The Expected Returns on Machine-Learning Strategies ⋆

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Abstract

We estimate the expected returns of machine learning-based anomaly trading strategies, accounting for three factors often overlooked in the previous literature: transaction costs, post-publication decay, and the post-decimalization era of high liquidity. Despite a cumulative performance reduction averaging about 50% when accounting for these three factors, we find that sophisticated machine learning strategies, particularly those employing Long Short-Term Memory (LSTM) models, remain profitable. We estimate that our most effective strategy, the one based on an LSTM model with one hidden layer, has an expected Sharpe Ratio of 0.97. Our findings contrast with previous literature suggesting that machine learning strategies are unprofitable after accounting for economic constraints, and demonstrate persistent return predictability that cannot be explained by common risk factors or limits to arbitrage.

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JEL classifications: G11, G12, G14, C45, C58.

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1. Introduction

A growing body of literature in finance documents the remarkable ability of machine learning techniques to enhance predictability in the cross-section of stock returns.[1] Studies that use these techniques to extract expected return signals routinely report annualized Sharpe ratios on trading strategies employing these signals in excess of 1.0, with extreme examples exceeding 2.0 (e.g., Freyberger et al., 2020; Chen et al., 2023; Cong et al., 2020), performance that corresponds to about five times the historical market Sharpe ratio of 0.43, estimated over the entire CRSP sample from 1925-2021.

Despite this impressive paper performance, the extent to which these strategies can be implemented in practice remains an ongoing debate. Avramov et al. (2022) argue that trading strategies based on machine learning models extract profitability from difficult-to-arbitrage stocks and during high limits-to-arbitrage market states, and their performance deteriorates in the presence of economic constraints because of high turnover. Blitz et al. (2023) advocate using longer prediction horizons to train the machine learning models and show that those can improve performance even after accounting for 25-basis-points-per-trade transaction costs. Jensen et al. (2022) go further and develop a framework that integrates trading-cost-aware portfolio optimization with machine learning.

Complicating the matters, simply accounting for transaction costs still does not answer the question of what the expected returns on machine learning strategies are and whether we can expect to see similar performance in the future. This question is more subtle, because many anomalies were not discovered for significant periods of the samples in which they are typically used for backtesting machine learning strategies. Thus, even if we make the optimistic assumption that the machine learning techniques were available for investors, the anomaly signals were not. Moreover, individual anomaly performance deteriorates post-publication (Mclean and Pontiff, 2016) and has further deteriorated in the more recent sample post-decimalization due to the new era of investment and trading technology (Chordia et al., 2014).

The issue of using pre-publication signals is further exacerbated because combining anomalies that are known to work well in-sample suffers from severe overfitting bias. Novy-Marx (2016) demonstrates that when anomalies that perform well in-sample are combined, the resulting strategy's backtested performance can be significantly overstated, as the individual signals may

[1] See, e.g., Gu et al. (2020); Leippold et al. (2022); Hanauer and Kalsbach (2023); Azevedo and Hoegner (2023); Chen et al. (2023); Azevedo et al. (2023); Cakici et al. (2023).

have worked solely due to chance. This issue is emphasized by Li et al. (2022) in the context of machine learning. They construct real-time machine learning strategies based on a universe of data-mined fundamental signals, and find significantly weaker performance compared to strategies that use anomaly signals from published studies.

In this study, we aim to quantify the effects of each of these issues in backtesting machine-learning based trading strategies discussed above and to arrive at a more realistic estimate of their expected returns. As a benchmark, we start with backtests that resemble what papers in the literature typically report. That is, we test nine different machine learning strategies that combine up to 320 anomaly signals from the Chen and Zimmermann (2022) dataset using out-of-sample backtesting period starting in 2000 and ignoring trading costs.[2] To arrive at our estimate of expected returns, we then sequentially 1) restrict the sample to more recent periods that more closely resemble current liquidity conditions, 2) restrict the anomaly signals to only published ones, and 3) subtract trading costs.

We find that the performance on the expected returns on machine learning strategies deteriorates significantly when compared to typical backtests. On average, the performance across all models declines by approximately 50% relative to that of a typical backtest. However, this reduction varies considerably among different models, ranging from a modest 24% decrease for the best-performing LSTM1 model to a substantial 80% decline for the FFNN5 model. After accounting for post-publication effects, the post-2005 sample period, and trading costs, most machine learning strategies yield Sharpe ratios that fail to surpass the market Sharpe ratio over the same timeframe. The impact on average monthly returns is also stark: what initially appeared as promising returns of 1-2% per month in typical backtests diminishes to a more modest range of 0.3-1.4% per month under more realistic investment conditions.

By far the biggest impact on performance comes from restricting the sample to post-2005, which due to decimalization and the advent of high-frequency trading is characterized by significantly better liquidity, lower trading costs, and lower anomaly strategy profitability. Averaging across the nine machine learning strategies, restricting the sample to post-2005 results in approximately 30% Sharpe ratio reduction. In terms of average returns, the typical machine-

[2]The machine learning techniques we use include Ordinary Least Square with Huber Loss Function (OLS-HUBER), an Elastic Net (ENET), which combines a Least Absolute Shrinkage and Selection Operator (LASSO) and ridge regression, Feedforward Neural Network (FFNN) with two to five hidden layers (FFNN2, FFNN3, FFNN4, and FFNN5), two variations on Long Short-Term Memory (LSTM) with one and two hidden layers (LSTM1 and LSTM2), and an ensemble model.

learning strategy loses about 39 basis points per month post-2005. Restricting anomalies to only published ones and trading costs cumulatively add another 47 basis points per month decline in the average machine learning strategy profitability relative to the benchmark case. The breakdown between the two effects is about 23 basis points per month decline due to the post-publication restriction and about 24 basis points per month due to trading costs.

Despite their subdued performance under this more realistic scenario, the machine learning based strategies cannot be subsumed by standard risk factors. While their expected returns and Sharpe ratios are in line with the market, all machine learning-based strategies exhibit positive generalized alphas, as defined by Novy-Marx and Velikov (2016), with six out of nine models showing statistically significant results. The LSTM1 model demonstrates the most impressive performance, achieving a generalized Fama and French (2018) six-factor alpha of 1.20% per month, with a t-statistic of 3.46. To put this in perspective, consider an investor who has access to the *net* returns of the Fama and French (2018) six factors. If this investor were also given access to the *net* returns of the LSTM1 strategy, the Sharpe ratio of their ex-post mean-variance efficient portfolio would increase substantially from 1.15 to 1.45. This improvement underscores the potential of sophisticated machine learning strategies, particularly LSTM models, to enhance investment performance beyond what can be achieved with traditional factor models alone.

We find that the Long Short-Term Memory (LSTM) models consistently outperform the other machine learning models. Their superior performance is particularly noteworthy during the Great Recession, a period when many well-known anomalies (such as momentum) and other machine-learning models typically underperform. The LSTM models managed to avoid significant losses during this time, contributing substantially to their overall impressive performance. Furthermore, we document that the LSTM models exhibit remarkably low correlation with other machine learning signals, implying that they capture unique patterns in the data. To rigorously evaluate the LSTM1 signal, we subject it to the comprehensive "Assaying Anomalies" protocol developed by Novy-Marx and Velikov (2023). Our signal demonstrated exceptional performance across nearly all dimensions of this stringent evaluation framework, further validating its robustness and efficacy in predicting stock returns.

In the final section of our study, we investigate how cost-mitigation techniques and economic constraints affect the expected returns of machine learning strategies. We draw on previous research by Avramov et al. (2022), who examined economic constraints such as excluding the smallest 20% of stocks by market cap or filtering out the 30% of stocks with the highest

recent trading costs. We also consider transaction cost mitigation techniques explored by Novy-Marx and Velikov (2016, 2019), including extending holding periods up to four months and implementing a Buy-Hold-Spread (BHS) approach, which involves buying stocks in the top/bottom decile but only selling when they fall out of the top/bottom quintile.

Our results show that these mitigation techniques significantly reduce both turnover and transaction costs. However, in most cases, these benefits are offset by a corresponding decrease in gross average returns. The only technique that consistently improves net performance across all machine learning strategies is extending the holding period to two months, but even this improvement is modest, averaging only five basis points. These findings suggest that while cost-mitigation strategies can effectively reduce trading costs, they may also diminish the strategies' ability to capture short-term price movements, resulting in minimal net benefit for most machine learning approaches in the current market environment.

Our paper contributes to a fast growing literature on using machine learning in asset pricing settings. Many papers demonstrate the impressive predictive power of machine learning signals in the cross-section of U.S. stock returns. For example, Freyberger et al. (2020) use adaptive group LASSO, Gu et al. (2020) survey and apply many machine learning techniques including elastic net, dimension reduction techniques (PCR and PLS), trees, and neural networks. Cong et al. (2020) apply a deep reinforcement learning model. Simon et al. (2022) use neural networks to optimize portfolio weights as a function of firm characteristics. Chen et al. (2023) apply both feedforward and recurrent neural networks with long short-term memory.

Our main contribution relative to all these studies is our focus on the expected returns of machine learning strategies through careful treatment of trading costs, post-publication decay, and the staleness of historical data. While most of the studies cited above attempt to address the issue of implementability, they do so indirectly through economic constraints in the spirit of Avramov et al. (2022) such as size or turnover cutoffs or using crude trading costs measures. For example, Freyberger et al. (2020) use the Brandt et al. (2009) trading costs imputation based on size and Blitz et al. (2023) use a flat 25 basis points per trade assumption. The Chen and Velikov (2023) effective bid-ask spread measure we employ presents a more realistic estimate of trading costs post-decimalization since it is based on high-frequency TAQ data. To the best of our knowledge, we are also the first to estimate the Novy-Marx and Velikov (2016) generalized alphas for machine learning strategies. Integrating these generalized alphas into machine learning approaches enables a more precise comparison and understanding of

risk-adjusted net returns. This method directly addresses the frequently overlooked impact of trading costs in asset pricing models, a factor often neglected in studies that use gross return asset pricing models to explain anomalies in net returns.

Furthermore, our study demonstrates that machine learning strategies can be profitable, even in the recent era of high liquidity and when using only discovered anomalies. This is in stark contrast to the conclusions in Avramov et al. (2022). While it is true that machine learning strategies concentrate on historically difficult-to-arbitrage stocks, value-weighting stocks in the portfolios and the sharp decline in trading costs over the past couple of decades combine to result in significant profits for these strategies.

Our study is also related to recent papers that use machine learning techniques to construct improved factor models (Feng et al., 2023), show that technical analysis works, though its profitability decreases through time (Brogaard and Zareei, 2023), explain the post-earnings announcement drift (Hansen and Siggaard, 2023; Meursault et al., 2023), measure firm complexity (Loughran and McDonald, 2023), and uncover sparsity and heterogeneity in firm-level return predictability (Evgeniou et al., 2023).

Finally, we also add to the debate on the virtue of complexity. Kelly et al. (2023) establish the rationale for using machine learning to model expected returns and theoretically show that "complex" models should outperform "simple" models. Consistent with their findings, our strongest results are obtained using the most sophisticated machine learning models - the LSTM. All of our machine learning strategies are stronger compared to the ones in Chen and Velikov (2023), who find that using simpler combination techniques results in measly expected returns for strategies based on sorts of individual stock return predictors.

2. Methodology and Data

This section describes our data and methodology.

2.1. Data Sources, Samples and Pre-Processing

Our anomaly data come from Chen and Zimmermann (2022), who provide the most comprehensive dataset of replicated anomalies.[3] We use the March 2022 version of their signals. Motivated by Kelly et al. (2023), we download all 320 anomalies to ensure we provide our machine learning signals with the largest set of characteristics possible.

[3]For a detailed description of the methodology and anomaly composition, as well as the corresponding code and dataset, see their website: https://www.openassetpricing.com

We follow common practice and include only common equity stocks (CRSP share code 10 or 11). Our sample is from March 1957 to December 2021, totaling ~3.4mn stock-month observations over nearly 65 years. The anomaly signals have varying ranges of values over which they are defined, making it more difficult for neural networks to estimate suitable parameters during training (Singh and Singh, 2020). Consequently, we follow the current literature by percent-ranking all anomaly features into the same range [-1;1] (Kelly et al., 2019; Freyberger et al., 2020; Gu et al., 2020). Missing values are replaced with 0.

We enrich the anomaly dataset with further relevant data points following Gu et al. (2020) and include eight key macroeconomic predictors of Welch and Goyal (2008), namely dividend-price ratio, earnings-price ratio, book-to-market ratio, net equity expansion, treasury-bill rate, term-spread, default spread, and stock variance. The objective is to inform our models about the current macroeconomic situation and make them capable of setting it into context for anomaly returns. Furthermore, we incorporate the 49 Fama and French (1997) industry classification indicators publicly available on the Kenneth R. French data library[4] into the feature set. We use one-hot encoding to ensure that our studied models do not suffer multi-collinearity issues, leading to 48 additional features. Next-month returns, market capitalization, and further metadata are obtained from the Center for Research in Security Prices (CRSP). This leads to $320 + 8 + 48 = 376$ features per observation for our models.

[Figure 1 about here.]

Figure 1 reports the number of input features we use over time. For the first year of our asset pricing tests, 2005, our models only use the 137 anomalies from Chen and Zimmermann (2022) with publication dates up to 2004, resulting in $137 + 48 + 8 = 193$ features. The Chen and Zimmermann (2022) database ends in 2016, at which point our list of features reaches its maximum of 376.

To train our machine learning models and tune hyperparameters without any data snooping or look-ahead bias, we follow standard machine learning practice and split the overall dataset into three subsets: a training-, validation-, and testing-set. To allow our model to learn from new information and adapt to the non-stationarity characteristics of stock return time series, we follow the latest literature using an expanding window approach for the training data (Gu et al.,

[4]For more information, see https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

2020; Azevedo and Hoegner, 2023). We re-train the models annually to include new data while keeping a fixed-length moving validation set of six years and a one-year out-of-sample test set. For example, for the out-of-sample year 2005, we use all available stock-month observations from Mar 1957 to Dec 1998 to train the model. We then tune hyperparameters based on the six-year validation set from Jan 1999 to Dec 2004 to ensure the temporal ordering of the observations in the training process. The out-of-sample test set, which we use to evaluate our models regarding machine learning metrics and long-short portfolio performance, contains predictions for January 2005 to December 2021 (i.e., each new year, we move this approach one year ahead, extending the training set).

The portfolio construction process follows common practices in anomaly research. We construct decile portfolios based on the models' next-month stock return predictions, calculating the long-short gross excess return on a monthly rebalancing frequency. Transaction costs are estimated using the composite high-frequency Chen and Velikov (2023) effective bid-ask spread estimator and applied in the calculation of the net excess return following Detzel et al. (2023). To ensure an openly accessible and thus replicable construction and testing protocol, we use the methodology and library of Novy-Marx and Velikov (2023). Further construction details can be found in their paper and on their website.[5]

2.2. Machine learning algorithms and evaluation methodology

Our choice of machine learning models is motivated by prior literature, which shows that neural networks outperform traditional linear regressions as well as penalized ones such as elastic nets (Gu et al., 2020; Chen et al., 2023; Azevedo and Hoegner, 2023; Azevedo et al., 2023; Avramov et al., 2022). We follow Gu et al. (2020), and include the non-constrained OLS-HUBER, a regularized linear model using ENET, and four feedforward neural networks (FFNNs) with hidden layers ranging from two to five.[6] We extend this core model set with two (one- and two-hidden layers, respectively) recurrent neural networks with long short-term memory (LSTMs), which are designed to capture long-term dependencies. Finally, we create an (ENSEMBLE) model by taking the average of all deep-learning models (FFNNs and LSTMs).

Following Chen et al. (2023), we use hyperparameter tuning to find the best set of parameters in each model. To optimize the tuning parameters for each model, we initially implemented a random search strategy. For efficiency and computational practicality, we selected a represen-

[5]Documentation available at `http://assayinganomalies.com/`.
[6]A more detailed explanation of the models can be found in the internet appendix of Gu et al. (2020).

tative subset of the data, comprising 20% of the total dataset, to fine-tune these parameters. Once established, these parameters were consistently applied throughout the expanding window estimation process. However, this approach was not feasible for neural network models due to their extensive computational demands and the wide variability in their parameter ranges. For the neural network approaches, we apply a fixed set of parameters. Like Gu et al. (2020), we use the geometric rule to derive the specific neuron configuration for our 2-, 3-, 4-, and 5-hidden-layer FFNNs, and similarly for our 1- and 2-hidden layer LSTMs. All neural networks use an ADAM optimizer with a learning rate of 0.01, the mean squared error (MSE) validation metric, and 200 epochs with a batch size of 10,000 observations. We apply dynamic learning rate shrinkage by factor 5 when our validation metrics have not improved for ten epochs of training. We also regularize the models through an early stopping callback, which stops training when validation metrics have not improved for 20 epochs.

We evaluate the actual out-of-sample trading strategy performance using common portfolio metrics, mainly gross and net excess return of the long-short portfolios, their statistical significance, Sharpe ratio, information ratio, turnover ratio, and transaction costs, as well as the R2. Also, we test the model against the most comprehensive factor model to date, the Fama and French (2018) six-factor model (FF6). To the best of our knowledge, we are the first to estimate the Novy-Marx and Velikov (2016) generalized alphas to machine-learning-based strategies in order to evaluate the ability of these strategies to expand the net-of-costs mean-variance efficient frontier based on the six factors alone.

2.3. Turnover and cost mitigation techniques

Novy-Marx and Velikov (2016) and Novy-Marx and Velikov (2019) study the impact of cost-mitigation techniques on the profitability of anomaly trading strategies after accounting for trading costs. In addition to their proposed three techniques, we add further variations and filters to test the effect of cost mitigation on the net performance of machine-learning-based strategies.

One major driver to reduce transaction costs is to reduce turnover rate. Most straightforwardly, this can be achieved using an increased holding period/reduced rebalancing frequency for the portfolio construction. Since we train our models based on monthly predictions, we extend the holding period mitigation to 2-, 3-, or 4-months (H2, H3, H4). Additionally, we create quintile- instead of decile portfolios (QUINTILE), hypothesizing that this could reduce turnover while keeping a significant signal-to-noise ratio. As a more complex variation of adapted

overall holding period and number of portfolios, we apply the Buy-Hold-Spread (BHS) technique outlined by Novy-Marx and Velikov (2016) to enter a position for the top-/bottom-decile but exit a position only if they fall out of the top-/bottom-quintile.

Furthermore, we test multiple stock universe filters and weightings. We follow Fama-French in creating a high-market cap filter that excludes the bottom 20% of stocks by market capitalization for our tradable stock universe (HMCAP20). We hypothesize that those high market capitalization stocks are more liquid, i.e., they should face lower transaction costs. In addition, we follow the approach of Novy-Marx and Velikov (2016) in directly filtering out stocks with high previous transaction costs, i.e., using only stocks in the bottom 70%-percentile transaction costs (LTC 70). As an alternative way to incorporate transaction costs into the portfolio construction process, we weight predictions to 25% with the transaction cost percentile (TCWEIGHTED75).[7] Finally, we create two combination strategies that use (a) both H2 and BHS (COMBO1) and (b) H2, BHS, and TCWEIGHTED75 (COMBO2).

3. Machine learning strategy performance decomposition

[Figure 2 about here.]

Figure 2 illustrates the out-of-sample Sharpe ratios for a various machine learning strategies applied to stock return prediction. These strategies span a range of complexity, from simpler models like Ordinary Least Squares (OLS) with Huber loss and elastic net, to more sophisticated approaches such as feedforward neural networks with varying depths (2-5 layers), long short-term memory (LSTM) models with 1-2 layers, and an ensemble model that combines the neural networks and LSTMs.

The figure presents the Sharpe ratios under different scenarios, with each successive scenario providing a more refined and realistic estimate of the strategies' expected performance. The baseline scenario uses the full set of anomalies and data starting in 2000. The next scenario restricts the anomalies to only those that have been published. The third scenario further restricts the sample to the post-2005 period. Finally, the last scenario incorporates trading costs. For comparison, the market's Sharpe ratio over this period is also shown, assuming no trading costs.

[7] Example: a prediction of 1% excess return but with a transaction cost at the 80% percentile will result in $75\% * 1.0\% + 25\% * (1 - 80\%) = 0.8\%$, while the same return prediction with lower transaction cost at the 20% percentile will result in a $75\% * 1.0\% + 25\% * (1 - 20\%) = 0.95\%$ signal for the models.

The key observations from Figure 2 reveal several important insights about the performance of machine learning strategies for stock return prediction. First, simpler strategies like OLS-Huber and elastic net significantly underperform the market when realistic constraints are considered. In contrast, the more sophisticated LSTM-based strategies demonstrate the most impressive performance, consistent with the idea that complexity can be beneficial for return prediction. Feedforward neural networks and the ensemble model combining deep learning approaches also perform well, though not as strongly as the LSTM models. Importantly, the figure illustrates how accounting for post-publication effects, focusing on the post-2005 period, and incorporating trading costs successively reduces the Sharpe ratios of all strategies. However, even under these more realistic conditions, the LSTM and ensemble strategies still manage to outperform the market, highlighting their potential value for investors.

Overall, the figure illustrates that while machine learning strategies can appear highly profitable in typical backtests, their expected performance is more muted once various real-world considerations are taken into account. Nonetheless, the more sophisticated LSTM and ensemble approaches still demonstrate potential to outperform the market even in the presence of these constraints.

[Table 1 about here.]

Table 1 provides a more detailed look at the average monthly returns and associated t-statistics (in brackets) of the machine learning strategies across the different scenarios introduced in Figure 2. The key findings from Table 1 provide a more nuanced understanding of the performance of machine learning strategies for stock return prediction across different scenarios. In the baseline scenario, most strategies generate average returns that are 2-3 times higher than the market. However, restricting the anomalies to only those that have been published reduces returns by an average of 23 basis points per month, while further restricting the sample to the post-2005 period subtracts another 39 basis points per month on average. After accounting for all three effects – post-publication, post-2005, and trading costs – the cumulative reduction in returns amounts to approximately eighty-six basis points per month. Despite these reductions, the LSTM models stand out by continuing to generate significant positive returns and alphas, even net of trading costs. These findings complement the insights from Figure 2, highlighting that while machine learning strategies' expected returns are more moderate when accounting for real-world considerations, the LSTM models, in particular, still demonstrate potential for meaningful outperformance.

4. The expected returns on machine learning based strategies

Figure 3 shows the cumulative *net* performance of the machine learning strategies. It reveals that the outperformance of the LSTM and ENSEMBLE strategies is largely due to their impressive performance during the Great Recession. All other machine learning strategies exhibited a severe drawdown in 2009, which the LSTM strategies completely and the ENSEMBLE strategy to some extent are able to avoid. The LSTM1 strategy also performed better in the aftermath of the COVID-19 pandemic at the end of 2020, while all other strategies suffered losses.

[Figure 3 about here.]

Figure 4 presents a correlation heatmap of the returns generated by our nine machine learning strategies. This visualization allows us to assess the degree of similarity in the return patterns across different approaches. The color scale ranges from dark blue (indicating strong positive correlation) to white (indicating weak or no correlation).

[Figure 4 about here.]

Several interesting patterns emerge from this heatmap. First, we can observe that the LSTM-based strategies (LSTM1 and LSTM2) exhibit relatively low correlations with the other approaches, particularly with the linear models (OLS-HUBER and ENET) and the simpler neural networks (FFNN2 and FFNN3). This suggests that the LSTM models are capturing unique patterns in the data that are not identified by the other techniques. The ENSEMBLE strategy, as expected, shows moderate correlations with most other strategies, reflecting its nature as a combination of multiple approaches. Interestingly, the feedforward neural networks (FFNN2 through FFNN5) show strong correlations with each other, indicating that increasing the number of layers beyond two may not substantially alter the patterns being captured. The linear models (OLS-HUBER and ENET) also show strong correlation with each other but lower correlations with the more complex models, highlighting the potential benefits of non-linear approaches in capturing complex market dynamics.

[Table 2 about here.]

Table 2 reports the performance metrics for our nine machine-learning-based strategies after accounting for trading costs. We can observe that costs have a significant impact on performance,

reducing average returns across the board. The trading costs for the strategies, reported in the last column, range between 19 and 26 basis points per month, rendering the average returns to FFNN2 and FFNN5 insignificant. The two-sided portfolio turnover, reported in the second-to-last column, varies between 120 and 140% per month, classifying the machine-learning strategies as high-turnover anomalies based on the Novy-Marx and Velikov (2016) taxonomy.

Nevertheless, we still observe economically and statistically significant average returns and generalized six-factor model alphas for the majority of the models. Both LSTM strategies continue to exhibit t-statistics on their average returns and alphas in excess of three. The feedforward neural network models perform slightly worse, but their average returns and alphas are still sizable. The clear winner is the LSTM1 strategy, which earns an impressive 1.42% per month (t-stat of 3.99) and a generalized FF6 alpha of 1.20% (t-stat of 3.46).

[Table 3 about here.]

Table 3 presents the ex-post mean-variance efficient (MVE) weights and Sharpe ratios for portfolios combining the Fama-French 6-factor model (FF6) with each of the machine learning strategies. The MVE weights represent the optimal allocation to each factor and the machine learning strategy that would have maximized the portfolio's Sharpe ratio over the sample period, based on the realized returns and covariances.

The table shows that the FF6 factors alone yield an in-sample Sharpe ratio of 1.15. The market factor (MKT) receives a consistent weight of around 30% across all portfolios, while the portfolios all short the value factor (HML). The MVE portfolios generally substitute the machine learning strategy for the profitability factor (RMW).

Consistent with the findings from Figure 2 and Table 1, the LSTM models demonstrate the most impressive improvement in the MVE portfolio's Sharpe ratio. The LSTM1 model increases the Sharpe ratio from 1.15 to 1.45, with a 25% allocation to the strategy. The ensemble model, which combines the deep learning approaches, also performs well, increasing the Sharpe ratio to 1.35 with a 21

The feedforward neural network strategies receive allocations ranging from 9% to 18%, with the 3-layer network (FFNN3) showing the best improvement in the Sharpe ratio among the FFNN models. The elastic net strategy receives a 20% allocation and increases the Sharpe ratio to 1.28.

These results suggest that the machine learning strategies, particularly the LSTM and ensemble models, have the potential to significantly enhance the mean-variance efficiency of

a factor-based portfolio. By combining these strategies with the FF6 factors, investors could have achieved higher risk-adjusted returns over the sample period compared to using the factors alone. However, it is important to note that these are ex-post results based on realized returns, and future performance may vary.

5. Zooming in on the LSTM1 strategy performance

The online appendix evaluates the performance of the LSTM1 strategy using the "Assaying Anomalies" protocol proposed by Novy-Marx and Velikov (2023). This protocol consists of five key steps: (1) providing basic signal diagnostics, (2) checking return predictability, (3) comparing performance to the universe of known anomalies, (4) finding closely related anomalies, and (5) controlling for the entire factor zoo.

In the first step, the LSTM1 signal demonstrates good coverage, with a consistent 25th to 75th percentile range and a stable mean. The second step reveals strong gross profitability for the LSTM1 strategy, with significant excess returns and alphas relative to various factor models. The strategy's performance is robust to alternative portfolio construction methods and trading costs, and a double sort on size and the LSTM1 signal shows that the strategy's returns are not concentrated among microcaps.

Comparing the LSTM1 strategy to the anomaly universe in the third step, the strategy exhibits an impressive Sharpe ratio and alpha relative to other anomalies, both gross and net of trading costs. In the fourth step, the LSTM1 strategy shows low correlations with other anomalies and does not cluster tightly with any specific group. Furthermore, the strategy's predictive power remains significant when controlling for the most closely related anomalies using Fama-MacBeth regressions and spanning tests. Finally, in the fifth step, the LSTM1 strategy's performance is evaluated in the context of combination strategies that pool information from the entire anomaly universe. While the results are mixed, the LSTM1 signal does not consistently enhance the combination strategies' performance, suggesting that its predictive power may already be captured by the collective information in the anomaly universe.

Overall, the LSTM1 strategy performs well across the majority of the "Assaying Anomalies" protocol tests. It demonstrates robust predictive power, generates significant returns and alphas, and appears to capture unique information not fully represented by existing anomalies. However, the strategy's incremental contribution to combination strategies that already exploit the full anomaly universe is less clear. These findings suggest that the LSTM1 strategy is a promising

approach for stock return prediction, but its value may be most pronounced when used in isolation or in combination with a more limited set of anomalies.

6. Applying turnover and transaction cost mitigation strategies

The machine-learning-based strategies examined thus far were designed without regard for trading costs. Recently, Blitz et al. (2023) show that training the models in a longer time horizon can lead to higher returns. Furthermore, Avramov et al. (2022) show that excluding firms with economic constraints can reduce the significance of machine learning models. In this section, we investigate different mitigation techniques and their impact on the net performance of machine learning-based strategies.

[Figure 5 about here.]

Table 4 and Figure 5 show the impact of the previously outlined mitigation approaches on our four classes of model architectures, namely linear models, FFNNs, LSTMs, and the ensemble model. We show the impact of absolute differences in the net excess return portfolio metrics and generalized FF6 alpha and relative changes in turnover and transaction costs. As the results show, most of the cost-mitigation techniques significantly reduce turnover and, as a result, transaction costs.

[Table 4 about here.]

This decrease in transaction costs, however, is only beneficial if it is not accompanied by a larger reduction in gross returns. As we can observe in Table 4, the average change net excess returns across the nine machine learning models is negative for all but one mitigation technique. This implies that the drop in the gross average returns due to the mitigation techniques more than compensates for the reduced trading costs. This is likely because our testing sample period, which consists of the last two decades, is marked by higher liquidity and significantly lower trading costs post-decimalization (Chordia et al., 2014; Chen and Velikov, 2023). The only technique that seems to marginally improve the net average returns across the nine machine learning strategies is the two-month holding period. Not surprisingly, the stock universe filters have a smaller impact on turnover but a similar impact on transaction costs, as they aim to reduce the weight of high-cost stocks. However, the change in net excess returns for these methods is similarly negative.

7. Conclusion

This study provides a comprehensive assessment of machine learning-based anomaly trading strategies, focusing on their expected returns in real-world conditions. By accounting for transaction costs, post-publication decay, and the post-decimalization era of high liquidity, we offer a more realistic estimate of these strategies' performance than typically reported in the literature.

Our findings challenge some prevailing notions in the field while confirming others. Contrary to claims in prior literature, we find that sophisticated machine learning strategies, particularly those employing Long Short-Term Memory (LSTM) models, remain profitable even after accounting for real-world constraints. The best-performing LSTM strategy earns net out-of-sample monthly returns of 1.42% and a six-factor generalized net alpha of 1.20%, with statistically significant t-statistics. This performance persists despite high turnover rates exceeding 50

A key contribution of our study is the decomposition of backtested returns to arrive at expected returns. We find that restricting the sample to the post-2005 period has the largest impact on performance, followed by post-publication effects and trading costs. Cumulatively, these factors reduce strategy performance by 25-80% compared to typical backtests. However, the fact that some strategies, particularly LSTM models, continue to outperform even under these constraints is a testament to the potential of advanced machine learning techniques in asset pricing.

Our analysis of cost-mitigation techniques reveals an interesting paradox. While these techniques successfully reduce turnover and trading costs, they generally do not improve net anomaly performance in the recent high-liquidity era. This finding suggests that the benefits of reduced costs are often offset by a decrease in the strategy's ability to capture short-term price movements.

The superior performance of LSTM models, especially during the Great Recession when many other strategies faltered, highlights the potential for machine learning to capture complex, non-linear relationships in financial data. The low correlation between LSTM models and other machine learning approaches further underscores their unique contribution to the field of quantitative finance.

Our rigorous evaluation of the LSTM1 signal using the "Assaying Anomalies" protocol demonstrates its robustness across multiple dimensions. This comprehensive testing framework provides strong evidence for the signal's validity and predictive power, even when controlling

for a wide array of known anomalies and risk factors.

These findings have significant implications for both academic research and practical investment strategy design. For academics, our results underscore the importance of considering real-world constraints when evaluating new predictive signals or models. They also highlight the potential for advanced machine learning techniques to uncover persistent sources of alpha in financial markets.

For practitioners, our study offers valuable insights into the realistic performance expectations for machine learning-based strategies. While the raw performance of these strategies may be less impressive than some backtests suggest, the persistent outperformance of certain models, particularly LSTM, indicates their potential value in investment portfolios.

Future research could extend this work in several directions. First, investigating the specific features or architectures that contribute to the superior performance of LSTM models could yield valuable insights. Second, exploring the integration of transaction cost models directly into the machine learning framework might lead to strategies that are inherently more robust to trading frictions. Finally, examining the performance of these strategies across different asset classes or international markets could provide a more comprehensive understanding of their efficacy.

In conclusion, while our study tempers some of the more optimistic claims about machine learning in asset pricing, it also provides compelling evidence for the continued relevance and potential of these techniques. As we venture deeper into the age of artificial intelligence and big data, research at the intersection of machine learning and finance will undoubtedly continue to yield valuable insights and innovative investment approaches.

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Table 1: Decomposition of Monthly Returns (%)

Model	Baseline	Post-Publication	Post-2005 Sample	Net of Costs	Total Change
Panel A: Market					
MKT	0.63	0.63	0.86	0.86	+0.23
	[2.25]	[2.25]	[2.82]	[2.82]	
Panel B: Linear Models					
OLS-Huber	0.94	0.84	0.48	0.29	-0.65
	[2.73]	[2.19]	[1.34]	[0.79]	
ENET	1.44	1.15	0.83	0.64	-0.80
	[5.27]	[4.22]	[3.16]	[2.43]	
Panel C: Neural Networks					
FFNN2	1.86	1.22	0.82	0.57	-1.29
	[5.28]	[3.79]	[2.50]	[1.75]	
FFNN3	2.01	1.58	1.01	0.75	-1.26
	[5.49]	[4.83]	[3.02]	[2.25]	
FFNN4	1.96	1.30	0.92	0.66	-1.30
	[5.08]	[3.59]	[2.67]	[1.92]	
FFNN5	1.98	1.13	0.66	0.40	-1.58
	[5.17]	[3.58]	[1.95]	[1.17]	
Panel D: LSTM Models					
LSTM1	1.87	2.06	1.63	1.42	-0.45
	[5.34]	[5.82]	[4.57]	[3.99]	
LSTM2	1.92	1.58	1.27	1.06	-0.86
	[5.45]	[4.35]	[3.94]	[3.30]	
Panel E: Ensemble					
ENSEMBLE	1.62	1.66	1.07	0.83	-0.79
	[4.89]	[4.94]	[3.30]	[2.56]	
Panel F: ML Average					
Average (ML)	1.62	1.39	1.00	0.76	-0.86

This table presents the decomposition of monthly returns (in percentages) for various machine learning strategies across different scenarios. The 'Baseline' scenario uses the full set of anomalies and data starting in 2000. 'Post-Publication' restricts anomalies to only those that have been published. 'Post-2005 Sample' further restricts the sample to the post-2005 period. 'Net of Costs' incorporates trading costs. 'Total Change' shows the difference between 'Net of Costs' and 'Baseline' returns. T-statistics are reported in square brackets below the corresponding returns. ML = Machine Learning, OLS = Ordinary Least Squares, ENET = Elastic Net, FFNN = Feedforward Neural Network, LSTM = Long Short-Term Memory.

Table 2: Out-of-Sample Net Performance of Machine-Learning Anomaly Strategies

Model architecture	Net monthly excess return (%)	Generalized alpha FF6 (%)	Two-sided turnover (%)	Transaction costs (%)
OLS-HUBER	0.29	0.38	122.11	0.20
	[0.79]	[1.47]		
ENET	0.64	0.55	139.76	0.19
	[2.43]	[2.20]		
FFNN2	0.57	0.57	129.14	0.24
	[1.75]	[1.90]		
FFNN3	0.75	0.68	128.40	0.26
	[2.25]	[2.25]		
FFNN4	0.66	0.58	127.27	0.26
	[1.92]	[1.78]		
FFNN5	0.40	0.38	130.25	0.26
	[1.17]	[1.20]		
LSTM1	1.42	1.20	129.56	0.22
	[3.99]	[3.46]		
LSTM2	1.06	0.95	129.07	0.21
	[3.30]	[3.07]		
ENSEMBLE	0.83	0.84	129.91	0.24
	[2.56]	[2.74]		

This table presents the out-of-sample net performance metrics for various machine learning anomaly strategies during the period from January 2005 to December 2021. All portfolios are constructed as value-weighted long-short decile portfolios with NYSE breakpoints and a 1-month holding period/rebalancing frequency. 'Net monthly excess return' and 'Generalized alpha FF6' (generalized alpha relative to the Fama-French 6-factor model) are reported in percentages, with t-statistics in square brackets. 'Two-sided turnover' is the average monthly two-sided portfolio turnover in percentages. 'Transaction costs' are estimated using the Chen and Velikov (2023) high-frequency combination effective bid-ask spread estimator and reported in percentages. OLS = Ordinary Least Squares, ENET = Elastic Net, FFNN = Feedforward Neural Network, LSTM = Long Short-Term Memory.

Table 3: Net Ex-Post Mean-Variance Efficient Weights and Sharpe Ratios

2*Model	Mean-Variance Efficient Weights (%)							Sharpe	ΔSharpe
	MKT	SMB	HML	RMW	CMA	UMD	ML	Ratio	
FF6	28	9	-22	59	22	3		1.15	
FF6+OLS-H	30	17	-24	47	18	0	12	1.21	+0.06
FF6+ENET	29	11	-27	49	17	0	20	1.28	+0.13
FF6+FFNN2	31	10	-26	53	17	0	15	1.25	+0.10
FF6+FFNN3	33	11	-32	51	19	0	18	1.28	+0.13
FF6+FFNN4	30	9	-30	54	23	0	13	1.24	+0.09
FF6+FFNN5	28	10	-28	57	24	0	9	1.19	+0.04
FF6+LSTM1	33	6	-27	38	23	2	25	1.45	+0.30
FF6+LSTM2	34	6	-26	44	18	0	25	1.39	+0.24
FF6+ENSEMBLE	33	9	-30	49	18	0	21	1.35	+0.20

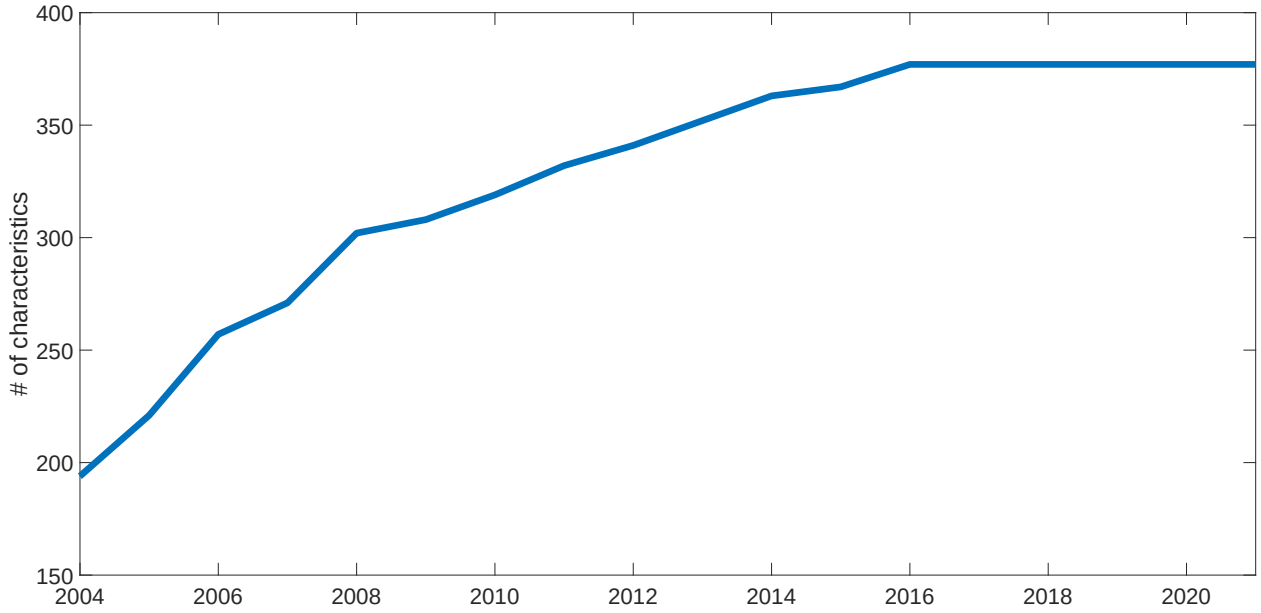
This table presents the net ex-post mean-variance efficient (MVE) weights and Sharpe ratios for portfolios combining the Fama-French 6-factor model (FF6) with each machine learning strategy. MKT = Market, SMB = Size, HML = Value, RMW = Profitability, CMA = Investment, UMD = Momentum, ML = Machine Learning strategy. ΔSharpe shows the improvement in Sharpe ratio compared to the FF6 baseline. OLS-H = Ordinary Least Squares with Huber loss, ENET = Elastic Net, FFNN = Feedforward Neural Network, LSTM = Long Short-Term Memory.

Table 4: Average mitigation technique effect across different model architectures

Mitigation technique	relative change in %		absolute average Δ to baseline			
	Two-sided turnover	Trans-action costs	Net excess return in %	[t-stat]	Generalized FF6 alpha in %	[t-stat]
H2	-46.10	-46.78	0.03	0.05	0.24	0.69
H3	-62.57	-63.26	-0.18	-0.54	-0.04	-0.17
H4	-71.28	-71.93	-0.15	-0.29	0.03	0.40
BHS	-33.46	-36.03	-0.08	0.06	-0.01	0.32
QUINTILE	-12.17	-34.16	-0.25	-0.04	-0.23	-0.12
HMCAP20	-1.78	-29.40	-0.10	-0.31	-0.08	-0.20
LTC70	3.04	-53.67	-0.12	-0.48	-0.10	-0.46
TCWEIGHTED75	-1.21	-17.51	-0.04	-0.06	-0.03	-0.01
COMBO1	-72.21	-73.20	-0.26	-0.62	-0.10	-0.11
COMBO2	-72.41	-77.28	-0.32	-0.81	-0.17	-0.30

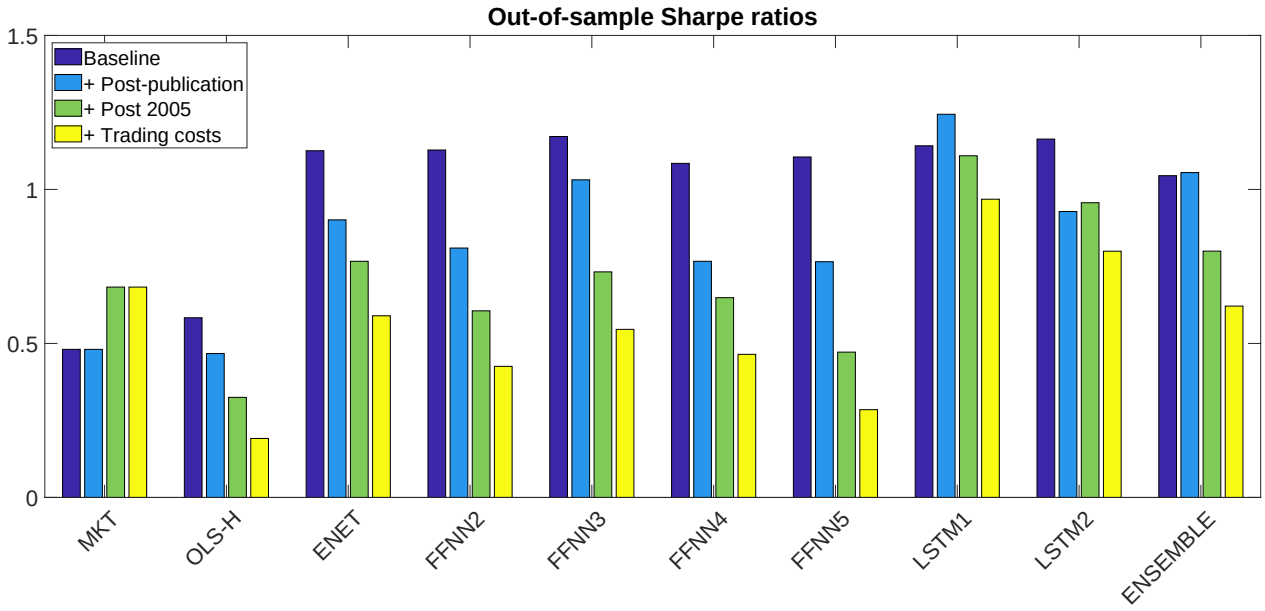
This table shows the average effect on the baseline model of applying each mitigation technique in our out-of-sample period from January 2005 to December 2021. We apply different techniques to our sample: Extended holding period/reduced rebalancing frequency (H2, H3, and H4 for 2-, 3-, and 4-month periods), a BHS, quintile instead of decile portfolios (QUINTILE), a high market cap filter (HMCAP20), a low transaction cost filter (LTC70), and a weighting of next month's predicted returns by estimated transaction costs (TCWEIGHTED75). We report the impact in % change of the respective variable compared to the baseline model without mitigations.

Figure 1: Number of characteristics used over time



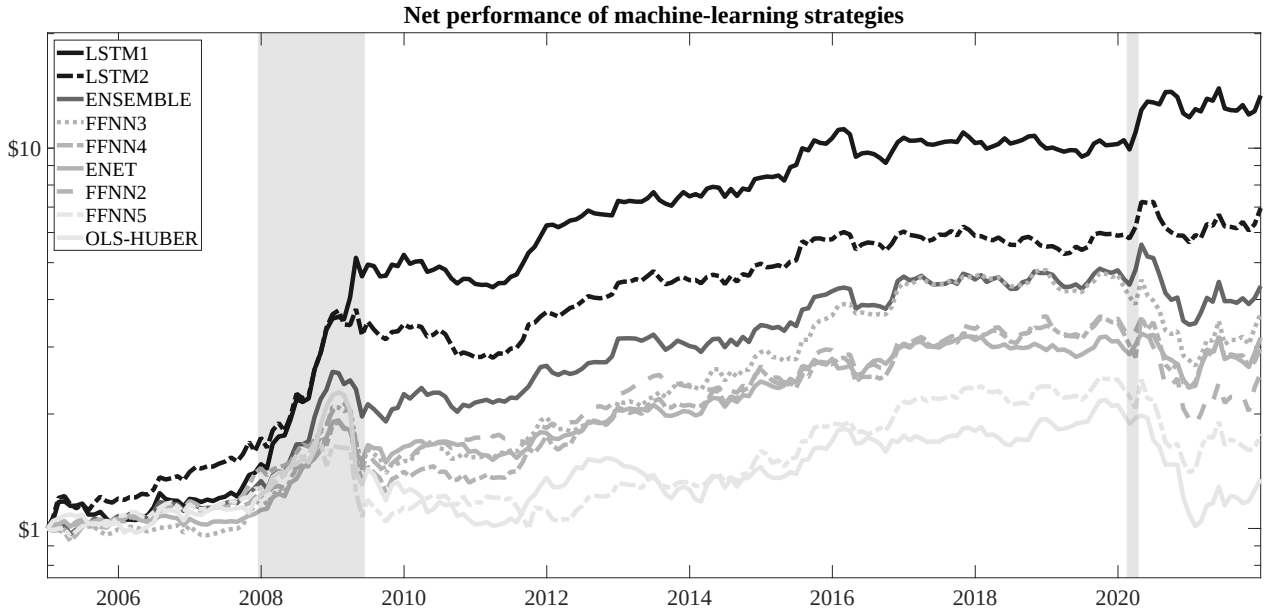
The figure plots the number of characteristics used in the construction of our machine-learning strategies over time.

Figure 2: Return decomposition



The figure describes the monthly performance of our baseline models gross and net of trading costs in the out-of-sample period from January 2005 to December 2021. The transaction costs are estimated using the Chen and Velikov (2023) high-frequency combination effective bid-ask spread estimator. The bars show the average monthly gross and net excess returns of the value-weighted long-short decile portfolios based on NYSE breakpoints. The t-statistic is in brackets. The labels below the columns show the respective relative drop in return in % due to the introduction of transaction costs.

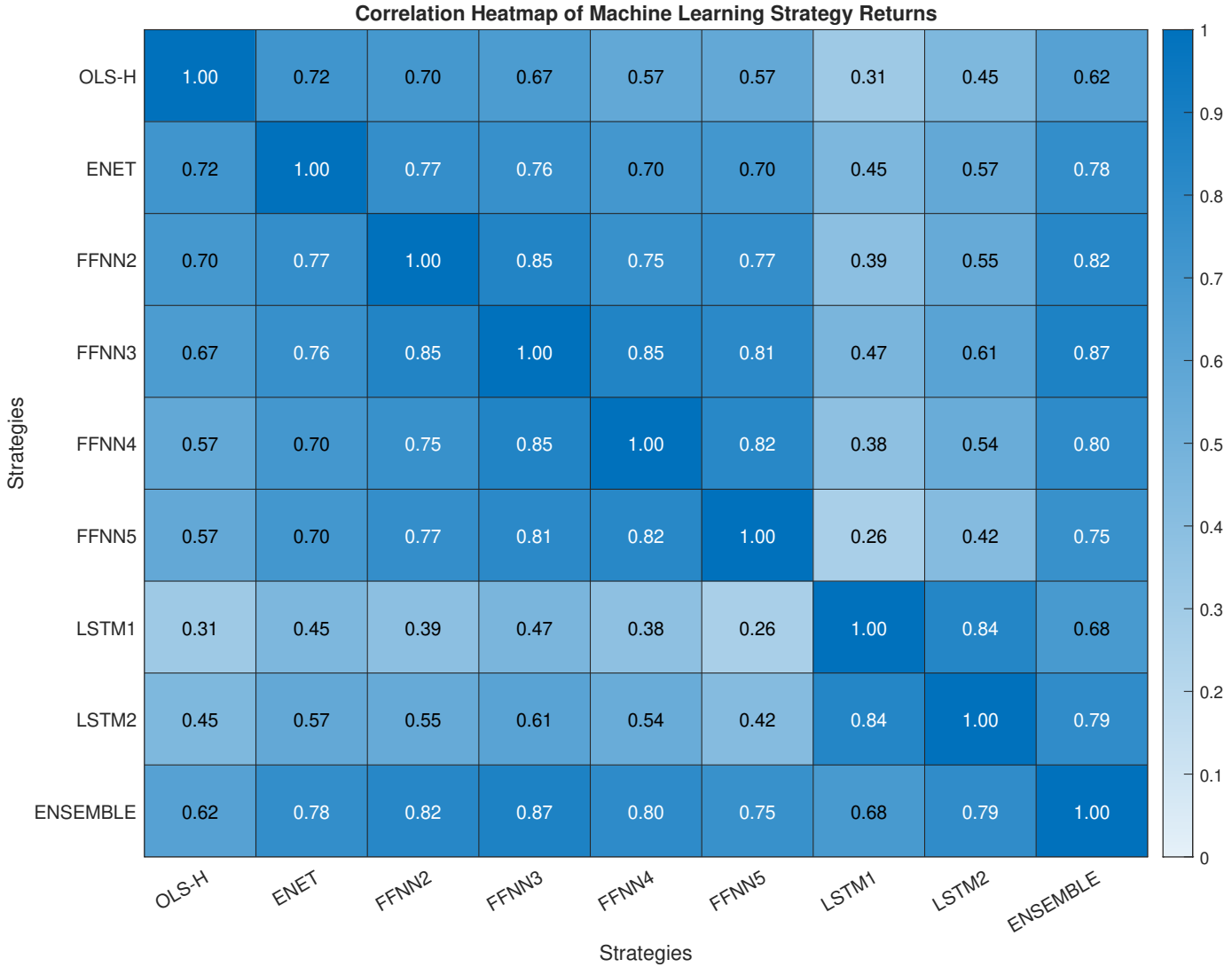
Figure 3: Net performance of machine-learning strategies



The figure describes the monthly performance of machine-learning anomaly strategies net of trading costs in the out-of-sample period from January 2005 to December 2021. The transaction costs are estimated using the Chen and Velikov (2023) high-frequency combination effective bid-ask spread estimator. The bars show the average monthly gross and net excess returns of the value-weighted long-short decile portfolios based on NYSE breakpoints.

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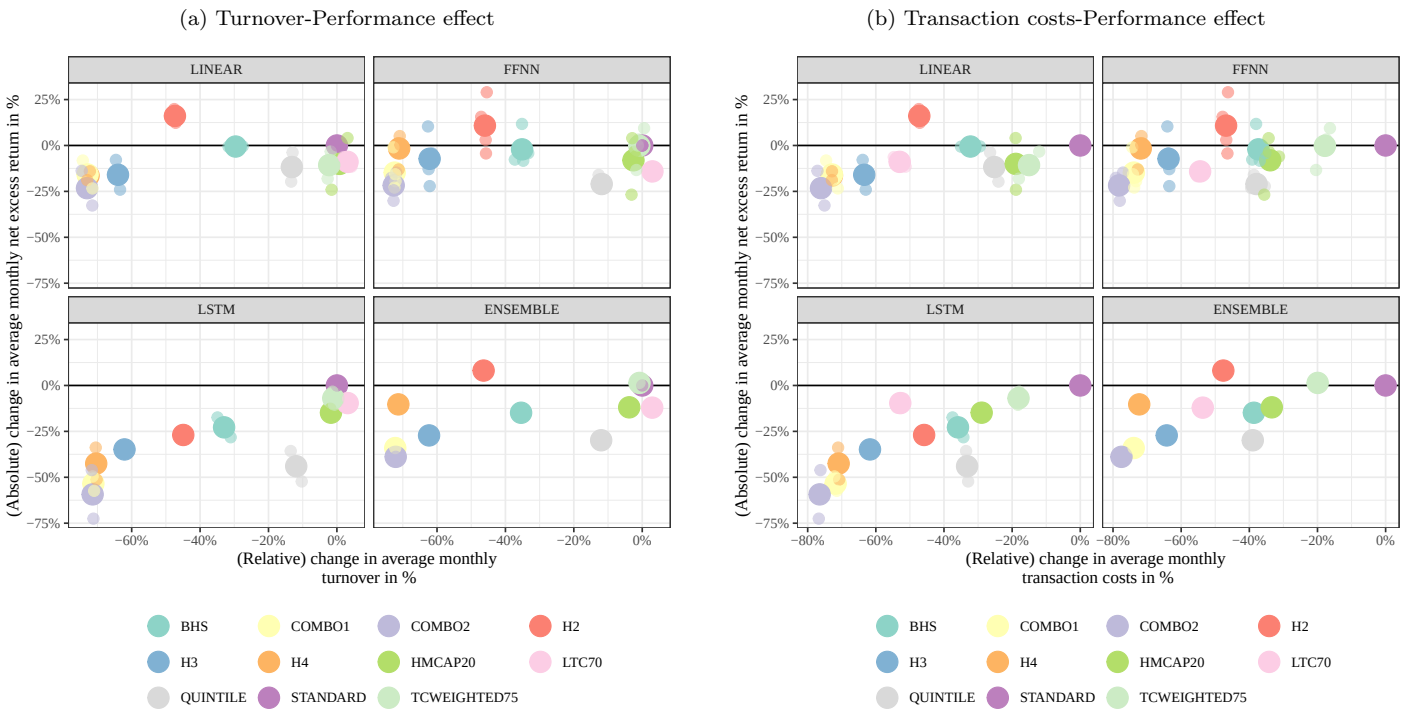
Figure 4: Correlation heatmap



This figure plots the pairwise correlations between the returns of nine different machine learning strategies. The color scale ranges from dark blue (strong positive correlation) to white (weak or no correlation). Strategies include Ordinary Least Squares with Huber Loss (OLS-H), Elastic Net (ENET), Feedforward Neural Networks with 2-5 layers (FFNN2-5), Long Short-Term Memory networks with 1-2 layers (LSTM1-2), and an ensemble model (ENSEMBLE). The heatmap reveals varying degrees of correlation between different strategies, with LSTM-based approaches showing the lowest correlations with other methods

.

Figure 5: Net return impact of different turnover and cost mitigation techniques on portfolio performance



The table illustrates the turnover and transaction cost relations to the respective change in net excess return in the out-of-sample period from Jan 2005 to Dec 2021. All changes are in % compared to the baseline model without mitigation techniques in a transaction cost environment.

Appendix A.

Elastic Net Regression

The Elastic Net is a linear regression model that combines the L1 and L2 regularization of the Lasso and Ridge regression methods. This approach is beneficial when dealing with highly correlated independent variables.

Mathematical Formulation

The objective function of the Elastic Net is:

$$\text{Minimize} \left(\sum_{i=1}^{n} (y_i - \sum_{j=1}^{p} x_{ij}\beta_j)^2 + \lambda \left(\alpha \sum_{j=1}^{p} |\beta_j| + \frac{1-\alpha}{2} \sum_{j=1}^{p} \beta_j^2 \right) \right) \tag{A.1}$$

where y_i represents the response variable, x_{ij} are the predictors, β_j are the coefficients, λ is the regularization parameter, and α is the mixing parameter between Lasso and Ridge penalties.

Overview of Feedforward Neural Networks

Architecture of FFNN with Two Hidden Layers

In this study, FFNNs with two to five hidden layers are used. The architecture of an FFNN with two hidden layers is as follows:

- Input Layer: Receives input features (e.g., stock market anomalies, macroeconomic predictors, and industry dummies).
- First Hidden Layer: Applies a non-linear transformation to the inputs using the ReLU activation function.
- Second Hidden Layer: Further processes the data from the first hidden layer, also using ReLU.
- Output Layer: Produces the final output (e.g., asset return predictions), typically using a linear activation function.

Mathematical Formulation with ReLU Activation

Consider an FFNN with two hidden layers. The mathematical formulation using the ReLU activation function is:

First Hidden Layer:

$$H_1 = ReLU(W_1 \cdot x + b_1) \tag{A.2}$$

Second Hidden Layer:

$$H_2 = ReLU(W_2 \cdot H_1 + b_2) \tag{A.3}$$

Output Layer:

$$\hat{y} = f(W_3 \cdot H_2 + b_3) \tag{A.4}$$

where:

- x is the input vector.
- W_1, W_2, W_3 are the weight matrices for the first hidden layer, second hidden layer, and output layer, respectively.
- b_1, b_2, b_3 are bias vectors for each corresponding layer.
- $ReLU(\cdot)$ is the Rectified Linear Unit activation function, defined as $ReLU(z) = max(0, z)$.
- $f(\cdot)$ is typically a linear activation function for regression tasks.

ReLU Activation Function

The Rectified Linear Unit (ReLU) activation function is defined as:

$$ReLU(z) = max(0, z) \tag{A.5}$$

ReLU introduces non-linearity in the model, enabling the network to learn complex patterns. It is computationally efficient and helps mitigate the vanishing gradient problem.

Backpropagation and Initialization of Weights and Biases

Backpropagation is a fundamental algorithm in supervised learning used for training Feed-forward Neural Networks (FFNNs). The core idea of backpropagation is to adjust the network's weights and biases to minimize the error between the predicted and actual outputs. Mathematically, this involves computing the gradient of the loss function with respect to each weight and bias in the network.

Let's consider a network with L layers, each with weights $W^{(l)}$ and biases $b^{(l)}$ for layer l. The process of backpropagation can be described as follows:

1. Forward Pass: Compute the output of the network for a given input. For layer l, the output $H^{(l)}$ is given by:

$$H^{(l)} = \sigma(W^{(l)} \cdot H^{(l-1)} + b^{(l)}) \tag{A.6}$$

where σ is the activation function and $H^{(0)}$ is the input to the network.

2. Compute Loss: Calculate the loss (error) $\mathcal{L}$ using a loss function which compares the predicted output $\hat{y}$ and the actual output y. For regression tasks, a common loss function is Mean Squared Error (MSE):

$$\mathcal{L} = \frac{1}{N}\sum_{i=1}^{N}(\hat{y}_i - y_i)^2 \tag{A.7}$$

3. Backward Pass: Compute the gradient of the loss function with respect to each weight and bias. For a weight $W_{ij}^{(l)}$ in layer l, the gradient is:

$$\frac{\partial \mathcal{L}}{\partial W_{ij}^{(l)}} = \frac{\partial \mathcal{L}}{\partial H_i^{(l)}} \cdot \frac{\partial H_i^{(l)}}{\partial W_{ij}^{(l)}} \tag{A.8}$$

4. Update Weights and Biases: Adjust the weights and biases in the direction that minimizes the loss. This is typically done using gradient descent:

$$W_{ij}^{(l)} = W_{ij}^{(l)} - \eta \frac{\partial \mathcal{L}}{\partial W_{ij}^{(l)}} \tag{A.9}$$

$$b_i^{(l)} = b_i^{(l)} - \eta \frac{\partial \mathcal{L}}{\partial b_i^{(l)}} \tag{A.10}$$

where η is the learning rate.

A common approach to initializing weights and biases is to use small random values for weights and zeros for biases. This breaks the symmetry in the learning process and allows the network to learn effectively. For example, weights can be initialized from a normal distribution with a mean of 0 and a small standard deviation, e.g., 0.01.

Long Short-Term Memory (LSTM) Networks

LSTMs are a special kind of Recurrent Neural Network (RNN) capable of learning long-term dependencies in sequential data, which is particularly useful in financial time series forecasting.

Core Components

An LSTM unit includes several gates to control the flow of information:

- Forget Gate: Decides what information to discard from the cell state.
- Input Gate: Updates the cell state with new information.

- Output Gate: Determines the next hidden state.

Mathematical Formulation of LSTM Cell

The operations inside an LSTM cell can be formulated as follows:

$$\text{Forget Gate:} \quad f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f) \tag{A.11}$$

$$\text{Input Gate:} \quad i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i) \tag{A.12}$$

$$\text{Cell State Update:} \quad \tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C) \tag{A.13}$$

$$\text{Final Cell State:} \quad C_t = f_t * C_{t-1} + i_t * \tilde{C}_t \tag{A.14}$$

$$\text{Output Gate:} \quad o_t = \sigma(W_o \cdot [h_{t-1}, x_t] + b_o) \tag{A.15}$$

$$\text{Output:} \quad h_t = o_t * \tanh(C_t) \tag{A.16}$$

Where f_t, i_t, and o_t are the activations of the forget, input, and output gates, respectively; C_t is the cell state; h_t is the hidden state; W and b are the weights and biases for each gate; and x_t is the input at time t.

Online Appendix for The Expected Returns on Machine-Learning Strategies: Long-Short Term Memory Model Signal and the Cross Section of Stock Returns

Vitor Azevedo Christopher Hoegner Mihail Velikov

September 12, 2024

Abstract

This report studies the asset pricing implications of Long-Short Term Memory Model Signal (LSTM), and its robustness in predicting returns in the cross-section of equities using the protocol proposed by Novy-Marx and Velikov (2023). A value-weighted long/short trading strategy based on LSTM achieves an annualized gross (net) Sharpe ratio of 1.15 (1.00), and monthly average abnormal gross (net) return relative to the Fama and French (2015) five-factor model plus a momentum factor of 99 (80) bps/month with a t-statistic of 4.34 (3.59), respectively. Its gross monthly alpha relative to these six factors plus the six most closely related strategies from the factor zoo (Momentum based on FF3 residuals, Earnings surprise streak, Industry return of big firms, Net external financing, Momentum in high volume stocks, Return on assets (qtrly)) is 72 bps/month with a t-statistic of 3.38.

1 Introduction

The following automatically generated report tests the asset pricing implications of Long-Short Term Memory Model Signal (LSTM), and its robustness in predicting returns in the cross-section of equities. It is produced using the methodology of Novy-Marx and Velikov (2023), from input data consisting of firm-month observations for the proposed predictor.[1]

2 Signal diagnostics

Figure 1 plots descriptive statistics for the LSTM signal. Panel A plots the time-series of the mean, median, and interquartile range for LSTM. On average, the cross-sectional mean (median) LSTM is 0.01 (0.01) over the 2004 to 2021 sample, where the starting date is determined by the availability of the input LSTM data. The signal's interquartile range spans -0.01 to 0.12. Panel B of Figure 1 plots the time-series of the coverage of the LSTM signal for the CRSP universe. On average, the LSTM signal is available for 92.64% of CRSP names, which on average make up 98.04% of total market capitalization.

3 Does LSTM predict returns?

Table 1 reports the performance of portfolios constructed using a value-weighted, quintile sort on LSTM using NYSE breaks. The first two lines of Panel A report monthly average excess returns for each of the five portfolios and for the long/short portfolio that buys the high LSTM portfolio and sells the low LSTM portfolio. The rest of Panel A reports the portfolios' monthly abnormal returns relative to the five most common factor models: the CAPM, the Fama and French (1993) three-factor

[1]It used version v0.4.1 of the publicly available code repository at https://github.com/velikov-mihail/AssayingAnomalies. See more details at http://AssayingAnomalies.com.

model (FF3) and its variation that adds momentum (FF4), the Fama and French (2015) five-factor model (FF5), and its variation that adds momentum factor used in Fama and French (2018) (FF6). The table shows that the long/short LSTM strategy earns an average return of 1.09% per month with a t-statistic of 4.77. The annualized Sharpe ratio of the strategy is 1.15. The alphas range from 0.99% to 1.21% per month and have t-statistics exceeding 4.34 everywhere. The lowest alpha is with respect to the FF6 factor model.

Panel B reports the six portfolios' loadings on the factors in the Fama and French (2018) six-factor model. The long/short strategy's most significant loading is 0.18, with a t-statistic of 3.23 on the UMD factor. Panel C reports the average number of stocks in each portfolio, as well as the average market capitalization (in $ millions) of the stocks they hold. In an average month, the five portfolios have at least 607 stocks and an average market capitalization of at least $2,996 million.

Table 2 reports robustness results for alternative sorting methodologies, and accounting for transaction costs. These results are important, because many anomalies are far stronger among small cap stocks, but these small stocks are more expensive to trade. Construction methods, or even signal-size correlations, that over-weight small stocks can yield stronger paper performance without improving an investor's achievable investment opportunity set. Panel A reports gross returns and alphas for the long/short strategies made using various different protfolio constructions. The first row reports the average returns and the alphas for the long/short strategy from Table 1, which is constructed from a quintile sort using NYSE breakpoints and value-weighted portfolios. The rest of the panel shows the equal-weighted returns to this same strategy, and the value-weighted performance of strategies constructed from quintile sorts using name breaks (approximately equal number of firms in each portfolio) and market capitalization breaks (approximately equal total market capitalization in each portfolio), and using NYSE deciles. The average return is lowest

for the quintile sort using cap breakpoints and value-weighted portfolios, and equals 80 bps/month with a t-statistics of 4.47. Out of the twenty-five alphas reported in Panel A, the t-statistics for twenty-five exceed two, and for twenty-five exceed three.

Panel B reports for these same strategies the average monthly net returns and the generalized net alphas of Novy-Marx and Velikov (2016). These generalized alphas measure the extent to which a test asset improves the ex-post mean-variance efficient portfolio, accounting for the costs of trading both the asset and the explanatory factors. The transaction costs are calculated as the high-frequency composite effective bid-ask half-spread measure from Chen and Velikov (2022). The net average returns reported in the first column range between 69-151bps/month. The lowest return, (69 bps/month), is achieved from the quintile sort using cap breakpoints and value-weighted portfolios, and has an associated t-statistic of 3.88. Out of the twenty-five construction-methodology-factor-model pairs reported in Panel B, the LSTM trading strategy improves the achievable mean-variance efficient frontier spanned by the factor models in twenty-five cases, and significantly expands the achievable frontier in twenty-five cases.

Table 3 provides direct tests for the role size plays in the LSTM strategy performance. Panel A reports the average returns for the twenty-five portfolios constructed from a conditional double sort on size and LSTM, as well as average returns and alphas for long/short trading LSTM strategies within each size quintile. Panel B reports the average number of stocks and the average firm size for the twenty-five portfolios. Among the largest stocks (those with market capitalization greater than the 80^{th} NYSE percentile), the LSTM strategy achieves an average return of 80 bps/month with a t-statistic of 4.03. Among these large cap stocks, the alphas for the LSTM strategy relative to the five most common factor models range from 69 to 80 bps/month with t-statistics between 3.41 and 4.01.

4 How does LSTM perform relative to the zoo?

Figure 2 puts the performance of LSTM in context, showing the long/short strategy performance relative to other strategies in the "factor zoo." It shows Sharpe ratio histograms, both for gross and net returns (Panel A and B, respectively), for 212 documented anomalies in the zoo.[2] The vertical red line shows where the Sharpe ratio for the LSTM strategy falls in the distribution. The LSTM strategy's gross (net) Sharpe ratio of 1.15 (1.00) is greater than 100% (100%) of anomaly Sharpe ratios, respectively.

Figure 3 plots the growth of a $1 invested in these same 212 anomaly trading strategies (gray lines), and compares those with the growth of a $1 invested in the LSTM strategy (red line).[3] Ignoring trading costs, a $1 invested in the LSTM strategy would have yielded $6.22 which ranks the LSTM strategy in the top 1% across the 212 anomalies. Accounting for trading costs, a $1 invested in the LSTM strategy would have yielded $4.38 which ranks the LSTM strategy in the top 0% across the 212 anomalies.

Figure 4 plots percentile ranks for the 212 anomaly trading strategies in terms of gross and Novy-Marx and Velikov (2016) net generalized alphas with respect to the CAPM, and the Fama-French three-, four-, five-, and six-factor models from Table 1, and indicates the ranking of the LSTM relative to those. Panel A shows that the LSTM strategy gross alphas fall between the 98 and 99 percentiles across the five factor models. Panel B shows that, accounting for trading costs, a large fraction of anomalies have not improved the investment opportunity set of an investor with access to the factor models over the 200412 to 202111 sample. For example, 47%

[2]The anomalies come from March, 2022 release of the Chen and Zimmermann (2022) open source asset pricing dataset.

[3]The figure assumes an initial investment of $1 in T-bills and $1 long/short in the two sides of the strategy. Returns are compounded each month, assuming, as in Detzel et al. (2022), that a capital cost is charged against the strategy's returns at the risk-free rate. This excess return corresponds more closely to the strategy's economic profitability.

(53%) of the 212 anomalies would not have improved the investment opportunity set for an investor having access to the Fama-French three-factor (six-factor) model. The LSTM strategy has a positive net generalized alpha for five out of the five factor models. In these cases LSTM ranks between the 99 and 99 percentiles in terms of how much it could have expanded the achievable investment frontier.

5 Does LSTM add relative to related anomalies?

With so many anomalies, it is possible that any proposed, new cross-sectional predictor is just capturing some combination of known predictors. It is consequently natural to investigate to what extent the proposed predictor adds additional predictive power beyond the most closely related anomalies. Closely related anomalies are more likely to be formed on the basis of signals with higher absolute correlations. Figure 5 plots a name histogram of the correlations of LSTM with 211 filtered anomaly signals.[4] Figure 6 also shows an agglomerative hierarchical cluster plot using Ward's minimum method and a maximum of 10 clusters.

A closely related anomaly is also more likely to price LSTM or at least to weaken the power LSTM has predicting the cross-section of returns. Figure 7 plots histograms of t-statistics for predictability tests of LSTM conditioning on each of the 211 filtered anomaly signals one at a time. Panel A reports t-statistics on β_{LSTM} from Fama-MacBeth regressions of the form $r_{i,t} = \alpha + \beta_{LSTM} LSTM_{i,t} + \beta_X X_{i,t} + \epsilon_{i,t}$, where X stands for one of the 211 filtered anomaly signals at a time. Panel B plots t-statistics on α from spanning tests of the form: $r_{LSTM,t} = \alpha + \beta r_{X,t} + \epsilon_t$, where $r_{X,t}$ stands for the returns to one of the 211 filtered anomaly trading strategies at a time. The strategies employed in the spanning tests are constructed using quintile sorts,

[4]When performing tests at the underlying signal level (e.g., the correlations plotted in Figure 5), we filter the 212 anomalies to avoid small sample issues. For each anomaly, we calculate the common stock observations in an average month for which both the anomaly and the test signal are available. In the filtered anomaly set, we drop anomalies with fewer than 100 common stock observations in an average month.

value-weighting, and NYSE breakpoints. Panel C plots t-statistics on the average returns to strategies constructed by conditional double sorts. In each month, we sort stocks into quintiles based one of the 211 filtered anomaly signals. Then, within each quintile, we sort stocks into quintiles based on LSTM. Stocks are finally grouped into five LSTM portfolios by combining stocks within each anomaly sorting portfolio. The panel plots the t-statistics on the average returns of these conditional double-sorted LSTM trading strategies conditioned on each of the 211 filtered anomalies.

Table 4 reports Fama-MacBeth cross-sectional regressions of returns on LSTM and the six anomalies most closely-related to it. The six most-closely related anomalies are picked as those with the highest combined rank where the ranks are based on the absolute value of the Spearman correlations in Panel B of Figure 5 and the R^2 from the spanning tests in Figure 7, Panel B. Controlling for each of these signals at a time, the t-statistics on the LSTM signal in these Fama-MacBeth regressions exceed 5.46, with the minimum t-statistic occurring when controlling for Momentum in high volume stocks. Controlling for all six closely related anomalies, the t-statistic on LSTM is 2.87.

Similarly, Table 5 reports results from spanning tests that regress returns to the LSTM strategy onto the returns of the six most closely-related anomalies and the six Fama-French factors. Controlling for the six most-closely related anomalies individually, the LSTM strategy earns alphas that range from 78-92bps/month. The minimum t-statistic on these alphas controlling for one anomaly at a time is 3.50, which is achieved when controlling for Momentum in high volume stocks. Controlling for all six closely-related anomalies and the six Fama-French factors simultaneously, the LSTM trading strategy achieves an alpha of 72bps/month with a t-statistic of 3.38.

6 Does LSTM add relative to the whole zoo?

Finally, we can ask how much adding LSTM to the entire factor zoo could improve investment performance. Figure 8 plots the growth of $1 invested in trading strategies that combine multiple anomalies following Chen and Velikov (2022). The combinations use either the 162 anomalies from the zoo that satisfy our inclusion criteria (blue lines) or these 162 anomalies augmented with the LSTM signal.[5] We consider six different methods for combining signals.

Panel A shows results using "Average rank" as the combination method. This method sorts stocks on the basis of forecast excess returns, where these are calculated on the basis of their average cross-sectional percentile rank across return predictors, and the predictors are all signed so that higher ranks are associated with higher average returns. For this method, $1 investment in the 162-anomaly combination strategy grows to $1.78, while $1 investment in the combination strategy that includes LSTM grows to $1.82.

Panel B shows results using "Weighted-Average rank" as the combination method. This method sorts stocks on the basis of forecast excess returns, where these are calculated as weighted-average cross-sectional percentile rank across return predictors, and the predictors are all signed so that higher ranks are associated with higher average returns and the weights are determined by the average returns over the past ten years to the long/short strategies based on the individual signals. For this method, $1 investment in the 162-anomaly combination strategy grows to $0.94, while $1 investment in the combination strategy that includes LSTM grows to $0.99.

Panel C shows results using "Fama-MacBeth" as the combination method. This method sorts stocks on the basis of forecast excess returns, where these are calculated from Fama and MacBeth (1973) regressions following Haugen and Baker (1996) and

[5]We filter the 207 Chen and Zimmermann (2022) anomalies and require for each anomaly the average month to have at least 40% of the cross-sectional observations available for market capitalization on CRSP in the period for which LSTM is available.

Lewellen (2015) using only data in the investor's information set at the time of portfolio formation. The estimation uses rolling ten years of data, so the actual strategies begin ten years later for this combination method. For this method, $1 investment in the 162-anomaly combination strategy grows to $1.39, while $1 investment in the combination strategy that includes LSTM grows to $1.44.

Panel D shows results using "Partial Least Squares" as the combination method. This method sorts stocks on the basis of forecast excess returns, where these are calculated from partial least squares (PLS) filtering procedure following Light et al. (2017) using only data in the investor's information set at the time of portfolio formation. The estimation uses rolling ten years of data, so the actual strategies begin ten years later for this combination method. For this method, $1 investment in the 162-anomaly combination strategy grows to $2.68, while $1 investment in the combination strategy that includes LSTM grows to $2.55.

Panel E shows results using "IPCA" as the combination method. This method sorts stocks on the basis of forecast excess returns, where these are calculated from the instrumented principal component analysis (IPCA) procedure of Kelly et al. (2019) using only data in the investor's information set at the time of portfolio formation. The estimation uses rolling ten years of data, so the actual strategies begin ten years later for this combination method. For this method, $1 investment in the 162-anomaly combination strategy grows to $1.02, while $1 investment in the combination strategy that includes LSTM grows to $0.80.

Panel F shows results using "LASSO" as the combination method. This method sorts stocks on the basis of forecast excess returns, where these are estimated by least absolute shrinkage and selection operator (LASSO) using only data in the investor's information set at the time of portfolio formation. Following Chen and Velikov (2022), LASSO penalty (λ) is selected by minimizing the mean squared error (MSE) estimated by 5-fold cross validation. The estimation uses rolling ten years of data,

so the actual strategies begin ten years later for this combination method. For this method, $1 investment in the 162-anomaly combination strategy grows to $1.26, while $1 investment in the combination strategy that includes LSTM grows to $0.95.

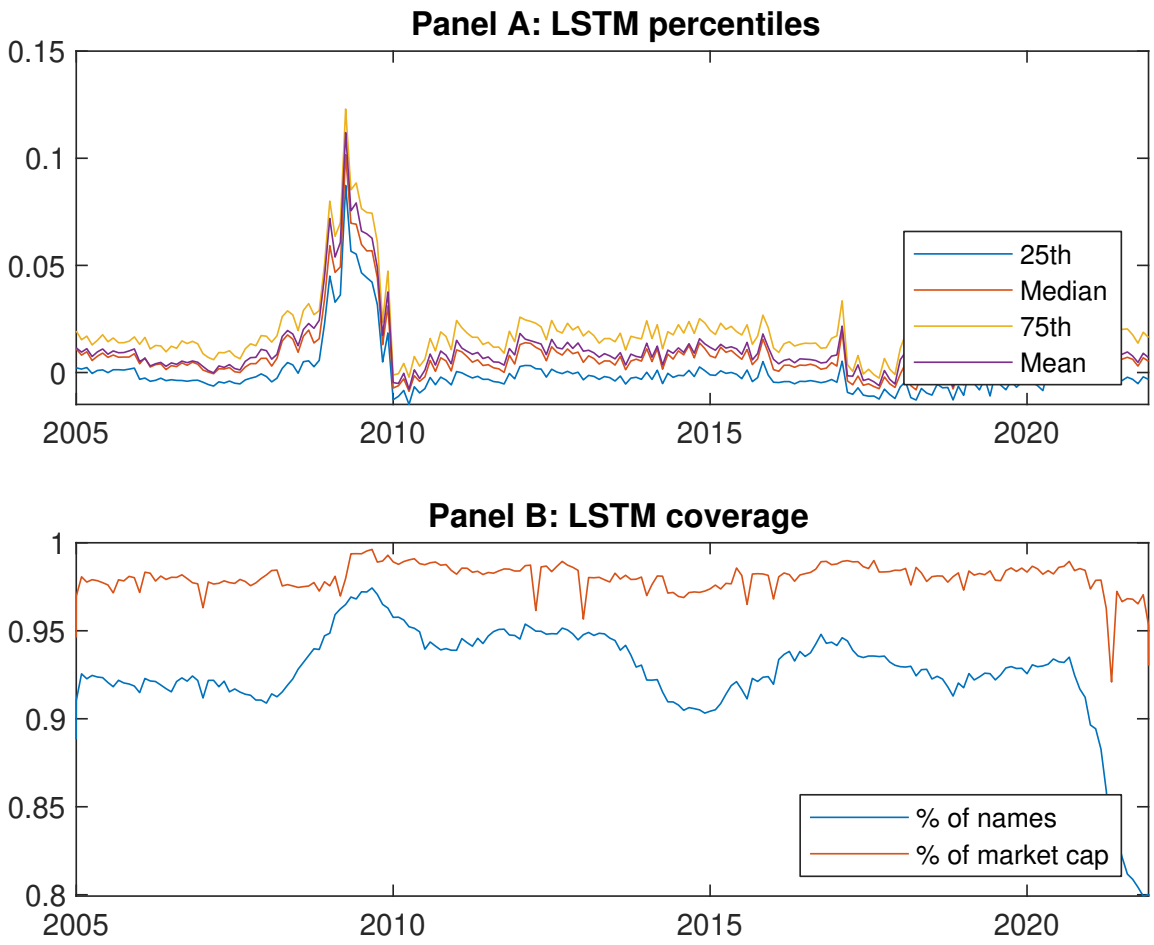


Figure 1: Times series of LSTM percentiles and coverage.
This figure plots descriptive statistics for LSTM. Panel A shows cross-sectional percentiles of LSTM over the sample. Panel B plots the monthly coverage of LSTM relative to the universe of CRSP stocks with available market capitalizations.

Table 1: Basic sort: VW, quintile, NYSE-breaks

This table reports average excess returns and alphas for portfolios sorted on LSTM. At the end of each month, we sort stocks into five portfolios based on their signal using NYSE breakpoints. Panel A reports average value-weighted quintile portfolio (L,2,3,4,H) returns in excess of the risk-free rate, the long-short extreme quintile portfolio (H-L) return, and alphas with respect to the CAPM, Fama and French (1993) three-factor model, Fama and French (1993) three-factor model augmented with the Carhart (1997) momentum factor, Fama and French (2015) five-factor model, and the Fama and French (2015) five-factor model augmented with the Carhart (1997) momentum factor following Fama and French (2018). Panel B reports the factor loadings for the quintile portfolios and long-short extreme quintile portfolio in the Fama and French (2015) five-factor model. Panel C reports the average number of stocks and market capitalization of each portfolio. T-statistics are in brackets. The sample period is 200412 to 202111.

Panel A: Excess returns and alphas on LSTM-sorted portfolios						
	(L)	(2)	(3)	(4)	(H)	(H-L)
r^e	0.21	0.75	0.88	1.02	1.30	1.09
	[0.53]	[2.48]	[2.94]	[3.24]	[3.65]	[4.77]
α_{CAPM}	-0.77	-0.05	0.09	0.20	0.40	1.17
	[-5.04]	[-0.76]	[1.42]	[2.41]	[3.06]	[5.03]
α_{FF3}	-0.75	-0.05	0.08	0.23	0.46	1.21
	[-4.92]	[-0.75]	[1.18]	[2.83]	[3.71]	[5.25]
α_{FF4}	-0.70	-0.04	0.08	0.22	0.47	1.16
	[-4.92]	[-0.60]	[1.21]	[2.71]	[3.73]	[5.16]
α_{FF5}	-0.58	-0.07	0.02	0.22	0.44	1.03
	[-3.90]	[-1.09]	[0.30]	[2.62]	[3.45]	[4.41]
α_{FF6}	-0.54	-0.06	0.02	0.21	0.45	0.99
	[-3.90]	[-0.97]	[0.33]	[2.53]	[3.47]	[4.34]
Panel B: Fama and French (2018) 6-factor model loadings for LSTM-sorted portfolios						
β_{MKT}	1.07	0.96	0.98	0.99	1.04	-0.03
	[30.12]	[58.25]	[59.61]	[47.21]	[31.31]	[-0.60]
β_{SMB}	0.01	-0.02	-0.02	-0.01	0.12	0.11
	[0.15]	[-0.75]	[-0.81]	[-0.34]	[2.11]	[1.11]
β_{HML}	-0.08	-0.01	-0.05	0.15	0.14	0.22
	[-1.30]	[-0.26]	[-1.77]	[4.28]	[2.66]	[2.30]
β_{RMW}	-0.39	0.06	0.14	0.02	0.02	0.41
	[-4.82]	[1.50]	[3.75]	[0.41]	[0.31]	[3.12]
β_{CMA}	-0.10	0.02	0.03	0.01	0.08	0.18
	[-0.97]	[0.49]	[0.72]	[0.10]	[0.89]	[1.10]
β_{UMD}	-0.20	-0.04	-0.01	0.04	-0.02	0.18
	[-5.84]	[-2.55]	[-0.50]	[2.12]	[-0.57]	[3.23]
Panel C: Average number of firms (n) and market capitalization (me)						
n	879	612	607	656	937	
me ($\$10^6$)	2996	4913	4982	4600	3010	

Table 2: Robustness to sorting methodology & trading costs

This table evaluates the robustness of the choices made in the LSTM strategy construction methodology. In each panel, the first row shows results from a quintile, value-weighted sort using NYSE break points as employed in Table 1. Each of the subsequent rows deviates in one of the three choices at a time, and the choices are specified in the first three columns. For each strategy construction methodology, the table reports average excess returns and alphas with respect to the CAPM, Fama and French (1993) three-factor model, Fama and French (1993) three-factor model augmented with the Carhart (1997) momentum factor, Fama and French (2015) five-factor model, and the Fama and French (2015) five-factor model augmented with the Carhart (1997) momentum factor following Fama and French (2018). Panel A reports average returns and alphas with no adjustment for trading costs. Panel B reports net average returns and Novy-Marx and Velikov (2016) generalized alphas as prescribed by Detzel et al. (2022). T-statistics are in brackets. The sample period is 200412 to 202111.

Panel A: Gross Returns and Alphas								
Portfolios	Breaks	Weights	r^e	α_{CAPM}	α_{FF3}	α_{FF4}	α_{FF5}	α_{FF6}
Quintile	NYSE	VW	1.09	1.17	1.21	1.16	1.03	0.99
			[4.77]	[5.03]	[5.25]	[5.16]	[4.41]	[4.34]
Quintile	NYSE	EW	2.06	2.06	2.06	2.08	1.90	1.92
			[10.35]	[10.16]	[10.24]	[10.41]	[9.38]	[9.55]
Quintile	Name	VW	1.56	1.63	1.68	1.70	1.45	1.47
			[5.12]	[5.27]	[5.46]	[5.54]	[4.66]	[4.74]
Quintile	Cap	VW	0.80	0.80	0.83	0.79	0.72	0.69
			[4.47]	[4.38]	[4.54]	[4.43]	[3.87]	[3.79]
Decile	NYSE	VW	1.72	1.90	1.92	1.91	1.53	1.52
			[4.70]	[5.15]	[5.25]	[5.20]	[4.25]	[4.20]
Panel B: Net Returns and Novy-Marx and Velikov (2016) generalized alphas								
Portfolios	Breaks	Weights	r^e_{net}	α^*_{CAPM}	α^*_{FF3}	α^*_{FF4}	α^*_{FF5}	α^*_{FF6}
Quintile	NYSE	VW	0.95	0.94	0.98	0.96	0.81	0.80
			[4.14]	[4.15]	[4.33]	[4.31]	[3.61]	[3.59]
Quintile	NYSE	EW	1.09	0.99	0.98	0.99	0.85	0.86
			[5.72]	[5.32]	[5.32]	[5.40]	[4.68]	[4.75]
Quintile	Name	VW	1.37	1.31	1.36	1.37	1.16	1.17
			[4.56]	[4.42]	[4.58]	[4.64]	[3.91]	[3.96]
Quintile	Cap	VW	0.69	0.63	0.66	0.64	0.56	0.55
			[3.88]	[3.51]	[3.66]	[3.64]	[3.10]	[3.08]
Decile	NYSE	VW	1.51	1.53	1.56	1.55	1.20	1.19
			[4.13]	[4.29]	[4.36]	[4.34]	[3.46]	[3.45]

Table 3: Conditional sort on size and LSTM

This table presents results for conditional double sorts on size and LSTM. In each month, stocks are first sorted into quintiles based on size using NYSE breakpoints. Then, within each size quintile, stocks are further sorted based on LSTM. Finally, they are grouped into twenty-five portfolios based on the intersection of the two sorts. Panel A presents the average returns to the 25 portfolios, as well as strategies that go long stocks with high LSTM and short stocks with low LSTM .Panel B documents the average number of firms and the average firm size for each portfolio. The sample period is 200412 to 202111.

Panel A: portfolio average returns and time-series regression results

		LSTM Quintiles					LSTM Strategies					
		(L)	(2)	(3)	(4)	(H)	r^e	α_{CAPM}	α_{FF3}	α_{FF4}	α_{FF5}	α_{FF6}
Size quintiles	(1)	-0.34	0.33	0.92	1.43	2.28	2.62	2.55	2.55	2.65	2.28	2.36
		[-0.62]	[0.76]	[2.17]	[3.31]	[3.83]	[7.80]	[7.47]	[7.61]	[8.33]	[6.75]	[7.44]
	(2)	0.27	0.70	1.03	1.12	1.41	1.14	1.25	1.29	1.29	1.03	1.03
		[0.51]	[1.63]	[2.54]	[2.70]	[2.94]	[4.42]	[4.82]	[5.23]	[5.21]	[4.35]	[4.33]
	(3)	0.39	0.87	1.06	1.02	1.09	0.71	0.78	0.76	0.67	0.49	0.41
		[0.81]	[2.25]	[2.70]	[2.54]	[2.54]	[2.91]	[3.14]	[3.09]	[2.97]	[2.04]	[1.89]
	(4)	0.49	0.78	0.88	1.00	1.19	0.70	0.76	0.78	0.72	0.61	0.56
		[1.16]	[2.29]	[2.47]	[2.83]	[3.03]	[3.39]	[3.67]	[3.82]	[3.71]	[2.99]	[2.90]
	(5)	0.34	0.81	0.90	0.94	1.14	0.80	0.76	0.80	0.76	0.72	0.69
		[1.06]	[2.71]	[3.13]	[3.17]	[3.45]	[4.03]	[3.75]	[4.01]	[3.89]	[3.51]	[3.41]

Panel B: Portfolio average number of firms and market capitalization

		LSTM Quintiles					LSTM Quintiles				
		Average n					Average market capitalization ($\$10^6$)				
		(L)	(2)	(3)	(4)	(H)	(L)	(2)	(3)	(4)	(H)
Size quintiles	(1)	384	384	384	384	384	68	76	74	68	45
	(2)	123	123	123	123	123	118	121	122	121	119
	(3)	87	87	87	87	87	202	208	208	207	205
	(4)	75	75	75	75	75	436	448	446	441	436
	(5)	69	69	69	69	69	2919	3175	3396	3483	3358

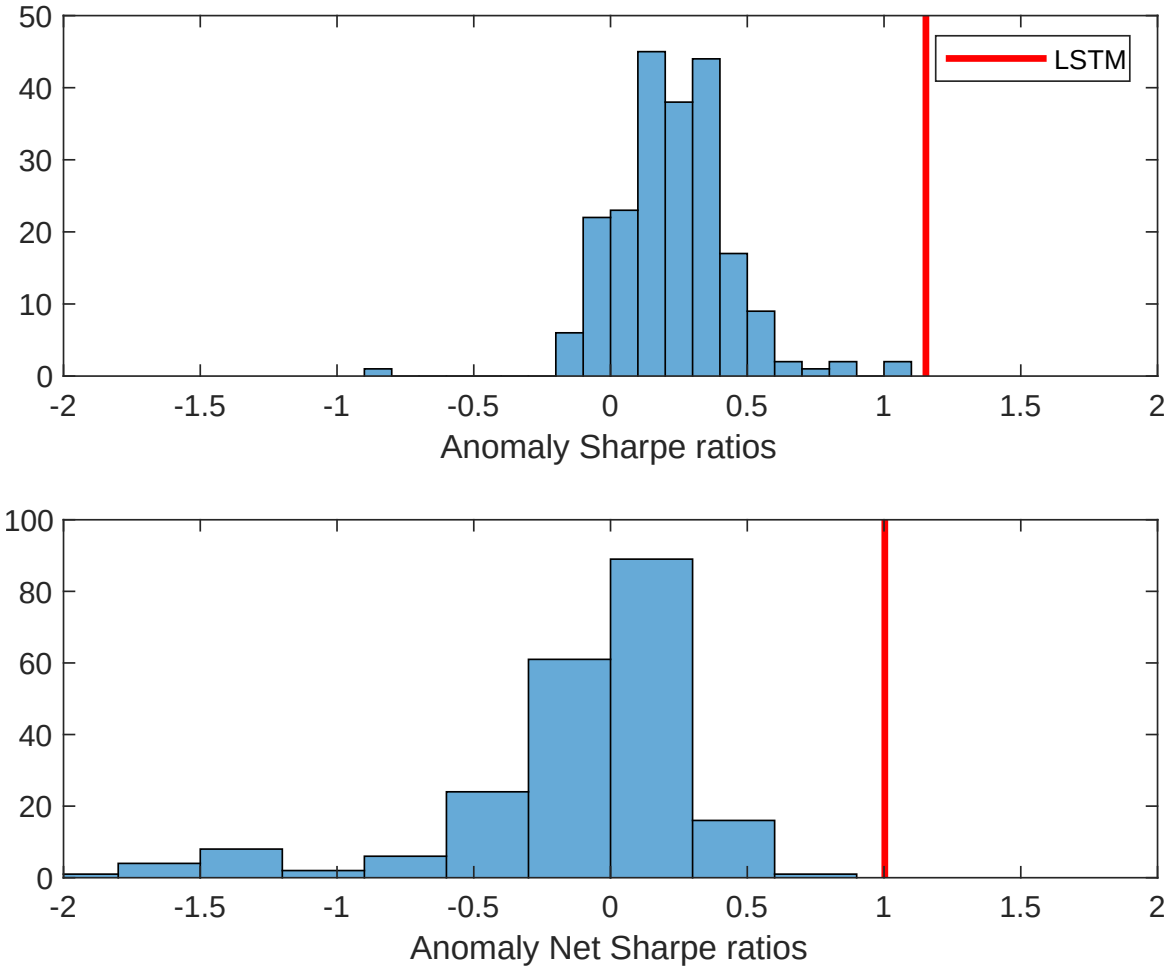


Figure 2: Distribution of Sharpe ratios.
This figure plots a histogram of Sharpe ratios for 212 anomalies, and compares the Sharpe ratio of the LSTM with them (red vertical line). Panel A plots results for gross Sharpe ratios. Panel B plots results for net Sharpe ratios.

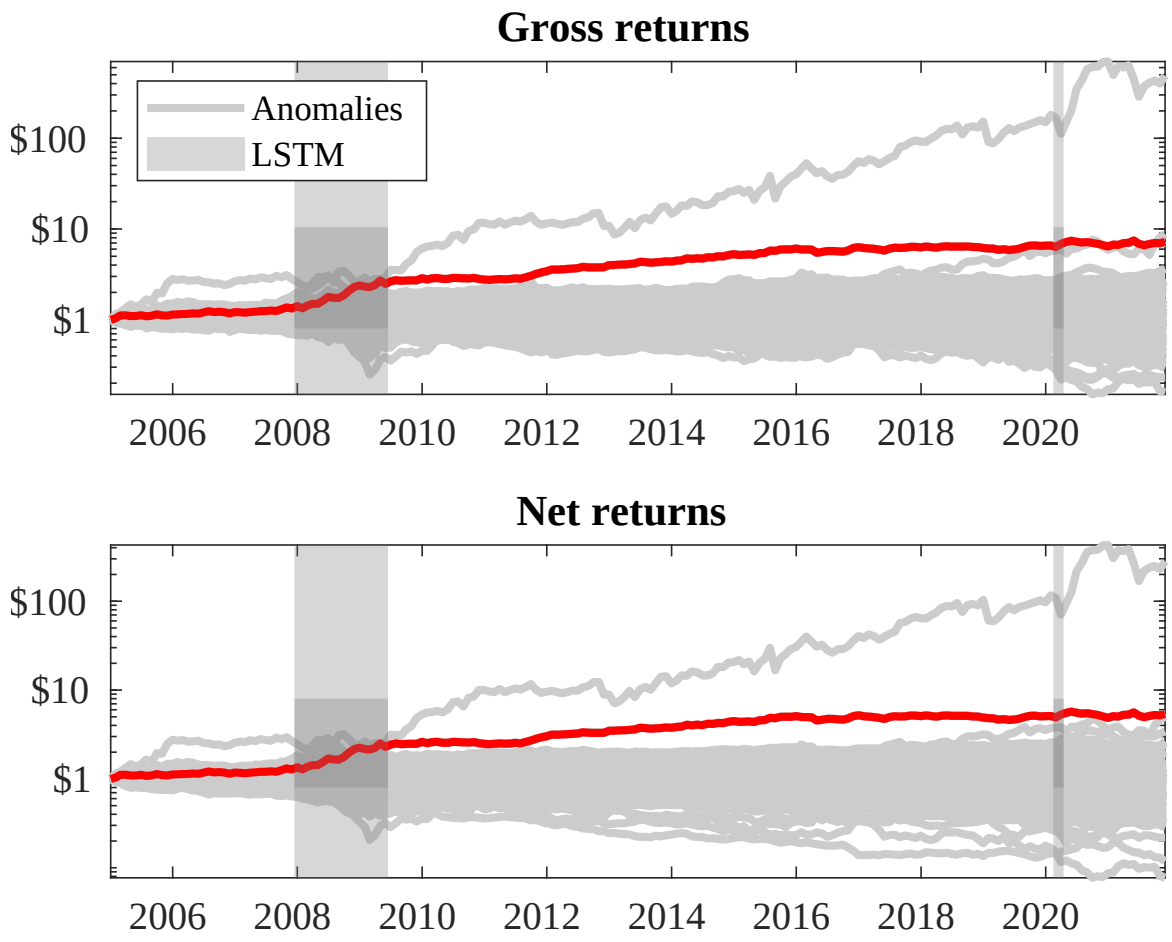


Figure 3: Dollar invested.
This figure plots the growth of a $1 invested in 212 anomaly trading strategies (gray lines), and compares those with the LSTM trading strategy (red line). The strategies are constructed using value-weighted quintile sorts using NYSE breakpoints. Panel A plots results for gross strategy returns. Panel B plots results for net strtaegy returns.

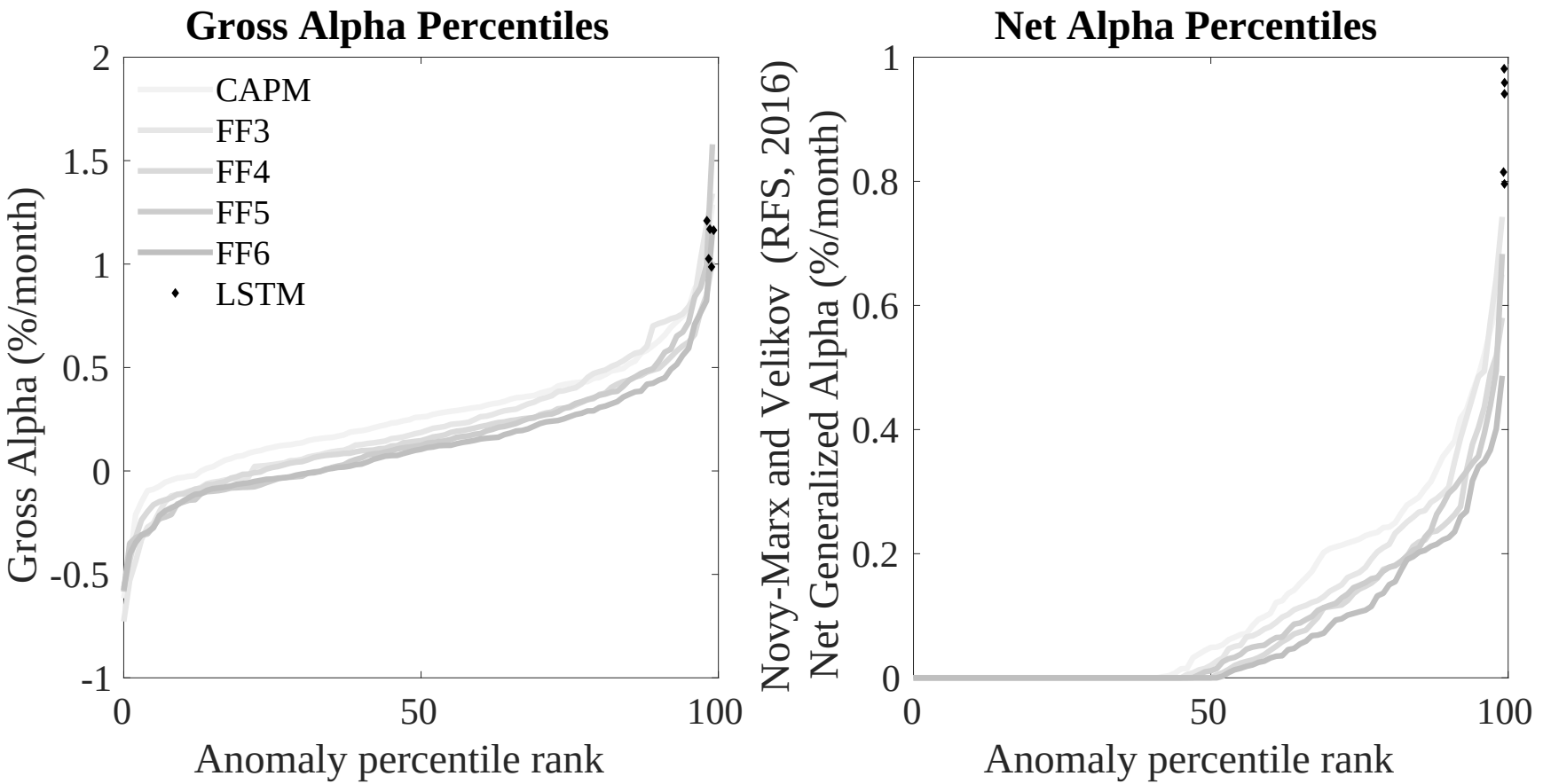
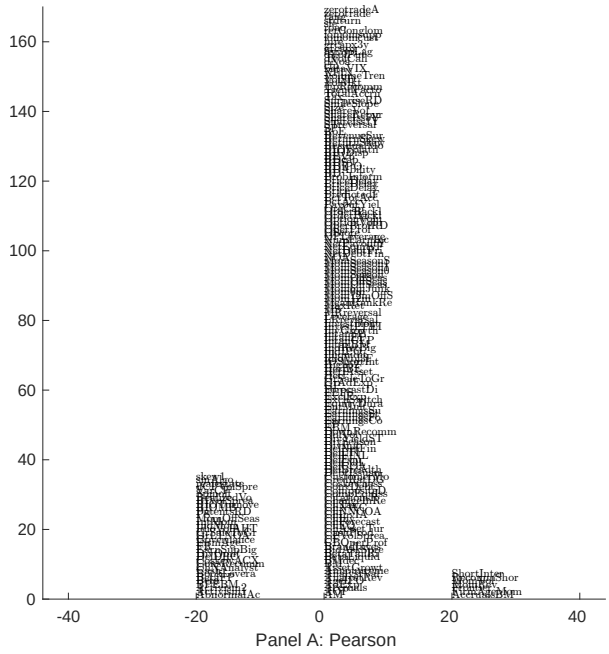


Figure 4: Gross and generalized net alpha percentiles of anomalies relative to factor models

This figure plots the percentile ranks for 212 anomaly trading strategies in terms of alphas (solid lines), and compares those with the LSTM trading strategy alphas (diamonds). The strategies are constructed using value-weighted quintile sorts using NYSE breakpoints. The alphas include those with respect to the CAPM, Fama and French (1993) three-factor model, Fama and French (1993) three-factor model augmented with the Carhart (1997) momentum factor, Fama and French (2015) five-factor model, and the Fama and French (2015) five-factor model augmented with the Carhart (1997) momentum factor following Fama and French (2018). The left panel plots alphas with no adjustment for trading costs. The right panel plots Novy-Marx and Velikov (2016) net generalized alphas.



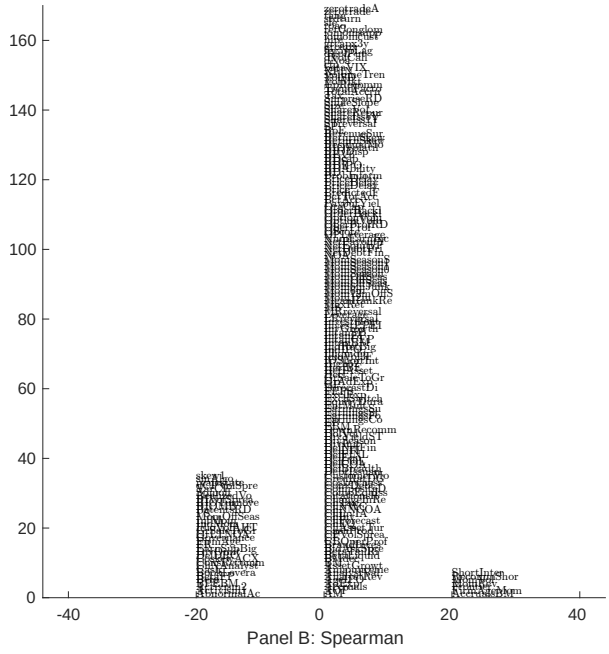


Figure 5: Distribution of correlations.
This figure plots a name histogram of correlations of 211 filtered anomaly signals with LSTM. The correlations are pooled. Panel A plots Pearson correlations, while Panel B plots Spearman rank correlations.

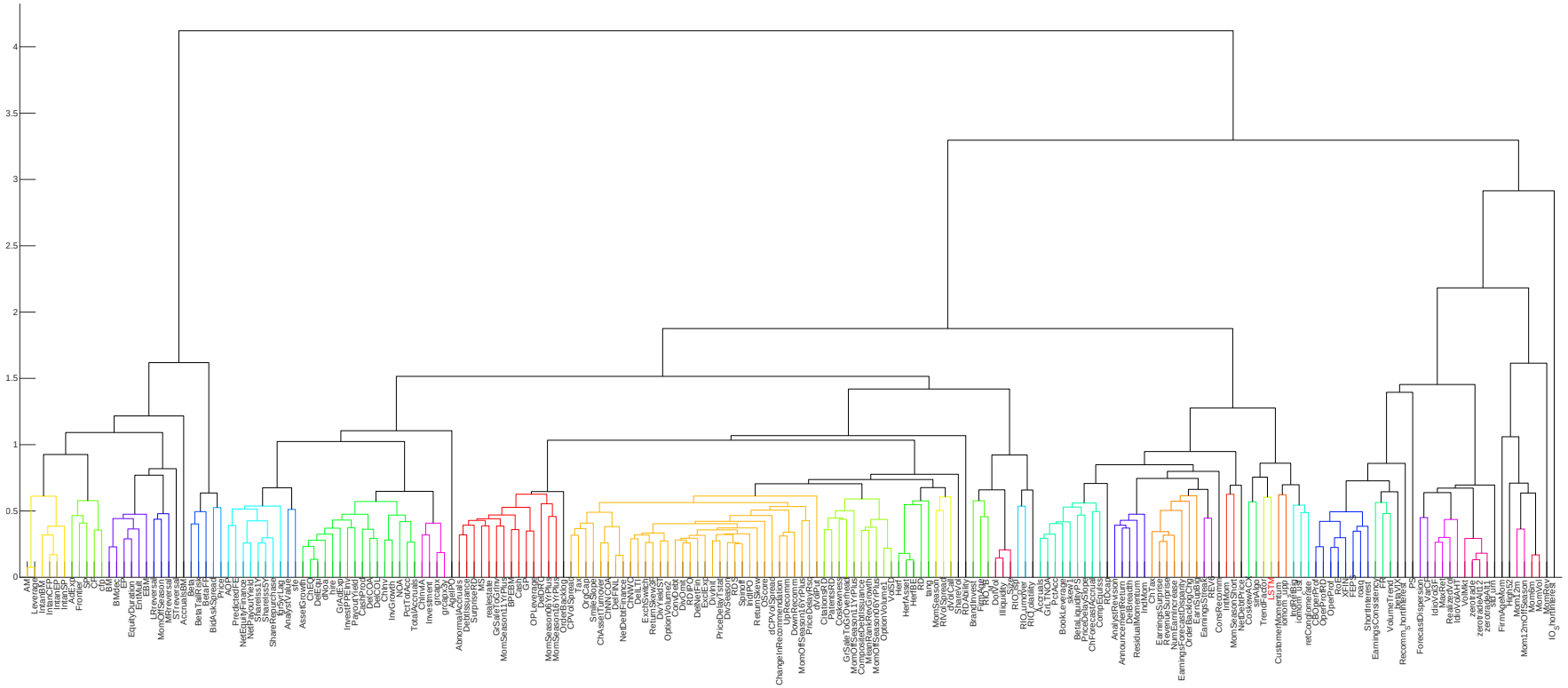


Figure 6: Agglomerative hierarchical cluster plot
This figure plots an agglomerative hierarchical cluster plot using Ward's minimum method and a maximum of 10 clusters.

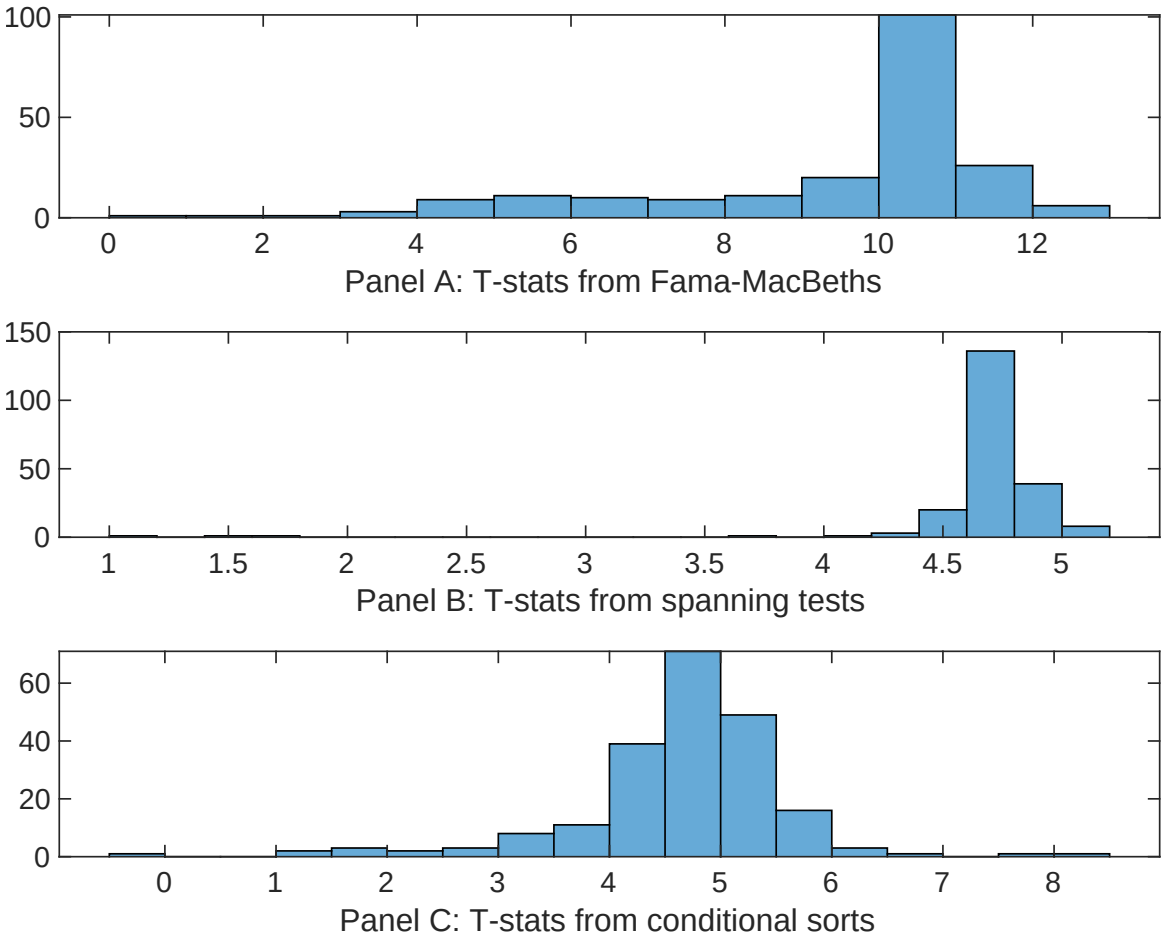


Figure 7: Distribution of t-stats on conditioning strategies
This figure plots histograms of t-statistics for predictability tests of LSTM conditioning on each of the 211 filtered anomaly signals one at a time. Panel A reports t-statistics on β_{LSTM} from Fama-MacBeth regressions of the form $r_{i,t} = \alpha + \beta_{LSTM} LSTM_{i,t} + \beta_X X_{i,t} + \epsilon_{i,t}$, where X stands for one of the 211 filtered anomaly signals at a time. Panel B plots t-statistics on α from spanning tests of the form: $r_{LSTM,t} = \alpha + \beta r_{X,t} + \epsilon_t$, where $r_{X,t}$ stands for the returns to one of the 211 filtered anomaly trading strategies at a time. The strategies employed in the spanning tests are constructed using quintile sorts, value-weighting, and NYSE breakpoints. Panel C plots t-statistics on the average returns to strategies constructed by conditional double sorts. In each month, we sort stocks into quintiles based one of the 211 filtered anomaly signals at a time. Then, within each quintile, we sort stocks into quintiles based on LSTM. Stocks are finally grouped into five LSTM portfolios by combining stocks within each anomaly sorting portfolio. The panel plots the t-statistics on the average returns of these conditional double-sorted LSTM trading strategies conditioned on each of the 211 filtered anomalies.

Table 4: Fama-MacBeths controlling for most closely related anomalies
This table presents Fama-MacBeth results of returns on LSTM. and the six most closely related anomalies. The regressions take the following form: $r_{i,t} = \alpha + \beta_{LSTM} LSTM_{i,t} + \sum_{k=1}^{s} ix\beta_{X_k} X_{i,t}^{k} + \epsilon_{i,t}$. The six most closely related anomalies, X, are Momentum based on FF3 residuals, Earnings surprise streak, Industry return of big firms, Net external financing, Momentum in high volume stocks, Return on assets (qtrly). These anomalies were picked as those with the highest combined rank where the ranks are based on the absolute value of the Spearman correlations in Panel B of Figure 5 and the R^2 from the spanning tests in Figure 7, Panel B. The sample period is 200412 to 202111.

Intercept	0.41	0.44	0.11	0.42	0.56	0.39	-0.95
	[1.04]	[1.04]	[0.26]	[1.05]	[0.87]	[0.98]	[-0.13]
LSTM	0.47	0.42	0.51	0.45	0.34	0.46	0.27
	[10.24]	[8.52]	[10.54]	[9.97]	[5.46]	[10.40]	[2.87]
Anomaly 1	-0.16						-0.30
	[-0.68]						[-0.05]
Anomaly 2		0.14					0.25
		[4.36]					[0.46]
Anomaly 3			0.39				0.67
			[1.32]				[1.25]
Anomaly 4				0.72			0.14
				[1.75]			[1.98]
Anomaly 5					-0.61		0.70
					[-0.11]		[1.15]
Anomaly 6						0.28	-0.31
						[1.67]	[-1.17]
# months	203	203	203	203	203	203	203
$\bar{R}^2(\%)$	1	1	1	1	2	2	0

Table 5: Spanning tests controlling for most closely related anomalies
This table presents spanning tests results of regressing returns to the LSTM trading strategy on trading strategies exploiting the six most closely related anomalies. The regressions take the following form: $r_t^{LSTM} = \alpha + \sum_{k=1}^{6} \beta_{X_k} r_t^{X_k} + \sum_{j=1}^{6} \beta_{f_j} r_t^{f_j} + \epsilon_t$, where X_k indicates each of the six most-closely related anomalies and f_j indicates the six factors from the Fama and French (2015) five-factor model augmented with the Carhart (1997) momentum factor. The six most closely related anomalies, X, are Momentum based on FF3 residuals, Earnings surprise streak, Industry return of big firms, Net external financing, Momentum in high volume stocks, Return on assets (qtrly). These anomalies were picked as those with the highest combined rank where the ranks are based on the absolute value of the Spearman correlations in Panel B of Figure 5 and the R^2 from the spanning tests in Figure 7, Panel B. The sample period is 200412 to 202111.

Intercept	0.92	0.78	0.83	0.89	0.91	0.81	0.72
	[4.25]	[3.50]	[3.71]	[4.04]	[4.04]	[3.68]	[3.38]
Anomaly 1	34.97						31.47
	[3.82]						[3.19]
Anomaly 2		27.95					19.93
		[3.36]					[2.45]
Anomaly 3			13.37				8.06
			[2.68]				[1.59]
Anomaly 4				34.39			27.86
				[2.53]			[1.89]
Anomaly 5					2.64		-10.23
					[0.41]		[-1.61]
Anomaly 6						37.07	18.11
						[3.62]	[1.62]
mkt	-1.38	0.95	0.94	1.86	-2.05	3.74	8.24
	[-0.25]	[0.17]	[0.16]	[0.31]	[-0.35]	[0.64]	[1.43]
smb	11.20	12.18	13.98	27.29	16.25	29.67	23.08
	[1.15]	[1.24]	[1.41]	[2.53]	[1.62]	[2.86]	[2.18]
hml	10.37	15.35	18.38	16.94	15.50	21.52	16.93
	[1.10]	[1.63]	[1.92]	[1.78]	[1.60]	[2.26]	[1.82]
rmw	36.55	38.26	41.32	28.38	44.62	9.63	1.09
	[2.81]	[2.93]	[3.15]	[1.94]	[3.35]	[0.60]	[0.07]
cma	9.26	9.11	7.26	-5.34	9.93	10.90	-3.83
	[0.58]	[0.56]	[0.44]	[-0.30]	[0.59]	[0.68]	[-0.23]
umd	1.94	8.97	18.83	14.51	15.40	8.32	0.13
	[0.28]	[1.47]	[3.40]	[2.55]	[1.82]	[1.37]	[0.01]
# months	203	203	203	203	203	203	203
$\bar{R}^2$(%)	16	14	12	12	9	15	23

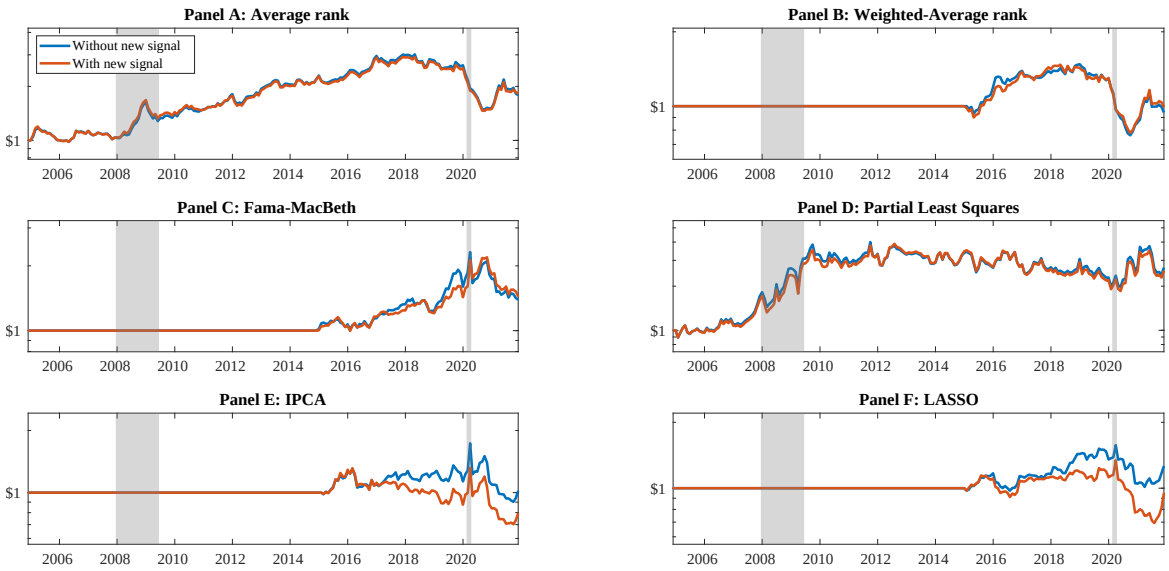


Figure 8: Combination strategy performance
This figure plots the growth of a $1 invested in trading strategies that combine multiple anomalies following Chen and Velikov (2022). In all panels, the blue solid lines indicate combination trading strategies that utilize 162 anomalies. The red solid lines indicate combination trading strategies that utilize the 162 anomalies as well as LSTM. Panel A shows results using "Average rank" as the combination method. Panel B shows results using "Weighted-Average rank" as the combination method. Panel C shows results using "Fama-MacBeth" as the combination method. Panel D shows results using "Partial Least Squares" as the combination method. Panel E shows results using "IPCA" as the combination method. Panel F shows results using "LASSO" as the combination method. See Section 6 for details on the combination methods.

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