A Miscalculated Risk:
Investment choices under exponential growth bias

Karna Basu*

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Abstract

"Exponential growth bias" – the tendency to underestimate exponential growth – has been shown to affect how much people save and borrow. I develop a simple theoretical framework to show that this bias, through its interaction with individual risk preferences, also affects *how* people save and borrow. Relatively risk-tolerant consumers will choose inefficiently safe investments, appearing more risk-averse than they are. In contrast, relatively risk-averse consumers will choose inefficiently risky investments, appearing less risk-averse than they are. I present survey data to support this hypothesis. The model introduces a new perspective on some apparent anomalies in intertemporal choice and raises new considerations for consumer protection and financial literacy programs.

Keywords: financial literacy, consumer protection, exponential growth bias, risk aversion.
JEL codes: D91, G41, G53

*Department of Economics, Hunter College and The Graduate Center, City University of New York (email: kbasu@hunter.cuny.edu). I am grateful to Matthew Baker, Saurabh Bhargava, Utteeyo Dasgupta, Jorgen Harris, Matthew Nagler, David Rosenkranz, seminar participants at Fordham University and Jindal Global Business School, and conference participants at ThRED for useful feedback. Hunter College IRB approval was obtained for the survey.

1 Introduction

In his itemization of obligatory human experiences, Benjamin Franklin should perhaps have changed "death and taxes" to "death, taxes, and exponential functions." All three are not only unavoidable but also hard to comprehend. We are surrounded by growth processes that follow exponential functions – procreation, contagion, and the evolution of savings and debt. In the case of savings and debt, exponential growth is a feature of compound interest, whereby interest accrued on a principal deposit or loan is itself subject to interest. Compound interest has been applied to loan contracts for thousands of years. In early civilizations, compound interest on cattle loans was likely based on their natural rates of reproduction (Lewin, 2019).

For a functional form so pervasive in the natural and social world and so central to financial literacy, exponential growth is remarkably difficult to estimate. The tendency to systematically underestimate exponential growth is known as *exponential growth bias* (henceforth "EGB"). This cognitive bias has been documented extensively and shown to persist across populations and financial contexts. Consumers tend to linearize the exponential compound interest function, often by anchoring on the linear simple interest function. This leads them to under-estimate compound interest accumulation, making future consumption appear more expensive than it really is. EGB can lead to sub-optimal financial choices and exploitation by firms, with possibly acute welfare consequences for low-income consumers or members of developing economies who lack access to computing and automation tools.

EGB has some self-evident effects – consumers will underestimate the costs of loans and the returns to saving while assessing intertemporal trade-offs. Teichman and Zamir (2022) show how legal frameworks can be designed to protect consumers from the consequences of such mistakes. Real-world attempts to protect consumers range from religious restrictions on usury to the contemporary regulation of mortgage contracts in the United States. The 2010 Dodd-Frank Act introduced sweeping changes to the regulation of US financial markets. Among these was the creation of the Consumer Financial Protection Bureau (CFPB), which expanded the government's role

in protecting consumers from not just fraudulent firm practices but also consumers' own biases like EGB (Corrigan, 2015). The CFPB's mandate, for example, includes regulating balloon payments in mortgage contracts and educating young consumers about the long-run benefits of saving.[1]

In this paper, I make the theoretical observation that EGB also leads to some less obvious mistakes that are not addressed by existing policies. I show that apart from distorting *how much* people save and borrow, EGB will also distort *which instruments* consumers use to save and borrow (for example, saving in stocks vs bonds, or borrowing at fixed vs floating rates). On the face of it, it is not obvious that EGB should have any effect on risk-taking. It turns out that the effect depends on the interplay of EGB with the consumer's risk preferences. EGB makes relatively risk-tolerant consumers appear more risk-averse than they are, pushing them towards inefficiently safe investments; with the opposite being true for relatively risk-averse consumers. This presents a new dimension over which EGB generates suboptimal behavior, and the fact that these mistakes are not monotonic in risk aversion correspondingly poses a new policy challenge.

I characterize EGB quite generally. As in prior literature, I assume the biased consumer uses a function that generates lower values and is less convex than the compound interest function. I then analyze how the consumer will choose from a set of savings instruments or investment opportunities. How does EGB affect the choice between, say, a safe and a risky investment; and if this choice were taken as an accurate reflection of a rational consumer's preferences, what would we infer about her risk tolerance? The answer is not transparent since EGB distorts the consumer's assessment of both risky and safe investments.

For intuition, consider an investment for which the rate of return is uncertain – it could be high or low. This investment matures at some fixed future date. What safe rate of return does the consumer view as equivalent to the risky investment? In other words, what is the certainty equivalent interest

[1] See https://www.consumerfinance.gov/rules-policy/regulations/1026/interp-37/ and https://www.consumerfinance.gov/consumer-tools/money-as-you-grow/teen-young-adult/explore-saving/.

rate, and how does it differ for the biased and unbiased consumer? Observe that EGB has two opposing effects. First, it lowers the expected utility of any risky investment ("EU effect"). Second, it raises the fixed interest rate required to attain a target utility ("required rate effect"). Whether the bias makes the consumer appear more or less risk-averse depends on which effect dominates.

Suppose the consumer is relatively risk-tolerant. She underestimates maturity values at both low and high interest rates, with underestimation most severe at the high rate (due to the reduced convexity of her maturity function). Since she is risk-tolerant, she is sensitive to changes in consumption even at high levels. So this underestimation of maturity value results in a significant drop in expected utility. In such cases, the EU effect is large relative to the required rate effect, resulting in a drop in the certainty equivalent interest rate. The risk-tolerant consumer will appear more risk-averse than she is. At the other extreme, consider a highly risk-averse consumer. Her utility function will then be highly concave, so her underestimation of the high interest rate maturity value matters relatively little in utility terms. Here, the EU effect is dominated by the required rate effect, and she will appear less risk-averse than she is.

Section 3 formalizes these points in a general setting, provides more intuition, and discusses some policy implications. Section 4 presents survey results that offer preliminary empirical evidence in support of the predictions of this model. I find that those with low levels of risk aversion appear more risk averse when choices are framed in terms of interest rates. The opposite pattern emerges for those with high levels of risk aversion.

This paper proposes that there are insights to be gained from introducing choices across saving and borrowing instruments into models of intertemporal choice. The effects of EGB on these choices depend on the shape of the utility function in some unexpected ways. By showing how inferences of consumer risk aversion might be confounded by a new factor, the model offers a partial explanation for apparent anomalies like the equity premium puzzle. It also yields some suggestions, and cautionary lessons, for the design of financial literacy programs that aim to help consumers optimize their engagement

with financial markets.

2 Related literature

Starting with Wagenaar and Sagaria (1975), several papers have documented the prevalence of exponential growth bias and its relation to financial outcomes.[2] Lusardi and Mitchell (2007) show that there is an overall low understanding of compound interest in the United States, and that this is correlated with poor retirement planning and low stock market participation. For a sample of Americans in their early 50s in 2004, of those who had basic numeracy skills, only 18% got compound interest right (43% simply used simple interest). Lusardi (2008) makes a broader case for treating an understanding of compound interest as an essential component of financial literacy. Mckenzie and Liersch (2011) present evidence of EGB among both college students and employees of Fortune 500 firms. Binswanger and Carman (2010) show that the bias is greater when respondents guess future consumption given current savings ("prospective bias") than when they guess current savings given future consumption ("retrospective bias"). More recently, Banerjee and Majumdar (2023) find evidence of EGB in individuals' predictions about the spread of COVID-19.

Considering the evidence of EGB, there is the possible skepticism about its relevance to consumer welfare. After all, spreadsheet software and calculators are widely available today, unlike, say, in 1400s Florence where one would have to travel in search of a copy of Pegolotti's interest tables (Lewin, 2019). But first, a consumer who rationally ignored their own intuition would have to be aware of their bias. Levy and Tasoff (2017) and Cordes et al. (2019) show that this is not the case. And second, even if consumers were aware of their bias, by definition they cannot estimate the extent to which they are off, and therefore might underuse computing tools and rely

[2]Some papers also raise cautionary points. Konigsheim et al. (2018) observe that structural estimates of EGB are sensitive to assumptions about the curvature of the consumer's utility function. Almenberg and Gerdes (2012) show how we might mis-attribute financial choices to EGB if we don't adequately control for other forms of financial literacy.

too much on intuition, including about when computing tools are necessary. Eisenstein and Hoch (2007) make this point well:

> "Even sophisticated consumers who understand the theory do not whip out calculators for every possible financial decision, and they certainly do not immediately turn to a computer when making judgments about inflation or price or wage fairness. Furthermore, to accurately choose an option in many of the financial domains outlined above would require greater spreadsheet modeling expertise than is possessed by the vast majority of consumers. All of these observations lead to the conclusion that there will be a substantial intuitive component to consumers' decision-making processes when problems are fundamentally based around compounding processes." (p. 2)

There is evidence that the bias is correlated with saving and borrowing decisions, and also that debiasing can cause behavioral change. Foltice and Langer (2018) find that EGB is positively and negatively correlated with debt and saving experience, respectively. Goda et al. (2019) show that EGB predicts retirement savings, even controlling for many possible confounding factors including general financial literacy. Goda et al. (2014) and Song (2019) find that improving understanding about compound interest raises savings among populations in the United States and China, respectively.

In contrast to the substantial bodies of theoretical work on other widespread biases, there is relatively little on EGB. Ng (1992) was a pioneering work that considered the effect of EGB on consumption-savings decisions. More recently, the seminal paper by Stango and Zinman (2009) built a framework for much continuing work on the topic by combining a set of theoretical predictions with empirical evidence on the implications of EGB. Levy and Tasoff (2016a) introduce EGB into a life cycle model to study its effects on intertemporal decisions and find that predictions on savings are borne out in data.

The papers cited above make a compelling case for EGB's effects on consumption-savings tradeoffs. To the best of my knowledge, there is only

one paper, Levy and Tasoff (2016b), that speaks to a topic related to this paper – that of within-future tradeoffs, or choices across different kinds of saving instruments. The authors make a novel and counterintuitive theoretical observation – the same error that leads a consumer to underestimate compound interest accumulation at a given interest rate (namely, EGB) can lead the consumer to *overestimate* accumulation when interest rates are time-varying. Consider the following example from their paper. A consumer is offered a 5-year investment that earns compound interest, with the respective interest rate in each year being: $40\%, 4\%, 1\%, -60\%, 70\%$. This investment earns a negative overall return. But the biased consumer (and also the casual reader) will fail to grasp the difference between the geometric mean of this sequence (which is negative) and the arithmetic mean (which is positive), and will mistakenly choose this investment over one that earns a zero rate of return. So EGB can make time-varying (but risk-free) returns appear more attractive than non-varying returns.

This leads us to the current model, which introduces risk. I compare risky returns to safe returns and find that the bias could make either type of instrument appear misleadingly attractive, for quite different reasons than above.

3 The model

The formal model is presented in Section 3.3 and key intuition in 3.4. The two sub-sections may be read in either order.

3.1 Assumptions

Consider a principal deposit, $X > 0$, in an account earning interest at a rate $r \geq -1$, compounded annually. The account matures after t years, where t is an integer greater than 1. The maturity value is denoted Y, so that:

$$Y(r) = X(1+r)^t. \tag{1}$$

An individual with exponential growth bias anticipates a maturity value of $Y_B(r)$.[3] Assume $Y_B(r)$ is continuously twice differentiable. Rather than impose a functional form on the bias, I place the minimal restrictions necessary to capture two essential features – under-estimation (Assumption 2) and linearization (Assumption 3).[4]

Assumption 1. $Y_B(0) = Y(0)$

Assumption 2. $\begin{array}{ll} 0 < Y_B'(r) < Y'(r), & \text{for } r > 0 \\ Y_B'(r) > Y'(r), & \text{for } r < 0 \end{array}$

Assumption 3. $0 \leq \frac{Y_B''(r)}{Y_B'(r)} < \frac{Y''(r)}{Y'(r)}$

The individual's utility from any anticipated maturity value is given by a twice differentiable function $u(\cdot)$, with $u' > 0$ and $u'' \leq 0$.[5]

Below, I analyze the individual's attitude towards a risky investment opportunity where the interest rate, r, is drawn from some interval with a probability distribution function $f(r)$.[6]

3.2 Choosing between investments and cash

First, a minor observation. Suppose the individual were asked to choose between the risky investment and a fixed cash amount in the future. It can easily be shown that the bias makes the individual appear more risk-averse than she is.

The unbiased individual's expected utility is

$$EU = \int u(Y(r)) f(r) \, dr \tag{2}$$

[3] Maturity values depend on X, r, and t. But for expositional convenience in the following analysis, I denote them as a functions of just r, the main variable of interest.

[4] This is consistent with the empirical evidence and with existing theoretical formulations of the bias, as in Stango and Zinman (2009) and Levy and Tasoff (2016a), and also with any weighted average of the true maturity value and the simple interest value ($\tilde{Y} = \alpha X(1+rt) + (1-\alpha) X(1+r)^t$, with $0 < \alpha \leq 1$).

[5] I assume there is no wealth other than from the investment. The model could easily be modified to include other sources of wealth.

[6] For simplicity, the model assumes no time-variation in interest rates. Alternatively we could interpret this as a setting where interest rates do vary over time, but consumers have information on the annualized interest rate associated with any time variation.

Since $Y_B(r) < Y(r)$ at any non-zero interest rate, the biased individual's expected utility from the investment, EU_B, is lower than EU. Therefore, the biased individual has a lower certainty equivalent cash value, and in this sense appears more risk averse than the unbiased individual. Conversely, given a choice between a safe investment and a lottery over cash values, EGB would lower the relative appeal of the investment, making the consumer appear less risk averse than she really is.

This observation, while analytically trivial, has implications for retirement savings decisions when some options are framed in terms of rates of return and others in terms of cash. An individual's choice of a defined benefits plan over a defined contribution plan, for example, suggests heightened risk aversion when in fact it could be a result of EGB.

3.3 Choosing across investments

I now turn to the main topic of the paper – the effect of exponential growth bias on implied risk attitudes when savings opportunities are denoted in terms of rates of return. We can analyze this by studying the coefficient of absolute risk aversion over the relevant range of maturity values.

3.3.1 Deriving coefficients of risk aversion

The unbiased individual's risk preferences around the maturity value generated by a rate r can be summarized by her coefficient of absolute risk aversion, denoted $A(Y)$. This is her true underlying risk aversion.

$$A(Y) \equiv \frac{u''(Y)}{u'(Y)} \tag{3}$$

Now, suppose this same individual were biased and perceived the yield as Y_B instead of Y. By her own assessment, her coefficient of risk aversion is $A(Y_B)$ – the same function as above but evaluated at a different (incorrect) maturity value. But the biased individual is not just mistaking the maturity value; she is also mistaking the change in maturity value associated with any change in interest rate. Therefore, when faced with a menu of investments

that are denoted in interest rates, $A(Y_B)$ is not the coefficient of risk aversion that would be implied by her choices.

To derive the risk preferences that would actually be implied by her choices, it will be helpful to reframe some expressions. Let's rewrite the biased individual's perceived maturity value in terms of the true maturity value Y:

$$\tilde{Y}(Y) \equiv Y_B(r) \tag{4}$$

The biased individual's perceived maturity value can now be interpreted as a distortion of the true maturity value Y.[7]

Define the following new utility function:

$$\tilde{u}(Y) \equiv u\left(\tilde{Y}(Y)\right) \tag{5}$$

We have reframed the biased individual as an unbiased individual who correctly estimates outcomes but evaluates them with a different utility function. We can now characterize the risk preferences implied by her choices. Around any maturity value Y, the biased individual's implied coefficient of risk aversion is $\frac{\tilde{u}''(Y)}{\tilde{u}'(Y)}$. Denote this $\tilde{A}(Y)$.

At any interest rate r, we wish to compare the risk-aversion coefficients $A(Y)$ and $\tilde{A}(Y)$ for the unbiased and biased individuals, respectively. Differentiating and twice differentiating both sides of Equation 5 with respect to Y, and rearranging terms, we get:

$$\tilde{A}(Y) = \left[A\left(\tilde{Y}\right)\right]\tilde{Y}' + \frac{\tilde{Y}''}{\tilde{Y}'} \tag{6}$$

3.3.2 Interpreting coefficients of risk aversion

To interpret Equation 6, observe that the bias has two distinct effects. First, the slope of $\tilde{Y}$ matters. If $\tilde{Y}' < 1$, $\tilde{u}(Y)$ is stretched horizontally relative to $u(Y)$ (the biased agent requires a larger rise in interest rate to experience a given rise in utility), linearizing her utility function and implying lower risk

[7] It follows from Equation 1 that $r = \left(\frac{Y}{X}\right)^{\frac{1}{t}} - 1$. So, any function of r can be rewritten as a function of Y.

aversion. If $\tilde{Y}' < 1$, the opposite is true. This effect is captured by the first term of the RHS of Equation 6. From Assumptions 1-3, $\tilde{Y}' < 1$ for positive interest rates and $\tilde{Y}' > 1$ for negative interest rates.

Second, the curvature of $\tilde{Y}$ matters. From Assumptions 1-3, $\frac{\tilde{Y}''}{\tilde{Y}'} < 0$. As $\tilde{Y}$ gets more concave, more concavity is induced in $\tilde{u}$. This is the effect of the second term of the RHS of Equation 6.

Taken together: at negative interest rates, the biased agent appears more risk averse; at positive interest rates, the bias has opposing effects on the consumer's apparent risk-aversion – underestimation itself makes her appear less risk averse, while increasing underestimation makes her appear more risk averse.

Consider the following examples. Suppose the consumer were risk-neutral. Risk-neutrality implies that the first term in Equation 6 is zero. When making investment decisions, then, her implied coefficient of risk aversion is $\tilde{A}(Y) = \frac{\tilde{Y}''}{\tilde{Y}'}$. As a result of EGB, the risk-neutral consumer appears risk-averse. This apparent risk-aversion stems from the concave relationship between the true yield of an investment and her perception of the yield. At the other extreme, a consumer who is highly risk-averse to begin with might appear less risk-averse in investment decisions because of the moderating effect that the bias has on the first term of Equation 6.

Here is another way to think about the opposing forces in the case of positive interest rates. $u(\cdot)$ is concave function and $\tilde{Y}(Y)$ is a concave function with slope less than 1. Is the composite utility function $u\left(\tilde{Y}(Y)\right)$more or less concave than $u(Y)$? Imagine that $\tilde{Y}$ were piecewise linear with declining slopes. The composition of u with $\tilde{Y}$ can be analyzed in two parts. First, within any linear segment, the slope being less than 1 would force the biased consumer's implied utility function to stretch horizontally (for the reason given above), lowering risk-aversion. Now consider two adjacent linear segments of $\tilde{Y}$. If the slope drops rapidly from the first to the second segment, the straightening effect on the second segment of the utility function is much more pronounced than the straightening effect on the first. Comparing across segments, $u\left(\tilde{Y}(Y)\right)$ could now exhibit a more rapid drop in slope than $u(Y)$.

3.3.3 The proposition

At any interest rate r, the biased individual will appear more risk averse than the unbiased individual if and only if $\tilde{A}(Y) < A(Y)$. Equation 6 helps generate some intuitive sufficient conditions for this. Define Z as:

$$Z \equiv \frac{\tilde{Y}''}{\tilde{Y}'}\left(\frac{1}{1-\tilde{Y}'}\right) \tag{7}$$

$\tilde{A}(Y) < A(Y)$ (i.e. the biased agent appears more risk-averse than the unbiased agent) if

$$min\left\{A(Y), A\left(\tilde{Y}\right)\right\} > Z \tag{8}$$

$\tilde{A}(Y) > A(Y)$ (i.e. the biased agent appears less risk-averse than the unbiased agent) if

$$max\left\{A(Y), A\left(\tilde{Y}\right)\right\} < Z \tag{9}$$

The left-hand side in the two conditions above can be interpreted as the general level of risk-aversion exhibited by the individual. The min and max operators are used to generate simple sufficient conditions that accommodate the possibility of varying coefficients of risk aversion over the domain of the utility function. In the special case of CARA utility, for example, the two terms on the left-hand side would be identical. The right-hand side depends only on the consumer's bias and not her preferences.

Proposition 1 summarizes conditions under which EGB makes a consumer appear more or less risk averse. When a condition is satisfied across the range of interest rates under consideration, EGB lowers or raises her certainty equivalent interest rate, respectively, for any risky investment.

Proposition 1. *At any negative interest rate, exponential growth bias makes the consumer appear more risk averse than she really is.*

At any positive interest rate:

(a) If the consumer has sufficiently low risk aversion (satisfying Equation 8), exponential growth bias makes her appear more risk averse than she really is.

(b) If the individual has sufficiently high risk aversion (satisfying Equa-

tion 9), exponential growth bias makes her appear less risk averse than she really is.

While the analysis above focuses on savings and investment, the framework can be used to generate similar results for choices over loans. This model could also easily be extended to account for risk-loving consumers. For positive interest rates, risk-loving consumers with EGB will appear less risk-loving than they are. For negative interest rates, EGB has opposing effects like in the analysis above.

3.4 An explanation with figures

I now illustrate a special case of the result above. Given a fixed principal deposit and maturity period, any variation in the maturity value, Y, can be attributed to the interest rate alone. So we can denote the biased prediction $\tilde{Y}(Y)$ – this is the biased consumer's predicted maturity value at the interest rate r that generates Y. The biased consumer's utility can then be written as a compound function of Y: $\tilde{u}(Y) \equiv u\left(\tilde{Y}(Y)\right)$. For any investment with a true maturity value Y, the biased consumer incorrectly anticipates a lower maturity value ($\tilde{Y}$) and therefore has a lower utility ($\tilde{u}(Y)$) than does the unbiased consumer. To determine how EGB affects implied risk preferences, we need to determine if $\tilde{u}$ is more or less concave than u.

Consider a risky investment that will earn a low rate of interest (r_L) with probability p or a high rate of interest (r_H) with probability $(1-p)$. For expositional purposes, it is convenient to think of p as close to .5. We can look for the safe investment that the unbiased consumer considers equivalent to this risky investment, i.e. a certainty equivalent interest rate r_C that satisfies:

$$pu(Y(r_L)) + (1-p)u(Y(r_H)) = u(Y(r_C)) \tag{10}$$

The biased consumer's certainty equivalent interest rate ($\tilde{r}_C$) must similarly satisfy:

$$p\tilde{u}(Y(r_L)) + (1-p)\tilde{u}(Y(r_H)) = \tilde{u}(Y(\tilde{r}_C)) \tag{11}$$

EGB has two opposing effects on Equation 10. First, the "EU effect" lowers the expected utility of the risky investment, making the LHS smaller, which means the RHS of Equation 11 needs to be smaller than in Equation 10. On the other hand, the "required rate effect" makes the biased consumer require a higher interest rate to achieve any target utility, or any given RHS value. Whether $\tilde{r}_C$ is larger or smaller than r_C depends on which of these effects dominates.

Figure 1 depicts a relatively risk-averse consumer. The risky investment gives him an expected utility denoted EU (indicated on the y-axis as the average of $u\left(Y\left(r_L\right)\right)$ and $u\left(Y\left(r_H\right)\right)$). The fixed quantity, $Y\left(r_C\right)$, that gives him the same utility can be found by tracking his utility function u. This in turn determines his certainty equivalent interest rate, r_C. Now consider the effects of EGB. When the true maturity value is $Y\left(r_L\right)$, the biased consumer makes a small mistake (the error is relatively small at low interest rates), so his utility is slightly below $u\left(Y\left(r_L\right)\right)$. When the true maturity value is $Y\left(r_H\right)$, the biased consumer makes a big mistake, but because the utility function is relatively insensitive to consumption changes at high values, the mistake matters little in terms of utility. His expected utility, $\tilde{EU}$, is therefore only slightly below the unbiased level. Suppose this consumer were offered the fixed interest rate r_C instead. Here, his error is of intermediate size but his utility function in this region is relatively sensitive to errors. Therefore, his utility from r_C is significantly lower than the true utility. To accept a safe investment, he must be offered an interest rate $\tilde{r}_C > r_C$. In such a case, the EU effect is smaller than the required rate effect, and he appears more risk tolerant than he is.

Figure 2 depicts a relatively risk-tolerant (in this example, risk-neutral) consumer. Now, the mistake at the high interest rate results in a large drop in expected utility, because the utility function is sensitive to changes even around the high value. Here, the EU effect is larger than the required rate effect and he appears more risk-averse than he is.

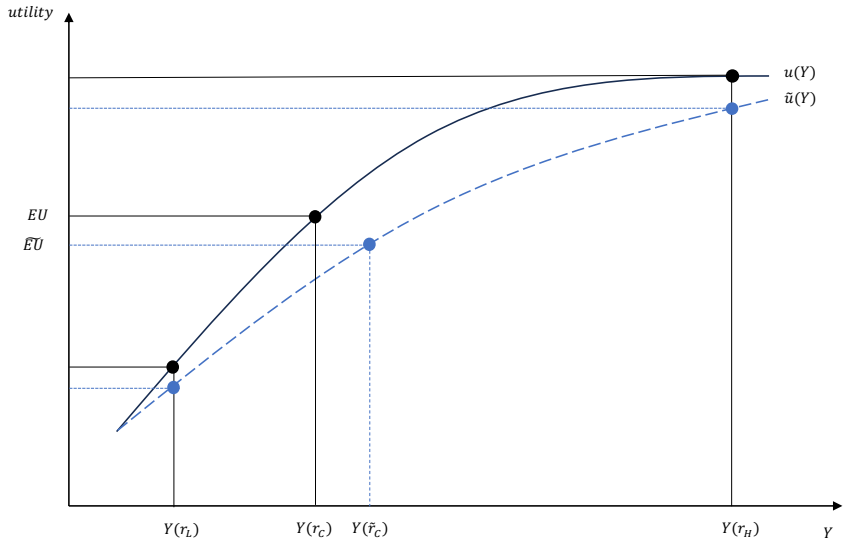


Figure 1: A comparison of the unbiased and biased utility functions (u and $\tilde{u}$) respectively, for a relatively risk-averse consumer. The biased consumer is more tolerant of risk.

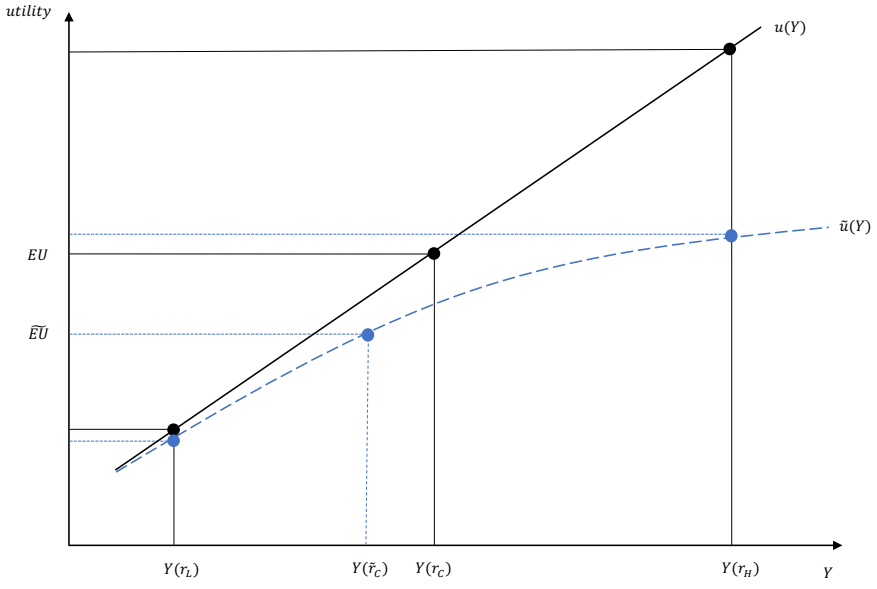


Figure 2: A comparison of the unbiased and biased utility functions (u and $\tilde{u}$) respectively, for a risk-neutral consumer. The biased consumer is less tolerant of risk.

3.5 Comparative statics

The model suggests some comparative statics exercises that could be used to refine empirical hypotheses. Consider Equation 6. The biased consumer's implied risk aversion ($\tilde{A}$) depends on her true risk aversion (A), the extent to which her biased perception tracks the true maturity value ($\tilde{Y}'$), and the concavity of the bias function ($\frac{\tilde{Y}''}{\tilde{Y}'}$). In principle, it is possible to compute how the relationship between the true risk aversion and implied risk aversion is mediated by exogenous factors X, t, r, and the bias function itself. However, the explicit characterization of this relationship can get messy and opaque under many functional form assumptions. I carry out this exercise using two plausible functional forms that lend themselves to a tractable analysis of comparative statics.

Consider a formulation of the bias similar to Stango and Zinman (2009). The biased consumer perceives the maturity value to be:

$$\tilde{Y} = X\left(1+r\right)^{(1-\alpha)t} \tag{12}$$

The extent of the bias is captured by a parameter $\alpha \epsilon [0,1]$, with $\alpha = 0$ representing the unbiased consumer. Replacing r by $Y\left(r\right)$ using Equation 1, this can be rewritten as:

$$\tilde{Y}\left(Y\right) = X^{\alpha}Y^{1-\alpha} \tag{13}$$

Assume a CARA utility function with coefficient of absolute risk aversion ρ:

$$u\left(Y\right) = \frac{-e^{-\rho Y}}{\rho} \tag{14}$$

We can now rewrite Equation 6 using these functions:

$$\tilde{A}\left(\tilde{Y}\left(r\right)\right) = \left[A\left(\tilde{Y}\left(Y\right)\right)\right]\tilde{Y}'\left(Y\right) + \frac{\tilde{Y}''\left(Y\right)}{\tilde{Y}'\left(Y\right)} \tag{15}$$

$$\Rightarrow \tilde{\rho} = \rho\left[\left(1-\alpha\right)\left(\frac{X}{Y}\right)^{\alpha}\right] + \frac{\alpha}{Y} \tag{16}$$

An increase in X, t, or r must result in a decrease in $\tilde{\rho}$, lowering the implied

risk aversion for the biased agent. An increase in α, however, has an ambiguous effect. The coefficient on ρ, $\tilde{Y}(Y)$, falls, lowering her implied risk aversion. On the other hand, her bias function gets more concave ($\frac{\tilde{Y}''(Y)}{\tilde{Y}'(Y)}$ rises), generating an opposing effect.[8]

3.6 Some implications

The model has some implications that could be investigated with further empirical work. For example, if savings tend to be risky and loans safe, EGB could create an additional bias towards loans from factors beyond the overall under-estimation of compound interest accumulation. Within loans, consider choices between fixed and floating interest rates. Americans' fondness for fixed-rate mortgages is well known,[9] and studies in multiple countries suggest a behavioral angle to mortgage choice.[10] The current model proposes an additional angle – the interaction of EGB and risk preferences – that might aid our understanding of consumer behavior in these settings.

Next, consider the equity premium puzzle. Mehra and Prescott (1985) postulated that the historical gap between stock and bond returns suggest an improbably high level of consumer risk aversion. The puzzle has been subsequently studied extensively, with notable attempts to rationalize it including Benartzi and Thaler (1995), who offer an explanation based on prospect theory and mental accounting. This model offers an additional partial explanation. If investors tend to have relatively high levels of wealth, and if utility functions generally get less risk-averse with wealth, then EGB will make investors appear more risk-averse than they actually are. So, some of the implied high risk aversion in the equity premium puzzle could be a result of systematic consumer mis-estimation of compound interest. Financial literacy programs that include de-biasing could raise investor demand for stocks.

[8] Similar exercises, with similar comparative statics, could be carried out for other utility functions (e.g. CRRA) and bias functions (e.g. a weighted average of the true value and simple interest): $\tilde{Y}(Y) = \alpha X(1+rt) + (1-\alpha) X(1+r)^t$.

[9] https://www.nytimes.com/2023/08/25/opinion/fixed-rate-mortgages-inflation.html

[10] See Bergstresser and Beshears (2010), Damen and Buyst (2013), and Mugerman et al. (2016).

Finally, the model generates some broad lessons for the design of financial literacy and consumer protection programs that aim to educate consumers about the long-term consequences of their financial choices. Many such programs emphasize the benefits of saving and costs of loans, either in general terms or through rules of thumb like the "rule of 72" or through the explicit provision of maturity values at specific interest rates. The focus across these approaches is on compounding growth over time. The current model suggests a need to also educate consumers on differences in compounding growth across interest rates, to improve consumer comprehension of how even small changes in interest rates can have large impacts on maturity values.

4 Empirical evidence

To test the basic predictions of the model, I conducted a short survey on students enrolled in a large introductory economics class at Hunter College. The questionnaire had three components. One, students were asked to estimate compound interest accumulation – if you deposit $1,000 in an account that earns 15% interest, compounded annually, how much will you have after 30 years?

Two, they were asked to make hypothetical choices between receiving a cash lottery ($6,000 or $30,000, with equal probability) and a series of fixed amounts, all arriving 30 years in the future. The fixed cash amounts are listed in the first column of Table 1. Three, they were asked to make hypothetical choices between a risky investment (deposit $1,000, which will be randomly allocated to an account earning 6% compound interest or to an account earning 12% compound interest) and a series of safe investments, all expressed in terms of interest rates. The choices are listed in the last column of Table 1.

I use these responses to infer risk preferences. A respondent's "baseline risk aversion" score is given by the lowest cash choice that they preferred to the cash lottery (see Table 1 for scores). Higher scores imply greater risk aversion. For example, someone who chose $13,000 over the lottery, but chose the lottery over all lower cash values, gets a score of 6. The respondent's

Cash lottery		Risk-aversion score		Investment lottery
$\left[6,000(\frac{1}{2}), 30,000(\frac{1}{2})\right]$ vs:				$\left[6\%(\frac{1}{2}), 12\%\left(\frac{1}{2}\right)\right]$ vs:
		0		
30,000		1		12%
23,000		2		11%
17,500		3		10%
13,000		4		9%
10,000		5		8%
7,500		6		7%
6,000		7		6%

Table 1: Safe and risky scenarios

"interest rate risk aversion" score is similarly assigned based on their choices between risky and safe investments.

Investment interest rates were chosen to generate cash values similar to those used in the cash questions.[11] The null hypothesis, then, is that a respondent will have identical baseline risk aversion and interest rate risk aversion scores. Given the simplicity of the survey, a discrepancy across scores can arguably be attributed to a faulty understanding of compound interest.

There were 180 respondents. I drop those who did not complete the survey or exhibited inconsistency across choices (example, choosing a low cash value over the lottery but rejecting a higher cash value against the same lottery). The analysis proceeds with the remaining 120 respondents. Most of this group (105) exhibited EGB. The distribution of estimates by those exhibiting EGB is in Figure 3. (For reference, the correct answer is \$66212 and the answer under simple interest would be \$5500.)

I now present some OLS regressions (Table 2). In Specification 1, I regress interest rate risk aversion scores on baseline risk aversion scores and find a positive relationship between the two.

Specification 2 reframes the question in Specification 1 to offer key evidence against the null hypothesis. Define "score difference" as the respon-

[11] A cash risk-aversion score of 3 and an interest rate risk-aversion score of 3, for example, both suggest approximate risk-neutrality.

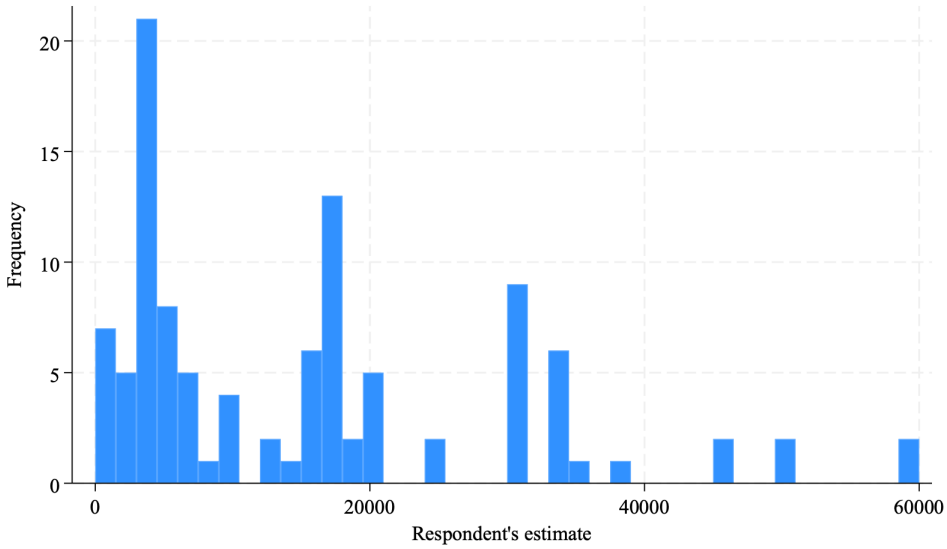


Figure 3: Distribution of compound interest estimates

dent's interest rate risk aversion score minus their baseline risk aversion score. I find that score difference is negatively correlated with the baseline risk aversion score. The coefficient and intercept (all significant at the 1% level) interpreted together show that, as predicted by the model, those who have low baseline risk aversion (or are risk-loving) appear more risk-averse when information is provided via interest rates; those who have high baseline risk aversion appear less risk-averse. Consider one illustration of this pattern: if we divide the sample into those with low (0-4) and high (5-7) baseline risk aversion scores, the mean score difference for the low baseline group is 0.47 while the mean score difference for the high baseline group is -1.00.[12]

In Specification 3, I check whether EGB intensity, defined as the percentage by which compound interest is underestimated, plays a role. The regression does not detect any significant effects of EGB intensity. This is consistent with the model, which shows how implied risk aversion is affected

[12]The low and high baseline groups have 64 and 56 observations, respectively.

	Interest rate risk aversion score (1)	Score difference (int. rate - baseline) (2)	Score difference (int. rate - baseline) (3)
Baseline risk aversion score	**0.475**	**-0.525**	**-0.510**
	(0.085)	(0.085)	(0.087)
EGB intensity			-0.000
			(0.001)
Baseline risk aversion score x EGB intensity			0.000
			(0.001)
Intercept	**2.098**	**2.098**	**2.025**
	(0.400)	(0.400)	(0.411)
Number of observations	120	120	120
Adjusted R-squared	0.20	0.24	0.23

Notes: The dependent variable in Specification 1 is the Interest rate risk aversion score. The dependent variable in Specifications 2 and 3 is the score difference. Standard errors are in parentheses. Coefficients in bold are significant at the 1% level.

Table 2: Regression results

by the *shape* of the bias function rather than the intensity of EGB at a particular point. This link could perhaps be investigated in future research with data that more fully captures the first and second derivatives of individual bias functions.[13]

5 Conclusion

This paper makes the broad point that exponential growth bias affects not just tradeoffs between present and future consumption, but also tradeoffs between safe and risky investments or between safe and risky loans. I show how the effects can reverse directions depending on risk preferences and the shape of the bias. Integrating the model into optimization exercises could yield new insights. For example, what are the implications for life-

[13]The regression results are similar using an ordered logit model and robust to expanding or contracting the sample based on milder or stricter notions of rationality, respectively. In particular, we might be concerned about those with extreme risk aversion scores of 0 or 7, who like or dislike risk to an extent that cannot be explained within an expected utility framework. Removing these respondents from the analysis does not affect the results.

cycle consumption, where income and substitution effects interact in some interesting ways? How does EGB under interest rate risk interact with EGB under time-varying interest rates which, as Levy and Tasoff (2016b) show, raise the attractiveness of investment? And does this create opportunities for arbitrage and exploitation of biased consumers? These questions remain relevant, especially for low-income and vulnerable consumers for whom the consequences of sub-optimal choices might be particularly severe.

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