Platform competition with intertwined network effects*

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Abstract

I model a firm acting as a matching platform and an aggregator that gives one user group (buyers) access to a competing matching platform's other user group (sellers). This creates "intertwined network effects" (INE) between the platforms. I study how INE influence platform participation, pricing, and user groups' welfare when platforms are independent and post-merger. I show that while INE increase consumer surplus, they reduce seller surplus if the platforms are sufficiently differentiated for sellers. In presence of INE, a non-consolidating merger can harm consumers and benefit sellers if the platforms are sufficiently homogeneous to consumers.

JEL Classification: D43, L13, L86, L41.

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1 Introduction

In many transaction two-sided markets, aggregator platforms serve as gateways to other matching platforms while providing a competing matching service. For example, Trivago allows travelers to be matched with hotels either directly or through a third-party lodging platform. In such a setting, all the users of one side of the market (e.g., travelers) participating in the aggregator platform (e.g., Trivago) can access all the users of the other side of the market (e.g., hotels) present in another "source" platform (e.g., Booking). As illustrated in Figure 1 below, this allows platform users from one side of the market to benefit from the presence of users from the other side of the market participating in a *competing* platform. When this is the case, I say that the two platfoms set-up *intertwined network effects* (INE, hereafter).

Examples of aggregator platforms with INE include retail marketplaces (Google Shopping, Bing Shopping), price comparison sites (Price Runner), real estate platforms (real estate online platforms hosting listings from real estate agencies, Jinka) and digital wallets aggregating debit and credit cards (Apple Pay, We Chat, Pay Pal, Google Pay). Importantly, aggregators and source platforms might be horizontally differentiated on both sides of the market. For example, generalist marketplaces and price comparison sites aggregate partially-competing specialized marketplaces (e.g., Milanuncios/Coches, Adverts/DoneDeal, Idealo/Otto). Moreover, across multiple industries, it is common for platforms having set-up intertwined network effects to merge without consolidating. Examples include eBay/Motors, Adevinta/Gumtree, Se Loger/Logic Immo, Rightmove/PrimeLocation and eBay/StubHub.

Intertwined network effects have been at the core of recent merger cases. In some of them, aggregators have merged with one of their source platforms without consolidating into a single platform. In others, platforms have (tried to) set-up INE post-merger, typically arguing this practice constitutes a merger efficiency (e.g., Trade Me/Property NZ). Interestingly, when faced with such cases, competition authorities have so far either not accounted for INE in their assessment of the merger's impact on welfare (e.g., Schibsted/Nettbil, Trade Me/Property NZ, viagogo/Stubhub) or judged them to be detrimental to at least one of the user groups (e.g., Booking/eTraveli, FDJ/Zeturf, Seek Asia Investments/JobStreet, Wedding Planner/Zank you) because they intensify network effects.

The extant literature has investigated the effects of platform competition and mergers under various settings. However, little attention has been paid to how intertwined network could alter established results. This article studies the effect of intertwined network effects on platform participation, prices and the welfare of the two sides of the market. It also shows how a non-consolidating merger[1] between the platforms affects these outcomes.

I develop a model in which two horizontally-differentiated platforms compete to facilitate transactions between two types of users benefiting from positive indirect network effects:

[1]For simplicity, I will hereafter say "merger" to refer to a *non-consolidating* merger.

buyers and sellers. I consider a competitive bottleneck setting in which buyers singlehome and sellers can multihome. Sellers are charged a per-transaction price, while buyers interact for free. If the platforms decide to set-up intertwined network effects, the buyers from the aggregator platform and the sellers of the source platform interact. For each of these cross-platform interactions, the aggregator platform charges an endogenously-determined per-transaction referral fee to the source platform. I first consider the scenario with intertwined network effects and compare it to one without intertwined network effects when the platforms are legally independent. I then compare a scenario in which the platforms set INE and merge without consolidating with one in which legally-independent platforms set INE. In both comparisons, I determine, for each platform and in the aggregate, the effects on both user groups' participation, prices and surpluses.

I show that INE benefit consumers from the aggregator platform and harms consumers of the source platform. However, the overall effect on consumer surplus is always positive. The gain in consumer surplus brought about by the aggregator platform consumers' capacity to reach all sellers always more than compensates the harm to the source platform's consumers triggered by a INE-driven drop in seller participation. As for sellers, if the platforms are sufficiently differentiated for them, INE decrease their surplus, and vice-versa. The reason is that INE increase participation and seller surplus in the aggregator platform, which is able to attract more consumers than without INE. The opposite happens in the source platform, leading some of its sellers to migrate to the aggregator. With sufficiently low seller transportation costs, enough sellers migrate from the source platform to the aggregator, leading to an increase in overall seller surplus, and vice-versa.

A non-consolidating merger between two platforms having set-up INE, in turn, can harm consumers if the platforms are not sufficiently differentiated to them. The reason is that, given that the merger eliminates referral fees, the price charged to sellers in the source platform decreases. This increases their and overall seller participation, which benefits both platforms' consumers. However, due to the presence of INE, the aggregator platform's consumers can interact with more sellers than consumers from the source platform irregardless of the ownership structure of the platforms. Hence, consumers switching from the aggregator to the source platform decreases consumer surplus. Moreover, if consumers' transportation costs are sufficiently high, the merger hurts sellers. The reason is that, if consumers are sufficiently captive, the pre-merger referral fee is high. Hence, post-merger, the aggregator platform has to increase its price substantially to compensate for that loss of revenue, leading to an overall decrease in seller surplus.

The remainder of the article is organized as follows. Section 2 discusses the related literature. Section 3 presents the model. Section 4 studies the impact of intertwined network effects on platform participation, prices and the welfare of buyers and sellers when the platforms are legally independent. Section 5 analyzes how a non-consolidating merger affects these outcomes. Section 6 concludes and discusses the findings.

2 Related literature

This article relates to the vast literature on price competition between platforms, notably that focusing on a competitive-bottleneck setting (Belleflamme and Peitz, 2019; Armstrong, 2006). More precisely, given its focus on intertwined network effects, it contributes to the literature on interoperability with network effects (Rasch et al., 2023; Shekhar et al., 2022; Rasch and Wenzel, 2014; Doganoglu and Wright, 2006; Crémer et al., 2000; Katz and Shapiro, 1985; Farrell and Saloner, 1985). Within this literature, the article is particularly relevant to the work on interoperability between competing platforms with indirect network effects. This article also contributes to the growing literature on platform mergers (Ivaldi and Zhang, 2022; Farronato et al., 2020; Correia-da Silva et al., 2019; Tan and Zhou, 2019; Baranes et al., 2014; Chandra and Collard-Wexler, 2009), notably to the strand focusing on non-consolidating horizontal mergers.

Interoperability between competing platforms with indirect network effects. This article relates to the literature studying the effect of interoperability (also referred to as "compatibility") in markets subject to network effects. Indeed, I model INE as an *asymmetric* form of compatibility between two platforms in which, for each platform, only one of the user groups can interact with the other user group present in another platform.

Specifically, this article relates to the strand of this literature that, building on the seminal contributions focusing on the compatibility between network goods subject to direct network effects (e.g., (Katz and Shapiro, 1985; Farrell and Saloner, 1985, 1986; Katz and Shapiro, 1994), shifted to studying the choice and welfare effects of compatibility between competing platforms enabling interactions between user groups subject to *indirect* network effects. The closest contribution to ours is Maruyama and Zennyo (2015). Building on Rasch and Wenzel (2014)'s results, they consider a setting in which two symmetric platforms intermediating between consumers and content providers can independently decide whether to be compatible with each other or not at a fixed cost. Consumers single-home while content providers might multi-home. When the fixed cost of interoperability is intermediate, the equilibrium is an "asymmetric case" in which one platform chooses compatibility while the other does not. This generates cross-platform network effects analogous to what I call "intertwined network effect" in this article. They find that the asymmetric case leads to a drop in consumer surplus. Moreover, as in this article, if the network effects enjoyed by content providers are sufficiently small, content providers' surplus increases, and vice-versa.

To the best of our knowledge, the rest of the contributions to this literature have focused on cases in which interoperability is symmetric across platforms. An established result in this setting is that symmetric platforms serving single-homing users have an excessive incentive to be compatible with respect to a social planner. This is because compatibility renders demand less elastic, which benefits platforms but might hurt users in absence of sufficient market expansion, as in Crémer et al. (2000). Doganoglu and Wright (2006) make this argument in their study of the interplay between platforms' compatibility decisions and

users' decisions to multi-home. When users can multi-home, compatibility leads to market contraction because it makes multi-homing unnecessary, as users can already benefit from all the network externalities without joining more than one platform (Doganoglu and Wright, 2006; Salim, 2010). It also makes demand more elastic (as consumers joining decisions are rival), which reduces prices and profits while increasing consumer surplus. Thus, platforms have insufficient incentives to be compatible with respect to the social optimum. Rasch and Wenzel (2014) encompass both cases in their study of a competitive-bottleneck model with single-homing users and multi-homing content developers, both subject to a membership fee charged by horizontally-differentiated symmetric platforms. They find that the private incentives to choose compatibility can be insufficient or excessive. The key mechanism here is the change in content provision. From a social-welfare perspective, compatibility is desirable if it increases content, which happens if content providers' network effects are sufficiently strong, and vice-versa.

This article contributes to this literature by focusing on the unexplored (with the exception of Maruyama and Zennyo (2015)) yet prevalent case of asymmetric compatibility (i.e., INE) between platforms enabling interactions between two user groups subject to indirect network effects. The main departure from this literature lies not only in the focus on INE, but also on some key modelling choices, namely per-transaction prices and referral fees, as well as differential transportation costs across user groups. Although the focus of this article is not determining when the choice of (asymmetric) interoperability is socially-optimal, from a consumer welfare perspective, it shows that platforms might or might not choose to be interoperable although it always benefits consumers. Moreover, I provide a new mechanism to explain the a priori ambiguous effect of compatibility on seller surplus that relies on the extent to which platform differentiation allows sellers to reallocate across platforms and increase multi-homing. With sufficiently low seller transportation costs, enough sellers leave the source platform (where prices increase) for the aggregator platform (where prices decrease), leading to an increase in overall seller surplus, and vice-versa

Non-consolidating horizontal platform mergers. The literature on non-consolidating horizontal platform mergers has explored the conditions under which the merged entity has incentives to lower prices and thus benefit at least one side of the market. Chandra and Collard-Wexler (2009) were the first to make this point in a model of mergers between newspapers intermediating between advertisers and readers. The latter are assumed to be heterogeneous and hence advertisers' value of being in the platform depends on the composition of consumers they can access. Because the platform cannot target consumers and price-discriminate, a marginal consumers' contribution might be negative. Therefore, price hikes might lead consumers to choose a newspaper to which their contribution to profit is negative. This can generate incentives to lower prices post-merger.

On of the main results that the literature subsequently established under various settings is that, if network effects are sufficiently strong, the merged entity has incentives to lower prices post-merger. Leonello (2010) makes this argument in a model of two platforms competing

à la Hotelling on both sides of the market. If indirect network effects are sufficiently strong in side 2, the price decreases and demand increases in side 1. Important to this article, post-merger, the platforms also become interoperable in that side-1 users in platform A can access side-2 users of both platforms. This reinforces the merged entity's incentives to lower prices. Baranes et al. (2014) extend Leonello (2010)'s model to four platforms equidistantly located on a Salop circle with linear externalities and full market coverage. Similarly, they conclude that mergers between adjacent platforms may lead to lower prices if externalities are sufficiently strong. Tan and Zhou (2019) reverse this argument with a model that includes the possibility of non-linear externalities and in which consumers have a random utility function. Assuming full market coverage, they show that the merged entity always has incentives to increase prices unless there are strong cost-related efficiency gains.

I contribute to this literature by exploring a new setting that generates an unstudied reason why a non-consolidating horizontal platform merger might harm users. In a competitive bottleneck model with INE and per-transaction pricing on the multi-homing side, the less differentiated the platforms are on the single-homing side, the more likely it is that the merger will harm both sides of the market. The reason is that, despite the overall increase in seller participation triggered by the merger, strong differentiation discourages single-homing users from sufficiently switching from the aggregator platform (where the number of sellers decreases) to the source platform (where the number of sellers increases).

3 The model

In this section I present the model in its two settings: with and without (i.e., the benchmark) intertwined network effects. In accordance with the observed features of matching platforms that set-up INE, the model considers i) a competitive bottleneck setting in which buyers single-home and sellers multi-home[2], ii) transaction prices on the seller side and zero-pricing on the buyer side and iii) when INE are set-up, an endogenously-determined (per-transaction) referral fee charged by the aggregator to the source platform.

Model overview. There are two symmetric platforms in the market that compete to enable interactions between a unit mass of sellers and a unit mass of buyers. This interaction generates homogeneous, positive indirect network effects to both user groups. Platform $i \in \{1, 2\}$ competes for sellers through the per-transaction price p_s^i it charges them. Moreover, they have constant marginal costs, which, for simplicity, are normalized to zero. Buyers perceive the platforms as horizontally differentiated. We model horizontal differentiation à la Hotelling, where platform 1 is located at $x = 0$ and platform 2 at $x = 1$. Both consumers and sellers are uniformly distributed on a unit interval and face an opportunity cost of joining a platform that increases linearly over the distance at rates τ_b and τ_s, respectively.

[2]Duch-Brown (2017) provides empirical evidence of the prevalence of seller multi-homing in Europe for a wider scope of platforms that includes the following categories: "marketplaces", "apps stores", "social networks" and "online advertising".

Buyers singlehome while sellers can choose to multihome. Each buyer purchases one unit from each of the seller present in the platform she has chosen to join. Each transaction generates a benefit α_b for the buyer and α_s for the seller. Additionally, buyers obtain the same stand-alone benefit v_b from joining a platform. We assume this benefit is sufficiently high for the market to be covered on the buyer side. Let n_b^i and n_s^i denote the mass of buyers and sellers active on platform i, respectively.

In the following subsections, two settings of the model are described: the benchmark (denoted with the superscript B) and the intertwined network effects setting (denoted with the superscript INE).

3.1 Benchmark setting

Utility and profit functions. In the benchmark, the utility of a buyer located at $x \in [0,1]$ is:

$$U_b^{iB}(x, p_s^1, p_s^2) \equiv v_b + \alpha_b n_s^i(p_s^1, p_s^2) - \tau_b|x^i - x| \tag{1}$$

The utility of a seller located at $x \in [0,1]$ is:

$$U_s^{iB}(x, p_s^1, p_s^2) \equiv n_b^i(p_s^1, p_s^2)(\alpha_s - p_s^i) - \tau_s|x^i - x| \tag{2}$$

The profit of platform $i \in \{1,2\}$ is thus given by:

$$\Pi^{iB} \equiv n_b^i(p_s^1, p_s^2) n_s^i(p_s^1, p_s^2) p_s^i \tag{3}$$

Timing. In the first period, platforms simultaneously set the per-transaction prices charged to sellers (p_s^1, p_s^2). In the second period, consumers and sellers simultaneously choose which platform(s) to join.

3.2 Intertwined network effects setting

Utility and profit functions. In a setting in which platforms choose to set-up intertwined network effects (INE, hereafter) at no cost, the buyers from platform 1 can access all the sellers that have joined platform 2 in addition to the sellers having joined platform 1. Conversely, the sellers that decide to join platform 2 can access not only the buyers from platform 2, but also those from platform 1. Then, we say that platform 1 is the *aggregator* and platform 2 the *source platform*. We assume that multi-homing sellers can interact with platform 2 (i.e., the source platform) twice.[3] Transactions between a platform 1 buyer and

[3]This assumption can represent the fact that a multi-homing seller has a higher probability of reaching the same buyer than a single-homing one. It also translates a stylized fact observed in transaction platforms, where it is common for sellers to be present both in an aggregator and a source platform.

a platform 2 seller take place in platform 2. Platform 1 charges a referral fee f to platform 2 for each of these transactions. Therefore, in the INE setting, the utilities of a buyer located at $x \in [0,1]$ joining platform $i \in \{1,2\}$ is given by:

$$U_b^{iINE}(x, p_s^1, p_s^2) \equiv \begin{cases} v_b + \alpha_b \left(n_s^1(p_s^1, p_s^2) + n_s^2(p_s^1, p_s^2) \right) - \tau_b |0 - x| & \text{if } i = 1 \\ v_b + \alpha_b n_s^2(p_s^1, p_s^2) - \tau_b |1 - x| & \text{if } i = 2 \end{cases} \tag{4}$$

The utilities of a seller located at $x \in [0,1]$ joining platform $i \in \{1,2\}$ is in turn given by:

$$U_s^{iINE}(x, p_s^1, p_s^2) \equiv \begin{cases} n_b^1(p_s^1, p_s^2)(\alpha_s - p_s^1) - \tau_s |0 - x| & \text{if } i = 1 \\ \left(n_b^1(p_s^1, p_s^2) + n_b^2(p_s^1, p_s^2) \right)(\alpha_s - p_s^2) - \tau_s |1 - x| & \text{if } i = 2 \end{cases} \tag{5}$$

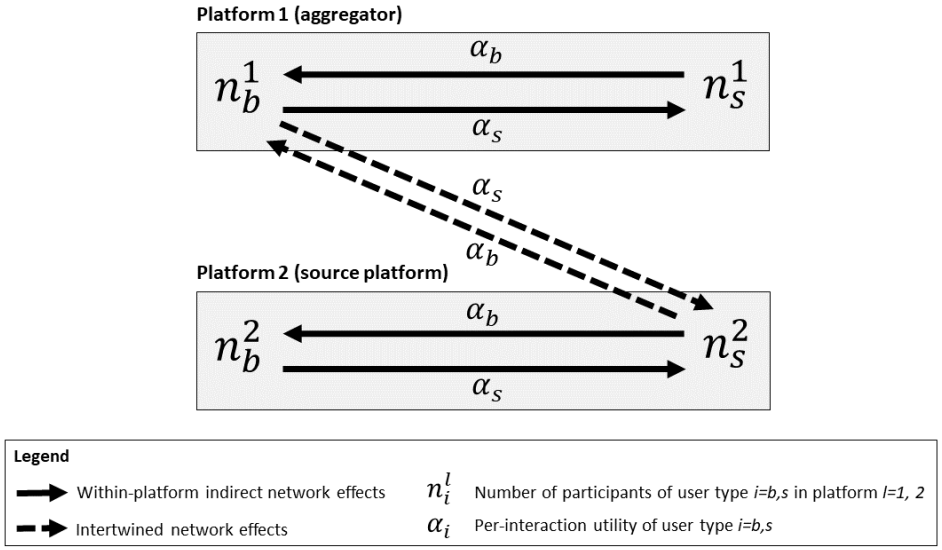
And the profit of platform $i \in \{1,2\}$ is given by:

$$\Pi^{iINE} \equiv \begin{cases} n_b^1(p_s^1, p_s^2) n_s^1(p_s^1, p_s^2) p_s^1 + n_b^1(p_s^1, p_s^2) n_s^2(p_s^1, p_s^2) f & \text{if } i = 1 \\ n_b^2(p_s^1, p_s^2) n_s^2(p_s^1, p_s^2) p_s^2 + n_b^1(p_s^1, p_s^2) n_s^2(p_s^1, p_s^2)(p_s^2 - f) & \text{if } i = 2 \end{cases} \tag{6}$$

Timing. In the first period, platform 1 sets the per-transaction referral fee f. In the second period, platforms simultaneously set the per-transaction prices charged to sellers (p_s^1, p_s^2). In the second period, consumers and sellers simultaneously choose which platform(s) to join.

Figure 1 illustrates the network effects that exist in each setting.

Figure 1: Indirect network effects with and without intertwined network effects



4 Intertwined network effects between legally-independent firms

In this section we characterize the equilibria of the two settings described in Section 3. We assume full information for all participants in the model, i.e., each participant observes all the price decisions and knows all the parameters of the model. The equilibrium concept is the Subgame Perfect Nash equilibrium.

4.1 Equilibrium without intertwined network effects (benchmark)

In this subsection we consider the benchmark setting described in Section 3.1. The aim is to characterize the equilibrium in which platforms are active.

In the second stage, the consumer indifferent between joining platform 1 and 2 is located at $\tilde{x}^B$ such that ${U_b^1}^B(\tilde{x}^B) = {U_b^2}^B(\tilde{x}^B)$. Thus, the number of consumers buying from platform 1 is equal to $\tilde{x}^B$ and the number of consumers buying from platform 2 is equal to $1 - \tilde{x}^B$. Sellers are divided into three sub-intervals on the unit interval. Sellers located "on the left" only join platform 1. Those located "on the right" join only platform 2. Those located "in the middle" join both platforms and cand thus interact with both platforms' buyer. The seller indifferent between joining platform 1 and not joining it is located at x_{10}^B such that ${U_s^1}^B = 0$. The seller indifferent between joining platform 2 and not joining it is located at x_{20}^B such that ${U_s^2}^B = 0$. Then, the sellers that singlehome in platform 1 are located in the $[0, x_{20}^B]$ sub-interval, those who singlehome in platform 2 in the $[x_{10}^B, 1]$ sub-interval and those who multihome in the (x_{20}^B, x_{10}^B) sub-interval. To focus on the interesting case in which there is multihoming in the benchmark, I assume for the time being that $0 < x_{20}^B < x_{10}^B < 1$ (I provide the necessary and sufficient conditions below), so that $n_s^1 = x_{10}^B$ and $n_s^2 = 1 - x_{20}^B$.[4] Then, the number of buyers and sellers in each platform is found by solving the following system of four equations and four unknowns:

$$
\begin{aligned}
n_b^1 &= \frac{\tau_b + n_s^1\alpha_b - n_s^2\alpha_b}{2\tau_b} \\
n_b^2 &= 1 - \frac{\tau_b + n_s^1\alpha_b - n_s^2\alpha_b}{2\tau_b} \\
n_s^1 &= \frac{n_b^1(\alpha_s - p_s^1)}{\tau_s} \\
n_s^2 &= 1 - \frac{n_b^2(p_s^2 + \tau_s - \alpha_s)}{\tau_s}
\end{aligned}
$$

Which yields buyers' and sellers' participation as a function of seller prices and the model's

[4] Note that the existence of multihoming on the seller side in the benchmark allows for an elastic demand on the money-making side of the market. This will be important to explain the effects of INE (cf. Section 4.3) and of a merger between platforms having set-up INE (cf. Section 5.2).

parameters.

$$
\begin{aligned}
n_b^1(p_s^1, p_s^2) &= \frac{\tau_b\tau_s + \alpha_b(p_s^2 - \alpha_s)}{2\tau_b\tau_s + \alpha_b(p_s^1 + p_s^2 - 2\alpha_s)} \\
n_b^2(p_s^1, p_s^2) &= \frac{\tau_b\tau_s + \alpha_b(p_s^1 - \alpha_s)}{2\tau_b\tau_s + \alpha_b(p_s^1 + p_s^2 - 2\alpha_s)} \\
n_s^1(p_s^1, p_s^2) &= \frac{(p_s^1 - \alpha_s)(\alpha_b(\alpha_s - p_s^2) - \tau_b\tau_s)}{\tau_s(2\tau_b\tau_s + \alpha_b(p_s^1 + p_s^2 - 2\alpha_s))} \\
n_s^2(p_s^1, p_s^2) &= \frac{(p_s^2 - \alpha_s)(\alpha_b(\alpha_s - p_s^1) - \tau_b\tau_s)}{\tau_s(2\tau_b\tau_s + \alpha_b(p_s^1 + p_s^2 - 2\alpha_s))}
\end{aligned}
\tag{7}
$$

In stage 1, each platform solves the maximisation program $\max_{p_s^i} \Pi^i(p_s^1, p_s^2)$. Solving the system of first-order conditions yields only one set of equilibrium symmetric prices that satisfy the second-order conditions and for which $0 < x_{20}^B < x_{10}^B < 1$ (i.e., for which there is multihoming on the seller side):

$$
p_s^i = \alpha_s - \frac{\tau_b\tau_s\sqrt{\tau_b\tau_s(\tau_b\tau_s - \alpha_b\alpha_s)}}{\alpha_b}
$$

Note that, in line with Amstrong's (2006) competitive bottleneck model, in the benchmark equilibrium, platform i charges sellers the marginal cost of an interaction (normalized to 0) plus the per-interaction benefit enjoyed by sellers (α_s), adjusted upward[5] by a term that depends positively on the two user groups' transportation costs and negatively on the the cross-group external effect that sellers exert on buyers.

Replacing these equilibrium prices in (7) gives the equilibrium participation on the buyer and the seller side:

$$
\begin{aligned}
n_b^i &= \frac{1}{2} \\
n_s^i &= \frac{\tau_b}{2\alpha_b} - \frac{\sqrt{\tau_b\tau_s(\tau_b\tau_s - \alpha_b\alpha_s)}}{2\tau_s\alpha_b}
\end{aligned}
$$

Consumer and seller surplus are calculated respectively as

$$
\begin{aligned}
CS^B &\equiv \int_0^{\tilde{x}^B} {U_b^1}^B(x)dx + \int_{\tilde{x}}^1 {U_b^2}^B(x)dx \\
SS^B &\equiv \int_0^{x_{10}^B} {U_s^1}^B(x)dx + \int_{1-x_{20}^B}^1 {U_s^2}^B(x)dx
\end{aligned}
$$

Then, the equilibrium values of the model in the benchmark setting (i.e., without intertwined network effects) are the following.

[5] The term $-\frac{\tau_b\tau_s\sqrt{\tau_b\tau_s(\tau_b\tau_s - \alpha_b\alpha_s)}}{\alpha_b}$ is strictly positive, as per the second-order condition on the seller side reported in Assumption 1.

Lemma 1 (Equilibrium without intertwined network effects). *In the benchmark setting equilibrium:*

$$
\begin{aligned}
{n_b^i}^B &= \frac{1}{2} \\
{n_b}^B &= 1 \\
{n^i}_s &= \frac{\tau_b}{2\alpha_b} - \frac{\sqrt{\tau_b\tau_s(\tau_b\tau_s - \alpha_b\alpha_s)}}{2\tau_s\alpha_b} \\
{n_s}^B &= \frac{\tau_b\tau_s - \sqrt{\tau_b\tau_s(\tau_b\tau_s - \alpha_b\alpha_s)}}{\tau_s\alpha_b} \\
{p_s^i}^B &= \alpha_s - \frac{\tau_b\tau_s\sqrt{\tau_b\tau_s(\tau_b\tau_s - \alpha_b\alpha_s)}}{\alpha_b} \\
{\Pi^i}^B &= \frac{(\tau_b\tau_s - \sqrt{\tau_b\tau_s(\tau_b\tau_s - \alpha_b\alpha_s)}(\alpha_b\alpha_s + \sqrt{\tau_b\tau_s(\tau_b\tau_s - \alpha_b\alpha_s)} - \tau_b\tau_s)}{4\tau_s{\alpha_b}^2} \\
\Pi^B &= \frac{\left(\tau_b\tau_s - \sqrt{\tau_b\tau_s(\tau_b\tau_s - \alpha_b\alpha_s)}\right)\left(\tau_b\tau_s - \alpha_b\alpha_s - \sqrt{\tau_b\tau_s(\tau_b\tau_s - \alpha_b\alpha_s)}\right)}{2\tau_s\alpha_b^2} \\
CS^B &= v_b - \frac{\sqrt{\tau_b\tau_s(\tau_b\tau_s - \alpha_b\alpha_s)}}{2\tau_s} \\
SS^B &= \frac{(\tau_b(4\tau_s\alpha_b + \alpha_b\alpha_s + 2\sqrt{\tau_b\tau_s(\tau_b\tau_s - \alpha_b\alpha_s)})) - (2{\tau_b}^2\tau_s + 2\alpha_b(t_s\alpha_b + 2\sqrt{\tau_b\tau_s(\tau_b\tau_s - \alpha_b\alpha_s)}))}{4{\alpha_b}^2}
\end{aligned}
$$

For the equilibrium reported in Lemma 1 to be valid, a series of conditions summarized in Assumption 1 have to hold. First, the second-order conditions of the platforms' maximization program are met if $\alpha_b\alpha_s < \tau_b\tau_s$. As it is usually the case in platform competition models, these conditions require that indirect network effects are sufficiently small compared to platforms' horizontal differentiation. They are also sufficient to ensure a stable and unique equilibrium in which both platforms are active. Second, to ensure that the market is covered on the buyer side, we need to verify that the consumer indifferent between the two platforms obtains a positive surplus, i.e., $U_b^{iB}(\tilde{x}^B) > 0$. This condition is more stringent than the first. Third, we impose that some sellers multihome at equilibrium, which in turn implies full coverage on the seller side. This implies that, in equilibrium, $0 < x_{20}^B < x_{10}^B < 1$. Collecting these conditions, we obtain:

Assumption 1 (Benchmark conditions)**.** *In the benchmark setting without intertwined network effects, the parameters satisfy the following conditions.*

$$
\begin{aligned}
&\alpha_b\alpha_s < \tau_b\tau_s && \textit{(SOCs}^B\textit{)} \\
&\alpha_b\alpha_s < \tau_b\tau_s \wedge \tau_b^2\tau_s < 4v_b^2\tau_s + \tau_b\alpha_b\alpha_s && \textit{(FPB}^B\textit{)} \\
&-\tau_b\tau_s + \tau_s\alpha_b + \sqrt{-\tau_b\tau_s(\tau_b\tau_s - \alpha_b\tau_s)} < 0 \ \wedge \\
&-\tau_b\tau_s + 2\tau_s\alpha_b + \sqrt{-\tau_b\tau_s(\tau_b\tau_s - \alpha_b\tau_s)} > 0 && \textit{(MHS}^B\textit{)}
\end{aligned}
$$

4.2 Equilibrium with intertwined network effects

We now turn to a setting in which platforms decide to set-up intertwined network effects and characterize its equilibrium. To present shorter mathematical expressions, let me introduce the following additional notation:

$$\Omega \equiv \sqrt{\tau_b\tau_s(16{\tau_b}^3{\tau_s}^3 - 8{\tau_b}^2{\tau_s}^2\alpha_b\alpha_s + 5\tau_b\tau_s\alpha_b{}^2{\alpha_s}^2 - 2{\alpha_b}^3{\alpha_s}^3)} \tag{8}$$

In the third stage, the consumer that is indifferent between joining platform 1 and 2 is located at $\tilde{x}^{INE}$ such that ${U_b^1}^{INE}(\tilde{x}^{INE}) = {U_b^2}^{INE}(\tilde{x}^{INE})$. Thus, the number of consumers buying from platform 1 is equal to $\tilde{x}^{INE}$ and the number of consumers buying from platform 2 is equal to $1 - \tilde{x}^{INE}$. The seller indifferent between joining platform 1 and not joining it is located at x_{10}^{INE} such that ${U_s^1}^{INE} = 0$. The seller indifferent between joining platform 2 and not joining it is located at x_{20}^{INE} such that ${U_s^2}^{INE} = 0$. Then, the sellers that singlehome in platform 1 are located in the $[0, x_{20}^{INE}]$ sub-interval, those who singlehome in platform 2 in the $[x_{10}^{INE}, 1]$ sub-interval and those who multihome in the $(x_{20}^{INE}, x_{10}^{INE})$ sub-interval. As in the previous setting, I assume for the time being that $0 < x_{20}^{INE} < x_{10}^{INE} < 1$ (I provide the necessary and sufficient conditions below), so that $n_s^1 = x_{10}^{INE}$ and $n_s^2 = 1 - x_{20}^{INE}$. Then, the number of buyers and sellers in each platform is found by solving the following system of four equations and four unknowns:

$$\begin{aligned}
n_b^1 &= \frac{\tau_b + n_s^1\alpha_b}{2\tau_b} \\
n_b^2 &= 1 - \frac{\tau_b + n_s^1\alpha_b}{2\tau_b} \\
n_s^1 &= \frac{n_b^1(\alpha_b - p_s^1)}{\tau_s} \\
n_s^2 &= 1 - \frac{n_b^1 p_s^2 + n_b^2 p_s^2 + \tau_s - n_b^1\alpha_s - n_b^2\alpha_s}{\tau_s}
\end{aligned}$$

Which yields buyers' and sellers' participation as a function of seller prices and the model's parameters.

$$\begin{aligned}
n_b^1(p_s^1) &= \frac{\tau_b\tau_s}{2\tau_b\tau_s + p_s^1\alpha_b - \alpha_b\alpha_s} \\
n_b^2(p_s^1) &= \frac{\tau_b\tau_s}{-2\tau_b\tau_s + \alpha_b(\alpha_b - p_s^1)} - 1 \\
n_s^1(p_s^1) &= \frac{\tau_b(\alpha_s - p_s^1)}{2\tau_b\tau_s - \alpha_b(\alpha_s - p_s^1)} \\
n_s^2(p_s^2) &= \frac{\alpha_s - p_s^2}{\tau_s}
\end{aligned} \tag{9}$$

Note that, while in (7) sellers and buyers' participation depend on the price charged to sellers in both platforms, this is not the case in (9). With INE, the only price implicitly considered by buyers when deciding their participation level on both platforms is the per-transaction price charged to sellers in the host platform (platform 1). Buyers know that, if they join platform 2, they can only interact with platform 2 sellers. If they join platform 1, they have access to both platforms' sellers and additional interactions with multihoming platform 1 sellers. Hence, the difference in the number of sellers they can expect to interact with depends only on platform 1 prices to sellers, which will determine the number of sellers joining also platform 1.

As for sellers, under INE, their participation in a platform does not depend on the rival platform's price charged to them. Sellers know that if they join platform 2 they will have access to all consumers at a p_s^2 per-interaction price, and that if they (also) join platform 1 they will have (additional) interactions with consumers from platform 1 at p_s^1 per interaction. This makes the decision to join each platform depend only of that platform's price. Additionally, note that, because platform 2 gives access to all consumers, sellers' decision to join it do not depend on consumers' transportation costs in (9).

In stage 2, each platform solves the maximisation program $\max_{p_s^i} \Pi^i(p_s^1, p_s^2, f)$. Solving the system of first-order conditions yields a unique set of equilibrium prices:

$$\begin{aligned} p_s^{1*} &= \frac{f^2\tau_b\tau_s\alpha_b + 2\tau_b\tau_s\alpha_s(2\tau_b\tau_s - \alpha_b\alpha_s) + f\alpha_b(\alpha_b\alpha_s - 2\tau_b\tau_s)}{8{\tau_b}^2{\tau_s}^2 - 2\tau_b\tau_s\alpha_b\alpha_s + f{\alpha_b}^2\alpha_s} \\ p_s^{2*} &= \frac{f^2\alpha_b^2\alpha_s + 4\tau_b\tau_s(2\tau_b\tau_s - \alpha_b\alpha_s) + f\tau_b\tau_s(4\tau_b\tau_s - \alpha_b\alpha_s)}{f^2\alpha_b^2 + 8\tau_b\tau_s(2\tau_b\tau_s - \alpha_b\alpha_s)} \end{aligned} \tag{10}$$

In stage 1, platform 1 sets the optimal per-transaction fee that it charges platform 2 for every interaction between a platform 1 user and a platform 2 seller. It therefore solves the maximisation program $\max_f \Pi^i(p_s^{1*}, p_s^{2*}, f)$. The only value of f that satisfies the first and second-order condition of this maximisation program is:

$$f^* = \frac{2\big(\tau_b\tau_s(\alpha_b\alpha_s - 4\tau_b\tau_s) + \Omega\big)}{\alpha_b^2\alpha_s} \tag{11}$$

Replacing (10) and (11) in (9) we obtain the quantities on both sides of the market and the locations of the indifferent consumer and sellers in equilibrium. Consumer and seller surplus are calculated in the same way as in the benchmark setting and using the corresponding utility functions (cf. Section 3.2) and equilibrium threshold values.

Then, the equilibrium values of the model in the intertwined network effects setting are the following.

Lemma 2 (**Equilibrium with intertwined network effects**). *In the intertwined network effects setting equilibrium:*

$$
\begin{aligned}
n_b^{1\,INE} &= \frac{4{\tau_b}^2{\tau_s}^2 - \tau_b\tau_s\alpha_b\alpha_s + \Omega}{16{\tau_b}^2{\tau_s}^2 - 8\tau_b\tau_s\alpha_b\alpha_s} \\
n_b^{2\,INE} &= \frac{12{\tau_b}^2{\tau_s}^2 - 7\tau_b\tau_s\alpha_b\alpha_s - \Omega}{16{\tau_b}^2{\tau_s}^2 - 8\tau_b\tau_s\alpha_b\alpha_s} \\
{n_b}^{INE} &= 1 \\
n_s^{1\,INE} &= \frac{\tau_b\tau_s(3\alpha_b\alpha_s - 4\tau_b\tau_s) + \Omega}{4\tau_s\alpha_b(2\tau_b\tau_s - \alpha_b\alpha_s)} \\
n_s^{2\,INE} &= \frac{\alpha_s}{4\tau_s} \\
{n_s}^{INE} &= \frac{\alpha_s\left(2\tau_b\tau_s - \alpha_b\alpha_s\right) + \tau_b\tau_s\left(-4\tau_b\tau_s + 3\alpha_b\alpha_s\right)}{4\tau_s\alpha_b\left(2\tau_b\tau_s - \alpha_b\alpha_s\right)} + \frac{\Omega}{4\tau_s\alpha_b\left(2\tau_b\tau_s - \alpha_b\alpha_s\right)} \\
p_s^{1\,INE} &= \frac{-16{\tau_b}^3{\tau_s}^3 + 4{\tau_b}^2{\tau_s}^2\alpha_b\alpha_s - 2\tau_b\tau_s{\alpha_b}^2{\alpha_s}^2 + {\alpha_b}^3{\alpha_s}^3 + 4\tau_b\tau_s\Omega}{\alpha_b^3\alpha_s^2} \\
p_s^{2\,INE} &= \frac{3\alpha_s}{4} \\
f^{INE} &= \frac{2\big(\tau_b\tau_s(\alpha_b\alpha_s - 4\tau_b\tau_s) + \Omega\big)}{\alpha_b^2\alpha_s} \\
\Pi^{1\,INE} &= \frac{(4\tau_b\tau_s - \alpha_b\alpha_s)(-4{\tau_b}^2{\tau_s}^2 + \tau_b\tau_s\alpha_b\alpha_s + \alpha_b^2 + \alpha_s^2 + \Omega)}{16\tau_s\alpha_b^2(2\tau_b\tau_s - \alpha_b\alpha_s)} \\
\Pi^{2\,INE} &= \frac{\alpha_s^2}{16\tau_s} \\
\Pi^{INE} &= \frac{\alpha_s^2 + (-4\tau_b\tau_s + \alpha_b\alpha_s)\left(-4\tau_b^2\tau_s^2 + \tau_b\tau_s\alpha_b\alpha_s + \alpha_b^2\alpha_s^2\right)}{16\tau_s\alpha_b^2\left(-2\tau_b\tau_s + \alpha_b\alpha_s\right)} + \frac{\Omega}{16\tau_s\alpha_b^2\left(-2\tau_b\tau_s + \alpha_b\alpha_s\right)} \\
CS^{INE} &= \frac{-16\tau_b^4\tau_s^4 + 32\tau_b^3\tau_s^3\alpha_b\alpha_s - 2\alpha_b^2\alpha_s^2\Omega + 8v_b\tau_s(2\tau_b\tau_s - \alpha_b\alpha_s)\big(\tau_b\tau_s(4\tau_b\tau_s - \alpha_b\alpha_s) + \Omega\big)}{32\tau_b\tau_s^2(\alpha_b\alpha_s - 2\tau_b\tau_s)} \\
&\quad + \frac{\tau_b^2\tau_s^2\big(13\alpha_b^2\alpha_s^2 + 4\Omega\big) + \tau_b\tau_s\alpha_b\alpha_s\big(\alpha_b^2\alpha_s^2 + 7\Omega\big)}{32\tau_b\tau_s^2(\alpha_b\alpha_s - 2\tau_b\tau_s)} \\
SS^{INE} &= \frac{1}{32}\Big(8\tau_s\left(\frac{\tau_b^2}{\alpha_b^2} - 2\right) - \frac{3\alpha_s^2}{\tau_s} + \frac{16\alpha_b^4\alpha_s^3 + 2\alpha_b^2\alpha_s\big(\alpha_b^2\alpha_s(32\tau_s + 3\alpha_s) - 4\Omega\big)}{\alpha_b^2(\alpha_b\alpha_s - 2\tau_b\tau_s)^2} \\
&\quad + 2\tau_b\alpha_b\alpha_s\big(-\alpha_b^2\alpha_s(32\tau_s + \alpha_s) + 3\Omega\big)\Big)
\end{aligned}
$$

For the equilibrium reported in Lemma 2 to be valid, a series of conditions summarized in Assumption 2 have to hold. These conditions are analogous to those presented in Assumption 1.

Assumption 2 (Intertwined network effects conditions)**.** *In the intertwined network effects setting, the parameters satisfy the following conditions.*

$$\alpha_b\alpha_s < 2\tau_b\tau_s \qquad \textit{(SOCs}^{INE}\textit{)}$$

$$\frac{4(4v_B-3\tau_b)\tau_b\tau_S^2+(-8v_B+11\tau_b)\tau_S\alpha_b\alpha_s-2\alpha_b^2\alpha_s^2}{8\tau_S\left(2\tau_b\tau_S-\alpha_b\alpha_s\right)}+$$
$$\frac{\sqrt{\tau_b\tau_S\left(16\tau_b^3\tau_S^3-8\tau_b^2\tau_S^2\alpha_b\alpha_s+5\tau_b\tau_S\alpha_b^2\alpha_s^2-2\alpha_b^3\alpha_s^3\right)}}{8\tau_S\left(2\tau_b\tau_S-\alpha_b\alpha_s\right)}>0 \qquad \textit{(FPB}^{INE}\textit{)}$$

$$0<1-\frac{\alpha_s}{4\tau_S}<\frac{\tau_b\tau_S\left(-4\tau_b\tau_S+3\alpha_b\alpha_s\right)+}{4\tau_S\alpha_b\left(2\tau_b\tau_S-\alpha_b\alpha_s\right)u}$$
$$\frac{\sqrt{\tau_b\tau_S\left(16\tau_b^3\tau_S^3-8\tau_b^2\tau_S^2\alpha_b\alpha_s+5\tau_b\tau_S\alpha_b^2\alpha_s^2-2\alpha_b^3\alpha_s^3\right)}}{4\tau_S\alpha_b\left(2\tau_b\tau_S-\alpha_b\alpha_s\right)u}<1 \qquad \textit{(MHS}^{INE}\textit{)}$$

4.3 Comparison between the equilibrium with and without intertwined network effects

In this subsection, we compare the equilibrium of the INE and the benchmark settings. Let us first introduce the following remark.

Remark 1. *Let $Cond^B$ and $Cond^{INE}$ be the parameter spaces defined by Assumptions 1 and 2, respectively.*

i) $\exists \quad S^B \subseteq Cond^B \quad | \quad S^B \cap Cond^{INE} = \varnothing$

ii) $\exists \quad S^{INE} \subseteq Cond^{INE} \quad | \quad S^{INE} \cap Cond^B = \varnothing$

iii) $\exists \quad S^{B\cap INE} = Cond^B \cap Cond^{INE} \neq \varnothing$

Remark 1 tells us that there are admissible parameter spaces in which i) only the benchmark case might take place, ii) only the INE case can take place and iii) both the benchmark and INE case can take place. Given that our focus is the effect of INE relative to the benchmark case, in the remainder of this section we will assume that Assumptions 1 and 2 hold, i.e., that we are in the parameter space defined as $S^{B\cap INE}$ in Remark 1. When this is the case, as per Lemma 3, platforms always have an incentive to set INE.

Lemma 3 (**Platforms' incentives to set out intertwined network effects**)**.** *If Assumptions 1 and 2 hold, setting-up intertwined network effects is a dominant strategy for the aggregator and the source platforms. Formally, $\Pi^{1^{INE}} > \Pi^{1^B}$ and $\Pi^{2^{INE}} > \Pi^{2^B}$.* ***Proof****: See Appendix A.1.*

Lemma 3 tells us that, whenever both setting and not setting up intertwined network effects can be an equilibrium, setting INE will be a dominant strategy for both platforms. Hence, in

the remaining of this section, we study the effects of setting up INE on platform participation, prices and participants' welfare.

4.3.1 Prices

Lemma 4 shows how INE affect prices.

Lemma 4 (**Effect of intertwined network effects on prices charged to sellers**). *When platforms set-up intertwined network effects, the prices paid by sellers to the source platform increase ($p_s^{2^{INE}} > p_s^{2^B}$). In contrast, the prices paid by sellers to the aggregator platform might increase or decrease. There exists a value $\tilde{\tau}_b(\alpha_b, \alpha_s, \tau_s)$ satisfying Assumptions 1 and 2 such that, if $\tau_b > \tilde{\tau}_b(\alpha_b, \alpha_s, \tau_s)$, they decrease ($p_s^{1^{INE}} < p_s^{1^B}$); otherwise (i.e., if $\tau_b < \tilde{\tau}_b(\alpha_b, \alpha_s, \tau_s)$), they increase ($p_s^{1^{INE}} > p_s^{1^B}$).*

***Proof**: See Appendix A.2.*

Platform 2 increases the price charged to sellers for two reasons. The first one is the increase in its marginal cost in matching consumers referred by platform 1 to platform 2 sellers by f^{INE}. The second one is the increase in quality of platform 2 to for sellers generated by INE, as, post-INE, platform 2 sellers can interact with all buyers. In platform 1, the price charged to sellers increases if consumers' transportation costs are sufficiently low, and decrease otherwise. The intuition is as follows. Post-INE, platform 1's quality to buyers increases, as they can now access both platforms' sellers through it. Hence, as shown in Lemma 5, post-INE, buyers' participation increases in platform 1. The lower transportation costs are, the stronger this increase is. With a sufficiently high increase in buyers' participation, platform 1 becomes so attractive to sellers that the platform maximises profits by increasing the price it charges them even if this reduces sellers' participation (and hence consumer's), leading to lower revenues through the referral fee. Conversely, if transportation costs are sufficiently low, the post-INE increase in buyers' participation in platform 1 is mild. Hence, the latter maximises profit by lowering the price charged to sellers. This increase their participation, and hence consumers', leading to higher revenues through the referral fee.

4.3.2 Platform participation

Lemma 5 shows how INE affect platform participation in equilibrium.

Lemma 5 (**Effect of intertwined network effects on platform participation**). *When platforms set-up intertwined network effects:*

i) The number of consumers increases in the aggregator platform ($n_b^{1^{INE}} > n_b^{1^B}$) and decreases in the source platform ($n_b^{2^{INE}} < n_b^{2^B}$)

ii) The number of sellers increases in the aggregator platform ($n_s^{1\,INE} > n_s^{1\,B}$) and decreases in the source platform ($n_s^{2\,INE} < n_s^{2\,B}$)

iii) The total number of sellers increases ($n_s^{INE} > n_s^B$) if and only if $\tau_b > \frac{(3+2\sqrt{3})\alpha_b\alpha_s}{6\tau_s}$ and decreases ($n_s^{INE} < n_s^B$) otherwise ($\tau_b < \frac{(3+2\sqrt{3})\alpha_b\alpha_s}{6\tau_s}$).

***Proof**: See Appendix A.3.*

The results regarding buyers' participation is intuitive. Post-INE, platform 1 becomes more attractive to buyers, as they can access all sellers through it. Given that buyers are assumed to singlehome, this results in an increase in platform 1 participation and a decrease in platform 2 participation on the buyer side.

Regarding sellers, in the aggregator platform (platform 1), the number of consumers increases, while the price might increase or decrease. However, the net effect on a seller's utility is always positive. Hence, seller participation platform 1 increases. In the source platform (platform 2), post-INE, the number of buyers increases by $\frac{1}{2}$, while the price also increases. The net effect on a platform 2 seller's utility is a priori ambiguous. However, the increase in platform 1's seller utility drives some platform 2 sellers to migrate to platform 1, leading to a decrease in platform 2 seller participation.

Hence, overall seller participation can only increase if seller multihoming increases. This is the case if consumers' transportation cost are sufficiently high. As it can be analytically verified, the threshold of τ_b above which seller multihoming increases is above that for which prices in platform 1 decreases prices. In other words, if seller prices decrease sufficiently in the aggregator platform, the number of sellers that switched from singlehoming in platform 2 to multihoming will exceed that of sellers singlemoning that start to singlehome in platform 1, leading to an overall increase in seller multihoming.

4.3.3 Participants' surpluses

Propositions 1 and 2 show how INE affects consumers' and seller surpluses, respectively.

Proposition 1 (**Effect of intertwined network effects on consumer surplus**). *When platforms set-up intertwined network effects, consumer surplus always increases ($CS^{INE} > CS^B$).*

***Proof**: See Appendix A.4.*

The intuition is as follows. In platform 1 (the aggregator), as per Lemma 5, consumer participation increases post-INE. This increases consumer surplus through an *extensive margin* effect. Moreover, the number of sellers participating in platform 1 increases, and

INE allow platform 1 consumers to interact with platform 2 sellers. This increases platform 1 consumer surplus through an *intensive margin effect*. Hence, consumer surplus increases in platform 1. In platform 2 (the source platform), as per Lemma 5, the number of buyers decreases post-INE. This makes the consumer surplus generated in platform 2 decrease through an *extensive margin effect*. Moreover, the number of sellers in platform 2 decreases, leading to a further decrease in consumer surplus through an *intensive margin* effect.

The increase in platform 1's consumer surplus always more than compensates the decrease in platform 2's. Given that consumers singlehome and the market is covered, the positive extensive margin effect in platform 1 cancels the negative extensive margin effect in platform 2. The positive intensive margin effect in platform 1 always countervails the negative intensive margin effect in platform 2. Hence, intertwined network effects between legally-independent firms benefits consumers.

Proposition 2 (**Effect of intertwined network effects on seller surplus**). *When platforms set-up intertwined network effects:*

i) If $\tau_s \leq \frac{5}{8}\alpha_s$*, seller surplus increases* ($SS^{INE} > SS^{B}$)

ii) If $\tau_s > \frac{5}{8}\alpha_s$*, seller surplus decreases* ($SS^{INE} < SS^{B}$)

***Proof**: See Appendix A.5.*

Proposition 2 shows that if the platforms are sufficiently differentiated for sellers, INE decrease their surplus, and vice-versa. As shown above, post-INE, the utility of a seller increases in platform 1 (the aggregator). Hence, some sellers leave platform 2 for platform 1, leading to an increase in the number of sellers in the former and a decrease in the latter. As a result, seller surplus increases in platform 1 and decreases in platform 2. The overall effect depends on which platforms' seller surplus increases the most, which in turn depends on how many sellers leave platform 2 for platform 1. With sufficiently low seller transportation costs, enough sellers leave the source platform for the aggregator platform, leading to an increase in overall seller surplus, and vice-versa.

5 Platform merger with intertwined network effects

In this section I analyze the impact of a non-consolidating merger between two platforms having set-up intertwined network effects on prices and the participation and surpluses of buyers and sellers. As in the previous section, we assume full information for all participants in the model, i.e., each participant observes all the price decisions and knows all the parameters of the model. The equilibrium concept is the Subgame Perfect Nash equilibrium.

5.1 Post-merger equilibrium with intertwined network effects

In this section we characterize the post-merger equilibrium in a setting with intertwined network effects. The superscript $INE-M$ is used to refer to this setting in which the platforms having set-up intertwined network effects merged.

As in Section 3.2, the utility function of buyers and sellers are given by (4) and (5), respectively. However, given that the platforms are under common ownership, the inter-platform fee f represents an internal transfer, and it is therefore not charged. Then, the profit of platform $i \in \{1,2\}$ is given by:

$$\Pi^{iINE-M} \equiv \begin{cases} n_b^1(p_s^1,p_s^2)n_s^1(p_s^1,p_s^2)p_s^1 & \text{if } i=1 \\ p_s^2\left(n_b^2(p_s^1,p_s^2)n_s^2(p_s^1,p_s^2)+n_b^1(p_s^1,p_s^2)n_s^2(p_s^1,p_s^2)\right) & \text{if } i=2 \end{cases} \tag{12}$$

Thus, in this setting, the game has two stages. In the first stage, the platforms simultaneously set the prices charged to sellers (p_s^1,p_s^2). In the second stage, consumers and sellers simultaneously choose which platform(s) to join.

Given that their utility functions remain unchanged by the merger, in stage 2, consumers' and sellers' participation as a function of sellers prices and the model's parameters are given by (9). In stage 1, the platforms solve the joint maximization problem $\max_{p_s^1,p_s^2} \Pi(p_s^1,p_s^2)$, where $\Pi \equiv \Pi^1+\Pi^2$. Solving the system of first-order conditions yields a unique set of equilibrium prices:

$$\begin{aligned} p_s^{1*} &= \frac{\alpha_s(\alpha_b\alpha_s-2\tau_b\tau_s)}{\alpha_b\alpha_s-4\alpha_b\alpha_s} \\ p_s^{2*} &= \frac{\alpha_s}{2} \end{aligned} \tag{13}$$

Replacing (13) in (9) we obtain the quantities on both sides of the market and the locations of the indifferent consumer and sellers in equilibrium. Consumer and seller surplus are calculated in the same way as in the benchmark setting and using the corresponding utility functions and equilibrium threshold values.

Then, the post-merger equilibrium values of the model in the setting with intertwined network effects are the following.

Lemma 6 (**Post-merger equilibrium with intertwined network effects**). *In the intertwined network effects setting equilibrium:*

$$
\begin{aligned}
n_b^{1\,INE-M} &= \frac{1}{4} + \frac{\tau_b\tau_s}{4\tau_b\tau_s - 2\alpha_b\alpha_s} \\
n_b^{2\,INE-M} &= \frac{3}{4} - \frac{\tau_b\tau_s}{4\tau_b\tau_s - 2\alpha_b\alpha_s} \\
n_b{}^{INE-M} &= 1 \\
n_s^{1\,INE-M} &= \frac{\tau_b\alpha_s}{4\tau_b\tau_s - 2\alpha_b\alpha_s} \\
n_s^{2\,INE-M} &= \frac{\alpha_s}{2\tau_s} \\
n_s{}^{INE-M} &= \frac{\alpha_s}{2\tau_s} + \frac{\tau_b\alpha_s}{4\tau_b\tau_s - 2\alpha_b\alpha_s} \\
p_s^{1\,INE-M} &= \frac{\alpha_s(-2\tau_b\tau_s + \alpha_b\alpha_s)}{-4\tau_b\tau_s + \alpha_b\alpha_s} \\
p_s^{2\,INE-M} &= \frac{\alpha_s}{2} \\
\Pi^{1\,INE-M} &= \frac{\tau_b\alpha_s^2}{16\tau_b\tau_s - 8\alpha_b\alpha_s} \\
\Pi^{2\,INE-M} &= \frac{\alpha_s^2}{4\tau_s} \\
\Pi^{INE-M} &= \frac{1}{4}\alpha_s^2\left(\frac{1}{\tau_s} + \frac{\tau_b}{4\tau_b\tau_s - 2\alpha_b\alpha_s}\right) \\
CS^{INE-M} &= \frac{(4\tau_b\tau_s - \alpha_b\alpha_s)\left(4\left(4r_B - \tau_b\right)\tau_b\tau_s^2 + \left(-8r_B + 11\tau_b\right)\tau_s\alpha_b\alpha_s - 4\alpha_b^2\alpha_s^2\right)}{16\tau_s\left(-2\tau_b\tau_s + \alpha_b\alpha_s\right)^2} \\
SS^{INE-M} &= \frac{1}{8}\left(\tau_s\left(-4 + \frac{\tau_b^2}{\alpha_b^2}\right) + 8\alpha_s - \frac{3\alpha_s^2}{\tau_s} + \frac{4\tau_b^4\tau_s^3}{\alpha_b^2\left(-2\tau_b\tau_s + \alpha_b\alpha_s\right)^2} + \frac{4\tau_b^3\tau_s^2}{\alpha_b^2\left(-2\tau_b\tau_s + \alpha_b\alpha_s\right)}\right)
\end{aligned}
$$

For the equilibrium reported in Lemma 6 to be valid, a series of conditions summarized in Assumption 3 have to hold. These conditions are analogous to those presented in Assumption 1, with some additional simplifications. The second order condition on the source platform's price ($-\frac{2}{\tau_s} < 0$) is not reported, as it is always met. In the same vein, the condition for the price vector that satisfies the first order conditions to constitute a maximum are always met when the second-order condition on the aggregator's price is verified. Hence, for the sake of simplification, we do not include it in Assumption 3.

Assumption 3 (Platform merger with intertwined network effects conditions). *In the setting in which platforms having set-up intertwined network effects merge, the parameters satisfy the following conditions.*

$$\alpha_b\alpha_s < 2\tau_b\tau_s \qquad \textit{(SOCs}^{\textit{INE-M}}\textit{)}$$

$$\frac{2\tau_b\tau_s}{\alpha_b} \neq \alpha_s \wedge 4v_B + \frac{2\alpha_b\alpha_s}{\tau_s} > \tau_b\left(3 + \frac{2\tau_b\tau_s}{-2\tau_b\tau_s + \alpha_b\alpha_s}\right) \qquad \textit{(FPB}^{\textit{INE-M}}\textit{)}$$

$$0 < 1 - \frac{\alpha_s}{2\tau_s} < \frac{\tau_b\alpha_s}{4\tau_b\tau_s - 2\alpha_b\alpha_s} < 1 \qquad \textit{(MHS}^{\textit{INE-M}}\textit{)}$$

5.2 Comparison between the pre- and post-merger equilibria

In this subsection, we compare the equilibrium with INE and legally-independent firms to a post-merger equilibrium with INE and firms subject to common ownership. Let us first introduce the following remark.

Remark 2. *Let $Cond^{INE}$ and $Cond^{INE-M}$ be the parameter spaces defined by Assumptions 2 and 3, respectively.*

$$i)\ \exists\ S^{INE} \subseteq Cond^{INE} \mid S^{INE} \cap Cond^{INE-M} = \varnothing$$

$$ii)\ \exists\ S^{INE-M} \subseteq Cond^{INE-M} \mid S^{INE-M} \cap Cond^{INE} = \varnothing$$

$$iii)\ \exists\ S^{INE \cap INE-M} = Cond^{INE} \cap Cond^{INE-M} \neq \varnothing$$

Remark 2 tells us that there are admissible parameter spaces in which i) only the case with INE and legally-independent firms might take place, ii) only the INE and post-merger case can take place and iii) both the cases with INE and legally-independent or merged firms can take place. Given that our focus is the effect of a non-consolidating merger relative to the case with INE and legally-independent firms, in the remainder of this section we will assume that Assumptions 2 and 3 hold, i.e., that we are in the parameter space defined as $S^{INE \cap INE-M}$ in Remark 2. When this is the case, as per Lemma 7, platforms always have an incentive to merge.

5.2.1 Platform incentives

Lemma 7 (**Merger dominance in presence of intertwined network effects**)**.** *Consider two platforms having set out intertwined network effects. If Assumptions 2 and 3 hold, merging is a dominant strategy for both. Formally,* ${\Pi^1}^{INE-M} + {\Pi^2}^{INE-M} > {\Pi^1}^{INE} + {\Pi^2}^{INE}$.

***Proof**: See Appendix A.6.*

Lemma 7 shows that platforms having set-up intertwined network effects always have incentives to merge. The reason is that this allows them to better internalize the cross-platform network effects by jointly maximising profit. This provides a rationale of why it is common to see INE between transaction platforms belonging to the same group.

5.2.2 Prices

Lemma 8 shows how the merger affects equilibrium prices.

Lemma 8 (**Merger effect on prices**)**.** *Consider two platforms having set-up intertwined network effects. After they merge, the price paid by sellers to the aggregator platform increases ($p_s^{1\,INE-M} > p_s^{1\,INE}$). In contrast, the prices paid by sellers to the source platform decrease ($p_s^{2\,INE-M} < p_s^{2\,INE}$).*

***Proof**: See Appendix A.7.*

Lemma 8 shows how the change in the inter-platform pricing structure brought about by the merger affects the prices charged to sellers. When the platforms are under common ownership, they maximize profit jointly. Hence, the referral fee f disappears. This lowers the marginal per-interaction cost of the source platform (platform 2) and hence the price it charges to sellers. Conversely, in absence of the referral fee, the aggregator (platform 1) loses a revenue stream that was allowing it to have lower prices, attract more sellers and increase its revenue through referral fees. Hence, post-merger, platform 1 increases the price charged to sellers.

5.2.3 Platform participation

Lemma 9 shows how the merger affects participation in equilibrium.

Lemma 9 (**Merger effect on platform participation**)**.** *Consider two platforms having set-up intertwined network effects. After they merge:*

i) The number of consumers decreases in the aggregator platform ($n_b^{1\,INE-M} < n_b^{1\,INE}$) and increases in the source platform ($n_b^{2\,INE-M} > n_b^{2\,INE}$)

ii) The number of sellers decreases in the aggregator platform ($n_s^{1\,INE-C0} < n_s^{1\,INE}$) and increases in the source platform ($n_s^{2\,INE-M} > n_s^{2\,INE}$)

iii) The total number of sellers always increases ($n_s^{INE-M} > n_s^{INE}$).

***Proof**: See Appendix A.8.*

The changes in platform participation induced by the merger steam directly from the change in prices analysed in Lemma 8. Note that, contrary to what happens when comparing the effect of INE on platform participation (cf. Lemma 5), the merger does not alter neither firms' nor consumers' utility functions. Hence, their changes in participation are only

explained by changes the changes prices shown in Lemma 8. In platform 1, the increase in the price charged to sellers drives some sellers to leave the platform, which in turn decreases consumers' participation. The opposite happens in platform 2.

Interestingly, the increase in seller participation in platform 2 is always stronger than the decrease in platform 1, leading to an more overall increase in seller participation led by more seller multihoming. This effect is analogous to the elimination of double marginalization effect in vertical mergers, which has been widely studied since Spengler (1950). Eliminating the referral fee results in a decrease in the price charged to sellers in the source platform (platform 2) that is stronger than the increase in the price charged to sellers in the aggregator platform (platform 1)[6]. This expands overall sellers' demand for interactions.

Consequently, the increase in the number of sellers platform 1 consumers can interact with (i.e., both platforms' sellers) is smaller than the increase in the number of sellers platform 1 consumers have access to. This drives some consumers switching from platform 1 to platform 2.

5.2.4 Participants' surpluses

To present shorter mathematical expressions, let me introduce the following notation.

$$\hat{\tau_b} \equiv 8\alpha_b \left(-4 + \frac{\alpha_b \left(8\tau_s + \alpha_b\right)}{\tau_s \left(4\tau_s - \alpha_b\right)} + \sqrt{\frac{256\tau_s^4 + 128\tau_s^3\alpha_b - 16\tau_s^2\alpha_b^2 + 24\tau_s\alpha_b^3 + \alpha_b^4}{\tau_s^2 \left(-4\tau_s + \alpha_b\right)^2}} \right)$$

$$\overline{\tau_b} \equiv \frac{1}{16} \left(\sqrt{\frac{\alpha_b^2 \left(256\tau_s^4 - 128\tau_s^3\alpha_s - 16\tau_s^2\alpha_s^2 + 40\tau_s\alpha_s^3 - 7\alpha_s^4\right)}{\tau_s^2 \left(-2\tau_s + \alpha_s\right)^2}} + \alpha_b \left(-8 + \frac{3\alpha_s}{\tau_s} - \frac{6\alpha_s}{-2\tau_s + \alpha_s} \right) \right)$$

Proposition 3 (**Merger effect on consumer surplus**). *Consider two platforms having set-up intertwined network effects. After they merge, consumer surplus decreases ($CS^{INE-M} < CS^{INE}$) if and only if $\tau_b < \overline{\tau_b}(\tau_s, \alpha_b, \alpha_s, v_b)$.*

***Proof**: See Appendix A.9.*

The intuition is as follows. *Ceteris paribus*, the increase in seller participation in platform 2 (which benefits platform 2 buyers) and overall (which benefits platform 1 buyers) brought about by the merger benefits consumers in both platforms. This increases consumer welfare. However, due to the presence of INE, platform 1 consumers can interact with more sellers than platform 2 consumers irregardless of the ownership structure of the platforms. Hence, consumers switching from platform 1 to platform 2 decreases consumer surplus. If transportation costs are sufficiently low, the merger harms consumers, and vice-versa.

[6] It can be shown that $(p_s^{1\,INE-M} - p_s^{1\,INE}) + (p_s^{2\,INE-M} - p_s^{2\,INE}) < 0$ under Assumption 3.

Proposition 4 (**Merger effect on seller surplus**). *Consider two platforms having set-up intertwined network effects. After they merge, if $\tau_b > \hat{\tau}_b$, seller surplus decreases.*

***Proof**: See Appendix A.10.*

The merger affects seller surplus through different channels. First, as per Lemma 9, total seller participation increases. This enlarges seller surplus. Second, in platform 2, the decrease in the price charged to sellers further increases seller surplus. Third, in platform 1, the increase in the price charged to sellers and the decrease in the number of consumers reduces seller surplus. Fourth, the migration of sellers from platform 1 to platform 2 mitigates this negative effect on seller surplus: the more sellers leave platform 1 for platform 2, the smaller the number of sellers affected by the drop in seller surplus in platform 1 will be. If consumers' transportation costs are sufficiently large, the only relevant effects have to do with the increase in the price to platform 1 sellers. As shown in (2) and (6), in equilibrium, the only price that depends on consumers' transportation cost is that of platform 1. The higher these are, the higher the referral fee that platform 1 could charge before the merger. Hence, the higher the increase in its prices will be post-merger to compensate for the loss of revenue from the referral fee.

6 Conclusion and discussion

This article studied how intertwined network effects influence platform participation, pricing, and user groups' welfare when platforms are independent and post-merger. It showed that while intertwined network effects increase consumer surplus, they reduce seller surplus if the platforms are sufficiently differentiated for sellers. Moreover, in presence of INE, a non-consolidating merger can harm consumers and benefit sellers if the platforms are sufficiently homogeneous to consumers.

These results extend the literature on interoperability between platforms to an asymmetric case referred to as "intertwined network effects" in this article, a direction that had only been explored by Maruyama and Zennyo (2015). As in the latter, when the network effects enjoyed by the multi-homing side of the market are sufficiently small, their surplus increases, and vice-versa. However, given this article's focus on platforms applying per-transaction pricing to the multi-homing side of the market, I find that asymmetric interoperability (i.e., INE) benefits the single-homing side. The effect of INE on the multi-homing user group, in turn, depends on a mechanism introduced in this article: the reallocation of the multi-homing user group across platforms. With sufficiently low seller transportation costs, enough sellers leave the source platform (where prices increase) for the aggregator platform (where prices decrease), leading to an increase in overall seller surplus, and vice-versa.

This article also extends the growing literature on platform mergers. It introduces per-transaction pricing to the multi-homing side of the market to the canonical bottleneck

setting to study a non-consolidating horizontal merger between platforms subject to INE. A novel mechanism through which a platform merger can harm users emerges from it: the reallocation of the single-homing user group across platforms. Despite the overall increase in the participation of the multihoming user group triggered by the merger, if platform differentiation is sufficiently strong, single-homing users fail to sufficiently switch from the aggregator platform (where the number of sellers decreases) to the source platform (where the number of sellers increases).

Future work might complement this article's results by extending the analysis of INE to other settings. How are results affected when there is multi-homing on both sides of the market? And when the multi-homing side is (also) charged a participation fee? One of the main limitations of this article's modelling is that it loses tractability when both sides of the market are charged a per-transaction price. It can therefore not provide insights in such cases, which are common in certain transaction platforms such as house rental platforms. Developing an alternative modelling of INE that allows to include per-transaction pricing to both sides of the market while preserving tractability might be a fruitful endeavour.

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A Proofs

A.1 Proof of Lemma 3

Subtracting the profits platforms 1 and 2 obtain in equilibrium with (cf. Lemma 2) and without (cf. Lemma 1) intertwined network effects yields the following simplified expressions, respectively.

$$\Pi^{1^{INE}} - \Pi^{1^{B}} = \frac{4\left(-\tau_b\tau_s + \sqrt{\tau_b\tau_s(\tau_b\tau_s - \alpha_b\alpha_s)}\right)\left(-\tau_b\tau_s + \alpha_b\alpha_s + \sqrt{\tau_b\tau_s(\tau_b\tau_s - \alpha_b\alpha_s)}\right)}{16\tau_s\alpha_b^2} + \frac{(4\tau_b\tau_s - \alpha_b\alpha_s)\left(-4\tau_b^2\tau_s^2 + \tau_b\tau_s\alpha_b\alpha_s + \alpha_b^2\alpha_s^2 + \sqrt{\tau_b\tau_s(16\tau_b^3\tau_s^3 - 8\tau_b^2\tau_s^2\alpha_b\alpha_s + 5\tau_b\tau_s\alpha_b^2\alpha_s^2 - 2\alpha_b^3\alpha_s^3)}\right)}{16\tau_s\alpha_b^2}$$

$$\Pi^{2^{INE}} - \Pi^{2^{B}} = \frac{(-2\tau_b\tau_s + \alpha_b\alpha_s + 2\sqrt{\tau_b\tau_s(\tau_b\tau_s - \alpha_b\alpha_s)})^2}{16\tau_s\alpha_b^2}$$

Given that all the parameters are positive by definition, the two expressions are non-negative. *Q.E.D.*

A.2 Proof of Lemma 4

Subtracting the prices platforms 1 and 2 charge sellers in equilibrium with (cf. Lemma 2) and without (cf. Lemma 1) intertwined network effects yields the following simplified expressions, respectively.

$$p_s^{1^{INE}} - p_s^{1^{B}} = -\frac{16\tau_b^3\tau_s^3 - 4\tau_b^2\tau_s^2\alpha_b\alpha_s + \alpha_b^2\alpha_s^2\sqrt{\tau_b\tau_s(\tau_b\tau_s - \alpha_b\alpha_s)}}{u^3\alpha_s^2} + \frac{\tau_b\tau_s(u^2\alpha_s^2 - 4\sqrt{\tau_b\tau_s(16\tau_b^3\tau_s^3 - 8\tau_b^2\tau_s^2\alpha_b\alpha_s + 5\tau_b\tau_s\alpha_b^2\alpha_s^2 - 2\alpha_b^3\alpha_s^3)})}{u^3\alpha_s^2}$$

$$p_s^{2^{INE}} - p_s^{2^{B}} = \frac{4\tau_b\tau_s - \alpha_b\alpha_s - 4\sqrt{\tau_b\tau_s(\tau_b\tau_s - \alpha_b\alpha_s)}}{4\alpha_b}$$

$p_s^{1^{INE}} - p_s^{1^{B}}$ is positive if and only if $\frac{\alpha_b\alpha_s}{\tau_s} < \tau_b < \tilde{\tau_b}(\alpha_b, \alpha_s, \tau_s)$ and negative otherwise, where $\tilde{\tau_b}$ is the second root of the following polynomial:

$$P(\tau_b) = -\alpha_b^4\alpha_s^4 + 80\tau_s^2\alpha_b^2\alpha_s^2\tau_b^2 - 384\tau_s^3\alpha_b\alpha_s\tau_b^3 + 256\tau_s^4\tau_b^4$$

The expression for $p_s^{2\,INE} - p_s^{2\,B}$, in turn, is always positive given that all the parameters are strictly positive. *Q.E.D.*

A.3 Proof of Lemma 5

Platform 1 buyers. Subtracting the equilibrium number of buyers in platform 1 with (cf. Lemma 2) and without (cf. Lemma 1) intertwined network effects yields the following simplified expression.

$$n_b^{1\,INE} - n_b^{1\,B} = \frac{\tau_b\tau_s(-4\tau_b\tau_s+3\alpha_b\alpha_s)+\sqrt{\tau_b\tau_s\left(16\tau_b^3\tau_s^3-8\tau_b^2\tau_s^2\alpha_b\alpha_s+5\tau_b\tau_s\alpha_b^2\alpha_s^2-2\alpha_b^3\alpha_s^3\right)}}{8\tau_b\tau_s(2\tau_b\tau_s-\alpha_b\alpha_s)}$$

This expression is non-negative when the second-order conditions with intertwined network effects (cf. Assumption 2), i.e., $\alpha_b\alpha_s < 2\tau_b\tau_s$.

Platform 2 buyers. Subtracting the equilibrium number of buyers in platform 2 with (cf. Lemma 2) and without (cf. Lemma 1) intertwined network effects yields the following simplified expression.

$$n_b^{2\,INE} - n_b^{2\,B} = \frac{-2\tau_b\tau_s+\alpha_b\alpha_s+2\sqrt{\tau_b\tau_s(\tau_b\tau_s-\alpha_b\alpha_s)}}{4\tau_s\alpha_b}$$

Given that all the parameters are strictly positive by definition, this expression cannot be positive.

Platform 1 sellers. Subtracting the equilibrium number of sellers in platform 1 with (cf. Lemma 2) and without (cf. Lemma 1) intertwined network effects yields the following simplified expression.

$$n_s^{1\,INE} - n_s^{1\,B} = \frac{2\sqrt{\tau_b\tau_s(\tau_b\tau_s-\alpha_b\alpha_s)}+\frac{\tau_b\tau_s(-8\tau_b\tau_s+5\alpha_b\alpha_s)+\sqrt{\tau_b\tau_s(16\tau_b^3\tau_s^3-8\tau_b^2\tau_s^2\alpha_b\alpha_s+5\tau_b\tau_s\alpha_b^2\alpha_s^2-2\alpha_b^3\alpha_s^3)}}{2\tau_b\tau_s-\alpha_b\alpha_s}}{4\tau_s\alpha_b}$$

Given that all the parameters are strictly positive by definition, this expression is non-negative.

Platform 2 sellers. Subtracting the equilibrium number of sellers in platform 2 with (cf. Lemma 2) and without (cf. Lemma 1) intertwined network effects yields the following simplified expression.

Given that all the parameters are strictly positive by definition, this expression cannot be positive.

$$n_s^{2\,INE} - n_s^{2\,B} = \frac{-2\tau_b\tau_s+\alpha_b\alpha_s+2\sqrt{\tau_b\tau_s(\tau_b\tau_s-\alpha_b\alpha_s)}}{4\tau_s\alpha_b}$$

Platforms 1 and 2 sellers. Subtracting the equilibrium number of sellers present in both platforms with (cf. Lemma 2) and without (cf. Lemma 1) intertwined network effects yields the following simplified expression.

$$n_s{}^{INE} - n_s{}^{B} = \frac{\alpha_s+\frac{4(-\tau_b\tau_s+\sqrt{\tau_b\tau_s(\tau_b\tau_s-\alpha_b\alpha_s)})}{\alpha_b}+\frac{\tau_b\tau_s(-4\tau_b\tau_s+3\alpha_b\alpha_s)+\sqrt{\tau_b\tau_s(16\tau_b^3\tau_s^3-8\tau_b^2\tau_s^2\alpha_b\alpha_s+5\tau_b\tau_s\alpha_b^2\alpha_s^2-2\alpha_b^3\alpha_s^3)}}{\alpha_b(2\tau_b\tau_s-\alpha_b\alpha_s)}}{4\tau_s}$$

Bearing in mind that all the parameters are strictly positive by definition, ${n_s}^{INE} - {n_s}^B < 0 \iff \frac{\alpha_b \alpha_s}{\tau_s} \leq \tau_b < \frac{(3+2\sqrt{3})\alpha_b \alpha_s}{6\tau_s}$ and ${n_s}^{INE} - {n_s}^B > 0 \iff \tau_b > \frac{(3+2\sqrt{3})\alpha_b \alpha_s}{6\tau_s}$.

A.4 Proof of Proposition 1

Denote Δn_i^l the absolute value of the variation in participation of user group $i = b, s$ in platform $l = 1, 2$ after the introduction of intertwined network effects (INE), compared to the non-INE benchmark. Given the uniform distribution of the mass 1 total number of consumers, the absolute change in consumer surplus generated by INE in platforms 1 (denoted ΔCS^1) and 2 (denoted ΔCS^2) can be expressed as:

$$\Delta CS^1 = (\frac{1}{2} + \Delta n_b^1)({n_s^1}^B + \Delta n_s^1 + {n_s^2}^B - \Delta n_s^2)$$
$$\Delta CS^2 = (\frac{1}{2} - \Delta n_b^2)\Delta n_s^2$$

Where the positive or negative signs are given by Lemma 5. The net effect of INE on consumer surplus on both platforms $\Delta CS \equiv \Delta CS^1 + \Delta CS^2$ is hence:

$$\Delta CS = \Delta \frac{1}{2}{n_s^1}^B + \frac{1}{2}\Delta n_s^1 + \frac{1}{2}{n_s^2}^B + \Delta n_b^1 {n_s^1}^B + \Delta n_b^1 \Delta n_s^1 + \Delta n_b^1 {n_s^2}^B - (\Delta n_b^1 + \Delta n_b^2)\Delta n_s^2$$

Denote $\Delta n_b^1 \equiv \Delta n_b$. Given that the market is covered on the consumer side, $\Delta n_b = -\Delta n_b^2$, and hence $-\Delta n_b^2 \equiv -\Delta n_b$. Then,

$$\Delta CS = \left(\frac{1}{2} + \Delta n_b\right){n_s^1}^B + \left(\frac{1}{2} + \Delta n\right)\Delta n_s^1 + \left(\frac{1}{2} + \Delta n_b\right){n_s^2}^B > 0.$$

Q.E.D.

A.5 Proof of Proposition 2

Subtracting seller surplus equilibrium with (cf. Lemma 2) and without (cf. Lemma 1) intertwined network effects yields the following simplified expression.

$$\begin{aligned}
SS^{INE} - SS^B = \frac{1}{32}\Bigg(&8\tau_s\left(-2 + \frac{\tau_b^2}{\alpha_b^2}\right) - \frac{3\pi_s^2}{\tau_s} - \frac{8}{\alpha_b^2}\Bigg(- 2\tau_b^2\tau_s \\
&- 2\alpha_b\left(\tau_s\alpha_b + 2\sqrt{\tau_b\tau_s(\tau_b\tau_s - \alpha_b\pi_s)}\right) + \tau_b\left(4\tau_s\alpha_b + \alpha_b\pi_s + 2\sqrt{\tau_b\tau_s(\tau_b\tau_s - \alpha_b\pi_s)}\right)\Bigg) \\
&+ \frac{1}{\alpha_b^2(-2\tau_b\tau_s + \alpha_b\pi_s)^2}\Bigg(16\alpha_b^4\pi_s^3 + 2\tau_b^2\tau_s\Big(\alpha_b^2\pi_s\left(32\tau_s + 3\pi_s\right) \\
&- 4\sqrt{\tau_b\tau_s\left(16\tau_b^3\tau_s^3 - 8\tau_b^2\tau_s^2\alpha_b\pi_s + 5\tau_b\tau_s\alpha_b^2\pi_s^2 - 2\alpha_b^3\pi_s^3\right)} \\
&+ 2\tau_b\alpha_b\pi_s\Big(- \alpha_b^2\pi_s\left(32\tau_s + \pi_s\right) + 3\sqrt{\tau_b\tau_s\left(16\tau_b^3\tau_s^3 - 8\tau_b^2\tau_s^2\alpha_b\pi_s + 5\tau_b\tau_s\alpha_b^2\pi_s^2 - 2\alpha_b^3\pi_s^3\right)}\Big)\Bigg)\Bigg)\Bigg)
\end{aligned}$$

To simplify the mathematical expressions, and without loss of generality in the proof[7], denote $R^n_{(d,r)}(x)$ a function describing the r^{th} root a polynomial of degree $d \in \mathbb{N}$ in variable x. Superscript $n = [I, II, ...]$ identifies the equation describing the polynomial. Roots are ordered in increasing order.

Bearing in mind that all the parameters are strictly positive by definition, $SS^{INE} - SS^B < 0$ if and only if one of the following sets of conditions, labelled as $C^{SS-}_{i\in\mathbb{N}}$ are met:

$$C^{SS-}_1 = \left\{ \frac{5\alpha_s}{8} < \tau_s \leq \frac{17\alpha_s}{16} - \frac{1}{8}\sqrt{11}\alpha_s \right\} \wedge \left\{ \tau_b > R^I_{(9,3)}\left(\tau_b\alpha_b, \alpha_s, \tau_s\right) \right\}$$

$$C^{SS-}_2 = \left\{ \frac{17\alpha_s}{16} - \frac{1}{8}\sqrt{11}\alpha_s < \tau_s < R^{II}_{(4,2)}\left(\tau_s(\alpha_s)\right) \right\} \wedge \left\{ \tau_b \geq \frac{\alpha_s\alpha_b}{\tau_s}) \right\}$$

$$C^{SS-}_3 = \left\{ R^{II}_{(4,2)}(\tau_s(\alpha_s)) \leq \tau_s < R^{III}_{(16,3)}(\tau_b(\alpha_b, \alpha_s, \tau_s)) \right\} \wedge \left\{ \left(\frac{\alpha_s\alpha_s}{\tau_s} \leq \tau_b < R^I_{(9,1)}\left(\tau_b\alpha_b, \alpha_s, \tau_s\right) \right) \vee \left(\tau_b > 2\alpha_s\right) \right\}$$

$$C^{SS-}_4 = \left\{ \tau_s = R^{III}_{(16,3)}\left(\alpha_s\right) \right\} \wedge \left\{ \left(\frac{\alpha_s\alpha_s}{\tau_s} \leq \tau_b < R^I_{(9,1)}\left(\tau_b(\alpha_b, \alpha_s, \tau_s)\right) \right) \right\}$$

$$C^{SS-}_5 = \left\{ \tau_s > R^{III}_{(16,3)}\left(\alpha_s\right) \right\} \wedge \left\{ \tau_b \geq \frac{\alpha_s\alpha_s}{\tau_s} \right\}$$

In the same vein, $SS^{INE} - SS^B > 0$ if and only if one of the following sets of conditions, labelled as $C^{SS+}_{i\in\mathbb{N}}$ are met:

$$C^{SS+}_1 = \left\{ 0 < \tau_s \leq \frac{5\alpha_s}{8} \right\} \wedge \left\{ \tau_b \geq \frac{\alpha_s\alpha_s}{\tau_s} \right\}$$

$$C^{SS+}_2 = \left\{ \frac{5\alpha_s}{8} < \tau_s \leq \frac{17\alpha_s}{16} - \frac{1}{8}\sqrt{11}\sqrt{\pi^2_s} \right\} \wedge \left\{ \frac{\alpha_b\alpha_s}{\tau_s} \leq \tau_b < R^I_{(9,1)}\left(\tau_b(\alpha_b, \alpha_s, \tau_s)\right) \right\}$$

[7]The full expressions are available upon request.

$$C_3^{SS+} = \left\{ \frac{17\alpha_s}{16} - \frac{1}{8}\sqrt{11}\sqrt{\pi_s^2} < \tau_s < R_{(4,2)}^{II} \right\} \wedge \left\{ R_{(9,1)}^{I}(\tau_b\alpha_b, \alpha_s, \tau_s)) < \tau_b < R_{(9,3)}^{I}(\tau_b\alpha_b, \alpha_s, \tau_s)) \right\}$$

$$C_4^{SS+} = \left\{ \tau_s = R_{(4,2)}^{II} \right\} \wedge \left\{ R_{(9,1)}^{I}(\tau_b\alpha_b, \alpha_s, \tau_s)) < \tau_b < 2\alpha_b \right\}$$

$$C_5^{SS+} = \left\{ R_{(4,2)}^{II} < \tau_s < R_{(16,3)}^{III}(\tau_s, \alpha_b, \alpha_s, u) \right\} \wedge \left\{ R_{(9,1)}^{I}(\tau_b\alpha_b, \alpha_s, \tau_s)) < \tau_b < R_{(9,2)}^{I}(\tau_b\alpha_b, \alpha_s, \tau_s)) \right\}$$

Note that none of the C^{SS-} sets of conditions imply $\tau_s \leq \frac{5\alpha_s}{8}$. Moreover, condition C_1^{SS+} shows that $\tau_s \leq \frac{5\alpha_s}{8}$ is a sufficient condition for seller surplus to decrease, as the second part of the condition, $\tau_b \geq \frac{\alpha_s\alpha_b}{\tau_s}$, corresponds to the second order conditions in the pre-INE benchmark reported in Assumption 1. This proves point i) of Proposition 2.

By comparing the C^{SS-} sets of conditions, we can see that they all imply $\tau_s > \tilde{\tau}_s(\alpha_s) > \frac{5}{8}\alpha_s$. This proves point ii) of Proposition 2.

Q.E.D.

A.6 Proof of Lemma 7

Denote $\Pi^{INE-M} \equiv \Pi^{1\,INE-M} + \Pi^{2\,INE-M}$ and $\Pi^{INE} \equiv \Pi^{1\,INE} + \Pi^{2\,INE}$. Subtracting Π^{INE} from Π^{INE-M} and simplifying yields:

$$\Pi^{INE-M} - \Pi^{INE} = \frac{\left(-2\tau_b\tau_s + \alpha_b\alpha_s + 2\sqrt{\tau_b\tau_s\left(\tau_b\tau_s - \alpha_b\alpha_s\right)}\right)^2}{8\tau_s\alpha_b^2} > 0$$

Q.E.D.

A.7 Proof of Lemma 8

Subtracting the equilibrium price charged to sellers in platform 1 when the platforms are legally-independent (cf. Lemma 2) from the post-merger equilibrium price charged to sellers

in platform 1 (cf. Lemma 6) and simplifying yields:

$$p_s^{1\,INE-M} - p_s^{1\,INE} = 2\tau_b\tau_s \left(\frac{1}{\alpha_b} - \frac{2\tau_b\tau_s}{\alpha_b^2\alpha_s} + \frac{\alpha_s}{-4\tau_b\tau_s + \alpha_b\alpha_s} - \frac{2\left(-4\tau_b^2\tau_s^2 + \sqrt{\tau_b\tau_s\left(16\tau_b^3\tau_s^3 - 8\tau_b^2\tau_s^2\alpha_b\alpha_s + 5\tau_b\tau_s\alpha_b^2\alpha_s^2 - 2\alpha_b^3\alpha_s^3\right)}\right)}{\alpha_b^3\alpha_s^2} \right)$$

Which is strictly positive under the second order conditions of the merger case set out in Assumption 3, $\alpha_b\alpha_s < 2\tau_b\tau_s$

Subtracting the equilibrium price charged to sellers in platform 2 with INE (cf. Lemma 2) from the equilibrium price charged to sellers in platform 2 post-merger (cf. Lemma 6) yields:

$$p_s^{2\,INE-M} - p_s^{2\,INE} = -\frac{\alpha_s}{4} < 0.$$

Q.E.D.

A.8 Proof of Lemma 9

Platform 1 consumers' participation. Subtracting the equilibrium participation on the consumer side in platform 1 with INE (cf. Lemma 2) from the post-merger one (cf. Lemma 6) and simplifying yields:

$$n_b^{1\,INE-M} - n_b^{1\,INE} = \frac{4\tau_b^2\tau_s^2 + \tau_b\tau_s\alpha_b\alpha_s - \alpha_b^2\alpha_s^2 - \sqrt{\tau_b\tau_s(16\tau_b^3\tau_s^3 - 8\tau_b^2\tau_s^2\alpha_b\alpha_s + 5\tau_b\tau_s\alpha_b^2\alpha_s^2 - 2\alpha_b^3\alpha_s^3)}}{8\tau_b\tau_s^2\alpha_b - 4\tau_s\alpha_b^2\alpha_s}$$

Which is strictly negative given that all the parameters are strictly positive by definition.

Platform 2 consumers' participation. Subtracting the equilibrium participation on the consumer side in platform 2 with INE (cf. Lemma 2) from the post-merger one (cf. Lemma 6) and simplifying yields:

$$n_b^{2\,INE-M} - n_b^{2\,INE} = \frac{\tau_b\tau_s(-4\tau_b\tau_s + \alpha_b\alpha_s) + \sqrt{\tau_b\tau_s(16\tau_b^3\tau_s^3 - 8\tau_b^2\tau_s^2\alpha_b\alpha_s + 5\tau_b\tau_s\alpha_b^2\alpha_s^2 - 2\alpha_b^3\alpha_s^3)}}{8\tau_b\tau_s(2\tau_b\tau_s - \alpha_b\alpha_s)}$$

Which is strictly positive given that all the parameters are strictly positive by definition.

Platform 1 sellers' participation. Subtracting the equilibrium participation on the seller side in platform 1 when the platforms are legally independent (cf. Lemma 2) from the post-merger one (cf. Lemma 6) and simplifying yields:

$$n_s^{1\,INE-M} - n_s^{1\,INE} = \frac{4\tau_b^2\tau_s^2 - \tau_b\tau_s\alpha_b\alpha_s - \sqrt{\tau_b\tau_s(16\tau_b^3\tau_s^3 - 8\tau_b^2\tau_s^2\alpha_b\alpha_s + 5\tau_b\tau_s\alpha_b^2\alpha_s^2 - 2\alpha_b^3\alpha_s^3)}}{8\tau_b\tau_s^2\alpha_b - 4\tau_s\alpha_b^2\alpha_s}$$

Which is strictly negative given that all the parameters are strictly positive by definition.

Platform 2 sellers' participation. Subtracting the equilibrium participation on the seller side in platform 2 when the platforms are legally independent (cf. Lemma 2) from the post-merger one (cf. Lemma 6) and simplifying yields:

$$n_s^{2\,INE-M} - n_s^{2\,INE} = \frac{\alpha_s}{4\tau_s} > 0$$

Overall sellers' participation. Subtracting the equilibrium participation on the seller side in both platforms when the platforms are legally-independent (cf. Lemma 2) from the post-merger one (cf. Lemma 6) and simplifying yields:

$$n_s{}^{INE-M} - n_s{}^{INE} = \frac{-4\tau_b^2\tau_s^2 + \tau_b\tau_s\alpha_b\alpha_s + \sqrt{\tau_b\tau_s(16\tau_b^3\tau_s^3 - 8\tau_b^2\tau_s^2\alpha_b\alpha_s + 5\tau_b\tau_s\alpha_b^2\alpha_s^2 - 2\alpha_b^3\alpha_s^3)}}{8\tau_b\tau_s^2\alpha_b - 4\tau_s\alpha_b^2\alpha_s}$$

Which is strictly negative given that all the parameters are strictly positive by definition.

Q.E.D.

A.9 Proof of Proposition 3

The parameter space in which $CS^{INE-M} < CS^{INE}$ and Assumptions 2 and 3 hold simultaneously is given by:

$$CS^{INE-M} < CS^{INE} \cap Assumption\ 2 \cap Assumption\ 3 = \alpha_b > 0 \wedge 2\tau_s > \alpha_s \wedge \alpha_s > 0 \wedge$$
$$\underline{v_b} \equiv \Bigg(\sqrt{\frac{(\tau_b(16\tau_b\tau_s - 5\alpha_b\alpha_s)^2(16\tau_b^3\tau_s^3 - 8\tau_b^2\tau_s^2\alpha_b\alpha_s + 5\tau_b\tau_s\alpha_b^2\alpha_s^2 - 2\alpha_b^3\alpha_s^3))}{\tau_s\alpha_b^2\alpha_s^2(-2\tau_b\tau_s + \alpha_b\alpha_s)^2}} +$$
$$\left(2 + \frac{32\tau_b\tau_s}{\alpha_b\alpha_s} + \frac{\alpha_b\alpha_s}{2\tau_b\tau_s - \alpha_b\alpha_s}\right) - \frac{4\alpha_b\alpha_s}{\tau_s}\Bigg)/16 < v_b + \wedge$$
$$\underline{\tau_b} \equiv \frac{1}{8}\left(\sqrt{\frac{\alpha_b^2(256\tau_s^4 + 128\tau_s^3\alpha_s - 16\tau_s^2\alpha_s^2 + 24\tau_s\alpha_s^3 + \alpha_s^4)}{\tau_s^2(-4\tau_s + \alpha_s)^2}} + \right.$$
$$\left.\alpha_b\left(-4 + \frac{\alpha_s(8\tau_s + \alpha_s)}{\tau_s(4\tau_s - \alpha_s)}\right)\right) < \tau_b <$$
$$\frac{1}{16}\left(\sqrt{\frac{\alpha_b^2(256\tau_s^4 - 128\tau_s^3\alpha_s - 16\tau_s^2\alpha_s^2 + 40\tau_s\alpha_s^3 - 7\alpha_s^4)}{\tau_s^2(-2\tau_s + \alpha_s)^2}} + \right.$$
$$\left.\alpha_b\left(-8 + \frac{3\alpha_s}{\tau_s} - \frac{6\alpha_s}{-2\tau_s + \alpha_s}\right)\right)$$

Note that $CS^{INE-M} > CS^{INE} \cap Assumption\ 2 \cap Assumption\ 3 \cap \tau_b < \underline{\tau_b} = \varnothing$ and that $CS^{INE-M} > CS^{INE} \cap Assumption\ 2 \cap Assumption\ 3 \cap \tau_b < \underline{\tau_b} \neq \varnothing$. Hence, $\tau_b > \underline{\tau_b}$ is a necessary condition for both $CS^{INE-M} > CS^{INE}$ and $CS^{INE-M} < CS^{INE}$ but $\tau_b < \overline{\tau_b}$ is only a necessary condition for $CS^{INE-M} < CS^{INE}$.

Furthermore,

$$CS^{INE-M} < CS^{INE} \cap Assumption\ 2 \cap Assumption\ 3 \cap v_b > \underline{v_b} =$$
$$\alpha_s > 0 \quad \wedge \quad \tau_s > \frac{\alpha_s}{2} \quad \wedge \quad \alpha_b > 0 \quad \wedge \underline{\tau_b} < \tau_b < \overline{\tau_b} \wedge$$
$$v_b > -\frac{64\tau_b^3\tau_s^3 - 28\tau_b^2\tau_s^2\alpha_b\alpha_s - 9\tau_b\tau_s\alpha_b^2\alpha_s^2 + 4\alpha_b^3\alpha_s^3}{16\tau_s\alpha_b\alpha_s(-2\tau_b\tau_s + \alpha_b\alpha_s)} +$$
$$\frac{1}{16}\sqrt{\frac{4096\tau_b^6\tau_s^5 - 4608\tau_b^5\tau_s^4\alpha_b\alpha_s + 2960\tau_b^4\tau_s^3\alpha_b^2\alpha_s^2 - 1512\tau_b^3\tau_s^2\alpha_b^3\alpha_s^3 + 445\tau_b^2\tau_s\alpha_b^4\alpha_s^4 - 50\tau_b\alpha_b^5\alpha_s^5}{\tau_s\alpha_b^2\alpha_s^2(2\tau_b\tau_s - \alpha_b\alpha_s)^2}}$$

Hence, $CS^{INE-M} < CS^{INE} \cap Assumption\ 2 \cap Assumption\ 3 \cap v_b > \underline{v_b} \implies \underline{\tau_b} < \tau_b < \overline{\tau_b}$. *Q.E.D.*

Thus, $CS^{INE-M} < CS^{INE} \cap Assumption\ 2 \cap Assumption\ 3 \iff \tau_b < \overline{\tau_b}$.

A.10 Proof of Proposition 4

By comparing the region in which $SS^{INE-M} < SS^{INE} \cap Assumption\ 2 \cap Assumption\ 3$ holds with that in which $SS^{INE-M} > SS^{INE} \cap Assumption\ 2 \cap Assumption\ 3$ holds, we see that

$$SS^{INE-M} > SS^{INE} \cap Assumption\ 2 \cap Assumption\ 3 \cap \tau_b > \hat{\tau_b} = \varnothing$$
$$SS^{INE-M} < SS^{INE} \cap Assumption\ 2 \cap Assumption\ 3 \cap \tau_b > \hat{\tau_b} \neq \varnothing$$

Therefore, $\tau_b > \hat{\tau_b} \implies SS^{INE-M} < SS^{INE}$.