2024

WEALTH INEQUALITY AND THE MERIT TRAP, A NEW PRISONER'S DILEMMA:

WHY COMPETITIVE MERIT WORKS FOR SURPRISINGLY FEW OF THE MOST TALENTED AND WHY THE ANSWER MAY BE MORE OF THE SAME

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Contents

ABSTRACT 1
INTRODUCTION 1
Wealth Inequality 1
Wealth's Statistical Distribution - Fat Tails make Fat Cats 2
A Distribution in Multiple Parts 2
Cutting the Tail and Fitting the Curves 3
Creating the Distributions in a Lab: Random Asset Exchange Models (RAEM) 3
Diving Deeper Into the RAEM 4
The Exchange (Competition) Stakes Format – Fixed vs Percentage, Fair vs Unbalanced 4
Probability Manipulation vs Random Exchange 5
RAEM Results: Condensation vs Equilibrium 6
The Theory Behind the Distribution 7
A Call to Arms 9
METHOD 9
Competition Based Exchange 9
Economic Agency: Profit Maximisation and Perfect Market Knowledge 13
Competition Model (The Stakes) 13
Population Sampling 14
RESULTS 14
Preliminary 14
Condensation vs. Equilibrium 15
Population Endpoint Effect 15
The Competition/Merit Index and its Effect on Wealth Distribution 16
AACM Model vs. Random Exchange Model 22
Wealth Distribution Shape Contrast 22
The Effect of Economic Agency 23
Macroeconomic Data – Results Comparison 24
Preliminary Note 24
Professional Tennis Ranking Points: A Pure, 2-Agent Competition 24
Company Market Cap Value vs. AACM Model 27

Individual Worldwide Wealth vs. AACM Model 28
Individual Income vs. AACM Model 31
DISCUSSION 31
The Competition/Merit Index and a Competition Equilibrium 32
Competitive Equilibrium or Maximum 32
Not By Chance 34
Not a Matthew Effect 35
Not a Percentage Effect 35
Further Speculation 36
A Lottery Economy 36
Participation Rates 36
Gini-max CM Breeds Exceptionality 37
Mechanistic Implications 37
Retain Participants where AA can fluctuate 38
CONCLUSION 38
REFERENCES 39
APPENDIX 42
Python Code for AACM program 42

ABSTRACT

Wealth inequality is both a long-standing and acute economic issue. Wealth distribution is characterized by asymmetry and a fat upper tail of extreme values: a few hold a lot, and a lot hold little. Random Asset Exchange Models (RAEM) have entered to mimic the mathematical/statistical distributions describing empirical wealth data with some success, but often requiring segmented multiple cause/distribution explanations and fundamentally lacking explanatory "*concrete system dynamics, common to all market economies, which generate the characteristic features of inequality"* (Greenberg and Gao, 2024, p.27).

We introduce the AACM Model, which adds (1) competition predicated on a [normally distributed] talent/ability/advantage (AA) metric and a competition/merit (CM) variable that translates the AA differential to win probability and (2) economic agency in the form of profit maximation and perfect market knowledge.

The AACM succeeds with uncanny accuracy in modeling the wealth distribution's entirety (not just tails) across the spectrum of economic and pseudo-economic competitive markets and actors: tennis ranking points, corporate market cap, individual wealth and income worldwide. In doing so, it reinforces the possibility of a singular fundamental economic/competitive mechanism.

The AACM demonstrates that while initial increases in CM competitively advantage the above-average AA creating increasing wealth inequality, further CM increases favor progressively fewer of only the highest AA until a reversal-point at maximum inequality (the Gini-max) is reached. Thereafter, additional CM increases reduce inequality.

Empirically, the current economic data all fit the AACM very near the Gini-max CM suggesting it may be an equilibrium point created out of a prisoner's dilemma style behavior.

INTRODUCTION

Wealth Inequality

Every paper on the subject begins the same, so it almost seems unnecessary to outline the case. "What is indisputable… is that in nearly every single developed market economy, the degree of stratification between the rich and everyone else is not only staggering, but is also increasing." (Greenberg and Gao, 2024, p.2)

This statement contains two elements worthy of highlighting. First, this is a phenomenon pertaining to market economies. One could instinctively understand high degrees of inequality in command economies and petrol kingdoms, but this is a capitalist problem. Second is the arguably ambiguous use of 'increasing'. Does this imply a continuation of a process or an escalating rate?

Wealth inequality is not a recent phenomenon, and its observation in 1897 led civil engineer Vilfredo Pareto to turn mathematical economist in order to explain: describing its distribution shape which would take his name. (Yakovenko and Rosser, 2009). As we will see, the Pareto distribution was valid then and probably remains. So, has something actually changed, and if so, what?

Today, the UBS Global Wealth Report 2023 lists that the bottom 52.5% of the population collectively holds only 1.2% of global wealth, the top 13.1% has 85.2%, and the top 1.1% holds 45.8% of the total (Davies, Lluberas and Shorrocks, 2023).

The 'top-down' process which creates this outcome is elucidated by a multi-decade progression studied by Atkinson (2009). Referring to his income data from 1976-2007, "[i]t shows that most of the changes in the top decile are due to *dramatic* [emphasis mine] changes in the top percentile…[T]he top 1% captured 58% of real economic growth per family during that period" (p.4-5). Using income, rather than the related wealth, he shows what appears to be a top 10% growth is primarily driven by the top 1%.

Wealth's Statistical Distribution - Fat Tails make Fat Cats

Were wealth distribution to take the shape of the world's most famous Gaussian distribution, it would be characterized by a central heavy loading with relatively few extreme, or 'tail' values. A tiny portion of the population would have little wealth (the left tail), and an equally small portion of the population would have great wealth (the "thin" right tail). Extreme wealth would have a vanishingly small occurrence.

Observing this was decidedly not at all representative of wealth, the godfathers of economic statistics, Pareto and Gibrat, sought to do better.

Studying **only** the upper tail of wealth, Pareto concluded what would become the Pareto or Power Law (PL), which is that the probability of occurrence of wealth (w) at a given value would decline as a power of α (the Pareto constant).

$$P(w) \propto w^{-(\alpha+1)}$$

Smaller α values create fatter tails, higher probability in the extremes, and therefore more wealth concentrated in those hands and greater wealth inequality (During et al., 2008).

Gibrat postulated a size-indifferent proportional growth (an essentially multiplicative process) would create a lognormal distributions and compared this to manufacturing firm size and incomes in the early 20th century with good results (Sutton, 1997).

A Distribution in Multiple Parts

It was not part of Pareto's undertaking, but it quickly became evident that while PL may describe the right tail, it did not represent the remainder of the distribution well. "Analysis of current economic

data seems to indicate that Pareto's law is valid for the high income strata of society, while for middle and low income classes the distribution appears to be a log-normal (Gibrat) distribution… [i]t is possible to make a power law fit for the richer countries, while the others seems to be better fitted by an exponential or log-normal distribution". (Iglesias, 2004)

A good quantity of research is dedicated to categorizing whether (model and observed) data is best fit by PL, exponential (Boltzmann-Gibbs), asymmetric Gaussian, log-normal, Gamma, Zipf (a PL where $\alpha = 1$), Lotka-Volterra or other mathematical distributions individually or in combination with another and for what range of the data. (see Yakovenko and Rosser, 2009; Bouchaud and Mezard, 2000; Chakraborti and Chakrabarti, 2000; Iglesias, 2004; Gabaix, 2008 and see also below The Theory Behind the Distributions)

Cutting the Tail and Fitting the Curves

The above raises a few complicating issues when 'fitting' curves to data which becomes the indicator of validity of a theorem (whether mathematical or economic).

Gabaix (2008) studies a related field of city-size scaling which in many respects mirrors wealth scaling. He highlights the problem of using only the arbitrary tail of a set. He asks, "[h]ow does one estimate a distributional PL? Take the example of cities. We order cities by size $S(1) \geq \ldots \geq S(n)$, stopping at a rank n that is a cutoff still 'in the upper tail.' There is not yet a consensus on how to pick the optimal cutoff." (p. 45 and see Beirlant et al., 2006).

He goes on to raise the issue that equations that have more manipulable parameters may have an easier time fitting a data set than the single-parameter Pareto, but without substantive relevance. He observes, "sometimes a lognormal fits better… The reason, broadly, is that adding more parameters (e.g. a curvature), as a lognormal permits, can only improve the fit… The extra degree of freedom allowed by a lognormal might be a distraction from the 'essence' of the phenomenon." (Gabaix, 2008, p.47) The Gamma distribution similarly has two parameters that make a profound difference in the overall curve shape (Wikipedia Contributors, 2020).

Creating the Distributions in a Lab: Random Asset Exchange Models (RAEM)

Almost each of these title terms is contradicted by a model within the genre, and it is not the present purpose to do complete justice to any particular model but rather to convey the basics to understand the background of the model we will propose.

The following offer excellent reviews of the early, mid, and latest status of the field: (Hayes, 2002; Yakovenko Rosser, 2009; Greenberg and Gao, 2024).

RAEM encompasses a diverse range of experimentation on similar lines: model a macro-economic outcome on the basis of a series of micro-economic interactions that are treated as physical or arbitrary in nature, like particle physics, giving way to observable phenomena. The macro-

economic outcome sought to be modeled or explained is typically wealth/income distribution among the participants.

A basic (game) model would be:

- 100 participants begin with an equal number of wealth units (coins).
- 2 participants are selected at random to 'trade/exchange' with each other with one designated [with equal chance] to give a coin of its own to the other.
- The experiment continues for a large number of exchanges in this zero-sum format.

The final distribution of wealth is tallied by organizing the participants by wealth (coins) from low to high and the distribution shape is characterized mathematically.

The surprising result from an initial equitable distribution and pure chance exchanges is that a very large proportion of the wealth of the system (coins) winds up in the hands of very few participants. (Dragulescu and Yakovenko, 2000; Ispolatov et al.,1998)

The similarities to wealth and other unequal distributions become apparent.

Within the modelling of this 'game' lies every permutation of each rule; the goal is to create a system whose process is both authentic to economic transactions and whose result is realized in empirical fact. Randomness is not a manifest conception but rather a tool to demonstrate a quasi-physical process that does not rely on structure to take its effect. There is no structure as among whom the exchanges are undertaken, but the large number effect should ensure reasonable equality of transactions among participants. Participants begin as identically situated equals, and exchanges occur randomly and with equal probability.

Diving Deeper Into the RAEM

Very simply and broadly speaking, we can divide the game rules into a few subsets according to the exchange format and the probability assignment.

The Exchange (Competition) Stakes Format – Fixed vs Percentage, Fair vs Unbalanced

Mostly implicit, the literature often skirts the nature of the micro-economic transactions which compose the money/income/wealth distributive result. In describing their methods, researchers characterize the participants as a 'loser' (a net transferor of wealth) and the other a 'winner' (the recipient) while framing the transaction as a trade or an exchange. This language, however, is more suggestive of a competition or a bet and pure economic competition or financial speculation is easily conceived. (see Moukarzel et al., 2007) More inclusively, John Angle describes the transfer as reflecting which agent got the better end of the deal (1986). For example, in a sale of a commodity, the purchaser recognizes a hidden value, and the seller undervalues its present worth. Alternatively, where the commodity is exchanged at correct, momentary present worth but subsequent appreciation makes the sale effectively a loss to the seller and gain to the acquirer.

Combining Yakovenko with Angle's notion that 'wealth' means surplus wealth rather than simple money exchange. In such a case, the wealth transfer transaction could represent, for example, an exchange of labor for salary by agent 1 and agent 2. If agent 2 were to realize value from the fruits of that labor in excess of the salary, there would be a corresponding wealth transfer in favor of agent 2. Conversely, if agent 2 hired agent 1 to perform futile work, failed research and development perhaps, agent 2 would reduce its wealth while agent 1 would still gain its salary.

We adopt all of the economic explanations and will refer to these interactions/exchanges in the language of competition/wager.

The first categorical permutation is between fixed stakes and percentage-based ones. Fixed stakes are defined as an absolute quantity and do not depend on the wealth holding of either participant. Percentage stakes are defined by reference to some percentage of the wealth of one of the participants (1%, 5%, 10%...). The Percentage Stakes method can then be subdivided into sub-categories on the basis of the percentage of which (winner, loser, wealthier, or poorer) agent/participant.

A further permutation would be to categorize the Stakes as Unfair or Fair based on whether the winner has as much to gain as the loser has to lose.

The Fixed Stakes are definitionally fair (with an exception to be discussed below). The winner and loser transfer/receive the same amount. (Dragulescu and Yakovenko, 2000)

The Percentage Stakes can be Fair or Unfair. Fair Percentage Stakes would occur when defining the stakes with reference to a percentage of either the wealth of the richer or poorer agent and transferring that amount from loser to winner. Each agent has the same amount at stake and in the context of random exchange, equal probability leading to a net zero expected transfer. Unfair Percentage Stakes would occur when defining the stakes with reference to percentage to (typically) the loser's wealth. In this model, the wealthier agent will always have more to lose than to win. 5% of high wealth agent vs. 5% of low wealth agent does not create parity of wager; it is an unfair bet. (see Caon et al., 2007; Moukarzel et al., 2007; Iglesias et al., 2004)

Probability Manipulation vs Random Exchange

Early in the RAEM evolution and appearing consistently, researchers have experimented with varying the exchange probability from neutral/random to bias probability in a particular direction. Iglesias et al. (2004) introduced a probability bias in favor of the poorer agent in an exchange as "a way to simulate the action of the state or of some type of regulatory policy that tries to redistribute the resources" (p.3). Moukarzel (2007) continued this line and found that setting probabilities even considerably above .5 for the poorer agent continued to result in wealth condensation (see below). Boghosian (2017) and continued by Devitt-Lee et al. (2018), introduces what he characterizes as 'wealth attained advantage' in which the wealthier agent is favored by a probability metric which is dependent on the wealth differential between the interacting agents, the larger the wealth

differential, the more the probability is tipped towards the wealthier agent. This is alternatively to simulate the economic advantages created by the position in a definitionally Matthew Effect structure (described below).

Pre-dating all of these, Angle (1986) made provision for both sides proposing that the wealthier agent would have a probability increase commensurate with those advantages but also that a poorer agent should have a probability increase in recognition of its increased interest in hanging on to remaining wealth (a marginal utility argument).

Pluchino et al. (2018) interestingly introduce a 'Talent vs. Luck Model' which is not an exchange model but possesses similar features and seeks to create and explain similar PL distributed success outcomes along the same lines as the asset exchange models. They create a population set with a normally (Gaussian) distributed 'talent' metric and use this metric to vary the possibility that the agent can capitalize on randomly occurring 'lucky' events. "A lucky event intercepts the position of the agent… as a consequence, agent doubles her capital/success with a probability proportional to her talent" (p.1850014-6). Conversely, unlucky events randomly (without regard to talent) intercept the agent and half the capital/success. This is a non-interactive model and sets agents on independent random walks and charts the wealth/success endpoints for each finding. They observe a "Pareto-like power-law distribution… whose tail is well fitted by the function y(C) ~ C^-1.27. (p.1850014-6)"

RAEM Results: Condensation vs Equilibrium

The term 'condensation' seems to have been coined by Bouchaud and Mezard (2000) to describe the outcome where all (or functionally all) wealth transitions to the hands of a single agent (or very finite number). It is a largely undesirable outcome from an economic rationale for two reasons. First, in the millennia of economic activity, all wealth has demonstrably not collapsed into singular ownership. So, its usefulness in predicting real-world outcomes is minimal. Second, any transitional state that may actually represent a real-world wealth distribution would be fleeting from a model perspective and an economic researcher would be left with the very difficult task of describing why that transitory snapshot is rationally represented at a given moment.

The other, more desirable outcome is that the model produces a stable distribution: an equilibrium state. The existence of a stable state gives credence to the notion that it represents a 'natural' phenomenon, and the end-state can then be compared to real-world economic data. The model (and economic data) can be compared to mathematically defined distributions.

In a literal sense, condensation is created by agents dropping out of the game by hitting zero wealth (the Bankrupt Agent), having nothing to trade, and leaving fewer agents to trade until the singularity is reached.

Very generally speaking, to avoid condensation the exchange model must violate fairness in one of the above-described respects. Dragulescu and Yakovenko (2000) avoid the Bankrupt Agent

problem and condensation in their Fixed Stakes type model by keeping Bankrupt Agents in the game; these agents can win, but they cannot lose. In this experiment, when a random pairing is made between, for example, agent 1 (no wealth) and agent 2 (with wealth). The transaction is simply disregarded if the random pairing stipulates agent 1 as the transferor. The exchange goes ahead if the random pairing stipulates agent 2 as the transferor. Though this does not disturb the authors, whose physics-based rationale is heat exchange in a closed system, it does rather disturb the economic analogy: why would agent 2 agree to an economic transaction in which it has only potential downside. This, memorably, led Hayes (2002) to dub this class the 'theft and fraud' model because it seems to allow an economic mugging.

Similar problems arise in the second camp which uses the Percentage Stakes models of competition. This approach sets the stakes as a fixed percentage of each agent's wealth. However, this raises similar rationality arguments as to why the wealthier agent would agree to risk a greater amount against a lesser (Hayes, 2002). Another sets the stakes as the percentage of the poorer agent against the same absolute amount of the richer agent. This solves the Bankrupt Agent problem, and creates stake-parity fairness, but does not functionally solve the condensation problem because a set of agents face what Hayes called a 'Zeno walk' (2002), where diminishing wealth prevents recovery by also diminishing gain prospects. It leaves them functionally, if not literally, bankrupt. This may or may not have a rational economic tie-in: a failed enterprise may have fewer opportunities to re-enter a large market (lost partnerships, bank lending, etc.) and may have the opportunity only to pursue smaller markets.

These models solve some problems but create others. A loss being capped at a percentage of assets eliminates the above Bankrupt Agent problem, provided the percentage cap is of the poorer agent's assets but this may not stop condensation.

The Theory Behind the Distribution

Well over a decade ago, Xavier Gabaix (2008) beckoned for a theoretical economic answer to these distribution formations, and Greenberg and Gao (2024) highlight the relative lack of response.

Empirical data matching a PL distribution "prompts [us] to seek theoretical explanations of why this should be true… Is there an explanation for the PL distribution of firms that is not based on a simple "mechanical" Gibrat's law, but instead comes from full efficiency maximization?... Is there any "natural" mechanism, perhaps random growth for the accumulation or detection of talent, that would generate that distribution?" (Gabaix, 2008, p.48)

"[M]ainstream economic theory has, broadly speaking, shied away from further attempts to impose a universal form to these distributions or to explain the processes responsible for their emergence. There are both normative and methodological reasons for this gap in the economics literature. The normative aversion, as voiced by Piketty, manifests as skepticism that a universal law governing the right tails of income distributions exists at all (Piketty, 2014). The methodological aversion, on the other hand, stems

from the fact that most macroeconomic models make use of single, representative agents, which are ill-suited for describing heterogeneity within a population." (Greenberg and Gao, 2024, p.5)

There have, however, been some explanatory attempts that span the purely physical, socio-political, economic, and even external causation, such as tax structures.

The pure physics explanation that "economic inequality is the result of luck and the inherently diffusive (entropy-increasing) nature of [heat] exchange itself" (Greenberg and Gao, 2024, p.26) is entirely hollow and unsatisfying from a substantive economics perspective.

Another commonly held view is socio-political: there is a rigged game, a Matthew Effect. A Matthew Effect is so termed based on the Bible verse indicating the rich will be given more and the poor will have taken from them. It requires not only a system in which the rich get richer, and the poor poorer, but that they do so *because* they are rich or poor. (Perc, 2014) From a model perspective, this could be incorporated by raising the win probability via the 'wealth attained advantage' in the research referenced above.

A Matthew Effect could easily create a heavily skewed distribution, but is a skewed distribution necessarily an indication of the Matthew Effect? Given the existence of PL distributions, certainly, economics would not enjoy an affirmative answer.

Yakovenko and Rosser (2009) make the mathematical argument that Fixed Stakes (our terminology) transfer models like the Dragulescu and Yakovenko (2000) will generate exponential Boltzmann-Gibbs distributions and that Percentage Stakes exchange models will generate 'Gamma-like' distributions which are closely related to PL.

They plot both UK data for wealth and US IRS data for income, finding and tracing the exponential nature of the lower wealth/income data and a PL in the upper tail. (Yakovenko and Rosser, 2009) Then, referencing also Levy and Levy (2003), they speculate that the "upper tail of wealth distribution is dominated by investment assets" (p.14) which would presumably produce returns on a multiplicative Percentage Stakes model. Whereas in the lower wealth/income "the corresponding people do not have other significant assets" (p.14), so their exchange would be appropriately modelled by the Fixed (or additive) Stakes, a wage analogy. The Fixed (additive) exchanges would reflect in a Boltzmann-Gibbs distribution in the lower ranges while the upper ranges would reflect the Gamma-like, PL of the Percentage Stakes (multiplicative) returns (Yakovenko and Rosser, 2009; Levy and Levy, 2003).

These arguments have a great deal of intuitive merit; the separation of (wage) labor and capital seems to tap into some economics fundamentals. But is it true?

A Call to Arms

> *"the literature should move in the direction of more explicit representations of economic structure and processes to acquire greater explanatory power … the literature has not yet been able to provide an adequate explanation for the emergence of these distributional features which it posits to be universal… it has not yet been able to* ***identify and describe the concrete system dynamics, common to all market economies, which generate the characteristic features of inequality.****" … "[D]etermining what policies would be required to create a fairer society and economy necessitates a clearer understanding of the principal processes responsible for generating inequality in the first place, and much work remains to be done before such a satisfactory understanding is reached."*
>
> (Greenberg and Gao, 2024, p.1,26-27 emphasis added)

> *"The availability of large new datasets makes it possible to discover new PLs, and test the models' predictions about microeconomic behavior. Times seem ripe for economists to use those PLs, and renew the tradition of Gibrat, Champernowne, Mandelbrot and Simon, and investigate old and new regularities with renewed models and data."*
>
> (Gabaix, 2008, p.50)

"Challenge Accepted" - Barney Stinson

METHOD

Competition Based Exchange

Competition is the fundamental element of the market economy. Paul J. McNulty (1968) analogizes that what "[Adam] Smith did for economics, through the principle of competition, precisely what he himself credited Newton …for physics and astronomy through the principle of gravity" (p.646); he goes on to highlight that even Smith was not the originator of this idea, but it was "already well developed in the economic literature of his time" (p. 647). Competition is the mechanism by which the superior element is selected at the expense of the inferior. Basic economic theory suggests this is the fundamental force that pushes the market towards more efficient use of resources and provides more, cheaper, superior, differentiated goods to consumers. It is as fundamental as gravity and older than economics' Newton.

We characterize these interactions/exchanges as micro-competitions whether in the nature of speculation (as might be in financial markets, participants wagering in opposite directions on current and future values), exchange of capital/labor, or price competition in the production of goods. A firm may enter a market looking to undersell and take market share (wealth) from another

firm to build volume, capitalize on economies of scale, improve efficiencies, and thereby accede to the competitor's wealth. Or, it may fail and, in doing so, yield its own wealth.

We propose that, as in any competition, the participants will vary in characteristics which would make them more or less likely to win relative to basic chance: 50/50 in a 2-participant competition.

We propose an 'arbitrary talent/ability/advantage' ("AA") metric as the singular, invariant, and normally distributed determinant of advantage/disadvantage among agents in the system which will vary the likelihood of prevailing in the competition away from its even baseline. Relating this to firms, it could reflect the quantification of institutional intelligence, efficiency, experience, adaptability or the aggregation of all of these factors to the single model norm. On an individual level, it could be considered ability or talent. It is chosen to be normally (Gaussian) distributed as this is a common and naturally occurring pattern. A similar approach has been taken by other researchers including, as mentioned, Pluchino et al. (2018).

Additionally, the normal probability distribution is centrally heavy and thin-tailed at the extremes. These features make it likely to be environmentally representative; a profoundly dominant advantage is an extreme outlier. As such, it would be unlikely that such a model would *a priori* skew the resultant distribution to the extremes.

Degree of competition ("CM"), how perfect it is, is something of a term of art. We propose a differentiation of the level of competition which measures the likelihood of the superior agent (as defined above) to succeed against the inferior agent. In this sense, 'perfect' competition indicates 100% certainty that the better agent wins (high CM). Degrees of imperfection increase until we reach the state of 50/50 randomness, where superiority has no advantage over the economic competition (low CM). In this way, merit and competition merge (hence "CM"). The reward of ability advantage is in the meritorious award of its benefit.

By way of illustration, chess would be considered (nearly) perfect competition because a worse player has virtually no chance of winning against the better. Backgammon would be a less perfect competition because the role of chance variation may overwhelm in any given game; the better player would win with regularity but not universality. In the economic domain, there is a substantial overlap with conventional usage. Consumer irrationality – sticky product or brand loyalty, market dominance, anti-competitive practices like product tying, barriers to entry of other sorts, and especially corruption are market occurrences that equally suit either definition in the same ways. These confounding economic behaviors counteract and reduce the 'perfection' of competitive forces.

To quantify the above, for AA, we take the common statistical z -score of each agent which is simply

$$z = \frac{x - \mu}{\sigma}$$

Where:

z is the z-score.
x is the value of the data point.
μ is the mean of the population.
σ is the standard deviation of the population.

the agent's distance from the mean in terms of the population's standard deviation. We compare the agents to each other $\Delta z_1 = z_1 - z_2$ to get the simple difference between them (the 'advantage gap'). The 'advantage gap' is conceptually identical to the "standardized mean difference known as Cohen's *d*" (LeCroy, 2007); it is a difference measure "void of measurement considerations" (p.244).

In order to translate the AA's Δz to a probability of the superior agent's likelihood of success, we take inspiration from the Binomial Effect Size Display (BESD) in the social sciences (LeCroy, 2007) which translates a statistical observation of difference into a probability. Therefore, we define:

$$r = \frac{\Delta z_1}{\sqrt{\Delta z_1^2 + CM}}$$

where

CM = Index of Competition/Merit (the "CM index")

and

$$P(\text{ Agent } 1 > \text{ Agent } 2) = .5 + \frac{r}{2}$$

Therefore, as the CM index approaches 0, $\lim_{CM \to 0}$, competition becomes increasingly perfect because $r \to 1$ and therefore $P(\text{ Agent } 1 > \text{ Agent } 2)$ =1. Even a small Δz will result in the superior agent's success. As the index of competition rises, r becomes a smaller fraction, and the probability advantage falls, ultimately approaching .5 random chance regardless of the magnitude of Δz.

By way of examples, Table 1 below shows the probability of success of the higher advantage/ability (AA) agent against a lower-tiered agent across competition/merit (CM) levels. The first row shows the probability of one of the highest AA agents succeeding against one of the lowest: the 95th percentile agent to win against the 5th percentile agent. At a CM index of 1000, the agent's probability of success is only somewhat raised relative to random chance: 55.17% vs. 50%. As the CM index decreases (competition/merit increase), that probability of success increases until practically reaching 100% certainty. The bottom row shows a competition between more average and closer AA agents: a 55th percentile vs. 45th. At CM=1000, the success probability barely elevates from basic chance at 50.4%. As CM increases by factors of 10, the success probability rises until becoming above 99%.

Table 1

Probability of Win

	CM Index						
Agent Percentile	**1000**	**100**	**10**	**1**	**0.1**	**0.01**	**0.001**
P95 v. P5	55.17%	65.62%	86.05%	97.84%	99.77%	99.98%	100.00%
P75 v. P25	52.13%	56.68%	69.62%	90.17%	98.68%	99.86%	99.99%
P55 v. P45	50.40%	51.26%	53.96%	62.19%	81.11%	96.46%	99.61%

Table 1: shows the probabilities of an agent of given percentile AA to succeed against the indicated lower AA percentile agent across several CM index levels (from low to high competition/merit).

Similarly, Fig. 1 below shows an agent's probability advantage [above chance] against its immediately below agent in a 100-agent population. The X-axis has the agent range in percentiles and the Y-axis has the probability advantage (above 50%). The 'U' shape does not change substantially for most measures of CM; rather, the Y-scale changes in magnitude.

Figure 1

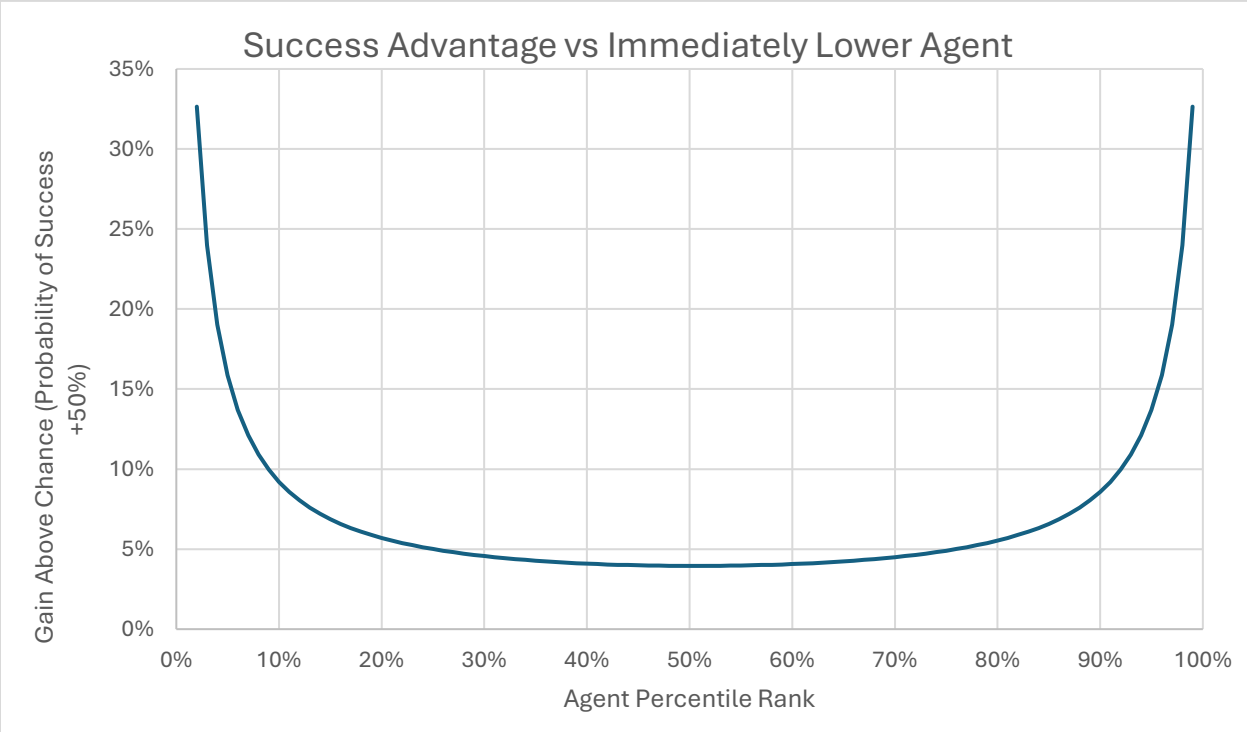


Figure 1: shows the advantage that an agent of a given percentile AA rank has over the agent of a single percentile lower expressed as a percent increase above 50% (CM=.1). The U-shape slope does not substantially change across the typical CM range but the Y-scale changes magnitude.

The model defines its participants as representing a normal distribution of abilities/advantages (AA) and varies the regularity and certainty by which ability/advantage are rewarded by an index variable which will be referred to as the competition/merit (CM) index.

Economic Agency: Profit Maximisation and Perfect Market Knowledge

We introduce economic agency via ordered trade initiation and rational challenge selection. As previously discussed, it appears that random agent pairings were introduced to ensure large-number equity of transactions among agents and emphasize the 'natural' cycle progression. As regards the first, this can be at least as fairly attained by using sequential cycling through the agent population, like turns in a board game. Regarding the second, we take inspiration from game theory and economic theory. Both routinely use rational profit-maximizing agents with perfect, complete market knowledge as a simple and verifiable baseline reasoning.

The method introduces a 'challenge' or 'king of the hill' system. Each 'round' is composed of a 'turn' for each agent. On the agent's turn (the "challenger"), it makes a challenge to another agent (the "target"). The challenger chooses the target based on the *maximum expected value* of all targets. Knowing the probability of success derived from the given (assigned) arbitrary advantage in conjunction with the above formula and multiplied by the stakes (discussed below), yields the expected gain.

It is here that the model allows for manipulation of decision-making. It can be set to maximize on the basis of all alternatives: the total expected value which is the expected value of a win, less the expected value of a loss (the "prudent" model). It can be alternatively manipulated only to take risky or risk-averse mentalities. A risky mentality would choose the highest expected value from a success without regard to the expected loss from a failure. The opposite consideration would prevail for the risk-averse. While it is possible to change this metric, all experiments here were conducted with 'prudence' considering the net expected value.

Competition Model (The Stakes)

The competition stakes are delineated as follows: a defined percentage of the target's wealth (in this case we have set this at 5%). The winning agent receives this absolute amount from the losing agent. Should the challenger lose and not possess enough wealth as the Stakes, it forfeits all it has to the target.

We recognize that this revives both of the criticisms of prior models. This allows for bankruptcy and the unequal bet. These are both acceptable in the economic analogy. First, it is conceptually feasible for an agent without wealth to challenge a wealthier agent. As 'king of the hill', the wealthiest agents don't really want to be challenged as they have little to gain, possessing most or all of the wealth of a given system. It is, however, very much in the nature of competitive markets that they will be forced to defend their possession of the wealth. Conceptually, a penniless programmer could write a better operating system to garner the wealth of Microsoft's market share. It doesn't require an economic mugging, and it does not require parity of wealth. Ford could challenge Tesla's wealth in the realm of EV vehicles. Should Ford fail, its loss would be capped at what it has, but its capacity for gain would not be. It's gain is a function of the target's wealth (e.g. market share) that is looks to take.

In this sense, labelling this approach as a Percentage Stakes model is probably inappropriate. It has parity of stakes not present in the Percentage Stakes denoted by each's own wealth (percent of the loser). It does not trap lower wealth into the Zeno walk as does the percent of poorer stakes.

Population Sampling

One hundred agents are used (with some exceptions as below) to make a representation of the population commensurate and are defined as the percentile ranges of the population with the concomitant, normally distributed 'arbitrary advantage'. This ensures a smooth sampling with equal cumulative probability density between agents.

This leaves the definition of the first and last agent. As the 100th and 0th percentiles yield undefined z-statistics, we use a 'population set' value to identify the hypothetical population size from which the 100th (top) agent sample is taken. The bottom percentile agent is defined as 1/(population sample size * hypothetical population size), and the top agent is defined as 1 minus that amount. The remaining agents are divided equally between those endpoints on a percentile(CDF – cumulative density function) basis.

A Python computer program (attached as an Appendix) runs the trials with the parameters designated. Each agent in turn will choose its opponent based on the maximum expected value of all opponents. In executing the competition, it uses a random number generator to select a value 0 to 1 with uniform probability. If the decimal lies at or below the challenger's probability of success, it wins, otherwise, the target wins. The above-defined stakes are transferred from loser to winner. A 'round' is a complete cycle of all agents acting as challenger in turn. The 'trial' is completed at the conclusion of a designated number of rounds. A set of trials (of determined number) is completed and each agent's results (the terminal wealth value) are averaged across trials and reported as the results.

We have chosen 250 rounds as the duration and smoothed the data set by averaging over 50 trials. We have chosen 100 agents (basically each representing a full percentile range) to demonstrate the model behaviors and 400 agents for empirical comparisons such that each agent would represent a more granular ¼ percentile range.

Taken together, we refer to this method as the AACM Model.

RESULTS

Preliminary

A principle difference to the random exchange results should be made clear as a preliminary reporting matter. The typical RAEM does not differentiate among agents but rather categorizes them

into percentiles based on the wealth attained over the trial. At the end, agents are put in rank order of wealth and statistics are aggregated on that rank order, per e.g.: the top 10 percentile held 50% of the total wealth, the bottom 10 percentile held 2%, etc. Because agents are equal and identical and exchanges are random, the terminal, vastly unequal wealth distribution is both notable and surprising. Moreover, all wealth inequality is an 'unjust' or non-meritorious result.

By contrast, in the AACM model, agents are defined on the percentile spectrum of Advantage/Ability and wealth distribution results for each are reported accordingly and not re-ordered by wealth result. Therefore, it is expected, and one may certainly argue justfied, that high AA agents will attain greater wealth. However, given random chance is still an element of the model (depending on CM), unexpected peaks and dips may appear in the results.

Condensation vs. Equilibrium

Wealth inequality and some degree of concentration of total wealth into a small minority of agents and large percentages of agents with negligible wealth has become nearly the expectation in the models. However, a model which rapidly transfers wealth to a single agent and where a large number of agents have null wealth or are dropped from interaction is seen as both theoretically and practically problematic.

Varying the number of rounds in the trial does not reveal a strong condensation effect in the AACM model. Table 2 shows the final percent of total wealth attained in the listed percentile ranges for increasing lengths of trials. Condensation would tend to predict the rapid rise of wealth in the upper percentiles while draining the lower and later middle percentiles. We do observe an evolving wealth redistribution to the upper percentiles over the course of the first 50,100, and 250 rounds. However, the trend markedly slows, plateaus or possibly reverses. While more specific investigation would be required to conclude an equilibrium state definitively, it is satisfying enough to say that the AACM model does not rapidly and problematically condense, and that even if an equilibrium state has not been reached, a 250-round trial should be largely similar to a more evolved 500-round state, and therefore a reasonably useful model duration.

Table 2

	Rounds in Trial (CM=1)			
Percentile Range	**50**	**100**	**250**	**500**
Bottom P10	0.75%	0.20%	0.16%	0.23%
P25-P75	36.34%	22.44%	11.69%	12.68%
Top P10	34.98%	53.15%	74.20%	74.62%

Table 2 shows the percentage of total wealth held in the top and bottom 10 percentile range and the middle 50 percent of AA for various lengths of trials. The data do not suggest a strong condensation effect in the AACM model.

Population Endpoint Effect

In composing the agent set, we were faced with the appropriate selection of the highest and lowest AA agent when modeling the full AA range, theoretical 0^{th} and 100^{th} percentiles. Given the asymptotic nature of the normal distribution, the z-score in either direction approaches the infinite,

so agents at either end must be designated as some definite value below: some P99.999... value. Choosing an especially high value might create an unreasonably infallible agent, potentially skewing the results. Choosing a value that is too low, P99.0, would functionally cut off an analysis of the distribution of AAs above this point.

A variable, the 'Population Set Size', within the AACM program was set to select where the highest (and lowest) agent lay with the remainder of agents divided evenly between these two endpoints. A population set size is the population size where the strongest agent would be found: A population set size of 1000 implies that the strongest agent is a '1 in a 1000' phenomenon or the P99.9. A population set size of 1,000,000 implies that the strongest agent (one in a million) is P99.9999.

Table 3 below demonstrates that the wealth attained by the top agent plateaus making only minor differences in percent held from increases by factors of 10: a single percentage point in wealth difference between the 'one in 10 million' or the 'one in a billion' top agent.

Table 3

1 in	100	1,000	100,000	10,000,000	100,000,000	1,000,000,000	10,000,000,000
held by top agent	24.43%	33.74%	40.38%	43.45%	44.50%	44.48%	45.58%

Table 3: shows a plateau of the percentage of wealth attained by the top agent of increasing outlier AA strength.

The Competition/Merit Index and its Effect on Wealth Distribution

The random exchange models have taught us to expect an uneven and concentrated wealth distribution. The AACM model introduces two concepts that might be expected to exacerbate the skew. First, the AACM model stratifies the agent population making some less likely to win/accrue and others more. Second, the AACM introduces 'agency' and target selection. As a rational profit-maximizer in a perfect-knowledge environment, each agent scans and chooses a target based on the maximum expected value from a competition. Thus, we might expect vulnerable agents with probabilistically unlikely wealth accumulation to be challenged more frequently and reduced. We may also expect very wealthy agents to be challenged even by low-probability challengers.

As Fig. 2 below shows, the competition element does introduce such an uneven wealth distribution, predictably along AA lines. At CM=100, the top 50th percentile holds greater than 80% of the wealth and the bottom half correspondingly less than 20%. This pattern repeats for each cumulative top and bottom percentile group being better/worse off. At CM100, the top 1% hold wealth nearly equal to the bottom 30 percent of agents.

Figure 2

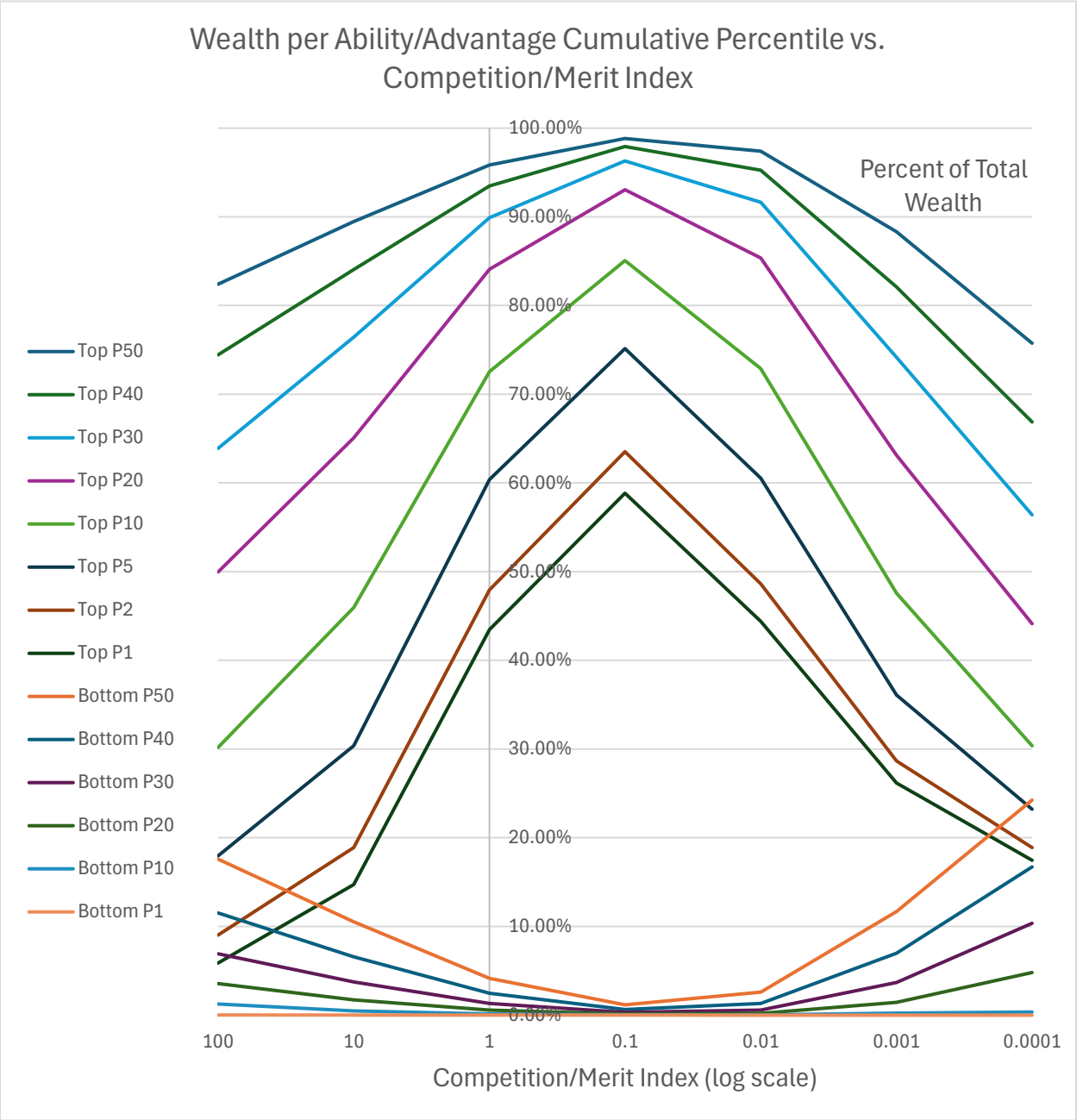


Figure 2: shows the percent of wealth attained by agents within specified talent/ability (AA) cumulative percentile ranges across the range of specified CM indices (shown in log scale). Wealth initially increases for all top cumulative percentile range groups and declines for all bottom cumulative percentile group ranges as competition/merit forces increase (CM Index decreases). After reaching a maximum at CM=.1, the trend reverses for all cumulative percentile groups.

Whether or not the wealth inequality would be greater or less a random exchange model is to be assessed, but what is evident is that introducing competition/ability/merit/agency quickly and reliably redistributes wealth on the AA parameter, the better agents accrue more and the worse accrue less and find themselves very reliably in wealth order matching AA order.

Unsurprisingly, more competition/merit (lower CM index) continues the trend of greater skew/wealth inequality. As competition/merit improves to 1 (index number reduces), the top half's wealth share is over 95%. Each top cumulative percentile group top P50/40/30/20/10/5/1 increases its wealth share, and each cumulative bottom percentile group loses.

Logic might suggest that this trend will continue indefinitely with more competition/merit favoring the more able/advantaged. However, the wealth attained distribution skew/wealth inequality reaches a peak near CM=.1 and then begins to reverse. As competition/merit continues to increase (CM index reduces to .01,.001, and .0001), each of these trends reverse. By CM=.0001, the top 50th percentile is now below 80% of total wealth, and the top 1 percentile is back to wealth roughly equivalent to the bottom 40% (similar levels to CM=100).

Closer scrutiny reveals further unexpected results. Fig. 3 below breaks the results into decile rather than cumulative ranges. It shows the opposite trend is actually prevailing for every AA decile range other than the top 10%. From competition/merit increase CM=100 to .1, every other decile's wealth percentage decreases until reaching a nadir and then increases thereafter.

Figure 3

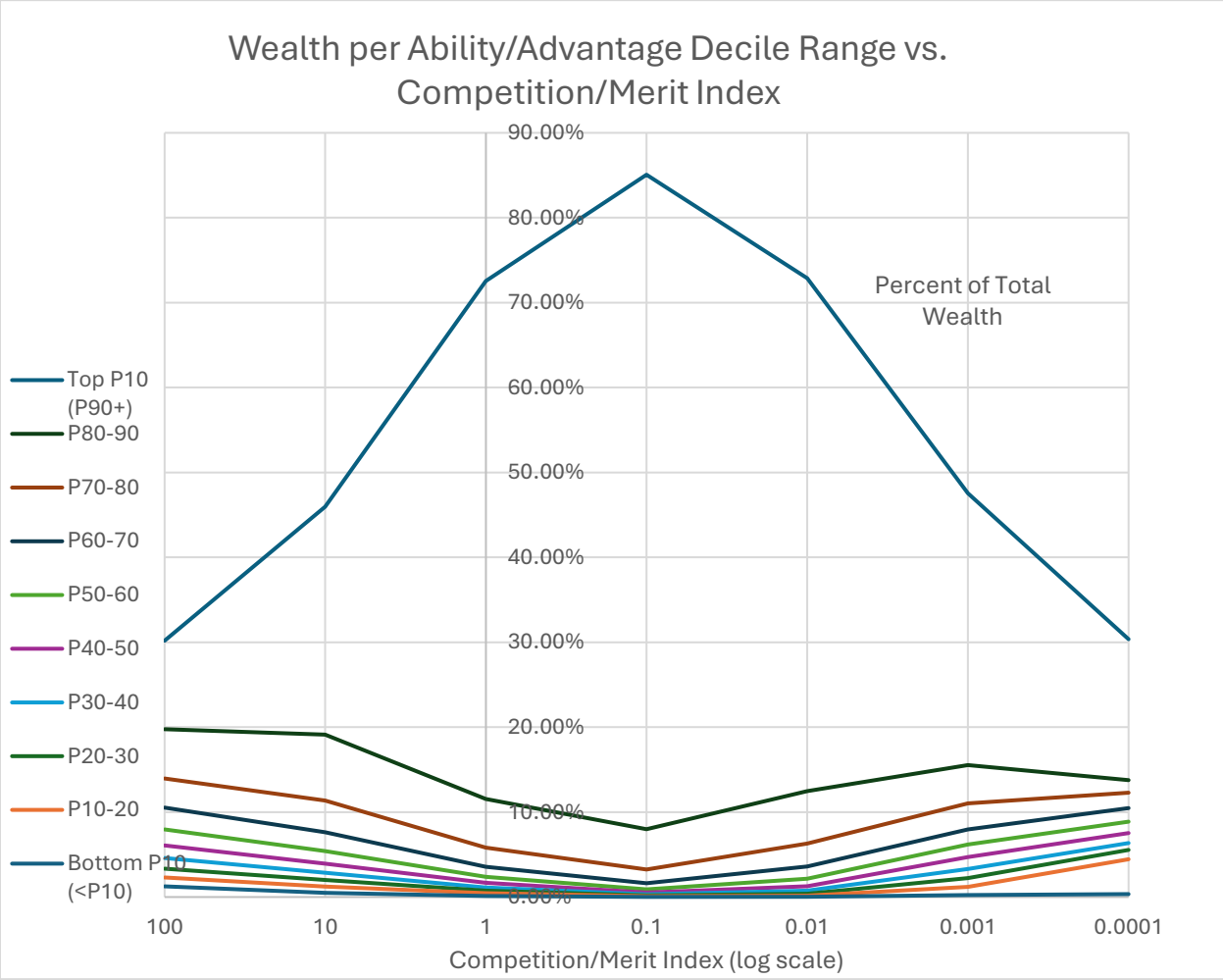


Figure 3: shows the percent of wealth attained by agents within specified talent/ability (AA) decile ranges across the range of specified CM indices (shown in log scale). Wealth initially increases **only** for the top 10th percentile range, and declines for all for all other decile ranges as competition/merit forces increase (CM Index decreases). After reaching a maximum at CM=.1, the trend reverses for each decile group.

Further breaking down the top 10 percent of AA into single percentile ranges in Fig. 4 below shows that the increase in competition/merit from CM=100 to .1 has radically favored only the top 1 percentile of ability/advantage.

Figure 4

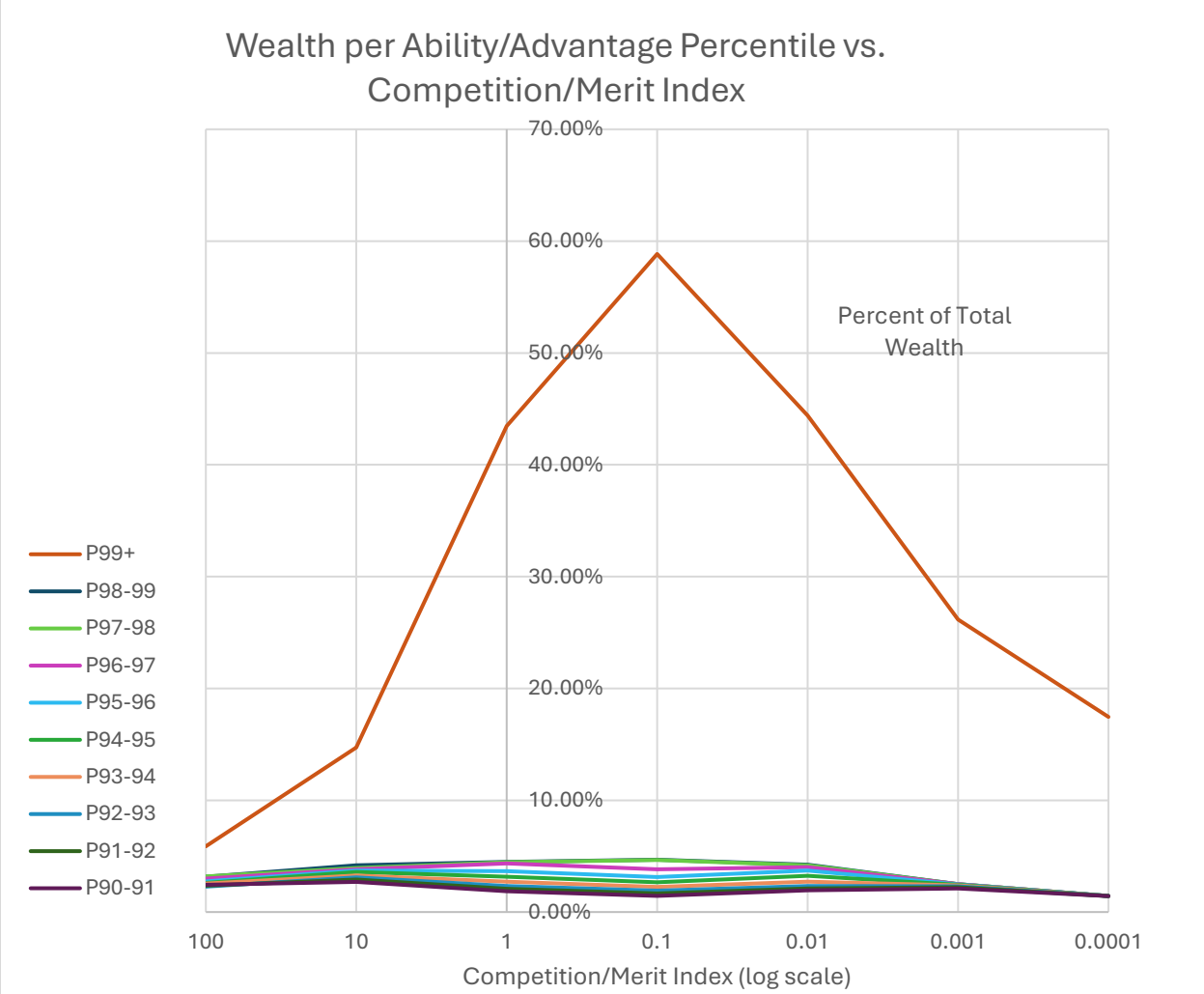


Figure 4: shows the percent of wealth attained by agents within specified talent/ability (AA) single percentile ranges across the range of specified CM indices (shown in log scale). Wealth initially increases dramatically for only the top one percentile range as competition/merit forces increase (CM Index decreases). After reaching a maximum at CM=.1, the trend reverses.

Finally, the subtlety of the driving effect is shown in Fig. 5 below, where P90-99 (excluding P99+) is shown in single percentile ranges on a smaller scale. From CM100 to 10, all agents in the range increase wealth and improve from the increased competition/merit. From CM10 to 1, while the top

3 percentile agents in the range continue to increase/benefit, the bottom six begin to decline. From the CM1 to .1, the winners shed another percentile, with only the top 2 gaining.

Figure 5

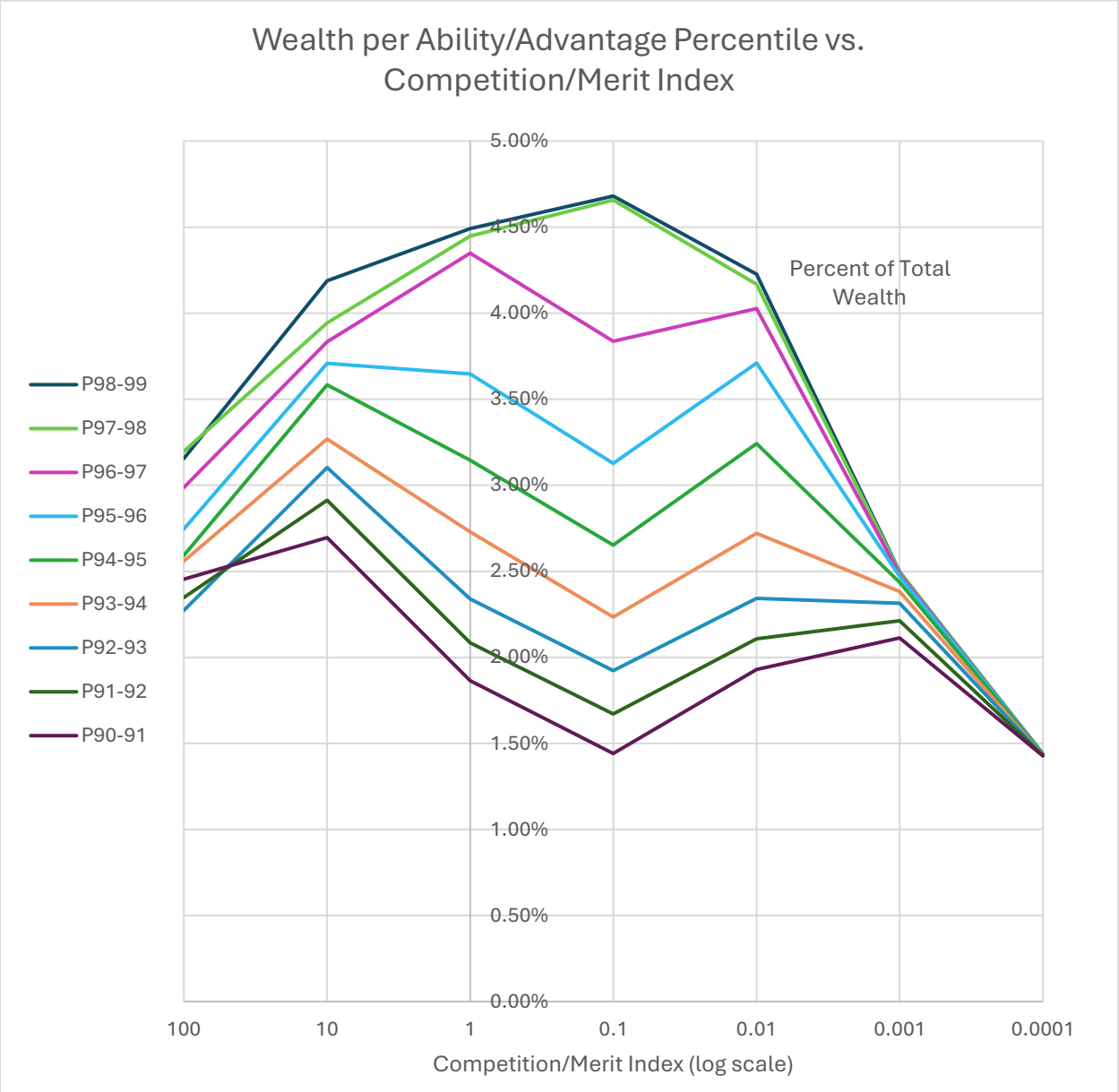


Figure 5: shows the percent of wealth attained by agents within specified talent/ability (AA) single percentile ranges across the range of specified CM indices (shown in log scale). Wealth initially increases for each of the percentile ranges shown as competition/merit forces increase (CM Index decreases). At each step in increasing competition/merit (lower CM index), more of the single percentile ranges begin to decline. After reaching a maximum at CM=.1, the trend reverses.

Therefore, 'initial' or early competition/merit increases benefit a wider, but still relatively narrow, subset of agents. Each subsequent increase in competition benefits a smaller and smaller subset until the maximum is reached at about CM=.1.

Indeed, as defined by the number of agents (in a 100-agent trial) which increased their wealth level from their initial allocation: at CM=1000, the total was 45% of agents; at CM=100, it was 36%; at CM=10, it was 28%; at CM=1, it was 16%; and at CM=.1, it was 12%.

We will declare the CM=.1 to be the Gini-maximum (or Gini-max). On or about this competition/merit index level produces the most unequal wealth distributions, so it will be a focus of discussion and analysis to come[1].

[1] This is a reference to the Gini coefficient. Ranging from 0 to 1, a higher number represents greater inequality. The Gini coefficient is defined by the ratio of the area betweeen the Lorenz Curve of distribution of wealth in a population and a line of perfect wealth equality (Hasell, 2023).

AACM Model vs. Random Exchange Model

Wealth Distribution Shape Contrast

Figure 6

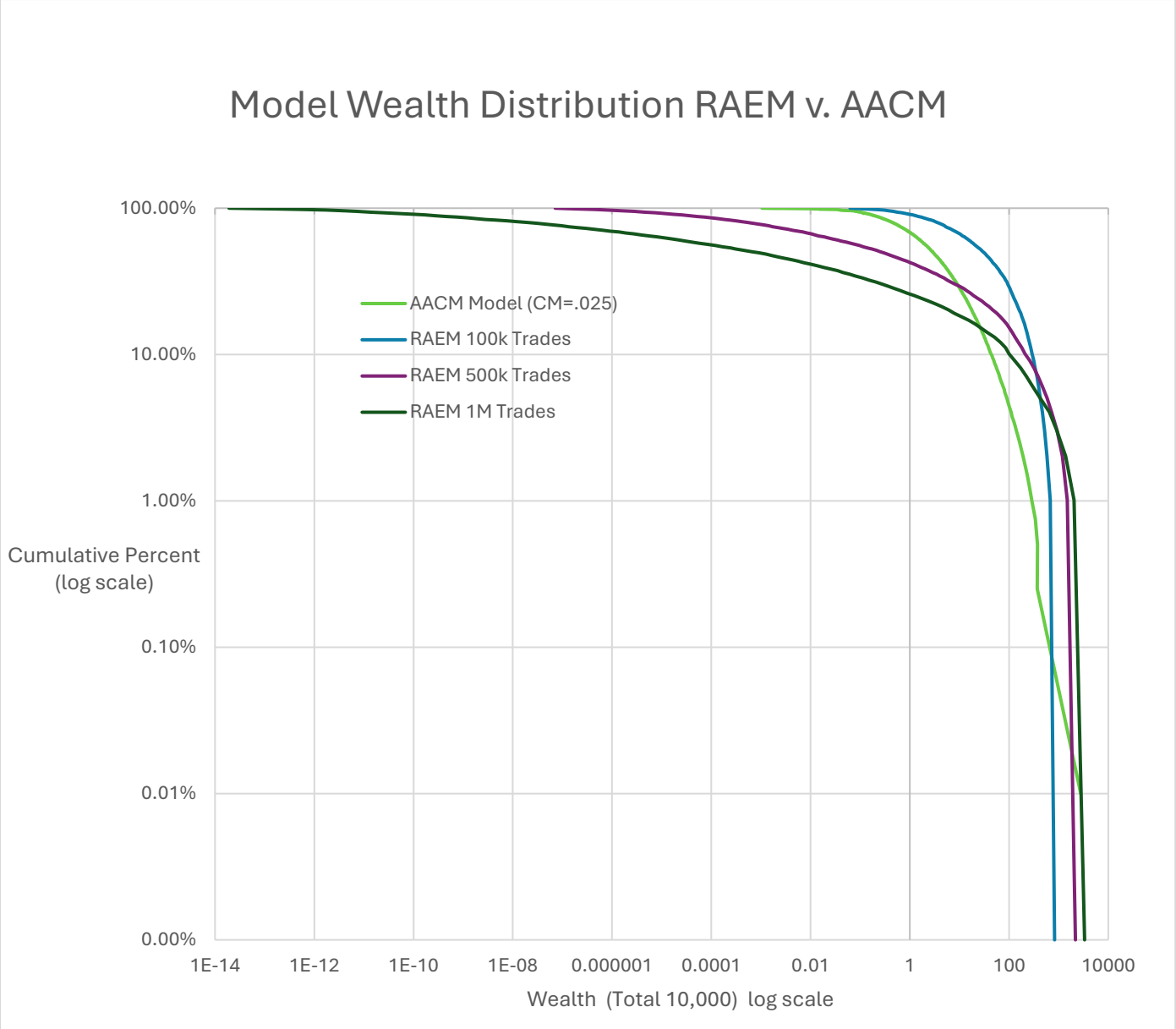


Figure 6: shows the wealth distribution for the AACM and RAEM (random asset exchange) models. The RAEM model conducted random exchanges of 5% of the poorer agent's wealth for the designated number of interactions. The AACM model conducted 100,000 interactions (400 agents x 250 rounds) with CM=.025. The AACM model shows different distribution characteristics than the RAEM for all trial lengths.

It is conventional in this area to compare the data on a Zipf chart. The AACM Model results are compared to another random exchange model (in this case, a 5% of poorer agent RAEM) to detect whether they fit the shape predicted by a PL distribution and how they differ from each other.

The Y-axis plots the cumulative frequency, the data set probability of lying at or above the value at the corresponding X-axis value. The X-axis has the wealth metric, which is units out of a total of 10,000 [percentage of total wealth x 10,000]. So, Fig. 5 above demonstrates what percentage of the population lies at or above this level for any given wealth level. Both axes are plotted on the log scale, which visually gives equal horizontal/vertical space to powers of 10 to make power law detection easier. Fat-tailed PL distributions characteristic of high wealth inequality will show as straight lines with negative slope on the log-log chart (Kakwani, 1980).

For the AACM model in the range of the top ten percentile and higher (lower on the Y-axis), the shape does appear relatively straight and certainly flattens from the higher cumulative percentile ranges. Compared to the random exchange model shown, the overall shape is quite different. Plotted here are 3 curves after various random transactions (100,000; 500,000, and 1,000,000 whereas the AACM here contained 100,000 exchanges) where the exchange is 5% of the poorer agent's wealth, a similar but not identical exchange construct in the AACM. The chart shows a definite progression as the number of transactions extends for the RAEM. At no point in the RAEM progression is there a suggestion that the curves would match up well. Generally, the random exchange model appears to chronically predict lower wealth at the low percentile range (high cumulative percent Y-axis) than the AACM, then higher percentages of some wealth, and a more linear (as opposed to log-linear) increase in wealth amongst the highest percentiles. There does not seem to be a transaction number where the curves would match distribution behavior among the RAEM and the AACM model.

The Effect of Economic Agency

When economic agency is introduced versus random exchange pairing, the capacity of random chance (equal win probabilities) to produce vastly skewed wealth distributions seems to be extinguished.

In parallel, any economic agency will leverage even the slightest probabilistic advantage. Setting the AACM to a 100-agent pool selected from the range P49.999 to P50.001 of AA and a CM=100,000 still results in a distributed wealth outcome. Selecting the agents from a minuscule range at the center of the AA distribution creates the flattest and smallest scale to minimize the AA difference between agents. The very high CM index (low competition/merit environment) ensures that the minimum AA difference between agents translates to the smallest possible elevation in probabilistic advantage. In this trial, the top agent enjoys only a .000008% advantage over the lowest agent, with all other differences vastly smaller. Even so, the top P50 achieves 63.65% of the wealth to the bottom half's 36.35%. The bottom P10 has an average outcome of 40.7 coins, a nearly 60% decline from its starting level. The top P10 has an average outcome of 128.1, more than one quarter up.

Compared to random chance/ random pairing of the RAEM model, the AACM with de minimis probability differences seems to create a substantially less skewed distribution, suggesting the absence of economic agency is an essential element of the RAEM in creating the fat-tailed

distributions. This finding, while fascinating, should be treated as preliminary rather than conclusive and worthy of further inquiry.

Macroeconomic Data – Results Comparison

Preliminary Note

The basic model consists of a closed universe of 100 agents, each possessing an initial 100 'coins' for a total of 10,000. This is a zero-sum model, and economic activity is characterized as competition for this wealth. It is also a zero-loss model, and wealth is also not consumed.[2]

Our comparison metric standard then is the 10,000-coin world from the model. It is noted that this indicates the percentage of the whole possessed by an agent x 10,000. This metric is maintained as standard across much of the study for consistency and in recognition that percentage vs percentage may be somewhat less intuitive than percentile against wealth 'score'.

In light of this, economic data compared to model data will be normalized to this basic standard: percentage of the whole x 10,000. [Additionally, all model data with different numbers of agents will be re-standardized to the same. E.g. 400 agents x 100 coins = 40,000 coins total should each be divided by 4 so the total remains 100% x 10,000.]

The economic data queried will be worldwide personal wealth and income data, corporate wealth data, and professional tennis ranking points. It will be compared to the AACM model where 400 agents, each representing a quarter percentile range, for additional granularity on the 100-agent, single percentiles, for each trial.

Professional Tennis Ranking Points: A Pure, 2-Agent Competition

Professional tennis ranking points have been chosen because they most clearly and simply represent a two-party, competitive endeavor over a fixed reward metric. It is a two-agent competition in its purest form with singularity of objective. Prior research has shown that the percent probability of a match win of player A over B can be derived from a single metric of player A's probability of winning an individual service point over B and extrapolated from there (see Newton and Keller, 2005; Riddle, 1988; Datagenetics.com, 2009).

[2] Perhaps neither is strictly accurate in a real-world setting, as we typically recognize value creation from economic activity and the consumption of economic wealth (food for example) occurs in fact. For a simple market analogy, we might assume that these opposing forces cancel. The loser of a competition can be seen to have gained a sustenance level of wealth which is promptly consumed. The winner gains a sustenance which is also consumed plus the surplus value-added wealth transferred allowing its wealth-holding to increase. This can be thought of (per Angle) as surplus wealth. It can be acknowledged as a theoretical simplification necessary to observe patterns in the most basic without layering of assumptions and consequences. It is noted that a productive, prisoner's dilemma element (two cooperative players) can be added to create two winners in a future study.

In this way, it most clearly mirrors the assumptions made in the AACM theoretical model of single quantifiable advantage translating to a single probability differential and repeated competitions. Ranking points are the metric of success; they are the wealth of the system. It also notably creates a true economic market as points directly translate to prize money amounts.

Association of Tennis Professional ranking points are the wealth metric in this market. End-of-year 2023 data was aggregated for the top 2,048 men's tennis players. (ATP Tour, Inc., 2023). This is understood to include all players to achieve a single point. A player's ranking points are the sum of his 19 or 20 (as below) best results from a rolling year's events (Wikipedia Contributors, 2024). The theoretical maximum of ranking points is therefore 21,000 points which would be achieved by winning all events including the highest point-yielding events of the four Grand Slams, Tour Finals (if qualify, the 20th event result), highest level regional events, and the maximum number of second highest ranking events.

In order to normalize ranking points to the '% of whole x 10,000' standard detailed above, the following transformations were done to account for the method of awarding and aggregating points. Each player's raw ranking points is divided by the 21,000 theoretical whole/max to yield a percentage-of-whole number. Aggregating all players' percent of max numbers sums to far greater than 100% because parallel events mean that players are competing for multiple pots. Therefore, a second normalization to the whole is performed by dividing each player's percentage by the aggregate summed percentage, which was on the order of 12.88. This is then multiplied by 10,000 to create a parallel basis for comparison to the model.

To get a direct comparison of the 'wealth per agent', aggregating among a few (5-6) tennis players results ("basketing') among the 2,048 player set is required to match the percentile range for each of the 400 AACM agents.

Fig. 7 below compares the AACM model with the professional tennis ranking points distribution. CM index levels were tested on the AACM model until the closest fit appeared at approximately the CM=.025 level shown. The wealth distributions are well matched among the tennis ranking points and the AACM model in all percentile ranges and exhibit very similar curve and slopes in all chart areas.

Figure 7

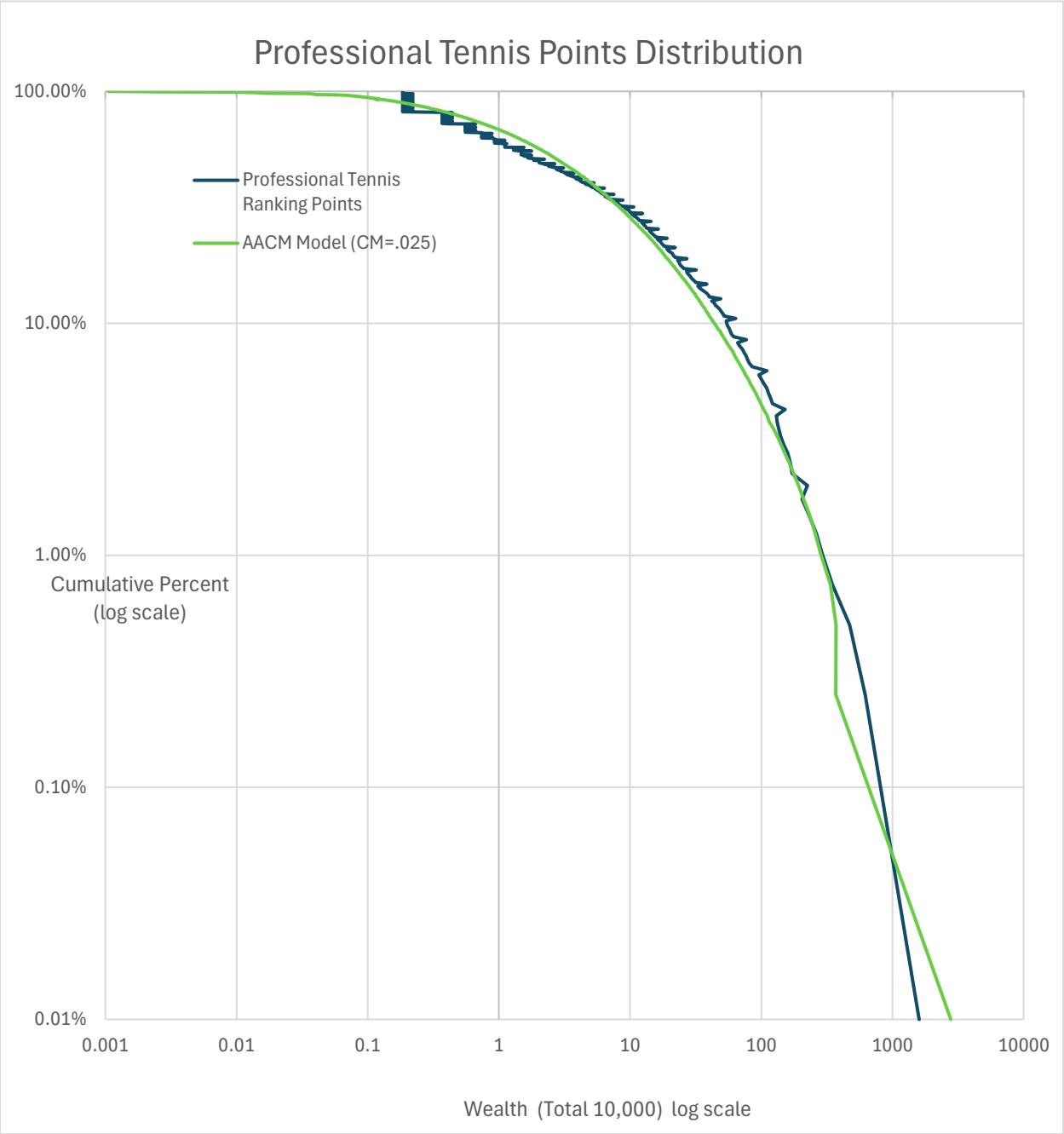


Figure 7 shows the substantial similarity of professional tennis ranking points distribution versus the AACM model wealth distribution. Professional tennis ranking points are year-end 2023 (ATP Tour, Inc., 2023). All players with any ranking points are included. Points are normalized to a percent of whole x 10,000 standard and basketed in ¼ percentile ranges to match the AACM model (see text). The block/step of the tennis data is a function of the integer nature of the tennis points award, and the spikes are a function of the periodic inclusion of an extra player in the baskets.

It should be noted that a couple of graphical aberrations in the tennis ranking points line are caused by the integer nature of the points system and the basketing of players into quarter percentile

ranges. First, the tennis points line's initial block step pattern down results from the indivisibility of single points. The first 375 players have a single point; the next 185 have 2; the next 121 have 3; and so on. The point system's integer nature and distribution create the block and step element of the tennis points line. The AACM wealth metric is infinitely subdividable, showing a smooth trend in the lower values at the top of the chart.

The tennis ranking points graph 'spikes' are caused by the basketing into quarter-percentile ranges. As mentioned above 2,048 players are allocated to 400 quarter-percentile baskets, causing 48 baskets to have an extra player's points included relative to the others. This shows as the graphical spikes followed by a reversion to the smooth trend line.

It should be further noted that tennis ranking points are acquired in an additive and not multiplicative format, although players at higher levels are able to compete for larger point prizes.

Company Market Cap Value vs. AACM Model

Fig. 8 compares the AACM model with the value of the broadest set of companies in order of market cap value. The data composes a list of 8,355 public companies worldwide (largest to smaller) with market cap calculated on April 5, 2024; they compose USD 103,212,649,673,672 in total value (CompaniesMarketCap.com, 2024). Market capitalization is considered the direct analog of wealth/value of the organization as an economic actor. All companies are traded in public markets such that a wealth value is available and reliable.

As with the tennis ranking points, the companies are aggregated to quarter percentile range baskets and their wealth normalized to 10,000 ((individual market cap / total market cap of all companies) x 10,000).

The table below shows a very tight fit in the AACM (CM=.025) Model distribution and the Company Market Value distributions in all percentile ranges.

Figure 8



Figure 8: shows the striking correspondence between the wealth distribution of the AACM model (CM=.025) with companies' market capitalizations. This data was aggregated on April 5, 2024, and composes globally 8,355 largest companies with a collective market cap of more than USD 103 trillion (CompaniesMarketCap.com, 2024). Market capitalization is considered the direct analog of wealth/value of the organization as an economic actor. Market cap was normalized to % of total x 10,000 and put in ¼ percentile baskets to match the AACM model data (see text).

Individual Worldwide Wealth vs. AACM Model

UBS Global Wealth Report 2023, sourcing data from James Davies, Rodrigo Lluberas and Anthony Shorrocks Global Wealth Databook 2023, estimate total global individual wealth at USD 454.4

trillion (Davies, Lluberas and Shorrocks, 2023). They break this down into four population subsets known as the global wealth pyramid: populations with <USD 10,000 net worth representing 52.5% of the global population, USD 10,000 – 100,000 representing 34.4% of the population, USD 100,000 – 1 million representing 12.2% of the population, and USD >1 million representing 1.1% of the population and corresponding net wealth held by each sector (Davies, Lluberas and Shorrocks, 2023).

Though no methodology is given in that publication, these researchers have been publishing a similar report databook for over a decade. Prior presentations give a conception of the methods used to derive these very large-scale wealth estimates (Davies et al., 2014).

They define wealth as real assets + financial assets – debts. They estimate household wealth levels per country depending on the available data from 'statistical organizations, central banks, ministries of finance,' etc. and conduct surveys in others. Thereafter, they estimate the 'shape' of the wealth distribution to place and extrapolate those statistics. They use published wealth distribution data where available, use income data and impute wealth using comparisons from other countries publishing both. They 'impute missing quantile shares using an 'ungrouping' program' and compare survey data to their methodically estimated data (Davies et al., 2014).

What emerges is experienced, diligent, well-supported, but still ultimately, estimate, a projection of actual wealth numbers and their distribution among the population set. The gross numbers afford only a 4-data point set by which to compare the corporate market cap and the AACM model.

Table 4 below aggregates these data showing the wealth held (as a percentage of total wealth) in each of the four percentile ranges detailed in the global wealth pyramid. We combine this with the corporate market cap data from above pooled into the same percentile ranges as the UBS individual wealth data. This wealth statistic is reported for each of the UBS Individual Wealth, Corporate Market Cap, their average, and the AACM model.

Table 4

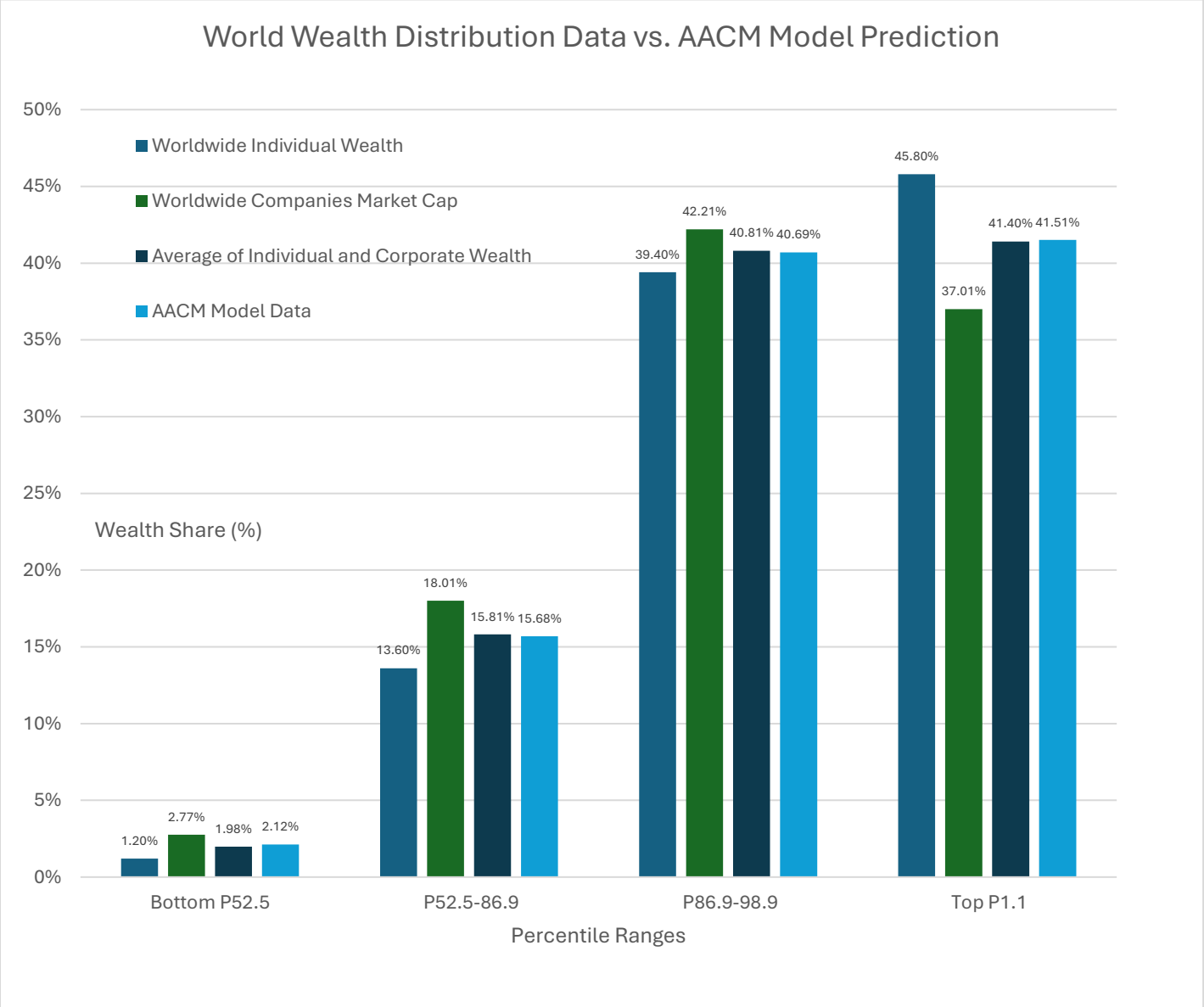


Table 4: Estimates of individual wealth percentage share held by each of 4 percentile ranges: bottom P52.5, next 34%, next 12%, and top 1.1% were collected by Davies, Lluberas and Shorrocks (2023). These are presented with the companies' market cap and AACM data from Figure 8 compiled in the same ranges. The AACM is reliably in between the two wealth estimates (company and individual) and, when compared to their average, shows a difference no greater than .14% for any category.

Having already viewed the tight fit of the market cap distribution to the AACM model on the Zipf diagram with its 400 - ¼ percentile range data points, we can see how this data appears when reduced to a 4-category range division. AACM data compared to the 4-category wealth pyramid divisions, the AACM achieves a per category average difference (absolute value/directionless) of 2.25% versus the wealth values of the worldwide company market cap wealth divided into the same percentile divisions. More importantly, it achieves a weighted average per percentile

difference of .0137% of model vs. market cap observed wealth value. Comparing this level of correspondence with the AACM vs. worldwide individual wealth calculation shows equal or better prediction. Compared to individual wealth, the AACM model exhibits a 2.15% average per category absolute difference and a nearly identical .0140%/ percentile weighted average difference. If the individual wealth data could be stratified with as much granularity as the corporate market cap, the graphs would probably exhibit an identically tight fit as the abovementioned AACM vs. corporate market cap graph.

Both individuals and companies equally fit the definition of 'economic agent' and are both worthy of review in how wealth is distributed among them. What is termed 'net worth' or wealth for the individual is paralleled by the market cap of the company. It is the deemed value of that agent as an economic actor. It is also noteworthy when they follow a very similar distribution pattern.

Most striking, the AACM model reliably predicts a per percentile range wealth-holding between the wealth estimation of the UBS Individual Wealth and the Worldwide Market Cap Corporate wealth. Under the rationale that each of these represents an estimate of the wealth of those percentile ranges and compares their average against the AACM model, we find that the percentage of wealth at each range differs by no more than .14% in total for any of the large percentile ranges. This exceptional correspondence demonstrates very effective modeling of wealth distribution throughout the entire percentile range from lowest wealth to the very highest.

Individual Income vs. AACM Model

It is also most worthy of mention that the percentage of total wealth attained by the P99+ percentile in the AACM model at the Gini-max (CM=.1) was 58.85% (see Fig. 4). This model prediction absolutely corresponds to the Atkinson (2009) data quoted above which found that over the period of 1976-2007 "the top 1% captured 58% of real economic growth per family during that period" (p.5).

DISCUSSION

The AACM Model is a simple model with assumptions rooted in the very basic tenants of economic theory. It produces a result distribution that closely matches the distributive results of tennis competition, which most closely resembles the AACM's micro-competition assumptions. It also accurately produces a result distribution that closely matches macro-economic data amongst individual (income and wealth) and corporate economic actors alike. As a single-parameter model (CM index), its fit is not artificially enhanced by manipulating multiple parameters, and the best fit to all the economic data is achieved close to a singular and mathematically meaningful point. Its fit to the various data is not confined to the probability tails; it is as adept in predicting the low and center of the wealth distributions, eliminating the requirement of a multiple cause/ multiple distribution paradigm. Doing so across conceptually different markets, and with theoretically coherent underpinnings, it reinforces the possibility of singular basic economic/competitive mechanism at work.

The Competition/Merit Index and a Competition Equilibrium

Methodical manipulation of the CM index across a broad range of levels has demonstrated that increased competition (decreasing CM index number) results in a more inequitable, top-heavy, wealth distribution until approximately CM=.1 (the "Gini-max" point of most unequal wealth distribution) at which point the trend reverses, and, though still skewed, the wealth concentration reduces.

Most remarkably, the CM index which forms the tightest fit across every economic (individual wealth and corporate market cap) as well as pseudo-economic (tennis ranking points) metrics, was the same at CM=.025, very close to the Gini-max. This raises two obvious and fascinating questions: why and why there?

Competitive Equilibrium or Maximum

While it is possible that each metric coincidentally forms around CM=.025, that they do coincide without obvious reasons suggests this competition level either represents an equilibrium condition or a practical maximum.

To consider which of equilibrium or maximum requires the determination of who/what sets the prevailing CM level. The possibilities are that the participants (the economic actors) themselves form this within the competitive market or that some outside agency sets the CM irrespective of the participants.

It must be remembered that, within the model's premises, CM designates only the regularity with which superior AA translates to a win and that agents do not have the capacity to alter their AA. So, CM can be considered the 'rules of the game' – the manipulation of the game towards chess where chance factors less or backgammon where chance has a greater role in each outcome. It designs the NBA Finals as a single championship game or a seven-game series. In an only slightly un-evenly matched event, the lesser team would have a better chance at winning if the format were a single game than to have to win four of seven. It is tennis' best-of-five grand slam format versus the ordinary best-of-three; it more consistently rewards the better player, even if the difference is small.

In the economic setting, who might act as the rule-setter laying the format for the economic competition? One may well imagine that it would be the high AA subset. The high AA would be naturally suited to become dominant in the political spheres which would set tax, trade, competition, and regulatory environments to create a 'game' that would suit their position atop the AA distribution. They move the CM needle to create the competition and victory formats to reward the range that maximizes their wealth share but not so far that it begins to reduce.

The other possibility is that the equilibrium evolves as more of an arms race driven by the top-down. It is in the nature of an arms race to be unilaterally initiated. If one participant engages in an arms race, all are engaged (similar to the defector in the Prisoner's Dilemma).

First, we observe that no amount of increase in competition/merit favors the bottom 50 percentile range. These agents are better served by random chance as they would have a better aggregate probability of economic wins versus a merit-based AA. The random exchange models instruct that wealth will be concentrated even in the absence of merit, but the AACM demonstrates the reliability with which the AA hierarchy can order the wealth distribution on its lines. Therefore, the bottom percentiles would be better suited to a chance-created re-ordering than a merit one. The higher the AA, the more rewarding the stricter CM is.

Since agents cannot raise their AA in search of their own competitive advantage, it could only take the shape of lowering the CM index. That is, alter the attributes of the competition so to raise the probability of their victory. In reality, this would mean reducing the variability of the expression of their own AA performance: being more focused, better prepared, equipped, streamlined, and studying the opponent more to reduce surprise. All these would be the economic equivalents of altering non-ability/advantage elements while raising the probability that the superior AA would more reliably translate to a win.

If one agent acts to increase competition/merit (lower the index), both are subject because their probabilities are tied together. And the higher AA agent is the one (individually) which will have the incentive to do so.

Each agent of above average AA would be motivated to move the CM needle to increase its own likelihood of victory over a lower AA agent that it could challenge and accrue wealth. So, the upper 50 percentile AA each purse a course to increase the CM (lower the index) in pursuit of their individual advantage over the no competition/random baseline. The process continues to be driven by the higher AA against the lower. The net wealth is being transferred up the AA hierarchy, and the wealth of the lower diminishes. As the CM rises (index number lowers), the reward for challenging up the AA distribution reduces, but each agent controls only to raise its expected gain against a lower AA target so continue to push the CM index lower. The CM movement arms race stops when no upper percentile AA further benefits from pushing it. The 'competitive equilibrium' is reached even if theoretically higher CM (lower index) is possible.

On the reciprocal side, higher AA agents increase the CM (lower the index) to improve the likelihood of warding off challenges from lower AA agents which may find a positive expected value from such a challenge. The lower AA agent's wealth may position it with little to lose (asymmetrically) and more to gain against a high-wealth agent, even given a low probability of success (a 'moonshot'). The lower CM (higher index number) raises its probability.

The progression recalls both the Prisoner's Dilemma and the Nash Equilibrium.

At each transaction, the higher agent seeks a proximal, episodic advantage by raising CM (lowering the index, either to target or to defend a lower AA agent). Much like the defectors in the Prisoner's Dilemma, it succeeds on an episodic basis but in the aggregate winds up in a worse position of lower wealth.

At the initiation of low competition versus random, with a CM=1000, close to the entire above-average AA range is better served. Breakeven wealth (finishing with at least the same as the starting wealth) is reached at approximately the 54th percentile AA agent, just above average. The AA upper half achieves 72.4% of the wealth. This is a broad benefit, a little shy of majority or parity. It also broadly satisfies conceptions of social and economic justice: the meritorious have been rewarded.

Adopting this as the new starting point, the arms race/Prisoner's Dilemma continues. Increasing competition, progressively dropping CM by factors of 10, breakeven wealth continues to shift to ever higher AA percentiles: the 63rd percentile (CM=100), 72nd percentile (CM=10), the 84th percentile (CM=1), before reaching its peak at the 87th percentile at CM=.1 the Gini-max, competition equilibrium. As seen in the above figures, each move to more competition benefits a quickly declining population set. The shift from CM=100 to 10 benefits only the top 10th percentile and CM=10 to 1 only the top 4%. Manifestly and steeply, the top percentile increases its wealth share across the range.

It should be noted that while de novo higher CM levels (index lower) than Gini-max competition equilibrium would create more distributed wealth, escalating CM once a concentrated wealth distribution exists would likely do nothing to reduce it. All competition/merit quickly allocates wealth along AA lines, and more competition/merit only buttresses wealth from challenges from below and reduces chance redistribution.

The Nash equilibrium is a fundamental concept in game theory that describes a stable state in a strategic game where no player can improve their outcome by unilaterally changing its strategy (Osborne and Rubinstein, 1994). The Prisoner's Dilemma demonstrates the basic construct in which individually maximizing an expected value can result in a collective sub-optimal result. In many ways, this is the antithesis of Adam Smith's theory that individual pursuit of profit maximization will result in the most efficient macro outcome.

To the extent that the agents can manipulate the CM at all, like the defecting prisoner, they may raise the CM (lower the index) for proximate advantage only to be trapped in a sub-optimal outcome. Then, no further increase in CM can improve it [remembering that as an arms race, a single agent has only the power to raise the CM but to lower it would require both].

Not By Chance

Vastly unequal wealth distributions can be created either by chance, as in the random exchange models, or by merit, as in the AACM model (albeit with somewhat different shapes). However, the RAEM's capacity to do so seems to depend on twin pillars of random probability and random action. The inequality-making potential is reduced when the RAEM's random pairing of exchanges is removed in favor of purposeful economic agency, profit maximization, and target selection while retaining basic (if not total) equal probability. This is a fascinating outcome which requires much more study, but intuitively it appears that much like the perception of advantage, or the selection

among the best options, even if vastly similar, alters the behavior of the agents in such a way as to alter the outcome distribution.

Not a Matthew Effect

As the AACM model very closely predicts real-world wealth distributions across percentile ranges, a Matthew effect is not required. The Matthew Effect is defined by wealth begetting wealth or wealth creating ever-expanding privilege and capacity to garner wealth. This is not required and the match to real-world data suggests that static merit alone can be the source. A further Matthew Effect on the AACM model would entail that the probability advantage of the high AA would increase as they gained wealth. This would lead to an even more accelerated and skewed wealth distribution (or even condensation) which is not borne out in the real-world data.

It raises the very likely and uncomfortable prospect that if indeed the world has become a more unequal place, it may well be the result of competition and meritorious forces at work. According to the AACM model, the remedy, but not the antidote, may be for more competition and merit in the system. However, such an increase would do nothing to reverse existing inequality but only close the avenues for the lower AA to win.

The most that can be said in favor of the Matthew Effect is perhaps that the high-AA population organize/set the game to maximize the return for their innate position, but we have argued that it is at least as likely the competition has been set on an arms race/Prisoner's Dilemma style individual basis.

Not a Percentage Effect

Pure percentage-based growth or decline could have a similar snowball effect to the Matthew Effect. A wealth/size indifferent, five percent growth model across the board would favor the larger wealth agents in absolute terms and lead to increasingly large differences. On the lower side of exchange/competition, a percentage effect could trap the agents in a Zeno walk of diminished capacity for gains.

However, the similarity between tennis ranking points and the economic wealth distributions suggests this percentage effect is not at work. While corporation size might be eligible for size-indifferent growth rates, or percent return on capital, tennis ranking points are purely additive. It is also not apparent that the AACM could be considered a Percent Based competition scheme because though the stakes are set at a percentage of target wealth, it is not the percentage which is relevant but rather the absolute size and success probability which determine the expected value, the selection and execution of potential transfer. The corresponding loss stakes are also set on an absolute basis, and all-in gambles for non-symmetric stakes are allowed.

The wealth scaling should not be a percentage effect (snowball) but could be the result of competition scaling which depends, in turn, on the number of participants to take its effect. If A beats B beats C, C's wealth will flow to B and the now aggregated quantity to be accessible by A

beating B. The effect is magnified by increasing the number of participants, so in a larger market with frequent transactions, this effect is increased without a self-reinforcing percentage effect.

Further Speculation

Two more intriguing strategic outcomes have been noticed and will require further analysis to determine their influence and possible modeling opportunities for the real-world economy.

A Lottery Economy

First, there may be the creation of a lottery economy – a gambler's trap. It does appear that lower AA agents with falling wealth values begin to be 'seduced' by a low (but not impossible) possibility of a large gain against a very wealthy high AA agent. As the low AA agent's wealth reduces so too does the loss element of the expected value calculation skew it towards the low probability/high reward 'moonshot' which seems to assist and accelerate the transfer of wealth from low to high AA agents. In higher CM (lower CM index) formats, these moonshots tend towards nil probability, so the agents have increasing motivation only to challenge down and not up, reducing the frequency of opportunities for the high AA to win. In lower CM (higher CM index) formats, the moonshot tendency would increase but is also a function of wealth concentration which in turn is accelerated by a rise of CM (lower CM index) above chance levels. This sweet-spot CM which concentrates wealth while not discouraging moonshots may be a contributing factor to the Gini-max equilibrium.

Participation Rates

The latter observation raises the possibility of modeling participation rates in the real economy. In a very high CM environment (low CM index), challenges to higher AA agents become virtually useless, and challenges down are the only choices with a positive expected value. The lowest AA would forego a challenge without a positive expected value and would be challenged from above and methodically stripped of wealth. If a de minimis stake was supposed, in a high CM (low CM index) environment, a creeping proportion of agents would decline to challenge, as an upward challenge would be futile and a challenge lower practically useless. So, the probable cost of a very high CM environment would be a declining participation rate.

In a zero-sum environment, a low participation rate is not a concern. However, it is likely that in real-world economics, we expect an economic interaction to contain a value-additive element. We expect that the wealth residue accumulated is the surplus wealth when some wealth to be consumable. The AACM models essentially allocation of wealth but not quantity. It is most probable that absolute economic wealth creation requires higher participation and economic transactions. This is why absolute wealth may increase (raising a higher proportion of the population above the poverty line, for example) while wealth inequality also simultaneously rises. A very high CM environment may ameliorate the wealth distribution while reducing participation and thus not optimizing the total economic interaction necessary for maximum system-wide wealth creation.

Gini-max CM Breeds Exceptionality

If we were to presume alternatively that agents can develop AA, the Gini-max CM would maximally reward, and therefore, arguably breed high-AA exceptionality, superstar outliers.

This somewhat counterintuitive result can be explained as follows. In the highest CM (lowest index) environment, high AA (large z -scores) are not rewarded any more than lower z -scores which are nonetheless marginally higher than the next highest. To assure 100% success against the next lower agent, any increase in z -score will suffice. Therefore, the highest percentiles would not be driven to the z -score skew of the normal distribution but would progress in a minimal stepwise fashion.

Therefore, if economic progress is determined to depend on exceptional superstars, a Gini-max CM would be the most advantageous, and wealth inequality could be tolerated or ameliorated by other means.

Mechanistic Implications

Whether in random exchange or meritorious competition, wealth inequality is created.

One tool afforded by the AACM model is to understand where the economy stands on the CM spectrum and how changes in the CM, which might be manipulated by regulation, may push it in one direction or another.

Another advantage is the de-stigmatization of economic non-participation and low outcomes. Non-competitive AA population sets are definitionally present in a competitive environment. As we have seen, even high AA populations can have modest outcomes in a fiercely competitive/meritorious environment.

Whether wealth (resource) inequality is problematic requires differentiation between firms and individuals as the economic actors.

When considering organizations as the economic agents, the considerations shift. From an efficiency perspective, it seems rational that wealth should be in the hands of the most able economic actors, as this suggests its most judicious use. The most productive firms would presumably be the most competitively/meritoriously advantaged (the highest AA) and therefore best able to use the resources. Concentration of wealth in the corporate world does not suggest the same social disruption. There is not the same social or economic mandate to support less able corporate agents as there is in the individual setting. With the following few exceptions, no perceivable economic utility is served by supporting definitionally less competent organizations.

In the real corporate world, however, there is a very strong risk for the development of a Matthew Effect (or percentage effect) where some institutions begin to command large portions of a market and vast percentages of wealth. The capacity to attract talent, access capital on favorable terms,

and influence the market in a self-serving or monopolistic manner is very real in a way that is not from the individual perspective.

An interesting extension of AACM model could be used to model/predict corporate combinations to elevate their competitive standing. For example, supposing the AA z -statistics could be aggregated and two z - 2.0s could unite to become a stronger, single z - 4.0 entity. A similarly simple and transparent maximization strategy could be used. One could imagine that as the wealth of viable targets dwindled, the shift to retaining half of a combined, higher competitive effort would become more rewarding for both and a merger would form. It may be possible to model the formations of oligopolies in this fashion.

Retain Participants where AA can fluctuate

In the model, we have defined AA as static. In the real world, we suspect and perhaps hope both that AA may be variable or that it can be acquired. Education is an obvious example that presumably would move an individual to a higher AA level and is within the agent's control to do so. Also, if AA fluctuates, it is in the interest of the economy to preserve as many economic actors as possible to ensure that the top agents are of a very high level. The wealth will tend to aggregate in the most competitive of a few agents; however, there will be no assurance that the absolute level of AA will be high.

CONCLUSION

The AACM has provided a singular economic mechanism, based solely on the most foundational of economic tenants, which has effectively modeled wealth distribution across a spectrum of competitive markets and the entirety of the distribution from lowest to highest income with comparable accuracy. The model itself is free from self-reinforcing cycles like the Matthew Effect and the Percentage Effect, so its correspondence to empirical data suggests these snowball effects are not responsible in the main for the high degree of wealth inequality. The AACM has revealed on a mechanistic level how and to what extent competition/merit interacts with talent/ability/advantage to create the outcome wealth/success distributions. The AACM has identified a CM index level which creates a maximally unequal distribution, the Gini-max. Below this level CM increases (index decrease) fuel inequality by favoring a declining top proportion of the highest AA, but beyond Gini-max, further CM increase (index decrease) reverses the effect. That the empirical data all fit the AACM at near Gini-max suggests that this level is an equilibrium or a maximum. We have argued it represents an equilibrium created by a game theory-inspired conception that individual profit maximization may lead to group-wise sub-optimal outcomes provided that CM can be increased (index decreased) unilaterally but only the opposite collectively. The Python program to run the model is provided for reviewers to confirm the results discussed here.

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APPENDIX

Python Code for AACM program

```
import scipy.stats as stats
import math
import random
import csv

# Define variables
num_participants = 100
starting_coins = 100
bottom_percentile = 0
top_percentile = 1
competition_index = .1    # converge to 0 for perfect competition, higher the number more chance
number_of_rounds = 250
trials = 50
population_set = 100000 # set this for the top/bottom participant (e.g. 0%/100% percentile) to
mirror this, THIS IS MULITPLIED BY NUMBER OF PARTICIPANTS TO GET HIGHEST AGENT AS 1 IN X

# Adjust range if needed
if bottom_percentile == 0:
   bottom_percentile = 1.0 / (num_participants * population_set)
if top_percentile == 1:
   top_percentile = 1 - (1.0 / (num_participants * population_set))

# Define the probability function
def calculate_prob_win(z_diff):
   r_stat = z_diff / math.sqrt(z_diff**2 + competition_index)
   prob_win = 0.5 + r_stat / 2
   return prob_win


# Initialize a list to record final coin totals for each trial
final_coins_per_trial = [[0] * num_participants for _ in range(trials)]

# Run multiple trials
for trial in range(trials):
   # Reinitialize participants data for each trial
   participants_data = [{'percentile': bottom_percentile + i * (top_percentile - bottom_percentile) /
(num_participants - 1),
                'z_statistic': stats.norm.ppf(bottom_percentile + i * (top_percentile -
bottom_percentile) / (num_participants - 1)),
                'coins': starting_coins} for i in range(num_participants)]
```

```
    print(f"Trial {trial}")

    # Simulate the rounds
    for _ in range(number_of_rounds):
        for i, participant in enumerate(participants_data):
            best_opponent = None
            highest_expected_value = 0 # set to 0, for all except only considering avoiding loss

            # Find the best opponent based on expected value (if no expected value > 0 is found,
participant does not challenge he passes)
            for j, other_participant in enumerate(participants_data):
                if i != j:
                    z_diff = participant['z_statistic'] - other_participant['z_statistic']
                    prob_win = calculate_prob_win(z_diff)
                    win_value = 0.05 * other_participant['coins']
                    lose_value = min(win_value, participant['coins'])
                    #expected_value = prob_win * win_value   # use this if only take into account the win
                    expected_value = (prob_win * win_value) - ((1-prob_win) * lose_value) # use this if
prudent, consider both sides
                    #expected_value = ((1-prob_win) * lose_value)*-1 # use this if consider only avoiding loss,
set above beginning highest_expected_value to be -1000000. Doesn't work only chooses zeros (first
guy)
                    if expected_value > highest_expected_value:
                        highest_expected_value = expected_value
                        best_opponent = j

            # Compete with the best opponent
            if best_opponent is not None:
                other_participant = participants_data[best_opponent]
                z_diff = participant['z_statistic'] - other_participant['z_statistic']
                prob_win = calculate_prob_win(z_diff)
                #print(f"Participant {i+1} challenges Participant {best_opponent+1} with an expected value
of {highest_expected_value:.2f} {z_diff} {prob_win}")  # Annotation added here
                win_value = 0.05 * other_participant['coins']
                rand_num = random.random()

                if rand_num < prob_win:  # Participant wins
                    win_amount = min(win_value, other_participant['coins'])
                    participant['coins'] += win_amount
                    other_participant['coins'] -= win_amount
                else:  # Participant loses
                    lose_amount = min(win_value, participant['coins'])
                    participant['coins'] -= lose_amount
                    other_participant['coins'] += lose_amount


    # Record the final coin totals for this trial
    for i, p in enumerate(participants_data):
```

```
        final_coins_per_trial[trial][i] = p['coins']

# Compute and print average final coins for each participant
for i in range(num_participants):
    average_coins = sum(final_coins_per_trial[trial][i] for trial in range(trials)) / trials
    # print(f"Participant {i+1}: Average Coins = {average_coins:.2f}")


# File to write results
output_file = 'participant_results.csv'

# Write average final coins for each participant to a CSV file
with open(output_file, mode='w', newline='') as file:
    writer = csv.writer(file)
    writer.writerow(['Participant', 'Average Coins'])

    for i in range(num_participants):
        average_coins = sum(final_coins_per_trial[trial][i] for trial in range(trials)) / trials
        percentile = participants_data[i]['percentile']
        writer.writerow([percentile, average_coins])

print(f"Results written to {output_file}")
```