The Emergence of a Platform Trap*

Andrei Hagiu† and Julian Wright‡

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Abstract

On platforms such as marketplaces and social networks, the existence of network effects can mean that not only do participating agents benefit when more agents join the platform, but agents' outside option gets worse. We show that in such a setting, by pricing dynamically, a monopoly platform can induce rational forward-looking agents to join even though participating agents ultimately end up worse off as a result. Agents face a dynamic collective action problem. We explore the limits of such a platform trap, considering factors such as whether agents can observe other agents' participation decisions and prices, whether the platform can price discriminate, and the relative bargaining power between the platform and individual agents.

Keywords: platforms, marketplaces, pricing, dynamics.

1 Introduction

Many businesses are growing more reliant on major platforms to reach customers, such as third-party sellers on Amazon, hotels on Booking.com and Expedia, and restaurants on DoorDash, Grubhub, and Uber Eats. As consumers shift toward shopping on these platforms instead of directly with sellers, concerns arise about the increasing dependence of sellers and the potential for their exploitation by the platforms. This can involve saddling sellers with higher or new types of fees, as well as an increasing requirement to advertise if they want to be discovered on the platforms they depend on. But why then would businesses join such platforms in the first place, knowing ultimately they will be held-up in this way?

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†Boston University Questrom School of Business, E-mail: ahagiu@bu.edu

‡Department of Economics, National University of Singapore, Singapore, E-mail: jwright@nus.edu.sg

In this paper we provide a general theory of why agents are induced to join a platform despite them ending up worse off as a result. We call this a platform trap. The theory applies to sellers participating on a marketplace which attracts consumers who previously searched directly for these sellers, but also to other settings with network effects: individuals participating on an online social network rather than relying on direct connections, and news organizations signing deals with Facebook and Google to provide their content which may lead these platforms to ultimately become consumers' preferred way to find information.

A key ingredient in our theory is that each agent's outside option is not constant, but is instead made worse as other agents participate on the platform, thereby making each agent more willing to join the platform. For example, in the case of sellers joining a marketplace, each seller's payoff from its outside option of selling directly (e.g. on its own website) can be negatively affected as more sellers list their products on the marketplace if this attracts more buyers to search on the platform rather than searching directly. As a result, agents face a collective action problem: each of them finds it individually attractive to join the platform but in doing so leads to a worsening of all agents' outside option. The platform exploits this by setting correspondingly higher prices, which can ultimately result in all agents being worse off. The problem is that while each individual agent takes into account the possibility that they may be better off not joining if that induces other agents to also not join, when they do join, they do not account for worsening the position of other agents who have not yet joined.

Our theory involves a finite number of agents each being made an offer and deciding sequentially whether to join the platform. The benefit each agent obtains from the platform is assumed to be some weakly increasing function of the number of agents participating on the platform, while the benefit each agent obtains from the outside option is some weakly decreasing function of the number of agents participating on the platform (and strictly decreasing at least at some point).

The sequential nature of the agents' participation decisions raises the possibility of an agent being pivotal—its participation decision affects whether subsequent agents can be profitably induced to participate. In this case, the platform needs to attract a critical mass of agents before it can ensure it offers more value for each subsequent agent than the outside option. Once a platform has reached this critical mass, each subsequent agent knows, regardless of what it decides, the platform will be able to induce all remaining agents to join by appropriately adjusting its prices, and so it is willing to join even when it is charged a price based on all other agents

joining.

In a baseline setting in which the platform has full pricing power and each agent can observe how many previous agents have joined the platform when making its decision, we provide a sufficient condition under which all agents are induced by the platform to participate and a platform trap arises. When no agents are pivotal, the platform extracts the maximum surplus from each agent in equilibrium by setting a single price that leaves each agent with the same low surplus as its outside option when all other agents participate. At the other extreme, when all agents are pivotal, provided the platform provides enough value, the platform can still attract all agents, but it does so with increasing prices, such that each agent is left with the outside option arising when only the agents joining before it participate. For intermediate cases, the platform sets increasing prices until the marginal agent joining is no longer pivotal, at which point it sets the same maximal price for all remaining agents as in the case without any pivotal agents.

In this baseline setting, whenever agents participate, all agents would be weakly better off and some strictly better off without the platform. In other words, they would be better off collectively committing to boycott the platform. However, when such a collective commitment is not possible, despite being forward-looking and rational, each agent participates because they realize that regardless of what they do, the platform will still induce all other agents to join with an appropriate adjustment in its prices. In short, agents are dammed if they do and dammed if they don't.

When payoffs are such that some or all agents are pivotal, the platform will sometimes need to subsidize the initial agents to induce them to participate. In case such subsidies are not possible, a similar result arises (agents would still be better off without the platform), but the conditions under which the platform can profitably attract agents (and so a platform trap arises) become tighter. We find that the condition for the platform to attract agents without subsidies is just that it is efficient for participating agents, i.e. the total surplus of the platform and participating agents is higher with the platform than without. By contrast, when the platform can use subsidies, it can attract agents even when it is inefficient.

After establishing the mechanism behind the platform trap, we explore the boundaries of when it arises: when would the platform make some or all agents better off? One key factor behind our platform trap is agents' ability to observe the participation decisions of agents that have decided before them. Once agents can no longer observe each others' participation decisions when deciding whether to join the platform, given that each agent's surplus from participating increases

in the number of other participating agents, there can be multiple equilibria. We characterize when the platform trap arises depending on equilibrium selection and depending on whether an agent can still observe the prices the platform has offered earlier agents. When prices are still observable and the equilibrium that is most favorable for the platform is selected, a platform trap emerges for a wider set of cases than in the full information baseline. This reflects that agents can no longer be pivotal when their decisions are not observed by other agents. On the other hand, in the equilibrium least favorable for the platform, the platform needs to run a divide-and-conquer strategy to overcome agents coordinating on an equilibrium in which no agents join. This involves the platform offering lower prices, thereby limiting the conditions under which a platform trap arises and resulting in some agents being better off because of the platform. However, in any such case, there are agents that participate even though they are worse off, implying a limited platform trap still emerges. When agents also don't observe prices, the favorable equilibrium still involves a maximum platform trap, but the platform can no longer run a divide-and-conquer strategy to undermine the most unfavorable equilibrium, and as a result it becomes possible that all agents will participate and will be better off due to the platform.

Another key factor driving our baseline results is the platform's ability to extract the full surplus that agents obtain from participating on the platform. This reflects that the platform makes take-it-or-leave-it offers and it can perfectly price discriminate (although sometimes it ends up setting a single price to all agents in equilibrium). If the platform is restricted to charge a single price to all agents, then whenever it still induces all agents to particiapte, it just leaves then indifferent between the platform existing or not. Without the ability to adjust prices based on what earlier agents decide, each agent becomes maximally pivotal and must be left with its surplus absent the platform. This highlights that a key part of the platform trap mechanism is the ability of the platform to adjust its price to future agents to induce them to participate if necessary. But this indifference result also suggests that once the platform can engage in some minimal level of price discrimination or there is some heterogeneity across agents, at least some agents will be worse off due to the platform and a platform trap re-emerges, something we show by way of a series of extensions.

Allowing agents to have some bargaining power in a setting in which the platform engages in bilateral bargaining with the sequence of agents can also lead some agents to be better off as a result of the platform. With linear externalities and focusing on

the case without any pivotal agents, we show that provided the agents' bargaining power is strong enough, all agents are better off with the platform, while provided the platform's bargaining power is strong enough, the reverse is true. For some intermediate range of bargaining power, agents that receive early offers are better off with the platform while those receiving later offers are worse off with the platform.

In the application to two-sided platforms (e.g. online marketplaces with buyers and sellers), there are potentially two different groups of agents who could benefit or be harmed by the platform. Throughout our main analysis, we focus on the benefit and harm to the strategic agents, with the decisions of any other non-strategic agents embedded in our payoff functions. In an Online Appendix we explicitly account for both buyers' and sellers' payoffs in a microfounded model of an online marketplace. We show how such a setting can map to our general payoffs provided buyers do not pay anything to the platform to participate and are not large enough to influence each other's decisions. We use this microfounded model to show how buyers, in addition to sellers, could be made worse off due to a platform trap.

How our theory applies to platform settings beyond marketplaces may not be obvious, and so we briefly outline two more applications here. The first is a social network. As more and more individuals among a certain demographic are active on a dominant social network, those in the demographic who are inactive on the social network (and instead rely on the outside option, e.g., direct connections) may experience social exclusion, fear of missing out, and therefore feel more compelled to join. In this example, the social network can extract surplus (via exposing users to advertising which it obtains revenue from) or offer subsidies (via free services for signed users). A second example is online news provision. Platforms like those offered by Facebook and Google provide alternative ways for users to access news (via snippets) rather than obtaining it directly from news providers. Signing agreements with such platforms for content distribution and revenue sharing could be a way for news organizations to obtain more exposure and obtain some additional revenue. But ultimately, if this leads more people to go to Facebook or Google to browse news rather than going to media sites directly, over time news providers will find they have little alternative but to distribute through Facebook and Google, and give up more of their advertising revenue to them.

2 Literature review

Our work is related to an old literature on network externalities, which shows that there can be inefficient adoption of standards. Most of the existing literature (e.g. Farrell and Saloner, 1985, and Katz and Shapiro, 1985) focuses on users being trapped in an inferior technology like the classic QWERTY keyboard example (excess intertia), while others (Farrell and Saloner, 1986, and Katz and Shapiro, 1986) study the possibility of excess momentum (the inefficient adoption of a new technology). From this literature, our mechanism is closest to that in Katz and Shapiro (1986). They study a market with technological change where there are two inherently incompatible (but symmetric) technologies subject to network externalities. They consider the possibility one or both technologies are sponsored by a firm that has property rights to the technology and hence is willing to make investments to promote it. In particular, they find when one of two rival technologies is sponsored, that technology has a strategic advantage and may be adopted even if it is less efficient. However, they do not consider whether all users are worse off as a result. As we show, the adoption of the platform can make all agents worse off even if it is efficient overall. Our model is also quite different from theirs: we allow for general and different benefit functions associated with the platform vs. the outside option, whereas in their model the two technologies offer the same benefit function to users but differ in terms of their per-user costs. Furthermore, in our model individual agents can be strategic (pivotal), a possibility which is not explored in Katz and Shapiro (1986).

Given network effects exist in our setting, our paper relates to previous work on platforms with network effects in dynamic contexts. Most of the related work involves some kind of lock-in of participants. For example, this can be the result of switching costs (Fudenberg and Tirole, 2000), an installed-base effect (Mitchell and Skrzypacz, 2006) or the selection among multiple participation equilibria to be influenced by participants' past choices (Halaburda et al., 2020). Our main focus is on settings without any such frictions. Also, in our baseline model, the platform approaches agents sequentially, and each agent observes the participation decisions of previous agents, so there is always a unique adoption equilibrium.

In the context of exclusive dealing, Segal and Whinston (2000) look at a setting which has some overlap with ours, particularly their analysis in which an incumbent firm makes sequential exclusive offers. By signing enough buyers, the incumbent is able to foreclose an entrant, which means even non-signing buyers would be worse

off. There are two key differences. One is that the decrease in the buyers' outside option only arises at the critical point where signing one more buyer denies the entrant the scale to profitably enter. In contrast, we allow both the benefit from signing and the value of the outside option to vary in more general ways, with each additional agent signing. The other is that the incumbent (which corrresponds to our platform) obtains a payoff from non-signing buyers as well as signing buyers once enough buyers have signed (its monopoly profit) whereas in our setting the platform only obtains payoffs from participating agents. As a result, Segal and Whinston obtain quite different characterizations from ours, even though the role of pivotal buyers is similar.

Segal (1999) provides a very general framework of a principal that contracts with multiple agents where there can be positive or negative externalities across agents. It covers a variety of applications, including those in Katz and Shapiro (1986) and Segal and Whinston (2000) discussed above. Segal (2003) extends this framework to look at the case with and without discrimination across agents, and in case there are multiple equilibria in agents decisions, the best and worst equilibria for the principal. One important difference between these papers and ours is that Segal assumes the principal makes its bilateral offers first and agents then decide whether to accept simultaneously, rather than the sequential structure of offers and decisions we focus on. More fundamentally, like the rest of the literature, he focuses on the possible inefficiencies arising from these externalities rather than whether agents would be better off without the principal.

The idea of collective traps has recently been explored in Bursztyn et al. (2024). They provide experimental evidence showing that total consumer welfare from the availability of social media platforms Instagram and TikTok is negative among their student sample, even though these users want to use the platforms. They propose a mechanism to explain these results in which users impose a negative externality on non-users (e.g. via social exclusion or a fear of missing out). In particular, as the total number of participating individuals increases, marginal users may participate to avoid the even worse outcomes arising from non-usage even though they still get negative overall utility from the platform. This underlying mechanism is consistent with our own, in which the value of the outside option decreases with increased participation of users on the platform. However, while Bursztyn et al. offer a conceptual framework to interpret their results, they don't provide a theory in which agents make individual choices to show how (or under what conditions) a collective trap emerges. They also do not consider how a platform can influence agents' choices

to induce such a trap, e.g. through dynamic pricing decisions. This reflects their different focus, which is on empirically demonstrating and measuring the size of the collective trap rather than providing a theory of such traps.

The literature on price parity clauses (platforms forbidding sellers from charging lower prices in their direct channels than the prices charged on the platform) also has the feature that the direct channel can be made less attractive because of the existence of the platform (Edelman and Wright, 2015; Wang and Wright, 2020; Gomes and Mantovani, 2024). This could result in consumers ending up worse off due to the existence of the platform. However, the mechanism is more direct. It works via the platform's fee directly raising the price in both channels because of the price parity clause, leading consumers to continue to use the platform despite prices being higher overall. Edelman and Wright (2015) highlight that there can be excessive participation of consumers on the platform in such a setting. However, all these papers are based on static models and work via the price parity clause which we do not require. Moreover, they do not focus on whether sellers are better or worse off due to the platform.

Finally, there is a literature that captures the idea that platforms intensify competition between sellers (e.g. Baye and Morgan, 2001; Galeotti and Moraga-González, 2009; Wang and Wright, 2020, 2024; Gomes and Mantovani, 2024; and Hagiu and Wright, 2024) compared to a direct channel alternative. In such settings, sellers could be collectively better off without the platform because the platform lowers their margins due to intensifying competition between them. Our mechanism does not rely on the commodization of sellers that underpins the harm to sellers arising in these settings. Moreover, these works close down any kind of dynamic decision making to explain the emergence of an equilibrium where sellers are worse off, which is one of our key contributions.

3 Baseline model

There is a single platform and many agents, some of whom can decide whether to participate on the platform or not. All agents have access to a non-strategic alternative, which we call the outside option (e.g. it could be a direct channel in the case of a marketplace, or an existing competitively provided product or network that is priced at cost). In our baseline model there is just one period (i.e. payoffs are only realized once) with agents making sequential participation decisions as defined below.

There are $N \geq 2$ active agents that can consider whether to participate on the platform. There may also be $N_0 \geq 0$ inactive agents who cannot participate on the platform, perhaps because they are not informed about it or because they would never consider joining the platform. Inactive agents just obtain the outside option.

We allow that the platform may exhibit network effects. We assume that the benefit to an (active) agent of participating on the platform when n total agents participate is written as $b(n)$, where the function $b(.)$ is always positive and is weakly increasing in its argument. Meanwhile, the payoff obtained by an agent that does not participate on the platform (i.e. its outside option) when n other agents participate on the platform is written as $u(n)$, where $u(.)$ is non-negative, weakly decreasing in its argument, and strictly decreasing over the full set of other agents participating, i.e. such that $u(N-1) < u(0)$. Note these assumptions imply $u(0) > 0$.

The platform makes an offer to each agent in some sequence. Since agents are symmetric in our baseline model, the order does not matter, and so can be exogenously determined or chosen by the platform.[1] An offer to agent $k \in \{1, .., N\}$ consists of a fixed price (or fee) P^k that the agent must pay in order to participate. There is no discounting. Thus, the payoff to agent k is

$$b(n+1) - P^k$$

from participating on the platform and is

$$u(n)$$

when it does not participate on the platform, in each case when $n \leq N$ other agents end up participating on the platform. Each agent makes it decision in the sequence it is offered, and payoffs are realized after all N agents have made their participation decisions.

We make two key assumptions for our baseline setting, both of which will be relaxed later: (i) each agent k can observe the decisions (participate or not) of the $k-1$ agents that were made offers prior to its decision[2], and (ii) the platform makes take-it-or-leave-it offers to each agent (these offers can differ across agents).

[1] The order will matter in one of our extensions, and in that case, we assume the platform chooses the order.

[2] Agent k can also observe the prices offered to these $k-1$ agents, but the only thing that we really need in this baseline model is that it can observe the number of prior agents that have chosen to participate.

Formally, this is a dynamic game of perfect information consisting of N stages, such that at each stage the platform offers an agent that hasn't already been selected the option to participate for a fixed price, and observing this offer and the full history of moves to this point, the agent chooses whether to participate or not. Payoffs are only realized at the end of these N stages.

We define the surplus function

$$\Delta(n+1) \equiv b(n+1) - u(n),$$

which given our assumptions is weakly increasing in n, with $\Delta(n+1) > \Delta(n)$ for some $1 \leq n \leq N-1$. This implies the propensity of the marginal agent to participate on the platform is weakly increasing in the number of agents that are participating, and strictly increasing at some point. We assume $\Delta(N) > 0$, so that each agent obtains a positive gross surplus from participating on the platform (i.e. before considering any price they pay) when all other agents participate. Without this, then it would not be possible for the platform to profitably attract any agents.[3]

3.1 Discussion

In some interpretations of our model, the agents who participate on the platform may still obtain the payoff from the outside option. In such cases, the function $b(.)$ encompasses any payoff agents obtain from doing so (e.g. if the platform is a marketplace, agents are sellers and the outside option is a direct channel, sellers that list on the platform may still sell in the direct channel). Moreover, the functions $b(n)$ and $u(n)$ could depend on N_0, the number of non-active agents, but since this is fixed, we do not write it explicitly as an argument in these functions. Indeed, in what follows, when we refer to "agents", we will be referring to only "active agents" since these are the only agents making decisions. In case we need to refer to inactive agents, we will do so explicitly.

There are two ways to interpret our single period model. Our preferred interpretation is that it captures just one of many (either finite or infinite) periods over which the game is repeated and over which payoffs are realized. In this interpretation, agents are free to change their participation decision each period (if they joined in a prior period, they still decide whether to continue participating in each subse-

[3]Even if the platform successfully attracted the first $N-1$ agents to participate, it would not be able to charge a positive price to the last agent, given the last agent would get $b(N) - P$ from participating and $u(N-1)$ from not participating. And since $\Delta(n)$ is increasing in n, repeating the same logic implies the platform will not attract any earlier agent either.

quent period), and there is discounting across periods. The key assumption then is that any long-lived agents remain free to switch their participation decision across periods. Since there is nothing that links one period to another, the equilibrium analysis can be done separately for any given period as we do in Section 4 below. Price dynamics would arise in a multi-period setting if there are new agents each period or more agents become active rather than inactive over time. These changes, together with any exogenous changes in payoffs, can be handled in our single-period setting as straightforward comparative static exercises.

An alternative interpretation is that the N stages we consider during which offers and participation decisions are made captures the full dynamics of the situation, with agents' participation decisions being irreversible. Payoffs are then realized over (possibly infinitely) many subsequent periods, the present discounted value of which is summarized in the single-period payoffs $b(n)$ and $u(n)$. The lack of discounting across different agents' decisions is an approximation to reflect the relatively short time frame during which the participation decisions are made compared to the long time frame over which payoffs would be realized.

The baseline setting is one where the platform can set a different price to each agent, including negative prices if necessary. We relax this in 5.2. Negative prices could alternatively be interpreted as the platform offering its services for free and giving agents additional fixed benefits for participation. Similarly, positive prices could be interpreted instead as the platform exposing agents to advertising (as would be relevant in the case of social media platforms). The baseline setting also assumes the platform cannot commit to its prices. As we will see, commitment does not help the platform except under very specific payoff structures, and then only if it can commit to contingent prices which involve offering negative prices (subsidies) to some agents in certain subgames. We also explore this possibility in Section 5.2.

4 Analysis of the benchmark model

To provide some intuition for how the equilibrium is determined in this model, it is helpful to consider two extreme cases first. The first is a setting where no agent can be pivotal and the second is where every agent (except the last) is pivotal. An agent is pivotal, if it is necessary to attract that agent in order to profitably induce subsequent agents to participate. To illustrate, suppose there are two agents ($N = 2$) and the platform needs both to join so it can offer positive value to each relative to the outside option (i.e. $\Delta(2) > 0$ and $\Delta(1) < 0$). Then the first agent to receive

the offer is pivotal. If it doesn't join, there is no way for the platform to profitably attract the other agent, whereas if the first agent joins, the platform can always profitably attract the second agent as well. On the other hand, under the same assumptions, if there are three agents to start with, then the first agent to receive the offer is no longer pivotal since it is still possible for the platform to profitably attract the remaining two agents if the first agent rejects its offer. However, if the first agent rejects its offer, we are in the situation equivalent to there being just two agents left, and the first of these to decide (the second agent) would become pivotal.

To consider the first extreme where no agent is pivotal, we assume

$$\Delta(1) \geq 0,$$

which implies $\Delta(n) \geq 0$ for all $n \geq 1$. This means the platform always creates more value for an individual agent than the outside option regardless of how many other agents are expected to participate, and as such, the platform can always profitably induce any agent to participate. Note this assumption would trivially hold in a standard setting where the outside option had payoff normalized to zero.

Since the last agent to decide can be induced to participate if no other agents before it have participated, and since there is more surplus to extract the more other previous agents have participated (i.e. $\Delta(n)$ is weakly increasing everywhere and sometimes strictly increasing), each agent expects that all other agents that follow it will participate. As a result, if some number $N' < N$ of prior agents have decided to participate, the k-th agent to decide will participate on the platform if and only if

$$P \leq P(N') = \Delta(N' + N - k + 1).$$

If the platform sets any higher price, then this agent would not participate and the platform would get no revenue from this agent (and it would also get less revenue from all subsequent agents that participate). With the price $P = P(N')$ at each stage, all N agents end up participating on the platform, so $N' = k - 1$ on the equilibrium path, and the platform obtains all agents to participate at the common price for all agents:

$$P^* = \Delta(N).$$

This is the same platform price that would arise if all agents decided simultaneously, and agents coordinate on the equilibrium most favorable for the platform (in which all agents participate whenever that is an equilibrium). Since each agent ex-

pects the remaining $N-1$ agents to participate in such an equilibrium, they expect to get $u(N-1)$ if they don't participate, while if they participate they expect to get $b(N)-P$. The price $P=\Delta(N)$, just ensures each agent is indifferent between participating and not, when they expect all others to participate.

Now we contrast this outcome to the one where every agent (except the last) is pivotal. Specifically, assume

$$\Delta(N-1)=b(N-1)-u(N-2)<0.$$

This means failure to attract one agent out of the available N agents would make it impossible for the platform to profitably attract any of the remaining $N-1$ agents (because even if each of the remaining agent assumes all other agents will participate, they would be better off not participating). Thus, every agent (except the last) is pivotal. The platform has to offer each agent in the sequence at least its outside option when no subsequent agents participate, because the agent can always obtain its current outside option by not participating (which induces all remaining agents to not participate). Thus, even though all agents are induced to participate in equilibrium, the price that just induces them to participate increases from the first agent, which faces the lowest price $P^1=b(N)-u(0)$, up till the last agent deciding, which faces the highest price $P^N=b(N)-u(N-1)=\Delta(N)$. This is profitable for the platform provided $b(N)\geq\frac{1}{N}\sum_{k=1}^{N}u(k)$, which is the condition (1) in the Proposition below. Otherwise, the platform will set any $P^k\geq 0$ and will not induce any participation.

In between these two extremes, i.e. when $\Delta(1)<0<\Delta(N-1)$, some (but not all) agents may be pivotal. Specifically, in the proof of the following proposition we show that if some agents are pivotal, then there exists some $k_0\geq 1$, such that the first $k\leq k_0$ agents are pivotal and will be charged prices $P^k=b(N)-u(k-1)$. Once the platform has attracted them, none of the remaining $N-k_0$ agents $k>k_0$ are pivotal, so they will all be charged $P^k=b(N)-u(N-1)=\Delta(N)$. These prices are consistent with the prices for the extreme cases discussed above. One can interpret a scenario with $1\leq k_0\leq N-1$ pivotal agents as one in which the platform must first attract a critical mass k_0 of agents before it can ensure it offers more value for each subsequent agent than the outside option (regardless of whether any subsequent agents participate).[4]

Proposition 1. *If no agents are pivotal, then the platform can always profitably*

[4]The proof, as with all proofs that are not covered in the text, is relegated to the Appendix.

attract all N agents with optimal prices $P^k = \Delta(N) > 0$ for all $k \in \{1, ..., N\}$. If $k_0 \in \{1, .., N-1\}$ agents are pivotal, then the platform's optimal prices are

$$P^k = \begin{cases} b(N) - u(k-1) & \text{if} \quad 1 \leq k \leq k_0 \\ b(N) - u(N-1) & \text{if} \quad k_0 + 1 \leq k \leq N \end{cases},$$

provided that $\sum_{k=1}^{N} P^k \geq 0$, in which case all N agents participate. If instead $\sum_{k=1}^{N} P^k < 0$, then the platform cannot profitably attract any agents to participate. In all cases where agents participate, they would all be weakly better off without the platform, with at least some agents strictly better off without the platform. A sufficient condition for all agents to participate is

$$b(N) \geq \frac{1}{N} \sum_{k=1}^{N} u(k-1). \tag{1}$$

We refer to the scenario in which agents participate despite being better off without the platform as a platform trap. A platform trap arises because when an individual agent is induced to participate on the platform, the value of all agents' outside option becomes less valuable, which makes all other agents even more likely to want to participate on the platform. Thus, the platform is able to exploit a collective action problem among agents. As Proposition 1 notes, when a platform trap does arise, at least some agents would be strictly better off without the platform (this includes, always, the last agent to join), with the rest weakly better off without the platform. In case $u(1) < u(0)$, this result would be stronger—all agents would be strictly better off without the platform (except the first agent in case it is pivotal).

Note that (1) is only a sufficient condition for a platform trap to arise. The complete set of necessary and sufficient conditions (in terms of model fundamentals) for the platform to be able to profitably attract agents (and so a platform trap to arise) involves many possible cases, the number of which increases with N. Nevertheless, our characterization shows all possible prices for all cases in which agents participate, which is also sufficient for drawing conclusions about a platform trap. And the two corollaries below fully spell out necessary and sufficient conditions for the platform to attract agents and so a platform trap to emerge in the two extreme cases discussed above.

Given the possible prices laid out in Proposition 1, the expected payoff of an agent that participates on the platform is either $u(N-1)$ if it is not pivotal, or is $u(k-1)$ if it is pivotal. Since $u(N-1) < u(0)$, at the very least, the last agent to

participate will be strictly better off without the platform (or equivalently, would be strictly better off if the agents collectively boycotted the platform).[5] Earlier agents to participate would also be better off without the platform to the extent $u(k-1) < u(0)$. In case, $u(1) < u(0)$, the only possible exception is when the agent in question is the first pivotal agent, since then $k = 1$ and $u(k-1) = u(0)$, and this agent would be indifferent. Since inactive agents are always strictly worse off whenever some agents participate on the platform (they end up getting $u(N)$ rather than $u(0)$), for brevity we do not state this result in our propositions.

The sufficient condition (1) in Proposition 1 is sometimes consistent with the platform attracting all agents even though $b(N) < u(0)$, i.e. even though the platform lowers welfare. In this case, not only are agents worse off as a result of the platform existing, but so is total welfare after accounting for the platform's profit.[6] Without the externality on the outside option, it would not be possible for the platform to attract participants when it is an inefficient equilibrium. This is a key novelty. It is different from the earlier network effects literature in which it is possible for participants to coordinate on an inefficient platform despite the lack of any such externality on the outside option.

We now provide the full characterization of the conditions under which the platform will profitably attract all agents for the two extreme cases discussed earlier.[7]

Corollary 1. *If $b(N-1) - u(0) \geq 0$, then the platform attracts all N agents and charges $P^k = \Delta(N) > 0$ for all $k \in \{1, .., N\}$. All agents would be strictly better off without the platform.*

Corollary 2. *If $b(N-1) - u(N-2) < 0$, then the platform attracts all N agents if and only (1) holds. If these conditions hold, the platform charges $P^k = b(N) - u(k-1)$ to agent $k \in \{1, .., N\}$ and all agents would be weakly better off without the platform, with at least some agents strictly better off without the platform.*

Corollary 1 characterizes the equilibrium prices when no agent is pivotal. Whenever $N > 2$, the relevant condition is weaker than the condition noted in the discussion above of $\Delta(1) \geq 0$. This is because provided $b(N-1) - u(0) \geq 0$, the

[5] Implicit in this approach is the assumption that the payoff each agent obtains without the platform is $u(0)$. To the extent the payoff each agent obtains without the platform actually differs from this (e.g. because the platform's existence affects the u function), then these two statements are not equivalent, and strictly speaking our results (which are based on a comparison with $u(0)$) measure whether agents are better off under a collective boycott of the platform.

[6] This is true even when not accounting for the negative effect on the N_0 non-active agents, which is an additional inefficiency associated with the platform attracting participants in this setting.

[7] Corollary 1 was proved in the text before Proposition 1.

possibility of pivotal agents only arises off equilibrium. Even if one agent does not participate, there are still enough remaining agents such that the platform can induce them to join. On the other hand, Corollary 2 characterizes the equilibrium prices when all agents are pivotal.

Proposition 1 sometimes relies on the platform setting negative prices to pivotal agents. In practice this could involve the platform providing its platform for free but providing additional benefits to the agent (that are costly for the platform to provide) to induce them to join. If we rule out negative price or any such additional benefits being offered, we can fully characterize the conditions under which each of the three cases (no pivotal agents, all pivotal agents, or some pivotal agents) arise.

Proposition 2. *If the platform cannot subsidize agents, it will induce all N agents to participate if*

$$b\left(N\right)-u\left(0\right)\geq 0, \tag{2}$$

and otherwise it attracts no agents. When (2) holds, there are three cases:

- *if $b(N-1)-u(0)\geq 0$, then no agent is pivotal and each agent $k\in\{1,...,N\}$ is charged the same price*
$$P^{k}=\Delta(N).$$

- *if $b(N-1)-u(N-2)<0$, then all agents are pivotal and the k-th agent to decide is charged*
$$P^{k}=b(N)-u(k-1).$$

- *if $b(N-1)-u(0)<0\leq b(N-1)-u(N-2)$, then there exists a unique $k_0\in\{1,..,N-2\}$, determined by $b\left(N-1\right)-u\left(k_0-1\right)<0\leq b\left(N-1\right)-u\left(k_0\right)$, such that agents $k\in\{1,...,k_0\}$ are pivotal and charged*
$$P^{k}=b(N)-u(k-1),$$
and agents $k\in\{k_0+1,...,N\}$ are charged
$$P^{k}=\Delta(N).$$

In all cases above, all agents would be weakly better off without the platform, with at least some agents strictly better off without the platform.

Assuming $b\left(N-1\right)-u\left(N-2\right)<0$, if the platform cannot subsidize agents,

the condition for the platform to attract agents (2) is now stronger than the corresponding condition (1) in Corollary 2. The condition requires that the platform creates more value for each participating agent than the outside option without the platform. In other words, without subsidies, the platform is able to attract all agents if and only if it is efficient for all agents to participate on the platform (from the perspective of the platform and those agents) rather than none doing so. This contrasts with the result in Corollary 2 (and Proposition 1 more generally) where we noted that the platform may be able to attract all agents even when it is inefficient to do so.

5 Extensions

In our baseline setting, we made a number of assumptions to obtain our main platform trap result. These included that each agent has full information (on the prices charged and decisions made by prior agents) and that the platform can make a different take-it-or-leave it offer to every agent. In the following subsections, we reexamine the logic of platform traps when these assumptions are relaxed.

5.1 Private information

In the baseline setting, we assumed full information so each agent can see everything that happened prior to it making its decision: which agents have been made offers, what those offers were and their decisions. In this subsection we relax this assumption. We consider two possibilities. We first assume each agent cannot observe the decisions of prior agents, but it can still observe which prior agents have been made offers and what those prior offers (i.e. prices) were. We then consider what happens when agents cannot observe anything about prior offers or agents' decisions.

Once agents cannot see each others' decisions, given that each agent's surplus from participating increases in the number of other participating agents, there can be multiple equilibria. In the following proposition, we characterize the best and the worst equilibrium for the platform.

Proposition 3. *Suppose each agent can observe the platform's prior price offers but not the decisions of the respective agents.*

1. *The best equilibrium for the platform involves the platform charging each agent the same price $P = \Delta(N)$, with all N agents participating, resulting in total*

platform profits $N\Delta(N)$*. All agents would be strictly better off without the platform.*

2. *The worst equilibrium for the platform involves the platform charging prices* $P^k = \Delta(k)$ *to the* k*-th agent (in the sequence of* N *agents), with all* N *agents participating, resulting in platform profits* $\sum_{k=1}^{N} \Delta(k)$ *provided either* $\Delta(1) \geq 0$ *if subsidies cannot be offered or* $\sum_{k=1}^{N} \Delta(k) \geq 0$ *if subsidies can be offered. In this case, there exists* $1 < \widehat{k} < N$ *such that agents* $k \leq \widehat{k}$ *would be (weakly) worse off without the platform, while agents* $k > \widehat{k}$ *would be strictly better off without the platform.*

The proposition characterizes the two most extreme equilibria that can arise from the platform's perspective when agents can observe the platform's prior price offers but not the decisions of prior agents. Equilibrium selection thus determines whether the platform trap applies even more widely than in the baseline setting or is more limited.

To understand why the platform trap may apply more widely, note agents can no longer be pivotal because their decision can't influence those of other agents. This is why the best equilibrium for the platform may be even better for it than in the case where agents' decisions are observable. This means the platform trap now arises in some situations where previously it did not. In particular, we don't need the extra conditions in Proposition 1 when subsidies are possible or in Proposition 2 when subsidies are not possible for the platform trap to arise. Moreover, even when such conditions hold so the platform can also induce agents to participate with observable decisions, agents are weakly worse off with unobserved decisions whenever prices are lower than $P = \Delta(N)$ in Proposition 1 (with subsidies) or in Proposition 2 (without subsidies).

To understand why the platform trap may be more limited with private information, note in case agents coordinate on the most unfavorable equilibrium for the platform, the platform needs to run a divide-and-conquer strategy to overcome agents coordinating on an equilibrium in which no agents join.[8] The highest price the platform can charge the first agent is $P^1 = b(1) - u(0) = \Delta(1)$, so agent 1 is willing to join no matter what it believes other subsequent agents will do. The platform is then able to increase the price offered the second agent to $P^2 = b(2) - u(1) = \Delta(2)$,

[8]The use of a divide-and-conquer strategy by a firm to overcome agents coordinating on an equilibrium that is bad for it has been explored in a number of previous works, such as Caillaud and Jullien (2003) and Segal (2003).

because agent 2 can infer from P^1 that agent 1 must have joined, so agent 2 joins as well. And so on. These prices are (weakly) lower than the lowest prices that the platform can ever charge in the baseline setting ($P^k = b(N) - u(k-1)$), and they may in fact be so low here that early agents benefit from the platform's existence: this is true for agent k whenever $b(N) - b(k) + u(k-1) > u(0)$. However, note the last agent is always worse off because $u(N-1) < u(0)$. Moreover, for this divide-and-conquer strategy to be feasible, the sum of all these prices must be non-negative. This may require offering subsidies for some of the initial agents when $\Delta(1) < 0$. If subsidies are not possible and $\Delta(1) < 0$, then the platform can no longer use a divide-and-conquer strategy, and there is an equilibrium where the platform cannot induce any agent to participate even if it charges nothing.

We next turn to the case where the agents' prior offers are also not observed.

Proposition 4. *Suppose each agent cannot observe anything but their own offer.*

1. *The best equilibrium for the platform involves the platform charging each agent the same price $P = \Delta(N)$, with all N agents participating, resulting in total platform profits $N\Delta(N)$. All agents would be strictly better off without the platform.*

2. *The worst equilibrium for the platform involves the platform charging each agent the same price $P = \Delta(1)$, with all N agents participating, resulting in platform profits $N\Delta(1)$ provided $\Delta(1) \geq 0$. In this case, all active agents would be weakly worse off without the platform. Otherwise the platform attracts no agents, in which case agents are indifferent between the platform existing or not.*

Once agents cannot observe the prices or decisions of other agents, the platform can no longer run a divide-and-conquer strategy. Without information on any other agents' prices or decisions, this case becomes equivalent to agents making simultaneous decisions. Moreover, there is no advantage of the platform ever setting different prices given they are not observable, which explains why the equilibrium prices are uniform in contrast to the result in Proposition 3. Indeed, it is straightforward to confirm the same extreme outcomes characterized in Proposition 4 also remain equilibria when the platform has to set a single price to all agents and all agents decide simultaneously whether to participate or not. Finally, it is worth noting that even in the worst equilibrium for the platform, provided $\Delta(1) \geq 0$ so the N agents

participate and are weakly better off for doing so, the N_0 non-active agents would still be better off without the platform.

5.2 Different pricing assumptions

Returning to the full information setting, we first consider what happens when the platform can no longer perfectly price discriminate across agents. We then consider the case it can set different prices to each agent but has to negotiate these prices with each agent rather than being able to set take-it-or-leave-it prices.

5.2.1 Absence of perfect price discrimination

Instead of perfect price discrimination, consider the opposite extreme in which the platform is restricted to make the same take-it-or-leave-it offer to all agents; i.e., specifically, whatever price it offers the first agent must also be offered to all subsequent agents. Given any uniform price must be positive for the platform to make a profit, we no longer have the possibility of the platform offering subsidies.

Proposition 5. *If the platform is restricted to charge a single price to all agents, then it will induce all N agents to participate with the price $P = b(N) - u(0)$ if $b(N) \geq u(0)$. Otherwise the platform attracts no agents. Agents who participate on the platform are indifferent between the platform existing or not.*

Although the condition for the platform to induce all N agents to participate is the same as with price discrimination and no subsidies, i.e. (2), the uniform price charged is (weakly) lower than in that case, and as a result, agents' payoffs are (weakly) higher. The platform trap disappears when the platform can only set a single fixed price that it charges to all agents. The only agents that are worse off due to the platform are the non-active agents; the active agents are indifferent about whether the platform exists or not. This reflects that each agent knows the platform cannot adjust its price to subsequent agents based on their decision, and so they become maximally pivotal (as if they alone can decide whether others will participate). The result highlights that a key part of the platform trap mechanism is the platform's ability to adjust its price to future agents if necessary (i.e. when agents deviate from the equilibrium path) in order to induce them to participate.

The fact that active agents are indifferent between the platform existing or not when the platform cannot price discriminate suggests that once we allow for some heterogeneity across agents or some limited amount of price discrimination, some

agents may again be better off without the platform. To illustrate the first possibility, suppose a set of N_z agents have an outside option that always gives them zero payoff (e.g. in the case of a marketplace, this could be sellers that do not have a direct channel, while the remaining $N - N_z$ agents are exactly as in the baseline model. In the appendix we prove the following result.

Proposition 6. *If N_z is such that $b(N) - u(N_z) \geq \frac{N_z}{N} b(N_z)$, then the platform sets $P = b(N) - u(N_z)$ and gets all agents of both types to join. The $N - N_z$ agents would be weakly better off without the platform, while the N_z agents would be weakly worse off without the platform. Alternatively, if $b(N) - u(N_z) < \frac{N_z}{N} b(N_z)$, then the optimal solution is to set $P = b(N_z)$ and only get N_z agents to join, all of whom are indifferent between the platform existing or not.*

If the first condition in the proposition holds for some $1 \leq N_z < N$, so the platform attracts all agents, then the N_z agents with zero outside option are weakly better off with the platform, obtaining $u(N_z)$ instead of zero, so they indeed want to join. On the other hand, the $N - N_z$ regular agents would be weakly better off without the platform (they get $u(N_z)$ instead of $u(0)$) although in equilibrium they still join.

To illustrate that allowing for some limited degree of price discrimination to the setting from Proposition 5 reinstates the platform trap, consider a very limited form of price discrimination: the platform can only change its price once, so it can set P^1 for the first N_1 agents that it makes offers to, followed by P^2 for the remaining $N_2 = N - N_1$ agents it makes offers to. Moreover, as in our baseline, the platform cannot commit to future prices, so its choice of P^2 is not known until it is offered to some agent. And the platform can choose N_1, i.e. the point at which it makes the price change.

With this set-up, if $b(N-1) \geq u(0)$, it turns out the platform can replicate the maximal platform trap with full price discrimination that we obtained in Corollary 1 (even though it has significantly less freedom to modify prices here). Indeed, the platform can offer the first agent the price $b(N) - u(N-1) = \Delta(N) > 0$. That agent knows from Proposition (5) that if it rejects, the platform will set a uniform price to all remaining agents that induces all of them to participate (that price is $P = b(N-1) - u(0)$). And if the first agent does participate, it is even easier for the platform to attract all remaining agents. Thus, anticipating that regardless of what it does, the remaining $N - 1$ agents will be induced to participate, the first agent participates at the price $\Delta(N)$. A similar logic applies for each subsequent

agent. In the end, all agents participate at the price $P = \Delta(N)$, which is the same outcome as in Corollary 1. Thus, we have:

Proposition 7. *If the platform is restricted to change its price only once but can choose when to do so, then provided $b(N-1) \geq u(0)$ it will induce all agents to participate with the prices $P^1 = P^2 = \Delta(N) > 0$. All agents would be strictly better off without the platform.*

Comparing this result with Proposition 5 highlights why the platform having some ability to adjust its price to future agents to induce them to participate if necessary is a key ingredient to the platform trap. This ability can arise from the platform having some discretion to adjust its price (Proposition 7), or via a commitment to do so, which we consider next.

Even if the platform cannot charge different agents different prices, the platform may make use of second-degree price discrimination to improve on the outcome compared to a single fixed price. Suppose the platform can commit to set a single contingent price $P(j)$, where j is the number of agents that end up participating, so all agents are offered the same pricing function in which the final price they pay will depend on how many agents participate in the end. In the following proposition, we consider both the case it can commit to offer a subsidy (which could arise off equilibrium in case $b(N-1) - u(0) < 0$) and the case it cannot.

Proposition 8. *If the platform can commit to a pricing function (but it must offer the same function for all agents), there are two cases, depending on whether the platform can offer subsidies or not.*

1. *If the platform can offer subsidies or if $b(N-1) - u(0) \geq 0$, the platform will induce all N agents to participate and pay $\Delta(N)$, resulting in the highest achievable platform profit of $N\Delta(N)$. All agents would be strictly better off without the platform.*

2. *If the platform cannot offer subsidies and $b(N-1) - u(0) < 0 \leq b(N) - u(0)$, the platform will induce all N agents to participate and pay $b(N) - u(0)$, resulting in the highest achievable platform profit of $N(b(N) - u(0))$. Otherwise, if $b(N) - u(0) < 0$, the attracts no agents with non-negative prices. Either way, the N active agents are indifferent between the platform existing or not.*

Provided the platform can commit to a pricing function (with subsidies if necessary), it can achieve maximal profits without the need for third-degree price discrimination. Indeed, the fact the platform is not allowed to offer different prices to different agents other than in a way that commits it to induce all subsequent agents to participate regardless of the consequences for its profit provides the platform with commitment value—the platform obtains even higher profits (and agents are even worse off) in case $b(N-1) - u(0) < 0$. Recall, that this is the case where in the baseline model some agents were pivotal and the platform needed to offer them lower prices as a result. With commitment to a single pricing function, regardless of whether some agents deviate from the proposed equilibrium and decide not to participate, the platform will still induce all subsequent agents to participate even if it means the platform has to subsidize those agents and make a loss. By ruling out agents from being pivotal, the platform can obtain the maximum possible profits $N\Delta(N)$.

On the other hand, if $b(N-1) - u(0) < 0$ and the platform cannot commit to offer a subsidy when necessary, then the platform obtains the same outcome as in the case with uniform pricing, which is to profitably induce agents to participate if and only if $b(N) - u(0) \geq 0$. The platform cannot do better than a uniform price equal to $b(N) - u(0)$ in this case. The idea is that given $b(N-1) - u(0) < 0$ and subsidies are not possible, the first agent is pivotal (consistent with Proposition 2) and will only participate if it ends up with a surplus of at least $u(0)$ which means it will not pay more than $b(N) - u(0)$. But since all agents must end up paying the same price, the platform cannot do better than charging all agents the fixed price $b(N) - u(0)$.

5.2.2 Negotiated prices

Consider the negotiation of prices with each agent when the platform no longer has all the bargaining power. Specifically, we assume that the platform engages in Nash bargaining with each agent (sequentially). As before, the platform can set a different price for each agent, and each agent can observe how many agents prior to itself in the sequence of offers have accepted their offers. We focus on the case that no agents can be pivotal, so $\Delta(1) \geq 0$, which ensures the platform can always induce all agents to participate in any equilibrium. This scenario leads to the strongest version of the platform trap, and so it serves as a useful benchmark to show that even in this case, when agents can negotiate, some may no longer suffer from the trap.

To get a sense for how this works formally, consider the last agent to decide whether to participate and suppose N' other agents have already decided to participate. When the last agent decides whether or not to participate, it expects to get $b(N'+1) - P^N$ if it participates and $u(N')$ if it doesn't. Meanwhile, the platform obtains an additional profit of P^N if the agent participates and no additional profit if it doesn't. Assuming each agent's bargaining power relative to the platform is measured by the parameter α, the Nash bargaining solution is the price P^N that maximizes

$$\left(\Delta(N'+1) - P^N\right)^{\alpha} \left(P^N\right)^{1-\alpha},$$

which implies

$$P^N(N') = (1-\alpha)\,\Delta(N'+1).$$

Since $\Delta(N'+1) \geq 0$, the platform will induce the last agent to participate because it can extract a non-negative price from doing so.

Things are more complicated for agents that decide earlier because whether an agent participates or not affects the platform's negotiation position vis-a-vis all subsequent agents. Nevertheless, we are able to to establish the following result.

Proposition 9. *Suppose $\Delta(1) \geq 0$ and the platform engages in Nash bargaining with each of the N agents in sequence, with $0 \leq \alpha \leq 1$ being each agent's bargaining power and $1-\alpha$ being the platform's bargaining power. The platform induces all N agents to participate and the price charged to the k-th agent is*

$$P^k = (1-\alpha)\left(\sum_{j=0}^{N-k} C_j^{N-k}\,(1-\alpha)^j\,\alpha^{N-k-j}\Delta(k+j)\right) \tag{3}$$

for all $k \in \{1,..,N\}$, where C_j^{N-k} is the binomial coefficient (the number of ways to choose j items from a set of $N-k$ items).

The price P^k is independent of k in the special cases of $\alpha = 0$ and $\alpha = 1$, so the platform sets the same price to all agents in equilibrium. When $\alpha = 0$, the platform has all the bargaining power, and we have $P^k = \Delta(N)$, so this is identical to Corollary 1 in which the plaform makes a take-it-or-leave-it offer to each agent. When $\alpha = 1$, agents have all the bargaining power, so we have $P^k = 0$ for all k. Whether agents would be better off without the platform in that case just depends on whether the platform is inefficient or not, i.e. whether $b(N) < u(0)$. Since we assumed $\Delta(1) \geq 0$, this implies $b(N) \geq u(0)$, so the platform is efficient and

participating agents are at least weakly better off with the platform, although the N_0 non-active agents are harmed by the platform still, obtaining $u(N)$ instead of $u(0)$.

The more interesting case arises when $0 < \alpha < 1$. Since $\sum_{j=0}^{N-k} C_j^{N-k} (1-\alpha)^j \alpha^{N-k-j} = 1$ for any k, the platform's optimal price to the k-th agent is $(1-\alpha)$ multiplied by a weighted average of the surplus obtained from joining the platform for each of the subsequent $N-k+1$ agents (including the k-th agent) deciding. That is, it is a weighted average of the $\Delta(n)$ terms, with n ranging from k to N. We wish to determine whether the platform's price is increasing across agents that sign up and whether agents are better or worse off due to the platform.

Since no agent is pivotal, by assumption, an agent expects all other agents to participate regardless of its decision. Thus, an agent's gross surplus $\Delta(N)$ from joining versus not doesn't change with k. Instead, any change in P^k over k happens due to changes in the additional payoff that the platform stands to gain on subsequent agents joining when agent k joins. Specifically, when agent k joins rather than doesn't, the additional profit that the platform stands to gain beyond the current price can be measured by the extent to which its negotiated price is expected to increase over subsequent agents. As k increases, with fewer potential agents left to join, the impact on the platform's profit from not signing an agent is felt over fewer subsequent agents signing up. This suggests the platform has less to lose from not attracting an agent as k increases, leading to P^k being increasing in k. But the additional profit that can be extracted from subsequent agents also depends on $\Delta(.)$. Given $\Delta(.)$ is an increasing function, as k increases, the average of the additional profit that can be extracted from the remaining agents increases, meaning the platform could actually have more to lose. The overall net effect on how much the platform stands to lose by not attracting an agent as k increases is ambiguous, and depends on the particular shape of $\Delta(.)$.

To proceed, we assume linear externalities so $b(n) = b_0 + b_1 n$ with $b_1 > 0$ and $u(n) = u_0 - u_1 n$ with $u_1 > 0$, which implies $\Delta(n) = \beta_0 + \beta_1 n$ where $\beta_0 = b_0 - u_0 - u_1$ and $\beta_1 = b_1 + u_1 > 0$. Substituting these expressions into (3) implies

$$P^k = (1-\alpha)(\beta_0 + \beta_1 (N - \alpha(N-k))) \tag{4}$$

for all $k \in \{1, .., N\}$. The monotonicity of P^k in k allows us to characterize when the platform trap arises.

Proposition 10. *Suppose* $\Delta(1) \geq 0$ *and the platform engages in Nash bargaining*

with each of $N \geq 2$ agents in sequence. If $\alpha = 0$, all agents would be strictly better off without the platform, while if $\alpha = 1$, all active agents would be strictly worse off without the platform. Now consider $0 < \alpha < 1$ and suppose externalities are linear as defined above. The price charged to the k-th agent, which is given in (4), is increasing in k. There exists $0 < \alpha_1 < \alpha_N < 1$ such that if $\alpha < \alpha_1$, all agents would be strictly better off without the platform, if $\alpha > \alpha_N$, all agents would be strictly worse off without the platform, while for $\alpha_1 \leq \alpha \leq \alpha_N$, agents with $k < \overline{k}$ would be strictly worse off without the platform and agents with $k > \overline{k}$ would be strictly better off without the platform, where $\overline{k}$ is defined as the unique level of k that makes $P^k = b(N) - u(0)$ for any given α in this range.

The proposition shows that under linear externalities, provided the platform's bargaining power is strong enough, all agents would be strictly better off without the platform, while provided the agents' bargaining power is strong enough, the reverse is true (for active agents). For some intermediate range of α, agents that receive early offers would be strictly worse off without the platform while those receiving later offers would be strictly better off without the platform.

6 Conclusion

Commentators have noted that far from being a boon for participants, some platforms may end up hurting them. In the case of online marketplaces, this could involve saddling sellers with new fees to reach the same customers they previously dealt with directly. We have provided a dynamic model that shows how a platform trap can emerge despite agents being rational and forward looking.

While we do not think our findings should directly translate into drastic policy recommendations (e.g. we don't propose banning platforms just because a platform trap may be theoretically possible), we do think platform traps are useful to consider when evaluating the impact of platforms and policies targeted at them. One reason to be careful about jumping to policy conclusions is that sometimes a platform trap may only apply to a subset of the agents, with other agents benefiting from the platform, as we showed in a number of extensions. The overall effect of the platform would then require an empirical exercise weighing up the costs and benefits. But even in such cases, the real impact of the platform on surplus would be quite different from the impact calculated simply based on comparing the *current* payoffs with the *current* outside option.

There are other ways a platform trap could arise which provide complementary theories to ours. A platform trap could arise in a more mechanical way in a multi-period setting if instead agents were myopic when deciding whether to participate (so they do not anticipate how their decision may affect future prices or other agents' joining decisions) or if they had to make an irrecoverable platform-specific investment, which increases the benefit they derive from the platform but also their switching cost. In a marketplace setting, the possible commoditization of sellers as the platform makes them compete more intensely could provide an additional mechanism through which sellers become worse off due to the existence of the platform. Future work should try to incorporate such an effect into a dynamic model of platform adoption like ours.

Finally, we focused on the case of a monopoly platform. An interesting question that remains to be explored is to what extent does platform competition undermine the platform trap? Can a platform trap still arise under limited platform competition? In a setting where the market ultimately tips to one platform or the other, will agents that join after the market tips to one of the platforms still be worse off due to the existence of platforms?

7 Appendix

In this appendix we provide the proofs of propositions not proven in the text.

7.1 Proof of Proposition 1

We prove the first part by induction. If $N = 2$, then we are in one of the two extremes explained in the text (i.e. whether agent 1 is pivotal or not), and the platform optimally charges

$$\begin{cases} P^1 = P^2 = b(2) - u(1) & \text{if} \quad b(1) - u(0) \geq 0 \\ P^1 = b(2) - u(0), P^2 = b(2) - u(1) & \text{if} \quad b(1) - u(0) < 0 \leq b(2) - \frac{u(0)+u(1)}{2} \end{cases},$$

where note $b(2) - u(1) > 0$ by assumption. If $b(2) - \frac{u(0)+u(1)}{2} < 0$, then the platform cannot profitably attract any agents, so it charges any $P^1 \geq 0$ and $P^2 \geq 0$ and attracts no agents.

We thus consider the following induction hypothesis for $N = n$:

- if the platform can profitably attract the last $n-1$ agents when the first agent

doesn't participate, then the platform can profitably attract all n agents with optimal prices $P^k = b(n) - u(n-1) > 0$ for all $k \in \{1, ..., n\}$

- if the platform cannot profitably attract the last $n-1$ agents when the first agent doesn't participate, then there exists $k_0 \in \{1, .., n-1\}$ such that the platform's optimal prices are

$$P^k = \begin{cases} b(n) - u(k-1) & \text{if} \quad 1 \leq k \leq k_0 \\ b(n) - u(n-1) & \text{if} \quad k_0 + 1 \leq k \leq n \end{cases}$$

if

$$\sum_{k=1}^{n} P^k \geq 0,$$

in which case all agents participate. If instead $\sum_{k=1}^{n} P^k < 0$, then the platform cannot profitably attract any agents.

This induction hypothesis holds for $N = 2$. Now suppose induction holds for $N = n \geq 2$ and let's show it also holds for $N = n + 1$.

First, suppose the platform can profitably attract n agents when the first agent doesn't participate. Then the platform will charge

$$P^1 = b(n+1) - u(n) > 0$$

to the first agent (out of $n+1$ agents) and attract it. Next consider the second agent, after the first agent has decided to participate. If the second agent does not participate, then when it faces the third agent (out of $n+1$ agents), the platform is effectively in the same position as if it were facing the second out of n agents, with the first agent having decided to participate. Since we know the platform can profitably attract n agents, this means the platform will attract all agents starting with the third one, even if the second agent does not participate. And if the second agent participates, then clearly the remaining agents (starting with the third one) will also participate. Thus, the platform can charge

$$P^2 = b(n+1) - u(n) > 0$$

to the second agent. By a very similar reasoning, this will also be the price it charges to all remaining agents. And total platform profits are obviously positive.

Second, suppose the platform cannot profitably attract n agents when the first agent doesn't participate. In this case, the highest price that the platform can charge to the first agent in order to attract them is

$$P^1 = b(n+1) - u(0).$$

Once the first agent participates, we can apply the induction hypothesis with $b'(k) = b(k+1)$ and $u'(k) = u(k+1)$ to the remaining n agents:

- If the platform can profitably attract the last $n-1$ agents when the first agent participates and the second agent doesn't, then the platform's optimal prices are $P^k = b(n+1) - u(n) > 0$ for all $k \in \{2, ..., n+1\}$ and the platform can profitably attract all n agents. So we have

$$P^k = \begin{cases} b(n+1) - u(0) & \text{if} \quad k = 1 \\ b(n+1) - u(n) & \text{if} \quad 2 \leq k \leq n+1 \end{cases}.$$

- If the platform cannot profitably attract the last $n-1$ agents when the first agent participates and the second agent doesn't, then there exists $k_0 \in \{2, .., n\}$ such that the platform's optimal prices for agents $k \in \{2, ..., n+1\}$ are

$$P^k = \begin{cases} b(n+1) - u(k) & \text{if} \quad 2 \leq k \leq k_0 \\ b(n+1) - u(n) & \text{if} \quad k_0 + 1 \leq k \leq n+1 \end{cases}$$

  if $\sum_{k=2}^{n+1} P^k \geq 0$. On the other hand, if $\sum_{k=2}^{n+1} P^k < 0$, then the platform cannot profitably attract the n agents $k \in \{2, ..., n+1\}$ (conditional on the first agent participating) so the platform will set any non-negative prices, resulting in zero profits. Thus, the platform's optimal prices with $n+1$ agents are

$$P^k = \begin{cases} b(n+1) - u(k) & \text{if} \quad 1 \leq k \leq k_0 \\ b(n+1) - u(n) & \text{if} \quad k_0 + 1 \leq k \leq n+1 \end{cases}$$

  provided

$$\sum_{k=1}^{n+1} P^k \geq 0.$$

  In particular, note that $\sum_{k=1}^{n+1} P^k \geq 0$ implies $\sum_{k=2}^{n+1} P^k \geq 0$ because P_k is weakly increasing in k.

Thus, we have proven the induction hypothesis holds in all cases for $N = n+1$.

And by induction, it holds for all $N \geq 2$. Moreover, the lowest possible prices that the platform can charge and still induce agents to participate are $P^k = b(n+1) - u(k)$ for all $k \in \{1, .., N\}$, which are the prices that arise when all agents are pivotal. Thus, if the sum of these prices is negative, then the platform could never attract all agents to participate, while provided it is non-negative, then the platform can always attract all N agents to participate. This explains (1) stated in the proposition.

Finally, note that agent k's payoff when all agents participate is $b(N) - P^k$, which is equal to either $u(N-1)$ (when agent k is not pivotal) or $u(k-1)$ (when agent k is pivotal). Thus, in all cases, all agents would be weakly better off without the platform (in which case they would all get $u(0)$), and at least one agent is strictly better off without the platform (this includes all non-pivotal agents, which at a minimum includes agent N).

7.2 Proof of Corollary 2

Suppose

$$b(N-1) - u(N-2) < 0, \tag{5}$$

which implies

$$b(k) - u(k-1) < 0$$

for all $k \in \{1, .., N-1\}$. This means all agents except the last one are pivotal, i.e. the failure to attract a single agent up until the last one makes the platform unable to profitably attract any subsequent agents. Using the logic from the proof of Proposition 1, we can then immediately conclude that the price charged to the k-th agent that participates is $P^k = b(N) - u(k-1)$, where $k = (1, ..., N)$, provided (1) holds.

7.3 Proof of Proposition 2

We prove the result by induction with respect to the numbers of agents N. If $N = 2$, there are two relevant cases:

- If $b(1) - u(0) \geq 0$, then following the logic in the text, the optimal prices are $P^1 = P^2 = b(2) - u(1)$, and both are positive
- If $b(1) - u(0) < 0$, then the first agent is pivotal, and the most that the platform can charge to the first agent is $P^1 = b(2) - u(0)$, which then allows

it to charge $P^2 = b(2) - u(1)$ to the second agent. Both prices are non-negative if and only if (iff) $b(2) - u(0) \geq 0$.

Thus, with $N = 2$, the platform can attract both agents with non-negative prices if

$$b(2) - u(0) \geq 0,$$

and no agents participate otherwise.

Suppose the result is true for $N = n \geq 2$, namely all agents participate in a period if

$$b(n) - u(0) \geq 0,$$

and no agents participate otherwise.

Now let's show the result is also true for $N = n+1$. If the first agent doesn't participate, then the induction hypothesis implies the remaining n agents all participate if

$$b(n) - u(0) \geq 0,$$

and no agents participate otherwise. If the first agent participates, then the situation when the platform faces the remaining n agents is equivalent to the one with $N = n$, except with payoff functions $(b'(.), u'(.))$, where

$$\begin{aligned} b'(n) &\equiv b(n+1) \\ u'(0) &\equiv u(1). \end{aligned}$$

Thus, we can apply the induction hypothesis for $N = n$ to $(b'(.), u'(.))$, implying that when the first agent has participated, all remaining n agents participate if

$$b(n+1) - u(1) \geq 0$$

and no agent participates otherwise.

Note that we always have

$$b(n+1) - u(1) \geq b(n) - u(0).$$

There are therefore three possibilities:

- If

$$b(n) - u(0) \geq 0,$$

then the last n agents are induced to participate at non-negative prices no matter what the first agent does. And therefore the first agent will also be induced to participate because

$$b(n+1) - u(n) > b(n) - u(0) \geq 0.$$

Thus, all $(n+1)$ agents are induced to participate in this case.

- If

$$b(n+1) - u(1) < 0,$$

then the platform cannot induce any agent to participate at non-negative prices because regardless of whether the first agent participates, the rest will not want to, and so the first will also not want to since

$$b(1) - u(0) \leq b(n+1) - u(1) < 0.$$

So no agents participate in this case.

- If

$$b(n) - u(0) < 0 \leq b(n+1) - u(1),$$

then the last n agents are induced to participate iff the first agent participates. This means that if the first agent participates, it gets $b(n+1) - P$, and if it doesn't participate it gets $u(0)$. So the platform induces all sellers to participate at non-negative prices in this case if

$$b(n+1) - u(0) \geq 0$$

and no agents participate otherwise.

Note that we always have

$$b(n) - u(0) \leq b(n+1) - u(0) \leq b(n+1) - u(1).$$

This implies that the three cases above can be summarized by stating that all $(n+1)$ agents are induced to participate if

$$b(n+1) - u(0) \geq 0$$

and no agents participate otherwise.

By induction on n, the result is true for all $N \geq 2$.

Now let us determine the prices when there are N agents. This is only relevant when

$$b(N) - u(0) \geq 0,$$

i.e. when the equilibrium involves all agents joining, since otherwise, there are no non-negative prices that can induce any agents to participate.

There are three cases

- if $b(N-1) - u(0) \geq 0$, then the prices will be those given in Corollary 1 because no agent is pivotal
- if $b(N-1) - u(N-2) < 0$, then the prices will be those given in Corollary 2 because all agents (except the last) are pivotal
- if $b(N-1) - u(0) < 0 \leq b(N-1) - u(N-2)$, then applying the logic from the proof of Proposition 1 and letting $k_0 \in \{1, N-2\}$ be uniquely defined by

$$b(N-1) - u(k_0 - 1) < 0 \leq b(N-1) - u(k_0),$$

we have the first k_0 agents are pivotal and the prices are

$$P^k = \begin{cases} b(N) - u(k-1) & \text{if} \quad 1 \leq k \leq k_0 \\ b(N) - u(N-1) & \text{if} \quad k_0 + 1 \leq k \leq N \end{cases}.$$

In all cases, the resulting prices are non-negative given $b(N) \geq u(0)$.

Agent k's payoff is thus

$$\begin{aligned} & b(N) - P^k \\ = & \begin{cases} u(k-1) & \text{if} \quad P^k = b(N) - u(k-1) \\ u(N-1) & \text{if} \quad P^k = b(N) - u(N-1) \end{cases} \\ \leq & u(0), \end{aligned}$$

where the last inequality is strict for agents $k > k_0$ (this always includes $k = N$). The conclusion for agents' payoffs follows the proof of Proposition 1.

7.4 Proof of Proposition 3

The case with $P = \Delta(N)$ offered to all agents is clearly the best possible equilibrium for the platform, with the platform obtaining a profit of $N\Delta(N)$. At any

higher price, even if agents expect all other agents to participate, they will not want to participate. Since an agent that rejects the offer cannot influence other agents' decisions (since its deviation would not be observed), no agent is pivotal, and we do not need any of the additional conditions that we had in Propositions 2–**??**. As in Proposition **??**, at these prices, all agents obtain the net payoff $u(N-1) < u(0)$, so they would be better off without the platform.

Next consider the worst possible equilibrium for the platform. Suppose first the platform can offer subsidies. The proposed equilibrium strategies are:

- The platform selects the agents in any arbitrary order and offers agent k from its selected order $P^k = \Delta(k)$.
- Agent 1 participates iff $P^1 \leq \Delta(1)$.
- For agent $k = 2, ..., N$, let j be the number of prior agents that agent k can infer will participate based on the prices they were offered. Then agent k participates iff $P^k \leq \Delta(j+1)$.

These equilibrium strategies are supported by the following off-equilibrium beliefs: if agent k receives any higher price $P^k > \Delta(j+1)$, it expects all agents $j > k$ will also receive $P^j > \Delta(j+1)$ and so will not participate. This is the worst possible equilibrium because the platform can guarantee obtaining all agents with these (or any lower) prices. Facing a price $P \leq b(1) - u(0) = \Delta(1)$ agent 1 will participate for sure regardless of what it expects the platform or subsequent agents will do. Since the platform maximizes profits, it will never set prices strictly less than $\Delta(1)$. Once it has set the first price equal to $\Delta(1)$, subsequent agents know the first agent participates for sure, so all other agents are willing to pay at least $b(2) - u(1) = \Delta(2)$. Based on the same logic, the platform sets the second price to equal $\Delta(2)$ and now all remaining agents are willing to pay at least $b(3) - u(2) = \Delta(3)$. Repeating this logic, the platform maximizes profits by setting the sequence of prices $P^k = \Delta(k)$ for $k = 1, .., N$. This implies all agents participate and agent k's net payoff is

$$b(N) - P^k = b(N) - b(k) + u(k-1).$$

Note that this payoff is decreasing in k, for $k = 1$ it is weakly higher than $u(0)$, and for $k = N$ it is strictly lower than $u(0)$. Thus, there exists $\widehat{k}$ such that $1 \leq \widehat{k} < N$ and agents $k \leq \widehat{k}$ would be weakly worse off without the platform, while agents $k > \widehat{k}$ would be strictly better off without the platform.

Since the platform obtains $\sum_{k=1}^{N} \Delta(k)$ in the end, provided this is non-negative, the worst the platform can do is to obtain this payoff. On the other hand, if subsidies are not possible, then we need $\Delta(1) \geq 0$, otherwise there is an equilibrium where no agents participates.

7.5 Proof of Proposition 4

In the best possible equilibrium for the platform, the platform prices at $P = \Delta(N)$ to all agents. Each agents expects this, and participates since it expects the equilibrium where the platform prices in this way to all other agents and all agents also participate. All agents receive net payoff $u(N-1)$, so they would all be strictly better off without the platform.

In the worst possible equilibrium for the platform, whenever the platform sets $P^k > \Delta(1)$ to any agent k, the agent expects that all other agents have received a price $P > \Delta(1)$ and will not participate because they do not expect any other agent to participate. This means the platform has to set $P = \Delta(1)$ to each agent for them to participate. Thus, if $\Delta(1) \geq 0$, the platform sets $P = \Delta(1)$ for all agents, the platform gets $N\Delta(1)$ and all agents participate. Otherwise, the platform sets $P = 0$ and no agents participate. Finally, note, when $\Delta(1) \geq 0$, all agents receive net payoffs $b(N) - b(1) + u(0)$, so all active agents would be weakly worse off without the platform.

7.6 Proof of Proposition 5

If $N = 2$, it is easily verified that both agents join if the platform sets $P \leq b(2) - u(0)$ and no agents join otherwise.

Suppose that when $N = n$, all agents join if $P \leq b(n) - u(0)$ and no agents join otherwise. Now let us look at the case with $N = n + 1$ agents.

If the platform sets $P \leq b(n+1) - u(0)$, then if the first agent joins, all remaining agents will also join. Indeed, we can apply the induction hypothesis to the remaining n agents with $b'(n) = b(n+1)$ and $u'(n) = u(n+1)$ and note that

$$P \leq b(n+1) - u(0) \leq b(n+1) - u(1) = b'(n) - u'(0).$$

Thus, if the first agent joins, its payoff will be at least

$$b(n+1) - P \geq u(0),$$

whereas if the first agent doesn't join, its payoff will be at most $u(0)$. So the first agent joins, and so will all remaining n agents.

If the platform sets $P > b(n+1) - u(0)$, then the first agent's payoff from joining will be less than $u(0)$. If the first agent doesn't join, then none of the remaining agents will join either. Indeed, we can simply apply the induction hypothesis to the remaining n agents by noting that

$$P > b(n+1) - u(0) \geq b(n) - u(0).$$

Thus, if the first agent doesn't join, its payoff will be $u(0)$, which is higher than the payoff from joining. So the first agent doesn't join and neither will any of the remaining n agents. The induction thus holds for $N = n + 1$.

By induction, the result holds for all $n \geq 2$. And we can then conclude that the platform's optimal price when $N = n$ is $P = b(n) - u(0)$ if $b(n) - u(0) \geq 0$, which attracts all agents, and any $P \geq 0$ if $b(n) - u(0) < 0$, which attracts no agents.

7.7 Proof of Proposition 6

At any price P, the best the platform can do is either attract all agents or only the zero-outside option agents (z-agents) since z-agents will always participate if P is such that "regular" agents participate.

Suppose the platform gets all agents to participate. The highest price it can charge while doing so is $P = b(N) - u(N_z)$. Indeed, if $P > b(N) - u(N_z)$, then the regular agents' payoff is less than $u(N_z)$. It is then easily verified that the first regular agent can deviate to not joining, which would induce at least all other regular agents to not join, resulting in a payoff of at least $u(N_z)$ for all regular agents. To get all agents to participate at $P = b(N) - u(N_z)$, the platform can first approach all z-agents and then all regular agents. Thus, the highest profit the platform can obtain by attracting all agents is $N(b(N) - u(N_z))$.

Suppose now the platform gets only the z-agents to participate at price P. Then we must have $P \leq b(N_z)$ for the z-agents to be willing to participate. Thus, if $b(N_z) \leq b(N) - u(N_z)$, then the platform's best option is clearly to charge $P = b(N) - u(N_z)$ and get all agents to join. Alternatively, if $b(N_z) > b(N) - u(N_z)$, then the platform chooses the best of two options:

- set $P = b(N) - u(N_z)$, attract all agents and obtain profits $N(b(N) - u(N_z))$
- set $P = b(N_z)$, attract N_z agents and obtain profits $N_z b(N_z)$

Comparing these two options, the platform prefers to attract all agents whenever

$$b(N) - u(N_z) < b(N_z) \leq \frac{N}{N_Z}(b(N) - u(N_z)).$$

Combining with the case $b(N_z) \leq b(N) - u(N_z)$, we obtain the result in the proposition.

7.8 Proof of Proposition 8

Start with the case where the platform can commit to offer a subsidy if necessary. Suppose the platform offers the pricing function $P(j) = \Delta(j)$ for all $j = 1, .., N$. Consider the last agent to decide. And suppose N' other agents have decided to participate at that point. Then the pricing function $P(j)$ implies the agent would face a price of $P(N'+1) = b(N'+1) - u(N')$ if it participates and would want to participate since it gets $u(N')$ either way. Consider the second-to-last agent to decide, and suppose N' other agents have decided at that point. If it participates, then it knows the last agent will also participate and in the end it will get $b(N'+2) - P(N'+2) = u(N'+1)$, while if it doesn't participate, then next period N' will remain unchanged, and the last agent will participate, which means the second-to-last agent will get $u(N'+1)$. Thus, this agent is also just willing to participate. By backwards induction, the same is true for all agents including the first. So all agents participate and along the equilibrium path they are all charged $P(N) = \Delta(N)$. This is the maximum price that the platform can charge while getting all agents to participate. Indeed, if it charged a higher price, then any agent would prefer to not participate. Thus, it must be that the pricing function $P(j) = \Delta(j)$ achieves maximum profits for the platform in this case.

Now consider the case the platform cannot commit to offer a subsidy. Suppose $b(N-1) - u(0) \geq 0$. And suppose the platform offers the pricing function

$$P(j) = \begin{cases} \Delta(N) & \text{if} \quad j = N \\ b(N-1) - u(0) & \text{if} \quad j = N-1 \\ 0 & \text{if} \quad j \leq N-2 \end{cases}. \tag{6}$$

Note this involves no subsidies. We will show the platform can still get all agents to join and therefore pay $P(N) = \Delta(N)$, which is the maximum price that the platform can extract while still getting all agents to participate.

To prove this, we first need to prove the following result. For any N such

that $b(N) - u(0) \geq 0$, any pricing function such that $P(N) \leq b(N) - u(0)$ and $P(j) = 0$ for all $j \leq N-1$ guarantees participation of all N agents. We prove it by induction. Start with $N = 2$ and suppose the platform sets a pricing function such that $P(2) \leq b(2) - u(0)$ and $P(1) = 0$. If agent 1 participates, then agent 2 gets at least $u(0)$ from participating and $u(1) \leq u(0)$ from not participating, so agent 2 participates as well. Thus, agent 1 gets at least $u(0)$ from participating, and it would get at most $u(0)$ from not participating (it would get $u(1) \leq u(0)$ if agent 2 still participates). Thus, both agents end up participating. Now suppose the result holds for $N = n$ and consider the case $N = n+1$, assuming $b(n+1) - u(0) \geq 0$, $P(n+1) \leq b(n+1) - u(0)$ and $P(j) = 0$ for all $j \leq n$. If agent 1 participates, then all subsequent agents participate. To see this, note that once agent 1 participates, we can apply the induction hypothesis with $b'(j) = b(j+1)$, $u'(j) = u(j+1)$ and $P'(j) = P(j+1)$ for all $j = 1, .., n$, because

$$\begin{aligned} P'(n) &= P(n+1) \leq b(n+1) - u(0) \leq b(n+1) - u(1) = b'(n) - u'(0) \\ P'(j) &= P(j+1) = 0 \text{ for } j \leq n-1. \end{aligned}$$

Thus, if agent 1 participates, its payoff will be $b(n+1) - P(n+1) \geq u(0)$. And if it doesn't participate, its payoff will be at most $u(0)$, so agent 1 and all subsequent agents participate. By induction, the result holds for all $N \geq 2$.

Now back to the case where the platform offers the pricing function (6). If the first agent doesn't participate, then from this pricing function and the result above, we know that all remaining $N-1$ agents will still participate. If the first agent does participate, then the remaining $N-1$ participants face a very similar game, with payoff functions $b'(j) = b(j+1)$ and $u'(j) = u(j+1)$ and pricing function

$$P'(j) = \begin{cases} \Delta'(N-1) = \Delta(N) & \text{if} \quad j = N-1 \\ b'(N-2) - u'(0) & \text{if} \quad j = N-2 \\ 0 & \text{if} \quad j \leq N-3 \end{cases}.$$

Note

$$b'(N-2) - u'(0) = b(N-1) - u(1) \geq b(N-1) - u(0) \geq 0$$

Thus, a straightforward induction argument guarantees that all remaining $N-1$ agents participate when the first agent participates. This means that the first agent's payoff when participating is $u(N-1)$, which is also the payoff when it doesn't participate. So the first agent is just willing to participate, which induces all other

$N-1$ agents to participate.

Finally, suppose subsidies are not possible and $b(N-1)-u(0)<0$. We will prove the platform can profitably get all N agents to participate iff $b(N)-u(0)\geq 0$. If this inequality holds, the platform will extract the highest possible profit $N(b(N)-u(0))$ with the price function $P(N)=b(N)-u(0)$ and $P(j)=0$ for $j<N$.

Start with $N=2$. Suppose first $b(2)-u(0)\geq 0$ and the platform sets the schedule $P(2)=b(2)-u(0)$ and $P(1)=0$. We have already shown above that both agents participate with this price schedule and each pays $b(2)-u(0)$. The platform cannot do better because if it tried to set $P(2)>b(2)-u(0)$, then agent 1 would prefer not to participate because that would induce agent 2 not to participate ($b(1)-u(0)<0$) so agent 1 would get $u(0)$, which is better than $b(2)-P(2)$. And certainly the platform cannot do better by attracting only one agent because $b(1)-u(0)<0$.

Suppose instead $b(2)-u(0)<0$. The platform must set $P(j)\geq 0$ for $j=1,2$ (no subsidies). However, in this case, agent 1 prefers not to join, because that induces agent 2 not to join ($u(0)>b(1)-P(1)$) so agent 1 gets $u(0)$, as opposed to getting either $b(1)-P(1)$ or $b(2)-P(2)$ when joining, both of which are lower than $u(0)$. So the platform cannot profitably attract any agents in this case.

Now suppose the following induction hypothesis holds for $N=n$:

- if $b(n)-u(0)\geq 0>b(n-1)-u(0)$, then the platform can attain its maximum profits with the price schedule $P(j)=\max\{b(j)-u(0),0\}$ for all $j=1,..,n$, which leads all agents to participate
- if $b(n)-u(0)<0$, then the platform cannot attract any agents with non-negative prices

Consider the case $N=n+1$. Suppose first $b(n+1)-u(0)\geq 0>b(n)-u(0)$ and the platform sets the schedule $P(j)=\max\{b(j)-u(0),0\}$ for $j=1,..,n+1$. If the first agent participates, then all subsequent agents will also participate. Indeed, we know from the induction hypothesis that in this case the last n agents would all be willing to participate at prices $P'(j)=\max\{b(j+1)-u(1),0\}$ for $j=1,..,n$, so they would all still participate at a schedule of prices which are uniformly lower, i.e. $P(j)=\max\{b(j)-u(0),0\}$ for $j=1,..,n$. And if the first agent does not participate, none of the remaining agents participate (by the induction hypothesis with $b(n)-u(0)<0$). Thus, the first agent is willing to pay exactly

$b(n+1) - u(0)$ to join, which is the price it will end up paying if it joins based on the price schedule proposed. This leads all agents to join and the platform 's profits are $(n+1)(b(n+1) - u(0))$.

We also need to make sure the platform cannot achieve higher profits. The only way it could do that is if it were to still attract all $n+1$ agents and have them all pay a price $P(n+1) > b(n+1) - u(0)$. However, at such a price, agent 1 would prefer not to join. Indeed, if agent 1 does not join, then none of the remaining agents join either because $b(n) - u(0) < 0$ (so we can apply the induction hypothesis). Thus, by not joining, agent 1 would obtain $u(0)$, which is now strictly higher than $b(n+1) - P(n+1)$.

Thus, we have proven the induction hypothesis also holds for $N = n+1$. So it holds for all $n \geq 2$.

7.9 Proof of Proposition 9

We first show by backwards induction that the price charged to the k-th agent to decide is

$$P^k = (1-\alpha)\left(\sum_{j=0}^{N-k} C_j^{N-k}(1-\alpha)^j \alpha^{N-k-j} \Delta(N'+j+1)\right)$$

and all subsequent agents (including the k-th agent) will participate, where N' is the number of agents that have decided to participate before k.

We have already shown in the main text that this holds for the last agent to decide ($k = N$).

Suppose the result holds for all $k \geq K$ for some $K \leq N$. Let us now show it also holds for $k = K-1$. To determine P^{K-1}, suppose N' other agents have already decided to participate. The $(K-1)$-th agent knows regardless of whether it decides to participate, the next $N-K+1$ agents will participate (by the induction hypothesis). Thus, the agent's increase in surplus from participating relative to not participating is

$$\Delta(N'+N-K+2) - P^{K-1}.$$

If the agent participates, the platform's profit from this and the next $N-K+1$

agents participating is

$$\begin{aligned} & P^{K-1} + \sum_{m=0}^{N-K} P^{K+m} \\ = \; & P^{K-1} + (1-\alpha) \sum_{m=0}^{N-K} \left(\sum_{j=0}^{N-K-m} C_j^{N-K-m} (1-\alpha)^j \alpha^{N-K-m-j} \Delta (N' + m + j + 2) \right). \end{aligned}$$

If the $(K-1)$-th agent does not participate, then the platform's profit from the subsequent agents is

$$(1-\alpha) \sum_{m=0}^{N-K} \left(\sum_{j=0}^{N-K-m} C_j^{N-K-m} (1-\alpha)^j \alpha^{N-K-m-j} \Delta (N' + m + 1 + j) \right).$$

Thus, the Nash bargaining solution is

$$\begin{aligned} P_t^{K-1} \; = \; & (1-\alpha) \Delta (N' + N - K + 2) \\ & -\alpha (1-\alpha) \sum_{m=0}^{N-K} \left(\sum_{j=0}^{N-K-m} C_j^{N-K-m} (1-\alpha)^j \alpha^{N-K-m-j} \begin{pmatrix} \Delta (N' + m + j + 2) \\ -\Delta (N' + m + 1 + j) \end{pmatrix} \right) \end{aligned}$$

After some re-arrangement of the various terms, this can be re-written

$$P_t^{K-1} = (1-\alpha) \begin{pmatrix} (1-\alpha)^{N-K+1} \Delta (N' + N - K + 2) + \alpha^{N-K+1} \Delta (N' + 1) \\ + \sum_{s=1}^{N-K} \alpha^{N-K+1-s} \Delta (N' + s + 1) \begin{pmatrix} \sum_{m=0}^{s} C_{s-m}^{N-K-m} (1-\alpha)^{s-m} \\ -\sum_{m=0}^{s-1} C_{s-1-m}^{N-K-m} \alpha (1-\alpha)^{s-1-m} \end{pmatrix} \end{pmatrix}$$

We wish to show this can be rewritten as

$$P_t^{K-1} = (1-\alpha) \left(\sum_{s=0}^{N-K+1} C_s^{N-K+1} (1-\alpha)^s \alpha^{N-K+1-s} \Delta (N' + s + 1) \right),$$

so we need to prove

$$\begin{aligned} & \sum_{m=0}^{s} \frac{(N-K-m)!}{(N-K-s)!\,(s-m)!} (1-\alpha)^{s-m} - \sum_{m=0}^{s-1} \frac{(N-K-m)!}{(N-K-s+1)!\,(s-1-m)!} \alpha (1-\alpha)^{s-1-m} \\ = \; & C_s^{N-K+1} (1-\alpha)^s \end{aligned}$$

for all $s \in \{1, .., N-K\}$ and $\alpha \in [0,1]$, which can be rewritten

$$\sum_{m=0}^{s} \frac{(N-K-s+m)!}{(N-K-s)!m!} (1-\alpha)^m - \sum_{m=0}^{s-1} \frac{(N-K-s+1+m)!}{(N-K-s+1)!m!} \alpha (1-\alpha)^m = C_s^{N-K+1} (1-\alpha)^s .$$

With the change of variable $x = 1-\alpha$, this is equivalent to proving

$$\begin{aligned} & \frac{1}{(N-K-s)!} \sum_{m=0}^{s} (N-K-s+m) \times ... \times (1+m) x^m \\ & - \frac{1}{(N-K-s+1)!} \sum_{m=0}^{s-1} (N-K-s+1+m) \times ... \times (1+m)(1-x) x^m \\ = & \frac{(N-K+1)!}{(N-K+1-s)!s!} x^s \end{aligned}$$

for all $s \in \{1, .., N-K\}$ and $x \in [0,1]$.

To do this, let

$$\begin{aligned} f_s(x) &= \frac{1}{(N-K-s)!} \sum_{m=0}^{s} (N-K-s+m) \times ... \times (1+m) x^m \\ & - \frac{1}{(N-K-s+1)!} \sum_{m=0}^{s-1} (N-K-s+1+m) \times ... \times (1+m) \left(x^m - x^{m+1}\right) \\ g_s(x) &= \frac{(N-K+1)!}{(N-K+1-s)!s!} x^s. \end{aligned}$$

Note that both $f_s(x)$ and $g_s(x)$ are polynomials in x with the highest power of x being s. We have

$$f_s(0) = 0 = g_s(0).$$

For all $k \in \{1, .., s-1\}$ we have

$$\begin{aligned} \frac{d^k f_s}{dx^k}(x=0) &= \frac{(N-K-s+k)!}{(N-K-s)!} - \frac{((N-K-s+1+k)! - (N-K-s+k)!k)}{(N-K-s+1)!} \\ &= \frac{(N-K-s+k)!}{(N-K-s)!} - \frac{(N-K-s+k)!}{(N-K-s)!} \\ &= 0 = \frac{d^k g_s}{dx^k}(x=0) \end{aligned}$$

Finally,

$$\begin{aligned}\frac{d^s f_s}{dx^s}(x=0) &= \frac{(N-K)!}{(N-K-s)!}+\frac{(N-K)!s}{(N-K-s+1)!}\\ &= \frac{(N-K+1)!}{(N-K-s+1)!}=\frac{d^s g_s}{dx^s}(x=0).\end{aligned}$$

Thus, we can conclude $f_s(x)=g_s(x)$ for all $x\in[0,1]$ and any $s\in\{1,..,N-K\}$.

So by induction we have proven

$$P_t^k(N')=(1-\alpha)\left(\sum_{j=0}^{N-k}C_j^{N-k}(1-\alpha)^j\alpha^{N-k-j}\Delta(N'+j+1)\right)$$

for any $k\leq N$, when N' agents have decided to participate prior to agent k.

Since all agents participate along the equilibrium path, we can conclude that in equilibrium the price charged to agent k is

$$P_t^k=(1-\alpha)\left(\sum_{j=0}^{N-k}C_j^{N-k}(1-\alpha)^j\alpha^{N-k-j}\Delta(k+j)\right).$$

7.10 Proof of Proposition 10

Define the additional payoff of agent k due to the existence of the platform as $\Omega=b(N)-P^k-u(0)$, where P^k is defined in (4). The assumptions that $\Delta(1)\geq 0$ and $N>1$ together with $b(n)$ being strictly increasing imply the platform is efficient, so $b(N)>u(0)$. This implies $\Omega>0$ if $\alpha=1$. Moreover, from Proposition **??** we have $\Omega<0$ if $\alpha=0$. Now consider $0<\alpha<1$. Since Ω is a quadratic function of α, with $\Omega''<0$, there exists a unique $\alpha(k)$ on $(0,1)$ such that $\Omega=0$ for $\alpha=\alpha(k)$, $\Omega<0$ for all $\alpha<\alpha(k)$ and $\Omega>0$ for all $\alpha>\alpha(k)$. And given P^k is clearly increasing in k, $\alpha(k)$ must be increasing in k. The rest of the proposition follows from these properties.

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Online Appendix: The Emergence of a Platform Trap

Andrei Hagiu[1] and Julian Wright[2]

In this online appendix, we provide a fully worked microfoundation for our general model based on the case of a two-sided marketplace.

Suppose there are $N \geq 2$ independent sellers (local monopolists) who are aware of the platform and $N_0 \geq 1$ other sellers who are either not aware or would not be willing to sell on the marketplace. There is a continuum of measure one of buyers. Each buyer wants to buy from one of the independent sellers, with the particular seller a buyer is matched to being randomly drawn with equal probability across all sellers.

We assume sellers always sell in the direct channel. For instance, this may be because there is no costs of doing so and there is no way for them to commit not to do so (we will relax this in an extension below). In addition, the N active sellers can sell on a platform if they decide to list on the platform. In either channel, sellers face a standard downward sloping demand from each buyer, denoted $q_D(p)$ in the direct channel and $q_P(p)$ on the platform, where p is the seller's price which can be different on each channel. Sellers are assumed to have a constant marginal cost of c.

Each buyer faces a search cost of s of using the direct channel (which allows them to see all $N + N_0$ sellers in the direct channel) and so find their preferred seller for sure, with s drawn from some strictly increasing distribution function $G(s)$ with full support over $[0, S]$, where $S > 0$. On the other hand, buyers face no search cost of searching on the platform (which allows them to see all sellers, if any, listed on the platform). However, buyers can only choose one channel.[3]

The platform sets a fee to each seller to participate, denoted P^k for seller k. After each of the N sellers has decided whether to participate on the platform or not, buyers observe how many sellers have decided to participate. Knowing their particular draw s, buyers then each decide (simultaneously) which channel to use (the platform or the direct channel). Since buyers do not know if the sellers they observe are a match until after they go to a channel and discover the sellers there,

[1]Boston University

[2]Department of Economics, National University of Singapore

[3]Equivalently, after searching on one channel, their search costs of searching in another channel becomes prohibitive.

buyers care about how many sellers they can reach on the platform. The more sellers participate on the platform, the greater the payoff a buyer gets from going to the platform relative to the direct channel, since the buyer is more likely to be able to find her preferred seller there.

If n sellers participate on the platform, let $m(n)$ be the measure of buyers that choose to go to the platform, so $1-m(n)$ is the measure of buyers that choose to go to the direct channel. A seller's per-buyer profit in the direct channel is then $\max_p \{(p-c) q_D(p)\}$, where each buyer that goes to the direct channel chooses how many (i.e. $q_D(p)$) transactions to make based on the price p in the direct channel. Let $p_D = \arg\max_p \{(p-c) q_D(p)\}$. Then $\pi_D = (p_D - c) q_D(p_D)$ is a seller's profit per buyer on the direct channel. A seller's expected profit in the direct channel is therefore $\frac{1}{N+N_0}(1-m(n))\pi_D$. Moreover, if the indirect utility function of buyers associated with the demand q_D is denoted $v_D(p)$, a buyer's indirect utility from purchasing from her matched seller in the direct channel is $v_D = v_D(p_D)$.

Similarly, a seller's per-buyer profit from sales on the platform (before the platform's fee) is $\max_p \{(p-c) q_P(p)\}$. Let $p_P = \arg\max_p \{(p-c) q_P(p)\}$. Then $\pi_P = (p_P - c) q_P(p_P)$ is a seller's profit per buyer on the platform. A seller's expected profit from sales on the platform is therefore $\frac{1}{N+N_0} m(n) \pi_P$. And given a buyer's indirect utility function $v_P(p)$, a buyer's indirect utility from purchasing from her matched seller on the platform is $v_P = v_P(p_P)$.

We assume $\pi_P > \pi_D$, so the platform generates higher demand for sellers from each buyer (i.e. $q_P(p) > q_D(p)$), reflecting various transactional benefits the platform adds.

If n sellers participate on the platform, a buyer will go to the platform if their expected utility $\frac{n}{N+N_0} v_P$ is greater than the expected utility of going to the direct channel, which is $v_D - s$. Thus, the measure of buyers going to the platform is

$$m(n) = 1 - G\left(v_D - \frac{n}{N+N_0} v_P\right).$$

To keep things as simple as possible, assume that $S + \frac{1}{N+N_0} v_P > v_D > \max\left\{S, \frac{N}{N+N_0} v_P\right\}$. The first inequality ensures some buyers would go to the platform provided at least one seller has participated. The second inequality ensures (i) no buyers will go to the platform if it attracts no sellers, and (ii) if it attracts all sellers, not all buyers will go to the platform. These assumptions imply $m(0) = 0$, $0 < m(n) < 1$ for all $1 \leq n \leq N$, and that m is strictly increasing in n over the full range of n.

Now we translate this microfounded model to our general setting. Each of the

N sellers is one of the (active) agents. Each of the N_0 sellers is one of the inactive agents. An agent's payoff from participating on the platform when n agents in total participate is $b(n) = \frac{1}{N+N_0}(m(n)\pi_P + (1-m(n))\pi_D)$. Since $\pi_P > \pi_D$, b is increasing in n. An agent's payoff if it does not participate on the platform but n agents do is $u(n) = \frac{1}{N+N_0}(1-m(n))\pi_D$, which is decreasing in n. This is also the payoff of non-active agents. Note

$$\Delta(n) = \frac{1}{N+N_0}(m(n)(\pi_P - \pi_D) + m(n-1)\pi_D),$$

so given $\pi_P > \pi_D$, in this model $\Delta(1) > 0$. This reflects that participating on the platform does not involve giving up on direct channel profits.

Since the payoffs fit our general framework, Corollary 1 implies all N sellers participate and face the platform fee

$$P^* = \Delta(N).$$

They are all worse due to the platform. Instead of getting a payoff of $\frac{1}{N+N_0}\pi_D$ without the platform, with the platform sellers get a payoff of $\frac{1}{N+N_0}m(N)\pi_P + \frac{1}{N+N_0}(1-m(N))\pi - P^* = \frac{1}{N+N_0}(1-m(N-1))\pi_D$, which is strictly lower.

Up till now, we have just focused on the payoffs of the sellers, which we interpreted as the relevant agents (they face the fees and make the strategic decisions). Clearly, buyers are always better off due to the existence of the platform. With the platform, buyers have the additional option to go to the platform, which is desirable if they draw a high search cost s for using the direct channel. Moreover, the value of that option increases as more sellers participate on the platform, reflecting that the platform fee does not feed back into buyer surplus (there is no pass-through in this model).

This raises the question of whether it is ever possible that the platform can induce sellers to participate even though both buyers and sellers could end up worse off? In order to consider this possibility, we modify the above model in two ways. First, sellers are assumed to be limited to one channel. If they participate on the platform, they no longer keep their direct channel. Second, we modify the model so there is passthrough from the fee a platform charges a seller to the seller's price. To do so we assume the platform charges ad-valorem fees (a percentage of a seller's revenue) instead of a fixed fee per seller. When deciding whether to search on the platform or in the direct channel, we continue to assume buyers just observe how

many sellers have listed on the platform, but they do not observe the fees the sellers are charged or their prices.

Let the ad-valorem fee charged to a seller be denoted τ, where $0 \leq \tau \leq 1$. Facing such a fee, a seller's per-buyer profit from sales on the platform is $\max_p \{((1-\tau)\, p - c)\, q_P(p)\}$. Let $p(\tau) = \arg\max_p \{((1-\tau)\, p - c)\, q_P(p)\}$, which is increasing in τ. Then $\pi_P(\tau) = ((1-\tau)\, p(\tau) - c)\, q_P(p(\tau))$ is a seller's profit per buyer, which is decreasing in τ. A seller's gross profit from sales on the platform is therefore $\frac{1}{N+N_0} m(n)\, \pi_P(\tau)$. And given a buyer's indirect utility function $v_P(p)$, a buyer's indirect utility from purchasing from her matched seller on the platform is $v_P(\tau) = v_P(p(\tau))$, which is decreasing in τ. Consistent with the case without any ad-valorem fee, we assume $q_P > q_D$, which implies $\pi_P(0) > \pi_D$.

When buyers decide which channel to go to, they only observe how many sellers are on each channel. Note there will always be at least N_0 sellers in the direct channel. The remaining N sellers will be split between the platform and the direct channel. Buyers do not observe the actual fees charged to different sellers on the platform. Suppose buyers expect a single equilibrium fee τ^*. If n sellers participate on the platform, a buyer will go to the platform if their expected utility $\frac{n}{N+N_0} v_P(\tau^*)$ is greater than the expected utility of going to the direct channel, which is $\frac{N-n+N_0}{N+N_0} v_D - s$. Thus, the measure of buyers going to the platform is

$$m(n) = 1 - G\left(\frac{N-n+N_0}{N+N_0} v_D - \frac{n}{N+N_0} v_P(\tau^*)\right). \tag{7}$$

We modify our previous assumptions to this new setting, so assume $S + \frac{1}{N+N_0} v_P(\tau^*) > \frac{N+N_0-1}{N+N_0} v_D$, $v_D > S$, and $\frac{N_0}{N+N_0} v_D > \frac{N}{N+N_0} v_P(\tau^*)$. These assumptions again imply $m(0) = 0$, $0 < m(n) < 1$ for all $1 \leq n \leq N$, with m strictly increasing in n over the full range of n (for given τ^*). Furthermore, to focus on the simplest case without any seller being pivotal, suppose $m(1)\, \pi_P(0) \geq \pi_D$.

Consider the last (active) seller deciding and facing the fee τ^N assuming N' other sellers have already joined the platform. If the seller joins it will get

$$\frac{m(N'+1)\, \pi_P\left(\tau^N\right)}{N+N_0}$$

while if it does not join, it will get

$$\frac{(1-m(N'))\, \pi_D}{N+N_0}.$$

The seller will participate provided τ^N is no more than the fee τ satisfying

$$\pi_P(\tau) = \frac{1 - m(N')}{m(N'+1)}\pi_D. \tag{8}$$

Since $m(1)\pi_P(0) > \pi_D$ and $m(1)\pi_P(1) = 0 < \pi_D$, there exists a unique value of τ solving (8), and the platform can always induce this last seller to join even if no other sellers have joined by setting this fee. Since the right-hand side of (8) is decreasing in N' but is always strictly positive, while the left-hand side of (8) goes to zero as $\tau \to 1$, there is always a unique τ solving (8) for any $N' \leq N-1$. Based on the same logic as in our baseline analysis, the platform can therefore induce all sellers to join by charging appropriate fees to each seller. The equilibrium fee is the unique τ solving (8) when $N' = N-1$, given each seller expects all other sellers to join in equilibrium.

In summary, the equilibrium τ^* is the τ solving (8) when $N' = N-1$ and $m(N-1)$ and $m(N)$ are determined by (7).

Active sellers end up with expected payoffs $\frac{1}{N+N_0} m(N)\pi_P(\tau^*)$, which after using the equilibrium characterization of τ^* is the same as what inactive sellers get, which equals $\frac{1}{N+N_0}(1 - m(N))\pi_D$. This compares to getting $\frac{1}{N+N_0}\pi_D$ in the absence of the platform. So clearly all sellers are worse off given $m(N) > 0$.

Now consider buyers' expected utility. Without the platform they get $v_D - s$, with s drawn on G. With the platform they get

$$\max\left\{\frac{N}{N+N_0}v_P(\tau^*), \frac{N_0}{N+N_0}v_D - s\right\}.$$

Buyers with $s < \frac{N_0}{N+N_0}v_D - \frac{N}{N+N_0}v_P(\tau^*)$ are clearly worse off, since they go to the direct channel still but now they only find their match a fraction of the time. Buyers with $s \geq \frac{N_0}{N+N_0}v_D - \frac{N}{N+N_0}v_P(\tau^*)$, get $\frac{N}{N+N_0}v_P(\tau^*)$ since they use the platform versus getting $v_D - s$ absent the platform. Clearly, the marginal buyer is worse off with the platform. Under our assumptions, it is possible that all buyers are also worse off, which arises if $\frac{N}{N+N_0}v_P(\tau^*) < v_D - S$. Otherwise, some buyers that use the platform will be worse off and others will be better off compared to the case without the platform, depending on their draw of s.