Markups and Entry in a Circular Hotelling Model*

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October 2024

Abstract

The Hotelling locational model and its adaptation to a circular city provide a core framework for industrial organization. The present paper expands the explanatory power of this model by incorporating a continuum of consumers with constant-elasticity demand functions along with stores that have constant marginal costs of production. The stores are evenly spaced in equilibrium. The model generates an approximate formula in which the markup of price over marginal cost depends on the spacing between stores and a transportation-cost parameter but is independent of the elasticity of demand. This result reflects pricing decisions by stores that factor in the threat of losing business entirely at the borders with neighboring stores. This model provides a theory of price markups that is an alternative to the familiar Lerner approach, which puts all the weight on the elasticity of demand.

*I have benefited from comments by Ed Glaeser, Elhanan Helpman, and Chad Syverson.

Hotelling (1929) constructed a model of monopolistic competition in which stores were located along a finite straight line and consumers purchased goods from their nearest store. Because of locational advantages, stores had some degree of monopoly power, and this power entered into their pricing decisions. The present analysis uses a circular version of the Hotelling model, usually labeled the Salop model and attributed to Salop (1979).[1] A major technical advantage of the circle framework is that it allows for symmetric outcomes by avoiding issues related to the end points of a finite straight line.[2]

The finite straight line is an attractive framework in some contexts because the end points represent extremes of characteristics such as preferences of political parties about the size of government (Downs [1957]) or of religions about the degree of strictness (Barro and McCleary [2005]). However, for most applications in industrial organization, these end-point features do not apply, and the circle model provides a more useful setting.

A central feature of the model is that each store's pricing behavior involves competition from neighboring stores. From the perspective of customers, the closest substitutes for each store are the adjacent ones. This interaction between stores means that the threat of losing business at borders, rather than the elasticity of demand, is the key force in the determination of price markups. These markups ultimately determine the extent of entry into the market and, thereby, the efficiency of market outcomes.

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[1]Salop (1979) cites Schmalensee (1978) for a prior application of the circular setting. Moreover, the circular framework was used earlier by Vickrey (1964, Ch. 8), who referred to "… a simple loop (which may or may not be circular) …"

[2]Salop (1979, p. 142) says: "The product space of the industry is taken to be an infinite line or the unit-circumference of a circle. While neither assumption is realistic, both allow the 'corner' difficulties of the original Hotelling model to be ignored and an industry equilibrium with identical prices by equally-spaced firms to obtain." Hay (1976, p. 243) relied on a very long straight line: "For most of the analysis we assume that the market is very long in comparison with the market size of the firm. This enables us to avoid the analytic complications of market intervals at the end points of the market."

I. Circular Hotelling Model

The economy, which can be viewed as a city, features consumers distributed uniformly around a circle with circumference H, as in Figure 1. The parameter H represents the size of the city. A number N of stores service the consumers. Each store j produces goods at the same constant marginal cost, c. (Hotelling [1929, p. 45] assumed that the marginal cost was zero.) Store j prices at P_j. In addition to paying P_j for each unit of goods, a customer pays an amount per unit that increases linearly with the distance, z, from its nearest store, as in Hotelling (1929, p. 45). This transportation cost per unit is represented by tz, where $t>0$ is a transport-cost parameter that depends on transportation technology and the value of customer time. As stressed by Hotelling (1929, p. 45), "transportation cost" should be viewed as a metaphor for many differentiating characteristics of goods and stores that cause customers to prefer one seller over another for a given price charged.[3]

As in Salop (1979), the stores are evenly spaced around the circle in Figure 1. This even spacing of stores is an equilibrium outcome in the present model. Store 1 is adjacent to stores 2 and N, as shown. Going to the right from its location, store 1 services customers out a distance h, which is the mid-point of the spacing, $2h$, from store 2. Similarly, going to the left, store 1 services customers out a distance h, the mid-point of its spacing from store N.

Customers located at distance h to the right of store 1 will be indifferent between buying from store 1 or store 2 and similarly to the left for store 1 versus store N. Behavior at these borders will be crucial for the analysis but is neglected initially for expository purposes. The spacing between stores, $2h$, will satisfy the condition

[3]Hotelling (1929, p. 45) says: "… there will be many causes leading particular classes of buyers to prefer one seller over another, but the ensemble of such consideration is here symbolized by transportation costs."

(1) $$h = \frac{1}{2}\frac{H}{N}.$$

For now, the analysis takes the number of stores, *N,* as given. However, in a full equilibrium, *N* and, hence, *h* will be determined from a free-entry condition.

Consider the choice of price, P_j, for store *j*, given the prices and locations of the other stores (as in a Bertrand analysis). In equilibrium, each store will charge the same price, $P_j=P$, and each customer will patronize its closest store. Without loss of generality, take $j=1$ and consider only the market going to the right where store 2 is the relevant alternative to store 1. (The results going to the left, toward store *N*, will be analogous.)

Denote by *q(z)* the quantity purchased of store 1's goods by customers located at distance *z* from the store. The effective price per unit faced by customers at position *z,* where $0 \leq z \leq h$, is assumed to add a linear transportation cost, *tz*, to the price set by store 1:

(2) $$P^*(z) = P + tz,$$

where the store subscript, 1 in this case, is omitted on $P^*(z)$ and *P*. An assumption is that store 1 charges each customer the same price regardless of their location.

Transportation costs can be reinterpreted as costs from consuming a good with characteristics that deviate from a customer's ideal type, as in Lancaster (1966) and Baumol (1967). As noted by Schmalensee (1978 p. 309), " … the formal correspondence between Lancastrian models with two characteristics and one-dimensional spatial models is almost exact." Helpman (1981, part 2) applied the Lancaster-type model to the Salop-style circle framework, with arc distances around the circle representing differences in product characteristics.

Equation (2) implies that transportation charges, which are *tz* per unit, are proportional to the quantity transported, *q(z)*. In some contexts, such as driving to and from a grocery store,

these charges might be independent of *q(z)* over some range.[4] However, it seems reasonable for most applications to view transportation charges as proportional to *q(z)*.

The quantity demanded of store 1's goods by customers located at distance z from the store is assumed to take a constant-elasticity form:[5]

$$q(z) = A \cdot [P^*(z)]^{-\eta}, \tag{3}$$

where $\eta \geq 0$.[6] The standard analysis requires $\eta > 1$, but this restriction is ultimately unnecessary in the present model. The parameter $A > 0$ represents the scale of each consumer's demand.

A modeling device that turns out to be highly convenient treats *q(z)* as contributing an infinitesimal amount to the total quantity of goods demanded, Q, from store 1; that is,

$$\frac{dQ}{dz} = q(z).$$

The total quantity, Q, sold by store 1 to customers to the right of the store's location equals the integral of *q(z)* for z going from *0* to h:

$$Q = \int_0^h q(z)dz.$$

Using the demand curve from Eq. (3) and making a change of variable from z to *P*(z),* the integral can be evaluated to get

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[4]Anderson and de Palma (2000) and Gu and Wenzel (2009) assume that transport charges are independent of the quantity shipped.

[5]Anderson and de Palma (2000) and Gu and Wenzel (2009) show that this form of demand function is implied by a utility function of the form $U = \frac{\eta}{\eta-1} q(z)^{(\eta-1)/\eta} + \hat{q}$, where $\hat{q}$ is the quantity of a numeraire good and gross income is $Y = P^*(z) \cdot q(z) + \hat{q}$. If all consumers have the same Y, customers located at higher z will effectively have lower net income because they spend more on transportation costs for a given *q(z)*. This net income difference would be exactly offset by lower residential rents if the only dimension of residential location that matters is its distance from the nearest store. This result accords with the standard insight in urban economics, summarized in Brueckner (1987), whereby differences in rents represent compensating differentials for differences in commuting costs.

[6]Hotelling (1929, pp. 45, 56) assumed $\eta = 0$. He says (p. 56): "The problem … might be varied by supposing that each consumer buys an amount of the commodity in question which depends on the delivered price. If one tries a particular demand function the mathematical complications will now be considerable … " Smithies (1941) and Hay (1976) extended the Hotelling analysis to allow for elastic consumer demand, though in the form of linear demand functions. Anderson and de Palma (2000) and Gu and Wenzel (2009) allowed for a constant elasticity of demand, as in Eq. (3), but their restriction to $\eta < 1$ limits the applicability of their setting.

(4) $$Q = \frac{A}{t(\eta-1)}[P^{1-\eta} - (P+th)^{1-\eta}],$$

where $\eta>1$ is assumed at this stage.

The marginal effect of P on Q follows from Eq.(4) as

(5) $$\frac{\partial Q}{\partial P} = -\left(\frac{A}{t}\right)\cdot\left[P^{-\eta} - (1+t\frac{\partial h}{\partial P})(P+th)^{-\eta}\right].$$

The essence of the Hotelling analysis is the term $\frac{\partial h}{\partial P}$, which indicates how an increment in P affects the location of the border between stores. However, to see the relation to standard results on markup pricing, the analysis assumes to begin that $\frac{\partial h}{\partial P} = 0$.

The profit flow for store 1 (for sales to the right in Figure 1) is

(6) $$\pi = (P-c)Q - f,$$

where $f > 0$ is the fixed cost of operating a store. (The fixed cost could correspond to the rent paid on the store's premises. In the subsequent analysis, this fixed cost also covers sales to the left of store 1 in Figure 1.) The first-order condition for choosing P to maximize π follows from Eq. (6) as

(7) $$\frac{\partial \pi}{\partial P} = Q + (P-c)\frac{\partial Q}{\partial P} = 0.$$

Substituting into Eq. (7) for Q from Eq. (4) and for $\frac{\partial Q}{\partial P}$ from Eq, (5) (assuming $\frac{\partial h}{\partial P} = 0$) leads, if $\eta>1$, to the first-order maximization condition:

(8) $$\left(\frac{1}{\eta-1}\right)\cdot[P^{1-\eta} - (P+th)^{1-\eta}] = (P-c)[P^{-\eta} - (P+th)^{-\eta}].$$

The results simplify if the maximal operative transport cost, th, is small compared to the price, P; that is, if the main component of the effective price, $P^*(z)$ in Eq. (2), is always the price

charged, P, rather than the transport cost, tz. Ultimately, the required condition will be that the marginal production cost dominates over transport cost, so that $th \ll c$ applies. This approximation, which is explored in detail in the next section, allows for simplifications of the terms involving $P+th$ in Eq. (8) by means of first-order Taylor approximations. These approximations deliver the standard result, described in Lerner (1934), whereby P is a constant markup on marginal cost, c:[7]

$$P \approx c \cdot \left(\frac{\eta}{\eta-1}\right), \tag{9}$$

so that the markup ratio is

$$\frac{P-c}{c} \approx \frac{1}{\eta-1}. \tag{10}$$

Therefore, when the effect of P on the location of the border is neglected, the markup ratio is determined in the usual way by the elasticity of demand, η, with a higher elasticity implying a lower markup ratio. Finite price, P, and profit, π, require $\eta>1$. This restriction on the elasticity of demand is familiar but also likely to be unrealistic.

The price, P, shown in Eq. (9) is the one chosen by store 1. However, the same price applies for each store around the circle in Figure 1 as long as the marginal cost, c, and the elasticity of demand, η, are the same in each territory. The analysis now turns to the crucial influence on pricing behavior from the interactions at the borders between stores.

II. Hotelling Effect at the Borders

At the border between stores 1 and 2, where $z=h$ in Figure 1, customers are indifferent between buying goods from store 1 or store 2 when the prices are the same. Starting from a

[7]The second-order condition for profit maximization is satisfied if $\eta>1$.

position of equal prices and holding fixed store 2's price, a positive increment in store 1's price, $dP>0$, would motivate customers located at the border to make all of their purchases from store 2. Since $q(h)$ is the quantity bought, the amount $q(h)\cdot dP$ is the extra cost imposed on buyers located at the border. In response, the border has to shift leftward toward store 1; that is, $dh<0$ applies. This change corresponds to a reduction by the magnitude $tq(h)\cdot dh$ in the amount paid by border customers for transport costs from store 1. Similarly, the amount paid by border customers for transport costs from store 2 rises by the magnitude $tq(h)\cdot dh$. Thus, the difference between transport costs from store 1 compared to those from store 2 falls by the magnitude $2tq(h)\cdot dh$. In order for persons located at the new border to be indifferent between buying from store 1 or store 2, the magnitude of $2tq(h)\cdot dh$ must equal that of $q(h)\cdot dP$. That is, $2tq(h)\cdot dh + q(h)\cdot dP=0$ must hold. Therefore, the key condition that connects the position of the border to the increment in price is:[8]

$$\frac{\partial h}{\partial P} = -\frac{1}{2t}. \tag{11}$$

Substituting the result from Eq. (11) into the expression for $\frac{\partial Q}{\partial P}$ in Eq. (5) implies

$$\frac{\partial Q}{\partial P} = -\left(\frac{A}{t}\right)\cdot[P^{-\eta} - \tfrac{1}{2}(P+th)^{-\eta}]. \tag{12}$$

Substitution of this result into the expression for $\frac{\partial \pi}{\partial P}$ in Eq. (7) yields the new first-order maximization condition:

$$\left(\frac{1}{\eta-1}\right)\cdot[P^{1-\eta} - (P+th)^{1-\eta}] = (P-c)\cdot[P^{-\eta} - \tfrac{1}{2}(P+th)^{-\eta}]. \tag{13}$$

[8]This analysis requires $N\geq 2$, so that $h\leq H/4$. Otherwise, there is no border competition, and the solution is the standard one given in Eqs. (9) and (10).

The results again simplify if the maximal operative transport cost, *th*, is small compared to the price, *P*. In this case, Eq. (13) simplifies using first-order Taylor approximations for the terms involving *P*+*th* to

$$P \approx c + 2th. \tag{14}$$

The markup ratio is then

$$\frac{P-c}{c} \approx \frac{2th}{c}. \tag{15}$$

The second-order condition for profit maximization is satisfied if $\eta \geq 0$. Note that the solution for the price markup in Eqs. (14) and (15) holds even when $\eta \leq 1$. In the standard, Lerner-type analysis, this range for η has to be excluded to avoid infinite profit by producers.[9]

Table A1 in the appendix examines the accuracy of the approximations that underlie the derivation of Eq. (14). The results can be expressed in terms of two parameters, η and $2th/c$, where the latter term is the ratio of markup to marginal cost in the formula in Eq. (15). Note first from Eq. (13) that Eq. (14) holds exactly if $\eta = 0$. Equation (14) also holds exactly if $th = 0$. More generally, Eq. (14) holds better as an approximation to the true solution for *P* when η and $2th/c$ are smaller. For example, as shown in Table A1, if $\eta \leq 5$ and $2th/c \leq 0.1$, the true solution for *P* differs from $c+2th$ by less than 3%. Further, if $\eta \leq 2$ and $2th/c \leq 0.2$, the true solution for *P* again differs from $c+2th$ by less than 3%. However, if η is very large, *P* can fall short of $c+2th$ by substantial amounts for reasonable values of $2th/c$. The bottom line from Table A1 is that, in plausible ranges for η and $2th/c$, such as $\eta \leq 5$ and $2th/c \leq 0.1$ or $\eta \leq 2$ and $2th/c \leq 0.2$, the formula for the markup ratio in Eq. (15) is a close approximation to the true solution. This result, where the

[9]The key mechanism in the Hotelling model is that a rise in *P* results in a contraction of the border, *h,* with *h* reaching zero for sufficiently high *P*. When $\eta \leq 1$, this mechanism prevents a store from achieving infinite profit by charging an infinite price.

markup ratio equals $2th/c$, contrasts with the standard Lerner formula in Eq. (10), which says that the markup ratio equals $1/(\eta-1)$ (when $\eta>1$).

In the literature on price markups, many studies mention the Lerner formula but make no use of it empirically.[10] Instead, these studies, exemplified by de Loecker, Eeckhout, and Unger (2020), emphasize ways to measure price markups, following the lead of Hall (1988). Although these results are useful, they do not relate observed price markups to "fundamentals," which might include differences in the elasticity of demand or in factors that affect the substitutability of neighboring products. In the Hotelling-type model, the latter group includes influences on the spacing between stores (or products), h, and the transportation-cost parameter, t.

There is a sense in which the results for the markup ratio in the present model accord with the standard formula based on the elasticity of demand in Eq. (10). The standard result holds if η is interpreted not as the parameter in the individual demand curve in Eq. (3) but rather as the full magnitude of the elasticity of Q with respect to P—including the extensive margin whereby a store loses all of the business at its borders by raising its price. In this setting, the micro-level elasticity of demand, η—effectively the intensive margin for quantity demanded—does not affect the markup ratio as an approximation in a range of reasonable parameter values.

Return now to the assumption that stores are evenly spaced around the circle in Figure 1. Suppose that store 1 takes as given the positions of stores 2 and N. Suppose, starting from a position equidistant between stores N and 2, that store 1 considers moving its location, say by a distance X to the right. The market border with store N would then shift from a distance h to a distance $h+X/2$ from store 1's location. The border with store 2 would shift from a distance h to a distance $h-X/2$. Therefore, the total distance covered by store 1 would remain at $2h$. However,

[10]See, for example, Hall (2018, p. 2), Bond, et al. (2021, p. 4), and Syverson (2024, p. 5).

there is a shift toward customers who are relatively far from the store (in the region toward store *N*) and away from those who are relatively close (in the region toward store 2). Because of the downward-sloping demand curve in Eq. (3), the more distant customers buy a smaller quantity and are less profitable for store 1. Hence, on net, store 1's profit declines. It follows that store 1 would not move and is best off remaining equidistant between stores 2 and *N*. That is, the equal-spacing pattern is an equilibrium outcome.[11] Note that this result depends on the downward-sloping demand curve—the result would not apply under the common assumption that each household buys exactly one unit of the good (for example, in the version of the Salop model described in Tirole [1988, Section 7.1.2]). The result also depends on the assumption that transportation charges are proportional to the quantity bought, *q(z),* rather than being independent of this quantity.

The results in Eqs. (14) and (15) are reminiscent of models of contestable markets, as described by Baumol, Panzar, and Willig (1982). In those models, an individual firm's markup is limited by the cost of production for potential entrants into a market, and marginal-cost pricing prevails under assumptions that include zero costs of entry and exit and equal access of all firms to all technologies. A difference in the present, Hotelling context is that the relevant competition depends on actual participants in the market—neighboring stores—rather than hypothetical entry by potential participants. Moreover, the Hotelling-type model does not predict marginal-cost pricing but rather price markups that depend on the distances between active stores in the market.

[11]This conclusion accords with Vickrey (1964, p. 330).

III. Free-Entry Condition

The analysis of entry in this section applies when Eq. (14) is an accurate formula for the price; that is, when $P \approx c + 2th$. The total quantity sold, Q, by store 1 (or any store) can then be determined from Eq. (4) to be:

$$Q \approx 2hAc^{-\eta}, \tag{16}$$

where the 2 reflects store 1's sales on both sides of its location in Figure 1. The associated profit is given from Eq. (6) by

$$\pi \approx 4tAh^2c^{-\eta} - f. \tag{17}$$

If $\pi > 0$, there is an incentive for new stores to enter the market, thereby raising N and lowering $h = \frac{1}{2}\frac{H}{N}$.[12] The decrease in h lowers π in Eq (17). This process continues as long as $\pi > 0$. If the integer constraint on N can be neglected, as will be satisfactory if N is large,[13] the free-entry condition will be $\pi \approx 0$. Equation (17) then implies

$$2h \approx \sqrt{\frac{f}{tAc^{-\eta}}}. \tag{18}$$

Therefore, under free entry, the spacing between stores, $2h$, follows a square-root rule, whereby the spacing is larger the lower the transport cost per unit, t, the higher the fixed cost f of operating a store, and the (approximate) quantity sold to each buyer, $Ac^{-\eta}$. The last two results apply because the fixed cost is effectively scaled by the quantity sold to each buyer. Note that the spacing does not depend on the overall size of the city, represented by H in Figure 1.

[12]The assumption here is that firms immediately relocate to preserve their equal spacing around the circle in Figure 1. More realistically, the model applies to a long-run, steady-state situation, rather than to the dynamics of entry and exit.

[13]For example, for New York City, with a central city population of 8 million in 2019, the New York State Comptroller reports that the number of retail businesses at that time was about 33,000, each servicing on average 242 domestic residents. In this context, where N is *33,000*, the integer constraint would be of no consequence. Mankiw and Whinston (1986) discuss integer constraints for a related model.

The solution for spacing, *2h,* in Eq. (18) implies that the number of firms is given from Eq. (1) by

$$N = \frac{H}{2h} \approx H \cdot \sqrt{\frac{tAc^{-\eta}}{f}}. \tag{19}$$

Therefore, N is proportional to city size, *H,* and rises with the square root of the transport cost per unit, *t,* and the quantity sold to each buyer, $Ac^{-\eta}$. The number *N* varies inversely with the square root of the fixed cost, f, of operating a store.

The free-entry result in Eq. (18) implies from Eq. (14) that the price charged will be

$$P \approx c + \sqrt{\frac{tf}{Ac^{-\eta}}}. \tag{20}$$

That is, the markup on marginal cost, *c*, is increasing with the transport-cost parameter, *t*, and the fixed cost f of operating a store and decreasing with the quantity sold to each buyer, $Ac^{-\eta}$. The markup does not depend on *H,* the overall size of the city. The markup also does not depend (as an approximation) on the elasticity of demand, η, except for a positive effect that involves the quantity sold, $Ac^{-\eta}$. Moreover, the results hold for any $\eta \geq 0$, including Hotelling's case where $\eta = 0$.

The theoretical results can be related to the empirical analysis of Chevalier, Kashyap, and Rossi (2003) concerning pricing patterns of a major supermarket chain. Their key finding is that markups for affected categories of products are relatively low at times of peak demand, notably during major holidays and events such as Christmas, Thanksgiving, and Lent. Their empirical results rule out an explanation for this pricing pattern based on the elasticity of demand being unusually high at these times of peak demand. This finding accords with the present model in that the price markup is not predicted to fall when the demand elasticity, η, rises in Eq.(14) for a given market size, *2h*, or in Eq. (20), where 2*h* is determined by a free-entry condition. In the

model, peak demand would be represented by a temporarily high value of the parameter A, which enters into the demand function in Eq. (3). High A has no effect on the price markup in Eq. (14) but reduces this markup in Eq. (20), which factors in the free-entry condition. An application of this last result to the setting of Chevalier, Kashyap, and Rossi (2003) depends on there being significant entry during peak demand periods. Possibly this entry can take the form not of new stores but of longer store hours or increased advertising (a margin emphasized by Chevalier, Kashyap, and Rossi).

IV. Socially-Optimal Entry

Heuristically, there are two elements in the model that may cause the free-entry choices of spacing, $2h$, and number of stores, N, from Eqs. (18) and (19) to deviate from socially-optimal values. The first distortion is the markup, approximated by $2th$ in Eq. (14), which generates an excess of the effective price, P^*, over social marginal cost, $c+tz$, in Eq. (3). This excessive price leads to quantities of goods consumed that fall short of socially-optimal values. That is, at the existing quantity sold, each household's willingness to pay, P^*, for an additional unit—corresponding to the inverse-demand function implied by Eq. (3)—exceeds the social marginal cost, $c+tz$. Therefore, the profit flow in Eq. (6), which is the signal for entry, is too low in the sense of falling short of the amount that corresponds to the socially-optimal quantity of goods produced. On this ground, one would expect the free-entry choice of the number of stores, N, to be too low, corresponding to the choice of spacing, $2h$, being too high.

The second distortion is that an entering firm's profit includes revenue that is transferred from incumbent firms. That is, there is a "business-stealing effect,"[14] which is a private reward

[14]This term appears in Mankiw and Whinston (1986, p. 49) and also in Tirole (1988, p. 284). The business-stealing effect plays a large role in growth models that feature creative destruction, as in Aghion and Howitt (1992). The

for entry that has no social benefit. (The distortion here can be viewed as generated by the lack of property rights for incumbent firms in their profit flows.) On this ground, one would expect the free-entry choice of the number of stores, N, to be too high, corresponding to a choice of spacing, $2h$, that is too low.

Tirole's (1988, Ch. 7, p. 284) analysis of the Salop-Hotelling model finds that entry is excessive in that model: "… we compare the free-entry equilibrium with the allocation selected by a social planner. We already know that the price charged by the firms is greater than the marginal cost. However, in this case, where consumers all receive the same utility from the good and each consumes only one unit, this price introduces no distortion." Hence, Tirole's conclusion that the free-entry equilibrium features too much entry applies because the analysis considers only the distortion from business-stealing.

Suppose in the present model that the social planner can dictate the spacing between stores, $2h$, and the quantity of goods purchased, $q(z)$, by each household. However, the planner is assumed to accept the uniform distribution of consumers around the circle in Figure 1. Optimality entails the equation of each customer's willingness to pay, corresponding to the demand price from the inversion of Eq. (3), to the social marginal cost, $c+tz$. The socially-optimal value of spacing, $2h$ can then be derived, following the approach of Mankiw and Whinston (1986, p. 50), by maximizing an expression for aggregate consumer surplus (netting out costs of producing and transporting goods and the fixed costs of operating stores). The result, derived in the appendix, is that the social planner's choice of store spacing, $2h$, exceeds

term creative destruction was used by Schumpeter (1942, p. 83): "This process of Creative Destruction is the essential fact about capitalism. It is what capitalism consists in and what every capitalist concern has got to live in." Schumpeter's analysis of creative destruction was heavily influenced in Schumpeter (1942, part I) by his reading of Karl Marx. It is unclear whether Schumpeter's vision was preceded by Picasso's famous quote: "Every act of creation begins with an act of destruction."

the free-entry choice (in Eq. [18]). Correspondingly, the social planner's choice of number of stores, N, falls short of the free-entry number (in Eq. [19]). In this sense, the model accords with the excess-entry result of Tirole (1988).

Recall that Tirole's calculations correspond to assuming $\eta=0$, which eliminates the distortion associated with inefficiently low quantities of goods. In the present model, η can be positive, but the main results also assume that the approximation $2th<<c$ is valid. This condition generates a quantity distortion that is nonzero but quantitatively minor. Therefore, the excess entry result still obtains. For very high values of η, the results from the approximate solution become unsatisfactory as shown in Table A1 in the appendix. In this range, the distortion from inefficiently low quantity becomes more important, and the excess-entry result need not apply.

Free entry and socially-optimal entry have also been studied in non-spatial models of monopolistic competition, which include Spence (1976), Dixit and Stiglitz (1977), and Mankiw and Whinston (1986). In this literature, free entry sometimes results in the socially optimal amount of entry, but this result is not general. However, an important feature of these models is that the various products are treated symmetrically, with each good equally substitutable with each alternative good. In contrast, the central feature of Hotelling-type models and related Lancaster-type models is the strong substitution with "neighboring" producers and products. This property underlies the markup-pricing result in Eq. (14), which is driven by the threat of losing business at the borders with adjacent stores. This formula for markup pricing is, in turn, a key ingredient in the analysis of free entry and socially-optimal entry.

V. Summary

In a circular Hotelling model, customers are uniformly distributed around a circle with circumference H and number of stores N. Equal spacing of stores applies in equilibrium. A customer's transportation cost per unit is tz, where z is the distance from the nearest store. The effective price paid per unit is $P^*=P+tz$, where P is the price at the store's location. The marginal cost of production is the constant c, and the magnitude of the elasticity of each customer's quantity demanded with respect to P^* is the constant η.

Stores serve customers out a distance $h=H/2N$. If production costs dominate transport costs, each store's equilibrium price approximates the markup $2th$ over marginal cost, independent of the elasticity of demand. The key element in a store's markup is the threat of losing all of the business at its borders. In a free-entry equilibrium, h is larger the lower t, the higher the fixed cost of operating a store, and the smaller the scale of quantity demanded by each customer. Given these factors, the number of stores, N, is proportional to city size, H, and the markup is independent of H.

Two distortions affect the equilibrium: the excess of price over marginal cost implies inefficiently low quantities of goods produced, and a business-stealing effect implies that a new entrant's profit includes revenue transferred from incumbent stores. In a plausible range for the parameters η and $2th/c$, the business-stealing effect dominates, and entry is excessive.

From the perspective of understanding price markups, the biggest contribution from Hotelling-type models is the emphasis on factors that influence the substitutability among neighboring producers and products. This focus contrasts with the standard Lerner formula, which puts all the weight on the elasticity of demand.

References

Aghion, Philippe and Peter Howitt (1992). "A Model of Growth through Creative Destruction," *Econometrica*, 60, 323-351.

Anderson, Simon P. and Andre de Palma (2000). "From Local to Global Competition," *European Economic Review*, 44, 423-448.

Barro, Robert J. and Rachel M. McCleary (2005). "Which Countries Have State Religions?" *Quarterly Journal of Economics*, 130, 1331-1370.

Baumol, William J. (1967). "Calculation of Optimal Product and Retailer Characteristics: The Abstract Product Approach," *Journal of Political Economy,* 75, 674-685.

Baumol, William J., John C. Panzar, and Robert D. Willig (1982). *Contestable Markets and the Theory of Industrial Structure*, New York, Harcourt Brace Jovanovich.

Bond, Steve, Arshia Hashemi, Greg Kaplan, and Piotr Zoch (2021). "Some Unpleasant Markup Arithmetic: Production Function Elasticities and their Estimation from Production Data," *Journal of Monetary Economics*, 121, 1-14.

Brueckner, Jan K. (1987). "A Unified Treatment of the Muth-Mills Model," in Edwin S. Mills, ed., *Handbook of Regional and Urban Economics*, Amsterdam, Elsevier.

Chevalier, Judith A., Anil K. Kashyap, and Peter E. Rossi (2003). "Why Don't Prices Rise during Periods of Peak Demand? Evidence from Scanner Data," *American Economic Review*, 93, 15-37.

De Loecker, Jan, Jan Eeckhout, and Gabriel Unger (2020). "The Rise of Market Power and the Macroeconomic Implications," *Quarterly Journal of Economics*, 135, 561-644.

Dixit, Avinash K. and Joseph E. Stiglitz (1977). "Monopolistic Competition and Optimum Product Diversity," *American Economic Review*, 67, 297-308.

Downs, Anthony (1957). "An Economic Theory of Political Action in a Democracy," *Journal of Political Economy*, 65, 135-150.

Gu, Yiquan and Tobias Wenzel (2009). "A Note on the Excess Entry Theorem in Spatial Models with Elastic Demand," *International Journal of Industrial Organization*, 27, 567-571.

Hall, Robert E. (1988). "The Relation between Price and Marginal Cost in U.S. Industry," *Journal of Political Economy*, 96, 921-947.

Hall, Robert E. (2018). "Using Empirical Marginal Cost to Measure Market Power in the US Economy," NBER working paper no. 25251, November.

Hay, Donald A. (1976). "Sequential Entry and Entry-Deterring Strategies in Spatial Competition," O*xford Economic Papers*, 28, 240-257.

Helpman, Elhanan (1981). "International Trade in the Presence of Product Differentiation, Economies of Scale and Monopolistic Competition," *Journal of International Economics*, 11, 305-340.

Hotelling, Harold (1929). "Stability in Competition," *Economic Journal*, 39, 41-57.

Lancaster, Kelvin (1966). "A New Approach to Consumer Theory," *Journal of Political Economy*, 74, 132-157.

Lerner, Abba P. (1934). "The Concept of Monopoly and the Measurement of Monopoly Power," *Review of Economic Studies*, 1, 157-175.

Mankiw, N. Gregory and Michael D. Whinston (1986). "Free Entry and Social Inefficiency," *Rand Journal of Economics*, 17, 48-58.

Salop, Steven C. (1979). "Monopolistic Competition with Outside Goods," *Bell Journal of Economics*, 10, 141-156.

Schmalensee, Richard (1978). "Entry Deterrence in the Ready-to-Eat Cereal Industry," *Bell Journal of Economics*, 9, 305-327.

Schumpeter, Joseph A. (1942). *Capitalism, Socialism, and Democracy*, New York, Harper & Brothers Publishers.

Smithies, Arthur A. (1941). "Optimum Location in Spatial Competition," *Journal of Political Economy*, 49, 423-439.

Spence, Michael (1976). "Product Selection, Fixed Costs, and Monopolistic Competition," *Review of Economic Studies*, 43, 217-235.

Syverson, Chad (2024). "Markups and Markdowns," NBER working paper no. 32871, August.

Tirole, Jean (1988). *The Theory of Industrial Organization,* Cambridge MA, MIT Press.

Vickrey, William S. (1964). *Microstatics*, New York, Harcourt, Brace and World.

Appendix

I. The Approximation in the Markup Formula

The result for the price markup, $P \approx c + 2th$ in Eq. (14), is an approximation to the solution for *P* in Eq. (13) based on the condition $th \ll P$. Recall that the price *P* is determined exactly from

$$(13) \quad \left(\frac{1}{\eta-1}\right) \cdot [P^{1-\eta} - (P+th)^{1-\eta}] = (P-c) \cdot [P^{-\eta} - \frac{1}{2}(P+th)^{-\eta}].$$

Table A1 reports solutions for *P* from Eq. (13) determined numerically for an array of settings for the two underlying parameters, which can be expressed as η and *2th/c*. The latter term is the ratio of the markup to marginal cost in the approximate solution in Eq. (15). The table shows the results for *P* expressed as a ratio to *c+2th*, which is the approximate result for *P* in Eq. (14). The approximate solution is exact if $\eta=0$ or $th=0$, and the approximation works better the smaller η and *2th/c*. For example, if $\eta \leq 5$ and 2th/*c* ≤ 0.1, the true solution for *P* differs from *c+2th* by less than 3%. Further, if $\eta \leq 2$ and 2th/*c*≤ 0.2, the true solution for *P* again differs from *c+2th* by less than 3%. If η is very large, *P* can fall short of *c+2th* by substantial amounts for reasonable values of *2th/c*. However, the bottom line, based on $\eta \leq 5$ and 2th/*c* ≤ 0 or $\eta \leq 2$ and 2th/*c*≤ 0.2 in plausible circumstances, is that the approximation in Eq. (14) will typically be reasonably good.

Table A1

Approximation for Price-Markup Solution

	η=0	**η=0.1**	**η=0.5**	**η=1**	**η=2**	**η=5**	**η=100**
2th/c	**P/(c+2th)**						
0	1	1	1	1	1	1	1
0.02	1	1	1	1	0.999	0.999	0.988
0.04	1	1	0.999	0.999	0.998	0.995	0.970
0.1	1	0.999	0.997	0.995	0.989	0.976	0.918
0.2	1	0.998	0.990	0.982	0.967	0.933	0.842
0.4	1	0.994	0.973	0.949	0.911	0.841	0.721
1	1	0.983	0.922	0.858	0.763	0.628	0.505
2	1	0.971	0.866	0.757	0.604	0.433	0.337
5	1	0.960	0.809	0.652	0.444	0.265	0.202

II. Social-Planner Problem

The social planner effectively prices goods at the social marginal cost, which is $c+tz$ for a household located at distance z from its nearest store. The associated socially optimal quantity of goods is $q(z) = A \cdot (c + tz)^{-\eta}$ from Eq. (3). The consumer surplus, $\Omega(z)$, for an agent at z is then

$$\Omega(z) = \int_0^{q(z)} \left[\left(\frac{A}{q}\right)^{1/\eta} - c - tz \right] dq, \tag{A1}$$

where $\left(\frac{A}{q}\right)^{1/\eta}$ is the demand price associated with q. The integral can be evaluated to get

$$\Omega(z) = \frac{A(c+tz)^{1-\eta}}{\eta-1}, \tag{A2}$$

where $\eta>1$ is assumed at this stage (but is ultimately an unnecessary restriction).

The consumer surplus given in Eq. (A2) applies for *z=(0,h)* to the right of store 1 in Figure 1. Including also the analogous term to the left of store 1, the total consumer surplus for all customers of store 1 is

(A3) $$\Omega = 2\int_0^h \frac{A(c+tz)^{1-\eta}}{\eta-1}dz.$$

Using a change of variable from *z* to *c+tz*, the integral can be evaluated to get

(A4) $$\Omega = \frac{2A}{t(\eta-1)(\eta-2)}[c^{2-\eta} - (c+th)^{2-\eta}].$$

The analysis at this stage assumes $\eta>2$, but this restriction (or $\eta>1$) is ultimately not needed for the results.

The consumer surplus in Eq. (A4) applies from Eq. (1) to *N=H/2h* markets. The fixed cost of operations is *Nf*. The social planner chooses *h* to maximize the overall net surplus:

(A5) $$Net\ consumer\ surplus = \frac{H}{h}\left\{\frac{A}{t(\eta-1)(\eta-2)}[c^{2-\eta} - (c+th)^{2-\eta}] - \frac{f}{2}\right\}.$$

Setting the derivative of Eq. (A5) with respect to *h* to zero to get the first-order maximization condition leads to

$$\frac{f}{2} = \frac{A}{t(\eta-1)(\eta-2)}[c^{2-\eta} - (c+th)^{2-\eta}] - \frac{Ah}{\eta-1}(c+th)^{1-\eta}.$$

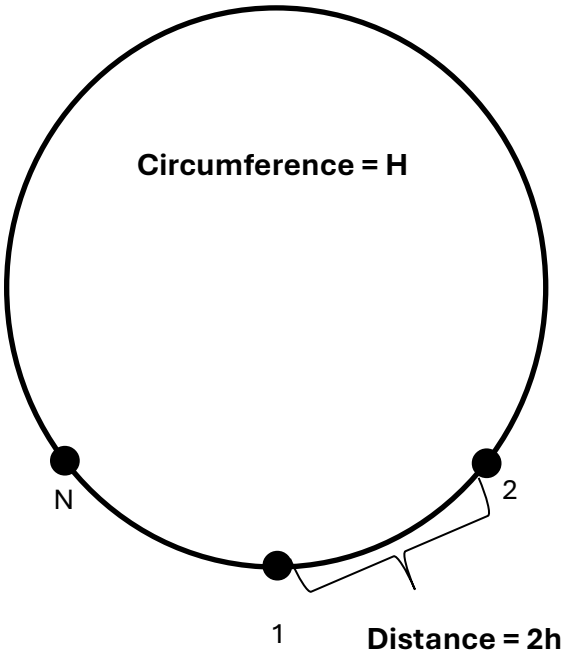
Using the condition from before, $th<<c$, the right-hand side simplifies using first-order Taylor approximations for the terms involving $c+th$ to $Ath^2c^{-\eta}$. Therefore, the approximate solution is

(A6) $$2h \approx \sqrt{\frac{2f}{tAc^{-\eta}}}.$$

The social planner's solution for store spacing in Eq. (A6) is the multiple by the square root of 2 of the free-entry solution in Eq. (18). That is, the social planner's spacing exceeds the free-entry one by about 40%, corresponding to having only about 70% of the number of stores, N.

Figure 1

Salop-Hotelling Circular City



Note: Consumers are uniformly distributed around a circle with circumference *H*. Stores are located at positions 1, 2, …, *N*, with an equal spacing of *2h.* Store 1's market extends out a distance *h* to the right and left of its location.