The Gas Trap:
Outcompeting Coal vs. Renewables*

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Abstract

We analyze a fundamental dilemma and time-inconsistency problem facing a climate coalition producing natural gas. In the short term, it is tempting to export more to outcompete coal. When this policy is anticipated, however, investments in renewables fall and emissions ultimately increase. When the coalition cannot pre-commit, its policies will be counterproductive. We discuss the robustness of this result and possible solutions. If the coalition can invest directly in renewables, for instance, the incentive to maintain a high price on exports can mitigate the temptation to reduce the price to outcompete coal. Under certain conditions, the commitment outcome can be implemented.

Keywords: climate change, carbon leakage, green transition, supply-side environmental policy, renewable energy, time inconsistency

JEL codes: F18, Q55, H23

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1 Introduction

- The best is the enemy of the good.
M. de Voltaire

The good can also be the enemy of the best.

Gas, coal, and renewables can be referred to as the good, the bad, and the best. The three sources of energy are competing and substitutable. To avoid continued increases in global temperatures, the usage of coal must end as soon as possible. The production of natural gas is good in that it can replace coal. New York Times (2024) writes that: "While nations have agreed on the necessity to transition away from fossil fuels as quickly as possible, almost all major economic powers are promoting gas as a "transition fuel."[1] After all, coal is twice as carbon-intensive, per unit of energy, as natural gas.[2] In the long run, however, in order to achieve widely-discussed targets for global warming, all types of fossil fuels – gas as well as coal – must be replaced by renewable energy sources. Welsby et al. (2021) find that of existing reserves, only 11 percent of coal, and 44 percent of gas, can be extracted to reach the 1.5°C target. Pellegrini et al. (2024) find the numbers to be 3 percent and 19 percent, respectively.

Climate-friendly gas producers are therefore facing a dilemma. By producing and exporting more gas, coal can be replaced. By producing less gas, investments in renewables pay off more.

Each effect travels through changes in the world price of energy. The extent to which a producer reacts to a change in the price, and thus to the supply of gas, depends on the price elasticity. This price elasticity measures the percentage increase in supply given a percentage increase in the energy price.

The starting point of our paper is that the price elasticities are not fixed numbers. Instead, the ability to change the supply, because of a change in the price, will depend on the time horizon. In the short term, the ability to change the supply is limited. With more time, the supply is likely to be more elastic. This qualitative insight is likely to hold for all sources of energy.

Nevertheless, there is a fundamental difference between coal and renewables. With the green transition, coal will be a declining industry. The capital that has been sunk risks being stranded assets. In other words, investments and capital that have accumulated over several decades will not be fully exploited when coal is being produced. Because the capacity to produce is not binding, production can increase or decrease on short

[1] https://www.nytimes.com/2024/05/31/climate/greece-europe-natural-gas-lng.html
[2] See, for example, https://www.eia.gov/environment/emissions/co2_vol_mass.php

notice if the energy price changes unexpectedly. The price elasticity of coal in the short term is relatively large.

Renewables, in contrast, are up-and-coming. We are seeing R&D and a lot of other investments that will gradually expand the capacity to produce renewable energy. This expansion takes time, however, so the short-term price elasticity is relatively small. With a longer time horizon in mind, investors can increase investments by building more solar panels, for example, if the investments are expected to be profitable. Thus, supply is likely to be elastic in the long run.

The fundamental difference between the two generates a time-inconsistency problem for climate-concerned gas producers. The optimal long-term policy is to limit gas production so that investments in renewables will materialize. In the short term, the stock of renewables is given, and it will be tempting to expand gas production to outcompete coal. This temptation is especially strong when the harm associated with climate change is large. But when the temptation to outcompete coal is anticipated in the market, investments in renewables fall and emissions ultimately increase. This is the *gas trap*.

The purpose of this paper is to analyze this time-inconsistency problem, its robustness, and potential solutions. We first develop an extremely simple model. The model is not intended to be general, but minimalistic in that it is just enough to capture the key mechanism driving the time-inconsistency problem. Thereafter, we explain that the results are robust to a number of realistic extensions. Finally, we discuss solutions. In the model we develop, climate policy is counterproductive unless commitment is possible. To prevent being counterproductive, the supply-side climate coalition can design a treaty that creates commmitment, so that a regulated supply side will be credible. The coalition can also tax and regulate activities such as search and exploration, because taxation and regulation will limit future gas supply and, anticipating the limits, investments in renewables will increase. Finally, we the coalition to invest directly in renewables. With enough of this type of investment, the coalition will export so much energy that the motivation to maintain a high price on that export will balance the temptation to reduce the price and outcompete coal.

Our findings are important because economists as well as practitioners are still emphasizing first-best policies, such as Pigou taxes, as optimal when policies can be committed to. At the same time, gas producers, such as Norway, are oftentimes emphasizing the need to replace coal by producing more gas. In our model, this replacement can reduce climate change in the short term when the stock of renewables is hard to change. But to avoid that investments in renewables fall in equilibrium, and thereby prevent the green transition from materializing, the supply-side coalition in our model must restrict their gas supply rather than subsidizing it.

A growing empirical literature supports the assumptions of our model. Ahlvik et al. (2024) show that the supply of fossil fuel reacts to taxes, and that suppliers not subject to the tax are also responding, leading to carbon leakage. Clean and dirty energy inputs are substitues (Papageorgiou et al., 2017; Brehm, 2019), coal production declines if the supply of wind energy is large (Fell & Kaffine, 2018) or the supply of gas is large: See Knittel et al. (2016), McGlade et al. (2018), or Coglianese et al. (2020). Reversely, Aghion et al. (2016) showed how energy prices affect clean innovation.

Technically, what we refer to as renewable energy can just as well represent nuclear energy or, in fact, energy economizing capacity, so that consumption can remain high without the need to emit. Jarvis et al. (2022) found that when investments in nuclear energy fell in Germany after the Fukushima accident, coal production increased. Colmer et al. (2024), studying the effect of carbon prices in Europe, find evidence in the support of energy economizing investments that firm pay for up front, so that marginal production costs do not necessarily increase after the carbon price was introduced. These findings are also in line with our model when we broaden the interpretation for what we refer to as investments in renewables.

The theoretical literature on supply-side environmental policies is also growing. When free riders trade with climate cooperators, Hoel (1994) showed that a climate coalition will, in general, benefit from regulating both the consumption and the production of fossil fuel. Harstad (2012a) found that when the coalition can take advantage of the international market for fossil fuel reserves, the first-best outcome can be implemented by supply-side policies alone. In the first best, the coalition purchases coal deposits in free-riding countries. In this paper, we acknowledge that purchasing deposits in other countries can be politically challenging. Instead, we depart from the more realistic assumption that, although free riders trade fossil fuels internationally, the coalition does not pay them directly for consuming or producing less.

The empirical fact that natural gas can replace coal and renewables has been analyzed in the theoretical literature: See, for example, Gerlagh & Smulders (2024) or Acemoglu et al. (2023). In these papers, the short term effect is that gas replaces coal, but the long-term effect includes a negative effect on renewables. Consequently, the US gas boom can delay the green transition to renewables. The delay is harmful for a climate-concerned coalition, which would like to commit to reduce emission in order to raise the investments in renewables (Harstad, 2012b), even in other countries (Holtsmark & Midttømme, 2021). Profit-maximizing fossil-fuel producers, in contrast, can take advantage of the fact that more fossil fuel delays the transition (Gerlagh & Liski, 2011).

Our contribution to this literature is to explore the consequences of the climate coalition's time inconsistency problem. We illustrate when the comparative statics are reversed (and the policy is counterproductive)

in equilibrium, relative to the outcome under commitment. The simplicity of our model makes it sufficiently flexible to allow for a large number of generalizations, including multiple types of fossil fuels, many periods, learning by doing, resources that are depleted over time, and search-and-exploration activities prior to the production stage. These extensions allow us to contribute to several strands of literatures.

More generally, our emphasis on time inconsistency connects our paper with the literature on macroeconomic policy going back to Kydland & Prescott (1977). Fischer (1980) discussed the temptation to set a positive tax on capital, after capital was sunk, even though a lower tax would have been optimal from the outset. If international tax competition reduces the tax, international cooperation to reduce the competition can be harmful (Rogoff, 1992; van der Ploeg, 1988; Kehoe, 1989). Karp & Newbery (1993) discuss several time inconsistency problem for resource extraction. More recently, Gerlagh & Liski (2018) study the value of commitment in a model with taxes on capital as well as emission and Acemoglu & Rafey (2023) explain why the optimal emission tax fails to be time consistent in the presence of geoengineering.

Our paper is organized as follows. In the next section, we present the simplest version of our model. In Section 3, we compare the outcome under commitment (Proposition 1) to the equilibrium when policies are not committed to (Proposition 2). Our main result is that short-term policies are counterproductive and that the comparative statics are reversed. In Section 4, we argue that this finding is robust and holds when we allow for many types of fossil fuels, many periods, time-varying parameters, learning by doing, and depletable resources. We discuss solutions in Section 5 and explain why it may be necessary to regulate upstream activities, such as search and exploration, why direct investments in renewables can mitigate the short-term temptation to flood the market with energy as a way to outcompete coal, and we discuss the importance of designing binding supply-side treaties. Section 6 concludes. The Appendix contains all proofs.

2 The Model

The model can be quite general, but it does not need to be complicated. To explain the results in a pedagogical way, in this section we present an especially simple model. Natural generalizations are analyzed in Sections 4 and 5.

There are two types of countries in the world. M is the set of members of a coalition that is actively regulating consumption and production of energy. N is the set of countries that do not.

M is producing natural gas, x_G. Gas competes with the supply of coal, x_C, and renewable energy, x_R, both produced by the rest of the world (i.e., by N). The three sources of energy are perfect substitutes and, in

the world market, they trade at the same price p. (We refer to gas instead of oil because coal and renewables are rather imperfect substitutes for oil.) The energy price is endogenous and ensures that the total supply is equal to the total demand. M's consumption of energy is y_M, while y_N is the energy consumed by N.

Payoffs. The representative consumer's benefit from consumption is given by the increasing and strictly concave function $b_i(y_i)$, $i \in \{M, N\}$. The production cost is $c_j(x_j)$, $j \in \{C, G, R\}$. M experiences harm $ah(e)$ from the total emissions:

$$e = e_C x_C + e_G x_G, \tag{1}$$

where e_C is the carbon content of coal, $e_G \in (0, e_C)$ is the carbon content of gas, and $h(.)$ is a weakly convex function. There are no emissions from renewable energy. The harm is written as $ah(e)$, so that we can consider a change in the harm (a) without changing the function $h(.)$. N might also experience harm from e, of course, but as mentioned, N does not regulate its consumption or production. Thus, N's harm plays no role in the game.

Policies. M regulates x_G and y_M or, equivalently, M imposes a production tax, τ_G, on domestic gas and a tax, τ_M, on M's consumption. With quasi-linear utility functions, that are linear in money, the representative consumer in M solves:

$$\max_{y_M} b(y_M) - (p + \tau_M) y_M \Rightarrow y_M = D_M(p + \tau_M) \equiv b_M'^{-1}(p + \tau_M). \tag{2}$$

The representative gas producer in M solves:

$$\max_{x_G} (p - \tau_G) x_G - c_G(x_G) \Rightarrow x_G = S_G(p - \tau_G) \equiv c_G'^{-1}(p - \tau_G). \tag{3}$$

Because there is no regulation in N, N's demand for energy is $y_N = D_N(p) \equiv b_N'^{-1}(p)$, and its supply of coal is $x_C = S_C(p) \equiv c_C'^{-1}(p)$, when we assume that all consumers and producers take the price as given. We will assume that there are interior solutions to all these quantities. Supply equals demand when:

$$S_C(p) + S_G(p - \tau_G) + x_R = D_N(p) + D_M(p + \tau_M). \tag{4}$$

The taxes are domestic transfers, so M's policy is set to maximize M's objective:

$$b_M(y_M) - c_G(x_G) + (x_G - y_M) p - ah(e), \tag{5}$$

subject to (1)–(4).

Timing. Our key assumption is that oil production is more price elastic than renewable energy production in the short run, but the reverse holds in the long run. As explained in the Introduction, this

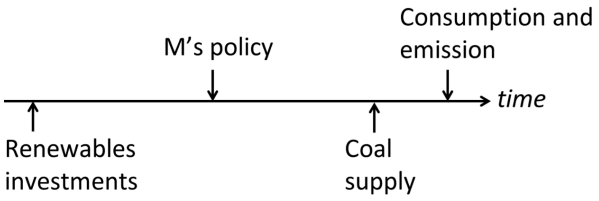


Figure 1: Timing

assumption is natural when coal is a declining industry, with spare capacity that can be taken advantage of whenever profitable. Renewable energy sources, in contrast, require new capacity to be developed and built. New constructions take more time.

We formalize this assumption as follows. First, the capacity to produce x_R must be determined Δ_R units of time before the energy can be consumed. The time required to produce and supply coal is given by $\Delta_C \in [0, \Delta_R)$. The inequality $\Delta_C < \Delta_R$ implies that after x_R has been determined, it is still possible to change x_C. Consequently, the supply of coal is more price elastic than that of gas – in the short run.

In the long run, however, the production of renewable energy is more price elastic than the supply of coal is. In fact, the supply of renewable energy can be increased to unprecedented levels if the world just builds enough windmills and solar panels. To capture this intuition in the simplest way, we assume that there is a constant cost for each unit of renewable energy capacity that is invested in: $c_R'(x_R) = k$. The cost of producing coal, in contrast, is given by the increasing and strictly convex function $c_C(x_C)$.

We do not need to make particular assumptions about the production cost and lag for gas. For simplicity, we assume that the production cost is an increasing and strictly convex function. In the main analysis, we suppose that the time it takes to deliver gas is $\Delta_G \in (\Delta_C, \Delta_R)$, as in Figure 1. However, at the beginning of Section 3, we will also consider $\Delta_G > \Delta_R$ and the case in which M can commit to policies before x_R is decided. We end Section 3 by discussing the consequence of assuming $\Delta_G < \Delta_C$.

Generalizations. The model is simple for pedagogical reasons. In Section 4, we explain how we can allow for many periods, an infinite time horizon, and parameters that change over time. In fact, k can be endogenous to take into account learning-by-doing effects. The rest of the world, N, can produce several types of coal and gas, with different emission contents. We explain that if M's stock of gas is depletable, the results are strengthened.

In Section 5, we distinguish between the decision to explore and search for natural gas, and the decision

to extract from existing reserves. There, we also allow M to invest in renewable energy sources directly. In the Appendix, we explain why M cannot do better by using trade policies (such as import taxes) in addition to (τ_G,τ_M).

3 The Results

Before deriving the subgame-perfect equilibrium (SPE) of the game above, we provide two benchmark results.

In the first best (FB), all quantities are decided to maximize benefits minus costs and harms. The FB quantities require that every marginal benefit equals the marginal cost plus the marginal harm. These quantities are implemented in a decentralized economy if all producers face Pigou taxes representing the marginal harms. (In the following, we simply write h' even when the marginal harm is a function of e.)

Proposition 0. *The FB is implemented by Pigou taxes on gas and coal, but no tax on consumption or renewables:*

$$\begin{aligned} \tau_G &= e_G a h', \, \tau_C = e_C a h', \text{ and} \\ \tau_M &= \tau_N = \tau_R = 0. \end{aligned}$$

Thus: The larger a is, the larger τ_G and x_R are, and the smaller x_C, x_G, and e are.

3.1 The Commitment Solution

We now characterize the SPE if M could, at the beginning of the game, commit to (x_G, y_M) or, equivalently, the pair (τ_G, τ_M) that implements (x_G, y_M).

Proposition 1. *If M could commit, M would set a Pigou tax on domestic gas production:*

$$\begin{aligned} \tau_G &= e_G a h' \text{ and} \\ \tau_M &= 0. \end{aligned}$$

Thus: The larger a is, the larger τ_G and x_R are, and the smaller x_G and e are.

Even if M could commit, the outcome would not be FB, of course, because there would be no tax on N's coal production. Thus, we refer to this outcome as the "second best" (SB).

But the SB is qualitatively similar to the FB. M finds it optimal to set a Pigou tax on gas production, which increases in ah'. Consequently, when ah' is large, the equilibrium x_G is small and, to satisfy (4), x_R will be large. The level of x_C is unchanged, and pinned down by $S_C(p)$, given that $p = k$, in equilibrium. When x_G is reduced, and x_C is unchanged, e is reduced.

Just as in the FB, there is no tax on domestic consumption. The intuition for this result is that regulating consumption would simply have reduced the investments in renewables. After all, the equilibrium price, $p = k$, pins down the equilibrium y_N, x_C, and, for a given τ_G, also x_G. So, committing to lower consumption does not reduce emissions, given the assumption that the supply of renewables is price elastic.

3.2 Equilibrium

Instead of assuming that M commits to policies before x_R is decided, as in Section 3.1, we now consider the equilibrium without long-term commitment. We assume that after x_R is decided, but before x_C has been set, M determines (x_G, y_M) or, equivalently, the pair (τ_G, τ_M) that implements (x_G, y_M).

Our main result is that if gas is relatively green, compared to coal, the comparative statics are reversed relative to the FB and the SB. M ends up with more emissions if ah' is large. M's policy is counterproductive and does more harm than good. This is the gas trap:

Proposition 2. *In equilibrium, $p = k$, and M's policy satisfies:*

$$\begin{aligned} \tau_G &= e_G ah' - \Omega = \left(e_G - \frac{e_C S_C'(k)}{S_C'(k) - D_N'(k)} \right) ah' - \frac{D_M(k + \tau_M) - S_G(k - \tau_G)}{S_C'(k) - D_N'(k)}, \\ \tau_M &= \Omega, \text{ where } \Omega \equiv \frac{e_C S_C'(k)}{S_C'(k) - D_N'(k)} ah' + \frac{D_M(k + \tau_M) - S_G(k - \tau_G)}{S_C'(k) - D_N'(k)}. \end{aligned} \tag{6}$$

(i) Thus, the larger a is, the larger τ_M is, and the smaller y_M is.

(ii) Suppose

$$\frac{e_G}{e_C} < \frac{S_C'(k)}{S_C'(k) - D_A'(k)}, \text{ where } D_A'(p) \equiv D_M'(p) - D_N'(p). \tag{7}$$

The larger a is, the smaller τ_G and x_R are, and the larger x_G and e are.

So, if (7) holds, the comparative statics from the FB and the SB are reversed: With a larger marginal harm, the amount of renewable energy will be smaller and the emission level will be higher. Perversely, M's well-meaning climate policy contributes to more harm, especially if a is large. Figure 2 illustrates the comparative statics and contrasts them to the FB and the SB. If M is a net exporter, $\tau_G > 0$ when $ah' = 0$

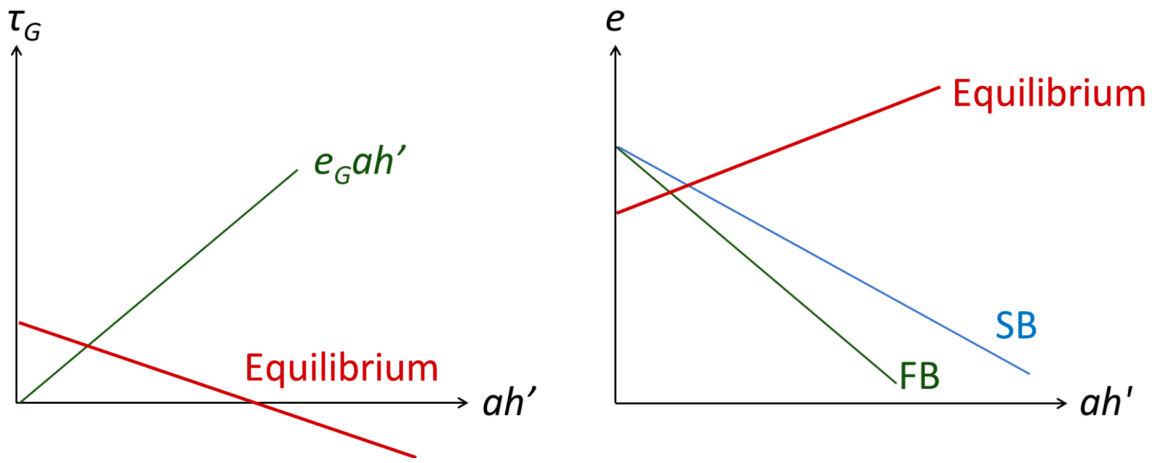


Figure 2: In contrast to the FB and the SB, if the marginal harm increases then the equilibrium gas production tax decreases and emission increases.

because regulating production raises M's export price (for any given x_R). The production tax is less than Pigouvian if and only if $\Omega > 0$.

Why is M designing a policy that increases emissions and, ultimately, reduces M's payoff?

The intuition is that M cannot help attempting to outcompete coal. For any given x_R, a larger x_G, and a lower y_M, will reduce the equilibrium energy price. As illustrated in Figure 3 (left panel), the lower price will reduce coal production. If coal is sufficiently dirty, relative to gas, total emissions will decline. In the subgame that starts with a given x_R, M's policy is sensible.

When M's policy is anticipated, however, investments in renewables will adjust. Rational expectations about the low τ_G, and the high τ_M, will imply that the expected p is low, for any given x_R. In equilibrium, x_R is reduced so much that the equilibrium price remains at $p = k$: See Figure 3 (right panel). The reduced x_R, and the maintained price, imply that equilibrium coal production does not fall, even if x_G increases. Because x_G is larger, equilibrium emissions increase when M attempts to outcompete coal. M's short-run solution – more gas – is a long-run trap.

Other comparative statics are also worth noticing. The larger the short-term price elasticity of coal is (i.e., $S_C'(p)$), the smaller τ_G is and the larger τ_M is, when ah' is large, because both these tax changes contribute to a smaller p and x_C, for any given x_R. Thus, it is in this situation when x_R falls the most in equilibrium, so that equilibrium x_G and e will actually increase.

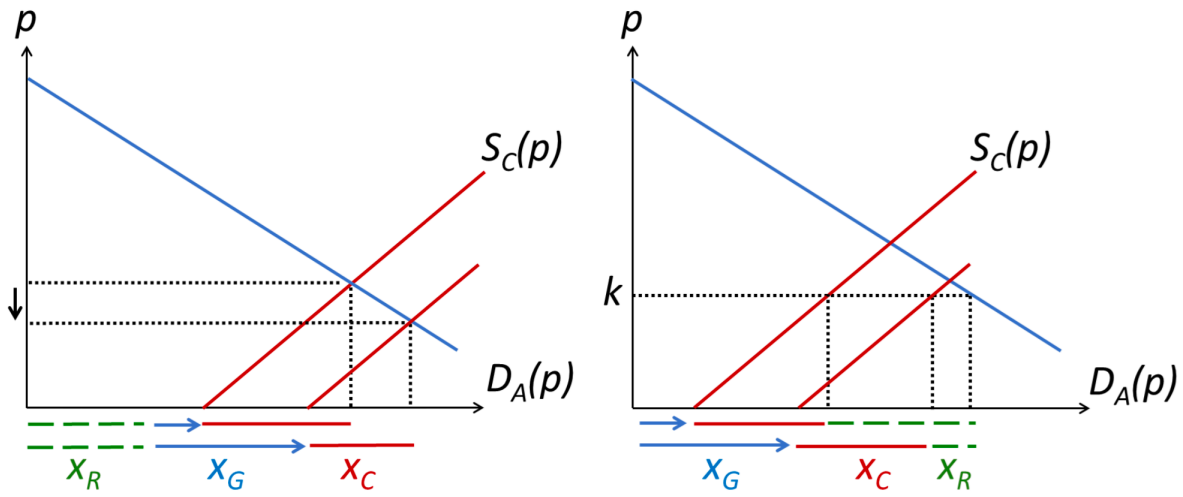


Figure 3: In the short term (left panel), the capacity to produce renewables (dashed line) is given, and more gas (arrow) reduces coal (solid line). In equilibrium (right panel), more gas replaces renewables, not coal.

Intriguingly, (7) is more likely to hold when e_G is small. That is, if gas has a lower emission intensity, relative to coal, it is more likely that gas will be subsidized and, thus, that M's policy contributes to a larger e when a is large.

Remark on $\Delta_G < \Delta_C$. Suppose M's policy were set after both x_C and x_R. (This would require $\Delta_G < \Delta_C$.) In this case, (6) would continue to hold if we just replaced S'_N by zero. M would still take into account the harm associated with more gas production, and also how M's terms of trade (i.e., p) would change because of the policies, given x_C and x_R. For instance, if M were a net importer, M would tax supply less, and tax consumption more, than in the SB. These policies would be anticipated so, as above, x_R would be small, relative to the SB, so that, in equilibrium, p would continue to equal k.

4 Generalizations and Robustness

We have designed the model to be simple so that the mechanism can be easily understood, and also because it is then straightforward to develop the framework in various directions. In this section, we explain why the main result is quite robust and, in fact, is strengthened by reasonable extensions. In Section 5, we study extensions that can contribute to a solution to M's time-inconsistency problem.

4.1 Multiple Fossil Fuel Types

As mentioned in Section 2, the proofs permit N to produce gas and, in fact, an arbitrary number of fuel types with various emission contents. Suppose e_j and $S_j(p)$ are the emission content and supply function, respectively, for N's fuel of type $j \in J$. The FB outcome is, as in Proposition 0, implemented by Pigou taxes on all types of fuels. If M can commit, but N has no policy, then M's policy is implemented by a Pigou tax on M's production, and no tax on M's consumption, exactly as in Proposition 1. If M cannot commit, equilibrium taxes satisfy:

$$\begin{aligned}\tau_G &= e_G a h' - \Omega = \left(e_G - \frac{\sum_{j\in J} e_j S_j'(k)}{S_N'(k) - D_N'(k)}\right) a h' - \frac{D_M(k+\tau_M) - S_G(k-\tau_G)}{S_N'(k) - D_N'(k)},\\ \tau_M &= \Omega \equiv \frac{\sum_{j\in J} e_j S_j'(k)}{S_N'(k) - D_N'(k)} a h' + \frac{D_M(k+\tau_M) - S_G(k-\tau_G)}{S_N'(k) - D_N'(k)},\end{aligned}$$

where $S_N'(k) \equiv \sum_{j\in J} S_j'(k)$. In equilibrium, a larger weight, a, on the harm, will lead to more emissions and harm if and only if:

$$e_G < \frac{\sum_{j\in J} e_j S_j'(k)}{S_N'(k) - D_A'(k)},$$

which generalizes the condition in Proposition 2. Golombek et al. (1995) find a similar result.

A special case is that N produces only gas, with supply function $S_{G,N}(p)$ and emission content e_G, in addition to coal. In this case, the inequality becomes:

$$e_G < \frac{e_G S_{G,N}'(k) + e_C S_C'(k)}{S_{G,N}'(k) + S_C'(k) - D_A'(k)} \Leftrightarrow \frac{e_G}{e_C} < \frac{S_C'(k)}{S_C'(k) - D_A'(k)},$$

which is exactly the same condition as in Proposition 2. Thus, allowing N to produce gas does not change the results. In the proofs of Propositions 1 and 2, we also permit M to produce multiple types of fossil fuels, including coal, and we derive expressions that must be satisfied by the equilibrium taxes.

4.2 Multiple Periods

The proofs and the results hold if there are many periods. In fact, if we simply add a variable superscript denoting time t, then the above results hold for every possible t if we abstract from the possibility that payoff-relevant stocks can change over time. In this case, the extensive-form stage game above is repeated. The SPE characterized in Section 3 is the unique Markov-perfect equilibrium (MPE) of the dynamic game. Other than M, the players are price takers, and if their strategies are Markov, M's strategy will be Markov

as well. (M cannot benefit from using history-dependent strategies when no one else does.)[3]

The exogenously given parameters can also be time-dependent, and thus change over time.

Furthermore, we can take into account that emissions and the investment in renewable energy contribute to stocks that change over time. Regarding renewables, suppose that $x_R^t = q_R x_R^{t-1} + I_R^t$, where $1 - q_R \in [0, 1]$ measures how much the stock at time $t-1$ has depreciated when we enter period t, while I_R^t measures the investments in renewable energy at time t. If K^t measures the unit cost of investment, the net cost of raising x_R^t at time t is $K^t - \delta q_R K^{t+1}$, where δ is the discount factor, because after one period of usage, the investor can cash in by selling the remaining capacity at price $q_R K^{t+1}$. With $k^t \equiv K^t - \delta q_R K^{t+1}$, the analysis above is unchanged if, on the equilibrium path, the inequality $I_R^t \geq 0$ does not bind.

Regarding emissions, suppose that the accumulated emissions at time t are $E^t = q_E E^{t-1} + e^t$, where $1 - q_E \in [0, 1]$ measures how much the greenhouse-gas stock at time $t-1$ has depreciated when we enter period t. The analysis above is unchanged if $q_E = 0$. Suppose, instead, that $q_E > 0$. If M's one-period harm from each unit of E^t can be approximated by some constant, H^t, then the present-discounted cost of a unit of emissions equals $h' \equiv \sum_{i=t}^{\infty} (\delta q_E)^{i-1} H^i$, which equals $H/(1 - \delta q_E)$ if $H^i = H$ happens to be invariant in time. With this definition of h', M can account for the marginal present-discounted cost immediately, and this accounting leaves the analysis above unchanged.

Proposition 3. *Suppose there are multiple periods, and the environmental harm is linear in the accumulated stock of emissions. The analysis leading up to consumption and emissions at time t holds for every such t, if, for each period, h' is replaced by $h^t \equiv \sum_{i=t}^{\infty} (\delta q_E)^{i-1} H^i$ and k by $k^t \equiv K^t - \delta q_R K^{t+1}$.*

Although the main result in Section 3 continue to hold, time-varying parameters can lead to additional results. Suppose, for example, that H^t is expected to be larger in the future, when t is large. That would increase the present-discounted cost of emitting at every earlier point in time also, and the contrast between the commitment solution (described in Proposition 1) and the equilibrium (Proposition 2) will be larger. Furthermore, this difference will be even larger in the future, because also h' will increase in t when H^t increases in t.

If the investment cost K^{t+1} is expected to decrease in t, then renewables are going to be more competitive relative to coal and gas. In the short term, however, x_R^t can decline, and emissions can increase. To see

[3]There can be multiple SPEs when the time horizon is infinite, of course. If we modelled the game between the multiple member countries in M, an infinite time horizon can actually help us to rationalize why the member countries comply with the policies we have derived, if defections are followed by some form of punishment by the other member countries.

this, note that $k^t \equiv K^t - \delta q_R K^{t+1}$ is larger when K^{t+1} is small, for any given K^t. The intuition is that the net cost of investing is higher at time t when the declining costs reduce the value of the stock that survives to time $t+1$. The larger k^t leads to lower investments in renewables at time t, a higher energy price, and more production of coal.

If both $q_R > 0$ and the harm is nonlinear in the stock, then (strategically) high emissions level today will reduce how much it will be optimal to emit in the future (Calvo & Rubio, 2012; Harstad, 2012b).

4.3 Learning by Doing

In Section 4.2, we considered the reasonable situation in which the cost of renewable energy investment can change over time. In reality, this change is likely to be endogenous. The cost of solar panels, for example, falls over time not by chance, but because of R&D and learning by doing: Baker et al. (2013) survey the evidence when it comes to solar panels; Covert & Sweeney (2022) document considerable learning by doing in wind turbine manufacturing.

To capture the effects of learning by doing simply, consider a two-period model where the first period is as in Sections 2 and 3. Here, we refer to the first-period cost as k^1. In the second period, the unit cost of renewable energy capacity is $k^2 = f\left(x_R^1\right)$, where f is a decreasing function of the amount of renewable energy in period 1, x_R^1. To simplify, we henceforth assume that the present-discounted harm from emissions is h^t at time t, and that renewables are not accumulating over time.[4]

If M could commit, M would benefit from a larger x_R^1 in order to reduce k^2. Although the reduction in k_2 harms M if M exports a lot, it also reduces the production of coal and gas, especially if the supplies of coal and gas are price elastic. This possibility may motivate M to raise the tax on gas even more and to subsidize domestic consumption, because both changes raise the demand for energy and, therefore, x_R^1 increases. The commitment solution coincides with the FB outcome, except that N's fossil fuel production is taxed in the FB, in line with Proposition 0.

[4]That is, $q_R = 0$, using the notation in the previous subsection. This helps us to abstract from the benefit of investing less now, because of lower investment costs in the future.

Proposition 4. *Suppose the first-period unit cost of x_R^1 is k^1, while the unit cost of x_R^2 is $k^2 = f\left(x_R^2\right)$.*
(i) If M could commit, M's first-period policy would be:

$$\begin{aligned}
\tau_G^1 &= e_G a h^1 + \Omega^2 \left(-f'\right)\left(S_N'\left(k^2\right) - D_N'\left(k^2\right)\right), \\
\tau_M^1 &= -\Omega^2 \left(-f'\right)\left(S_N'\left(k^2\right) - D_N'\left(k^2\right)\right), \text{ where} \\
\Omega^t &\equiv \frac{e_C S_C'\left(k^t\right) a h^t - \left(x_G^t - y_M^t\right)}{S_N'\left(k^t\right) - D_N'\left(k^t\right)},\ t \in \{1,2\}, \qquad (8)
\end{aligned}$$

while M's second-period policy would be in line with Proposition 1:

$$\tau_G^t = e_G a h^t \text{ and } \tau_M^t = 0, \text{ for } t = 2.$$

(ii) When M cannot commit, M's policies in both periods are in line with Proposition 2:

$$\tau_G^t = e_G a h^t - \Omega^t \text{ and } \tau_M^t = \Omega^t,\ t \in \{1,2\}.$$

When M cannot commit, however, x_R^1 is already given at the time when M decides on its policy. In this case, M cannot influence k^2. Therefore, M's policy is characterized by Proposition 2, just as before.

The contrast between the commitment solution and the equilibrium under noncommitment is even starker than in Section 3. When M cannot commit, M may be tempted to reduce τ_G in order to outcompete coal, and to tax consumption to further reduce the demand for energy, especially when a is large. If M could commit, then M would set a Pigou tax on gas extraction, and no tax on consumption, in the absence of any learning-by-doing effect. With learning by doing, M would further raise τ_G^1, and M would subsidize domestic consumption, especially if h^t were large, because both these policy changes would raise the investments in renewables, and k^2 would decline. A smaller k^2 generates a positive externality on M's payoff because N produces less coal as a result. With this result, we broaden the justification for policy intervention in the presence of learning by doing: van Benthem et al. (2008), Borenstein (2017), and Baker et al. (2013), for example, argue that learning by doing justifies policy intervention only when there are knowledge spillovers to other firms.

4.4 Depleting Resources

Fossil fuel stocks are exhaustible and shrink over time when they are exploited. In this subsection, we argue that our main result is strengthened when we incorporate exhaustibility into the model. We show that when M cannot commit, M is further reducing its tax (or raising its subsidy) on the production of gas, because of

exhaustibility. In the short term, investments in renewables will fall, emissions will increase, and the stock will be depleted faster than if M could commit. The intuition for this result is that M seeks a way to commit to reducing gas production in the future. If this reduction is credible, investments in renewables will increase and emissions will decrease. When M controls a limited amount of natural gas, M will indeed produce less in the future if M reduces the tax and produces more today: See Cruz & Rossi-Hansberg (2024), for instance.

We illustrate this insight in a simple two-period model. The world is not going to run out of coal anytime soon, but we will assume that M's marginal cost of producing x_G^t at time $t = 2$ increases in M's first-period production level: M's production cost is $c\left(x_G^t\right)$ and time $t = 1$, as before, but $c^2\left(x_G^2, x_G^1\right)$ at $t = 2$, where the derivatives are $c_1^2 > 0$, $c_{11}^2 > 0$, $c_2^2 > 0$, and $c_{12}^2 > 0$. This way of capturing exhaustibility is more general and reasonable than a fixed constraint on $x_G^1 + x_G^2$, because gas production will not end, in reality, when there is not a single unit left in the reserves. Instead, it will end when it is too costly to extract the next unit. To abstract from the possibility that the marginal harm is endogenous, we henceforth assume h^t to be a constant for each $t \in \{1, 2\}$.

In the second period, when x_G^1 is fixed, the model and the analysis are no different from those in Sections 2 and 3. Policies would still be characterized by Proposition 1 if M could commit, and by Proposition 2 when M cannot. The model extension has bite only in period 1, because the choice of x_G^1 will influence the cost, the policies, and the quantities in the second period. The larger x_G^1 is, the larger the marginal cost of x_G^2 is, and the smaller we may expect x_G^2 to be.

The desire to influence second-period policies would be absent if M could commit. In that case, M's second-period policy would be just fine, even from the first-period point of view. The fact that x_G^1 raises the cost of x_G^2 will be taken into account by the market participants. Thus, if M could commit to policies at the beginning of each period, $t \in \{1, 2\}$, M would set a Pigou tax on x_G^t, but no tax on consumption, just as in Proposition 1. In line with Proposition 0, the commitment solution coincides with the FB outcome except that even N's fossil fuel production is taxed in the FB.

When M cannot commit to future policies, M is in each period setting quantities or, equivalently, taxes, after x_R^t has been determined. As before, M is tempted to outcompete coal but when this temptation is anticipated, x_R^t falls. M's second-period policy is in line with Proposition 2, but in the first period, M takes into account that the second-period policies and quantities will depend on x_G^1. By contributing to a larger x_G^1, equilibrium x_G^2 falls, and x_R^2 increases. Thus, M prefers to set a smaller tax (or a larger subsidy) on the production of gas in the first period in order to make it credible that x_G^1 will be smaller.

Proposition 5. *Suppose the second-period cost is* $c^2\left(x_G^2, x_G^1\right)$*, with* $c_1^2 > 0$*,* $c_2^2 > 0$*,* $c_{11}^2 > 0$*,* $c_{12}^2 > 0$*.*
(i) If M could commit to policies for period $t \in \{1, 2\}$ *at the beginning of period* t*, M's policy would be in line with Proposition 1:* $\tau_G^t = e_G a h^t$ and $\tau_M^t = 0$*,* $t \in \{1, 2\}$*.*
(ii) When M cannot commit, M's second-period policy is in line with Proposition 2, M's first-period consumption tax is in line with Proposition 2, but M's first-period production tax is reduced if and only if $c_{12}^2 > 0$*:*

$$\begin{aligned}\tau_G^1 &= e_G a h^1 - \Omega^1 - \delta \frac{\left(e_G a h^2 - \tau_G^2\right)\left[S_C'(k^2) - D_A'(k^2)\right] + \tau_M^2 D_M'(k^2)}{S_A'(k^2) - D_A'(k^2)} \frac{c_{12}^2}{c_{11}^2} \\ &= e_G a h^1 - \Omega^1 - \delta\Omega^2 \frac{S_C'(k^2) - D_N'(k^2)}{S_A'(k^2) - D_A'(k^2)} \frac{c_{12}^2}{c_{11}^2}\end{aligned}$$

As before, Ω^t is given by (8) for $t \in \{1, 2\}$.

If the model permits an infinite number of periods, the desire to commit will be present in every period. In that case, τ_G^t will always be less than the one in Proposition 2 when M cannot commit, $\Omega^t > 0$, and the future marginal production cost increases in x_G^t. As a consequence, the stock is being depleted faster when M cannot commit.

5 Solutions

We now consider three extensions that can shed light on how M can mitigate its time-inconsistency problem. As for the extensions in Section 5, each can be read in isolation, because they do not build on one another.

5.1 Upstream Regulation

The gas trap arises because M can adjust x_G after x_R is sunk. We believe this timing is reasonable but, in reality, x_G is not decided at a single point in time (Venables, 2014). It takes several years to decide on search-and-exploration policies, find and develop reserves, and exploit the reserves: See Anderson et al. (2018) or Ahlvik et al. (2024). Arezki et al. (2017) find lags from 4 to 6 years lag between discovery and production for oil and gas. Bornstein et al. (2023) estimate the average lag between investments in exploration and subsequent production in a dataset containing all oil fields with production between 1971 and 2015, collected by Rystad Energy. Their estimate implies an average time lag between investment and extraction of oil of 12 years. These fields contain gas also. By regulating the early stages of this process, M can influence the market's beliefs about how much M will end up producing.

To analyze this possibility in a simple model, we now modify the game in Section 2 by including a search-and-exploration stage at the very beginning of the game. We let s measure the expenses of search and

exploration, and we account for the fact that when s is small, the production cost will be high. In particular, the cost of producing gas will be given by the function $c^x(x_G, s)$, where the derivatives satisfy $c^x_1 > 0$, $c^x_{11} > 0$, $c^x_2 < 0$, $c^x_{22} < 0$, and, crucially, $c^x_{12} < 0$.

If M could commit, M would prefer that s be set to take into account the benefit $-c^x_2$. In the competitive market, those paying for s will indeed be able to cash in the benefit $-c^x_2$ when the services are sold to the gas producers. Empirically, Ahlvik et al. (2024) document that search-and-exploration activities respond to a production tax. Thus, the market's decision on s would be in line with M's preferred s, and M does not benefit from regulating s: $\tau_s = 0$. As above, the commitment solution coincides with the FB outcome, except that even N's fossil fuel production is taxed in the FB, in line with Proposition 0.

When M cannot commit, M will be tempted to outcompete coal and, when this temptation is anticipated, x_R declines. M's short-term policy is suboptimal from the long-term point of view. Thus, M benefits from making it credible that x_G will be small because, then, x_R will be large. To gain this credibility, M benefits from reducing or taxing search and exploration – especially when equilibrium τ_M will be small relative to the Pigouvian level (i.e., when Ω is large).

Proposition 6. *Suppose M can regulate search-and-exploration activities, s, at the beginning of the game.*
(i) If M could also commit to (τ_G, τ_M) at the beginning of the period, M's policy would involve $\tau_s = 0$ and otherwise be in line with Proposition 1.
(ii) When M cannot commit, equilibrium (τ_G, τ_M) is in line with Proposition 2 and M imposes the following tax on search and exploration:

$$\tau_s = \frac{(e_G a h' - \tau_G)\left[S'_C(k) - D'_A(k)\right] + \tau_M D'_M}{S'_A(k) - D'_A(k)} \left(-\frac{c^x_{12}}{c^x_{11}}\right) = \Omega \frac{S'_C(k) - D'_N(k)}{S'_A(k) - D'_A(k)} \left(-\frac{c^x_{12}}{c^x_{11}}\right).$$

As before, Ω is as defined in Proposition 2. When ah' is large, Ω is large, and M will tax production less and consumption more. To mitigate the discouraging effect on renewables, M benefits from taxing search-and-exploration activities more when ah' is large.

5.2 Credibility by Investing in Renewables

In this section, we consider the possibility that also M can invest (or incentivize domestic investments) in renewables. Let $x_{R,M}$ denote M's level of investment. Because $x_{R,M}$ is a result of M's policy, suppose $x_{R,M}$ must be decided before N's investors determine $x_{R,N}$.

We let M's cost of investing in renewables be the same as for N, k. When the investment cost is, for simplicity, linear, then it may be reasonable to consider constraints on how much it might be possible to invest to avoid "bang-bang" corner solutions. We will assume that M can invest at most $\overline{x}_{R,M}$, where $\overline{x}_{R,M}$ is so small that, in equilibrium, there will be a strictly positive level of investments in renewables also by the investors in N.[5] We discuss the consequence of relaxing this assumption in the proof of Proposition 7.

If M could commit to (τ_G, τ_M), M would not strictly benefit from investing in renewables. The cost coincides with the price, and the investment will not alter M's payoff or any of the variables of interest.

When M cannot commit, then a larger $x_{R,M}$ will influence M's equilibrium policies. As Proposition 2 uncovers, if M exports more (or imports less) then M will raise τ_G and reduce τ_M, attempting to raise the price and thus M's terms of trade. Although M will not succeed with raising p, when x_R is endogenous, M's desire to raise τ_G and reduce τ_M can change M's short-term policy in the direction that M would have preferred if M could commit. In fact, M's policies under commitment and noncommitment coincide if M just produces so much energy that M's desire to outcompete coal is balanced by the desire to improve the terms of trade (i.e., $\Omega = 0$). This follows from a comparison of Propositions 1 and 2.

Proposition 7. *Suppose M can invest $x_{R,M} \leq \overline{x}_{R,M}$ directly in renewables at the beginning of the game.*
(i) If M could commit, it would be optimal for M to not invest in renewables.
(ii) When M cannot commit, M implements the commitment solution if and only if M invests:

$$x_{R,M} = e_C S_C'(k)\, ah' + D_M(k) - S_G(k - e_G h'). \tag{9}$$

In other words, when M faces a temptation to outcompete coal, with the result that investments in renewables will fall, M gains credibility by investing directly (or by incentivizing domestic investments) in renewables. When these investments are sunk, and (9) holds, M finds it optimal to set $\tau_G = e_G ah'$ and $\tau_M = 0$ even in the short run. With this, M gains credibility in that M will no longer be expected to produce a lot of gas. Equilibrium x_R ends up being the same as if M could commit to (τ_G, τ_M). This result might rationalize the puzzle that just when the US has raised its production of (shale) gas, it also raises subsidies on investments in renewables with the 2022 Inflation Reduction Act.

[5] That is,

$$x_{R,M} \leq \overline{x}_{R,M} := D_A(k) - S_C(k) - S_G\left(k - e_G ah'\right).$$

5.3 Coalition Size and Composition

Our main result is that if the climate coalition M cannot commit to future policies, the anticipation of its climate policies will lead to more emissions, especially when emissions are harmful.

The straightforward solution to M's problem is to find a way to commit. In general, it is difficult for policymakers to commit, because the freedom to change policies is crucial for democratic systems to work. That said, international treaties are sometimes used and referred to as a way to commit. If the treaty is binding and defection politically costly, then it can influence the decisions of future governments. A binding supply-side treaty, with significant sanctions being imposed on defectors, can help the members commit to limiting the future production of gas. One interpretation of our main result is to illustrate just how important such a commitment may be. Without it, well-meaning climate policies can be counterproductive.

The size and composition of the climate coalition will also influence the temptation to outcompete coal. But a larger coalition will not necessarily help. On the one hand, a larger coalition will imply that the harm facing a larger number of members will be internalized. In the model, this effect can be captured by a larger $ah(.)$. As noted above, a larger $ah(.)$ will enlarge the difference between the commitment solution and the noncommitment equilibrium. The larger the coalition is, the larger the equilibrium emission level will be. On the other hand, a larger coalition might be helpful for the ability to commit to future policies. If the size comes along with larger penalties on defectors, then a large coalition can be more successful in making production cuts credible.

Regarding the composition of the climate coalition, it will be helpful to include members that export a lot of energy, even coal. If the coalition as a whole is a large net exporter, then the incentive to maintain a high price on exports will counter the temptation to reduce the price (by producing more gas) as a way to outcompete coal. In fact, equilibrium policies coincide with those under commitment if M exports so much that $\Omega = 0$. That is, if:

$$x_M - y_M = \sum_{j \in J} e_j S_j'(k)\, ah'. \tag{10}$$

Here, J is the set of fossil fuels supplied by N (as in Section 4). Thus, if large exporters are included in M, the left-hand side of (10), $x_M - y_M$, can be large. In addition, note that $\sum_{j \in J} e_j S_j'(k)$, on the right-hand side, declines if the number of coal producers in N is reduced because some of them are included in M. The Appendix permits M to produce coal as well as gas, and it confirms that it can be helpful to include coal producers in M (so that N's $\sum_{j \in J} e_j S_j'(k)$ declines), especially those that are more concerned with profit than with the climate (so that ah' is limited). This inclusion will help ensure that the right-hand side of (10)

is no larger than the left-hand side. Intuitively, a stronger desire to maintain a high price on exports will help the coalition's credibility for setting a climate policy that makes the investments in renewables worthwhile.[6]

If countries can decide whether or not to be a member of M, the decision may depend on how much a country exports, what it exports, and its concern for climate change. It is not at all clear that the right types of countries will be those that in equilibrium benefit from being members. Suppose, however, that a potential member country considers how its membership will influence the possibility to commit or to ensure that (10) holds. Then, the benefit of being a member will be especially large for those countries that can contribute to a policy that is beneficial rather than damaging. Whether this effect is sufficient to ensure that the coalition will be the right one remains an open question.

6 Conclusions

Is gas green? Natural gas is a substitute for black energy (coal), on the one hand, and green energy (renewables), on the other. Coal consumption must be replaced for there to be any hope of getting close to the UN's 1.5°C target. Gas can substitute for coal but, at the same time, gas will be competing with renewable energy which, ultimately, should replace all kinds of fossil energy.

In this paper, we begin by arguing that the extent to which gas outcompetes coal vs. renewables depends on the time horizon. When gas production increases, the energy price declines, and the supply of other energy sources declines – depending on how price elastic the various sources are. The price elasticity is not a fixed number, but it depends on the time horizon. The more time one has to adjust, the more supply can increase in response to a change in the price. Coal and renewables are fundamentally different, however: Renewable energy is limited by the capacity that has been invested in so far. This capacity can increase, but that requires investments in advance. Coal, in contrast, is a declining industry. The green transition will ensure that there will be stranded assets in the coal sector, implying that the capacity to supply coal might not be binding in the short run. Thus, coal supply is likely to be more elastic in the short run, but the supply of renewable energy is likely to be more price elastic in the long term.

The difference means that governments concerned with climate change face a time-inconsistency problem. A long-term solution would be to set Pigou taxes on all energy sources one can control. These taxes will

[6]See the surveys by Calvo & Rubio (2012) or Buchholz & Sandler (2021). Battaglini & Harstad (2016) analyze how the equilibrium coalition depends on whether or not investments in renewables can be contracted on in addition to the emission levels.

make renewable energy more competitive, and investments in renewables will increase. In the short term, however, when the stock of renewable energy is given and inelastic, then it is more tempting to raise the production of natural gas in order to outcompete coal. The larger the environmental harm is, the larger the temptation to outcompete coal will be. When this temptation is anticipated, before the policy is set, investments in renewables will be held back. The overall effect is that total emissions will increase when policymakers cannot commit to future policies. The outcome is a gas trap. The effects of larger environmental harm, and of the lower emission content in natural gas, are exactly the reverse in the noncommitment equilibrium of those in the scenario where policymakers can commit.

We have explained why these negative findings are robust to the introduction of many types of fuels, many periods, learning by doing, and resources that are depletable. To overcome the time-inconsistency problem, we have emphasized the importance of committing through international treaties, and through regulation at the search-and-exploration stages of gas. We have also shown that if the regulators of natural gas production can invest directly in renewables, then the temptation to flood the market with gas, and thereby reduce the price of energy, will be weakened. By investing a sufficient amount, the commitment solution can be implemented even if there is no commitment regarding the regulation of gas and consumption.

Future research should study whether our assumptions hold and how they can be relaxed. Empirically, we need more research on how the price elasticities of various types of fuels change with the time horizon. Theoretically, our multiple-period treatment of learning by doing and resource depletion is promising but preliminary. We have also taken the existence of a climate coalition as given, even though the incentive to participate is likely to depend on the time-inconsistency problem that we have discussed. Further research along all these lines is necessary to deepen our understanding of how to escape the gas trap.

7 Appendix: Proofs

The proofs allow N to have a set J of different fossil fuel types with supply function $S_j(p)$ and emission content e_j, $j \in J$. We write $S_N(p) \equiv \sum_{j\in J} S_j(p)$. Similarly, we begin by allowing M to supply multiple types of fossil fuels, L, with cost $c_l(x_l)$ and $S_l(p) \equiv c_l'^{-1}(p)$ for $l \in L$, and we write $S_M(p) \equiv \sum_{l\in L} S_l(p)$. The proofs consider a general supply function of renewables from N equal to $x_{R,N} = S_R(p)$. We allow M to be endowed with fixed $x_{R,M}$ units of renewables (to be endogenized at the end), in addition to the $x_{R,N}$ units in N. We write $x_R = x_{R,M} + x_{R,N}$ and $x_M = S_M(p) + x_{R,M}$. We write $A = M \cup N$ as the set of all countries or fossil fuel supply functions, so $D'_A = D'_M + D'_N$ and $S'_A = S'_M + S'_N$. Unless discussed, the second-order conditions will hold, thanks to our assumptions.

Proof of Proposition 1.

(i) *Effect of quotas:* If we differentiate foreign supply and demand conditions, we get

$$dx_j = S'_j dp,\ dx_{R,N} = S'_R dp,\ dy_N = D'_N dp. \tag{11}$$

When these equations are combined with the differentiation of the market clearing condition:

$$\sum_{j\in J} S_j(p) + x_{R,N} + x_M = D_N(p) + y_M, \tag{12}$$

we get:

$$dp = \frac{dy_M - dx_M}{S'_N - D'_N}. \tag{13}$$

Effect of taxes: With taxes, for every $l \in L$ produced by M,

$$dx_l = S'_l(dp - d\tau_l) \text{ and } dy_M = D'_M(dp + d\tau_M).$$

Combined with (11) and the differentiation of (12), we get:

$$dp = \frac{D'_M(dp + d\tau_M) - \sum_{l\in L} S'_l(dp - d\tau_l)}{S'_N + S'_R(p) - D'_N} = \frac{\sum_{l\in L} S'_l d\tau_l + D'_M d\tau_M}{S'_A + S'_R(p) - D'_A}. \tag{14}$$

M's optimal quantities maximize

$$b_M(y_M) - \sum_{l\in L} c_l(x_l) + p(x_M - y_M) - ah(e),$$

subject to (11)-(13). The solution is (in line with Lemma 4 in Harstad, 2012):

$$\begin{aligned} p - c'_l &= \left(e_l - \frac{\sum_{j\in J} e_j S'_j(p)}{S'_N(p) + S'_R(p) - D'_N(p)}\right) ah' - \frac{y_M - x_M}{S'_N(p) + S'_R(p) - D'_N(p)}, \\ b'_M - p &= \frac{\sum_{j\in J} e_j S'_j(p)}{S'_N(p) + S'_R(p) - D'_N(p)} ah' + \frac{y_M - x_M}{S'_N(p) + S'_R(p) - D'_N(p)}. \end{aligned}$$

And, because $c'_l = p - \tau_l$ and $b'_M = p + \tau_M$, M's optimal outcome can be implemented by taxes that satisfy:

$$\begin{aligned} \tau_l &= \left(e_l - \frac{\sum_{j\in J} e_j S'_j(p)}{S'_N(p) + S'_R(p) - D'_N(p)}\right) ah' - \frac{y_M - x_M}{S'_N(p) + S'_R(p) - D'_N(p)}, \\ \tau_M &= b'_M - p = \frac{\sum_{j\in J} e_j S'_j(p)}{S'_N(p) + S'_R(p) - D'_N(p)} ah' + \frac{y_M - x_M}{S'_N(p) + S'_R(p) - D'_N(p)}. \end{aligned} \tag{15}$$

When $x_{R,N}$ is endogenous and $S'_R \to \infty$, (15) gives:

$$\tau_l \to e_l ah' \text{ and } \tau_M \to 0.$$

So, in equilibrium, when $x_{R,N} > 0$, $p = k$, $x_j = S_j(k)$, and $x_l = S_l(k - e_l ah')$, then:

$$\begin{aligned} \frac{\partial x_l}{\partial a} &= -e_l h' S'_l(k - e_l ah') < 0, \\ x_{R,N} &= D_A(k) - S_N(k) - \sum_{l\in L} S_l(k - e_l ah') - x_{R,M}, \text{ so} \\ \frac{\partial x_{R,N}}{\partial a} &= \sum_{l\in L} e_l h' S'_l(k - e_l ah') > 0, \text{ and} \\ e &= \sum_{j\in J} e_j S_j(p) + \sum_{l\in L} e_l S_l(k - e_l ah'), \text{ so} \\ \frac{\partial e}{\partial a} &= -\sum_{l\in L} e_l^2 h' S'_l(k - e_l ah') < 0. \end{aligned}$$

Remark on trade policies. M cannot do better than the SB by adding trade policies among its instruments. If, for example, M imposes a tariff on energy imports, then M's domestic energy price will be $p_M = p + \tau_I$, M's producers supply according to $c'_l = p_M - \tau_l = p + \tau_I - \tau_l$, and M's consumers consume according to $b'_M = p_M + \tau_M = p + \tau_I + \tau_M$. Clearly, M implements (15) by any two taxes as long as:

$$\begin{aligned} \tau_l - \tau_I &= \left(e_l - \frac{\sum_{j\in J} e_j S'_j(p)}{S'_N(p) + S'_R(p) - D'_N(p)}\right) ah' - \frac{y_M - x_M}{S'_N(p) + S'_R(p) - D'_N(p)}, \\ \tau_M + \tau_I &= b'_M - p = \frac{\sum_{j\in J} e_j S'_j(p)}{S'_N(p) + S'_R(p) - D'_N(p)} ah' + \frac{y_M - x_M}{S'_N(p) + S'_R(p) - D'_N(p)}. \end{aligned}$$

QED

Proof of Proposition 2.

(i) At the time when the taxes are decided on, $x_{R,N}$ and $x_{R,M}$ are fixed, so (15) holds with $S'_R = 0$. In equilibrium, $p = k$ when $x_{R,N} > 0$, so (15) becomes

$$\begin{aligned} \tau_l &= \left(e_l - \frac{\sum_{j\in J} e_j S'_j(k)}{S'_N(k) - D'_N(k)}\right) ah' - \frac{D_M(k + \tau_M) - \sum_{l\in L} S_l(k - \tau_l)}{S'_N(k) - D'_N(k)}, \\ \tau_M &= \frac{\sum_{j\in J} e_j S'_j(k)}{S'_N(k) - D'_N(k)} ah' + \frac{D_M(k + \tau_M) - \sum_{l\in L} S_l(k - \tau_l)}{S'_N(k) - D'_N(k)}, \end{aligned} \tag{16}$$

which, in the main model, simplifies to (6).

(ii) If we differentiate (15), we must take into account that $I = y_M - x_M$ is endogenous:

$$\begin{aligned} d\tau_l &= \left(e_l - \frac{\sum_{j\in J} e_j S_j'(p)}{S_N'(p) - D_N'(p)}\right) h' da - \frac{dI}{S_N'(p) - D_N'(p)}, \\ d\tau_M &= \frac{\sum_{j\in J} e_j S_j'(p)}{S_N'(p) - D_N'(p)} h' da + \frac{dI}{S_N'(p) - D_N'(p)}. \end{aligned} \tag{17}$$

The above proofs, where M can have multiple types of fuels, are referred to in Section 5. From now on, however, we can restrict attention to the setting (in Sections 2-3) in which M is endowed with only gas.

When $p = k$ is given, we have $dI = D_M' d\tau_M + S_G' d\tau_G$. With this, (17) becomes:

$$\begin{aligned} d\tau_G &= \left(e_G - \frac{\sum_{j\in J} e_j S_j'(p)}{S_N'(p) - D_N'(p)}\right) h' da - \frac{S_G' d\tau_G + D_M' d\tau_M}{S_N'(p) - D_N'(p)}, \\ d\tau_M &= \frac{\sum_{j\in J} e_j S_j'(p)}{S_N'(p) - D_N'(p)} h' da + \frac{S_G' d\tau_G + D_M' d\tau_M}{S_N'(p) - D_N'(p)}. \end{aligned} \tag{18}$$

So,

$$d\tau_M = \frac{\sum_{j\in J} e_j S_j'(p)}{S_N'(p) - D_A'(p)} h' da + \frac{S_G' d\tau_G}{S_N'(p) - D_A'(p)}$$

which we can use to find $d\tau_G$:

$$\begin{aligned} d\tau_G = \left(e_G - \frac{\sum_{j\in J} e_j S_j'(p)}{S_N'(p) - D_N'(p)}\right) h' da - \frac{S_G' d\tau_G}{S_N'(p) - D_N'(p)} \\ - \frac{D_M'}{S_N'(p) - D_N'(p)} \left[\frac{\sum_{j\in J} e_j S_j'(p)}{S_N'(p) - D_A'(p)} h' da + \frac{S_G' d\tau_G}{S_N'(p) - D_A'(p)}\right] \quad \Leftrightarrow \\ d\tau_G \left[1 + \frac{S_G'}{S_N'(p) - D_N'(p)} \left(1 - \frac{-D_M'}{S_N'(p) - D_A'(p)}\right)\right] \\ = \left(e_G - \frac{\sum_{j\in J} e_j S_j'(p)}{S_N'(p) - D_N'(p)}\right) h' da - \frac{D_M'}{S_N'(p) - D_N'(p)} \left[da \frac{\sum_{j\in J} e_j S_j'(p)}{S_N'(p) - D_A'(p)} h'\right]. \end{aligned}$$

The bracket on the l.h.s. is positive. Thus, $d\tau_G/da < 0$ iff

$$\begin{aligned} & e_G - \frac{\sum_{j\in J} e_j S_j'(p)}{S_N'(p) - D_N'(p)} - \frac{D_M'}{S_N'(p) - D_N'(p)} \left[\frac{\sum_{j\in J} e_j S_j'(p)}{S_N'(p) - D_A'(p)}\right] \\ = \; & e_G - \frac{1}{S_N'(p) - D_N'(p)} \frac{\sum_{j\in J} e_j S_j'(p)}{S_N'(p) - D_A'(p)} \left[S_N'(p) - D_A'(p) + D_M'\right] \\ = \; & e_G - \frac{\sum_{j\in J} e_j S_j'(p)}{S_N'(p) - D_A'(p)} < 0. \end{aligned}$$

Under this condition, if a increases, τ_G decreases, and thus $x_G = S_G(p - \tau_G)$ increases when $p = k$ is fixed. Consequently, e must increase, because $x_j = S_j(p)$ and $x_N = S_N(p)$ are given by $p = k$. QED

Proof of Proposition 4.

(i) Suppose that M can commit. M's equilibrium payoff level in the second period is:

$$\max_{x_M^2, y_M^2} b_M\left(y_M^2\right) - c_G\left(x_G^2\right) + \left(x_G^2 - y_M^2\right) k^2 - ah^2 \left(\sum_{j \in J} e_j S_j\left(k^2\right) + e_G x_G^2\right),$$

whose first-order conditions (f.o.c.'s) are as in Proposition 1. Using the Envelope theorem, the benefit of a larger k^2 is:

$$\left(x_G^2 - y_M^2\right) - \left(\sum_{j \in J} e_j S_j'\left(k^2\right)\right) ah^2.$$

With the decreasing function $k^2 = f\left(x_R^1\right)$, where:

$$x_R^1 = D_N\left(k^1\right) + y_M^1 - S_C\left(k^1\right) - x_G^1,$$

M's first-period problem is to solve:

$$\max_{x_G^1, y_M^1} b_M\left(y_M^1\right) - c_G\left(x_G^1\right) + \left(x_G^1 - y_M^1\right) k^1 - ah^1 \left(\sum_{j \in J} e_j S_j'\left(k^1\right) + e_G x_G^1\right) + \delta V\left(k^2\right),$$

where $V\left(k^2\right)$ is the continuation value, as a function of k^2. The f.o.c.'s are:

$$\begin{aligned}
0 &= k^1 - c_G'\left(x_G^1\right) - e_G ah^1 + \delta V'\left(k^2\right) \frac{\partial k^2}{\partial x_G^1} \Leftrightarrow \\
0 &= k^1 - c_G'\left(x_G^1\right) - e_G ah^1 + \delta \left[\left(x_G^2 - y_M^2\right) - \sum_{j \in J} e_j S_j'\left(k^2\right) ah^2\right] \left(-f'\right) \Leftrightarrow \\
\tau_G^1 &= k^1 - c_G'\left(x_G^1\right) = e_G ah^1 + \delta \left[\sum_{j \in J} e_j S_j'\left(k^2\right) ah^2 - \left(x_G^2 - y_M^2\right)\right] \left(-f'\right),
\end{aligned}$$

and

$$\begin{aligned}
0 &= b_M'\left(y_M^1\right) - k^1 + \delta V'\left(k^2\right) \frac{\partial k^2}{\partial y_M^1} \Leftrightarrow \\
0 &= b_M'\left(y_{M,1}\right) - k^1 + \delta \left[\left(x_G^2 - y_M^2\right) - \sum_{j \in J} e_j S_j'\left(k^2\right) ah^2\right] f' \Leftrightarrow \\
\tau_M^1 &= b_M'\left(y_M^1\right) - k^1 = -\delta \left[\sum_{j \in J} e_j S_j'\left(k^2\right) ah^2 - \left(x_G^2 - y_M^2\right)\right] \left(-f'\right).
\end{aligned}$$

where the last bracket is positive if $\Omega^2 > 0$. Under this condition, M prefers to commit to $\tau_G^1 > e_G ah^1$ and $\tau_M^1 < 0$ as a way to encourage investments in renewables, especially if the learning-by-doing effect is large.

(ii) If M cannot commit, M sets policies for period t at the time when x_R^t and thus the future k^{t+1} are given. Therefore, M's optimal policy does not take into account any learning-by-doing effect, and M's policy satisfies Proposition 2, as if this effect were absent. QED

Proof of Proposition 5.

Second period: To accommodate the next proof, let the second-period extraction cost be $c^x\left(x_G^2, s\right)$, and the first-period extraction level be s, instead of $c^2\left(x_G^2, x_G^1\right)$ and x_G^1. In the second period, when $t = 2$, M's equilibrium payoff is:

$$\begin{aligned} V(s) &= b_M\left(y_M^t\right) - c^x\left(x_G^t, s\right) + \left(x_G^t - y_M^t\right)k^t - ah^t e, \text{ so} \\ V'(s) &= \left(b'_M\left(y_M^t\right) - k^t\right)\frac{y_M^t}{ds} - \left(c_1^x - k^t + e_G a h^t\right)\frac{dx_G^t}{ds} - c_2^x. \end{aligned} \tag{19}$$

Lemma 1

$$\frac{dy_M^t}{ds} = \frac{D'_M\left(k^t\right)}{S'_A(k^t) - D'_A(k^t)}\frac{c_{12}^x}{c_{11}^x} \text{ and } \frac{dx_G^t}{ds} = -\left(\frac{S'_C(k^t) - D'_A(k^t)}{S'_A(k^t) - D'_A(k^t)}\right)\frac{c_{12}^x}{c_{11}^x} \text{ for } t = 2. \tag{20}$$

Proof. Given the production tax τ_G^t in period $t \in \{1, 2\}$, the representative gas producer would like to maximize:

$$\left(k^1 - \tau_G^1\right)x_G^1 - c_G^1\left(x_G^1\right) + \delta\left[\left(k^2 - \tau_G^2\right)x_G^2 - c^x\left(x_G^2, x_G^1\right)\right], \text{ so}$$

$$\partial c_G^1\left(x_G^1\right)/\partial x_G^1 = k^1 - \tau_G^1 - \delta c_2^x, \text{ for the first period quantity, and} \tag{21}$$

$$c_1^x\left(x_G^2, s\right) = k^2 - \tau_G^2, \text{ for the second period quantity,} \tag{22}$$

which implicitly defines second-period supply x_G^2 as a function of s and $k^t - \tau_G^t$, written $x_G^t = S^x\left(k^t - \tau_G^t, s\right)$. By differentiating (22), we get:

$$c_{11}^x\left(x_G^t, s\right)dx_G^t + c_{12}^x\left(x_G^t, s\right)ds = -d\tau_G^t, \tag{23}$$

making it easy to confirm that $S_1^x = 1/c_{11}^x$ and $S_2^x = -c_{12}^x/c_{11}^x$. By differentiating (6), we get for $t = 2$:

$$\begin{aligned}
d\tau_y^t &= \frac{1}{S'_C(k^t) - D'_N(k^t)} \left[D'_M(k^t + \tau_y^t) d\tau_y^t + S_1^x d\tau_x^t - S_2^x ds \right] \\
&= \frac{1}{S'_C(k^t) - D'_A(k^t)} \left[S_1^x d\tau_x^t - S_2^x ds \right], \\
d\tau_x^t &= -\frac{1}{S'_C(k^t) - D'_N(k^t)} \left[D'_M(k^t + \tau_y^t) d\tau_y^t + S_1^x d\tau_x^t - S_2^x ds \right] \\
&= -\frac{1}{S'_C(k^t) - D'_N(k^t)} \left[D'_M(k^t + \tau_y^t) \frac{1}{S'_C(k^t) - D'_A(k^t)} \left[S_1^x d\tau_x^t - S_2^x ds \right] + S_1^x d\tau_x^t - S_2^x ds \right] \\
&= -\frac{1}{S'_C(k^t) - D'_N(k^t)} \left[\frac{S'_C(k^t) - D'_N(k^t)}{S'_C(k^t) - D'_A(k^t)} \left[S_1^x d\tau_x^t - S_2^x ds \right] \right] \Leftrightarrow \\
\frac{d\tau_x^t}{ds} &= \frac{S_2^x}{S'_C(k^t) + S_1^x - D'_A(k^t)}, \text{ so} \\
\frac{d\tau_y^t}{ds} &= -\frac{d\tau_x^t}{ds} = \frac{-S_2^x}{S'_C(k^t) + S_1^x - D'_A(k^t)}.
\end{aligned}$$

From the f.o.c. for M's consumption level, we get $dy_M^t = D'_M d\tau_M^t$, so, at $t = 2$:

$$\frac{dy_M^t}{ds} = \frac{D'_M d\tau_M^t}{ds} = \frac{-D'_M S_2^x}{S'_C(k^t) + S_1^x - D'_A(k^t)}.$$

From (23), we get at $t = 2$:

$$\begin{aligned}
c_{11}^x \left(x_G^t, s \right) dx_G^t + c_{12}^x \left(x_G^t, s \right) ds &= -d\tau_G^t \Leftrightarrow \\
c_{11}^x \left(x_G^t, s \right) \frac{dx_G^t}{ds} &= -\frac{d\tau_G^t}{ds} - c_{12}^x \left(x_G^t, s \right) \Leftrightarrow \\
\frac{dx_G^t}{ds} &= -\frac{1}{c_{11}^x} \frac{d\tau_G^t}{ds} - \frac{c_{12}^x}{c_{11}^x} \\
&= -\frac{1}{c_{11}^x} \frac{S_2^x}{S'_C(k^t) + S_1^x - D'_A(k^t)} - \frac{c_{12}^x}{c_{11}^x}.
\end{aligned}$$

Because $S_2^x = -c_{12}^x/c_{11}^x$ and $S_1^x = 1/c_{11}^x$, at $t = 2$ we have:

$$\begin{aligned}
\frac{dx_G^t}{ds} &= -\frac{1}{c_{11}^x} \frac{-c_{12}^x/c_{11}^x}{S'_A(k^t) - D'_A(k^t)} - \frac{c_{12}^x}{c_{11}^x} = \left(\frac{S_1^x(k^t)}{S'_C(k^t) + S_1^x - D'_A(k^t)} - 1 \right) \frac{c_{12}^x}{c_{11}^x} \\
&= -\left(\frac{S'_C(k^t) - D'_A(k^t)}{S_1^A(k^t, s) - D'_A(k^t)} \right) \frac{c_{12}^x}{c_{11}^x} \text{ where } S_1^A \left(k^t, s \right) \equiv S'_C(k^t) + S_1^x \left(k^t, s \right). \| \qquad (24)
\end{aligned}$$

Substituting (20) into (19), we get, with $t = 2$:

$$\begin{aligned}
V'(s) + c_2^x &= \frac{(b'_M - k) D'_M + (c_1^x - k^t + e_G a h^t) \left[S'_C(k^t) - D'_A(k^t) \right]}{S_1^A(k^t, s) - D'_A(k^t)} \frac{c_{12}^x}{c_{11}^x} \\
&= \frac{\tau_M^t D'_M + (e_G a h^t - \tau_G^t) \left[S'_C(k^t) - D'_A(k^t) \right]}{S_1^A(k^t, s) - D'_A(k^t)} \frac{c_{12}^x}{c_{11}^x},
\end{aligned}$$

which would be zero if M could commit. When M cannot commit, taxes at $t = 2$ are given by (6), so:

$$\begin{aligned} V'(s) + c_2^x &= \left[\frac{e_C S_C'(k^t)\, ah^t}{S_C'(k^t) - D_N'(k^t)} + \frac{y_M^t - x_G^t}{S_C'(k^t) - D_N'(k^t)}\right] \frac{S_C'(k^t) - D_N'(k^t)}{S_1^A(k^t, s) - D_A'(k^t)} \frac{c_{12}^x}{c_{11}^x} \\ &= \frac{e_C S_C'(k^t)\, ah^t + y_M^t - x_G^t}{S_1^A(k^t, s) - D_A'(k^t)} \frac{c_{12}^x}{c_{11}^x}. \end{aligned} \tag{25}$$

First period: Compared to Section 3, the difference is the continuation value $\delta V(s)$, here discounted. This continuation value depends only on first-period extraction and not on M's first-period consumption. The f.o.c.'s for M's policy will satisfy:

$$k^1 - c_l' = \left(e_l - \frac{\sum_{j\in J} e_j S_j'(k^1)}{S_N'(k^1) + S_R'(k^1) - D_N'(k^1)}\right) ah^1 - \frac{y_M^1 - x_M^1}{S_N'(k^1) + S_R'(k^1) - D_N'(k^1)} - \delta V'(s), \tag{26}$$

$$b_M' - k^1 = \frac{\sum_{j\in J} e_j S_j'(k^1)}{S_N'(k^1) + S_R'(k^1) - D_N'(k^1)} ah^1 + \frac{y_M^1 - x_M^1}{S_N'(k^1) + S_R'(k^1) - D_N'(k^1)}. \tag{27}$$

(i) If M could commit in period 2, $\delta V'(s) = -\delta c_2^x$ and, when M sets its policies, $S_R'(k^1) \to \infty$. Note that (21) and (26) then coincide if $\tau_G^1 = e_l ah^1$. With $\tau_M^1 = 0$, the representative consumer ensures that $b_M' = k^1$, which is in line with (27) when $S_R'(k^1) \to \infty$. Thus, Proposition 1 holds for both periods.

(ii) When M cannot commit, M's continuation value follows from (25), and, in the first-period, M sets policies given that $S_R' = 0$. Both equations can be substituted into (26) to determine M's optimal quantities. Combined with (21), the necessary production tax is:

$$\begin{aligned} \tau_G^1 &= \left(e_G - \frac{e_C S_C'(k^1)}{S_C'(k^1) - D_N'(k^1)}\right) ah^1 - \frac{D_M(k^1 + \tau_M^1) - S_G(k^1 - \tau_G^1)}{S_C'(k^1) - D_N'(k^1)} - \delta V'(s) - \delta c_2^x \\ &= \left(e_G - \frac{e_C S_C'(k^1)}{S_C'(k^1) - D_N'(k^1)}\right) ah^1 - \frac{D_M(k^1 + \tau_M^1) - S_G(k^1 - \tau_G^1)}{S_C'(k^1) - D_N'(k^1)} - \delta \frac{e_C S_C'(k^2)\, ah^2 + y_M^2 - x_G^2}{S_A'(k^2) - D_A'(k^2)} \frac{c_{12}^x}{c_{11}^x}, \end{aligned}$$

when $S_A'(k^2) \equiv S_1^A(k^2, s)$ is naturally defined as the slope of the supply curve given the equilibrium s. The necessary consumption tax is simply (27) with $S_R' = 0$. Compared to the model without depletion, τ_G^1 is further reduced when $c_{12}^x > 0$ when M cannot commit. In equilibrium, a lower τ_G^1 increases x_G^1 and decreases x_R^1, for the same reasons as in the proof of Proposition 2. QED

Proof of Proposition 6.

This proof draws on the proof of Proposition 5. At the beginning of the period, M's objective is to maximize:

$$b_M(y_M) - c^x(x_G, s) + (x_G - y_M)k - ah(e) - c_s(s),$$

where $c_s(s)$ is the cost of s. This problem gives the f.o.c.:

$$(b'_M - k)\frac{y_M}{ds} - c_2^x - (c_1^x - k + e_G a h')\frac{dx_G}{ds} = c'_s,$$

while, in equilibrium, the market invests in s according to:

$$c'_s = -c_2^x - \tau_s.$$

Thus, M's optimal τ_s is:

$$\tau_s = (e_G a h' - \tau_G)\frac{dx_G}{ds} - \tau_M \frac{y_M}{ds}. \tag{28}$$

(i) From Proposition 1, it follows that if M could commit, $e_G a h' - \tau_G = \tau_M = 0$ and, therefore, $\tau_s = 0$.

(ii) Lemma 1 and (20) present $\frac{dx_G}{ds}$ and $\frac{y_M}{ds}$ for the last period. (In that proof, the last period is $t = 2$, but here there is only one period.) When the two expressions are substituted into (28), we get:

$$\begin{aligned}\tau_s &= (e_G a h' - \tau_G)\left(-\left(\frac{S'_C(k) - D'_A(k)}{S_1^A(k,s) - D'_A(k)}\right)\frac{c_{12}^x}{c_{11}^x}\right) - \tau_M D'_M\left(\frac{c_{12}^x/c_{11}^x}{S_1^A(k,s) - D'_A(k)}\right) \\ &= \frac{(e_G a h' - \tau_G)[S'_C(k) - D'_A(k)] + \tau_M D'_M\left(-\frac{c_{12}^x}{c_{11}^x}\right)}{S_1^A(k,s) - D'_A(k)}.\end{aligned}$$

When M cannot commit, we can substitute in with (6) to get:

$$\begin{aligned}\tau_s &= \left[\frac{e_C S'_C(k)}{S'_C(k) - D'_N(k)} a h' + \frac{y_M - x_G}{S'_C(k) - D'_N(k)}\right]\frac{S'_C(k) - D'_N(k)}{S_1^A(k,s) - D'_A(k)}\left(-\frac{c_{12}^x}{c_{11}^x}\right) \\ &= \frac{e_C S'_C(k) a h' + y_M - x_G}{S'_A(k) - D'_A(k)}\frac{-c_{12}^x}{c_{11}^x},\end{aligned}$$

when $S'_A(k) \equiv S_1^A(k,s)$ is naturally defined as the slope of the supply curve given the equilibrium s. QED

Proof of Proposition 7.

The above proofs permit M to produce $x_{R,M}$ units of renewable energy. We believe that it is unrealistic to permit M to invest so much that no-one else in the world finds it attractive to invest in renewables, so we will assume away this possibility:

Assumption R. *M can invest in $x_{R,M}$ at unit cost k, but M cannot invest so much that M crowds out all investment in renewables by N. I.e.:*

$$x_{R,M} \leq \overline{x}_{R,M} \equiv D_A(k) - S_C(k) - S_G(k - e_G a h').$$

With this assumption, in equilibrium, $x_{R,N} > 0$. If the assumption failed, $x_{R,M}$ could be so large that the energy price would be less than k, and then the larger $x_{R,M}$ would be gradually crowding out coal and gas. In this case, M can produce so much renewables that the equilibrium energy price is below the cost of investing, and coal can be outcompeted without having to worry about reduced investments in renewables in other countries.

(i) If M could commit to policies, M would set $\tau_G = e_{_G} ah'$ and $\tau_M = 0$, regardless of $x_{R,M}$. If M increased $x_{R,M}$, then $x_{R,N}$ would decline by the same amount, so that the market clearing condition holds:

$$x_{R,M} + x_{R,N} = D_A(k) - S_C(k) - S_G(k - e_G ah').$$

Thus, M would change neither policies, nor quantities, by increasing $x_{R,M}$. The cost would coincide with the revenue, so it is optimal for M to invest nothing (or anything) in $x_{R,M}$, and M does not strictly benefit from subsidizing these investments.

(ii) When M cannot commit, M would prefer to commit to policies that are in line with Proposition 1, but, in equilibrium, M's policies are in line with Proposition 2. The two are equal only if (9) holds. Equation (9) does not need to violate Assumption R as long as:

$$\sum_{j \in J} e_j S_j'(k_2)\, ah' < D_N(k) - S_C(k) - S_G(k).$$

Thus, with enough investment, M implements its commitment solution at no cost, since the equlibrium price of energy equals the unit cost of $x_{R,M}$. It is not necessary to subsidize investments to achieve this goal, since it is an equilibrium that private investors invest (9) as long as $p = k$. (There are multiple equilibria when it comes to where the investors are located.) It is easy to check that an even larger investment by M is suboptimal, as long as Assumption R holds. QED

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