Global Trade Shocks and Local Agricultural Policy in Rwanda*

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Abstract

We study how the dramatic fertilizer price shock in late 2021 caused by the Russian invasion of Ukraine affects Rwandan farmers. We do so by collecting household-level panel data from 2020-2024 from 444 rural Rwandan villages and traders in the markets where these households buy and sell products. Fertilizer prices paid by farmers rise 70 percent in the 6 months between late 2021 and early 2022. Villages that used more fertilizer before the shock experience larger drops in harvest and villagers return to farming from non-agricultural employment. We structurally estimate a general equilibrium model of farmers subject to financial frictions using our empirical results, and show how the empirics inform elasticities that determine not only the size of optimal fertilizer subsidy but even whether the optimal subsidy should rise or fall in response to higher prices. We highlight under what conditions other (equally blunt) policy tools can be used more effectively than fertilizer subsidies when production takes place across technologically heterogeneous villages.

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1 Introduction

Few markets in the developing world are subject to more government intervention than the fertilizer market, with subsidies alone making up a quarter of agricultural budgets in many African countries (Jayne et al., 2018; Holden, 2019). The rationale is straightforward. Developing country agriculture is characterized by low productivity and a high share of aggregate employment (Gollin et al., 2014), due in large part to the lack of input intensification, including critical intermediate inputs like fertilizer (Restuccia et al., 2008; Boppart et al., 2023). To the extent these facts are explained by market failures – credit, risk, etc. – there is, at least in principle, a role for subsidies.

A second, perhaps less appreciated, feature of the fertilizer market in Sub-Saharan Africa is that it is entirely imported, exposing these economies to fluctuations in international prices. In this paper, we study how farmers are affected a large shock to the international price of fertilizer and how government fertilizer policy should respond. We take on these issues in Rwanda. We first document empirically how the Russian invasion of Ukraine generated an unexpected shock to global fertilizer prices in 2021 and filtered down to Rwandan farmers using primary micro data covering 444 villages from 2020-2024. We then use those results to estimate and quantify how fertilizer subsidy policy should respond in a general equilibrium model.

Rwanda, like other Sub-Saharan African countries, imports the entirety of their fertilizer. The Russian invasion of Ukraine had two important effects. First, it restricted fertilizer exports from Belarus and Russia, two of the most important global fertilizer exporters. Second, the E.U.'s attempted to replace Russian oil with American natural gas increased gas prices. This raised the cost of ammonia, a key input into many fertilizers to which natural gas is a critical input. International fertilizer prices more than doubled in just 6 months as a consequence of these global economic forces outside of Rwanda's control.

The impact in Rwanda was felt immediately. The Rwandan government, who controls the domestic price of fertilizer via their national subsidy scheme, was forced into an unprecedented off-schedule price increase of about 40 percent in January 2022, despite a historical precedent of adjusting prices annually each July. Prices offered by fertilizer vendors in our data rise by the same amount (we discuss data collection below). This January price increase corresponds to one of the most important fertilizer-buying months on the agricultural calendar: one of the two main cropping seasons begins in February. For a sense of scale, the program's cost increased from 5 billion Rwandan francs in 2019 to 13 billion in 2022.

We start by detailing the empirical impact of this shock using household-level

panel data we collected around most of Rwanda. This annual panel runs from 2020 - 2024 and contains four waves of data, covering about 45,000 individuals from 15,000 households across 444 villages. We create plausibly exogenous variation in the impact of the fertilizer price shock with a shift-share approach. We use baseline village-level fertilizer intensity (total fertilizer expenditures per acre of farmland in production) as a measure of exposure and will refer to it as such throughout. The rationale for village-level exposure instead of individual-level exposure is due to within-village general equilibrium spillovers we document empirically.

We find that farmers in more exposed villages decrease fertilizer use substantially in the aftermath of the shock. The most exposed villages decrease fertilizer use by 28 percentage points more than those in the least exposed villages. This impact translates into harvests, with a gap of 19 percentage points between the most and least exposed. These output changes require a reallocation of inputs in the economy. They do not come from land, as we observe no changes in land rental, either in or out, suggesting difficulty of adjusting input structure in the short run due to land market frictions highlighted elsewhere (Chen et al., 2022; Acampora et al., 2023). Instead, the reallocation operates primarily through labor. High fertilizer villages begin to more seriously specialize in agriculture. Total wage earnings decrease by 17 percent, driven by a substantial 33 percent decline in earnings outside the village, which is dominated by non-agricultural jobs. Price changes are consistent with these patterns of pulling employment back into agriculture. Both village agricultural wages and crop sales prices rise in fertilizer intensive villages, the latter of which is concentrated among fertilizer-intensive crops.

These effects imply both a large average impact and substantial heterogeneity across space, which has distributional consequences for the Rwandan economy. The Rwandan government responded to the shock by increasing its per-unit fertilizer subsidy from 30 to 45 percent. To complement the empirics, we develop and estimate a general equilibrium model of rural Rwanda to investigate critically how fertilizer subsidies should evolve in response to a fertilizer price shock. We do so not just because of changes in Rwanda, but because the design of such policies have found renewed interest in both the popular press (Shah, 2022) and among policymakers (Malpass, 2022) in light of the global fertilizer contraction studied here and the willingness of governments to involve themselves in this market.

The model is one of household-farmers subject to limited borrowing and financial frictions, similar to canonical models reviewed in Buera et al. (2015) and Kaboski (2023). The model has three core features, all of which are empirically justified. First, farmers have to pay inputs from savings before harvest payoffs are realized

and have limited ability to fund inputs via borrowing. Second, villages differ in their underlying technologies, varying specifically in the technological intensity of fertilizer relative to labor. Third, village production may not be perfectly substitutable in the consumption market.

The first assumption creates scope for policy. Because the fertilizer price enters directly into the financial friction, a price shock like the one observed in 2021 makes it more difficult for farmers to fund production. Thus, the policymaker attempts to undue the distortion created by these higher prices (subject to funding the increased cost). The second assumption creates different options by which to accomplish this goal. While villages with more fertilizer-intensive technology are most directly affected by the higher fertilizer price, technological heterogeneity across villages implies that the policymaker can induce more production from other, relatively unscathed villages to make up for lost production. The third assumption determines whether or not this is a viable strategy. If village production is complementary, increasing production in other locations to replace lost production may not be optimal. In this case, the policymaker would instead prefer to incentivize production from exactly those high-fertilizer-intensity villages that were most directly affected by the shock.

We formalize this intuition and show that the policymaker's role is to help achieve the productive redistribution that would have occurred in a frictionless world via general equilibrium price movements. At issue is that financially constrained villages cannot respond fully to output price changes, thus muting equilibrium price effects. Government policy then attempts to do the rest. When a fertilizer subsidy is the only policy lever available, this leads to a stark characterization. If villages are substitutes, the government should *lower* the fertilizer subsidy when the fertilizer price rises. This allows the government to push production toward less affected, low fertilizer intensity villages that can substitute the lost production. The opposite is true if villages are complements. Both financial frictions and technological heterogeneity are critical to this result: if either is removed, the fertilizer subsidy does not vary with the price.

We then quantify the implications of our model for government policy, adding additional quantitative richness relative to our analytic model. One take-away from the theoretical results, however, is that parameters other than the tightness of financial frictions determine both quantitative and qualitative features of the optimal subsidy schedule. We estimate the model using our empirical results and show how they inform key parameters in the model. We find that village production is substitutable in producing the agricultural consumption good. Thus, in line with our analytic results, the optimal fertilizer subsidy falls after a fertilizer price shock. While the sign of this change does not depend on the tightness of the financial frictions, they are

central to quantitative magnitudes. We show how to map changes in distortions to our observable empirical results, and thus can target them directly.

To measure the quantitative impact, we start the economy in its stationary equilibrium, motivated by the stability of fertilizer prices in the decade leading up to this 2021 shock. We then introduce a so-called "MIT shock," where the fertilizer price unexpectedly rises by 70 for one period only and trace the impulse response of the economy back to steady state. We consider two exercises related to the fertilizer subsidy. The first is that we restrict the government to adjust only the subsidy in the period of the shock, requiring them to fix it at its baseline level for all future periods. Second, we trace out the optimal path of the subsidy after the one-time shock.

Quantitatively, we find that the optimal fertilizer subsidy falls by 60 percent on impact. That the subsidy falls is a function of fact that our empirics point us toward villages that are substitutable.[1] The quantitative magnitude depends on the sustainability between labor and fertilizer in the production function and the tightness of financial frictions, both of which we use our empirical results to estimate.

Finally, motivated by the underlying mechanisms that drive changes to the optimal fertilizer subsidy, we explore how other policies may be better suited to respond to higher fertilizer prices. We consider two: subsidies to production and subsidies to labor. As in our baseline case, we assume these subsidies are uniform and cannot target specific villages.[2] We find that increasing labor subsidies are the best response to the fertilizer price shock. Because we find that villages are substitutes, the policy maker wants to shift production toward low-fertilizer-intensity or, identically, high-labor-intensity villages. Thus, increasing a labor subsidy has the same qualitative properties as lowering the fertilizer subsidy, but can achieve the same goal more cost-effectively. At a basic level, the result says that subsidizing an input with a rising price is not cost-effective when subsidizing other, more stable inputs achieves the same goal. On the other hand, because financial frictions make it difficult for villages to respond to higher output prices, price subsidies offer limited benefit.

1.1 Related Literature

This paper relates to a number of different literatures. First, it relates to a substantial literature that has documented the importance of intermediate inputs in the development process (Jones, 2011; Casal and Caunedo, 2023) and specifically the role of

[1] The optimal subsidy calculated in the baseline equilibrium of our model is lower than the observed subsidy in Rwanda. Our results are generated under the assumption that the policymaker maximizes utilitarian welfare, but there do exist welfare weights such that the optimal subsidy equals the observed level in Rwanda.

[2] This is consistent with realized Rwandan policy and likely due to political economy constraints. District mayors sign *imihigo* contracts with the national government, which are essentially *ex ante* development performance contracts. Each year the results are posted publicly and districts ranked. Thus, subsidizing specific villages at the expense of others in response to an *ex post* shock creates a myriad of political issues.

fertilizer in agriculture (Restuccia et al., 2008; Boppart et al., 2023). We use detailed micro data to trace the impact of a global shock to intermediates and show that it can have damaging effects for the poor, especially in the short run where the ability to substitute to alternative technologies or inputs is limited (Barrot and Sauvagnat, 2016; Boehm et al., 2019).

We also connect to related literature on government interventions into critical markets in developing countries, especially those also concerned with fertilizer (Diop, 2023; Garg and Saxena, 2023; Mazur and Tetenyi, 2024) and others like education (Fujimoto et al., 2023) and energy (Ryan, 2022). Our focus on understanding how a subsidy should evolve with its corresponding input price is new to this literature. However, like many of these papers, we find that taking seriously the role of general equilibrium effects, including their distributional consequences (e.g. Donovan, 2021; Falcao Bergquist et al., 2023), is central to understanding the quantitative gains (or lack thereof) from policies that subsidize some of the population.

The importance of fertilizer to agricultural production coupled with the possibility of exogenous price shocks implies that fertilizer policy has a role in social insurance policy. This literature tends to focus on agricultural insurance initiatives (as reviewed in Cole and Xiong, 2017; Nshakira-Rukundo et al., 2021) and we extend this to fertilizer subsidy programs, though we find that the smoothing aspect of the program is a smaller concern than its productivity implications.

Finally, our results trace out the long reach of conflict though international trade and add causal, micro evidence to the patterns seen in global trade flows (Martin et al., 2008). Adamopoulos and Leibovici (2024) highlight how import risk of final goods affects welfare through food security in Ethiopia.

2 Setting, Data, and Fertilizer Price Shock

Global inorganic fertilizer production is concentrated in a small number of countries, because of the importance of comparative advantage in key inputs like natural gas. Russia is the largest exporter globally, implying that shocks to their ability to export play an important role in global price fluctuations. In this section, we detail how global fertilizer prices translated into changes in Rwandan policy, after a brief detour to discuss the data we use in this paper.

2.1 Data

Our main data sources for the empirical results are primary data. It includes household- and vendor-level data, which we discuss in turn. We collect household data from 444

villages across most of Rwanda. This dataset was originally designed to measure the impact of an RCT on last-mile infrastructure and as such covers all of Rwanda except the relatively flat east part of the country.[3] The dataset began in 2020 with a baseline and has been collected annually since, implying 5 visits per household (2020 – 2024, inclusive). We exclude all treated villages from the RCT and focus exclusively on control villages. Despite our description as a household-level panel, these data are an individual-level panel. We collect household rosters and ask individual-level questions on demographics, earnings, and health and education outcomes, along with household-level questions about farming practices, savings, and other related decisions.

Starting in 2021, we began collecting data from traders in the markets associated with these villages. In the 2020 baseline we asked households to tell us about the markets that they bought and sold goods. Any market mentioned by more than 2 households is included in the study, though in practice there are rarely more than one or two markets per village. These data are collected collected monthly by in-person enumeration. This high-frequency collection requires surveys to be relatively short so that enumerators can travel to many markets over the short time horizon. This survey asks questions about goods stocked, inventory, and prices.

2.2 Fertilizer in Rwanda and the 2022 Price Changes

The agricultural calendar in Rwanda is made up of two main cropping seasons. Season A begins with planting in September and ends with harvesting in January and February, while Season B runs from March to June. These seasons are generally referenced by the year of Season A's harvest, so that the 2022 season consists of Season A from September 2021 – February 2022 and Season B from March 2022 – June 2022.

Types of Fertilizer and Link to Global Markets The main fertilizers used in Rwanda all include nitrogen. These include DAP (a combination of nitrogen and phosphorus used primarily for beans, maize and wheat), a balanced nitrogen-phosphorus-potassium (NPK) 17-17-17 (used for rice and potatoes), and urea (primarily nitogren) which is used as a topdressing for all of the above crops. These three fertilizers make up 90 percent of the value of all fertilizers used in Rwanda in 2020 (National Institute of Statistics Rwanda, 2021). Nitrogen-based fertilizers use ammonia to deliver the nitrogen to plants, which is produced primarily with natural gas.[4] The centrality of

[3] See Macharia et al. (2022) for the pre-analysis plan and other details about the RCT.

[4] Ammonia (NH_3) is easier for plants to break down than the nitrogen in the air (N_2) due to the triple-covalent bond between the two nitrogen atoms. The Haber-Bosch process subjects natural gas to extremely high pressure, causing its hydrogen atoms to bind with nitrogen atoms in the air. The induced reaction, $N_2 + 3H_2 \rightleftharpoons 2\ NH_3$, creates

natural gas in the production process implies two relevant features. First, shocks to natural gas prices will affect fertilizer prices. Second, countries with natural gas have a comparative advantage in production. Because of this, production is concentrated in a handful of countries, none of which are in Sub-Saharan Africa.

Market Structure Fertilizer is not freely traded in Rwanda, instead managed by the Ministry of Agriculture and Animal Resources (MINAGRI). The government offers licenses for organizations to import fertilizer from abroad (usually 4 – 10 private firms and large NGOs), who then sell fertilizer via a set of licensed retailers scattered throughout the country. This involves the government setting two prices: the market price and subsidy level. The market price is the price received by an importer, and the consumer pays market price net subsidy. Technically, this involves importers selling to agro-dealers (i.e., the retail vendors) at the subsidized price. The importers are then reimbursed the difference between their negotiated price and the subsidized price by the government.[5] The normal price-setting involves the government announcing prices on or around July 1 preceding fertilizer purchases for Season A, then held fixed for the crop year and reset only in the next July. We say "normal" here because we will shortly show how price announcements adjusts after the fertilizer price shock.

One natural question when dealing with non-market prices is the equilibrium possibility of quantity restrictions. In practice this seems to be of limited concern, as farmers can almost always access the fertilizer they demand. By law, any fertilizer importer is required to service all agro-dealers in the country. Thus, the fertilizer market is national instead of a collection of closed local fertilizer markets, limiting the possibility of local shocks driving a local mis-match of supply and demand. Second, even when there is the possibility of national under-supply, the government induces imports to correct the projected imbalance. Both our farmer survey evidence and discussions with importers and agro-dealers support the view that this margin is relatively unimportant, though could play an indirect role through the government's budget.

2.2.1 Shock to Global Fertilizer Prices

One of the most important nitrogen-based fertilizer producers is Russia. In 2020, they produced 32 percent of the world's NPK and 10 percent of urea, contributing 13 percent of all nitrogen exports in the world (FAOSTAT, 2024).

the ammonia used in fertilizer.

[5]This process is managed by the Rwandan government's Smart Nkunganire System (link here). There are no subsidized products sold outside this system which facilitates repayment.

In 2022, the Russia-Ukraine war created two main issues that caused fertilizer prices to spike. First, fertilizer and ammonia exports declined, the latter of which made it difficult to produce fertilizer in other, non-natural gas producing countries like Morocco. Recent estimates suggest Russian ammonia exports fell 63 percent between 2022 and 2021, and urea fell 23 percent (Glauber and Laborde, 2022). Belarusian potash exports also collapsed, which is the "K" in NPK fertilizer. While Rwanda primarily imports fertilizer from Morocco and Saudi Arabia, the decline in global supply impacted global prices.[6]

The second implication of the war was its impact on the natural gas market. Russian natural gas exports fell dramatically in 2022, at the same time as the European market attempted to replace Russian oil demand with U.S.-based gas (Gil Terte, 2023). This redirected American gas exports toward Europe. Thus, not only was fertilizer is relatively short supply globally, its most important input was increasing in price for non-agricultural reasons. In the Appendix, we show that monthly global fertilizer prices closely track natural gas prices, with correlations of 0.67 and 0.69 for DAP and urea.

The relative (but not total) importance of these shocks remains subject to debate, but clearly none are related to Rwandan policy decisions. Figure 1 shows they were also unanticipated. It plots the global prices of DAP and urea from 2015 to 2023 using the World Bank Commodity Price Outlook (World Bank, 2024). The realized price shows the sharp rise from a period of relative stability, with DAP and urea rising to 2.5 and 3 times their 2020 level by 2022.[7] The dashed lines in Figure 1 plot the forecasted prices from each October release from 2018 to 2022. The 2018 - 2020 forecasts of continued stable prices were clearly not realized.

2.2.2 Response in Rwanda

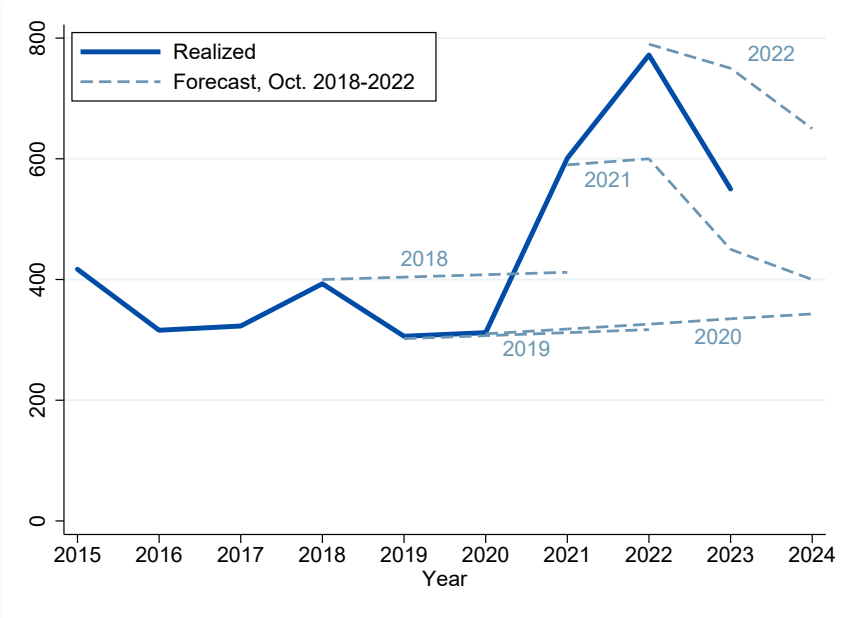
As mentioned above, the Rwandan government had historically updated fertilizer prices one per year. This policy-induced price stickiness was irrelevant when global fertilizer prices were stable during most of the 2000s. But the Rwandan government was forced to respond to the higher prices with an unprecedented off-schedule price change in January 2022 before the start of Season B. The market price for DAP was adjusted upward by 40 percent. For some sense of scale, the average annual price

[6] In 2020, 58 percent of Rwanda DAP imports came from Morocco, 28 percent from Saudi Arabia and 14 percent from Russia. By 2022, Russian DAP fell to zero and was partially made up for by an increase in Saudia Arabian fertilizer. The total quantity of Rwandan DAP imports fell from 24 million tons in 2020 to 15.6 million tons in 2022.

[7] The fertilizer price actually starts to rise in mid-2021. This was due to an issue of low U.S. coal inventories putting upward pressure on natural gas prices in 2021. That pressure was released at the end of 2021 on the backs of higher gas production and a mild winter (EIA, 2022). Thus, the proper non-war counterfactual would have been a temporary and relatively minor increase in natural gas prices (and thus fertilizer prices) in 2021 only. This is consistent with the 2021 forecast in Figure 1.

Figure 1: Global Fertilizer Prices and Projected Prices (Current USD/metric ton)

(a) DAP



(b) Urea

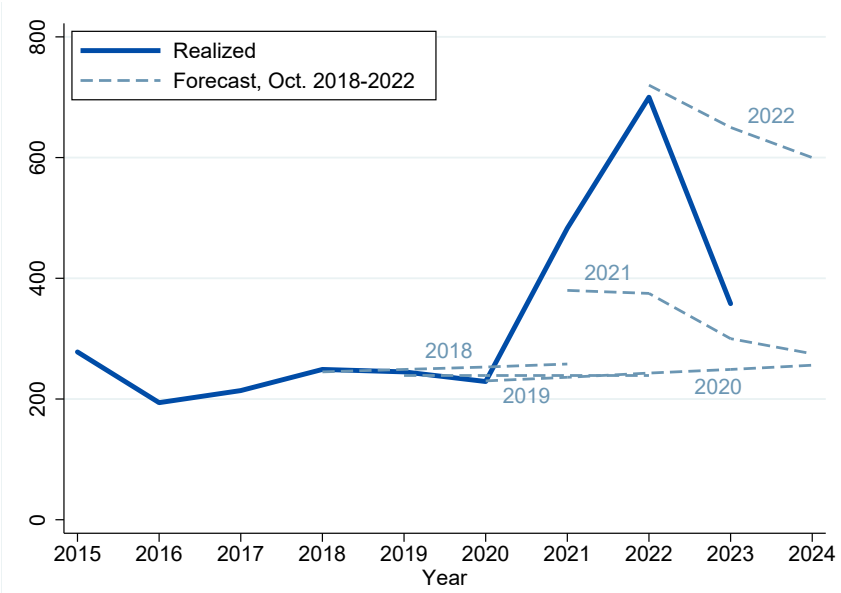


Figure Notes: The solid line plots the realized price for DAP and urea from 2015 - 2024. The dashed lines plot the projected prices each year. Both realized and projected prices are derived from the World Bank Commodity Price Outlook (World Bank, 2024).

change between 2018 and 2021 was 4 percent. Similar adjustments were made for urea and NPK.

In Figure 2, we use government documents to plot the government-defined market

price for 3 main fertilizers along with the subsidy rate. The farmers then pay (1-subsidy) $\times$ the market price. In addition, the figure includes markers at any point when the government releases an updated price (even if that price is identical to the previous one). The discussion above is reflected in these figures. First, prices and subsidy rates remain stable until 2022. Second, through 2021, all price adjustments occur in Season A and are held fixed in B. However, in 2022-B, both the market price and subsidy rates are updated due to the stark rise in international prices.

Figure 2 shows that the government made two adjustments in 2022. The first was an increase in the market price. The second was that they shared the cost of that increase with farmers by increasing the subsidy rate. As such, program jumped from 4 to 19 percent of MINAGRI's total budget. In raw values, the cost of the program rose from 5 billion francs in 2019 to 13 billion in 2021. Figure 3 then plots the implied prices for farmers, with raw prices in Figure 3a and the implied percentage growth in Figure 3b. Figure 3b also plots the implied path of fertilizer prices if the subsidy levels had been held at 2017 levels.

Our data show the same patterns, as should be expected given that we survey these same licensed dealers. For some preliminary comparisons, Figure 4 plots the raw log per-unit prices for various intermediate inputs and agricultural outputs over time derived from the vendor survey. The 95 confidence intervals are constructed via standard errors clustered at the market level. Each contains a dashed line at January 2022, where prices are normalized to zero. The top row plots two inputs: the most common type of fertilizer (DAP) and fungicide (Detan) purchased from markets in our study. Fertilizer prices rise by almost 40 percent, consistent with the evolution of the price set by the Rwandan government. Fungicide prices remain relatively stable. The latter is more easily sourced locally and thus not subject to the same shocks. The bottom row of Figure 4 plots the same log prices but for two main crops produced in Rwanda, beans and sweet potatoes. Neither of these crops are imported from abroad but see a stark increase in prices with a few months lag, suggesting an important equilibrium spillover onto food prices from the fertilizer price shock. We return to this issue in the next section.

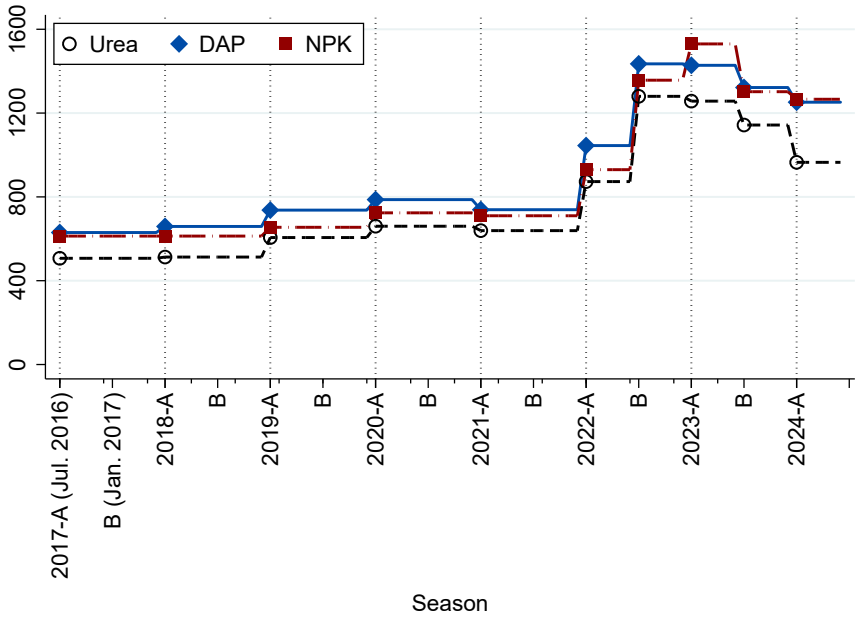
To study these patterns more systematically we exploit the panel dimension of the data and run a series of regressions

$$\log(p^j_{imt}) = \alpha + \beta Post_t + \theta_i + \eta_m + \nu \log(t) + \varepsilon_{it} \tag{2.1}$$

where, for each product j, we regress the log price on a dummy $Post_t = 1$ if post-January 2022. We control for vendor i's fixed effect θ_i, month effects γ_m for seasonality, and $\log(t)$ is a linear time trend. The results are in Table 1 and confirm the

Figure 2: Government Fertilizer Prices

(a) Nominal Market Price (Rwandan Francs)



(b) Subsidy Level

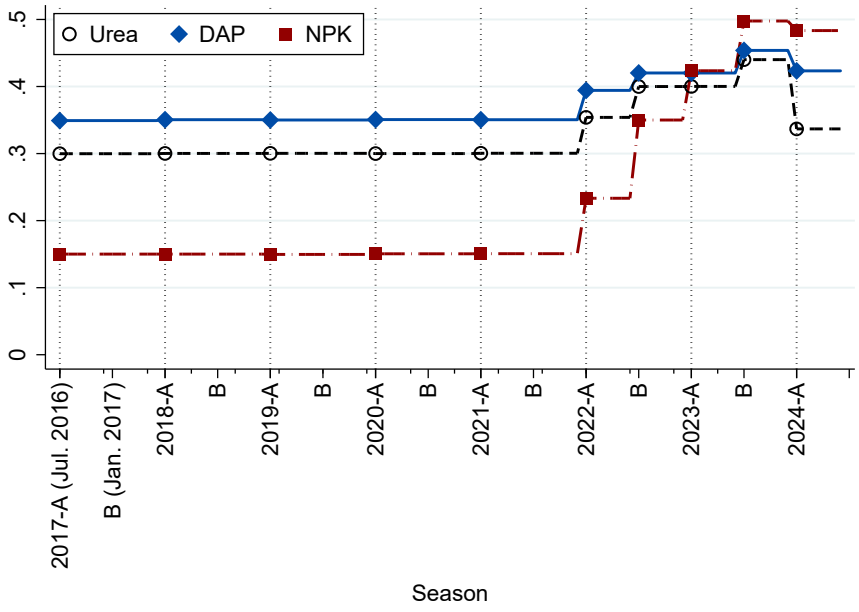
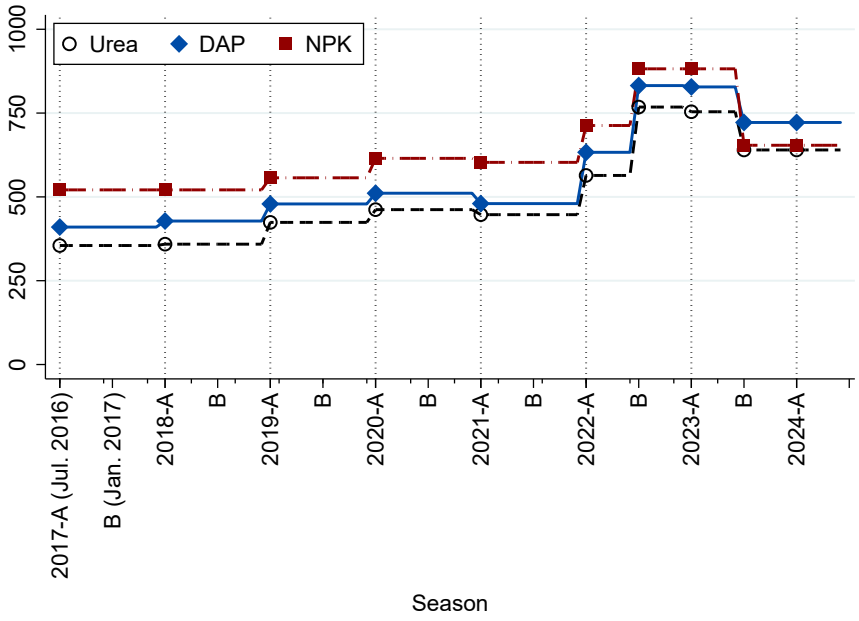


Figure Notes: Panel (a) plots the market price defined by the Rwandan government. Panel (b) plots the subsidy level. The farmer price is (1-Subsidy) × Market Price. In both figures, a marker is included when a new prices is released even if the price is not updated. Notice that before 2022, all price updates were in Season A.

patterns from the raw data in Figure 4.

Figure 3: Implied Prices for Farmers

(a) Nominal Farmer Price (Rwandan Francs)



(b) Normalized Farmer Price and Counterfactual Paths

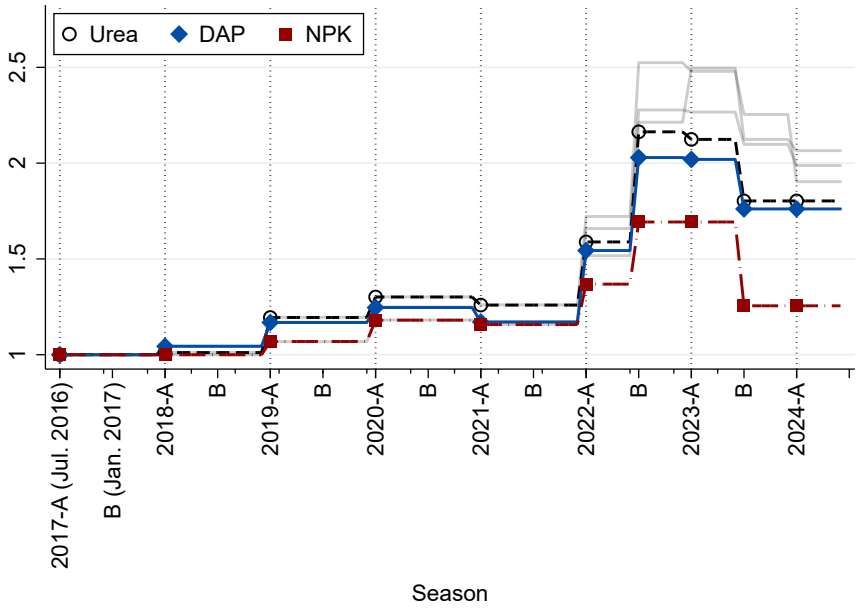


Figure Notes: Panel (a) plots the farmer price defined by the Rwandan government. It is equal to (1-subsidy rate) × market price. Those two moments are in Figure 2. Panel (b) plots the farmer price with 2017-A normalized to one. The grey lines are the counterfactual price series if the subsidy rate was held constant at the 2017-A level. A marker on the series indicates a new price release by the government even if the price itself does not change.

Figure 4: Raw Prices for Inputs (2021-2024)

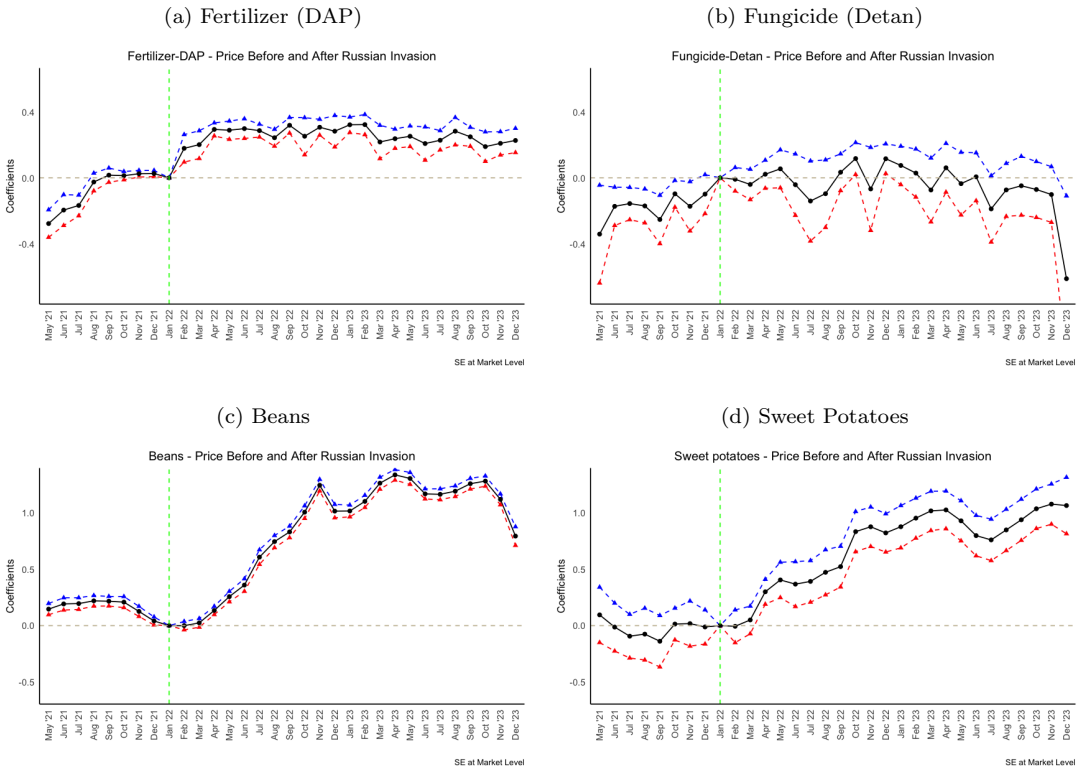


Figure Notes: This figure plots log prices for listed item over time as averages over all vendors. Dashed lines indicate 95 percent confidence interval with standard errors clustered by market.

Table 1: Price Changes Post-January 2022 for Intermediate Inputs

Inputs	Fertilizer			Other Intermediates		
	DAP	NPK	Urea	Roket	Detan	Ridomil
Post-January 2022	0.35***	0.39***	0.33***	0.41	0.14*	-0.18
	(0.03)	(0.05)	(0.08)	(0.26)	(0.07)	(0.24)
Observations	1,658	1,227	1,605	3,172	2,623	1,192
R-squared	0.44	0.43	0.14	0.61	0.27	0.41

Table notes: Standard errors clustered by market are in parentheses. Ridomil is a fungicide and Rocket and Detan are insecticides used in Rwanda. All regressions include trader and month fixed effects and a linear time trend. Statistical significance at 0.10, 0.05, and 0.01 is denoted by *, **, and, ***.

3 Empirical Results

To estimate the impact of this shock across villages, we take advantage of the fact that villages differ substantially in their fertilizer intensity, measured here as baseline fertilizer expenditure per acre of land in agricultural production.[8] We focus on village-

[8]Because prices do not vary across space, expenditures here do not pick up any spatial price variation. We use expenditures only to aggregate different fertilizers via their prices.

level intensity instead of farmer-level intensity due to general equilibrium spillovers that will become clear below. The fertilizer intensity of village v at survey wave t is

$$\widetilde{f}_{vt} = \log\left(\frac{\sum_{h\in v} f_{hvt}}{\sum_{h\in v} \ell_{hvt}}\right).$$

where f is household level fertilizer use and ℓ is land used in production. We then consider the following regression

$$y_{hvmt} = \alpha + \beta Post_t + \eta\left(Post_t \times \tilde{f}_{v0}\right) + \delta_m + \theta_v + \zeta_t + \varepsilon_{hvmt}, \tag{3.1}$$

Here, y is the outcome for household h in village v in month m in year t. $Post_t$ equals one after January 2022, and we control for village, month, and survey wave dummies.[9] Our interest here is the parameter η, which measures how outcomes vary with baseline fertilizer intensity $\widetilde{f}_{v0}$ after January 2022.

We also include the simpler "single difference" regression to measure the average change. This is somewhat more tenuously interpreted causally, but given the unexpected nature of the shock, offers a helpful benchmark that we will use when calibrating the model. This regression takes the form

$$y_{hvmt} = \alpha + \beta Post_t + \delta_m + \theta_v + \varepsilon_{hvmt}, \tag{3.2}$$

In all cases, we trim continuous outcomes at 1 percent to eliminate potential misreporting and run (3.1) as a poisson regression to interpret as percentages with zeros (e.g. Chen and Roth, 2024). Outcomes in Rwandan francs are deflated by the monthly Rwandan CPI for rural consumers.

3.1 Headline Household Earnings Channels

We begin in Table 2 with some basic household outcomes, including fertilizer expenditures, harvests and total household wage earnings.

Column (1) shows that fertilizer use declines more in places with higher baseline usage, showing that these villages are indeed more exposed to the shock (along with the mechanical aspect of the result that it bounded by zero below). For some sense of scale of this and the other results below, note that the difference between the fifth and ninety-fifth percentile village in terms of baseline fertilizer intensity is 3.74 log points (from -1.75 to 1.72). Thus, the decline in fertilizer intensity is about $-0.08 \times 3.47 = 28$ percentage points larger between the top and bottom of the distribution.

This decline in fertilizer impacts harvest. We value crops at the median sale

[9] Because of the number of villages covered by our study, survey waves are conducted over multiple months.

Table 2: Agricultural Production

			Harvest Value					
	Fertilizer Expenditures		Total		Cash Crops		Staple Crops	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Post	-0.067***	-0.052*	-0.149***	-0.059**	-0.013	-0.192***	-0.198***	-0.003
	(0.021)	(0.031)	(0.021)	(0.029)	(0.028)	(0.049)	(0.022)	(0.029)
Post x Village Fertilizer Intensity		-0.089***		-0.060***		-0.050**		-0.065***
		(0.020)		(0.015)		(0.021)		(0.016)
Observations	40,070	40,003	40,456	40,389	40,456	40,389	40,456	40,389

Table notes: Standard errors clustered by village are in parentheses. The regression is given in (3.1) and is run as a Poisson regression. Statistical significance at 0.10, 0.05, and 0.01 is denoted by *, **, and, ***.

Table 3: Labor Market Earnings

	Total		Outside Village		Inside Village	
	(1)	(2)	(3)	(4)	(5)	(6)
Post	-0.228***	-0.184***	-0.253***	-0.258***	-0.238***	-0.121**
	(0.020)	(0.034)	(0.035)	(0.060)	(0.030)	(0.051)
Post x Village Fertilizer Intensity		-0.049***		-0.108***		0.014
		(0.017)		(0.028)		(0.026)
Observations	40,060	39,997	40,040	39,978	40,061	39,999

Table notes: Standard errors clustered by village are in parentheses. The regression is given in (3.1) and is run as a Poisson regression. Statistical significance at 0.10, 0.05, and 0.01 is denoted by *, **, and, ***.

price at baseline, so columns 2 – 4 do not include price changes induced by the shock. Column (2) shows that a one log point increase in baseline fertilizer intensity causes a 5 percent decline in harvests. Both cash and staple crops exhibit a similar variation across villages, but the level is quite different. On net, the lowest fertilizer intensive villages increase staple production and decrease cash production. High fertilizer intensity villages, on the other hand, decrease cash crop production but do not change staple production ($0.08 - 0.05 \times 1.72 = 0.01$).

Columns 5 – 7 consider a second critical source of income: wage earnings. Again, fertilizer intensive villages exhibit a much larger decline in wage income, entirely accounted for by variation in wage earnings outside the village.

3.2 Output Price Changes

The household results above suggest a retrenchment of farm activity after the shock. We show that this is consistent with general equilibrium forces inherent in models of structural change. Table 4 shows that the price of agricultural output rises after the shock. To show this, we run a crop-household regression

$$p_{chvmt} = \alpha + \beta Post_t + \eta \left(Post_t \times \tilde{f}_{v0} \right) + \delta_{cm} + \theta_v + \zeta_t + \varepsilon_{chvmt},$$

where c is the crop and the remaining subscripts are as before. Unlike our previous regression, we include crop-month fixed effects to take into account crop-specific seasonal price patterns. Table 4, Column (2) shows that the price of agricultural output rises after the shock.

Table 4: Crop Prices

VARIABLES	All (1)	All (2)	Low Market (3)	High Market (4)
Post	0.023***	-0.074***	-0.078***	-0.061***
	(0.009)	(0.014)	(0.020)	(0.023)
Post x Village Fertilizer Intensity		0.032***	0.032***	0.012
		(0.006)	(0.009)	(0.010)
Observations	59,150	59,145	30,524	28,621

Table notes: Standard errors clustered by village are in parentheses. All outcome variables are in logs and regressions are run as OLS. Statistical significance at 0.10, 0.05, and 0.01 is denoted by *, **, and, ***.

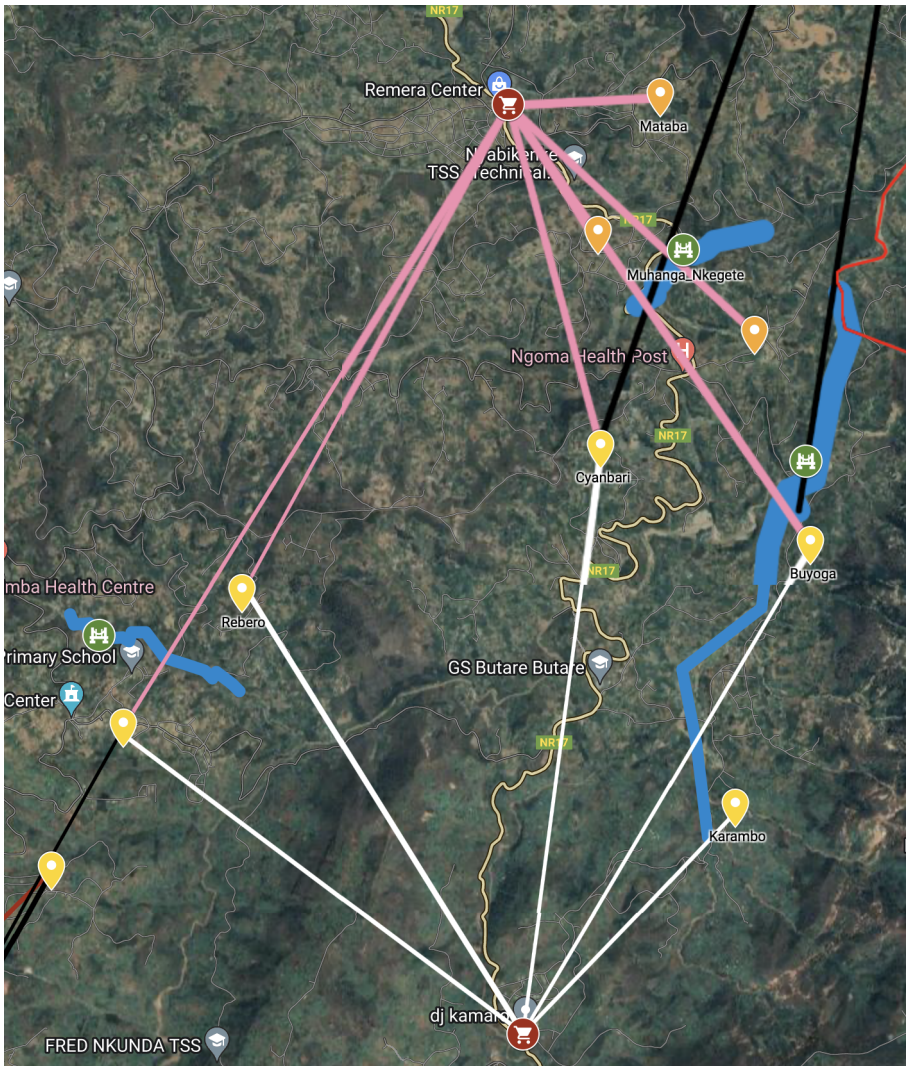
Villages do not each sell to unique markets. Figure 5 shows 9 villages (the yellow pins) that interact in two different markets (the red shopping carts). We would only expect the market price to respond to a village's own fertilizer intensity if it made up an important share of the market-level fertilizer intensity. As a robustness check, we therefore measure the residual fertilizer intensity of the market by computing the average fertilizer intensity of market-connected villages, given by (3.3), and test how prices adjust.

$$\tilde{f}_{Mv0} = \log\left(\frac{\overbrace{\sum_{\hat{v}\in M_v}\sum_{h\in\hat{v}} p_{x0}f_{h\hat{v}0}}^{\text{fert. for hhs that trade in same market}} - \overbrace{\sum_{h\in v} p_{x0}f_{hv0}}^{\text{own fert.}}}{\underbrace{\sum_{\hat{v}\in M_v}\sum_{h\in\hat{v}} \ell_{h\hat{v}0}}_{\text{land for hhs that trade in same market}} - \underbrace{\sum_{h\in v} \ell_{hv0}}_{\text{own land}}}\right) / |M_v| \tag{3.3}$$

(3.3) takes total baseline fertilizer for all households that trade in the same market as village v net of that fertilizer from v, and similarly for land, to compute a net fertilizer intensity for the market. If $\tilde{f}_{Mv0}$ is high, then village v makes up only a small portion of the market's fertilizer intensity. We would not expect output prices to respond to village v's fertilizer intensity in this case. On the other hand, if $\tilde{f}_{Mv0}$ is low, village v plays an important role in market M.

Columns (3) and (4) replicate the same regression as Column (2) of Table 4, but breaks the sample into those with high market-connected fertilizer intensity and those with low market-connected fertilizer intensity. As expected, the output price in v responds to v's fertilizer intensity only when v is a sufficiently large share of the

Figure 5: Example of Village Links to Markets



market.

3.3 Wages

We next study the impact on wages. We break them into two groups: inside-village agricultural jobs and all others. We run these regressions at the individual level and include individual fixed effects. We also continue to use the Poisson regression for consistency with our earlier results, though it is not critical given that wages are positive for all workers. Regardless, these decisions have no impact on the results.

The results are in Table 5.

Table 5: Log Daily Wages

	Inside Agr.		Other Jobs	
	(1)	(2)	(3)	(4)
Post	0.192***	-0.032***	0.259***	-0.045
	(0.009)	(0.012)	(0.015)	(0.028)
Post x Village Fertilizer Intensity		0.012*		-0.003
		(0.006)		(0.012)
Observations	13,970	13,968	16,875	16,848

Table notes: Standard errors clustered by village are in parentheses. Regressions include individual fixed effects. Statistical significance at 0.10, 0.05, and 0.01 is denoted by *, **, and, ***.

Agricultural wages inside the village change as well. They fall by 7 percent in low fertilizer intensity villages and exhibit no change on net in high fertilizer intensity villages. There is no change to non-agricultural wages.[10,11] To summarize, low fertilizer intensive villages see output prices and village wages fall. High fertilizer intensive villages see output prices rise and agricultural wages remain approximately the same.

To recap, villages using more baseline fertilizer (1) decrease fertilizer use and harvests, (2) see higher sales prices for crops, (3) see within-village agricultural wages rise, and (4) pull workers from other jobs into agricultural employment.

4 Model

As discussed in Section 2, the Rwandan government responded to the exogenous fertilizer price shock with a change to their subsidy level. Our goal now is to understand the underlying economics of how fertilizer subsidies should respond to price

[10] While we do not ask households for a detailed price listing of non-agricultural purchases (only aggregated expenditures), we do collect those data from market traders. The two main non-agricultural products sold in local markets are cooking oil and salt. There is no change in the price of either. We similarly see no adjustment in fungicide prices. So to that extent, one should interpret the changes reported here as changes to the price of these inputs or goods relative to the non-agricultural output price as would be required in a general equilibrium model.

[11] Another possibility is that households are selling less of their harvest to the market and thus the average sold unit is of higher quality. We find no difference in the share of crops sold to the market, stored for own consumption, or gifted to others.

fluctuations. To do so, we develop a general equilibrium model that we can use as a laboratory to study government policy.

Sectors and Village Structure The model is cast in discrete time with two sectors: non-agriculture, or "manufacturing" m for short, and agriculture a. Within the economy are a continuum of villages, denoted $j \in \mathcal{J}$. Each village contains a measure one of *ex ante* identical households that live forever. Each household derives utility from each of the two sectors' final goods, and utility flow is Cobb-Douglas with discount rate β,

$$\sum_{t=0}^{\infty} \beta^t \Big[\zeta \log(c_{at}) + (1-\zeta) \log(c_{mt}) \Big].$$

Households save s denominated in the agricultural final good with exogenous gross return R (which may be less than one due to storage costs or depreciation) and are restricted to savings, $s \geq 0$.

Villages differ in two ways: their technological fertilizer intensity α_j and the severity of their working capital constraint ϕ_j, both of which we will formalize below.

Manufacturing Production The manufacturing sector produces the final consumption good according to the production function $Y_{mt} = AN_{mt}$ where N_{mt} is labor hired in a centralized, competitive market that draws from all villages. We normalize the output price of the manufacturing sector to $p_m = 1$. In equilibrium, the wage $w_{mt} = A$ for all t.

Agricultural Production Each household i in village j produces a village-specific crop with the production function

$$y_{ij} = e^{z_i} \left(\alpha_j^{\frac{1}{\sigma}} x^{\frac{\sigma-1}{\sigma}} + (1-\alpha_j)^{\frac{1}{\sigma}} n^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\gamma\sigma}{\sigma-1}}$$

where x is fertilizer and n is labor. α_j is sub-scripted, meaning that it differs across villages. Note that the returns to scale are equal to γ everywhere. Input choices are made after the realization of z, an i.i.d. productivity shock drawn from a distribution with c.d.f $F(z)$.

Input choices are constrained by a working capital constraint that varies across villages,

$$p_x x + w_a n \leq \phi_j s$$

where p_x is the price of fertilizer and w_a is the wage per unit of labor. This constraint says that a household can pay for inputs up to a multiple ϕ_j of its savings s. Thus,

a village is defined by its technological fertilizer intensity α_j and the tightness of its working capital constraint ϕ_j. The distribution across villages is summarized by the c.d.f. $G(\alpha, \phi)$.

The village-specific crops are transformed into the final agricultural consumption good by a CES aggregator

$$Y_a = \left(\int_j y_j^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}}$$

so that a stand-in firm has the profit function

$$\max_{y_{aj}} \; p_{ac} \left(\int_j y_j^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}} - \int_j p_{aj} y_{aj} dj.$$

Here, p_{ac} is the price of the final agricultural consumption good paid by consumers and p_j is the output price received by households in village j.

Notice that the continuum of households within any village implies no household market power for the differentiated products.

Household Labor Allocation Each worker can work inside the village at the local market clearing wage w_{aj} or in the centralized manufacturing market for wage w_m. We assume that each individual gets productivity draws (ξ_a, ξ_m) and these map into earnings with a parameter θ. The earnings of a worker are

$$\text{Earnings}(\xi_a, \xi_m) = \max\{\xi_a^{\frac{-1}{\theta}} w_a, \xi_m^{\frac{-1}{\theta}} w_m\}$$

We assume throughout that $\xi_a \sim Exp(T_a)$ and $\xi_m \sim Exp(T_m)$ which when combined with the continuum of individuals within a household lets us characterize the total household i labor earnings as

$$\mathcal{W}_{ij} = (T_a w_{aj}^\theta + T_m w_m^\theta)^{\frac{1}{\theta}} \Gamma\left(\frac{\theta - 1}{\theta}\right). \tag{4.1}$$

where Γ is the gamma function. This is identical for all households in village j, so we drop the i subscript and write $\mathcal{W}_j$ going forward.

Fertilizer, Subsidies, and Government Budgets The fertilizer system is designed to mimic the Rwandan system.

Fertilizer is entirely imported into Rwanda and farmers pay price p_x for it. p_x is made up of an exogenous market price p_x^{market} and the government subsidy τ_x, so that $p_x := (1 - \tau_x) p_x^{\text{market}}$. The government buys at p_x^{market} and sells to farmers at $(1 - \tau_x) p_x^{\text{market}}$ incurring a cost of $\tau_x p_x^{\text{market}}$ on each unit.

That subsidy is financed by taxes, subject to a balanced budget constraint. We design the tax system to mimic the relevant features of the Rwandan code. Expenditures on agricultural goods – both inputs and outputs – are exempt from VAT. And while there is in principle income taxes on agricultural profits, the thresholds exempt nearly all non-commercial-scale farming. Farm incomes must be above 12 million RWF (9,200 USD in 2024, though the franc has steadily depreciated over the study period) to incur any tax burden. Other taxes are designed to capture revenue from the agricultural sector, but play a minor role.[12] Instead, the main tax faced by rural Rwandans is non-agricultural transactions subject to VAT.

We therefore model government revenue as a tax τ_m on manufacturing consumption. The government earns τ_m on each unit given the normalized manufacturing output price.

Household Problem in Village j The timing of the model works as follows. At $t-1$, households choose savings s_t to bring into period t. Some of that depreciates they enter t with Rs_t. Then the shock z is realized. Households then make decisions on farm inputs and the allocation of labor, income is realized, and consumption and savings decisions are made.

The individual state of the household is therefore (z, s), and the aggregate state is the distribution μ across individual states and villages. Recursively, the problem is

$$
\begin{aligned}
v_j(z,s,\mu) \;=\; & \max_{c_a,c_m,x,n_a,s'} \; \zeta \log(c_a) + (1-\zeta)\log(c_m) + \beta \int_{z'} v(z',s',\mu') dF(z') \\
s.t. \quad & p_a^c c_a + (1+\tau_m) c_m + p_a^c (s' - Rs) = p_{aj} e^z \left(\alpha_j^{\frac{1}{\sigma}} x^{\frac{\sigma-1}{\sigma}} + (1-\alpha_j)^{\frac{1}{\sigma}} n^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\gamma\sigma}{\sigma-1}} - p_x x - w_{aj} n + \mathcal{W}_j \\
& p_x x + w_{aj} n \leq \phi_j R s \\
& s' \geq 0
\end{aligned}
$$

The first constraint is the budget constraint, the second is the financial friction, and the third is the no-borrowing constraint. Recall that $p_x = (1-\tau_x) p_x^{\text{market}}$ so that the subsidy enters directly into the budget constraint and financial friction. A household's income is made up of agricultural profits $\pi_j(z,s)$ and wage earnings $\mathcal{W}_j$.

We note here that when we introduce a shock to the fertilizer price p_x^{market}, we do so with a so-called "MIT shock." That is, we unexpectedly shock the stationary equilibrium with some baseline price p_{x0}^{market} with a deterministic sequence of prices

[12] For example, market fees are paid by traders who operate stalls in markets, but these are small revenue generators and we exclude them for simplicity. Similarly, non-agricultural earnings are subject to income taxes (above a threshold), but given that much of this work is informal in our setting, we refrain from modeling this tax on non-agricultural wages.

$\{p_{xt}^{\text{market}}\}_{t=1}^{t=\infty}$. Hence, the recursive program above includes perfect foresight of the evolution of the aggregate state μ. We discuss this more in the quantitative results.

4.1 Recursive Competitive Equilibrium

Given a subsidy level τ_x and market price of fertilizer p_x^{market}, a competitive equilibrium of this model is a set of decision rules for households $(c_{aj}, c_{mj}, x_j, n_{aj}, s')$, the manufacturing firm N_m, and the final agricultural good producer y_{aj}, prices $\{p_{aj}\}, p_{ac}, w_a$ and tax rate τ_m such that (i) the household's decision rules are consistent with its optimization problem given prices and taxes, (ii) N_m solves the manufacturing firm's decision problem, (iii), y_{aj} solves the agricultural final goods firm decision problem, (iv) the law of motion for the aggregate state, $\mu' := \Lambda(\mu)$, is consistent with the decision rules, and (v) the government balances its budget and markets clear:

1. Government budget balance:

$$\tau_x p_x^{market} \int_j \int_{s,z} x_j(s,z) d\mu_j dj = \tau_m \int_j \int_{s,z} c_{mj}(s,z) d\mu_j dj$$

2. Market clearing:

   (a) Agricultural intermediate market: for each j,

$$y_{aj} = \int_{s,z} e^z \left(\alpha_j^{\frac{1}{\sigma}} x_j(s,z)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_j)^{\frac{1}{\sigma}} n_j(s,z)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\gamma\sigma}{\sigma-1}} d\mu_j$$

   (b) Agricultural final goods market:

$$\left(\int_j y_j^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}} = \int_j \int_{(s,z)} c_{aj}(s,z) d\mu_j dj$$

   (c) The agricultural labor market: for each j,

$$\int_{s,z} n_j(s,z) d\mu_j = T_a w_{aj}^{\theta-1} \Gamma \left(\frac{\theta-1}{\theta} \right) (T_a w_{aj}^{\theta} + T_m w_m^{\theta})^{\frac{1-\theta}{\theta}}$$

   (d) The manufacturing labor market:

$$N_m = T_m w_m^{\theta-1} \Gamma \left(\frac{\theta-1}{\theta} \right) \int_j (T_a w_{aj}^{\theta} + T_m w_m^{\theta})^{\frac{1-\theta}{\theta}} dj$$

   which amounts to the restriction that $w_m = A$.

4.2 Analytic Results

The goal of the model is to understand how the optimal subsidy τ_x responds to changes in the market fertilizer price p_x^{market}. The model developed above includes several margins that can play an important role in the quantitative answer to this question.

To clarify the underlying economics of the problem, we first provide some analytic results. To make these results as sharp as possible, we analyze the static version of the model (or, identically, $R = 0$). Doing so necessitates a small change in the financial friction which we re-write as

$$p_x x + w_a n \leq \phi_j$$

where now ϕ_j is a maximum cost level in village j instead of a multiple of savings (with zero savings, this would imply the agricultural sector would not exist).

We make three additional assumptions. The first is purely for notational convenience: we make the idiosyncratic shocks degenerate with $Pr(z = 0) = 1$. This eliminates the need to carry distributions of idiosyncratic shocks through the analysis and has no bearing on the results. The latter two will look familiar to readers of standard macro-development models with an agricultural sector, such as Restuccia et al. (2008). We let $\sigma \to 1$ so that the production function becomes Cobb-Douglas

$$y_j = x^{\alpha_j} n^{\gamma - \alpha_j}.$$

We also assume $\theta \to \infty$, which makes labor supply infinitely elastic and guarantees that local agricultural wages are pinned down by the manufacturing sector in equilibrium, $w_{aj} = w_m = A$ for all j.

In this simplified model, a household in village j solves

$$\begin{aligned} & \max_{c_a, c_m, x, n_a} \quad \zeta \log(c_a) + (1 - \zeta) \log(c_m) \\ s.t. \quad & p_a^c c_a + (1 + \tau_m) c_m = p_{aj} x^{\alpha_j} n_a^{\eta_j} - p_x x - wn + w \\ & p_x x + w_a n \leq \phi_j \end{aligned}$$

We define the competitive equilibrium of this simplified economy in the Appendix for interested readers, though it differs only slightly from the one defined above.

4.3 The Frictionless Equilibrium

To understand how τ_x should respond to fertilizer price changes, it is helpful to start from the frictionless economy with $\phi_j = \infty$ for all villages j.[13] Characterizing this economy allows us to show how equilibrium forces work to reallocate production across heterogeneous villages. We then use this to benchmark against the economy with binding financial frictions.

Because we are interested in changes between two equilibria, our strategy will generally be to characterize the new equilibrium (with higher p_x^{market}) in terms of the baseline (with low p_x^{market}). To start, Lemma 1 characterizes the new equilibrium in this way. All proofs are in the Appendix.

Lemma 1. *Assume that the farmer fertilizer price rises from p_{x1} to p_{x2}. Define*

$$\mathcal{Y} = \frac{\int_j y_{j1}^{\frac{\nu-1}{\nu}} dj}{\int_j \left(\frac{p_{x2}}{p_{x1}}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{j1}^{\frac{\nu-1}{\nu}} dj}.$$

Then the new equilibrium prices can be characterized as

$$\frac{p_{ac2}}{p_{ac1}} = \mathcal{Y}^{\frac{\nu-(\nu-1)\gamma}{\nu-1}} \qquad \textit{and} \qquad \frac{p_{j2}}{p_{j1}} = \mathcal{Y}^{\frac{\nu-(\nu-1)\gamma-1}{\nu-1}} \left(\frac{p_{x2}}{p_{x1}}\right)^{\frac{\alpha_j}{\nu-(\nu-1)\gamma}}$$

The term $\mathcal{Y}$ is ratio of baseline aggregate output an adjusted version of itself, where the adjustment depends on the change to the fertilizer price p_x. In fact, in the proof of Lemma 1, we show that $Y_{a2}/Y_{a1} = 1/\mathcal{Y}^{\frac{\nu}{\nu-1}-\gamma}$. Thus, it summarizes the aggregate decline in production from the higher fertilizer price. The key feature of $\mathcal{Y}$ is that it depends on no endogenous values from the new equilibrium with p_{x2}^{market}. Thus, it fully characterizes the new equilibrium in terms of the baseline one.

Notice also that α_j affects how village j's output price changes. This fact alone should not be too surprising – villages have different fertilizer cost shares in production, and thus are affected differently. But *where* the new equilibrium wants to redistribute lost production is at the heart of this problem. To make this more explicit, we combine Lemma 1 with a bit of algebra to characterize the change in household farm profit (we derive this explicitly in the proof of Lemma 1 for completeness),

[13] We refer to this economy as "frictionless" instead of "efficient" because with fluctuating prices and heterogeneous technology villages would benefit from *ex ante* insurance contracts. We rule these out because our interest lies elsewhere in understanding how GE forces redistribute production. In this sense, the economy is constrained efficient.

$$\pi_{j2} = \underbrace{\mathcal{Y}\left(\frac{p_{x2}}{p_{x1}}\right)^{\frac{\alpha_j(1-\nu)}{\nu-(\nu-1)\gamma}}}_{\equiv \text{ adjustment to } \pi_{j1}} \underbrace{(1-\gamma)p_{j1}y_{j1}}_{\equiv\ \pi_{j1}}. \tag{4.2}$$

We can re-write this profit adjustment to give it a more natural economic interpretation, where profit rises in village j if and only if

$$\pi_{j2} > \pi_{j1} \Leftrightarrow \mathcal{Y}^{\frac{\nu-(\nu-1)\gamma}{(\nu-1)(1-\gamma)}} > \left(\frac{p_{x2}}{p_{x1}}\right)^{\frac{\alpha_j}{1-\gamma}}.$$

The right-hand side is the partial equilibrium decline in village j profit from the higher fertilizer price. Naturally, this effect is larger for villages with high α: these villages use more fertilizer and thus their costs are more affected. The left-hand side is the equilibrium increase in profit that results from the change to j's output price. Unlike the PE effect, the GE price response depends on the complementarity across villages, ν. Thus, understanding the total effect requires understanding how the direct, PE effect of the shock compares to the GE price response across locations. Armed with Lemma 1, this turns out to be quite simple. Re-writing (4.2) in log differences gives

$$\Delta \log(\pi_j) = \log(\mathcal{Y}) + \frac{\alpha_j(1-\nu)}{\nu-(\nu-1)\gamma}\Delta \log(p_x),$$

and leads to a characterization of these effects summarized in Proposition 1.

Proposition 1. *The change in profit associated with a change in the market fertilizer price is given by*

$$\frac{\partial \Delta \log(\pi_j)}{\partial \Delta \log(p_x^{market})} = \frac{\partial \log(\mathcal{Y})}{\partial \Delta \log(p_x^{market})} + \frac{\alpha_j(1-\nu)}{\nu-(\nu-1)\gamma}$$

which means that

$$\frac{\partial^2 \Delta \log(\pi_j)}{\partial \Delta \log(p_x^{market})\partial \alpha_j} = \frac{(1-\nu)}{\nu-(\nu-1)\gamma}\begin{cases} < 0, & \text{if } \nu > 1 \\ = 0, & \text{if } \nu = 1 \\ > 0, & \text{if } \nu < 1 \end{cases}$$

Proposition 1 characterizes the evolution of profit across villages, showing that low-α villages see the largest changes in profit when villages are substitutes ($\nu > 1$), with the opposite pattern holding when villages are complements ($\nu < 1$). While the proof is in the Appendix, the intuition relies on understanding how village substitutability affects patterns of equilibrium redistribution. After the shock, equilibrium forces

work to redistribute that lost production to other villages. This is why ν matters. If villages are substitutes ($\nu > 1$), the equilibrium tries to source more production from the relatively unscathed (in the PE sense) low-α villages. On the other hand, with complementarity ($\nu < 1$) this is no longer viable. The decline in production from high-α villages needs to be replaced with production from those same villages. Thus, the equilibrium shifts the price changes toward higher-α villages to cushion the blow of the higher input price.

These results offer a benchmark to show what *should* occur in the absence of financial frictions. As we will show, the inability of the frictional economy to replicate the redistribution highlighted in Proposition 1 will be central to the government's ability to increase welfare with a fertilizer subsidy.

4.4 Frictional Equilibrium

We now return to the frictional equilibrium, where each village is subject to

$$p_x x + wn \leq \phi_j.$$

To make the results as sharp as possible, we assume that ϕ_j is tight enough that it binds with equality for all j at baseline price p_{x1} (which also guarantees it binds at $p_{x2} > p_{x1}$). Thus, we study an economy in which financial frictions are extremely tight, but note that there is still heterogeneity in the tightness of financial frictions across villages. This implies that input choices are

$$x_j^c = \frac{\alpha_j \phi_j}{\gamma p_x} \qquad \text{and} \qquad n_j^c = \frac{\eta_j \phi_j}{\gamma w} \tag{4.3}$$

where the superscript c is a reminder that these come from the "constrained" frictional equilibrium (and we will sometimes use superscript u as a reminder of the frictionless or "unconstrained" economy).

It will be useful to define the village j deviation of fertilizer (x) and output (y) from its frictionless benchmark,

$$\kappa_j^x := \frac{x_j^u}{x_j^c} \qquad , \qquad \kappa_j^y := \frac{y_j^u}{y_j^c}$$

A high κ_j^x, for example, implies that x_j^c is far below its unconstrained level x_j^u. We focus our discussion in terms of the fertilizer but note that the two move one-for-one

(see footnote 14). Plugging in the relevant input choices yields

$$\kappa_j^x = p_x^{\frac{-\alpha_j}{1-\gamma}} (p_j^u)^{\frac{1}{1-\gamma}} \left(\frac{\gamma \left(\frac{\eta_j}{w}\right)^{\frac{\eta_j}{1-\gamma}} \alpha_j^{\frac{\alpha_j}{1-\gamma}}}{\phi_j} \right) \tag{4.4}$$

Notice that the equilibrium price p_j^c does not enter (4.4), which follows directly from frictional input choices (4.3). This is the sense in which financial frictions limit villages' ability to respond to equilibrium output price changes, thus inhibiting the redistribution highlighted in the previous section.

One approach to make some sense of this economy would be to characterize κ^x. This unfortunately depends not only on (α_j, ϕ_j) but on the distribution $G(\alpha, \phi)$ and various other parameters that determine equilibrium prices, making characterization difficult or at least a complicated non-linear combination of model features. But we are interested in changes to κ, not its level. This offers a way to sidestep many of these issues. Defining $\Delta_t x = x_2 - x_1$ as the difference between the p_{x2} and p_{x1} equilibria, (4.4) becomes

$$\Delta_t \log(\kappa_j^x) = \frac{-\alpha_j}{1-\gamma} \Delta_t \log(p_x) + \frac{1}{1-\gamma} \Delta_t \log(p_j^u) \tag{4.5}$$

The second term is the change in village j's equilibrium price from the frictionless equilibrium, which we characterized in Lemma 1. Applying it then taking a second difference with village k yields

$$\Delta_t \log(\kappa_j^x) - \Delta_t \log(\kappa_k^x) = (\alpha_k - \alpha_j)(\nu - 1) \Delta_t \log(p_x). \tag{4.6}$$

(4.6) offers a "diff-in-diff" characterization of how the fertilizer distortion evolves between any two villages.[14] To extract some economic intuition it helps to rewrite $\kappa_j^x = x_j^u / x_j^c$ as the components of the fraction. If we consider two villages with $\alpha_H > \alpha_L$, then writing (4.6) in this manner implies

$$\overbrace{\Delta_t \log(x_H^c) - \Delta_t \log(x_L^c)}^{\equiv \Delta \text{ frictional equilibrium}} > \overbrace{\Delta_t \log(x_H^u) - \Delta_t \log(x_L^u)}^{\equiv \Delta \text{ frictionless benchmark}} \quad \text{if } \nu > 1 \tag{4.7}$$

$$\Delta_t \log(x_H^u) - \Delta_t \log(x_L^u) = \Delta_t \log(x_H^c) - \Delta_t \log(x_L^c) \quad \text{if } \nu = 1 \tag{4.8}$$

$$\Delta_t \log(x_H^c) - \Delta_t \log(x_L^c) < \Delta_t \log(x_H^u) - \Delta_t \log(x_L^u) \quad \text{if } \nu < 1. \tag{4.9}$$

The left-hand side measures the relative fertilizer changes in the frictional equilibrium and compares them to their frictionless counterparts on the right, which

[14] (4.5) and (4.6) link directly to the output because $\Delta_t \log(\kappa_j^y) = \gamma \Delta_t \log(\kappa_j^x)$. Thus, discussing the results in terms of inputs is equivalent to discussing in terms of output.

were characterized in Proposition 1. The main result is that the frictional economy under-delivers the necessary redistribution for any value of ν except the knife-edge Cobb-Douglas case. What differs, however, is the location in which the economy wants to incentivize more production. With complementary ($\nu < 1$), Proposition 1 showed that the frictionless equilibrium tries to compensate high-α villages for its higher cost. (4.9) shows this gap between high- and low-α villages is too small with financial frictions. On the other hand with $\nu > 1$, Proposition 1 shows that the frictionless equilibrium wants to shift resources from high-α to low-α villages. Again, (4.7) shows that the frictional economy keeps too much fertilizer, and thus too much production, concentrated in high-α villages relative to this benchmark.

The result follows because financially constrained households cannot adjust inputs fully to output price changes. This unrealized reallocation opens scope for a subsidy. How then should the subsidy respond to deliver it? A seemingly reasonable intuition is that the government should increase its subsidy to counter the tighter financial frictions. But this intuition fails to account for the fact that cross-village technological heterogeneity offers the government an alternative to recoup lost production: shift production to other villages. Proposition 2 shows that the subsidy response depends only on how easily the government can utilize other, less exposed villages to replace lost production.

Proposition 2. *Suppose the government chooses its subsidy τ_x to maximize utilitarian welfare,*

$$\max_{\tau_x} \quad \int_j \Big[\zeta \log(c_{aj}) + (1-\zeta)\log(c_{mj})\Big] dj$$

subject to τ_x being consistent with government budget balance and the resulting competitive equilibrium of the frictional economy. For any non-degenerate distribution of $G(\alpha, \phi)$, the optimal subsidy is such that

$$\frac{\partial \tau_x^*}{\partial p_x^{market}} \begin{cases} < 0, & \text{if } \nu > 1 \\ = 0, & \text{if } \nu = 1 \\ > 0, & \text{if } \nu < 1 \end{cases}$$

If the marginal distribution of α is degenerate so that all villages have identical α, then $\frac{\partial \tau_x^*}{\partial p_x^{market}} = 0$ for all values of ν.

Proposition 2 shows that the government increases the subsidy in response to higher fertilizer prices only when $\nu < 1$. The intuition is entirely based on where the government needs to incentivize production, per the discussion above. When high-α villages decrease production, substitutability ($\nu > 1$) implies that low-α villages can

replace that production. But financial frictions limit the ability of prices to properly incentivize this redistribution. The government therefore lowers its subsidy, shifting production toward less affected villages.

Doing the same with complementarity ($\nu < 1$) would imply a large negative impact on aggregate availability of the agricultural consumption good. Thus, the government seeks to subsidize the same affected villages to increase production. While doing so simultaneously increases subsidization to low-α villages, the cost to the government's budget is relatively small given the technological irrelevance of fertilizer in these places, and thus still worthwhile.[15] That the subsidy remains constant when all villages have the same α emphasizes the centrality of heterogeneity in the argument.

These results come from a simplified model. Allowing villages to more easily substitute labor for capital, or inducing wage effects via a finite labor supply elasticity will affect the magnitude of the government's chosen subsidy level and allow us to study the role of alternative policy tools rendered trivially uninteresting by our simplifications here (e.g., wage subsidies). But the main message will remain the same: the government uses its subsidy to induce reallocation across heterogeneous production units that is otherwise limited by financial frictions. Parameters other than just the tightness of financial friction affect both the sign and magnitude of how the subsidy evolves with fertilizer prices. Thus, answering this question both quantitatively *and* qualitatively depends on credibly measuring them. Our goal now is to link these model parameters back to the causal empirical estimates in the full dynamic model, to quantify how the subsidy should change in response to the fertilizer price shock.

5 Quantitative Exericse and Model Estimation

5.1 Quantitative Exercise

Our main quantitative exercise works as follows. We start the model with a baseline market fertilizer price p_{x0}^{market} and subsidy level τ_{x0} that agents assume will remain fixed forever. This generates a baseline stationary equilibrium. We then shock the economy with a deterministic sequence $\{p_{xt}^{\text{market}}, \tau_{xt}\}_{t=1}^{t=\infty}$ that eventually converges back to its baseline values $(p_{x0}^{\text{market}}, \tau_{x0})$ and trace out the transition path of the economy.[16] Under this assumption, we calibrate the model using the empirical counterparts $\{\widehat{p}_{xt}^{\text{market}}, \widehat{\tau}_{xt}\}_{t=0}^{t=\infty}$, but restrict ourselves to model outcomes from $t \in \{0, 1, 2, 3\}$

[15] Proposition 2 does not depend on how the subsidy is financed. In the Appendix, we provide the proof of under alternative financing schemes in which all income is taxed and the case in which only labor market income is taxed.

[16] The international fertilizer price and Rwandan subsidy level were extremely stable in the decade leading up to 2021 (see Section 2). While this is of course just one realization of a random variable, the expert forecasts in Figure 1 are consistent with beliefs that the price stability would continue. Beliefs consistent with this result would closely approximate our assumption. This comes at the quantitative cost of missing any increased uncertainty on the evolution of fertilizer prices after the shock.

to be consistent with our empirics. Once calibrated, we will impose different time paths depending on the question in which we are interested.

5.2 Estimating Model

The model includes a number of parameters. On the production side, we require the CES aggregator for agricultural production ν. For village production, we need to estimate the joint distribution of fertilizer intensity α and financial frictions ϕ across villages, $G(\alpha, \phi)$ along with the elasticity between fertilizer and labor σ and returns to scale γ, along with the idiosyncratic shock distribution $F(z)$. In the labor market, we need values for the exponential distributions (T_a, T_m) and the parameter θ that governs the returns to skill. Finally, we require the return to savings R and utility function parameters including the discount rate β.

While we set some of these directly outside the model, many will be estimated via the empirical results from Section 3. These results were derived from regressions of the form given in (3.1),

$$y_{hvmt} = \alpha + \beta Post_t + \eta \left(Post_t \times \tilde{f}_{v0} \right) + \delta_m + \theta_v + \zeta_t + \varepsilon_{hvmt}.$$

We will make use of two moments from this regression. The first uses the usual triple-diff intuition, implying

$$\frac{\partial \Delta_t y_{hvmt}}{\partial \tilde{f}_{v0}} = \hat{\eta}.$$

When we derive estimating equations below, we will generally do so in the same derivative terms. Second, it will sometimes be helpful to match the post outcomes for the average village specifically, for which we will use the estimated coefficient $\hat{\beta}$.

5.2.1 Parameters Set Exogenously

We normalize manufacturing productivity to $A = 1$ and set two parameters outside the model, the discount rate $\beta = 0.96$ and the returns to scale $\gamma = 0.8$.

5.2.2 Parameters that Match to Empirical Results Directly

We then match three critical parameters directly to the empirical results. These are the complementarity between village production, ν, the skill parameter θ that governs the labor supply elasticity, and the CES parameter in the production technology σ.

Substitutability Between Village Production, ν The market clearing condition for the final agricultural good aggregator implies that that village j output price is a function

of its production share and the consumption price for agriculture,

$$p_{jt} = \left(\frac{Y_{at}}{y_{jt}}\right)^{\frac{1}{\nu}} p_{act}.$$

Some algebra implies

$$(1-\nu)\Delta_t \log(p_{jt}) = \Delta_t \log(p_{jt}y_{jt}) - (\Delta_t \log(Y_{at}) + \nu \log(p_{act}))$$

Notice that the last term in parenthesis is constant across villages, which means that

$$\underset{[+0.032]}{(1-\nu)\frac{\partial \Delta_t \log(p_{jt})}{\partial x_{j0}}} = \underset{[-0.060]}{\frac{\partial \Delta_t \log(p_{jt}y_{jt})}{\partial x_{j0}}}$$

The parameter ν depends on the relative movements of village j production to its output price across villages. The relevant values from Section 3 (listed below the equation) give $\nu = 2.88$.

Our theoretical results in Section 4 highlight the importance of this parameter. The natural experiment combined with micro data allows us to infer it independently of the remaining model structure. This is our first, and most important, result suggesting that the model will imply a decline in the optimal fertilizer subsidy after the fertilizer price rises.

Joint Estimation of Production Elasticity σ and Labor Supply Elasticity θ Each household i in village j has a cost ratio of

$$\frac{p_{xt}x_{ijt}}{w_{ajt}n_{ijt}} = \frac{\alpha_j}{1-\alpha_j}\left(\frac{w_{ajt}}{p_{xt}}\right)^{\sigma-1} \tag{5.1}$$

A brief look at (5.1) might suggest that it can be estimated directly from the prices and inputs. The issue is that this wage bill also includes the opportunity cost of own-farm work and the unobservable labor skills. To solve this problem, note first that because the cost ratio is identical for every household, this also represents the village-level cost ratio. Second, because internal agricultural labor markets clear village-by-village, $w_{ajt}\int_i n_{ijt}di$ is equal to total agricultural wage earnings for households in village j. This implies that we can re-write (5.1) as

$$\underset{[-0.067]}{\Delta_t \log(p_{xt}x_{jt})} = \Delta_t \log(\text{Agr. Earnings}_j) + (\sigma-1)[\underset{[+0.012]}{\Delta_t \log(w_{ajt})} - \underset{[+0.752]}{\Delta_t \log(p_x)}] \tag{5.2}$$

or, noting that the fertilizer price p_x does not vary across villages,

$$\underset{[-0.089]}{\frac{\partial \Delta_t \log(p_{xt} x_{jt})}{\partial x_{j0}}} = \frac{\partial \Delta_t \log(\text{Agr. Earnings}_j)}{\partial x_{j0}} + (\sigma - 1) \underset{[+0.012]}{\frac{\partial \Delta_t \log(w_{ajt})}{\partial x_{j0}}}. \tag{5.3}$$

Our strategy here will be to match both the average time variation in (5.2) and the additional cross-village variation in (5.3) with parameters σ (the CES parameter of the production function) and θ (the skill parameter that controls labor supply elasticity) because we cannot read agricultural earnings off the data in a model-consistent way.

However, we can replace $\log(\text{Agr. Earnings}_j)$ in (5.2) and $\partial \Delta_t \log(\text{Agr. Earnings}_j)/\partial x_{j0}$ in (5.3) by taking advantage of the properties of the exponential distribution. Specifically, the earnings ratio in village j is

$$\frac{\text{Total Manufacturing Earnings}}{\text{Total Agricultural Earnings}} = \frac{T_m}{T_a} \left(\frac{w_{mt}}{w_{ajt}} \right)^{\theta}.$$

which means that the relevant terms are

$$\begin{aligned} \Delta_t \log(\text{Agr. Earnings}) &= \underset{[-0.253]}{\Delta_t \log(\text{Man. Earnings})} + \underset{[+0.012]}{\theta \Delta_t \log(w_{ajt})} \\ \frac{\partial \Delta_t \log(\text{Agr. Earnings})}{\partial x_{j0}} &= \underset{[-0.108]}{\frac{\partial \Delta_t \log(\text{Man. Earnings})}{\partial x_{j0}}} + \underset{[+0.012]}{\theta \frac{\partial \Delta_t \log(w_{ajt})}{\partial x_{j0}}} \end{aligned}$$

Plugging this back into (5.2) and (5.3) gives two equations and two unknowns.

$$\begin{aligned} 0.19 &= 0.01\theta - 0.74(\sigma - 1) \\ 0.02 &= 0.01\theta + 0.01(\sigma - 1) \end{aligned}$$

Together, these imply $\sigma = 0.78$ and $\theta = 1.81$. Thus, fertilizer and labor are slightly less substitutable than Cobb-Douglas. That σ is near one is a useful sanity check on the results. We observe a large increase in fertilizer prices but a small change in the fertilizer cost share of revenue, a result intuitively consistent with approximately Cobb-Douglas production.

5.2.3 Parameters Set to Match Equilibrium Moments

What remains are parameters that require solving the impulse response of the model to match moments. There are the two shape parameters from the exponential distribution of skills, T_a and T_m, the gross interest rate R, and the joint distribution of

technology and financial frictions across villages, $G(\alpha, \phi)$.

T_a, T_m, R, σ_z to Match Baseline Cross-Sectional Moments We set T_a, T_m to match the average baseline wage premium in the manufacturing sector and the share of earnings derived from manufacturing. To measure the wage premium, we run the cross-sectional regression on baseline data,

$$\log(w_{i\ell v0}) = \beta_0 + \beta_1 \mathbb{1}[\ell = \text{outside}] + \beta_2 X_{i\ell v0} + \theta_v + \varepsilon_{i\ell v0}$$

for individual i working in location ℓ in village v at baseline. We control for individual characteristics like age and education in X and village fixed effects in θ. The outcome variable $\log(w)$ is the log daily wage rate. This yields $\hat{\beta}_1 = 0.23$ (s.e. $= 0.009$). Second, we note that in the average village at baseline, 62 percent of all wage earnings come from outside-village activities. We match these two moments.

We set the interest rate R such that we match the total savings/harvest ratio of the average village.

Finally, we set the standard deviation of the idiosyncratic shocks σ_z to match the average within-village variation in log yields.

Joint Estimation of Technological and Financial Friction Heterogeneity, $G(\alpha, \phi)$ We parameterize the joint distribution by assuming the marginal distribution of $\alpha \sim \text{Beta}(p_\alpha, q_\alpha)$ and $\phi \sim \text{Fréchet}(\theta_\phi, s_\phi, 1)$ where the parameters are shape, scale and location ($= 1$ so that villages can always self-finance). We then combine the two with a Frank copula, which imposes the joint c.d.f.

$$G(\alpha, \phi) = \frac{-1}{\rho} \log \left[1 + \frac{(e^{-\rho H_\alpha} - 1)(e^{-\rho H_\phi} - 1)}{e^{-\rho} - 1} \right]$$

where H_α and H_ϕ are the marginal c.d.f.s for technology and the financial friction. This leaves us with five parameters. Two from each of the marginals and ρ, which governs co-movement between the two.

Critical for quantifying the model is to make sure the changes in distortions we derive are consistent with the data. If the model predicts (overly) large changes to distortions, the model will also want to generate large swings in fertilizer subsidies. This is a particularly complicated issue here, because we do not know whether villages using little fertilizer do so because of technology α or frictions ϕ.

We solve this by re-casting the theoretical distortions as functions of our empirical results and thus target changes in distortions directly in the estimation. While all 5 parameters are jointly estimated (along with other parameters of the model), our

strategy here is roughly the following. We first choose the parameters of the marginal financial frictions distribution to match the evolution of distortions after the fertilizer price shock. We then set the parameters of the marginal technology distribution to capture any remaining variation in fertilizer cost shares. We finally estimate the co-variation of frictions and technology based on how changes to distortions vary with baseline fertilizer intensity.

We start by measuring the evolution of distortions. By distortions, we we measure deviations from undistorted fertilizer cost share of revenue. With a slight abuse of notation from our analytic results, we denote this as the parameter κ in the equation below.

$$\kappa_{ijt}^{\sigma} p_{xt} x_{ijt} = p_{ajt}^{\frac{\sigma-1}{\gamma}} p_{xt}^{\frac{1-\sigma}{\sigma}} \gamma^{\sigma} \alpha_j z_{ij}^{\frac{1-\gamma}{\gamma}(\sigma-1)} (p_{ajt} y_{ijt})^{\sigma-\frac{\sigma-1}{\gamma}} \tag{5.4}$$

κ measures a household's deviation from its undistorted cost share.[17] Taking log differences yields

$$\begin{aligned} \Delta_t \log(\kappa_{ij}) = & \underbrace{-\frac{1}{\sigma}\Delta_t \log(p_x x_{ij})}_{\text{[-0.067]}} - \underbrace{\frac{\sigma-1}{\sigma\gamma}\Delta_t \log(p_{aj})}_{\text{[+0.023]}} \\ & + \underbrace{\left(1-\frac{\sigma-1}{\sigma\gamma}\right)\Delta_t \log(p_{aj} y_{aj})}_{\text{[-0.149]}} + \underbrace{(1-\sigma)\Delta_t \log(p_x)}_{\text{[+0.752]}} \end{aligned} \tag{5.5}$$

(5.5) tells us how the distortions in household i from village j changes after the price shock. Notice that we can use the variation induced by the empirical price shock, combined with our values of σ and γ to measure the average value of (5.5). We therefore target the two parameters of the marginal financial frictions distribution θ_ϕ and s_ϕ to match the mean and variance of (5.5). This is the sense in which we target the evolution of distortions directly. Matching the distortion changes guarantees we are not needlessly biasing the model for or against large policy changes.

We then choose the parameters of the technology distribution p_α and q_α to make sure our baseline variation in fertilizer cost shares is consistent with the data. Therefore, we target the mean and variance of fertilizer cost shares across villages.

Finally, we require some way to estimate the co-movement of α and ϕ across villages. Taking the derivative of (5.5) noting that p_x is constant across villages

[17] While the CES assumption makes this equation somewhat unintelligible, note that with $\sigma = 1$ it simplifies to

$$\frac{p_{xt} x_{ijt}}{p_{ajt} y_{aijt}} = \frac{\gamma \alpha_j}{\kappa}$$

where $\gamma\alpha_j$ is the exponent on fertilizer in the production function $y = z(x^{\alpha} n^{1-\alpha})^{\gamma}$. (5.4) is just the CES generalization of this familiar Cobb-Douglas result.

implies

$$\frac{\partial \Delta_t \log(\kappa_{ij})}{\partial x_{j0}} = -\frac{1}{\sigma}\frac{\partial \Delta_t \log(p_x x_{ij})}{\partial x_{j0}} - \frac{\sigma-1}{\sigma\gamma}\frac{\partial \Delta_t p_{aj}}{\partial x_{j0}} + \left(1 - \frac{\sigma-1}{\sigma\gamma}\right)\frac{\partial \Delta_t \log(p_{aj} y_{aj})}{\partial x_{j0}} \tag{5.6}$$

[-0.089] [+0.032] [-0.060]

Intuitively, this moment determines whether villages with low baseline fertilizer intensity or high baseline fertilizer intensity see their distortion grow the most. Roughly, if this value is positive, it implies that places with high α also have low ϕ (i.e., tight financial frictions).

6 Quantitative Results

Our calibration is still in progress because it requires solving the transition path for each parameter set. Therefore, we provide results for a set of parameters that match most, but not all, of our moments, though all of them are in the right ballpark and we do not expect huge swings in the quantitative results.

With those caveats noted, we find that on impact the optimal fertilizer subsidy rate falls by 60 percent, from $\tau_x = 0.05$ to $\tau_x = 0.02$. The baseline level is quite a bit lower than the subsidy observed in Rwanda, though we can adjust welfare weights to guarantee the two match.[18] While this changes the baseline level, it will not change the qualitative patterns that the optimal fertilizer subsidy falls upon impact.

Figure 6 begins by plotting the decline in village-level production in the technology-financial friction space (α, ϕ) immediately upon impact, taking into account the decline in the optimal fertilizer subsidy. The impact varies substantially, ranging from 5 percent in financially unconstrained, fertilizer un-intensive villages (that is, high ϕ and low α) to almost 60 percent in low-ϕ, high-α villages. While there is some variation by the level of financial frictions, the impact is mostly driven by technology α. All told, this leads to a 20% decline in production of the agricultural consumption good.

We then study the relative importance of the subsidy change. Specifically, we compare two economies: one in which the fertilizer subsidy falls on impact to its optimal 2% level and another in which it is held fixed at its baseline 5% level. Figure 7 plots village-level outcomes relative to this baseline of an unchanged subsidy. We report the percentage change between the two. Figure 7a shows that production falls for all villages, a natural consequence of the lower subsidy rate. However, it

[18] The simplest way to do this is to assume that the policymaker maximizes a generalized mean of individual-level welfare. Then utilitarian and Rawlsian welfare are special cases. We have verified that this procedure works, but report the utilitarian results here in the interest of simplicity.

Figure 6: Impact of Optimal Subsidy Level on Village-Level Outcomes

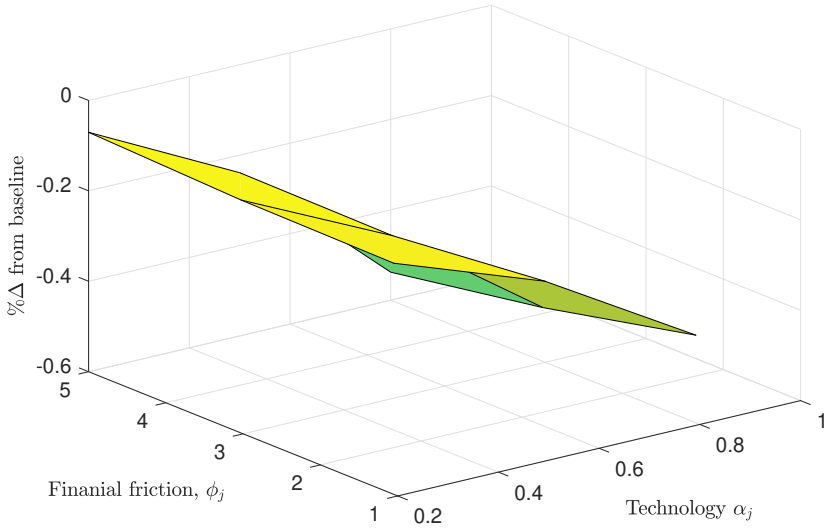
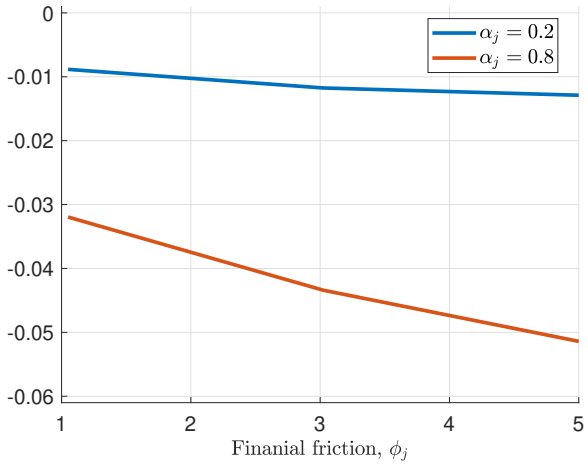


Figure Notes: This figure reports the percentage decline in village production from baseline to the first period after the price shock, taking into account the evolution of the optimal fertilizer subsidy.

falls substantially more for high-α villages. The intuition follows from our theoretical results earlier. Because villages are substitutes, the optimal subsidy attempts to re-direct production toward low-α villages after the price shock. Holding the subsidy fixed at its baseline level keeps too much production in high-α villages.

Figure 7: Impact of Optimal Subsidy Level on Village-Level Outcomes

(a) Production



(b) Village Output Price

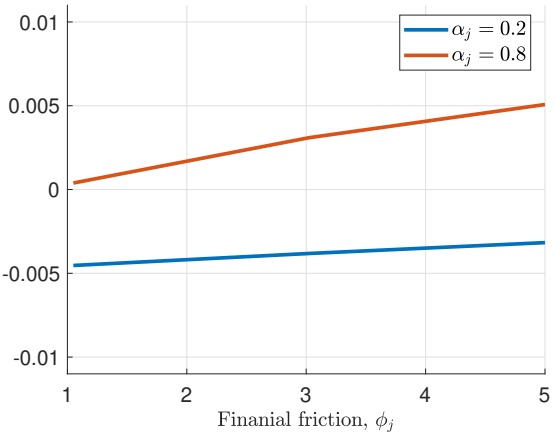
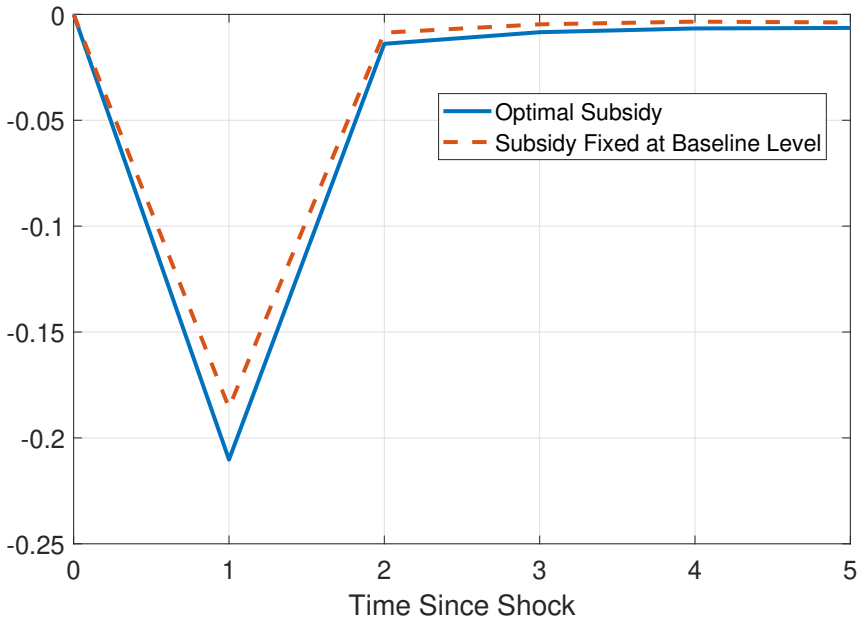


Figure Notes: These figures report the outcome in the period of initial impact, measured as as the log difference between the optimal subsidy and the economy in which the subsidy is held fixed at its baseline level.

Interestingly, the net effect of the lower fertilizer subsidy is in fact a decline in production of the agricultural consumption good, which we show in Figure 8. In the optimal subsidy case, aggregate production falls by 21% but only 18% if the subsidy is held at its baseline level. Intuitively, the higher fertilizer subsidy keeps sectoral production higher. However, it comes at the cost of taxing households at a higher rate given the increased cost to the government of supporting the subsidy. As such, for welfare purposes, the government prefers the lower subsidy. These results highlight important distinctions between welfare and production maximizing policies in general equilibrium.

Figure 8: Agricultural Consumption Good Production Under Two Subsidies



7 Conclusion

Agriculture has low labor productivity relative to other sectors in developing countries. This fact, along with its implications for the distribution of employment across sectors, have been well appreciated. Less appreciated is the fact that farmers are subject to global forces affecting the price of one of their most critical inputs, fertilizer, perhaps due to its historical price stability. This paper shows that these fluctuations can be quite damaging for rural households but that the government can offer stability in terms of adjusting already-existing policy levers. We show this in the context of Rwanda.

An interesting angle for future work would be to consider the long-run implications

of these shocks. We know, for example, that long run elasticities of substitution for intermediates can be substantially different from short-run elasticities (Peter and Ruane, 2022). In fact, in late 2023 the Rwandan government created a joint venture with a Moroccan fertilizer company to create a fertilizer processing plant in Rwanda, suggesting a similar divergence in this context (MINAGRI, 2023). These types of adjustments are rare, but perhaps more importantly, do not necessarily solve issues related to the procurement of inputs like natural gas. Thus, future policy will likely require a combination of both short run stability and longer run adaptation.

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A Natural Gas Prices and Global Fertilizer Prices

Figure 9 plots the relationship between natural gas prices and fertilizer prices since 2014 from the World Bank Commodity Price Outlook (World Bank, 2024). The correlation between the gas prices and the two fertilizer prices are 0.67 and 0.69 for DAP and urea. Figure 9 also shows that two key fertilizer prices more than double in just two years between 2020 and 2022.

Figure 9: Fertilizer Prices and Natural Gas Prices

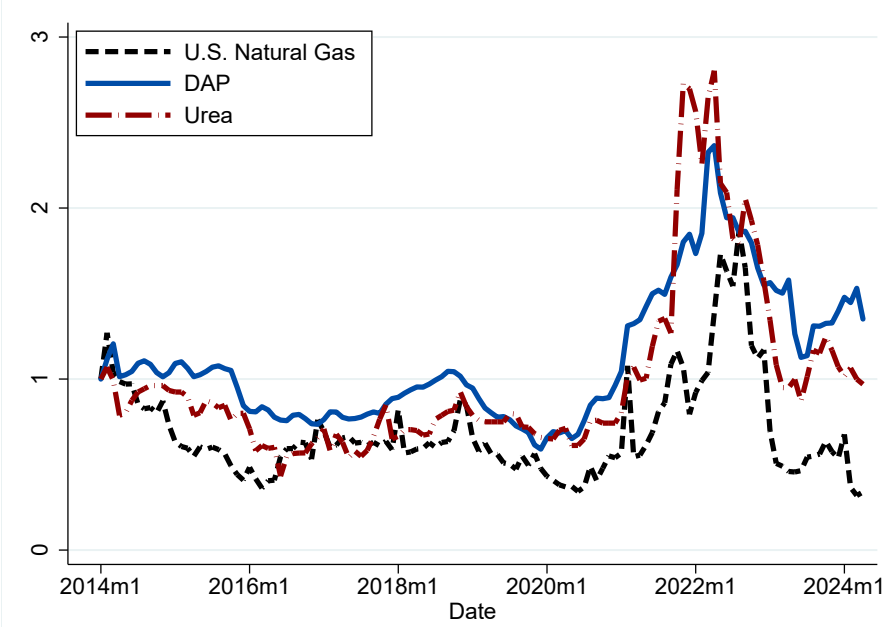


Figure Notes: Nominal monthly prices of DAP, urea, and U.S. natural gas from World Bank (2024). 2014m1 normalized to one.

B Competitive Equilibrium of Simplified Model

Given a subsidy level τ_x and market price of fertilizer p_x^{market}, a competitive equilibrium of this model is a set of decision rules for households $(c_{aj}, c_{mj}, x_j, n_{aj})$, the manufacturing firm N_m, and the final agricultural good producer y_{aj}, prices $\{p_{aj}\}, p_{ac}, w_a$ and tax rate τ_m such that (i) the household's decision rules are consistent with its optimization problem given prices and taxes, (ii) N_m solves the manufacturing firm's decision problem, (iii), y_{aj} solves the agricultural final goods firm decision problem, and (iv) the government balances its budget and markets clear:

1. Government budget balance: $\tau_x p_x^{market} \int_j x_j dj = \tau_m \int_j c_{mj} dj$.
2. Market clearing:
   (a) Agricultural intermediate goods market: for each j, $y_{aj} = x_j^{\alpha_j} n_j^{\eta_j}$
   (b) Agricultural final goods market: $\left(\int_j y_j^{\frac{\nu-1}{\nu}} dj\right)^{\frac{\nu}{\nu-1}} = \int_j c_{aj} dj$
   (c) The manufacturing labor market: $N_m = 1 - \int_j n_j dj$, which amounts to the condition that $w_a = w_m = A$.

C Proofs

C.1 Characterization of Frictionless Equilibrium and Proof of Lemma 1

Proof. The proof of Lemma 1 proceeds essentially by guess-and-verify. Specifically, we hypothesize that there exists some value $\mathcal{Y}$ such that the following results hold:

$$\frac{p^2_{ac}}{p^1_{ac}} = \frac{Y^1_a}{Y^2_a} = \mathcal{Y}$$

We then show that there is only one value for $\mathcal{Y}$ that satisfies the market clearing conditions and optimality conditions.

Constructing the Candidate $\mathcal{Y}$ From market clearing for village production

$$p_{aj} = \left(\frac{Y_a}{y_{aj}}\right)^{\frac{1}{\nu}} p_{ac}$$

Therefore,

$$\frac{p^2_{aj}}{p^1_{aj}} = \left(\frac{Y^2_a}{Y^1_a}\right)^{\frac{1}{\nu}} \left(\frac{y^2_{aj}}{y^1_{aj}}\right)^{\frac{-1}{\nu}} \left(\frac{p^2_{ac}}{p^1_{ac}}\right) = \mathcal{Y}^{\frac{\nu-1}{\nu}} \left(\frac{y^2_{aj}}{y^1_{aj}}\right)^{\frac{-1}{\nu}}. \tag{C.1}$$

Using this price ratio in combination with the village farm first order conditions implies

$$\frac{y^2_{aj}}{y^1_{aj}} = \left(\frac{p^2_{aj}}{p^1_{aj}}\right)^{\frac{\gamma}{1-\gamma}} \left(\frac{p^2_x}{p^1_x}\right)^{\frac{-\alpha_j}{1-\gamma}} = \mathcal{Y}^{\frac{(\nu-1)\gamma}{\nu(1-\gamma)}} \left(\frac{y^2_{aj}}{y^1_{aj}}\right)^{\frac{-\gamma}{\nu(1-\gamma)}} \left(\frac{p^2_x}{p^1_x}\right)^{\frac{-\alpha_j}{1-\gamma}}$$

which, re-arranging, gives

$$\frac{y^2_{aj}}{y^1_{aj}} = \mathcal{Y}^{\frac{(\nu-1)\gamma}{\nu-(\nu-1)\gamma}} \left(\frac{p^2_x}{p^1_x}\right)^{\frac{-\alpha_j\nu}{\nu-(\nu-1)\gamma}}. \tag{C.2}$$

and plugging this result into (C.1) gives the village price ratio of

$$\begin{aligned} \frac{p^2_{aj}}{p^1_{aj}} &= \mathcal{Y}^{\frac{\nu-1}{\nu}} \mathcal{Y}^{\frac{-(\nu-1)\gamma}{\nu(\nu-(\nu-1)\gamma)}} \left(\frac{p^2_x}{p^1_x}\right)^{\frac{\alpha_j}{\nu-(\nu-1)\gamma}} \\ &= \mathcal{Y}^{\frac{\nu-(\nu-1)\gamma-1}{\nu-(\nu-1)\gamma}} \left(\frac{p^2_x}{p^1_x}\right)^{\frac{\alpha_j}{\nu-(\nu-1)\gamma}} \end{aligned} \tag{C.3}$$

Aggregate to get total production

$$Y_2 = \left(\int y_{2j}^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}} = \mathcal{Y}^{\frac{(\nu-1)\gamma}{\nu-(\nu-1)\gamma}} \left(\int_j \left(\frac{p_x^2}{p_x^1} \right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{aj1}^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}}.$$

Under the maintained hypothesis that $Y_{a2} = Y_{a1}\mathcal{Y}^{-1}$, then

$$\mathcal{Y}^{-1} \left(\int_j y_{a1j}^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}} = \mathcal{Y}^{\frac{(\nu-1)\gamma}{\nu-(\nu-1)\gamma}} \left(\int_j \left(\frac{p_x^2}{p_x^1} \right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{aj1}^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}}$$

which can be re-arranged to solve for the candidate $\mathcal{Y}$

$$\mathcal{Y} = \left(\frac{\int_j y_{a1j}^{\frac{\nu-1}{\nu}} dj}{\int_j \left(\frac{p_x^2}{p_x^1} \right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{aj1}^{\frac{\nu-1}{\nu}} dj} \right)^{\frac{\nu-(\nu-1)\gamma}{\nu-1}}. \tag{C.4}$$

The construction of $\mathcal{Y}$ in (C.4) uses two conditions. First, it uses the relevant farm first order conditions. Thus, it already satisfies the optimality conditions of the village farms. Second, it uses the market clearing condition from the stand-in firm that aggregates village production into the agricultural final good. Thus, it satisfies this market clearing condition by construction. The only remaining non-trivial market clearing condition is that of the demand and supply for the final agricultural good.

Confirming that Agricultural Market Clearing Condition is Satisfied Combining the characterizations of village output quantity and price from equations (C.2) and (C.3) gives farm revenue

$$\frac{p_{a2j} y_{a2j}}{p_{a1j} y_{a1j}} = \mathcal{Y}^{\frac{\nu-1}{\nu-(\nu-1)\gamma}} \left(\frac{p_x^2}{p_x^1} \right)^{\frac{\alpha_j(1-\nu)}{\nu-(\nu-1)\gamma}}$$

and therefore farm profit can be written as (we highlight this equation in red because it is used in the main text)

$$\begin{aligned}
\pi_{2j} &= (1-\gamma)p_{a2}y_{2j} \\
&= (1-\gamma)\mathcal{Y}^{\frac{\nu-1}{\nu-(\nu-1)\gamma}}\left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} p_{a1}y_{1j} \\
&= \underbrace{\left(\frac{\int_j y_{a1j}^{\frac{\nu-1}{\nu}}dj}{\int_j\left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{aj1}^{\frac{\nu-1}{\nu}}dj}\right)\left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}}}_{\equiv \text{ adjustment to } \pi_{1j}} \underbrace{(1-\gamma)p_{a1}y_{1j}}_{\equiv\ \pi_{1j}}
\end{aligned}$$

Using the fact that $p_{aj} = (Y_a/y_{aj})^{\frac{1}{\nu}}p_{ac}$, total baseline income is

$$\begin{aligned}
\int_j \pi_{j1}dj + w &= (1-\gamma)\int_j p_{a1j}y_{a1j}dj + w = (1-\gamma)Y_{a1}^{\frac{1}{\nu}}p_{ac1}\int y_{a1j}^{\frac{\nu-1}{\nu}}dj + w \\
&= (1-\gamma)p_{ac1}\left(\int_j y_{a1j}^{\frac{\nu-1}{\nu}}\right)^{\frac{\nu}{\nu-1}} + w = (1-\gamma)p_{a1c}Y_{a1} + w
\end{aligned}$$

which remains constant after the price shock

$$\begin{aligned}
\int \pi_{j2}dj + w &= (1-\gamma)\left(\frac{\int_j y_{a1j}^{\frac{\nu-1}{\nu}}dj}{\int_j\left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{aj1}^{\frac{\nu-1}{\nu}}dj}\right)\left(\int_j\left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} p_{a1j}y_{1j}dj\right) + w \\
&= (1-\gamma)\left(\frac{\int_j y_{a1j}^{\frac{\nu-1}{\nu}}dj}{\int_j\left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{aj1}^{\frac{\nu-1}{\nu}}dj}\right)\left(Y_{a1}^{\frac{1}{\nu}}p_{ac1}\int_j\left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{1j}^{\frac{\nu-1}{\nu}}dj\right) + w \\
&= (1-\gamma)\left(\int_j y_{a1j}^{\frac{\nu-1}{\nu}}dj\right)Y_{a1}^{\frac{1}{\nu}}p_{ac1} + w \\
&= (1-\gamma)p_{a1c}Y_{a1} + w.
\end{aligned}$$

That the market clearing condition is satified follows quickly.

$$D_{a2} = \frac{\zeta}{p_{ac2}}\left(\int_j \pi_{j2}dj + w\right) = \frac{\zeta}{\mathcal{Y}p_{a1c}}\left(\int_j \pi_{j1}dj + w\right) = \frac{D_{a1}}{\mathcal{Y}}$$

Since $Y_{a2} = Y_{a1}/\mathcal{Y}$, this implies $Y_{a2} = D_{a2}$ as required. ■

C.2 Proof of Proposition 1

Proof. Using the results from the proof of Lemma 1, farm profit rises

$$\pi_{j2} > \pi_{j1} \quad \Leftrightarrow \quad \left(\frac{\int_j y_{a1j}^{\frac{\nu-1}{\nu}} dj}{\int_j \left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{aj1}^{\frac{\nu-1}{\nu}} dj} \right) \left(\frac{p_x^1}{p_x^2}\right)^{\frac{\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} (1-\gamma) p_{a1j} y_{a1j} > (1-\gamma) p_{a1j} y_{a1j}$$

$$\Leftrightarrow \quad \frac{\int_j y_{a1j}^{\frac{\nu-1}{\nu}} dj}{\int_j \left(\frac{p_x^1}{p_x^2}\right)^{\frac{\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{aj1}^{\frac{\nu-1}{\nu}} dj} > \left(\frac{p_x^2}{p_x^1}\right)^{\frac{\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} \tag{C.5}$$

$$\Leftrightarrow \quad \left(\frac{\int_j y_{a1j}^{\frac{\nu-1}{\nu}} dj}{\int_j \left(\frac{p_x^1}{p_x^2}\right)^{\frac{\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{aj1}^{\frac{\nu-1}{\nu}} dj} \right)^{\frac{\nu-(\nu-1)\gamma}{\nu-1}} > \left(\frac{p_{x2}}{p_{x1}}\right)^{\alpha_j} \tag{C.6}$$

This last line is used in the main text. If you raise both sides to the $1/(1-\gamma)$ power, the RHS is the partial equilibrium cost to farm profit. The LHS is the increase in farm profit from higher prices.

From here, we break the results into three cases depending on the value of ν.

If $\nu = 1$ If $\nu = 1$, then (C.5) boils down to $1 = 1$. So all villages receive exactly the same income in the two states.

If $\nu > 1$ Set $\alpha = 0$. Then (C.5) is

$$\frac{\int_j y_{a1j}^{\frac{\nu-1}{\nu}} dj}{\int_j \left(\frac{p_x^1}{p_x^2}\right)^{\frac{\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{aj1}^{\frac{\nu-1}{\nu}} dj} > 1$$

This is true, because $p_{x1}/p_{x2} < 1$, so that the RHS is always bigger than one.

If $\alpha = \gamma$, then note that

$$\int_j \left(\frac{p_x^1}{p_x^2}\right)^{\frac{\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{aj1}^{\frac{\nu-1}{\nu}} dj > \int_j \left(\frac{p_x^1}{p_x^2}\right)^{\frac{\gamma(\nu-1)}{\nu-(\nu-1)\gamma}} y_{aj1}^{\frac{\nu-1}{\nu}} dj = \left(\frac{p_x^1}{p_x^2}\right)^{\frac{\gamma(\nu-1)}{\nu-(\nu-1)\gamma}} \int_j y_{aj1}^{\frac{\nu-1}{\nu}} dj$$

Therefore,

$$\frac{\int_j y_{a1j}^{\frac{\nu-1}{\nu}} dj}{\int_j \left(\frac{p_x^1}{p_x^2}\right)^{\frac{\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{aj1}^{\frac{\nu-1}{\nu}} dj} < \frac{\int_j y_{a1j}^{\frac{\nu-1}{\nu}} dj}{\left(\frac{p_x^1}{p_x^2}\right)^{\frac{\gamma(\nu-1)}{\nu-(\nu-1)\gamma}} \int_j y_{aj1}^{\frac{\nu-1}{\nu}} dj} = \left(\frac{p_x^2}{p_x^1}\right)^{\frac{\gamma(\nu-1)}{\nu-(\nu-1)\gamma}}$$

so that profit falls when $\alpha = \gamma$. The intermediate value theorem implies that there is an $\hat{\alpha}$ such that $\alpha < \hat{\alpha}$ have profit rises and $\alpha > \hat{\alpha}$ where profit falls.

If $\nu < 1$ This is identical to the $\eta > 1$ case with the results flipped. If $\alpha = 0$, then because the coefficient on the p_x ratio is negative,

$$\frac{\int_j y_{a1j}^{\frac{\nu-1}{\nu}} dj}{\int_j \left(\frac{p_x^1}{p_x^2}\right)^{\frac{\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{aj1}^{\frac{\nu-1}{\nu}} dj} < 1$$

Therefore, $\alpha = 0$ villages see their profit fall. Using the same strategy as above (set $\alpha_j = \gamma \; \forall j$ in the p_x ratio) $\alpha = \gamma$ villages increase profit. The result is monotone, as is the case with $\nu > 1$, but the change in profit is monotonically increasing in α instead of decreasing.

■

C.3 Proof of Proposition 2

C.3.1 An Additional Lemma

Before proving this result, it is helpful to characterize a few results from the frictional equilibrium. We do those in Lemma 2.

Lemma 2. *Two results are true in any equilibrium of the frictional economy.*

1. *Total economy-wide expenditures $E := \int_j \pi_j dj + A$ do not depend on the market price p_x^{market} or the subsidy τ_x.*
2. *The budget-balancing manufacturing tax τ_m does not depend on market price p_x^{market}.*

Proof.

Claim 1 Denoting $e_j = \pi_j + A$ as total expenditures in village j and $E = \int_j e_j dj$ as total economy-wide expenditures, the market clearing condition for agricultural

consumption implies

$$p_{ac} = \frac{\zeta E}{Y_a}.$$

This means the village j output price is

$$p_{aj} = \left(\frac{Y_a}{y_{aj}}\right)^{\frac{1}{\nu}} p_{ac} = Y_a^{\frac{1-\nu}{\nu}} y_{aj}^{\frac{-1}{\nu}} \zeta E.$$

Total economy-wide expenditures are therefore

$$\begin{aligned} E &= \int_j \Big[(Y_a^{\frac{1-\nu}{\nu}} y_{aj}^{\frac{-1}{\nu}} \zeta E) y_{aj} - \phi_j + A \Big] dj \\ &= Y_a^{\frac{1-\nu}{\nu}} \Big(\int_j y_{aj}^{\frac{\nu-1}{\nu}} dj \Big) \zeta E - \mathbb{E}[\phi] + A \\ &= Y_a^{\frac{1-\nu}{\nu}} Y_a^{\frac{\nu-1}{\nu}} \zeta E - \mathbb{E}[\phi] + A \\ &= \zeta E - \mathbb{E}[\phi] + A \end{aligned}$$

which implies $E = \frac{A - \mathbb{E}[\phi]}{1-\zeta}$, which is independent of the fertilizer price and subsidy.

Claim 2 The government's budget constraint is

$$\tau_x p_x^{\text{market}} \int_j x_j dj = \tau_m \int_j c_{mj} dj.$$

Optimality conditions imply

$$\begin{aligned} x_j &= \frac{\alpha_j \phi_j}{\gamma p_x^{\text{market}} (1 - \tau_x)} \\ c_{mj} &= \frac{(1-\zeta) e_j}{1 + \tau_m} \end{aligned}$$

where $e_j := \pi_j + A$ is total expenditures in village j. This implies that the government budget constraint is

$$\left(\frac{\tau_x}{1-\tau_x}\right) \frac{1}{\gamma} \Big(\int_j \alpha_j \phi_j dj \Big) = \frac{\tau_m}{1+\tau_m} (1-\zeta) E$$

where $E = \int_j \pi_j dj + A$ is economy-wide expenditures. By the first result that E does not depend on p_x^{market}, this proves the second claim. ■

C.3.2 Proof of Proposition 2

Proof. Denoting $e_j = \pi_j + w$ as the expenditures of village j, we can write the consumption decisions $p_{ac}c_a = \zeta e_j$ and $(1+\tau_m)c_m = (1-\zeta)e_j$ which gives the indirect utililty function for village j,

$$\Omega - \zeta \log(p_{ac}) + (\zeta - 1)\log(1+\tau_m) + \log(e_j)$$

with constant Ω. Manipulating the agricultural consumption market clearing condition implies

$$p_{ac} = \frac{\zeta E}{Y_a}$$

where $E := \int_j e_j dj$ is total expenditures. The utilitarian government problem is therefore

$$S := \max_{\tau_x} \; e^{\Omega} Y_a^{\zeta} (1+\tau_m)^{\zeta-1} E^{1-\zeta}$$

or

$$\log(S) = \max_{\tau_x} \; \Omega + \zeta \log(Y_a) + (\zeta - 1)\log(1+\tau_m) + (1-\zeta)\log(E).$$

By Lemma 2, E does not respond to τ_x. Therefore, the first order condition is

$$\zeta \frac{\partial \log(Y_a)}{\partial \tau_x} + (\zeta - 1)\frac{\partial \log(1+\tau_m)}{\partial \tau_x} = 0. \tag{C.7}$$

By the implicit function theorem, we are interested in

$$\frac{\partial \tau_x^*}{\partial p_x^{\text{market}}} = -\frac{\frac{\partial^2 \log(S)}{\partial \tau_x \partial p_x^{\text{market}}}}{\frac{\partial^2 \log(S)}{\partial \tau_x^2}}$$

Since $\log(S)$ is concave,

$$\text{sign}\left(\frac{\partial \tau_x^*}{\partial p_x^{\text{market}}}\right) = \text{sign}\left(\frac{\partial^2 \log(S)}{\partial \tau_x \partial p_x^{\text{market}}}\right). \tag{C.8}$$

Taking the derivative of (C.7) and using the results of Lemma 2 to note that $\partial \tau_m / \partial p_x^{\text{market}} = 0$, the sign restriction in (C.8) is equivalent to

$$\text{sign}\left(\frac{\partial \tau_x^*}{\partial p_x^{\text{market}}}\right) = \text{sign}\left(\frac{\partial^2 \log(Y_a)}{\partial \tau_x \partial p_x^{\text{market}}}\right). \tag{C.9}$$

Since

$$y_{aj} = \frac{\phi_j}{\gamma^\gamma} \left(\frac{\alpha_j}{p_x^{\text{market}}(1-\tau_x)} \right)^{\alpha_j} \left(\frac{\eta_j}{A} \right)^{\eta_j},$$

then

$$\log(Y) = \left(\frac{\nu}{\nu - 1} \right) \log \left(\int_j \left[\Delta_j (p_x^{\text{market}})^{-\alpha_j} (1-\tau_x)^{-\alpha_j} \right]^{\frac{\nu-1}{\nu}} dj \right)$$

where Δ_j summarizes the remaining terms from y_{aj} that do not include p_x^{market} or τ_x. Taking the derivative with respect to the market fertilizer price,

$$\frac{\partial Y_a}{\partial p_x} = \frac{-1}{p_x^{\text{market}}} \int_j \alpha_j \frac{\left[\Delta_j (p_x^{\text{market}})^{-\alpha_j} (1-\tau_x)^{-\alpha_j} \right]^{\frac{\nu-1}{\nu}}}{\int_k \left[\Delta_k (p_x^{\text{market}})^{-\alpha_k} (1-\tau_x)^{-\alpha_k} \right]^{\frac{\nu-1}{\nu}} dk}.$$

Define the probability measure

$$\Gamma_j = \frac{\left[\Delta_j (p_x^{\text{market}})^{-\alpha_j} (1-\tau_x)^{-\alpha_j} \right]^{\frac{\nu-1}{\nu}}}{\int_k \left[\Delta_k (p_x^{\text{market}})^{-\alpha_k} (1-\tau_x)^{-\alpha_k} \right]^{\frac{\nu-1}{\nu}} dk}$$

so that we can re-write

$$\frac{\partial Y_a}{\partial p_x} = \frac{-\mathbb{E}_\Gamma[\alpha; \tau_x]}{p_x^{\text{market}}}$$

where $\mathbb{E}_\Gamma$ is the expectation of α taken using Γ, which depends (in part) on τ_x. This implies

$$\text{sign} \left(\frac{\partial^2 \log(Y_a)}{\partial \tau_x \partial p_x^{\text{market}}} \right) = - \text{ sign} \left(\frac{\partial \mathbb{E}_\Gamma[\alpha; \tau_x]}{\partial \tau_x} \right). \tag{C.10}$$

Note that

$$\frac{\partial \log(\Gamma_j / \Gamma_k)}{\partial \tau_x} = \frac{\nu - 1}{\nu} (\alpha_j - \alpha_k) \left(\frac{1}{1-\tau_x} \right).$$

If $\nu > 1$, this is positive for $\alpha_j > \alpha_k$, implying that weight is shifting toward higher α, and therefore $\partial \mathbb{E}_\Gamma[\alpha; \tau_x] / \partial \tau_x > 0$. (C.10) therefore implies $\frac{\partial^2 \log(Y_a)}{\partial \tau_x \partial p_x^{\text{market}}} < 0$. The same exercise gives the required result that $\frac{\partial^2 \log(Y_a)}{\partial \tau_x \partial p_x^{\text{market}}} = 0$ when $\nu = 1$ and $\frac{\partial^2 \log(Y_a)}{\partial \tau_x \partial p_x^{\text{market}}} > 0$ when $\nu < 1$. By (C.9), $\partial \tau_x^* / \partial p_x$ shares the same signs.

■