output function, $z_{i,\omega}(w_i/t_i, q)$.[13]

Trade – Firms face an iceberg trade cost τ_{ni} and potentially a fixed cost f_{ni} when

[12] We allow the case where firms cannot abate emissions.

[13] Because $l_{i,\omega}(w_i/t_i, q)$ and $z_{i,\omega}(w_i/t_i, q)$ are factor demand functions, they only depend on the factor price ratio and the quantity.

exporting. Specifically, as variable trade costs, firms in country i have to produce τ_{ni} amount of goods to sell one unit to country n. For a fixed cost, firms have to pay f_{ni} amount of labor in the destination country. We denote the shipped quantity and the value of export as $q_{ni}(\omega)$ and $x_{ni}(\omega)$, respectively, and denote associated emissions as $z_{ni}(\omega)$.

Market Structure – The market structure can be either (i) perfect competition or (ii) monopolistic competition. In the case of perfect competition, anyone can produce variety ω, and there are large numbers of potential firms and consumers. Firms and consumers take the prices, wages, and emission taxes as given. In this case, there are no fixed costs of exporting ($f_{ni} = 0$).

For monopolistic competition, we consider both free and fixed entry. In free entry, each firm can produce a unique variety ω by paying a fixed entry cost $F_i > 0$ in terms of labor. This fixed entry cost is not associated with emissions. Firms enter until their expected profit from entry is zero. We denote the mass of firms in country i by M_i. In the case of fixed entry, each country has a fixed mass of firms, each producing a unique variety ω. The profits earned by the firms in i in the fixed entry case are given to consumers in the country. Firms take the aggregate price index and wage as given in both free and fixed entry and maximize their profit.

Equilibrium – The carbon tax in country i is exogenously given, and tax revenue is repatriated to consumers in the country. In equilibrium, the labor and goods markets clear.

Before discussing specific assumptions on how emissions are related to production, we briefly state how the four canonical trade models we consider throughout the paper fit into the general model structure outlined thus far.[14]

Armington – The Armington model arises as a special case of the general model structure if we assume perfect competition and a fixed discrete number of varieties, namely one per country.

Krugman – In the Krugman case, firms engage in monopolistic competition; the num-

[14]We detail the formulations of these models in the Online Appendix.

ber of firms in each country is endogenous, and each firm produces its unique variety. The production technology is the same for all firms within a country. We assume there is no emission associated with fixed costs, but we can allow entry cost payments involving abatement and emissions as in production.[15]

EK – The EK model is the second perfect competition special case with a fixed set of varieties. In this case, the set of varieties is a unit interval, and every variety can be produced in every country. Countries differ in their efficiency of producing these varieties and draw their productivities from a Fréchet distribution with location parameter T_i and dispersion parameter θ.

Melitz – The Melitz model is similar to Krugman's, but firms differ in their productivity. Firms in every country draw their productivities from a Pareto distribution with scale parameter T_i and shape parameter θ. We assume export fixed costs are paid in terms of the labor of the importing country.[16] We assume there is no emission associated with entry costs and export fixed costs, but we can allow these cost payments involving abatement and emissions as in the production setting.[17]

3.2 Emissions linked to values

Up until now, we have kept the model specification deliberately general. In order to pin down if and how different trade models with and without firm heterogeneity have different emission implications, we will have to take a more specific stand on how emissions arise in the production of goods. We will consider three alternative modeling choices and start in this section with the most common approach, characterized by the following restriction.

[15]Notice that in the Krugman model, the fixed cost payment is a constant fraction of revenue. If the emission per entry cost, in monetary terms, is constant across varieties (similar to Restriction **??** below), then the emissions associated with entry costs only depend on the aggregate revenue. Therefore, Proposition **??** below will remain to hold.

[16]This is to make the model consistent with the ACR assumptions.

[17]For the entry cost, see the footnote on the Krugman model. Notice that in the Melitz model, the export fixed cost payments are a constant fraction of the importer's revenue. If we assume that emission per monetary value of export fixed cost is constant across varieties (similar to Restriction **??** below), then the emissions associated with export fixed costs only depend on the aggregate revenue of importers. Therefore, Proposition **??** below will remain to hold.

Restriction 1. *Denote the export value of a variety ω from country i to country n as $x_{ni}(\omega)$. The first assumption is that the emission embodied in the export of variety ω from country i to country n is proportional to the export value, and the proportion is common across varieties:*

$$z_{ni}(\omega) = \frac{\beta_i(w_i/t_i)}{t_i} x_{ni}(\omega), \tag{3}$$

where β_i can be fixed or depend on the country's wages relative to the emission cost.

A simple interpretation of this formula is that the proportion of emissions embodied in trade is common across firms and can be written as the emission cost share divided by the emission cost. First, we discuss three assumptions that achieve this restriction when combined. Notice that these assumptions are slight generalizations of the example we discussed in the previous section:

Common cost share – In country i, for any variety ω, the emission (tax) cost share is common and constant for different quantities:

$$\frac{t_i z_{i,\omega}(w_i/t_i, q)}{C_{i,\omega}(w_i, t_i, q)} = \alpha_i(w_i/t_i),$$

where $\alpha_i(w_i/t_i)$ is a cost share function that only depends on the relative wages.

Constant marginal cost – In any country and any variety, the marginal cost is constant:

$$C_{i,\omega}(w_i, t_i, q) = c_{i,\omega}(w_i, t_i)q,$$

where $c_{i,\omega}(w_i, t_i)$ is the marginal cost function, which differs across the country of production and depends on the factor prices.

Constant markup – The factory gate price of variety ω in country i is as follows:

$$p_{i,\omega}(w_i, t_i) = \tilde{\sigma} c_{i,\omega}(w_i, t_i),$$

where $\tilde{\sigma}$ is a markup ratio in the economy.

Constant markup and constant marginal cost jointly imply that the price of variety ω

produced in country i and sold in country n is:

$$p_{ni,\omega}(w_i, t_i) = \tilde{\sigma}\tau_{ni}c_{i,\omega}(w_i, t_i)$$

These assumptions indicate that the emissions per monetary unit of export sales from i to n are constant across varieties:

$$\begin{aligned}\frac{z_{ni}(\omega)}{x_{ni}(\omega)} &= \frac{z_{ni}(\omega)}{\tilde{\sigma}c_{i,\omega}(w_i, t_i)q_{in}(\omega)} \\ &= \frac{\alpha_i(w_i/t_i)}{\tilde{\sigma}t_i}.\end{aligned}$$

These common assumptions in the literature result in Restriction **??** with $\beta_i(w_i/t_i) = \alpha_i(w_i/t_i)/\tilde{\sigma}$. Examples include the endogenous abatement modeling a la **?** in **?** and the Cobb-Douglas or CES energy input with proportional emissions in **?** and **?**.[18]

3.2.1 Aggregate implications

Denote Z_{ni} as the total emissions associated with the exports from country i to country n. This can be written as

$$Z_{ni} = \int_{\omega\in\Omega_{ni}} z_{ni}(\omega)d\omega = \int_{\omega\in\Omega_{ni}} \frac{\beta_i(w_i/t_i)}{t_i}x_{ni}(\omega)d\omega = \frac{\beta_i(w_i/t_i)}{t_i}X_{ni}$$

where $X_{ni} \equiv \int_{\omega_{ni}\in\Omega} x_{ni}(\omega)d\omega$ is the total export from country i to country n. This equation suggests that the emissions associated with trade do not depend on the micro-composition of emissions.

Building on ACR, we introduce the notion of trade shocks and explore their impact on aggregate emissions.

Definition 1. *A trade shock with constant emission tax is a change from* $\{\boldsymbol{\tau}\}$ *to* $\{\boldsymbol{\tau'}\}$*, while fixing the relative price of emission* $(\boldsymbol{t}/\boldsymbol{w} = \boldsymbol{t'}/\boldsymbol{w'})$*.*

We choose a constant relative emission price to fix the environmental policy and focus

[18]The treatment of emissions in **??** does not exactly fall under Restriction **??**, because there is no emission price in these models. Instead, emissions are assumed to be generated accoroding to $z_{ni}(\omega) = \beta_i x_{ni}(\omega)/c_i$. Crucially, this also links emissions to the export value with a common intensity across varieties. All results derived for models under Restriction **??** below, therefore, also apply to the models in **??**.

on trade liberalization.[19,20] We fix the emission tax relative to the wages because t_i is a nominal variable, and $\boldsymbol{t} = \boldsymbol{t}'$ is not a meaningful policy rule.[21] We denote the economic variable v after trade shock as v', and denote $\hat{v} \equiv v'/v$ as the relative changes in the economic variable v following **??**. Using the trade shocks, we show that the changes in the aggregate emissions ($Z_i = \sum_{n=1}^{N} Z_{ni}$) can be derived by only tracking the aggregate trade flows, wages, and emission taxes:

Proposition 1. *Suppose Restriction* **??** *is satisfied. The change in the aggregate emission for country i through a trade shock with constant emission tax can be written as*

$$\hat{Z}_i = \frac{\hat{R}_i}{\hat{w}_i} \tag{4}$$

where $R_i \equiv \sum_{n=1}^{N} X_{ni}$ *is the total revenue in country i .*

Proof. By Restriction **??**, we showed that:

$$Z_{ni} = \frac{\beta_i (w_i/t_i)}{t_i} X_{ni},$$

and this implies

$$Z_i = \sum_{n=1}^{N} Z_{ni} = \frac{\beta_i (w_i/t_i)}{t_i} R_i.$$

Notice we fix the relative wages (w_i/t_i), and the emission tax moves proportional to the wages ($\hat{w}_i = \hat{t}_i$). Taking the ratio between the emissions of the baseline equilibrium and the equilibrium after a trade shock yields:

$$\hat{Z}_i = \frac{\hat{R}_i}{\hat{t}_i} = \frac{\hat{R}_i}{\hat{w}_i}.$$

□

[19] Similar assumptions are discussed in **?**, pp. 112, 146

[20] Note that we formulate the analytical insights generally in terms of not further specified emissions and environmental policy. In the quantification exercise, we will specifically focus on carbon emissions.

[21] When $\boldsymbol{t} = \boldsymbol{t'}$, the emission outcome crucially depends on the normalization of wages. Note that this rules out any factor price-driven within-firm emission intensity changes. Any technique effect will, therefore, come from reallocation between firms with different productivities (see **?**, for a discussion of these two different sources of the technique effect). To differentiate from the within-firm technique effect, **?** denote the reallocation-induced technique effect as a "rationalization effect".

3.2.2 Combining the restriction with ACR

The results of Theorem **??** are complemented with the restrictions imposed in ACR. These are macro restrictions that models may satisfy.[22] We slightly modify their restriction to introduce an emission tax and state the restrictions similar to R1, R2, and R3' in ACR.

ACR R1: Trade balance – For any country i, trade is balanced:

$$\sum_{i=1}^{N} X_{ni} = \sum_{i=1}^{N} X_{in}.$$

This assumption implies that the total revenue R_i is the same as total expenditure $Y_i \equiv \sum_{n=1}^{N} \int_{\omega \in \Omega} x_{in}(\omega) d\omega$.[23] ACR further imposes a constant aggregate profit share for each country:

ACR R2: Constant aggregate profit – Denote the aggregate profit of country i by $\Pi_i \equiv \int_{\omega \in \Omega} \pi_i(\omega) d\omega$, and the aggregate revenue of country i by R_i. For any country i, Π_i / R_i is constant.

For the third assumption, we introduce a slightly modified version of R3' in ACR to incorporate carbon taxes:

ACR R3': CES import demand – The import demand system is such that for any exporting country i and importing country n, the expenditure share of country n on goods from country i is:

$$\lambda_{ni} = \frac{\chi_{ni} \cdot M_i \cdot (\tilde{c}_i(w_i, t_i)\tau_{ni})^{\varepsilon}}{\sum_{k=1}^{N} \chi_{nk} \cdot M_k \cdot (\tilde{c}_k(w_k, t_k)\tau_{nk})^{\varepsilon}}.$$

where χ_{nk} is a parameter, M_i is a number of varieties, and $\tilde{c}_i(w_i, t_i)$ is a function that combines wages and the emission tax, which is homogeneous degree of one. The trade elasticity is denoted by ε.

We can show that as in ACR, the changes in trade flows associated with trade shocks are the same across models. Combining this result with our Proposition **??**, we obtain the following proposition:

[22] In all the models we discuss, these restrictions are satisfied.

[23] We could include trade deficits as an exogenous transfer between countries.

Proposition 2. *Consider a model that satisfies Restriction* **??***, ACR R1, ACR R2, and ACR R3'. The relative change in emissions associated with trade shocks with constant carbon tax is*

$$\hat{Z}_i = 1.$$

Trade share, revenue, and wage changes are determined by:

$$\hat{\lambda}_{ni} = \frac{\left(\hat{\tau}_{ni}\hat{w}_i\right)^{\varepsilon}}{\sum_{k=1}^{N} \lambda_{nk} \left(\hat{\tau}_{nk}\hat{w}_k\right)^{\varepsilon}}, \tag{5}$$

$$R_i^{'} = \hat{w}_i R_i, \tag{6}$$

$$\hat{w}_i = \frac{1}{R_i} \sum_{n=1}^{N} \hat{\lambda}_{ni} \lambda_{ni} R_n^{'}, \tag{7}$$

where R_i is the total revenue of country i.

Proof. See Appendix **??**. □

The result is striking; the proposition shows that global emissions remain constant following any trade shocks.[24] Our result highlights that in a single-industry trade model with emissions proportional to values, trade shocks do not change the aggregate emissions. We briefly discuss the rationale behind the absence of an emission effect in the Armington, Krugman, EK, and Melitz models.

Armington – In the single-industry Armington model, trade only affects where goods are sold and leaves production unaffected. As there are no heterogeneous producers, the same quantity is produced using the same technology (and hence emission intensity), irrespective of trade openness. In terms of the emission decomposition discussed in the primer, there is a zero scale effect and a zero technique effect (and — in the absence of a sectoral structure for now — a zero composition effect).

Krugman – As in the Armington case, trade only affects the destination products are sold to, not the overall quantity produced. In the Krugman case, this is less obvious

[24]While it is possible to generate changes in emissions by fixing $\boldsymbol{t}$ and changing $\boldsymbol{w}$, the aggregate effect on emissions crucially depends on the wage normalization scheme.

because each firm features a production firm with an increasing return to scale. However, if the ACR Restrictions hold, the mass of active firms does not change with trade, and the scale economies hence are not affected, and the overall quantities and production techniques stay the same.

EK – In EK, trade allows countries to focus on goods they can produce efficiently. Hence, trade increases the overall quantity produced. Simultaneously, specialization in goods for which a country has high productivity implies specialization in goods that the country can produce at a low emission intensity. These two effects — a positive scale effect and a negative emission effect — perfectly offset each other.

Melitz – In Section **??**, we discussed that in the Melitz model, trade leads to a reallocation towards highly productive firms and, hence, to higher quantities but lower emission intensities. Proposition **??** implies that, as in the EK case, the scale and technique effects cancel each other out exactly. Notably, while EK and Melitz share this property, the magnitudes of the scale and technique effects do not coincide across the two models.

3.3 Emissions linked to quantities

In an environmental context, physical units play a crucial role, as we are interested in how much firms pollute rather than how much they spend on pollution. A clear alternative for introducing emissions into trade models is to link them directly to physical output.[25] This quantity-based emission modeling approach, for example, is used by **?**, who embeds it into an Armington trade model.

While the value-based approach is more common and allows flexibility in substituting emissions/energy against other production inputs, there are contexts where focusing on quantities is more appropriate. Examples include transportation emissions (where the physical quantity shipped between countries matters) and process-related emissions (which, unlike emissions from fossil fuel combustion, cannot be avoided by shifting inputs away from fossil fuels). We formalize this setting in the following restriction:

[25]We can also rationalize the results below with potential abatement with $z_i(\omega) = g_{i,\omega}(q_{ni}, 0) = \mu_i q_{ni}$. When we assume $t_i = 0$, there is no abatement, and the emissions are proportional to quantity.

Restriction 2. *Denote the export quantity of a variety ω from country i to country n as $q_{ni}(\omega)$. Emissions embodied in the export of variety ω from country i to country n are proportional to the export quantity, and the proportion is common across varieties:*

$$z_{ni}(\omega) = \mu_i q_{in}(\omega), \tag{8}$$

where μ_i is fixed.

The aggregate implication is similar to the common value assumption case:

$$Z_{ni} = \int_{\omega \in \Omega_{ni}} z_{ni}(\omega) d\omega = \int_{\omega \in \Omega_{ni}} \mu_i q_{ni}(\omega) d\omega = \mu_i Q_{ni}, \tag{9}$$

where $Q_{ni} \equiv \int_{\omega \in \Omega_{ni}} q_{ni}(\omega) d\omega$ is the aggregate *quantity* produced to export from country i to country n.[26] The aggregate changes in the emissions following a trade shock is a weighted sum of the change in quantities.

Proposition 3. *Suppose Restriction* **??** *is satisfied. The change in the aggregate emission for country i in response to a trade shock can be written as:*

$$\hat{Z}_i = \sum_{n=1}^{N} \iota_{ni} \hat{Q}_{ni}, \tag{10}$$

where $\iota_{ni} \equiv \frac{Z_{ni}}{Z_i}$, and in this specific case, we also have $\iota_{ni} = \frac{Q_{ni}}{\sum_k Q_{ki}}$.

Proof. From the Restriction **??**, the aggregate emission Z_i is:

$$Z_i = \mu_i \sum_{n=1}^{N} Q_{ni}.$$

Taking the ratio between counterfactual Z_i' and Z_i and expressing $Q_{ni}' = \hat{Q}_{ni} Q_{ni}$ yields:

$$\hat{Z}_i = \frac{\sum_{n=1}^{N} Q_{ni}'}{\sum_{n=1}^{N} Q_{ni}} = \frac{\sum_{n=1}^{N} \hat{Q}_{ni} Q_{ni}}{\sum_{n=1}^{N} Q_{ni}} = \sum_{n=1}^{N} \iota_{ni} \hat{Q}_{ni},$$

where $\iota_{ni} = \frac{Q_{ni}}{\sum_{k=1}^{N} Q_{ki}} = \frac{Z_{ni}}{\sum_{k=1}^{N} Z_{ki}}$. □

[26] To be clear, the quantity includes the amount of goods used to pay the iceberg costs.

The change in export quantity in the Armington and Krugman models is derived as $\hat{Q}_{ni} = \hat{X}_{ni}/\hat{p}_{ni}$. However, as discussed, it is not trivial to derive changes in the quantity in the EK and Melitz models.

We adopt another refinement to Restriction **??** so that the model is compatible with ACR:

No emission tax – Producers do not pay a price for their emissions:

$$t_i = 0.$$

This assumption implies that labor is the only relevant cost in production.[27] We combine these assumptions with the ACR restrictions:

Proposition 4. *Consider a model that satisfies Restriction* **??***, ACR R1, ACR R2, and ACR R3'. The percentage change in emissions associated with any change in variable trade costs in country i can be expressed as*

$$\hat{Z}_i = \sum_{n=1}^{N} \iota_{ni} \cdot \hat{Q}_{ni}.$$

Trade share changes, counterfactual revenues, and wage changes are determined by:

$$\begin{aligned}
\hat{\lambda}_{ni} &= \frac{\left(\hat{\tau}_{ni}\hat{w}_i\right)^{\varepsilon}}{\sum_{k=1}^{N} \lambda_{nk} \left(\hat{\tau}_{nk}\hat{w}_k\right)^{\varepsilon}}, \\
R_i^{'} &= \hat{w}_i Y_i, \\
\hat{w}_i &= \frac{1}{R_i} \sum_{n=1}^{N} \hat{\lambda}_{ni}\lambda_{ni}R_n^{'}.
\end{aligned}$$

The change in the quantity, $\hat{Q}_{ni}$, and hence the emission change varies across different models (e.g., Armington, Krugman, EK, and Melitz).

Proof. The proof for $\hat{Z}_i$ is already provided in the proof of Proposition **??**. The trade share changes follow from the ACR restrictions. □

The key insight is that although trade values are common in the ACR class of models,

[27] With positive emission taxes, the model cannot be reduced to the ACR class of trade models.

trade quantities differ across models. Combining Restriction **??** with ACR restrictions gives rise to specific and common emission predictions. Meanwhile, combining Restriction **??** with ACR restrictions is insufficient to pin down the emission response to a trade shock. There is no general solution for $\hat{Q}_{ni}$, and a closed-form solution may not exist. Fortunately, we can show that there are closed-form solutions for the canonical models we consider (i.e., Armington, Krugman, EK, and Melitz).

Armington – In the case of the Armington model, we have

$$\hat{Q}_{ni} = \frac{\hat{X}_{ni}}{\hat{w}_i}. \tag{11}$$

and $\iota_{ni} = \frac{X_{ni}}{Y_i}$. The equation implies $\hat{Z}_i = \hat{R}_i/\hat{w}_i = 0$, which coincides with the case of emissions associated with values. In the Armington model, emissions do not change with trade shocks. Without a technology change, the production quantity remains unchanged. Trade shocks only change the allocation of goods used for iceberg transportation costs and consumption.

Krugman – The expressions for the quantity change $\hat{Q}_{ni}$ and the emission share ι_{ni} in the Krugman model coincide with the Armington case. Intuitively, as all producers use the same technology, the sales share coincides with the emission share, as there is no selection of exporters. Also, as trade liberalization does not affect the mass of active firms or their prodution technologies, aggregate emissions do not change.

EK – For the EK model, the quantity of production is not only a function of trade values but also of the import share $\lambda_{ni} = \frac{X_{ni}}{X_n}$. Specifically, changes in the production quantities associated with trade shocks are:

$$\hat{Q}^{EK}_{ni} = \frac{\hat{X}_{ni}}{\hat{w}_i}\hat{\lambda}^{-1/\theta}_{ni}, \tag{12}$$

where θ is a parameter of the Fréchet distribution. The quantity share is given by:

$$\iota_{ni} = \frac{X_{ni}\lambda^{-1/\theta}_{ni}}{\sum_{k=1}^{N} X_{ki}\lambda^{-1/\theta}_{ki}}.$$

Appendix **??** provides the derivation of these results. When comparing to the Armington model, it's important to note that the EK model accounts for endogenous changes in productivity due to the selection of more productive varieties. In the EK framework, λ_{ni} represents the import share and the proportion of varieties that country i exports to country n. A key implication is that a lower λ_{ni} indicates a higher quantity for a given trade value X_{ni}, reflecting the export of only highly productive varieties charging lower prices. Furthermore, when calculating changes in quantity (and consequently emissions), it is essential to recognize that value shares do not precisely conincide with quantity shares. This distinction is critical and is incorporated into the computation of ι_{ni}.

Melitz – For the Melitz model, the quantity of production is a function of the export value, domestic productivity cutoff φ^*_{ii}, and share of exporting firms to a particular destination, $S_{ni} = M_{ni}/M_i$:

$$\hat{Q}^{Melitz}_{ni} = \frac{\hat{X}_{ni}}{\hat{w}_i}\hat{S}^{-1/\theta}_{ni}\hat{\varphi}^*_{ii} = \frac{\hat{X}_{ni}}{\hat{w}_i}\hat{\lambda}^{-1/\theta}_{ni}, \tag{13}$$

where θ is a shape parameter of the Pareto distribution and the quantity share is:

$$\iota_{ni} = \frac{S^{-1/\theta}_{ni}X_{ni}}{\sum_{k=1}^{N} S^{-1/\theta}_{ki}X_{ki}}$$

The derivation of $\hat{Q}^{Melitz}_{ni}, \hat{S}_{ni}$ and $\hat{\varphi}^*_{ii}$ are delegated to Appendix **??**. Compared to the homogenous firm's models, changes in quantities in the Melitz model must again consider endogenous productivity changes due to selection. Changes in S_{ni} represent the productivity changes due to the selection in country i of being an exporter to country n. In addition to the changes in S_{ni}, we must consider the change in the domestic productivity distribution of producers, which is represented by the changes in φ^*_{ii}. Ultimately, these two forces are summarized by the change in the bilateral expenditure share. In the Melitz-Pareto model, the change in expenditure share with respect to iceberg trade costs is entirely driven by the extensive margin. Hence, an increase in the expenditure share is directly linked to a higher number of exporting firms, which indicates that also less productive firms can serve this destination leading to a lower average productivity. Like in the EK model, we need to consider that the quantity share differs from the value

share. Note that while the source of productivity changes differ across EK and Melitz, their magnitude coincides in a single-industry economy. The differences between emission effects across these two models originate entirely in differences in the bilateral emission shares ι_{ni}.

Mimicking ACR's expression for the welfare effects of a move to autarky, we can derive a common formula for the emission effects of a move to autarky in the two models with firm heterogeneity considered here:[28]

$$\hat{Z}_i^A = \iota_{ii} \lambda_{ii}^{(1-\theta)/\theta}. \tag{14}$$

Note two key differences from ACR's welfare formula. First, even though the formula is identical for EK and Melitz, it does not deliver the same quantitative results. This is because even for identical domestic *trade* shares, domestic *emission* shares differ across these models. Second, even though it only contains domestic shares, to quantify (**??**), one has to observe all of a country's trade relations. This is because the domestic emission share depends on all of a country's trade shares (and shares of exporting firms in the Melitz case) rather than only the domestic sales share.

Using the insights from Proposition **??** and Equations (**??**) to (**??**), we derive the emission effects of trade opening:

Definition 2. *Trade opening is a change in trade costs from autarky (infinite iceberg trade costs) to finite trade costs.*

Proposition 5. *Consider a model that satisfies Restriction* **??***, ACR R1, ACR R2, and ACR R3'. Trade opening*

(i) leaves aggregate emissions in Armington and Krugman models unaffected

(ii) increases emissions in the EK and the Melitz models.

Proof. See Appendix **??**. □

[28]The derivation is provided in the Online Appendix.

3.4 A flexible emission specification

The two emission modeling approaches considered so far have very different implications. In the quantity-based setting, more productive firms are not at the same time cleaner than less productive firms. Hence, nothing counteracts the scale effect in the case of trade liberalization. In the value-based setting, more productive firms are cleaner firms, introducing a counteracting reallocation effect on emissions. However, for the value case, the magnitude of this effect turns out to be identical to the scale effect and, therefore, neutralizes the role of firm heterogeneity. In this section, we follow **?** and consider an emission-generating process linked to the produced quantities and relate emission intensity to productivity with a technology parameter.

Restriction 3. *Assume that the production function is a linear one-factor function and emissions are generated according to the following expression:*

$$q_i(\omega) = \varphi_i(\omega) l_i(\omega), \tag{15}$$

$$z_i(\omega) = \frac{\mu_i q_i(\omega)}{[\varphi_i(\omega)]^{\gamma_i}}, \tag{16}$$

where $\varphi_i(\omega)$ denotes the productivity for variety ω in country i.

The relationship between the physical emission intensity and firm productivity now depends on the value of γ_i. If $\gamma_i < 0$, more productive firms are dirtier than less productive firms. Conversely, $\gamma_i > 0$ relates to the empirically more relevant case in which highly productive firms produce less emission-intensively. Whether the associated reallocation effect suffices to offset the scale effect entirely depends on the precise value of γ_i rather than just on its sign. Emissions become proportional to firms' physical output as in Restriction **??** if $\gamma_i = 0$ and, in case of constant markups, proportional to firms' sales as in Restriction **??** if $\gamma_i = 1$.

The change in the aggregate emissions is shown in terms of changes in bilateral emissions weighted by the initial shares:

$$\hat{Z}_i = \sum_{n=1}^{N} \iota_{ni} \hat{Z}_{ni}. \tag{17}$$

With $t_i = 0$ and ACR assumptions, trade share changes, counterfactual revenues, and wage changes are as Proposition **??**.

Unlike in Sections **??** and **??**, as emission intensity across firms is constant neither in value nor quantity terms, we cannot write useful general expressions for aggregate national emissions or their relative change. Instead, we directly consider the implications of the third emission modeling approach in the four trade models.[29]

Armington – The change in emissions is equal to the change in the quantity of production, which is given by

$$\hat{Z}_{ni}^{Armington} = \frac{\hat{X}_{ni}}{\hat{w}_i}, \tag{18}$$

and the emission share again equals the sales share ($\iota_{ni} = \frac{X_{ni}}{Y_i}$).

Krugman – Any differences in the KR case from the quantity case arise due to the heterogeneity of producers. Since firms are homogenous in the Krugman framework, emission changes again simply coincide with the Arimington model ($\hat{Z}_{ni}^{Armington} = \hat{Z}_{ni}^{Krugman}$).

EK – The change in emissions associated with trade shocks is:

$$\hat{Z}_{ni}^{EK} = \frac{\hat{X}_{ni}}{\hat{w}_i} \hat{\lambda}_{ni}^{-(1-\gamma_i)/\theta}, \tag{19}$$

and the emission share, ι_{ni}^{EK}, is

$$\iota_{ni}^{EK} = \frac{X_{ni} \lambda_{ni}^{-(1-\gamma_i)/\theta}}{\sum_{k=1}^{N} X_{ki} \lambda_{ki}^{-(1-\gamma_i)/\theta}}.$$

Melitz – The change in emissions associated with trade shocks is::

$$\hat{Z}_{ni}^{Melitz} = \frac{\hat{X}_{ni}}{\hat{w}_i} \hat{S}_{ni}^{-(1-\gamma_i)/\theta} \left(\hat{\varphi}_{ii}^{*}\right)^{1-\gamma_i} = \frac{\hat{X}_{ni}}{\hat{w}_i} \hat{\lambda}_{ni}^{-(1-\gamma_i)/\theta}, \tag{20}$$

[29] We derive the exact expressions in Appendix **??**

and the emission share, ι_{ni}^{Melitz}, is

$$\iota_{ni}^{Melitz} = \frac{S_{ni}^{-(1-\gamma_i)/\theta} X_{ni}}{\sum_{k=1}^{N} S_{ki}^{-(1-\gamma_i)/\theta} X_{ki}}.$$

As in the quantity emission case, we can write the changes in emission by moving to autarky for the two models with firm heterogeneity:[30]

$$\hat{Z}_i^A = \iota_{ii} \lambda_{ii}^{(1-\theta-\gamma_i)/\theta}. \tag{21}$$

Note that this expression nests the emission responses for the alternative emission modelling choices represented by Restriction **??** if $\gamma_i = 1$ and by Restriction **??** if $\gamma_i = 0$.

Using these results, we state the following proposition for the emissions effect of trade openings:

Proposition 6. *Consider a model that satisfies Restriction* **??**, *ACR R1, ACR R2 and ACR R3'. Trade opening*

(i) leaves aggregate emissions in Armington and Krugman models, as well as in EK and Melitz models with $\gamma_i = 1$ *unaffected*

(ii) increases emissions in EK and Melitz models with $\gamma_i < 1$

(iii) lowers emissions in EK and Melitz models with $\gamma_i > 1$.

Proof. See Appendix **??**. □

3.5 Decomposing emission changes

As discussed above, the different effects in models with and without firm heterogeneity can be intuitively linked to emission decompositions. We discuss the four cases by decomposing the changes in emissions into scale and technique effects. Log-linearizing the emissions embodied in trade yields:

$$d \ln Z_{ni} = \underbrace{d \ln Q_{ni}}_{\text{scale effect}} + \underbrace{d \ln \left(\frac{Z_{ni}}{Q_{ni}} \right)}_{\text{technique effect}}.$$

[30]The derivation is provided in the Online Appendix.

In the value case, we can show that:

$$d\ln Z_{ni}^{\text{Arm/Krug}} = \underbrace{d\ln X_{ni} - d\ln w_i}_{\text{scale effect}} + \underbrace{0}_{\text{technique effect}},$$
$$d\ln Z_{ni}^{\text{EK/Mel}} = \underbrace{-\frac{1}{\theta}d\ln\lambda_{ni} + d\ln X_{ni} - d\ln w_i}_{\text{scale effect}} + \underbrace{\frac{1}{\theta}d\ln\lambda_{ni}}_{\text{technique effect}},$$

and using $\sum_{i=1}^{N}\iota_{ni}\left(dlnX_{ni} - dlnw_i\right) = 0$,[31] we obtain:

$$d\ln Z_{i}^{\text{Arm/Krug}} = \underbrace{0}_{\text{scale effect}} + \underbrace{0}_{\text{technique effect}},$$
$$d\ln Z_{i}^{\text{EK/Mel}} = \underbrace{\sum_{n=1}^{N}\iota_{ni}\left(-\frac{1}{\theta}d\ln\lambda_{ni}\right)}_{\text{scale effect}} + \underbrace{\sum_{n=1}^{N}\iota_{ni}\frac{1}{\theta}d\ln\lambda_{ni}}_{\text{technique effect}}.$$

The result indicates that in the Armington and Krugman models, the scale and technique effects are zero, whereas for the other two, the effects cancel out exactly. For the flexible emission specification, the decomposition is expressed as follows:

$$dlnZ_i^{Arm/Krug} = \sum_{n=1}^{N}\iota_{ni}\left(dlnX_{ni} - dlnw_i\right) + 0 = 0 + 0,$$
$$dlnZ_i^{EK/Mel} = \sum_{n=1}^{N}\iota_{ni}\left(-\frac{1}{\theta}dln\lambda_{ni} + dlnX_{ni} - dlnw_i\right) - \gamma_i\sum_{n=1}^{N}\iota_{ni}\frac{1}{\theta}dln\lambda_{ni},$$

where the sign and the magnitude of the technique effect now depend on the parameter γ_i.[32] When $\gamma_i = 0$, the model is equivalent to the quantity case, where no technique effect exists, and only the scale effect exists. When $\gamma_i = 1$, the model collapses to the value case, where the technique and scale effect cancel out. Note that while the decomposition expressions look identical for EK and Melitz, the decompositions differ quantitatively because ι_{ni} differs across these models.

Our decomposition also sheds light on the conditions for **?**'s Pollution Reduction by Rationalization (PRR) Hypothesis to hold in the considered models. The hypothesis refers to the case in which the reallocation across firms induces a technique effect (or –

[31] This is from $\sum_{i=1}^{N}\iota_{ni}\left(dlnX_{ni} - dlnw_i\right) = \sum_{i=1}^{N}\frac{X_{ni}}{R_i}dlnX_{ni} - dlnw_i = dlnR_i - dlnw_i = 0$

[32] Also, ι_{ni} depends on γ_i.

in their words – a "rationalization effect") sufficiently strong to outweigh the scale effect associated with the same reallocation. The PRR Hypothesis hence holds only in our flexible emission specification if $\gamma_i < 1$.

4 Multi-industry extension

4.1 General model set-up

We extend the model in Section **??** to multiple sectors, where we denote the sectors as $s = (1, ..., S)$. The production technology and market structure in each industry are the same as in the aggregate consideration above.[33] Additionally, we follow **?**, **?**, and **?** and assume that consumers have a three-tier nested CES utility. The upper-tier is Cobb-Douglas across sectors with spending share κ_{ns}. The second tier is CES across different origins within industries, with the elasticity of substitution η_s, and the last tier aggregates varieties within a country of origin with the elasticity of substitution σ_s.[34] This model implies the following expenditure share of consumers in n on products from i, sector s (see the Online Appendix for the derivation):

$$\lambda_{nis} = \frac{\left(\frac{L_{is}}{\tilde{\beta}_{is}(w_i/t_i)}\right)^{\delta_s} \left(\tilde{c}_{is}(w_i, t_i)\tau_{nis}\right)^{-\varepsilon_s} \xi_{nis}}{\sum_{k=1}^{N} \left(\frac{L_{ks}}{\tilde{\beta}_{ks}(w_k/t_k)}\right)^{\delta_s} \left(\tilde{c}_{ks}(w_k, t_k)\tau_{nks}\right)^{-\varepsilon_s} \xi_{nks}}, \tag{22}$$

where ε_s is the trade elasticity, ψ_s is the scale elasticity, and $\delta_s = \psi_s\varepsilon_s$ is the product of the scale and trade elasticities, $\tilde{\beta}_{is}(w_i/t_i) = 1 - \beta_{is}(w_i/t_i)$ and ξ_{nis} is a constant. We now briefly outline how the four canonical models deliver special cases of Equation (**??**).

Armington – In the multi-industry Armington model, the three-tier nested utility function reduces to two nests because every country produces a single variety per country. The trade elasticity simply relates to the elasticity of substitution across varieties within one sector from different countries ($\varepsilon_s = \sigma_s - 1$). $\delta_s = 0$ because there are no scale economies in this framework.

[33] However, we follow **?** and focus on free entry cases for multiple industry settings.

[34] We introduce this nest structure compared to Section **??** to ensure a unique equilibrium in the monopolistic competition versions of the model (see **?**, for a discussion).

EK – In the multi-industry EK model, the three-tier nest also reduces to two tiers because firms from different countries produce varieties from the same unit interval per sector rather than distinct varieties. The trade elasticity coincides with the sectoral Fréchet dispersion parameter ($\varepsilon_s = \theta_s$). As in the other perfect competition framework, there are no scale economies, and hence $\delta_s = 0$. Note that for a given trade elasticity, trade adjustments will be identical in the multi-sector EK and Armington model because they only differ in the constant ξ_{ins}, which does not affect the equilibrium in changes.

Krugman – In the multi-sector Krugman model, substitutability has to differ between varieties from different firms, countries, and sectors to ensure a unique equilibrium. The trade elasticity is directly linked to the elasticity of substitution across different varieties in a given sector from the same country ($\varepsilon_s = \eta_s - 1$). Further, with free entry in this monopolistically competitive framework, scale economies are present and linked to the elasticity of substitution between varieties from different countries ($\delta_s = (\eta_s - 1)/(\sigma_s - 1)$).

Melitz – In the multi-industry Melitz model, substitutability again has to differ across the three layers of firms, countries, and sectors. The trade elasticity is linked to both the Pareto shape parameter θ_s and the elasticities of substitution between varieties from both different firms and countries $\left(\varepsilon_s = \theta_s / \left(1 + \theta_s \left(\frac{1}{\eta_s - 1} - \frac{1}{\sigma_s - 1}\right)\right)\right)$. Since the firms are monopolistically competitive, there are scale economies if firms can freely enter the market. The scale economy parameteri is linked to the Pareto shape parameter, specifically $\delta_s = 1/\left(1 + \theta_s \left(\frac{1}{\eta_s - 1} - \frac{1}{\sigma_s - 1}\right)\right)$. Note that trade responses are identical in the multi-sector Melitz and Krugman models for given values of trade and scale elasticities.

4.2 Emissions linked to values

Assume that Restriction **??** holds on a sectoral level. Following the same lines as in **??**, it is straightforward to show that emissions in sector s, country i are given by:

$$Z_{is} = \frac{\beta_{is}(w_i/t_i)}{t_i} R_{is}, \tag{23}$$

where revenues are equal to:

$$R_{is} = \sum_{n=1}^{N} \lambda_{nis} E_{ns},$$

and the sector-level expenditure is defined as[35]:

$$E_{ns} = \kappa_{ns} \left(w_n L_n + t_n Z_n \right). \tag{24}$$

Proposition 7. *Suppose we have a model where Equation (??) and Restriction ?? hold. Following a trade shock as in Definition ??, emission changes in country i are:*

$$\hat{Z}_i = \sum_{s=1}^{S} \iota_{is} \hat{L}_{is}, \tag{25}$$

where $\iota_{is} = Z_{is}/Z_i$ is the sectoral emission share. Changes in the labor allocations, trade shares, expenditures, and wages are given by:

$$\hat{L}_{is} = \frac{1}{\hat{w}_i} \frac{\sum_{n=1}^{N} \hat{\lambda}_{nis} \lambda_{nis} \hat{E}_{ns} E_{ns}}{\sum_{n=1}^{N} \lambda_{nis} E_{ns}}, \tag{26}$$

$$\hat{\lambda}_{nis} = \frac{\left(\hat{L}_{is} \right)^{\delta_s} \left(\hat{w}_i \hat{\tau}_{ins} \right)^{-\varepsilon_s}}{\sum_{k=1}^{N} \lambda_{nks} \left(\hat{L}_{ks} \right)^{\delta_s} \left(\hat{w}_k \hat{\tau}_{nks} \right)^{-\varepsilon_s}}, \tag{27}$$

$$\hat{E}_{ns} E_{ns} = \kappa_{ns} \hat{w}_n \left(w_n L_n + \hat{Z}_n t_n Z_n \right), \tag{28}$$

and

$$\hat{w}_i = \sum_{s=1}^{S} (1 - \beta_{is}(w_i/t_i)) \frac{\sum_{n=1}^{N} \hat{\lambda}_{nis} \lambda_{nis} \hat{E}_{ns} E_{ns}}{\sum_{k=1}^{N} \lambda_{kis} E_{ks}}, \tag{29}$$

where ε_s is the trade elasticity, ψ_s is the scale elasticity, and $\delta_s = \psi_s \varepsilon_s$ is the product of the scale and trade elasticities.

Proof. See Appendix **??**. □

Corollary 1. *Suppose the model structure that satisfies equation (??) and that Restriction ?? holds. We further assume a common trade elasticity across the four models and a common scale elasticity for Melitz and Krugman (which is zero for Armington and EK). Following a trade shock as in Definition ??,*

(i) the canonical Armington and EK model lead to the same emission changes, $\hat{Z}_i$.

[35] Because we assume free entry of firms, profits are completely absorbed by the fixed cost payment and thus included in the labor income.

(ii) the Melitz and Krugman model lead to the same emission changes, $\hat{Z}_i$, which is different to the emission changes for EK and Armington due to scale economies.

(iii) $\hat{Z}_i = 1$ *for all models if* $\frac{Z_{is}}{Z_i} = \frac{L_{is}}{L_i} \quad \forall i \in N \; s \in S.$

Proof. See Appendix **??**. □

In the multi-industry case, a trade shock affects emissions even when the emission price is fixed relative to the wage rate. The reallocation of labor across sectors fully explains these emission changes. As is evident from Equation **??**, the trade response, which determines the magnitude and the direction of labor reallocation, differs across models. The canonical Armington and Eaton-Kortum (EK) models lack economies of scale, so changes in expenditure share are determined solely by variations in iceberg trade costs and wage rates. In contrast, the monopolistic competition models of Krugman and Melitz introduce external economies of scale. Consequently, in these models, the expenditure share is also influenced by sectoral employment. These divergent trade responses have been documented in previous research, such as the work by (see e.g. **?**). Moreover, Proposition **??** shows that emission responses are solely driven by the shift in the industry composition. heterogeneity. A special case arises when the emission per employed labor is common across industries within the country, $Z_{is}/L_{is} = Z_i/L_i$. In this case, the composition effect is absent because the reallocation of labor does not alter the relative importance of emission-intensive and cleaner sectors. Hence, trade shocks do not affect emissions as in the single industry economy, regardless of the model considered.

Note that part (i) of Corollary **??** is of practical relevance because for a range of models used in the literature (including e.g., **????????**), it implies that they could switch back and forth between an Armington representation and an EK model without any changes to their aggregate outcomes. The same is not quite true for the Melitz models by **?** and **?**: Switching them to Armington models would change aggregate emission outcomes, however, *only because* the trade responses are different in the Melitz case, not because of the within-industry reallocation between more or less emission-intensive producers. Hence, moving them to Krugman models would leave emission outcomes unaffected for given scale and trade elasticities, in spite of the loss of firm heterogeneity.[36]

[36]Vice versa, adding firm heterogeneity à la Melitz to the Krugman model version of **?** would not make

4.3 Flexible emission specification

The basic model structure is similar to Section **??**. Moreover, Restriction **??** is assumed to hold for each sector, with sector-varying emissions intensities and elasticity μ_{is} and γ_{is}.[37] The emission changes due to a trade shock according to Definition **??** is given by:

$$\hat{Z}_i = \sum_{s=1}^{S} \iota_{is} \sum_{n=1}^{N} \iota_{nis} \hat{Z}_{nis}. \tag{30}$$

For the Armington, Krugman, and EK models, the changes in emissions are given by the sector-level equivalents of (**??**) and (**??**) as well as the corresponding embodied emission shares ι_{nis}. In the Melitz case, we additionally have to take into account sector-level scale economies leading to additional average productivity changes, which are captured via changes in sectoral employment shares:

$$\hat{Z}_{nis}^{\text{Melitz}} = \frac{\hat{X}_{nis}}{\hat{w}_i} \left(\hat{L}_{is}\right)^{\psi_s(1-\gamma_s)} \hat{\lambda}_{nis}^{-(1-\gamma_s)/\theta_s}. \tag{31}$$

The remaining equilibrium conditions in changes are given by Equation **??** and the following equations:

$$\hat{E}_{ns} E_{ns} = \kappa_{ns} \hat{w}_n w_n L_n, \tag{32}$$

$$\hat{w}_i = \frac{1}{w_i L_i} \sum_{s=1}^{S} \sum_{n=1}^{N} \hat{\lambda}_{nis} \lambda_{nis} \hat{E}_{ns} E_{ns}, \tag{33}$$

$$\hat{L}_{is} = \frac{1}{\hat{w}_i w_i L_{is}} \sum_{n=1}^{N} \hat{\lambda}_{nis} \lambda_{nis} \hat{E}_{ns} E_{ns}. \tag{34}$$

When $\gamma = 0$, the formulation collapses to the quantity case with multiple industries. Unlike in the single industry case, the flexible model with $\gamma = 1$ differs from the model that links emissions to values. This is because collected emission taxes, which respond to the trade shock in multiple industry cases, are absent in the flexible model. If one assumes that environmental tax revenues are lost in rent-seeking as, e.g., **?**, the two models are again isomorphic.

a difference either. Note that for their model, **?** point out this equivalence in Appendix E of their paper. The equivalence between Melitz and Krugman only holds for the assumptions of fixed costs being paid in terms of labor of the destination country and the absence of tariff revenue and trade deficits.

[37] As noted above, the quantity case is a special case of Restriction **??**

The multi-sector model features composition effects, and the role of firm heterogeneity may differ across industries. As a result, it becomes more challenging to make general statements about how trade shocks and trade opening affect emissions, as well as about the overall role of firm heterogeneity. Nevertheless, we still reach a joint expression for the emission effects of autarky for the four canonical trade models:

$$\hat{Z}_i^A = \sum_{s=1}^{S} \iota_{is} \left(\frac{\kappa_{is}}{s_{is}} \right)^{1+\mathbb{1}\psi_s(1-\gamma_s)} \frac{\iota_{iis}}{s_{iis}} \left(\lambda_{iis}\right)^{\mathbb{1}(1-\gamma_s)/\theta_s}, \tag{35}$$

where $s_{is} = \sum_{n=1}^{N} X_{nis} / (\sum_{n=1}^{N} \sum_{s'=1}^{N} X_{nis'})$ and $s_{nis} = X_{nis} / (\sum_{k=1}^{N} X_{kis})$ are sectoral and bilateral sales shares and $\mathbb{1}$ is an indicator variable equal to one for models with firm heterogeneity (EK and Melitz) and zero otherwise (Armington and Krugman).

Additionally, we reach sharp conclusions for the *relative* emission effects between models (mostly between EK and Armington) for a set of interesting special cases concerning the range of values that γ_{is} takes, summarized in the following proposition:[38]

Proposition 8. *Consider a model that satisfies Equation* **??** *and Restriction* **??**. *Trade opening*

(i) leads to equal aggregate emissions in Krugman and Melitz models if $\gamma_{is} = 1 \ \forall i, s$.

(ii) leads to equal aggregate emissions in Armington and EK models if $\gamma_{is} = 1 \ \forall i, s$.

(iii) increases emissions in EK relative to Armington models if $\gamma_{is} < 1 \ \forall i, s$.

(iv) lowers emissions in EK relative to Armington models if $\gamma_{is} > 1 \ \forall i, s$.

Proof. See Appendix **??**. □

Note that generally, γ_{is} will be above one in some sectors and below one in others. Whether firm heterogeneity leads to relatively higher or lower emissions associated with trade liberalization then has to be answered by quantitative simulations.

[38] Comparing Krugman and Melitz models is not straightforward, since trade liberalization may lower the productivity of certain industries in a multi-industry setting.

5 Quantification

5.1 Data and parameters

Our main data source is the World Input-Output Database (WIOD) from **?**, which contains trade flows and expenditure shares. CO_2 emissions consistent with WIOD classifications are provided by **?**. To quantify emission changes in the Melitz model, we require data on the shares of exporting firms. We gather these data from various sources. The number of active firms is given by the OECD Structural Statistics of Industry and Services database. For the number of exporting firms by partner country, we rely on the OECD Trade by Enterprise Characteristics Database, Eurostat, and the World Bank Exporter Dynamics Database. We limit the number of exporting/active firms to those operating in manufacturing.[39]

We rely on estimates from the literature for the standard model parameters. We obtain the demand substitution parameters, σ_s and η_s, from **?**. We use the Pareto shape parameter estimate θ_s from **?**. The translation of these parameters into the trade and scale elasticities, ε_s and δ_s, is model-specific. We calculate ε_s and δ_s in line with the multi-sector Melitz model structure. We then fix the resulting trade elasticities across all four canonical models. Similarly, we fix δ_s across the two monopolistically competitive models but put it equal to zero for the two perfectly competitive models as they do not feature scale economies. While this procedure implicitly assumes different values for underlying parameters (such as σ_s and θ_s) across models, it ensures maximal comparability across models in the quantitative results. We want all differences to stem from *model* differences rather than from differently calibrated elasticities. Each calibrated parameter is shown in appendix **??**.

The key parameter for how firm heterogeneity affects the effects of trade on emissions is the productivity elasticity of emissions. We estimate γ_s using administrative German firm-level data. We combine different modules of the official German manufacturing census *AFiD-Panel (Amtliche Firmendaten für Deutschland)*. The main module is the *AFiD Panel Industrial Firms* (**?**). It covers the universe of German manufacturing and mining

[39]Unfortunately, we do not have data on the share of exporting firms for all trading pairs. We impute the remaining export shares to retain as many countries as possible. For details, see Appendix **??**.

firms with 20 or more employees and provides data on sales and employed labor. For a representative sample, we further observe material expenditure, different costs of the firm, and investments. We complement this module with the module *Energy Usage* (**?**), which contains the energy inputs in physical quantity. We combine these data with fuel-specific emission factors from **?**, which enables us to calculate firm-level CO_2 emissions from fuel combustion.[40] As a result, our definition of emissions focuses on direct emissions and those embodied in the electricity usage in our emission intensity measure. Finally, we combine these two modules with the *AFiD Panel Foreign Trade Statistics* (**?**), to obtain firm-level exports.

5.2 Estimation of productivity elasticity of emissions

Hereafter, we assume γ_{is} is common across countries but varies across sectors. We estimate γ_s by exploiting the relationship between domestic sales and the emission intensity. Here, we provide a derivation through the perspective of the Melitz model, while the estimation is also consistent with the EK model.[41] Expressing emissions in terms of sales leads to:

$$z_{is}(\omega) = (\varphi(\omega))^{-\gamma_s}\mu_{is}\frac{\sum_{n=1}^{N}\tau_{nis}x_{nis}(\omega)}{p_{nis}(\omega)} = (\varphi(\omega))^{1-\gamma_s}\mu_{is}\left(\frac{\sigma_s}{\sigma_s-1}w_i\right)^{-1}\sum_{n=1}^{N}x_{nis}(\omega),$$

and the emission intensity is then given by:

$$e_{is}(\omega) = \frac{z_{is}(\omega)}{x_{is}(\omega)} = (\varphi(\omega))^{1-\gamma_s}\mu_{is}\left(\frac{\sigma_s}{\sigma_s-1}w_i\right)^{-1}.$$

Firm productivity is linked to the sales in the domestic market as follows:

$$\varphi^{\sigma_s-1}(\omega) = x_{iis}(\omega)\left(\frac{\sigma_s}{\sigma_s-1}w_i\right)^{\sigma_s-1}B_{iis}^{-1},$$

[40] This method of calculating carbon emissions has been applied in several other studies such as **?** or **?**.

[41] However when we assume common trade elasticity between the model, the model-consistent σ_s will be different across two models.

where B_{iis} is the real market size of sector s in country i. This leads to the following expression of the emission intensity in terms of domestic sales:

$$e_{is}(\omega) = (x_{iis}(\omega))^{\frac{1-\gamma_s}{\sigma_s-1}} \left(\frac{\sigma_s}{\sigma_s - 1} w_i\right)^{-\gamma_s} B_{iis}^{\frac{\gamma_s-1}{\sigma_s-1}}. \tag{36}$$

Hence, we can estimate γ_s by regressing emission intensity on the domestic sales:

$$\ln e_{\omega,is} = \beta_0 + \beta_1 \ln x_{\omega,iis} + \epsilon_{\omega,is}, \tag{37}$$

where $\epsilon_{\omega,is}$ is an idiosyncratic component capturing measurement error. With an additional estimate of σ_s, we can obtain an estimate for γ_s from the estimated coefficient:

$$\hat{\gamma}_s = 1 - \hat{\beta}_1(\sigma_s - 1).$$

We estimate (**??**) by the whole sample (Mining and Manufacturing) and each WIOD sector. Table **??** shows the estimation results and the parameters sourced from the literature. Note that we calculate $\hat{\gamma}_s$ based on the Melitz-consistent σ_s values from **?** and then fix γ_s across all models considered — again ensuring that quantitative differences across models stem from model differences, rather than differences in the calibration of key elasticities.

The aggregate γ, estimated using manufacturing and mining firms, is 1.2. This value is close but above unity, which implies that the technique effect is stronger than the scale effect in the models with firm heterogeneity. In individual industries, while γ_s is always positive (more productive firms are not dirtier), the value of γ_s widely differs across sectors. In industries like the manufacture of basic metals (C24) and the manufacture of paper and paper products (C17), the value of γ_s is close to zero, which indicates that these industries exhibit emissions almost proportional to quantity. Conversely, industries like the manufacture of motor vehicles, trailers, and semi-trailers (C29) and repair and installation of machinery and equipment (C33) have γ_s above 2, indicating a strong positive relationship between productivity and emission intensity.[42] The relative strength of the scale and the technique effect differs by industry, and the aggregate implications

[42]Note that, as mentioned in Section **??**, this is calculated using only combustion emissions.

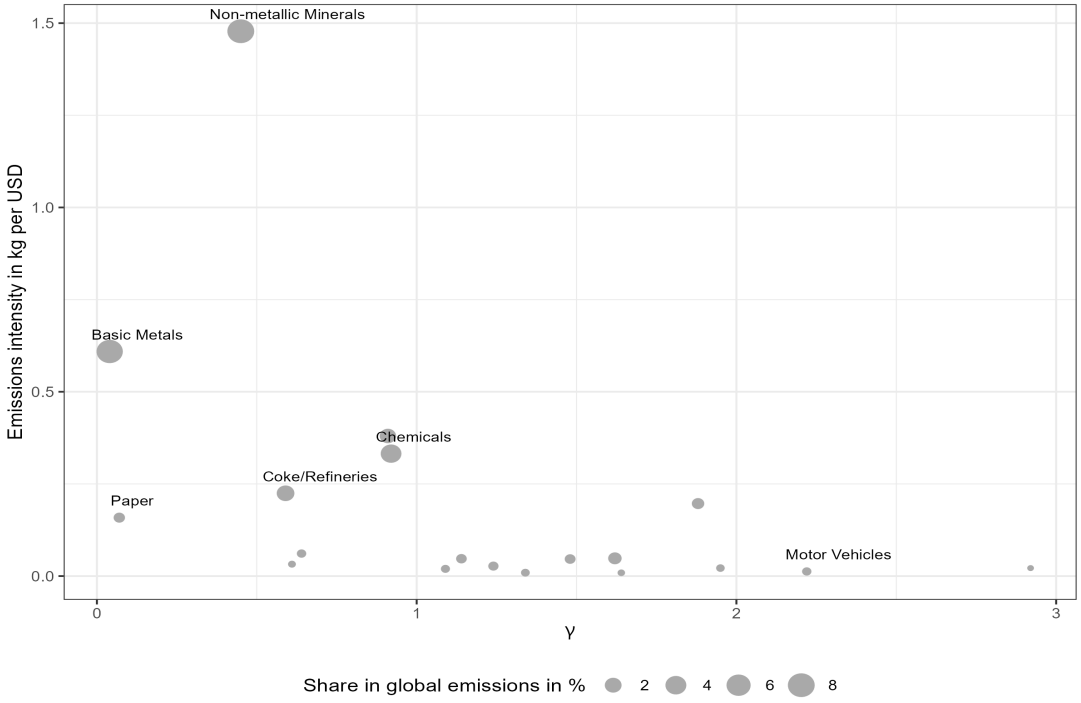
are a-priori unclear. To further investigate the industry heterogeneity, Figure **??** plots the emission intensity measured in kg of CO_2 per USD (y-axis) against the estimated γ_s (x-axis). Each point is a manufacturing sector and the size of the marker corresponds to the share in global emissions. In general, the estimated $gamma_s$ is lower for sectors with a high emission intensity and a large share of overall global emissions. This negative relationship suggests that firm heterogeneity may increase the aggregate emission effects of trade liberalization.

Table 1: Parameter Estimates

Sector	Estimate	Std. Err.	γ
C10-C12	-0.09	(0.008)	1.62
C13-C15	-0.19	(0.027)	1.48
C16	0.08	(0.033)	0.64
C17	0.21	(0.026)	0.07
C18	0.12	(0.02)	0.61
C19	0.13	(0.122)	0.59
C20	0.09	(0.024)	0.92
C21	-0.15	(0.042)	1.64
C22	0.02	(0.014)	0.91
C23	0.08	(0.023)	0.45
C24	0.16	(0.024)	0.04
C25	-0.03	(0.01)	1.14
C26	-0.07	(0.019)	1.34
C27	-0.05	(0.017)	1.09
C28	-0.13	(0.01)	1.24
C29	-0.15	(0.02)	2.22
C30	-0.12	(0.038)	1.95
C31-C32	-0.11	(0.016)	1.88
C33	-0.29	(0.028)	2.92
Total	-0.05	(0.004)	1.20

Note: Estimation of γ according to (**??**). The data source for the estimation is **???**. The first column presents the estimation result. The second column presents the corresponding standard error. The implied γ is calculated based on the value of σ from **?**.

Figure 1: Emission intensity and γ_s



Note: The figure shows the estimated γ and the emission intensity for all manufacturing sectors. The data source for the estimation of γ is **???**. Gross output is calculated from WIOD and emissions from **?**. Each point is a manufacturing sector. The size of the points reflects the sector's share in global emissions.

5.3 Results

5.3.1 Single industry

To quantitatively illustrate the role of firm heterogeneity in shaping the emissions effect of international trade, we simulate a uniform 40% decrease in iceberg trade costs. We need a multi-industry model to fully understand the effects of trade on emissions. We nevertheless start with the single-industry model as it can accentuate the role of firm heterogeneity within the industry.

Figure **??** shows the global emission response for the different models and varying values for γ. Recall that γ captures the relationship between emission intensity and productivity: The larger γ, the cleaner the more productive firms are. The figure displays the effects for $\gamma \in [0, 3]$. The left end ($\gamma = 0$) corresponds to the quantity-based emission

case in which the physical emission intensity is independent of the productivity. The two dashed vertical lines indicate the special case when emissions are proportional to value, i.e., $\gamma = 1$, and when γ lies at our estimated aggregate value of 1.2. The red line shows the emission effect in the aggregate Armington (and, by equivalence, Krugman) model. The blue and green curves refer to the Melitz and EK models.[43]

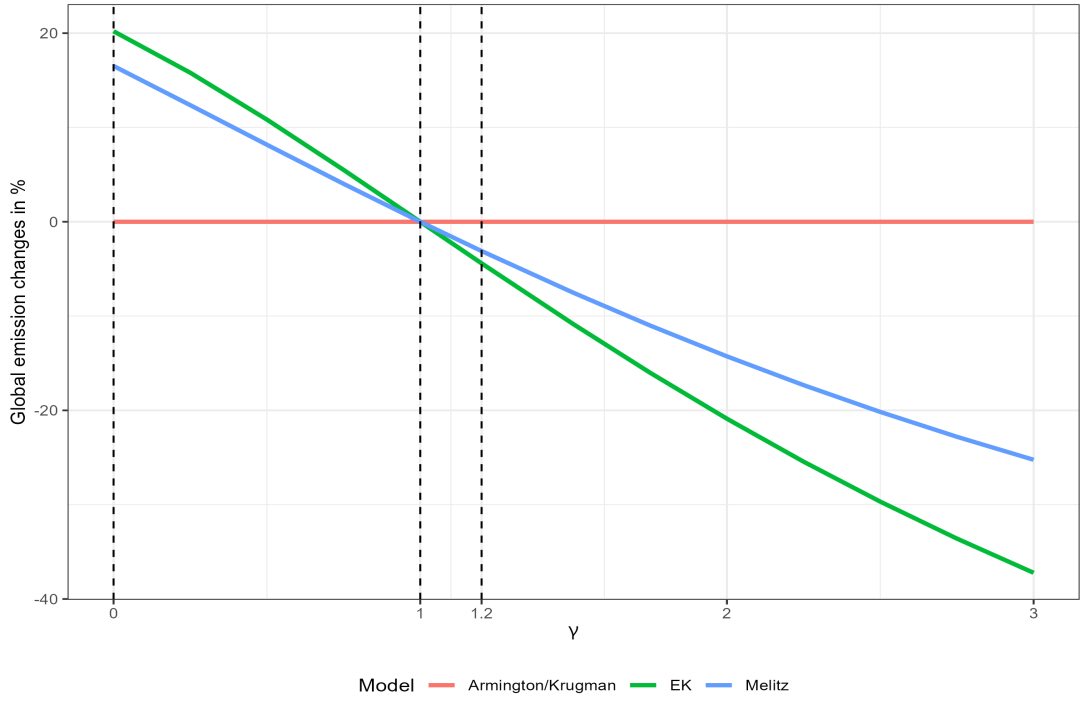
In line with the intuition from Propositions **??** and **??**, emissions do not change in response to the trade liberalization shock in the Armington and Krugman models for any value of γ, as well as in the EK and Melitz models if $\gamma = 1$. In the quantity case ($\gamma = 0$), consistent with the intuition from Proposition **??**, emissions increase in the EK and Melitz models. The increase in global emissions is considerable: Around a fifth in both EK and Melitz. Global emissions increase in the two models with firm heterogeneity for all $\gamma < 1$. However, as γ increases, the emission increase becomes weaker because the scale effect (the only active effect in the quantity case) gets increasingly offset by the technique effect.

For all $\gamma > 1$, global emissions decrease in the EK and Melitz models in response to trade liberalization. While the production scale increases, reallocating production towards more productive firms significantly reduces the average emissions intensity. This technique effect is stronger than the scale effect, which reduces the aggregate emissions. For the estimated $\gamma = 1.2$, global emissions decrease by approximately 7% in Melitz and 5% in EK. However, it is premature to conclude that firm heterogeneity positively affects the emission effects of trade or that trade generally lowers global carbon emissions. First, the previous section shows that the aggregate estimated γ hides considerable heterogeneity across sectors. Second, a full quantification of the trade effects needs to consider sectoral reallocations — which can alter global emissions irrespective of sectoral differences in γ.

The Melitz model's emission response is slightly stronger than EK for all values of γ. However, the difference between this group of two models with producer heterogeneity and the two homogenous producer models with a zero-emission response is much more pronounced than the difference within the heterogenous producer model group.

[43] In the Online Appendix, we additionally show the *national* emission changes in EK and Melitz for the two cases in which emissions are assumed to be proportional to the physical quantity produced ($\gamma = 0$) or γ taking the estimated value of 1.2.

Figure 2: Changes in emissions for different γ



Note: The figure shows the changes in global emissions due to a 40% uniform trade liberalization for different values of γ.

5.3.2 Multiple Industries

As discussed in Section **??**, the emission response to trade liberalization can differ across models in multi-industry settings for two distinct and potentially interacting reasons: firm heterogeneity as in the single-industry models and differential trade responses due to scale economies. We compare the multi-industry Armington, Krugman, EK, and Melitz models, and for the models with firm heterogeneity, consider four cases with $\gamma_s = 0$, $\gamma_s = 1$, $\gamma_s = 1.2$, and estimated γ_s. As with the aggregate quantification, we simulate a uniform cut in international trade costs by 40%.

The results are illustrated in Figure **??**. Just as in the aggregate consideration, the choice of γ_s does not affect the results in the Armington and Krugman models because there is no firm heterogeneity. Therefore, only one result bar is shown for each of these models. In contrast to the single-industry case, global emissions are affected by trade liberalization. The global emissions increase mildly by less than one percent (Armington) and 1.33% (Krugman). Even though both the Armington and the Krugman models

feature no heterogenous productivity, the results are not identical across these two models. This difference is because trade outcomes and the associated emission effects from sectoral reallocation are not identical, as the Krugman model features scale economies.

These increased emissions are driven by a global composition effect, i.e., trade liberalization induces countries with emission-intensive production technologies to specialize in more emission-intensive industries. However, these findings suggest that these compositional shifts play a relatively minor role in global emissions for this specific counterfactual.

In line with Proposition **??**, if $\gamma_s = 1$, the emission response is identical within the two groups of perfect competition models without scale economies (Armington and EK) and monopolistic competition models with scale economies (grey bars in Figure **??**). Within every industry, productivity heterogeneity does induce additional scale and technique effects. Still, as in the single industry case, these cancel out if the KR elasticities are unity.

Further focussing on the EK model, when we use the estimated γ_s for each industry, a 40% reduction in trade costs leads to a 8.59% increase in global emissions (red bar). Accounting for productivity heterogeneity, hence, in this case, leads to a more than eight times higher global emission increase associated with the trade liberalization shock. Note that this contrasts with the result of a single industry, where the trade liberalization resulted in a decrease in global emissions. This stresses the importance of taking the heterogeneity in the relationship between productivity and emission intensity across sectors. We can further illustrate this by using the aggregate γ estimate of 1.2 for all sectors in the multi-sector EK model (turquoise bar). In this case, within-sector reallocations across firms induce a net emission reduction and emissions decrease in response to the trade liberalization shock by 2.46%.

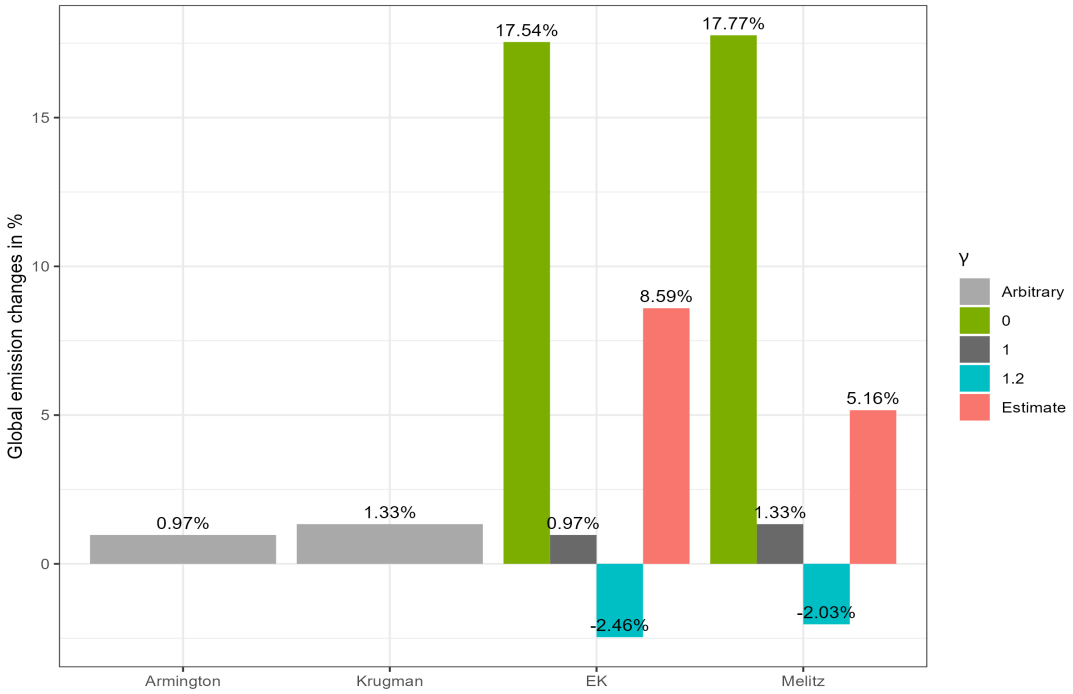
As a final multi-sector EK consideration, assuming emissions are directly proportional to output ($\gamma_s = 0$), the trade liberalization shock increases global emissions by over 17%. Failure to account for differences in emission intensity between firms with different productivity hence leads to an overestimation of the emission-increasing impact of firm heterogeneity.

The emission response of the multi-industry Melitz model behaves qualitatively in line with the EK model. Thus, global emissions increase following trade liberalization when

emissions are proportional to quantities (i.e. $\gamma_s = 0$) and for the estimated γ, while they decrease for $\gamma = 1.2$. Quantitatively, the Melitz model leads to slightly higher emissions than the EK model when considering a uniform γ. For the estimated γ_s, the emission increase of the trade liberalization shock is smaller for Melitz than for EK.

The stark differences in global emissions prediction across models underscore the quantitative significance of firm heterogeneity and the importance of accurately modeling the emissions-productivity relations. Both the exact model specification and the parametrization make a sizable difference in the quantitative results.[44]

Figure 3: Changes in global emissions by model and γ_s



Note: The figure shows the changes in global emissions due to a 40% uniform trade liberalization for Armington and EK. Values are shown for the quantity case $\gamma_s = 0$, the value case $\gamma_s = 1$, and the estimated values of γ_s from Table **??**.

6 Conclusions

In their seminal paper, ACR raise the question of how much the rise of new trade models has altered the answer to the field's central question of how large the gains from trade

[44]The country-level results are shown in the Online Appendix.

are. Their answer is: "So far, not much." A decade later, we investigate how strongly new trade models have affected the key question in the trade and environment subfield: What are the effects of trade on emissions?

We initially arrive at a conclusion similar to ACR's. While new trade models add interesting mechanisms via which trade affects environmental outcomes, they tend to be incorporated into quantitative models in a way that leaves aggregate emission effects unaltered. If emissions are linked to production value, the emission-saving effect of reallocating to cleaner producers is perfectly offset by an emission-increasing impact of higher overall production.

However, upon closer inspection, the answer is more nuanced. Even in cases where the ACR's trade equivalence holds, emissions equivalence breaks whenever emissions are linked to quantities rather than values. We further generalize the relationship between quantity and emissions to be productivity-dependent. Depending on how clean the productive firms are, accounting for firm heterogeneity can shed a better or worse light on the environmental consequences of international trade.

The basic intuition holds for multiple industries but requires additional caution because ACR's trade equivalence no longer generally holds in multi-industry settings. In the model of multiple industries with emissions linked to value, the Armington and EK models still yield identical aggregate emission effects because the ACR-type trade equivalence continues to hold. A multi-industry Melitz model yields different aggregate emission effects, however, only because the trade effects and associated global changes in sectoral composition differ due to the presence of scale economies. It, therefore, coincides with a multi-sector Krugman model.

We also provide a generalized multi-industry case in which emissions are flexibly linked to quantities, and firm heterogeneity matters for aggregate emission outcomes. Our quantitative exercises illustrate the importance of the elasticity with which emission intensity depends on productivity. If the link is relatively weak, firm heterogeneity worsens the emission effects of trade. If it is strong, firm heterogeneity can make trade environmentally beneficial. We estimate the key elasticity using German firm-level data and quantify our multi-industry model. Given our estimated elasticity, we show that firm heterogeneity increases the emission effect of trade liberalization considerably.

So far, new trade models with firm heterogeneity have had "not much" effect on the quantification of the emission effects of international trade. Yet, this may be driven more by the way firm heterogeneity was incorporated in the models than by the heterogeneity actually not making a difference. Our model, which has a flexible link between productivity and emission intensity, is one way of allowing firm heterogeneity to matter.

Appendix

A Proof for the exact hat algebra

We start with the change in the trade shares $\hat{\lambda}_{ni}$. For all i. We take the ratio of trade values:

$$\begin{aligned}\hat{\lambda}_{ni} &= \frac{\chi_{ij} \cdot N_i \cdot (\tilde{c}_i(w_i, t_i)\tau_{ni})^{\varepsilon}}{\sum_{k=1}^{N} \chi_{kj} \cdot N_k \cdot (\tilde{c}(w_k, t_k)\tau_{ki})^{\varepsilon}} \cdot \frac{\sum_{k=1}^{N} \chi_{kj} \cdot N'_k \cdot (\tilde{c}(w'_k, t'_k)\tau'_{ki})^{\varepsilon}}{\chi_{ij} \cdot N'_i \cdot (\tilde{c}_i(w'_i, t'_i)\tau_{ni'})^{\varepsilon}} \\ &= \frac{\hat{N}_i \left(\hat{\tau}_{ni}\hat{c}_i\right)^{\varepsilon}}{\sum_{k=1}^{N} \lambda_{nk}\hat{N}_k \left(\hat{\tau}_{nk}\hat{c}_k\right)^{\varepsilon}}\end{aligned}$$

where $\hat{c}_i = \frac{\tilde{c}_i(w'_i, t'_i)}{\tilde{c}_i(w_i, t_i)}$. Notice that $\tilde{c}_i$ is homogeneous degree of one, and by the assumption, $\hat{t}_i = \hat{w}_i$. Therefore, we can simply calculate the change in costs as follows:

$$\hat{c}_i = \frac{\tilde{c}_i(w'_i, t'_i)}{\tilde{c}_i(w_i, t_i)} = \hat{w}_i \frac{\tilde{c}_i(w'_i/\hat{w}_i, t'_i/\hat{w}_i)}{\tilde{c}_i(w_i, t_i)} = \hat{w}_i \frac{\tilde{c}_i(w_i, t_i)}{\tilde{c}_i(w_i, t_i)} = \hat{w}_i.$$

Furthermore, as discussed in ACR, the first two restrictions imply $\hat{M}_i = 1$. Using these relationships, we have:

$$\hat{\lambda}_{ni} = \frac{\left(\hat{\tau}_{ni}\hat{w}_i\right)^{\varepsilon}}{\sum_{k=1}^{N} \lambda_{nk} \left(\hat{\tau}_{nk}\hat{w}_k\right)^{\varepsilon}}.$$

Now we discuss the goods market clearing. The goods market clearing assumption implies:

$$\hat{R}_i R_i = \sum_{n=1}^{N} \hat{\lambda}_{ni}\lambda_{ni}R'_k.$$

The change in the revenue (output) can be written as

$$\begin{aligned}\hat{R}_i &= \hat{w}_i \frac{w_i l_i}{w_i l_i + t_i Z_i + \Pi_i} + \hat{t}_i \hat{Z}_i \frac{t_i Z_i}{w_i l_i + t_i Z_i + \Pi_i} + \hat{\Pi}_i \frac{\Pi_i}{w_i l_i + t_i Z_i + \Pi_i} \\ &= \hat{w}_i \frac{w_i l_i}{w_i l_i + t_i Z_i + \Pi_i} + \hat{R}_i \frac{t_i Z_i}{w_i l_i + t_i Z_i + \Pi_i} + \hat{R}_i \frac{\Pi_i}{w_i l_i + t_i Z_i + \Pi_i}\end{aligned}$$

where the second equality uses the fact that $\hat{Z}_i = \hat{R}_i/\hat{t}_i$ and $\hat{\Pi}_i = \hat{R}_i$. This derivation implies $\hat{R}_i = \hat{w}_i$.

B Deriving emission for the quantity case and the flexible specification case

Since the quantity case is a special case of the flexible specification with $\gamma_i = 0$, we omit the proof and only exhibit the proof of the flexible specifications.

B.1 EK

We start from the fact that the price of goods exported from country i to country n also follows a Fréchet distribution ((**?**)). Specifically, the price distribution of goods exported from country i to country n ,$G_{ni}(p)$, only depends on the destination country:

$$\begin{aligned} &Pr\left(p_{ni}(\omega) = p | p_{ni}(\omega) \leq min_{k=1,...,N} p_{ki}(\omega)\right) \\ &= G_{ni}(p) = exp(-\Phi_n p^{\theta}), \end{aligned}$$

where $\Phi_n = \Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)^{\theta/(1-\sigma)} P_n^{-\theta}$ is a function of the price index in country n. The productivity of a product with price p_{ni} is $\frac{w_i \tau_{ni}}{p_{ni}}$, and hence the emissions of a particular producer with price p_{ni} is

$$z_{ni}(p_{ni}(\omega)) = \mathbf{1}(p_{ni}(\omega) \leq min_k p(\omega)_{nk}) \mu_i \left(\frac{p_{ni}}{w_i \tau_{ni}}\right)^{\gamma_i} \frac{p_{ni}(\omega)^{-\sigma}}{P_n^{1-\sigma}} \tau_{ni} X_i.$$

Hence, the aggregate emissions are

$$\begin{aligned} Z_{ni} &= \int_0^1 z_{ni}(\omega) d\omega \\ &= \int_0^1 \mathbf{1}(p_{ni}(\omega) \leq min_k p_{nk}(\omega)) \mu_i \left(\frac{p_{ni}}{w_i \tau_{ni}}\right)^{\gamma_i} \frac{\mu_i p_{ni}(\omega)^{-\sigma}}{P_n^{1-\sigma}} X_n d\omega \\ &= \lambda_{ni} \left(w_i \tau_{ni}\right)^{-\gamma_i} \int_0^{\infty} \frac{\mu_i p^{\gamma_i - \sigma}}{P_n^{1-\sigma}} \tau_{ni} X_n dG_{ni}(p) \\ &= X_{ni} \mu_i \tau_{ni} \left(w_i \tau_{ni}\right)^{-\gamma_i} \frac{1}{\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)} \Phi_n^{(\sigma-1)/\theta} \int_0^{\infty} p^{\gamma_i - \sigma} dG_{ni}(p) \\ &= X_{ni} \mu_i \tau_{ni} \left(w_i \tau_{ni}\right)^{-\gamma_i} \frac{1}{\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)} \Phi_n^{(\sigma-1)/\theta} \Phi_n^{(\gamma_i-\sigma)/\theta} \Gamma\left(\frac{\theta + \gamma_i - \sigma}{\theta}\right) \\ &= X_{ni} \mu_i \tau_{ni} \left(w_i \tau_{ni}\right)^{-\gamma_i} \frac{\Gamma\left(\frac{\theta+\gamma_i-\sigma}{\theta}\right)}{\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)} \Phi_n^{(\gamma_i-1)/\theta} \\ &= X_{ni} \mu_i \left(w_i^{-\theta} \tau_{ni}^{-\theta} T_i\right)^{(\gamma_i-1)/\theta} w_i^{-1} T_i^{(1-\gamma_i)/\theta} \frac{\Gamma\left(\frac{\theta+\gamma_i-\sigma}{\theta}\right)}{\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)} \Phi_n^{(\gamma_i-1)/\theta} \\ &= \frac{X_{ni}}{w_i} \mu_i \left(\frac{T_i}{\lambda_{ni}}\right)^{(1-\gamma_i)/\theta} \frac{\Gamma\left(\frac{\theta+\gamma_i-\sigma}{\theta}\right)}{\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)}. \end{aligned}$$

This leads to the following exact hat algebra:

$$\hat{Z}_{ni}^{EK} = \frac{\hat{X}_{ni}}{\hat{w}_i} \hat{\lambda}_{ni}^{(\gamma_i-1)/\theta},$$

and the corresponding emission share ι_{ni} is given by

$$\iota_{ni}^{EK} = \frac{\lambda_{ni}^{(\gamma_i-1)/\theta} X_{ni}}{\sum_{k=1}^{N} \lambda_{ki}^{(\gamma_i-1)/\theta} X_{ki}}.$$

B.2 Melitz model

We start by expressing the physical quantity in terms of revenues, i.e.:

$$q_{ni}(\omega) = \frac{x_{ni}\tau_{ni}}{p_{ni,\omega}(w_i, t_i)}$$

Then emissions embodied in the trade flow of variety ω from i to n can be expressed in terms of

$$z_{ni}(\omega) = \frac{\mu_i \tau_{ni} x_{ni}(\omega)}{\tau_{ni}\tilde{\sigma} w_i (\varphi(\omega))^{\gamma_i}}.$$

In Melitz, the only firm-specific component of the unit costs is the productivity, $\varphi(\omega)$, i.e.

$$c_{i,\omega} = \frac{w_i}{\varphi(\omega)}.$$

Thus, emissions embodied in trade flows from i to n:

$$Z_{ni} = \frac{\mu_i}{\tilde{\sigma} w_i} M_{ni} \int_{\omega \in \Omega_{ni}} (\varphi(\omega))^{1-\gamma_i} x_{ni}(\omega) dG(\varphi(\omega)|\omega \in \Omega_{ni}).$$

With exports of firm ω from i to n of

$$x_{ni}(\omega) = p_{ni}^{1-\sigma} w_n L_n P_n^{\sigma-1} = (\varphi(\omega))^{\sigma-1} \left(\tilde{\sigma} w_i \tau_{ni}\right)^{1-\sigma} w_n L_n P_n^{\sigma-1},$$

where the second equality follows from constant markup-pricing. To enter the foreign market, firms have to pay a fixed cost of $w_n f_{ni}$, which leads to a unique productivity cut-off of

$$(\varphi_{ni}^*)^{\sigma-1} = \frac{f_{ni}}{L_n P_n^{\sigma-1}} \left(\tilde{\sigma} w_i \tau_{ni}\right)^{\sigma-1}.$$

Because $\varphi(\omega)$ is the only parameter that varies at the firm level, we henceforth only index firms by their productivity. The embodied emissions are given by

$$Z_{ni} = \frac{\mu_i}{\tilde{\sigma} w_i} w_n L_n P_n^{\sigma-1} M_{ni} \int_{\varphi_{ni}^*}^{\infty} \varphi^{\sigma-\gamma_i} dG(\varphi|\varphi \geq \varphi_{ni}^*).$$

With the assumption of Pareto distributed productivity, i.e. $G_i(\varphi) = \left(\frac{T_i}{\varphi}\right)^{\theta}$ we get:

$$Z_{ni} = \frac{\mu_i}{\tilde{\sigma} w_i} w_n L_n P_n^{\sigma-1} \frac{\theta}{\theta + \gamma_i - \sigma} M_{ni} (\varphi_{ni}^*)^{\sigma-\gamma_i}.$$

Expressing $(\varphi_{ni}^*)^{\sigma-\gamma_i}$ in terms of revenues and the share of exporting firms:

$$S_{ni} = \left(\frac{\varphi_{ni}^*}{\varphi_{ii}^*}\right)^{-\theta},$$

yields:

$$Z_{ni} = \frac{\mu_i}{\tilde{\sigma} w_i} S_{ni}^{-(1-\gamma_i)/\theta} (\varphi_{ii}^*)^{1-\gamma_i} M_{ni} \frac{\theta}{\theta + \gamma_i - \sigma} x_{ni}(\varphi_{ni}^*).$$

Noting that

$$M_{ni}\frac{\theta}{\theta+1-\sigma}x_{ni}(\varphi^*_{ni}) = M_{ni}E[x_{ni}|\varphi \geq \varphi^*_{ni}] = X_{ni},$$

yields:

$$Z_{ni} = \frac{\mu_i}{\tilde{\sigma}w_i}\frac{\theta+1-\sigma}{\theta+\gamma_i-\sigma}S_{ni}^{-(1-\gamma_i)/\theta}(\varphi^*_{ii})^{1-\gamma_i}X_{ni}. \tag{38}$$

This leads to the exact hat algebra expression of:

$$\hat{Z}^{\text{Melitz}}_{ni} = \frac{\hat{X}_{ni}}{\hat{w}_i}\hat{S}_{ni}^{-(1-\gamma_i)/\theta}(\widehat{\varphi^*_{ii}})^{1-\gamma_i},$$

which yields the first part of (**??**) and for $\gamma = 0$ the first part of (**??**). For the second part of (**??**) and (**??**), we first express the changes in emissions solely in terms of the bilateral productivity cut-offs:

$$\hat{Z}^{\text{Melitz}}_{ni} = \frac{\hat{X}_{ni}}{\hat{w}_i}(\widehat{\varphi^*_{ni}})^{1-\gamma_i}, \tag{39}$$

which follows directly from the definition of the expenditure share. Second, note that the expenditure of n on varieties from i is given by:

$$\lambda_{ni} = \frac{M_{ni}\int_{\omega\in\Omega_{ni}}x_{ni}(\omega)d\omega}{w_nL_n} = \frac{M_{ni}\frac{\theta}{\theta+1-\sigma}x_{ni}(\varphi^*_{ni})}{w_nL_n} = \frac{M_i}{L_n}T_i^{\theta}(\varphi^*_{ni})^{-\theta}\frac{\theta}{\theta+1-\sigma}f_{ni},$$

where the first equaltity follows from the relationship between marginal and average revenue with pareto distributed productivity. The second equality follows from the free entry condition into exporting and the number of exporting firms given by:

$$M_{ni} = (1-G(\varphi^*_{ni})M_i = T_i^{\theta}(\varphi^*_{ni})^{-\theta}.$$

Hence, the change in the productivity cut-off can be expressed as:

$$\widehat{\varphi^*_{ni}} = \hat{\lambda}_{ni}^{-1/\theta}\hat{M}_i^{1/\theta} = \hat{\lambda}_{ni}^{-1/\theta}\hat{L}_i^{1/\theta},$$

where the last equality follows from the fact that the number of entering firms is proportional to the labor endowment in the Melitz-Pareto model with labor being the only factor of production. Because the labor endowment is fixed and thus the change in the productivity cut-off is fully determined by the change in the expenditure share. We still keep it here to illustrate the parallel to the multi-sector economy when the labor allocation across sectors is not fixed. Plugging back into (**??**) directly yields the second part of (**??**) and for $\gamma = 0$ the second part of (**??**). From (**??**) the corresponding weights to calculate aggregate emissions are given by

$$\iota^{Melitz}_{ni} = \frac{S_{ni}^{-(1-\gamma_i)/\theta}X_{ni}}{\sum_{k=1}^{N}S_{ki}^{-(1-\gamma_i)/\theta}X_{ki}}.$$

C Proof for Proposition ?? and ??

We discuss the Armington, Krugman, EK, and the Melitz model sequentially. The proof of the quantity case (Proposition **??**) is implicitly provided as the special case with $\gamma_i = 0$. We only explicitly prove the case for $\gamma_i < 0$ since we can prove the case of $\gamma_i = 0$, $\gamma_i > 0$, and $\gamma_i = 1$ in a similar manner.

C.1 Armington model

Because there is only a single variety in each country, we omit the notion of ω. The total emission of country i is proportional to the quantity produced:

$$Z_i = \mu_i \varphi^{-\gamma_i} Q_i,$$

and the produced quantity is:

$$Q_i = \varphi_i L_i.$$

Notice that the trade shocks that move the economy from autarky to trade economy do not alter the labor endowment L_i and the productivity φ_i. Therefore, the overall emissions of country i remain constant after the trade opening:

$$Z_i^{TRADE} = \mu_i \varphi_i^{1-\gamma_i} L_i = Z_i^{AUT}.$$

C.2 Krugman model

In the Krugman model, there is not just one variety per country but a continuum of firms M_i producing distinct varieties. As they have all the same productivity and emission intensity, the expression for aggregate emissions only depends on the number of firms M_i and the amount of labor input per firm. Under ACR restrictions, both the M_i and labor input per firm do not change, which makes the emission unchanged.

C.3 EK model

For the EK model, we utilize the expression for emissions embodied in trade:

$$Z_{ni} = \mu_i \frac{X_{in}}{w_i} \left(\frac{T_i}{\lambda_{ni}} \right)^{(1-\gamma_i)/\theta} \frac{\Gamma\left(\frac{\theta+\gamma_i-\sigma}{\theta}\right)}{\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)}.$$

and the aggregate emissions are

$$\begin{aligned} Z_i &= \mu_i \frac{Y_i T_i^{1/\theta}}{w_i} \sum_{n=1}^{N} \frac{X_{ni}}{Y_i} \lambda_{in}^{(\gamma_i-1)/\theta} \frac{\Gamma\left(\frac{\theta+\gamma_i-\sigma}{\theta}\right)}{\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)} \\ &= \mu_i L_i T_i^{1/\theta} \sum_{n=1} s_{ni} \lambda_{ni}^{(\gamma_i-1)/\theta} \frac{\Gamma\left(\frac{\theta+\gamma_i-\sigma}{\theta}\right)}{\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)}, \end{aligned}$$

where $s_{ni} = \frac{X_{ni}}{Y_i}$. Since λ_{ii} and s_{ii} are unity in autarky, we have

$$Z_i^{AUT} = \mu_i L_i T_i^{1/\theta} \frac{\Gamma\left(\frac{\theta+\gamma_i-\sigma}{\theta}\right)}{\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)}.$$

Notice that with finite trade costs, $0 < \lambda_{ni} < 1$. If $\gamma_i < 1$, we have hence $\lambda_{ni}^{(\gamma_i-1)/\theta} > 1$. Combining these formula and $\sum_{n=1}^{N} s_{ni} = 1$, we have:

$$\begin{aligned} Z_i^{AUT} &= \mu_i L_i T_i^{1/\theta} \frac{\Gamma\left(\frac{\theta+\gamma_i-\sigma}{\theta}\right)}{\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)} = \mu_i L_i T_i^{1/\theta} \sum_{n=1}^{N} s_{ni} \frac{\Gamma\left(\frac{\theta+\gamma_i-\sigma}{\theta}\right)}{\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)} \\ &< \mu_i L_i T_i^{1/\theta} \sum_{n=1}^{N} s_{ni} \lambda_{in}^{(\gamma_i-1)/\theta} \frac{\Gamma\left(\frac{\theta+\gamma_i-\sigma}{\theta}\right)}{\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)} = Z_i^{TRADE}. \end{aligned}$$

We can prove the case for $\gamma_i = 1$ and $\gamma_i > 1$ in a similar manner.

C.4 Melitz model

We start by rewriting emissions embodied in trade flows as

$$Z_i = \frac{\mu_i L_i}{\tilde{\sigma}} \frac{\theta}{\theta + \gamma_i - \sigma} \left(\varphi_{ii}^*\right)^{1-\gamma_i} \sum_{n=1}^{N} S_{ni}^{(1-\gamma_i)/\theta} \frac{X_{ni}}{w_i L_i}. \tag{40}$$

By the zero profit condition of exporting the domestic productivity cut-off is

$$\varphi_{ii}^* = \frac{f_{ii}}{P_i} w_i \tilde{\sigma} \tau_{ii}.$$

Hence $\varphi_{ii}^* \propto \frac{w_i}{P_i}$. As shown by **?** trade opening increases the real wage and thus must rise the domestic productivity cut-off. Hence $\varphi_{ii}^{*,AUT} < \varphi_{ii}^{*,TRADE}$. Further, under Autarky $\frac{X_{ii}}{w_i L_i} = S_{ii} = 1$ and under conventional parameter restrictions, $S_{ni} < 1$. As a consequence, if $\gamma_i < 1$, $S_{ni}^{(\gamma_i-1)/\theta} > 1$. In combination with $\sum_{n\in N} \frac{X_{ni}}{w_i L_i} = 1$ we get:

$$\begin{aligned} Z_i^{AUT} = \frac{\mu_i L_i}{\tilde{\sigma}} \frac{\theta}{\theta + \gamma_i - \sigma} \left(\varphi_{ii}^{*,AUT}\right)^{1-\gamma_i} &= \frac{\mu_i L_i}{\tilde{\sigma}} \frac{\theta}{\theta + \gamma_i - \sigma} \left(\varphi_{ii}^{*,AUT}\right)^{1-\gamma_i} \sum_{n\in N} \frac{X_{ni}}{w_i L_i} \\ &< \frac{\mu_i L_i}{\tilde{\sigma}} \frac{\theta}{\theta + \gamma_i - \sigma} \left(\varphi_{ii}^{*,TRADE}\right)^{1-\gamma_i} \sum_{n\in N} S_{ni}^{(1-\gamma_i)/\theta} \frac{X_{ni}}{w_i L_i}. \end{aligned}$$

The proof works similarly for $\gamma_i = 0$ and $\gamma_i = 1$.

D Proof of Proposition ??

The change in country level emissions is $\hat{Z}_i = \sum_{s \in S} \frac{Z_{is}}{Z_i} \hat{Z}_{is}$. From (**??**), the counterfactual change in emissions in sector s country i is given by

$$\hat{Z}_{is} = \frac{\hat{\beta}_{is}(w_i/t_i)}{\hat{t}_i} \hat{R}_{is} \tag{41}$$

Fixing $\hat{t}_i = \hat{w}_i$ and noting that from the labor market clearing condition $\hat{R}_{is} = \hat{w}_i \hat{L}_{is}$ yields:

$$\hat{Z}_{is} = \hat{L}_{is} \tag{42}$$

and summing over all sectors yields (**??**). Labor market on the sector level clearing implies, $w_i L_{is} = (1 - \beta(w_i/t_i))R_{is}$. With $\hat{t}_i = \hat{w}_i$, the counterfactual labor allocation is given by:

$$\hat{L}_{is} = \frac{1}{\hat{w}_i} \hat{R}_{is}$$

Noting that aggregate revenues are given by $R_{is} = \sum_{n=1}^{N} \lambda_{nis} E_{ns}$ directly yields (**??**). Applying the exact hat algebra to (**??**) directly yields (**??**). Applying hte hat algebra to (**??**) and using again $\hat{t}_i = \hat{w}_i$ yields (**??**). Finally, aggregate labor market clearing implies that

$$w_i L_i = \sum_{s=1}^{S} \tilde{\beta}_{is}(w_i/t_i)) R_{is} \tag{43}$$

As the total labor endowment is fixed, expressing this equation in changes yields (**??**).

E Proof of Corollary ??

Part (i) From the derivation of (**??**), the change in the trade share for an Armington and EK model is obtained by setting $\delta_s = 0$ in (**??**). Hence, the trade share response is exactly the same across models. From Proposition **??**, the difference in emissions across models is fully captured via differential responses in the trade share. Thus, the emission response is the same for Armington and EK.

Part (ii) The derivation of equation (**??**) shows that the change in expenditure shares in Krugman is obtained by setting $\delta_s = \frac{\eta_s - 1}{\sigma_s - 1}$, which is larger than zero because $\eta_s > 1$ and $\sigma_s > 1$. Hence, there is an additional term affecting expenditure changes. From (**??**), these scale economies are also operative in Melitz. Moreover, ζ_s is generally different from zero.

Part (iii) If $\frac{Z_{is}}{Z_i} = \frac{L_{is}}{L_i}$, the change in emission is given by:

$$\hat{Z}_i = \sum_{s \in S} \frac{L_{is}}{L_i} \hat{L}_{is} \tag{44}$$

Because the labor endowment is fixed, for any trade shock $\hat{L}_i = 1 \forall i$. Hence,

$$\sum_{s \in S} \frac{L_{is}}{L_i} \hat{L}_{is} = \hat{L}_i = 1$$

F Derivation of (??)

Similar to the single-sector case, the expenditure share can be written as:

$$\lambda_{nis} = \frac{M_{nis} \frac{\theta_s}{\theta_s + 1 - \sigma} x_{nis}(\varphi^*_{ni})}{E_{ns}} = \frac{M_{is}}{\kappa_s w_n L_n} T_i^{\theta_s} (\varphi^*_{nis})^{-\theta_s} \frac{\theta_s}{\theta_s + 1 - \sigma_s} f_{nis} w_n.$$

Recalling that the number of entering firms is given by (see the only appendix):

$$M_{is} = \frac{L_{is}}{f^e_{is}} \frac{1}{1 - \frac{\theta_s + 1 - \sigma_s}{\theta_s \sigma_s}}$$

, the change in the productivity cut-off can be expressed as:

$$\widehat{\varphi^*_{nis}} = \left(\frac{\hat{L}_{is}}{\hat{\lambda}_{nis}} \right)^{1/\theta_s}$$

Similar to the one sector case emissions can be expressed as:

$$\hat{Z}^{\text{Melitz}}_{nis} = \frac{\hat{X}_{nis}}{\hat{w}_i} (\widehat{\varphi^*_{nis}})^{1-\gamma_s}.$$

With the change in the productivity cut-off and setting $\psi_s = 1/\theta_s$, we obtain (**??**).

G Proof of Proposition ??

For propositions (i) and (ii), we obtain the result directly from Equation (**??**). We only prove the statement (iii), since proving (iv) is a straightforward extension.

We prove that if $\gamma_s < 1$, trade opening implies higher emissions in the EK than in the Armington model. We start from the same observed autarky equilibrium with Z^{AUT}_{is} and R^{AUT}_{is}. Denote the industry's emissions per revenue in autarky as $\tilde{\mu}^{AUT}_{is} = Z^{AUT}_{is} / R^{AUT}_{is}$ where Z^{AUT}_{is} and R^{AUT}_{is} are emissions and revenue in autarky for industry s in country i, respectively. Since emission per revenue in the Armington model is constant, the emissions under trade opening in the Armington model are:

$$Z^{ARM}_{is} = \tilde{\mu}^{AUT}_{is} R_{is},$$

where R_{is} is the revenue of industry s in country i after the trade opening. Notice that R_{is} is common across the two models. In the EK model, the emission per sales after trade

opening is $\tilde{\mu}_{nis}^{EK} = \tilde{\mu}_{is}^{AUT} \lambda_{nis}^{(\gamma_s - 1)/\theta}$, which implies:

$$Z_{is}^{EK} = \sum_{n=1}^{N} \tilde{\mu}_{nis}^{EK} X_{nis} = \sum_{n=1}^{N} \tilde{\mu}_{is}^{AUT} \lambda_{nis}^{(\gamma_s - 1)/\theta} X_{nis} > \sum_{n=1}^{N} \tilde{\mu}_{is}^{AUT} X_{nis} = \tilde{\mu}_{is}^{AUT} R_{is} = Z_{is}^{ARM}.$$

The inequality holds since $0 < \lambda_{nis} < 1$ and $(\gamma_s - 1)/\theta < 0$, and hence $\lambda_{nis}^{(\gamma_s - 1)/\theta} > 1$. Aggregating this over industries will result in Proposition **??** (iii).

H Imputation of the exporting share

We impute the share of exporting firms based on a model-driven gravity equation. Recall that the share of exporters from n to i in sector s is given by

$$S_{nis} = \frac{M_{nis}}{M_{iis}} \tag{45}$$

The number of exporters can be expressed as the ratio of total to average exports, i.e. :

$$M_{nis} = \frac{X_{nis}}{\bar{x}_{nis}}, \tag{46}$$

where $\bar{x}_{nis}$ is the average export value. This implies:

$$S_{nis} = \frac{X_{nis}}{X_{iis}} \frac{\bar{x}_{iis}}{\bar{x}_{nis}}. \tag{47}$$

In the Melitz-Chaney model, the ratio of the average sales is equal to the ratio of fixed costs:

$$\frac{\bar{x}_{iis}}{\bar{x}_{nis}} = \frac{f_{nis}}{f_{iis}}. \tag{48}$$

We proxy the relative fixed costs by the distance between n and i, $dist_{ni}$. Hence, we get the following estimation equation:

$$log(S_{nis}) = log(\lambda_{nis}^{x}) + log(dist_{ni}) + \xi_n, \tag{49}$$

where $\lambda_{ni}^{x} = \frac{X_{nis}}{X_{iis}}$, and ξ_n are destination fixed effects. We drop all internal trade flows for the estimation and estimate the equation for each WIOD sector and the entire economy.

I Parameter calibration

Calibration of parameters for the multiple industries is done as follows. We take σ_s and η_s from **?** and θ_s from **?**. Using these parameters, we construct trade elasticity and scale elasticity using the formula from the Melitz model ($\varepsilon_s = \frac{\theta_s}{(1+\theta_s\left(\frac{1}{\eta_s - 1} - \frac{1}{\sigma_s - 1}\right)}$, $\delta_s = \frac{\varepsilon_s}{\theta_s}$). We use the trade elasticity derived from the Melitz model for other models. For the scale elasticity, we use the derived scale elasticity for the Melitz model while setting the parameter to zero for the Armington and the EK. The parameter γ_s is calculated from the estimate in column 9 and the same σ_s we use to calculate elasticities. Apart from γ_s, the

counterfactual emissions in EK and Melitz depend on the value of θ_s, which is different for these two models. For Melitz, we use the value presented in Table **??**, whereas for EK, θ_s equals the trade elasticity, i.e., we set $\theta_s^{EK} = \varepsilon_s$.

We acknowledge that this is not internally consistent. If we take the parameters seriously, we should have different trade elasticity across the model. If we fix the trade elasticity, we should use the implied σ_s and θ_s to calculate γ_i^s and the counterfactual emissions, which will differ across models. We keep our calibration this way to highlight the key differences from the firm heterogeneity while keeping the elasticity intact.

Table 2: Parameters

Sector	δ	ϵ	θ	η	σ
A01	0.00	1.85		7.23	7.99
A02	0.00	0.52		7.23	7.99
A03	0.00	11.98		7.23	7.99
B	0.00	13.97		7.23	7.99
C10-C12	0.86	3.35	3.89	3.30	3.54
C13-C15	0.74	3.55	4.8	4.36	5.46
C16	0.85	5.29	6.2	4.90	5.37
C17	0.77	4.00	5.21	3.65	4.12
C18	0.77	4.00	5.21	3.65	4.12
C19	0.22	2.21	9.91	1.64	1.82
C20	0.93	3.27	3.5	4.97	5.31
C21	0.93	3.27	3.5	4.97	5.31
C22	0.80	3.70	4.62	6.16	8.14
C23	0.92	3.71	4.05	6.28	6.99
C24	0.45	4.47	10.01	4.00	5.78
C25	0.63	3.01	4.8	4.00	5.78
C26	0.42	2.24	5.32	2.24	2.81
C27	0.45	2.14	4.77	2.24	2.81
C28	0.96	4.04	4.19	8.75	9.33
C29	0.44	2.46	5.6	3.81	8.75
C30	0.53	2.06	3.87	3.81	8.75
C31-C32	0.96	3.61	3.75	7.17	7.58
C33	0.96	3.61	3.75	7.17	7.58
D-E	0.00	4.00		4.00	5.00
F	0.00	4.00		4.00	5.00
G	0.00	4.00		4.00	5.00
H	0.00	4.00		4.00	5.00
I-U	0.00	4.00		4.00	5.00
Total		4.00			5.00

Note:
Parameters used in the analysis. σ_s and η_s are sourced from citeLashkaripour2023 and $theta_s$ from citeShapiro2018. The remaining parameters are calculated based on these estimates and consistent with the Melitz model, i.e $\epsilon_s = \frac{\theta_s}{(1+\theta_s\left(\frac{1}{\eta_s-1}-\frac{1}{\sigma_s-1}\right)}$, $\delta_s = \frac{\epsilon_s}{\theta_s}$. For sectors outside manufacturing and without an estimate of θ_s we take the estimated trade elasticity from **?**.