Foreign Exchange Interventions and Intermediary Constraints

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This version: 17th September 2024

Abstract

We study the impact of foreign exchange interventions and their efficacy during periods of tight credit constraints. Expanding on the Gabaix and Maggiori (2015) model, we predict that long-lived spot interventions have larger effects on exchange rates than short-lived swaps, unanticipated interventions are more impactful, and tighter credit constraints amplify all effects. Using high-frequency data on Brazilian Central Bank interventions from 1999 to 2023, we find that unanticipated spot sales of USD reserves lead to significant domestic currency appreciation and reduced covered interest rate parity deviations. Spot interventions outperform swaps, especially when global intermediaries are constrained, and enhance market efficiency by lowering USD borrowing costs.

JEL classification: E44; E58; F31; G14
Keywords: Exchange Rate; Central Bank; Interventions; Yield Curve; Asset Pricing

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We thank discussants Ozer Karagedikli and Małgorzata Walerych and seminar participants at the Brazilian Central Bank, the CEPR Workshop on Macroeconomic Policy in Emerging Markets, the 2024 RCEA International Conference, the 24th Brazilian Finance Annual Meeting, the 2024 CEBRA Annual Meeting, and the 2024 Warsaw Macro-Finance Conference for helpful comments. Gustavo Sung provided excellent research assistance with the dataset. We thank Patricia Dias who helped us understand the Brazilian Central Bank's institutional framework of FX management.

1 Introduction

An important and long-standing question in international finance is how foreign exchange interventions (FXI) influence currency markets. While it is widely accepted that non-sterilized interventions can impact exchange rates by altering the money supply, the effectiveness of sterilized interventions and the mechanisms through which they operate remain subjects of ongoing debate (see Sarno and Taylor, 2001, for a review that remains relevant today).

This debate persists largely because identifying the causal effects of FXI on financial and macroeconomic variables poses two significant empirical challenges. Specifically, clean identification requires interventions that are unanticipated by market participants and not driven by the central bank's endogenous responses to current market conditions (see Fratzscher et al., 2019, for a discussion).

While we do not fully resolve both of these empirical challenges, we make significant progress toward addressing the first by employing a recently released high-frequency dataset of interventions by the Brazilian Central Bank (BCB) from 1999–2023. This dataset enables us to distinguish between interventions that were announced ahead of time and those that were not, thereby surprising market participants. By focusing on these unanticipated events, which can be viewed as news relative to the information set of market participants, we can trace the effects of interventions on market variables, following a methodology similar to that used in event studies on monetary policy surprises (see Gürkaynak et al., 2021, for a discussion).[1]

Our main contribution is to show that the impact of FXI depends on the degree to which global FX dealers are constrained in supplying USD liquidity. When intermediaries are constrained, FXI has a greater capacity to affect spot and forward rates, as the inelastic supply of cross-border capital makes relative quantity changes more impactful.

[1]From an econometric point of view, the use of market surprises in event studies is widely accepted, as these surprises are considered news relative to the information sets of market participants. However, identifying structural policy shocks and their causal effects requires that the instrument capture innovations that are both news to market participants and orthogonal to the economic state. This assumption may be violated if the information sets of policymakers and market agents diverge. We acknowledge this caveat in our methodology and leave further exploration of this issue to future studies.

We identify this mechanism as the *dollar intermediation* channel, which can be seen as a version of the standard *portfolio balance channel* discussed in the literature. The channel arises from the imperfect sustainability of domestic currency and USD, and from the existence of financial frictions limiting access to dollar liquidity.[2]

To shed light on the dollar intermediation channel, we study the interaction between currency and interest rate markets through the covered interest rate parity (CIP) condition. CIP, an important no-arbitrage condition in international finance, can indicate a relative scarcity of dollar liquidity in cross-border financial markets when violated. FXI can alleviate dollar scarcity and reduce friction in dollar intermediation in FX markets. We exploit the history of BCB interventions to test whether unanticipated sales of USD reserves by the BCB lead to systematic domestic currency appreciation and a decline in CIP violations.

To motivate our empirical framework, we extend the Basic Gamma Model of Gabaix and Maggiori (2015). In our extension, we distinguish between spot and swap interventions and consider the possibility of interventions being either partially or fully anticipated. The model involves two countries, Home and Foreign, each with a representative household, domestic tradable and non-tradable goods, and a domestic bond. Households consume domestic and international goods but only hold domestic bonds due to the lack of access to international bond markets.

A foreign exchange Financier intermediates bond trade, constructing a zero-cost portfolio of Home and Foreign bonds to maximize expected profit subject to a credit constraint. The Central Bank conducts short-lived swap interventions and long-lived spot interventions, intervening when trade shocks exceed set thresholds. These thresholds are communicated as distributions to manage anticipation of interventions.

The model yields three key empirical predictions: long-lived spot interventions have larger exchange rate effects than short-lived swaps; unanticipated interventions have larger effects than anticipated ones; and tighter credit constraints amplify the effects of all interventions.

[2]The BCB does not have any active foreign-currency liquidity swap line with the Fed and hence has to commit its reserves to supply dollars to domestic financial firms.

To test model predictions, we exploit a large historical database on BCB interventions. The database includes a high-frequency timestamp of each intervention, the intervention type (spot or swap), and the sign (buy or sell of USD) and size of the intervention. We classify interventions as either anticipated or unanticipated depending on whether the announcement of the intervention occurs on the same day as the auction that implements the intervention.

We combine our database on interventions with tick level data on spot and futures prices at the 5-minute level from Thomson Reuters Tick history. Using this data, we measure the effects of FX buy and sell interventions on prices, interest rates and CIP violations. Our analysis uses the local projections method in Jordà (2005), a procedure that controls for feedback effects in prices and the intervention amount.

First, we test for the impact of unanticipated buy and sell interventions. The domestic currency appreciates by approximately 150 basis points per USD Billion for a sell intervention. In contrast, we observe smaller transitory effects (10 basis points) on the spot exchange rate from FX swap interventions. We find insignificant impact for buy interventions, suggesting the dollar intermediation channel is important in explaining the asymmetric effect between USD buy and sell interventions.

Second, we examine the effects on cross-border funding by measuring the difference in borrowing costs between BRL and USD. In practice, borrowing dollars synthetically by converting BRL to USD in the forward market is more expensive. This inefficiency is reflected in CIP violations, which measure the difference in BRL and USD interest rates after hedging exchange rate risk with a forward contract. CIP violations reflect several factors, including the scarcity of dollar liquidity, given dollars are the reserve currency and there are global supply constraints. We test whether FXI impacts CIP violations and find that unanticipated sell interventions lead to a decline in the magnitude of these violations. The decline improves cross-border market efficiency by reducing the relative cost of borrowing USD through forward markets, aligning with our model's predictions that FXI alleviates dollar intermediation constraints.

Third, we test whether the impact of FXI on spot prices and CIP violations is more

pronounced during periods of tighter intermediary constraints. When dollar liquidity is limited, the central bank conducts USD sale operations, anticipating stronger FXI price effects due to the inelastic dollar supply. We use the dealer capital ratio from He et al. (2017) as a proxy for intermediary constraints, which measures the capital-to-assets ratio for key global FX dealers. Our findings show that spot sales have significant effects primarily during periods of tighter intermediary constraints. Specifically, while swap FXI generally does not significantly impact the spot rate, it does have significant effects during periods of tight intermediary constraints.

Finally, we explore an alternative channel through which FXI can impact the market: signaling future interest rates. Using high-frequency and daily interest rate data, we analyze the immediate and long-term effects of unanticipated FXI on interest rates. Our findings suggest that unanticipated spot sales of USD result in an economically insignificant short-term increase in interest rates based on intra-day data. Longer-term analysis provides weak evidence of a systematic effect on interest rates following both spot and swap FXI. Overall, our evidence supports the dollar intermediation channel as the more relevant theory explaining our findings.

Related Literature. This paper contributes to a literature on understanding the channels through which FXI affects exchange rates (e.g. Sarno and Taylor, 2001; Neely, 2005; Vitale, 2006; Menkhoff, 2010). These studies discuss primarily two channels through which sterilized FXI may operate: the portfolio balance and signaling channels. In a standard sterilized intervention, the Central Bank sells foreign currency while simultaneously purchasing domestic bonds. In the portfolio balance channel, if foreign currency is perceived as riskier than domestic bonds, private agents will absorb more foreign currency in their portfolios only if its relative price decreases. In the signalling channel, selling foreign currency reserves signals an anticipated increase in domestic interest rates. In our paper, the *dollar intermediation* channel, a variation of the portfolio balance channel, emphasizes how financial constraints affect intermediaries' ability to supply USD liquidity. We find strong empirical support for the dollar intermediation channel of FXI in Brazil, but weak support for the signaling channel.

Our paper relates to prior studies on the impact of FXI on exchange rates and other markets (e.g. Kearns and Rigobon, 2005; Menkhoff et al., 2021; Fratzscher et al., 2019, 2020; Naef, 2023; Dominguez et al., 2013; Arango-Lozano et al., 2020), as well as studies on BCB interventions (Nakashima, 2012; Kohlscheen and Andrade, 2013; Janot and Macedo, 2016; Santos, 2021). For example, Kohlscheen and Andrade (2013) found an impact of 29 basis points per billion using intra-day spot and futures prices, while Nedeljkovic and Saborowski (2019) reported up to 100 basis points per billion USD with a GMM approach. Other studies such as Barroso (2019) estimated impacts between 51 to 118 basis points using IV analysis, and Santos (2021) found a peak effect of 30 basis points using intra-day data on futures. Turning to swap interventions, da Costa Filho (2021) identified a smaller effect of 3-5 basis points per billion USD.

We contribute to the literature on FXI as a macro-prudential tool (e.g. Ferreira et al., 2019; Fratzscher et al., 2019, 2020; Levy-Yeyati and Gómez, 2022; Naef, 2023; Rodnyansky et al., 2023). Fratzscher et al. (2019) show that central banks stabilize exchange rates by counteracting private market trades. Rodnyansky et al. (2023) find that FXI mitigates spillovers from foreign monetary policy, particularly for firms with high foreign currency debt.

We also engage with theoretical work on FXI in models with financial frictions (see Gabaix and Maggiori, 2015; Fanelli and Straub, 2021; Candian et al., 2024; Agénor et al., 2020; Agénor and da Silva, 2023; Pelin, 2024; Beltran and He, 2023). Beltran and He (2023) use the Gabaix and Maggiori (2015) model to estimate FXI needed in EMEs during currency demand shocks. On the effectiveness of FXI, Candian et al. (2024) develop a framework in which central banks use opaque FX interventions to manage expectations and prevent overreactions to information about economic fundamentals. Similarly, Yago (2024) propose a model where FX interventions are optimally combined with monetary policy to mitigate the inflation-output trade-off. Lastly, Agénor et al. (2020); Agénor and da Silva (2023) highlight real macroeconomic effects of sterilized interventions through a bank portfolio channel.

Relative to these empirical and theoretical studies, our contribution is twofold: we

combine an intra-day analysis of BCB auctions over a longer sample and employ a high-frequency identification methodology that leverages the precise timing of interventions. Additionally, we differentiate between spot and swap instruments, distinguish between anticipated and unanticipated interventions, and test the role of intermediation constraints in determining impact, thereby supporting models that incorporate financial frictions to explain the effects of FXI.

Finally, we contribute to the literature on emerging market CIP violations (Du and Schreger, 2016; Hartley, 2020; Bahaj and Reis, 2020; Cerutti and Zhou, 2023; Hertrich and Nathan, 2023), which examines the determinants of CIP violations, sovereign risk, and the role of Central Bank policies like swap lines. We show how FXI can improve FX market efficiency using CIP violations as a proxy for dollar scarcity. Our paper complements Hertrich and Nathan (2023), who find that spot FXI by Israel's central bank reduces CIP violations for the Shekel/USD pair. We contribute to this discussion and show similar effects on the magnitude of CIP violations using Brazil as a case study.

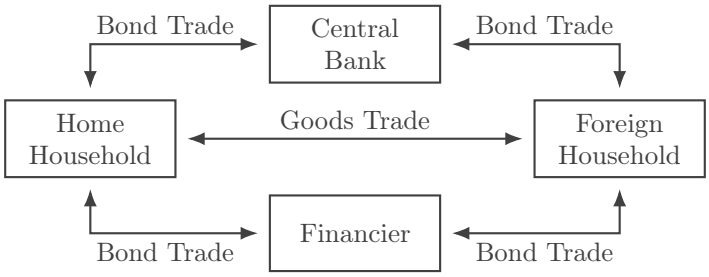
The remainder of the paper is structured as follows. In Section 2, we introduce a theoretical model with testable predictions on the effects of FXI on spot rates and CIP violations. In Section 3, we introduce the institutional setting of FXI conducted by the BCB and describe our data. In Section 4, we present our findings on the effects of FXI on the exchange rate, CIP violation, and test for the signaling channel. In Section 5, we conclude.

2 Theoretical Model

We extend the Basic Gamma Model of Gabaix and Maggiori (2015) and derive predictions that motivate our empirical investigation of Central Bank interventions in foreign exchange markets. In our extension, we introduce a rudimentary distinction between spot and swap interventions and we allow interventions to be partially or fully anticipated.

There are two countries, Home and Foreign, each with a representative Household,

Figure 1: Structure of the Gamma-Eta Model.



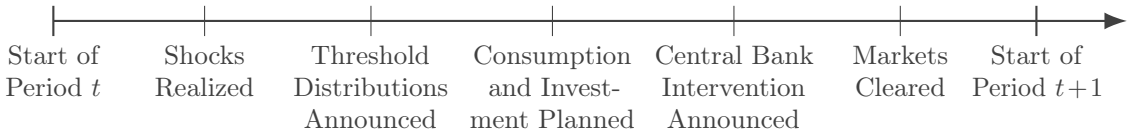
Notes. The figure depicts a simplified structure for the Gamma-Eta Model. Rectangles depict the four decision makers: the Home Household, the Foreign Household, the Financier, and the Central Bank. The decision makers transact in markets for Home and Foreign goods and Home and Foreign bonds. Arrows in the figure depict external trade flows. The figure omits internal trade flows and profit or loss transfers. Figure A.1 in Appendix A.1 provides a more complete picture of the model, showing individual markets and distinguishing between real flows and currency flows.

a domestic tradable and non-tradable good, and a domestic bond. Households receive endowments of domestic tradable and non-tradable goods and consume baskets of domestic and international goods. Households have access to international goods markets but no access to international bond markets, and therefore hold only domestic bonds.

A representative FX dealer intermediates bond trade, known as a Financier in Gabaix and Maggiori (2015). The Financier begins each period with zero assets and constructs a zero-cost portfolio of Home and Foreign bonds to maximize expected single-period profit. By trading in both bond markets, the Financier absorbs imbalances in demand for Home and Foreign bonds. However, creditors impose constraints on the Financier that limit its ability to intermediate optimally.

A Central Bank conducts two types of sterilized FXI: short-lived swap interventions that are unwound after one period and long-lived spot interventions that are maintained for two periods. The Central Bank intervenes when trade shocks exceed exogenous thresholds that it sets. The Central Bank withholds its exact threshold values from the Household and Financier and instead communicates distributions of possible values. The Central Bank sets the parameters of the distributions (through a unmodeled communication policy) to control the anticipation of its interventions. Households and the Financier form *a prior* plans based on the probability of an intervention, before the Central Bank announces its intervention decision each period.

Figure 2: Phases of Periods Zero and One in the Gamma-Eta Model.



Notes. Periods zero and one are divided into five phases. In the first phase, preference shocks are realized and observed by all. In the second phase, the Central Bank announces distributions for its intervention thresholds. If trade shocks exceed the thresholds, the Central Bank will carry out a foreign exchange intervention. In the third phase, the Home and Foreign Households and the Financier form consumption and investment plans. In the fourth phase, the Central Bank reveals its foreign exchange intervention. In the fifth phase, prices are determined and markets clear.

The model makes three central empirical predictions: first, that long-lived spot interventions have larger exchange rate effects than short-lived swap interventions; second, that unanticipated interventions have larger exchange rate effects than anticipated interventions; and third, that all interventions have larger exchange rate effects when credit constraints are tighter. Figure 1 provides an overview of the model structure, and Figure 2 illustrates the timing assumptions we make in the model. We present the model and state empirical predictions below and provide additional derivations in Appendix A.

The Home Household. The Home Household solves a three-period intertemporal utility maximization problem,

$$\max_{\{C_{NTs},C_{Hs},C_{Fs}\}_{s=t}^{2}} \sum_{s=t}^{2} \beta^s \, \mathrm{E}_t^{(\cdot)}[\varphi_s \ln C_s] \,, \tag{1}$$

where $t \in \{0,1,2\}$, $\mathrm{E}_t^{(\cdot)}[\cdot]$ denotes a conditional expectation operator that includes period-t information on realized preference shocks but excludes information on the exact intervention thresholds the Central Bank sets in period t, β denotes the subjective discount factor, $\varphi_t \equiv \chi_t + \alpha_t + \iota_t$ denotes a sum of stochastic preference shocks, and where

$$C_t \equiv \left[(C_{NTt})^{\chi_t}(C_{Ht})^{\alpha_t}(C_{Ft})^{\iota_t}\right]^{\frac{1}{\varphi_t}} \tag{2}$$

denotes the Home Household consumption basket, a sub-utility function composed of Home non-tradable goods C_{NTt}, Home tradable goods C_{Ht}, and Foreign tradable goods

C_{Ft}. The intertemporal maximization is subject to period budget constraints of the form

$$Q_{Ht} + p_{Ht}C_{Ht} + p_{Ft}C_{Ft} + C_{NTt} = Y_{NTt} + p_{Ht}Y_{Ht} + R_{Ht}Q_{Ht-1}\,, \tag{3}$$

where Q_{Ht} denotes the real value of Home holdings of the Home bond, with $Q_{H2} = 0$, where p_{Ht} and p_{Ft} denote relative prices of the Home and Foreign tradable goods, and where Y_{NTt} and Y_{Ht} denote stochastic endowments of the Home non-tradable and tradable goods, respectively. The Home non-tradable good is the numéraire for the Home economy.

The Home Household's intertemporal utility maximization problem yields the following optimality conditions,

$$\mathrm{E}_t^{(-)}[\lambda_s] = \mathrm{E}_t^{(-)}\left[\chi_s/C_{NTs}\right] \quad \text{and} \quad \mathrm{E}_t^{(-)}[\lambda_s] = \beta\,\mathrm{E}_t^{(-)}[\lambda_{s+1}R_{Hs+1}]\,, \tag{4}$$

where λ_s denotes the Lagrange multiplier on the Home Household's period-t budget constraint and where R_{Hs+1} denotes the gross real return on the Home bond, with $t \leq s < 2$.

The Home Household also solves an intratemporal utility maximization problem, allocating expenditure across domestic and international goods. Specifically, the Home Household maximizes the logarithm of its sub-utility function in (2),

$$\max_{C_{NTt},C_{Ht},C_{Ft}} \chi_t \ln C_{NTt} + \alpha_t \ln C_{Ht} + \iota_t \ln C_{Ft}\,, \tag{5}$$

subject to a consumption expenditure constraint,

$$p_tC_t = C_{NTt} + p_{Ht}C_{Ht} + p_{Ft}C_{Ft}\,, \tag{6}$$

where p_t is the Home price index in terms of the Home non-tradable numéraire, and where the Home Household takes total consumption expenditure p_tC_t as fixed in the intratemporal problem. The Home Household's intratemporal problem yields the following

optimality conditions for consumption expenditure on Home and Foreign tradable goods,

$$p_{Ft}C_{Ft} = (\chi_t / C_{NTt})\iota_t \quad \text{and} \quad p_{Ht}C_{Ht} = (\chi_t / C_{NTt})\alpha_t \,. \tag{7}$$

We maintain several simplifying assumptions that Gabaix and Maggiori (2015) make in their Basic Gamma Model: namely, that the Home non-tradable endowment adjust proportionally to fluctuations in the Home preference for non-tradable goods, $Y_{NTt} = \chi_t$, and that the Home Household does not discount future utility, $\beta = 1$. These assumptions simplify the model and focus our investigation on understanding the impact of Central Bank interventions on the real exchange rate.

The maximization problems of the Foreign Household are analogous to those of the Home Household, and we maintain a similar set of simplifying assumptions for the Foreign Household. We present details of the Home Household's maximization problems and our specification of the Foreign Household in Appendices A.2 and A.3.

The Financier. The Financier intermediates bond trade and absorbs cross-country imbalances in Home and Foreign Household demand for bonds. While Households borrow and lend only in their respective domestic bonds, they trade in international goods markets where cross-country imbalances can arise. The Financier maximizes single-period firm value,

$$V_t = \beta \, \mathrm{E}_t^{(\cdot)} \left[R_{Ht+1} Q_{Ht}^f + \frac{e_{t+1}}{e_t} R_{Ft+1}^* Q_{Ft}^f \right], \tag{8}$$

where V_t denotes the Financier's value. The Financier's holdings of Home and Foreign bonds are denoted Q_{Ht}^f and Q_{Ft}^f, respectively, and the real return on the Foreign bond is denoted R_{Ft+1}^*. The real exchange rate is denoted e_t and defined in units of the Home numéraire per unit of the Foreign numéraire. The Financier transfers any gains or losses to the Foreign Household each period.

The Financier has limited capacity to bear risk. This limitation derives from a

constraint that creditors impose on the Financier to prevent it from diverting funds. The constraint aligns incentives such that the value to the Financier of ceasing to intermediate and diverting funds never exceeds the value of continuing to intermediate bond trade. To motivate the functional form of the constraint, Gabaix and Maggiori (2015) suggest that the Financier can more easily divert funds when its portfolio is larger. The constraint is given by

$$\frac{V_t}{\mathrm{E}_t^{(\cdot)}[e_t]} \geq \underbrace{\left|\frac{Q_{Ht}^f}{\mathrm{E}_t^{(\cdot)}[e_t]}\right|}_{\substack{\text{Creditor} \\ \text{Claims}}} \times \Gamma \underbrace{\left|\frac{Q_{Ht}^f}{\mathrm{E}_t^{(\cdot)}[e_t]}\right|}_{\substack{\text{Divertible} \\ \text{Fraction}}}, \tag{9}$$

where the parameter $\Gamma > 0$ governs the Financier's ability to divert funds, and therefore governs the risk-bearing capacity of the Financier. When Γ approaches zero, the Financier's risk-bearing capacity approaches infinity. When Γ approaches infinity, the Financier's risk-bearing capacity approaches zero and the economy approaches financial autarky.[3]

We assume the Financier begins each period with zero assets, so its balance sheet constraint is given by

$$Q_{Ht}^f + Q_{Ft}^f = 0\,, \tag{10}$$

and any bond position the Financier takes must cost nothing to construct.[4]

The Financier's profit maximization problem yields the following optimality condition, which pins down the amount of bond trade the Financier chooses to intermediate,

$$\frac{Q_{Ht}^f}{\mathrm{E}_0^{(\cdot)}[e_t]} = \frac{1}{\Gamma}\,\mathrm{E}_0^{(\cdot)}\left[R_{Ht+1} - \frac{e_{t+1}}{e_t} R_{Ft+1}^*\right]. \tag{11}$$

The expression shows how higher Γ corresponds to lower risk-bearing capacity and less intermediation.

[3] Gabaix and Maggiori allow Γ to depend positively on the variance of the real exchange rate. We find limited empirical support for a positive relationship between financial constraints on intermediaries and the variance of the real exchange rate outside of crisis periods. See the online Appendix B.2 for more details. For this reason, we treat Γ as an exogenous parameter in our model.

[4] We assume the simple microfoundation in Gabaix and Maggiori (2015) to be operating in the background. In their setup, a measure of individual households populates each economy, and new households are randomly selected to serve as financiers each period. Households enter this occupation with zero assets, and maximize their single-period profits. Our representative agent model is isomorphic to the micro-founded model.

Central Bank. The Central Bank conducts sterilized foreign exchange interventions by constructing zero-cost portfolios of Home and Foreign bonds. The Central Bank has no currency liabilities and transfers any gains or losses from its interventions to the Foreign Household each period. Accordingly, the Central Bank's balance sheet satisfies the following condition,

$$Q_{Ht}^{cb} + Q_{Ft}^{cb} = 0\,, \tag{12}$$

where Q_{Ht}^{cb} and Q_{Ft}^{cb} denote the real values of the Central Bank's holdings of Home and Foreign bonds.

The Central Bank intervenes when trade shocks rise above or fall below intervention thresholds that the Central Bank sets exogenously. In particular, the Central Bank's interventions take the form of bond purchases or sales defined by the following equation,

$$Q_{Ht}^{cb} = q\big[H(\Delta\iota_t - \bar{\iota}_t) - H(\underline{\iota}_t - \Delta\iota_t)\big] = q\Delta H_t\,, \qquad \underline{\iota}_t \leq \bar{\iota}_t\,, \tag{13}$$

where $q > 0$ governs the scale of the intervention, $H(\cdot)$ denotes a Heaviside function defined to equal one when its argument is strictly positive and zero otherwise, $\Delta\iota_t \equiv \iota_t - \iota_t^*$ defines a trade shock as the cross-country difference between Home and Foreign preferences for imports, and $\bar{\iota}_t$ and $\underline{\iota}_t$ denote the upper and lower intervention thresholds that the Central Bank sets exogenously each period. The Heaviside functions govern the sign of the Central Bank's interventions, and to lighten notation we define their difference as $\Delta H_t \equiv H(\Delta\iota_t - \bar{\iota}_t) - H(\underline{\iota}_t - \Delta\iota_t) \in \{-1, 0, 1\}$. Because the intervention thresholds are chosen by the Central Bank, we treat ΔH_t as an exogenous policy variable.

Our model allows for two stylized intervention types: swap and spot interventions. In a swap intervention, the Central Bank takes a bond position in period zero and unwinds the position in period one. The payoff of a swap position is equal to the expected currency return on holding a long position in the home currency bond. In a spot intervention, the Central Bank takes a bond position in period zero and maintains the position in period one. These stylized interventions capture two features of real-world foreign exchange interventions: first, because they operate through bond markets and cost nothing to

construct, they are sterilized; second, because the swap intervention is unwound after one period while the spot intervention persists, the spot intervention is longer-lived.

Both interventions can take either sign. That is, the Central Bank responds to sufficiently large positive trade shocks by buying Home bonds and selling Foreign bonds, or to sufficiently large negative trade shocks by selling Home bonds and buying Foreign bonds. In the former case Q_{Ht}^{cb} is positive, in the latter case Q_{Ht}^{cb} is negative. The balance sheet constraint then implies the opposite sign for Q_{Ft}^{cb}. The Central Bank does not intervene when trade shocks remain within its intervention thresholds.

To gain intuition for the cause and effect of a Central Bank intervention, consider the currency flows leading to and resulting from an intervention.[5] A relative rise in the Home preference for Foreign goods (a positive trade shock) induces the Home Household to import more. The Home Household exchanges Home currency for Foreign currency and Home currency weakens. The Central Bank responds by buying Home bonds and selling Foreign bonds. The Central Bank exchanges Foreign currency for Home currency and Home currency strengthens. Thus, the Central Bank buys Home bonds to strengthen Home currency in response to a positive trade shock. The parallel construction also holds for a negative trade shock.

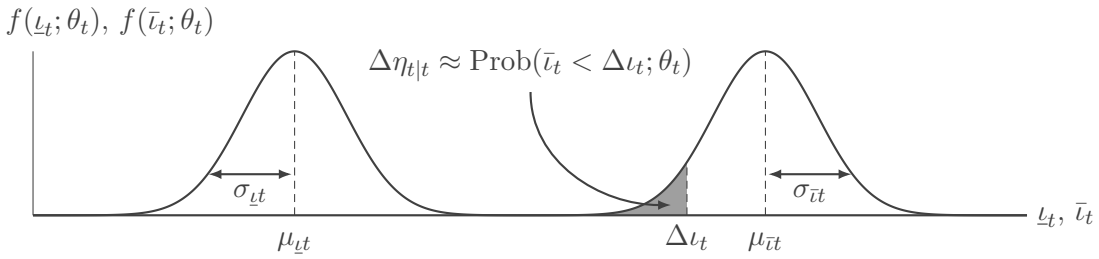
The Household and the Financier have imperfect information about the Central Bank's intervention thresholds. In period zero, the Central Bank communicates distributions with parameters θ_0 for its thresholds, $F(\bar{\iota}_t;\theta_0)$ and $F(\underline{\iota}_t;\theta_0)$ for $t \in \{0,1\}$, but not the exact values of the thresholds. In period one, the Central Bank can update the distributions for its period-one thresholds, $F(\bar{\iota}_1;\theta_1)$ and $F(\underline{\iota}_1;\theta_1)$. Withholding information about its intervention thresholds allows the Central Bank to conduct unanticipated interventions with larger exchange rate effects, as we later show.[6]

The parameters θ_0 and θ_1 govern the ability of the Household and the Financier to

[5]When we refer to currency flows, we abuse the word currency to mean a claim to the numéraire of the economy, mirroring the language in Gabaix and Maggiori (2015), as described in their footnote 5.

[6]The Central Bank can intervene a maximum of two times, so we impose no requirement that the Central Bank's actions be consistent with its communications and we abstract from credibility considerations. However, the model's empirical predictions are most sensible when the Central Bank's communications are reasonably consistent with its actions, e.g. when it avoids communicating that it will buy Home bonds before actually intervening to sell them.

Figure 3: Distributions of Intervention Thresholds.



Notes. The figure shows normal density functions $f(\underline{\iota}_t;\theta_t)$, $f(\bar{\iota}_t;\theta_t)$ for the Central Bank's intervention thresholds $\underline{\iota}_t$ and $\bar{\iota}_t$, as perceived by the Household and Financier in period t. The distributions are characterized by means and standard deviations, $\theta_t = (\,\mu_{\underline{\iota}t},\, \mu_{\bar{\iota}t},\, \sigma_{\underline{\iota}t},\, \sigma_{\bar{\iota}t},\, \ldots\,)$, which the Central Bank communicates to the Household and Financier. The shaded region shows the probability, as perceived by the Household and Financier, that the Central Bank will intervene positively by purchasing Home bonds and selling Foreign bonds to strengthen Home currency. The value $\Delta\eta_{t|t}$ approximately equals the shaded probability in the figure, because the probability of a negative intervention is approximately zero. That is, $\Delta\eta_{t|t} \approx \text{Prob}(\bar{\iota}_t < \Delta\iota_t;\theta_t) - 0$.

anticipate interventions when forming consumption and investment plans. In period t, the expected intervention in period $s \geq t$ is

$$\mathrm{E}_t^{(\sim)}\left[Q_{Hs}^{cb}\right] = q\,\mathrm{E}_t^{(\sim)}[\Delta H_s] = q\Delta\eta_{s|t}\,, \tag{14}$$

where we define $\Delta\eta_{s|t} \equiv \text{Prob}(\bar{\iota}_s < \Delta\iota_s;\theta_t) - \text{Prob}(\Delta\iota_s < \underline{\iota}_s;\theta_t) \in [-1,1]$, and treat $\Delta\eta_{s|t}$ as an exogenous policy variable controlled by the Central Bank.[7] Figure 3 depicts distributions of the Central Bank's upper and lower intervention thresholds and the probability that the Household and the Financier assign to the event that the Central Bank intervenes positively in response to an observed trade shock. The figure assumes normally distributed thresholds with means and standard deviations such that the probability of a negative

[7]The expectation of the Heaviside function equals the probability that it's argument is positive. For example, the probability of a positive intervention in $t = s$ is given by

$$\mathrm{E}_t^{(\sim)}\big[H(\Delta\iota_t - \bar{\iota}_t)\big] = \int_{-\infty}^{\infty} H(\Delta\iota_t - \bar{\iota}_t)\; d\,F(\bar{\iota}_t;\theta_t) = \int_{-\infty}^{\Delta\iota_t} 1\; d\,F(\bar{\iota}_t;\theta_t) = \text{Prob}(\bar{\iota}_t < \Delta\iota_t;\theta_t)\,, \tag{15}$$

where the trade shock $\Delta\iota_t$ is known and the Central Bank controls the parameters in θ_t through communication policy that we do not model. A similar expression holds for the expectation of a negative intervention in the current period. For the expectation of interventions in future periods ($t < s$), the trade shock $\Delta\iota_s$ is random and the joint distribution of $\Delta\iota_s$ and $\bar{\iota}_s$ must be considered. But the joint distribution will also depend on θ_t and can therefore be controlled by the Central Bank. Hence, we treat $\Delta\eta_{s|t}$ as an exogenous policy variable for $t \leq s < 2$.

intervention is approximately zero.

Equilibrium Real Exchange Rate. We find it convenient to work with linearized equations in deviations from the non-stochastic steady state of the economy for the remainder of our analysis. We use hats to denote deviations from steady state.

Bond markets clear in equilibrium. With bonds in zero net supply, market clearing for the Home bond requires that the demands of the Home Household, the Financier, and the Central Bank sum to zero,

$$\hat{Q}_{Ht} + \hat{Q}^{f}_{Ht} + \hat{Q}^{cb}_{Ht} = 0\,. \tag{16}$$

Bond holdings are zero for all parties in the non-stochastic steady state, so we linearize bond holdings in levels and divide by the non-stochastic steady state value of the Household preference for imports as a normalization.[8] To obtain an expression for the real exchange rate, we combine linearized demand equations for the Home Household, the Financier, and the Central Bank with the Home bond market clearing condition in (16).

Home Household demand for Home bonds derives from the period budget constraints in (3). Under our simplifying assumptions, after imposing market clearing for the Home good, the budget constraints take the following forms,

$$\begin{aligned}
\hat{Q}_{H0} &= \hat{e}_0 - \Delta\hat{\iota}_0 + O\big(\epsilon^2\big) \\
\hat{Q}_{H1} &= \hat{e}_0 + \hat{e}_1 - \Delta\hat{\iota}_0 - \Delta\hat{\iota}_1 + O\big(\epsilon^2\big) \\
0 &= \hat{e}_0 + \hat{e}_1 + \hat{e}_2 - \Delta\hat{\iota}_0 - \Delta\hat{\iota}_1 - \Delta\hat{\iota}_2 + O\big(\epsilon^2\big)\,,
\end{aligned} \tag{17}$$

where the third constraint requires that total variation in the real exchange rate equals total variation in trade shocks and gives rise to the "boomerang" effect of Central Bank interventions that Gabaix and Maggiori (2015) find. For a given sequence of trade shocks, interventions that shifts the real exchange rate in one period must be offset by

[8] For example, for the Home Household, we define $\hat{Q}_{Ht} \equiv (Q_{Ht} - Q_H)/\iota$, where $Q_H = 0$ is the Home Household's steady-state bond demand and $\iota \equiv 1$ is the Home Household's steady-state preference for imports. Because bond holdings are linearized in levels, the clearing condition in (16) is an exact condition rather than a first-order approximation. We linearize most other variables in logs, and the bond demand expressions are first-order approximate.

compensating shifts in future periods. The strength of this effect depends on the model's finite horizon.

The Financier's demand for Home bonds derives from the Financier's optimality condition in (11). Linearizing this condition leads to the following demand for Home bonds,

$$\hat{Q}^f_{Ht} = \frac{1}{\Gamma} \mathrm{E}^{(-)}_t [\hat{e}_t - \hat{e}_{t+1}] + O(\epsilon^2) \,, \tag{18}$$

which again shows that higher Γ reduces the intermediation undertaken by the Financier, all else equal. The gross real returns on Home and Foreign bonds that appear in (11) are zero to a first-order approximation; this result derives from our assumption that non-tradable endowments adjust proportionally to fluctuations in preference parameters for non-tradable goods.

Finally, we rewrite the Central Bank's policy rule in (16) as follows,

$$\hat{Q}^{cb}_{Ht} = q \Delta H_t \,, \tag{19}$$

where we use the fact that the Central Bank's intervention is zero in the non-stochastic steady state.[9] We provide derivations of the linearized demand equations in (16)–(19) in Appendix A.4.

Combining the linearized demand equations with the market clearing condition for the Home bond, we obtain solutions for the path of expected and realized real exchange rates in periods zero, one, and two. Our primary interest lies in the response of the real exchange rate in period zero to Central Bank interventions in the anticipated and unanticipated cases when financial constraints are tight or loose. For this reason, we relegate the full set of solutions and a brief analysis of exchange rate dynamics to Appendices A.5 and A.6.

[9] Recall that we linearize bond holdings in levels. For the Central Bank, we have $\hat{Q}^{cb}_{Ht} \equiv (Q^{cb}_{Ht} - Q^{cb}_{H})/\iota = Q^{cb}_{Ht}$, where the last equality follows from $Q^{cb}_{H} = 0$ and $\iota = 1$. Hence, (19) is an exact expression, not an approximation of (13).

The real exchange rate in period zero is given by

$$\begin{aligned}\hat{e}_0 = \Delta\hat{\iota}_0 - \frac{2\Delta\hat{\iota}_0 - \mathrm{E}_0^{(-)}[\Delta\hat{\iota}_1] - \mathrm{E}_0^{(-)}[\Delta\hat{\iota}_2]}{3+\Gamma} + \Gamma\frac{\Delta\hat{\iota}_0 - \mathrm{E}_0^{(-)}[\Delta\hat{\iota}_2]}{(1+\Gamma)(3+\Gamma)} \\ - q\frac{\Delta H_0 - \Delta\eta_{0|0}}{1+\Gamma} - \Gamma q\frac{\Delta H_0}{1+\Gamma} - \Gamma q\frac{\Delta\eta_{1|0} - \Delta\eta_{0|0}}{(1+\Gamma)(3+\Gamma)} + O\big(\epsilon^2\big)\,.\end{aligned} \tag{20}$$

To build intuition for the terms that appear in (20), consider separately the first and second lines on the right-hand side of the expression, focusing on the extreme cases of zero and infinite credit constraints.

If the Central Bank is passive ($q = 0$), the real exchange rate has three determinants in the first line on the right-hand side of (20): the pure effect of the trade shock, the intertemporal smoothing effect of bond trade, and a credit constraint friction. With severe credit constraints ($\Gamma \to \infty$), only the first term survives and trade shocks drive the real exchange rate in autarky. Without credit constraints ($\Gamma \to 0$), the first and second terms survive and bond trade smooths the real exchange rate optimally. With limited credit constraints ($0 < \Gamma < \infty$), all three terms survive and the real exchange rate deviates from the frictionless ideal. If trade shocks follow a random walk, zero bond holdings are optimal and the second and third terms again vanish.

If the Central Bank is active ($q > 0$), interventions operate through three channels in the second line on the right-hand side of (20): a surprise channel, a private intermediation replacement channel, and a private intermediation support channel. Without credit constraints ($\Gamma \to 0$), only the first term survives and Central Bank interventions are effective when they surprise the Household and Financier. With severe credit constraints ($\Gamma \to \infty$), only the second terms survives; the Central Bank completely replaces the Financier and intermediates all bond trade. With limited credit constraints ($0 < \Gamma < \infty$), all three terms survive and the Central Bank supports the Financier by reducing the Financier's intermediation burden.

Empirical Predictions. The Gamma-Eta Model yields an intuitive expression for the real exchange rate that captures the interplay between Central Bank interventions, market

anticipation, and credit constraints. We now derive qualitative predictions from the model to test empirically in the sections that follow. Specifically, we derive predictions on the direction of intervention effects, on the effects of swap versus spot interventions, on the effects of anticipated versus unanticipated interventions, the effects of interventions on the quantity of private intermediation, and on the effect of interventions during periods of looser and tighter credit constraints. We provide proofs for the predictions in Appendix A.7.

Prediction 1 (Direction of Intervention Effects)**.** *Consider a Central Bank intervention* $\Delta H_0 \neq 0$*, whereby the Central Bank constructs a portfolio of Home and Foreign bonds. The real exchange rate responds to the currency flows arising from the portfolio construction. The direction of the response depends on the direction of the intervention as follows,*

$$\operatorname{sign} \frac{\partial \hat{e}_0}{\partial q} = -\operatorname{sign} \Delta H_0 \,. \tag{21}$$

A positive intervention ($\Delta H_0 > 0$)*, whereby the Central Bank buys Home bonds and sells Foreign, strengthens Home currency relative to Foreign and the real exchange rate falls. A negative intervention* ($\Delta H_0 < 0$)*, whereby the Central Bank sells Home bonds and buys Foreign bonds, weakens Home currency relative to Foreign and the real exchange rate rises.*

Prediction 2 (Anticipated Spot and Swap Interventions)**.** *Consider a fully anticipated Central Bank intervention in period one such that* $\Delta \eta_{0|0} = \Delta H_0 \neq 0$*. If the Financier faces a credit constraint* ($\Gamma > 0$)*, then*

$$\left| \frac{\partial \hat{e}_0}{\partial q} \right|_{\Delta \eta_{1|0} = \Delta H_0} > \left| \frac{\partial \hat{e}_0}{\partial q} \right|_{\Delta \eta_{1|0} = 0} . \tag{22}$$

A fully anticipated long-lived spot intervention ($\Delta \eta_{1|0} = \Delta H_0$) *has a greater effect on the real exchange rate than a fully anticipated short-lived swap intervention* ($\Delta \eta_{1|0} = 0$)*.*

If the Financier faces no credit constraint ($\Gamma = 0$)*, then*

$$\frac{\partial \hat{e}_0}{\partial q} = 0 \tag{23}$$

and fully anticipated interventions have no effect. In this case, the Central Bank can build foreign exchange reserves without disturbing the real exchange rate.

Prediction 3 (Unanticipated Spot and Swap Interventions)**.** *Consider a Central Bank communication that raises the expectation of a positive intervention in period zero* ($\Delta\eta_{0|0} > 0$)*. The Financier anticipates stronger Home currency in period zero and sells Home bonds to buy Foreign bonds. The anticipation weakens Home currency and the real exchange rate rises,*

$$\frac{\partial^2 \hat{e}_0}{\partial q \partial \Delta\eta_{0|0}} > 0\,. \tag{24}$$

If the Central Bank carries out a positive intervention in period zero, the real exchange rate will fall by less if the Financier anticipates the intervention in period zero.

Consider a Central Bank communication that raises the expectation of a positive intervention in period one — e.g. to maintain an on-going spot intervention. The Financier anticipates stronger Home currency in period one and sells Foreign bonds to buy Home bonds. Thus, the anticipation strengthens Home currency and the real exchange rate falls,

$$\frac{\partial^2 \hat{e}_0}{\partial q \partial \Delta\eta_{1|0}} < 0\,. \tag{25}$$

If the Central Bank carries out a positive intervention in period zero, the real exchange rate will fall by more if the Financier anticipates a long-lived spot intervention that will be maintained in period one.

Prediction 4 (Private Intermediation)**.** *Consider a Central Bank communication that raises the expectation of an intervention in period zero, such that the Financier believes the period-zero intervention to be at least half as likely as a period-one intervention of the same sign* ($\Delta\eta_{0|0}/\Delta\eta_{1|0} > 1/2$)*. The Financier responds to the Central Bank's communication by adjusting the amount of its private intermediation as follows,*

$$\operatorname{sign} \frac{\partial \hat{Q}^f_{H0}}{\partial q} = -\operatorname{sign} \Delta\eta_{0|0}\,. \tag{26}$$

Suppose the Financier expects the Central Bank to intervene positively by buying Home

bonds and selling Foreign, such that $\Delta\eta_{0|0} > \frac{1}{2}\Delta\eta_{1|0} \geq 0$*. The Financier will then expect to earn a reduced carry trade spread, defined as the excess return on the home bond, and will therefore reduce the amount of private intermediation it undertakes.*

Prediction 5 (Credit Constraints)**.** *Consider a tightening of credit constraints. If the Central Bank is passive* ($q = 0$)*, tighter credit constraints lower the amount of intermediation the Financier undertakes,*

$$\operatorname{sign} \frac{\partial \hat{Q}^f_{H0}}{\partial \Gamma} = -\operatorname{sign} \hat{Q}^f_{H0}\,. \tag{27}$$

The Financier's portfolio positions in Home and Foreign bonds shrink when credit constraints tighten, limiting the ability of Households to smooth consumption intertemporally.

If the Central Bank is active ($q > 0$) *and its communications are consistent with its actions* ($\operatorname{sign}\Delta\eta_{0|0} = \operatorname{sign}\Delta\eta_{1|0} = \operatorname{sign}\Delta H_0 \neq 0$)*, then tighter credit constraints amplify the effects of interventions,*

$$\operatorname{sign} \frac{\partial^2 \hat{e}_0}{\partial q \partial \Gamma} = \operatorname{sign} \frac{\partial \hat{e}_0}{\partial q}\,, \tag{28}$$

if $\Delta\eta_{0|0}/\Delta\eta_{1|0} > 1/2$*. This regularity condition admits the anticipation of period-zero swap interventions* ($\Delta\eta_{0|0} > \Delta\eta_{1|0} = 0$) *and the anticipation of spot interventions* ($\Delta\eta_{0|0} = \Delta\eta_{1|0} > 0$).[10]

To map these predictions to the empirical analysis that follows, we treat Brazil as Home, with currency BRL, and the U.S. as Foreign, with currency USD. In this setup, the central bank is the BCB, and a positive intervention refers to an intervention to sell USD, either through a spot or swap transaction. In the following sections, we describe the data and present the results of our empirical analysis testing the predictions of the Gamma-Eta Model.

[10]Define a positive spot intervention as $\Delta H_0 = 1 \cap \Delta H_1 = 1$. If the Financier expects only spot interventions, then $\text{Prob}(\Delta H_1 = 1 \mid \Delta H_0 = 1) = \text{Prob}(\Delta H_0 = 1 \mid \Delta H_1 = 1) = 1$. It follows that

$$\begin{aligned}\text{Prob}(\Delta H_0 = 1 \cap \Delta H_1 = 1) &= \text{Prob}(\Delta H_0 = 1) \times \text{Prob}(\Delta H_1 = 1 \mid \Delta H_0 = 1) = \text{Prob}(\Delta H_0 = 1) \\ &= \text{Prob}(\Delta H_1 = 1) \times \text{Prob}(\Delta H_0 = 1 \mid \Delta H_1 = 1) = \text{Prob}(\Delta H_1 = 1)\,.\end{aligned}$$

The same holds for negative spot interventions. Definitions of $\Delta\eta_{0|0}$ and $\Delta\eta_{1|0}$ then imply $\Delta\eta_{0|0} = \Delta\eta_{1|0}$.

3 Datasets

In our empirical analysis, we focus on the Brazilian case and employ a public dataset released by the BCB in 2023, along with a number of supporting datasets on foreign exchange, interest rates, intermediary constraints, and macroeconomic aggregates. We briefly describe the Brazilian market structure and the BCB's policy framework before introducing the data and defining key variables.

3.1 Policy Framework

Since early 1999, Brazil's monetary system has operated under an inflation-targeting framework established by Presidential Decree No. 3,088. The National Monetary Council, composed of the finance and planning ministers and the BCB governor, sets the inflation target based on the Extended National Consumer Price Index (IPCA).

The BCB uses the Selic rate as its primary tool to achieve this inflation target. The Selic rate is the weighted average interest rate of overnight interbank loans, which are collateralized by federal government securities and settled through the Special System for Settlement and Custody (Selic, in Portuguese). The BCB's Monetary Policy Committee (Copom) meets regularly to set the target for the Selic rate, with the goal of keeping the IPCA inflation rate within a predefined range, specified by upper and lower bands, over the relevant monetary policy horizon.

Regarding exchange rate policy, the BCB operates a floating regime, which the IMF defines as largely market-determined rates without a predictable path, where intervention is permitted to prevent excessive volatility and manage liquidity but not to maintain a specific rate. To be classed as *free*-floating, authorities must have intervened at most three times in the previous six months, with each intervention lasting no more than three business days (Nowzad et al., 2021). Brazil's exchange rate regime falls short of this definition, and would be better described as a managed-floating regime.

3.2 BCB FX Interventions

The primary goals of BCB's FXI are to maintain an orderly market, control volatility, and ensure liquidity, while also accumulating reserves during periods of capital inflows (Bank for International Settlements, 2019). Spot interventions are more frequently employed when market imbalances are severe. In contrast, swap interventions are often used to hedge currency mismatches on bank balance sheets; for example, the currency swap interventions helped mitigate the spillovers of the US Taper Tantrum on bank balance sheets (Chamon and Magud, 2019; Gonzalez et al., 2021; da Costa Filho, 2021).

To execute interventions, the BCB's International Reserve Department transacts with 14 accredited financial institutions known as Dealers-14. These institutions, mainly large banks, operate in an inter-dealer market and are authorized by the BCB. They play a crucial role in supplying USD liquidity to customers in the Brazilian FX market.[11]

In 2023, the BCB released a comprehensive public dataset of its historical foreign exchange interventions since 1999.[12] The dataset details the sign of each intervention (USD purchase or sale), the auction amount, the announcement date and high-frequency announcement timestamp, the operation date, and the intervention type. Four intervention types are reported: spot purchases, spot sales, traditional swaps, and reverse swaps.

In *spot operations*, the BCB buys or sells USD in exchange for BRL in the interbank foreign exchange market for settlement in two business days.

In *traditional swap operations*, the BCB takes a buying position in swap contract auctions. While the BCB classifies these derivative auctions as swaps, they typically function as non-deliverable futures contracts settled in BRL (Nedeljkovic and Saborowski, 2019). Unlike a standard swap contract, there is no exchange of principals. Instead, the BCB pays the dealer the variation in the BRL/USD exchange rate, plus an onshore USD rate known as the *cupom cambial*, and receives the domestic BRL Selic rate for the same period.[13] Therefore, the traditional swap protects the dealer in the event of a USD

[11] Besides trading with the central bank during FX interventions, the Dealers-14 also participate in PTAX consultations to establish the daily market reference price and join conference calls with the BCB to ensure coordinated market operations.

[12] The database can be found on the BCB's Dados Abertos portal `https://www.bcb.gov.br/conteudo/dadosabertos/BCBDepin/historico-atuacoes-mercado-cambio.csv`.

[13] The *cupom cambial*, which serves as the onshore dollar interest rate, is equal to the spread between

appreciation and is useful for hedging net USD liabilities.

In *reverse swap operations*, the BCB takes a selling position in swap contracts. Similar to a traditional swap, there is no exchange of principals. In this case, the BCB pays the BRL Selic rate, and the dealer pays the *cupom cambial*, plus the variation in the BRL/USD exchange rate. This setup protects the dealer against a depreciation of the USD and can be used to hedge net USD asset positions.[14]

The intervention announcement dates and high-frequency timestamps in the database indicate when the BCB informed the public about a future foreign exchange intervention or, in the case of auctions in interbank spot or forward markets, when an intervention had begun. The intervention operation date specifies when the intervention was conducted, but it does not include a high-frequency timestamp for the operation. The announcement and operation dates allow us to classify interventions as unanticipated or anticipated. We classify interventions as unanticipated if the operation date equals the announcement date and as anticipated if the operation date occurs after the announcement date.

Figure 4 illustrates the size distribution of interventions by type. The mean unanticipated spot sale intervention in the database is 0.17 billion USD with a standard deviation of 0.22 billion USD, and the mean unanticipated traditional (reverse) swap intervention is 0.43 (0.35) billion USD with a standard deviation of 0.41 (0.45) billion USD. Unanticipated interventions are generally larger than anticipated interventions for all intervention types. There are 385 unanticipated and 87 anticipated spot sale interventions in the database, while there are 345 unanticipated and over 5000 anticipated traditional swap interventions. All spot purchases are unanticipated. Table B.1 in the Online Appendix provides detailed summary statistics on the BCB's foreign exchange interventions by type.

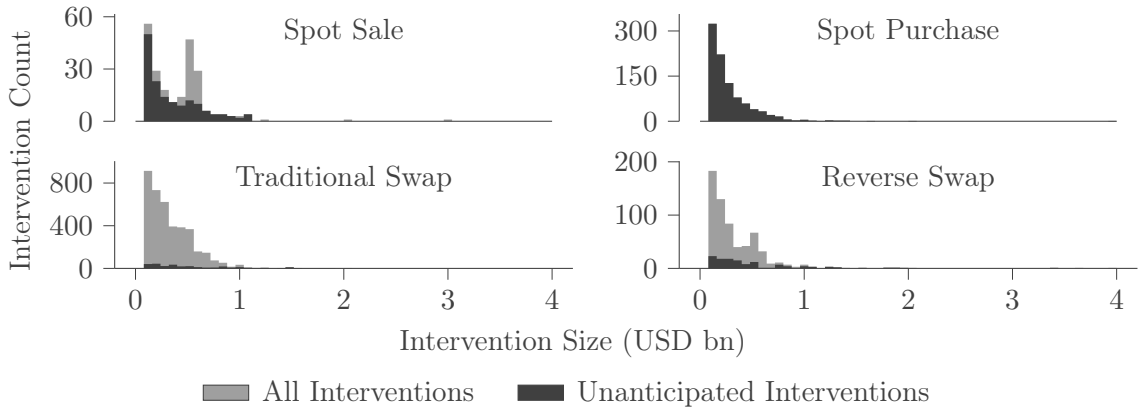
Figure 5 illustrates patterns in BCB interventions over time, showing a shift towards swaps as its principal policy instrument in recent years and suggesting two distinct periods of FXI use, as observed by Nagy et al. (2019).[15] From 2005 to 2012, the BCB primarily

the overnight interbank deposit rate and the expected exchange rate variation in the BRL/USD. See Nedeljkovic and Saborowski (2019) for more details.

[14]For more information, we refer readers to the institutional details of traditional and reverse swap operations on the BCB website `https://www.bcb.gov.br/estabilidadefinanceira/swapcambial`.

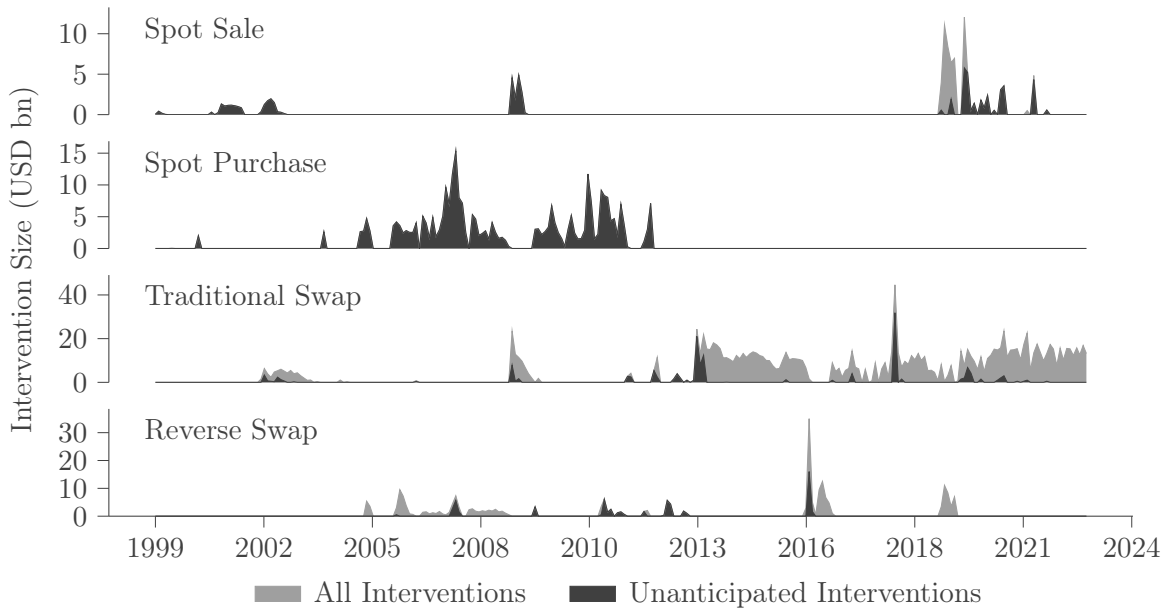
[15]Figure B.1 in the Online Appendix shows cumulative intervention amounts by intervention type over time.

Figure 4: Histograms of BCB FX Market Interventions



Notes. These figures show 50-bin histograms of BCB's interventions in the FX markets, by intervention type. The horizontal axes show the intervention size in USD billions, after aggregating at daily frequency. The lighter shade shows all interventions, both anticipated and unanticipated, while the darker shade shows the subset of unanticipated interventions. Unanticipated interventions are defined as interventions with an announcement equal to the operation date, while anticipated interventions are defined as interventions with an announcement date that precedes the operation date. Traditional swaps involve the sale of USD while reverse swaps involve the purchase of USD at the spot leg of the swap contract. The sample period runs from 1999-01-22 to 2023-04-27.

Figure 5: Time Series of BCB FX Market Interventions



Notes. These figures show BCB FX market interventions over time, by intervention type. The horizontal axes show the intervention size in USD billions, after aggregating at monthly frequency. The lighter shade shows all interventions, both anticipated and unanticipated, while the darker shade shows the subset of unanticipated interventions, defined as interventions with an announcement equal to the operation date.

used FXI to lean against capital flows and accumulate foreign reserves through USD spot purchases, except during the Global Financial Crisis in 2008 and 2009, when it conducted spot sales to provide USD liquidity to the FX market. Since 2012, the BCB has utilized FX swaps as its primary tool to manage price asymmetries, providing hedging options when market supply is limited or abundant (Gonzalez et al., 2021; Nedeljkovic and Saborowski, 2019). In contrast, spot auctions are reserved for more severe imbalances.

FX swaps are attractive instruments because the derivatives market is larger and more liquid than the spot market, which may explain their frequent use by the BCB. Additionally, the balance sheet effects of swaps are short-lived: with a traditional swap, the BCB effectively extends a collateralized loan in USD that is repaid upon swap maturity, automatically restoring the BCB's USD reserves. In contrast, spot sales of USD have potentially longer-lived effects, reducing the BCB's precautionary savings in USD and weakening its insurance against periods of tight intermediary constraints (Cheng, 2015; Jeanne and Sandri, 2020). While the BCB infrequently used USD spot sales between 2010 and 2019, it employed these operations more frequently in 2020 during the COVID-19 pandemic. We posit that the BCB adjusted its strategy in response to dollar scarcity in cross-border markets and higher dollar funding costs for intermediaries, as measured by CIP violations.

The BCB does not provide an official framework distinguishing between anticipated and unanticipated swaps. However, unanticipated swaps typically have short maturities (less than 2 days), while anticipated swaps have a broader range of maturities from overnight to 3 months. This difference in maturity may offer insights into the BCB's motivations. We speculate that the BCB uses unanticipated short-term swaps to provide short-term liquidity to specific dealers, addressing roll-over risk and ensuring the orderly functioning of interbank money markets. Conversely, the BCB may employ anticipated long-term swaps to provide long-term liquidity, addressing maturity and currency mismatches on bank balance sheets. Figure B.2 in the Online Appendix shows the maturity breakdown of traditional and reverse currency swaps for both anticipated and unanticipated interventions.

3.3 Additional Data Sources

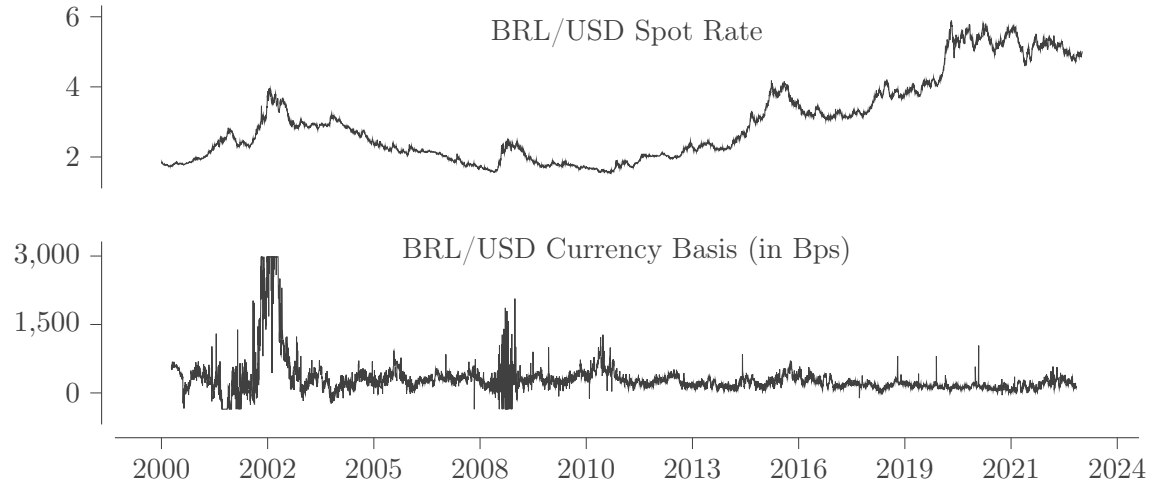
B3 Exchange Tick Database. We use tick-by-tick data on futures and interest rates, obtained from Brazil's main stock exchange, B3, via an FTP link that was available for a limited time. The dataset covers transactions from 28/02/2019 to 23/01/2020, spanning 224 trading days, and includes trading prices, volumes, aggressor buy-sell signals, market participant trading codes, and bid-ask quotes. For this study, we focus on the BRL/USD WDO contract, a mini U.S. Dollar futures contract traded on B3.[16] Each contract represents an exposure of 10,000 U.S. Dollars and is quoted in BRL per USD. WDO contracts expire monthly and allow investors to take positions on the future BRL/USD exchange rate. An alternative contract, the DOL futures contract, offers similar characteristics but represents a larger USD 50,000 exposure. We use the WDO contract instead of the DOL contract for our analysis due to its higher liquidity and trading activity within our sample.

Reuters Benchmark Spot and Forward Data. We supplement the B3 data with high-frequency spot and forward indicative quotes from Thomson Reuters Tick History. The quotes are given at 5-minute intervals, include bid and ask prices, and cover the period 1999 to 2023. While the Reuters quotes are lower frequency than the B3 tick-by-tick data, the Reuters quotes offer a longer time series that better aligns with the historical intervention dataset from the BCB. For this reason, we use the Reuters quotes for our primary econometric analysis in Section 4. In our analysis, we use the mid-quote of the spot and forward prices.

Figure 6 plots the BRL/USD spot rate and currency basis. We express the spot rate in units of BRL per USD, so an increase indicates a BRL depreciation. The USD has strengthened against BRL since 2008, rising from below 2 BRL per USD in 2008 to above 5 BRL per USD in 2020.

[16] For more details, see the B3 official website: `https://www.b3.com.br/en_us/products-and-services/trading/exchange-rates/mini-u-s-dollar-futures.htm`

Figure 6: BRL/USD Spot Rate and Currency Basis



Notes. The upper panel plots the spot exchange rate S_t expressed in units of BRL per USD. An increase in S_t denotes a depreciation of BRL and an appreciation of USD. The lower panel plots the daily 1-month maturity BRL/USD currency basis measured in basis points. A positive currency basis indicates a premium to swap BRL into USD in forward and swap markets. To improve legibility in the plot, we winsorize the BRL/USD currency basis at the 1% level. The sample period runs from 1999-01-22 to 2023-04-27.

Interest Rates. We obtain daily interest rates from the Brazilian Financial and Capital Markets Association (ANBIMA) through the Institute of Applied Economic Research (IPEA) data repository. For our baseline analysis, we use the 1-month maturity, based on the yield curve of the National Treasury Bill (LTN) with a term of 21 business days[17]

Covered Interest Rate Parity (CIP). The CIP condition states that equivalent assets denominated in different currencies should earn equal returns after hedging exchange rate risk with a forward contract. To illustrate, an investor can invest one USD at t to receive a risk-free payment in USD at $t+h$, or exchange one USD for BRL and invest in BRL at t to receive a risk-free payment in BRL at $t+h$. In the latter case, the investor can hedge exchange rate risk by entering a forward contract at t to exchange BRL for USD at $t+h$. The CIP condition states that both strategies should offer the same return.

[17]ANBIMA uses Svensson (1994) to estimate the term structure of interest rates from Treasury securities. See `https://www.anbima.com.br/data/files/12/D0/11/B0/C7CFD71028DFACD76B2BA2A8/Metodologia_estrutura-termo_out21_ingles.pdf` for more details

That is,

$$\left(1 + r^{USD}_{t,t+h}\right)^h = \left(1 + r^{BRL}_{t,t+h}\right)^h \frac{S_t}{F_{t,t+h}},$$

where $r^{USD}_{t,t+h}$ denotes the U.S. nominal risk-free rate in USD, $r^{BRL}_{t,t+h}$ denotes the Brazilian nominal risk-free rate in BRL, S_t denotes the nominal spot exchange rate expressed in BRL per USD, and $F_{t,t+h}$ denotes the nominal h-period forward exchange rate in BRL per USD.

Empirically, the CIP condition is frequently violated, and the cross-currency basis, which we denote $x_{t,t+h}$, quantifies the violation. Following Du et al. (2018), we define the cross-currency basis as a wedge between the *synthetic* and direct USD risk-free rate,

$$\left(1 + r^{USD}_{t,t+h}\right)^h = \left(1 + r^{BRL}_{t,t+h} - x_{t,t+h}\right)^h \frac{S_t}{F_{t,t+h}}.$$

Using lower-case variables to denote logs of the spot and forward exchange rates, the cross-currency basis can be approximated as

$$x_{t,t+h} \approx r^{BRL}_{t,t+h} - r^{USD}_{t,t+h} - \rho_{t,t+h} \qquad \text{with} \qquad \rho_{t,t+h} \equiv \frac{1}{h}\left(f_{t,t+h} - s_t\right), \tag{29}$$

where $\rho_{t,t+h}$ denotes the forward premium.

For the BRL/USD currency pair, a positive cross-currency basis ($x_{t,t+h} > 0$) indicates that borrowing BRL and swapping for USD in the forward market is more expensive than borrowing USD directly. The currency basis is measured in basis points and is persistently positive over the sample, as shown in Figure 6. The positive currency basis indicates that market participants require a premium to swap BRL into USD, and suggests a relative scarcity of USD funding in cross-border markets.

Intermediary Constraints. The Gamma-Eta model predicts that foreign exchange interventions have greater effects when intermediaries are constrained, as measured by the model's parameter Γ. To measure intermediary constraints empirically, we adopt the *intermediary capital ratio* of He et al. (2017) as an empirical proxy for Γ. The ratio,

denoted HKM_t, is defined as

$$HKM_t = \frac{\sum_i \text{Market Equity}_{it}}{\sum_i (\text{Market Equity}_{it} + \text{Book Debt}_{it})}, \tag{30}$$

where book debt is computed by subtracting total common equity from total assets, and where i indexes individual intermediaries. We follow Cerutti and Zhou (2023) and construct a daily measure of HKM_t using the subset of primary dealers that deal with emerging market currencies based on the Euromoney FX survey.[18]

A decline in HKM_t indicates lower risk-bearing capacity at the largest emerging-market currency dealers. The intuition is that intermediaries with lower capital ratios face tighter borrowing constraints and demand fewer risky assets to comply with regulatory capital requirements. The capital ratio HKM_t is an important indicator of the ability of primary dealer banks to intermediate USD transactions in cross-border financial markets. If Prediction 5 of the Gamma-Eta model holds, we would expect tighter intermediary constraints to reduce USD intermediation and strengthen the exchange rate response to Central Bank interventions.

Macroeconomic Variables. Finally, we include monetary announcements of the Federal Reserve and BCB, NBER recession dates, and measures of global and sovereign risk. For global risk we use the VIX index. To measure sovereign risk of emerging markets, we obtain the JPMorgan EMBI+ (Emerging Markets Bond Index Plus) from the IPEA data repository. The EMBI+ estimates the daily performance of debt securities from emerging markets relative to United States Treasury bonds. The index is based on the bonds (debt securities) issued by this group of countries and shows the financial returns obtained each day by a selected portfolio of securities.

[18]The listed entities we use for our analysis include BNP Paribas, Barclays, Bank of America, Citigroup, Credit Suisse, Deutsche Bank, Goldman Sachs, HSBC, JP Morgan, Morgan Stanley, Societe Generale, Standard Chartered, State Street, and UBS.

4 Empirical Analysis

We divide our empirical analysis into four sections. In Section 4.1, we use tick-by-tick data to closely examine two dates where the BCB conducted both unanticipated and anticipated spot sales of USD. This case study provides preliminary support for the Gamma-Eta Model's Prediction 1 on the direction of FXI's effects on the exchange rate, as well as Predictions 2 and 3 regarding the relative impact of anticipated and unanticipated FXI. In Section 4.2, we systematically analyze nearly twenty-five years of BCB interventions using the BCB's historical foreign exchange intervention database, further validating and expanding our case study findings. We also test Prediction 4, examining how FXI influences private intermediation, with CIP violations serving as a proxy for dollar supply by Financiers. In Section 4.3, we explore the conditional effects of BCB interventions during periods of tighter and looser intermediary constraints. When constraints are tighter, we find stronger responses in the exchange rate and cross-currency basis, in line with Prediction 5. Finally, in Section 4.4, we investigate an alternative to the dollar intermediation channel by testing whether FXI signals changes in future interest rates.

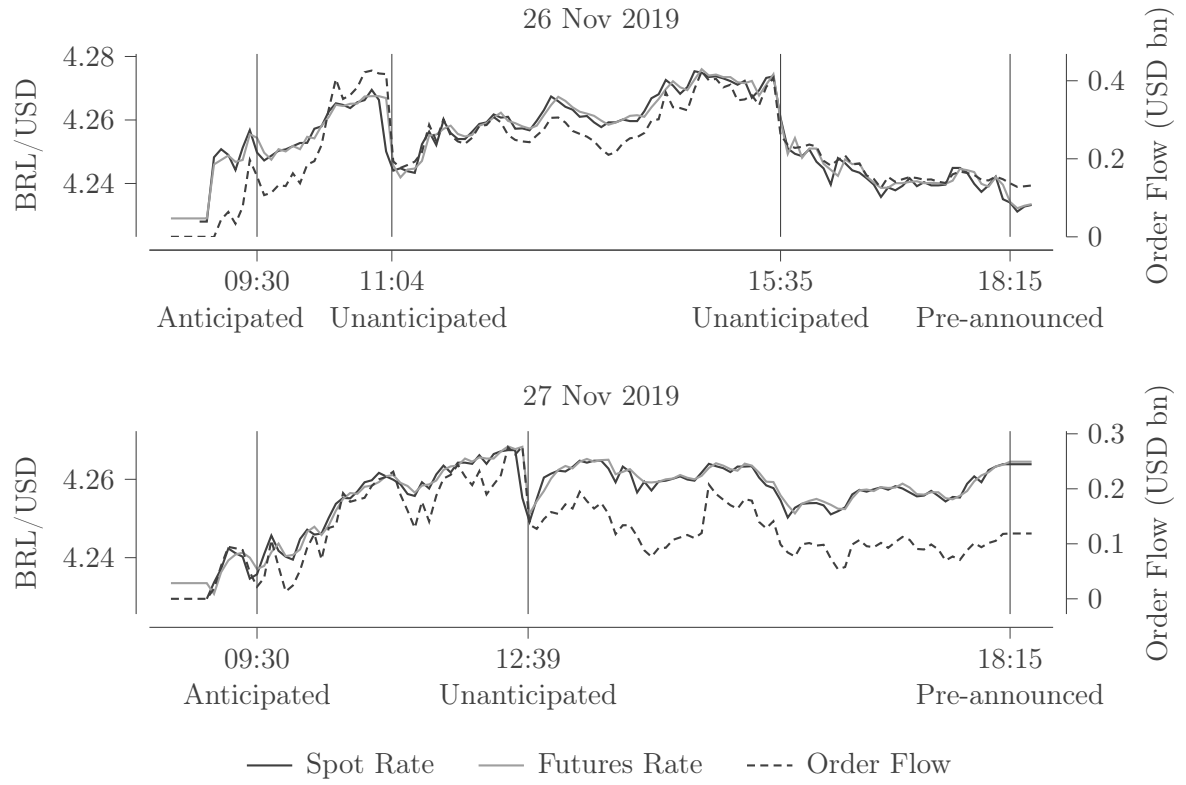
4.1 A High-frequency Case Study

To initiate our analysis, we examine the impact of interventions on 26 and 27 November 2019, when the BCB carried out both unanticipated and anticipated spot sales of USD. These dates serve as illustrative examples. We utilize intra-day data on transaction prices and order flow for the liquid BRL/USD futures contracts on the B3 exchange, focusing on these interventions due to the availability of detailed data from the B3 exchange.[19]

Figure 7 depicts intra-day BRL/USD spot and futures prices, as well as cumulative order flow on these dates. In the plots, the left vertical axes measure spot and futures prices and the right vertical axes measure cumulative order flow. Vertical lines mark intervention times. Order flow refers to net initiated buy orders over sell orders, a measure of buying pressure derived from aggressor buy-sell indicators in our data. An increase in order flow indicates an increase in net buying pressure for USD. On 26 November, the

[19]For the full sample analysis, we use historical indicative quotes from Reuters in Section 4.2.

Figure 7: High-Frequency BRL/USD Case Study on Two Intervention Days with BCB Spot Sales.



Notes. The figures show the BRL/USD exchange rate and cumulative order flow on two dates when the BCB conducted anticipated and unanticipated spot sales of USD. The vertical axes on the left show the BRL/USD exchange rate, and the vertical axes on the right show cumulative order flow in USD billions. Order flow is defined as the net of buyer initiated transaction volume in the 1 month BRL/USD futures market. The upper figure plots the exchange rate and order flow for 26 November 2019, when the BCB conducted one anticipated and two unanticipated interventions and pre-announced an intervention to be conducted on the following day. The lower figure plots the exchange rate and order flow for 27 November 2019, when the BCB conducted one anticipated and one unanticipated intervention and pre-announced an intervention to be conducted on the following day.

BCB implemented an anticipated spot sale of USD at 09:30, unanticipated spot sales of USD at 11:04 and 15:35, and pre-announced an intervention at 18:15 that it planned to implement the following morning. On the following day, the BCB implemented an anticipated (pre-announced) spot sales of USD at 09:30, and an unanticipated spot sale of USD at 12:39, before pre-announcing a future intervention at 18:15.

The plots show a sharp appreciation of BRL spot and futures prices around unanticipated interventions and an equally sharp decrease in order flow, indicating an accelerated execution of seller-initiated orders in the minutes following unanticipated announcements.

In contrast, the figures display muted responses to anticipated interventions and pre-announcements of future interventions. In these cases, the BCB appears to be reacting to BRL depreciation. In the lead-up to these interventions, BRL volatility had increased and the BRL had weakened against the USD. For instance, on 25 November 2019, the PTAX reference rate for dealers in the B3 currency futures market rose by nearly 12%. This depreciation may be attributable to a reduction in Brazil's benchmark Selic interest rate from 6.40% to 4.90% or to trade tensions between the U.S. and China.

This case study provides initial evidence at tick-by-tick frequency that the BCB can strengthen the BRL by intervening in spot markets to sell USD and purchase BRL, consistent with Prediction 1 of the Gamma-Eta Model. In the following section, we examine the BCB's historical interventions database, where we strengthen and extend the preliminary findings from our case study.

4.2 Historical BCB FXI Analysis

The BCB's database of historical FXI allows us to examine a greater variety of interventions over a longer time series than our B3 data. Specifically, we analyze the BCB's unanticipated and anticipated spot and swap interventions in both buy and sell directions between 1999 and 2023, utilizing indicative quotes of the BRL/USD spot exchange rate from Thomson Reuters.

For our baseline empirical specification, we adopt the local projections method of Jordà (2005). This procedure is useful for identifying the impulse response of FXI controlling for feedback effects through lags of the outcome variable and appropriate controls. Specifically, we test the effects of FXI on outcome variables y_{t+h} at horizon h,

$$\begin{aligned} y_{t+h} - y_{t-1} = {} & \beta_h^z INT_t^z \times SAD_{t+h} + \gamma_h^z INT_t^z \times (1 - SAD_{t+h}) \\ & + \delta_h SAD_{t+h} + \theta_h HKM_t \\ & + \text{Daily-Freq Controls}_t + \text{High-Freq Controls}_t + u_{t+h}\,, \end{aligned} \tag{31}$$

where the outcome variables are BRL/USD spot prices, forward premia, and the level

of CIP violations. Both spot and forward prices are expressed in logarithms.[20]

In this specification, HKM_t denotes the intermediary capital ratio, INT_t^z denotes the BCB intervention amount in USD, with z indicating the intervention type (spot sale, spot purchase, traditional swap, or reverse swap), and SAD_{t+h} indicates whether y_{t+h} is on the same day as y_t, when the intervention was announced. For example, if an intervention occurs at 10am, SAD_{t+h} takes a value of one for values of h until the close of trading on that day. This allows us to control for variation in interventions depending on its occurence during the trading day. Specifically, INT_t^z and SAD_{t+h} are defined as

$$INT_t^z \equiv \begin{cases} \text{USD Amount} & \text{if intervention } z \text{ announced at } t \\ 0 & \text{otherwise}\,, \end{cases} \tag{32}$$

$$SAD_{t+h} \equiv \begin{cases} 1 & \text{if } t+h \text{ on the same day as } t \\ 0 & \text{otherwise}\,. \end{cases} \tag{33}$$

Daily frequency controls include interest rates, the term interest rate spread, spot rate volatility, the total amount of interventions in USD at date t across all instruments, and the Brazilian Emerging Markets Bond Index Plus (EMBI+) that measures the sovereign risk of Brazil. Up to 10 days of lagged values are included. High-frequency controls include lags of the outcome variable and lagged spot rate bid-ask spreads. For all controls, up to 10 lags are included.

We begin by studying the direction and strength of the exchange rate response to interventions. Our empirical findings relate to Predictions 1 through 3 of the Gamma-Eta Model. We then examine the role of intermediaries in the BRL/USD foreign exchange market and how interventions affect the quantity of intermediation supplied in this market. Our findings with respect to intermediation relate to Prediction 4 of the Gamma-Eta Model.

We focus our analysis on unanticipated interventions, which are news with respect to

[20] In Section 4.3, we study FXI conditional on intermediary constraints using a modification of the baseline specification in (31).

the information set of market participants and not yet reflected in market prices, providing a clear identification of market responses. Importantly, high-frequency announcement timestamps are available for unanticipated interventions, but not for anticipated interventions. Anticipated interventions are pre-announced before markets close on the preceding day, allowing exchange rates to adjust before the auctions take place the following day. Furthermore, anticipated interventions are expected to have weaker effects, due to the forward looking behaviour of rational agents, and in line with Prediction 2 of the Gamma-Eta Model. We document empirical support for this prediction in Online Appendix B.4, using Brazilian market opening times as event dates for anticipated interventions.

Exchange Rate Response to Unanticipated FXI. To understand how different types of unanticipated interventions impact the exchange rate, we categorize the BCB's historical interventions based on type (spot or swap) and direction (buy or sell). We then estimate our baseline local projections specification in (31) for each category separately.

The upper group of plots in Figure 8 displays our results. Unanticipated spot sale interventions have the most significant effect on spot prices, leading to an approximate 1.5 percentage point appreciation within 7 hours of the announcement. In contrast, unanticipated spot purchase interventions have much weaker effects. After a spot purchase, we observe a 0.1 percentage point depreciation 5 hours post-intervention, with little statistical significance. For unanticipated swap interventions, we find weaker and more transitory effects. Unanticipated traditional swap interventions result in a 10 basis point BRL appreciation over a 1-hour period, while unanticipated reverse swap interventions lead to a 15 basis point BRL depreciation over the same horizon, with little statistical significance.

Taken together, our results suggest that USD sale interventions appreciate BRL, while purchase interventions depreciate BRL. Spot interventions have stronger effects than swap interventions, and unanticipated interventions have stronger effects than anticipated interventions. These empirical findings are broadly consistent with Predictions 1 through 3 of the Gamma-Eta Model.

The disaggregation of interventions shows clear heterogeneity in the effects. Permanent

effects on the spot rate appear to arise solely from spot sale interventions, while effects from swap interventions are transitory. The Gamma-Eta Model, with its distinction between long-lived spot interventions and short-lived swap interventions, offers a consistent theoretical explanation for this empirical finding, albeit in a stylized way.[21]

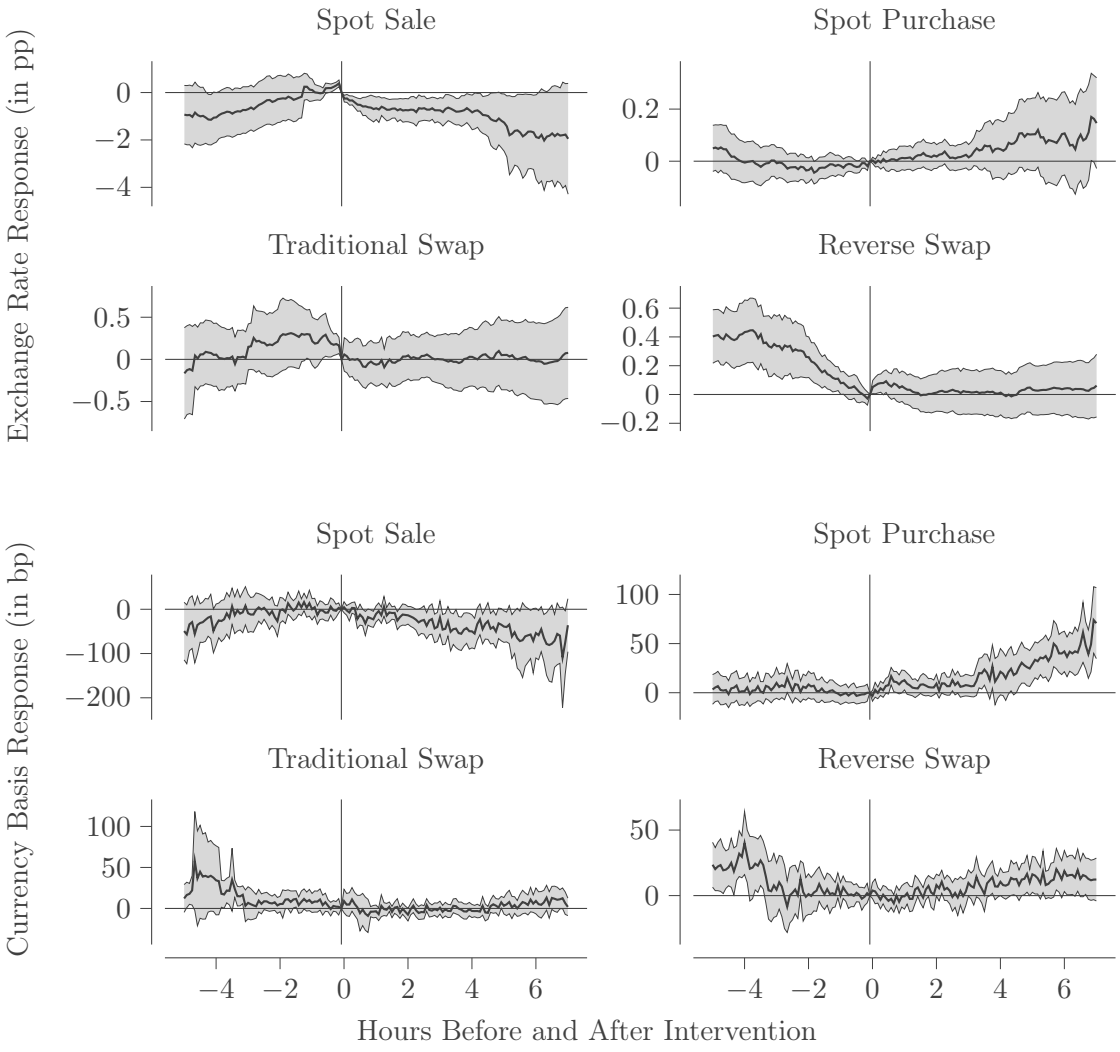
Interestingly, we find that spot purchase interventions are less effective than sale interventions. One reason for this asymmetry may be that USD purchase interventions are typically conducted during periods of relaxed intermediary constraints. During these periods, the Gamma-Eta Model predicts weaker effects on spot exchange rates, as intermediaries with loose constraints are well-positioned to absorb the excess supply of BRL without assistance from the Central Bank. We examine the role of intermediary constraints in more detail in the following section.

We also observe that unanticipated interventions tend to follow intra-day periods of depreciation. This is consistent with the case study of unanticipated interventions on 26-27 November 2019 in Section 4.1. The BCB reacts to developments that lead to a weakening of the domestic currency. Comparing the different types of instruments, we find the strongest pre-trends for spot sales, which is intuitive because spot sale interventions historically occur in periods with scarce USD liquidity in Brazilian FX markets. For spot sales, we find an approximate 100 basis point depreciation in the hours leading up to the intervention. Therefore, there is a clear reversal following the intervention, suggesting FXI is a useful stabilization tool.

We briefly compare our results to the findings of prior literature on the exchange rate effects of BCB interventions (Kohlscheen and Andrade, 2013; Nedeljkovic and Saborowski, 2019; Barroso, 2019; Santos, 2021). These studies typically find estimates in the range of 30 to 100 basis points in response to a sale of 1 billion USD in reserves by the BCB. We reconcile differences due to variations in the time period of analysis (with studies typically focusing on 2011-2013 and shorter sub-samples) and methodological differences in identification. Importantly, we separate out the effects of swap and spot instruments,

[21] Figure A.2 in the Online Appendix illustrates the dynamic response of the exchange rate to Central Bank interventions as implied by the Gamma-Eta Model for a variety of calibrations. The figure shows, among other things, that the exchange rate response to an unanticipated swap intervention is shorter-lived than the response to an unanticipated spot intervention.

Figure 8: BRL/USD Exchange Rate and Currency Basis Responses to Unanticipated BCB Interventions.



Notes. The upper group of figures show BRL/USD exchange rate responses to unanticipated BCB interventions measured in percentage points (pp), while the lower group figures show BRL/USD currency basis responses to unanticipated BCB interventions measured in basis points (bp). Within each group, the figures show responses to the BCB's spot sale, spot purchase, traditional swap, and reverse swap interventions. Traditional swaps involve the sale of USD while reverse swaps involve the purchase of USD at the spot leg of the swap contract. Shading indicates a 95% confidence interval using White's heteroscedasticity-robust standard errors. The sample period runs from 1999-01-22 to 2023-04-27.

as well as unanticipated and anticipated interventions. Indeed, our impact estimates are attenuated if we pool buy and sell interventions to create net measures for spot and swap FXI, as we show in Appendix B.3.

Intermediation Response to Unanticipated FXI. Next, we examine the effects of BCB foreign exchange intervention on the quantity of private intermediation supplied in

the BRL/USD foreign exchange market. Prediction 4 of the Gamma-Eta Model states that the amount of private USD intermediation provided decreases following a USD spot sale by the BCB. To test this prediction, we require an empirical proxy for the supply of intermediation.

We use a measure of CIP violations called the cross-currency basis for this purpose. The cross-currency basis measures the premium that market participants pay to swap currencies in forward markets. We reason that the Financier's supply of intermediation in (11) is a function of the carry trade return made by going short in the USD bond and long in the BRL bond. If forward markets are efficient, the future spot rate is predicted by the forward rate and the carry trade return is equivalent to the cross-currency basis. By this logic, the intermediation that Financiers supply should be proportional to the cross-currency basis.[22]

To test Prediction 4, we use the baseline local projection specification in (31) to estimate the intra-day effect of FXI, with the cross-currency basis as the outcome variable, serving as a proxy for the quantity of intermediation in foreign exchange markets. The results, shown in the lower group of plots in Figure 8, suggest that spot sales have the most persistent effects on the cross-currency basis among all intervention types. The negative coefficients indicate that FXI reduces CIP violations, lowering the relative cost of swapping BRL into USD through forward and swap contracts, thereby reducing USD liquidity costs for Financiers. This is consistent with Prediction 4 from the Gamma-Eta Model.

We find weak and insignificant effects for swap FXI (both traditional and reverse) and a temporary widening of CIP violations by about 50 basis points over a 5-hour horizon following spot purchase FXI. While our theory predicts that traditional swaps should affect CIP violations, we note that these swaps in our sample generally occur during periods when intermediaries are unconstrained, which may reduce their effectiveness, as per Prediction 5 of the Gamma-Eta Model.[23]

[22] Formally, we refer to model equation (11) which states the amount of bond trade the Financier chooses to intermediate. Under the expectations hypothesis $\mathrm{E}[e_{t+h}] = F_{t,t+h}$, the amount of intermediation undertaken by the financier is proportional to the cross-currency basis.

[23] Additional results on the forward premium can be found in Section B.5 of the Online Appendix. The

4.3 FXI and Intermediary Constraints

Prediction 5 from the Gamma-Eta Model states that tighter intermediary constraints strengthen the exchange rate response to foreign exchange interventions and lower the quantity of intermediation that Financiers supply. Applied to the Brazilian case, the Gamma-Eta Model predicts that constrained intermediaries will limit USD liquidity to the Brazilian economy, and BCB operations to sell USD and counteract the constraints will have larger exchange rate effects.

To test Prediction 5, we augment the baseline specification in (31) by including dummy variables that indicate the state of intermediary constraints as either tight or loose. The intermediary capital ratio HKM_t defined in (30) serves as our measure of intermediary constraints, with low HKM_t values indicating a tight constraint. The outcome variables we examine are the BRL/USD exchange rate and cross-currency basis.

We adopt the following augmented version of our baseline specification in (31),

$$\begin{aligned} y_{t+h} - y_{t-1} = {} & \beta_h^z INT_t^z \times SAD_t \times D_{HKM,t} \\ & + \gamma_h^z INT_t^z \times SAD_{t+h} \times (1 - D_{HKM,t}) + \delta_h SAD_{t+h} + \theta_h HKM_t \\ & + \text{Interactions}_t + \text{Daily-Freq Controls}_t + \text{High-Freq Controls}_t + u_{t+h}\,, \end{aligned} \tag{34}$$

where the outcome variable y_{t+h} is either the BRL/USD exchange rate or the cross-currency basis, and $D_{HKM,t}$ denotes an indicator for intermediary constraints, taking a value of one when the intermediary capital ratio measure HKM_t lies below its 50th percentile and zero otherwise. We interact $D_{HKM,t}$ with intervention size INT_t^z for intervention type z.[24]

The upper group of plots in Figure 9 illustrate how FXI and intermediary constraints interact to influence the exchange rate. We plot the conditional exchange rate response to unanticipated BCB interventions using both spot and swap instruments. For spot

forward premium behaves in line with the observed response of CIP violations, which links CIP behavior to the relative demand for currency forwards. If FXI increases USD liquidity in FX swap markets, it raises the forward premium (in BRL per USD). This effect is most pronounced for spot FXI sales of USD reserves.

[24] Additional interactions in the specification are between $1 - SAD_t$ with periods of tight ($D_{HKM,t} = 1$) and loose ($D_{HKM,t} = 0$) constraints.

USD sales under tight intermediary constraints ($D_{HKM,t} = 1$), we find that the BCB can statistically significantly appreciate the BRL against the USD by 2 percentage points over a 7-hour horizon per USD 1 billion sold. In comparison, the unconditional effect reported in Section 4.2 was smaller, with a 1.5 percentage point appreciation over the same horizon.

For traditional swaps under tight constraints ($D_{HKM,t} = 1$), the BCB can appreciate the BRL by approximately 20 basis points over a 7-hour window per USD 1 billion. In contrast, no statistically significant effect is observed for traditional swaps under loose constraints, although we do detect a noisy 40 basis point depreciation of the BRL against the USD. For spot purchases of USD and reverse swaps, we find no statistically significant effects.

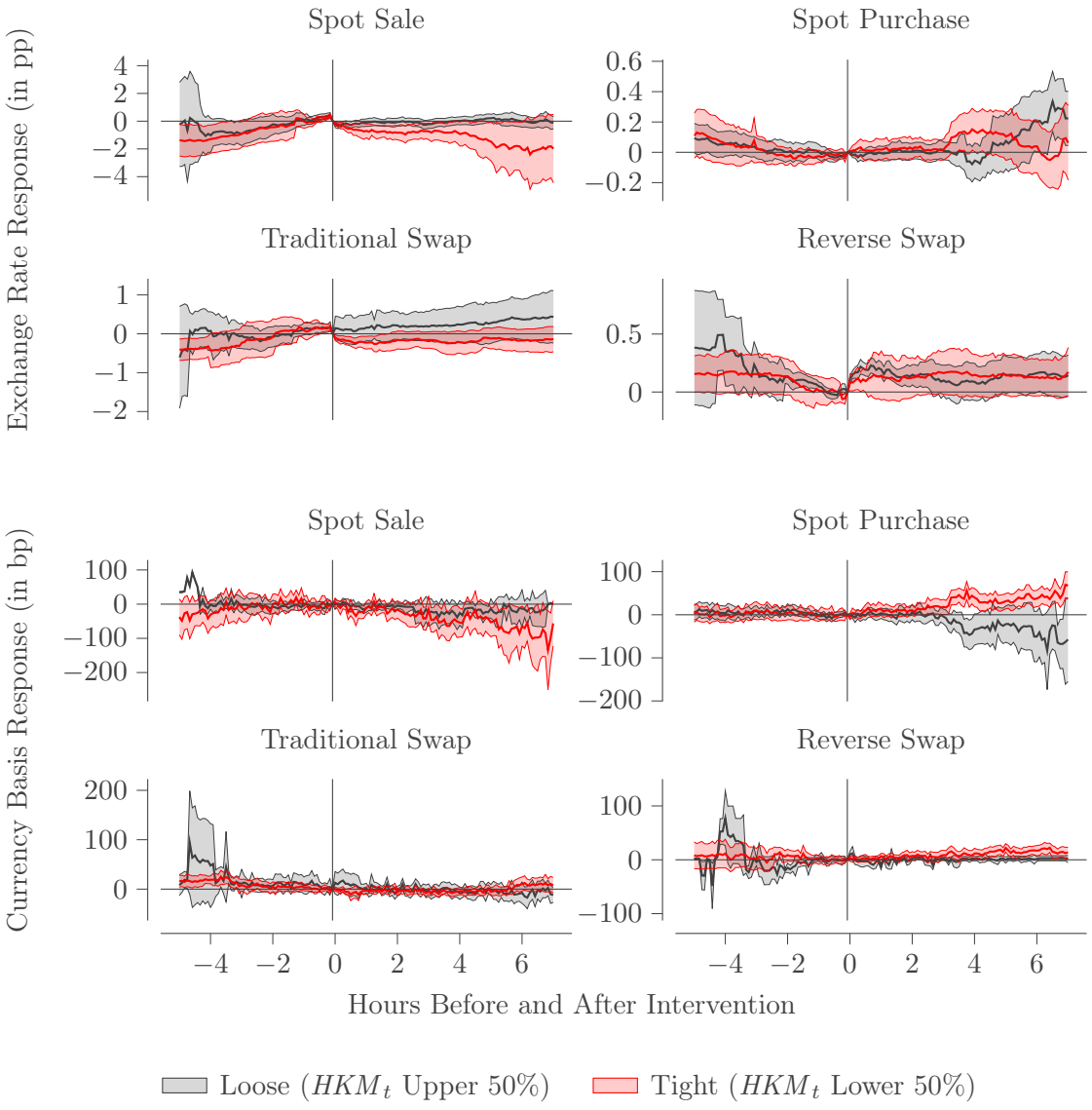
Next, we examine how intermediary constraints affect the quantity of intermediation, using the BRL/USD cross-currency basis as a proxy. According to Prediction 5, FXI should relax intermediation constraints and reduce the cost of providing USD liquidity, which would be reflected in a decline in the BRL/USD cross-currency basis. While we found no unconditional effects of swap FXI on CIP violations, significant effects emerge when accounting for intermediary constraints. The lower group of plots in Figure 9 shows the conditional response of the cross-currency basis to unanticipated BCB interventions.[25]

Similar to the exchange rate results, we find that the impact of spot USD sales on the cross-currency basis is amplified during periods of tight intermediary constraints ($D_{HKM,t} = 1$), leading to an approximate 50 basis point reduction in CIP deviations. In contrast, spot USD purchases during tight constraints widen CIP deviations by 70 basis points, with no significant effects for traditional or reverse swaps.

In summary, our empirical findings support Prediction 5 of the Gamma-Eta Model. The data show that unanticipated interventions are more effective when intermediaries are constrained, whereas their effectiveness diminishes when constraints are looser. The economic intuition behind this result is clear: the BCB's spot USD sales and traditional

[25] A potential concern with our measure of intermediary constraints is that it might reflect a structural break in our sample, as dealer capital levels tend to be lower after 2008. To address this, in Section B.7 of the Online Appendix, we use the magnitude of CIP violations as an alternative proxy for intermediary constraints. Using this proxy, we find similar results: spot sales and traditional swap interventions during periods of above-median CIP violations lead to BRL appreciation and a narrowing of the cross-currency basis.

Figure 9: Conditional BRL/USD Exchange Rate and Currency Basis Responses to Unanticipated BCB Interventions.



Notes. The upper group of figures show BRL/USD exchange rate responses to unanticipated BCB interventions in percentage points (pp), while the lower group figures show BRL/USD currency basis responses to unanticipated BCB interventions in basis points (bp). Within each group, the figures show responses to the BCB's spot sale, spot purchase, traditional swap, and reverse swap interventions. Each figure plots responses conditional on the state of FX dealers' balance sheet constraints on the intervention day. We measure balance sheet constraints by constructing the intermediary capital ratio HKM_t in equation (30). Red indicates tight constraints, defined as periods with HKM_t in the lower 50% of values in our sample. Gray indicates loose constraints, defined as periods with HKM_t in the upper 50% of values in our sample. Shading indicates a 95% confidence interval using White's heteroscedasticity-robust standard errors. The sample period runs from 2003-11-24 to 2023-04-27.

swaps increase USD liquidity. When intermediaries face tighter constraints, this added liquidity enhances FX market intermediation and causes larger exchange rate movements.

However, when intermediaries are unconstrained, or during spot USD purchases or reverse swaps, the existing USD liquidity is sufficient for efficient intermediation, leading to no significant exchange rate effects.

4.4 Signaling Channel

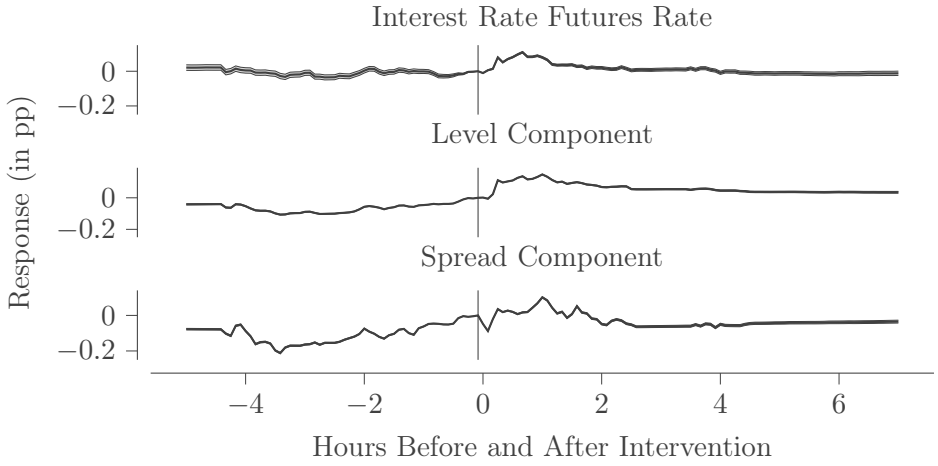
So far, we have found evidence for the dollar intermediation channel, which is a specific case of the portfolio balance models with constrained financial intermediaries. We now test the effects of FXI on the future path of interest rates. This contributes to the debate on whether the signaling or portfolio balance channel is quantitatively important for the transmission of FXI (Sarno and Taylor, 2001). The signaling channel of monetary policy posits that FXI can provide information to market participants on the future path of interest rates. For example, a spot sale of USD reserves can signal that the central bank intends to strengthen the domestic currency in the future, which, all else being equal, requires higher future domestic interest rates.

Intra-day Interest Rate Futures. We first test the signaling channel using intra-day interest rate futures data from B3. Figure 10 plots the response of interest rate futures to the unanticipated spot sale interventions that occur during the period of our sample of B3 data coverage.[26] The 1-month interest rate futures increase by up to 15 basis points 2 hours after the announcement, and then drop to approximately 2.5 basis points 6 hours after the announcement.

Additionally, we plot the level component (average across maturities) of interest rate futures contracts and the spread between 5-year and 1-month interest rates. The average interest rate increase reaches a peak of 20 basis points and declines to 10 basis points after 7 hours, while the spread between the 1-month and 5-year yields increases in the immediate hours but is close to zero over the day. The effects are quantitatively and economically insignificant: a 1 USD billion change in USD increases interest rates by between 2.5 basis points for the 1-month and up to 10 basis points when averaging across

[26]Similar to our B3 case study in Section 4.1, this includes unanticipated interventions on 27 August, 26 November, and 27 November 2019, for which we have intra-day transaction data from the B3 exchange.

Figure 10: High-Frequency Interest Rate Response to BCB Unanticipated Spot Sales

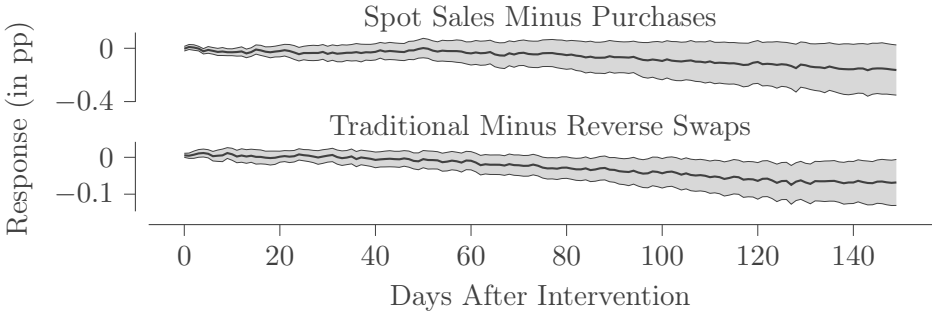


Notes. The figures show high-frequency responses of Brazilian interest rates to BCB spot sale interventions in the BRL/USD FX market measured in percentage points (pp). The upper figure shows the overall response of B3 interest rate futures for the most liquid contract. The middle figure shows the response of the level component of interest-rate futures rates. The bottom figure shows the response of the spread component of interest rate futures rates. The sample period of B3 interest rate futures runs from 2019-02-08 to 2020-01-23. Four unanticipated spot sale interventions occurred during the sample period, in total 2.481 billion USD. Shading indicates a 95% confidence interval using White's heteroscedasticity-robust standard errors.

maturities. In comparison, a 1 USD billion spot sale leads to a 1.5% (150 basis points) change in spot appreciation and a 50-100 basis point narrowing of the CIP basis.

Daily Interest Rates. In addition to using intra-day data, we use daily interest rates to test the signalling channel in our longer historical sample. In our baseline specification in Eq. (31), we estimate the effects using interest rates as the outcome variable. At a daily frequency, we control for potential endogeneity by identifying a reaction function of FXI. We use macroeconomic variables to estimate the reaction function, including the level and volatility of exchange rates, interest rates, recession dates, the HKM dealer capital ratio, and the VIX index. The measure of FXI that we use, denoted $F\tilde{X}I$, is the residual component unexplained by macroeconomic fundamentals and is therefore plausibly exogenous to interest rates in our analysis. This approach addresses the potential feedback from interest rates to FXI in our daily-frequency analysis. For supplementary information on the construction of our residualized FXI, we refer readers to Appendix

Figure 11: High-Frequency Interest Rate Response to BCB Unanticipated Net Spot and Swap Interventions.



Notes. The figures show 1-month interest rate responses to unanticipated BCB net spot and swap interventions at daily frequency, in percentage points (pp). Unanticipated interventions are measured in USD Billion. Net unanticipated spot intervention is the amount of unanticipated spot sales minus the amount of unanticipated spot purchases. Net unanticipated swap intervention is the amount of unanticipated traditional swaps minus the amount of unanticipated reverse swaps. Shaded indicates a 95% confidence interval using White heteroscedasticity-robust standard errors.

B.8.

For spot and swap FXI at daily frequency, we examine the effect of net (sell-buy) interventions. We find no significant effects of FXI on daily interest rates, as shown in Figure 11. Interest rates decline weakly by up to 25 basis points following (net) spot FXI over 6 months, however the effects are statistically insignificant. In summary, we find weak evidence for the central bank signaling changes in interest rates in response to both spot and swap interventions.

5 Conclusion

In this paper, we investigate the importance of the *dollar intermediation* channel of FXI, hypothesizing that the effects of FXI are more pronounced during periods of tight intermediary constraints.

To motivate our empirical framework, we extend the Basic Gamma Model of Gabaix and Maggiori (2015), distinguishing between spot and swap interventions and considering both anticipated an unanticipated interventions. The model involves two countries with Households that consume domestic and international goods but hold only domestic bonds.

A foreign exchange Financier intermediates bond trade, constrained by creditors. The Central Bank uses interventions in the FX market to respond to trade shocks, with thresholds communicated as distributions to manage intervention anticipation. The model predicts that long-lived spot interventions and unanticipated interventions have larger exchange rate effects, and that interventions are more impactful under tighter credit constraints.

Our analysis utilizes an extensive dataset on interventions by Brazil's Central Bank from 1999-2023, combined with tick-level data on spot and forward prices, enabling us to examine the intra-day effects of FXI. We highlight three key findings. First, unanticipated sell interventions of USD reserves lead to an appreciation of the domestic currency by approximately 150 basis points per USD billion, whereas buy interventions have insignificant effects. Second, unanticipated sell interventions reduce CIP violations, indicating improved market efficiency. Third, the impact of FXI on spot prices and CIP violations is more pronounced during periods of tighter intermediary constraints, with significant effects observed primarily during these times.

Finally, we test whether FXI works through an alternative channel in signaling changes in the stance of monetary policy. Our findings indicate small and transitory effects on interest rates following the intervention, with minimal long-term effects, suggesting that the portfolio balance channel, rather than central bank signaling, is more important in explaining the observed effects of FXI.

In summary, our findings have salient policy recommendations for central banks using FXI as a tool to improve the functioning of dollar intermediation and FX markets. We contribute to the debate on the relative importance of the portfolio balance and signaling channel of FXI, and the relative efficacy of spot and swap instruments. We show that FXI can alleviate constraints on global financial intermediaries, enhancing market efficiency, with unanticipated spot sales of USD reserves being the most effective instrument.

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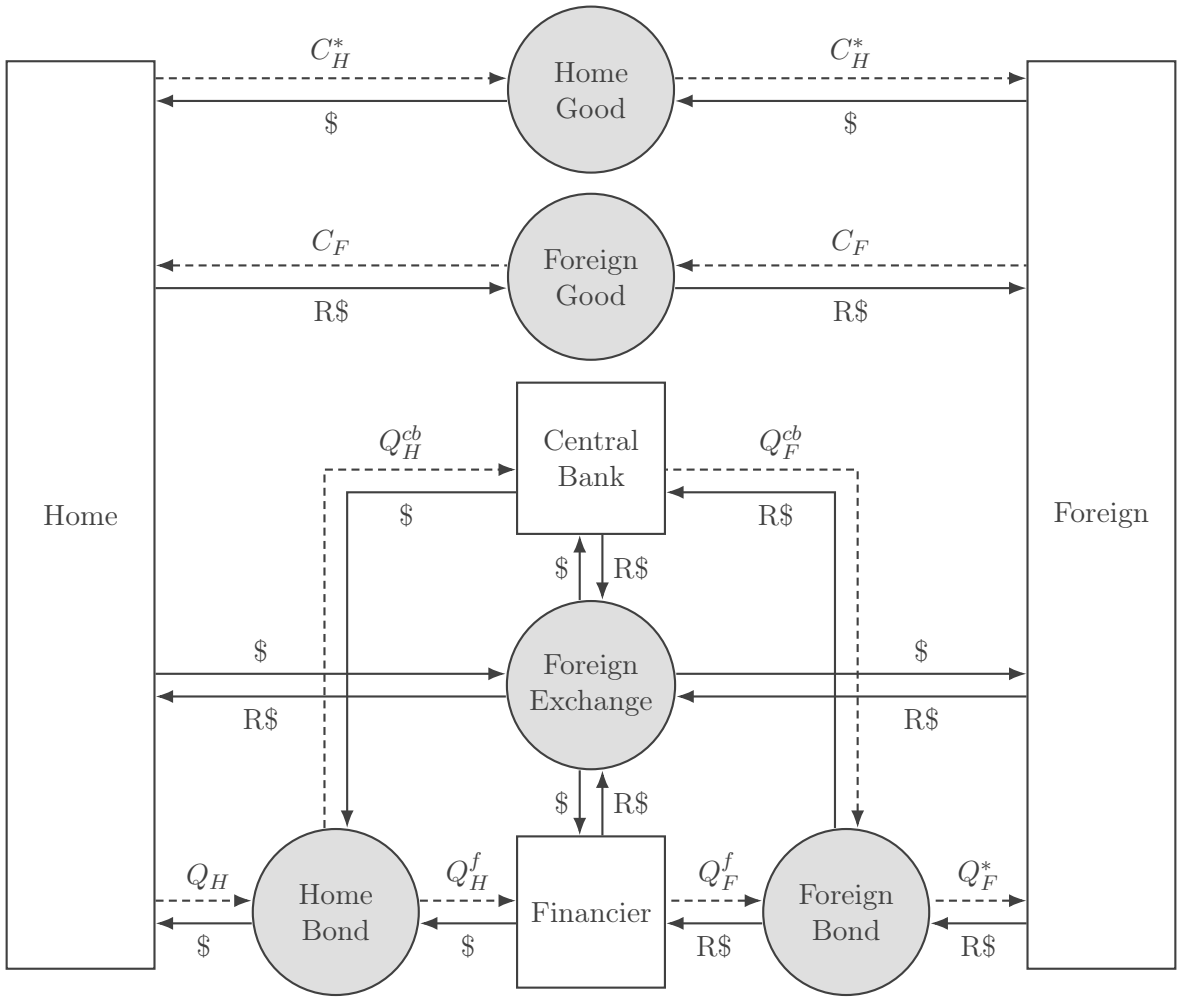
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Appendix

A Theory

A.1 Model Structure

Figure A.1: Real and financial flows in the two-country model.



Notes. The figure depicts trade flows resulting from a positive trade shock $\Delta\iota_t$ that induces the Home Household to sell bonds and the Financier and Central Bank to buy Home bonds. Decision makers are depicted as rectangles, markets are depicted as circles, and trade flows are depicted as arrows. An arrow into a market depicts a supply flow, and an arrow out of a market depicts a demand flow. Real flows are shown as dashed lines, currency flows are shown as solid lines with labels "$" for USD and "R$" for BRL. Within-country goods trade, as well as profit or loss transfers from the Financier and the Central Bank to the Foreign Household are omitted.

A.2 Home Household Problem

The Lagrangian formulation of the Home Household's intertemporal objective function for $t = 0$ is given by

$$\begin{aligned}\mathcal{L}_0 = &\sum_{s=t}^{2} \beta^s \, \mathrm{E}_0^{(\cdot)}[\chi_s \ln C_{NTt} + \alpha_s \ln C_{Ht} + \iota_s \ln C_{Fs}] \\ &+\beta^0 \lambda_0 \, \mathrm{E}_0^{(\cdot)}[Y_{NT0} + p_{H0} Y_{H0} - C_{NT0} - p_{H0} C_{H0} - p_{F0} C_{F0} - Q_{H0}] \\ &+\beta^1 \lambda_1 \, \mathrm{E}_1^{(\cdot)}[Y_{NT1} + p_{H1} Y_{H1} - C_{NT1} - p_{H1} C_{H1} - p_{F1} C_{F1} - Q_{H1} + R_{H1} Q_{H0}] \\ &+\beta^2 \lambda_2 \, \mathrm{E}_2^{(\cdot)}[Y_{NT2} + p_{H2} Y_{H2} - C_{NT2} - p_{H2} C_{H2} - p_{F2} C_{F2} + R_{H2} Q_{H1}] \,,\end{aligned} \tag{35}$$

and derivatives $\partial \mathcal{L}_0 / \partial C_{NTs}$ and $\partial \mathcal{L}_0 / \partial Q_{Hs}$ lead to the optimality conditions in (4) for $s < 2$. Similar Lagrangian formulations can be written for the Household's intertemporal problems in $t = 1, 2$.

The Lagrangian formulation of the Home Household's intratemporal objective function is given by

$$\mathcal{L}_t = \chi_s \ln C_{NTt} + \alpha_s \ln C_{Ht} + \iota_s \ln C_{Fs} + \lambda_t (p_t C_t - C_{NTt} - p_{Ht} C_{Ht} - p_{Ft} C_{Ft}) \,, \tag{36}$$

and combining the derivative $\partial \mathcal{L} / \partial C_{NTt}$ with the derivatives $\partial \mathcal{L} / \partial C_{Ht}$ and $\partial \mathcal{L} / \partial C_{Ft}$ yields the optimality conditions in (7).

A.3 Foreign Household Problem

The Foreign Household solves a three-period intertemporal utility maximization problem,

$$\max_{\left\{C^*_{NTs}, C^*_{Hs}, C^*_{Fs}\right\}_{s=t}^{2}} \sum_{s=t}^{2} \beta^{*s} \, \mathrm{E}_t^{(\cdot)}[\varphi^*_s \ln C^*_s] \,, \tag{37}$$

β^* denotes the Foreign Household's subjective discount factor, $\varphi_t \equiv \chi^*_t + \iota^*_t + \alpha^*_t$ denotes a sum of stochastic preference shocks, and where

$$C^*_t \equiv \left[(C^*_{NTt})^{\chi^*_t} (C^*_{Ht})^{\iota^*_t} (C^*_{Ft})^{\alpha^*_t} \right]^{\frac{1}{\varphi^*_t}} \tag{38}$$

denotes the Foreign Household consumption basket, a sub-utility function composed of Foreign non-tradable goods C^*_{NTt}, Home tradable goods C^*_{Ht}, and Foreign tradable goods C^*_{Ft}. The intertemporal maximization is subject to period budget constraints of the form

$$Q^*_{Ft} + p^*_{Ht}C^*_{Ht} + p^*_{Ft}C^*_{Ft} + C^*_{NTt} = Y^*_{NTt} + p^*_{Ft}Y^*_{Ft} + R^*_{Ft}Q^*_{Ft-1} + \pi^*_t + \tau^*_t\,, \tag{39}$$

where Q^*_{Ft} denotes the real value of Foreign holdings of the Foreign bond, with $Q^*_{F-1} = Q^*_{F2} = 0$, where p^*_{Ht} and p^*_{Ft} denote relative prices of the Home and Foreign tradable goods, where Y^*_{NTt} and Y^*_{Ft} denote stochastic endowments of the Foreign non-tradable and tradable goods, respectively, and where π^*_t and τ^*_t denote profits and taxes transferred to the Foreign HOusehold by the Financier and the Central Bank, respectively. The Foreign non-tradable good is the numéraire for the Foreign economy.

The Lagrangian formulation of the Foreign Household's intertemporal objective function for $t = 0$ is given by

$$\begin{aligned}
\mathcal{L}^*_0 = &\sum_{s=t}^{2} \beta^{*s}\, \mathrm{E}^{(\cdot)}_0[\chi^*_s \ln C^*_{NTt} + \iota^*_s \ln C^*_{Ht} + \alpha^*_s \ln C^*_{Fs}] \\
&+\beta^{*0}\lambda^*_0\, \mathrm{E}^{(\cdot)}_0[Y^*_{NT0} + p^*_{F0}Y^*_{F0} - C^*_{NT0} - p^*_{H0}C^*_{H0} - p^*_{F0}C^*_{F0} - Q_{H0}] \\
&+\beta^{*1}\lambda^*_1\, \mathrm{E}^{(\cdot)}_1[Y^*_{NT1} + p^*_{F1}Y^*_{F1} - C^*_{NT1} - p^*_{H1}C^*_{H1} - p^*_{F1}C^*_{F1} - Q^*_{H1} + R^*_{F1}Q^*_{F0}] \\
&+\beta^{*2}\lambda^*_2\, \mathrm{E}^{(\cdot)}_2[Y^*_{NT2} + p^*_{F2}Y^*_{F2} - C^*_{NT2} - p^*_{H2}C^*_{H2} - p^*_{F2}C^*_{F2} + R^*_{F2}Q^*_{F1}]\,,
\end{aligned} \tag{40}$$

and derivatives $\partial\mathcal{L}^*_0/\partial C^*_{NTs}$ and $\partial\mathcal{L}^*_0/\partial Q^*_{Fs}$ lead to the following optimality conditions,

$$\mathrm{E}^{(\cdot)}_t[\lambda^*_s] = \mathrm{E}^{(\cdot)}_t\left[\chi^*_s/C^*_{NTs}\right] \quad \text{and} \quad \mathrm{E}^{(\cdot)}_t[\lambda^*_s] = \mathrm{E}^{(\cdot)}_t\left[\lambda^*_{s+1}R^*_{Fs+1}\right], \tag{41}$$

where λ^*_s denotes the Lagrange multiplier on the Foreign Household's period-t budget constraint and where R^*_{Fs+1} denotes the gross real return on the Foreign bond, with $t \leq s < 2$.

The Foreign Household also solves an intratemporal utility maximization problem, allocating expenditure across domestic and international goods. Specifically, the Foreign

Household maximizes the logarithm of its sub-utility function in (38),

$$\max_{C^*_{NTt}, C^*_{Ht}, C^*_{Ft}} \chi^*_t \ln C^*_{NTt} + \iota^*_t \ln C^*_{Ht} + \alpha^*_t \ln C^*_{Ft} \,, \tag{42}$$

subject to a consumption expenditure constraint,

$$p^*_t C^*_t = C^*_{NTt} + p^*_{Ht} C^*_{Ht} + p^*_{Ft} C^*_{Ft} \,, \tag{43}$$

where p^*_t is the Foreign price index in terms of the Foreign non-tradable numéraire, and where the Foreign Household takes total consumption expenditure $p^*_t C^*_t$ as fixed in the intratemporal problem.

The Lagrangian formulation of the Foreign Household's intratemporal objective function is given by

$$\mathcal{L}^*_t = \chi^*_s \ln C^*_{NTt} + \iota^*_s \ln C^*_{Ht} + \alpha^*_s \ln C^*_{Fs} + \lambda^*_t (p^*_t C^*_t - C^*_{NTt} - p^*_{Ht} C^*_{Ht} - p^*_{Ft} C^*_{Ft}) \,, \tag{44}$$

and combining the derivative $\partial \mathcal{L}^* / \partial C^*_{NTt}$ with the derivatives $\partial \mathcal{L}^* / \partial C^*_{Ht}$ and $\partial \mathcal{L}^* / \partial C^*_{Ft}$ yields the following optimality conditions for consumption expenditure on Home and Foreign tradable goods,

$$p^*_{Ft} C^*_{Ft} = (\chi^*_t / C^*_{NTt}) \alpha^*_t \quad \text{and} \quad p^*_{Ht} C^*_{Ht} = (\chi^*_t / C^*_{NTt}) \iota^*_t \,. \tag{45}$$

As we did for the Home Household, we make two simplifying assumptions for the Foreign Household: the Foreign non-tradable endowment adjusts proportionally to fluctuations in the Foreign preference for non-tradable goods, $Y^*_{NTt} = \chi^*_t$, and the Foreign Household does not discount future utility, $\beta^* = 1$.

A.4 Deviations from Steady State

To solve the model, we write key equations in terms of deviations from the non-stochastic steady state. We first derive the steady-state equilibrium, and then derive the expressions

in deviations from the steady state for bond demands and bond market clearing that stated in (16)–(19) in the main text.

Steady State. To indicate non-stochastic steady-state values, we omit time subscripts from variables. Under our simplifying assumptions, the Household optimality conditions in (4) and (41) imply that steady-state gross real returns on Home and Foreign bonds equal one,

$$R_H = R_F^* = 1\,. \tag{46}$$

The Financier's value function (8) is linear in the Financier's bond holdings, so the Financier's the credit constraint (9) holds with equality. Using the Financier's value function, credit constraint, and balance sheet constraint (10), we obtain the optimality condition in (11). The Financier's optimality condition in (11) then implies zero steady-state bond holdings for the Financier.

The Central Bank's steady-state bond holdings in (13) depend on the steady-state value of the trade shock and the steady-state values of the Central Bank's intervention thresholds. We assume that preferences for imports equal one in the non-stochastic steady state, $\iota = \iota^* = 1$, which implies a steady-state trade shock of zero, $\Delta\iota = 0$. We assume the Central Bank sets its intervention thresholds to zero in the non-stochastic steady-state, $\underline{\iota} = \bar{\iota} = 0$. The Heaviside function is defined to take a value of zero when its argument is zero, so $\Delta H = H(\Delta\iota - \bar{\iota}) - H(\underline{\iota} - \Delta\iota) = 0$. The Central Bank's steady state bond holdings are therefore zero.

Bond market clearing then implies zero bond holdings for the Home Household, so that

$$Q_H = Q_H^f = Q_H^{cb} = 0\,. \tag{47}$$

Market clearing for Home tradable and non-tradable goods requires that

$$Y_{Ht} = C_{Ht} + C_{Ht}^* \quad \text{and} \quad Y_{NTt} = C_{NTt}\,. \tag{48}$$

Combining goods market clearing conditions in (48) with the Home Household period

budget constraint in (3) evaluated at the non-stochastic steady state, using goods expenditures in (7) and (45) with our simplifying assumptions, using steady-state bond holdings in (47), and using the Law of One Price for the Home tradable good ($p_{Ht} = e_t p^*_{Ht}$), we obtain the following steady state real exchange rate,

$$e = 1\,, \tag{49}$$

where we have used $\iota = \iota^* = 1$. The steady-state results in (46), (47), and (49) suffice for the derivations of bond demands that we turn to next.

Bond Demands. We begin by deriving the Home Household's Home bond demand in (17). The Home Household's period zero budget constraint can be rewritten using market clearing conditions for the Home tradable and non-tradable in (48), the Law of One Price for the Home tradable good, goods expenditures in (7) and (45), and our simplifying assumptions on preferences to obtain

$$Q_{H0} + \iota_0 = e_0 \iota^*_0\,.$$

Linearizing around the non-stochastic steady state, in logs with respect to e_t, ι_t, and R_{Ht+1} and levels with respect to Q_{Ht}, we obtain

$$(Q_{H0} - Q_H) + \iota \hat{\iota}_0 = e\iota^* \hat{e}_0 + e\iota^* \hat{\iota}^*_0 + O\big(\epsilon^2\big)\,,$$

where we have used $Q_H = 0$, and where we define $\hat{e}_t \equiv \ln e_t - \ln e$, $\hat{\iota}_t \equiv \ln \iota_t - \ln \iota$ and $\hat{\iota}^*_t \equiv \ln \iota^*_t - \ln \iota^*$. Letting $\hat{Q}_{Ht} \equiv (Q_{Ht} - Q_H)/\iota$ and $\Delta\hat{\iota}_t \equiv \hat{\iota}_t - \hat{\iota}^*_t$, and using $e = \iota = \iota^* = 1$, we obtain

$$\hat{Q}_{H0} = \hat{e}_0 - \Delta\hat{\iota}_0 + O\big(\epsilon^2\big)\,. \tag{50}$$

Following a similar procedure for period one budget constraint, we first obtain

$$Q_{H1} + \iota_1 = e_1 \iota^*_1 + R_{H1} Q_{H0}\,,$$

which we linearize around the non-stochastic steady state to obtain

$$\begin{aligned}(Q_{H1} - Q_H) + \iota\hat{\iota}_1 &= e\iota^*\hat{e}_1 + e\iota^*\hat{\iota}_1^* + R_H Q_H \hat{R}_{H1} + R_H(Q_{H0} - Q_H) + O\big(\epsilon^2\big) \\ \Leftrightarrow \quad \hat{Q}_{H1} &= \hat{e}_1 - \Delta\hat{\iota}_1 + \hat{Q}_{H0} + O\big(\epsilon^2\big)\,,\end{aligned}$$

where we have used $e = \iota = \iota^* = 1$. We combine the linearized expression with (50) to obtain

$$\hat{Q}_{H1} = \hat{e}_0 + \hat{e}_1 - \Delta\hat{\iota}_0 - \Delta\hat{\iota}_1 + O\big(\epsilon^2\big)\,. \tag{51}$$

For the period two budget constraint, we first obtain

$$\iota_2 = e_2\iota_2^* + R_{H2}Q_{H1}\,,$$

which we linearize around the non-stochastic steady state to obtain

$$\begin{aligned}\iota\hat{\iota}_2 &= e\iota^*\hat{e}_2 + e\iota^*\hat{\iota}_2^* + R_H Q_H \hat{R}_{H2} + R_H(Q_{H1} - Q_H) + O\big(\epsilon^2\big) \\ \Leftrightarrow \quad 0 &= \hat{e}_2 - \Delta\hat{\iota}_2 + \hat{Q}_{H1} + O\big(\epsilon^2\big)\,,\end{aligned}$$

where we have used $e = \iota = \iota^* = 1$. We combine the linearized expression with (51) to obtain

$$0 = \hat{e}_0 + \hat{e}_1 + \hat{e}_2 - \Delta\hat{\iota}_0 - \Delta\hat{\iota}_1 - \Delta\hat{\iota}_2 + O\big(\epsilon^2\big)\,. \tag{52}$$

The expressions in (50)–(52) appear in (17) in the main text.

The Financier's linearized Home bond demand derives from the Financier's optimality condition in (11). We linearize (11) as follows,

$$\frac{1}{e}\hat{Q}_{Ht}^f - \frac{Q_H^f}{e}\,\mathrm{E}_t^{(\cdot)}[\hat{e}_t] = \frac{1}{\Gamma}\,\mathrm{E}_t^{(\cdot)}\left[R_H\hat{R}_{Ht+1} + R_F^*\left(\hat{e}_{t+1} - \hat{e}_t + \hat{R}_{Ft+1}^*\right)\right] + O\big(\epsilon^2\big)\,, \tag{53}$$

where we define $\hat{Q}_{Ht}^f \equiv (Q_{Ht}^f - Q_H^f)/\iota$. This expression simplifies because $Q_H^f = 0$ and $e = R_H = R_F^* = 1$, and because $\mathrm{E}_t^{(\cdot)}\left[\hat{R}_{Ht+1}\right] = \mathrm{E}_t^{(\cdot)}\left[\hat{R}_{Ft+1}^*\right] = 0$. The latter result derives from linearizations of the Home and Foreign Household intertemporal optimality conditions

in (7) and (45). Linearizing (7), assuming $\chi_t = C_{NTt}$ and $\beta = 1$,

$$\mathrm{E}_t^{(\cdot)}\left[\hat{\lambda}_s\right] = \mathrm{E}_t^{(\cdot)}\left[\hat{\chi}_s - \hat{C}_{NTs}\right] = 0 + O\left(\epsilon^2\right) \quad \Rightarrow \quad \mathrm{E}_t^{(\cdot)}\left[\hat{R}_{Hs+1}\right] = 0 + O\left(\epsilon^2\right),$$

and linearizing (45), assuming $\chi_t^* = C_{NTt}^*$ and $\beta^* = 1$,

$$\mathrm{E}_t^{(\cdot)}\left[\hat{\lambda}_s^*\right] = \mathrm{E}_t^{(\cdot)}\left[\hat{\chi}_s^* - \hat{C}_{NTs}^*\right] = 0 + O\left(\epsilon^2\right) \quad \Rightarrow \quad \mathrm{E}_t^{(\cdot)}\left[\hat{R}_{Fs+1}^*\right] = 0 + O\left(\epsilon^2\right).$$

These results combine with (53), using $Q_H^f = 0$ and $e = R_H = R_F^* = 1$, to yield the Financier's linearized Home bond demand in (18).

The Central Bank's bond demand written in deviations from steady state derives from the policy rule in (13). We can rewrite the left-hand side of (13) as follows,

$$Q_{Ht}^{cb} = Q_{Ht}^{cb} - Q_H^{cb} = (Q_{Ht}^{cb} - Q_H^{cb})/\iota = \hat{Q}_{Ht}^{cb}, \tag{54}$$

where we define $\hat{Q}_{Ht}^{cb} = (Q_{Ht}^{cb} - Q_H^{cb})/\iota$ and use $Q_H^{cb} = 0$ and $\iota = 1$. We leave the right-hand side of the Central Bank's policy rule unchanged, and use (54) to obtain the linearized policy rule in (19).

Finally, we write the Home bond market clearing condition in deviations from steady state using the definitions of $\hat{Q}_{Ht}$, $\hat{Q}_{Ht}^f$, and $\hat{Q}_{Ht}^{cb}$ stated above. The expression (16) is exact, given that bond holdings are linearized in levels.

A.5 Real Exchange Rate Solutions

To solve for the real exchange rate, we combine the linearized bond demands of the Home Household in (17), the Financier in (18), and the Central Bank in (19) with the market clearing condition in (16) to obtain a system of three linear equations,

$$0 = \hat{e}_0 - \Delta\hat{\iota}_0 + q\Delta H_0 + \frac{1}{\Gamma}\,\mathrm{E}_0^{(\cdot)}[\hat{e}_0 - \hat{e}_1] + O\left(\epsilon^2\right) \tag{55}$$

$$0 = \hat{e}_0 + \hat{e}_1 - \Delta\hat{\iota}_0 - \Delta\hat{\iota}_1 + q\Delta H_1 + \frac{1}{\Gamma}\,\mathrm{E}_1^{(\cdot)}[\hat{e}_1 - \hat{e}_2] + O\left(\epsilon^2\right) \tag{56}$$

$$0 = \hat{e}_0 + \hat{e}_1 + \hat{e}_2 - \Delta\hat{\iota}_0 - \Delta\hat{\iota}_1 - \Delta\hat{\iota}_2 + O\left(\epsilon^2\right), \tag{57}$$

in the endogenous real exchange rates $\hat{e}_0$, $\hat{e}_1$, and $\hat{e}_2$ and their conditional expected values. Because conditional expected values appear in the system of three equations, we need additional conditions to pin down equilibrium real exchange rates.

Taking the expectations of (55)–(57) conditional on period-zero information and the expectations of (56) and (57) conditional on period-one information, we obtain five additional conditions, which gives us a system of eight linear equations in total and allows us to solve for $\hat{e}_0$, $\hat{e}_1$, $\hat{e}_2$, $\mathrm{E}_0^{(\cdot)}[\hat{e}_0]$, $\mathrm{E}_0^{(\cdot)}[\hat{e}_1]$, $\mathrm{E}_0^{(\cdot)}[\hat{e}_2]$, $\mathrm{E}_1^{(\cdot)}[\hat{e}_1]$, and $\mathrm{E}_1^{(\cdot)}[\hat{e}_2]$. Standard methods can be used to solve the system. We omit the intermediate algebra and provide final solutions for realized real exchange rates,

$$\begin{aligned}\hat{e}_0 = \Delta\hat{\iota}_0 &- \frac{2\Delta\hat{\iota}_0 - \mathrm{E}_0^{(\cdot)}[\Delta\hat{\iota}_1] - \mathrm{E}_0^{(\cdot)}[\Delta\hat{\iota}_2]}{3+\Gamma} - \frac{\Gamma\big(\mathrm{E}_0^{(\cdot)}[\Delta\hat{\iota}_2 - \Delta\hat{\iota}_0]\big)}{(1+\Gamma)(3+\Gamma)} \\ &- q\frac{\Delta H_0 - \eta_{0|0}}{1+\Gamma} - \Gamma q\frac{\Delta H_0}{1+\Gamma} - \Gamma q\frac{\eta_{1|0} - \eta_{0|0}}{(1+\Gamma)(3+\Gamma)} + O\Big(\epsilon^2\Big)\,,\end{aligned} \tag{58}$$

$$\begin{aligned}\hat{e}_1 = \Delta\hat{\iota}_1 &- \frac{2\Delta\hat{\iota}_1 - \Delta\hat{\iota}_0 - \mathrm{E}_1^{(\cdot)}[\Delta\hat{\iota}_2]}{3+\Gamma} + \frac{(1+\Gamma)\big(\Delta\hat{\iota}_1 - \mathrm{E}_0^{(\cdot)}[\Delta\hat{\iota}_1]\big)}{(2+\Gamma)(3+\Gamma)} + \frac{\mathrm{E}_1^{(\cdot)}[\Delta\hat{\iota}_2] - \mathrm{E}_0^{(\cdot)}[\Delta\hat{\iota}_2]}{(2+\Gamma)(3+\Gamma)} \\ &- q\frac{\Delta H_1 - \eta_{1|1}}{2+\Gamma} - q\frac{(\Delta H_1 - \eta_{1|1}) - (\Delta H_0 - \eta_{0|0})}{2+\Gamma}\,, \\ &- \Gamma q\frac{\Delta H_1 - \Delta H_0}{3+\Gamma} - \Gamma q\frac{(\Delta H_1 - \eta_{1|0}) - (\Delta H_0 - \eta_{0|0})}{(2+\Gamma)(3+\Gamma)} + O\Big(\epsilon^2\Big)\,, \quad \text{and}\end{aligned} \tag{59}$$

$$\begin{aligned}\hat{e}_2 = \Delta\hat{\iota}_2 &- \frac{2\Delta\hat{\iota}_2 - \Delta\hat{\iota}_0 - \Delta\hat{\iota}_1}{3+\Gamma} + \frac{\Gamma(\Delta\hat{\iota}_2 - \Delta\hat{\iota}_0)}{(1+\Gamma)(3+\Gamma)} + \frac{2\Delta\hat{\iota}_2 - \mathrm{E}_0^{(\cdot)}[\Delta\iota_2] - \mathrm{E}_1^{(\cdot)}[\Delta\hat{\iota}_2]}{(1+\Gamma)(3+\Gamma)} \\ &+ \frac{\Gamma\big(\Delta\hat{\iota}_2 - \mathrm{E}_1^{(\cdot)}[\Delta\iota_2]\big)}{(1+\Gamma)(3+\Gamma)} + \frac{\Delta\hat{\iota}_1 - \mathrm{E}_0^{(\cdot)}[\Delta\hat{\iota}_1]}{(2+\Gamma)(3+\Gamma)} - \frac{\mathrm{E}_1^{(\cdot)}[\Delta\iota_2] - \mathrm{E}_0^{(\cdot)}[\Delta\iota_2]}{(2+\Gamma)(3+\Gamma)} \\ &+ q\frac{\Delta H_0 - \eta_{0|0}}{1+\Gamma} + \Gamma q\frac{\Delta H_0}{1+\Gamma} + \Gamma q\frac{\eta_{1|0} - \eta_{0|0}}{(1+\Gamma)(3+\Gamma)} \\ &+ q\frac{\Delta H_1 - \eta_{1|1}}{2+\Gamma} + q\frac{(\Delta H_1 - \eta_{1|1}) - (\Delta H_0 - \eta_{0|0})}{2+\Gamma} \\ &+ \Gamma q\frac{\Delta H_1 - \Delta H_0}{3+\Gamma} + \Gamma q\frac{(\Delta H_1 - \eta_{1|0}) - (\Delta H_0 - \eta_{0|0})}{(2+\Gamma)(3+\Gamma)} + O\Big(\epsilon^2\Big)\,.\end{aligned} \tag{60}$$

A.6 Real Exchange Rate Dynamics

We limit our theoretical model to three periods because we seek interpretable closed-form expressions that deliver intuitive qualitative predictions on the real exchange rate effects of Central Bank interventions. In our model, interventions are only possible in the first

two periods, when bond trading occurs. This timeline gives us enough flexibility to differentiate between short-lived swap and long-lived spot interventions, but not more. The constrained timeline of the model precludes a nuanced study of economic dynamics; however, limited insights can still be gained from analyzing the three-period path of the real exchange rate in response to trade shocks and interventions.

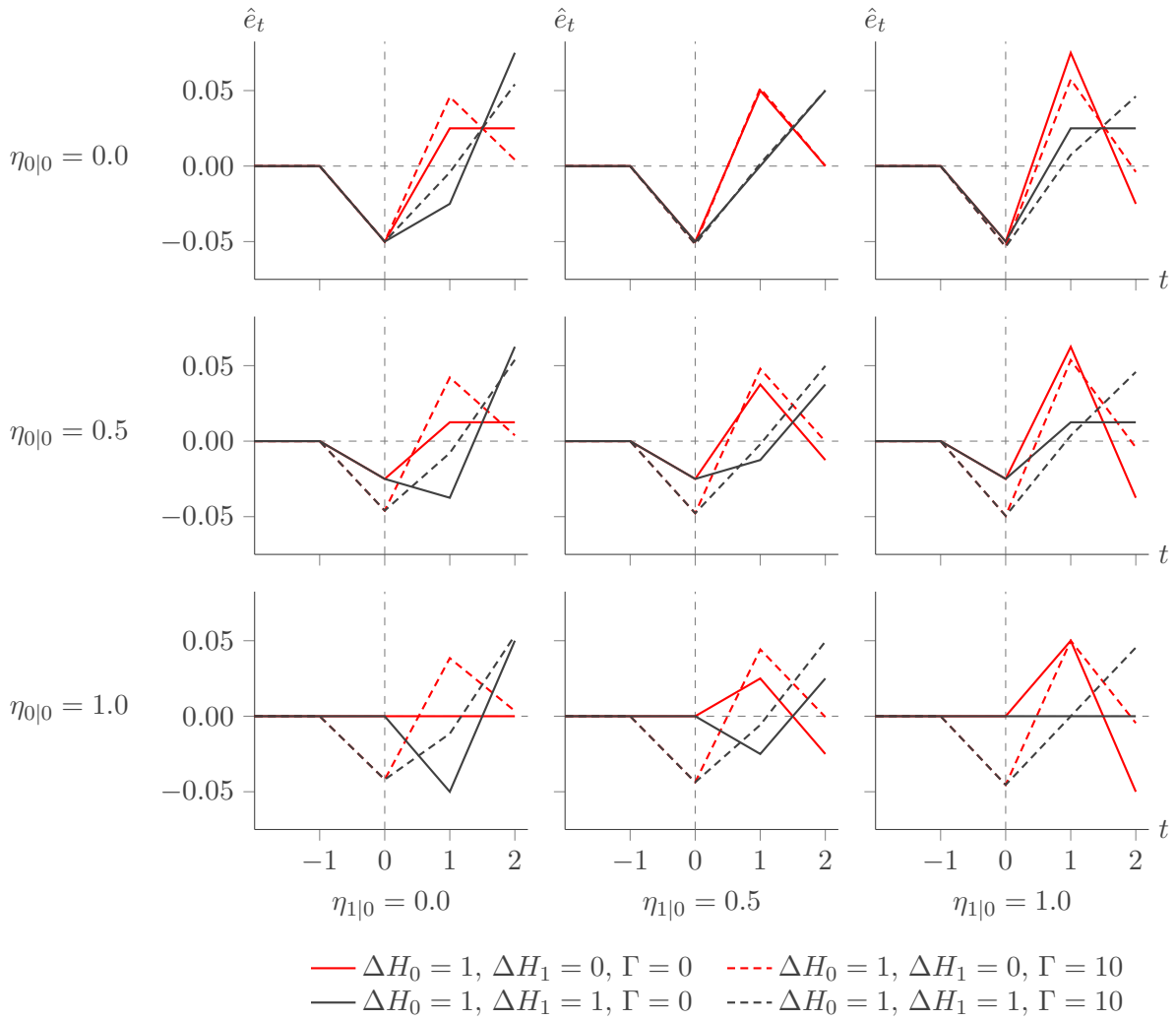
Figure A.2 plots the path of the real exchange rate in response to positive Central Bank interventions at scale $q = 0.05$. To isolate the effects of interventions, we set trade shocks to zero, $\Delta\hat{\imath}_t = 0$. In the figure, the Central Bank intervenes positively to strengthen Home currency and lower the real exchange rate $\hat{e}_0$. In a positive intervention, the Central Bank sells Foreign bonds and buys Home bonds, exchanging Foreign currency for Home currency in the process. The Home currency demand caused by the Central Bank's bond trade strengthens Home currency and put downward pressure on $\hat{e}_0$. The model's equation (17) produces what Gabaix and Maggiori (2015) describe as a "boomerang" effect.

Figure A.2 shows how the path of the real exchange rate depends on the type of intervention (swap versus spot), the degree of anticipation (unanticipated versus partially or fully anticipated), and the credit constraint (tight or loose). The initial real exchange rate response is generally larger when the intervention is unanticipated, as shown in the top row of the figure. The effect is largest when the intervention is fully unanticipated and the credit constraint is loose, as shown in the upper left figure. When fully anticipated with loose credit constraints, interventions have no effect, as shown in the bottom left and right figures. In between these extremes, the Central Bank has can shape the path of the real exchange rate in many ways by varying the type of intervention and manipulating Household and Financier expectations.

A.7 Prediction Proofs

Proof 1 (Direction of Interventions)**.** *We now prove Prediction 1. Consider a Central Bank intervention whereby the Central Bank constructs a portfolio of Home and Foreign bonds. The real exchange rate responds to the currency flows arising from the portfolio*

Figure A.2: Real Exchange Rates Dynamics.



Notes. The figure plots the path of the real exchange rate in response to Central Bank interventions at scale $q = 0.05$. To isolate the effects of interventions, we set trade shocks to zero, $\Delta\hat{\iota}_t = 0$. Red lines represent short-lived swap interventions. Black lines represents long-lived spot interventions. Solid lines represent loose credit constraints. Dashed lines represent tight credit constraints. Each row corresponds to a value for the probability of a positive period-zero intervention as perceived by Households and the Financier. Each column corresponds to a value for the probability of a positive period-one intervention as perceived by Households and the Financier. Probability values capture three cases: unanticipated ($\eta_{s|t} = 0$), partially anticipated ($\eta_{s|t} = 0.5$), and fully anticipated ($\eta_{s|t} = 1$).

construction as follows,

$$\frac{\partial \hat{e}_0}{\partial q} = -\frac{\Gamma \Delta H_0}{1+\Gamma} - \frac{\Delta H_0 - \mathrm{E}_0^{(\cdot)}[\Delta H_0]}{1+\Gamma} - \frac{\Gamma\, \mathrm{E}_0^{(\cdot)}[\Delta H_1 - \Delta H_0]}{(1+\Gamma)(3+\Gamma)}\,. \tag{61}$$

The direction of the response depends on the direction of the intervention. Recall that $\Delta H_0 \in \{-1, 0, 1\}$ *and consider two cases: 1) positive intervention, and 2) negative intervention.*

- ***Case 1:*** *Consider a* ***positive intervention****,* $\Delta H_0 = 1$*. In this case,*

$$\begin{aligned}
\frac{\partial \hat{e}_0}{\partial q} = -1 + \frac{(3+2\Gamma)\,\mathrm{E}_0^{(-)}[\Delta H_0]}{(1+\Gamma)(3+\Gamma)} - \frac{\Gamma\,\mathrm{E}_0^{(-)}[\Delta H_1]}{(1+\Gamma)(3+\Gamma)} &\overset{!}{<} 0 \\
\Leftrightarrow \quad \frac{(3+2\Gamma)\,\mathrm{E}_0^{(-)}[\Delta H_0]}{(1+\Gamma)(3+\Gamma)} - \frac{\Gamma\,\mathrm{E}_0^{(-)}[\Delta H_1]}{(1+\Gamma)(3+\Gamma)} &< 1 \\
\Leftarrow \quad \frac{3+2\Gamma}{(1+\Gamma)(3+\Gamma)} + \frac{\Gamma}{(1+\Gamma)(3+\Gamma)} &< 1 \\
\Leftrightarrow \quad \frac{3}{3+\Gamma} &< 1 \\
\Leftrightarrow \quad 0 &< \Gamma\,.
\end{aligned}$$

- ***Case 2:*** *Consider a* ***negative intervention****,* $\Delta H_0 = -1$*. In this case,*

$$\begin{aligned}
\frac{\partial \hat{e}_0}{\partial q} = 1 + \frac{(3+2\Gamma)\,\mathrm{E}_0^{(-)}[\Delta H_0]}{(1+\Gamma)(3+\Gamma)} - \frac{\Gamma\,\mathrm{E}_0^{(-)}[\Delta H_1]}{(1+\Gamma)(3+\Gamma)} &\overset{!}{>} 0 \\
\Leftrightarrow \quad \frac{(3+2\Gamma)\,\mathrm{E}_0^{(-)}[\Delta H_0]}{(1+\Gamma)(3+\Gamma)} - \frac{\Gamma\,\mathrm{E}_0^{(-)}[\Delta H_1]}{(1+\Gamma)(3+\Gamma)} &> -1 \\
\Leftarrow \quad -\frac{3+2\Gamma}{(1+\Gamma)(3+\Gamma)} - \frac{\Gamma}{(1+\Gamma)(3+\Gamma)} &> -1 \\
\Leftrightarrow \quad -\frac{3}{3+\Gamma} &< -1 \\
\Leftrightarrow \quad 0 &< \Gamma\,.
\end{aligned}$$

Hence, for $\Delta H_0 \neq 0$ *and* $\Gamma > 0$*,*

$$\operatorname{sign} \frac{\partial \hat{e}_0}{\partial q} = -\operatorname{sign} \Delta H_0\,.$$

In words, a positive intervention strengthens Home currency and lowers the real exchange rate, while a negative intervention weakens Home currency and raises the real exchange rate.

Proof 2 (Anticipated Spot and Swap Interventions)**.** *We now prove Prediction 2. Consider*

four cases of fully anticipated Central Bank interventions: 1) a positive spot intervention, 2) a positive swap intervention, 3) a negative spot intervention, and 4) a negative swap intervention.

- ***Case 1:*** *Consider a fully anticipated* ***positive spot*** *intervention in period one such that* $\Delta H_0 = \eta_{0|0} = \eta_{1|0} = 1$. *In this case, from* (61),

$$\frac{\partial \hat{e}_0}{\partial q} = -\frac{\Gamma}{1+\Gamma}.$$

- ***Case 2:*** *Consider a fully anticipated* ***positive swap*** *intervention in period one such that* $\Delta H_0 = \eta_{0|0} = 1$, $\eta_{1|0} = 0$. *In this case, from* (61),

$$\frac{\partial \hat{e}_0}{\partial q} = -\frac{\Gamma}{1+\Gamma} \times \frac{2+\Gamma}{3+\Gamma}.$$

For $\Gamma > 0$, *cases one and two imply that*

$$\left.\frac{\partial \hat{e}_0}{\partial q}\right|_{\Delta H_0 = \eta_{0|0} = \eta_{1|0} = 1} < \left.\frac{\partial \hat{e}_0}{\partial q}\right|_{\Delta H_0 = \eta_{0|0} = 1,\, \eta_{1|0} = 0},$$

where the subscripts indicate the values at which the derivative is evaluated.

- ***Case 3:*** *Consider a fully anticipated* ***negative spot*** *intervention in period one such that* $\Delta H_0 = \eta_{0|0} = \eta_{1|0} = -1$. *In this case, from* (61),

$$\frac{\partial \hat{e}_0}{\partial q} = \frac{\Gamma}{1+\Gamma}.$$

- ***Case 4:*** *Consider a fully anticipated* ***negative swap*** *intervention in period one such that* $\Delta H_0 = \eta_{0|0} = -1$, $\eta_{1|0} = 0$. *In this case, from* (61),

$$\frac{\partial \hat{e}_0}{\partial q} = \frac{\Gamma}{1+\Gamma} \times \frac{2+\Gamma}{3+\Gamma}.$$

For $\Gamma > 0$, cases three and four imply that

$$\left.\frac{\partial \hat{e}_0}{\partial q}\right|_{\Delta H_0 = \eta_{0|0} = \eta_{1|0} = -1} > \left.\frac{\partial \hat{e}_0}{\partial q}\right|_{\Delta H_0 = \eta_{0|0} = -1,\, \eta_{1|0} = 0},$$

where the subscripts indicate the values at which the derivative is evaluated. Hence, for $\Delta H_0 = \Delta\eta_{0|0} \neq 0$ and $\Gamma > 0$, cases one through four imply that

$$\left|\frac{\partial \hat{e}_0}{\partial q}\right|_{\Delta\eta_{1|0} = \Delta H_0} > \left|\frac{\partial \hat{e}_0}{\partial q}\right|_{\Delta\eta_{1|0} = 0}.$$

In words, a fully anticipated long-lived spot intervention ($\Delta\eta_{1|0} = \Delta H_0$) has a greater effect on the real exchange rate than a fully anticipated short-lived swap intervention ($\Delta\eta_{1|0} = 0$), for both positive and negative interventions. If $\Gamma = 0$ and $\Delta H_0 = \eta_{0|0}$, the right-hand size of (61) equals zero as can be seen by inspection.

Proof 3 (Unanticipated Spot and Swap Interventions)**.** *We now prove Prediction 3. Consider a Central Bank communication that raises the expectation of a positive intervention in period zero ($\Delta\eta_{0|0} > 0$). The second partial derivative of (61) with respect to the parameter that $\eta_{0|0}$ that governs the expectation of a period zero intervention is given by*

$$\frac{\partial^2 \hat{e}_0}{\partial q \partial \Delta\eta_{0|0}} = \frac{3 + 2\Gamma}{(1+\Gamma)(3+\Gamma)} > 0\,.$$

Consider a Central Bank communication that raises the expectation of a positive intervention in period one — e.g. to maintain an on-going spot intervention. The second partial derivative of (61) with respect to the parameter that $\eta_{1|0}$ that governs the expectation of a period one intervention is given by

$$\frac{\partial^2 \hat{e}_0}{\partial q \partial \Delta\eta_{1|0}} = -\frac{\Gamma}{(1+\Gamma)(3+\Gamma)} < 0\,.$$

In words, if the Central Bank carries out a positive intervention in period zero, the real exchange rate will fall by less if Households and the Financier anticipate the intervention in period zero. If the Central Bank carries out a positive intervention in period zero, the

real exchange rate will fall by more, however, if Households and the Financier anticipates a long-lived spot intervention that will be maintained in period one.

Proof 4 (Private Intermediation)**.** *We now prove Prediction 4. From the Financier's linearized bond demand in (18), using the real exchange rate solutions for periods zero and one in (58) and (58), we obtain*

$$\begin{aligned}\hat{Q}^f_{H0} = &\frac{\Delta\hat{\imath}_0 - \mathrm{E}^{(-)}_0[\Delta\hat{\imath}_1]}{3+\Gamma} + \frac{\Delta\hat{\imath}_0 - \mathrm{E}^{(-)}_0[\Delta\hat{\imath}_2]}{(1+\Gamma)(3+\Gamma)} \\ &- q\left[\frac{\Delta\eta_{0|0}}{1+\Gamma} - \frac{\Gamma(\Delta\eta_{1|0} - \Delta\eta_{0|0})}{(1+\Gamma)(3+\Gamma)}\right] + O\big(\epsilon^2\big)\end{aligned}$$

Taking the derivative of $\hat{Q}^f_{H0}$ with respect to q, we obtain

$$\frac{\partial\hat{Q}^f_{H0}}{\partial q} = \frac{\Gamma\Delta\eta_{1|0}}{(1+\Gamma)(3+\Gamma)} - \frac{(3+2\Gamma)\Delta\eta_{0|0}}{(1+\Gamma)(3+\Gamma)}\,.$$

This derivative is negative if

$$\frac{\Delta\eta_{1|0}}{\Delta\eta_{0|0}} < 2 + \frac{3}{\Gamma}\,, \tag{62}$$

which holds for any $\Gamma > 0$ as long as $\Delta\eta_{0|0}/\Delta\eta_{1|0} > 1/2$.

In words, the condition $\Delta\eta_{0|0}/\Delta\eta_{1|0} > 1/2$ requires the probability of a period-zero intervention to exceed half the probability of a period-one intervention of the same sign, conditional on information available to the Financier in period zero. In the previous sentence, probability refers to the "net" probability of a positive intervention, because $\Delta\eta_{t|0}$ gives the probability of a positive intervention minus the probability of a negative intervention in period t, conditional on information available in period zero.

Proof 5 (Credit Constraints)**.** *We now prove Prediction 5. From the Financier's bond demand in (18) and the real exchange rate solutions in (58) and (59),*

$$\begin{aligned}\hat{Q}^f_{H0} = &\frac{\Delta\hat{\imath}_0 - \mathrm{E}^{(-)}_0[\Delta\hat{\imath}_1]}{3+\Gamma} + \frac{\Delta\hat{\imath}_0 - \mathrm{E}^{(-)}_0[\Delta\hat{\imath}_2]}{(1+\Gamma)(3+\Gamma)} \\ &- q\frac{\Delta\eta_{0|0}}{1+\Gamma} + \Gamma q\frac{\Delta\eta_{1|0} - \Delta\eta_{0|0}}{(1+\Gamma)(3+\Gamma)} + O\big(\epsilon^2\big)\,.\end{aligned} \tag{63}$$

Suppose the Central Bank is passive $(q = 0)$ *and consider a tightening of credit constraints,*

$$\frac{\partial \hat{Q}^f_{H0}}{\partial \Gamma} = -\frac{\Delta \hat{\iota}_0 - \mathrm{E}_0^{(-)}[\Delta \hat{\iota}_1]}{(3+\Gamma)^2} - 2(2+\Gamma)\frac{\Delta \hat{\iota}_0 - \mathrm{E}_0^{(-)}[\Delta \hat{\iota}_2]}{(1+\Gamma)^2(3+\Gamma)^2}\,. \tag{64}$$

By inspection of (63) *and* (64)*, tighter credit constraints lower the amount of intermediation the Financier undertakes if the Central Bank is passive* $(q = 0)$*,*

$$\operatorname{sign}\frac{\partial \hat{Q}^f_{H0}}{\partial \Gamma} = -\operatorname{sign}\hat{Q}^f_{H0}\,.$$

The Financier's portfolio positions in Home and Foreign bonds shrink when credit constraints tighten, limiting the ability of Households to smooth consumption intertemporally.

Suppose the Central Bank is active $(q > 0)$ *and consider the impact that tighter credit constraints have on the real exchange rate effect of an intervention. From* (61)*,*

$$\frac{\partial^2 \hat{e}_0}{\partial q \partial \Gamma} = \frac{\left[(3-\Gamma^2) - (3+\Gamma)^2\right]\Delta\eta_{0|0}}{(1+\Gamma)^2(3+\Gamma)^2} - \frac{(3-\Gamma^2)\Delta\eta_{1|0}}{(1+\Gamma)^2(3+\Gamma)^2}\,. \tag{65}$$

To establish the sign of the partial derivative in (65)*, we evaluate four cases.*

- ***Case 1:*** *Let* $\Delta\eta_{0|0} > 0$*,* $\Delta\eta_{1|0} > 0$*, and* $3 - \Gamma^2 > 0$*. From* (65)*,*

$$\begin{aligned}\frac{\partial^2 \hat{e}_0}{\partial q \partial \Gamma} \overset{!}{<} 0 \quad &\Leftrightarrow \quad \frac{(3-\Gamma^2) - (3+\Gamma)^2}{(3-\Gamma)^2} < \frac{\Delta\eta_{1|0}}{\Delta\eta_{0|0}} \\ &\Leftrightarrow \quad -\frac{6+6\Gamma+2\Gamma^2}{(3-\Gamma)^2} < \frac{\Delta\eta_{1|0}}{\Delta\eta_{0|0}}\,,\end{aligned}$$

  which is satisfied under the assumptions in Case 1.

- ***Case 2:*** *Let* $\Delta\eta_{0|0} > 0$*,* $\Delta\eta_{1|0} > 0$*, and* $3 - \Gamma^2 < 0$*. From* (65)*,*

$$\begin{aligned}\frac{\partial^2 \hat{e}_0}{\partial q \partial \Gamma} \overset{!}{<} 0 \quad &\Leftrightarrow \quad \frac{(3-\Gamma^2) - (3+\Gamma)^2}{(3-\Gamma)^2} > \frac{\Delta\eta_{1|0}}{\Delta\eta_{0|0}} \\ &\Leftrightarrow \quad \frac{(3+\Gamma)^2}{3-\Gamma^2} > \frac{\Delta\eta_{1|0} - \Delta\eta_{0|0}}{\Delta\eta_{0|0}} \\ &\Leftarrow \quad 1 > \frac{\Delta\eta_{1|0} - \Delta\eta_{0|0}}{\Delta\eta_{0|0}} \quad \Leftrightarrow \quad \Delta\eta_{0|0} > \frac{1}{2}\Delta\eta_{1|0}\,.\end{aligned}$$

  Hence, the inequality is satisfied in Case 2 when $\Delta\eta_{0|0} > \frac{1}{2}\Delta\eta_{1|0}$*.*

- ***Case 3:*** *Let* $\Delta\eta_{0|0} < 0$, $\Delta\eta_{1|0} < 0$, *and* $3 - \Gamma^2 > 0$. *From* (65),

$$\frac{\partial^2 \hat{e}_0}{\partial q \partial \Gamma} \overset{!}{>} 0 \quad \Leftrightarrow \quad \frac{(3-\Gamma^2) - (3+\Gamma)^2}{(3-\Gamma)^2} < \frac{\Delta\eta_{1|0}}{\Delta\eta_{0|0}}$$
$$\Leftrightarrow \quad -\frac{6 + 6\Gamma + 2\Gamma^2}{(3-\Gamma)^2} < \frac{\Delta\eta_{1|0}}{\Delta\eta_{0|0}},$$

  which is satisfied under the assumptions in Case 3.

- ***Case 4:*** *Let* $\Delta\eta_{0|0} < 0$, $\Delta\eta_{1|0} < 0$, *and* $3 - \Gamma^2 < 0$. *From* (65),

$$\frac{\partial^2 \hat{e}_0}{\partial q \partial \Gamma} \overset{!}{>} 0 \quad \Leftrightarrow \quad \frac{(3-\Gamma^2) - (3+\Gamma)^2}{(3-\Gamma)^2} > \frac{\Delta\eta_{1|0}}{\Delta\eta_{0|0}}$$
$$\Leftrightarrow \quad \frac{(3+\Gamma)^2}{3-\Gamma^2} > \frac{\Delta\eta_{1|0} - \Delta\eta_{0|0}}{\Delta\eta_{0|0}}$$
$$\Leftarrow \quad 1 > \frac{\Delta\eta_{1|0} - \Delta\eta_{0|0}}{\Delta\eta_{0|0}} \quad \Leftrightarrow \quad \Delta\eta_{0|0} < \frac{1}{2}\Delta\eta_{1|0}.$$

  Hence, the inequality is satisfied in Case 4 when $\Delta\eta_{0|0} < \frac{1}{2}\Delta\eta_{1|0}$.

From (61) *and* (65) *and Cases 1–4, under the regularity condition that* $|\Delta\eta_{0|0}| > \frac{1}{2}|\Delta\eta_{1|0}|$, *and assuming the Central Bank's communications are consistent with its actions* ($\operatorname{sign}\Delta\eta_{0|0} = \operatorname{sign}\Delta\eta_{1|0} = \operatorname{sign}\Delta H_0 \neq 0$), *we have*

$$\operatorname{sign}\frac{\partial^2 \hat{e}_0}{\partial q \partial \Gamma} = \operatorname{sign}\frac{\partial \hat{e}_0}{\partial q},$$

and tighter credit constraints amplify interventions.

A.8 Sterilized Intervention

Central banks can manage exchange rates without altering the money supply through sterilized intervention. Consider a spot purchase of Foreign currency as an example intervention. In a non-sterilized intervention, the Central Bank purchases Foreign currency with Home currency, increasing the Home money supply. To "sterilize" the impact, the Central Bank sells Home-currency bonds to absorb the excess Home currency. If the sterilization is perfect, the money supply remains constant, while the ratio of Home-currency and Foreign-currency bonds held by the public and the central bank changes.

Table A.1: Central Bank Balance Sheet View of Sterilized Intervention.

1. Purchase Foreign Currency

Assets	Liabilities
Assets in Foreign Currency (+1)	Circulating Home Currency (+1)

2. Sell Home Bonds

Assets	Liabilities
Assets in Home Currency (−1)	Circulating Home Currency (−1)

3. Net Effect

Assets	Liabilities
Assets in Foreign Currency (+1)	Circulating Home Currency (±0)
Assets in Home Currency (−1)	

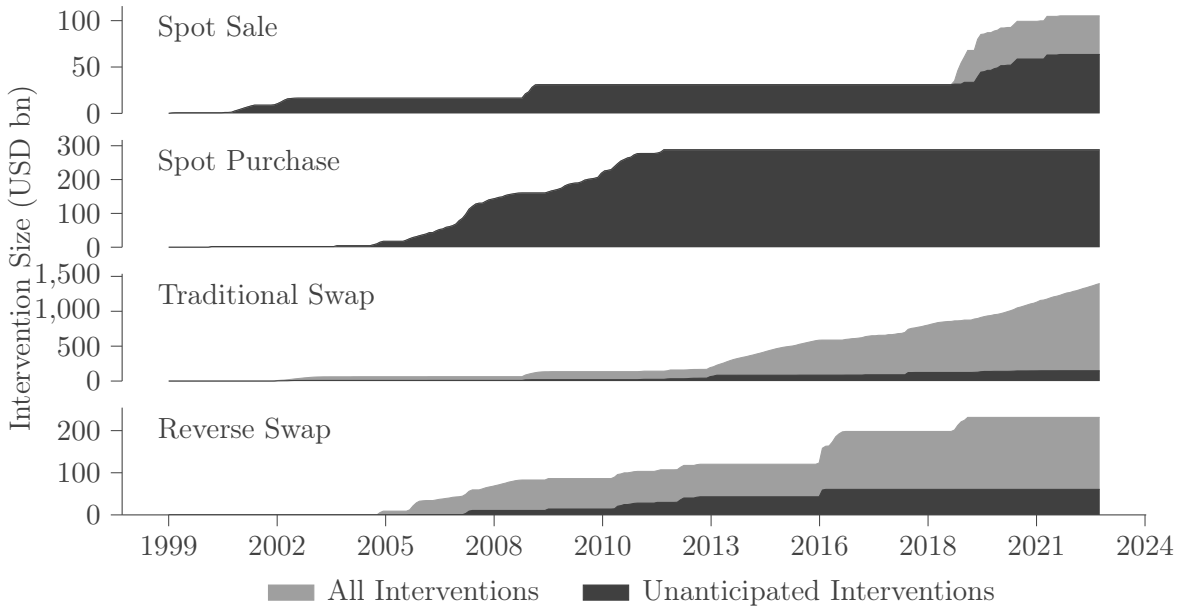
Notes. The tables illustrate the process of sterilizing a Central Bank intervention with respect to Home currency. Step 1 shows the Central Bank purchasing Foreign currency with Home currency, increasing the Home currency in circulation. Step 2 shows the Central Bank selling Home-currency bonds to offset the rise in Home currency from Step 1. Step 3 shows the net effect of Steps 1 and 2: the Home money supply remains unchanged, while the Central Bank's Foreign currency assets increase and its Home currency assets decrease. This sterilized intervention keeps the supply of Home currency constant but alters the composition of the central bank's assets.

A sterilized intervention can be viewed as a combination of two transactions. First, in the FX market, the Central Bank conducts a non-sterilized intervention by purchasing Foreign currency (or Foreign-currency bonds), first issuing Home currency to fund the purchase. Second, in the money market, the Central Bank "sterilizes" the effect by selling an equivalent amount of Home bonds to absorb the initial increase in the Home money supply. The net effect of a sterilized spot purchase is neutral regarding the Home currency in circulation, but there is a portfolio change in assets, with an increase (decrease) in the share of Foreign-currency assets held by the central bank (public). Table A.1 illustrates these steps in a stylized Central Bank balance sheet.

B Empirics

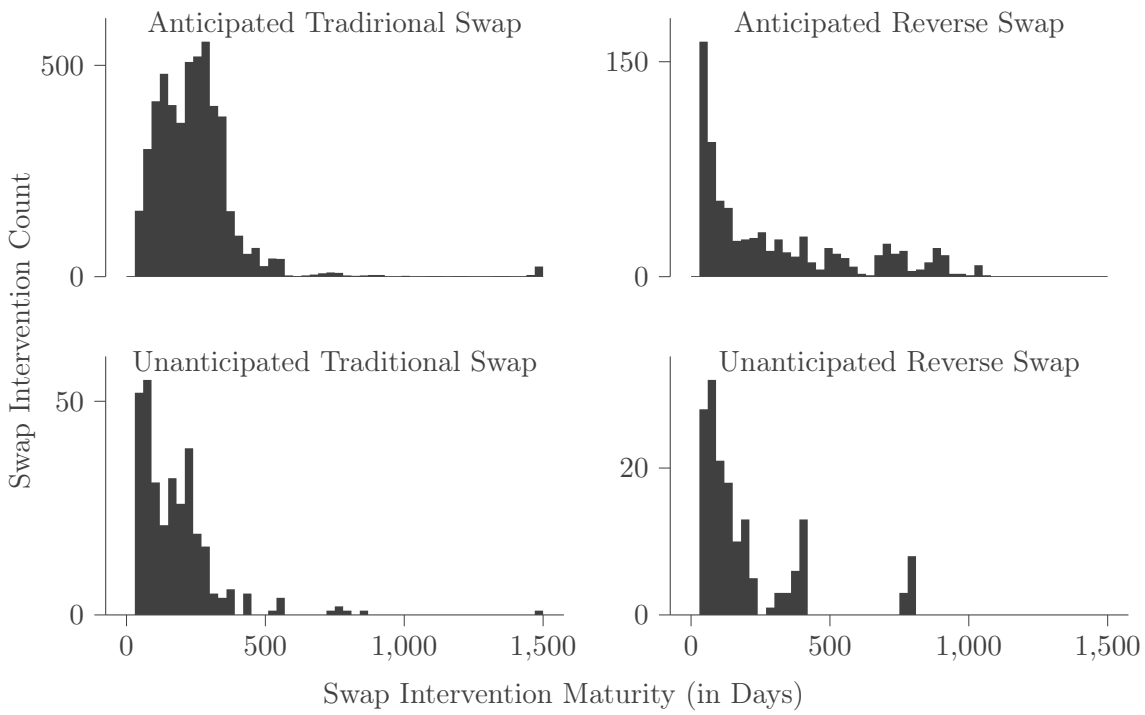
B.1 Description of BCB FXI

Figure B.1: The Cumulative BCB's Intervention in FX markets since 2000



Notes. These figures show the cumulative UCB FX market interventions over time, by intervention type. The horizontal axes show the intervention size in USD billions, after aggregating at monthly frequency. The lighter shade shows all interventions, both anticipated and unanticipated, while the darker shade shows the subset of unanticipated interventions, defined as interventions with an announcement equal to the operation date.

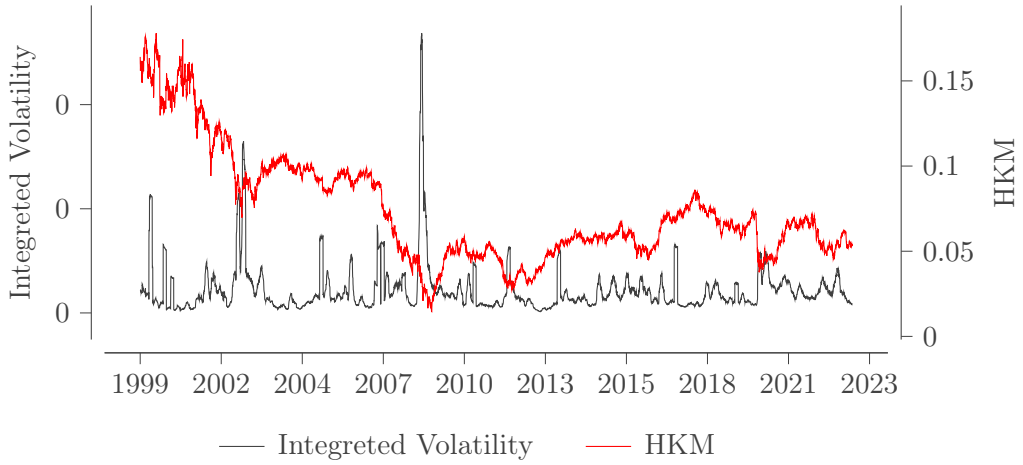
Figure B.2: Maturity breakdown of BCB's Swap Line Interventions



Notes. These figures show 50-bin histograms of the maturities of BCB's interventions in the FX markets, by intervention type. The horizontal axes show the maturity in days. Unanticipated interventions are defined as interventions with an announcement equal to the operation date, while anticipated interventions are defined as interventions with an announcement date that precedes the operation date. Traditional swaps involve the sale of USD while reverse swaps involve the purchase of USD at the spot leg of the swap contract. The sample period runs from 1999-01-22 to 2023-04-27.

B.2 Variance of real exchange rate and intermediary constraints

Figure B.3: Integrated volatility of BRL/USD exchnage rate and HKM factor



Notes. This figure plots the 30-day moving averages of the BRL/USD spot rate integrated volatility (blue) and the HKM factor (red). Integrated volatility is calcuated using high frequency spot quotes from Thomson Reuters Tick History. The HKM factor is constructed using the subset of primary dealers that deal with emerging market currencies following Cerutti and Zhou (2023). The sample period is from 1999-01-22 to 2023-04-27.

Table B.1: BCB's FXI Summary Statistics

	Spot Sale		Spot Purchase	
	Unanticipated	Anticipated	Unanticipated	Anticipated
Mean	0.17	0.48	0.19	NaN
S.D	0.22	0.39	0.24	NaN
Max	1.10	3.00	4.64	NaN
Count	385	87	1483	NaN
	Traditional Swap		Reverse Swap	
	Unanticipated	Anticipated	Unanticipated	Anticipated
Mean	0.43	0.25	0.35	0.20
S.D	0.41	0.24	0.45	0.28
Max	1.85	3.50	3.38	4.00
Count	345	5094	174	846

Notes. This table shows the mean, standard deviation, maximum, and total number of counts for BCB's FXI. For unanticipated interventions, the announcement date is equal to the operation date. For anticipated interventions, the announcement date precedes the operation date. Traditional (reverse) swap is the sale (purchase) of USD at the spot leg of the swap contract. The sample period runs from 1999-01-22 to 2023-04-27. Mean, standard deviation (S.D) , and max are denominated in USD billion.

Table B.2: Number of Days with Single or multiple FXIs

	Spot Sale		Spot Purchase	
	Unanticipated	Anticipated	Unanticipated	Anticipated
$= 1$	252	85	1157	NaN
$= 2$	54	1	157	NaN
≥ 3	8	0	4	NaN
	Traditional Swap		Reverse Swap	
	Unanticipated	Anticipated	Unanticipated	Anticipated
$= 1$	27	67	8	84
$= 2$	48	1055	10	35
≥ 3	48	805	33	175

Notes. This table shows the number of days with single or multiple BCB's FXIs for each type of intervention. For unanticipated interventions, the announcement date is equal to the operation date. For anticipated interventions, the announcement date precedes the operation date. Traditional (reverse) swap is the sale (purchase) of USD at the spot leg of the swap contract. The sample period runs from 1999-01-22 to 2023-04-27.

Table B.3: Summary Statistics for BRL/USD Spot rate, Currency Basis, and Forward Premium.

	BRL/USD Spot Rate	1m BRL/USD Currency Basis	1m BRL/USD Forward Premium
Mean	2.773	-220.106	186.341
S.D	1.105	133.919	111.107
Min	1.207	-639.128	-79.337
25%	1.916	-302.106	101.390
75%	3.268	-124.770	275.808
Max	5.905	147.176	638.495

Notes. This table shows the mean, standard deviation (S.D), minimum, 25% percentile, 75% percentile, and maximum for Spot rate, currency basis, and forward premium. Spot rate is expressed in units of Brazil Real per U.S. dollar. Currency basis and forward premium are expressed in basis points. The sample period for spot rate is from 1999-01-22 to 2023-04-27. The sample period for currency basis and forward premium is from 2003-11-24 to 2023-04-27.

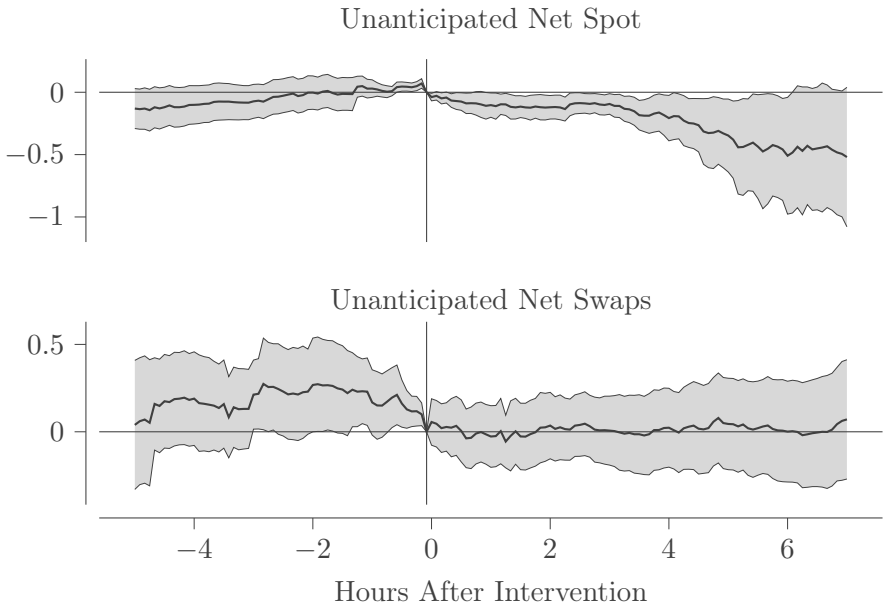
Table B.4: Summary Statistics for control variables

	(1) HKM	(2) $EMBI$	(3) i^{US}	(4) i^{BR}	(5) i_s^{US}	(6) i_s^{BR}	(7) INT^{TOL}	(8) S^{VOL}
Mean	0.074	0.042	1.531	11.712	0.614	0.551	0.644	1.253×10^{-6}
S.D	0.032	0.033	1.679	5.103	0.523	2.121	0.785	2.273×10^{-5}
Min	0.014	0.014	0.051	1.888	-3.485	-17.167	0	0
25%	0.051	0.023	0.149	8.249	0.298	-0.614	0.137	2.238×10^{-7}
75%	0.093	0.046	2.315	14.135	0.881	1.198	0.750	8.197×10^{-7}
Max	0.178	0.244	6.875	37.333	2.820	14.175	8.850	2.051×10^{-3}

Notes. This table shows the mean, standard deviation, minimum, 25% percentile, 75% percentile, and maximum for daily frequency control variables used in Eq. (31) and Eq. (34). The variables include (1) HKM, intermediary capital ratio, (2) $EMBI$, emerging Markets Bond Index Plus for Brazil from JP Morgan, (3)i^{US} one-day US Libor rate, (4) i^{BR} one-day Brazil inter-bank rate, (5) i_s^{US}, one-year minus one-day US Libor rate spread, (6) i_s^{BR} one-year minus one-day Brazil inter-bank rate spread, (7) INT^{TOL}, the total amount of interventions in USD (of all instruments) , (8) S^{VOL}, spot market volatility. (3), (4), (5), and (6) are denominated in percentage points. (7) is denominated in billion USD. The sample period runs from 1999-01-22 to 2023-04-27.

B.3 Net (Sell-Buy) Interventions

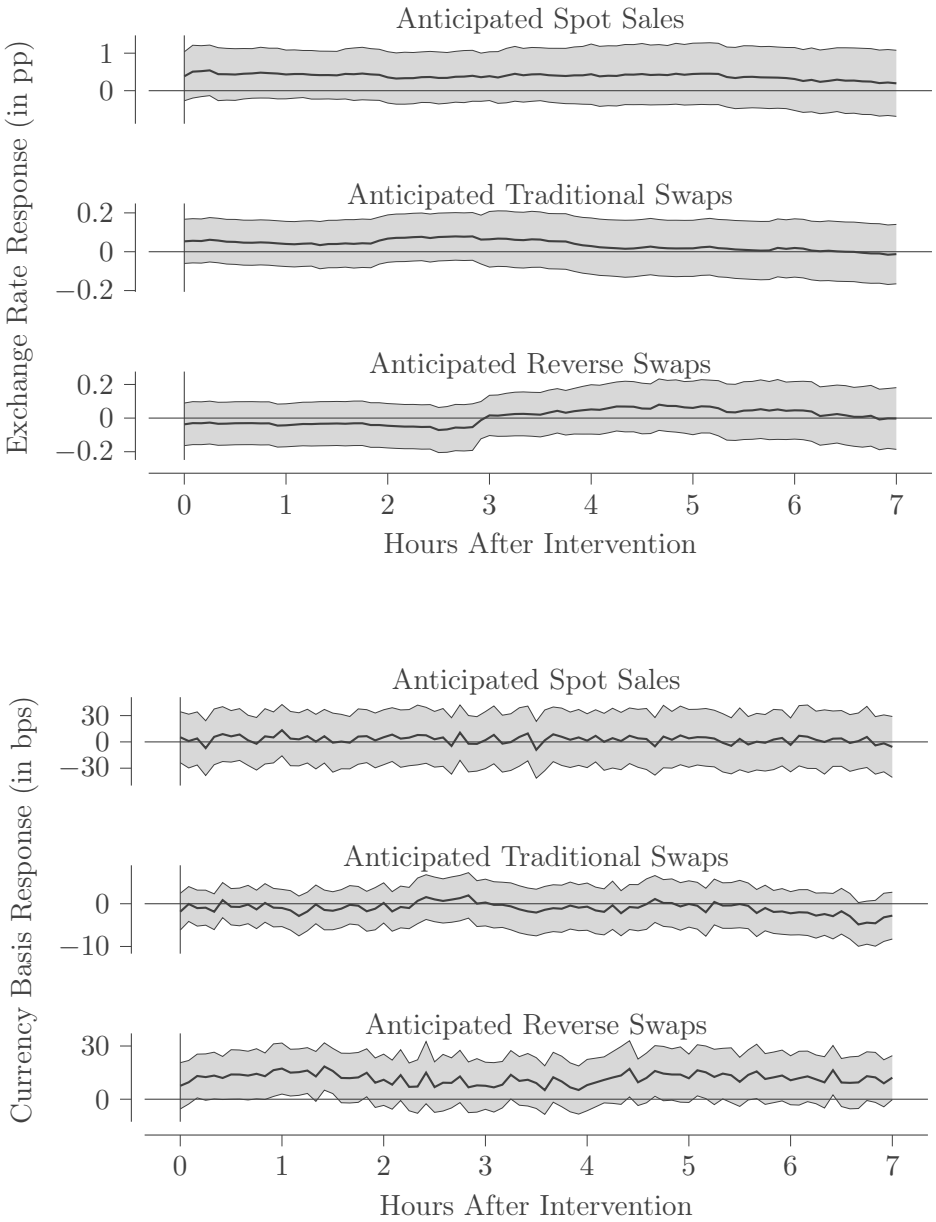
Figure B.4: Dynamic Response to Unanticipated Net (Sell-Buy) Interventions.



Notes. Net Spot FXI (top), Net Swap FXI (bottom). Net spot FXI is calculated as spot sale minus spot purchase. Net swap FXI is calculated as traditional swap minus reverse swap. Unanticipated interventions are measured in USD Billion. Traditional (reverse) swap is the sale (purchase) of USD at the spot leg of the swap contract. The dotted lines denote a 95% confidence interval using White's heteroscedasticity-robust standard errors. The sample period is from 1999-01-22 to 2023-04-27.

B.4 Anticipated Interventions

Figure B.5: BRL/USD Exchange Rate and Currency Basis Responses to Unanticipated BCB Interventions on Operational Date.



Notes. The upper group of figures show BRL/USD exchange rate responses to anticipated BCB interventions on operational date measured in percentage points (pp), while the lower group figures show BRL/USD currency basis responses to anticipated BCB interventions on operational date measured in basis points (bps). Within each group, the figures show responses to the BCB's spot sale, spot purchase, traditional swap, and reverse swap interventions. Traditional swaps involve the sale of USD while reverse swaps involve the purchase of USD at the spot leg of the swap contract. Shading indicates a 95% confidence interval using White's heteroscedasticity-robust standard errors. The sample period runs from 1999-01-22 to 2023-04-27.

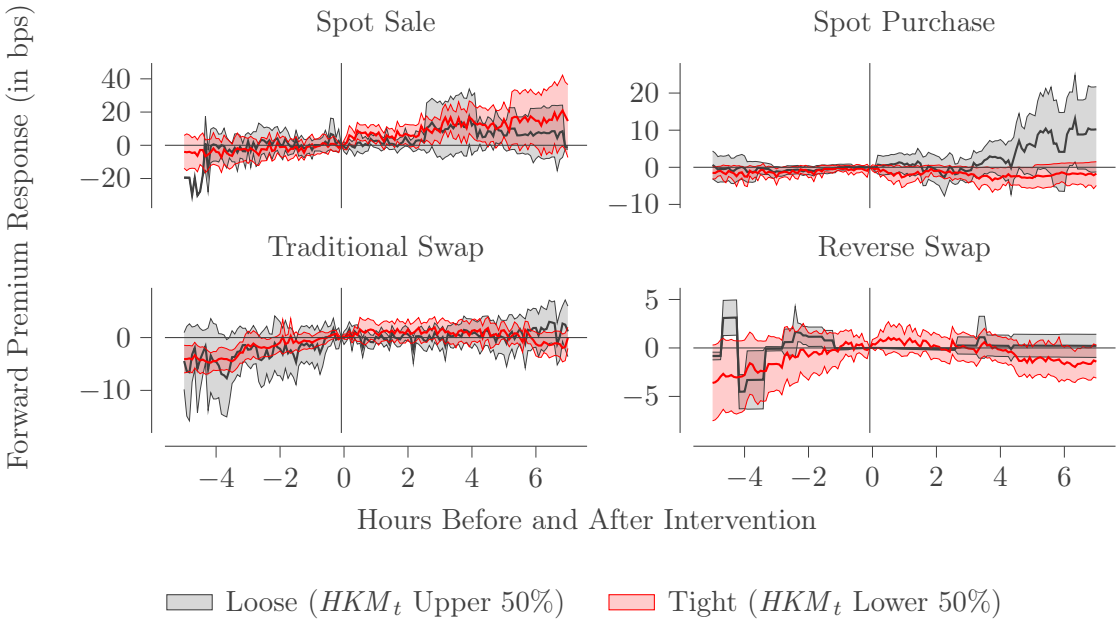
B.5 FXI Impact: Forward Premium

Figure B.6: Dynamic Response of forward premium to BCB's Unanticipated Interventions.



Notes. Spot sales(top left), spot purchase(top right), traditional Swap(bottom left), and reverse Swap(bottom Right). Forward premium is the difference between forward and spot BRL/USD rates, measured in basis points. Traditional (reverse) swap is the sale (purchase) of USD at the spot leg of the swap contract. The shaded area denotes a 95% confidence interval using White heteroscedasticity-robust standard errors. The sample is from 2003-11-24 to 2023-04-27.

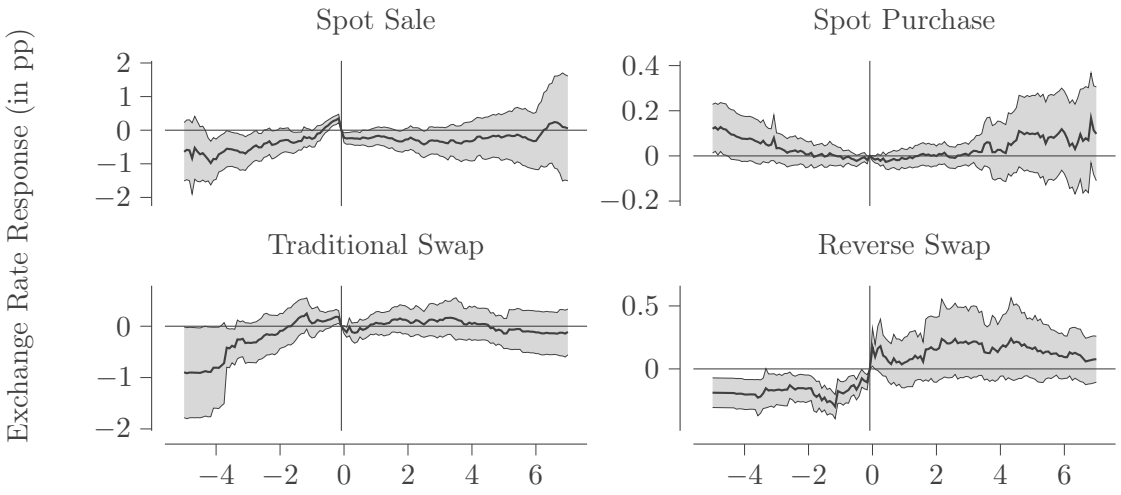
Figure B.7: Heterogeneous Response of forward premium to BCB's Unanticipated Interventions, HKM Ratio



Notes. The figures show BRL/USD 1-month forward premium rate responses to unanticipated BCB interventions in basis points (bps). The figures show responses to the BCB's spot sale, spot purchase, traditional swap, and reverse swap interventions. Each figure plots responses conditional on the state of FX dealers' balance sheet constraints on the intervention day. We measure balance sheet constraints by constructing the intermediary capital ratio HKM_t of He et al. (2017) for the FX dealer banks listed in Cerutti and Zhou (2023). Red indicates tight constraints, defined as periods with HKM_t in the lower 50% of values in our sample. Gray indicates loose constraints, defined as periods with HKM_t in the upper 50% of values in our sample. Shading indicates a 95% confidence interval using White's heteroscedasticity-robust standard errors. The sample period runs from 2003-11-24 to 2023-04-27.

B.6 FXI Impact: one Intervention versus Multiple Interventions

Figure B.8: Dynamic Response of Spot rate to BCB's Unanticipated Interventions in Days with One FXI.



Notes. Spot sales(top left), spot purchase(top right), traditional Swap(bottom left), and reverse Swap(bottom Right). Forward premium is the difference between forward and spot BRL/USD rates, measured in basis points. Traditional (reverse) swap is the sale (purchase) of USD at the spot leg of the swap contract. The shaded area denotes a 95% confidence interval using White heteroscedasticity-robust standard errors. The sample is from 2003-11-24 to 2023-04-27.

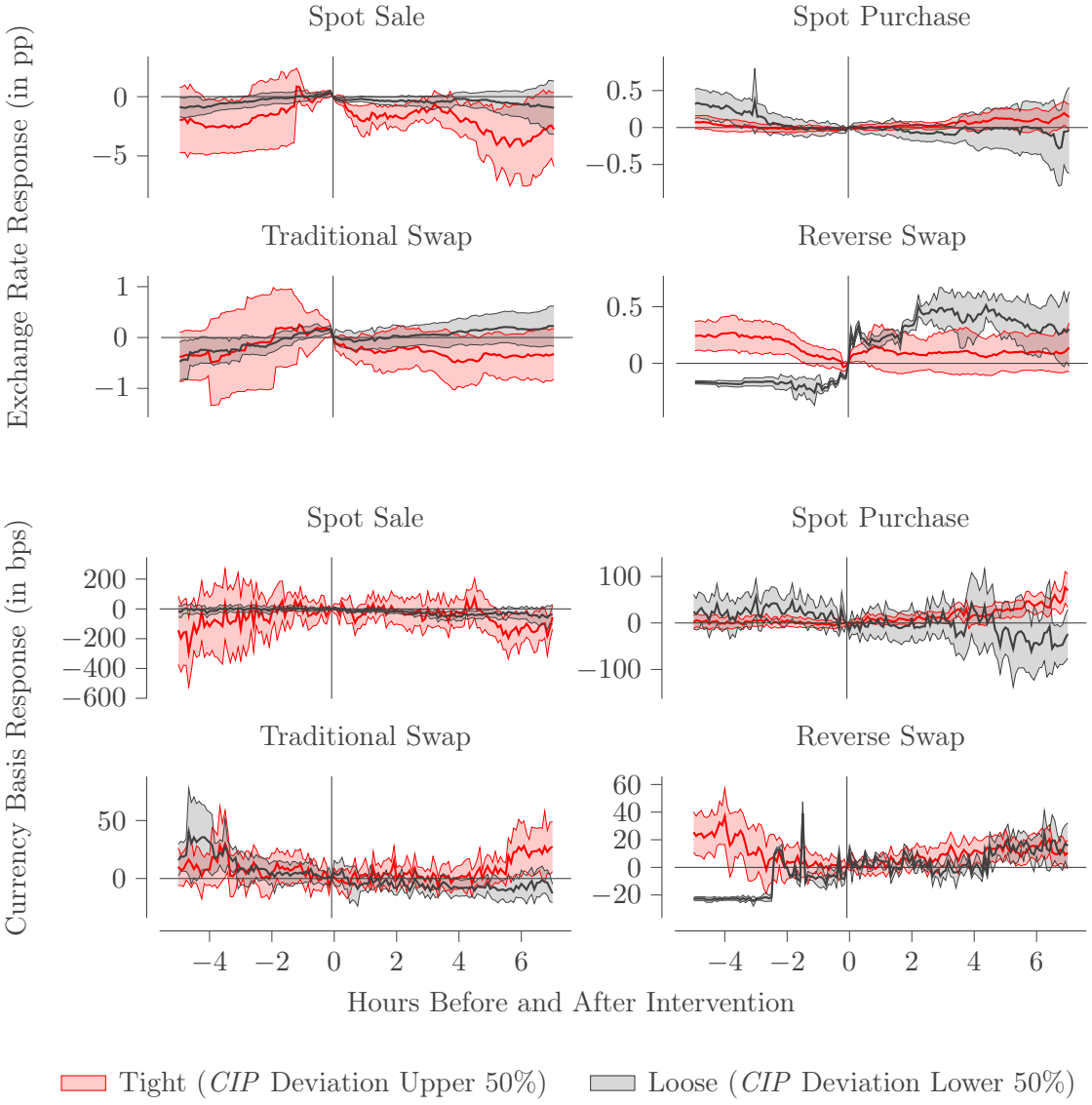
Figure B.9: Dynamic Response of Spot rate to BCB's Unanticipated Interventions in Days with Multiple FXIs.



Notes. Spot sales(top left), spot purchase(top right), traditional Swap(bottom left), and reverse Swap(bottom Right). Forward premium is the difference between forward and spot BRL/USD rates, measured in basis points. Traditional (reverse) swap is the sale (purchase) of USD at the spot leg of the swap contract. The shaded area denotes a 95% confidence interval using White heteroscedasticity-robust standard errors. The sample is from 2003-11-24 to 2023-04-27.

B.7 Heterogeneity effects: level of CIP violation

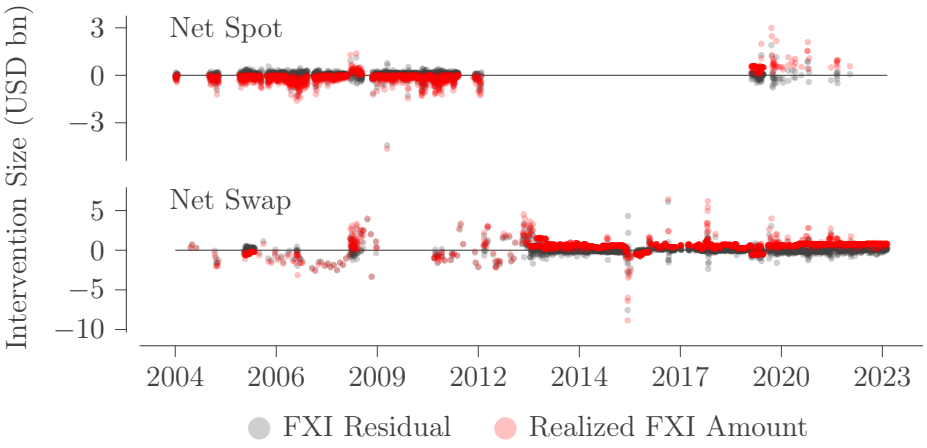
Figure B.10: Heterogeneous Response of BRL/USD Exchange Rate and Currency Basis Responses to Unanticipated BCB Interventions, Absolute Value of CIP violation.



Notes. The upper group figures show BRL/USD exchange rate responses to unanticipated BCB interventions in percentage points (pp), while the lower group figures show BRL/USD currency basis responses to unanticipated BCB interventions in basis points (bps). Within each group, the figures show responses to the BCB's spot sale, spot purchase, traditional swap, and reverse swap interventions. Each figure plots responses conditional on the level of CIP deviation on intervention day. Red indicates tight constraints, defined as periods with level of *CIP* deviation in the upper 50% of values in our sample. Gray indicates loose constraints, defined as periods with level of *CIP* deviation in the upper 50% of values in our sample. Shading indicates a 95% confidence interval using White's heteroscedasticity-robust standard errors. The sample period runs from 2003-11-24 to 2023-04-27.

B.8 Residualized FXI measure

Figure B.11: FXI Residual.



Notes. Figure plots realized values of *FXI* in red, along with prediction errors in grey, for BCB foreign exchange interventions 1999-2023. Top panel shows realized net spot sale interventions and residuals, and the bottom panel shows net swap sale interventions and residuals. The linear FXI prediction model we use to obtain the residuals is defined in equation (66).

To construct a residualized measure of FXI, we regress the amount of intervention on controls, then take the residual as an exogenous component of FXI. This is a similar procedure to the identification of FXI in Rodnyansky et al. (2023). We define the intervention about FXI_t as positive for net sales and negative for net purchase, and write the regression model we use to obtain residuals as

$$FXI_t = \alpha + \beta z_{t-1} + \epsilon_t \,, \tag{66}$$

for all dates on which FXI was non-zero, i.e. for all t such that $FXI_t \neq 0$, where FXI_t denotes the dollar amount of FXI in USD bn. The vector of control variables z_{t-1} includes lagged Brazilian interest rate expectations, the BRL/USD spot rate, the HKM intermediary capital ratio, sovereign default risk, exchange rate volatility, U.S. and Brazilian interest rates, monetary policy announcements, US recessions, forward premia, and FX interbank trading volume. We plot the realized FXI values and residuals in Figure B.11.