The Design of Platform Traffic and Data Services

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Abstract

This paper proposes a model for studying the design of traffic and data services for a platform. The platform acts as both an advertiser, driving traffic to sellers, and a data intermediary that facilitates market segmentation. Traffic and data are complementary; the value of providing both is strictly greater than the sum of providing each. When the platform is uninformed about the seller's marginal cost, incentive problems arising from lower-cost types result in reduced service availability for higher-cost types in the optimal menu. The optimal menu offers three categories of services: traffic and data, data only, or no service. In general, the inclusion of each category in the optimal menu is determined by the associated information rent. With a uniform distribution of cost types, the majority of platform's expected revenue comes from the lowest cost types that benefit from both traffic and data services, and the platform loses about half of its expected revenue due to information rents. Welfare analyses suggest that vertical integration will benefit both the platform and consumers, while the seller may prefer the opposite.

1 Introduction

In the context of a digital economy, traffic refers to consumers' attention measured by metrics such as views and clicks. To date, the business models of platforms have focused on generating traffic and allocating it among sellers. The sales of online merchants heavily rely on the volume of traffic they are able to receive. According to this business model centered around traffic, the departments responsible for determining traffic allocation rules or algorithms have been the most profitable ones within platform companies. Examples of such departments include Google Ads and Meta Ads from the United States, as well as Ocean Engine by ByteDance and Alimama by Alibaba from China. Although among the top platforms, the allocation rule may be based on more elaborate and targeted criteria, most platforms tend to indiscriminately drive traffic to their users. This has resulted in an increasing number of consumers complaining about bombardments of insertion ads and recommendations. In

recent years, significant advancements in digital technology have also facilitated extensive collection and utilization of data on consumer activities by platforms for various purposes, such as making personalized recommendations, enabling price discrimination, and building decision support tools for sellers. The role of traffic and data on the platforms, both separately and combined, is a question of particular interest. Also of interest is how the various design of platform's traffic and data service offerings may impact equilibrium prices and their implications for welfare.

In practice, platforms have different endowments in terms of traffic or data, and they provide traffic and data services in various ways. For instance, Google, Meta and ByteDance often charge for traffic services based on clicks or impressions while providing data as a complementary service. Specifically, Ocean Engine offers market-level data to small and medium businesses while also offering tailored data for qualified individual sellers, both free of charge. On the other hand, Alimama provides combined traffic and data services for toC e-commerce and charges per order placement. Alibaba's toB ecommerce platform 1688 offers three annual service plans that include varying levels of combined traffic and data services as well as separate à la carte options for additional traffic.

To capture these features, this paper proposes a model in which the platform acts as both an advertiser and a data intermediary. Acting as an advertiser, the platform drives traffic towards sellers to increase their sales. This can also be interpreted as enhancing the visibility of sellers and their products to consumers, with α denoting the level of visibility. In its role as a data intermediary, the platform provides data services that facilitate market segmentation, enabling sellers to implement price discrimination across different segments, with x denoting the point of segmentation. Together, this model allows us to consider the value of traffic and data within the same framework, understand the trade-off that the platform faces in providing traffic and data services, and characterize the optimal design for the platform service menu. Ultimately, this study aims to provide further insights into the evolving business models of platforms and the data economy, as well as related policies governing traffic services, online advertising, data ownership, and the sale of data.

The model includes a continuum of consumers with private values, a seller for a single product with private and linear marginal cost, and a monopoly platform. The platform perfectly knows consumer values and has some belief about the seller's marginal cost, whereas the seller only has some belief about consumer values. Initially, the seller has a certain level of inherent or natural visibility, so that in the absence of the platform, consumers with values exceeding the price have a strictly positive but less than one probability of purchasing the product. The platform can provide a menu that offers traffic and data services for the seller to increase the visibility of the product and reveal partial information on consumer values in the form of market segmentation.

1.1 Results

We first find that in the symmetric information benchmark, the data and traffic services complement each other. That is, the value of both services combined exceeds the sum of their individual values, making it optimal to include both on the platform service menu as frequently as possible. Whenever the platform provides a data service, however, it faces a trade-off between generating more revenue from a smaller group of high-value consumers and allowing more middle-value consumers to make purchases.

When there is asymmetric information between the platform and the seller, the results show that types with lower marginal costs have a higher profit margin, so they have an incentive to pretend to be the higher-cost types and retain positive information rents from the platform. This suggests that in the revenue maximization problem of the platform, there is no distortion at the bottom: the lowest cost type receives the same service as in the menu with symmetric information benchmark; and there is no rent at the top: the highest cost type has to make a payment that is equal to their willingness to pay.

Specifically, the platform offers three categories of service plans: data and traffic services, data service only, and no service. We illustrate this feature in detail using binary cost types as an example and explain that the inclusion of each category in the optimal menu is determined by the associated information rent. More generally for a continuum of types, the platform offers each category of service based on the range in which each cost type falls. Firstly, both traffic and data services are provided for the lowest cost types. Secondly, only data service is offered to the intermediate types. Lastly, no service is offered to the highest cost types. Meanwhile, only the cost type at the lower bound receives the benchmark service plan, and as the marginal cost increases, the point of market segmentation moves to the right, corresponding to a reduction in the value of data service provided. Again, traffic service is never provided alone in platform's optimal menu. In the example with a uniform distribution of costs, the platform's expected revenue mainly comes from the set of lowest cost types who obtain both traffic and data services from the platform, while the middle set of types who only receive the data service contribute very little. Additionally, the platform loses approximately half of its expected revenue to the seller's information rents.

Welfare analyses imply that the introduction of traffic and data services always increases total surplus and, in most cases, also increases consumer surplus but to a lesser extent. When there is asymmetric information between the platform and the seller, the seller obtains the largest share of surplus, while consumers and the platform divide the remaining half. In other words, the platform and consumers have partially aligned interests and will benefit if there is vertical integration between the platform and the seller, or if the seller is more likely to have a low marginal cost.

For extensions, we examine the design of the optimal service menu with contractible prices and endogenous choice on traffic service provision. The results indicate that when the

platform can specify product prices while providing services, it cannot improve the screening, reduce information rent, or generate more revenue. This is because the equilibrium prices for the seller with and without contractibility coincide due to a continuum of types. However, for binary types, the contractibility of product price does allow the platform to better screen the types and generate more revenue from the low-cost type by reducing information rents. More importantly, as natural visibility becomes sufficiently high and the cost difference between seller types becomes small, the platform will be able to offer the benchmark menu and extract all additional surplus from the seller.

In the second extension, when the platform endogenously determines the intensity of traffic service with a cost, we find that the composition of the optimal menu remains unchanged from the exogenous traffic case, except that eligibility criteria for each category of services have become more restrictive. The intuition is that as traffic acquisition becomes costly for the platform, it becomes less profitable to provide traffic service to seller types with a low willingness to pay. As a result, it restricts the provision of traffic service for types that have excessively high marginal costs.

1.2 Related Literature

This paper is related to the literature on mechanism design for the sale of information and advertising technology by intermediaries such as platforms. In one stream of the literature, platforms provide information to facilitate advertising. Bergemann and Bonatti (2015) analyze linear pricing of data queries with targeted marketing, as well as the interaction between markets for data and advertising space. More recently, Bergemann and Bonatti (2023) study a nonlinear pricing model for managed advertising campaigns. Platforms can also be considered as matchmakers that reduce search frictions, improve matching between sellers and consumers, thus increasing the probability of trade, as in Gomes and Pavan (2016, 2024). In contrast, this paper abstracts the advertising or matching function of the platform as a change in the probability of trade between sellers and consumers, which is referred to as visibility on the platform. Additionally, this paper examines optimal menu design within a nonlinear pricing framework.

Another body of literature regards platforms as data intermediaries that enable market segmentation or inform decision making in general. Bergemann, Bonatti, and Smolin (2018) derive the optimal menu for providing data appends used in predictions. Yang (2022) studies the optimal market segmentation when consumer data can be sold and utilized for price discrimination. Ali, Lewis, and Vasserman (2023) investigate the combination of personalized pricing with verifiable data disclosure by consumers and determine the circumstances under which certain disclosure technology can benefit consumers. Fang and Kim (2023) examine the provision of data services by the platform in the presence of vertical differentiation and competition among sellers. In this paper, we introduce a traffic service that increases

the probability of trade, along with a data service that facilitates market segmentation. Consequently, we are able to investigate and compare various combinations of services that a platform can provide for optimal menu design.

The mechanism design problem and the findings of this study are also related to Mussa and Rosen (1978) as well as Myerson (1981). There have been numerous variations and extensions of the classical screening problem, as well as in its application to nonlinear pricing, as summarized in Bolton and Dewartripont (2005). Maskin and Riley (1982) investigate the optimal bundling problem faced by a monopolist who incorporates both quality and quantity into its selling strategy, and they demonstrate that the quantity discount schedule remains the same across quality levels. Laffont, Maskin, and Rochet (1987) determine the optimal nonlinear price schedule when a monopolist faces two dimensions of asymmetric information. Baron and Myerson (1982) provide optimal regulatory policies under asymmetric information for a monopolistic producer in terms of the probability of obtaining operational permission, the regulated price and quantity of output, and a subsidy to cover fixed costs if applicable. Our study is most similar to theirs in that the regulator in their model is somewhat analogous to the platform in ours. However, their objective function focuses on social welfare, which includes both consumer and producer surplus, while our focus is on revenue maximization of the monopolist platform. Additionally, unlike our solution, their solution does not feature categories.

We build upon these foundational frameworks to derive the optimal design of the platform service menu primarily for a continuum of types and briefly for binary types. In particular, we propose a solution to the nonlinear pricing problem that incorporates both category and quantity as screening instruments, which constitutes a novel contribution to the existing literature. By employing a relatively straightforward model setup, we further investigate the welfare implications for different market participants as well as explore various extensions of the model.

2 Model

2.1 The setup

There is a unit mass of consumers, a seller for a single product and a monopolist platform. Consumers value the product differently with $v \in X = [\underline{v}, \bar{v}]$ and v follows a distribution function $F(v)$. The seller produces with a constant marginal cost $c \in C = [\underline{c}, \bar{c}]$. The platform knows the exact value for each consumer, and believes that the seller's marginal cost c is drawn from a distribution function $G(c)$. The seller knows their own cost and believes that each consumer's valuation v is drawn from $F(v)$.

The seller can enter the platform and sell goods free of charge; however, the platform provides the seller with options for additional traffic and data services at a fee. The traffic

service will increase the visibility of the seller's product to consumers, while the data service will facilitate market segmentation on the platform. In the absence of any traffic service, the seller has a limited visibility to consumers. Specifically, consumers will see the product with a probability of $\alpha < 1$. When the platform provides the seller with additional traffic, the product can achieve full visibility. The data service sorts consumers into market segments $[\underline{v}, x)$ and $[x, \bar{v}]$ with $x \in X$, and enables the seller to post a price for each market segment. Consumers will make a purchase whenever the product is available and affordable. This paper will focus on the case in which a maximum of two market segments is being allowed on the platform.

2.2 Timing of the events and equilibrium

First, the platform proposes a menu of traffic and data services with transfers according to its prior belief about the distribution of the seller's marginal cost. Then, the seller decides whether to pay for the platform services. If the seller chooses not to use any paid service, they operate in the unsegmented market with limited visibility. Otherwise, the demand for the product depends on the visibility and segmentation specified by the service plan, and the seller posts the profit-maximizing price for each market segment accordingly. Finally, consumers makes purchasing decisions based on their values and the price they see.

Consumers whose values exceed the price will purchase the product. Therefore, for the unsegmented market, the demand can be denoted as $D : [\underline{v}, \bar{v}] \to [0, 1]$, where $D(p) = 1 - F(p)$. Similarly, the demand for two market segments is denoted as $D_0^x : [\underline{v}, x) \to [0, 1]$ with $D_0^x(p_0) = F(x) - F(p_0)$ for the segment on the left and $D_1^x : [x, \bar{v}] \to [0, 1]$ with $D_1^x(p_1) = 1 - F(p_1)$ for the segment on the right.

For a given c, the seller posts prices for each market segment in order to maximize profits. For the unsegmented market, the seller sets the price at the monopoly level and earns a profit of $\alpha(p-c)D(p)$ with limited visibility, or $(p-c)D(p)$ with full visibility through traffic service. When the seller receives both data and traffic services from the platform, they earn profits from each market segment that add up to

$$\pi(x, c) = \max_{\{p_n\}} \sum_n (p_n - c) D_n^x(p_n). \tag{1}$$

If only the data service is provided, the seller's profit is calculated by multiplying (1) with α. In particular, for any given x, the seller charges $p_1(x, c) = \max\{x, p(c)\}$ in the market segment $[x, \bar{v}]$, where $p(c) = \frac{1-F(p(c))}{f(p(c))} + c$, and charges $p_0(x, c) = \max\{c, \frac{F(x)-F(p_0)}{f(p_0)} + c\}$ in segment $[\underline{v}, x)$.

The platform maximizes its expected revenue by offering a variety of traffic and data services, and charging τ for each service plan σ based on the cost type c. Define $\sigma : [\underline{c}, \bar{c}] \to \{0, 1\} \times \{0, 1\} \times [\underline{v}, \bar{v}]$ as the service plan for cost type c. For example, $\sigma(c) = (0, 1, x)$

means that the seller with a marginal cost c will receive data service with segmentation at x but no boost in traffic. Define $\tau : [\underline{c}, \bar{c}] \to \mathbb{R}$ as the payment made by the seller. In the asymmetric information setting, the platform designs a menu $\mathcal{M} = [\sigma, \tau]$ in order to maximize the expected revenue

$$R = \max_{\sigma,\tau} \mathbb{E}[\tau], \tag{2}$$

subject to incentive compatibility and individual rationality constraints. Note that by the revelation principle, we restrict our attention to direct mechanisms here.

2.3 The combination value of traffic and data

Before we delve into to the problem of platform service design, it might be useful to examine the combination value of traffic and data. Such analysis can enhance our understanding of the underlying trade-offs faced by the platform. It may also be relevant to potential regulations regarding the for-profit usage of traffic or data by platforms. The value of different combinations of traffic and data services, or the seller's willingness to pay for them, is determined by the service type, market segmentation, and the seller's marginal cost of production. The seller's values are summarized in Table 1.

Table 1: The combination value of traffic and data

Service Type σ	Value $V_\sigma(c)$
No traffic or data $(0, 0, \bar{v})$	0
Traffic Alone $(1, 0, \bar{v})$	$(1-\alpha)(p(c)-c)D(p(c))$
Data Alone $(0, 1, x)$	$\alpha \sum_{n=0}^{1} (p_n(x,c)-c)D_n^x(p_n(x,c)) - \alpha(p(c)-c)D(p(c))$
Traffic and Data $(1, 1, x)$	$\sum_{n=0}^{1} (p_n(x,c)-c)D_n^x(p_n(x,c)) - \alpha(p(c)-c)D(p(c))$

Proposition 1. *Suppose $v \sim U(0,1)$. For any $\alpha \in (0,1)$ and $c \in [0,1)$, the maximization of the combination value of traffic and data has the following properties.*

1. $V_{(1,1,x)} > V_{(1,0,\bar{v})} + V_{(0,1,x)}$, i.e. traffic and data are complements, and it is optimal for the platform to provide both.

2. $p(c) < x_{(1,1,x)}(c) = x_{(0,1,x)}(c)$, data service segments the market at a consumer value higher than the monopoly price in the unsegmented market, while traffic service has no impact on the price of the good.

3. The value of data $V_{(0,1,x)}$ goes to zero when $\alpha \to 0$, the value of traffic $V_{(1,0,\bar{v})}$ goes to zero as $\alpha \to 1$, and $V_{(1,0,\bar{v})} > V_{(0,1,x)}$ when $\alpha > \frac{3}{4}$.

In the symmetric information setting, the platform knows the true cost of the seller, so it will simply offer the service that maximizes the value of traffic and data for the seller and extracts all additional surplus. In this setting, assuming that consumers have uniformly distributed values in the range $[0, 1]$, the maximum value of the combination of traffic and data $V_{(1,1,x)} = \frac{4-3\alpha}{12}(1-c)^2$ is strictly greater than the sum of the values of traffic service alone $V_{(1,0,\bar{v})} = \frac{1-\alpha}{4}(1-c)^2$ and data service alone $V_{(0,1,x)} = \frac{\alpha}{12}(1-c)^2$. Therefore, traffic and data are complementary to each other, making it optimal for the platform to provide both.

The use of data increases the price for consumers with higher values from $p(c) = \frac{1+c}{2}$ to $p_1(c) = x(c) = \frac{2+c}{3}$, while lowering the price in the rest of the market to $p_0(c) = \frac{x+c}{2} = \frac{1+2c}{3}$. On the other hand, the use of traffic has no impact on the price of the good. The value of data alone goes to zero as $\alpha \to 0$, and the value of traffic alone goes to zero as $\alpha \to 1$. Additionally, when $\alpha > \frac{3}{4}$, the value of data alone exceeds the value of traffic alone. Furthermore, the values of all different combinations of traffic and data are proportional to $(1-c)^2$, indicating that the seller with a lower marginal cost tends to value every type of service more, and these values are all convex in c.

More generally, the provision of data services enables the seller to increase profits from previously excluded consumers, while driving traffic to the seller increases their profits from all purchasing consumers. As a result, the platform faces a trade-off when choosing the optimal market segmentation characterized by x: a lower x allows more consumers to buy the good but lowers revenue from consumers with higher values.

3 A First Example

In more realistic contexts, the platform often lacks knowledge of the true marginal cost of the seller and must design the service menu under asymmetric information to specify which services to provide and how much the seller has to pay. In similar screening problems, in order to incentivize types that generate higher revenue to participate, the designer may either provide partial services to opposites types or exclude opposites types from the market.

We find that the optimal menu should provide both data and services to the seller with the lowest marginal cost, while offering no service to the seller with the highest cost. In other words, there should be "no distortion at the bottom" and "no rent at the top". This is because the lower cost types value the same service plan more than higher cost types do; they tend to have an incentive to imitate higher cost types in order to obtain a less desirable service plan but pay even less. As a result, the platform is compelled to offer lower prices for low-cost types due to information rents, so that they pay less than the value of services they receive. Additionally, the platform reduces the value of services provided to high-cost types in order to decrease the information rent of low-cost types, while leaving no surplus for the highest highest cost type.

The platform also offers a data-only service for intermediate types, which is worth noting.

This special feature is associated with the optimal menu of platform traffic and data services, as well as more general screening or nonlinear pricing problems that incorporate various categories. In the context of this paper, the data-only service is included in the optimal menu rather than the traffic-only service because, over a certain range of cost types, the information rent is strictly lower with data-only service.

We illustrate these results using the example of binary cost types. Consider that there are two types of marginal cost for the seller, namely $c \in \{c^L, c^H\}$. Let γ represent the prior belief for being the type with a lower marginal cost, where $\gamma = \Pr(c = c^L)$. The platform faces the revenue maximization problem, which can be written as

$$\max_{\sigma,\tau} \mathbb{E}[\tau] = \max_{\sigma,\tau} \gamma \tau_{\sigma(c^L)}(c^L) + (1-\gamma)\tau_{\sigma(c^H)}(c^H), \tag{3}$$

subject to individual rationality constraints

$$V_{\sigma(c)}(c) - \tau_{\sigma(c)}(c) \geqslant 0, \tag{4}$$

and incentive compatibility constraints,

$$V_{\sigma(c)}(c) - \tau_{\sigma(c)}(c) \geqslant V_{\sigma(c')}(c) - \tau_{\sigma(c')}(c'), \tag{5}$$

for each c. Henceforth, we assume that the consumer values follow a uniform distribution $v \sim U(0,1)$.

Note that the seller with a lower marginal cost has the incentive problem and retains positive information rent. Therefore, IR^H binds, so $\tau_{\sigma(c^H)}(c^H) = V_{\sigma(c^H)}(c^H)$, which corresponds to "no rent at the top". Also, IC^L binds, so $\tau_{\sigma(c^L)}(c^L) = V_{\sigma(c^H)}(c^H) - V_{\sigma(c^H)}(c^L) + V_{\sigma(c^L)}(c^L)$. Since the low-cost type is offered the same service plan as in the symmetric information benchmark, $\sigma(c^L) = (1, 1, \frac{2+c^L}{3})$, which corresponds to "no distortion at the bottom". Thus the problem of the platform can be reformulated as

$$\max_{\sigma} V_{\sigma(c^H)}(c^H) - \gamma V_{\sigma(c^H)}(c^L) + \gamma V_{\sigma(c^L)}(c^L). \tag{6}$$

The solution demonstrates that for the low-cost seller, the optimal menu should include both traffic and data services. With $c^L = 0, c^H = c > 0$, we have market segmentation at $x(0) = \frac{2}{3}$ and the value of services $V_{\sigma(0)}(0) = \frac{4-3\alpha}{12}$ for the low-cost type. For the high-cost seller, the optimal service plan depends on the prior probability of being the low-cost type γ. In particular, there exist two thresholds $\bar{\gamma}(c)$ and $1-c$ across the range of γ, where $\bar{\gamma}(c) = \frac{1}{8}(3c^2 - 8c + 8 - \sqrt{3c^2(3c^2 - 16c + 16)})$ and $\bar{\gamma}(c) < 1 - c$ for all $c < 1$. If $\gamma < \bar{\gamma}(c)$, the platform provides both traffic and data to the high-cost seller, but with less value than in the symmetric information benchmark. That is, segmentation occurs at $x(c) = \frac{2}{3} + \frac{c}{3(1-\gamma)}$. If $\bar{\gamma}(c) \leqslant \gamma < 1 - c$, only data is provided for the high-cost seller, also with segmentation at

$x(c) = \frac{2}{3} + \frac{c}{3(1-\gamma)}$. Otherwise, if $\gamma \geqslant 1 - c$, no service is provided for the high-cost seller.

As a numerical example, let $c = \frac{1}{3}$ and $\alpha = \frac{1}{2}$. The optimal menu provides traffic and data services to the low-cost type with $x(0) = \frac{2}{3}$. It also provides both services with $x(c) = \frac{2}{3} + \frac{1}{9(1-\gamma)}$ to the high cost type if $\gamma < \frac{17-\sqrt{33}}{24} \doteq 0.469$, as opposed to $x(c) = \frac{7}{9} < \frac{2}{3} + \frac{1}{9(1-\gamma)}$ with symmetric information; only data service is provided if $\frac{17-\sqrt{33}}{24} \leqslant \gamma < \frac{2}{3}$; no service is provided if $\gamma \geqslant \frac{2}{3}$.

Notice that the low-cost type enjoys information rent given by $V_{\sigma(c^H)}(c^L) - V_{\sigma(c^H)}(c^H)$. The strictly positive information rent leads to distortion in the service plan for the high-cost type, which is proportional to values of γ and c. The distortion serves to reduce the low-cost type's incentive to mimic the other type by reducing the value of services offered to the high-cost type. Specifically, as γ increases, the traffic service is turned off all at once for the high-cost type when $\gamma \geqslant \bar{\gamma}(c)$. Meanwhile, the data service is distorted from segmentation at $\frac{2+c}{3}$ to $\frac{2}{3} + \frac{c}{3(1-\gamma)}$, and then it is completely turned off at a second threshold $\gamma \geqslant 1 - c$. When $\gamma \geqslant 1 - c$, which means that the seller is sufficiently likely to be of the low-cost type, the low-cost type also has to transfer all additional gains from services to the platform and cannot retain any information rent.

In other words, the platform stops providing traffic service sooner because it leaves a higher information rent for the low-cost type. For $\bar{\gamma}(c) \leqslant \gamma < 1 - c$, the information rent with data service alone is less than that with traffic service alone. Algebraically, this can be expressed and simplified as

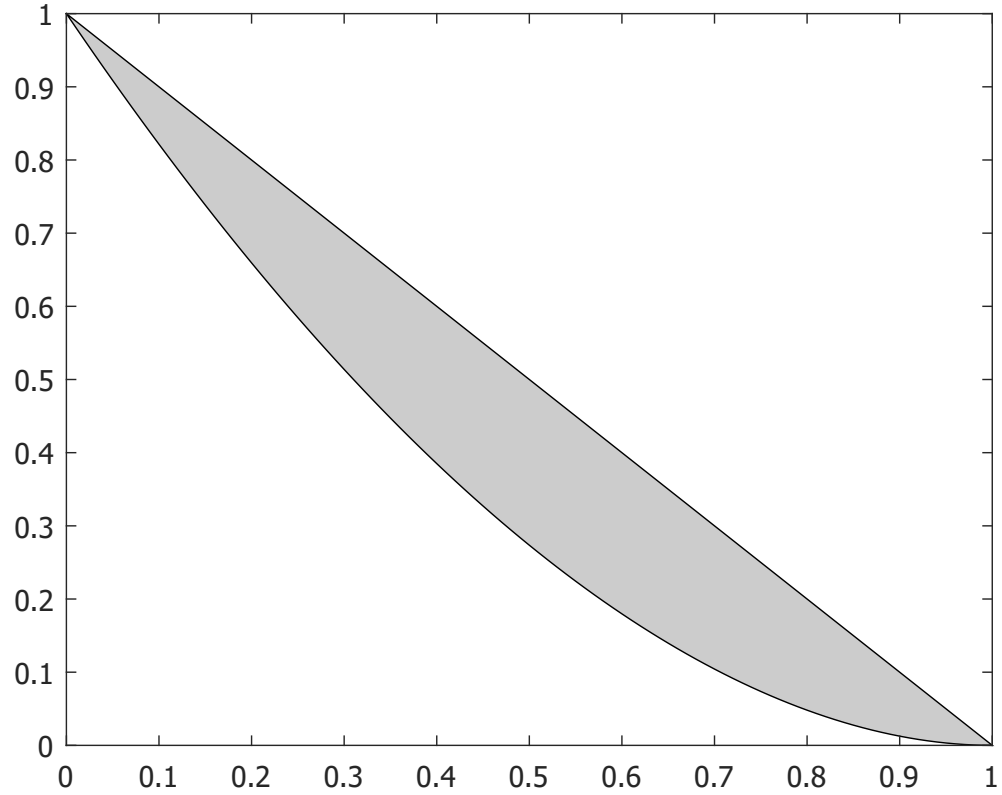
$$\alpha \left(x(1-x) + \frac{x^2}{4} - (x-c)(1-x) - \frac{(x-c)^2}{4} \right) < \frac{1}{4} - \frac{(1-c)^2}{4}, \tag{7}$$

where $x = \frac{2}{3} + \frac{c}{3(1-\gamma)}$. Intuitively, data service is more tailored to each cost type, so it would be less likely that the low-cost type will obtain a satisfactory return by purchasing one designed for the other type.

Figure 1 demonstrates the situations in which the optimal menu provides partial service to the high-cost type. The horizontal axis represents c and the vertical axis represents γ. The shaded area corresponds to the cases where the probability of having a low marginal cost γ lies between $1 - c$ and $\bar{\gamma}$(c), and the optimal menu offers only data service to the high-cost type. The lower left area represents cases with a sufficiently low probability of having a low marginal cost, allowing the high-cost type to benefit from both services from the platform.

The graph also indicates that the data-only option applies to a wider range of γ when the cost difference between types is not at either extreme. This suggests that the difference in the information rents between data and traffic services depends on two factors: the disparity in marginal costs of both types, as well as the value of services to the high-cost type. On one

Figure 1: The relationship between $\bar{\gamma}$ and $1 - c$



hand, as c approaches 0, the information rent for either service goes to zero as the difference between types disappears. On the other hand, as c approaches 1, the value of any service for the high-cost type also diminishes to zero, thereby eliminating the difference in information rents. Consequently, the difference in information rents becomes most apparent when the impact from both factors is moderate.

Just as one will see later in the case of a continuum of types, the data-only service plan may contribute only modestly to the platform's expected revenue. Therefore, in practice, the platform may either provide both services or no service at all to reduce the number of different categories on its menu for convenience and clarity. A comprehensive analysis of the optimal platform service design with binary cost types can be found in the appendix.

4 Platform Service Design

4.1 Continuum of cost types

Consider the more general case where there is a continuum of seller types with marginal cost c, which is drawn from a distribution function $G(c)$. Assume that $G(c)$ is regular. Now the platform chooses a mechanism $\mathcal{M} = [\sigma, \tau]$ to maximize expected revenue

$$\mathbb{E}\left[\tau_\sigma(c)\right] = \int_C \tau_\sigma(c) dG(c),$$

subject to the incentive compatibility constraint

$$V_{\sigma(c)}(c) - \tau_{\sigma(c)}(c) = \max_{c' \in C} V_{\sigma(c')}(c) - \tau_{\sigma(c')}(c'), \tag{8}$$

and individual rationality constraint

$$V_{\sigma(c)}(c) - \tau_{\sigma(c)}(c) \geqslant 0, \tag{9}$$

for every $c \in C$.

As shown in Table 1, the value of traffic and data services for the seller with marginal cost c is defined as

$$V_{\sigma(c)}(c) = a\sum_n (p_n(c) - c) D_n^x(p_n(c)) - \alpha(p(c) - c) D(p(c)), \tag{10}$$

where $a = 1$ if traffic is provided and $a = \alpha$ if not; $n = 0, 1$ and $x \in X$ if data is provided, $n = 0$ and $x = \bar{v}$ if not. The first term $a\sum_n (p_n(c) - c) D_n^x(p_n(c))$ represents the profit of the seller from receiving platform service $\sigma(c)$, and the second term $\alpha(p(c) - c) D(p(c))$ represents the seller's payoff from the outside option.

Symmetric information benchmark

To begin with, we briefly examine the case of symmetric information, which is equivalent to solving the problem above while relaxing the incentive compatibility constraints. With the monopolistic platform, the transfer will be set equal to the seller's willingness to pay. Therefore, the benchmark menu is characterized by $\mathcal{M}^* = [\sigma^*, \tau^*]$ with $\sigma^*(c) = (1, 1, \frac{2+c}{3})$ and $\tau^*(c) = \frac{4-3\alpha}{12}(1-c)^2$ for all $c \in C$. If we assume that the marginal cost follows a uniform distribution $c \sim U(0, 1)$, then the expected revenue of the platform is $\mathbb{E}\left[\tau_\sigma^*(c)\right] = \frac{4-3\alpha}{36}$.

Optimal menu with information asymmetry

With asymmetric information, the revenue-maximizing menu offered by the platform differs from the benchmark menu due to incentive problems. From the IC constraints (8), define $U(c) = V_{\sigma(c)}(c) - \tau_{\sigma(c)}(c) = \max_{c' \in C} V_{\sigma(c')}(c) - \tau_{\sigma(c')}(c')$. By the envelop theorem, there is $U'(c) = -a\sum_n D_n^x(p_n(c)) + \alpha D(p(c))$. Then, according to the Fundamental Theorem of Calculus, the transfer rule can be determined for each cost type as follows.

Lemma 1. *The transfer rule of a revenue-maximizing service menu is given by*

$$\tau_{\sigma(c)}(c) = V_{\sigma(c)}(c) - \int_c^{\bar{c}} \left[a\sum_n D_n^x(p_n(z)) - \alpha D(p(z)) \right] dz - U(\bar{c}), \tag{11}$$

where $V_{\sigma(c)}(c) = a\sum_n (p_n(c) - c) D_n^x(p_n(c)) - \alpha(p(c) - c)D(p(c))$.

Define the virtual marginal cost for all $c \in C$,

$$\phi(c) = c + \frac{G(c)}{g(c)}. \tag{12}$$

The distribution function $G(c)$ is assumed to be regular so that $\phi(c)$ is increasing in c.

From lemma 1, the objective function of the platform can then be written as

$$\mathbb{E}[\tau(c)] = \int_C \left[a\sum_n (p_n(c) - \phi(c)) D_n^x(p_n(c)) - \alpha(p(c) - \phi(c))D(p(c)) \right] dG(c) - U(\bar{c}). \tag{13}$$

The expression inside the integral can be regarded as the "virtual value of service" for the seller. Let $\nu(c)$ represents the virtual value of service for cost type c. Therefore, the revenue-maximizing service menu with asymmetric information is considered as the solution to an optimization problem, where the marginal cost c in the symmetric information problem is replaced by the virtual cost $\phi(c)$.

To determine the optimal choice of σ for each c, it is important to note that the IC constraints imply that the seller should truthfully report the marginal cost type when selecting from the optimal menu. This provides us with a monotonicity condition for σ in the optimal menu as follows,

$$V_{\sigma(c)}(c) - V_{\sigma(c')}(c) \geqslant V_{\sigma(c)}(c') - V_{\sigma(c')}(c'). \tag{14}$$

Assuming that the consumer values follow a uniform distribution $v \sim U(0,1)$ and $\frac{G(c)}{g(c)(1-c)}$ is an increasing function of c, the optimal menu is summarized in the following proposition.

Proposition 2. *For a continuum of cost types, the optimal menu provides both traffic and data to the seller with marginal cost* $c < \hat{c}$*, and data service to the seller with marginal cost* $\hat{c} \leqslant c < \tilde{c}$*, where* $\hat{c}$*,* $\tilde{c}$ *are determined by* $\frac{G(\hat{c})}{g(\hat{c})} = (1 - \hat{c})(4 - 2\sqrt{3})$ *and* $\frac{G(\tilde{c})}{g(\tilde{c})} = 1 - \tilde{c}$*. The*

corresponding market segmentation is specified by a non-decreasing function of c, denoted as $x(c) = min\{\frac{2+\phi(c)}{3}, 1\}$. *The platform does not offer any services to the remaining types.*

Additionally, the type with the lowest cost receives the same segmentation as in the symmetric information benchmark.

Proof. To solve the problem, we first take the first-order condition with respect to x for cases where there is data service, i.e. $\sigma = (1, 1, x)$ or $(0, 1, x)$, which is given by

$$a\sum_n \left[\frac{\partial p_n(\cdot)}{\partial x} D_n^x(p_n(c)) + (p_n(c) - \phi(c))\frac{\partial D_n^x(\cdot)}{\partial x}\right] = 0 \tag{15}$$

Then we can solve for the optimal $x(c) \in X$ and substitute it into the virtual value of service $\nu(c)$. Compare these virtual values with the values obtained when there is only traffic service or no service at all. Finally, pin down the service type with the highest virtual value $\nu(c)$ for each c, and integrate over the entire range of c to obtain expected revenue under the optimal menu. The transfer plan for this menu can be determined according to Lemma 1.

Since the consumer values are uniformly distributed on $[0, 1]$, the virtual value of service becomes

$$v(c) = a\sum_n (p_n(c) - \phi(c))D_n^x(p_n(c)) - \alpha(p(c) - \phi(c))D(p(c)), \tag{16}$$

where $p_0(c) = \frac{x(c)+c}{2}$, $p_1(c) = p(c) = x(c)$, and $x(c) = min\{\frac{2+\phi(c)}{3}, 1\}$. In particular, the first part of the right-hand side of the equation above corresponds to the virtual profit with data service, and it is given by

$$a\sum_n (p_n(c) - \phi(c))D_n^x(p_n(c)) = \frac{a}{3}(1 - c - \frac{G(c)}{g(c)})^2 - \frac{a}{4}(\frac{G(c)}{g(c)})^2. \tag{17}$$

The second part, which corresponds to the virtual profit without data service, is given by

$$a(p(c) - \phi(c))D(p(c)) = \frac{a(1-c)}{4}(1 - c - 2\frac{G(c)}{g(c)}), \tag{18}$$

where $a = 1$ with traffic service and $a = \alpha < 1$ without.

Notice that the virtual value of the traffic and data services together exceeds the virtual value of data service alone whenever there is a strictly positive virtual profit with data service (17), that is when $\frac{G(c)}{g(c)} > (1-c)(4+2\sqrt{3})$ or $\frac{G(c)}{g(c)} < (1-c)(4-2\sqrt{3})$. Also, offering both services weakly dominates offering traffic only for all c. Since the optimal market segmentation is given by $x(c) = min\{\frac{2+\phi(c)}{3}, 1\}$, the platform does not offer data service for types with $\phi(c) = c + \frac{G(c)}{g(c)} \geqslant 1$. Only the type that satisfies $\frac{G(c)}{g(c)} < 1 - c$ will possibly receive data service from the optimal menu. Since $4 + 2\sqrt{3} > 1$, the platform prefers providing both traffic and data rather than providing data service alone for the seller whose marginal cost

satisfies

$$\frac{G(c)}{g(c)} < (1-c)(4-2\sqrt{3}). \tag{19}$$

Assume $\frac{G(c)}{g(c)(1-c)}$ to be an increasing function of c, then the inequality (19) establishes an upper threshold $\hat{c}$ for receiving the traffic and data services. Next, we can verify that the virtual values are positive within this range. The virtual value of combined services is strictly positive if $\frac{G(c)}{g(c)} > (1-c)(4-3\alpha+\sqrt{(3-3\alpha)(4-3\alpha)})$ or $\frac{G(c)}{g(c)} < (1-c)(4-3\alpha-\sqrt{(3-3\alpha)(4-3\alpha)})$. Given the parameter range $\alpha \in (0,1)$, we have $4-3\alpha+\sqrt{(3-3\alpha)(4-3\alpha)} > 1$ and $4-2\sqrt{3} < 4-3\alpha$ *i.e.*$\sqrt{(3-3\alpha)(4-3\alpha)} < 1$. Therefore, the optimal menu provides both traffic and data to the seller whose marginal cost satisfies (19).

For types with $c \geqslant \hat{c}$, we can rule out the traffic service alone since it is always dominated by services combined. On the other hand, the data service alone can yield a higher virtual value than services combined as along as $\phi(c) < 1$, and the virtual value for data service alone is strictly positive for $\frac{G(c)}{g(c)} < 1-c$. Moreover, the virtual value of traffic service alone becomes negative whenever $\frac{G(c)}{g(c)(1-c)} > \frac{1}{2}$. Note that $4-2\sqrt{3} > \frac{1}{2}$. Therefore, in the optimal menu, only the data service will be provided to the seller with marginal cost c such that

$$4-2\sqrt{3} \leqslant \frac{G(c)}{g(c)(1-c)} < 1. \tag{20}$$

Under the same assumption for $\frac{G(c)}{g(c)(1-c)}$, this establishes another upper threshold $\tilde{c}$ for receiving only data service. □

Notice that $\hat{c}$, $\tilde{c}$ are independent of α, and $\hat{c}$ is always smaller than $\tilde{c}$, leaving a non-empty set of cost types for receiving only data service in the optimal menu. The size of this set is proportional to the flatness of $\frac{G(c)}{g(c)(1-c)}$.

In comparison with the scenario when types are binary, the traffic service is again more sensitive to the incentive problem than the data service. This is because the incentive problem simply increases the virtual marginal cost, putting downward pressure on the platform revenue from the traffic service. Specifically, the virtual profit from the traffic service decreases from $\frac{1}{4}(1-c)^2$ to $\frac{1}{4}(1-c)(1-c-2\frac{G(c)}{g(c)})$, as shown in (18). However, for expected revenue from providing data service, it simultaneously increases the virtual cost, raises the product price for higher-valued consumers while reducing the price for lower-valued consumers, as well as increases the total sales of the product, thereby mitigating its downward effect. In particular, the virtual profit decreases from $\frac{\alpha}{3}(1-c)^2$ to $\frac{\alpha}{3}(1-c-\frac{2+2\sqrt{3}}{2}\frac{G(c)}{g(c)})(1-c-\frac{2-2\sqrt{3}}{2}\frac{G(c)}{g(c)})$ as shown in (17). Intuitively, traffic service has no impact on the price of the product, so it benefits all cost types more uniformly, whereas data service is customized to each cost type individually, which provides less benefit for anyone attempting to imitate other types. As a result, offering only data service helps the platform better screen the seller's type and reduce information rents compared to offering only traffic service.

An example with uniform distribution for c

As in the context of symmetric information, we use the example where c follows a uniform distribution $U(0,1)$ for illustration. Here, $G(c) = c$ and $\phi(c) = 2c$. The virtual value of both services for the cost type c, i.e. $\sigma = (1, 1, min\{\frac{2+2c}{3}, 1\})$, becomes

$$\begin{aligned} v(c) &= \sum_n (p_n(c) - \phi(c)) D_n^x(p_n(c)) - \alpha(p(c) - \phi(c)) D(p(c)) \\ &= \frac{1}{12}(2 - (4+\sqrt{3})c)(2 - (4-\sqrt{3})c) - \frac{\alpha}{4}(1-3c)(1-c). \end{aligned} \tag{21}$$

To maximize revenue, the platform will offer both traffic and data services as long as $\frac{1}{12}(2-(4+\sqrt{3})c)(2-(4-\sqrt{3})c) = \frac{13}{12}c^2 - \frac{4}{3}c + \frac{1}{3} > 0$, which is the case when the marginal cost is below the threshold $c < \frac{8-2\sqrt{3}}{13}$. In addition, the platform will offer only data service to types with marginal cost c such that $\frac{8-2\sqrt{3}}{13} \leqslant c < \frac{1}{2}$. Again, the possibility of providing only traffic service has been ruled out. Finally, it is not optimal for the platform to provide any service for the remaining types whose $c \geqslant \frac{1}{2}$.

Thus, the optimal menu $\mathcal{M}^* = [\sigma^*, \tau^*]$ for this numerical example is given by

$$\sigma^*(c) = \begin{cases} (1, 1, \frac{2+2c}{3}) & \text{if } c < \hat{c} \\ (0, 1, \frac{2+2c}{3}) & \text{if } \hat{c} \leqslant c < \tilde{c}\,, \\ (0, 0, 1) & \text{o.w.} \end{cases} \tag{22}$$

$$\tau^*(c) = \begin{cases} \frac{(2-c)(2-3c)-3\alpha(1-c)^2}{12} - \int_c^{\hat{c}} \left[\frac{2}{3} - \frac{5z}{6} - \frac{\alpha(1-z)}{2}\right] dz - \int_{\hat{c}}^{\tilde{c}} \alpha \left[\frac{2}{3} - \frac{5z}{6} - \frac{1-z}{2}\right] dz & \text{if } c < \hat{c} \\ \frac{\alpha(2-c)(2-3c)-3\alpha(1-c)^2}{12} - \int_c^{\tilde{c}} \alpha \left[\frac{2}{3} - \frac{5z}{6} - \frac{1-z}{2}\right] dz & \text{if } \hat{c} \leqslant c < \tilde{c}\,, \\ 0 & \text{o.w.} \end{cases} \tag{23}$$

$$= \begin{cases} \frac{56+12\sqrt{3}}{507} - \frac{93+32\sqrt{3}}{1352}\alpha - \frac{c^2}{6} & \text{if } c < \hat{c} \\ \frac{\alpha}{24} - \frac{\alpha c^2}{6} & \text{if } \hat{c} \leqslant c < \tilde{c} \\ 0 & \text{o.w.} \end{cases} \tag{24}$$

where $\hat{c} = \frac{8-2\sqrt{3}}{13}$ and $\tilde{c} = \frac{1}{2}$.

Moreover, the expected revenue of the platform is

$$\begin{aligned}\mathbb{E}\left[\tau_\sigma^*(c)\right] =& \int_0^{\hat{c}} \left[\frac{1}{12}(2-(4+\sqrt{3})c)(2-(4-\sqrt{3})c) - \frac{\alpha}{4}(1-3c)(1-c)\right] dc + \\ & \int_{\hat{c}}^{\tilde{c}} \left[\frac{\alpha}{12}(2-(4+\sqrt{3})c)(2-(4-\sqrt{3})c) - \frac{\alpha}{4}(1-3c)(1-c)\right] dc \qquad (25)\\ =& \frac{56+12\sqrt{3}}{1521} - \frac{88+43\sqrt{3}}{4394}\alpha + \frac{100\sqrt{3}-153}{52728}\alpha \qquad (26)\\ =& \frac{56+12\sqrt{3}}{1521} - \frac{93+32\sqrt{3}}{4056}\alpha. \qquad (27)\end{aligned}$$

The expected revenue $\mathbb{E}\left[\tau_\sigma^*(c)\right]$ is linear in α. The first two terms of the expected revenue in (26) decrease with α. This part approaches $\frac{56+12\sqrt{3}}{1521} \doteq 0.0505$ as $\alpha \to 0$ and $\frac{664-75\sqrt{3}}{39546} \doteq 0.0135$ as $\alpha \to 1$. The last term of the expected revenue in (26) increases with α, which becomes zero as as $\alpha \to 0$ and reaching 3.83×10^{-4} as $\alpha \to 1$. This implies that the main source of expected revenue for the platform comes from providing both traffic and data services to low-cost types. Providing only data service to intermediate types contributes very modestly to the platform's expect revenue.

Overall, the expected revenue decreases with α. Remember that α represents the level of natural visibility or the desirability of the outside option. Compared to the symmetric information context where the platform can generate a revenue of $\frac{4-3\alpha}{36}$, which approaches $\frac{1}{9} \doteq 0.1111$ as $\alpha \to 0$ and $\frac{1}{36} \doteq 0.0278$ as $\alpha \to 1$, in this case, the platform loses about half of its revenue due to information rents for the low-cost types of seller.

4.2 Welfare

We consider four scenarios to investigate the welfare implications of the platform traffic and data services. First, the efficient allocation is achieved when all consumers with a higher value than the marginal cost of production receive the good, which we will use as the benchmark for welfare analysis. In the second scenario, where there is a monopolist seller with limited visibility and no platform, the total surplus in equilibrium will be less than the efficient level. Thirdly, when there is a platform that can offer traffic and data services to increase product visibility and facilitate market segmentation, the platform can extract all additional surplus from the seller if it has complete information about the seller's marginal cost. Lastly, if the seller's marginal cost is private information, then the platform restricts access to its services for certain cost types and leaves a higher expected surplus for the seller.

Proposition 3. *When there is symmetric information between the platform and the seller, the introduction of a platform always increases total surplus. It also increases consumer surplus as long as the natural visibility is not too high, i.e., $\alpha < \frac{8}{9}$. However, the increase in consumer surplus is smaller than the increase in the total surplus.*

Without a platform, the market can achieve a maximum amount of total surplus equal to $\frac{3\alpha}{4}$ of the efficient amount, with two-thirds, or $\frac{\alpha}{2}$, going to the seller and one-third, or $\frac{\alpha}{4}$, going to consumers. After the introduction of a platform, the total surplus can be increased to at most $\frac{8}{9}$ of the efficient level, with three-fourths, or $\frac{2}{3}$, allocated to the platform and the seller, and one-fourth, or $\frac{2}{9}$, allocated to consumer. This is the scenario when there is symmetric information between the platform and the seller, which can also been seen as a form of vertical integration between the two parties. These results hold regardless of the distribution of cost types, whether they are binary or continuous.

When there is information asymmetry between the platform and the seller, we first examine the case with binary types and then consider the case with a continuum of types.

Asymmetric information and a continuum of types

For a continuum of types with c follows a uniform distribution $U(0,1)$, the platform offers both services for $c < \frac{8-2\sqrt{3}}{13}$, only data service for $\frac{8-2\sqrt{3}}{13} \leqslant c < \frac{1}{2}$, and no service for $c \geqslant \frac{1}{2}$.

Proposition 4. *With asymmetric information and a continuum of types, the introduction of traffic and data services benefits all parties and nearly doubles the total surplus.*

The seller receives approximately half of the total surplus, while consumers and the platform divide the remaining part, with the platform receiving slightly more.

Table 2 presents the welfare results as ratios to the efficient allocation for continuous types, assuming $\alpha = \frac{1}{2}$. The total surplus in the efficient allocation is equal to $\frac{1}{6}$, which is normalized to 1. When there is no platform, the seller makes a profit of 0.25 and consumers keep a surplus of 0.125. When the platform offers traffic and data services under asymmetric information, the seller's profit increases substantially to 0.357, consumer surplus increases to 0.183, while the platform can generate revenue of 0.193. Compared to when the platform has perfect knowledge of the seller's type, both the total surplus and consumer surplus decrease while the producer surplus increases due to information rents. When comparing the case with a continuum of types to that with binary types, a continuous distribution of types tends to introduce more distortions into the platform's optimal service menu, resulting in higher surplus allocated for the seller, less for the platform, and slightly less for consumers.

Table 2: Welfare Analysis with a Continuum of Types, $\alpha = \frac{1}{2}$

Scenario	Total Surplus	Producer Surplus	Consumer Surplus	Platform Revenue
Efficient Allocation (Ratio)	1	-	-	-
No Platform (Ratio)	0.375	0.25	0.125	0
Symmetric Info. (Ratio)	0.889	0.25	0.222	0.417
Asymmetric Info. (Ratio)	0.734	0.357	0.183	0.193

Note: Total surplus of efficient allocation in this scenario is equal to $\frac{1}{6}$, which has been normalized to 1 in this table.

Moreover, the total surplus, producer surplus and consumer surplus are all linear and increasing in α. The platform revenue is also linear but decreasing in α. More details are shown in Table 3.

Table 3: Welfare Analysis with a Continuum of Cost Types

Scenario	Total Surplus	Producer Surplus	Consumer Surplus	Platform Revenue
Efficient Allocation	$\frac{1}{6}$	-	-	-
No Platform	$\frac{\alpha}{8}$	$\frac{\alpha}{12}$	$\frac{\alpha}{24}$	0
Symmetric Info.	$\frac{4}{27}$	$\frac{\alpha}{12}$	$\frac{1}{27}$	$\frac{1}{9} - \frac{\alpha}{12}$
Asymmetric Info.	$\frac{7127-537\sqrt{3}}{59319} + \frac{12591+5728\sqrt{3}}{632736}\alpha$	$\frac{964-228\sqrt{3}}{19773} + \frac{4379+1216\sqrt{3}}{105456}\alpha$	$\frac{2051-321\sqrt{3}}{59319} + \frac{851+3424\sqrt{3}}{632736}\alpha$	$\frac{56+12\sqrt{3}}{1521} - \frac{93+32\sqrt{3}}{4056}\alpha$

Implications

The takeaway message from these welfare analyses is threefold. First, consumers will always welcome the traffic and data services offered by the platform, as well as the vertical integration of the platform with the monopolist seller, in comparison to dealing solely with the seller. Second, the seller makes the most profit and retains the largest share of total surplus in the presence of asymmetric information between the seller and the platform. As shown in the appendix, the seller can benefit even more when the platform believes that there is a low chance of having a low marginal cost for binary cost types. However, this comes at the expense of others' welfare. Finally, the platform can generate the highest revenue

through vertical integration, as well as a higher probability density of low marginal cost seller. The platform's incentive partly aligns with both total surplus and consumer interests in this context, potentially motivating it to assist the seller in reducing the marginal cost of production.

5 Extensions

In this section, we consider several extensions based on the model with a continuum of types and examine how the optimal menu would vary in these situations.

5.1 Contractible price

Suppose the price of the product sold on the platform can be contracted based on traffic and data services. Therefore, for each service plan, only a specific price is allowed for each market segment on the platform, depending on the seller's cost type. Intuitively, this should allow the platform to better screen the seller's type and therefore increase platform revenue. While this holds true for binary types, it does not hold for a continuum of types, as the platform cannot improve its revenue by making price contractible.

Corollary 1. *When product prices can be contracted with platform services and there is a continuum of cost types, the optimal menu remains unchanged compared to when prices cannot be contracted.*

Proof. From incentive compatibility constraints (8), define

$$\begin{aligned} U(c) &= V_{\sigma(c)}(c) - \tau_{\sigma(c)}(c) \\ &= \max_{c' \in C} V_{\sigma(c')}(c) - \tau_{\sigma(c')}(c') \\ &= \max_{c'} \left(a \sum_n (p_n(c') - c) D_n^{x(c')}(p_n(c')) - \max_p \alpha(p - c) D(p) - \tau(c') \right), \end{aligned}$$

where $a = 1$ if traffic service is provided and $a = \alpha$ if not; $n = 0, 1$, $x \in X$ if data is provided, $n = 0$, $x = 1$ if not. Moreover, $p_n(c')$ is defined as $\arg\max_{\{p_n\}} \sum_n (p_n - c') D_n^{x(c')}(p_n)$. Notice that the seller with a marginal cost c is not allowed to choose prices that maximizes its profit when misreporting as the type with a marginal cost c'.

By the envelop theorem, we obtain

$$U'(c) = -a \sum_n D_n^x(p_n(c)) + \alpha D(p(c)), \tag{28}$$

which is the same as in the previous case with non-contractible prices. The rest of the proof follows that of Lemma 1 and Proposition 2. □

The reason is that the incentive compatibility constraint for each c only relies on the slope of the service value to the seller at its true type, where profit-maximizing prices coincide regardless of contractibility. Therefore, whether or not prices can be contracted does not affect equilibrium prices for the seller. In other words, unless the platform is able to acquire better information regarding the seller's cost types, it cannot improve its screening for a continuum of types by making price contractible.

However, for binary types, the contractibility of product price does make a difference. In particular, if the marginal cost of the higher cost type is smaller than a constant, offering the symmetric information benchmark menu maximizes revenue for the platform. Otherwise, the optimal menu offers three categories of services just like when price is not contractible, except for a higher transfer from the low-cost type and wider availability of both services for the high-cost type, resulting in increased platform revenue. Further details can be found in the appendix.

5.2 Endogenous traffic

Suppose the platform can determine the level of visibility it provides in traffic service while incurring a cost to acquire such traffic. Since attracting and maintaining consumers' attention on the platform is often expensive in practice, this setup can make the model more realistic.

Now the platform chooses a mechanism $\mathcal{M} = [\sigma, \tau]$ to maximize expected revenue

$$\max_{\sigma,\tau} \mathbb{E}\left[\tau - b\right], \tag{29}$$

where the service plan σ is now defined as $\sigma : [\underline{c}, \bar{c}] \to \{0,1\} \times [\underline{\alpha}, 1] \times \{0,1\} \times [\underline{v}, \bar{v}]$. For example, $\sigma(c) = (1, \alpha(c), 1, x(c))$ means that the seller with a marginal cost of c will receive traffic service with a visibility of $\alpha(c)$ as well as data service with segmentation at $x(\mathrm{c})$. The cost function for traffic acquisition is defined as $b : [\underline{\alpha}, 1] \to \mathbb{R}_+$ with $b(\underline{\alpha}) = 0$, $b'(\cdot) > 0$ and $b''(\cdot) > 0$. For every $c \in C$, the incentive compatibility and individual rationality constraints remain unchanged as (8) and (9).

The menu from the symmetric information benchmark can be described as

$$\sigma(c) = \begin{cases} (1,1,1,\frac{2+c}{3}) & \text{if } (b')^{-1}(\frac{1}{3}(1-c)^2) \geqslant 1 \\ (1,(b')^{-1}(\frac{1}{3}(1-c)^2),1,\frac{2+c}{3}) & \text{if } \underline{\alpha} < (b')^{-1}(\frac{1}{3}(1-c)^2) < 1 \\ (0,\underline{\alpha},1,\frac{2+c}{3}) & \text{o.w.} \end{cases}. \tag{30}$$

Notice that the benchmark choice of data service or market segmentation has not changed. Endogenizing traffic acquisition and provision only affects the design of traffic service in the platform's optimization problem. Due to the cost of traffic, the platform would like to

reduce the addition of visibility as the marginal cost c increases, and restrain traffic service provision for the seller with excessively high c.

When there is information asymmetry, the objective function of the platform can be written as

$$\int_C \left[\alpha(c) \sum_n (p_n(c) - \phi(c)) D_n^x(p_n(c)) - \underline{\alpha}(p(c) - \phi(c)) D(p(c)) - b(\alpha(c)) \right] dG(c) - U(\bar{c}). \quad (31)$$

As before, the expression inside the integral represents the virtual value of service $\nu(c)$ for the seller of type c. The revenue-maximizing service menu is still the solution to the modified symmetric information problem, where the marginal cost c is replaced by the virtual cost $\phi(c)$.

Then we can determine the services offered in the optimal menu. The first-order condition obtained from differentiating $\nu(c)$ with respect to x is the same as with exogenous traffic, which leads to $x(c) = min\{\frac{2+\phi(c)}{3}, 1\}$ whenever data service is provided. Differentiating $\nu(c)$ with respect to α yields the tangent condition $\sum_n (p_n(c) - \phi(c)) D_n^x(p_n(c)) = b'(\alpha(c))$. This implies that the marginal virtual value of providing traffic service is equal to the marginal cost. As a result, the provision of visibility in the optimal menu is given by

$$\alpha(c) = (b')^{-1}(\sum_n (p_n(c) - \phi(c)) D_n^x(p_n(c))). \quad (32)$$

Take $c \sim U(0,1)$ and $b(\alpha) = \beta\alpha$ with $\beta > 0$ as an example. Here we have

$$\alpha(c) = \begin{cases} 1 & \text{if } \sum_n (p_n(c) - \phi(c)) D_n^x(p_n(c)) > \beta \\ \underline{\alpha} & \text{if } \sum_n (p_n(c) - \phi(c)) D_n^x(p_n(c)) \leqslant \beta \end{cases}. \quad (33)$$

In particular, given that data service is provided, $\alpha(c) = 1$ if $c < \frac{8-2\sqrt{3+39\beta}}{13}$; otherwise, $\alpha(c) = \underline{\alpha}$. And given that no data service is provided, $\alpha(c) = 1$ if $c < \frac{2-\sqrt{1+12\beta}}{3}$; otherwise, $\alpha(c) = \underline{\alpha}$. For any $\beta > 0$, we have $\frac{2-\sqrt{1+12\beta}}{3} < \frac{8-2\sqrt{3+39\beta}}{13} < \frac{1}{2}$. This indicates that it is never optimal for the platform to provide only traffic service, since data service will be provided for any $c < \frac{1}{2}$. This is consistent with the previous results in a sense that when traffic become more costly, it becomes less desirable for the platform to include it in the menu.

As before, it is optimal for the platform to offer both traffic and data services as long as the virtual value for such a plan is strictly positive. This condition holds true when the marginal cost is below the threshold $c < \frac{8-6\alpha-\sqrt{(3-3\alpha)(4-3\alpha)+\beta(13-9\alpha)}}{13-9\alpha}$. The threshold is now lower than $\hat{c}$ in the earlier case, due to the additional positive term within the square root. This means that the platform only provides traffic and data services to a more limited set of lower cost types. For the remaining cost types, the platform will still offer data service to

those with a marginal cost $c < \frac{1}{2}$.

The corollary below summarizes the optimal menu considering endogenous traffic and a continuum of types.

Corollary 2. *Suppose $c \sim U(0,1)$. When the traffic is endogenous with a linear cost function $b(\alpha) = \beta\alpha$, the optimal menu provides both traffic and data to the seller with marginal cost $c < c^* = \frac{8-6\alpha-\sqrt{(3-3\alpha)(4-3\alpha)+\beta(13-9\alpha)}}{13-9\alpha}$, and only data service to the seller with marginal cost $c^* \leqslant c < \frac{1}{2}$. The platform excludes the remaining types from any service.*

Additionally, the type with the lowest cost receives the same segmentation as in the symmetric information benchmark.

That is, $\mathcal{M}^* = (\sigma^*, \tau^*)$ such that

$$\sigma^*(c) = \begin{cases} (1,1,1,\frac{2+2c}{3}) & \text{if } c < c^* \\ (0,\underline{\alpha},1,\frac{2+2c}{3}) & \text{if } c^* \leqslant c < \frac{1}{2} \\ (0,\underline{\alpha},0,1) & \text{o.w.} \end{cases}, \tag{34}$$

$$\tau^*(c) = \begin{cases} \frac{(2-c)(2-3c)-3\underline{\alpha}(1-c)^2}{12} - \int_c^{c^*} \left[\frac{2}{3} - \frac{5z}{6} - \frac{\underline{\alpha}(1-z)}{2}\right] dz & \text{if } c < c^* \\ \frac{\underline{\alpha}(2-c)(2-3c)-3\underline{\alpha}(1-c)^2}{12} - \int_c^{\frac{1}{2}} \left[\frac{2\underline{\alpha}}{3} - \frac{5\underline{\alpha}z}{6} - \frac{\underline{\alpha}(1-z)}{2}\right] dz & \text{if } c^* \leqslant c < \frac{1}{2} \\ 0 & \text{o.w.} \end{cases}. \tag{35}$$

The results suggest that the linear cost of traffic does not affect the design of service plans on the optimal menu, but rather modifies the set of marginal cost types accessible to each service plan.

6 Conclusion

We studied a platform that provides traffic and data services to a monopoly seller in order to enhance the sales of their product among a continuum of consumers. Our simple model incorporates the functions of the platform both as an advertiser and a data intermediary, with minimal assumptions imposed. The symmetric information benchmark suggests that traffic and data are complementary services for the seller, which makes it efficient for the platform to provide both together. However, when there is information asymmetry between the platform and the seller, the resulting screening problem suggests that it is no longer optimal to always provide both services, since types with lower marginal costs tend to have an incentive to misreport their types.

The optimal menu always include three categories of service plans: traffic and data, data only, and no service. When there is a continuum of cost types, each category corresponds to an interval on the continuum, ranging from the lower end to the higher end. With a uniform

distribution of costs, we find that the lower cost types that receive both services contribute the majority of the platform's expected revenue. Moreover, the platform loses half of its expected revenue as information rents to these lower cost types.

In one extension, we find that the platform cannot perform better by making the product price contractible as the optimal menu remains unchanged. However, for binary cost types, it is more likely that the optimal menu will offer both services to both types of the seller and generate higher revenue for the platform when price is contractible. In another extension, we allow the platform to determine the level of visibility provided in traffic service and introduce a constant marginal cost for acquiring traffic. Our findings indicate that there are no changes in the service categories appearing in the optimal menu, except for adjustments made to the cost type interval associated with each service category. Specifically, costly traffic leads to reduced availability in the first two service categories.

It is worth noting that traffic service alone never appears as an option in the optimal menu, regardless of the level of natural visibility on which the value of traffic service relies. This is primarily due to the higher information rent associated with traffic service compared to data service, making it desirable for the platform to offer traffic service only for a smaller range of cost types. Intuitively, the data service is more customized for each cost type, reducing the incentive for other types to deviate from truth-telling.

Our analyses provide further insights into the policies concerning platforms and the data economy. These policies including regulations that govern online advertising, traffic services, data ownership and data sales. In future research, one could introduce platform competition, incorporate consumer-platform interactions into the model, explore different market structures among sellers, and generalize the model with respect to the number of market segments.

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Appendix

A Platform service design for binary cost types

Consider that the platform believes there are only two types of marginal cost for the seller, namely $c \in \{c^L, c^H\}$. Let γ represent the prior belief for being the type with a lower marginal cost, where $\gamma = \Pr(c = c^L)$. In order to maximize its expected revenue, the platform offers a range of traffic and data services with corresponding payments on a menu $\mathcal{M} = [\sigma, \tau]$, subject to incentive compatibility and individual rationality constraints for each type c.

Symmetric information benchmark

In the relaxed problem with symmetric information, the platform can set the transfer equal to the seller's value of services and extract all additional surplus from the seller of each type. Thus, by providing the seller with services that help them make the most profit, the platform maximizes its revenue.

The platform's problem with binary cost types can be written as

$$\max_{\sigma,\tau} \mathbb{E}[\tau] = \max_{\sigma,\tau} \gamma \tau_{\sigma(c^L)}(c^L) + (1-\gamma)\tau_{\sigma(c^H)}(c^H), \tag{36}$$

subject to

$$V_{\sigma(c)}(c) - \tau_{\sigma(c)}(c) \geqslant 0, \tag{37}$$

for both c. Observe that for any service plan, the value of service $V_{\sigma(c)}(c)$ decreases in c, while the point of segmentation x always increases in c.

The benchmark menu with binary cost types is defined as follows: the platform provides traffic and services to both types with $\sigma(c) = (1, 1, \frac{2+c}{3})$ and transfer $\tau(c) = \frac{4-3\alpha}{12}(1-c)^2$. Note that the optimal segmentation associated with data service is given by $x(c) = \frac{2+c}{3} > p(c)$, which is greater than the monopoly price in the unsegmented market and independent of the level of α. In addition, the total demand for the seller's product increases from $\frac{\alpha(1-c)}{2}$ to $\frac{2(1-c)}{3}$, indicating a rise at a rate of $\frac{4-3\alpha}{3\alpha}$ compared to the outside option. This suggests that a larger number of consumers are capable of purchasing the product with the introduction of a platform.

Optimal menu with information asymmetry

In the following analysis, we assume that $c^L = 0$, $c^H = c > 0$. The platform's problem with information asymmetry is also defined as in (36) and (37), along with incentive compatibility constraints for both c,

$$V_{\sigma(c)}(c) - \tau_{\sigma(c)}(c) \geqslant V_{\sigma(c')}(c) - \tau_{\sigma(c')}(c'). \tag{38}$$

Due to information asymmetry, the low-cost type has an incentive to imitate the high-cost type when presented with the benchmark menu. Specifically, for any $c^H < 1$, the incentive compatibility constraint for the low-cost type IC^L is violated. Consequently, the revenue-maximizing menu for the platform must differ from the benchmark menu. In particular, we want to determine whether or not the platform offers any services to the high-cost type when faced with this incentive problem. e explore this latter idea in more detail as an extension in a subsequent section.

Lemma 2. *With binary cost types, the seller with a lower marginal cost has the incentive problem and retains positive information rent. Therefore,*

1. IR^H binds, so $\tau_{\sigma(c^H)}(c^H) = V_{\sigma(c^H)}(c^H)$, which means "no rent at the top".

2. IC^L binds, so $\tau_{\sigma(c^L)}(c^L) = V_{\sigma(c^H)}(c^H) - V_{\sigma(c^H)}(c^L) + V_{\sigma(c^L)}(c^L)$.

3. The low-cost type is offered the same service plan as in the symmetric information benchmark, so $\sigma(c^L) = (1, 1, \frac{2+c^L}{3})$, which means "no distortion at the bottom".

Intuitively, the monopolist seller with a lower marginal cost is able to make a higher profit margin, thus benefiting more from the platform services and exhibiting a greater willingness to pay. However, since the cost type is private information, the seller with a lower marginal cost also enjoys a higher level of information rent. Therefore, in designing the optimal menu, the platform faces a trade-off between providing more valuable services to the low-cost seller and reducing their information rent.

Based on Lemma 2, the problem of the platform can be reformulated as

$$\max_{\sigma} V_{\sigma(c^H)}(c^H) - \gamma V_{\sigma(c^H)}(c^L) + \gamma V_{\sigma(c^L)}(c^L), \tag{39}$$

With $c^L = 0, c^H = c > 0$, we have $x(0) = \frac{2}{3}$ and $V_{\sigma(c^L)}(c^L) = \frac{4-3\alpha}{12}$. The last term in the expression above becomes a constant and does not affect the optimal choice of σ.

Proposition 5. *With binary cost types, the optimal menu includes both traffic and data services for the low-cost seller, while for the high-cost seller, the optimal service depends on the prior probability of being the low-cost type γ. In particular, there exist two thresholds $\bar{\gamma}(c)$ and $1 - c$ in $(0, 1)$ such that*

1. If $\gamma < \bar{\gamma}(c)$, the platform provides both traffic and data to the high-cost seller, but with less value than in the symmetric information benchmark.

2. If $\bar{\gamma}(c) \leqslant \gamma < 1 - c$, only data is provided for the high-cost seller.

3. Otherwise, no service is provided for the high-cost seller.

More specifically, the optimal menu $\mathcal{M}^* = [\sigma^*, \tau^*]$ is given by

$$\sigma^*(0) = (1, 1, \frac{2}{3}) \tag{40}$$

$$\tau^*(0) = \begin{cases} \frac{c^2(5-3\gamma)}{12(1-\gamma)} + \frac{1-2c}{3} - \frac{\alpha(1-c)^2}{4} & \text{if } \gamma < \bar{\gamma}(c) \\ \frac{4-3\alpha}{12} - \frac{\alpha c(1-c-\gamma)}{6(1-\gamma)} & \text{if } \bar{\gamma}(c) \leqslant \gamma < 1-c\,, \\ \frac{4-3\alpha}{12} & \text{o.w.} \end{cases} \tag{41}$$

for the type with zero marginal cost, and

$$\sigma^*(c) = \begin{cases} (1, 1, \frac{2}{3} + \frac{c}{3(1-\gamma)}) & \text{if } \gamma < \bar{\gamma}(c) \\ (0, 1, \frac{2}{3} + \frac{c}{3(1-\gamma)}) & \text{if } \bar{\gamma}(c) \leqslant \gamma < 1-c\,, \\ (0, 0, 1) & \text{o.w.} \end{cases} \tag{42}$$

$$\tau^*(c) = \begin{cases} \frac{c^2(2-\gamma)(2-3\gamma)}{12(1-\gamma)^2} + \frac{1-2c}{3} - \frac{\alpha(1-c)^2}{4} & \text{if } \gamma < \bar{\gamma}(c) \\ \frac{\alpha c^2(2-\gamma)(2-3\gamma)}{12(1-\gamma)^2} + \frac{\alpha(1-2c)}{3} - \frac{\alpha(1-c)^2}{4} & \text{if } \bar{\gamma}(c) \leqslant \gamma < 1-c\,, \\ 0 & \text{o.w.} \end{cases} \tag{43}$$

for the type with a higher marginal cost $c > 0$, where $\bar{\gamma}(c) = \frac{1}{8}(3c^2 - 8c + 8 - \sqrt{3c^2(3c^2 - 16c + 16)})$.

In summary, in the scenario with binary cost types, the revenue-maximizing menu for the platform offers traffic and data to both types, but provides less value than in the symmetric information benchmark to the high-cost type if there is a high probability of being the high-cost type (i.e., $\gamma \to 0$). It only offers data service to the high-cost type if there is a moderate probability of being such (i.e., $\bar{\gamma}(c) \leqslant \gamma < 1 - c$). The platform completely excludes the high-cost type from its services if it believes that it is unlikely for the seller to be the high-cost type (i.e., $\gamma \to 1$). The low-cost type in the last case is also paying all additional gains from services to the platform, leaving no information rent for the seller.

Overall, the traffic service is more affected by the incentive problem from the seller with a low marginal cost than the data service. Specifically, for $\bar{\gamma}(c) \leqslant \gamma < 1 - c$, information rent is smaller for data service compared to traffic service. Therefore, it results in a wider margin for platform revenue by offering data-only instead of traffic-only service.

On the other hand, if the platform can better retain information about the seller's marginal cost or make the seller's prices contractible, it may be more likely to offer more valuable services to a wider range of seller types albeit leaving less surplus for the seller. We delve into this latter idea in greater detail following the welfare analysis.

Welfare

With the assistance of numerical examples, we examine the welfare implications for different levels of platform's prior beliefs about the probability of the seller being the low-cost type. Assume $\alpha = \frac{1}{2}$, $c^L = 0$ and $c^H = \frac{1}{3}$, the platform always offers both traffic and data services to the low-cost type. Meanwhile, the prior beliefs of $\gamma = \frac{2}{5}$, $\frac{3}{5}$ and $\frac{4}{5}$ correspond to the cases when the platform offers both services, data service only, and no service to the high-cost

type respectively.

Proposition 6. *With asymmetric information and binary types, the introduction of traffic and data services benefits all parties, except for the seller when the platform believes the seller is highly likely to have a low marginal cost.*

When belief is low, the total surplus is closest to the efficient allocation, and the seller receives the most surplus.

When belief is high, the total surplus is highest, benefiting both consumers and the platform more than the seller, although the platform retains a larger share of the additional surplus.

When the prior probability of being the low-cost type γ is low, the total surplus can still be very close to the level when the platform perfectly knows the seller's type, or $\frac{8}{9}$ of the efficient level, because the incentive problem only mildly distorts the optimal menu. The seller is able to enjoy the largest share of the total surplus, while consumers retain the smallest share. Nonetheless, consumer surplus has increased by 70 percent compared to when there is no platform.

When the prior probability of being the low-cost type γ is moderate or high, it deviates further from each corresponding efficient allocation,despite an increase in total surplus compared to before. In these cases, the platform retains the majority of surplus while consumers still receive a minimal share. Particularly, when γ is high, the seller's expected outcome is no better than if they were not using the platform. However, both the platform and consumers benefit from all additional surplus generated by traffic and data services.

Overall, a higher probability of low marginal cost leads to an increase in the level of consumer surplus, regardless of the uncertain direction of changes in its ratio to the efficient allocation. Conversely, it results in an uncertain direction of changes at the level of producer surplus but a steady reduction in its ratio to the efficient level. Additionally, it leads to increases in both the level and ratio of platform revenue. The total surplus always increases with γ, but its ratio to the surplus in the efficient allocation initially rises and then declines again. More details on the results can be found in Table 4.

Table 4: Welfare Analysis with Binary Cost Types, $\alpha = \frac{1}{2}$, $c = \frac{1}{3}$

A. Both services to the high-cost type: $\gamma < \bar{\gamma}$, $\gamma = \frac{2}{5}$, $\sigma(c) = (1, 1, \frac{2}{3} + \frac{c}{3(1-\gamma)})$

Scenario	Total Surplus	Producer Surplus	Consumer Surplus	Platform Revenue
Efficient Allocation	$\frac{1}{3}$	-	-	-
(Ratio)	1	-	-	-
No Platform	$\frac{1}{8}$	$\frac{1}{12}$	$\frac{1}{24}$	0
(Ratio)	0.375	0.25	0.125	0
Symmetric Info.	$\frac{8}{27}$	$\frac{1}{12}$	$\frac{2}{27}$	$\frac{5}{36}$
(Ratio)	0.889	0.25	0.222	0.417
Asymmetric Info.	$\frac{707}{2430}$	$\frac{49}{405}$	$\frac{173}{2430}$	$\frac{8}{81}$
(Ratio)	0.873	0.363	0.214	0.296

B. Only data service to the high-cost type: $\bar{\gamma} \leqslant \gamma < 1 - c$, $\gamma = \frac{3}{5}$, $\sigma(c) = (0, 1, \frac{2}{3} + \frac{c}{3(1-\gamma)})$

Scenario	Total Surplus	Producer Surplus	Consumer Surplus	Platform Revenue
Efficient Allocation	$\frac{7}{18}$	-	-	-
(Ratio)	1	-	-	-
No Platform	$\frac{7}{48}$	$\frac{7}{27}$	$\frac{7}{144}$	0
(Ratio)	0.375	0.25	0.125	0
Symmetric Info.	$\frac{28}{81}$	$\frac{7}{72}$	$\frac{7}{81}$	$\frac{35}{216}$
(Ratio)	0.889	0.25	0.222	0.417
Asymmetric Info.	$\frac{3911}{12960}$	$\frac{1}{10}$	$\frac{989}{12960}$	$\frac{271}{2160}$
(Ratio)	0.776	0.257	0.196	0.323

C. No service to the high-cost type: $\gamma \geqslant 1 - c$, $\gamma = \frac{4}{5}$, $\sigma(c) = (0, 0, 1)$

Scenario	Total Surplus	Producer Surplus	Consumer Surplus	Platform Revenue
Efficient Allocation	$\frac{4}{9}$	-	-	-
(Ratio)	1	-	-	-
No Platform	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{18}$	0
(Ratio)	0.375	0.25	0.125	0
Symmetric Info.	$\frac{32}{81}$	$\frac{1}{9}$	$\frac{8}{81}$	$\frac{5}{27}$
(Ratio)	0.889	0.25	0.222	0.417
Asymmetric Info.	$\frac{67}{180}$	$\frac{1}{9}$	$\frac{17}{180}$	$\frac{1}{6}$
(Ratio)	0.838	0.25	0.213	0.375

Contractible price

Consider the setting with binary types, making the prices of the product sold on the platform contractible enables better screening of the seller's type and increase platform revenue. Suppose $(c^L, c^H) = (0, c)$. With information asymmetry, the low-cost type has an incentive to imitate the high-cost type when presented with the symmetric information menu.

Corollary 3. *Suppose the price of the product is contractible in the platform service menu with binary cost types* $\{0, c\}$. *If* $c \geqslant \frac{8-6\alpha}{8-3\alpha}$, *the symmetric information menu with services* $\sigma^*(0) = (1, 1, \frac{2}{3})$, $\sigma^*(c) = (1, 1, \frac{2+c}{3})$, *and transfers* $\tau^*(0) = \frac{4-3\alpha}{12}$, $\tau^*(c) = \frac{4-3\alpha}{12}(1-c)^2$ *maximizes revenue for the platform.*

Otherwise, the optimal menu includes the same service plans and transfer from the high-cost type as in the non-contractible price setting. That is,

$$\sigma^*(0) = (1, 1, \frac{2}{3}), \tag{44}$$

and

$$\sigma^*(c) = \begin{cases} (1, 1, \frac{2}{3} + \frac{c}{3(1-\gamma)}) & \text{if } \gamma < \bar{\gamma}(c) \\ (0, 1, \frac{2}{3} + \frac{c}{3(1-\gamma)}) & \text{if } \bar{\gamma}(c) \leqslant \gamma < 1 - c \\ (0, 0, 1) & \text{o.w.} \end{cases}, \tag{45}$$

where $\bar{\gamma}(c) = \frac{2(1-c)(2-\sqrt{3}c)}{4-3c^2}$. *The transfer from the low-cost type increases by* $\frac{c^2}{4}$ *compared to the non-contractible case if* $\gamma < \bar{\gamma}(c)$ *and by* $\frac{\alpha\gamma c^2}{4}$ *if* $\bar{\gamma}(c) \leqslant \gamma < 1 - c$. *Consequently, the platform's expected revenue increases by either* $\frac{\gamma c^2}{4}$ *or* $\frac{\alpha\gamma c^2}{4}$.

If the prices on platform are contractible, the incentive problem only arises when $c < \frac{8-6\alpha}{8-3\alpha}$. That is, when $\alpha = 0$, IC^L is violated for all $c < 1$, as in the non-contractible price setting. However, when $\alpha = 1$, IC^L is violated only for $c < \frac{2}{5}$. In other words, as α approaches 1 and the marginal cost of the high type becomes small, it is possible for the platform to offer a menu similar to that in the symmetric information benchmark. Therefore, as long as IC^L is not violated, offering the benchmark menu also maximizes revenue for the platform.

In cases where IC^L is violated, the service plans offered by the optimal menu remains the same to the baseline case with binary cost types: the platform offers both traffic and data services for the low-cost seller, while the services for the high-cost seller depend on γ, the prior probability for being a low-cost type (see (40)-(43)). The only changes are in the threshold for γ and in the transfer rule for the low-cost seller. In particular, an additional charge of $\frac{c^2}{4}$ is now imposed on the low-cost seller when both services are provided, or $\frac{\alpha c^2}{4}$ if only data is provided. As a result, there is an increase in expected revenue for the platform by a level of $\frac{\gamma c^2}{4}$ or $\frac{\alpha\gamma c^2}{4}$, respectively. The increase in platform revenue is equivalent to the reduction in information rents from the low-cost seller, and it is positively related to the cost difference

between types, the prior probability of being a low-cost type, and the attractiveness of the outside option. Moreover, $\bar{\gamma}(c)$ is now strictly higher than before, resulting in a broader range of γ where the optimal menu offers both services for the high-cost type.

As expected, making the product price contractible reduces the information rents of the low-cost seller, increases platform revenue, and extends the accessibility of services to the high-cost seller.

B Proofs

Proposition 1

Proof of Proposition 1. When consumer values follow a uniform distribution $U(0,1)$, the monopoly price for the product is $p(c) = \frac{1+c}{2}$ in the unsegmented market; the prices are $p_0(x,c) = max\left\{c, \frac{x+c}{2}\right\}$ and $p_1(x,c) = max\left\{\frac{1+c}{2}, x\right\}$ with a segmentation at x.

The value of traffic alone is given by $V_{(1,0,1)} = \frac{1}{4}(1-\alpha)(1-c)^2$, independent of x. The maximum value of data alone is $\max_x \alpha(x-c)(1-x) + \frac{1}{4}\alpha(x-c)^2 - \frac{1}{4}\alpha(1-c)^2$ if $x > \frac{1+c}{2}$, or $\max_x \frac{1}{4}\alpha(x-c)^2$ if $x \leqslant \frac{1+c}{2}$. By using FOCs and comparing the cases where $x \leqslant \frac{1+c}{2}$ and $x > \frac{1+c}{2}$, we obtain the result $V_{(0,1,x)} = \frac{1}{12}\alpha(1-c)^2$ with $x = \frac{2+c}{3}$. The maximum value of traffic and data combined is $\max_x (x-c)(1-x) + \frac{1}{4}\alpha(x-c)^2 - \frac{1}{4}\alpha(1-c)^2$ if $x > \frac{1+c}{2}$, or $\max_x \frac{1}{4}(1-\alpha)(1-c)^2 + \frac{1}{4}\alpha(x-c)^2$ if $x \leqslant \frac{1+c}{2}$, which results in $V_{(1,1,x)} = \frac{4-3\alpha}{12}(1-c)^2$ with $x = \frac{2+c}{3}$. Since $\frac{4-3\alpha}{12} > \frac{1-\alpha}{4} + \frac{\alpha}{12}$ for $\alpha < 1$, we have $V_{(1,1,x)} > V_{(1,0,1)} + V_{(0,1,x)}$, i.e., traffic and data are complements. In a symmetric information setting, the platform will maximize its revenue by offering both services to the seller and choosing $x = \frac{2+c}{3}$.

Also, $\frac{2+c}{3} > \frac{1+c}{2}$ for all $c < 1$, thus $p(c) < x_{(1,1,x)}(c) = x_{(0,1,x)}(c)$. Furthermore, the value of data alone $V_{(0,1,x)} \to 0$ when $\alpha \to 0$, the value of traffic alone $V_{(1,0,1)} \to 0$ as $\alpha \to 1$, and the value of traffic exceeds the value of data when $\frac{1-\alpha}{4} > \frac{\alpha}{12}$ or when $\alpha < \frac{3}{4}$. □

Lemma 1

Proof of Lemma 1. From IC, define $U(c) = V_{\sigma(c)}(c) - \tau_{\sigma(c)}(c) = \max_{c' \in C} V_{\sigma(c')}(c) - \tau_{\sigma(c')}(c')$.

$$U(c) = \max_{c'} \left(\max_{\{p_n\}} a \sum_n (p_n - c) D_n^x(p_n) - \max_p \alpha(p-c)D(p) - \tau(c') \right), \tag{46}$$

where $a = 1$ if traffic service is provided and $a = \alpha$ if not; $n = 0, 1$, $x \in X$ if data is provided, $n = 0$, $x = 1$ if not.

By the envelop theorem, there is

$$U'(c) = -a\sum_n D_n^x(p_n(c)) + \alpha D(p(c)). \tag{47}$$

Then by the Fundamental Theorem of Calculus,

$$\begin{aligned} U(\bar{c}) - U(c) &= \int_c^{\bar{c}} U'(z)dz \\ &= \int_c^{\bar{c}} \left[-a\sum_n D_n^x(p_n(z)) + \alpha D(p(z)) \right] dz \\ &= U(\bar{c}) - V_{\sigma(c)}(c) + \tau_{\sigma(c)}(c), \end{aligned} \tag{48}$$

for each cost type c. By rearranging the terms, one can derive the transfer rule of a revenue-maximizing service menu as

$$\begin{aligned} \tau_{\sigma(c)}(c) &= V_{\sigma(c)}(c) - \int_c^{\bar{c}} \left[a\sum_n D_n^x(p_n(z)) - \alpha D(p(z)) \right] dz - U(\bar{c}) \\ &= a\sum_n (p_n(c) - c) D_n^x(p_n(c)) - \alpha(p(c) - c)D(p(c)) - \\ &\quad \int_c^{\bar{c}} \left[a\sum_n D_n^x(p_n(z)) - \alpha D(p(z)) \right] dz - U(\bar{c}). \end{aligned} \tag{49}$$

Define the virtual marginal cost for all $c \in C$,

$$\phi(c) = c + \frac{G(c)}{g(c)}. \tag{50}$$

The distribution function $G(c)$ is assumed to be regular so that $\phi(c)$ is increasing in c.

The objective function of the platform can then be written as

$$\begin{aligned}
\mathbb{E}\left[\tau(c)\right] &= \int_C \left[a\sum_n (p_n(c)-c)D_n^x(p_n(c)) - \alpha(p(c)-c)D(p(c)) - \right. \\
&\quad \left. \int_c^{\bar{c}} \left(a\sum_n D_n^x(p_n(z)) - \alpha D(p(z)) \right) dz \right] dG(c) - U(\bar{c}) \\
&= \int_C \left[\left(a\sum_n (p_n(c)-c)D_n^x(p_n(c)) - \alpha(p(c)-c)D(p(c)) \right) g(c) - \right. \\
&\quad \left. \left(a\sum_n D_n^x(p_n(c)) - \alpha D(p(c)) \right) G(c) \right] dc - U(\bar{c}) \\
&= \int_C \left[a\sum_n (p_n(c)-c-\frac{G(c)}{g(c)})D_n^x(p_n(c)) - \alpha(p(c)-c-\frac{G(c)}{g(c)})D(p(c)) \right] dG(c) - U(\bar{c}) \\
&= \int_C \left[a\sum_n (p_n(c)-\phi(c))D_n^x(p_n(c)) - \alpha(p(c)-\phi(c))D(p(c)) \right] dG(c) - U(\bar{c}). \qquad (51)
\end{aligned}$$

The second equality is derived through integration by parts. Specifically, $\int_C \int_c^{\bar{c}} \left(a\sum_n D_n^x(p_n(z)) - \alpha D(p(z)) \right)$ since $G(\underline{c}) = 0$ and $\int_{\bar{c}}^{\bar{c}} \left(a\sum_n D_n^x(p_n(z)) - \alpha D(p(z)) \right) dz = 0$. □

Proposition 2

Proof of Proposition 2. See the main text. □

Proposition 3

Proof of Proposition 3. For a continuum of seller types, the total surplus from the efficient allocation is $\int_0^1 \int_c^1 (v-c)dF(v)dG(c) = \frac{1}{6}$.

Without a platform, the consumer surplus is now given by $\alpha \int_0^1 \int_{\frac{1+c}{2}}^1 (v - \frac{1+c}{2})dvdc = \frac{\alpha}{24}$ and the producer surplus is $\alpha \int_0^1 \int_{\frac{1+c}{2}}^1 (\frac{1+c}{2} - c)dvdc = \frac{\alpha}{12}$, so the total surplus is $\frac{\alpha}{8}$.

With the introduction of a platform and symmetric information between the platform and the seller, the consumer surplus is given by $\int_0^1 [\int_x^1 (v-x)dv + \int_{\frac{x+c}{2}}^x (v - \frac{x+c}{2})dv]dc = \frac{1}{27}$, where $x = \frac{2+c}{3}$. Similarly, the producer surplus is $\frac{\alpha}{12}$ and the platform revenue is $\int_0^1 [\int_x^1 (x - c)dv + \int_{\frac{x+c}{2}}^x (\frac{x+c}{2} - c)dv - \alpha \int_{\frac{1+c}{2}}^1 (\frac{1+c}{2} - c)dv]dc = \frac{1}{9} - \frac{\alpha}{12}$. The total surplus with the platform is now $\frac{4}{27}$, which is higher than that without the platform but lower than the efficient allocation. The consumer surplus is higher with the introduction of a platform if $\alpha < \frac{8}{9}$.

For binary cost types, the total surplus from the efficient allocation is given by $\gamma \int_0^1 vdv + (1-\gamma) \int_c^1 (v-c)dF(v) = \frac{1}{2}(\gamma + (1-\gamma)(1-c)^2)$.

Without a platform, the consumer surplus is $\alpha\gamma\int_{\frac{1}{2}}^{1}(v-\frac{1}{2})dv+\alpha(1-\gamma)\int_{\frac{1+c}{2}}^{1}(v-\frac{1+c}{2})dv=\frac{\alpha}{8}(\gamma+(1-\gamma)(1-c)^2)$ and the producer surplus or the profit of the seller is $\frac{\alpha\gamma}{4}+\alpha(1-\gamma)\frac{(1-c)^2}{4}=\frac{\alpha}{4}(\gamma+(1-\gamma)(1-c)^2)$. The total surplus without a platform is therefore $\frac{3}{8}\alpha(\gamma+(1-\gamma)(1-c)^2)$.

After the platform is introduced, for the symmetric information benchmark, the consumer surplus when the seller's marginal cost is c can be calculated as $\int_x^1(v-x)dv+\int_{\frac{x+c}{2}}^{x}v-\frac{x+c}{2}dv$, where $x=\frac{2+c}{3}$. In expectation, it becomes $\frac{1}{9}(\gamma+(1-\gamma)(1-c)^2)$. The producer surplus remains at $\frac{\alpha}{4}(\gamma+(1-\gamma)(1-c)^2)$ as the platform retains all the value of traffic and data services from the seller. When the marginal cost is c, the value is given by $(x-c)(1-x)+\frac{(x-c)^2}{4}-\frac{\alpha(1-c)^2}{4}$, which in expectation is $\frac{4-3\alpha}{12}(\gamma+(1-\gamma)(1-c)^2)$. Thus the total surplus with a platform is $\frac{4}{9}(\gamma+(1-\gamma)(1-c)^2)$, which is higher than that without the platform. The consumer surplus with the platform is also higher when $\alpha<\frac{8}{9}$. □

Proposition 4

Proof of Proposition 4. With asymmetric information and continuous types, consumer surplus is given by

$$\begin{aligned}
&\int_0^{\hat{c}}\left(\int_x^1(v-x)dv+\int_{\frac{x+c}{2}}^{x}(v-\frac{x+c}{2})dv\right)dc+\alpha\int_{\hat{c}}^{\tilde{c}}\left(\int_x^1(v-x)dv+\int_{\frac{x+c}{2}}^{x}(v-\frac{x+c}{2})dv\right)dc+\\
&\alpha\int_{\tilde{c}}^{1}\int_{\frac{1+c}{2}}^{1}(v-\frac{1+c}{2})dvdc\\
=&\int_0^{\frac{8-2\sqrt{3}}{13}}\frac{1}{18}(2-5c+\frac{17}{4}c^2)dc+\int_{\frac{8-2\sqrt{3}}{13}}^{\frac{1}{2}}\frac{\alpha}{18}(2-5c+\frac{17}{4}c^2)dc+\int_{\frac{1}{2}}^{1}\frac{\alpha}{8}(1-c)^2dc\\
=&\frac{2051-321\sqrt{3}}{59319}+\frac{825+3424\sqrt{3}}{632736}\alpha. \qquad (52)
\end{aligned}$$

Given $(x-c)(1-x)+\frac{(x-c)^2}{4}=\frac{1}{12}(2-c)(2-3c)$, the platform revenue and producer surplus together are given by

$$\begin{aligned}
&\int_0^{\hat{c}}\left((x-c)(1-x)+\frac{(x-c)^2}{4}\right)dc+\alpha\int_{\hat{c}}^{\tilde{c}}\left((x-c)(1-x)+\frac{(x-c)^2}{4}\right)dc+\alpha\int_{\tilde{c}}^{1}\frac{(1-c)^2}{4}dc\\
=&\int_0^{\frac{8-2\sqrt{3}}{13}}\frac{1}{12}(2-c)(2-3c)dc+\int_{\frac{8-2\sqrt{3}}{13}}^{\frac{1}{2}}\frac{\alpha}{12}(2-c)(2-3c)dc+\int_{\frac{1}{2}}^{1}\frac{\alpha}{4}(1-c)^2dc\\
=&\frac{188-8\sqrt{3}}{2197}+\frac{1961+384\sqrt{3}}{105456}\alpha. \qquad (53)
\end{aligned}$$

Since the platform revenue is $\frac{56+12\sqrt{3}}{1521}-\frac{93+32\sqrt{3}}{4056}\alpha$ according to (27), the producer surplus is then $\frac{964-228\sqrt{3}}{19773}+\frac{4379+1216\sqrt{3}}{105456}\alpha$. Furthermore, the total surplus is $\frac{7127-537\sqrt{3}}{59319}+\frac{12591+5728\sqrt{3}}{632736}\alpha$. The results are also summarized in Table 3. The ratios relative to the efficient allocation are

reported in Table 2.

As a numerical example, suppose $\alpha = \frac{1}{2}$. The consumer surplus is approximately 0.031, the platform revenue is 0.032, and the producer surplus is about 0.060, which sum up to 0.122. Even under asymmetric information, the total surplus nearly doubles with the platform compared to the total surplus without it, which is 0.0625. □

Corollary 2

Proof of Corollary 2. When the traffic is endogenous, the platform problem becomes

$$\begin{aligned}
&\max_{\sigma,\tau} \mathbb{E}\left[\tau - b\right] \\
&\text{s.t.} V_{\sigma(c)}(c) - \tau_{\sigma(c)}(c) = \max_{c' \in C} V_{\sigma(c')}(c) - \tau_{\sigma(c')}(c') \\
&\quad V_{\sigma(c)}(c) - \tau_{\sigma(c)}(c) \geqslant 0, \forall c, c'.
\end{aligned}$$

For the symmetric information benchmark, $\tau(c) - b(c) = V_{\sigma(c)}(c) - b(c) = \alpha(c)(p_0(c) - c)(x(c)) + \underline{\alpha}(p(c) - c)(1 - p(c)) - b(\alpha(c))$. The revenue maximization problem gives $\sigma^*(c)$ as in (30). For $b(\alpha(c)) = \beta\alpha(c)$, which is linear in $\alpha(c)$, the optimal menu has $\alpha(c) = \begin{cases} 1 & \text{if } c < 1 - \sqrt{3\beta} \\ \underline{\alpha} & \text{if } c \geqslant 1 - \sqrt{3\beta} \end{cases}$.

For the asymmetric information case, define $U(c) = V_{\sigma(c)}(c) - \tau_{\sigma(c)}(c) = \max_{c' \in C} V_{\sigma(c')}(c) - \tau_{\sigma(c')}(c')$. Then

$$U(c) = \max_{c'} \alpha(c') \sum_n (p_n(c') - c) D_n^x(p_n(c')) - \underline{\alpha}(p(c') - c) D(p(c')) - \tau(c'), \tag{54}$$

where $n = 0, 1$, $x \in X$ if data is provided, $n = 0$, $x = 1$ if not.

By the envelop theorem, there is

$$U'(c) = -\alpha(c) \sum_n D_n^x(p_n(c)) + \underline{\alpha} D(p(c)). \tag{55}$$

By the Fundamental Theorem of Calculus,

$$\begin{aligned}
U(\bar{c}) - U(c) &= \int_c^{\bar{c}} \left[-\alpha(z) \sum_n D_n^x(p_n(z)) + \underline{\alpha} D(p(z)) \right] dz \\
&= U(\bar{c}) - V_{\sigma(c)}(c) + \tau_{\sigma(c)}(c),
\end{aligned} \tag{56}$$

for each cost type c. Then the transfer rule of a revenue-maximizing service menu is

$$\tau_{\sigma(c)}(c) \;=\; V_{\sigma(c)}(c) - \int_c^{\bar{c}} \left[\alpha(z) \sum_n D_n^x(p_n(z)) - \underline{\alpha} D(p(z)) \right] dz - U(\bar{c}). \tag{57}$$

Define the virtual marginal cost for all $c \in C$, $\phi(c) = c + \frac{G(c)}{g(c)}$, assuming that the distribution function $G(c)$ satisfies regularity conditions, so that $\phi(c)$ is increasing in c.

The objective function of the platform is given by

$$\begin{aligned}
&\mathbb{E}\left[\tau(c) - b(c)\right] \\
&= \int_C \Bigg[\alpha(c) \sum_n (p_n(c) - c) D_n^x(p_n(c)) - \underline{\alpha}(p(c) - c) D(p(c)) - \\
&\quad \int_c^{\bar{c}} \left(\alpha(z) \sum_n D_n^x(p_n(z)) - \underline{\alpha} D(p(z)) \right) dz - b(\alpha(c)) \Bigg] dG(c) - U(\bar{c}) \\
&= \int_C \Bigg[\left(\alpha(c) \sum_n (p_n(c) - c) D_n^x(p_n(c)) - \underline{\alpha}(p(c) - c) D(p(c)) - b(\alpha(c)) \right) g(c) - \\
&\quad \left(\alpha(c) \sum_n D_n^x(p_n(c)) - \underline{\alpha} D(p(c)) \right) G(c) \Bigg] dc - U(\bar{c}) \\
&= \int_C \left[\alpha(c) \sum_n (p_n(c) - \phi(c)) D_n^x(p_n(c)) - \underline{\alpha}(p(c) - \phi(c)) D(p(c)) - b(\alpha(c)) \right] dG(c) - U(\bar{c}).
\end{aligned} \tag{58}$$

The second equality is derived through integration by parts.

Taking first order condition with respect to x and α leads to $x(c) = min\{\frac{2+\phi(c)}{3}, 1\}$ and $\alpha(c) = (b')^{-1}(\sum_n (p_n(c) - \phi(c)) D_n^x(p_n(c)))$, respectively. For a linear cost of traffic $b(\alpha) = \beta\alpha$,

$$\alpha(c) = \begin{cases} 1 & \text{if } \sum_n (p_n(c) - \phi(c)) D_n^x(p_n(c)) > \beta \\ \underline{\alpha} & \text{if } \sum_n (p_n(c) - \phi(c)) D_n^x(p_n(c)) \leqslant \beta \end{cases}. \tag{59}$$

Notice that, for any service plan that includes data service,

$$\begin{aligned}
\sum_n (p_n(c) - \phi(c)) D_n^x(p_n(c)) &= \frac{1}{3}(1 - c - \frac{G(c)}{g(c)})^2 - \frac{1}{4}(\frac{G(c)}{g(c)})^2 \\
&= \frac{13}{12}c^2 - \frac{4}{3}c + \frac{1}{3},
\end{aligned} \tag{60}$$

where $c < \frac{1}{2}$. For any service plan that does not include data service,

$$\begin{aligned}\sum_n (p_n(c) - \phi(c)) D_n^x(p_n(c)) &= \frac{1}{4}(1-c)(1-c-2\frac{G(c)}{g(c)}) \\ &= \frac{1}{4}(1-c)(1-3c). \end{aligned} \tag{61}$$

Expression in (60) is strictly greater than that in (61), so providing data is always prefereable to not providing it.

Given that data service is provided, $\alpha(c) = 1$ if $\frac{13}{12}c^2 - \frac{4}{3}c + \frac{1}{3} > \beta$, which is $c < \frac{8-2\sqrt{3+39\beta}}{13}$ for $c < \frac{1}{2}$; otherwise, $\alpha(c) = \underline{\alpha}$. And given that no data service is provided, $\alpha(c) = 1$ if $\frac{1}{4}(1-c)(1-3c) > \beta$, which is $c < \frac{2-\sqrt{1+12\beta}}{3}$; otherwise, $\alpha(c) = \underline{\alpha}$. For any $\beta > 0$, we have $\frac{2-\sqrt{1+12\beta}}{3} < \frac{8-2\sqrt{3+39\beta}}{13} < \frac{1}{2}$. This indicates it is never optimal for the platform to provide only traffic service.

As before, it is optimal for the platform to offer both traffic and data services as long as the virtual value for such a plan $\frac{13}{12}c^2 - \frac{4}{3}c + \frac{1}{3} - \frac{1}{4}\underline{\alpha}(1-c)(1-3c) - \beta > 0$. This gives $c < c^* = \frac{8-6\alpha-\sqrt{(3-3\alpha)(4-3\alpha)+\beta(13-9\alpha)}}{13-9\alpha}$. Otherwise, only data service is provided as long as the virtual value $\underline{\alpha}\left(\frac{13}{12}c^2 - \frac{4}{3}c + \frac{1}{3} - \frac{1}{4}(1-c)(1-3c)\right) > 0$, which is $c < \frac{1}{2}$.

Finally, the transfer rule can be derived according to (57) with $x = \frac{2+2c}{3}$ and $\sum_n (p_n(c) - c) D_n^x(p_n(c)) = \frac{1}{12}(2-c)(2-3c)$. □

Lemma 2

Proof of Lemma 2. When the platform perfectly knows the cost of the seller, it offers traffic and data services with $x = \frac{2+c}{3}$, and requires a payment of $\frac{4-3\alpha}{12}(1-c)^2$ for $c = c^L$ or $c = c^H$. When there is asymmetric information, IC^L is violated. In order to maximize revenue, the platform will offer a menu that both IC^L and IR^H bind. Suppose not, the platform can simply raise $\tau_{\sigma(c^L)}(c^L)$ without affecting revenue from the high type. Moreover, the platform revenue is proportional to the value of services provided to the low-cost type; therefore, the low-cost type will have the most valuable service plan, just like in the symmetric information benchmark. □

Proposition 5

Proof of Proposition 5. From Lemma 2, the problem of the platform can be written as

$$\max_{\sigma,\tau} \mathbb{E}[\tau] = \max_{\sigma} V_{\sigma(c)}(c) - \gamma V_{\sigma(c)}(0) + \gamma V_{\sigma(0)}(0), \tag{62}$$

where $\sigma^*(0) = (1, 1, \frac{2}{3})$ and $V_{\sigma^*(0)}(0) = \frac{4-3\alpha}{12}$.

We establish the optimal service plan for the high-cost type, as shown in (42), by comparing the expected revenue of each possible combination of services according to Table 1.

Suppose $\sigma(c) = (1, 1, x)$. If $x > \frac{1+c}{2}$, then $p_0(x, c) = \frac{x+c}{2}$ and $p_1(x, c) = x$ for the high-cost type, while $p_0(x, 0) = \frac{x}{2}$ and $p_1(x, 0) = x$ for the low-cost type when choosing this plan. Then, $x = \arg\max(x-c)(1-x) + \frac{(x-c)^2}{4} - \frac{\alpha(1-c)^2}{4} - \gamma(x(1-x) + \frac{x^2}{4} - \frac{\alpha}{4}) + \frac{\gamma(4-3\alpha)}{12} = \frac{2}{3} + \frac{c}{3(1-\gamma)}$. It can be checked that $SOC < 0$, and $x < 1$ when $\gamma < 1-c$. This implies that when $\gamma \geqslant 1-c$, there will be $x = 1$ and $\sigma^*(c) = (1, 0, 1)$ or $(0, 0, 1)$, meaning that no data will be provided for the high-cost type within this range of values for γ. Moreover, the expected revenue of the platform is

$$\mathbb{E}[\tau(\sigma(c) = (1, 1, \frac{2}{3} + \frac{c}{3(1-\gamma)}))] = \frac{c^2 + (1-\gamma)((2-3c)(2-c) - 3\alpha(1-c)^2)}{12(1-\gamma)}. \tag{63}$$

If $max\left\{\frac{1}{2}, c\right\} < x \leqslant \frac{1+c}{2}$, $p_0(x, c) = \frac{x+c}{2}$ and $p_1(x, c) = \frac{1+c}{2}$ for the high-cost type, while $p_0(x, 0) = \frac{x}{2}$ and $p_1(x, 0) = x$ for the low-cost type. Then $x = \arg\max \frac{(1-c)^2}{4} + \frac{(x-c)^2}{4} - \frac{\alpha(1-c)^2}{4} - \gamma(x(1-x) + \frac{x^2}{4} - \frac{\alpha}{4}) + \frac{\gamma(4-3\alpha)}{12}$. Notice that $SOC > 0$, so the expected revenue of the platform is convex. In this case, if $x > \frac{c+2\gamma}{1+3\gamma}$, then $x = \frac{1+c}{2}$; if $x < \frac{c+2\gamma}{1+3\gamma}$, then $x = \frac{1}{2}$ for $\frac{1-\gamma}{2} < c \leqslant \frac{1}{2}$, or $x = c$ for $\frac{1}{2} < c < \frac{2}{3}$; or if $x = \frac{c+2\gamma}{1+3\gamma}$ for $\frac{1-\gamma}{2} < c \leqslant \frac{1}{2}$ or $\frac{1}{2} < c < \frac{2}{3}$.

The corresponding expected revenues are

$$\mathbb{E}[\tau(\sigma(c) = (1, 1, \frac{1+c}{2}))] = \frac{3(5-4\alpha)(1-c)^2 + \gamma(1-3c)^2}{48}, \tag{64}$$

$$\mathbb{E}[\tau(\sigma(c) = (1, 1, \frac{1}{2}))] = \frac{3(5-4\alpha) + 12(2\alpha-3)c + 12(2-\alpha)c^2 + \gamma}{48}, \tag{65}$$

$$\mathbb{E}[\tau(\sigma(c) = (1, 1, c))] = \frac{3(1-\alpha)(1-c)^2 + \gamma(2-3c)^2}{12}, \tag{66}$$

$$\mathbb{E}[\tau(\sigma(c) = (1, 1, \frac{c+2\gamma}{1+3\gamma}))] = \frac{3c^2(1+6\gamma) - 3\alpha(1-c)^2(1+3\gamma) - 6c(1+5\gamma) + 13\gamma + 3}{12(1+3\gamma)}. \tag{67}$$

Since the inequalities $\mathbb{E}[\tau(\sigma(c) = (1, 1, \frac{1+c}{2}))] > \mathbb{E}[\tau(\sigma(c) = (1, 1, \frac{1}{2}))]$, $\mathbb{E}[\tau(\sigma(c) = (1, 1, \frac{1+c}{2}))] > \mathbb{E}[\tau(\sigma(c) = (1, 1, c))]$, $\mathbb{E}[\tau(\sigma(c) = (1, 1, \frac{1+c}{2}))] > \mathbb{E}[\tau(\sigma(c) = (1, 1, \frac{c+2\gamma}{1+3\gamma}))]$, and $\mathbb{E}[\tau(\sigma(c) = (1, 1, \frac{2}{3} + \frac{c}{3(1-\gamma)}))] > \mathbb{E}[\tau(\sigma(c) = (1, 1, \frac{1+c}{2}))]$ always hold within corresponding parameter range, it must be concluded that $x = \frac{2}{3} + \frac{c}{3(1-\gamma)}$ when considering the optimal service plan for the high-cost type which includes both data and traffic, i.e. $\sigma(c) = (1, 1, x)$, for $\gamma < 1-c$.

Next, we would like to compare providing data service alone $\sigma(c) = (0, 1, x)$, traffic service alone $(1, 0, 1)$, or no service $(0, 0, 1)$ with both services $\sigma(c) = (1, 1, x)$ in order to determine the optimal service plan for $\gamma < 1 - c$. Following that, we will then compare between $\sigma(c) = (0, 1, 1)$ and $(0, 0, 1)$ for $\gamma \geqslant 1 - c$.

Suppose only data service is provided, $\sigma(c) = (0, 1, x)$. If $x > \frac{1+c}{2}$, both types of

seller set the prices $p_n(x,c)$ and $p_n(x,0)$ as in the previous case with both services, so $x = \arg\max \alpha[(x-c)(1-x) + \frac{(x-c)^2}{4} - \frac{(1-c)^2}{4}] - \alpha\gamma(x(1-x) + \frac{x^2}{4} - \frac{1}{4}) + \frac{\gamma(4-3\alpha)}{12}$. Again, $x = \frac{2}{3} + \frac{c}{3(1-\gamma)}$ for $\gamma < 1-c$. The expected revenue is

$$\mathbb{E}[\tau(\sigma(c) = (0,1,\frac{2}{3} + \frac{c}{3(1-\gamma)}))] = \frac{\alpha(1-\gamma-c)^2}{12(1-\gamma)} + \frac{\gamma(4-3\alpha)}{12}. \tag{68}$$

If $max\left\{\frac{1}{2}, c\right\} < x \leqslant \frac{1+c}{2}$, prices are the same as in the case with both services, and $x = \arg\max \frac{\alpha(x-c)^2}{4} - \alpha\gamma(x(1-x) + \frac{x^2}{4} - \frac{1}{4}) + \frac{\gamma(4-3\alpha)}{12}$. Since $SOC > 0$, if $x > \frac{c+2\gamma}{1+3\gamma}$, then $x = \frac{1+c}{2}$; if $x < \frac{c+2\gamma}{1+3\gamma}$, then $x = \frac{1}{2}$ for $\frac{1-\gamma}{2} < c \leqslant \frac{1}{2}$, or $x = c$ for $\frac{1}{2} < c < \frac{2}{3}$; or if $x = \frac{c+2\gamma}{1+3\gamma}$ for $\frac{1-\gamma}{2} < c \leqslant \frac{1}{2}$ or $\frac{1}{2} < c < \frac{2}{3}$. Similarly, it can be checked that $x = \frac{2}{3} + \frac{c}{3(1-\gamma)}$ yields the highest expected revenue given $\sigma(c) = (0,1,x)$.

Notice that $\mathbb{E}[\tau(\sigma(c) = (1,1,x))] > \mathbb{E}[\tau(\sigma(c) = (0,1,x))]$ if $\gamma < \frac{1}{8}(3c^2 - 8c + 8 - \sqrt{3c^2(3c^2 - 16c + 16)})$. Thus, this is the threshold below which providing both data and traffic for the high-cost type becomes more lucrative than providing data only, denoted as $\bar{\gamma}(c)$. Also note that $\bar{\gamma}(c) < 1-c$ for all $c \in (0,1)$.

Furthermore, when only traffic service for the high-cost type,

$$\mathbb{E}[\tau(\sigma(c) = (1,0,1))] = \frac{(1-\alpha)((1-c)^2 - \gamma)}{4} + \frac{\gamma(4-3\alpha)}{12}, \tag{69}$$

and when there is no service provided,

$$\mathbb{E}[\tau(\sigma(c) = (0,0,1))] = \frac{\gamma(4-3\alpha)}{12}. \tag{70}$$

Since $\mathbb{E}[\tau(\sigma(c) = (0,0,1))] > \mathbb{E}[\tau(\sigma(c) = (1,0,1))$ if $\gamma > (1-c)^2$ and $1-c > (1-c)^2$, the expected revenue from providing no service to the high-cost type is higher than that from providing traffic only for $\gamma \geqslant 1-c$.

Notice that for $\bar{\gamma}(c) < \gamma < 1-c$, $\mathbb{E}[\tau(\sigma(c) = (0,1,x))] > \mathbb{E}[\tau(\sigma(c) = (1,0,1))]$, while for $\gamma < \bar{\gamma}(c)$, $\mathbb{E}[\tau(\sigma(c) = (1,1,x))] > \mathbb{E}[\tau(\sigma(c) = (1,0,1))]$. As a result, the optimal menu offers both services to the high-cost type if $0 < \gamma < \bar{\gamma}(c)$, data service only to the high-cost type if $\bar{\gamma}(c) < \gamma < 1-c$, and no service if $\gamma \geqslant 1-c$.

The payments for each cost type's optimal service plan can be computed accordingly using formulas provided in Lemma 2. □

Proposition 6

Proof of Proposition 6. With asymmetric information and binary types, the optimal menu depends on the value of γ. For $\gamma < \bar{\gamma}(c)$, consumer surplus is $\gamma[\int_{\frac{2}{3}}^{1}(v - \frac{2}{3})dv + \int_{\frac{1}{3}}^{\frac{2}{3}}(v - \frac{1}{3})dv] + (1-\gamma)[\int_{\frac{2}{3}+\frac{c}{3(1-\gamma)}}^{1}(v - \frac{2}{3} - \frac{c}{3(1-\gamma)})dv + \int_{\frac{1}{3}+\frac{c}{6(1-\gamma)}+\frac{c}{2}}^{\frac{2}{3}+\frac{c}{3(1-\gamma)}}(v - \frac{1}{3} - \frac{c}{6(1-\gamma)} - \frac{c}{2})dv] = \frac{\gamma}{9} + \frac{1-\gamma}{2}[(1-x)^2 +$

$\frac{(x-c)^2}{4}]$,where $x = \frac{2}{3} + \frac{c}{3(1-\gamma)}$. The platform revenue and producer surplus together are given by $\frac{\gamma}{3} + (1-\gamma)[\frac{(x-c)^2}{4} + (x-c)(1-x)]$, where the platform revenue is $\frac{c^2(4-3\gamma)}{12(1-\gamma)} + \frac{1-2c}{3} - \frac{\alpha(1-c)^2}{4}$. Given $\alpha = \frac{1}{2}$, $c = \frac{1}{3}$ and $\gamma = \frac{2}{5} < \frac{17-\sqrt{33}}{24} = \bar{\gamma}(c)$, the consumer surplus is $\frac{173}{2430} \doteq 0.071$, the producer surplus is $\frac{49}{405} \doteq 0.121$, and the platform revenue is $\frac{8}{81} \doteq 0.099$. The total surplus, therefore, is $\frac{707}{2430} \doteq 0.291$. According to the previous proposition, in this numerical example, the total surplus in the efficient allocation would be $\frac{1}{3} \doteq 0.333$, while it would be $\frac{1}{8} = 0.125$ in the market equilibrium without a platform and $\frac{8}{27} \doteq 0.296$ in the market equilibrium with the platform and symmetric information. The consumer surplus is $\frac{1}{24} \doteq 0.042$ without a platform and $\frac{2}{27} \doteq 0.074$ with the platform and symmetric information, while the producer surplus is $\frac{1}{12} \doteq 0.083$ in both cases. The distribution of surplus among different parties in each scenario is reported in Panel A of the Table 4.

For $\bar{\gamma}(c) \leqslant \gamma < 1-c$, consumer is given by $\frac{\gamma}{9} + \frac{1-\gamma}{2}\alpha[(1-x)^2 + \frac{(x-c)^2}{4}]$, where $x = \frac{2}{3} + \frac{c}{3(1-\gamma)}$. The platform revenue and producer surplus together are $\frac{\gamma}{3} + (1-\gamma)\alpha[\frac{(x-c)^2}{4} + (x-c)(1-x)]$, and the platform revenue now becomes $\gamma[\frac{4-3\alpha}{12} - \frac{\alpha c(1-c-\gamma)}{6(1-\gamma)}] + (1-\gamma)[\frac{\alpha c^2(2-\gamma)(2-3\gamma)}{12(1-\gamma)} + \frac{\alpha(1-2c)}{3} - \frac{\alpha(1-c)^2}{4}]$. For the numerical example with $\gamma = \frac{3}{5}$, the consumer surplus is $\frac{989}{12960} \doteq 0.0763$, the producer surplus is $\frac{1}{10} = 0.1$, the platform revenue is $\frac{271}{2160} \doteq 0.125$, and the total surplus is $\frac{3911}{12960} \doteq 0.302$. According to the previous proposition, the total surplus is $\frac{7}{18} \doteq 0.389$ in the efficient allocation, $\frac{7}{48} \doteq 0.146$ without a platform, and $\frac{28}{81} \doteq 0.346$ with the platform and symmetric information. The consumer surplus is $\frac{7}{144} \doteq 0.049$ without a platform and $\frac{7}{81} \doteq 0.086$ with the platform and symmetric information, while the producer surplus is $\frac{7}{72} \doteq 0.097$ in both cases. The distribution of surplus among different parties is reported in Panel B of Table 4.

For $\gamma \geqslant 1-c$, and suppose $\gamma = \frac{4}{5}$, consumer surplus becomes $\frac{\gamma}{9} + (1-\gamma)\alpha \int_{\frac{1+c}{2}}^{1} (v - \frac{1+c}{2}) dv = \frac{\gamma}{9} + \frac{1-\gamma}{2} \cdot \frac{\alpha(1-c)^2}{4} = \frac{17}{180} \doteq 0.094$. The platform revenue and producer surplus together are $\frac{\gamma}{3} + (1-\gamma)\frac{\alpha(1-c)^2}{4} = \frac{5}{18} \doteq 0.278$, where the platform revenue is $\frac{\gamma(4-3\alpha)}{12} = \frac{1}{6}$ and the producer surplus is $\frac{1}{9} \doteq 0.111$. The total surplus is now $\frac{67}{180} \doteq 0.372$. According to the previous proposition, the total surplus is $\frac{4}{9} \doteq 0.444$ in the efficient allocation, $\frac{1}{6} \doteq 0.167$ without a platform, and $\frac{32}{81} \doteq 0.395$ with the platform and symmetric information. The consumer surplus is $\frac{1}{18} \doteq 0.056$ without a platform and $\frac{8}{81} \doteq 0.099$ with the platform and symmetric information, while the producer surplus is $\frac{1}{9} \doteq 0.111$ in both cases. Notice that within this range of γ, the producer surplus remains unchanged with and without the platform. The distribution of surplus among different parties is reported in Panel C of Table 4. □

Corollary 3

Proof of Corollary 3. When the platform can specify the product price set by the seller in the data and traffic service contract, the low-cost type will receive a value of $V_{\sigma(c)}(0) = \sum_{n=0}^{1} p_n(x,c) D_n^x(p_n(x,c)) - \alpha p(0) D(p(0)) = x(1-x) + \frac{1}{4}(x^2 - c^2) - \frac{1}{4}\alpha$, where $x = \frac{2+c}{3}$ instead of $\frac{2}{3}$, by misreporting as a high-cost type. In this case, IC^L is violated only when $V_{\sigma(c)}(c) < V_{\sigma(c)}(0)$,

or equivalently $\frac{4-3\alpha}{12}(1-c)^2 < \frac{4-4c^2-3\alpha}{12}$, which is $c < \frac{8-6\alpha}{8-3\alpha}$. Since $\frac{8-6\alpha}{8-3\alpha}$ decreases in α, there is more likely to be incentive problems when α is closer to 0. Again, IC^H is never violated.

When there is asymmetric information between the platform and the seller and $c < \frac{8-6\alpha}{8-3\alpha}$, the optimal menu will feature $\tau_{\sigma(c)}(c) = V_{\sigma(c)}(c)$ (IR^H binds) and $\tau_{\sigma(0)}(0) = V_{\sigma(c)}(c) - V_{\sigma(c)}(0) + V_{\sigma(0)}(0)$ (IC^L binds). The platform's revenue maximization problem is again $\max_{\sigma} V_{\sigma(c)}(c) - \gamma V_{\sigma(c)}(0) + \gamma V_{\sigma(0)}(0)$.

If the platform provide both services to the high-cost type, $x = \frac{2}{3} + \frac{c}{3(1-\gamma)}$ and $\mathbb{E}[\tau(\sigma(c) = (1, 1, \frac{2}{3} + \frac{c}{3(1-\gamma)}))] = \frac{c^2+(1-\gamma)((2-3c)(2-c)-3\alpha(1-c)^2+3\gamma c^2)}{12(1-\gamma)}$ for $\gamma < 1 - c$. In comparison to the non-contractible price case, the expected revenue in this case includes an additional term of $\frac{\gamma c^2}{4}$. If only data service is provided, $x = \frac{2}{3} + \frac{c}{3(1-\gamma)}$ and $\mathbb{E}[\tau(\sigma(c) = (0, 1, \frac{2}{3} + \frac{c}{3(1-\gamma)}))] = \frac{\alpha(1-\gamma-c)^2}{12(1-\gamma)} + \frac{\alpha\gamma c^2}{4} + \frac{\gamma(4-3\alpha)}{12}$ for $\gamma < 1 - c$. The expected revenue increases by $\frac{\alpha\gamma c^2}{4}$ compared to the case where prices cannot be contracted.

Moreover, the threshold below which providing both services for the high-cost type generates a higher expected revenue than providing data service only is given by $\bar{\gamma}(c) = \frac{2(1-c)(2-\sqrt{3}c)}{4-3c^2}$. This threshold is strictly larger than the one when prices cannot be contracted, thereby narrowing down the range of possible values for γ in which offering only data service is optimal. Once again, it can be confirmed that $\bar{\gamma}(c) < 1 - c$ for all c. □