Introducing Predictive Equity Analytics: New Empirical Findings in Asset Pricing

Mark Zanecki ASA, MAAA

IHA Consultants

(August 2024, v 1.0.0)

Abstract: This paper contains material that is empirical, and exploratory in nature which is collectively referred to as "predictive equity analytics." Predictive equity analytics' focus is the understanding of equity time-series close return dynamics resulting from point-wise deviations of the standard independent and identically distributed "(i.i.d.") assumption. A goal of predictive equity analytics is to measure the effect of point-wise deviation of "i.i.d." Our ML and AI-enabled analysis is underway building new tools to measure time-varying dynamics, which lead to new results that are so significant that at this juncture we would like to share. The new findings are supported by linear algebra (row space, column space), principle component analysis (pca), statistically robust T-tests, and random matrix theory. Predictive equity analytics holds the potential to significantly transform many aspects of current asset pricing by fundamentally altering the risk return trade-off paradigm. Depth and breadth includes concepts such as the pricing of systemic risk, Q-measure, mutual fund theorem, portfolio optimization, efficient frontier, efficient portfolio static diversification. Sharpe ratio and risk aversion.

Significant findings:

1) When volatility level is nominal, the eigenvalues and eigenvectors of proxy drift/meu $\widetilde{proxy}(\mu_t)$ and $\Delta\widetilde{proxy}(\mu_t)$ are predictive statistically with significance(alpha = 0.05, 0.1) and time varying in intensity for equities, ETFs, mutual funds and target date funds.
2) Retrospective detection of change-point occurrence in as little as 1 – 3 trade days using new finding that the coefficients for each element of principle eigenvector are simultaneous multiplied by "-1" relative to prior measurement indicates change-point with 100% accuracy for sufficient differential for equities, ETFs, mutual funds, and target date funds.
3) When volatility is "heavy or extreme, forecast of a change-point occurrence in next 1 – 3 trade-days is highly likely for equities, ETFs, mutual funds, and target date funds. Evidence in support only change-point is common point with forward -time signal set and time reversed signal set. We conjecture (3) holds for time reverse principle eigenvector.

Keywords: Predictive equity analytics, portfolio optimization, Sharpe ratio, time-varying stochastic diffusion with drift, discrete Fokker-Planck approximation.

1. Introduction

Predictive equity analytics, at its core, concerns inference methods for diffusion processes observed at discrete points in time. First attempts focused on the feasibility of creating sufficiently accurate discrete approximations using Gaussian distribution assumption. (See Florens-Zmirou, D. (1989, 1993).) Research continued along two paths: a) optimal maximum likelihood approximation techniques and b) non-parametric techniques. (See Poulsen (1999).) The majority of research has since focused on various versions of dispersion coefficient estimation. In this paper, we focus attention on and is our main contribution, to the predictive use of a sufficiently accurate discrete approximation for the unobservable μ_t, referred to as $\widetilde{proxy}(\mu_t)$ as well as $\Delta proxy(\mu_t)$ for portfolio investment purposes.[1]

Building on core capabilities, predictive equity analytics seeks to predict change-point likely occurrence in next 1 - 3 trade days that has a "buy action" (expected mean > 1, negative skewness) as indicators of $sign_t(R_{t+1}) > 0$ and if owned, a "sell action" (expected mean < 1, positive skewness) as indicators of $sign_t(R_{t+1}) < 0$. Predictive equity analytics extends Markowitz (1952, 1959) (expected return, volatility) to (expected return, volatility, skewness, and predictive signals of future 1 – 3 trade day change-point occurrence). One can point to Wald's theorem which states that a stationary time-series must decompose into a predictable component and an unpredictable component must apply to equity close return time- series for the period commencing after market close and prior to market open on the next trade day. Also, French and Roll 1985, found that variance per unit of time of markets over periods when they exchanges were closed is smaller than when open. To my knowledge, no formal proof has been put forward asserting impossibility of predicting short-run close return time-series and convincing demonstrating it.[2]

---

[1] Signal set creation of a discrete approximation for the unobservable μ_t and referred to as $\widetilde{proxy}(\mu_t)$ is found at Zanecki (2020, 2021).

[2] Short-run equity close return prediction problem has been characterized as "exceedingly difficult." Merton (1980, 1992)points to measurement with error of the drift/meu term as an obstacle since drift/meu Note:

Predictive equity analytics at the highest level, is a multi-faceted framework that seeks predictive signals in a) close price return series r_t, b) investment factors vector θ_t , c) micro-economics, and d) macro-economics for all major global exchanges such as NYSE, Nasdaq, TSE, LSE, Euronext, etc. See Figure 1.

The scope of this paper is restricted to uses of private predictive signals for the sign of the close return process r_t for equities, ETFs, and mutual funds. In a future paper, firms with favorable private signals are analyzed in terms of investments factor that best explain the change in sign of the favorable return process and using these identified active investment factors other firms are identified for possible investment.[3] Figure 2 depicts 5 equities (stock, ETF, mutual fund) where when favorable sign prediction prior to market open purchase and when unfavorable sign prediction prior to market open sell. This process is called dynamic diversification and has significantly less risk than static diversification with faster and higher wealth accumulation. Market downturns or single equity downturns are predicted in advance and updated daily prior to market open each trade day with accuracy exceeding 95% excluding news events.

A natural starting point is the next trade day sign of a time-varying close return process which is in the spirits of (Christoffersen and Diebold 2002) and (Kelly, Malamud, and Zhou 2022 paper albeit with changes.[4]

$$sign_t(R_{t+1}) = f(\theta_t) + \varepsilon_{t+1} \qquad (1)$$

---

although the drift/meu magnitude is exceedingly small, proxy drift/meu signals are detectable, time varying in intensity and statistically significant.

[3] This is the familiar APT model, only market analytics using machine learning techniques identify likely investment factors correlated to the predicted change-point.

[4] This setup is slightly different from Kelly, Malamud, Zhou paper where $R_{t+1} = f(G_t) + \epsilon_{t+1}$, G_t is a vector of predictive signals derived from investment factors and the focus of the paper is complexity and ML focused. Our starting point is $sign_t(R_{t+1}), and\ f(\theta_t)$, where θ_t is a vector of variables representing investment factors. We look for discovery of predictive signals directly from the left-hand side of (1), $(\Omega_t, sign_t(R_{t+1}))$ as first step and in second step use machine learning to identity factors , factor levels and changes in factor levels that explain $sign_t(R_{t+1})$ in the principle component sense using first conditional mean dependence and second by variance dependence. The scope of this paper is signal discovery of left-hand side of (1).

where θ_t is a vector of variables representing investment factors, $f(\square)$ is an unknown continuous function that can be approximated by piece-wise linear framework or modeled by a sufficiently wide neural network e.g., (Hornik, Stinchcombe, White (1990)). Each factor in $\theta_t$is measurable and effect on R_{t+1} is known or supported by subjective beliefs. So, measure θ_t , then infer effect on future $sign_t(R_{t+1})$ behavior. The analysis typically proceeds in cross-sectional form focusing on the eigenvalues and eigenvectors of the variance-covariance matrix. Predictive equity analytics will show that at nominal volatility levels, the eigenvalues and eigen vectors of proxy drift/meu are tier 1 predictive and when volatility is "heavy" the eigenvalues and eigenvectors of variance process are tier 1predictive.[5] Change-points occur at epochs of heavy volatility and are predicted 1 – 3 trade days in advance with high reliability.

Conceptually, **predictive equity analytics (PEA)** reverses (1) by finding the inverse function, $f^{-1}(\square)$,"at a sufficiently sensitive level to be predictive but overall coarse level to mitigate any measurement with error effects." Next, classify r_t in terms of next trade-day sign prediction for each equity time series and then associate first at the firm, then sector, and industry levels the set of factors as well as changes in the factor levels that are responsible for the close return time-series price behavior.[6] In this way, the market points out which factors, factor levels and changes in factor levels are relevant which removes subjective opinion and renders the process 100% data analytics. As a result, the value of security research is significantly increased and also is now focused on "securities on the move" as first order priority and general research as a secondary priority. Accuracy assessment moving in each direction provides the fullest information set available.[7]

[5] Scope is at single equity level, cross-sectional analysis is out of scope for this paper. The effect of news events subsequent to market close is also out of scope.
[6] There is no pre-conceived list of factors as found in the literature and testing of the standard factor list via machine learning would yields new insights.
[7] Predictive equity analytics predicts sets of equities with favorable/unfavorable change-points. Identifying the factors, factor levels and change in factor levels is performed by machine learning using neural nets and data lakes. In this way predictive equity analytics sharpens the value-add of ML, AI, and data lakes.

The inverse function is assumed to be an unknown continuous, time-varying, multi-variate function that generates a finite set of signals $\{\boldsymbol{Signal_t(1)}, \boldsymbol{Signal_t(2)}, \boldsymbol{Signal_t(3)}\}$ against an overall noisy background and is private information not known to other investors or the public due to complexity, cost and operations resource requirements. We will refer to time-varying public information set, Ω_t , which contains both close return time-series and factor information. The signal set at time t will be used as a filter to partition the noisy time-varying information set Ω_t into 3 partitions $\{A_t$= Poisson point process, B_t= heavy variance point process, C_t= filtered close return point process$\}$.[8] The noisy time-varying information set Ω_t is recovered as $\{A_t B_t C_t\}$. The framework is applied separately to the close return time-series and to factor investment set θ_t. For the purposes of this paper, we will focus on the close return process and partitioning and leave the investment factors as a future paper.

The rest of this paper is organized as follows. Sections 2 and 3 introduce more precise definitions for time-varying measurement involving signal sets. Section 4 informs of the signal set details including 3 x 3 series of matrices and mapping of data points. Section 5 discusses validation of the signal sets using random matrix theory, robust t-test, and principle component analysis. Section 6 outlines using time-reversed signal set is different from time forward signal set for all points except change-points. This is further supported by linear algebra row space and column space arguments. Section 7 contrasts single distribution vs matrix $A_t B_t C_t$ view of same sample data. Section 8 is a refinement to (Christoffersen and Diebold 2002) finding that sign dynamics follows from volatility dynamic. Section 9 uses principle component analysis to specify a new retrospective test for change-point occurrence. Section 10 gives examples of signal sets for selected ETFs, mutual funds and target date funds include retrospective eigen analysis detection. Section 11 considers 9 different input sources that can be used to predict expected return for the next trade day distribution. Section 12 describes the discrete approximate Fokker-Planck available information and how time 0 signal of highest strength is detected. Section 13 is a refinement to

[8] This follows in spirit Merton's 1971, 1976 and 1980 papers and is also found in Continuous -time Finance, chapters 1 – 5.

prior findings. Section 14 highlights the impact of introducing predictability into the classic riskless and risky asset portfolio allocation. Section 15 Applies predictive equity analytics to fund manager portfolio optimization. Relevant extensions are listed in Section 16. Section 17 concludes with a summary of discussion and results. In the Appendix are additional stock illustrations and robust T-tests.

2. A More Precise Stationary Distribution Definition

An augmented stationary distribution concept is needed to address presence or absence of predictive signals generated by continuous stochastic process:

$$r_t = \mu_t + f(\sigma_t^2) + \varepsilon_t \qquad (2)$$

where μ_t represents instantaneous drift/meu and $f(\sigma_t^2)$ instantaneous dispersion/volatility. A discrete approximation consisting of a proxy for drift/meu, μ_t and volatility is utilized. The framework is needed to address time-varying signals for proxy drift/meu, changes in proxy drift/meu level, volatility and stability of maximal eigenvalue and associated eigenvector. The first component is filtering of the raw close return process into three components: a) Poisson outlier point process, b) heavy variance, and c) filtered close return process. Each of the three components is represented by a 3 x 3 matrix with columns representing thee levels of proxy drift/meu signal level (Signal(3)): {Signal(3)<0.95, Signal(3) ∈ [0.95, 0.97], Signal(3)>0.97}. Rows represent three levels of change in proxy meu signal level (drift/meu momentum): {momentum(-), momentum(0), momentum(+)}. Volatility is measured by Signal(1) at three levels: {Signal(1)<0.6, Signal(1) ∈ [0.6, 0.99], Signal(1)=1} which are referred to as nominal volatility, heavy volatility and Poisson outlier point process. The original noisy close return process, $A_t B_t C_t$ is recovered as sum of matrices: {A_t= Poisson point process, B_t= heavy variance point process, C_t= filtered close return point process}. In all there are 27 cells a.k.a. "states" with descriptive statistics: counts, average, standard deviation, skewness, kurtosis with drill-down to sample set data points.

	Signal(3)<0.95	Signal(3) ∈ [0.95, 0.97]	Signal(3)>0.97
momentum(+)			
momentum(0)			
momentum(-)			

Signal(1) ∈ [0, 1]

Figure 3 – Original Noisy Close Return, Matrix $A_t B_t C_t$

	Signal(3)<0.95	Signal(3) ∈ [0.95, 0.97]	Signal(3)>0.97
momentum(+)			
momentum(0)			
momentum(-)			

Signal(1) = 1

Figure 4 – Poisson Outlier Point Process, Matrix A_t

	Signal(3)<0.95	Signal(3) ∈ [0.95, 0.97]	Signal(3)>0.97
momentum(+)			
momentum(0)			
momentum(-)			

Signal(1) ∈ [0.6, 0.99]

Figure 5 – Heavy Volatility, Matrix B_t

	Signal(3)<0.95	Signal(3) ∈ [0.95, 0.97]	Signal(3)>0.97
momentum(+)			
momentum(0)			
momentum(-)			

Signal(1) < 0.6

Figure 6 – Filtered Close Return, Matrix C_t

Predictive equity analytics concept of stationary distribution is as follows: an equity close return time series such that:

1. Signal(1) – Volatility < 0.6 for all samples,
2. Signal(3) – Proxy meu signal strength is unchanging for all samples,
3. Meu momentum – Change in proxy drift/meu signal strength is unchanging for all samples,
4. the original noisy close return matrix $A_t B_t C_t$ = Filtered close return, Matrix C_t (Matrices $A_t = 0, B_t$=0),
5. the $\binom{3}{2}$ row combinations all fail to reject H_0 , no difference in means under robust Yuen-Welch T-test,
6. the $\binom{3}{2}$ column combinations all fail to reject H_0 , no difference in means under robust Yuen-Welch T-test,
7. the set of eigenvalues (eigen spectrum) is constant for all samples,
8. and the associated set of eigenvectors is component-wise constant for all samples.

Succinctly said, “the predictive equity analytics framework produced no new additional predictive information to the original equity close return time-series.” This referred to “predictive equity analytics graceful fallback to standard theory property."

Additionally, if the set of original data points is retained but the order altered even by 1 transposition the signal set $\{(signal_t(1),\ signal_t(2), signal_t(3))\}$ would require regeneration starting at point in time of the earliest of the two transposed data points and for all subsequent points. If any set of original data points is excluded, the original signal set $\{(signal_t(1), signal_t(2), signal_t(3))\}$ is predictively uninformative to the transformed original data set. In this sense, predictive equity analytics is "rather precise" and sets it apart from distributional or time series which are informative even if have missing or transformed points at sufficiently low levels.

3. Quantifying Independent and Identically Distributed Assumption on Sample Data

Our focus is on testing identically distributed and assume independence. We will characterize identically distributed to mean that matrix $A_t B_t C_t$ = matrix C_t and perform robust Yuen-Welch T-tests for all $\binom{9}{2}$ cell combinations + $\binom{3}{2}$ row combinations + $\binom{3}{2}$ row combinations + $\binom{6}{2}$ row vs column combinations + 6 total matrix vs columns and rows = 54 and neglect the remaining combinations since that will be of lower magnitude. Ideally identically distributed means that all robust T-tests failed to reject null hypothesis H_0: means are similar or were not performed pairwise due to insufficient data, but it is sufficient that all row combinations and all column combinations fail to reject the null hypothesis.

4. Signal Set Description and Generation

We desire to discover the existence of predictive signals for the sign of the discrete approximation to the standard continuous stochastic diffusion:

$$sign_t(r_t) = \mu_t + h(\sigma_t^2) \qquad (3)$$

commencing just after exchange close through just prior to market open of the subsequent trade day. This is the only epoch available to apply Wald's Theorem for a slowly evolving or semi-stationary/stationary time-series combined with French and Roll (1980) low variance while exchange is closed observation. If Wald's predictive component can be calculated in sufficient time to act prior to subsequent exchange open, then a dominating investment strategy with comparative advantage is created with superior returns and mitigated risk e.g., significantly higher Sharpe ratios. If successful, we would then proceed to the creation of discrete, time-varying Fokker-Planck approximation.

The critical issue centers on the concept of the "standard finding of conditional mean independence and validity thereof." The standard finding of variance conditional mean dependence is assumed. We find that conditional mean behavior using $\widetilde{proxy}(\mu_t)$ is dynamic, at times independent and at other times dependent with light intensity or dependent with significant intensity. Now, we seek a proxy $\tilde{\mu}_t$ for the unobservable μ_t based on $\tilde{\mu}_t = M(\mu_t)$. We are hoping for the existence of a discrete linear algebra transform upon a sufficiently slowly evolving / semi-stationary close return time series. Said another way, we do know of the existence of a discrete approximation to the Fokker-Planck equation? Answer: "yes. See Lo (1988)."

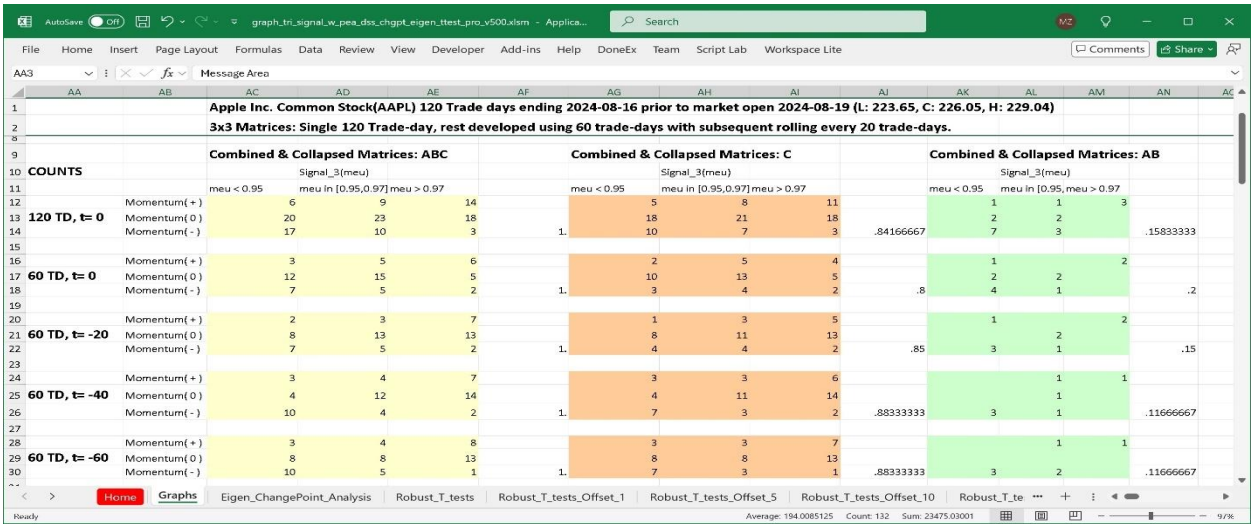
Now a significant new insight, the proxy $\tilde{\mu}_t$ and variance are used in a classification data tagging framework that allows complete mapping of each data point to a unique state space spanned by $\{A_t$= Poisson point process, B_t= heavy variance point process, C_t= filtered close return point process$\}$. The mapping will consist of twenty-seven unique states derived from a base class of 9 base states and further refined by 2 additional states of variance: heavy and Poisson point process. Visually we have a 3D filter in the form of 3 x 3 x 3 cube. See Figure 7.

What follows concerns measurability of the proxy $\tilde{\mu}_t$ which is of exceedingly small magnitude with an error in measurement issue. The second new insight is intentionally making coarse partitions to mitigate measurement error but sufficiently predictive for each of the three

levels in each of the planes. This was also necessitated by the need to control the number of states to a manageable number. In trying for ever more precise measurement of proxy $\breve{\mu}_t$, the state space expands as the square for the probability transition matrices. Moreover, as the number of states increases the matrix becomes sparse. Through clever ordering of states, collapsing from sparse matrix to a dense full rank matrix is utilized.

Sample size was motivated by considerations of 27 unique collapsed states and option pricing horizons which typically are 60 trade days, so a 60 trade-day sliding window is utilized starting from *t - 120* which means 121 tagged data points with daily update of most distant close return rolling off and t = 0 close return rolling on.

Our interest lies in conducting analysis of the combined signal plus noise view which consists of $\{A_t B_t C_t\}$ versus filtered signal view $\{C_t\}$ caused by combined variance matrix $A_t B_t$.



Apple Inc. Common Stock(AAPL) 120 Trade days ending 2024-08-16 prior to market open 2024-08-19 (L: 223.65, C: 226.05, H: 229.04)

3x3 Matrices: Single 120 Trade-day, rest developed using 60 trade-days with subsequent rolling every 20 trade-days.

		Combined & Collapsed Matrices: ABC				Combined & Collapsed Matrices: C				Combined & Collapsed Matrices: AB			
COUNTS		Signal_3(meu)				Signal_3(meu)				Signal_3(meu)			
		meu < 0.95	meu in [0.95,0.97]	meu > 0.97		meu < 0.95	meu in [0.95,0.97]	meu > 0.97		meu < 0.95	meu in [0.95, meu > 0.97		
	Momentum(+)	6	9	14		5	8	11		1	1	3	
120 TD, t= 0	Momentum(0)	20	23	18		18	21	18		2	2		
	Momentum(-)	17	10	3	1.	10	7	3	.84166667	7	3		.15833333
	Momentum(+)	3	5	6		2	5	4		1		2	
60 TD, t= 0	Momentum(0)	12	15	5		10	13	5		2	2		
	Momentum(-)	7	5	2	1.	3	4	2	.8	4	1		.2
	Momentum(+)	2	3	7		1	3	5		1		2	
60 TD, t= -20	Momentum(0)	8	13	13		8	11	13			2		
	Momentum(-)	7	5	2	1.	4	4	2	.85	3	1		.15
	Momentum(+)	3	4	7		3	3	6			1	1	
60 TD, t= -40	Momentum(0)	4	12	14		4	11	14			1		
	Momentum(-)	10	4	2	1.	7	3	2	.88333333	3	1		.11666667
	Momentum(+)	3	4	8		3	3	7			1	1	
60 TD, t= -60	Momentum(0)	8	8	13		8	8	13					
	Momentum(-)	10	5	1	1.	7	3	1	.88333333	3	2		.11666667

Figure 8

Apple (AAPL) 120 trade days ending 8/16/2024 matrix filtering representation of original time-series counts and average trends with 120 and sliding 60 trade day window. The original series(signal + noise) is label ABC and shown as first entry on left. Filtered close return is label matrix C and is the middle entry. Combined Poisson point process matrix A and heavy variance matrix B is show on right hand side.



	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO
1			Apple Inc. Common Stock(AAPL) 120 Trade days ending 2024-08-13 prior to market open 2024-08-14 (L: 216.26, C: 221.27, H: 224.34)												
2			3x3 Matrices: Single 120 Trade-day, rest developed using 60 trade-days with subsequent rolling every 20 trade-days.												
31															
32			Combined & Collapsed Matrices: ABC				Combined & Collapsed Matrices: C				Combined & Collapsed Matrices: AB				
33	AVERAGE TRENDS			Signal_3(meu)				Signal_3(meu)				Signal_3(meu)			
34			meu < 0.95	meu in [0.95,0.97]	meu > 0.97		meu < 0.95	meu in [0.95,0.97]	meu > 0.97		meu < 0.95	meu in [0.95,	meu > 0.97		
35		Momentum(+)	.99676108	1.00759155	1.00283443		1.00041589	1.00949238	1.00117419		.97848707	.9942857	1.00836853		
36	120 TD, t= 0	Momentum(0)	.99960749	1.00689602	1.00460584		.99846935	1.00609167	1.00462683		1.00985074	1.01534176	1.00418603		
37		Momentum(-)	.99354684	.99755833	1.00280285	1.0016426	.99871055	.99312607	1.00280285	1.00217772	.9861701	1.00790026		.99896704	
38															
39		Momentum(+)	.99512404	1.01464227	1.0028344		1.00344253	1.01464227	.99805242		.97848707	.	1.01000738		
40	60 TD, t= 0	Momentum(0)	.99509802	1.01227056	1.005661		.99214748	1.01179807	1.005661		1.00985074	1.01534176	.		
41		Momentum(-)	.99643028	.98985077	.99290347	1.00243009	1.00644022	.98586069	.99290347	1.00282117	.98892282	1.0058111	.	1.00086577	
42															
43		Momentum(+)	.99425068	1.01267344	1.00522248		1.0100143	1.01880268	1.00333484		.97848707	.9942857	1.00836853		
44	60 TD, t= -20	Momentum(0)	1.00828834	1.01199966	1.00693554		1.00828834	1.01144264	1.00693554		.	1.01534176	.		
45		Momentum(-)	1.00191529	.98726747	1.00280285	1.00570495	1.00235632	.98263156	1.00280285	1.00611715	1.00103322	1.0058111	.	1.00364394	
46															
47		Momentum(+)	.99839813	.99677726	1.00215736		.99839813	.99802303	1.00166844		.	.9942857	1.00509083		
48	60 TD, t= -40	Momentum(0)	1.00946492	1.01078541	1.00547806		1.00946492	1.01162236	1.00547806		.	1.00157893	.		
49		Momentum(-)	.99374725	.99925451	1.00280285	1.00343162	.99539784	.98458236	1.00280285	1.00343945	.98797017	1.04327095	.	1.00336113	
50															
51		Momentum(+)	.99839813	1.00054082	1.00283444		.99839813	1.00262586	1.0025121		.	.9942857	1.00509083		
52	60 TD, t= -60	Momentum(0)	1.00637168	.99681877	1.0039565		1.00637168	.99681877	1.00393737		.	.	1.00418603		
53		Momentum(-)	.99152843	1.00526588	1.01270223	1.00085512	.99539784	1.00281324	1.01270223	1.00158376	.9824998	1.00894484	.	.99611896	

Home | Graphs | Eigen_ChangePoint_Analysis | Robust_T_tests | Robust_T_tests_Offset_1 | Robust_T_tests_Offset_5 | Robust_T_tests_Offset_10 | Robust_T_tes

Ready

Figure 8 (continued)

Apple (AAPL) 120 trade days ending 8/16/2024 matrix filtering representation of original time-series counts and average trends with 120 and sliding 60 trade day window. The original series(signal + noise) is label ABC and shown as first entry on left. Filtered close return is label matrix C and is the middle entry. Combined Poisson point process matrix A and heavy variance matrix B are show on right hand side.

5. Signal Set Validation

How do you determine if the {volatility, proxy $\breve{\mu}_t$, Δproxy $\breve{\mu}_t$} is valid? The first thing to note is that each of the 60 or 120 trade day data points is mapped into one of the 27 state spaces. Using a view from the top of our 3 x 3 x 3 cube we view the time series $\{A_t B_t C_t\}$ in two dimensions (proxy $\breve{\mu}_t$, Δproxy $\breve{\mu}_t$). We know the mapping is bijective and a matrix with non-zero determinant is the result. For each state space (cell), we can calculate all the descriptive statistics such as counts, average, standard deviation, skewness, and kurtosis provided sufficient sample size. A first test is visual inspection and seeks to answer the question, "is there a coherent, persistent structure to the matrices?" Answer: "yes," there is definite structural pattern around column(1) and also around row(1) for all time offsets. A second test would assess the presence of a random matrix which uses the spacing between eigenvalues and is known as Wigner's surmise. Eigenvalues of a random matrix repel each other. We find no evidence of eigenvalue repulsion. Moreover, a series of random matrices would not have a similar maximal eigenvalue and eigenvector in common across five iterations. See Figure 9.

The third test is a robust Student T-test where care is taken to adjust for outliers and check normality assumptions. A Yuen-Welch with Tukey outlier removal was implemented at significance levels 0.10 and 0.05. We apply the robust T-test to the rows and columns: all row combinations taken 2 at a time, all column combinations taken 2 at a time. The robust T-test is performed on the sliding 60 trade day window for offsets { t-60, t-40, t-10, t-5, t-3, t-2, t-1} and well as for complete 120 trade day sample set. The results conclusively support the filtering using {proxy $\breve{\mu}_t$, Δproxy $\breve{\mu}_t$} for $\{A_t, B_t, C_t\}$ and { C_t} produces row states with statistically different means for at least 1 out of 3 trials and likewise produces column states with statistically different means for at least 1 out of 3 trials. (See Tables 1,2 with time offset 20 and Tables 3,4 with time offset 10.). This is evidence in support of r_t and $sign_t(r_{t+1})$ conditional dependence proxy $\breve{\mu}_t$, Δproxy $\breve{\mu}_t$ which means $sign_t(r_{t+1})$ is statistically predictable under Wald's Theorem. Moreover, the signals vary over the rows and columns as we slide through the 60-trade day window. The same holds true for filtered signal $\{C_t\}$ 60- trade day sliding matrices. See Figures 10 and 11.

Lastly, we found that the process is a balancing act. If the coarse thresholds are not appropriate you tend to have matrices that barely reject the random matrix test, have significantly varying maximal eigenvalues, and fail the robust T-tests for all time offsets. If you have too many states, the result will be spare matrices with determinant = 0 and usually not enough data to perform meaning statistical tests since the state space is larger than the sample size. This modeling balancing effect finding is significant in and of itself.

Of note, predictive equity analytics possess a graceful fallback in that should no new additional information be revealed therefore all combinations of Yuen-Welch T-test are not significant , and fallback process is to revert to standard time series and distribution theory using any cell statistics since they have no significant statistical difference.

Predictive equity analytics is not limited to Apple(AAPL) and has been implemented for U.S. exchanges covering 11,000+ symbols since July 2018. Predictive equity data sets available prior to market open each trade day by symbol count include:

- Equity: 3,650
- ETFs: 2,100
- Mutual Funds: 5,600
- Total: 11,000+

Only equity symbols that have significant degree of structure for predictability purposes are included in daily prior to market open data sets. There are many stocks (low volume or illiquid and not included in any index) which are processed and fail to have persistent structure suitable for predictability. For instance, the starting US equity file symbol count is 6,000 of which only 3,650 (60.8%) have significant persistent predictive structure. Mutual funds enjoy 93.3% significant persistent structure for prediction, ETFs have 70% persistent predictable structure.

Table 1: AAPL Matrix ABC Average Trends Yuen-Welch with Tukey oulter robust T-tests for 120 and sliding 60 trade-day windows at offsets {t-60, t-40, t-20, t-0}

Combined & Collapsed Matrices: ABC		Significance: 0.05	Significance: 0.1	
120 TD, t= 0	**Probability**	**Accept/Reject Null Hypothesis means**	**Accept/Reject Null Hypothesis means**	
Rows				
(momentum(+), momentum(-))	0.08025	Accept	Reject	
(momentum(+), momentum(0))	0.51046	Accept	Accept	
(momentum(-), momentum(0))	0.12859	Accept	Accept	
Columns				
(Signal(3)<0.95, Signal(3)>0.97)	0.01670	Reject	Reject	
(Signal(3)in[0.95,0.97], Signal(3)>0.97)	0.10981	Accept	Accept	
(Signal(3)<0.95, Signal(3)in[0.95,0.97])	0.49679	Accept	Accept	
60 TD, t= 0				
Rows				
(momentum(+), momentum(-))	0.04417	Reject	Reject	
(momentum(+), momentum(0))	0.46265	Accept	Accept	
(momentum(-), momentum(0))	0.07446	Accept	Reject	
Columns				
(Signal(3)<0.95, Signal(3)>0.97)	0.04386	Reject	Reject	
(Signal(3)in[0.95,0.97], Signal(3)>0.97)	0.04342	Reject	Reject	
(Signal(3)<0.95, Signal(3)in[0.95,0.97])	0.84945	Accept	Accept	
60 TD, t= -20				
Rows				
(momentum(+), momentum(-))	0.00167	Reject	Reject	
(momentum(+), momentum(0))	0.05755	Accept	Reject	
(momentum(-), momentum(0))	0.03465	Reject	Reject	
Columns				
(Signal(3)<0.95, Signal(3)>0.97)	0.01122	Reject	Reject	
(Signal(3)in[0.95,0.97], Signal(3)>0.97)	0.10081	Accept	Accept	** right on boundary
(Signal(3)<0.95, Signal(3)in[0.95,0.97])	0.68494	Accept	Accept	
60 TD, t= -40				
Rows				
(momentum(+), momentum(-))	0.17892	Accept	Accept	
(momentum(+), momentum(0))	0.86297	Accept	Accept	
(momentum(-), momentum(0))	0.09375	Accept	Reject	
Columns				
(Signal(3)<0.95, Signal(3)>0.97)	0.04483	Reject	Reject	
(Signal(3)in[0.95,0.97], Signal(3)>0.97)	0.28732	Accept	Accept	
(Signal(3)<0.95, Signal(3)in[0.95,0.97])	0.42357	Accept	Accept	
60 TD, t= -60				
Rows				
(momentum(+), momentum(-))	0.49865	Accept	Accept	
(momentum(+), momentum(0))	0.73250	Accept	Accept	
(momentum(-), momentum(0))	0.59461	Accept	Accept	
Columns				
(Signal(3)<0.95, Signal(3)>0.97)	0.06427	Accept	Reject	
(Signal(3)in[0.95,0.97], Signal(3)>0.97)	0.89673	Accept	Accept	
(Signal(3)<0.95, Signal(3)in[0.95,0.97])	0.05122	Accept	Reject	

Table 1

AAPL Matrix ABC 120 Trade-day and sliding 60 Trade-day windows at time offsets{t-60, t-40, t-20, t-0} showing time-varying significant robust T-tests.

Table 2: AAPL Matrix C Average Trends Yuen-Welch with Tukey oulter robust T-tests for 120 and sliding 60 trade-day windows at offsets {t-60, t-40, t-20, t-0}

Combined & Collapsed Matrices: C		Significance: 0.05	Significance: 0.1
120 TD, t= 0	**Probability**	**Accept/Reject Null Hypothesis means**	**Accept/Reject Null Hypothesis means**
Rows			
(momentum(+), momentum(-))	0.2234	Accept	Accept
(momentum(+), momentum(0))	0.32928	Accept	Accept
(momentum(-), momentum(0))	0.2234	Accept	Accept
Columns			
(Signal(3)<0.95, Signal(3)>0.97)	0.06935	Accept	Reject
(Signal(3)in[0.95,0.97], Signal(3)>0.97)	0.18647	Accept	Accept
(Signal(3)<0.95, Signal(3)in[0.95,0.97])	0.64377	Accept	Accept
60 TD, t= 0			
Rows			
(momentum(+), momentum(-))	0.12082	Accept	Accept
(momentum(+), momentum(0))	0.3423	Accept	Accept
(momentum(-), momentum(0))	0.26504	Accept	Accept
Columns			
(Signal(3)<0.95, Signal(3)>0.97)	0.14753	Accept	Accept
(Signal(3)in[0.95,0.97], Signal(3)>0.97)	0.04761	Reject	Reject
(Signal(3)<0.95, Signal(3)in[0.95,0.97])	0.55421	Accept	Accept
60 TD, t= -20			
Rows			
(momentum(+), momentum(-))	0.00416	Reject	Reject
(momentum(+), momentum(0))	0.03477	Reject	Reject
(momentum(-), momentum(0))	0.04926	Reject	Reject
Columns			
(Signal(3)<0.95, Signal(3)>0.97)	0.03392	Reject	Reject
(Signal(3)in[0.95,0.97], Signal(3)>0.97)	0.22472	Accept	Accept
(Signal(3)<0.95, Signal(3)in[0.95,0.97])	0.48251	Accept	Accept
60 TD, t= -40			
Rows			
(momentum(+), momentum(-))	0.36766	Accept	Accept
(momentum(+), momentum(0))	0.93593	Accept	Accept
(momentum(-), momentum(0))	0.93593	Accept	Accept
Columns			
(Signal(3)<0.95, Signal(3)>0.97)	0.19557	Accept	Accept
(Signal(3)in[0.95,0.97], Signal(3)>0.97)	0.51653	Accept	Accept
(Signal(3)<0.95, Signal(3)in[0.95,0.97])	0.5685	Accept	Accept
60 TD, t= -60			
Rows			
(momentum(+), momentum(-))	0.8006	Accept	Accept
(momentum(+), momentum(0))	0.70330	Accept	Accept
(momentum(-), momentum(0))	0.57508	Accept	Accept
Columns			
(Signal(3)<0.95, Signal(3)>0.97)	0.25345	Accept	Accept
(Signal(3)in[0.95,0.97], Signal(3)>0.97)	0.92772	Accept	Accept
(Signal(3)<0.95, Signal(3)in[0.95,0.97])	0.14541	Accept	Accept

Table 2

AAPL Matrix C 120 Trade-day and sliding 60 Trade-day windows at time offsets{t-60, t-40, t-20, t-0} showing time-varying significant robust T-tests.

Table 3: AAPL Matrix ABC Average Trends Yuen-Welch with Tukey oulter robust T-tests for 120 and sliding 60 trade-day windows at offsets {t-30, t-20, t-10, t-0}

Combined & Collapsed Matrices: ABC		**Significance:** 0.05	**Significance:** 0.1	
120 TD, t= 0	**Probability**	**Accept/Reject Null Hypothesis means similar**	**Accept/Reject Null Hypothesis means similar**	
Rows				
(momentum(+), momentum(-))	0.08025	Accept	Reject	
(momentum(+), momentum(0))	0.51046	Accept	Accept	
(momentum(-), momentum(0))	0.12859	Accept	Accept	
Columns				
(Signal(3)<0.95, Signal(3)>0.97)	0.01670	Reject	Reject	
(Signal(3)in[0.95,0.97], Signal(3)>0.97)	0.10981	Accept	Accept	
(Signal(3)<0.95, Signal(3)in[0.95,0.97])	0.49679	Accept	Accept	
60 TD, t= 0				
Rows				
(momentum(+), momentum(-))	0.04417	Reject	Reject	
(momentum(+), momentum(0))	0.46265	Accept	Accept	
(momentum(-), momentum(0))	0.07446	Accept	Reject	
Columns				
(Signal(3)<0.95, Signal(3)>0.97)	0.04386	Reject	Reject	
(Signal(3)in[0.95,0.97], Signal(3)>0.97)	0.04342	Reject	Reject	
(Signal(3)<0.95, Signal(3)in[0.95,0.97])	0.84945	Accept	Accept	
60 TD, t= -10				
Rows				
(momentum(+), momentum(-))	0.01078	Reject	Reject	
(momentum(+), momentum(0))	0.15575	Accept	Accept	
(momentum(-), momentum(0))	0.04186	Reject	Reject	
Columns				
(Signal(3)<0.95, Signal(3)>0.97)	0.00740	Reject	Reject	
(Signal(3)in[0.95,0.97], Signal(3)>0.97)	0.05453	Accept	Reject	
(Signal(3)<0.95, Signal(3)in[0.95,0.97])	0.75652	Accept	Accept	
60 TD, t= -20				
Rows				
(momentum(+), momentum(-))	0.00167	Reject	Reject	
(momentum(+), momentum(0))	0.05755	Accept	Reject	
(momentum(-), momentum(0))	0.03465	Reject	Reject	
Columns				
(Signal(3)<0.95, Signal(3)>0.97)	0.01122	Reject	Reject	
(Signal(3)in[0.95,0.97], Signal(3)>0.97)	0.10081	Accept	Accept	*** right at boundary
(Signal(3)<0.95, Signal(3)in[0.95,0.97])	0.68494	Accept	Accept	
60 TD, t= -30				
Rows				
(momentum(+), momentum(-))	0.07703	Accept	Reject	
(momentum(+), momentum(0))	0.71665	Accept	Accept	
(momentum(-), momentum(0))	0.05451	Accept	Reject	
Columns				
(Signal(3)<0.95, Signal(3)>0.97)	0.00807	Reject	Reject	
(Signal(3)in[0.95,0.97], Signal(3)>0.97)	0.05698	Accept	Reject	
(Signal(3)<0.95, Signal(3)in[0.95,0.97])	0.85853	Accept	Accept	

Table 3

AAPL Matrix ABC 120 Trade-day and sliding 60 Trade-day windows at time offsets{t-30, t-20, t-10, t-0} showing time-varying significant robust T-tests.

Table 4: AAPL Matrix C Average Trends Yuen-Welch with Tukey oulter robust T-tests for 120 and sliding 60 trade-day windows at offsets {t-30, t-20, t-10, t-0}

Combined & Collapsed Matrices: C		**Significance:** 0.05	**Significance:** 0.1
120 TD, t= 0	**Probability**	**Accept/Reject Null Hypothesis means similar**	**Accept/Reject Null Hypothesis means similar**
Rows			
(momentum(+), momentum(-))	0.2234	Accept	Accept
(momentum(+), momentum(0))	0.32928	Accept	Accept
(momentum(-), momentum(0))	0.2234	Accept	Accept
Columns			
(Signal(3)<0.95, Signal(3)>0.97)	0.06935	Accept	Reject
(Signal(3)in[0.95,0.97], Signal(3)>0.97)	0.18647	Accept	Accept
(Signal(3)<0.95, Signal(3)in[0.95,0.97])	0.64377	Accept	Accept
60 TD, t= 0			
Rows			
(momentum(+), momentum(-))	0.12082	Accept	Accept
(momentum(+), momentum(0))	0.3423	Accept	Accept
(momentum(-), momentum(0))	0.26504	Accept	Accept
Columns			
(Signal(3)<0.95, Signal(3)>0.97)	0.14753	Accept	Accept
(Signal(3)in[0.95,0.97], Signal(3)>0.97)	0.04761	Reject	Reject
(Signal(3)<0.95, Signal(3)in[0.95,0.97])	0.55421	Accept	Accept
60 TD, t= -10			
Rows			
(momentum(+), momentum(-))	0.01244	Reject	Reject
(momentum(+), momentum(0))	0.25503	Accept	Accept
(momentum(-), momentum(0))	0.03337	Reject	Reject
Columns			
(Signal(3)<0.95, Signal(3)>0.97)	0.02411	Reject	Reject
(Signal(3)in[0.95,0.97], Signal(3)>0.97)	0.1627	Accept	Accept
(Signal(3)<0.95, Signal(3)in[0.95,0.97])	0.61152	Accept	Accept
60 TD, t= -20			
Rows			
(momentum(+), momentum(-))	0.00416	Reject	Reject
(momentum(+), momentum(0))	0.03477	Reject	Reject
(momentum(-), momentum(0))	0.04926	Reject	Reject
Columns			
(Signal(3)<0.95, Signal(3)>0.97)	0.03392	Reject	Reject
(Signal(3)in[0.95,0.97], Signal(3)>0.97)	0.22472	Accept	Accept
(Signal(3)<0.95, Signal(3)in[0.95,0.97])	0.48251	Accept	Accept
60 TD, t= -30			
Rows			
(momentum(+), momentum(-))	0.08025	Accept	Reject
(momentum(+), momentum(0))	0.48475	Accept	Accept
(momentum(-), momentum(0))	0.10953	Accept	Accept
Columns			
(Signal(3)<0.95, Signal(3)>0.97)	0.12373	Accept	Accept
(Signal(3)in[0.95,0.97], Signal(3)>0.97)	0.41211	Accept	Accept
(Signal(3)<0.95, Signal(3)in[0.95,0.97])	0.67752	Accept	Accept

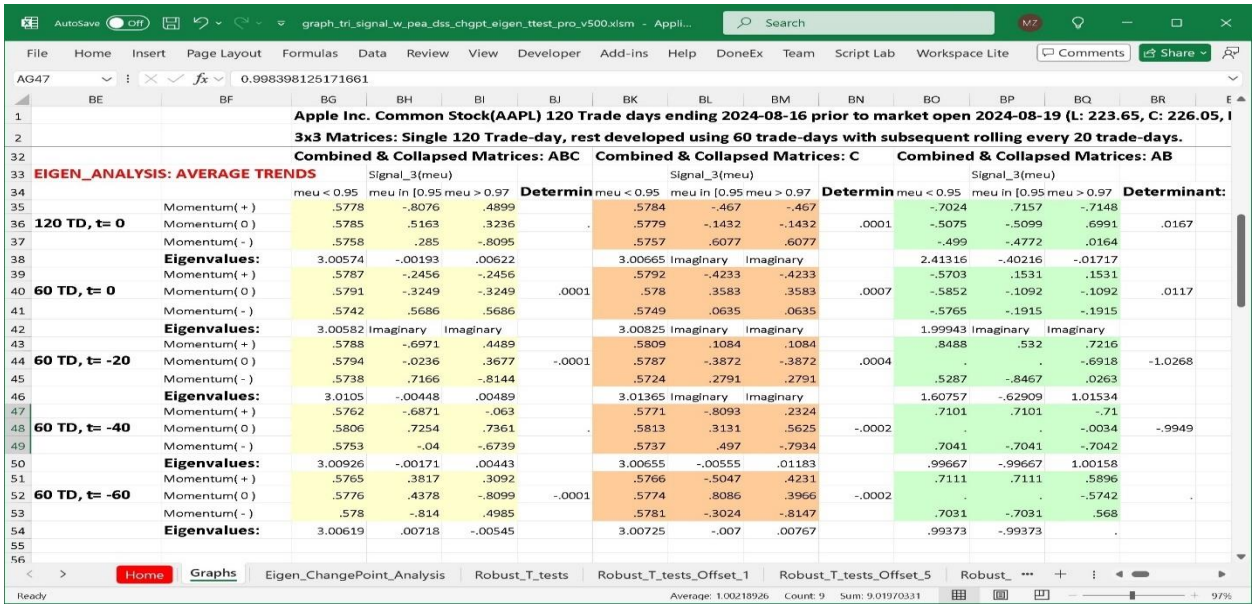
Table 4

AAPL Matrix C 120 Trade-day and sliding 60 Trade-day windows at time offsets{t-30, t-20, t-10, t-0} showing time-varying significant robust T-tests.

The above argument strongly supports predictive equity analytics new findings that predictive signals exist and are significant for sign(conditional mean), conditional mean as well as for conditional variance.

6. Sample Data Signal Set, Time Reversibility and Change-Point Detection

Significant new finding: the Signal set $\{(signal_t(1), signal_t(2), signal_t(3))\}$ moving forward in time differs from reversing in time signal set $\{(signal_{-t}(1), signal_{-t}(2), signal_{-t}(3))\}$ which is expected since the time reversed close return equity time series is the mirror image of the forward in time close return equity time series for all points **except change-point occurrence.** Later in the paper we show that retrospective change-point confirmation is shown definitely by the simultaneous -1 multiplication of all principle eigenvector coefficients relative to prior measurement, we conjecture that time reversal principle eigenvectors likewise displays simultaneous -1 multiplication of all principle its eigenvector coefficients measure relative to most recent prior measurement. We know that the 60 trade-day sliding window at offset t-1 rolls the most distant in time point off and rolls on the most recent for both moving forward in time and time reversal. From linear algebra there are only three types of movement: a) within row space, b) within column space, and c) using both row and column. This is further strong support that {volatility, proxy $\breve{\mu}_t$, Δproxy $\breve{\mu}_t$} are predictive of change-point likely occurrence in next 1- 3 trade days.



			BG	BH	BI	BJ	BK	BL	BM	BN	BO	BP	BQ	BR
1			Apple Inc. Common Stock(AAPL) 120 Trade days ending 2024-08-16 prior to market open 2024-08-19 (L: 223.65, C: 226.05, I											
2			3x3 Matrices: Single 120 Trade-day, rest developed using 60 trade-days with subsequent rolling every 20 trade-days.											
32			Combined & Collapsed Matrices: ABC				Combined & Collapsed Matrices: C				Combined & Collapsed Matrices: AB			
33	EIGEN_ANALYSIS: AVERAGE TRENDS		Signal_3(meu)				Signal_3(meu)				Signal_3(meu)			
34			meu < 0.95	meu in [0.95	meu > 0.97	Determin	meu < 0.95	meu in [0.95	meu > 0.97	Determin	meu < 0.95	meu in [0.95	meu > 0.97	Determinant:
35		Momentum(+)	.5778	-.8076	.4899		.5784	-.467	-.467		-.7024	.7157	-.7148	
36	120 TD, t= 0	Momentum(0)	.5785	.5163	.3236	.	.5779	-.1432	-.1432	.0001	-.5075	-.5099	.6991	.0167
37		Momentum(-)	.5758	.285	-.8095		.5757	.6077	.6077		-.499	-.4772	.0164	
38		Eigenvalues:	3.00574	-.00193	.00622		3.00665	Imaginary	Imaginary		2.41316	-.40216	-.01717	
39		Momentum(+)	.5787	-.2456	-.2456		.5792	-.4233	-.4233		-.5703	.1531	.1531	
40	60 TD, t= 0	Momentum(0)	.5791	-.3249	-.3249	.0001	.578	.3583	.3583	.0007	-.5852	-.1092	-.1092	.0117
41		Momentum(-)	.5742	.5686	.5686		.5749	.0635	.0635		-.5765	-.1915	-.1915	
42		Eigenvalues:	3.00582	Imaginary	Imaginary		3.00825	Imaginary	Imaginary		1.99943	Imaginary	Imaginary	
43		Momentum(+)	.5788	-.6971	.4489		.5809	.1084	.1084		.8488	.532	.7216	
44	60 TD, t= -20	Momentum(0)	.5794	-.0236	.3677	-.0001	.5787	-.3872	-.3872	.0004	.	.	-.6918	-1.0268
45		Momentum(-)	.5738	.7166	-.8144		.5724	.2791	.2791		.5287	-.8467	.0263	
46		Eigenvalues:	3.0105	-.00448	.00489		3.01365	Imaginary	Imaginary		1.60757	-.62909	1.01534	
47		Momentum(+)	.5762	-.6871	-.063		.5771	-.8093	.2324		.7101	.7101	-.71	
48	60 TD, t= -40	Momentum(0)	.5806	.7254	.7361	.	.5813	.3131	.5625	-.0002	.	.	-.0034	-.9949
49		Momentum(-)	.5753	-.04	-.6739		.5737	.497	-.7934		.7041	-.7041	-.7042	
50		Eigenvalues:	3.00926	-.00171	.00443		3.00655	-.00555	.01183		.99667	-.99667	1.00158	
51		Momentum(+)	.5765	.3817	.3092		.5766	-.5047	.4231		.7111	.7111	.5896	
52	60 TD, t= -60	Momentum(0)	.5776	.4378	-.8099	-.0001	.5774	.8086	.3966	-.0002	.	.	-.5742	.
53		Momentum(-)	.578	-.814	.4985		.5781	-.3024	-.8147		.7031	-.7031	.568	
54		Eigenvalues:	3.00619	.00718	-.00545		3.00725	-.007	.00767		.99373	-.99373	.	

Figure 9

Using AAPL average trends matrix series, the eigenvalue and eigen vectors are displayed for all sliding window offsets as well as static 120 trade day matrix. Note complex valued eigenvalues are displayed and must occur as conjugate pairs. The stability of the maximal eigenvalue and associated eigenvector for matrix ABC and for matrix C series is undeniable. There is no evidence of eigenvalue repulsion.

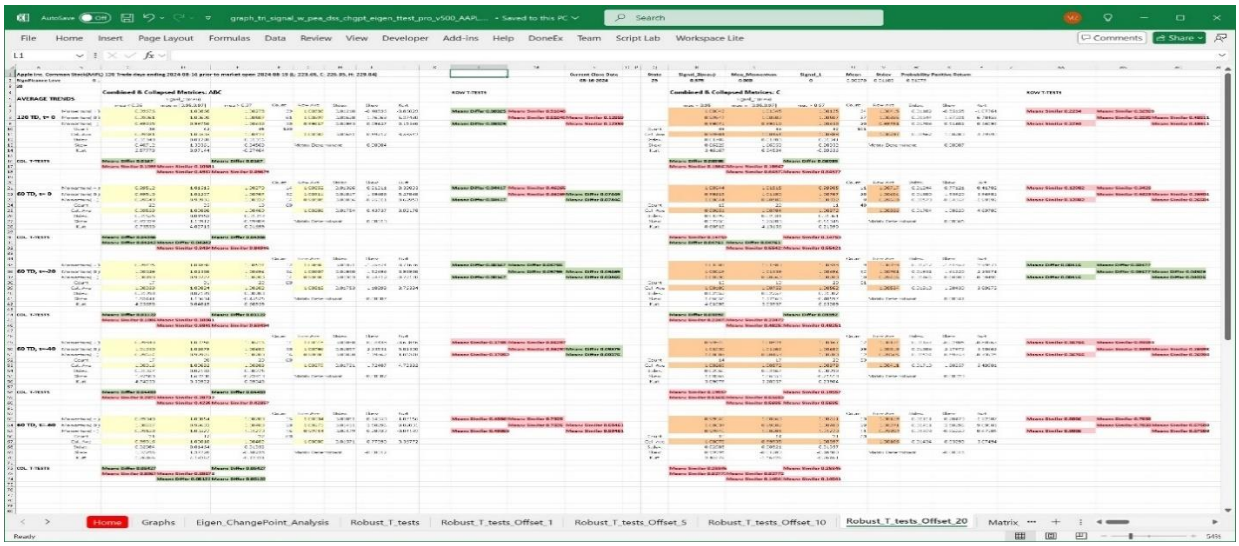


Figure 10

robust t-tests at **significance level = 0.10** for APPL matrix series ABC and matrix series C are displayed. For ABC matrix series there were at least 2 pairs of tests which rejected null hypothesis of similar distributions (shaded green). For filtered close return matrix series C, all but t-40 and t-60 have at least 1 significant test result. This is evidence in support of rejection of the assumption of independence and identically distributed (iid).

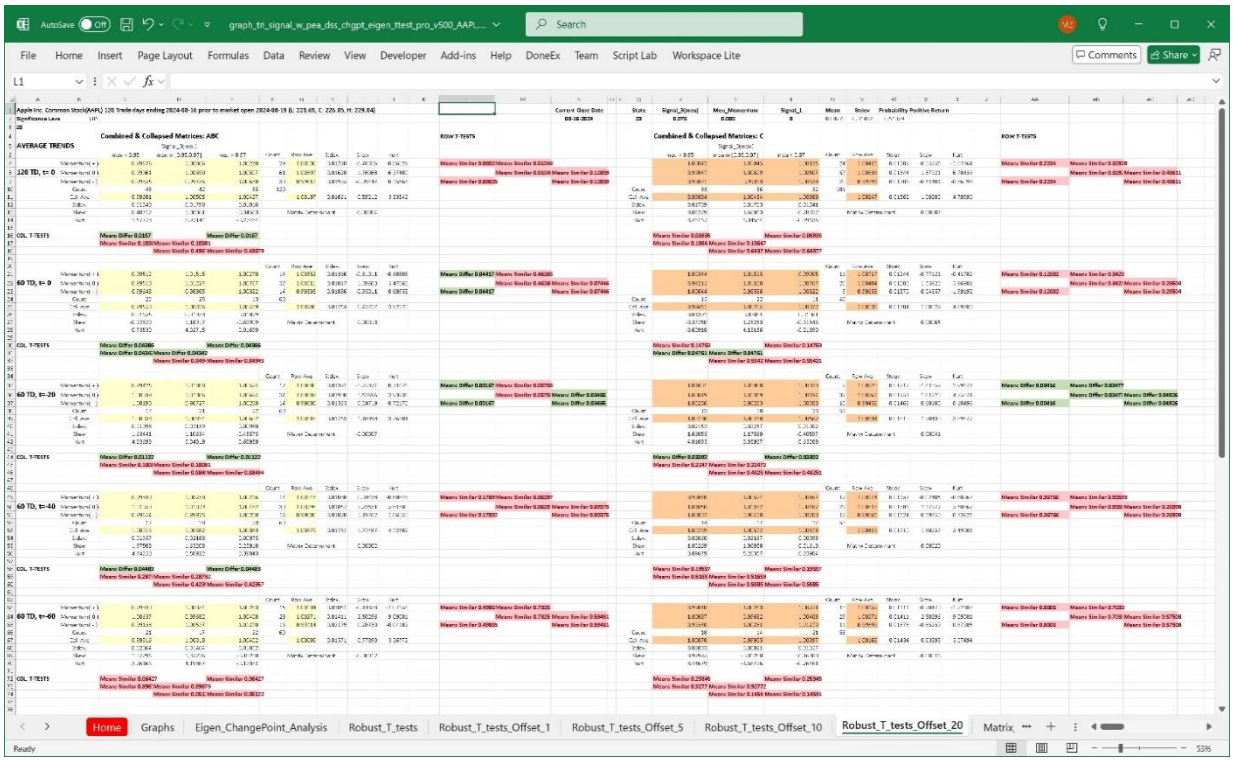


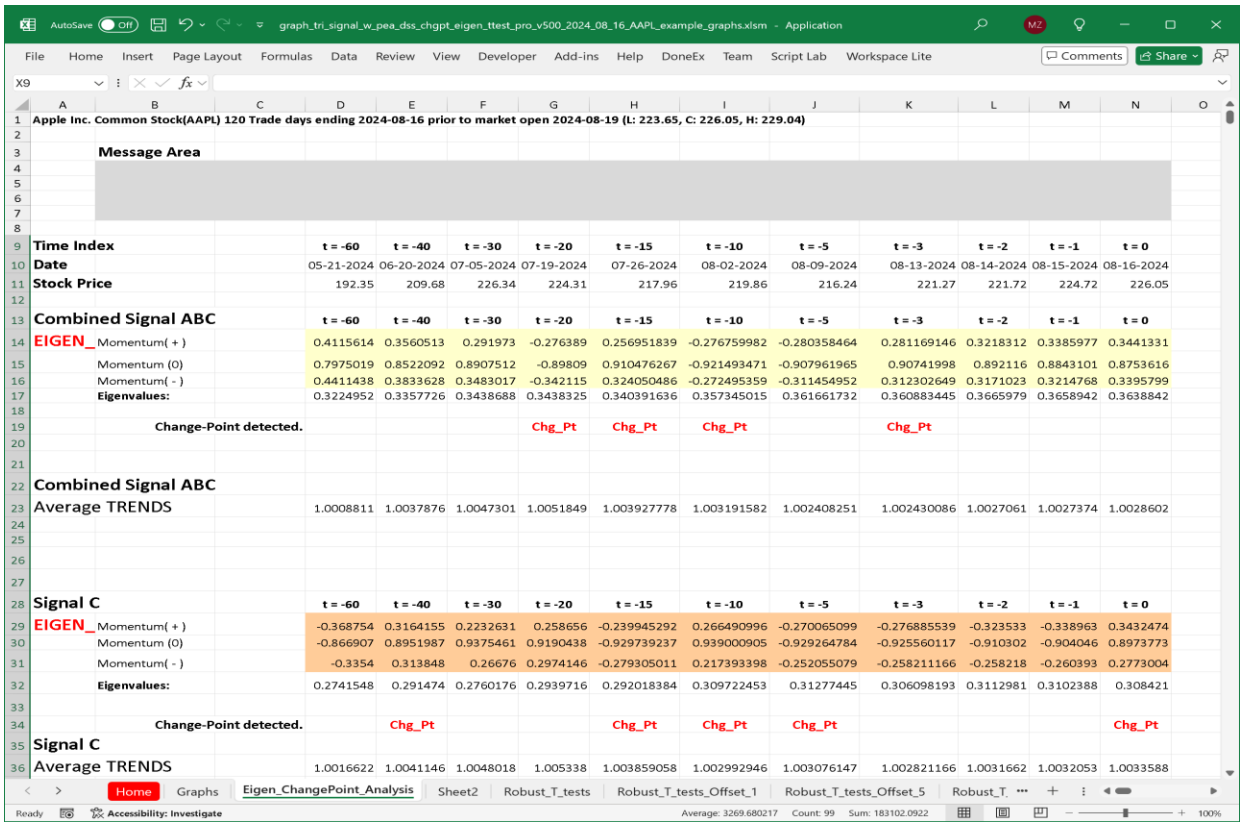
Figure 11

Robust t-tests at **significance level = 0.50** for APPL matrix series ABC and matrix series C are displayed. For ABC matrix series there were at least 1 test which rejected null hypothesis of similar distributions (shaded green) except for t-60. For filtered close return matrix series C, 120 trade day, , t-40 and t-60 failed to have at least 1 significant t-test result. However, comparing t-40 to t- 60 there was a change-point in col(1) and a robust t-test performed over the joint matrices would have at least 1 or more significant tests. This is evidence in support of rejection of the assumption of independence and identically distributed (iid).

7. Comparing Predictive Equity Analytics view to Standard Time-series/Distributional view

Predictive equity analytics applied to original noisy close return process $A_t B_t C_t$ and to C_t is visually compared in Figures 13 – 19. Traditional time-series and descriptive distributional graphs are illustrated in Figures 13,14 where a single distribution is assumed. This is contrasted with predictive equity analytics noisy signal matrix $A_t B_t C_t$ representing an amalgamation of 27 state spaces into a 3 x 3 matrix with 9 combined state spaces with color coding of close return correlation by columns to Signal(3) level in Figure 15. There is a definitive correlation of APPL

close return price series up-behavior to a) epoch persistent Proxy Drift/meu signal(3) > 0.97 or b) presence of short epoch of Heavy Volatility matrix $A_t B_t$. Changes in Proxy Drift/meu (meu momentum) depicted in Figure 17 are not as strong as Proxy Drift/meu signal(3) level. Prospective likely change-point occurrence is correlated with Heavy Volatility matrix $A_t B_t$ signal(1) > 0.7 as shown in Figure 18. Figure 19 graphically depicts Predictive Equity Analytics new finding: retrospective detection in shortest amount of time of change-point occurrence using maximal eigenvalue and associated eigenvector analysis. The maximal eigenvalue varies little, but the coefficients of the associated eigenvectors are simultaneously multiplied by -1. See Figure 12. Significant T-test results are found in Figure 20.



Apple Inc. Common Stock(AAPL) 120 Trade days ending 2024-08-16 prior to market open 2024-08-19 (L: 223.65, C: 226.05, H: 229.04)

Message Area

	t = -60	t = -40	t = -30	t = -20	t = -15	t = -10	t = -5	t = -3	t = -2	t = -1	t = 0
Time Index											
Date	05-21-2024	06-20-2024	07-05-2024	07-19-2024	07-26-2024	08-02-2024	08-09-2024	08-13-2024	08-14-2024	08-15-2024	08-16-2024
Stock Price	192.35	209.68	226.34	224.31	217.96	219.86	216.24	221.27	221.72	224.72	226.05
Combined Signal ABC	**t = -60**	**t = -40**	**t = -30**	**t = -20**	**t = -15**	**t = -10**	**t = -5**	**t = -3**	**t = -2**	**t = -1**	**t = 0**
EIGEN_ Momentum(+)	0.4115614	0.3560513	0.291973	-0.276389	0.256951839	-0.276759982	-0.280358464	0.281169146	0.3218312	0.3385977	0.3441331
Momentum (0)	0.7975019	0.8522092	0.8907512	-0.89809	0.910476267	-0.921493471	-0.907961965	0.90741998	0.892116	0.8843101	0.8753616
Momentum(-)	0.4411438	0.3833628	0.3483017	-0.342115	0.324050486	-0.272495359	-0.311454952	0.312302649	0.3171023	0.3214768	0.3395799
Eigenvalues:	0.3224952	0.3357726	0.3438688	0.3438325	0.340391636	0.357345015	0.361661732	0.360883445	0.3665979	0.3658942	0.3638842
Change-Point detected.				Chg_Pt	Chg_Pt	Chg_Pt		Chg_Pt			
Combined Signal ABC											
Average TRENDS	1.0008811	1.0037876	1.0047301	1.0051849	1.003927778	1.003191582	1.002408251	1.002430086	1.0027061	1.0027374	1.0028602
Signal C	**t = -60**	**t = -40**	**t = -30**	**t = -20**	**t = -15**	**t = -10**	**t = -5**	**t = -3**	**t = -2**	**t = -1**	**t = 0**
EIGEN_ Momentum(+)	-0.368754	0.3164155	0.2232631	0.258656	-0.239945292	0.266490996	-0.270065099	-0.276885539	-0.323533	-0.338963	0.3432474
Momentum (0)	-0.866907	0.8951987	0.9375461	0.9190438	-0.929739237	0.939000905	-0.929264784	-0.925560117	-0.910302	-0.904046	0.8973773
Momentum(-)	-0.3354	0.313848	0.26676	0.2974146	-0.279305011	0.217393398	-0.252055079	-0.258211166	-0.258218	-0.260393	0.2773004
Eigenvalues:	0.2741548	0.291474	0.2760176	0.2939716	0.292018384	0.309722453	0.31277445	0.306098193	0.3112981	0.3102388	0.308421
Change-Point detected.		Chg_Pt			Chg_Pt	Chg_Pt	Chg_Pt				Chg_Pt
Signal C											
Average TRENDS	1.0016622	1.0041146	1.0048018	1.005338	1.003859058	1.002992946	1.003076147	1.002821166	1.0031662	1.0032053	1.0033588

Figure 12

AAPL 120 Trade-day ending 8/15/2024 Stock Price and Close Return time-series maximal eigenvalue and associated eigenvector analysis sheet. **Predictive Equity Analytics NEW Findings:** retrospective change-point detection occurs with "flipping of sign for each eigenvector coefficient relative to prior measurement." Presence of $A_t B_t$ volatility alters the point in time of retrospective change-point detection as seen in filtered close return matrix C_t as compared with noisy signal matrix $A_t B_t C_t$.

The time-varying dynamics are fairly complex and range from high correlation to Proxy Drift/meu signal(3) level and low volatility $A_tB_t$occurence which was the case from 2/25/2024 thru 6/5/2024 with subsequent shift to Change in Proxy Drift/meu (meu momentum) and volatility $A_tB_t$greater occurrence and a small intensity increase for trade dates starting 6/20/2024.

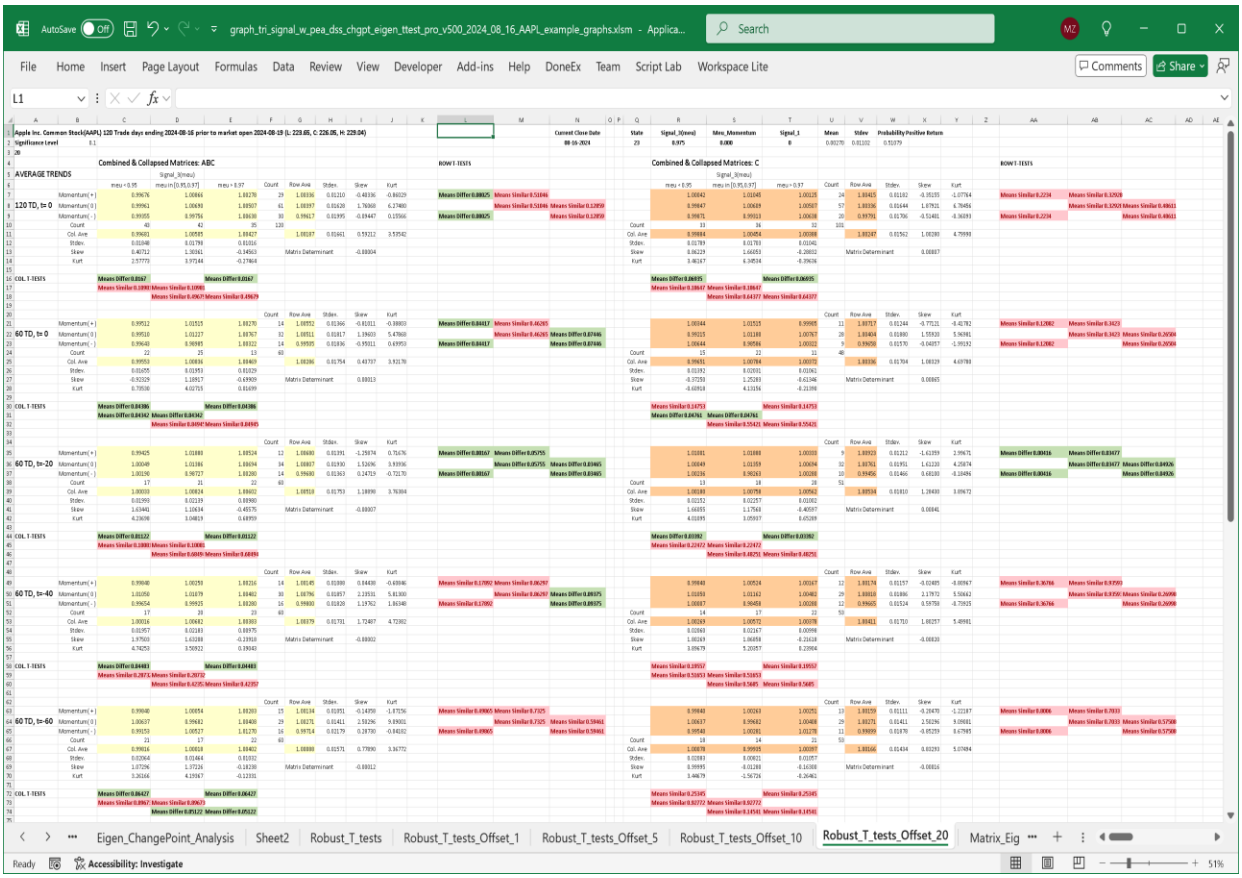


Figure 20

Predictive Equity Analytics: AAPL 120 Trade-day ending 8/15/2024 60 Trade-day sliding window at offsets t-60, t – 40, t -20 and t-0. Predictive Equity Analytics NEW finding: time-varying Proxy Drift/meu Signal(3) level and Change in Proxy Drift/meu (meu momentum) using robust Yeun-Welch T-tests at significance levels 0.05 and 0.1. Also depicted is effect on T-test of inclusion of volatility matrix A_tB_t by contrasting results for noisy signal matrix $A_tB_tC_t$ on the left with filtered close return matrix C_t. Starting at t-60, noisy signal matrix $A_tB_tC_t$ has Signal(3) columns with different means and morphs over to time to having Signal(3) and meu momentum (rows) with different means. Starting at t-60, filtered signal matrix C_t has no significant column nor row effects and by t-20, there are both significant column and significant row effects.

8. Prospective Predictive Signals for $Sign_t(R_{t+1})$

For each coming trade-day, $sign_t(R_t)$ prediction is a weighted combination consisting of the current state, row level descriptive statistics, column level descriptive statistics, presence of heavy or Poisson outlier, 52-week boundary and which signal dominates the Fokker-Planck time dimension approximation at time 0. Using the mean and standard deviation and the same basis an estimate of the complete return distribution is available including expected value.

Optimally using $sign_t(R_t)$, we are interested in earliest prospective detection of a favorable future 1 - 2 - 3 trade day change-point for an equity not currently owned. For owned equities we are interested in earliest possible future 1 - 2 - 3 trade day detection of unfavorable change-point. If shorting is not ruled out, then short, predicted assets with unfavorable change points.

Volatility signal is also used since when volatility spikes and clusters, the $sign_t(R_t) < 0$ with 100% accuracy this is correlated to $Signal_t(1)$ with remarkably high significance. Signal(1) levels exceeding 80% predict future change-point occurrence is very, very probable.

Predictive equity analytics revises (Christoffersen and Diebold 2002) finding that sign dynamics follows from volatility dynamics to "when volatility is high, this is sufficient for $sign_t(R_t)$ prediction, when volatility is nominal, $sign_t(R_t)$ prediction follows conditional mean dependence of $\widetilde{proxy}(\mu_t)$."

9. Earliest Possible Retrospective Change-Point Detection/Prospective Confirmation

One of predictive equity analytics' new findings is change-point detection with 100% accuracy in as little as 1 trade day. Using principle component analysis applied to each of the sliding windows matrices we found that retrospective change-points are detected by a unique pattern in that the signs of the coefficients of the principle eigenvector will all simultaneously be multiplied by "-1" while the maximal eigenvalue did vary that much. Where the maximal eigenvalue varied, the eigenvector also varied. These two tests, when combined and time reversibility detection of change-point, renders predictive equity analytics retrospective and prospective almost surly 100% accurate.

When the matrix rows and columns have statistically significantly different means the change in current state from prior state is observed as moving among the rows or column or both via moving along the diagonal or a pure jump from prior state to current state.

A third confirming view of retrospective change point is found in discrete Fokker-Planck approximation in the time dimension where a signal change has occurred, or the signal average intensity x signal probability has degraded.

In Figure 22 we see predictive signal(1)s: On July 12, 2024, Signal(1) level is 0.80 which is seldom seen. When combined with Signal(3) a significant drop from prior level is 98%+ for prospective change-point prediction. Retrospectively, the change point was detected on July 19, 2024, after exchange close. On August 5, 2024, signal(1) spiked indicating a change-point. This was retrospective confirmed with eigen change point detection in Fokker-Planck state space recorded on August 5, 2024, and August 9, 2024. See Figure 23.

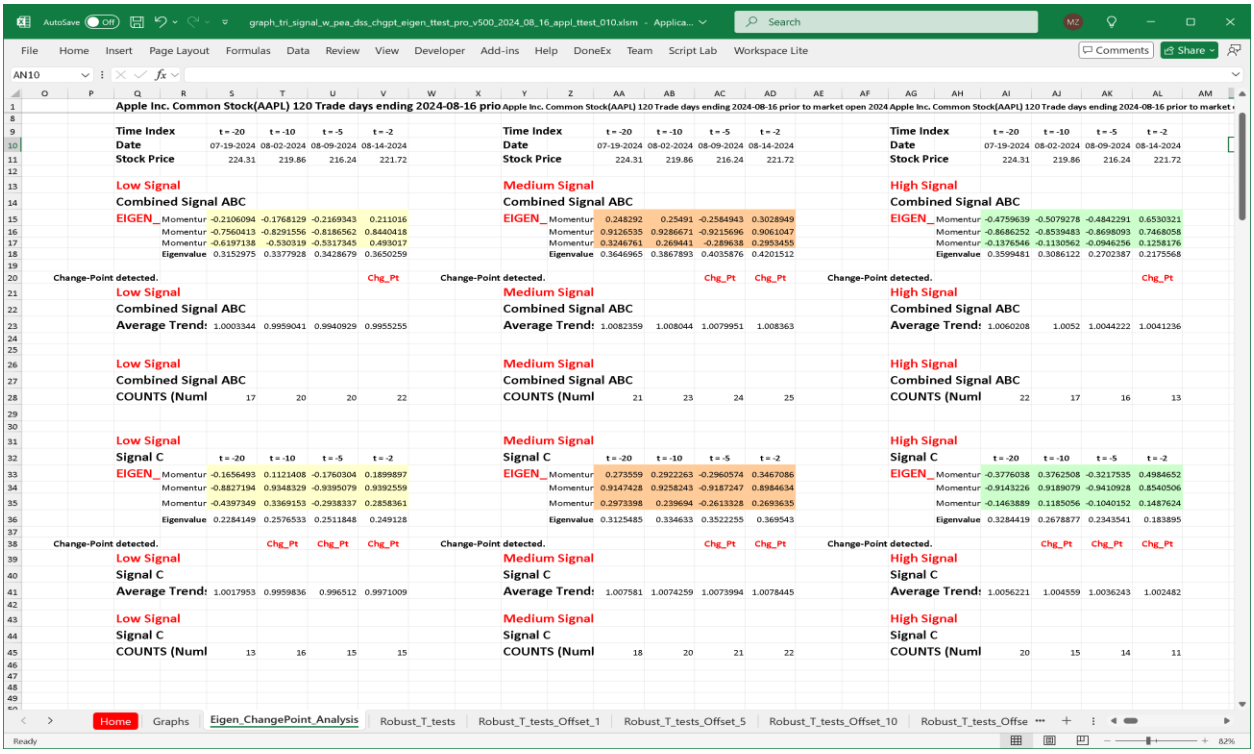


Figure 23

Predictive equity analytics new finding: discrete Fokker-Planck time dimension detects change-points across $proxy(\tilde{\mu}_t)$ signal levels. Shown above multiple simultaneous change-points are taking place in the neighborhood/recent close to t = 0.

10. Predictive Equity Analytics Proxy Drift/meu Signal(3) and Change in Proxy Drift/meu (meu momentum) for ETFs, Mutual Funds and Target Date Funds

Do predictive signals exist for ETFS, mutual funds, and target date funds? In this section, predictive equity analytic signals are generated and briefly discussed for ETF: Invesco QQQ, mutual fund: Fidelity FCNTX -Contra Fund, mutual fund: Fidelity FCPVX – Small Cap. Value Fund, and target date fund: Fidelity FIJOX (2035 class Z).

Invesco QQQ

QQQ has strong change-point prediction correlation with Signal(1) > 0.90 level. Signal(3) level is also predictive in that the closer to level 1 and the greater the time spent at or near level 1, the stock price time-series increases, as illustrated in Figure 24.

Retrospective change-points confirmed by eigen analysis are illustrated in Figure 25. Predictive equity analytics optimal timing was purchase on 6/7/2024 @ $463.53 and sell on 7/15/2024 @ $491.93 for a gross return of 6.12%. Purchase on 8/8/2024 @ $446.07, with retrospective change-point confirmation made on 8/9/2024. All predictive signals are prior to market open. Time-varying statistically significant robust T-tests using sliding 60 trade-day window for offsets {t-60, t-40, t-20, t-0} are located in Figure 26.

Fidelity FCNTX – Contra Fund

FCNTX has strong change-point prediction correlation with Signal(1) > 0.80 level. Signal(3) level is also predictive in that the closer to level 1 and the greater the time spent at or near level 1, the stock price time-series increases. See Figure 27.

Retrospective change-points confirmed by eigen analysis are illustrated in Figure 28. Predictive equity analytics optimal timing was purchase on 4/29/2024 @ $18.35 and sell on 7/8/2024 @ $20.71 for a gross return of 12.86%. Purchase on 8/9/2024 @ $19.55, with retrospective change-point confirmation made on 8/9/2024. All predictive signals are prior to market open. Time-varying statistically significant robust T-tests using sliding 60 trade-day window for offsets {t-15, t-10, t-5, t-0} are located in Figure 29.

Fidelity mutual fund FCPVX – Small Cap Value Fund

FCPVX is 100% has strong change-point prediction correlation with Signal(1) >= 0.65 level. Signal(3) level is also predictive in that the closer to level 1 and the greater the time spent at or near level 1, the stock price time-series increases. See Figure 30.

Retrospective change-points confirmed by eigen analysis are illustrated in Figure 31. Predictive equity analytics optimal timing was sell on 4/11/2024 @ $20.17, purchase on 4/22/2024 @ $17.90 and sell on 5/20/2024 @ $20.64 for a gross return of 15.30%. Purchase on 6/5/24 @ $19.83, followed by sell on 8/1/2024 @ $21.37 for a gross return of 7.76%. Purchase on 8/8/2024 @ $20.39 with retrospective change-point confirmation made on 8/9/2024. All predictive signals are prior to market open. Time-varying statistically significant robust T-tests using sliding 60 trade-day window for offsets {t-60, t-40, t-20, t-0} are located in Figure 32.

Fidelity FIJOX (2035 class Z)

FCNTX s 100% Proxy Drift/meu signal(s) > 0.97 and has strong change-point prediction correlation with Signal(1) >= 0.70 level which is due to change in meu momentum. Signal(3) level is also predictive in that the closer to level 1 and the greater the time spent at or near level 1, the stock price time-series increases. See Figure 33.

No retrospective change-points were confirmed by eigen analysis and are illustrated in Figure 34. This is likely due to close price is narrow[$14.00, 15.00] Predictive equity analytics optimal timing was purchase on 4/15/2024 @ $14.07 and sell on 7/29/2024 @ $14.72 for a gross return of 4.61%. Purchase on 8/12/2024 @ $14.60 and hold through 8/16/2024. All predictive signals are prior to market open. Time-varying statistically significant robust T-tests using sliding 60 trade-day window for offsets {t-60, t-40, t-20, t-0} are located in Figure 35.

11. Next Trade Day Close Return Distribution Estimation

Prior to market open, predictive equity analytics is able to develop a forecast of return distribution by polling available metrics. Starting at the micro-level, we have two sets of inputs: noisy signal matrix $A_tB_tC_t$ and filtered close return process matrix C_t. For each input we start at the current state space with descriptive statistics: mean, standard deviation, skewness, and kurtosis. The next level consists of Proxy Drift/meu signal(3) level (column effect) descriptive statistics and Change in Proxy Drift/meu signal(3) (meu momentum) (row effect) descriptive statistics. The highest aggregate level is noisy signal matrix $A_tB_tC_t$ and filtered close return process matrix C_t. There is also a machine learning based next trade-day high and low range. In all there are 9 separate inputs available for weighting to form an estimate of the next trade day return distribution.

At time 0, Friday August 16, 2024, the collapsed state is 23 which is contained noisy signal matrix $A_tB_tC_t$in filtered close return matrix C_t (nominal variance) at middle row (no momentum), last column (high signal). We have 10 inputs to derive expected return and return distribution.

Matrix ABC	**State 23**
Matrix C	**State 23**
mean	1.0051
stddev	0.0095
skewness	0.1928
kurt	-0.2547

From 60 trade-day, t=0 matrix $A_tB_tC_t$ row statistics:

mean	1.0051
stddev	0.0181
skewness	1.3960
kurt	5.4786

From 60 trade-day, t=0 matrix $A_tB_tC_t$ column statistics:

mean	1.0046
stddev	0.0102
skewness	-0.6990

kurt	0.0169

From 60 trade-day, t=0 matrix C_t row statistics:

mean	1.0040
stddev	0.0188
skewness	1.5590
kurt	5.9698

From 60 trade-day, t=0 matrix C_t column statistics:

mean	1.0037
stddev	0.0106
skewness	-0.6134
kurt	-0.2139

From 60 trade-day, t=0 matrix $A_t B_t C_t$ statistics:

mean	1.0028
stddev	0.0175
skewness	0.4373
kurt	3.9217

From 60 trade-day, t=0 matrix C_t statistics:

mean	1.0033
stddev	0.0170
skewness	1.0832
kurt	4.6978

From machine learning:

High	$229.03	1.0131
Low	$223.65	0.9893

Fokker-Planck Approximation Time Dimension

matrix $A_t B_t C_t$ medium signal 1.0083

matrix C_t medium signal 1.0078

12. Discrete Fokker-Planck Approximation with Poisson Jumps

The sliding 60 trade-day matrix $A_t B_t C_t$ and matrix C_t at offsets t-60, t-40, t-20, t-15, t-10, t-5, t-3, t-2, t-1 can partition to $proxy\ signal_t(3)$ over time by forming 3 x 3 matrices for each of the signals over the time dimension. Eigen analysis for principle eigenvector and change-point testing is conducted in the usual manner. By comparing results derived from matrix $A_t B_t C_t$ as compared to filtered close return matrix C_t, the effect of heavy variance and Poisson outliers may be estimated. As we move in time from distant to current time 0, we can deduce at t= 0 the dominant signal using principle component analysis techniques and use the associated descriptive statistics as basis for return distribution forecast. Using the right-hand cross product rule an estimate of the average height is available and thus changes over time are recorded. When the estimated height decreases (increases), we know the state dimension curvature must decrease (increase). This is in contrast with Lo 1988, numerically solving Fokker-Planck partial differential equation or simulating a large number of paths. (See Pedersen 1995).

13. Time-Varying Conditional Skewness and Kurtosis

Predictive equity analytics confirms (Christoffersen and Diebold 2002),finding that predictability of $sign_t(R_t)$ is related to time-varying skewness and kurtosis and points to drivers {proxy $\tilde{\mu}_t$, Δproxy $\tilde{\mu}_t$} when volatility is elevated. When volatility is nominal then $sign_t(R_t)$ predictable using {proxy $\tilde{\mu}_t$, Δproxy $\tilde{\mu}_t$}.

14. Implications for Optimal Consumer Asset Allocation

What are the implications to an individual investor of using predictive equity analytics on the standard two assets, one risky and other riskless asset allocation. Using Ingersoll's (1987), p 125 presentation as a starting point.

For CRRA utility, $v(r) = {(1+r)^{1-\gamma}}/{(1-\gamma)}$ and $b_n = (-1)^{n-1}(n!)^{-1}\gamma(\gamma+1)..(\gamma+n-2)$, for $n \geq 2$. There are two assets, a risk-free asset with return $R_{f_t} = 1 + r_{f_t} t$ and a risky asset

with drift and gross returns H_t and L_t with probabilities π_{H_t} and π_{L_t}. Define net realized rate of returns in excess of the risk-free rate and moments as follows:

$$x_{H_t} \equiv H_t - R_{f_t} = (\mu_t - r_{f_t})t + \sigma_t\sqrt{\pi_{L_t}t/\pi_{H_t}} \text{ and}$$

$$x_{L_t} \equiv L_t - R_{f_t} = (\mu_t - r_{f_t})t - \sigma_t\sqrt{\pi_{H_t}t/\pi_{L_t}}$$

$$\overline{x_t}t = E_t[\tilde{x}_t] = \pi_{H_t}x_{H_t} + \pi_{L_t}x_{L_t}$$

$$\sigma_t^2 t = var[\tilde{x}_t] = \pi_{H_t}\sigma_t^2\left(\frac{\pi_{L_t}}{\pi_{H_t}}\right)t + \pi_{L_t}\sigma_t^2\left(\frac{\pi_{H_t}}{\pi_{L_t}}\right)$$

$$E_t[(\tilde{x}_t - \overline{x_t}t)^n] = \pi_{H_t}\sigma_t^n\left(\frac{\pi_{L_t}}{\pi_{H_t}}\right)^{n/2}t^{n/2} + \pi_{L_t}(-\sigma_t)^n\sigma_t^n\left(\frac{\pi_{H_t}}{\pi_{L_t}}\right)^{n/2}t^{n/2}$$

$$= \sigma_t^n t^{n/2}(\pi_{H_t}\pi_{L_t})^{1-(n/2)}\left[\pi_{L_t}^{n-1} + (-1)^n\pi_{H_t}^{n-1}\right] = \omega_n^x t^{n/2}$$

$$\text{where } \omega_n^x = \sigma_t^n(\pi_{H_t}\pi_{L_t})^{1-(n/2)}\left[\pi_{L_t}^{n-1} + (-1)^n\pi_{H_t}^{n-1}\right]$$

At beginning of time period t, the portfolio holds the fraction w_t of the risky asset and $(1 - w_t)$ of the risky asset. The portfolio time t expected rate of return is

$$\left[(1 - w_t)r_{f_t} + w_t(\mu_t - r_{f_t})\right]t$$

with portfolio higher moments or order n defined by $w_t^n\omega_n^x t^{n/2}$.

First order condition satisfies

$$\pi_{H_t}\left[(W_0 - c_0)\left[w_tH_t + (1 - w_t)R_{f_t}\right]\right]^{-\gamma}(H_t - R_{f_t}) + \pi_{L_t}\left[(W_0 - c_0)\left[w_tL_t + (1 - w_t)R_{f_t}\right]\right]^{-\gamma}(L_t - R_{f_t}) = 0$$

The **exact time *t* optimal portfolio risky asset weight**

$$w_t^* = R_{f_t} \frac{\left[\pi_{L_t}\left(R_{f_t} - L_t\right)\right]^{(-1/\gamma)} - \left[\pi_{H_t}\left(H_t - R_{f_t}\right)\right]^{(-1/\gamma)}}{\pi_{H_t}^{-1/\gamma}\left[H_t - R_{f_t}\right]^{1-(1/\gamma)} + \pi_{L_t}^{-1/\gamma}\left[R_{f_t} - L_t\right]^{1-(1/\gamma)}}$$

With the benefit of predictive equity analytics, the individual investor's risky asset weighting is significantly increased as contrasted to the standard optimal risky weight. The asset allocation with the aid of predictive equity analytics affects the optimal weighting in many ways:

1) Increased probability of π_{H_t}
2) Increase gross return H_t
3) Decrease probability of π_{L_t}
4) Decrease gross return L_t
5) Prospective prediction of future change-point
6) Retrospective change-point confirmation in 1 to 3 trade days
7) Introduction of re-investment compounding

This is illustrated in Figures 15-18. Scanning for initial favorable change-point occurs just prior to 5/29/2024 equity price approximately \$190 and ABC, C and AB matrices report average trend > 1.003 for the first time. Using this predictive information starting 5/31/2024 through 7/25/2024 the investor would buy more AAPL stock. Starting with 7/26/2024, the investor would decrease weight or liquidate position as the average return of matrix AB is now below 1.003. A change-point occurred on 7/19/2024 with price approximately \$225 this was reinforcing signal. Return = 1.184 = 225/190 for 36 trade days is an exceptionally good return. See Figures 15-18. On 8/5/2024, Signal(1) spiked indicating coming change-point price is \$209 change-point eigenvalue confirmation is on 8/9/2024. On 8/16/2024 there is another change-point retrospective detection which means liquidate position at price = \$226 for return of 1.081 = 226/209. Assuming full reinvest of funds compound gross return of 1.28 in 59 trading days was realized. No ETF or mutual fund can match this return or even come close.

15. For Fund Managers Predictive based Dynamic Diversification is Superior to Static Diversification

Given a set of stocks, without predictive equity analytics a static diversification based on highest Sharpe ratio is optimal. Static diversification drawback is that using historic data means that for success, the near future must be similar to the past sampling epoch. An optimal weight asset allocation is used that spreads 100% of initial funds over a broad swath of equities. Of course, static allocation can benefit by monitoring the eigenvalues and eigenvectors for matrices ABC and C and tactically re-weight.

Contrast to predictive equity analytics framework. Prior to market open a search is made for stocks where a favorable change-point is predicted to occur in next 1-3 trade days. Use one-half of initial funds and purchase a weight portfolio. Invest the remaining one-half of initial capital into an interest-bearing account. The result is "no dead money," all initial funds are predicted to earn positive returns, and the portfolio level of risk is orders of magnitude lower than optimal static portfolio. The maintenance of a significant large liquid position acts like a reserve for random adverse events. Most importantly, the ability to predict and avoid downturns without options and their cost is priceless. If shorting stocks is permitted, then those stocks with predicted next 1 -3 trade unfavorable change-point to add additional investment return is another significantly valuable feature. Predictive equity analytics when used to focus security research or equities on the run to identify factor casualty is a blueprint to identify other stock or sectors or/industries. Of course, could do full scan of 11,00 symbols but that takes time and being familiar with a set of stocks vs new unknown, take familiar every time.[9]

With predictive equity analytics, fund managers will find it optimal in every respect to switch to dynamic diversification or lose out to competitors.

[9] Exception IPOs who are typically mispriced when first offered.

16. Extensions

Predictive equity analytics extensions:

1) Advanced option pricing techniques using predictive equity analytics.
2) Retirement defined contribution plan 401(k)/403(b) prevention of account value downturn with market as well as faster accumulation rate that is superior to target date fund concept.
3) Life variable annuity/ variable life insurance policy holder - prevention of account value downturn with market as well as faster accumulation.
4) Life variable annuity/ variable life: introduce predictive equity analytics as tool for risk mitigation of Equity Benefit Guarantee that can be used with options and other hedging.

17. Conclusion

In this paper we have put forth strong evidence that $\widetilde{proxy}(\mu_t)$ and $\Delta\widetilde{proxy}(\mu_t)$ are predictive statistically with significance(alpha = 0.05, 0.1) and time varying in intensity for equities, ETFs, and mutual funds. Further support utilized row space and column space combined with principle component analysis. Retrospective detection of change-point occurrence in as little as 1 – 3 trade days using new finding that the coefficients for each element of principle eigenvector are simultaneous multiplied by "-1" relative to prior measurement indicates change-point is 100% accurate. Lastly time-forward signal set differs from time reversed signal set for all points except change-point is strong evidence. Finding that the time reversed principle eigen vector coefficients exhibit the same simultaneous multiplication by "-1" relative to prior measurement would "finish the proof," and is being explored.

Predictive equity analytics effect on asset return predictability has nontrivial implications for finances core tenet: risk versus return trade-off, enabling significant increase in return with simultaneous reduction in risk that is almost costless. The concept of risk aversion plays only an indirect role rather than a required input through the use of dynamic diversification, prospective change-point occurrence and retrospective change-point confirmation. Dynamic diversification's

natural re-balancing using predicted and retrospectively confirmed change-points makes for "extremely high Sharpe Ratios and very steep efficient frontiers. With the new view provided by predictive equity analytics of the true data generating process further research is warranted that will yield more significant results.

References

[1] Albano G., Giorno V. (2020) Inferring time non-homogeneous Ornstein Uhlenbeck type stochastic process. Computational Statistics and Data Analysis, 150, 107008

[2] An, Li, and Bronson Argyle, 2021. Overselling winners and losers: How mutual fund managers' trading behavior affects asset prices, Journal of Financial Markets 55, 100580.

[3] Ang, A., and Bekaert, G.. 2007. "Stock Return Predictability: Is It There?" Review of Financial Studies, 20 , 651–707

[4] Ascione G., D'Onofrio G., Kostal L., Pirozzi E. (2020) An optimal Gauss-Markov approximation for a process with stochastic drift and applications. Stochastic Processes and their Applications. 130(11), 6481-6514.

[5] Barberis, N., 2000. Investing for the long run when returns are predictable.

[6] Barndorff-Nielsen, Ole E. and Neil Shephard, 2001. Non-Gaussian Ornstein–Uhlenbeck-based models and some of their uses in financial economics. Journal of the Royal Statistical Society: Series B (Statistical Methodology), Volume 63 Issue 2, Pages 167-241

[7] Barndorff-Nielsen, Ole E. and Neil Shephard, 2002. Econometric analysis of realized volatility and its use in estimating stochastic volatility models, Journal of the Royal Statistical Society: Series B (Statistical Methodology), Volume 64, Issue 2, May 2002, Page 253-280

[8] Bartlett, Mark S.. "The Characteristic Function of a Conditional Statistic." Journal of The London Mathematical Society-second Series (1938): 62-67.

[9] Bawa, V. S., Brown, S. J., and Klein, R. W. (1979) Estimation Risk and Optimal Portfolio Choice, Amsterdam: North Holland

[10] Billingsley, P. Convergence of Probability Measures, John Wiley & Sons Inc. New York, NY, 1999

[11] Bollserov, T. and Mikkelson, H., Modeling and pricing long memory in stock market volatility, Journal of Econometrics 1996 Volume 72, Issue 1, pages 151-184.

[12] Bossaerts, P. and P. Hillion, 1999, Implementing Statistical Criteria to Select Return Forecasting Models:what do we learn? Review of Financial Studies 12, 405-428.

[13] Breeden. D.T., 1979, An intertemporal asset pricing model with stochastic consumption and investment opportunities, Journal of Financial Economics 7, 265-96.

[14] Brennan, M. J., 1998, The Role of Learning in Dynamic Portfolio Decisions, European Economic Review 1, 295-306.

[15] Brennan, M.J. and Y. Xia, 1998a, Stock Price Volatility, Learning, and the Equity Premium, Working Paper, Anderson School of Management, UCLA.

[16] Brennan, M.J. and Y. Xia, 1998b, Resolution of an Asset Allocation Puzzle, Working Paper, Anderson School of Management, UCLA.

[17] Brennan, Michael John, and Xia, Yihong, Dynamic Asset Allocation Under Inflation (October 17, 2000). AFA 2002 Atlanta Meetings, Available at SSRN: https://ssrn.com/abstract=248693 or http://dx.doi.org/10.2139/ssrn.248693

[18] Brodsky, B. E., 1993, Nonparametric Methods in Change-Point Problems, Kluver Academic Publishers.

[19] Chung, K. L., A Course in Probability Theory, 2nd Ed., Academic Press 1974.

[20] Collin-Dufresne, Pierre, Kent D Daniel, Ciamac C Moallemi, and Mehmet Saglam, "Dynamic asset allocation with predictable returns and transaction costs," Available at SSRN 2618910, 2015

[21] Cox, D.R., Miller, H. D., 2017, The Theory of Stochastic Process, Chapmann & Hall

[22] Cox, J., and S, Ross, "The Valuation of Options for Alternative Stochastic Processes," Journal of Financial Economics, 1976.

[23] Crowder, M. J., "Maximum Likelihood Estimation for Dependent Observations." Journal of the Royal Statistical Society, Series B, 38(1976), 45-53

[24] Dai, Min & Duan, Jinqiao & Hu, Jianyu & Wen, Jianghui & Wang, Xiangjun. (2022). Variational inference of the drift function for stochastic differential equations driven by Lévy processes. Chaos: An Interdisciplinary Journal of Nonlinear Science. 32. 061103. 10.1063/5.0095605.

[25] Christoffersen, P., and Diebold, F., 2002, "Financial Asset Returns, Market Timing, and Volatility Dynamics," NBER 2003 Conference, available at https://users.nber.org/~confer/2003/si2003/papers/efww/diebold.pdf

[26] Dupire, Bruno, 1993. Pricing and Hedging with Smiles, Paribas Capital Markets.

[27] Dupire, Bruno, 2009. Functional Ito Calculus.

[28] Campbell, J., Lo, A., MacKinlay, C., 1997. The Econometrics of Financial Markets. Princeton University Press, Princeton, N.J.

[29] Emanuel, David. "Optimal Financial Policy and Firm Valuation: Discussion." The Journal of Finance, vol. 39, no. 3, 1984, pp. 607–09. JSTOR, https://doi.org/10.2307/2327918. Accessed 16 May 2024.

[30] Fama, E. F., and French, K. R.. 1992. “The Cross-Section of Expected Stock Returns.” Journal of Finance, 47 , 427–465

[31] Fama, E.F. and K.R. French, 1988. Permanent and Temporary Components of Stock Prices, Journal of Political Economy 96, 246-273.

[32] Fleming, J.; Kirby, C.; and Ostdiek, B.. 2001. “The Economic Value of Volatility Timing.” Journal of Finance, 56 , 329–352

[33] Florens-Zmirou, D. (1989) Approximate discrete-time schemes for statistics of diffusion processes. Statistics, 20, 547–557.

[34] Florens-Zmirou, D. (1993) On estimating the diffusion coefficient from discrete observations. J. Appl. Probabilty., 30, 790–804.

[35] French, K., and Roll, R. 1985 Stock Returns variances, the arrival of new information and the reaction of traders, UCLA working paper.

[36] Gibbons, Michael R, Stephen A Ross, and Jay Shanken, "A test of the efficiency of a given portfolio," Econometrica: Journal of the Econometric Society, 1989, pp. 1121-1152.

[37] Gnedenko, B . V.and Kolmogorov, A . N. Kolmogorov, Limit distributions for sums of independent random variables (translated from the Russian) . Addison-Wesley Publishing Co., Inc ., Reading, Mass ., 1954

[38] Henriksson, R.D., and R.C. Merton (1981), "On Market Timing and Investment Performance. II. Statistical Procedures for Evaluating Forecasting Skills", Journal of Business, 54, 513-533

[39] Hirshleifer, J. and Riley, J., The Analytics of Uncertainty and Information, Cambridge University Press. 1994.

[40] Hornik, K., Stinchcombe, M., and White, H., “Universal approximation of an unknown mapping and its derivatives using multilayer feedforward networks," Neural networks, 1990, 3 (5), 551-560.

[41] Ingersoll, J.E.,Jr., 1987. Theory of Financial Decision Making, Rowman and Littlefield Publishers, Inc.

[42] Johannes, M. ,Korteweg G., Polson N., Sequential learning, predictability, and optimal portfolio returns.

[43] Kandel, S., and Stambaugh, R. F. 1996. “On the Predictability of Stock Returns: An Asset-Allocation Perspective.” Journal of Finance, 51 , 385–424.

[44] Karatzas, I., Shreve, S., 1991. Brownian Motion and Stochastic Calculus, Second Edition. Springer- Verlag, New York.

[45] Kelly, B., Malamud, S., Zhou, K., 2022, "The Virtue of Complexity in Return Prediction," NBER Working Paper 30217, http://www.nber.org/papers/w30217.

[46] Keim, D.B. and R. F. Stambaugh, 1986. Predicting Returns in the Stock and Bond Markets, Journal of Financial Economics 17, 357-390.

[47] Krylov,N. V., Controlled Diffusion Processes, Springer, 2000 (reprint 1980).

[48] Large, S., (2024) Fokker-Planck: A Dynamic Integrator for Python v1.0.17 (Juypter Lab/Notebook) , MIT, https://github.com/StevenJLarge/FokkerPlanck

[49] Lo, A.W. (1988), "Maximum Likelihood Estimation of Generalized Ito Processes with Discretely Sampled Data" Econometric Theory, 4, 231-237.

[50] Lo, A.W. and A.C. MacKinlay, 1988. Stock Market Prices Do Not Follow Random Walks: Evidence from a Simple Specification Test, Review of Financial Studies 1, 41-66.

[51] Lo, A. W., and MacKinlay, A. C..1997. "Maximizing Predictability in the Stock and Bond Markets." Macroeconomic Dynamics, 1 , 102–134.

[52] Lo., A., and Wang, W., "Implementing Option Pricing Models When Asset Returns Are Predictable." Journal of Finance, VOL. L, NO. 1, March 1995

[53] Markowitz, H.M. 1952, Portfolio selection. Journal of Finance 7, March, 77-91.

[54] Markowitz, H.M. 1959, Portfolio selection: Efficient Diversification of Investments. New York: Wiley.

[55] Marquering, W., Verbeek, M., The Economic Value of Predicting Stock Index Returns and Volatility, The Journal of Financial and Quantitative Analysis, Vol. 39, No. 2 (Jun. 2004), pp. 407-429

[56] Merton, R., 1992. Continuous-time Finance. Blackwell, Cambridge, MA.

[57] Merton, Robert C., 1980, On estimating the expected return on the market: An exploratory investigation, Journal of Financial Economics 8, 323-361.

[58] Merton, R. C. (1976) "Option Pricing When underlying stock returns are discontinuous", Journal of Financial Economics, 3, 125-144

[59] Merton, R. C. (1971) Optimal consumption and portfolio rules in a continuous-time model, Journal of Economic Theory 3, 373–413.

[60] Merton, R. C. Lifetime portfolio selection under uncertainty: the continuous time case, Rev. Econ. Statist. LI (August 1969), 247-257.

[61] Pavliotis, G., 2014, Stochastic Processes and Applications: Diffusion Process, the Fokker-Planck and Langevin Equations, Springer

[62] Pedersen, A.R. 1995, "A New Approach to Maximum Likelihood Estimation for Stochastic Differential Equations based on Discrete Observations." Scandinavian Journal of Statistics, 22, p 55-71.

[63] Poulsen, R. (1999) Approximate maximum likelihood estimation of discretely observed diffusion processes. Working paper 29, Centre for Analytical Finance, Aarhus.

[64] RATS - Regression Analysis for Time Series v10.0, Estima Inc., Evanston, Ill.

[65] Robinson, P.,"Some Problems in the Identification and Estimation of Continuous Time Systems from Discrete Time Series," System Identification: Advances and Case Studies, Academic Press 1976.

[66] Rosenfeld, E., (1980), "Stochastic Processes of Common Stock Returns: An Empirical Examination, PhD. dissertation A.P. Sloan School of Management, Massachusetts Institute of Technology. Cambridge, MA.

[67] Rossi, P., 1996. Modeling Stock Market Volatility: Bridging the Gap to Continuous Time, Academic Press.

[68] Roy, S., Annunziato, M., Borzì, A., and Klingenberg, C. (2018). A Fokker–Planck approach to control collective motion. Computational Optimization and Applications, 69(2), 423-459.

[69] Samuelson, P. A,, "Lifetime Portfolio Selection by Dynamic Stochastic Programming," (Aug. 1969)

[70] Stambaugh, R., 1999. Predictive Regressions. Journal of Financial Economics 54, 375–421.

[71] Tu, J., Zhou, G., 2004. Data-generating process uncertainty: What difference does it make in portfolio decisions? Journal of Financial Economics 72, 385–421.

[72] Xia, Y., 2001. Learning about predictability: the effects of parameter uncertainty on dynamic asset allocation. Journal of Finance 55, 205–246.

[73] Vasicek, Oldrich, 1977, An Equilibrium Characterization of the Term Structure, Journal of Financial Economics, 5, 177-188.

[74] Zanecki, Mark, Empirical Stock Return Prediction as an Extension of Merton's Asset Return Model (June 28, 2021). Available at SSRN: https://ssrn.com/abstract=3875632 or http://dx.doi.org/10.2139/ssrn.3875632

[75] Zanecki, Mark, Predictive Equity Analytics: Portfolio Crisis Proofing and Superior Returns (September 3, 2020). Available at SSRN: https://ssrn.com/abstract=3703297 or http://dx.doi.org/10.2139/ssrn.3703297

Figures

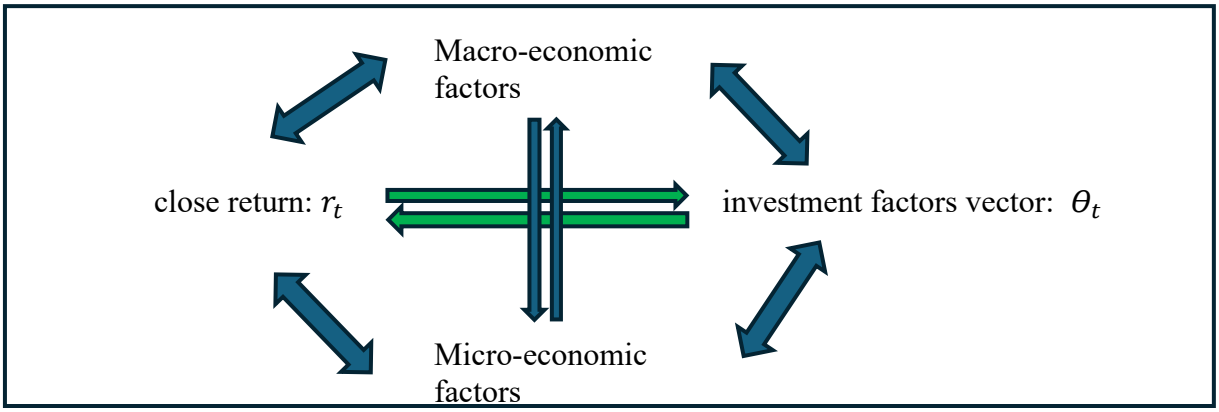


Figure 1

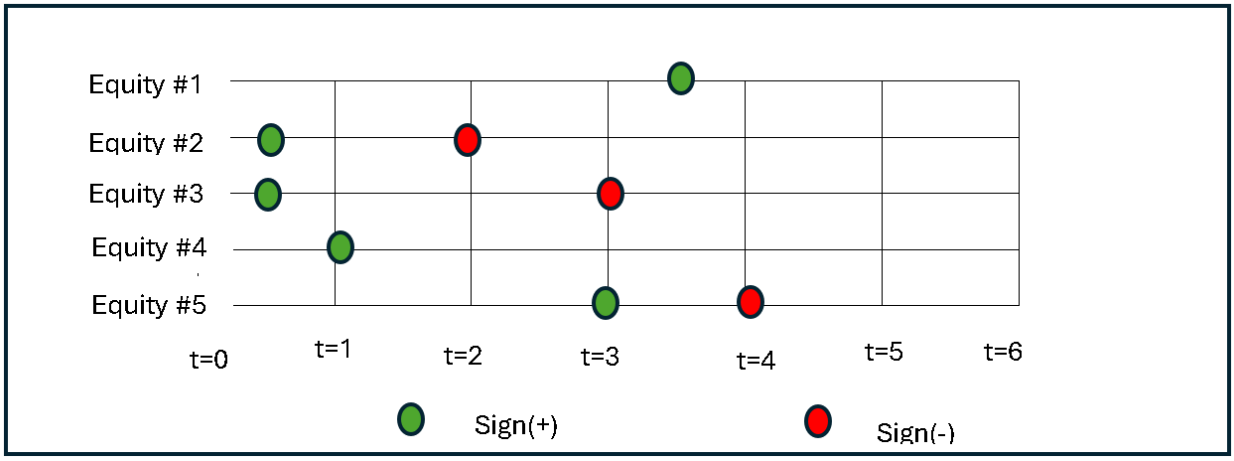


Figure 2

Predictive equity analytics enables dynamic diversification by which purchase the set of stocks/equities with predicted change-points in next 1 – 3 trade days and sell with prediction of upcoming unfavorable change-point, all prior to market open. Allocation weights are on "hot stocks on the run" and no allocation to "dead money stocks."

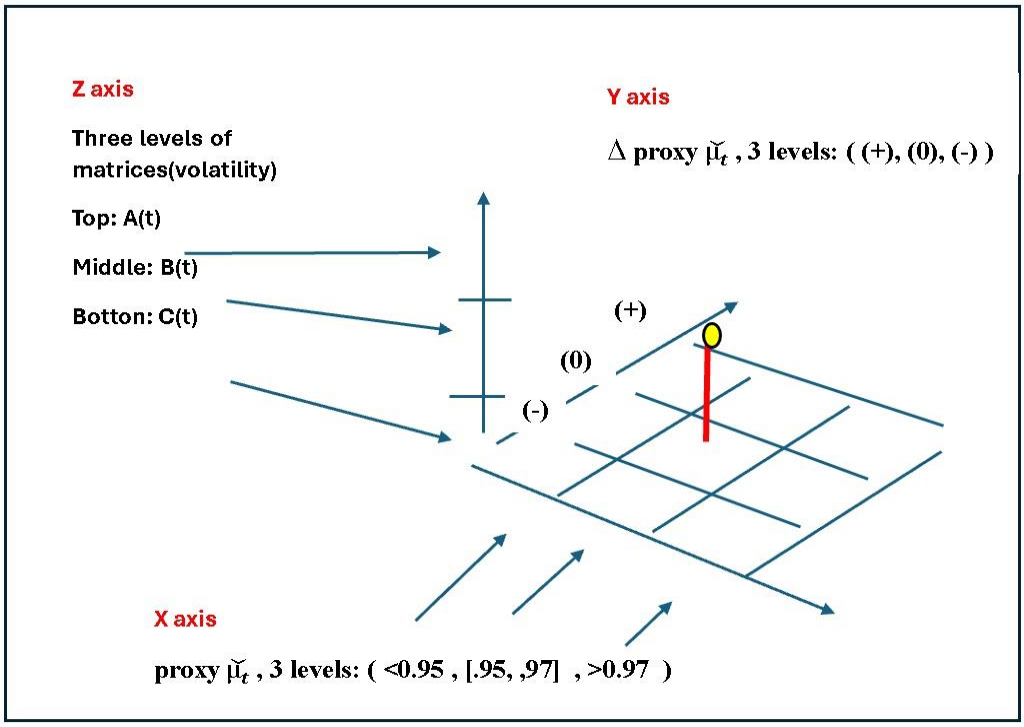


Figure 7

Each close return data point is tagged with a unique combination of a) variance level on z-axis, b) proxy (μ_t)ˇlevel on x-axis and c) Δ proxy (μ_t)ˇon y-axis. The data point would be assigned matrix B using volatility as first criteria then row(1) for no meu proxy momentum and col(1) proxy meu signal level [0.95, 0.97] and labeled as collapsed state space 13.

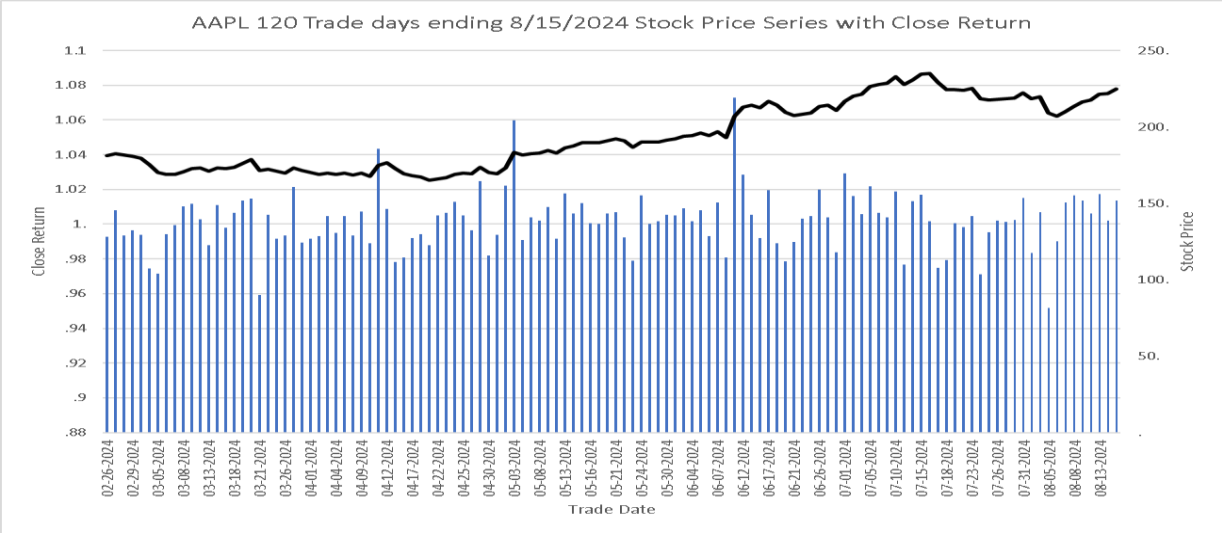


Figure 13

AAPL 120 Trade-day ending 8/15/2024 Stock Price and Close Return time-series.

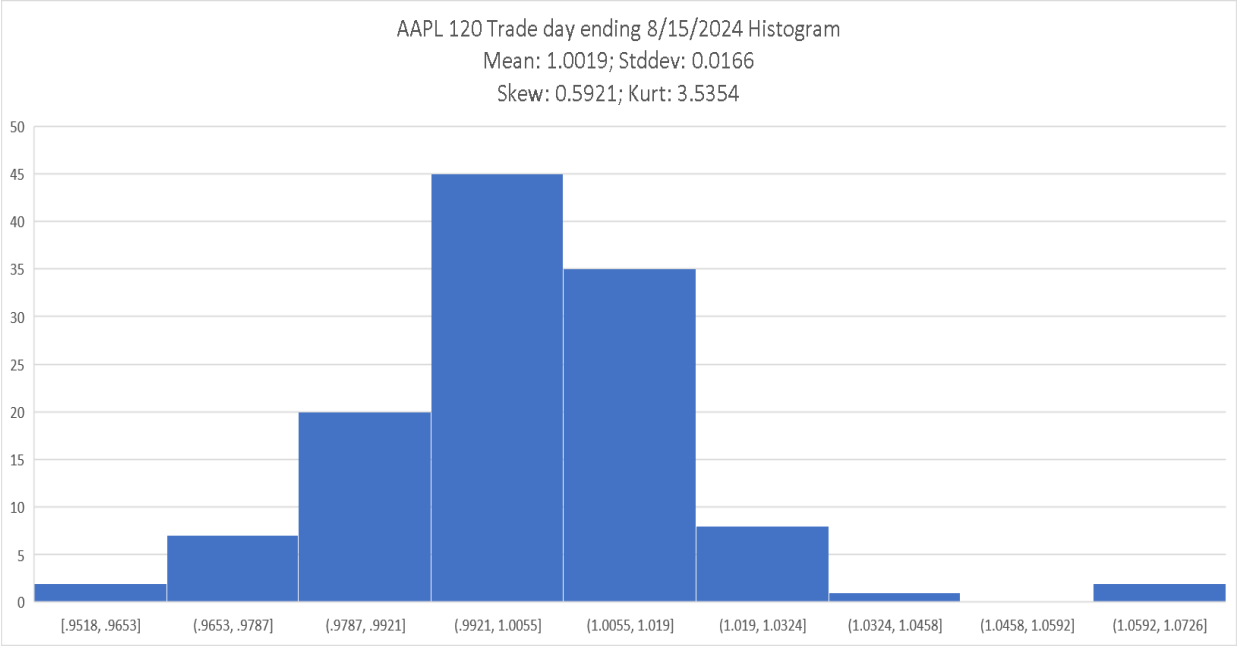


Figure 14

AAPL 120 Trade-day ending 8/15/2024 Close Return Histogram.

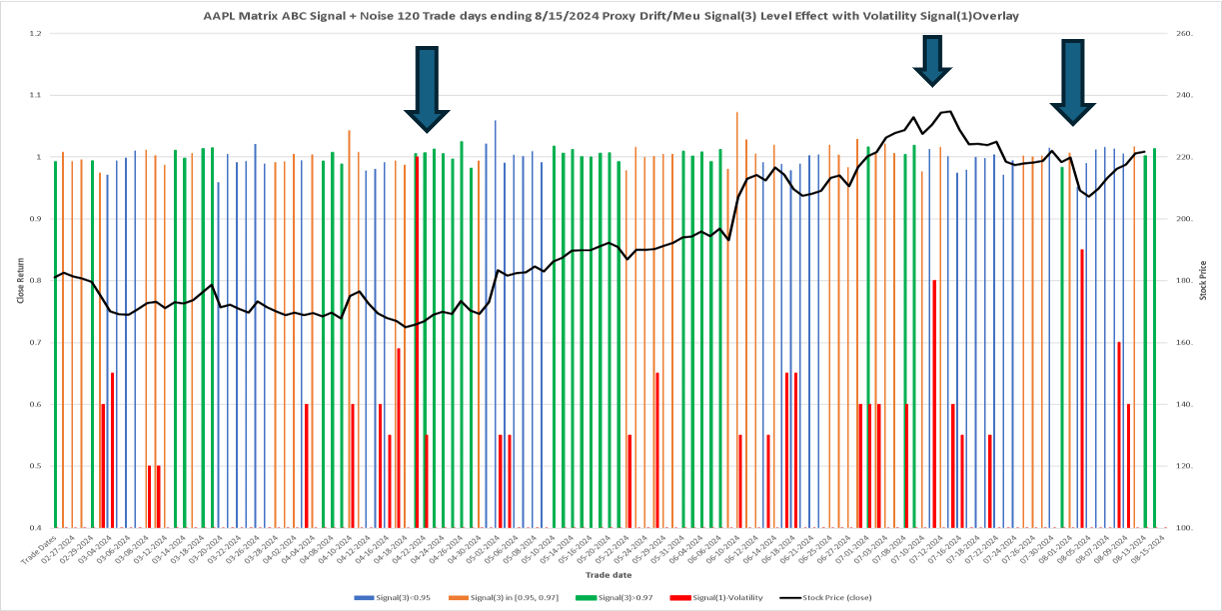


Figure 15

Predictive Equity Analytics: AAPL Matrix $A_t B_t C_t$ Proxy Drift/meu Signal(3) Level 120 Trade-day ending 8/15/2024 Stock Price, Close Return time-series with overlay Volatility Matrix $A_t B_t$.Notice the volatility clustering displayed by Volatility Matrix $A_t B_t$. Volatility Matrix $A_t B_t$ signal $\in$ [near 0.7, 1] is associated with change-points. Also notice where AAPL time-series behavior changes, the Signal(3) Proxy drift/meu changed. Lastly, notice that Signal(3)>0,97 (green) correlates increasing behavior.

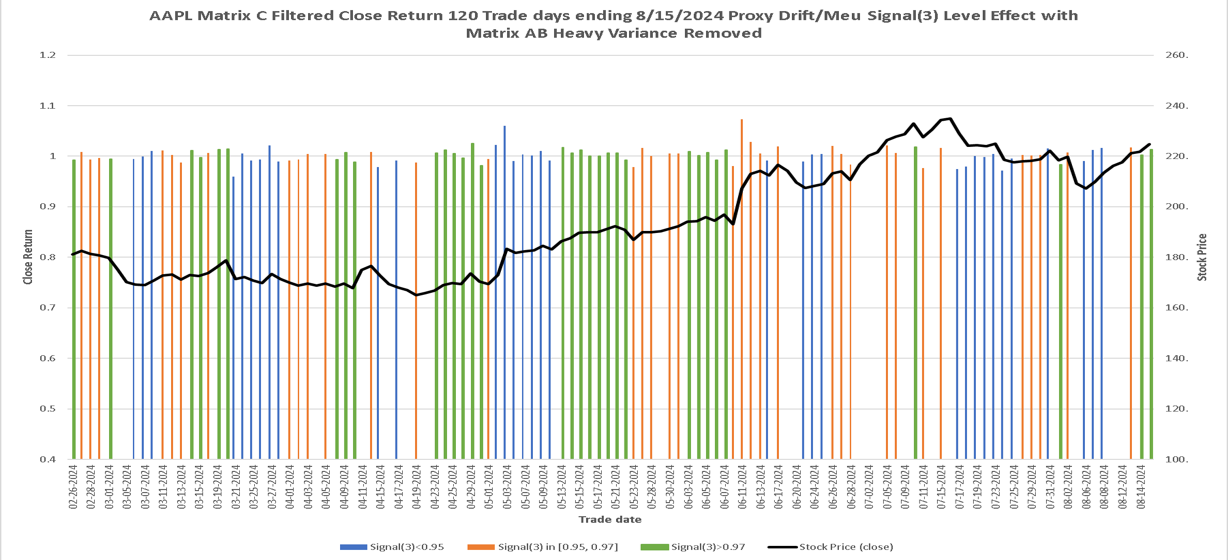


Figure 16

Predictive Equity Analytics: AAPL Filtered Close Return Matrix C_t Proxy Drift/meu Signal(3) Level 120 Trade-day ending 8/15/2024 Stock Price, Close Return time-series with Volatility Matrix $A_t B_t$ excluded.

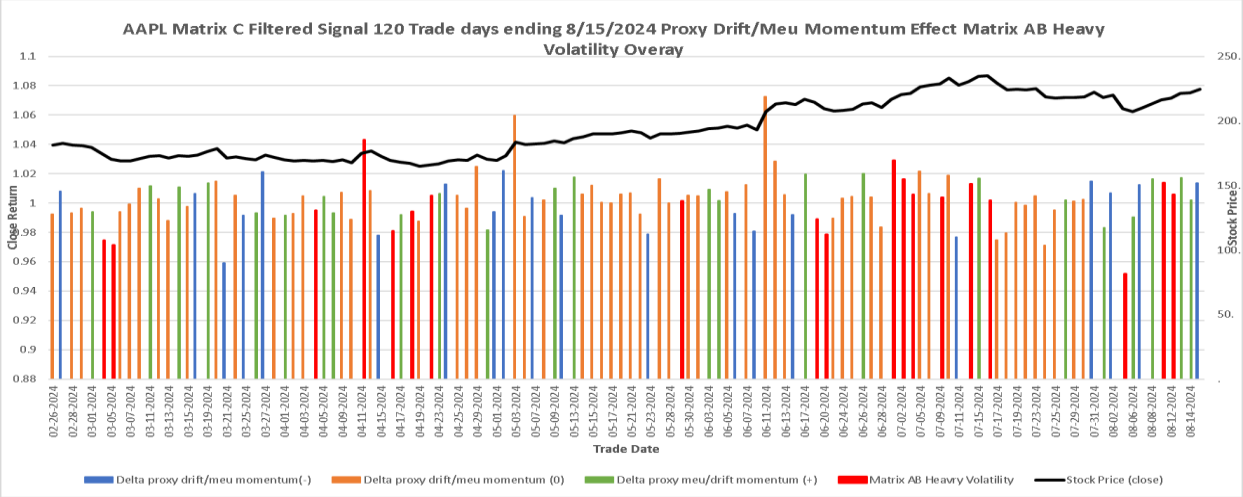


Figure 17

Predictive Equity Analytics: AAPL Filtered Close Return Matrix C_t Proxy Drift/meu Momentum 120 Trade-day ending 8/15/2024 Stock Price, Close Return time-series with Volatility Matrix A_tB_t overlay. There is a definite Proxy Drift/meu Momentum effect but not as strong as Proxy Drift/meu Signal(3) strength.

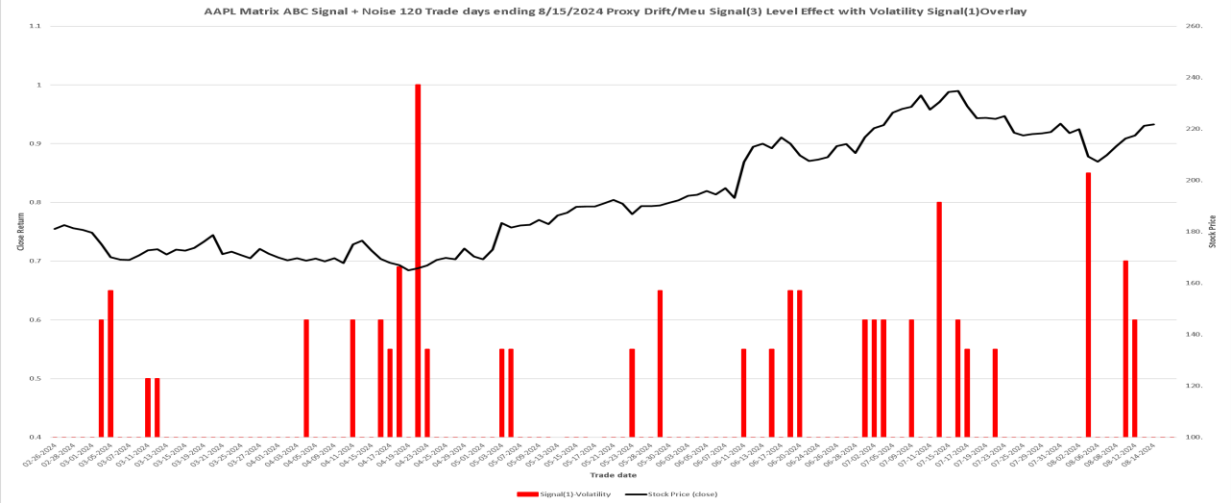
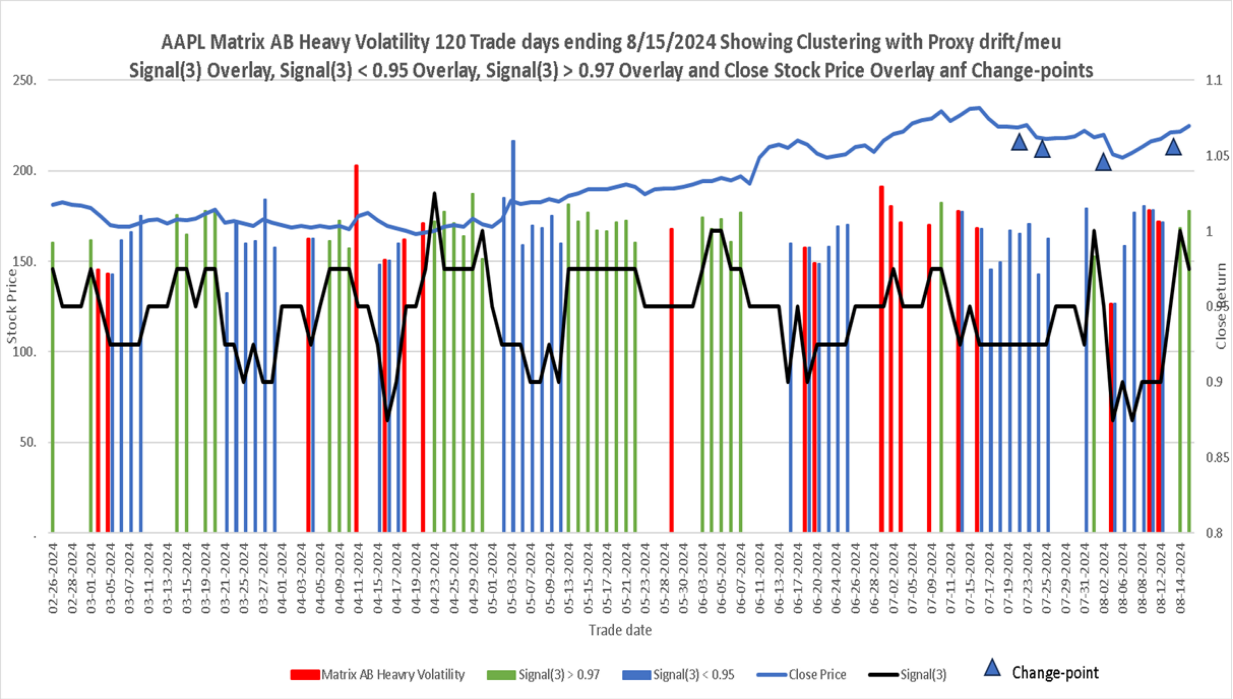


Figure 18

Predictive Equity Analytics: AAPL Heavy Volatility Matrix A_tB_t 120 Trade-day ending 8/15/2024 with Stock Price Close Return time-series. Notice Volatility Matrix A_tB_t Signal(1) > 0.7 indicates a change-point and change in $sign_t(r_t)$. Signals are available prior to market open each trade-day so buy/sell prediction based on change-point likely occurrence is now a reality for most symbols. It is sufficient to have a suitably large number of symbols with known predictability than to attempt to discern every symbol.



AAPL Matrices ABC, C 8/1-8/15/2024 Close Return using Proxy Drift/meu Signal(3) and Change in Proxy Drift/meu Signal(3)

Trade Dates	08-01-2024	08-02-2024	08-05-2024	08-06-2024	08-07-2024	08-08-2024	08-09-2024	08-12-2024	08-13-2024	08-14-2024	08-15-2024
Matrix ABC											
Signal(3)>0.97	0.98325									1.00203	1.01353
Signal(3)[0.95, 0.97]		1.00687							1.01719		
Signal(3)<0.95			0.95183	0.99025	1.01250	1.01663	1.01374	1.00597			
Matrix ABC											
momentum(+)	0.98325			0.99025		1.01663			1.01719	1.00203	
momentum(0)							1.01374	1.00597			
momentum(-)		1.00687	0.95183		1.01250						1.01353
Matrix C											
Signal(3)>0.97	0.98325									1.00203	1.01353
Signal(3)[0.95, 0.97]		1.00687							1.01719		
Signal(3)<0.95				0.99025	1.01250	1.01663					
Matrix C											
momentum(+)	0.98325			0.99025		1.01663			1.01719	1.00203	
momentum(0)											
momentum(-)		1.00687			1.01250						1.01353

Figure 19

Predictive Equity Analytics: AAPL 120 Trade-day ending 8/15/2024 with Stock Price time-series, Close Return time-series, Signal(3)<0.95, Signal(3)>0.97, Volatility Matrix $A_t B_t$ and (new finding) retrospective detection of change-points using maximal eigenvalue and associated eigenvector. Below is 8/1-8/15 Partition of close return time series.

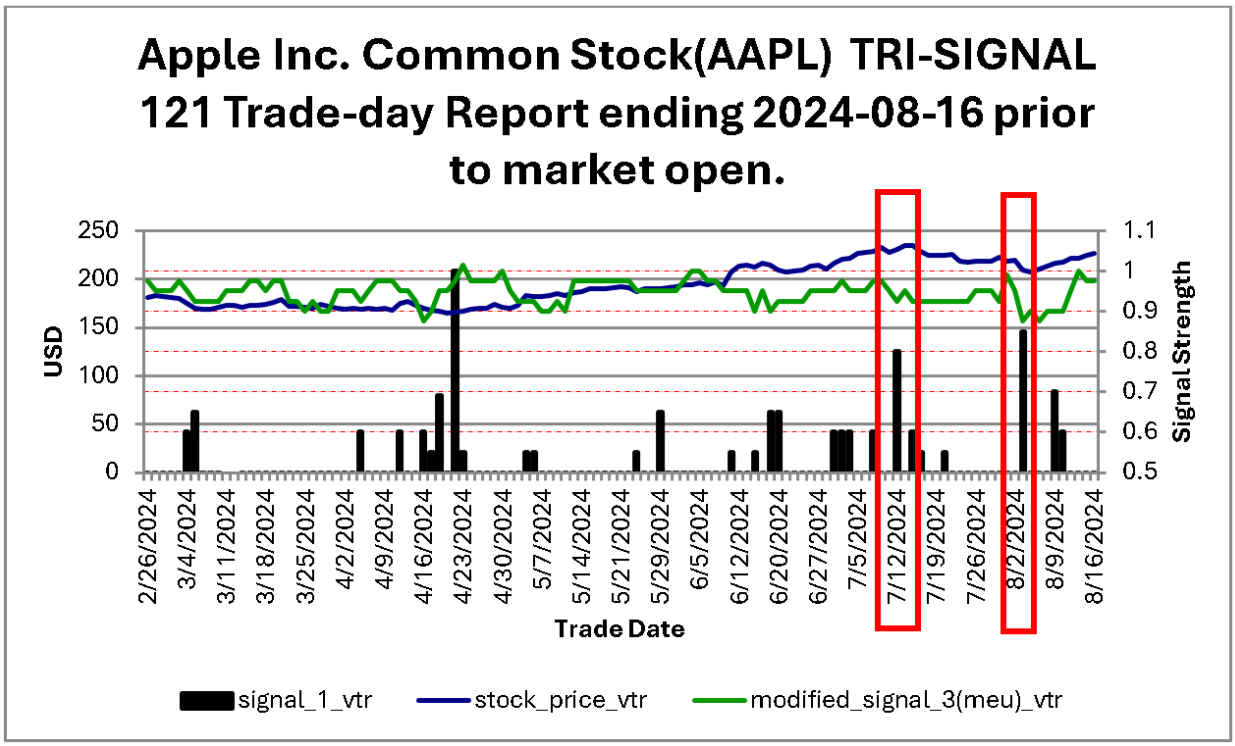


Figure 22

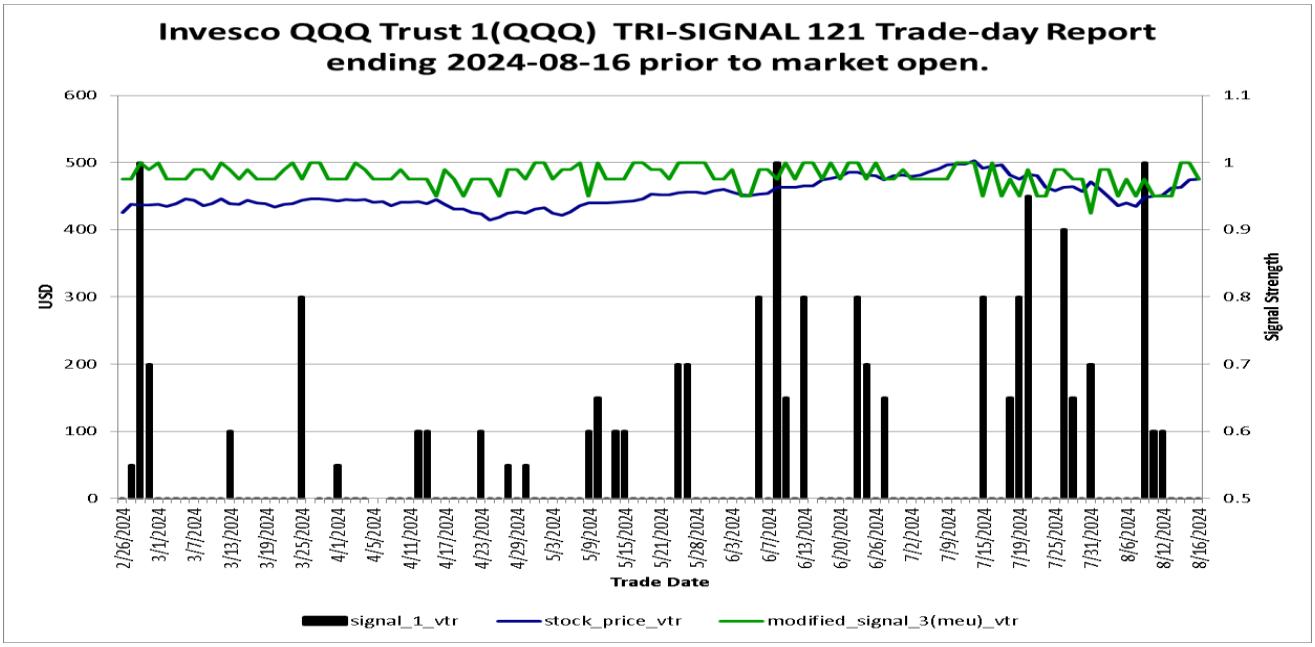


Figure 24

Predictive equity analytics signals{1,3} for Invesco QQQ.

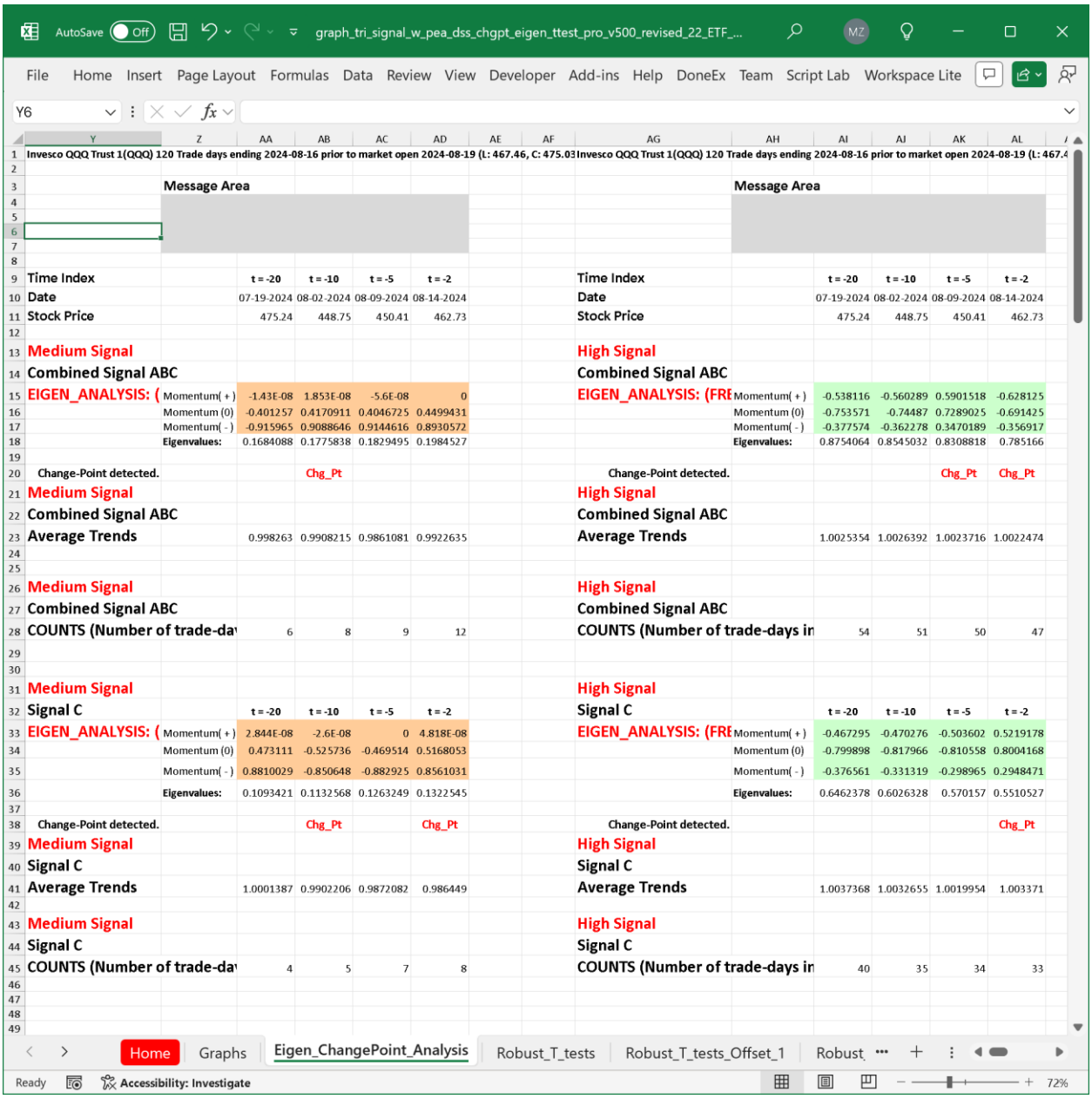


Figure 25

Invesco QQQ change points detected in the time dimension of the discrete Fokker-Planck approximation for Proxy Drift/meu over time, $Signal_t(3) \in [0.95, 0.97]$ "medium signal" and for Proxy Drift/meu over time, $Signal_t(3) > 0.97$ "high signal" for trade dates {08-02-2024, 08-09-2024, 08-14-2024}

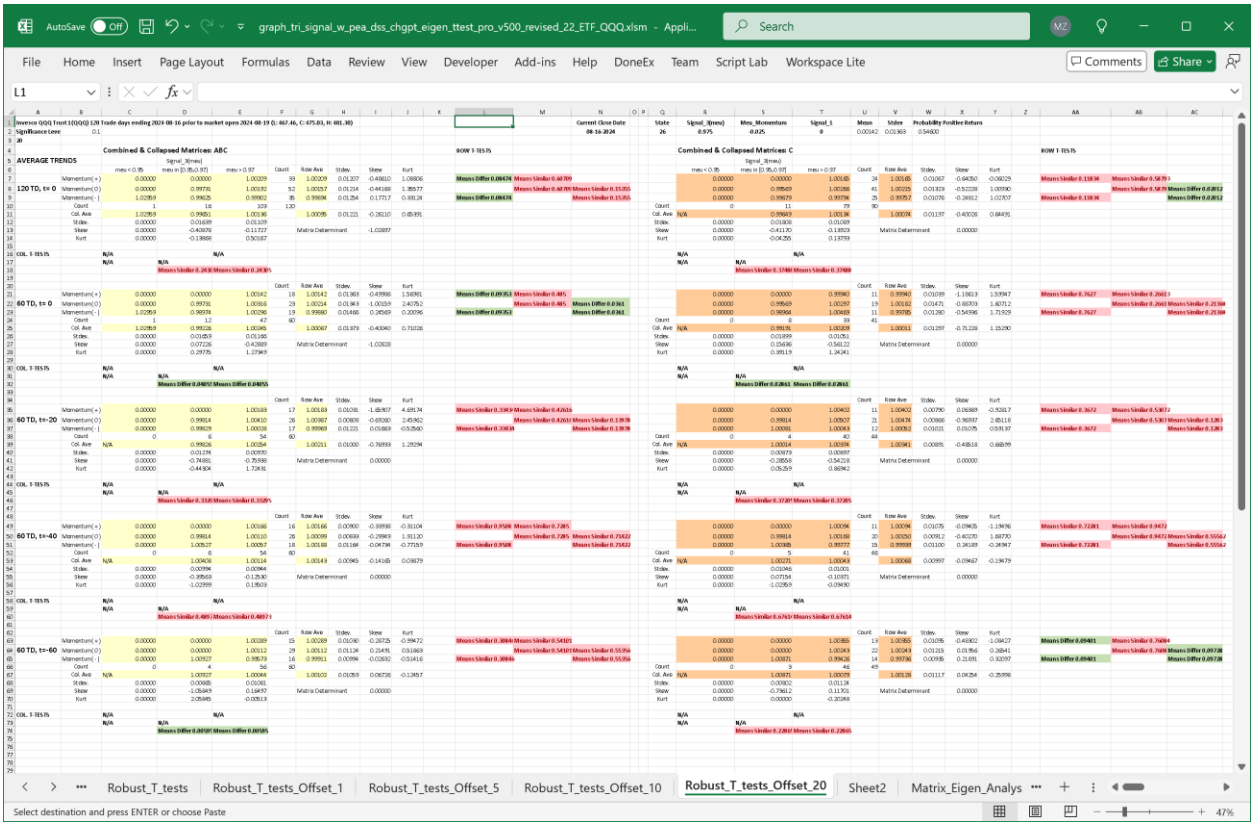


Figure 26

Invesco QQQ Sliding 60 Trade-day using offset 20, robust T-tests show time varying in Proxy Drift/meu signal(3) level (columns) and Change in Proxy Drift/meu (meu momentum) (rows) ranging from activity to disappearing and then reappearing starting from t-60 to t-0.

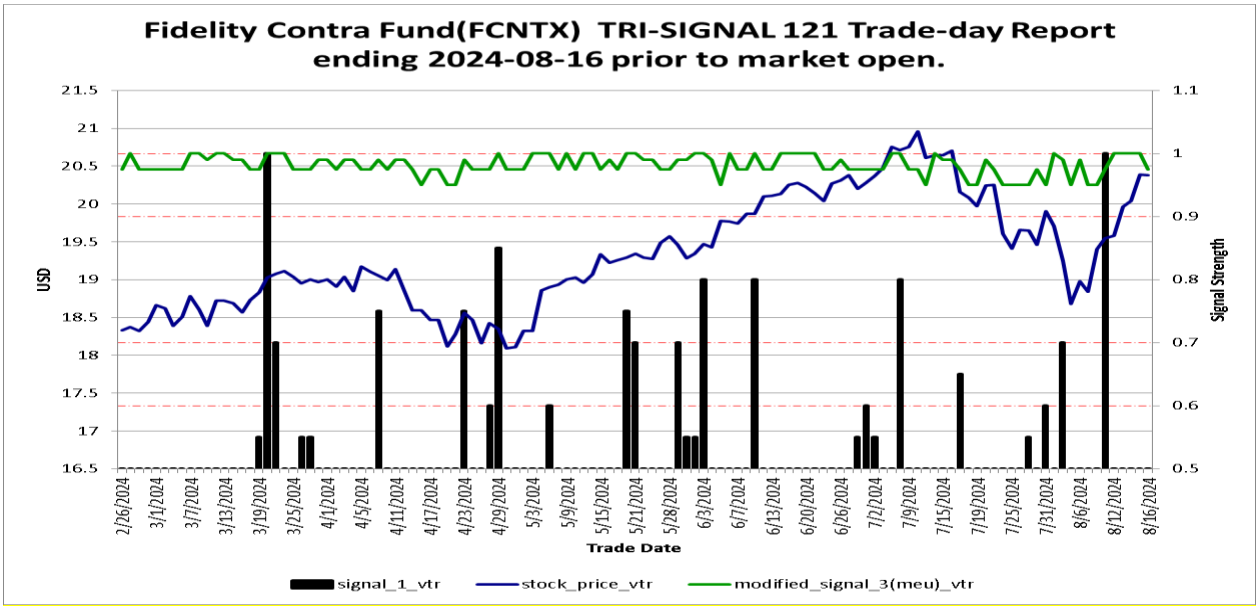


Figure 27

Predictive equity analytics signals{1,3} for Fidelity FCNTX.

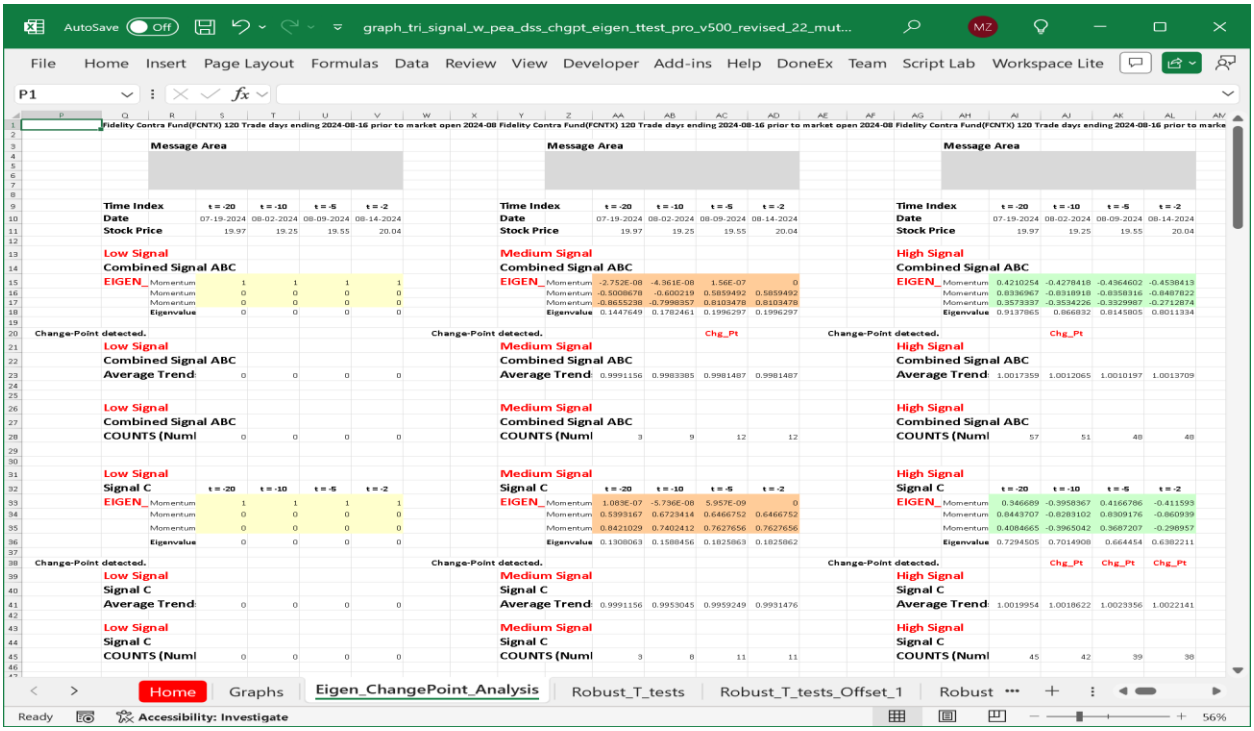


Figure 28

Fidelity FCNTX change points detected in the time dimension of the discrete Fokker-Planck approximation for Proxy Drift/meu over time, $Signal_t(3) \in [0.95, 0.97]$ "medium signal" and for Proxy Drift/meu over time, $Signal_t(3) > 0.97$ "high signal" for trade dates {08-02-2024, 08-09-2024, 08-14-2024}

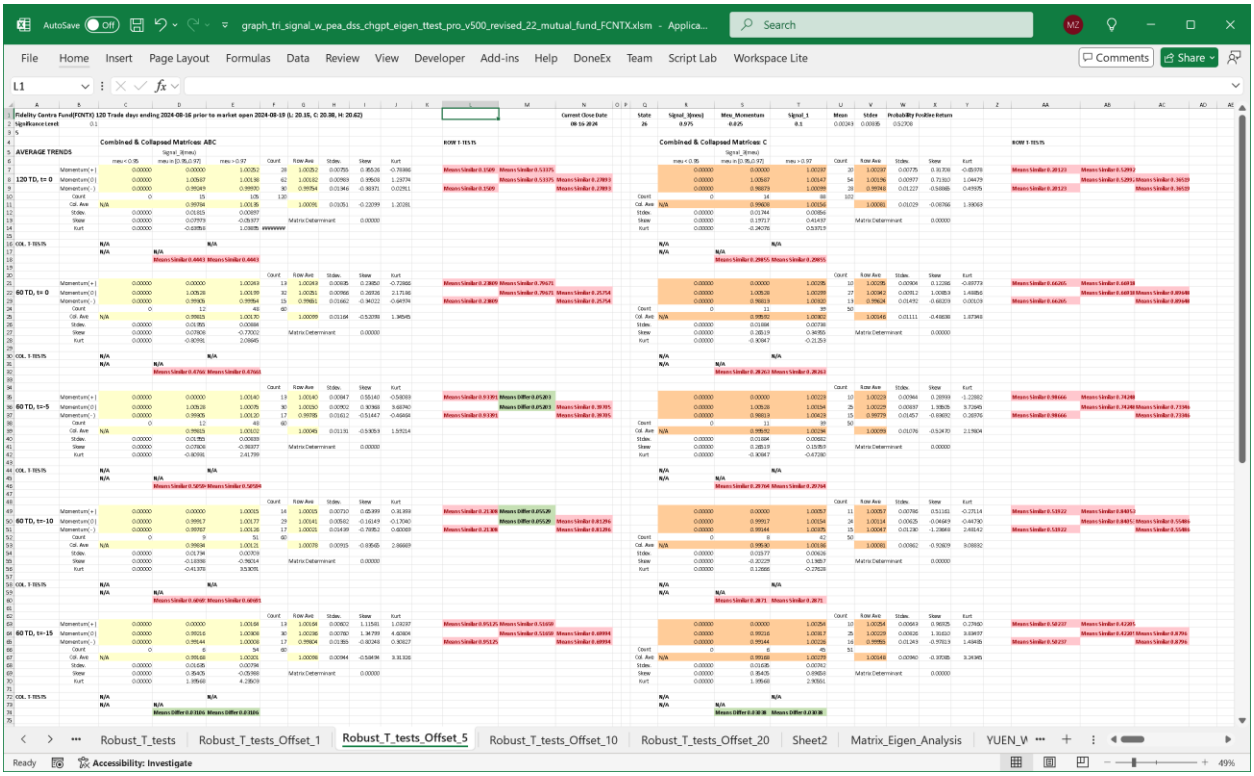


Figure 29

Fidelity FCNTX sliding **60 Trade-day using offset 5,** robust T-tests show time varying in Proxy Drift/meu signal(3) level (columns) and Change in Proxy Drift/meu (meu momentum) (rows) ranging from activity to disappearing starting from t-60 to t-0 for noisy matrix $A_t B_t C_t$ with filtered close return matrix C_t active at t-60 and then disappears for all subsequent offsets.

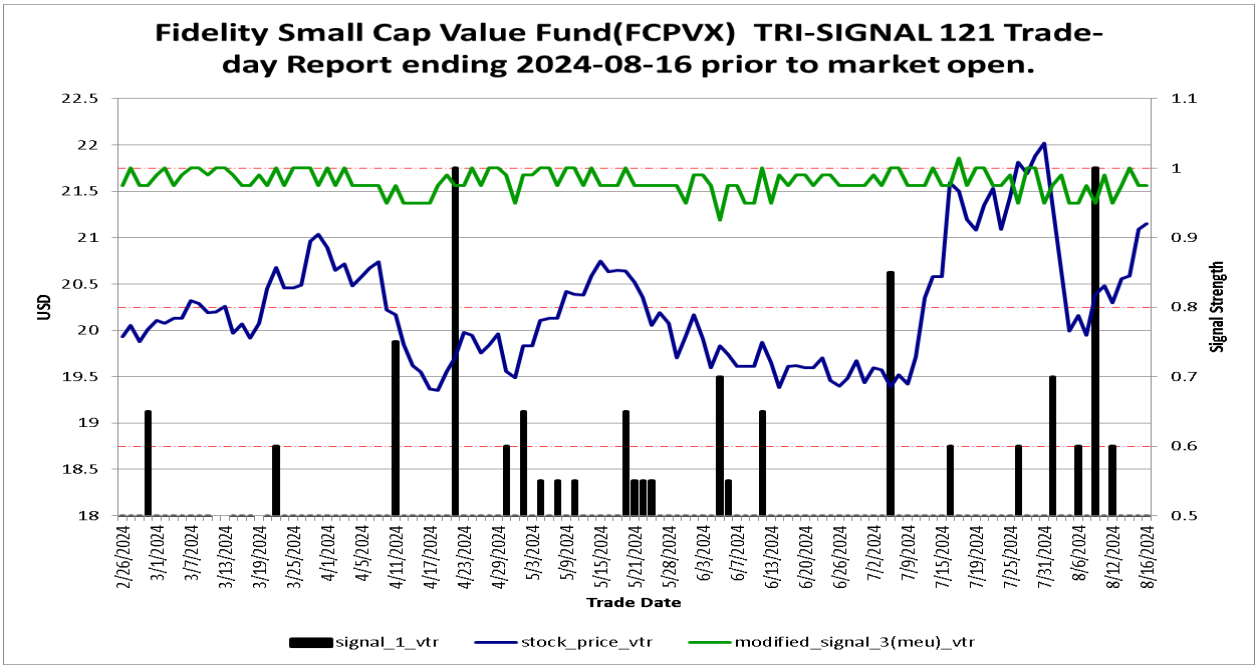


Figure 30

Predictive equity analytics signals{1,3} for target date fund: Fidelity FCPVX.

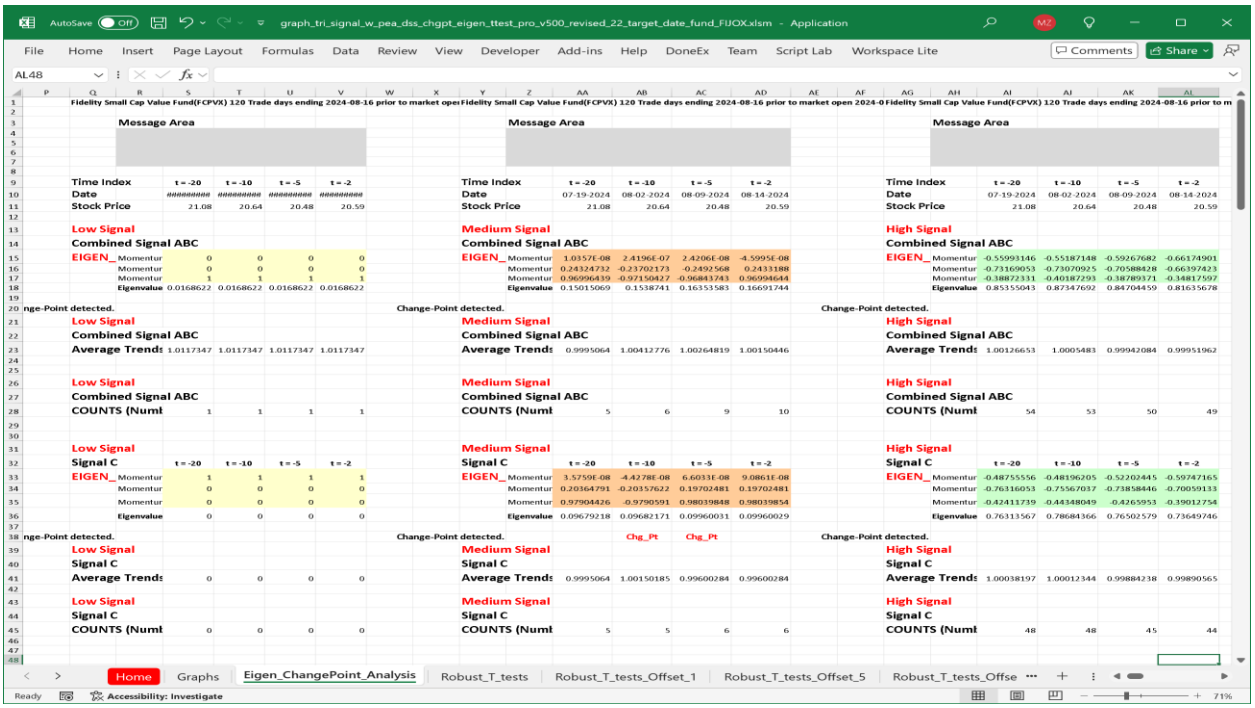


Figure 31

Fidelity FCPVX change points detected in the time dimension of the discrete Fokker-Planck approximation for Proxy Drift/meu over time, $Signal_t(3) \in [0.95, 0.97]$ "medium signal" for trade dates {08-02-2024, 08-09-2024}

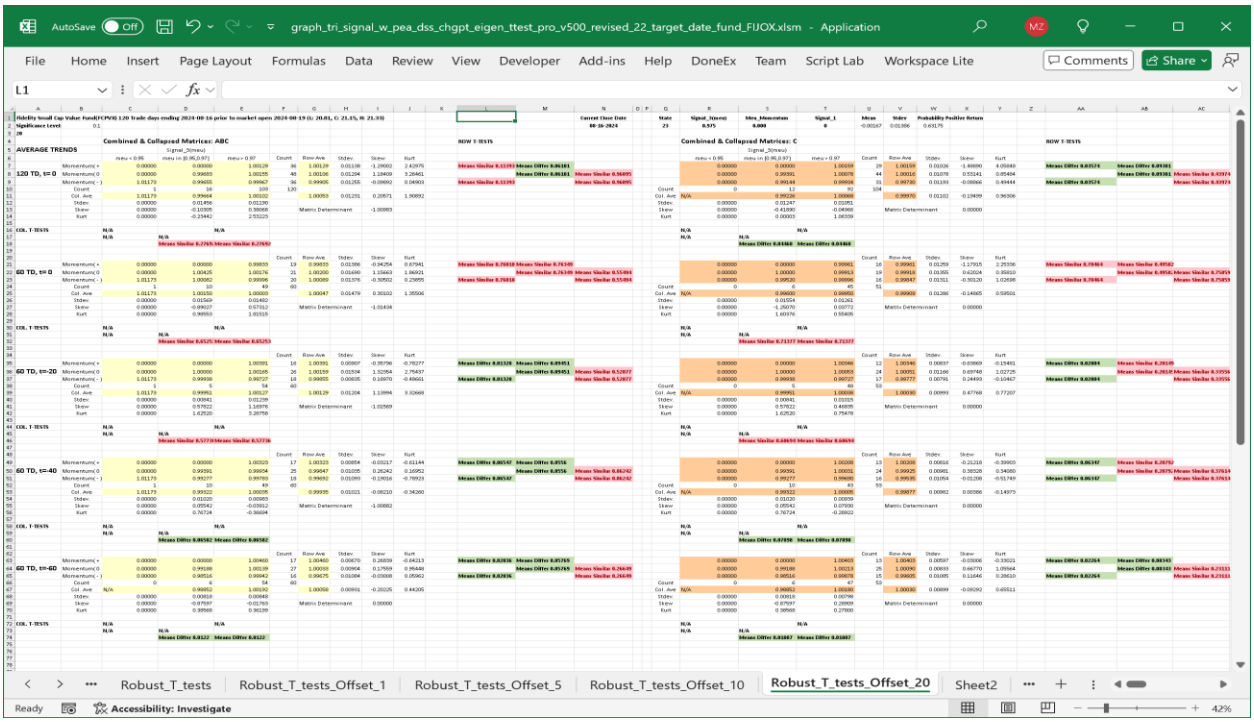


Figure 32

Fidelity FCPVX sliding **60 Trade-day using offset 20.** robust T-tests show time varying in Proxy Drift/meu signal(3) level (columns) and Change in Proxy Drift/meu (meu momentum) (rows) ranging from activity to disappearing starting from t-60 to t-0 for noisy matrix $A_t B_t C_t$ and close return matrix C_t.

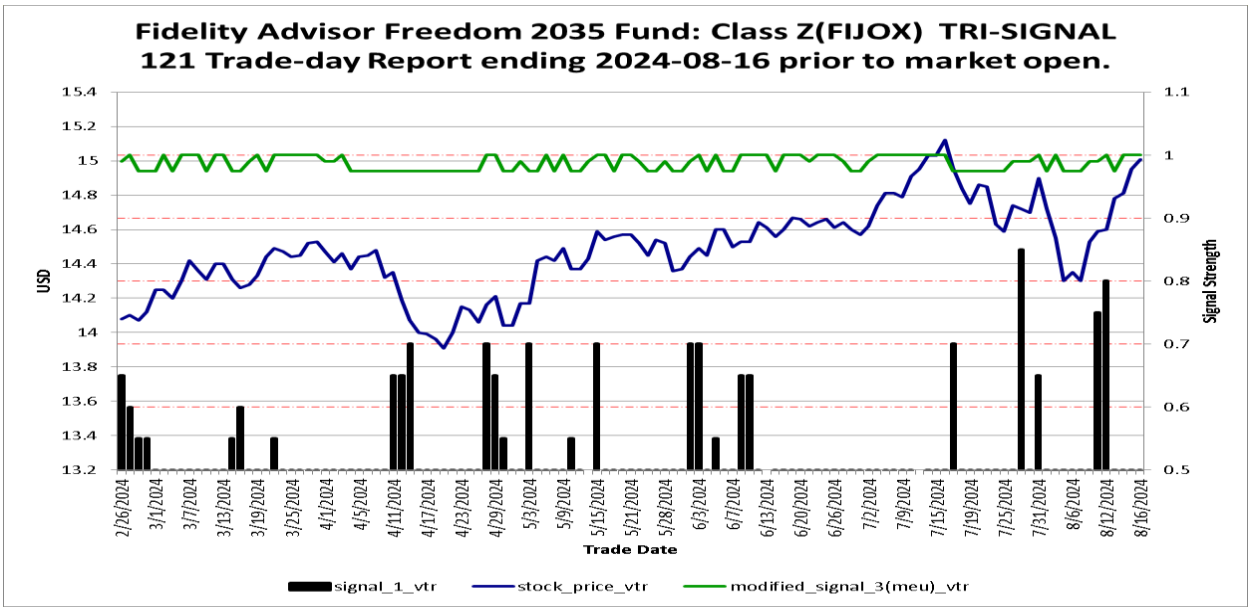


Figure 33

Predictive equity analytics signals{1,3} for target date fund: Fidelity FIJOX.

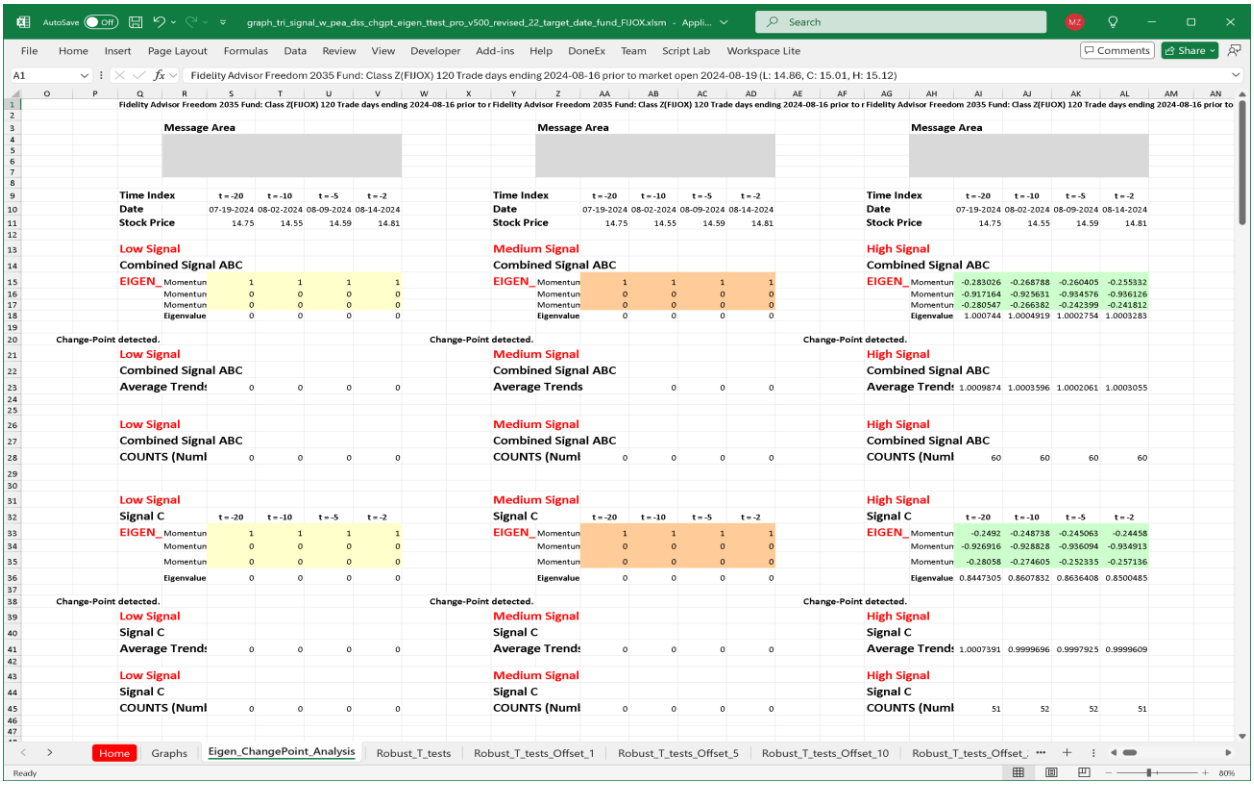


Figure 34

Fidelity FIJOX **no retrospective change points were detected in the time dimension of the discrete Fokker-Planck approximation for Proxy Drift/meu over time nor in the Fokker -Planck State space dimension.**

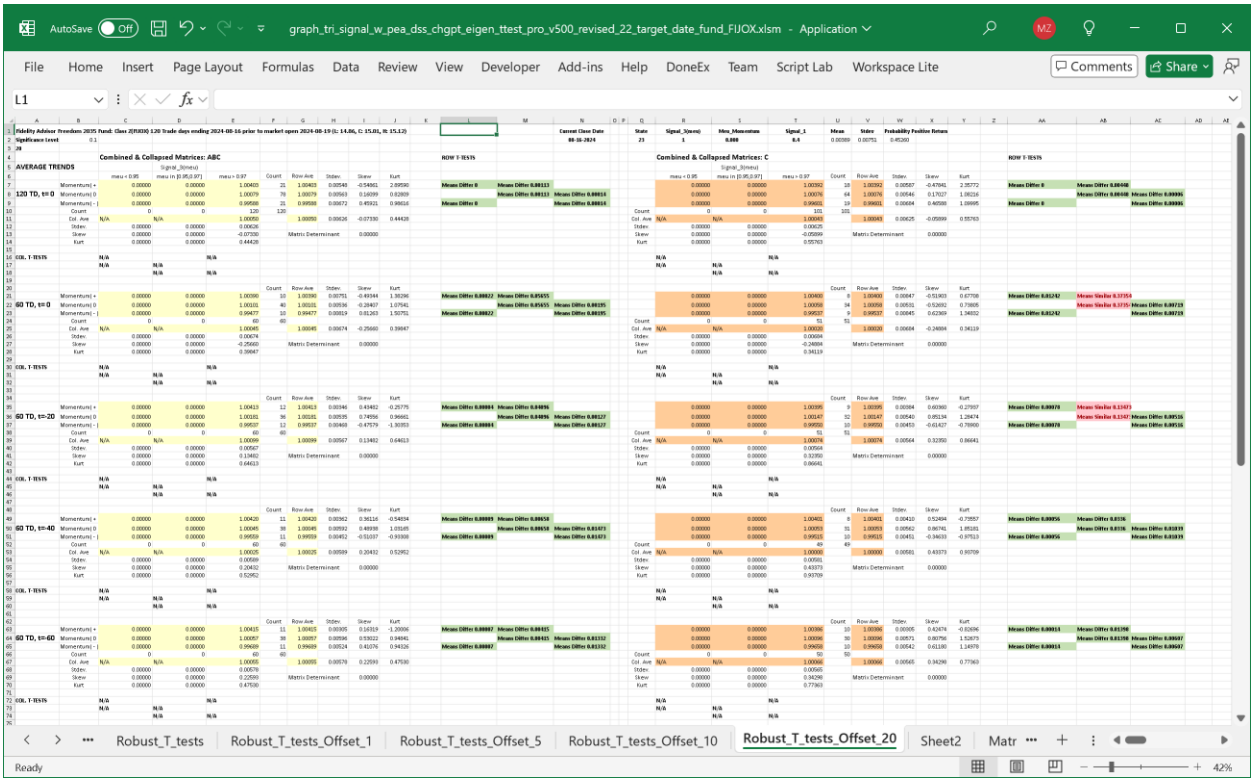


Figure 35

Fidelity FCPVX sliding **60 Trade-day using offset 20,** robust T-tests show time **no varying in Proxy Drift/meu signal(3) level (columns) and only Change in Proxy Drift/meu (meu momentum) (rows)** starting from t-60 to t-0 for noisy matrix $A_t B_t C_t$ and close return matrix C_t.

Appendix

This is a continuation of Section 8. Predictive Equity Analytics Proxy Drift/meu Signal(3) and Change in Proxy Drift/meu (meu momentum) for additional stocks: Goldman Sachs(GS) and Amazon (AMZN). Current coverage is 3,600 stocks, 2,100 ETFs and 5,600 mutual funds including target date funds, all available prior to market open each trade day.

Goldman Sachs(GS)

GS has strong change-point prediction correlation with Signal(1) > 0.90 level. Signal(3) level is also predictive in that the closer to level 1 and the greater the time spent at or near level 1, the stock price time-series increases, as illustrated in Figure 36.

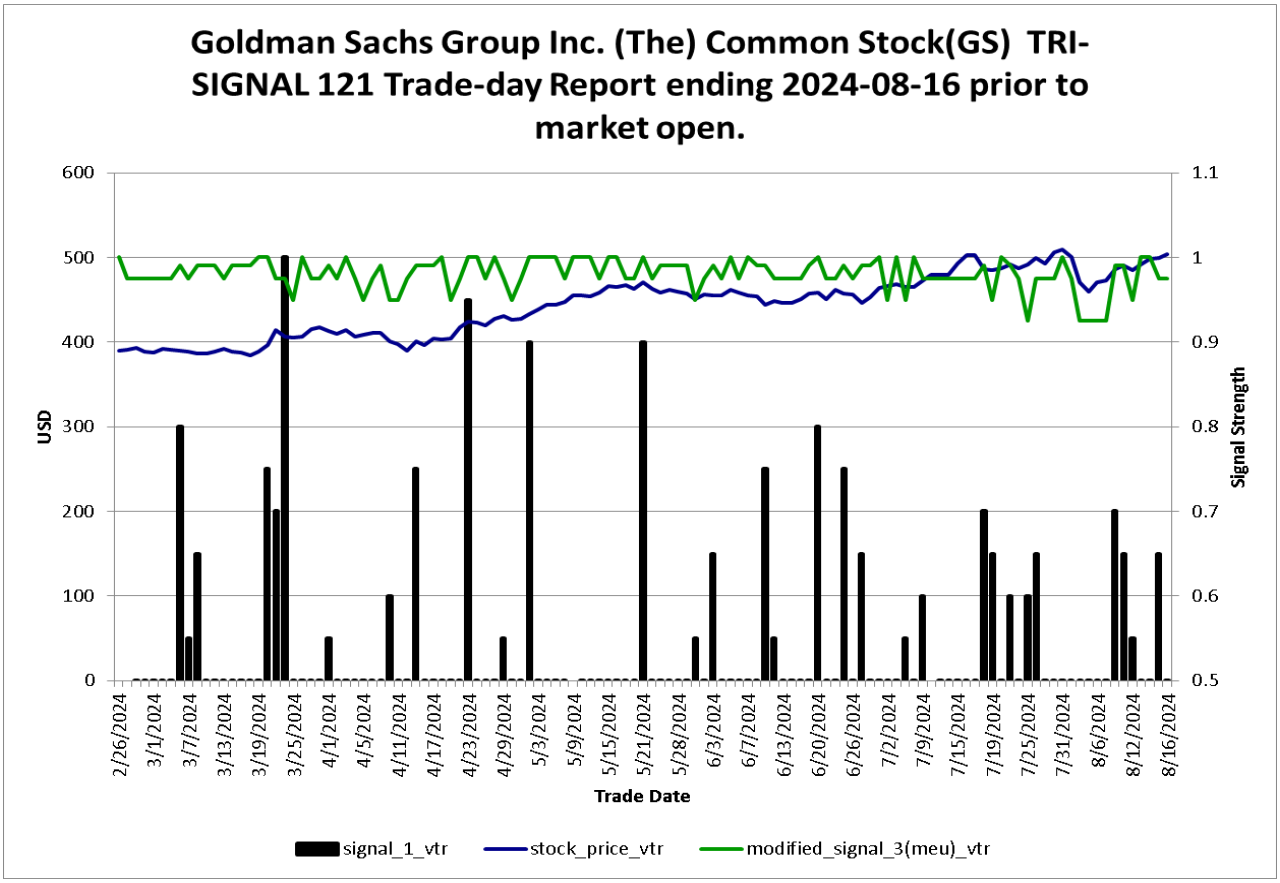


Figure 36

Predictive equity analytics signals{1,3} for GS.

Retrospective change-points confirmed by eigen analysis are illustrated in Figure 37. Predictive equity analytics optimal timing was purchase on 4/23/2024 @ $424.00, sell on 5/21/2024 @ $470.41 for gross return of 10.94%. Purchase on 6/20/2024 @ $458.05 and sell on change-point detection 8/9/2024 @ $490.26 for a gross return of 7.03%. All predictive signals are prior to market open. Time-varying statistically significant robust T-tests using sliding 60 trade-day window for offsets {t-60, t-40, t-20, t-0} are located in Figure 38.

Goldman Sachs Group Inc. (The) Common Stock(GS) 120 Trade days ending 2024-08-16 prior to market open 2024-08-19 (L: 497.09, C: 504.26, H: 507.41)

Message Area												
Time Index		t = -60	t = -40	t = -30	t = -20	t = -15	t = -10	t = -5	t = -3	t = -2	t = -1	t = 0
Date		05-21-2024	06-20-2024	07-05-2024	07-19-2024	07-26-2024	08-02-2024	08-09-2024	08-13-2024	08-14-2024	08-15-2024	08-16-2024
Stock Price		470.41	458.05	464.75	484.93	499.03	470.64	490.26	491.94	498.7	499.02	504.26
Combined Signal ABC		t = -60	t = -40	t = -30	t = -20	t = -15	t = -10	t = -5	t = -3	t = -2	t = -1	t = 0
EIGEN_	Momentum(+)	-0.46581697	-0.60021567	-0.55466145	-0.49611941	-0.46773192	-0.46995103	-0.47295254	-0.44126943	-0.47140974	-0.46440774	-0.47118601
	Momentum (0)	-0.71844423	-0.59846598	-0.6724636	-0.71707296	-0.71724594	-0.71999204	-0.71847612	-0.73476291	-0.71554887	-0.73451507	-0.71587086
	Momentum(-)	-0.51657772	-0.53064078	-0.49004441	-0.48956296	-0.51651263	-0.51064402	-0.51000792	-0.51517457	-0.51552165	-0.4947859	-0.51527917
	Eigenvalues:	0.31638905	0.31201193	0.29655972	0.29711404	0.31354725	0.30866221	0.30646005	0.31200984	0.31072581	0.30272377	0.31072712
Change-Point detected.												
Combined Signal ABC												
Average TRENDS		1.00289714	1.00203106	1.00226742	1.00262614	1.00228194	1.00208126	1.00127149	1.00080629	1.00108351	1.00119772	1.00137874
Signal C		t = -60	t = -40	t = -30	t = -20	t = -15	t = -10	t = -5	t = -3	t = -2	t = -1	t = 0
EIGEN_	Momentum(+)	-0.37649563	-0.51502717	-0.42626178	-0.35329807	-0.36764348	-0.37027329	0.35214788	-0.32105032	0.3610031	-0.35378361	-0.36457881
	Momentum (0)	-0.76317912	-0.64427733	-0.72980082	-0.76891023	-0.76798618	-0.77265882	0.77502495	-0.77779889	0.75836122	-0.78126585	-0.77016717
	Momentum(-)	-0.52517492	-0.56537914	-0.53450119	-0.53287667	-0.52443826	-0.51565123	0.52471715	-0.54032928	0.54273832	-0.51425755	-0.52337831
	Eigenvalues:	0.28053281	0.29464358	0.27378753	0.27871823	0.26327905	0.25717038	0.24984878	0.25368741	0.25200644	0.2436548	0.24063362
Change-Point detected.								Chg_Pt	Chg_Pt	Chg_Pt	Chg_Pt	
Signal C												
Average TRENDS		1.00195932	1.0018009	1.00280368	1.00362586	1.00330046	1.00301369	1.00147011	1.00069434	1.00105593	1.0012049	1.00145879
Signal C												
COUNTS (Number of trade-da		51	53	50	50	48	48	47	46	46	46	45

Figure 37

GS change points detected in the state dimension of the discrete Fokker-Planck approximation for Proxy Drift/meu over time, for trade dates {08-09-2024, 08-13-2024, 08-14-2024, 08-15-2024}

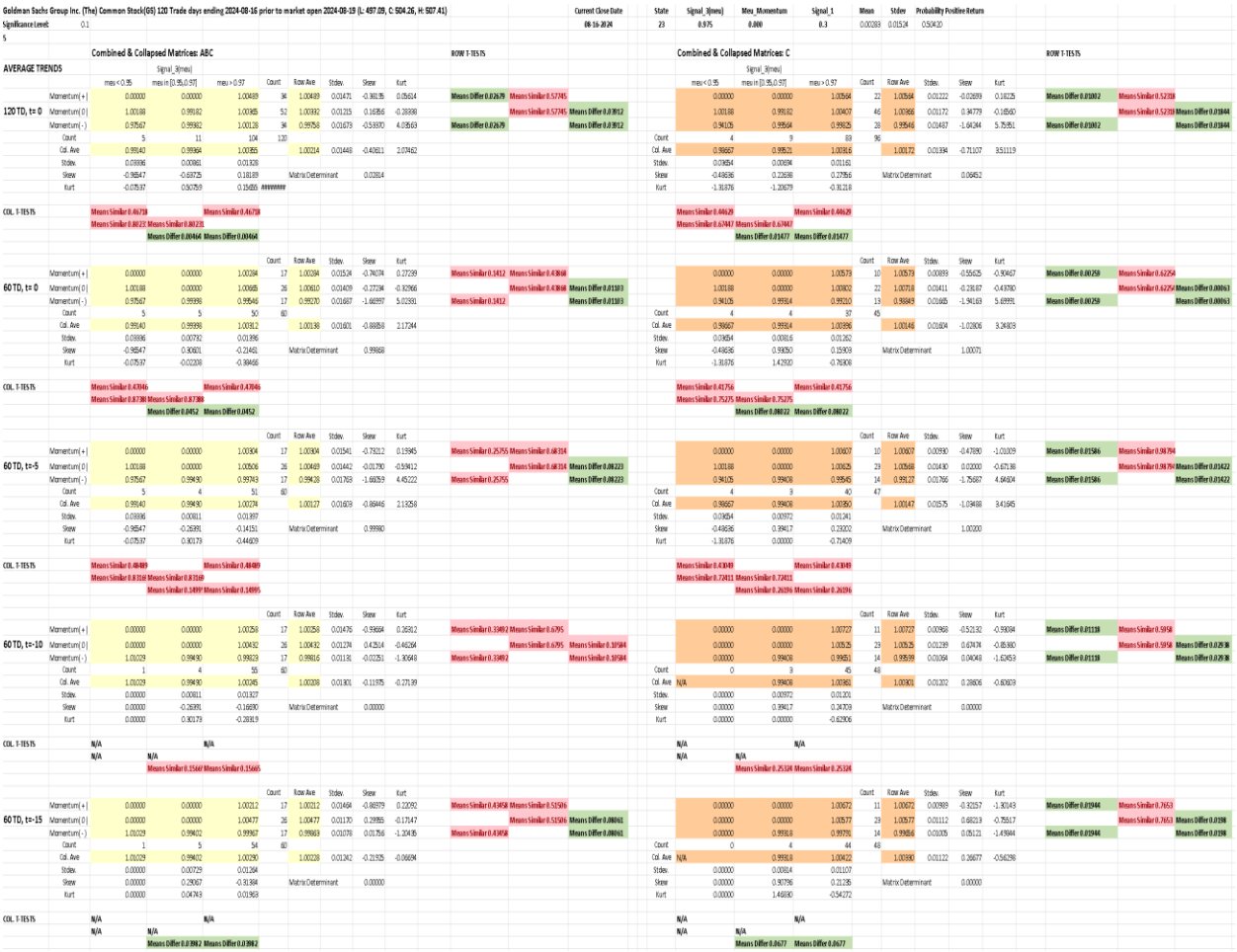


Figure 38

GS Sliding 60 Trade-day using offset 20, robust T-tests show time varying in Proxy Drift/meu signal(3) level (columns) and Change in Proxy Drift/meu (meu momentum) (rows) ranging from activity to disappearing and then reappearing starting from t-15 to t-0.

Amazon (AMZN)

AMZN has strong change-point prediction correlation with Signal(1) > 0.85 level. Signal(3) level is also predictive in that the closer to level 1 and the greater the time spent at or near level 1, the stock price time-series increases. See Figure 39. Retrospective change-points confirmed by eigen analysis are illustrated in Figure 40. Predictive equity analytics optimal timing was purchase on 4/30/2024 @ $175.00 and sell on 5/21/2024 @ $183.15 for a gross return of 4.65%. Purchase on 6/12/2024 @ $186.69 and sell on 7/17/2024 @ $187.93 for a gross return of 0.6%. Purchase pm 8/2/2024 or 8/5/2024 @ [$161, $158] with retrospective change-point confirmation

made on 8/2/2024. All predictive signals are prior to market open. Time-varying statistically significant robust T-tests using sliding 60 trade-day window for offsets {t-60, t-40, t-20, t-0} are located in Figure 41.

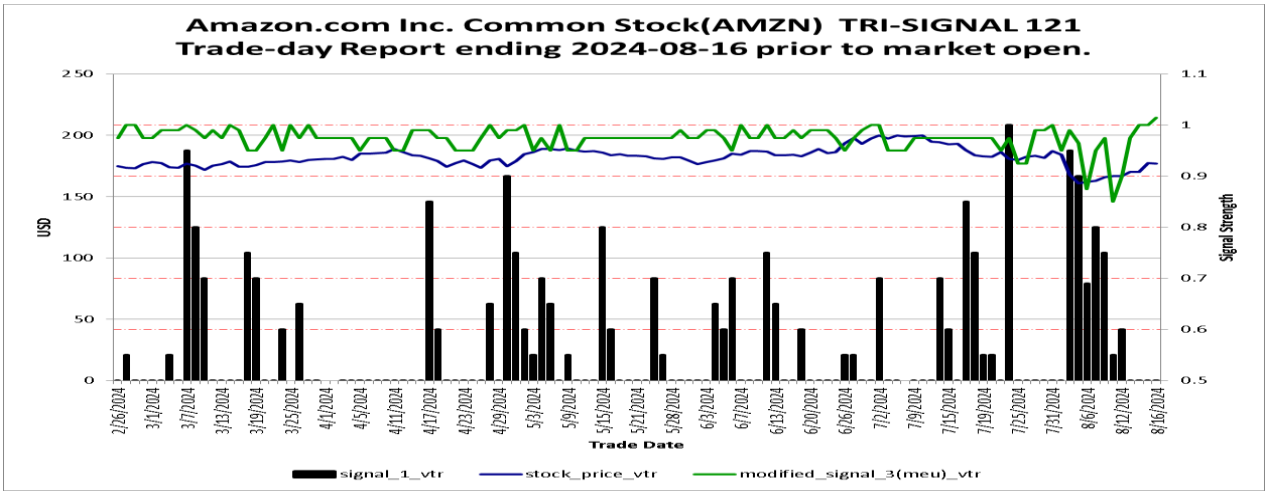


Figure 39

Predictive equity analytics signals{1,3} for AMZN.

Amazon.com Inc. Common Stock(AMZN) 120 Trade days ending 2024-08-16 prior to market open 2024-08-19 (L: 175.05, C: 177.06, H: 181.14)

Message Area

		t = -60	t = -40	t = -30	t = -20	t = -15	t = -10	t = -5	t = -3	t = -2	t = -1	t = 0
Time Index												
Date		05-21-2024	06-20-2024	07-05-2024	07-19-2024	07-26-2024	08-02-2024	08-09-2024	08-13-2024	08-14-2024	08-15-2024	08-16-2024
Stock Price		183.15	186.1	200.	183.13	182.5	167.9	166.94	170.23	170.1	177.59	177.06
Combined Signal ABC		t = -60	t = -40	t = -30	t = -20	t = -15	t = -10	t = -5	t = -3	t = -2	t = -1	t = 0
EIGEN_	Momentum(+)	-0.3915385	-0.3752578	-0.3743594	-0.3419442	0.3344532	0.3085743	0.3606146	0.3918313	0.4225871	-0.4531817	-0.4526773
	Momentum (0)	-0.7805468	-0.7798198	-0.7757801	-0.836197	0.8212744	0.832038	0.8161857	0.790199	0.7725993	-0.7536644	-0.7541536
	Momentum(-)	-0.4872825	-0.5010614	-0.507957	-0.4287759	0.4622224	0.4609713	0.4514399	0.4712259	0.4738252	-0.4760427	-0.4757474
	Eigenvalues:	0.3323835	0.3337303	0.3391966	0.3137516	0.3216697	0.3233877	0.3076565	0.309568	0.3079489	0.3062149	0.3063931
	Change-Point detected.					Chg_Pt					Chg_Pt	
Combined Signal ABC Average TRENDS		1.0008913	1.0004488	1.0011907	1.0007142	1.0000052	0.9996812	0.9982125	0.9983646	0.9989188	0.998809	0.9996476
Signal C		t = -60	t = -40	t = -30	t = -20	t = -15	t = -10	t = -5	t = -3	t = -2	t = -1	t = 0
EIGEN_	Momentum(+)	-0.4267858	-0.3563353	-0.3706951	0.3507752	0.350753	0.3451175	0.3101493	-0.3213389	-0.3522211	-0.3946786	-0.3939835
	Momentum (0)	-0.7925851	-0.8246634	-0.796663	0.8465145	0.819606	0.8354816	0.855525	-0.8405907	-0.8296813	-0.8078804	-0.8084358
	Momentum(-)	-0.4355026	-0.4392671	-0.4774025	0.4004619	0.4530105	0.4276265	0.4145893	-0.4360603	-0.4330926	-0.4376734	-0.4372738
	Eigenvalues:	0.2212585	0.225662	0.2358109	0.2093008	0.2156107	0.227583	0.2011613	0.2042117	0.2059534	0.2042103	0.2043837
	Change-Point detected.				Chg_Pt				Chg_Pt			
Signal C Average TRENDS		1.0006628	1.000126	1.000661	1.0009316	1.001473	1.0016984	1.0021573	1.0022622	1.0027086	1.0025478	1.003775
Signal C COUNTS (Number of trade-da		43	42	42	40	40	44	40	40	41	41	41

Figure 40

AMZN change points detected in the state dimension of the discrete Fokker-Planck approximation for Proxy Drift/meu over time, for trade dates {07-19-2024, 07-26-2024, 08-14-2024, 08-13-2024, 08-15-2024}

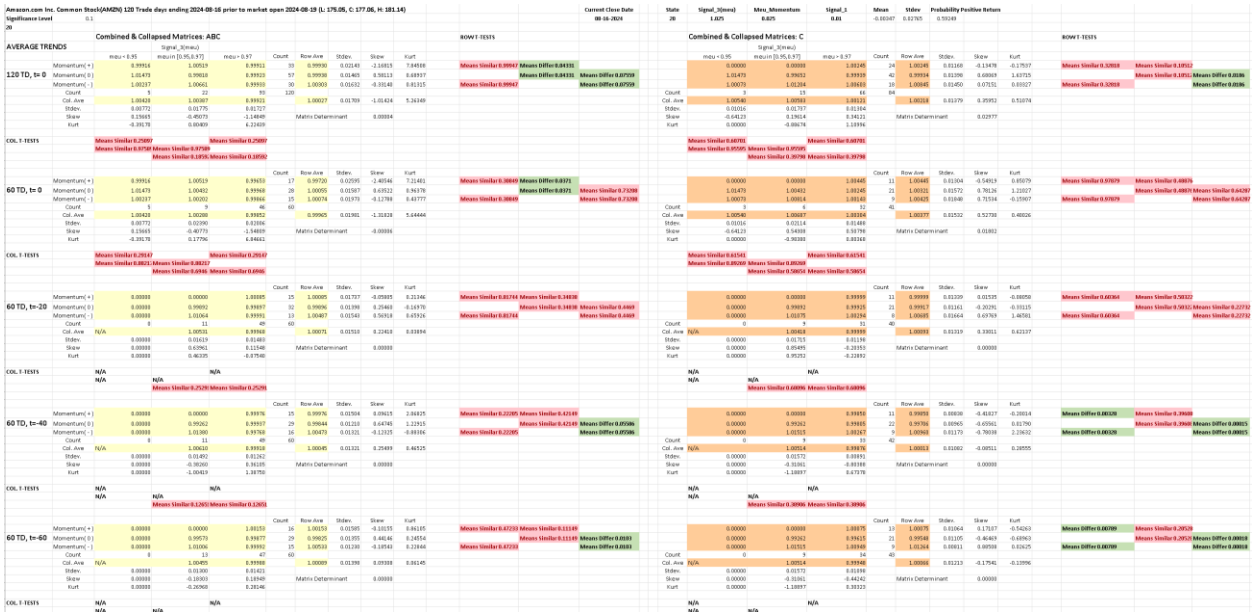


Figure 41

AMZN Sliding 60 Trade-day using offset 20, robust T-tests show time varying in Proxy Drift/meu signal(3) level (columns) and Change in Proxy Drift/meu (meu momentum) (rows) ranging from activity to disappearing and then reappearing starting from t-60 to t-0.