Sovereign Default Risk, Firm Investment and Debt Overhang[1]

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Abstract

This paper investigates the macroeconomic implications of sovereign risk on firms' investment decisions both empirically and quantitatively. Using highly granular loan-level data, we first empirically show that an increase in sovereign risk premium leads to a rise in borrowing costs, which then features a decline in firm investment. Moreover, higher sovereign risk premium curtails firm investment by intensifying the "debt overhang" i.e., a negative correlation between sovereign indebtedness and firm investment induced by the sovereign's lack of commitment to repay debt. Our results mostly apply for domestic borrowing, that is, firms with a more diverse credit portfolio (including cross-border lending) are able to substitute out their domestic credit and thus remain unaffected. We then construct a quantitative endogenous sovereign default model featuring these stylized facts and offer countercyclical lax capital regulations to mitigate the transmission of sovereign's default risk on firms' funding costs.

Keywords: Debt overhang, sovereign default, investment, crowding out, crowding in
JEL Codes: E32, E34, F32, F34

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1 Introduction

The quantitative literature to date is still rather silent about the mechanisms and effects of the debt overhang i.e., a negative correlation between sovereign indebtedness and firm investment. This paper attempts to fill this gap by first presenting empirical findings with granular and administrative loan level data followed by a quantitative model explaining these facts. We further complement the model by evaluating the impact of lax lending regulations to mitigate the adverse transmission of sovereign risk on firm funding costs during economic downturns.

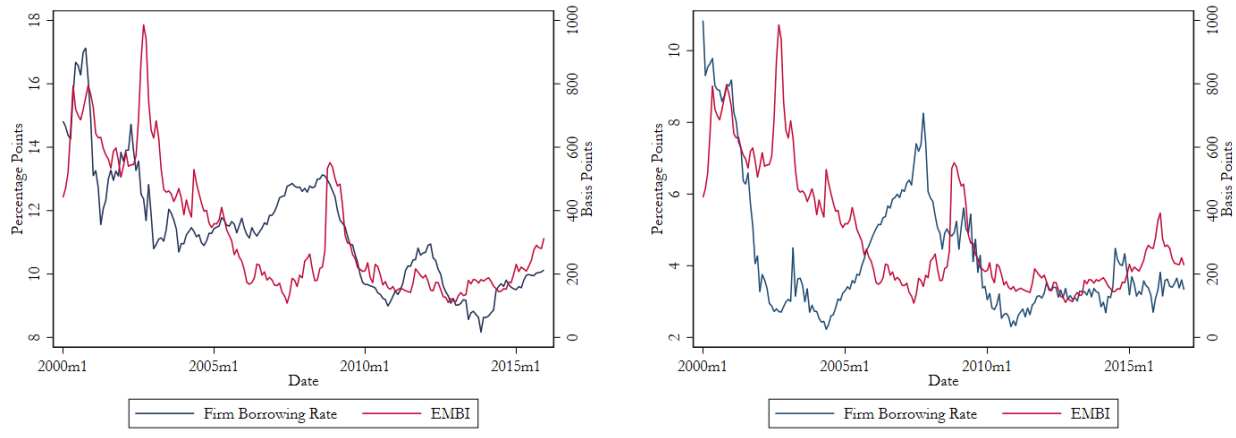
We highlight three stylized facts from granular loan level micro-data as follows:

FACT 1. *Higher levels of sovereign debt raise the cost of credit to private firms.* Numerous papers find robust evidence of a large and significant drop in the supply of foreign credit to the private sector during and in the aftermath of sovereign debt crises (**????**). By and large, this result holds for non-crisis episodes as well. If a government faces higher difficulty in meeting its debt obligations, it is more likely to impose higher taxes, tighter foreign exchange controls, or even a partial seizure of firm assets; all of which lower firms' repayment capability. Consequently, higher levels of sovereign debt may impair private firms' access to credit. Along this line, firm borrowing costs are intrinsically related to the sovereign risk premium, since it carries adverse effects over the value of sovereign bond prices (**??**). We argue that while this applies to both foreign and domestic credit, firms that only have access to domestic credit are more severely affected, as they are usually smaller in size and more vulnerable to a sudden change in funding (**?**). In Figure 1, we present visual evidence on this relationship using the entire Colombian Credit Registry database: panel (a) shows domestic credit rates and panel (b) shows foreign credit rates. As observed, newly originated loans command, on average, a higher interest rate when the sovereign risk premium is higher (differentially higher for domestic loans). In Section 2, we present firm-level evidence of this relationship at a micro level.

FACT 2. *Higher cost of credit is associated with lower investment.* A fundamental tenet of investment theory is that the user cost of capital is central to investment decisions. Among a variety of factors, market interest rates (or financing costs) are emphasized as a key

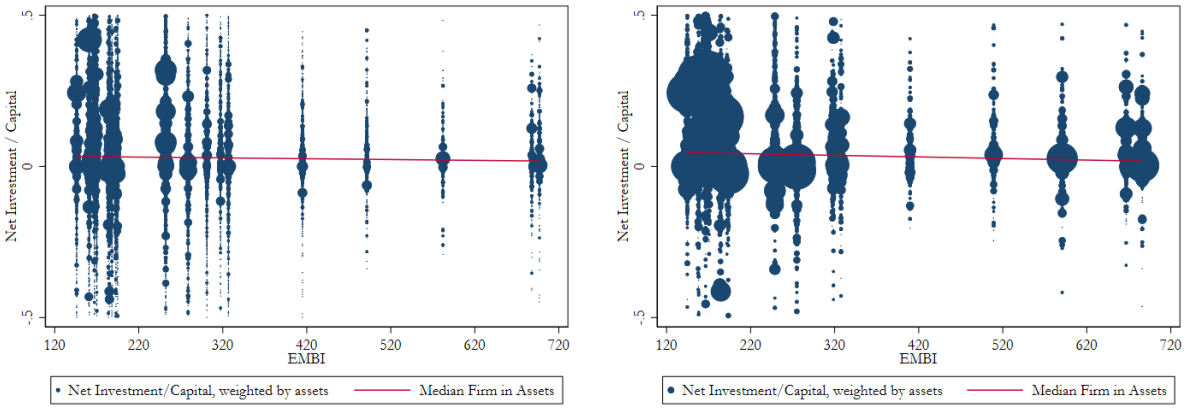
building block of user cost of capital (e.g. **???**; and references therein). In particular, a higher market interest rate, particularly when it is sizable, is predicted to lower investment expenditures. In Section 2, we take this evidence further to a small open economy setting, and show that higher cost of foreign credit is predicted to lower investment, also using firm-level data.

FACT 3. *Higher sovereign risk premium curtails corporate investment by intensifying the debt overhang*. In the finance literature, debt overhang refers to when high levels of debt inhibit firms' ability to finance future profitable projects (**?**). In the macroeconomic literature, it is broadly defined as the limited commitment of a sovereign on its public debt, which features a negative correlation between government debt and investment (**?**). In this paper, we focus on the latter channel which has mainly remained unexplored. Similar to **?**, in Figure 2 we show that a higher probability of sovereign default is associated with lower investment. While visually this relationship appears to be weak, in Section 2 we empirically corroborate a significant negative relationship (between country risk and and firm investment) but only for firms with domestic funding. Namely, larger firms with a more diverse credit portfolio (including cross-border lending) are able to substitute out their domestic credit and thus remain unaffected.



(a) Domestic loan rate **(b)** Foreign loan rate

Figure 1: Borrowing rate on domestic currency (Panel a) and foreign-currency (Panel b) newly originated loans. EMBI refers to the Colombian country risk index. The firm-level borrowing rates are aggregated using loan principal amounts. Frequency is monthly. Sample period: 2000-2015. Source: Colombian Credit Registry.



(a) Firms with domestic loans **(b)** Firms with foreign loans

Figure 2: Net investment/capital for private non-financial firms borrowing in domestic currency (Panel a) and foreign currency (Panel b) versus Colombian EMBI. Sample period is 2000-2015 and the frequency is yearly. Net investment corresponds to the change in gross fixed capital formation. The solid line corresponds to the median firm (in terms of asset size) each year. The circle sizes are weighted according to firm total assets (larger circles correspond to larger firms). Data Source: Colombian Credit Registry, Corporate Registry, and Central Bank's International Exchange Department Database.

The joint explanation of these stylized facts is proven to be difficult in international macroeconomics. One strand of literature models the interest rate as an exogenous variable which varies with the assumed functional form. On the other hand, quantitative sovereign default models are abstracted away from modeling the investment decision of firms. The literature mostly relies on the classical set up of **?** and models the income process exogenously. Thus, a joint explanation is still missing.

In this paper, we construct a quantitative sovereign default model consistent with these facts. In particular, the model allows for a dynamic interaction between firm investment and sovereign borrowing decisions. In the model, domestic and imported foreign investment goods are used to accumulate capital. These investment goods are only imperfectly substitutable, and therefore, a sovereign default –which results in temporary exclusion from international credit markets for both firms and the government– causes an efficiency loss in capital accumulation and production. A salient ingredient of our model is that firms need working capital financing to pay for imports of a subset of foreign investment goods. Importantly, and broadly inline with **Fact 1** (see above), firms borrow from international markets at the sovereign borrowing rate.

The dynamic link between sovereign default risk and firm investment hinges on two key features of our model: (i) firms need working capital financing and borrow at the sovereign borrowing rate; and (ii) both the government and domestic firms are excluded from international markets after a default decision. Based on these features, the model's dynamics show that the higher the sovereign debt holdings –and therefore higher sovereign default risk–, the higher the financing cost of importing foreign investment goods, and the stronger the need for firms to substitute away from imported foreign investment goods. This entails an efficiency loss since domestic and imported foreign investment goods are only imperfectly substitutable. As a result, and consistent with the empirical facts above, higher levels of sovereign debt results in lower investment.

In contrast to the Eaton-Gersovitz class of models, default costs are not incurred by sharp declines in the realized endowment. In the literature it is common to use either quadratic endowment loss functions as in **?** and proportional endowment loss as in **?** or a fix percentage loss in endowment as in **?**. The default costs in our model occur following a

decline in investment. The assumption of firms being excluded from international capital markets during exclusion periods is crucial for this outcome. Endogenous default costs in our framework preserves the features of asymmetric cost structure that is provided by **?**. That is, we do not have an episode where the sovereign opts defaulting during economic upturns in our simulations.

Our paper utilizes long-term debt when a sovereign makes its borrowing decision while firms undertake implied one-period rate for their working capital loans. Hence, we contribute to the sharpening and understanding of long-term debt dynamics. In times when the spread for one-period debt is zero, the spread for long-term debt lies in the positive territory since the pricing of these assets also relies on future borrowing and default rules. Our model also features cases of crowding-in and crowding-out. When the likelihood of default is low, the sovereign borrows more (at cheaper rates) and firms increase their capital and investment (crowding in). Alternatively, when default risk is high, then the borrowing rate for the sovereign is also high, which curtails private investment (crowding-out).

Lastly, we complement our analysis with a policy that is in the spirit of lax lending conditions during economic downturns. In particular, when the TFP shocks are smaller than its historical mean, the government allows firms to have access to a higher share of working capital loans to finance investment expenditures. We show that this policy: (i) mitigates the default risks, (ii) stems debt overhang, (iii)features higher debt-to-GDP ratio while lowering spreads, and (iv) generates significant welfare gains.

Related Literature:

There is a budding literature that explores the link between sovereign default risk and private economic activity. **?** introduce endogenous cost of default via the financing cost of working capital (which is then used to pay for imports of a subset of intermediate goods). While their model provides important insights on how sovereign default entails endogenous output costs, it abstracts away capital expenditures, a key input for production, and by modeling choice, do not address how fluctuations in sovereign risk affect firm investment cycles. The link between sovereign risk and firm investment, in addition to the

empirical evidence that we present in this paper, is recognized in a recent theoretical work by **?**. In particular, **?** theoretically show that governments' lack of commitment to policy and being impatient relative to the market induces an endogenous debt overhang effect: higher sovereign debt distorts investment and induces investment cycles that prolong the effect of even i.i.d shocks. Introducing dynamics of domestic credit supply into the picture, **?** studies the effect of sovereign defaults on domestic banks and its influence on government's default incentives in a framework with endogenous sovereign default with one-period debt and **?** introduces capital to standard Eaton-Gersovitz setting with one period debt contracts while **?** investigates the behavior of private credit during a heightened sovereign risk. **?** examine a channel through government debt and bank balance sheets which may inhibit banks' ability to finance firms.

On empirical front, this paper contributes to the existing literature by exploring the link debt overhang phenomena by establishing the negative correlation between sovereign riskiness and firm's investment.

Our paper improves upon the existing literature by (i) establishing key empirical facts on the joint dynamics of sovereign debt and firm investment cycles using highly granular micro-level data; and (ii) explicitly incorporating firm investment decision and providing a direct link between sovereign default and firm investment cycles in a quantitative endogenous sovereign default model.

The paper proceeds as follows: In Section 2, we present micro-level evidence on the link between sovereign risk premium, cost of foreign-currency borrowing and corporate investment and we show the model dynamics explaining these features in Section 3. In Section 4, we present the calibration strategy with which we obtain the model results in Section 5. Section 6 proposes a policy to mitigate the impact of crisis and Section 7 concludes.

2 Empirical Evidence

What do micro-level data tell about sovereign risk premium, cost of foreign borrowing and investment? In this section, we provide micro-level evidence on the joint dynamics of

investment and sovereign risk premium. In particular, using the entire Colombian *Credit Registry* database that contains extensive information on virtually all loan transactions, and the Colombian *Corporate Registry* that includes balance sheet and income statement for a comprehensive set of firms, we test the following three hypotheses:

H1. Higher levels of sovereign risk premium raise the cost of credit to private firms.
H2. Higher cost of borrowing leads to lower investment.
H3. Higher sovereign risk premium curtails firm investment by intensifying the debt overhang.

The Colombian credit registry data includes extensive details on all domestic loans, starting from the year 2000. Details include, and not limited to: loan principal amount, interest rate charged, date of loan origination, currency of denomination, together with unique identifiers for each party engaging in the transaction. The database is housed at the Financial Superintendence. We note that all banks are required to report details of each and every loan transaction with firms and households. In the empirical analyses below, we use interest rate charged on newly originated foreign-currency denominated loans to private non-financial firms. In addition, we use the Central Bank's International Exchange Department Database to observe all foreign-currency funding from banks or other financial entities located in foreign jurisdictions (we observe the same variables as for the domestic loans).

On the other hand, the *Corporate Registry* includes details on both balance sheet and income statement variables, thus enabling us to relate real economic outcomes with financing decisions, of over 30,000 firms. The database is housed and maintained at the Corporate Superintendence. Since debt overhang is more likely to be prevalent for small firms, using a large sample coverage is particular essential for our analysis. Indeed, the firm coverage in this database is comprehensive and goes beyond publicly-listed companies as in most related studies. Moreover, the database provides some key qualitative characteristics such as the sector in which a firm operates. In our empirical analysis, we use 4-digit level sector codes to control for sectoral or sector-year effects.

We match the Credit Registry with the Corporate Registry using unique firm and bank identifiers. In doing so, we make some aggregation. The credit register is highly granular,

is reported monthly, and includes extensive details on each loan transaction. The Corporate Registry, on the other hand, is reported annually. Therefore, when the dependent variable is loan related, we keep the monthly frequency and repeat yearly information for each firm within a year. When the dependent variable is firm related, we take a yearly weighted average of loan rates (weighted by loan amounts) for each firm across its different lenders. Moreover, we exclusively focus on newly originated loans to avoid over-representation of firms that (possibly) alter original credit terms and conditions or those that have access to already existing credit lines.

To this end, we provide empirical tests related to the hypotheses above. Since each hypothesis is a topic of its own, and in fact can command a stand-alone paper, here we keep the discussion brief and concise for the first two hypotheses, and refer to the related literature for brevity. We should note that our empirical results for the first two hypothesis confirm the existing studies. Yet, unlike many existing studies, we identify these channels with a highly granular data which sets the stage for our quantitative analysis in the next section.

Our third hypothesis, on the other hand, is an avenue which has not been explored much in the literature and allows for the interplay between sovereign risk and firm investment. Even though there has been a number of empirical studies exploring the debt overhang phenomenon within the corporate finance literature, we note that the literature is scant regarding how the indebtedness of a firm impedes its investment decision, that is, regarding the link between debt overhang and sovereign risk.

2.1 First Hypothesis

H1. Higher levels of sovereign risk premium raise the cost of foreign credit to private firms.

Determinants of firm borrowing costs are extensively studied in the literature. Among those, firm size (**?**; **?**; **?**), age (**?**; **?**), indebtedness (**?**; **?**), liquidity or fixed tangible assets (**?**; **?**) are emphasized as the key demand-side factors that determine firm borrowing costs. The common view is that smaller, highly leveraged or relatively less liquid firms are generally perceived more likely to face difficulty in repaying their debts. Moreover, younger firms, having less of a chance to establish long-standing credit relationship with banks, generally experience stronger asymmetric information with banks. In turn, banks, on average, set higher lending rates for such firms.

We take the analysis to a small open economy setting where we focus on determinants of both *local-currency* and *foreign-currency* borrowing costs and in particular ask whether sovereign risk premium has an effect on the cost of credit. Our empirical specification is given by:

$$r^*_{i,t} = \alpha EMBI_t + \beta Z_{i,t-1} + \rho r^*_{i,t-1} + \mu_i + \eta_t + \epsilon_{it} \tag{1}$$

where Z_{it} denotes for firm-specific control variables (lagged one year), μ_i and η_t are firm and year fixed effects, respectively. For Z, we include firm size in assets, total equity over assets (reflecting firm's indebtedness), cash flow over total financial debt (reflecting the degree of firm's ability to pay the debt back), and share of fixed tangible assets in total assets (reflecting firm's pledgable assets). Cash flow is measured by ebitda (net earnings before interest, taxes and depreciation allowances). Last, we include sovereign risk premium measured by the Colombian EMBI spread.

The results are reported in Table 1. Controlling for a large set of firm-specific variables, the results suggest that an increase in sovereign risk premium raises the cost of borrowing. The effect is stronger for firms with only domestic borrowing (columns 3 and 4), given their limited capability to substitute out credit with other lenders (especially with cross-border lenders). In particular, a 1 standard deviation increase in the EMBI is estimated to increase foreign and domestic loan rates by up to 30 and 99 basis points, respectively. As expected,

Table 1: Cost of Foreign-Currency Borrowing and Sovereign Risk Premium
H1. Higher levels of sovereign risk premium raise the cost of foreign credit to private firms.
Dependent Variables: Costs of Foreign-Currency and Domestic-Currency Borrowing

VARIABLES	(1) Foreign Loan Rate	(2) Foreign Loan Rate	(3) Domestic Loan Rate	(4) Domestic Loan Rate
EMBI	0.159***	0.294***	0.990***	0.809***
	(0.009)	(0.014)	(0.017)	(0.027)
Log(Real Assets)$_{t-1}$	-1.254***	-0.0558**	-1.481***	-1.040***
	(0.021)	(0.024)	(0.026)	(0.036)
Equity/Assets$_{t-1}$	-0.308***	-0.198***	-1.858***	-0.725***
	(0.054)	(0.042)	(0.118)	(0.118)
EBITDA/Fin.Debt$_{t-1}$	-3.55e-7*	-9.56e-8	1.93e-7	2.37e-7
	(2.12e-7)	(1.65e-7)	(1.88e-7)	(1.85e-7)
Fixed Assets/Assets$_{t-1}$	1.114***	-0.151	-0.694***	-0.497***
	(0.133)	(0.106)	(0.153)	(0.151)
Constant	25.01***	4.855***	39.60***	32.06***
	(0.367)	(0.402)	(0.421)	(0.584)
Observations	47,343	47,343	419,225	419,225
R-squared	0.552	0.729	0.291	0.312
Firm Fixed Effects	Y	Y	Y	Y
Year Fixed Effects	N	Y	N	Y

Notes. The dependent variables are the interest rate charged on newly originated foreign-currency or domestic-currency denominated loans for private non-financial firms. All independent variables are lagged by one period except EMBI, which is standardized. Robust standard errors are in parantheses. ***, **, and * denote significance levels at 1%, 5%, and 10%, respectively. Sample Period: 2000-2015. Frequency is monthly. Source: Colombian Credit Register, Colombian Superintendence of Corporations DataBase and Colombian Central Bank's International Exchange Department Database.

larger firms receive lower cost of borrowing. Less indebted firms (lower equity/assets) or relatively liquid firms (higher Ebitda/Fin.Debt) experience a lower interest rate on their loans. Perhaps surprisingly, firms with higher pledgable assets (as measured by fixed tangible assets to total assets) are estimated to experience a higher borrowing cost.

2.2 Second Hypothesis

H2. Higher cost of foreign borrowing is predicted to decrease investment.

The response of investment to changes in user cost of capital is widely studied in the literature (**?**; **?**; **?**). Changes in financial borrowing costs (market interest rates) or fiscal-related costs (e.g. changes in tax codes or tax incentives) are by and large emphasized as important determinants of user cost of capital. Here we follow **?** in forming the empirical specification. In particular, our set of control variables include a variable that proxies firm's marginal product of capital, a variable related to cost of capital along with key firm characteristics and a rich set of fixed effects. To proxy for marginal product of capital (MPK), we have used current net sales to long-term assets ratio inline with **?** (MPK^f). As a proxy for user cost of capital we use firm cost of borrowing for newly originated loans (the same variable used in $H1$). Lastly, we use firm- and year- fixed effects to control for unobserved heterogeneity.

In sum, our empirical specification is read as:

$$\left[\frac{I}{K}\right]_{i,t} = \alpha r^*_{i,t-1} + \beta MPK_{i,t-1} + \gamma Z_{i,t-1} + \rho \left[\frac{I}{K}\right]_{i,t-1} + \mu_i + \eta_t + \epsilon_{it} \tag{2}$$

where investment (I) is the annual change in fixed tangible assets, $r^*_{i,t-1}$ is the interest rate on foreign-currency denominated newly-originated loans, $MPK_{i,t-1}$ is the firm-specific current net sales to long-term assets ratio, and $Z_{i,t-1}$ is a vector of firm characteristics (firm size –total real assets– and firm indebtedness –equity over assets–).

The results are reported in Table 2. The negative and significant estimated coefficient for the 'Cost of Foreign Currency Borrowing' indicates that higher interest rate on firm-level newly-originated loan is predicted to decrease firm investment in the subsequent period. As in the previous exercise, we observe a stronger effect for firms with only domestic borrowing. A rise in the MPK (that proxies firm's current and future investment opportunities) is estimated to increase investment. Moreover, smaller or indebted firms are estimated to have lower levels of investment.

It is worth noting that any correlation uncovered does not rely on the variations in EMBI. All our regressions include year fixed effects and thus identification strategy relies on variation across firms in a given year; all firms are subject to the same EMBI.

Table 2: Firm Investment and Cost of Foreign-Currency Borrowing
H2. Higher cost of foreign borrowing is predicted to decrease investment.
Dependent Variable: Investment/Capital

VARIABLES	(1) Foreign Loan Rate	(2) Foreign Loan Rate	(3) Domestic Loan Rate	(4) Domestic Loan Rate
Log(Cost of Borrowing)	-0.0146***	-0.0252*	-0.382***	-0.383***
	(0.004)	(0.014)	(0.053)	(0.053)
Log(MPK)_{t-1}	-0.0173	-0.244***	-0.530***	-0.526***
	(0.020)	(0.068)	(0.040)	(0.039)
$\text{Investment/Capital}_{t-1}$	-0.935***		-0.00794***	
	(0.002)		(0.001)	
$\text{Log(Real Assets)}_{t-1}$	-0.074***	-1.275***	-2.750***	-2.733***
	(0.021)	(0.069)	(0.046)	(0.046)
$\text{Equity/Assets}_{t-1}$	0.106*	0.187	-0.582***	-0.573***
	(0.055)	(0.184)	(0.140)	(0.139)
Constant	1.253***	21.86***	44.64***	44.36***
	(0.359)	(1.191)	(0.762)	(0.756)
Observations	19,401	19,529	130,816	131,569
R-squared	0.957	0.506	0.202	0.201
Year Fixed Effects	Y	Y	Y	Y
Firm Fixed Effects	Y	Y	Y	Y

Notes. The dependent variable is the ratio of net investment (annual change in gross fixed capital) to capital. All independent variables are lagged by one period. Our choice for the independent variables is similar to **?**. 'Cost of Borrowing' is the interest rate charged on newly originated foreign-currency or domestic-currency denominated loans for private non-financial firms. Robust standard errors are in parantheses. ***, **, and * denote significance levels at 1%, 5%, and 10%, respectively. Sample Period: 2000-2015 for foreign-currency credits and 2000-2020 for domestic credits. Frequency is annual. Source: Colombian Credit Register, Colombian Superintendence of Corporations DataBase and Colombian Central Bank's International Exchange Department Database.

2.3 Third Hypothesis

H3. Higher sovereign risk premium curtails corporate investment by intensifying the debt overhang.

The empirical model is as follows:

$$\left[\frac{I}{K}\right]_{i,t} = \beta_1 \text{Overhang}_{i,t-1} + \beta_2 \text{Overhang}_{i,t-1} \text{Sov.Crisis}_t + \gamma_1 Z_{i,t-1} + \gamma_2 Z_{i,t-1} \text{Sov.Crisis}_t + \mu_i + \eta_{s,t} + \epsilon_{it} \quad (3)$$

where investment (I) is the annual change in fixed tangible assets, $Z_{i,t-1}$ is a vector of variables that potentially reflect firm-level determinants of investment (debt rollover risk and a set of firm characteristics). μ_i captures unobserved time-invariant heterogeneity across firms. $\eta_{s,t}$s are the sector-year fixed effects at four-digit level sector codes and absorb time-varying sector-specific demand conditions. Our focus variable, Overhang, is measured as the ratio of EBITDA (earnings before interest, tax, and depreciation allowances) to short-term total debt. In essence, the variable Overhang is the reciprocal of debt-overhang measure, and captures debt service capacity. Namely, a higher Overhang implies lower debt overhang via higher debt service capacity. The variable Sov.Crisis is a dummy variable that takes a value one if the sovereign risk premium (measured by the EMBI) is significantly high by historical standards (if two-standard-deviation higher) and zero otherwise.

Identification Strategy: An important challenge is to address the changes in credit demand along with unobserved bank and firm characteristics. A researcher has to account for the possibility that firms reduce their investment following idiosyncratic demand or productivity shocks. Alternatively, it can be that investment may have fallen because a firm's relationship with a weak bank whose lending capacity may have deteriorated during times of stress. We know firms, particularly small ones, are highly dependent on bank finance. Our rich dataset allows us to account for these possible shortcomings in an estimation strategy. We account for the changes in demand and productivity by incorporating four-digit industry-country-year fixed effects. Firm and bank fixed effects

account for the impact of unobserved time-invariant firm and bank specific conditions while controlling for the transmission of balance sheet effect of the bank that our firms obtains its loans from.

Table 3: Firm Investment and Sovereign Debt Overhang
H3. Higher sovereign risk premium intensifies the debt overhang.
Dependent Variable: Firm Invesment/Capital

VARIABLES	(1) Foreign Loan Rate	(2) Foreign Loan Rate	(3) Domestic Loan Rate	(4) Domestic Loan Rate
EBITDA/ST.Debt$_{t-1}$	-0.000	9.43e-5	-0.003**	-0.002**
	(0.001)	(0.007)	(0.001)	(0.001)
Sov. Crisis		0.039		-2.138**
		(0.497)		(1.013)
EBITDA/ST.Debt*Sov. Crisis$_{t-1}$	0.001	0.000	**0.003***	**0.003****
	(0.007)	(0.026)	(0.002)	(0.001)
Equity/Assets$_{t-1}$	0.097*	0.187	-0.292	0.169
	(0.056)	(0.215)	(0.189)	(0.145)
Equity/Assets*Sov.Crisis$_{t-1}$	-0.031	-0.071	-0.052	-0.168
	(0.054)	(0.207)	(0.150)	(0.131)
Log(Assets)$_{t-1}$	0.056***	0.012	-2.818***	-1.335***
	(0.016)	(0.042)	(0.608)	(0.294)
Log(Assets)*Sov.Crisis$_{t-1}$	-0.002	0.008	0.164**	0.139**
	(0.007)	(0.029)	(0.069)	(0.062)
I/K$_{t-1}$	-0.935***			
	(0.0019)			
Constant	-0.997***	-0.298	43.77***	21.04***
	(0.279)	(0.721)	(9.461)	(4.630)
Observations	19,319	19,443	145,287	145,287
R-squared	0.967	0.500	0.187	0.172
Year Fixed Effects	Y	N	Y	N
Firm Fixed Effects	Y	Y	Y	Y

Notes. The dependent variable is the ratio of net investment (annual change in gross fixed capital) to capital. Ebitda stands for earnings before interest payments, tax outlays and depreciation allowances. ST-Debt stands for short-term debt, including debt to financial institutions and non-financial debt (e.g. trade credit), all with a maturity less than a year. Therefore, ebitda-to-debt reflects the (inverse) degree of debt overhang (see also **?**). Sov.Crisis is a dummy variable that takes a value 1 if Colombian EMBI exceeds its historic mean, and 0 otherwise. All independent variables are lagged by one period to mitigate potential endogeneity. Robust standard errors are in parantheses. ***, **, and * denote significance levels at 1%, 5%, and 10%, respectively. Sample Period: 2000-2015. Frequency is annual. Source: Source: Colombian Credit Register, Colombian Superintendence of Corporations DataBase and Colombian Central Bank's International Exchange Department Database.

The results supports the debt overhang effect on firm investment when the EMBI spread is significantly high, but only for loans denominated in local currency (Table 3, columns 3-4). That is, firms with high levels of local debt in excess of earnings under-invest when sovereign risk premium is high, suggesting that these firms suffer from debt overhang.

Moreover, control variables have expected signs. Firms with high levels of short-term debt experience higher rollover risk and thus under-invest during crisis times. In other words, the benefit of having shorter debt maturity diminishes for such episodes. Last, larger firms or firms with higher sales growth experience milder debt overhang. In sum, controlling for debt maturity, firm characteristics and industry-specific demand conditions, firms with higher debt to earnings do under-invest when sovereign risk premium is higher.[1]

3 The Model

3.1 Households

Households choose how much to consume and supply labor to maximize expected discounted utility streams, $E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, L_t)$, where $0 < \beta < 1$ is the subjective discount factor, and c_t and L_t denote consumption and labor, respectively. $u(.)$ is the period utility function which is continuous, strictly increasing in consumption, strictly decreasing in labor, and strictly concave in both arguments. E_t is the expectation operator conditional on the information set available at t.

Household receive real wage per labor hour, w_t, profits paid by the final-goods producers, π_t, and transfers from the government (T_t). Formally, the household solves

$$\max_{c_t, L_t} E_t \left[\sum_{t=0}^{\infty} \beta^t u(c_t, L_t) \right] \tag{4}$$

subject to the period budget constraint

$$c_t = w_t L_t + \pi_t + T_t \tag{5}$$

[1] In an independent study, **?** has documented a stronger debt overhang effect and rollover risk for European firms during the European sovereign crisis. In particular, total debt (in excess of earnings) exerts a negative influence on firm investment, and this effect is more pronounced during sovereign crisis episodes. Moreover, firms located in countries with weak sovereigns (or those working with banks that hold risky sovereign debt instruments) cut back investment more if they enter the crisis with a higher share of short-term debt (that is, if they face a heightened rollover risk). Here we follow a similar strategy in formulating the empirical model and choosing the control variables, except that we abstract away bank-sovereign linkage as in **?**) to remain conformable with the theoretical model.

Household preferences are governed by a **?**-type utility function, which ensures no wealth effect on labor supply.[2] In particular, we use the utility function of the form

$$u(c_t, L_t) = \frac{\left(c_t - \frac{L_t^{1+v}}{1+v}\right)^{1-\sigma} - 1}{1-\sigma} \tag{6}$$

where $\sigma > 0$ is the constant relative risk aversion, and $v > 0$ governs the (inverse) Frisch elasticity of labor supply. The optimal labor supply is given by $L_t^v = w_t$.

3.2 Final-Goods Producers

We formulate the final goods producers problem following **?**. In particular, these firms maximize profits, π_t, net of depreciation and discounted at the country borrowing rate:

$$E_0\left[\sum_t \left(\frac{1}{1+r_t^*}\right)\left(\pi_t^f - p_t^I\left(K_t - (1-\delta)K_{t-1}\right)\right)\right] \tag{7}$$

where p_t^I is the price of investment good, $I_t \equiv (K_t - (1-\delta)K_{t-1})$ denotes investment, and π_t^f is the output net of labor costs ($\pi_t^f = A_t K_t^{\alpha^K} L_t^{\alpha^L} - w_t L_t$). The production technology is governed by a Cobb-Douglas function given by

$$Y_t = A_t K_t^{\alpha^K} L_t^{\alpha^L} \tag{8}$$

α^K is the output elasticity of capital, α^L the output elasticity of labor, and A_t the shock to level of total factor productivity. They operate under decreasing returns to scale, i.e. $0 < \alpha^K + \alpha^L < 1$.[3] Final-goods producers take factor prices, w_t and p_t^I, and the country borrowing rate, r_t^*, as given, and maximize their profits given by equation (7).

We assume that firms perceive the TFP shock as i.i.d., which then renders an optimal level of capital as a function of country borrowing rate (**?**). That is, for a given r^*, there is a unique and constant level of optimal K_t. The simplification, without loss of generality, is

[2] This formulation for preferences removes wealth effect on labor supply and helps explain key business cycle facts for small open economies. See, e.g., **?**.

[3] Under decreasing-returns-to-scale production technology, the firm makes positive economic profits (which are then transferred to households). Such transfer of profit helps match consumption dynamics better for default episodes.

that final goods producers are myopic, and therefore, their maximization problem satisfies $K \equiv K(r^*)$ and $I(r^*) = \delta K(r^*)$. This assumption is not as restrictive as it might appear, since in the sovereign's dynamic problem that we are going to postulate below, r^* fluctuates endogenously due to fluctuations in sovereign external borrowing and fluctuations in TFP, which then results in endogenously time-varying K_t and I_t.[4]

Profit maximization yields optimal factor demand schedules:

$$\alpha^K A_t K_t^{\alpha^K - 1} L_t^{\alpha^L} = \delta P_t^I \tag{9}$$

and

$$\alpha^L A_t K_t^{\alpha^K} L_t^{\alpha^L - 1} = w_t \tag{10}$$

3.3 Capital Accumulation and Investment Goods Producers

The importance of foreign investment goods in accumulating capital is well recognized in the literature (e.g. **?** and **?**). Along with this literature, we assume that investment good, I_t, is produced via a CES-aggregate of domestic and imported foreign investment goods, I_t^d and I_t^f, respectively. The aggregator satisfies

$$I_t = \left[\lambda (I_t^d)^\epsilon + (1-\lambda)(I_t^f)^\epsilon\right]^{\frac{1}{\epsilon}} \tag{11}$$

where $\lambda \in [0,1)$ is the weight of I_t^d, and $\epsilon < 1$ governs the (inverse) elasticity of substitution.

Moreover, I_t^f is composed of imperfectly substitutable varieties, for reasons that we discuss below. In particular, I_t^f is a CES-aggregate of foreign investment goods varieties,

$$I_t^f = \left(\int_{j\in[0,1]} (I_{i,t}^f)^\nu\right)^{\frac{1}{\nu}} \tag{12}$$

[4]Our assumption also boils down to the *non-contractibility of investment* as laid out by **?** that the firm cannot commit to an exact investment level when signing the contract, and, afterwards, it is free to choose investment to maximize its own objective function. Or, similar in notion to **?**, firms, in the face of realization of shock in the next period, moves quickly to bring investment to the new level, given the new endogenous borrowing constraint due to a given (r_{t+1}, A_{t+1}).

where $I^f_{i,t}$ is the foreign investment good variety i, and the elasticity of substitution across the varieties is given by $|\frac{1}{\nu-1}|$.

Investment goods producers minimize the cost of producing a certain level of investment, $\overline{I_t}$, given by

$$\min_{I^d_t, I^f_t} I^d_t + P^f_t I^f_t \tag{13}$$

subject to $\overline{I_t} = \left[\lambda (I^d_t)^\epsilon + (1-\lambda)(I^f_t)^\epsilon\right]^{\frac{1}{\epsilon}}$. At the second stage, given their optimal demand for aggregate foreign investment good, they choose their demand for the each variety.

The optimality conditions for the first-stage are

$$I^d_t = \left(\frac{1-\lambda}{\lambda}\frac{1}{P^f_t}\right)^{\frac{1}{\epsilon-1}} I^f_t \tag{14}$$

$$I^f_t = \left(\lambda \left(\frac{1-\lambda}{\lambda}\frac{1}{P^f_t}\right)^{\frac{\epsilon}{\epsilon-1}} + (1-\lambda)\right)^{-\frac{1}{\epsilon}} I_t \tag{15}$$

and given the zero profit condition $P^I_t I_t = I^d_t + P^f_t I^f_t$, the price of investment goods satisfies

$$P^I_t = \left(\lambda^{\frac{1}{1-\epsilon}} + (P^f_t)^{\frac{\epsilon}{\epsilon-1}}(1-\lambda)^{\frac{1}{1-\epsilon}}\right)^{\frac{\epsilon-1}{\epsilon}} \tag{16}$$

At the second stage, investment goods producers choose optimal demand for each variety i, subject to a working capital constraint. In particular, they need to finance a θ-fraction of their purchase of foreign investment good. One can show that the demand for each variety i satisfies

$$I^f_{it} = \begin{cases} \left(\frac{p^f_i(1+r^*_t)}{P^f(r^*_t)}\right)^{-\frac{1}{1-\nu}} I^f_t, & \text{for } i \in [0,\theta]; \\ \left(\frac{p^f_i}{P^f(r^*_t)}\right)^{-\frac{1}{1-\nu}} I^f_t, & \text{for } i \in [\theta,1]. \end{cases} \tag{17}$$

Moreover, when the country is in default, the investment goods producing firms cannot access to working capital loans, and therefore

$$I_{it}^{f} = \begin{cases} 0, & \text{for } i \in [0, \theta]; \\ \left(\frac{P_{aut}^{f}}{p_{i}^{f}} \right)^{-\frac{1}{1-\nu}} I_{t}^{f}, & \text{for } i \in [\theta, 1]. \end{cases} \tag{18}$$

where P_{aut}^{f} is the rental rate of foreign investment during autarky. Given the CES aggregator for foreign capital (equation (12)), and given that θ fraction of varieties needs to be financed by working capital loans, then $P^{f}(r_t^*)$ satisfies the following:

$$P_{t}^{f}(r_t^*) = \left[\int_{0}^{\theta} (p_{i}^{f}(1 + r_t^*))^{\frac{\nu}{\nu-1}} di + \int_{\theta}^{1} (p_{i}^{f})^{\frac{\nu}{\nu-1}} di \right]^{\frac{\nu-1}{\nu}} \tag{19}$$

and where, in autarky,

$$P_{aut}^{f} = \left[\int_{\theta}^{1} (p_{i}^{f})^{\frac{\nu}{\nu-1}} \right]^{\frac{\nu-1}{\nu}}. \tag{20}$$

When a sovereign defaults, similar to the assumption in **?**, the investment goods producers cannot have access to working capital financing and the demand function system is computed with the limit of that system as $r_t^* \to \infty$.

The firms' problem depends on the level of total factor productivity and the sovereign's implied one-period borrowing rate r_t^* and our interest rate function maps aggregate states to the government's implied one-period borrowing rate

$$\mathcal{S}(A_t, b_{t+1}) = \mathcal{H}_s(A_t, b_{t+1}). \tag{21}$$

Given the interest rate function, optimal factor allocations characterize a competitive equilibrium in factor markets.

3.4 Sovereign's problem

This section describes the benevolent government's optimization problem. The government lacks a commitment technology, thus cannot commit to its future default and borrowing decisions, and decides how much non-state contingent long-term debt to issue

each period after repayment.[5] The timing of our paper is as in **?** to rule out the multiplicity dynamics that is present in **?** and **?** where they assume that the government makes the default decision after borrowing.[6]

We introduce an endogenous link between sovereign default and private economic activity. This endogenous link hinges on two assumptions: First, there is one-to-one mapping between the government's implied one-period borrowing rate and the firm's short-term working capital loan rate; and (ii) both the government and domestic firms are excluded from the financial markets after a default decision. The first is instrumental to capture the debt overhang: a rise in government debt holdings increases the likelihood of government default and thus amplifies the bond spreads as well as the loan rates for domestic firms. The second facilitates the significant investment drop during financial exclusion since lack of access to credit markets inhibits firms to find necessary funds to finance investment expenditures and leads to an efficiency loss.

Long-term debt contracts. The government issues long-term non-state contingent debt in real goods which are traded at a price q_t by infinitely many risk neutral international lenders. Similar to **?** and **?**, a debt issued at time t promises a stream of geometrically decreasing coupons κ which depreciates at a rate $\delta_b \in (0, 1]$. Thus, a government promises to pay $\kappa(1-\delta_b)^{n-1}$ units of consumption good in period $t+n$ for $n \geq 1$. Coupon payment κ is computed such that in the absence of default risk, price of non-state contingent long-term debt is equal to the price of average one-period debt and it reads $\frac{r+\delta_b}{1+r}$. This is a common formulation for long-term debt contracts to avoid keeping track of the entire maturity distribution. Hence, the dynamics of the long-term can be represented as follows:

$$b_{t+1} = (1-\delta_b)b_t + m_t \tag{22}$$

where b_t and b_{t+1} are total outstanding debt obligations at the beginning of period t and $t+1$ and m_t is the amount of debt that is issued or purchased back by the government in

[5] Allowing for long-term maturity for sovereign debt helps obtain more realistic sovereign bond spreads and default frequencies. Note also that, under certain conditions, long-term debt incorporates short-term debt as a special case (**?**, **?**).

[6] **?** documents that the models and the extensions of Eaton-Gersovitz type of sovereign default models are not prone to multiple equilibria.

period t. Note that one-period debt is a special case of long-term debt where $\delta_b = 1$. The budget constraint of a government conditional on its repayment decision reads as

$$c_t = A_t f(K_t, L_t) - I_t^f P_t^f - \kappa b_t + q(b_{t+1}, A_t) m_t. \tag{23}$$

Defaults. It is common in the literature to assume that the government is excluded from the credit markets for a stochastic period of time upon its default decision and gains access to the credit markets next period with constant probability $\psi \in [0, 1]$. Bond contracts typically include acceleration and collective action clauses, thus all debt obligations become due when a government defaults.

In order to regain access to the credit markets, the government needs to repay α fraction of its defaulted debt. This is a simple way of introducing recovery to the model. However, in a model with long-term debt the government issues infinite amount of debt prior to its default decision which may distort the moments in the simulations. To account for that it is assumed the government cannot issue bonds with a price lower than $\underline{q}$ as in **?**. We confirm that the constraint never binds in model simulations. In default, total output is used for consumption and imports of foreign investment goods at the price P_{aut}^f. Therefore, the budget constraint of an economy in default reads as

$$c_t = A_t f(K_t, L_t) - I_t^f P_{aut}^f. \tag{24}$$

International lenders. The international risk-neutral lenders invest in non-state contingent long-term debt and within-period private working capital loans in a perfectly competitive market, taking decision and borrowing rules given. Thus, lenders' supply schedule reflects a no-arbitrage condition with an opportunity cost of funds equals to the world's risk-free rate r. We denote the pricing schedule of bonds that are repaid with $q_t(b_{t+1}, A_t)$ and for bonds that are in arrears with $q_t^D(b_t, A_t)$.

3.5 Recursive representation

We now formulate the government's optimization problem recursively. Let V be the value function of the government that has the option of defaulting. The government

chooses to repay if value of repayment V^R is greater than the value of default V^D. The function V satisfies the following functional equation:

$$V(b, A) = \max\left\{V^R(b, A), V^D(b, A)\right\}, \tag{25}$$

where the government's continuation value for repayment reads

$$\begin{aligned} V^R(b, A) &= \underset{b'\geq 0,\, c\geq 0,\, L\geq 0,\, I^f\geq 0,\, I^d\geq 0}{Max} \left\{u(c, L) + \beta E_A\left[V(b', A')\right]\right\}, \\ s.t. \quad & c = Af(K, L) - I^f P^f(r^*) - b\kappa + q(b', A)\left[b' - (1-\delta_b)b\right], \\ q(b', A) &\geq \underline{q} \text{ if } b' > (1-\delta_b)b, \end{aligned} \tag{26}$$

where $f(K, L) = K_t^{\alpha^K} L_t^{\alpha^L}$. I^f and $P^f(r^*)$ denote the imported foreign investment goods and the price of these goods, respectively. The term r^* is the government's implied interest rate on one-period bonds.

The government stays in default at least for one period and regains access to the credit markets with constant probability ψ. The continuation value of default takes into account the positive repayment when the government regains access to the credit markets. We assume that debt in arrears grows at the international risk free rate r during the sovereign's exclusion from the credit markets. Along these lines, the value of default is:

$$\begin{aligned} V^D(b, A) &= \underset{c\geq 0,\, L\geq 0,\, I^f\geq 0,\, I^d\geq 0}{Max} \left\{u(c, L) + \beta E_A\left[\psi V(\alpha b(1+r), A) + (1-\psi)V^D(b(1+r), A)\right]\right\} \\ s.t. \quad & c = Af(K, L) - I^f P^f_{aut}. \end{aligned} \tag{27}$$

The solution to the government's optimization problem yields decision rules for default $\hat{d}(b, A)$ and borrowing $\hat{b}(b, A)$. A default decision rule is denoted by the binary variable $\hat{d} \in \{0, 1\}$; $\hat{d} = 1$ if the government finds value of defaulting higher than value repayment and 0 otherwise. In equilibrium, defined in Section 3.6, lenders use these decision rules for contract pricing. The price of a bond satisfies

$$
\begin{aligned}
q(b', A) &= \frac{1}{1+r} E_A \left[\left(1 - \hat{d}\left(b', A'\right)\right) \left(\kappa + (1-\delta_b)\, q\left(b'', A'\right)\right) + \right. \\
&\quad \left. \hat{d}\left(b', A'\right) q^D\left((1+r)\, b', A'\right) \right], \\
b'' &= \hat{b}(b', A').
\end{aligned} \tag{28}
$$

Equation (28) indicates that with no-arbitrage condition, selling a bond and investing it in a risk-free asset today at an international risk-free rate r (left hand-side of the equation) is equal to the value of keeping the bond (right hand-side of the equation). If the lender keeps the bond, he will receive one unit of coupon κ and can sell the unamortized portion of its assets $(1-\delta_b)$ at tomorrow's prevailing price $q\left(b'', A'\right)$ conditional on government's repayment decision ($\hat{d}\left(b', A'\right) = 0$). If the government defaults, then lenders can still trade their bonds at the secondary market at a price q^D. The value of a bond in default satisfies

$$
q^D(b', A) = \frac{1}{1+r} E_A \left[\begin{array}{l} (1-\psi)\,(1+r) q^D\left((1+r) b', A'\right) + \\ \alpha\psi \hat{d}(b''^{DD}, A') q^D\left(b''^{DD}, A'\right) + \\ \alpha\psi \left(1 - \hat{d}(\alpha(1+r) b', A')\right) \left[\kappa + (1-\delta_b) q\left(b''^{DR}, A'\right)\right] \end{array} \right],
$$

where $b''^{DD} = \alpha(1+r)b'$ denotes the next-period end-of-period total debt if the government defaults after exiting the current default, $b''^{DR} = \hat{b}(\alpha b')$ denotes the next-period end-of-period debt obligations chosen by a government that repays after exiting the default.

The short-term interest rate r^* that is used by the investment good producers in the economy is computed by setting $\delta_b = 1$. Notice that when $\delta_b = 1$, equation (28) boils down to the price of one-period debt which is determined by tomorrow's default probability and the recovery rate.

3.6 Definition of equilibrium

This paper focuses on Markov Perfect Equilibrium. The government cannot commit to any future (repayment and borrowing) decisions. Hence, government's strategies depend only on the payoff relevant state variables.

Definition 1 (Markov perfect equilibrium) *A Markov perfect equilibrium is characterized by value functions* $V(b, A), V^D(b, A), V^R(b, A)$*, bond pricing function* $q(b', A)$*, transfer rule* $\hat{T}$*, default rule* $\hat{d}$ *and borrowing rule* $\hat{b}$ *such that*

1. *Given the bond pricing function, government policy rules* $\left\{\hat{d}, \hat{b}\right\}$ *and transfers rule* $\hat{T}$ *solve the utility maximization problem defined in equations (25), (26) and (27).*
2. *Given government policy rules* $\left\{\hat{d}, \hat{b}\right\}$*, the pricing function* q *satisfies condition (28).*
3. *The transfers rule* $\hat{T}$ *satisfies the sovereign's budget constraint*

$$T(b, A) = q(b', A)(b' - (1 - \delta_b)b) - b\kappa.$$

4. *These allocations are consistent with a competitive equilibrium in factor markets and the interest rate function* $\mathcal{H}_s$ *is consistent with government policies.*

A solution to the Markov Perfect Equilibrium consists of the solutions for the optimality conditions of the private sector including the factor allocations as well as production with and without having an access to the credit markets. Solutions for equilibrium wages and profits follow from the optimality conditions characterized earlier.

3.7 Numerical Solution

The solution to this problem is obtained using global solution methods. This section briefly sketches the main numerical algorithm, relegating the details of the implementation to the Appendix. Solving the model relies on iterating the value functions V^R and V^D and price function q as well as an approximation scheme to the private sector's allocation problem. To avoid a potential multiplicity problem outlined in **?**, we first solve the

equilibrium of the finite-horizon economy. We start with an initial guess of the terminal value and iterate backwards until the differences in value and price functions for two subsequent periods are less than 10^{-5}. We then use obtained values as the equilibrium of the infinite horizon economy.

4 Calibration

We calibrate the model economy at a quarterly frequency. For most parameters, we use conventional estimates reported in the literature. For the remaining, we resort to the data for Colombia. Table 4 presents the calibrated parameter values.

Table 4: Parameter values

	Parameter	Value	Target
Risk aversion	γ	2	Standard RBC value
Risk-free rate	r	1%	Standard RBC value
Probability of reentry after default	ψ	0.083	**?**, **?**
Recovery rate	α	0.55	**?**
Bond decay rate	δ_b	0.033	Average duration of 5 years
Lower bound for bond prices	$\underline{q}$	0.5	-
Labor supply curvature parameter	ω	1.455	Frisch wage elasticity (2.2)
Armington weight of domestic investment	λ	0.62	Data
Armington curvature parameter	ϵ	0.62	**?**
Dixit-Stiglitz curvature parameter	ν	0.59	**?**
Upper bound to use working capital	θ	0.7	**?**
Share of capital	α^K	0.45	**?**; **?**
Share of labor	α^L	0.45	**?**; **?**
Depreciation rate of capital	δ	0.035	
Calibrated			
Discount factor	β	0.94	Default frequency (2%)
Income autocorrelation coefficient	ρ	0.97	GDP autocorrelation (0.752)
Standard deviation of innovations	σ_ϵ	2.0%	GDP std. deviation (0.025)

The risk aversion parameter is set to 2 and the quarterly world interest rate is set to 1%, the conventional values used in the literature. The labor supply curvature parameter, ω, is set to 1.455 as in **?**. This value yields a Frisch elasticity of labor supply, $\frac{1}{\omega-1}$, equal to 2.2,

and is well within the range reported in the literature.[7] The probability of re-entry after default, ψ, is set to 0.083, which implies that the country stays in default about three years before regaining access to international markets (**?**, **?**).

The weight of domestic investment goods in the Armington-aggregate of investment is set based on OECD Trade-in Value Added (TiVA) database. In particular, about 38% of gross fixed capital formation in Colombia is due to imports. We accordingly set λ equal to 0.62. Moreover, we set $1 - \alpha^K - \alpha^L$ equal to 0.1 to have a reasonable profits-to-GDP ratio in the model, consistent with the data. The factors shares, α^K and α^L are then set equal to 0.45, inline with **?** and **?**. The quarterly depreciation rate, δ, is set to 0.035, slightly higher than estimates for developed economies. For the remaining parameters for production, ϵ, ν,θ, we follow **?**, and set them equal to 0.62, 0.59 and 0.70, respectively.

We target debt-to-GDP ratio of 35.6 percent using the rich public debt databaset provided by **?**. The productivity shocks in the model follow an AR(1) process:

$$log(A_t) = \rho log(A_{t-1}) + \varepsilon_t$$

with with $|\rho| < 1$, and $\varepsilon_t \sim N\left(0, \sigma_\epsilon^2\right)$. Due to lack of actual TFP series, we estimate the GDP process of Colombia, using HP detrended seasonally adjusted quarterly real GDP series for the period 1987Q1 to 2015Q4. This is the longest time frame during which the quarterly GDP data is available.

For the remaining parameters, the subjective discount rate, β, persistence of TFP process, ρ, and standard deviation of innovations to the TFP, σ_ε, we have followed a joint calibration strategy. Namely, we target the annualized default frequency of 2%, persistence in the GDP of 0.752 and standard deviation of GDP of 0.025. The resulting values for β, ρ and σ_ϵ are 0.94, 0.97, and 0.02, respectively. While discount factor β is smaller than the regular business cycle models, it is still much larger than the existing corresponding studies. For instance, **?** takes β to be 0.88 and generate a debt-to-GDP ratio of 23 percent. Similarly, **?** uses a discount factor of 0.80 which helps featuring a debt-to-GDP ratio of 13 percent and **?** uses 0.86 for β.

[7] See, e.g., **?**, **?** and references therein.

We set δ_b to be 3.3 percent. With that value, the maturity of long-term debt becomes 5 years which is roughly the average bond duration estimated for emerging economies documented by **?**[8]. **?** definition of duration is standard to calculate the long-term debt duration. Duration D is the weighted average maturity of future cash flows. A bond issued at time t makes periodic payments κ for the subsequent periods with a geometrically decay rate δ_b.[9] Observe that equation (30) is 1 for $\delta = 1$. The sovereign spread r_s is defined as the difference between yield i and the risk free rate r. Annualized spread reported in the tables is computed as

$$1 + r_s = \left(\frac{1+i}{1+r}\right)^4.$$

The debt levels obtained from the simulations are equivalent to the present value of future debt obligations which are discounted at the risk-free rate and computed as $\frac{b(1+r)}{\delta+r}$.

5 Quantitative Results

This section presents the baseline results. Table 5 compares the data moments from Colombian data with the one obtained from the model. The model features plausible

[8] **?** reports an average debt duration of 4.8 years with a standard deviation of 1.5 in the year 2000 for emerging economies.

[9] Duration D satisfies,

$$D(\kappa) = \frac{1}{q}\left(\sum_{j=1}^{J} j\frac{\kappa(1-\delta_b)^{j-1}}{(1+i)^j}\right)$$

where i is the periodic yield an investor would earn if the bond is held to maturity without any defaults and it satisfies

$$q = \sum_{j=1}^{\infty} \frac{\kappa(1-\delta)^{j-1}}{(1+i)^j},$$

and the periodic yield i reads as

$$i = \frac{\kappa}{q} - \delta_b$$

with which D becomes

$$D = \frac{1}{q(1-\delta_b)} \lim_{J\to\infty} \sum_{j=1}^{J} j\left(\frac{1-\delta}{1+i}\right)^j \tag{29}$$

$$= \frac{1+i}{i+\delta_b}. \tag{30}$$

moments and matches both the sovereign debt statistics as well as the business cycle moments reasonably well.

The model generates debt-to-GDP ratio of 31 percent on average while matching mean default rate. This result is mainly the byproduct of endogenous decline in investment and thus output during default which increases the cost of defaulting. Part of the reason why our study does a better job in matching debt-to-GDP ratio and the spreads lies in the modeling choice. Inclusion of long-term maturity of sovereign debt and partial recovery after default are important ingredients. The role of long-term debt in quantitative sovereign default models are thoroughly discussed in **?**, **?** and **?**.

As expected and in line with the data, the model generates negative correlation between interest rate spreads and GDP. Quantitative models of sovereign default as well as the business cycles in emerging markets yield countercyclical interest rates. However, in the former one output is an exogenous process and in the latter one the interest rate spreads are exogenous. In contrast, this paper introduces an endogenous income process and endogenously determined interest rate spreads both of which are influenced from each other. We should note that **?** also generate, even though it is weak, negative correlation between interest rate spreads and income.

The model generates positive correlation between investment-to-GDP ratio and debt-to-GDP ratio. However, as our empirical model shows, debt overhang intensifies when the government debt is high. To account for that, we also report the correlation of these two moments for the instances where debt-to-GDP ratio exceeds its long-run mean for two standard deviations. And the model produces the debt overhang we noted in the introduction. This result hinges on the link between private sector and the country risk. Higher levels of government debt induces endogenous country risk which leads a decline in the private sector's investment levels. Figure 3 displays the dynamics of debt-to-GDP and investment-to-GDP ratios. From the figure it is visible that investment-to-GDP is not responsive for low levels of debt-to-GDP. This follows because of the government's implied short-term borrowing rate. For low levels of borrowing amount, the country risk is small and almost zero for low enough debt levels (see Figure 4). Recall that the private sector's investment depends on the imperfectly substitutable foreign investment

Table 5: Long-run Statistical Moments

	(1) Data	(2) Model	(3) Policy
Long-term debt statistics[a] (%)			
Mean debt-to-GDP	35.6	31	41.9
Mean r_s	3.84	2.1	1.52
Default rate	2	2.07	1.5
Debt duration (years)	5	5	5.3
Mean investment-to-GDP	22	26.4	26.3
Inv decline during exclusions(%)	35	25	25
Volatilities (%)			
$\sigma(c)/\sigma(GDP)$	1.1	1.1	1.1
$\sigma(tb/GDP)$	3.1	1.71	1.48
$\sigma(r_s)$	2.1	0.41	0.36
Correlations			
$\rho(c, GDP)$	0.85	0.98	0.99
$\rho(tb/GDP, GDP)$	-0.51	0.72	-0.68
$\rho(r_s, GDP)$	-0.40	-0.83	-0.85
$\rho(inv, GDP)$	0.92	0.97	0.97
$\rho(r_s, inv)$	-0.47	-0.80	-0.77
$\rho(r_s, tb/GDP)$	0.46	0.94	0.92
$\rho(inv/GDP, b/GDP)$	-0.35	0.63	0.63
$\rho(inv/GDP, b/GDP)^a$	-0.9	-0.60	-0.66

Long-term statistics are annualized. Consumption (c), investment (inv), trade balance (tb) are in real terms. GDP excludes government expenditures (inline with the model). All series are de-seasonalized (X13), and HP filtered with $\lambda = 1600$. EMBIs (r_s) are quarterly averaged. Public debt (b) is reported annual, so the correlations that involves b are based on annual data.
$\sigma(h)$ denotes the standard deviation of a variable h. $\rho(h, m)$ denotes the coefficient correlation between variables h and m.
[a] Computes the correlations of instances where b/GDP is (one-/two-) standard deviations larger than the long-run mean.

goods whose cost depends on the country's short-term risk. As the government's total borrowing increase, the default risk and thus the short-term borrowing rate as well as the price of foreign investment goods rise. This in turn reduces the per period equilibrium investment and this explains the intuition of why we have debt overhang for higher levels of sovereign indebtedness. In addition, the model also features a negative correlation between sovereign spreads and private investment which is byproduct of debt overhang.

Our model also features both crowding in and crowding out. For lower default probabilities, the sovereign borrows more and investment is amplified because the borrowing rate is cheaper for firms as well (crowding in effect). At the same time, there is crowding out effect because for higher default probabilities borrowing becomes costlier for the sovereign and thus for the firms as well which curtails the investment.

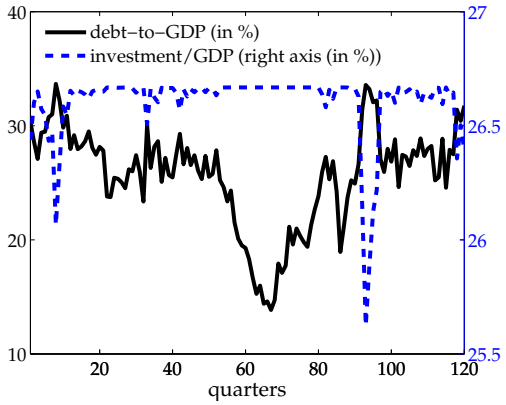


Figure 3: Dynamics of debt-to-GDP and investment-to-GDP in the model without the samples of default.

It is visible in Figure 4 that there is one-to-one mapping between investment-to-GDP ratio and the short-term borrowing rate. As noted in the model, investment goods producers use within period working capital loans and therefore, only the short-term country risk matters.

Crucially, the dynamics of one-period debt and long-term debt are not identical. In contrast to the pricing of short-term debt, the price of long-term debt depends not only on the current borrowing amount but also on the future borrowing and default decisions. This paper also adds to our understanding of quantitative endogenous sovereign default models with long-term debt. Figure 5 plots the spreads for long-term debt and the implied spreads for one-period debt. Observe that long-term debt spreads never fall to zero while one-period debt spreads do. This follows with the feedback mechanism inherently present with long-term debt: future default and borrowing decisions influence the current price dynamics. This is important because in a model where price of the foreign goods depends

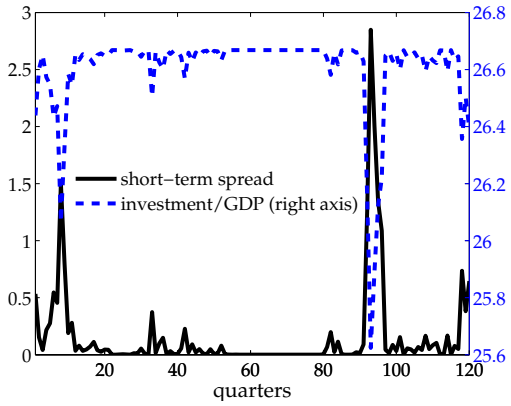


Figure 4: Dynamics of short term country risk spread and investment-to-GDP in the model without the samples of default.

on long-run borrowing rate, the implications would be boosted since spreads under one period debt are significantly smaller than the one with long-term debt.

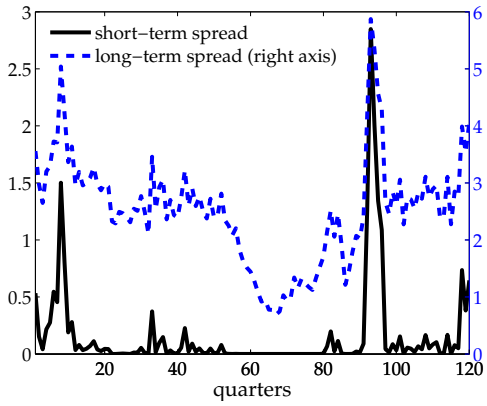


Figure 5: Dynamics of long-term (left-axis) and implied short-term (right-axis) borrowing rates without the samples of default.

Our model also accounts for the sharp fall in GDP, consumption, labor and investment goods during government's exclusion from the credit markets that are consistent with the existing empirical literature on sudden stop events. For the vast majority of the sudden stop literature, these dynamics are generated as an outcome of the exogenous shock; in contrast, in our model sudden stop occurs with an endogenous default following a decline in firm investment. Upon government's default decision, the firms cannot access to the necessary resources for foreign investment goods which is an imperfect substitute for

production. Our model slightly underestimates the decline in investment during financial exclusion. Total investment drops 25 percent in our model whereas this value corresponds to 35 percent in the data.

6 Lax capital regulation

The fact that firms can finance only θ fraction of their imports is always a drag on investment and this amplifies during economic downturns. In this section, we propose a policy that addresses this friction. Our proposed policy has the flavor of a lax capital regulation during an economic downturn. In particular, we assume that sovereign ensures, under full commitment and in a time-consistent manner, that firms finance a larger portion of imports of foreign investment goods during unfavorable macroeconomic conditions. Namely, we set a state-contingent rule for θ: When the TFP is below its long-run trend, the sovereign ensures that θ, the fraction of imports of foreign investment goods that needs to be financed, is higher and is equal to 0.90. The value 0.90 for θ is set mainly for expositional purposes, and ideally, the sovereign could set θ equal to one to improve aggregate welfare further. However, it would not be feasible for a firm to be able to finance all of its projects through loans ($\theta = 1$) in terms of policy proposal and in reality it is more likely that the banks are more reluctant to provide these loans during economic downturns. To this end, we explore the impact of lax capital regulations imposed by the sovereign without modeling the banks and the firm delinquency. The proposed policy, thus, will not study the optimal rate of θ and the goal of this section is to study a normative analysis of this proposed policy.

Here it is worth noting that θ has been part of the physical environment and not a regulation parameter. Yet, it can be embodied into the model with a price-tax or a quantity-tax interpretation. Thus, a sovereign can sustain such a policy. Alternatively, a sovereign can also legislate such contracts for firms which trade with foreign suppliers.

Policy column of Table 5 presents the outcome of this exercise. With the policy in place, the government defaults less in equilibrium and can generate higher debt-to-GDP ratio as well as lower borrowing costs. The mechanism that features these results works

primarily through the cost of defaulting and the cost of foreign investment goods. Notice that the price of foreign investment goods is now higher when government is excluded from the markets during economic downturns which reduces the total investment and makes defaulting costlier. As the severity of default heightened, the government is now going to avoid defaulting more which lowers the government's cost of borrowing. Figure 6 shows the government's equilibrium price schedule for mean TFP shocks. The price schedule with the policy is higher for all levels of debt holdings.

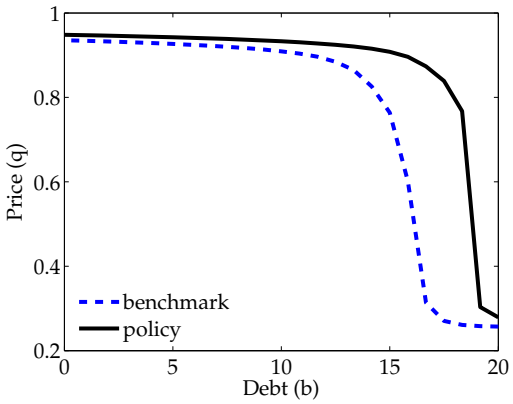


Figure 6: Price function for mean TFP shock.

The notion that sovereign needs to take into account the effect of its borrowing decision on firms' investment to improve aggregate welfare is by and large recognized (or hinted) in the empirical literature and the policy spheres. For instance, we show in Section 2 for Colombia, **?** and **?** for Europe that higher sovereign risk premium is a drag on corporate economic activity. Our proposed policy ensures that sovereign borrowing is perceived less risky for all states of nature and in turn makes firms borrow at more favorable terms. Note also that here we do not study explicitly the role of banks in transmitting fluctuations in sovereign risk to firm investment (e.g. by considering that banks with a higher exposure to riskier government debt are more likely to cut back lending more). In this regard, our policy of lax capital requirements policy can be read as a reduced form way of capturing regulatory practices that ensure easier lending standards during economic downturns, e.g. by allowing for easier collateral standards, modifying risk weights for certain type of investment loans, etc. during turbulent times.

6.1 Welfare gains

In this section, we now investigate the welfare implications of this policy proposal and analyze whether agents prefer living in an economy with the proposed policy. We compute the welfare gains as the constant proportional change in consumption that would leave a household indifferent between staying in an economy with the policy proposal in place and staying in an economy without the proposal. Welfare can be computed in terms of equilibrium value functions evaluated at particular debt-income doubles as:

$$\left[\frac{V_p(b,A)}{V_n(b,A)}\right]^{\frac{1}{1-\gamma}} - 1$$

where $V_p(b,A)$ denotes the economy with policy proposal and $V_n(b,A)$ denotes the economy without the proposal. Thus, a positive welfare gains means that agents prefer living in an economy with the policy in place.

Figure 7 plots the welfare gains as a function of TFP shock and zero debt. We find that switching to an economy where firms are required to have more working capital for TFP shocks lower than its long-run mean is welfare improving. This result mainly hinges on the reduction in cost of borrowing, spreads. Earlier, we noted that with increased cost of defaulting, the government defaults less and thus spreads decline. Notice that the price of foreign investment goods relies on the country risk which declines with the policy. So the government produces more investment.

The figure shows that the gains are larger for lower levels of TFP and this stems from the non-linearity of price schedules. For positive TFP shocks, the price schedule is more flat and thus the gains are lower. To better understand the welfare gains, we will also study the consumption dynamics within our transition exercises.

6.2 Transition dynamics of policy introduction

We now carry out the transition dynamics of this policy modeled as a date-zero unanticipated announcement such that the government is now committed to increasing θ for TFP shocks lower than its long-run mean. The figures presented below undertakes two-

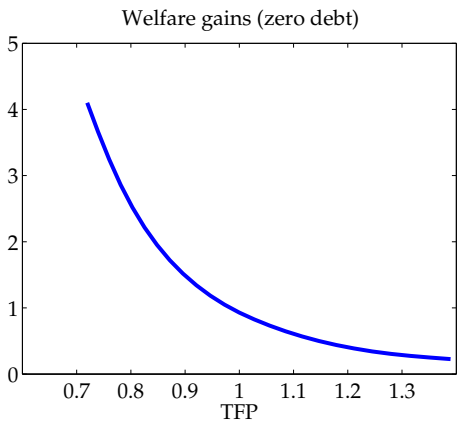


Figure 7: Percentage welfare gains from living in an economy with the policy.

different risk levels as an initial condition: one with mean TFP shock (blue lines) and one with one standard deviation below the mean TFP level (black lines). Solid lines correspond to the impact of policy whereas dashed lines correspond to the the transition dynamics without the policy.

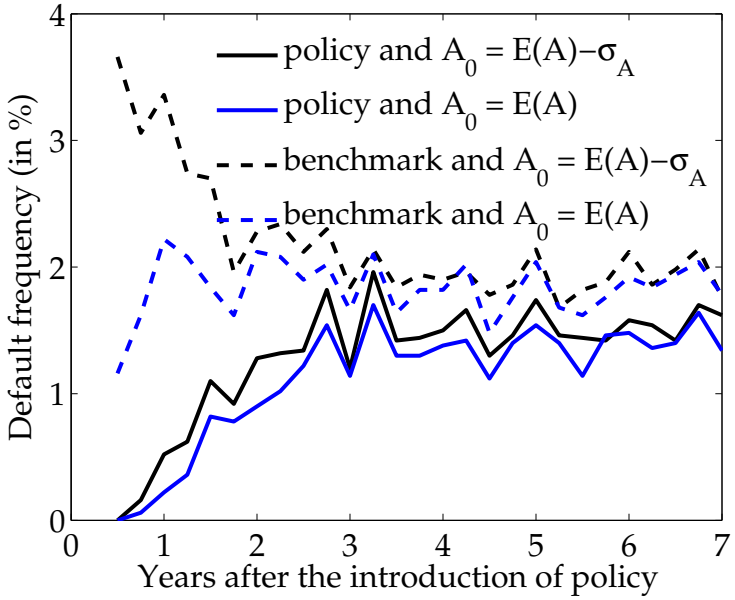


Figure 8: Default after the introduction of policy. The panel present default frequency during the transitions that follow the arrangement of policy. Solid lines represent the impact of policy whereas dashed lines represent the transition dynamics without the policy.

The transition dynamics for default is presented in Figure 8. Quite remarkably, the policy eliminates the default risk almost entirely at the time of policy introduction for both risk levels. As government starts accumulating more debt following the decline in default risks (see left panel of Figure 9), the default frequency rises but its long-run frequency still stays lower than the economy without the policy.

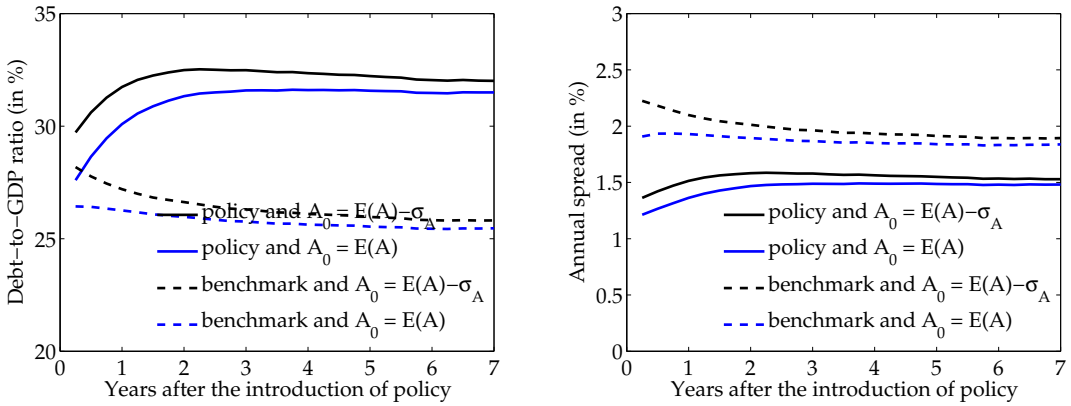


Figure 9: Debt and spread after the introduction of policy. The left and right panels present the debt-to-GDP ratio and spread during the transitions that follow the arrangement of policy for samples without default and financial exclusion. Solid lines represent the impact of policy whereas dashed lines represent the transition dynamics without the policy.

Figure 9 plots both the debt and spread dynamics following the introduction of policy excluding the default episodes. Following a decline in default risks, the government finds raising its current debt holdings more optimal. Importantly, since we model the debt as long-term maturity, the current spread does not depend only on current debt level but also on future borrowing and default decisions. This explains why the spreads do not drop to zero while default risks do at time of policy announcement.

Figure 10 sheds light on the welfare gains we discussed earlier. Welfare gains heavily relies on the consumption gains in the first 10 quarters following the decline in borrowing costs. With eye-balling, the figure also shows that the welfare gains are higher for lower TFP levels (the are between black lines are bigger than the blue ones) and confirms our Figure 7. After 10-11 quarters, consumption in an economy without the policy slightly goes over the consumption in an economy with the policy following a rise in total debt

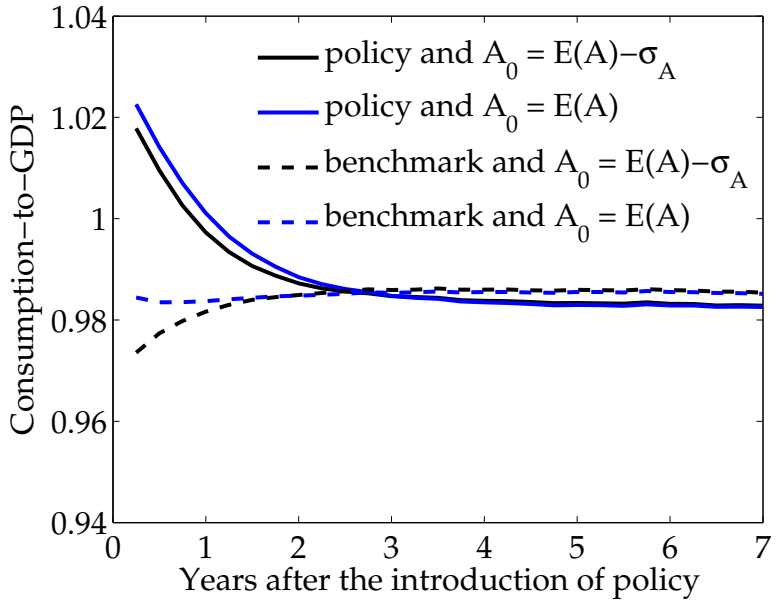


Figure 10: Consumption-to-GDP after the introduction of policy. The panel present the consumption dynamics during the transitions that follow the arrangement of policy for samples without default and financial exclusion. Solid lines represent the impact of policy whereas dashed lines represent the transition dynamics without the policy.

and subsequently the spreads. However, the present discounted value of consumption stays higher in an economy with the policy.

In here, it is worthwhile to note that the samples used in computing Table 5 are not identical with the ones used in plotting the transition dynamics. The samples providing the outcome of Table 5 does not include the periods of default, however, includes the periods leading to default and each sample includes 120 quarters (30 years). Typically, and in line with the data, these periods are characterized by higher debt-to-GDP ratios and higher spreads. However, with the transition exercises, each quarter is averaged from 20000 samples from which we exclude the samples with default and thus any sample that leads to a default is excluded. This explains why the long-run values in the transition figures are slightly below the ones reported in Table 5. If one reports the moments while excluding the periods leading to default for Table 5, then the long-run values for transition figures would be identical.

7 Conclusion

This paper features an equilibrium model of endogenous sovereign default and investment cycles. Using highly granular loan-level data for Colombia, we first empirically document that (i) a rise in sovereign indebtedness increase the cost of credit to private firms, (ii) a higher cost of credit is linked with lower investment and (iii) higher sovereign risk reduces the firm investment with augmented debt overhang. Our empirical regularities are in line with the existing empirical literature and the model presented here provides a joint explanation for these stylized facts.

Our framework provides a financial amplification mechanism such that an increase in sovereign borrowing raises the cost of foreign investment goods which reduces the total investment since domestic and imported foreign investment goods are only imperfectly substitutable. An important ingredient of our mechanism is that to pay for a subset of foreign investment goods, firms need within period working capital financing which relies on country risk that is broadly in line with the observed empirical regularities. We show that with the heightened country risk, debt overhang intensifies.

Our paper also features an endogenous default cost in contrast to the existing literature which depends on an exogenous endowment process with ad hoc default costs during its exclusion. Our results rely on the assumption that both the sovereign and the firms are excluded from international credit markets which inhibits firms' ability to produce investment. Our model accounts for the 71 percent of the decline in investment that is observed empirically (which is also the source endogenous default costs) during market exclusions.

Our framework also provides a new perspective by analyzing a policy which relaxes the lending conditions during economic downturns. With that policy in place, we show that the country risk as well as the spreads drop and the sovereign can now hold more debt. Importantly, debt overhang which is a drag on investment is subdued and the policy generates welfare gains.

Lastly, the model presented here features both crowding in (for lower default risks the sovereign borrows more while firms amplify their investment because of the lower

borrowing costs) and crowding out (during the episodes of high default risks, typically when debt-to-GDP ratio is high, firms' investment decline because of high interest rates) effects.

We acknowledge that further research is needed for the interactions between sovereign default and private sector credit and the efficiency loss generated by these forces. For instance, introducing risk averse lenders and global liquidity conditions which has been an important factor driving the country risks for the recent financial crisis can be a promising future research agenda.

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A Appendix - Computation algorithm

The computational algorithm works through iterating value functions (V^R, V^D) and price function q until a convergence is obtained [10]. As explained in the text, we first set the terminal values of the finite horizon economy and iterate the economy backwards until we obtain convergence.

For a given short-term borrowing rate r_t^* and TFP shock A_t, one can compute the optimal factor allocations (I_t^f, I_t^d, L) with w_t satisfying the shadow prices given by the Lagrange multipliers for labor. To obtain the allocations of the private sector, 100 grid points set for both short-term borrowing rate r_t^* and TFP shock A_t. One can solve this problem separately outside of the value-function iteration procedure and use the equilibrium outcomes of this procedure within the iteration routine using approximation schemes.

The numerical algorithm that is engineered to solve the problem can be explained in steps as follows:

1. Solve the equilibrium factor allocations of the private sector for a given TFP shock A and one-period loan rate r^*.

2. Set the initial guesses for the terminal values of value and pricing functions (V^R, V^D and q) in the finite-horizon economy as follows:

   - $V^R(b, A) = u(Af(K, L) - I^f P^f - \kappa b, L)$,
   - $V^D(b, A) = u(Af(K, L) - I^f P^f_{aut}, L)$,
   - and $q = 0$.

3. Solve for the optimization problem defined in equation (26) and equation (27) for each grid. We take 40 equally-spaced grid points for b' to obtain the optimal borrowing policy and find the candidate maximixing the optimization problem. We feed this

[10] Convergence criteria is 10^-5.

candidate grid point into one-dimensional DUVMIF routine of the IMSL library for Fortran as the initial guess of our optimal b'[11].

4. Iterate the procedure defined in step 3 over equations (26) to (28) until the difference in two subsequent iterations are less than the convergence criterion.

After obtaining the converged equilibrium functions, decision rules and factor allocations; I simulate the economy. For that, I fix the number of samples to be 300 and each sample has 1501 periods from which initial 1000 periods are trimmed to eliminate the influence of any arbitrarily chosen initial states. The initial states of A_0 and b_0 are set to be mean A and zero, respectively. It is assumed that the government does not default in the first period. We initially draw sequences of TFP shocks and re-entry probability ψ using random number generator.

The moments in the paper are calculated using the averages of 250 simulates sample paths of 100 quarters (25 years) without any default and each sample path starts at least five years after a default incident. The business cycle moments are detrended with HP-filter using a smoothing parameter of 1600.

B Appendix - Data

In this section, Table 6 reports the summary statistics for the variables used in our empirical analysis.

[11] Another alternative would be to increase the grid points for b', however, this methodology is faster than using discrete state space method as outlined in **?**.

Table 6: Descriptive Statistics

variable	mean	p50	sd	p25	p75	> N
Borrowing Firms						
Investment/Capital	0.38	0.01	1.28	-0.10	0.31	36052.
EBITDA / ST-Debt	0.01	0.00	1.34	0.00	0.00	39953
LT-Debt / Debt	0.24	0.15	0.26	0.00	0.40	42277
Equity / Assets	0.34	0.33	0.24	0.19	0.49	42279
Sales Growth	0.16	0.07	0.50	-0.09	0.27	35712
Foreign Borrowing Rate	4.78	4.62	2.08	3.20	6.17	42279
Net Sales / LT-Assets	1.26	1.25	1.88	0.46	2.11	41691
log(Real Assets)	12.02	11.90	1.40	11.07	12.85	42279
log(Age)	2.81	2.89	0.89	2.40	3.22	40966
log(Employment)	4.58	4.64	1.42	3.74	5.52	41827
Exports / Sales	0.33	0.21	0.34	0.01	0.59	41770
Int. Coverage	0.00	0.00	0.10	0.00	0.00	39707
Fixed Tangible Assets / Assets	0.28	0.24	0.22	0.10	0.41	42278
All firms						
Investment/Capital	0.47	-0.01	1.67	-0.17	0.37	202376
EBITDA / ST-Debt	0.05	0.00	6.17	0.00	0.00	222655
LT-Debt / Debt	0.17	0.01	0.26	0.00	0.26	307409
Equity / Assets	0.32	0.30	0.34	0.13	0.52	308037
Sales Growth	0.19	0.07	0.62	-0.15	0.33	198377

Source: Firm Level FX Credit