Human-machine interactions in the OTC market

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Abstract

I examine trading venue choice and price markups in over-the-counter markets. Clients have trading relationship with a dealer but can can choose to "defect" and trade electronically. I show that clients switch to the electronic venue when the expected market thickness on this venue is sufficiently high, and that dealer markups on bilateral trades fall following the introduction of an electronic venue. I confirm these predictions empirically using corporate bond transactions by national insurance companies. Following an insurer's initial transaction on an electronic venue, the frequency of bilateral trades between insurers and dealers in their network declines despite lower dealer markups. The decline in insurer-dealer relationship intensities is strongest for smaller insurers, insurers with small dealer networks and insurers in trading relationships with smaller dealers. Overall, this study highlights the tradeoffs involved in venue choice in OTC markets.

Keywords: OTC markets, electronic trading, fixed income.

Introduction

Over-the-Counter (OTC) markets for financial assets have traditionally involved off-exchange, bilateral negotiations between customers and dealers. As a result, these markets are opaque and transactions can be subject to significant markups by a select number of participants who intermediate a large share of the trading volume. These features were common contributors to financial crises such as the 1997-1998 Long-Term Capital Management crisis and the 2008-2009 financial crisis, both of which jeopardized the global financial system. In both cases, lack of price transparency and concentrated trading volume in a few dealers with impaired capital prevented the orderly exchange of risky securities. Similar issues arose in March-April 2020, when investment dealers were unable to supply liquidity in corporate and US Treasury bonds

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to investors as a result of the ongoing COVID epidemic (Kargar et al. 2021) leading to calls for greater price transparency in this market (SEC 2020).

Technological innovations on trading venues offer the promise to address some or all these issues by disintermediating trade of risky debt securities. Advances involving human-to-machine interactions allow investors in OTC markets to route trades to electronic trading venues instead of trading bilaterally via by "voice" with an investment dealer. Because participation on these venues is open to all, and the identity of the participants is public knowledge to other participants, such human-to-machine interactions can reduce trading costs, increase liquidity, and improve financial stability. However, a persistent puzzle for market regulators and financial market participants is why the market for corporate bonds has not evolved to one of centralized trading like the stock market. Large trades continue to be routed through dealers (O'Hara and Xing 2021), and trading volumes on electronic venues remain low relative to total daily volume (Hendershott and Madhavan 2015). What are the differences between human-human interactions and human-machine interaction in electronic venues that result in lower usage of electronic venues?

I explore this question in the OTC market for corporate bonds, which provides a rich set of trading data for testing predictions on the trading behaviour of investors and dealers. I show theoretically and empirically that market thickness is a determinant of venue choice. Clients trading bilaterally will trade on an electronic venue only when the market depth or "thickness" on this venue is high enough. A consequence of this result is that markups by a dealer on the voice venue decline once a client in a relationship with the dealer begins to trade on the electronic venue. I confirm both predictions empirically using a sample of over 1.1 million U.S. corporate bond transactions by national insurance companies between mid 2014 and 2021. I also find that insurer-dealer specific relationship (i.e., trading) intensities decline following the insurer's first electronic transaction. This decline in trading intensity is greatest for smaller insurers, insurers with smaller dealer networks and insurers in relationships with smaller dealers.

My analysis focuses on the market for corporate bonds, a US$8 trillion-dollar market that involves debt securities issued by large corporations. A unique feature of corporate bonds is that they are not traded on a traditional exchange but exchanged in the OTC market. The traditional trading venue has been through a dealer or network of dealers. Indeed, it is not straightforward to buy/sell assets in an OTC market. Typically, investors contact dealers to

seek quotes. A dealer then decides whether to take on the transaction on its own or to contact other dealers/investors to arrange a trade. The process can be time-consuming and even result in failed trading attempts. Corporate bonds can also trade on electronic trading venues via human-to-machine interactions. This mechanism promises to reduce overall trading costs (BIS 2016), increase price transparency and potentially reduce unfair pricing (Pagano and Roëll 1996).

In the first part of the paper I extend the seminal search model of Duffie, Garleanu, and Pedersen (2005) (DGP) with two features. The first feature involves the dealer-client trading relationship. I assume that clients in a trading relationship with a dealer benefit from occasional preferential pricing at inter-dealer prices. The greater the intensity of the relationship, the more often clients can obtain preferential pricing. The second feature is a competing trading venue in which clients can post electronic requests for quotes. The mechanics of this feature are based on the market exchange and simultaneous multilateral search theories developed in Kranton (1996) and Glebkin, Yueshen, and Shen (2023), respectively. I link the two features by assuming that when clients "defect" and trade on the electronic venue, dealers temporarily increase the average markup on bilateral transactions with defecting customers. It follows that clients will trade on the electronic venue as long as the "thickness" of this market is high enough. When thickness is sufficient to trade electronically, the penalty from defecting imposed by the dealer is not enough to offset the better pricing terms and liquidity on the electronic venue. A corollary is that once a customer trades electronically, average dealer markups on the voice venue decrease.

In the second part of the paper, I test these predictions using corporate bond transactions reported by the National Association of Insurance Commissioners (NAIC). This data is unique in that the identify to both parties to a secondary-market bond transaction are reported in the insurers Schedule D filings, including whenever a transaction is executed on an electronic venue.[1] When this is the case, the name of the venue, e.g. MarketAxess, is provided. Consistent with studies that use non-publicly available transaction data on electronic market transactions, I find that the proportion of electronic-venue transactions is steadily increasing over time. Moreover, I find that the increase in electronic trading is strongest among small insurers, as measured by the size of their corporate bond holdings.

[1]NAIC Schedule D filings data has been used in several other studies such as Bessembinder, Maxwell, and Venkataraman (2006), Ellul, Jotikasthira, and Lundblad (2011) and O'Hara, Wang, and Zhou (2018). Bessembinder, Spatt, and Venkataraman (2020) provide a survey of the principal database used in fixed-income research, including the NAIC dataset.

To test my predictions, I form nine portfolio classes defined by rating size and credit rating. I define market thickness by bond portfolio class as the ratio of the number of unique insurers that trade on any electronic venue to the total number of unique insurers who trade in a given calendar quarter. As might be expected, market thickness on the electronic venue is higher for larger bond issues, and bonds with a credit rating. I find that venue choice is determined by the market thickness of the electronic venue. The economic effect of market thickness on venue choice is similar to that of the size of the insurer's dealer network.

In the third part of the paper, I examine how the dealer-insurer trading relationship changes after the insurer's first electronic venue transaction. I evaluate the state of the trading relationship along two dimensions: price and trading frequency. Using the granular nature of the dataset, I form trading pairs between dealers and insurers for each dealer in an insurer's network. I then measure markups and trading intensities at the insurer-dealer pair before and after the first electronic transaction by the insurer. Because many corporate bonds do not trade often, I group insurer-dealer pairs by bond portfolio class. This pairing and grouping allow me to control for unobserved aspects of the insurer-dealer relationship as well as bond characteristics that might determine the markup on the bond and the frequency at which it trades. Consistent with model predictions, I find that dealer markups in bilateral transactions with insurers decline following the first electronic transaction by the insurer. Moreover, the decline in markups is greatest when market thickness is higher, and for insurers with larger dealer networks. The initiation of an electronic transaction by an insurer also weakens bilateral dealer-insurer trading intensities. Following the first electronic transaction by the insurer, the frequency of all dealer-insurer pair transactions within the insurer's network declines. The decline in trading intensity is greater for smaller insurers. For example, insurers in the bottom third of the size distribution are 30% less likely to trade bilaterally following their first electronic transaction, compared with only 18% less likely for medium or large insurers.

This work makes three contributions to the literature on search frictions in OTC markets. First, I use data on venue choice that contains the identity of both counterparties to each transaction. Previous studies on electronic venues in OTC market rely on privately available transaction data with anonymized customer identities. This restricts the analysis of trading venue choice in OTC markets to features of the bond being traded, such as the trade size, the bond issuer's

credit rating, etc. However, much of the choice to trade on the electronic venue is related to unobserved aspects of the relationship a client has with his dealer, and possibly to the size of the dealer's network. Clients with strong relationships with their dealer or clients with large dealer networks may be less inclined to switch to electronic trading venues, predictions that my data allows me to test. Knowing the identities of who trades on the electronic venues also allows me to measure the thickness of the electronic venue, something that would not be possible without knowing the identities of the parties to each transaction.

Second, I highlight a feature of electronic venues that is important to practitioners, namely the market depth, or lack thereof, of electronic trading platforms for corporate bonds. Because corporate bonds trade seldomly (Goldstein and Hotchkiss 2020), bilateral trading with a dealer can be more advantageous when there are few parties willing to trade on the electronic venue. Indeed, when using the most prevalent trading protocol on these venues, the RFQ protocol, clients do not know in advance how many dealers will respond on the platform. Advertising one's liquidity need to the world with an RFQ without the certainty of this need being satisfied may expose a liquidity-seeking client to adverse trading behavior on the part of dealers and other market participants. Moreover, several studies show that dealers that are more central in a network charge a centrality premium (Di Maggio, Kermani, and Song 2017, Li and Schürhoff 2019). Thus, in order to compete with central dealers, electronic trading venues must have a minimum number of participants, and the expected market thickness of these venues should be large enough that it offsets penalties for "defecting" to electronic trading. I find strong support for this prediction in the NAIC dataset.

Third, I assess the extent to which dealers modify their behavior in response to the introduction of electronic venues. Due to the difficulty in obtaining data on prices by venue, there is limited evidence on what happens to bilateral prices after clients begin to trade bonds on the electronic venue. The closest evidence is in O'Hara and Xing (2021) who find that transaction costs for *dealers* fall after the dealer first begins to trade electronically. In contrast, I look at markups charged by dealers to clients after clients first trade electronically. Against this backdrop I offer two pieces of evidence that shed light on the evolution of dealer-insurer relationships after the start of electronic trading. First, dealer markups on bilateral transactions with the same insurer fall after the insurer begins to trade electronically. Second, the inten-

sity of the dealer-insurer trading relationship declines following the first electronic transaction. These results control for unobserved time-invariant) aspects of the dealer-insurer relationship. The decline in trading intensity is strongest among small insurers. Lower markups but sustained trading frequencies on the voice venue post electronic trading for larger insurers suggest that these insurers continue to value certain non-trade aspects of their relationship with the dealer after transacting electronically.

Overall, I contribute to the the wider literature on search frictions in de-centralized markets. My focus is on the relationship aspect of these markets and what happens when one party defects on this relationship. In this sense my work is similar to Riggs et al. (2020) who highlight the importance of customer-dealer relationships in explaining the use of electronic trading mechanisms in the context of index CDS markets. A large volume of work focuses on search frictions and the mechanics of decentralized vs centralizing exchanges in OTC markets. Search frictions and their effect on liquidity are explored in Lagos, Rocheteau, and Weill (2011) and Lagos, Rocheteau, and Weill (2011). Lester, Rocheteau, and Weill (2015) explore the role of pre-trade transparency in reducing search frictions. Rocheteau and Weill (2011) and Weill (2020) survey the literatures on search frictions and the impact on liquidity. A substantial empirical and theoretical literature examines the role of networks to overcome search frictions in OTC markets. Babus and Kondor (2018) highlight the importance of dealer centrality on trading costs. Neklyudov (2019) shows how a centrality discount can arise out of differences in search technology. Sambalaibat (2023) shows that dealers specializing in specific client trading needs can become more or less central in a dealer network. Colliard, Foucault, and Hoffmann (2021) argue that core-peripheral networks arise to minimize inventory holding costs across dealers. Allen and Wittwer (2021) consider the customer welfare implications of the opposite extreme, namely a market with completely centralized trading.

1 Model

The objects of interest are the markups on the voice venue and the client's choice of trading venue. To understand how both are determined and interact, I use the search-and-matching model Duffie, Garleanu, and Pedersen (2005) (DGP) and add an electronic trading venue with

"market thickness" proportional to the number of dealers who participate in electronic trading. The electronic trading venue is based on simultaneous multilateral search concept developed in Glebkin, Yueshen, and Shen (2023). The thickness concept is based on the thickness of the Kranton (1996) market-exchange economy in her analysis of reciprocal exchanges. To these elements, I add preferential pricing for clients by dealers with whom clients have a trading relationship. Such pricing is akin to "loyalty" discounts in the consumer retail industry; see for example (Liu 2007). Turning on or off preferential pricing allows dealers to penalize customers who "defect" by trading on the electronic venue, thus jeopardizing the trading relationship when a client trades electronically.

There is a bond that pays a coupon C per year. Customers have either a high or a low valuation of the bond. As in DGP, low valuation owners discount the cash flows by δ per year. Valuation shocks occur at the rate λ_l for high- and low-valuation types. When a high-valuation owner switches to a low valuation state he will initiate a sale of the bond. Likewise when a low-valuation customer switches to a high valuation state, he will initiate a buy transaction. Buyers and sellers can only trade through an intermediary, i.e. a dealer. I assume that having a relationship with a dealer implies periodic access to more favorable pricing with frequency λ_s. The intensity of the client-dealer relationship increases in λ_s.

There are two trading technologies through which customers can contact intermediaries. Under the first technology, customers trade bilaterally with dealers in their network at a marked-up price with frequency λ_v, and trade at the inter-dealer price M at frequency λ_s. When using the second technology, customers trade on an electronic platform at rate λ_e. On this platform, dealers compete for the client business by responding to a client request for quote with a price. The quality of this technology is determined by the market thickness μ on this venue. Depending on μ, transacting through this venue may or may not be cheaper than bilateral trades within the client's dealer network.

1.1 Value of trading relationships

Let τ_i be a stopping time governed by a Poisson process with arrival rate λ_i. Let ΔV_h denote the difference in valuation between ownership and non-ownership for a high-valuation client. Likewise, denote by ΔV_l the difference in valuation between ownership and non-ownership by a

low-type client. The value S of the trading relationships in the customer's network for a customer seeking to trade is given by,

$$S = E_t\left[e^{-r(\tau_s - t)}\theta(\Delta V_h - M)\mathbf{1}_{(\tau=\tau_s)}\right] + E_t\left[e^{-r(\tau_s - t)}\theta(M - \Delta V_l)\mathbf{1}_{(\tau=\tau_s)}\right] \quad (1)$$

The first term is the value S_{hn} of the relationship to a client seeking to purchase the bond, i.e., a hn type. The second term is the value S_{lo} of the relationship to a client seeking to sell the bond, i.e., of type lo. The terms $\theta(\Delta V_h - M)$ and $\theta(M - \Delta V_l)$ are the dealer markups relative to the inter-dealer price M on client buy and sell orders, respectively. The proportion θ is dealers' bargaining power when trading bilaterally at non-relationship prices. I assume that whenever the client trades at the inter-dealer price, she keeps all of the surplus. These two terms reflect the loyalty discount for clients with a trading relationship. Intuitively, the value of the relationship is the present value of what the client would have otherwise incurred in markups had she traded at non-relationship prices on the voice venue. When I consider the client's choice to trade on the electronic venue below, she will consider the impact this has on the value of the trading relationship with dealers in her network.

1.2 Trading on voice only

I first start by specifying prices and the value of a trading relationship when there is only bilateral trading, i.e., voice only (vo). Let $\tau = Min(\tau_l, \tau_v, \tau_s)$. Let V_{lo} denote the value of holding the bond by a low-type. The terms V_{ho}, V_{ln}, and V_{hn} represent the values of high-type owners, low-type non-owners and high-type non-owners respectively. The steady state values are given by,

$$
\begin{aligned}
V_{lo} &= E_t\left[\int_t^{\tau}(C-\delta)e^{-r(u-t)}du + e^{-r(\tau_l-t)}V_{ho}\mathbf{1}_{(\tau=\tau_l)}\right] \\
&\quad + E_t\left[e^{-r(\tau_v-t)}(V_{ln}+P^{vo}_{bid})\mathbf{1}_{(\tau=\tau_v)} + e^{-r(\tau_s-t)}(V_{ln}+M)\mathbf{1}_{(\tau=\tau_s)}\right] \\
V_{ln} &= E_t\left[e^{-r(\tau_l-t)}V_{hn}\right] \\
V_{ho} &= E_t\left[\int_t^{\tau_l} Ce^{-r(u-t)}du + e^{-r(\tau_l-t)}V_{lo}\right] \\
V_{hn} &= E_t\left[e^{-r(\tau_l-t)}V_{ln}\mathbf{1}_{(\tau=\tau_l)} + e^{-r(\tau_v-t)}(V_{ho}-P^{vo}_{ask})\mathbf{1}_{(\tau=\tau_v)}\right] \\
&\quad + E_t\left[e^{-r(\tau_s-t)}(V_{ho}-M)\mathbf{1}_{(\tau=\tau_s)}\right] \\
S^{vo}_{hn} &= E_t\left[e^{-r(\tau_s-t)}\theta(\Delta V_h - M)\mathbf{1}_{(\tau=\tau_s)}\right] \\
S^{vo}_{lo} &= E_t\left[e^{-r(\tau_s-t)}\theta(M-\Delta V_l)\mathbf{1}_{(\tau=\tau_s)}\right]
\end{aligned}
\tag{2}
$$

The first term in the expression for V_{lo} is the value of owning bond for a low type until the first stopping time. The second term in the first integral is the value of switching to a high-type. Inside the second expectation for V_{lo} is the value of selling the bond to a dealer via a bilateral trade with a dealer in the network at marked-up price P^{vo}_{bid}. The second term in this expectation is the value of selling the bond to a dealer with a customer loyalty premium at price M. This last outcome is more likely, the higher the relationship intensity λ_s. The expressions S^{vo}_{hn} and S^{vo}_{lo} represent the value of the trading relationship to high-value non-owner and low-value owner types respectively. The expressions for V_{ln}, V_{ho}, and V_{hn} have similar interpretations to V_{lo}, where P^{vo}_{ask} is the price paid by a high-value non-owner type in a bilateral trade with a dealer at a non-relationship price.

1.2.1 Prices and dealer markups

Denote by M the price in the inter-dealer market (i.e., a dealer's outside option). Let θ bet the dealer's bargaining power when trading with a client bilaterally. Following standard arguments with Nash bargaining, dealer bid and ask prices when clients can only trade bilaterally are below and above the inter-dealer price M, respectively,

$$P^{vo}_{bid} = M - \theta(M - \Delta V_l) = (1-\theta)M + \theta\Delta V_l \tag{3}$$

$$P^{vo}_{ask} = M + \theta(\Delta V_h - M) = (1-\theta)M + \theta\Delta V_h \tag{4}$$

The expectations in (2) are derived in the Appendix using these prices.

Define the dealer markup on a transaction as $H^{vo} = 0.5(P^{vo}_{ask} - P^{vo}_{bid})$. I show in the Appendix that the markup on the voice venue is,

$$H^{vo} = \frac{\theta\delta(r+\psi_l)}{(r+\psi_l-\lambda_l)(r+\psi_l+\lambda_l)} - \frac{\theta(C-rM)}{r+\psi_l-\lambda_l} \tag{5}$$

where ψ_l is a function of λ_l, λ_v, and λ_s and increasing in λ_s. Based on (5), markups are declining in the intensity of the relationship, i.e., clients who access the inter-dealer market more frequently will require better prices to trade with non-relationship dealers. Intuitively, conditional on contacting a dealer in their network who offers to trade at *vo*-prices (3) or (4), clients are indifferent between trading immediately at these prices or waiting to trade at preferential relationship price M in $1/\lambda_s$ periods in expectation.

Under these assumptions, the total value S^{vo} to a client of a bilateral trading relationship with a dealer is proportional to the markup H^{vo} on the voice venue. The value of this relationship is,

$$S^{vo} = \frac{2\lambda_s H^{vo}}{(r+\lambda_l+\lambda_v+\lambda_s)} = \frac{\lambda_s\theta\delta}{(r+\lambda_l+\lambda_v+\lambda_s)(r+2\lambda_l+\lambda_v(1-\theta)+\lambda_s)} \tag{6}$$

Examination of this expression shows that the value of the trading relationship is decreasing with the client's trading frequency λ_l, holding λ_s constant.

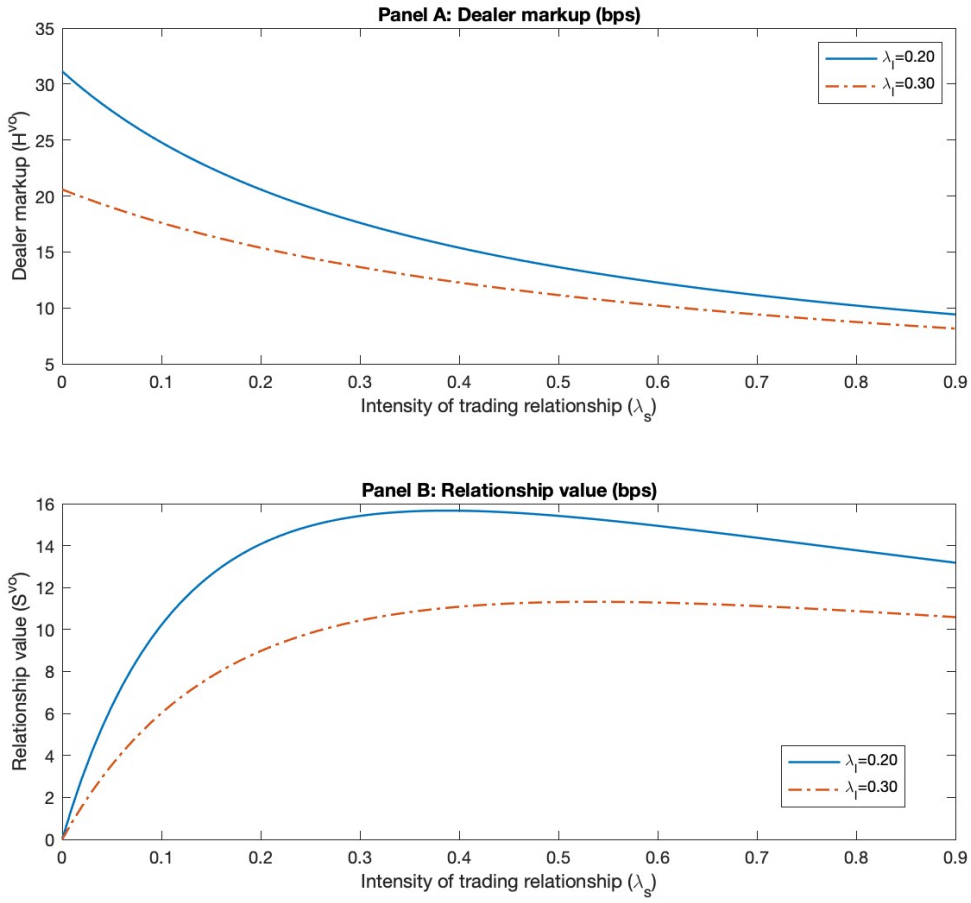


Figure 1: **Dealer markups and trading relationships**.
This figure reports the value in basis points (bp) of dealer markups and of the client-dealer trading relationship for different levels of the intensity of the relationship (λ_s) and the client's trading frequency (λ_l).

Figure 1 plots dealer markups and the relationship value as a function of the intensity λ_s of the relationship, assuming parameter values of $\theta = 85\%$, $r = 1\%$, $\lambda_v = 20\%$, $N = 1$, $\delta = 0.25$ and $C = 1$. Panel A shows that markups decline with the client trading frequency λ_l (orange vs blue line), which is consistent with evidence that clients that trade more often trade at better prices (O'Hara, Wang, and Zhou 2018). Markups are also declining in the intensity of the relationship λ_s, i.e., clients who access the inter-dealer market more frequently will require better prices to trade at non-relationship prices.

Panel B shows that the value of the trading relationship is decreasing with the client's trading frequency λ_l (orange vs blue line). The reason is that when clients trade more frequently, dealer

markups on non-relationship prices decline and preferential pricing at rate λ_s (and price M) becomes less valuable. Panel B also shows that the value of the relationship is initially increasing in the intensity λ_s of the relationship, and then declining. This is because of two opposing forces. First, trading more often at preferential price M is more valuable. However, trading more often at the inter-dealer price reduces markups H^{vo}, which reduces the value of the relationship, as the relationship value is proportional to the markup.

1.3 Trading on voice and electronic platforms

I next examine the impact on venue choice and markups of introducing an electronic trading venue with an auction-like trading protocol. I show 1), that there is a minimum market thickness on the electronic venue below which clients will not trade electronically, and 2), that prices on the voice venue fall after the introduction of electronic trading as long as the thickness of this venue is large enough.

I make two assumptions. First, dealers penalize clients who "defect" and trade on an electronic venue by withholding access to inter-dealer prices from the client for a finite period of time. This assumption is motivated by the literature on trading relationships. In Macchiavello and Morjaria (2015), Kenyan rose exporters lose access to bilateral exchanges once they start trading on an organized exchange. In Harris and Nguyen (2023), shippers who break a bilateral contract to secure shipping on the spot market are penalized by their counterparty. Second, I assume clients either trade only bilaterally with dealers or trade on both electronic and voice venues. The second assumption is based on the fact that in the trading data used in the empirical analysis, customers never trade exclusively on the electronic venue.

Let the stopping time τ_e be governed by a Poisson process with rate λ_e. Denote by μ the proportion of all market participants (clients and dealers) that engage in transactions on the electronic venue, or, equivalently, the "thickness" of the market. The market thickness concept used here is similar to that developed by Kranton (1996) in a gift exchange setting. Let P^v_{ask}, P^v_{bid}, P^e_{ask}, P^e_{ask} denote the expected prices paid and received by customers on the voice and electronic venues, respectively.

I assume that dealers penalize clients who trade on the alternative electronic venue. More specifically, after trading electronically, *lo* and *hn* type clients temporarily lose access to the inter-

dealer market for $\frac{1}{\lambda_s}$ periods in expectation upon changing to high type or low type, respectively. This assumption requires that, following an electronic transaction by a client, all dealers with whom a client has a relationship stop providing "loyalty" pricing a the inter-dealer price from stopping time τ_l to the time of the client's next transaction. This trigger strategy also requires knowledge by all dealers in the client's network that the client traded on an electronic venue. The former requirement is a strong assumption, the latter is weaker as RFQ quotes are advertised to everyone participating on the electronic platform. On the plus side, the penalty is finite and so avoids unrealistic "grim trigger" type strategies used in the relationship-literature whereby a client would be shut-out of the voice channel in perpetuity.[2]

Given the dealer's strategy, the steady-state values are given by,

$$
\begin{aligned}
V_{lo} &= E_t\left[\int_t^{\tau}(C-\delta)e^{-r(u-t)}du + e^{-r(\tau_l-t)}V_{ho}\mathbf{1}_{(\tau=\tau_l)} + e^{-r(\tau_s-t)}(V_{ln}+M)\mathbf{1}_{(\tau=\tau_s)}\right] \\
&\quad + E_t\left[e^{-r(\tau_e-t)}(V_{ln}+P^e_{bid}-S_{hn})\mathbf{1}_{(\tau=\tau_e)} + e^{-r(\tau_v-t)}(V_{ln}+P^v_{bid})\mathbf{1}_{(\tau=\tau_v)}\right] \\
V_{ln} &= E_t\left[e^{-r(\tau_l-t)}V_{hn}\right] \\
V_{ho} &= E_t\left[\int_t^{\tau_l} Ce^{-r(u-t)}du + e^{-r(\tau_l-t)}V_{lo}\right] \\
V_{hn} &= E_t\left[e^{-r(\tau_l-t)}V_{ln}\mathbf{1}_{(\tau=\tau_l)} + e^{-r(\tau_e-t)}(V_{ho}-P^e_{ask}-S_{lo})\mathbf{1}_{(\tau=\tau_e)}\right] \\
&\quad + E_t\left[e^{-r(\tau_v-t)}(V_{ho}-P^v_{ask})\mathbf{1}_{(\tau=\tau_v)} + e^{-r(\tau_s-t)}(V_{ho}-M)\mathbf{1}_{(\tau=\tau_s)}\right] \\
S_{hn} &= E_t\left[e^{-r(\tau_l-t)}S^{vo}_{hn}\right] \\
S_{lo} &= E_t\left[e^{-r(\tau_l-t)}S^{vo}_{lo}\right]
\end{aligned}
\tag{7}
$$

These expressions are similar to (2) except for the addition of extra terms with stopping time τ_e. For instance, the first term in the second expectation in the expression for V_{lo} captures what happens when a low-value owner type trades electronically. The client become a V_{ln} type, receives price P^e_{bid} but loses a part of the value of the trading relationship equal to S_{hn}. The expression for S_{hn} is a function of S^{vo}_{hn} defined above. The loss in value of the trading relationship reflects the fact that after selling on the electronic venue, the ln-type client client does not have

[2] A grim trigger strategy is explored in the Appendix. However, this type of strategy does not seem plausible nor enforceable as it would require coordination by all of the dealers in the client's network in perpetuity.

access to the inter-dealer market upon switching to a hn type at stopping time τ_l. The loss in value is temporary as the value of the relationship becomes zero as soon as she switches type again or transacts after switching to a high-type.

1.4 Prices on the electronic venue

Prices P^v_{bid} and P^v_{ask} on the voice venue when there is electronic trading can be expressed as (3) and (4), except that the values ΔV_h and ΔV_l change as a result of the introduction of electronic trading. To determine prices on the electronic venue, I assume that clients have the following beliefs about the participation of dealers on the electronic venue. Suppose that μ is the proportion of total dealers m who participate on the electronic trading venue, i.e., the "thickness of the market". Clients know μ but do not know which dealers participate on the electronic venue. Instead, clients observe a noisy signal of whether dealer i participates on the electronic platform ($\sigma = 1$) or not ($\sigma = 0$). Clients beliefs are such that a type hn or lo client who observes $\sigma = 1$ enters a request for quotes on the electronic venue with probability $P(client\ request|\sigma = 1) = P(client\ no\ request|\sigma = 0) = \phi$. Given these beliefs, Bayes rule implies that dealer i will respond to a quote request with probability $\pi(\mu)$, where

$$\pi(\mu) = \frac{\phi\mu}{\phi\mu + (1-\phi)(1-\mu)}.$$

Expected prices on the electronic venue are then given by,

$$P^e_{bid} = [1 - \alpha(\mu)]M + \alpha(\mu)\Delta V_l \tag{8}$$

$$P^e_{ask} = [1 - \alpha(\mu)]M + \alpha(\mu)\Delta V_h \tag{9}$$

where $\alpha(\mu) := \frac{m\mu(1-\pi(\mu))^{m\mu-1}}{1-(1-\pi(\mu))^{m\mu}}$ and $\alpha(\mu)$ is decreasing in μ. The proof of this results follows from arguments developed in Glebkin, Yueshen, and Shen (2023) and is left to the Appendix. Comparing expressions (3)-(4) with (8)-(9), it follows immediately that markups on the electronic venue are lower than markups on the voice venue whenever $\alpha(\mu) < \theta$.

Dealer markups on the voice and electronic venues are then,

$$H^v = \frac{\theta\delta(r+\psi)}{(r+\psi+\psi_s-\lambda_l)(r+\psi-\psi_s+\lambda_l)} - \frac{\theta(C-rM)}{r+\psi_\alpha+\psi_s} \tag{10}$$

$$H^e = \frac{\alpha\delta(r+\psi)}{(r+\psi+\psi_s-\lambda_l)(r+\psi-\psi_s+\lambda_l)} - \frac{\alpha(C-rM)}{r+\psi_\alpha+\psi_s} \tag{11}$$

where the expressions for ψ and ψ_s are derived in the Appendix. The value of the trading relationship when there is an electronic venue is,

$$S = \frac{\lambda_l\lambda_s}{(r+\lambda_l)(r+\lambda_l+\lambda_e+\lambda_v+\lambda_s)}\frac{\theta\delta}{(r+\psi-\psi_s+\lambda_l)}$$

1.4.1 When do clients trade electronically?

The following proposition characterizes the decision to trade on the electronic venue in terms of the the market thickness μ of this venue. It describes the minimum amount of market thickness for which clients consider trading on both venues.

Proposition 1: Suppose there are m dealers across the voice and electronic venues, π is as described above, and $\alpha(\mu) = \frac{m\mu(1-\pi(\mu))^{m\mu-1}}{1-(1-\pi(\mu))^{m\mu}}$. There then exists a threshold electronic venue market thickness μ^* given by,

$$\mu^*(r,m,\phi,\theta,\lambda_s,\lambda_v,\lambda_l) = \alpha^{-1}\left(1 - \frac{\lambda_l\lambda_s\theta}{(r+\lambda_l)(r+\lambda_l+\lambda_v+\lambda_s)}\right)$$

above which clients will trade on both voice and electronic venues. This threshold is independent of the frequency λ_e of trade on the electronic venue.

Proof: See Appendix.

The proof of this results compares client welfare under two scenarios. In the first the client only trades on the voice venue. In the second the client trades on both the voice and electronic venues where a proportion μ of dealers choose to participate on the electronic venue.

The expression for the market thickness threshold is increasing in the trading frequency λ_l only for low trading frequencies. For low trading frequencies, the threshold μ^* is low because the client does not have much to lose from defecting and trading electronically. As λ_l increases, so does the threshold, at least initially: the loss from defection is greater than the benefit from trading electronically. When λ_l is high, it becomes more advantageous to trade electronically

despite less frequent access to inter-dealer pricing from the client's relationship dealers due to defection penalties. At this point, the electronic venue trading threshold is declining with λ_l. The reason is that dealers compete in price on the electronic venue, and competitive pricing is more valuable the more often the client trades.

The next result describes the effect on dealer markups on the voice venue subsequent to the introduction of the electronic venue.

Corollary: Following the introduction of the electronic venue, markups on the voice venue fall for any market thickness above a threshold $\tilde{\mu}$ given by,

$$\tilde{\mu}(r, m, \phi, \theta, \lambda_s, \lambda_v, \lambda_l, \lambda_e) = \alpha^{-1}\left(1 - \frac{\lambda_l \lambda_s \theta}{(r + \lambda_l)(r + \lambda_l + \lambda_e + \lambda_v + \lambda_s)}\right) \tag{12}$$

Because $\tilde{\mu} < \mu^* \; \forall \; \lambda_e$, markups on the voice venue always decline following the introduction of the electronic venue with any μ greater than μ^*. Moreover, since μ^* is independent of λ_e this result holds for any trading frequency $\lambda_e \, \epsilon \, (0, \infty)$.

Proof: See Appendix.

Intuitively, to protect trading volumes on the more profitable voice venue, dealers adjust their pricing on bilateral trades by reducing markups following the introduction of an electronic venue.

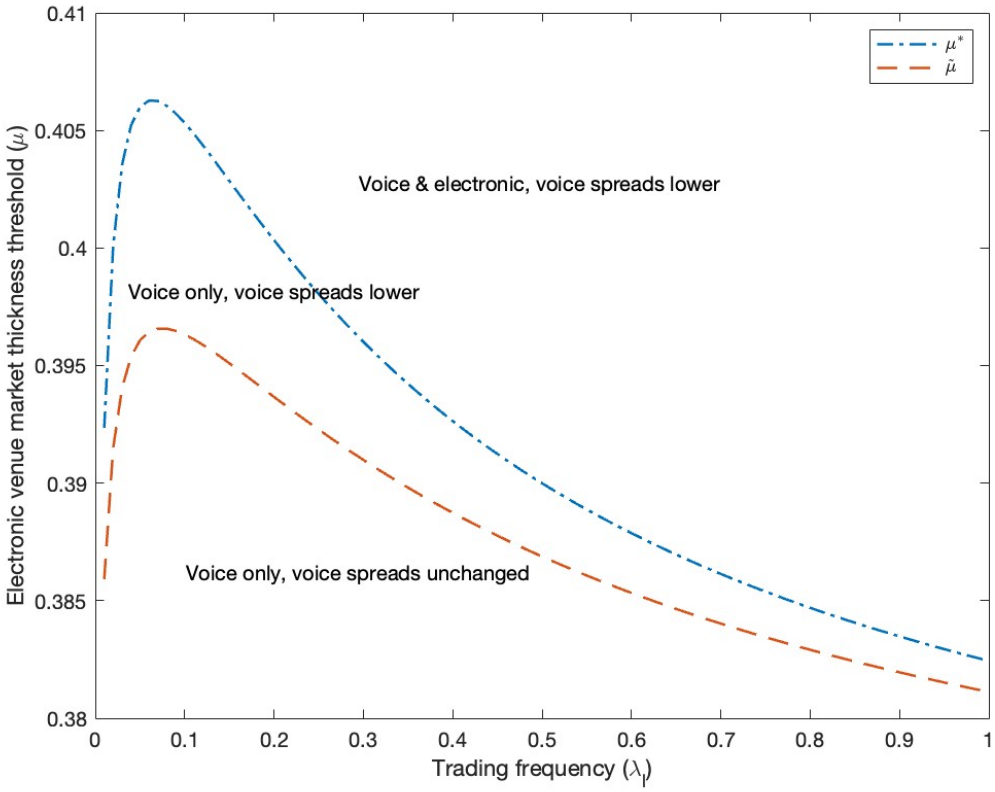


Figure 2: **Market thickness thresholds.**
This figure plots the market thickness thresholds $\mu^*$and $\tilde{\mu}$. The thresholds are defined in equations (A.48) and (A.50) in the text. The different regions indicate when clients trade on the electronic exchange, and when voice spreads decline as a consequence of the introduction of electronic trading.

The intuition behind the corollary can be understood from the case when $\tilde{\mu} < \mu < \mu^*$. Figure 2 plots $\mu^*$and $\tilde{\mu}$. Given λ_e, clients who currently trade on the voice venue will not switch to voice and electronic trading for μ in the middle region indicated as "Voice only, voice spreads lower" in the figure. For μ in this region, markups on the voice venue fall subsequent to the introduction of the electronic venue. However, the thickness of the electronic venue is not sufficient to offset the loss in the value of the trading relationship from trading on the electronic venue, and clients continue to trade bilaterally. Dealer markups are just low enough in this intermediate region that clients choose not to defect to electronic trading.

When $\tilde{\mu} < \mu^* < \mu$, price competition on the electronic venue is sufficiently high that the value of trading on both venues net of the penalty for defecting is greater than the value of trading bilaterally and not paying a defection penalty. The reason is that in this region dealer markups

on both the electronic venue and the voice venue are lower than in the voice-only case. As a result, when market thickness lies in this region, the penalty for defection is implicitly lower and the gains from electronic trading are higher.

I next test these two predictions, namely 1), client trading on the electronic venue increases with market thickness on this venue, and 2), following a client's first transaction on an electronic venue, non-relationship prices for this client on the voice venue fall.

2 Data sources and summary statistics

I use data on insurer transactions reported to the National Association of Insurance Commissioners, a self-regulatory organization that collects state-level regulatory filings by North American insurance companies. The transaction-level data is reported quarterly and date-stamped. Each insurer is required to report the counterparty to each transaction: a small but significant promotion of these transactions are reported with a electronic venue as the counterparty (usually, but not always MarketAxess). I combine these data with bond issue characteristics from Mergent-FISD and inter-dealer prices from TRACE. The merged transaction data covers 1,177,857 corporate bond transactions between January 2014 and December 2021. I also create a subset of transactions for insurers for whom I can match corporate bond holdings at a quarterly frequency obtained from eMaxx bond holdings. Because matching is done manually by insurance company name, this dataset is smaller and is comprised of 568,821 corporate bond transactions.

The variable of interest is the dealer markup. I measure markups as,

$$Markup(bps) = 10,000 \times ln\left(\frac{Price}{Interdealer\ price}\right) \times SIGN \tag{13}$$

where the price is expressed as a percentage of par and SIGN equals +1 or -1 depending on whether the trade is a buy or a sell by the client. This expression is the empirical counterpart to expressions (5), (10) and (11) in the preceding section.

Table 1 reports basic summary statistics on the full dataset of corporate bond transactions. As shown, the average markup is 9.8 bps and the median markup is 0.004 basis points. Markups can be negative and there is substantial variation in markups, as the standard deviation is 78 bps. I measure dealer network size as the number of distinct dealers each insurer trades with during a

calendar quarter. Based on this measure, the average network size is 13.7 dealers. Insurer trading frequency is 0.85 bond trades per day, measured quarterly. There are three electronic platforms used by insurers in the NAIC database: MarketAxess, TMC, and Tradeweb. About 2% of trades between 2014 and 2021 are executed on one of these platforms (over 22,000 electronic platform transactions in all). By far the largest platform in terms of volume is MarketAxess. Average markups on the voice venue are slightly larger than electronic venue (9.8 bps vs 8.9 bps), yet the median markups are large on the electronic venue, suggesting some sampling variation on the electronic venue due to the much smaller number of transactions.

I further explore the differences in markups across venues by grouping transactions by rating class and issue size in Table 2. Bonds are classified according to whether they are investment-grade, high yield or unrated and whether the issue size is less than \$500M, between \$500M and \$1B or greater than \$1B. Markups are lower on the electronic venue except for small bond issue sizes. For example, transacting a small issue size investment grade bond on the voice venue costs 11.7 bps compared with 16 on the electronic venue. In comparison, transacting a large issue size investment grade bond on the voice venue costs 7.5 bps compared with 6.4 on the electronic venue.

As I do not observe the number of dealers who are active on a given electronic platform, I assume that this number is proportional to the number of insurers who trade electronically. Thus to measure thickness, I count the number of insurers who use the platform in a given quarter and divide this number by the total number of insurers who trade in the quarter. Based on this measure, the average market thickness is approximately 6.5% over the entire sample.

Holdings data are obtained from eMaxx Bond Holders database for each fund. Whenever several funds belong to the same NAIC entity, I aggregate the holdings up to the insurer level. The last two rows of 1 report average bond-level holdings and aggregate quarterly portfolio holdings for each insurer. The average insurer has \$5.3B in corporate bond holdings in any given quarter. I create terciles based on this measure to classify insurers by size (small, medium and large).

3 Electronic market thickness and venue choice

I plot the monthly proportion of electronic venue transactions on any of the three electronic venues observed in the sample. The monthly proportion is measured across all types of bonds and is relative to the total number of bond transactions that occur in a month. These frequencies are reported in Figure 3. The proportion of electronic-venue trades increases over time. As shown, the fraction of total trades that take place on an electronic exchange is low (below 1%) at the beginning of the sample, but steadily increasing near the end of the sample. These frequencies are lower than those reported in O'Hara and Xing (2021). The primary reason is that I use a different dataset that is restricted to regulated insurance companies. My sample excludes electronic trading by exchange traded funds, hedge funds and other institutional fund managers.

It may be more advantageous for smaller insurers to trade electronically as their transactions are smaller on average and transaction costs decrease with transaction size (Schultz 2001). I split insurance companies for whom I have valid matches with the eMaxx Bond Holders dataset into three groups based on the total size of their bond portfolios. I then plot the monthly proportion of trades that occur on an electronic venue for each category of insurer and plot them in Figure 4. As expected, a larger proportion of insurer transactions occur on a electronic venue for small insurers compared with large insurers. This difference is most notable in the last two years of the sample, when small insurers trade electronically 5% of the time versus about half that rate for large insurers.

The theoretical outline in section 2 suggests that market thickness is important for understanding insurers' decision to trade electronically. To test this idea, while taking into account that certain types of bonds trade more frequently on an electronic venue (Hendershott and Madhavan 2015), I borrow the approach of Bessembinder, Jacobsen, et al. (2018) and form bond portfolios according to the nine bond issue size and credit rating classes described in the preceding section. I measure (electronic venue) market thickness separately for each of these nine bond portfolios in each calendar quarter. Figure 5 reports quarterly market thickness by portfolio in each calendar quarter of the sample. The market thickness is increasing for each of the nine bond portfolio classes. Some bond portfolio classes have greater thickness than others, notably large-issue investment grade bond and high-yield issues. Unrated bonds have the thinnest electronic

market thickness overall.

3.1 Venue choice

I next examine the trading venue choice by insurers. Proposition 1 predicts that insurers will choose to trade electronically the greater the market thickness on the venue. I test this prediction with the bond portfolio-level market thickness measure developed above, lagged one calendar quarter. Lagging avoids simultaneity bias between market thickness and venue choice. I also include calendar quarter fixed effects (d_q) in some of the regression specifications.

I estimate the following logit model of venue choice at the bond trade level,

$$y^*_{ijt} = a + bMkt.thickness_{jq-1} + cX_{jt} + dY_{iq-1} + d_q + e_j + u_{ijt} \tag{14}$$

where i indicates the insurer, j the bond portfolio, t indexes the trade date, and q indexes the calendar quarter of the trade date. The variable X indicates a vector of bond and trade characteristics, and the vector Y indicates a vector of insurer characteristics.

Table 3 shows that market thickness is significantly positive in all four versions of this regression model. Based on model (1), taking the derivative of the logit function using the coefficient estimates indicates that the economic significance of market thickness is similar to that of the size of the insurer's dealer network. Moving from the 25th to the 75th percentile of market thickness implies an 1.1% change in the probability of choosing an electronic venue, compared with 0.9% change from a 25th to 75th percentile increase in dealer network size. In comparison, trade size has an economically larger effect. Relative to the smallest trade size, trading over $1M par amount reduces the probability of choosing the electronic venue by 2.9%. The sign of the other covariates implies that 1), insurers with larger dealer networks are more likely to trade electronically, 2) insurers who trade more frequently are less likely to trade electronically and 3), larger transactions are less likely to be executed on the electronic venue, controlling for dealer network size.

The magnitude of the coefficient on market thickness declines when monthly time fixed effects are included (Model 2), likely because of the constant upward trend in electronic venue usage in the sample (Figure 5). Model (3) restricts the sample to observations with insurer bond holdings.

Controlling for insurer size does not affect the coefficient estimates on market thickness (columns 1 vs 3). However, larger insurers (in the top third of the holdings based on the previous quarter) are more likely to trade electronically than smaller insurers, controlling for trade size and dealer network size. The economic effect of insurer size is similar to that of market thickness, with a marginal effect of 0.5% of being a large insurer relative to a small insurer on the probability of trading electronically. The positive association between size and electronic venue choice suggests that the higher frequency of electronic venue transactions by small insurers observed in Figure 4 is due to the smaller transaction size made by these institutions, rather than the size of the insurer.

In unreported tests, I examine whether the effect of market thickness is moderated by trading frequency or dealer network size by including interaction effects between market thickness and dealer network size and market thickness and trading frequency. Neither interaction term is significant.

3.2 Markups

I next investigate whether dealer markups fall following trading by insurers on an electronic venue. To do this, I compare markups before and after an insurer first trades on an electronic venue. The variable of interest is a dummy variable equal to one after the first trade on an electronic venue. I define $Dummy : Post - elec_{ijt}$ separately for each insurer-bond portfolio combination.

The following regression specification controls for insurer-dealer relationship effects that affect markups by a dealer with respect to an insurer:

$$Markup(bps)_{ikjt} = a_{ik} + bDummy : Post - elec_{ijt} + cX_{it} + dZ_{kt} + d_k + u_{ikjt} \qquad (15)$$

where k indexes the dealer, i the insurer and j the bond class. The variable a_{ik} measures the time-invariant (unobserved) component of the trading relationship between insurer i and dealer k. $Dummy : Post - elec_{ijt}$ takes on a value of 1 after any bond from portfolio j is first traded electronically by insurer i . This variable captures the time at which an insurer begins to trade on the electronic venue. Corollary 1 implies that the coefficient b is negative: markups

on the voice venue fall after an insurer starts trading on the electronic venue. Because of the high number of multi-dimensional fixed effects, I lose some observations depending on which combination of relationship, time and portfolio fixed effects are included in any given regression model specification.

Ordinary least squares estimates are reported in Table 4. Model (1) reports estimates of equation (15). Model (2) add an interaction term between $Dummy : Post - elec_{ijt}$ and market thickness. Model (3) adds an additional interaction between $Dummy : Post - elec_{ijt}$ and dealer network size. Model (4) adds month time effects, and model (5) replaces the insurer-dealer fixed effects with insurer-dealer-portfolio fixed effects. This last specification uses the subset of observations for which there are at least two trades by the same insurer-dealer pair for each portfolio.

All four regression models imply that dealer markups for insurers in a given dealer-insurer trading relationship decline following the first transaction on an electronic venue. Based on model (1), trading electronically has a statistically significant negative effect on subsequent trades over the voice venue, controlling for unobserved effects relating to the insurer-dealer relationship. The coefficient estimate implies that trading electronically reduces subsequent dealer markups on the voice venue by approximately 0.74 bps. Given an average markup of 9 bps, this is equivalent to a 7.5% decrease in average trading costs.

Table 5 reports the effects of electronic trading on bilateral dealer markups based on different value of market thickness and dealer network size. Model (2) implies that the effect of trading electronically on dealer markups on the voice venue is increasing in market thickness. At the 25th percentile of market thickness and the median number of dealers, trading electronically subsequently decreases voice-venue markups by 0.22bps. When the market thickness is at the 75th percentile, dealer markups decline by 1.2 bps subsequent to the first electronic trade by an insurer.

Model (3) indicates that markup reductions post electronic trading are greater the larger the dealer network size. For example, setting market thickness to its median value and the number of dealers at the 25th percentile implies a 0.14 bp reduction in dealer markups (Table 5, column 3). When dealer network size is at the 75th percentile, dealer markups decline by 0.81 bps. The negative effect of dealer network size implies that insurers with large dealer networks pay lower

markups on the voice venue post electronic-trading than insurers with small dealer networks. This result is surprising as insurers who trade with a large number of dealers on the voice venue should be more competitive, pay lower markups, and thus benefit less from electronic trading. A possible explanation is that the number of dealers who respond to a request for quotes on the electronic venue is a function of the size of the dealer network of the insurer making the request on the electronic venue.

3.3 Bilateral trading intensities

I next examine the evolution of insurer-dealer trading intensities following the first electronic venue transaction by an insurer. More specifically, I count the number of voice transactions per quarter involving each insurer-dealer-portfolio combination and test if the quarterly transaction count between the insurer and the dealer declines following the first electronic venue transaction by the insurer for a given portfolio. The regression model is,

$$n_{ikjq} = a_j + bDummy : Post - elec_{ijkq} + cDummy : Post - elec_{ijt} \times nb.dealers(t-1) \quad (16)$$
$$+ dX_{iq} + eZ_{jq} + fW_{kq} + g_q + u_{ikjq}$$

where n_{ikjq} is the number of transactions between insurer i and dealer k involving bonds in portfolio j, i indexes the insurer, j indexes the portfolio, k indexes the dealer and q indexes the calendar quarter. The estimation dataset is comprised of quarterly counts per insurer-dealer-portfolio combination. The size of the dataset grows to 2,456,854 quarterly observations due to the creation of quarters with zero counts.

Equation (16) is estimated with a negative binomial model due to over-dispersion in the count variance relative to a Poisson model.[3] The exposure variable in the count regression equals the quarterly count of the number of transactions by the insurer in a bond portfolio. The binomial model estimates the determinants of the proportion of these transactions that are executed with a particular dealer in a given quarter. The coefficient b estimates the effect of the electronic venue transactions on trading intensities with each of the insurer's dealers in his dealer network.

[3]See Stata Base Reference Manual 16 for a discussion of the two types of count regression models.

Estimates are reported in Table 6. Model (1) estimates equation (16). Models (2) and (3) include an interaction term with the number of dealers in the insurer's dealer network. This interaction term controls for the fact that insurer-dealer pair level trading intensities may be lower for insurers with larger dealer networks. Model (3) also includes dummy variables for insurer size based on its aggregate bond holdings. All three regression models imply that bilateral trading intensities with dealers decline following the first electronic venue transactions.

3.3.1 Trading intensities and insurer dealer network size

The positive interaction effect with the number of dealers in model (2) implies that the effect of electronic venue trading is attenuated in dealers with larger dealer networks. To quantify this effect, I estimate the incidence ratio at the 25th, median and 75th percentiles of dealer network size. The incidence-rate ratios (IRRs) is,

$$IRR = exp\{b + c \times nb.dealers\}$$

Intuitively, IRR captures the proportion of all transactions in the quarter executed with an individual dealer.

Incidence ratios are reported in Table 7. Based on Model 1, the baseline IRR is 0.75, implying that insurers are 25% less likely to trade bilaterally with a given dealer following the insurer's first electronic venue transaction (panel A, column 1).

Model (2) allows the effect of electronic venue trading to vary with the number of dealers. To capture the effect of dealer network size, I set the number of dealers to 6 (25th pct), 13 (median) and 20 (75th pct). Based on Model 2, the IRR is 0.56 when the dealer network size is 6, compared to 0.73 and 0.94 for dealer network sizes of 13 and 20, respectively (panel A, column 2). This result is to be expected if insurers spread out a decrease in voice trading equally across all dealers in their network. When the dealer network size is large, the effect of electronic trading on the trading intensity with any given dealer in the network will be proportionally smaller, the larger the number of dealers. Thus, for example, an insurer with a dealer network of 6 dealers will trade 27% less with that dealer following the insurer's first electronic trade. In comparison, an insurer with a dealer network of 20 dealers will trade 65 less with that dealer following the

insurer's first electronic trade.

3.3.2 Trading intensities and dealer size

Trading intensities with larger dealers may be less affected by electronic venue trading. Model (3) examines this question with an interaction term between $Dummy : Post\,elec$ and a dummy variable for large dealers. This variable equals one for all of the insurer-dealer pairs involving a large dealer, measured by volume. These incidence ratios assume a median sized dealer network. Based on Model 3, the IRR is 0.76 when the dealer is large, compared with 0.25 when the dealer is medium or small (panel B, column 3). Trading volumes with smaller dealers decline proportionally more (75%) after an insurer's first electronic transaction than large-dealer trading volumes (24%).

3.3.3 Trading intensities and insurer size

To see whether the effect of electronic venue trading affects trading intensities differently for smaller insurers, I include insurer size fixed effects and interaction terms in model (4). Large insurers may be more inclined to consider alternative trading venues if they are more sophisticated and less dependent on any one dealer. On the other hand, bilateral transaction costs are higher for smaller trade sizes and smaller insurers may have more to benefit from switching to an electronic venue. The negative sign on the interaction term between $Dummy : Post\,elec$ and the small insurer dummy supports the later hypothesis. The corresponding incidence ratios for small and medium to large insurers are 0.70 and 0.82, respectively (panel C, column 4). This implies that small insurers are 30% less likely to trade bilaterally with a dealer after the first electronic venue transaction compared with an 18% reduction for medium and large insurers. Overall, the decline in bilateral trading following an insurer's first electronic transaction is strongest for the smaller insurers.

4 Conclusion

Critics of the US corporate bond market complain that it is opaque and costly and potentially unfair to smaller institutional clients. Electronic trading venues promise lower execution costs

and high liquidity to corporate bond investors at the expense of privacy. Despite this promise, adoption rates of electronic venue trading in this market have been low. Proposed explanations for low adoption rates relate to non-trade related services provided by traditional bond dealers, lack of market depth on the competing electronic venues, and the lack of readily available market quotations (i.e., poor pre-trade transparency).

I examine this question theoretically and empirically. I consider two extensions to the seminal paper by DGP on OTC markets: 1) I explicitly model the value of trading relationships, and (2) add an alternative electronic trading venue. I assume that dealers strategic response to electronic venue trading is to penalize clients for defecting by charging higher average markups on the voice venue. This strategy works as long as the market thickness on the electronic venue is not too "thick". When market thickness is high enough, clients defect and trade on both venues, which subsequently reduces dealer markups on the voice venue.

I document empirical support for both predictions. Using regulatory transaction data by insurance companies that discloses the parties to bond transactions, I find that electronic venue choice is more likely when electronic-venue market thickness is greater. By analyzing different insurer-dealer trading pairs, I also find that dealers in a relationship with an insurer charge lower markups to the insurer once the insurer begins to trade on the electronic venue. The post electronic trading decline in markups is more pronounced when market thickness is higher and when the insurer has a larger dealer network. In the last part of the paper, I show that bilateral trading frequencies decline (as might be expected) once an insurer begins to trade electronically. The decline is strongest for insurers with small dealer networks, insurers in relationships with smaller dealers, and smaller insurers.

Overall, this work contributes to our understanding of the tradeoffs involved in choosing to trade electronically instead of bilaterally. I focus on the tradeoff insurers must make between two unobserved features, namely the penalty they incur for defecting, and the gains in trading costs from trading on an electronic venue with sufficient market thickness. My findings suggest that smaller insurers with fewer dealer relationships have the most to gain from switching venues. The switch is particularly costly to smaller dealers in insurer-dealer relationships with a switching insurer. Overall, this work suggests that a fruitful avenue for future research should focus on dealer's strategic reaction to electronic venue trading such as non-trade aspects of dealer-client

relationships.

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Tables and Figures

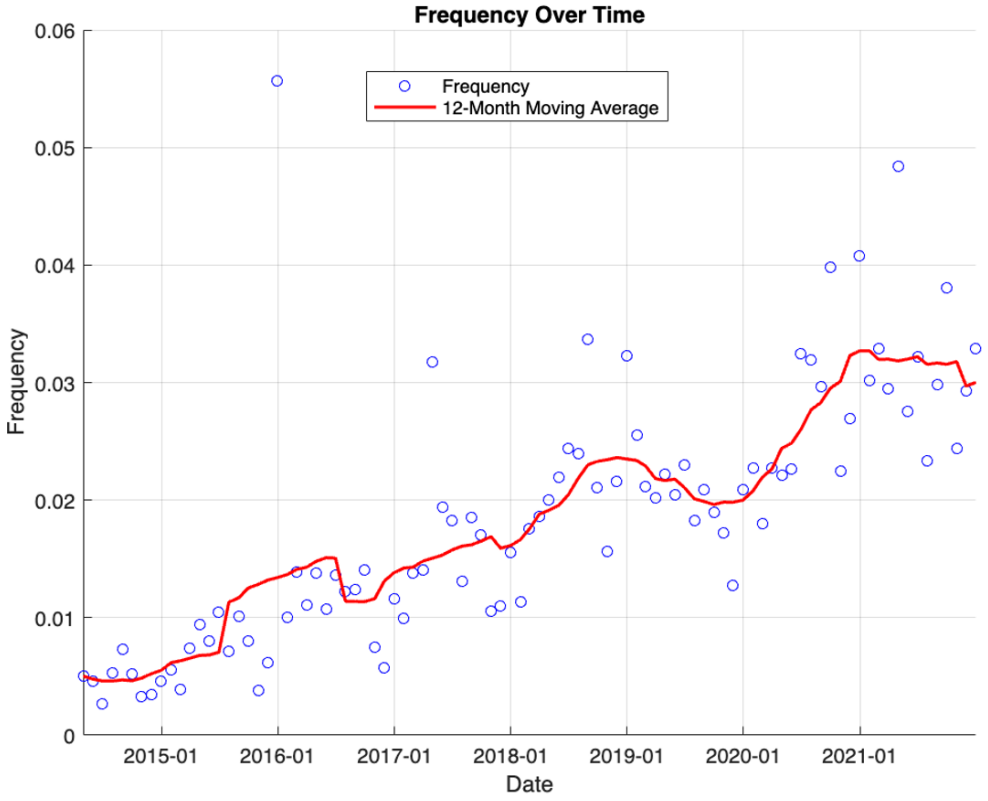


Figure 3: **Proportion of electronic bond trades by month.** Figure 1 reports the monthly frequencies of trading on an electronic trading venue in US corporate bonds. Electronic trades primarily occur on MarketAxess, but some also occur on Tradeweb and TMC bonds. Frequencies are based on transaction data reported by insurance companies to the NAIC, i..e. Schedule D forms part 3, part 4 and part 5.

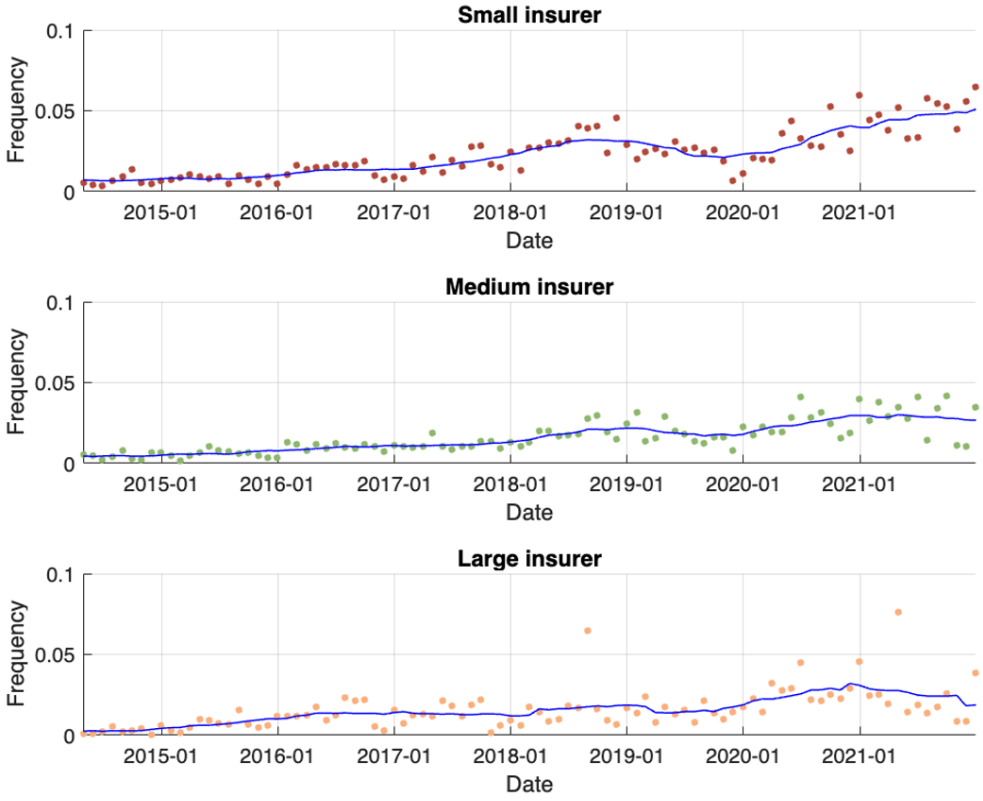


Figure 4: **Electronic trading by insurer size.** Electronic venue trading frequency by insurer size. The monthly trading frequency on electronic venues is reported by insurer size category. The frequency equals the ratio of the number of corporate bond transaction on an electronic venue to the total number of transactions in that month, by insurers in a given size group. Insurers with non-missing bond holdings at the end of the previous quarter are classified into tercile and labelled accordingly.

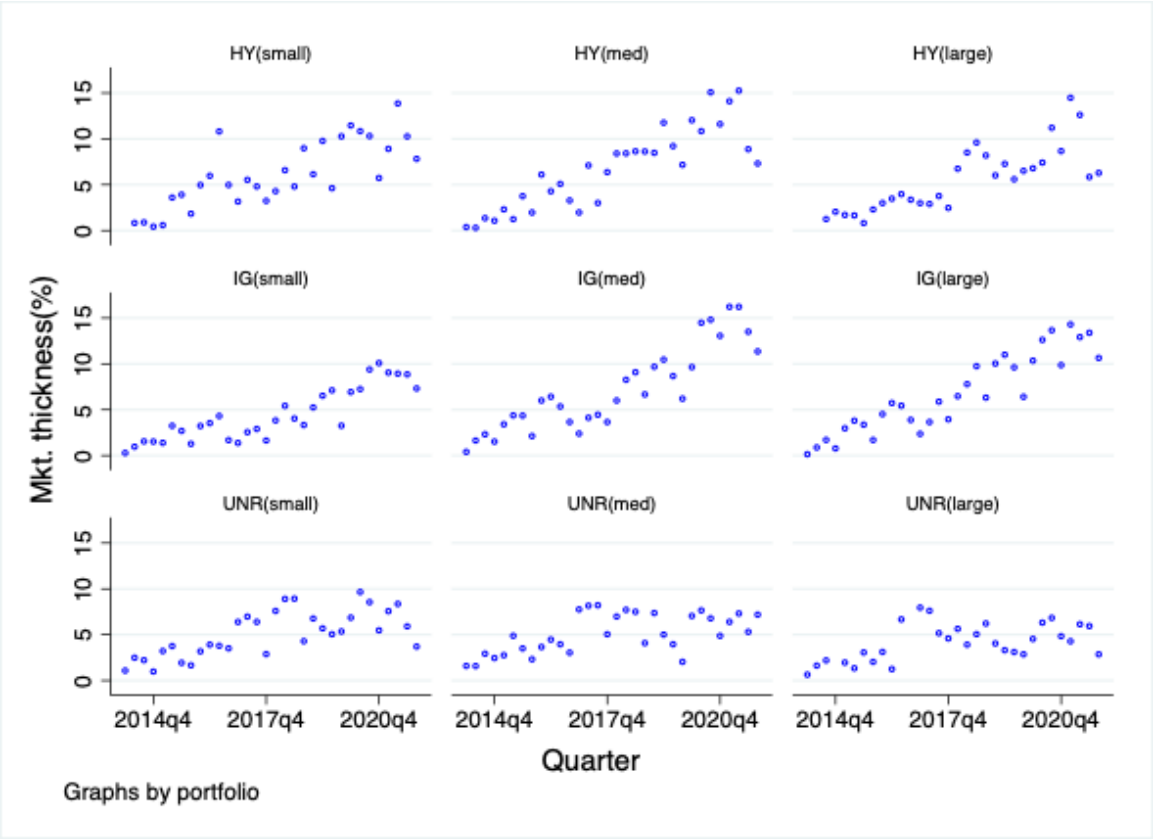


Figure 5: **Market thickness by calendar quarter and bond portfolio.** Market thickness is defined as the ratio of the number of unique insurers that trade a bond electronically to the total number of insurers who trade that bond in a quarter where the bond belongs to a rating class - issue size portfolio times 100. HY indicates a below investment grade credit rating. IG indicates an investment grade credit rating and UNR indicates the bond is unrated. Small indicates an issues size less than $500M, medium indicates an issue size between $500M and $1B, and large indicates an issues size greater than $1B.

Table 1: **Summary statistics.** This table reports summary statistics on corporate bond transactions by insurance companies between January 2014 and December 2021. Transactions are reported to the National Insurance Commissioners (NAIC). Bond purchaser and bond vendors are manually classified as either investment dealers, insurance companies, electronic trading platforms or other investment companies (e.g., a mutual fund). Information on bond characteristics, historical credit rating and amount outstanding are obtained from FISD/MERGENT. Entries involving primary issue transactions, corporate vents such as redemptions and calls are excluded, as are transactions for which the counterparty is not identifiable. Trading frequencies, the number of dealers in the client's network and the market thickness are measured quarterly. The trading frequency equals the number of insurance company transactions in a quarter divided by the number of days in the quarter.

Variable	mean	sd	p25	p50	p75	min	max	N
Markup	9.80	78.37	-6.68	0.00	26.71	-306.84	343.64	1177857
Markup (voice)	9.81	78.41	-6.57	0.00	26.68	-306.84	343.64	1155566
Markup (electronic)	8.87	76.24	-10.89	5.26	27.61	-306.84	343.64	22291
Dummy: Elec. trading	0.02	0.14	0.00	0.00	0.00	0.00	1.00	1177857
Nb. dealers (t-1)	13.69	9.29	6.00	13.00	20.00	0.00	50.00	1177857
Trad. Freq. (t-1)	0.86	1.25	0.13	0.41	1.08	0.01	15.74	1177857
Mkt. thickness (t-1)	6.56	4.10	3.37	6.00	9.65	0.14	16.22	1177857
Dummy: Buy	0.63	0.48	0.00	1.00	1.00	0.00	1.00	1177857
Dummy: Odd lot	0.44	0.50	0.00	0.00	1.00	0.00	1.00	1177857
Dummy: Round lot	0.28	0.45	0.00	0.00	1.00	0.00	1.00	1177857
Dummy: Maximum lot	0.10	0.31	0.00	0.00	0.00	0.00	1.00	1177857
Dummy: Investment grade	0.76	0.43	1.00	1.00	1.00	0.00	1.00	1177857
Dummy: High yield	0.11	0.32	0.00	0.00	0.00	0.00	1.00	1177857
Dummy: Unrated	0.13	0.34	0.00	0.00	0.00	0.00	1.00	1177857
Dummy: Financial	0.24	0.43	0.00	0.00	0.00	0.00	1.00	1177857
Dummy: Utility	0.08	0.27	0.00	0.00	0.00	0.00	1.00	1177857
Dummy: Last trading day of month	0.04	0.19	0.00	0.00	0.00	0.00	1.00	1177857
Dummmy: Most liquid bond issue	0.43	0.50	0.00	0.00	1.00	0.00	1.00	1177857
Dummy: Monday	0.19	0.39	0.00	0.00	0.00	0.00	1.00	1177857
Dummy: Friday	0.13	0.34	0.00	0.00	0.00	0.00	1.00	1177857
Bondmaturity (days)	3403.2	3039.1	1589.0	2553.0	3659.0	0.0	36615.0	1177857
Bondage (days)	895.0	1217.3	4.0	467.0	1343.0	0.0	22977.0	1177857
Issue size ($M)	1131.2	1055.1	500.0	800.0	1400.0	1.0	15000.0	1177857
Other issue size ($M)	14380.4	20380.3	2050.0	6450.0	16550.0	0.0	119232.9	1177857
AbsRet	0.0141	0.0187	0.0040	0.0089	0.0175	0.0000	0.7459	1040727
Par amount held ($M)(t-1)	2.8	5.9	0.2	0.8	2.8	0.0	172.8	271298
Agg. Amount ($M)(t-1)	5270.4	11673.7	95.5	693.6	4354.5	0.0	199722.1	568821

Table 2: **Markups by venue and bond issue size.** This table reports average markups in basis points on each venue grouped by credit rating and bond issue size. HY designates below investment grade bonds. IG designates investment grade bonds and UNR designates bonds that are unrated. Small designates bond issue sizes less than $500M, medium designates issues sizes between $500M and $1B and large designates issue sizes greater than $1B.

	Voice		Electronic	
Portfolio	mean(bps)	N	mean(bps)	N
HY(small)	11.7	27,864	19.9	681
HY(med)	11.9	70,906	1.7	1,538
HY(large)	14.4	30,068	-7.0	681
IG(small)	11.7	135,039	16.0	2,445
IG(med)	8.4	407,469	8.6	7,211
IG(large)	7.5	333,543	6.4	5,882
UNR(small)	18.1	51,300	14.3	1,503
UNR(med)	14.5	65,339	13.3	1,596
UNR(large)	10.0	34,038	7.1	754
Total	9.8	1,155,566	8.9	22,291

Table 3: **Venue choice regressions.** This table reports logit estimates of trading venue choice at the transaction level from April 2014 to December 2021. Market thickness equals the number of unique insurers trading on any electronic exchange by the total number of insurers that trade, lagged on quarter. Market thickness is computed separately for each of the nine issue-size by rating class bond portfolios. All specifications have portfolio fixed effects. Standard errors clustered by insurers are in parentheses.

	All trades		With insurer holdings
	(1)	(2)	(3)
Mkt. thickness (t-1)	0.096***	0.026***	0.101***
	(0.008)	(0.007)	(0.007)
Nb. dealers (t-1)	0.036***	0.037***	0.040***
	(0.010)	(0.010)	(0.010)
Log(Trad. Freq. (t-1))	-0.247***	-0.247***	-0.302***
	(0.058)	(0.054)	(0.055)
Dummy: medium insurer			0.077
			(0.117)
Dummy: large insurer			0.297**
			(0.127)
Dummy: Buy	-0.291***	-0.303***	-0.503***
	(0.071)	(0.063)	(0.074)
Dummy: Odd lot	-0.139	-0.136	-0.187*
	(0.100)	(0.099)	(0.096)
Dummy: Round lot	-0.669***	-0.675***	-0.771***
	(0.117)	(0.116)	(0.129)
Dummy: Maximum lot	-1.624***	-1.633***	-1.843***
	(0.150)	(0.147)	(0.178)
Dummy: Financial	-0.02	-0.013	-0.035
	(0.026)	(0.025)	(0.037)
Dummy: Utility	-0.066*	-0.078**	-0.094*
	(0.034)	(0.034)	(0.055)
Dummy: Last trading day of month	0.034	0.022	0.124
	(0.075)	(0.075)	(0.106)
Dummmy: Most liquid bond issue	-0.202***	-0.196***	-0.157***
	(0.027)	(0.028)	(0.043)
Dummy: Monday	0.022	0.017	-0.004
	(0.039)	(0.036)	(0.043)
Dummy: Friday	0.03	0.017	-0.041
	(0.052)	(0.051)	(0.045)
Log(Bondmaturity (days))	-0.209***	-0.207***	-0.283***
	(0.018)	(0.018)	(0.027)
Log(Bondage (days))	0.328***	0.330***	0.294***
	(0.011)	(0.012)	(0.012)
Log(Other issue size ($M))	-0.022***	-0.025***	-0.017**
	(0.007)	(0.006)	(0.007)
N	1177857	1177857	568821
Pseudo Rsquare	0.10	0.11	0.11
Calendar quarter FE	No	Yes	No

Table 4: **Markup regressions.** Markup regressions are reported controlling for dealer-insurer trading relationships. The dependent variable is the markup in basis points on bilateral transactions between dealers and insurers. Dummy: post elec. pf equals one following the first date at which an insurer trades a bond belonging to one of the nine rating by issue size portfolios. Standard errors are clustered by bond cusip. Control variables used in Table 3 relating to trade, bond issue and bond issuers are included but not reported to save on space. Note: 21,011 singleton observations are dropped from sample in columns (1)-(4), and 92,638 singleton observations are dropped from the sample in column (5). Standard errors in parentheses are clustered by insurer.

	(1)	(2)	(3)	(4)	(5)
Dummy: Post elec	-0.742***	0.291	1.082*	1.458**	0.381
	(0.274)	(0.493)	(0.617)	(0.642)	(0.768)
Mkt. thickness (t-1)	-0.121***	-0.051	-0.049	0.022	-0.009
	(0.036)	(0.043)	(0.043)	(0.097)	(0.120)
Dummy: Post elec X Mkt. thickness(t-1)		-0.153***	-0.156***	-0.136**	-0.114*
		(0.056)	(0.056)	(0.060)	(0.068)
Nb. dealers (t-1)	-0.029	-0.029	0.000	0.005	0.010
	(0.030)	(0.030)	(0.033)	(0.033)	(0.036)
Dummy: Post elec X Nb. Dealers (t-1)			-0.048*	-0.068***	-0.065**
			(0.025)	(0.025)	(0.029)
Log(Trad. Freq. (t-1))	0.014	0.012	-0.014	-0.212	-0.120
	(0.182)	(0.182)	(0.183)	(0.178)	(0.192)
N	1134555	1134555	1134555	1134555	1062928
AdjR2	0.061	0.061	0.061	0.062	0.081
Portfolio FE	Yes	Yes	Yes	Yes	No
Dealer X insurer FE	Yes	Yes	Yes	Yes	No
Dealer X insurer X portfolio FE	No	No	No	No	Yes
Month FE	No	No	No	Yes	Yes

Table 5: **Economic effects of electronic venue choice on dealer markups.** This table reports the total effect of the various interactions reported in Table 4. Total effects are obtained by combining the raw effect of *Dummy* : *Post elec* reported in Table 4 with interaction terms with respect to market thickness and the nb. dealers from each of the regression models.

		Post elec. pf total effect (bps)				
	Value	(1)	(2)	(3)	(4)	(5)
Mkt.thickness(t-1)						
25th pct	3.4	-0.74	-0.22	-0.07	0.12	-0.85
Median	6.0	-0.74	-0.63	-0.48	-0.24	-1.15
75 pct	9.6	-0.74	-1.18	-1.05	-0.74	-1.56
Difference (75th-25th pct)		0.00	-0.96	-0.98	-0.85	-0.72
Nb dealers(t-1)						
25th pct	6		-0.63	-0.14	0.23	-0.69
Median	13		-0.63	-0.48	-0.24	-1.15
75 pct	20		-0.63	-0.81	-0.72	-1.60
Difference (75th-25th pct)			0.00	-0.67	-0.95	-0.91

Table 6: **Trading intensity.** This table reports negative binomial model estimates of the quarterly trade count for each insurer-dealer-portfolio combination. Trading volume is measured at the portfolio level each quarter. *Dummy : Post elec.pf* equals one in quarters after the first instance an insurer trades a bond in portfolio j ($j = 1, .., 9$) on an electronic trading venue. Dealer size is measured according to terciles of dealer volume in the preceding quarter. Insurer size is based on the size of the insurer's corporate bond holdings in the previous quarter. Standard errors are clustered by insurer-dealer-pf combination. The sample includes quarters in which there are zero counts of insurer-dealer-portfolio transactions.

	(1)	(2)	(3)	(4)
Dummy: Post elec	-0.293***	-0.801***	-1.863***	-1.786***
	(0.008)	(0.015)	(0.084)	(0.104)
Nb. dealers (t-1)	-0.052***	-0.068***	-0.068***	-0.061***
	(0.001)	(0.001)	(0.001)	(0.001)
Dummy: Post elec X Nb. Dealers (t-1)		0.037***	0.036***	0.033***
		(0.001)	(0.001)	(0.001)
Dummy: large dealer			0.090***	0.170***
			(0.029)	(0.036)
Dummy: Post elec X Dummy: large dealer			1.119***	1.153***
			(0.078)	(0.099)
Dummy: small insurer				0.318***
				(0.010)
Dummy: Post elec X Dummy: small insurer				-0.148***
				(0.019)
Log(PFAmountout(t-1))	-0.080***	-0.110***	-0.111***	-0.112***
	(0.016)	(0.016)	(0.016)	(0.022)
Log(PFTradevolume(t-1))	0.056***	0.093***	0.095***	0.095***
	(0.017)	(0.017)	(0.017)	(0.023)
Dummy: medium insurer	-0.113**	-0.104**		
	(0.047)	(0.048)		
Dummy: large insurer	0.347***	0.378***		
	(0.046)	(0.047)		
Constant	-0.998***	-0.842***	-0.582***	-0.860***
	(0.061)	(0.062)	(0.049)	(0.066)
Log(alpha)	0.387***	0.362***	0.351***	0.234***
	(0.007)	(0.007)	(0.008)	(0.011)
N	2456854	2456854	2456854	1260819
AdjR2	0.044	0.047	0.049	0.053

Table 7: **Incidence ratios of electronic venue trading on bilateral trading intensities.**
This table reports the total effect of the various interactions reported in Table 6. Total effects are obtained by combining the raw effect of *Dummy* : *Post elec* with interaction terms with respect to the nb. dealers, dealer size and insurer size from each of the regression models. Each of columns (1) - (5) corresponds to a regression model in the previous table. Each entry in the table reports the incidence ratio (IRR), defined as $IRR = exp\{b + c \times z\}$ where z represents either dealer network size (panel A), dealer size (panel B), or insurer size (panel C).

Panel A: Varying the size of the dealer network

	Incidence rate ratios			
Nb dealers	(1)	(2)	(3)	(4)
6 (25th pct)	0.75	0.56	0.19	0.20
13 (median)	0.75	0.73	0.25	0.26
20 (75th pct)	0.75	0.94	0.32	0.32

Panel B: Varying dealer size

	Incidence rate ratios			
Dealer type	(1)	(2)	(3)	(4)
Small or medium	0.75	0.73	0.25	0.26
Large	0.75	0.73	0.76	0.82

Panel C: Varying insurer size

	Incidence rate ratios			
Insurer type	(1)	(2)	(3)	(4)
Small	0.75	0.45	0.76	0.70
Med or large	0.75	0.45	0.76	0.82

Appendix

First note that the memoryless property of the exponential distribution implies that,

$$\begin{aligned} E_t\left[e^{-r(\tau_l - t)}\right] &= \int_t^\infty \lambda_l e^{-(r+\lambda_l)(u-t)} du \\ &= \frac{\lambda_l}{r+\lambda_l} \end{aligned}$$

The following results on stopping times are derived in Feldhütter (2012). Using a similar argument and the fact that the poisson frequency of the minimum of several poisson processes is the sum of the frequencies,

$$\begin{aligned} E_t\left[e^{-r(\tau_l - t)}\mathbf{1}_{\{\tau=\tau_1\}}\right] &= \int_t^\infty \int_t^\infty \mathbf{1}(w>u)\lambda_l e^{-(r+\lambda_l)(u-t)}(\lambda_e+\lambda_v)e^{-(\lambda_v+\lambda_e)(w-t)} du dw \\ &= \int_t^\infty \lambda_l e^{-(r+\lambda_l)(u-t)} \int_u^\infty (\lambda_e+\lambda_v)e^{-(\lambda_v+\lambda_e)(w-t)} dw du \\ &= \int_t^\infty \lambda_l e^{-(r+\lambda_l)(u-t)} \left(\int_{-\infty}^{-(\lambda_v+\lambda_e)(u-t)} e^w dw \right) du \\ &= \int_t^\infty \lambda_l e^{-(r+\lambda_l)(u-t)} e^{-(\lambda_e+\lambda_v)(u-t)} du \\ &= \frac{\lambda_l}{r+\lambda_l+\lambda_v+\lambda_e} \end{aligned}$$

For stopping time $\tau = min\{\tau_l, \tau_e, \tau_v\}$ define $\lambda := \lambda_l + \lambda_e + \lambda_v$. Then

$$\begin{aligned} E_t\left[\int_t^\tau e^{-r(u-t)}(1-\delta)du\right] &= (1-\delta)\int_t^\infty \left(\int_t^x e^{-r(s-t)}ds\right)\lambda e^{-\lambda(x-t)}dx \\ &= (1-\delta)\int_t^\infty \frac{1}{r}\left(1-e^{-r(x-t)}\right)\lambda e^{-\lambda(x-t)}dx \\ &= \frac{(1-\delta)}{r}\left(1-\int_t^\infty \lambda e^{-(\lambda+r)(x-t)}dx\right) \\ &= \frac{(1-\delta)}{r}\left[1-\frac{\lambda}{(\lambda+r)}\right] \\ &= \frac{(1-\delta)}{r+\lambda} = \frac{(1-\delta)}{r+\lambda_l+\lambda_e+\lambda_v} \end{aligned}$$

Trading on voice venue

Let $\tau = Min(\tau_l, \tau_v, \tau_s)$ where τ_s is the event that the dealer passes all of the surplus to the client in the event of a transaction, i.e. the dealer transacts the security at the inter-dealer price M. For clients, having a relationship with a dealer implies periodic access to the inter-dealer market

with frequency λ_s. The steady state values are given by,

$$
\begin{aligned}
V_{lo} &= E_t\left[\int_t^{\tau}(C-\delta)e^{-r(u-t)}du + e^{-r(\tau_l-t)}V_{ho}\mathbf{1}_{(\tau=\tau_l)}\right] \\
&\quad + E_t\left[e^{-r(\tau_v-t)}(V_{ln}+P^{vo}_{bid})\mathbf{1}_{(\tau=\tau_v)} + e^{-r(\tau_s-t)}(V_{ln}+M)\mathbf{1}_{(\tau=\tau_s)}\right] \\
V_{ln} &= E_t\left[e^{-r(\tau_l-t)}V_{hn}\right] \\
V_{ho} &= E_t\left[\int_t^{\tau_l} Ce^{-r(u-t)}du + e^{-r(\tau_l-t)}V_{lo}\right] \\
V_{hn} &= E_t\left[e^{-r(\tau_l-t)}V_{ln}\mathbf{1}_{(\tau=\tau_l)} + e^{-r(\tau_v-t)}(V_{ho}-P^{vo}_{ask})\mathbf{1}_{(\tau=\tau_v)}\right] \\
&\quad + E_t\left[e^{-r(\tau_s-t)}(V_{ho}-M)\mathbf{1}_{(\tau=\tau_s)}\right] \\
S_{hn} &= E_t\left[e^{-r(\tau_s-t)}\theta(\Delta V_h - M)\mathbf{1}_{(\tau=\tau_s)}\right] \\
S_{lo} &= E_t\left[e^{-r(\tau_s-t)}\theta(M-\Delta V_l)\mathbf{1}_{(\tau=\tau_s)}\right]
\end{aligned}
$$

Prices on the voice venue

Define $\Delta V_h = V_{ho} - V_{hn}$ and $\Delta V_l = V_{lo} - V_{ln}$. Prices on the voice venue are the result of Nash bargaining between clients and dealers with bargaining power θ,

$$P^{vo}_{bid} = M - \theta(M - \Delta V_l) = (1-\theta)M + \theta\Delta V_l \tag{A.1}$$
$$P^{vo}_{ask} = M + \theta(\Delta V_h - M) = (1-\theta)M + \theta\Delta V_h \tag{A.2}$$

Value functions with voice

Taking the expectations above,

$$V_{lo} = \frac{(C-\delta)}{r+\lambda_l+\lambda_v+\lambda_s} + \frac{V_{ho}\lambda_l}{r+\lambda_l+\lambda_v+\lambda_s} + \frac{(V_{ln}+P^{vo}_{bid})\lambda_v}{r+\lambda_l+\lambda_v+\lambda_s} + \frac{(V_{ln}+M)\lambda_s}{r+\lambda_l+\lambda_v+\lambda_s} \tag{A.3}$$

$$V_{ln} = \frac{\lambda_l V_{hn}}{r+\lambda_l} \tag{A.4}$$

$$V_{ho} = \frac{C}{r+\lambda_l} + \frac{V_{lo}\lambda_l}{r+\lambda_l} \tag{A.5}$$

$$V_{hn} = \frac{\lambda_l V_{ln}}{r+\lambda_l+\lambda_v+\lambda_s} + \frac{\lambda_v(V_{ho}-P^{vo}_{ask})}{r+\lambda_l+\lambda_v+\lambda_s} + \frac{(V_{ho}-M)\lambda_s}{r+\lambda_l+\lambda_v+\lambda_s} \tag{A.6}$$

$$S_{hn} = \frac{\theta(\Delta V_h - M)\lambda_s}{r+\lambda_l+\lambda_v+\lambda_s} \tag{A.7}$$

$$S_{lo} = \frac{\theta(M-\Delta V_l)\lambda_s}{r+\lambda_l+\lambda_v+\lambda_s} \tag{A.8}$$

The associated system of HJB equations is,

$$rV_{lo} = (C-\delta) + \lambda_l(V_{ho}-V_{lo}) + \lambda_v(V_{ln}+P^{vo}_{bid}-V_{lo}) + \lambda_s(V_{ln}+M-V_{lo}) \tag{A.9}$$

$$rV_{ln} = \lambda_l(V_{hn}-V_{ln}) \tag{A.10}$$

$$rV_{ho} = C + \lambda_l(V_{lo}-V_{ho}) \tag{A.11}$$

$$rV_{hn} = \lambda_l(V_{ln}-V_{hn}) + \lambda_v(V_{ho}-P^{vo}_{ask}-V_{hn}) + \lambda_s(V_{ho}-M-V_{hn}) \tag{A.12}$$

$$rS_{hn} = \lambda_s\theta(\Delta V_h - M - S_{hn}/\theta) - (\lambda_l+\lambda_v)S_{hn} \tag{A.13}$$

$$rS_{lo} = \lambda_s\theta(M-\Delta V_l - S_{lo}/\theta) - (\lambda_l+\lambda_v)S_{lo} \tag{A.14}$$

One can rewrite (A.11) as,

$$(r+\lambda_l)V_{ho} = C + \lambda_l V_{lo} \tag{A.15}$$

One can also rewrite (A.12) as,

$$(r+\lambda_l)V_{hn} = \lambda_l V_{ln} + \lambda_v\Delta V_h + \lambda_s\Delta V_h - \lambda_v P^v_{ask} - \lambda_s M \tag{A.16}$$

Subtracting (A.12) from (A.11),

$$\begin{aligned}
(r+\lambda_l)\Delta V_h &= C + \lambda_l\Delta V_l - \Delta V_h(\lambda_v+\lambda_s) \\
&\quad + \lambda_v P^v_{ask} + \lambda_s M \\
(r+\lambda_l)\Delta V_h &= C + \lambda_l\Delta V_l - \Delta V_h(\lambda_v+\lambda_s) \\
&\quad + \lambda_v(1-\theta)M + \lambda_v\theta\Delta V_h + \lambda_s M \\
[r+\lambda_l+\lambda_v(1-\theta)+\lambda_s]\,\Delta V_h - \lambda_l\Delta V_l &= C + [\lambda_v(1-\theta)+\lambda_s]\,M
\end{aligned}$$

One can rewrite (A.10) as,

$$(r+\lambda_l)V_{ln} = \lambda_l V_{hn} \tag{A.17}$$

One can also rewrite (A.9) as,

$$(r+\lambda_l)V_{lo} = (C-\delta) + \lambda_l V_{ho} - \lambda_v \Delta V_l + \lambda_v P^v_{bid} - \lambda_s \Delta V_l + \lambda_s M \tag{A.18}$$

Subtracting (A.17) from (A.9),

$$\begin{aligned}
(r+\lambda_l)\Delta V_l &= (C-\delta) + \lambda_l \Delta V_h - \Delta V_l \lambda_v + \lambda_v P^v_{bid} \\
&\quad - \lambda_s \Delta V_l + \lambda_s M \\
(r+\lambda_l)\Delta V_l &= (C-\delta) + \lambda_l \Delta V_h - \Delta V_l(\lambda_v + \lambda_s) \\
&\quad + \lambda_v(1-\theta)M + \lambda_v \theta \Delta V_l + \lambda_s M \\
\left[r + \lambda_l + \lambda_v(1-\theta) + \lambda_s\right]\Delta V_l - \lambda_l \Delta V_h &= (C-\delta) + \left[\lambda_v(1-\theta) + \lambda_s\right] M
\end{aligned}$$

Then,

$$\begin{aligned}
\left[r+\psi\right]\Delta V_h - \lambda_l \Delta V_l &= C + \psi_\alpha M \\
\left[r+\psi\right]\Delta V_l - \lambda_l \Delta V_h &= (C-\delta) + \psi_\alpha M \\
S_{hn} &= \theta(\Delta V_h - M)\psi_r \\
S_{lo} &= \theta(M - \Delta V_l)\psi_r
\end{aligned}$$

Define

$$\begin{aligned}
\psi_v &= \lambda_v(1-\theta) + \lambda_s \\
\psi_r &= \frac{\lambda_s}{(r+\lambda_l+\lambda_v+\lambda_s)} \\
\psi_l &= \lambda_l + \psi_v
\end{aligned}$$

This system has the following representation,

$$\begin{bmatrix} r+\psi_l & -\lambda_l \\ -\lambda_l & r+\psi_l \end{bmatrix} \begin{bmatrix} \Delta V_h \\ \Delta V_l \end{bmatrix} = \begin{bmatrix} C + \psi_v M \\ C - \delta + \psi_v M \end{bmatrix} \tag{A.19}$$

Let,

$$\Sigma = \begin{bmatrix} r+\psi_l & -\lambda_l \\ -\lambda_l & r+\psi_l \end{bmatrix}$$

The adjoint of Σ is,

$$adj\Sigma = \begin{bmatrix} r+\psi_l & \lambda_l \\ \lambda_l & r+\psi_l \end{bmatrix}$$

The determinant of Σ is,

$$det\Sigma = (r+\psi_l)^2 - \lambda_l^2$$

So the inverse of Σ is,

$$\Sigma^{-1} = \frac{1}{det\Sigma} adj\Sigma = \frac{1}{\left[(r+\psi_l)^2 - \lambda_l^2\right]} \begin{bmatrix} r+\psi_l & \lambda_l \\ \lambda_l & r+\psi_l \end{bmatrix}$$

Multiplying both sides of (A.19) by Σ^{-1} gives,

$$\begin{bmatrix} \Delta V_h \\ \Delta V_l \end{bmatrix} = \frac{1}{\left[(r+\psi_l)^2 - \lambda_l^2\right]} \begin{bmatrix} r+\psi_l & \lambda_l \\ \lambda_l & r+\psi_l \end{bmatrix} \begin{bmatrix} C+\psi_v M \\ C-\delta+\psi_v M \end{bmatrix}$$

$$\begin{aligned} \begin{bmatrix} \Delta V_h \\ \Delta V_l \end{bmatrix} &= \begin{bmatrix} \frac{(r+\psi_l)}{(r+\psi_l)^2-\lambda_l^2} & \frac{\lambda_l}{(r+\psi_l)^2-\lambda_l^2} \\ \frac{\lambda_l}{(r+\psi_l)^2-\lambda_l^2} & \frac{r+\psi_l}{(r+\psi_l)^2-\lambda_l^2} \end{bmatrix} \begin{bmatrix} C+\psi_v M \\ C-\delta+\psi_v M \end{bmatrix} \\ &= \begin{bmatrix} \frac{(r+\psi_l)(C+\psi_\alpha M)+\lambda_l(C-\delta+\psi_v M)}{(r+\psi_l)^2-\lambda_l^2} \\ \frac{\lambda_l(C+\psi_\alpha M)+(r+\psi_l)(C-\delta+\psi_v M)}{(r+\psi)^2-\lambda_l^2} \end{bmatrix} \\ &= \begin{bmatrix} \frac{(r+\psi_l+\lambda_l)(C+\psi_v M)-\lambda_l\delta}{(r+\psi-\lambda_l)(r+\psi+\lambda_l)} \\ \frac{(r+\psi_l+\lambda_l)(C+\psi_v M)-(r+\psi_l)\delta}{(r+\psi_l-\lambda_l)(r+\psi_l+\lambda_l)} \end{bmatrix} \end{aligned}$$

$$\begin{bmatrix} \Delta V_h \\ \Delta V_l \end{bmatrix} = \begin{bmatrix} \frac{(C+\psi_v M)}{(r+\psi_l-\lambda_l)} - \frac{\lambda_l\delta}{(r+\psi_l-\lambda_l)(r+\psi_l+\lambda_l)} \\ \frac{(C+\psi_v M)}{(r+\psi_l-\lambda_l)} - \frac{(r+\psi_l)\delta}{(r+\psi_l-\lambda_l)(r+\psi_l+\lambda_l)} \end{bmatrix}$$

The interdealer price is $M = q\Delta V_h + (1-q)\Delta V_l$ for some $q \epsilon [0,1]$ and is set between the high and low type reservation values (i.e., $q = 1/2$),

$$\begin{aligned} M &= \frac{\Delta V_h + \Delta V_l}{2} \\ M &= \frac{(C+\psi_v M)}{(r+\psi_l-\lambda_l)} - \frac{(r+\psi_l+\lambda_l)\delta}{2(r+\psi_l-\lambda_l)(r+\psi_l+\lambda_l)} \\ M &= \frac{(C+\psi_v M)}{(r+\psi_l-\lambda_l)} - \frac{\delta}{2(r+\psi_l-\lambda_l)} \\ (r+\psi_l-\lambda_l)\, M &= C+\psi_v M - \delta/2 \\ M &= \frac{C-\delta/2}{r} \end{aligned}$$

Substituting these expressions into equations (3) and (4) gives,

$$\begin{aligned} P_{bid}^{vo} &= (1-\theta)M + \theta\Delta V_l \\ &= \frac{\left[(1-\theta)(r+\psi_v)+\theta\psi_v\right] M}{r+\psi_v} - \frac{\theta\delta(r+\psi_l)}{(r+\psi_l-\lambda_l)(r+\psi_l+\lambda_l)} \\ &+ \frac{\theta C}{r+\psi_l-\lambda_l} \\ &= M + \frac{\theta(C-rM)}{r+\psi_l-\lambda_l} - \frac{\theta\delta(r+\psi_l)}{(r+\psi_l-\lambda_l)(r+\psi_l+\lambda_l)} \end{aligned}$$

and,

$$
\begin{aligned}
P^{vo}_{ask} &= (1-\theta)M + \theta\Delta V_h \\
&= \frac{\left[(1-\theta)(r+\psi_v)+\theta\psi_v\right]M}{r+\psi_v} - \frac{\theta\delta\lambda_l}{(r+\psi_l-\lambda_l)(r+\psi_l+\lambda_l)} \\
&+ \frac{\theta C}{r+\psi_l-\lambda_l} \\
&= M + \frac{\theta(C-rM)}{r+\psi_l-\lambda_l} - \frac{\theta\delta\lambda_l}{(r+\psi_l-\lambda_l)(r+\psi_l+\lambda_l)}
\end{aligned}
$$

Note that if the inter-dealer price is set halfway between the bid and the ask,

$$
\begin{aligned}
M &= \frac{P^{vo}_{bid}+P^{vo}_{ask}}{2} \\
M &= M + \frac{\theta(C-rM)}{r+\psi_l-\lambda_l} - \frac{\theta\delta(r+\psi_l+\lambda_l)}{2(r+\psi_l-\lambda_l)(r+\psi_l+\lambda_l)} \\
0 &= \frac{\theta(C-rM)}{r+\psi_l-\lambda_l} - \frac{\theta\delta}{2(r+\psi_l-\lambda_l)} \\
rM &= C-\delta/2
\end{aligned}
$$

The bid-ask spread is,

$$
\begin{aligned}
P^{vo}_{ask} - P^{vo}_{bid} &= \frac{\theta\delta(r+\psi_l-\lambda_l)}{(r+\psi_l-\lambda_l)(r+\psi_l+\lambda_l)} \\
&= \frac{\theta\delta}{(r+\psi_l+\lambda_l)} \\
&= \frac{\theta\delta}{(r+2\lambda_l+\lambda_v(1-\theta)+\lambda_s)}
\end{aligned}
$$

Noting that $\psi_v = \psi_l - \lambda_l$, the values of the trading relationship are given by,

$$
\begin{aligned}
S^{vo}_{hn} &= \frac{\theta(\Delta V_h - M)\lambda_s}{r+\lambda_l+\lambda_v+\lambda_s} \\
&= \psi_r\theta\left[\frac{(C+\psi_v M)}{(r+\psi_l-\lambda_l)} - \frac{\lambda_l\delta}{(r+\psi_l-\lambda_l)(r+\psi_l+\lambda_l)} - M\frac{(r+\psi_v)}{(r+\psi_l-\lambda_l)}\right] \\
&= \frac{\psi_r}{}\left[\frac{\theta\,(C-rM)}{(r+\psi_l-\lambda_l)} - \frac{\theta\lambda_l\delta}{(r+\psi_l-\lambda_l)(r+\psi_l+\lambda_l)}\right] \\
&= \frac{\psi_r}{}(P^{vo}_{ask} - M) \\
&= \frac{\lambda_s(P^{vo}_{ask} - M)}{(r+\lambda_l+\lambda_v+\lambda_s)}
\end{aligned}
$$

$$
\begin{aligned}
S_{lo}^{vo} &= \frac{\theta(M-\Delta V_l)\lambda_s}{r+\lambda_l+\lambda_v+\lambda_s} \\
&= \psi_r\theta\left[\frac{(r+\psi_v)M}{(r+\psi_l-\lambda_l)} - \frac{(C+\psi_v M)}{(r+\psi_l-\lambda_l)} + \frac{(r+\psi_l)\delta}{(r+\psi_l-\lambda_l)(r+\psi_l+\lambda_l)}\right] \\
&= \frac{\psi_r}{}\left[\frac{\theta\,(rM-C)}{(r+\psi_l-\lambda_l)} + \frac{\theta(r+\psi_l)\delta}{(r+\psi_l-\lambda_l)(r+\psi_l+\lambda_l)}\right] \\
&= \frac{\psi_r}{}(M-P_{bid}^{vo}) \\
&= \frac{\lambda_s(M-P_{bid}^{vo})}{(r+\lambda_l+\lambda_v+\lambda_s)}
\end{aligned}
$$

Summing,

$$S^{vo} = \frac{\psi_r}{}\frac{\theta(r+\psi_l-\lambda_l)\delta}{(r+\psi_l-\lambda_l)(r+\psi_l+\lambda_l)} \tag{A.20}$$

$$S^{vo} = \frac{\psi_r\theta\delta}{(r+\psi_l+\lambda_l)} \tag{A.21}$$

$$S^{vo} = \frac{\lambda_s}{(r+\lambda_l+\lambda_v+\lambda_s)}\frac{\theta\delta}{(r+\lambda_v(1-\theta)+\lambda_s+2\lambda_l)} \tag{A.22}$$

Regression estimates of the value of the trading relationship

Define the half-spread as $H^{vo} = 0.5(P_{ask}^{vo} - P_{bid}^{vo})$. Re-arranging equation,

$$
\begin{aligned}
S^{vo} &= \frac{\lambda_s(P_{ask}^{vo}-P_{bid}^{vo})}{(r+\lambda_l+\lambda_v+\lambda_s)} \\
S^{vo} &= \frac{2\lambda_s H^{vo}}{(r+\lambda_l+\lambda_v+\lambda_s)} \\
(r+\lambda_l+\lambda_v+\lambda_s)\frac{S^{vo}}{2\lambda_s} &= H^{vo} \\
H^{vo} &= \frac{S^{vo}}{2\lambda_s}(r+\lambda_l+\lambda_v+\lambda_s) \\
H^{vo} &= \left(\frac{S^{vo}}{2\lambda_s}\right)(r+\lambda_l) + \left(\frac{S^{vo}}{2\lambda_s}\right)(\lambda_v+\lambda_s) \\
H^{vo} &= \left(\frac{S^{vo}(r+\lambda_l)}{2\lambda_s}\right) + \left(\frac{S^{vo}}{2\lambda_s}\right)\lambda_s + \left(\frac{S^{vo}}{2\lambda_s}\right)\lambda_v
\end{aligned}
$$

The regression counterpart to this relation is then,

$$H^{vo} = a + b\lambda_s + c\lambda_v + u$$

where u is an error term that is independent of λ_s and the trading frequency λ_v. Given a regression estimate $\hat{b}$, an estimate of the value of the relationship is given by $2\lambda_s\hat{b}$.

Trading on voice and electronic venues

I assume that dealers penalize clients who trade on the alternative electronic venue. The penalty is finite and so avoids unrealistic "grim trigger" type strategies whereby a client would be shut-out of the voice channel in perpetuity. Instead, *lo* and *hn* type clients temporarily lose access to the inter-dealer market for $\frac{1}{\lambda_s}$ periods in expectation. The steady-state values are given by,

$$
\begin{aligned}
V_{lo} &= E_t\left[\int_t^{\tau}(C-\delta)e^{-r(u-t)}du + e^{-r(\tau_l-t)}V_{ho}\mathbf{1}_{(\tau=\tau_l)} + e^{-r(\tau_s-t)}(V_{ln}+M)\mathbf{1}_{(\tau=\tau_s)}\right] \\
&\quad + E_t\left[e^{-r(\tau_e-t)}(V_{ln}+P^e_{bid}-S^N_{hn})\mathbf{1}_{(\tau=\tau_e)} + e^{-r(\tau_v-t)}(V_{ln}+P^v_{bid})\mathbf{1}_{(\tau=\tau_v)}\right] \\
V_{ln} &= E_t\left[e^{-r(\tau_l-t)}V_{hn}\right] \\
V_{ho} &= E_t\left[\int_t^{\tau_l} Ce^{-r(u-t)}du + e^{-r(\tau_l-t)}V_{lo}\right] \\
V_{hn} &= E_t\left[e^{-r(\tau_l-t)}V_{ln}\mathbf{1}_{(\tau=\tau_l)} + e^{-r(\tau_e-t)}(V_{ho}-P^e_{ask}-S^N_{lo})\mathbf{1}_{(\tau=\tau_e)}\right] \\
&\quad + E_t\left[e^{-r(\tau_v-t)}(V_{ho}-P^v_{ask})\mathbf{1}_{(\tau=\tau_v)} + e^{-r(\tau_s-t)}(V_{ho}-M)\mathbf{1}_{(\tau=\tau_s)}\right] \\
S^N_{hn} &= E_t\left[e^{-r(\tau_l-t)}S_{hn}\right] \\
S^N_{lo} &= E_t\left[e^{-r(\tau_l-t)}S_{lo}\right] \\
S_{hn} &= E_t\left[e^{-r(\tau_s-t)}\theta(\Delta V_h - M)\mathbf{1}_{(\tau=\tau_s)}\right] \\
S_{lo} &= E_t\left[e^{-r(\tau_s-t)}\theta(M-\Delta V_l)\mathbf{1}_{(\tau=\tau_s)}\right]
\end{aligned}
$$

The assumed trigger strategy is that the client who trades electronically temporarily loses access to the inter-dealer market whenever they switch type from a *ln* to a *hn* type or from a *ho* to a *lo* type in the future. Such a strategy requires coordination by all dealers in the client's network starting from time τ_l until the client's next transaction. It also requires knowledge by all N dealers in the client's network that the client traded on an electronic venue. The former requirement is a strong assumption, the latter is weaker as RFQ quotes are advertised to everyone participating on the electronic platform.

Prices on the electronic venue

I assume that clients have the following beliefs about the participation of dealers on the electronic venue. Suppose that μ is the proportion of total dealers n who participate on the electronic trading venue, i.e., the "thickness of the market". Clients know μ but do not know which dealers participate on the electronic venue. Instead, clients observe a noisy signal of whether dealer i participates on the electronic platform ($\sigma = 1$) or not ($\sigma = 0$). The clients beliefs are such that a client wanting to trade with put in a request or not on the electronic venue with probability $P(client\,request|\sigma = 1) = P(client\,no\,request|\sigma = 0) = \phi$. Clients believe that any given dealer will respond to a quote request with probability p, where

$$\pi = \frac{\phi\mu}{\phi\mu + (1-\phi)(1-\mu)}$$

I assume that dealers employ a mixed pricing strategy on the electronic venue and that prices on this electronic venue are given by,

$$\tilde{P^e}_{bid} = M - x(M - \Delta V_l) = (1-x)M + x\Delta V_l \tag{A.23}$$

$$\tilde{P^e}_{ask} = M + x(\Delta V_h - M) = (1-x)M + x\Delta V_h \tag{A.24}$$

where x is a random variable.

Proposition A1: Given total dealers m across both venues and electronic market thickness μ, the random variable x is positive, has CDF $F(x; m, \mu, \pi) = \frac{1-(1-\pi)x^{\frac{-1}{m\mu-1}}}{\pi}$ and is bounded above by 1 and below by $(1-\pi)^{mu-1}$.

Proof: The proof follows Glebkin, Yueshen, and Shen (2023), Proposition S3 of their Internet Appendix. Let $F(x)$ be the CDF of x. The probability of a given dealer i who asks $x$of winning an auction on the electronic venue given n other dealers is,

$$\begin{aligned} Pr(Win) &= [(1-\pi) + \pi(1-F(x))]^{n-1} \\ &= [1 - \pi F(x)]^{n-1} \end{aligned}$$

Dealer i must be indifferent between asking $x = 1$ and any other value of x. Since his expected profit when $x = 1$ is $[1-\pi]^{n-1}(\Delta V_h - M)$,

$$\begin{aligned} [1-\pi F(x)]^{n-1} x (\Delta V_h - M) &= [1-\pi]^{n-1}(\Delta V_h - M) \\ [1-\pi F(x)]^{n-1} x &= [1-\pi]^{n-1} \\ [1-\pi F(x)] x^{1/(n-1)} &= [1-\pi] \\ 1 - \pi F(x) &= (1-\pi)x^{-1/(n-1)} \\ 1 - (1-\pi)x^{-1/(n-1)} &= \pi - F(x) \\ F(x) &= \frac{1-(1-\pi)x^{-1/(n-1)}}{\pi} \end{aligned}$$

Setting $F(x) = 0$,the lower bound is given by,

$$\begin{aligned} 0 &= \frac{1-(1-\pi)x^{-1/(n-1)}}{\pi} \\ x^{-1/(n-1)}\left(\frac{1-\pi}{\pi}\right) &= \frac{1}{\pi} \\ x^{-1/(n-1)}(1-\pi) &= 1 \\ 1-\pi &= x^{\frac{1}{n-1}} \\ x &= (1-\pi)^{n-1} \end{aligned}$$

Similarly, setting $x = 1$ implies that $F(x) = 1$.

The next proposition characterizes the winning ask price on the electronic venue.

Proposition A2: The winning ask and bid prices in an auction on the electronic venue are given by $\tilde{P^e}_{ask} = M + \underline{X}(\Delta V_h - M)$ and $\tilde{P^e}_{bid} = M - \underline{X}(M - \Delta V_l)$ where $\underline{X}$ has the cumulative distribution function $G(x; m, \pi, \mu)$,

$$G(x; n, \pi, \mu) = \frac{1 - (1-\pi)^{m\mu} x^{\frac{-m\mu}{m\mu-1}}}{1 - (1-\pi)^{m\mu}}$$

The expectation of $\underline{X}$ is,

$$E(\underline{X} |\ N >= 1) = \frac{m\mu(1-\pi)^{m\mu-1}}{1 - (1-\pi)^{m\mu}}$$

Proof:

The proof follows Glebkin, Yueshen, and Shen (2023), Proposition S3 and Section 2 of Kranton (1996). Given N of n dealers participate in an auction on the electronic venue, the probability that the minimum ask price $\underline{X}$ across N dealers is less than x is,

$$Pr(\underline{X} <= x \mid N) = 1 - [1 - F(x)]^N$$

Using Bayes Rule, the probability that the minimum price is less than X given $N >= 1$ dealers participate in the auction is,

$$Pr(\underline{X} <= x \mid N >= 1) = \frac{Pr(\underline{X} <= x \ \&\ N >= 1)}{Pr(N >= 1)}$$

where $Pr(N >= 1) = 1 - (1-\pi)^n$. The numerator of the above expression has a binomial expression,

$$\begin{aligned}
Pr(\underline{X} <= x \ \&\ N >= 1) &= \sum_{j=1}^{n} Pr(N = j) Pr(\underline{X} <= x \mid N = j) \\
&= \sum_{j=1}^{n} \left(\begin{array}{c} n \\ j \end{array} \right) \pi^j (1-\pi)^{n-j} \left(1 - [1 - F(x)]^j\right) \\
&= \sum_{j=0}^{n} \left(\begin{array}{c} n \\ j \end{array} \right) \pi^j (1-\pi)^{n-j} \left(1 - [1 - F(x)]^j\right) \\
&- (1-\pi)^n [1 - [1 - F(x)]^0] \\
&= \sum_{j=0}^{n} \left(\begin{array}{c} n \\ j \end{array} \right) \pi^j (1-\pi)^{n-j} - \sum_{j=0}^{n} \left(\begin{array}{c} n \\ j \end{array} \right) \pi^j (1-\pi)^{n-j} \left(\frac{\pi - 1 + (1-\pi) x^{-1/(n-1)}}{\pi} \right)^j \\
&= 1 - \sum_{j=0}^{n} \left(\begin{array}{c} n \\ j \end{array} \right) (1-\pi)^{n-j} \left(\pi \left[\frac{\pi - 1 + (1-\pi) x^{-1/(n-1)}}{\pi} \right] \right)^j \textit{ by the Binomial Thm.} \\
&= 1 - \left[1 - \pi + \pi - 1 + (1-\pi) x^{-1/(n-1)} \right]^n \textit{ by the Binomial Thm.} \\
&= 1 - (1-\pi)^n x^{\frac{-n}{n-1}}
\end{aligned}$$

It follows that,

$$Pr(\underline{X} <= x \mid N >= 1) = \frac{1-(1-\pi)^n x^{\frac{-n}{n-1}}}{1-(1-\pi)^n}.$$

The density $g(x)$is,

$$\begin{aligned}
g(x \mid N >= 1) &= \frac{d}{dx} G(x \mid N >= 1) \\
&= \frac{d}{dx}\left(\frac{1-(1-\pi)^n x^{\frac{-n}{n-1}}}{1-(1-\pi)^n}\right) \\
&= \left(\frac{1}{1-(1-\pi)^n}\right)\left((1-\pi)^n\left(\frac{n}{n-1}\right)x^{\frac{-n}{n-1}-1}\right) \\
&= \frac{(1-\pi)^n\left(\frac{n}{n-1}\right)}{1-(1-\pi)^n}\left(x^{\frac{1-2n}{n-1}}\right)
\end{aligned}$$

Define,

$$\begin{aligned}
H &= \frac{(1-\pi)^n\left(\frac{n}{n-1}\right)}{1-(1-\pi)^n} \\
w &= \frac{1-2n}{n-1}
\end{aligned}$$

Taking the expectation of $\underline{X}$,

$$\begin{aligned}
E[\underline{X}|\ N >= 1] &= \int_{(1-\pi)^{n-1}}^{1} x g(x \mid : N >= 1) dx \\
&= \int_{(1-\pi)^{n-1}}^{1} x^{w+1} H dx \\
&= \frac{H x^{w+2}}{w+2}\Big|_{(1-\pi)^{n-1}}^{1} \\
&= \frac{H}{w+2} - \frac{H}{w+2}\left((1-\pi)^{(n-1)(w+2)}\right) \\
&= \left(\frac{H}{w+2}\right)\left[1-(1-\pi)^{-1}\right] \\
&= \left(\frac{(1-\pi)^n}{1-(1-\pi)^n}\right)\left(\frac{n}{n-1}\right)\left(\frac{n-1}{-1}\right)\left(1-\frac{1}{1-\pi}\right) \\
&= \left(\frac{(1-\pi)^n}{1-(1-\pi)^n}\right) n \left(\frac{1}{1-\pi}-1\right) \\
&= \frac{n(1-\pi)^{n-1}}{1-(1-\pi)^n}
\end{aligned}$$

Let $n = m\mu$, where m is the total number of dealers across both voice and electronic venues and μ is the thickness on the electronic venue. Denote the expectation $\alpha(\mu) := E(\underline{X}|\ N >= 1; m, \pi, \mu) = \frac{m\mu(1-\pi)^{m\mu-1}}{1-(1-\pi)^{m\mu}}$. Expected bid and ask prices on the electronic venue are then given

by $P^e_{bid} = (1-\alpha)M + \alpha\Delta V_l$ and $P^e_{ask} = (1-\alpha)M + \alpha\Delta V_h$.

Value functions with electronic and voice

Taking the expectations above,

$$V_{lo} = \frac{(C-\delta)}{r+\lambda_l+\lambda_e+\lambda_v+\lambda_s} + \frac{V_{ho}\lambda_l}{r+\lambda_l+\lambda_e+\lambda_v+\lambda_s} + \frac{(V_{ln}+P^e_{bid}-S^N_{hn})\lambda_e}{r+\lambda_l+\lambda_e+\lambda_v+\lambda_s} \tag{A.25}$$

$$+ \frac{(V_{ln}+P^v_{bid})\lambda_v}{r+\lambda_l+\lambda_e+\lambda_v+\lambda_s} + \frac{(V_{ln}+M)\lambda_s}{r+\lambda_l+\lambda_e+\lambda_v+\lambda_s} \tag{A.26}$$

$$V_{ln} = \frac{\lambda_l V_{hn}}{r+\lambda_l} \tag{A.27}$$

$$V_{ho} = \frac{C}{r+\lambda_l} + \frac{V_{lo}\lambda_l}{r+\lambda_l} \tag{A.28}$$

$$V_{hn} = \frac{\lambda_l V_{ln}}{r+\lambda_l+\lambda_e+\lambda_v+\lambda_s} + \frac{\lambda_e(V_{ho}-P^e_{ask}-S^N_{lo})}{r+\lambda_l+\lambda_e+\lambda_v+\lambda_s} \tag{A.29}$$

$$+ \frac{\lambda_v(V_{ho}-P^v_{ask})}{r+\lambda_l+\lambda_e+\lambda_v+\lambda_s} + \frac{(V_{ho}-M)\lambda_s}{r+\lambda_l+\lambda_e+\lambda_v+\lambda_s} \tag{A.30}$$

$$S^N_{hn} = \frac{\lambda_l}{(r+\lambda_l)}\frac{\theta(\Delta V_h - M)\lambda_s}{(r+\lambda_l+\lambda_e+\lambda_v+\lambda_s)} \tag{A.31}$$

$$S^N_{lo} = \frac{\lambda_l}{(r+\lambda_l)}\frac{\theta(M - \Delta V_l)\lambda_s}{(r+\lambda_l+\lambda_e+\lambda_v+\lambda_s)} \tag{A.32}$$

Summing,

$$S^N_{hn} + S^N_{lo} = \frac{\theta(\Delta V_h - \Delta V_l)\lambda_s\lambda_l}{(r+\lambda_l)(r+\lambda_l+\lambda_e+\lambda_v+\lambda_s)}$$

$$S^N = \frac{\theta(\Delta V_h - \Delta V_l)\lambda_s\lambda_l}{(r+\lambda_l)(r+\lambda_l+\lambda_e+\lambda_v+\lambda_s)}$$

The associated system of HJB equations is,

$$rV_{lo} = (C-\delta) + \lambda_l(V_{ho}-V_{lo}) + \lambda_e(V_{ln}+P^e_{bid}-S^N_{hn}-V_{lo}) \tag{A.33}$$

$$+ \lambda_v(V_{ln}+P^v_{bid}-V_{lo}) + \lambda_s(V_{ln}+M-V_{lo}) \tag{A.34}$$

$$rV_{ln} = \lambda_l(V_{hn}-V_{ln}) \tag{A.35}$$

$$rV_{ho} = C + \lambda_l(V_{lo}-V_{ho}) \tag{A.36}$$

$$rV_{hn} = \lambda_l(V_{ln}-V_{hn}) + \lambda_e(V_{ho}-P^e_{ask}-S^N_{lo}-V_{hn}) \tag{A.37}$$

$$+ \lambda_v(V_{ho}-P^v_{ask}-V_{hn}) + \lambda_s(V_{ho}-M-V_{hn}) \tag{A.38}$$

Define

$$\psi_s := \frac{\lambda_l\lambda_s\lambda_e\theta}{(r+\lambda_l)(r+\lambda_l+\lambda_e+\lambda_v+\lambda_s)}$$

One can rewrite (A.36) as,

$$(r+\lambda_l)V_{ho} = C + \lambda_l V_{lo} \tag{A.39}$$

One can also rewrite (A.37) as,

$$(r+\lambda_l)V_{hn} = \lambda_l V_{ln} + (\lambda_v+\lambda_e+\lambda_s)\Delta V_h - \lambda_e P^e_{ask} - \lambda_v P^v_{ask} - \lambda_e S^N_{lo} - \lambda_s M \tag{A.40}$$

Subtracting (A.37) from (A.36),

$$\begin{aligned}
(r+\lambda_l)\Delta V_h &= C + \lambda_l \Delta V_l - \Delta V_h(\lambda_v+\lambda_e+\lambda_s) \\
&\quad + \lambda_e P^e_{ask} + \lambda_v P^v_{ask} + \lambda_e S^N_{lo} + \lambda_s M \\
(r+\lambda_l)\Delta V_h &= C + \lambda_l \Delta V_l - \Delta V_h(\lambda_v+\lambda_e+\lambda_s) + \lambda_e P^e_{ask} \\
&\quad + \lambda_v(1-\theta)M + \lambda_v\theta\Delta V_h + \lambda_e S^N_{lo} + \lambda_s M \\
[r+\lambda_l+\lambda_e+\lambda_s+\lambda_v(1-\theta)]\,\Delta V_h - \lambda_l\Delta V_l &= C + \lambda_v(1-\theta)M + \lambda_e P^e_{ask} + \lambda_e S^N_{lo} + \lambda_s M \\
[r+\lambda_l+\lambda_s+\lambda_v(1-\theta)+\lambda_e(1-\alpha)]\,\Delta V_h &= C + [\lambda_v(1-\theta)+\lambda_e(1-\alpha)+\lambda_s]\,M \\
&\quad -\lambda_l\Delta V_l + \psi_s(M-\Delta V_l) \\
[r+\lambda_l+\lambda_s+\lambda_v(1-\theta)+\lambda_e(1-\alpha)]\,\Delta V_h &= C + [\lambda_v(1-\theta)+\lambda_e(1-\alpha)+\lambda_s+\psi_s]\,M \\
&\quad -(\lambda_l-\psi_s)\Delta V_l
\end{aligned}$$

One can rewrite (A.35) as,

$$(r+\lambda_l)V_{ln} = \lambda_l V_{hn} \tag{A.41}$$

One can also rewrite (A.33) as,

$$(r+\lambda_l)V_{lo} = (C-\delta) + \lambda_l V_{ho} - (\lambda_v+\lambda_e+\lambda_s)\Delta V_l + \lambda_e P^e_{bid} + \lambda_v P^v_{bid} - \lambda_e S^N_{hn} + \lambda_s M \tag{A.42}$$

Subtracting (A.35) from (A.33),

$$\begin{aligned}
(r+\lambda_l)\Delta V_l &= (C-\delta) + \lambda_l \Delta V_h - \Delta V_l(\lambda_v+\lambda_e+\lambda_s) \\
&\quad + \lambda_e P^e_{bid} + \lambda_v P^v_{bid} - \lambda_e S^N_{hn} + \lambda_s M \\
(r+\lambda_l)\Delta V_l &= (C-\delta) + \lambda_l \Delta V_h - \Delta V_l(\lambda_v+\lambda_e+\lambda_s) \\
&\quad + \lambda_e P^e_{bid} + \lambda_v(1-\theta)M + \lambda_v\theta\Delta V_l - \lambda_e S^N_{hn} + \lambda_s M \\
[r+\lambda_l+\lambda_s+\lambda_e+\lambda_v(1-\theta)]\,\Delta V_l - \lambda_l\Delta V_h &= (C-\delta) + \lambda_v(1-\theta)M + \lambda_e P^e_{bid} - \lambda_e S^N_{hn} + \lambda_s M \\
[r+\lambda_l+\lambda_s+\lambda_v(1-\theta)+\lambda_e(1-\alpha)]\,\Delta V_l &= (C-\delta) + [\lambda_v(1-\theta)+\lambda_e(1-\alpha)+\lambda_s]\,M \\
&\quad -\lambda_l\Delta V_h - \psi_s(\Delta V_h - M) \\
[r+\lambda_l+\lambda_s+\lambda_v(1-\theta)+\lambda_e(1-\alpha)]\,\Delta V_l &= (C-\delta) + [\lambda_v(1-\theta)+\lambda_e(1-\alpha)+\lambda_s+\psi_s]\,M \\
&\quad -(\lambda_l-\psi_s)\Delta V_h
\end{aligned}$$

Then,

$$
\begin{aligned}
[r+\psi]\,\Delta V_h - (\lambda_l - \psi_s)\Delta V_l &= C + (\psi_\alpha + \psi_s)M \\
[r+\psi]\,\Delta V_l - (\lambda_l - \psi_s)\Delta V_h &= (C-\delta) + (\psi_\alpha + \psi_s)M \\
\lambda_e S_{hn} &= (\Delta V_h - M)\psi_s \\
\lambda_e S_{lo} &= (M - \Delta V_l)\psi_s
\end{aligned}
$$

where

$$
\begin{aligned}
\psi_\alpha &= \lambda_v(1-\theta) + \lambda_e(1-\alpha) + \lambda_s \\
\psi_s &= \frac{\lambda_l \lambda_s \lambda_e \theta}{(r+\lambda_l)\,(r+\lambda_l+\lambda_e+\lambda_v+\lambda_s)} \\
\psi &= \lambda_l + \psi_\alpha
\end{aligned}
$$

This system has the following representation,

$$
\begin{bmatrix} r+\psi & -(\lambda_l - \psi_s) \\ -(\lambda_l - \psi_s) & r+\psi \end{bmatrix}
\begin{bmatrix} \Delta V_h \\ \Delta V_l \end{bmatrix}
=
\begin{bmatrix} C + (\psi_\alpha + \psi_s)\,M \\ C - \delta + (\psi_\alpha + \psi_s)M \end{bmatrix}
\tag{A.43}
$$

Let,

$$
\Sigma = \begin{bmatrix} r+\psi & -(\lambda_l - \psi_s) \\ -(\lambda_l - \psi_s) & r+\psi \end{bmatrix}
$$

The adjoint of Σ is,

$$
adj\Sigma = \begin{bmatrix} r+\psi & (\lambda_l - \psi_s) \\ (\lambda_l - \psi_s) & r+\psi \end{bmatrix}
$$

The determinant of Σ is,

$$
det\Sigma = (r+\psi)^2 - (\lambda_l - \psi_s)^2
$$

So the inverse of Σ is,

$$
\Sigma^{-1} = \frac{1}{det\Sigma} adj\Sigma = \frac{1}{[(r+\psi)^2 - (\lambda_l - \psi_s)^2]}
\begin{bmatrix} r+\psi & (\lambda_l - \psi_s) \\ (\lambda_l - \psi_s) & r+\psi \end{bmatrix}
$$

Multiplying both sides of (A.43) by Σ^{-1} gives,

$$
\begin{bmatrix} \Delta V_h \\ \Delta V_l \end{bmatrix}
= \frac{1}{[(r+\psi)^2 - (\lambda_l - \psi_s)^2]}
\begin{bmatrix} r+\psi & (\lambda_l - \psi_s) \\ (\lambda_l - \psi_s) & r+\psi \end{bmatrix}
\begin{bmatrix} C + (\psi_\alpha + \psi_s)\,M \\ C - \delta + (\psi_\alpha + \psi_s)\,M \end{bmatrix}
$$

$$\begin{bmatrix} \Delta V_h \\ \Delta V_l \end{bmatrix} = \begin{bmatrix} \frac{r+\psi}{(r+\psi)^2-(\lambda_l-\psi_s)^2} & \frac{\lambda_l-\psi_s}{(r+\psi)^2-(\lambda_l-\psi_s)^2} \\ \frac{\lambda_l-\psi_s}{(r+\psi)^2-(\lambda_l-\psi_s)^2} & \frac{r+\psi}{(r+\psi)^2-(\lambda_l-\psi_s)^2} \end{bmatrix} \begin{bmatrix} C+(\psi_\alpha+\psi_s)M \\ C-\delta+(\psi_\alpha+\psi_s)M \end{bmatrix}$$

$$= \begin{bmatrix} \frac{(r+\psi)(C+(\psi_\alpha+\psi_s)M)+(\lambda_l-\psi_s)(C-\delta+(\psi_\alpha+\psi_s)M)}{(r+\psi)^2-(\lambda_l-\psi_s)^2} \\ \frac{(\lambda_l-\psi_s)(C+(\psi_\alpha+\psi_s)M)+(r+\psi)(C-\delta+(\psi_\alpha+\psi_s)M)}{(r+\psi)^2-(\lambda_l-\psi_s)^2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{(r+\psi-\psi_s+\lambda_l)(C+(\psi_\alpha+\psi_s)M)-(\lambda_l-\psi_s)\delta}{(r+\psi+\psi_s-\lambda_l)(r+\psi-\psi_s+\lambda_l)} \\ \frac{(r+\psi-\psi_s+\lambda_l)(C+(\psi_\alpha+\psi_s)M)-(r+\psi)\delta}{(r+\psi+\psi_s-\lambda_l)(r+\psi-\psi_s+\lambda_l)} \end{bmatrix}$$

$$\begin{bmatrix} \Delta V_h \\ \Delta V_l \end{bmatrix} = \begin{bmatrix} \frac{(C+(\psi_\alpha+\psi_s)M)}{(r+\psi+\psi_s-\lambda_l)} - \frac{(\lambda_l-\psi_s)\delta}{(r+\psi+\psi_s-\lambda_l)(r+\psi-\psi_s+\lambda_l)} \\ \frac{(C+(\psi_\alpha+\psi_s)M)}{(r+\psi+\psi_s-\lambda_l)} - \frac{(r+\psi)\delta}{(r+\psi+\psi_s-\lambda_l)(r+\psi-\psi_s+\lambda_l)} \end{bmatrix}$$

Noting that $\psi_\alpha = \psi - \lambda_l$ and substituting these expressions into equations (3) and (4) gives,

$$\begin{aligned} P^v_{bid} &= (1-\theta)M + \theta\Delta V_l \\ &= \frac{\left[(1-\theta)(r+\psi_\alpha+\psi_s)+\theta\left(\psi_\alpha+\psi_s\right)\right]M}{r+\psi_\alpha+\psi_s} \\ &- \frac{\theta\delta(r+\psi)}{(r+\psi+\psi_s-\lambda_l)(r+\psi-\psi_s+\lambda_l)} + \frac{\theta C}{r+\psi+\psi_s-\lambda_l} \\ &= M + \frac{\theta(C-rM)}{r+\psi_\alpha+\psi_s} - \frac{\theta\delta(r+\psi)}{(r+\psi+\psi_s-\lambda_l)(r+\psi-\psi_s+\lambda_l)} \end{aligned}$$

and,

$$\begin{aligned} P^v_{ask} &= (1-\theta)M + \theta\Delta V_h \\ &= \frac{\left[(1-\theta)(r+\psi_\alpha+\psi_s)+\theta\left(\psi_\alpha+\psi_s\right)\right]M}{r+\psi_\alpha+\psi_s} \\ &- \frac{\theta\delta(\lambda_l-\psi_s)}{(r+\psi+\psi_s-\lambda_l)(r+\psi-\psi_s+\lambda_l)} + \frac{\theta C}{r+\psi+\psi_s-\lambda_l} \\ &= M + \frac{\theta(C-rM)}{r+\psi_\alpha+\psi_s} - \frac{\theta\delta(\lambda_l-\psi_s)}{(r+\psi+\psi_s-\lambda_l)(r+\psi-\psi_s+\lambda_l)} \end{aligned}$$

Prices on the electronic venue can be recovered similarly implying,

$$P^e_{bid} = M + \frac{\alpha(C-rM)}{r+\psi_\alpha+\psi_s} - \frac{\alpha\delta(r+\psi)}{(r+\psi+\psi_s-\lambda_l)(r+\psi-\psi_s+\lambda_l)}$$

$$P^e_{ask} = M + \frac{\alpha(C-rM)}{r+\psi_\alpha+\psi_s} - \frac{\alpha\delta(\lambda_l-\psi_s)}{(r+\psi+\psi_s-\lambda_l)(r+\psi-\psi_s+\lambda_l)}$$

The interdealer price is $M = q\Delta V_h + (1-q)\Delta V_l$ for some $q\,\epsilon\,[0,1]$ and is set between the high and low type reservation values (i.e., $q = 1/2$),

$$
\begin{aligned}
M &= \frac{\Delta V_h + \Delta V_l}{2} \\
M &= \frac{(C + (\psi_\alpha + \psi_s)\, M)}{(r + \psi + \psi_s - \lambda_l)} - \frac{(r + \psi + \lambda_l - \psi_s)\delta}{2(r + \psi + \psi_s - \lambda_l)(r + \psi - \psi_s + \lambda_l)} \\
M &= \frac{(C + (\psi_\alpha + \psi_s)\, M)}{(r + \psi + \psi_s - \lambda_l)} - \frac{\delta}{2(r + \psi + \psi_s - \lambda_l)} \\
(r + \psi_\alpha + \psi_s)\, M &= C + (\psi_\alpha + \psi_s)\, M - \delta/2 \\
M &= \frac{C - \delta/2}{r}
\end{aligned}
$$

Spreads by venue

Noting that $r + \psi = r + \lambda_l + \psi_\alpha = r + \lambda_l + \lambda_v(1-\theta) + \lambda_e(1-\alpha) + \lambda_s$, the bid-ask spreads on each venue are,

$$
\begin{aligned}
P^v_{ask} - P^v_{bid} &= \frac{\theta\delta(r + \psi - \lambda_l + \psi_s)}{(r + \psi + \psi_s - \lambda_l)(r + \psi - \psi_s + \lambda_l)} \\
&= \frac{\theta\delta}{r + \psi + \lambda_l - \psi_s} \\
&= \frac{\theta\delta}{r + 2\lambda_l + \lambda_v(1-\theta) + \lambda_e(1-\alpha) + \lambda_s - \psi_s} \\
P^e_{ask} - P^e_{bid} &= \frac{\alpha\delta(r + \psi - \lambda_l + \psi_s)}{(r + \psi + \psi_s - \lambda_l)(r + \psi - \psi_s + \lambda_l)} \\
&= \frac{\alpha\delta}{r + \psi - \psi_s + \lambda_l} \\
&= \frac{\alpha\delta}{r + 2\lambda_l + \lambda_v(1-\theta) + \lambda_e(1-\alpha) + \lambda_s - \psi_s}
\end{aligned}
$$

So for spreads to be smaller on the electronic venue, it must be that,

$$
\begin{aligned}
\alpha &< \theta \\
\frac{n\mu(1-\pi)^{n\mu-1}}{1-(1-\pi)^{n\mu}} &< \theta
\end{aligned}
$$

Value of trading relationships

Substituting into the expressions for S_{lo} and S_{hn} and noting that $r + \psi + \psi_s - \lambda_l = r + \psi_\alpha + \psi_s$,

$$
\begin{aligned}
S_{hn} &= \frac{\theta(\Delta V_h - M)\lambda_s\lambda_l}{(r+\lambda_l)(r+\lambda_l+\lambda_v+\lambda_s+\lambda_e)} \\
&= \frac{\psi_s\theta}{\theta\lambda_e}\left[\frac{(C+(\psi_\alpha+\psi_s)M)}{r+\psi_\alpha+\psi_s} - \frac{(\lambda_l-\psi_s)\delta}{(r+\psi+\psi_s-\lambda_l)(r+\psi-\psi_s+\lambda_l)} - M\frac{(r+\psi_\alpha+\psi_s)}{(r+\psi_\alpha+\psi_s)}\right] \\
&= \frac{\psi_s}{\theta\lambda_e}\left[\frac{\theta(C-rM)}{r+\psi_\alpha+\psi_s} - \frac{\theta(\lambda_l-\psi_s)\delta}{(r+\psi+\psi_s-\lambda_l)(r+\psi-\psi_s+\lambda_l)}\right] \\
&= \frac{\psi_s}{\theta\lambda_e}(P^v_{ask}-M) \\
&= \frac{\lambda_l\lambda_s(P^v_{ask}-M)}{(r+\lambda_l)(r+\lambda_l+\lambda_e+\lambda_v+\lambda_s)}
\end{aligned}
$$

Likewise,

$$
\begin{aligned}
S_{lo} &= \frac{\theta(M-\Delta V_l)\lambda_s\lambda_l}{(r+\lambda_l)(r+\lambda_l+\lambda_v+\lambda_s+\lambda_e)} \\
&= \frac{\psi_s\theta}{\theta\lambda_e}\left[-\frac{(C+(\psi_\alpha+\psi_s)M)}{r+\psi_\alpha+\psi_s} + \frac{(r+\psi)\delta}{(r+\psi+\psi_s-\lambda_l)(r+\psi-\psi_s+\lambda_l)} + M\frac{(r+\psi_\alpha+\psi_s)}{(r+\psi_\alpha+\psi_s)}\right] \\
&= \frac{\psi_s}{\theta\lambda_e}\left[\frac{\theta(rM-C)}{r+\psi_\alpha+\psi_s} + \frac{\theta(r+\psi)\delta}{(r+\psi+\psi_s-\lambda_l)(r+\psi-\psi_s+\lambda_l)}\right] \\
&= \frac{\psi_s}{\theta\lambda_e}(M-P^v_{bid}) \\
&= \frac{\lambda_l\lambda_s(M-P^v_{bid})}{(r+\lambda_l)(r+\lambda_l+\lambda_e+\lambda_v+\lambda_s)}
\end{aligned}
$$

So the value of the trading relationships can be measured based on observed prices and frequencies on trade on each venue. Summing S_{hn} and S_{lo} and defining the half spread $H = P^v_{ask} - M = M - P^v_{bid} = 0.5\lambda_s(P^v_{ask} - P^v_{bid})$,

$$
\begin{aligned}
S_{hn} + S_{lo} &= \frac{\lambda_l\lambda_s(P^v_{ask}-P^v_{bid})}{(r+\lambda_l)(r+\lambda_l+\lambda_e+\lambda_v+\lambda_s)} \\
S &= \frac{2\lambda_l\lambda_s H}{(r+\lambda_l)(r+\lambda_l+\lambda_e+\lambda_v+\lambda_s)}
\end{aligned} \tag{A.44}
$$

Alternatively,

$$
\begin{aligned}
S &= \frac{\psi_s}{\theta\lambda_e}\frac{\theta(r+\psi+\psi_s-\lambda_l)\delta}{(r+\psi+\psi_s-\lambda_l)(r+\psi-\psi_s+\lambda_l)} \\
S &= \frac{\psi_s}{\lambda_e}\frac{\delta}{(r+\psi-\psi_s+\lambda_l)} \\
S &= \frac{\lambda_l\lambda_s}{(r+\lambda_l)(r+\lambda_l+\lambda_e+\lambda_v+\lambda_s)}\frac{\theta\delta}{(r+\psi-\psi_s+\lambda_l)}
\end{aligned}
$$

Venue Choice

This section details the proof to Proposition 1.

Clients who are of type hn prefer to trade exclusively over the voice venue as long as,

$$\begin{aligned}
rV_{hn}^{vo} &> rV_{hn} \\
\lambda_l(V_{ln}^{vo} - V_{hn}^{vo}) + \lambda_v(V_{ho}^{vo} - P_{ask}^{vo} - V_{hn}^{vo}) &> \lambda_l(V_{ln} - V_{hn}) + \lambda_e(V_{ho} - P_{ask}^{e} - S_{lo}^{N} - V_{hn}) \\
+\lambda_s(V_{ho}^{vo} - M - V_{hn}^{vo}) + \lambda_v(V_{ho} - P_{ask}^{v} - V_{hn}) + \lambda_s(V_{ho} - M - V_{hn}) & \\
\lambda_l(V_{ln}^{vo} - V_{hn}^{vo}) + \lambda_v\Delta V_h^{vo} - \lambda_v P_{ask}^{vo} &> \lambda_l(V_{ln} - V_{hn}) + \lambda_v\Delta V_h - \lambda_v P_{ask}^{v} + \lambda_s\Delta V_h \\
+\lambda_s\Delta V_h^{vo} - \lambda_s M + \lambda_e\Delta V_h - \lambda_e P_{ask}^{e} - \lambda_s M - \lambda_e S_{lo}^{N} & \\
\lambda_l\left[(V_{ln}^{vo} - V_{hn}^{vo}) - (V_{ln} - V_{hn})\right] &> \lambda_e(\Delta V_h - P_{ask}^{e}) - (M - \Delta V_l)\psi_s \\
+(\lambda_v + \lambda_s)(\Delta V_h^{vo} - \Delta V_h) - \lambda_v(P_{ask}^{vo} - P_{ask}^{v}) & \\
\lambda_l\left[(V_{ln}^{vo} - V_{ln}) - (V_{hn}^{vo} - V_{hn})\right] &> \lambda_v(P_{ask}^{vo} - P_{ask}^{v}) - \lambda_e P_{ask}^{e} - (M - \Delta V_l)\psi_s \\
+(\lambda_v + \lambda_s)(\Delta V_h^{vo} - \Delta V_h) - \lambda_e\Delta V_h & \\
\lambda_l\left[(V_{ln}^{vo} - V_{ln}) - (V_{hn}^{vo} - V_{hn})\right] &> \lambda_e\left(\Delta V_h - P_{ask}^{e}\right) + \lambda_v(P_{ask}^{vo} - P_{ask}^{v}) \\
+(\lambda_v + \lambda_s)(\Delta V_h^{vo} - \Delta V_h) + (M - \Delta V_l)\psi_s &
\end{aligned}$$

Likewise, clients who are type lo prefer to purchase exclusively over the voice venue whenever,

$$\begin{aligned}
rV_{lo}^{vo} &> rV_{lo} \\
(C - \delta) + \lambda_l(V_{ho}^{vo} - V_{lo}^{vo}) + \lambda_v(V_{ln}^{vo} + P_{bid}^{vo} - V_{lo}^{vo}) &> (C - \delta) + \lambda_l(V_{ho} - V_{lo}) + \lambda_e(V_{ln} + P_{bid}^{e} - S_{hn}^{N} - V_{lo}) \\
+\lambda_s(V_{ln}^{vo} + M - V_{lo}^{vo}) + \lambda_v(V_{ln} + P_{bid}^{v} - V_{lo}) + \lambda_s(V_{ln} + M - V_{lo}) & \\
\lambda_l(V_{ho}^{vo} - V_{lo}^{vo}) - \lambda_v\Delta V_l^{vo} + \lambda_v P_{bid}^{vo} &> \lambda_l(V_{ho} - V_{lo}) - \lambda_v\Delta V_l + \lambda_v P_{bid}^{v} - \lambda_s\Delta V_l + \lambda_s M \\
-\lambda_s\Delta V_l^{vo} + \lambda_s M - \lambda_e\Delta V_l + \lambda_e P_{bid}^{e} - \lambda_e S_{hn}^{N} & \\
\lambda_l\left[(V_{ho}^{vo} - V_{lo}^{vo}) - (V_{ho} - V_{lo})\right] - (\lambda_v + \lambda_s)(\Delta V_l^{vo} - \Delta V_l) &> \lambda_e(P_{bid}^{e} - \Delta V_l) - (\Delta V_h - M)\psi_s \\
+\lambda_v(P_{bid}^{vo} - P_{bid}^{v}) & \\
\lambda_l\left[(V_{ho}^{vo} - V_{ho}) - (V_{lo}^{vo} - V_{lo})\right] &> \lambda_e P_{bid}^{e} - \lambda_v(P_{bid}^{vo} - P_{bid}^{v}) - (\Delta V_h - M)\psi_s \\
-(\lambda_v + \lambda_s)(\Delta V_l^{vo} - \Delta V_l) + \lambda_e\Delta V_l & \\
\lambda_l\left[(V_{ho}^{vo} - V_{ho}) - (V_{lo}^{vo} - V_{lo})\right] &> \lambda_e\left(P_{bid}^{e} - \Delta V_l\right) + \lambda_v(P_{bid}^{v} - P_{bid}^{vo}) \\
-(\lambda_v + \lambda_s)(\Delta V_l^{vo} - \Delta V_l) + (\Delta V_h - M)\psi_s &
\end{aligned}$$

Adding both inequalities,

$$\begin{aligned}
\lambda_l\left[\Delta V_l - \Delta V_h + \Delta V_h^{vo} - \Delta V_l^{vo}\right] &> \lambda_e\left(\Delta V_h - P_{ask}^{e} + P_{bid}^{e} - \Delta V_l\right) \\
+(\lambda_v + \lambda_s)(\Delta V_h^{vo} - \Delta V_h - \Delta V_l^{vo} + \Delta V_l) + \psi_s(\Delta V_h - \Delta V_l) + \lambda_v\left(P_{ask}^{vo} - P_{ask}^{v} + P_{bid}^{v} - P_{bid}^{vo}\right) & \\
(\lambda_v + \lambda_s + \lambda_l)\left[\Delta V_h^{vo} - \Delta V_l^{vo}\right] &> (\lambda_v + \lambda_s + \lambda_l + \lambda_e - \psi_s)\left[\Delta V_h - \Delta V_l\right] \\
&\quad - \lambda_e\left(P_{ask}^{e} - P_{bid}^{e}\right) + \lambda_v\left[(P_{ask}^{vo} - P_{bid}^{vo}) - (P_{ask}^{v} - P_{bid}^{v})\right] \\
(\lambda_v + \lambda_s + \lambda_l)\left[\Delta V_h^{vo} - \Delta V_l^{vo}\right] &> \lambda_v\left[(P_{ask}^{vo} - P_{bid}^{vo}) - (P_{ask}^{v} - P_{bid}^{v})\right] \\
-(\lambda_v + \lambda_s + \lambda_l + \lambda_e - \psi_s)\left[\Delta V_h - \Delta V_l\right] - \lambda_e\left(P_{ask}^{e} - P_{bid}^{e}\right) &
\end{aligned}$$

Noting that,

$$\begin{aligned}\Delta V_h^{vo} - \Delta V_l^{vo} &= \frac{(r+\psi)\delta}{(r+\psi_l-\lambda_l)(r+\psi_l+\lambda_l)} \\ &\quad - \frac{\lambda_l\delta}{(r+\psi_l-\lambda_l)(r+\psi_l+\lambda_l)} \\ &= \frac{\delta}{r+\psi_l+\lambda_l}\end{aligned}$$

and,

$$\begin{aligned}\Delta V_h - \Delta V_l &= \frac{(r+\psi)\delta}{(r+\psi+\psi_s-\lambda_l)(r+\psi-\psi_s+\lambda_l)} \\ &\quad - \frac{(\lambda_l-\psi_s)\delta}{(r+\psi+\psi_s-\lambda_l)(r+\psi-\psi_s+\lambda_l)} \\ &= \frac{\delta}{r+\psi-\psi_s+\lambda_l}\end{aligned}$$

We have,

$$\begin{aligned}(\lambda_v+\lambda_s+\lambda_l)\left[\Delta V_h^{vo} - \Delta V_l^{vo}\right] &> \lambda_v\left[(P_{ask}^{vo} - P_{bid}^{vo}) - (P_{ask}^{v} - P_{bid}^{v})\right] \\ -(\lambda_v+\lambda_s+\lambda_l+\lambda_e-\psi_s)\left[\Delta V_h - \Delta V_l\right] - \lambda_e\left(P_{ask}^{e} - P_{bid}^{e}\right) &\qquad \text{(A.45)} \\ \frac{(\lambda_v+\lambda_s+\lambda_l)\delta}{r+\psi_l+\lambda_l} &> \lambda_v\left(\frac{\theta\delta}{r+\psi_l+\lambda_l} - \frac{\theta\delta}{r+\psi-\psi_s+\lambda_l}\right) \\ -\frac{(\lambda_v+\lambda_s+\lambda_l+\lambda_e-\psi_s)\,\delta}{r+\psi-\psi_s+\lambda_l} - \lambda_e\left(\frac{\alpha\delta}{r+\psi-\psi_s+\lambda_l}\right) & \\ \frac{\delta[(\lambda_v+\lambda_s+\lambda_l)-\lambda_v\theta]}{r+\psi_l+\lambda_l} &> \frac{\delta\,(\lambda_v+\lambda_s+\lambda_l+\lambda_e-\psi_s-\lambda_v\theta-\lambda_e\alpha)}{r+\psi-\psi_s+\lambda_l} \\ \frac{[(\lambda_v+\lambda_s+\lambda_l)-\lambda_v\theta]}{r+\psi_l+\lambda_l} &> \frac{(\lambda_v+\lambda_s+\lambda_l+\lambda_e-\psi_s-\lambda_v\theta-\lambda_e\alpha)}{r+\psi-\psi_s+\lambda_l} \\ \frac{[\lambda_v(1-\theta)+\lambda_s+\lambda_l]}{r+2\lambda_l+\lambda_v(1-\theta)+\lambda_s} &> \frac{(\lambda_v+\lambda_s+\lambda_l+\lambda_e-\psi_s-\lambda_v\theta-\lambda_e\alpha)}{r+2\lambda_l+\lambda_v(1-\theta)+\lambda_s+\lambda_e(1-\alpha)-\psi_s} \qquad \text{(A.46)}\end{aligned}$$

When both sides are equal, clients are indifferent between trading exclusively on the voice venue and trading on both the voice and electronic venues. This equality implicitly defines a threshold market thickness μ^* under which there is no incentive for clients to participate on the voice venue.

Define,

$$K := \frac{[\lambda_v(1-\theta)+\lambda_s+\lambda_l]}{r+2\lambda_l+\lambda_v(1-\theta)+\lambda_s}$$

Then setting equation (A.46) to zero implicitly defines a value α^*,

$$
\begin{aligned}
K &= \frac{(\lambda_v + \lambda_s + \lambda_l + \lambda_e - \psi_s - \lambda_v\theta - \lambda_e\alpha^*)}{r + 2\lambda_l + \lambda_v(1-\theta) + \lambda_s + \lambda_e(1-\alpha^*) - \psi_s} \\
K(r + 2\lambda_l + \lambda_v(1-\theta) + \lambda_s + \lambda_e(1-\alpha^*) - \psi_s) &= (\lambda_v(1-\theta) + \lambda_s + \lambda_l + \lambda_e(1-\alpha^*) - \psi_s) \\
K\left[\lambda_e(1-\alpha^*) - \psi_s\right] - \left[\lambda_e(1-\alpha^*) - \psi_s\right] &= (\lambda_v(1-\theta) + \lambda_s + \lambda_l) - K(r + 2\lambda_l + \lambda_v(1-\theta) + \lambda_s) \\
K\left[\lambda_e(1-\alpha^*) - \psi_s\right] - \left[\lambda_e(1-\alpha^*) - \psi_s\right] &= (\lambda_v(1-\theta) + \lambda_s + \lambda_l) - (\lambda_v(1-\theta) + \lambda_s + \lambda_l) \\
(K-1)\left[\lambda_e(1-\alpha^*) - \psi_s\right] &= 0 \\
\lambda_e(1-\alpha^*) &= \psi_s \\
\lambda_e(1-\alpha^*) &= \frac{\lambda_s\lambda_e\lambda_l\theta}{(r+\lambda_l)(r+\lambda_l+\lambda_v+\lambda_s)} \\
(1-\alpha^*) &= \frac{\lambda_s\lambda_l\theta}{(r+\lambda_l)(r+\lambda_l+\lambda_v+\lambda_s)} \\
\alpha^* &= 1 - \frac{\lambda_s\lambda_l\theta}{(r+\lambda_l)(r+\lambda_l+\lambda_v+\lambda_s)}
\end{aligned} \tag{A.47}
$$

So clients will avoid the electronic venues whenever $\alpha > \alpha^*$. Define the following functions $\alpha(\mu)$ and $\pi(\mu)$,

$$
\begin{aligned}
\alpha(\mu; r, n, \phi) &:= \frac{n\mu(1-\pi(\mu))^{n\mu-1}}{1-(1-\pi(\mu))^{n\mu}} \\
\pi(\mu) &:= \frac{\phi}{\phi\mu + (1-\phi)(1-\mu)}
\end{aligned}
$$

Relation (A.47) can be re-written in terms of μ^* by taking the inverse of the function $\alpha(\mu)$,

$$
\mu^* = \alpha^{-1}(\alpha^*) \tag{A.48}
$$

Because α is decreasing in μ, clients will avoid the electronic venue whenever $\mu < \mu^*$, i.e., whenever market thickness is below a threshold value given by equation (A.48). The trading threshold is independent of the frequency of trade (λ_e) on the electronic venue.

Figure A.1 plots the market thickness threshold μ^* as a function of the clients trading frequency λ_l for different values of λ_s. The greater the relationship intensity, the higher the market thickness threshold as the penalty for defection is higher.

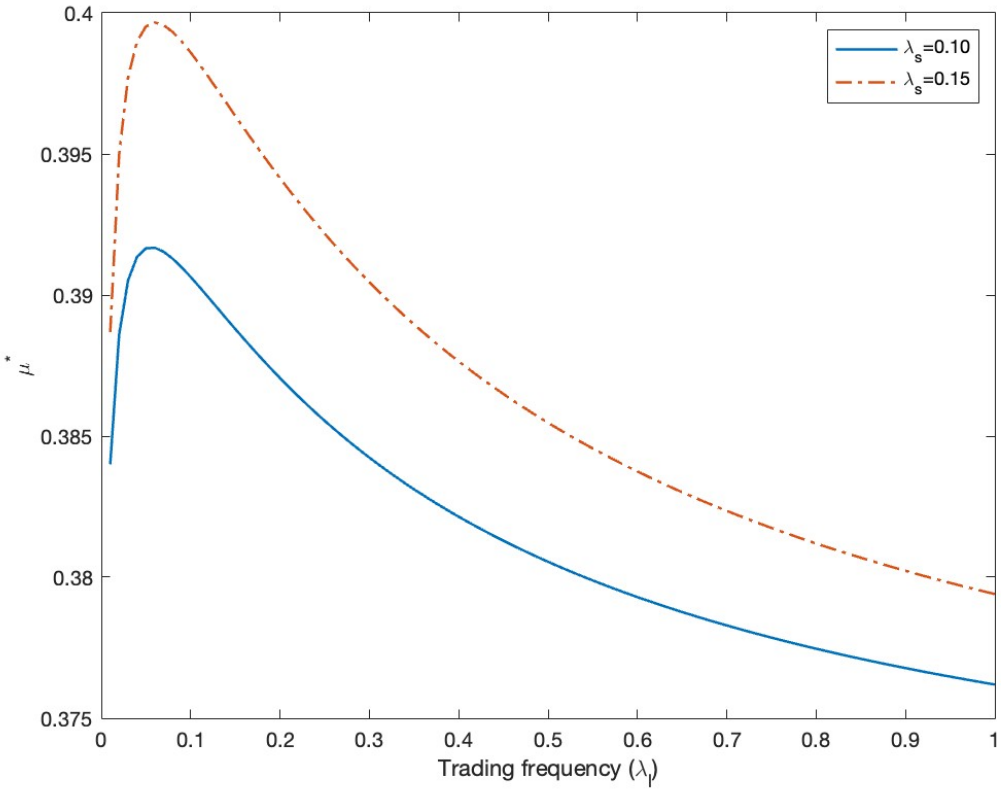


Figure A.1: **Market thickness thresholds and client trading frequencies**.
This figure depicts the market thickness threshold μ^* as a function of the client's trading frequency λ_l for different values of the dealer-client relationship intensity λ_s. All other parameters are set as: $r = 0.01, \theta = 0.85, \lambda_v = 0.20, m = 10, \phi = 0.55$.

Figure A.2 plots the value of this threshold for different parameter values. In regions below the line, clients avoid trading on the electronic venue. As shown, the voice-only region is larger, the larger the relationship intensity λ_s. This is intuitive: clients are less likely to trade on the voice venue the more they can trade at inter-dealer prices. The voice-only region initially expands as the frequency of liquidity shocks λ_l increases, but then shrinks as this frequency becomes larger and larger. The voice-only region also shrinks the larger the number of dealers. The voice only region also shrinks as the quality of the signal ϕ about dealers' propensity to participate on the electronic venue increases.

Counterintuitively, the voice only region expands, the greater dealer bargaining power. The reason is that for these clients, having intermittent access to the inter-dealer market through their relationship with the dealer implies a high trading relationship value. These clients are loath to jeopardize this relationship by putting their dealer in competition with other dealers in an auction on the electronic venue.

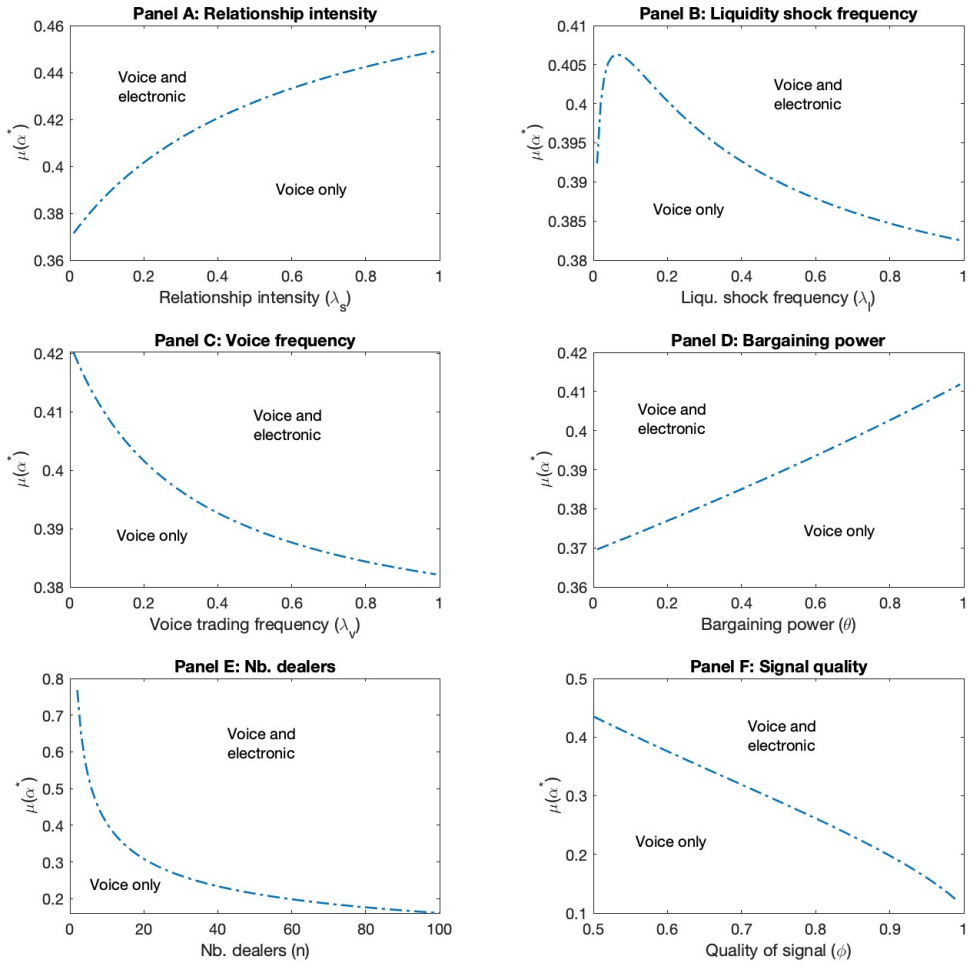


Figure A.2: **Market thickness thresholds.**
This figure reports the market thickness threshold μ^* above which clients will trade on both the electronic and voice venues. The baseline parameters correspond to $r = 0.01, \theta = 0.85, \lambda_l = 0.175, \lambda_s = 0.20, \lambda_v = 0.20, n = 10, \phi = 0.55$.

Impact of introduction of electronic trading on spreads in the voice venue

This section offers a proof of the corollary to Proposition 1. The proof is as follows. Note that,

$$
\begin{aligned}
\psi_\alpha &= \psi_v + \lambda_e(1-\alpha) \\
\psi &= \psi_l + \lambda_e(1-\alpha) \\
\psi_v &= \lambda_v(1-\theta) + \lambda_s \\
\psi_l &= \lambda_l + \psi_v \\
\psi_v &= \lambda_v(1-\theta) + \lambda_s \\
\psi_s &= \frac{\lambda_s \lambda_e \lambda_l \theta}{(r+\lambda_l)\,(r+\lambda_l+\lambda_v+\lambda_s)} \\
\psi_l &= \lambda_l + \psi_v
\end{aligned}
$$

And the bid-ask spread,

$$
P^v_{ask} - P^v_{bid} = \frac{\theta\delta}{r + 2\lambda_l + \lambda_v(1-\theta) + \lambda_s + \lambda_e(1-\alpha) - \psi_s}
$$

Comparing with the voice only venue,

$$
P^{vo}_{ask} - P^{vo}_{bid} = \frac{\theta\delta}{r + 2\lambda_l + \lambda_v(1-\theta) + \lambda_s}
$$

So the bid-ask spread on the voice venue narrows subsequent to the introduction of the electronic venue as long as,

$$
\begin{aligned}
\lambda_e(1-\alpha) - \psi_s &> 0 \\
\lambda_e(1-\alpha) &> \psi_s \\
1 - \frac{\psi_s}{\lambda_e} &> \alpha \\
1 - \frac{\lambda_l \lambda_s \theta}{(r+\lambda_l)\,(r+\lambda_l+\lambda_e+\lambda_v+\lambda_s)} &> \alpha
\end{aligned}
$$

Setting both sides equal implicitly defines threshold above which the half spread on the voice venue falls subsequent to the introduction of electronic trading with thickness $\mu > \mu(\tilde{\alpha})$, where

$$
\tilde{\alpha} := 1 - \frac{\lambda_l \lambda_s \theta}{(r+\lambda_l)\,(r+\lambda_l+\lambda_e+\lambda_v+\lambda_s)} \tag{A.49}
$$

That is, (A.49) can be re-written in terms of $\tilde{\mu}$ by taking the inverse of the function $\alpha(\mu)$,

$$
\tilde{\mu} = \mu(\tilde{\alpha}) = \alpha^{-1}(\tilde{\alpha}). \tag{A.50}
$$

Regression model with two venues

Define the half-spread as $H = 0.5(P_{ask} - P_{bid})$. Re-arranging equation (A.44),

$$S = \frac{2\lambda_l\lambda_s H}{(r+\lambda_l)(r+\lambda_l+\lambda_e+\lambda_v+\lambda_s)}$$

$$(r+\lambda_l)(r+\lambda_l+\lambda_e+\lambda_v+\lambda_s)\frac{S}{2\lambda_s\lambda_l} = H$$

$$H = \frac{S}{2\lambda_s\lambda_l}(r+\lambda_l)(r+\lambda_l+\lambda_e+\lambda_v+\lambda_s)$$

$$H = \left(\frac{S}{2\lambda_s\lambda_l}\right)(r+\lambda_l)(r+\lambda_l) + \left(\frac{S}{2\lambda_s\lambda_l}\right)(r+\lambda_l)\left(\lambda_e+\lambda_v+\lambda_s\right)$$

$$H = \left(\frac{S(r+\lambda_l)^2}{2\lambda_l}\right)\left(\frac{1}{\lambda_s}\right) + \left(\frac{S(r+\lambda_l)}{2\lambda_l}\right)\left(\frac{\lambda_e+\lambda_v}{\lambda_s}\right) + \left(\frac{S}{2\lambda_s\lambda_l}\right)(r+\lambda_l)\lambda_s$$

$$H = \left(\frac{S(r+\lambda_l)^2}{2\lambda_l}\right)\left(\frac{1}{\lambda_s}\right) + \left(\frac{S(r+\lambda_l)}{2\lambda_l}\right)\left(\frac{\lambda_e+\lambda_v}{\lambda_s}\right) + \left(\frac{S(r+\lambda_l)}{2\lambda_l\lambda_s}\right)\lambda_s$$

$$H = \left(\frac{S(r+\lambda_l)^2}{2\lambda_l\lambda_s}\right) + \left(\frac{S(r+\lambda_l)}{2\lambda_l\lambda_s}\right)\lambda_s + \left(\frac{S(r+\lambda_l)}{2\lambda_l\lambda_s}\right)\lambda_e + \left(\frac{S(r+\lambda_l)}{2\lambda_l\lambda_s}\right)\lambda_v$$

The regression counterpart to this relation is then,

$$H = a + b\lambda_s + c\lambda_e + d\lambda_v + u$$

where u is an error term that is independent of λ_s and the trading frequencies λ_e, λ_v.

Alternative trigger strategy

An alternative trigger strategy, would be for the client to be excluded from the inter-dealer market in perpetuity. For example, with such a grim trigger strategy,

$$V_{lo} = E_t\left[\int_t^{\tau}(C-\delta)e^{-r(u-t)}du + e^{-r(\tau_l-t)}V_{ho}\mathbf{1}_{(\tau=\tau_l)} + e^{-r(\tau_s-t)}(V_{ln}+M)\mathbf{1}_{(\tau=\tau_s)}\right] + E_t\left[e^{-r(\tau_e-t)}(V_{ln}^{grim}+P_{bid}^{e}-S_{hn})\mathbf{1}_{(\tau=\tau_e)} + e^{-r(\tau_v-t)}(V_{ln}+P_{bid}^{v})\mathbf{1}_{(\tau=\tau_v)}\right]$$

$$V_{ln} = E_t\left[e^{-r(\tau_l-t)}V_{hn}\right]$$

$$V_{ho} = E_t\left[\int_t^{\tau_l}Ce^{-r(u-t)}du + e^{-r(\tau_l-t)}V_{lo}\right]$$

$$V_{hn} = E_t\left[e^{-r(\tau_l-t)}V_{ln}\mathbf{1}_{(\tau=\tau_l)} + e^{-r(\tau_e-t)}(V_{ho}^{grim}-P_{ask}^{e})\mathbf{1}_{(\tau=\tau_e)}\right] + E_t\left[e^{-r(\tau_v-t)}(V_{ho}-P_{ask}^{v})\mathbf{1}_{(\tau=\tau_v)} + e^{-r(\tau_s-t)}(V_{ho}-M)\mathbf{1}_{(\tau=\tau_s)}\right]$$

where,

$$
\begin{aligned}
V_{lo}^{grim} &= E_t\left[\int_t^{\tau}(C-\delta)e^{-r(u-t)}du + e^{-r(\tau_l-t)}V_{ho}^{grim}\mathbf{1}_{(\tau=\tau_l)}\right] \\
&\quad + E_t\left[e^{-r(\tau_e-t)}(V_{ln}^{grim}+P_{bid}^{e}-S_{hn})\mathbf{1}_{(\tau=\tau_e)} + e^{-r(\tau_v-t)}(V_{ln}^{grim}+P_{bid}^{v})\mathbf{1}_{(\tau=\tau_v)}\right] \\
V_{ln}^{grim} &= E_t\left[e^{-r(\tau_l-t)}V_{hn}^{grim}\right] \\
V_{ho}^{grim} &= E_t\left[\int_t^{\tau_l}Ce^{-r(u-t)}du + e^{-r(\tau_l-t)}V_{lo}^{grim}\right] \\
V_{hn}^{grim} &= E_t\left[e^{-r(\tau_l-t)}V_{ln}^{grim}\mathbf{1}_{(\tau=\tau_l)} + e^{-r(\tau_e-t)}(V_{ho}^{grim}-P_{ask}^{e})\mathbf{1}_{(\tau=\tau_e)}\right] \\
&\quad + E_t\left[e^{-r(\tau_v-t)}(V_{ho}^{grim}-P_{ask}^{v})\mathbf{1}_{(\tau=\tau_v)}\right]
\end{aligned}
$$

However, such a strategy does not seem plausible nor enforceable, as it would require coordination in perpetuity among all of the dealers in the client's network.