Why the Political World is Flat: An Endogenous "Left" and "Right" in Multidimensional Elections

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September 10, 2024

Abstract

This paper offers an explanation for why multidimensional politics reduce so consistency to a one-dimensional liberal-conservative continuum, using a common-interest election model with endogenous policy platforms. Logical relationships between issues induce correlation between voter opinions across issues. Anticipating voter information, candidates position themselves on the principal component axis of the underlying space of policy possibilities, an axis that is unlikely to vary with time, location, and institutional details. Orthogonal equilibria can arise, partly validating a common fear that policy issues might be bundled inefficiently, but these equilibria are unstable and may not exist.

JEL Classification Numbers: D72, D82

Keywords: Information Aggregation, Voting, Elections, Jury Theorem, Multiple Dimensions, Public Opinion

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1 Introduction

The world of public policy is complex and multifaceted. Elected officials must decide tax policy, foreign policy, health policy, education policy, immigration policy, social policy, and numerous narrower issues that are themselves complex and multifaceted. To model this properly, the number of dimensions should be enormous. Indeed, every sentence of legislation could be viewed as a separate dimension in which policy could be adjusted. With two sides to each issue on D policy dimensions, there are 2^D policy combinations, plus a continuum of intermediate positions between each pair of extremes.

With so many policy configurations to choose from, politics seem shockingly one dimensional, simply ranging from liberal to conservative, with moderates in between. Poole and Rosenthal (1985, 1997, 2001) report that a one-dimensional model of policy preference correctly predicts almost 90% of the roll call votes cast in the history of the U.S. House and Senate. They cite similar findings for the European Parliament, the U.N. General Assembly, and several European national parliaments. Grofman and Brazill (2002) and Shor and McCarty (2011) find the same for the U.S. supreme court and state legislatures. Such unidimensionality effectively ignores all but *two* of the 2^D orthants of the policy space: one consisting of liberal positions on each of the D issues, and the other consisting of conservative positions on every issue. In the words of Converse (1964, p. 207), "...if a person is opposed to the expansion of social security, he is probably a conservative and is probably opposed as well to any nationalization of private industries, federal aid to education, sharply progressive income taxation, and so forth." Converse (1964) ascribed such unidimensionality only to political elites, but Shor (2014) estimates that a single dimension also explains 80% of the individual policy opinions expressed by ordinary citizens.[1]

Existing election models offer no explanation for such unidimensionality. Beginning with Hotelling (1928) and Downs (1957), most simply *constrain* policy choices to be one dimensional.[2] Models that allow multiple dimensions, beginning with Plott (1967), often exhibit no pure strategy equilibrium.[3] Probabilistic voting models, beginning with Hinich (1978), have an equilibrium but predict counterfactually that candidates should adopt identical policies and exhibit no ideological differences.[4] Polarized equilibria exist in the "citizen candidate" models of Osborne and Slivinski (1996) and Besley and Coate (1997), but there are many such equilibria, and many non-polarized equilibria as well, leaving the analysis indeterminate.[5] In particular, this leaves open the possibility that candidates in one election polarize in one direction but candidates in

[1] See also Ares et al. (2024). Tausanovitch and Warshaw (2014) find that voter preferences correlate on local and national issues, as well.

[2] One dimensional models sometimes attribute ideological differences to wealth, which affects the demand both for redistribution (Romer, 1975; Meltzer and Richard, 1981) and for public goods (Bergstrom and Goodman, 1973). This could potentially explain unidimensionality, as the same individuals favor multiple redistributive policies or multiple public goods. As I discuss in McMurray (2017a), however, many policy preferences cannot easily be traced to wealth. Empirically, for example, wealthy voters favor redistribution almost as much as the poor. As another example, defense spending and environmental protection are both textbook examples of public goods but tend to draw voter support from opposite ends of the political spectrum.

[3] More recently, see Duggan and Fey (2005). Mixed-strategy equilibria exist more generally (Duggan and Jackson, 2006), but have unclear empirical relevance in the context of political campaigns (Austen-Smith and Banks, 2005).

[4] More recently, see Xefteris (2017). Bafumi and Herron (2010) and Shor (2011) show empirically that candidates polarize substantially.

[5] As the latter authors write, "basically *any* pair of candidates who split the voters evenly can be an equilibrium."

another place or time polarize in a different direction, contrary to the empirical finding that candidates everywhere polarize in essentially the *same* direction. Indeed, McDonald, Mendes, and Kim (2007) report that the same, single dimension categorizes nearly 90% of the policy positions of eighty political parties in seventeen countries, over twenty-five years.

The models above attribute voter disagreements to exogenous tastes. In McMurray (2017a) I propose another possibility, which is that voters share a preference for promoting social welfare but have different *information* about the welfare consequences various policies will have.[6] In that case, the fundamental hope of democracy is to aggregate collective wisdom to determine what is truly best, as in Condorcet's (1785) classic "jury" theorem.[7] This paper extends that model to multiple dimensions but shows that candidates settle endogenously on a single dimension, which does not vary across elections.

In a common interest environment, rational voters partly anticipate other voters' information, restricting attention to the *pivotal* event of changing the election winner. As I show in McMurray (2022), rational candidates partly anticipate voters' information as well, restricting attention to the pivotal event of winning the election. This leads candidates to polarize, each reasoning that if she wins, it will be because truth is on her side.

In an information model with uncorrelated policy issues, candidates could polarize in myriad ways, each bundling policy issues differently, so that no pattern would emerge across jurisdictions or over time. The central observation underlying this paper's analysis is that logical links between policy issues naturally correlate voters' opinions across issues. To end an economic recession, for example, fiscal stimulus might prove effective while monetary stimulus does not, or vice versa, but ex ante it is more likely either that the economy functions more or less as Keynesian models predict, so that both forms of stimulus are helpful, or that the economy functions as in more classical models, and both are wasteful. More broadly, belief in market efficiency or distrust of government regulators might feed opposition to a host of regulations, while worldviews about the relative importance of luck and effort in creating wealth shape support (or lack thereof) for multiple forms of redistribution.[8] If policy opinions are correlated, voters' and candidates' best response incentives rotate in the direction of correlation, as shown below.

Of course, not all issues are as tightly connected as fiscal and monetary stimulus. As Shor (2014) expresses, for example, "it is not clear why environmentalism necessarily hangs together with a desire for more union prerogatives, but [empirically] it does." Logical connections do exist, but may be weak. For example, support for environmental regulations and workers' unions could both stem from a view of businesses as overly ruthless, foregoing employee wellbeing and public health in their pursuit of profit. In fact, this

[6] As that paper explains, a welfare objective can arise as large elections amplify voter altruism, and is consistent with numerous empirical instances of voters who put social objectives ahead of their own narrow interests. An information model of elections also explains key features of voter behavior, such as why uninformed voters tend to remain ideologically moderate and participate less in politics and why an extreme candidate still expects to win when on the side of truth.

[7] The jury theorem states that majority opinion in a large electorate favors whichever of two policies is truly superior, as long as individual opinions are correlated with the truth and, conditional on the truth, are mutually independent.

[8] Brunner, Ross, and Washington (2011) show empirically that economic conditions have a causal impact on both economic and non-economic vote choices.

perspective could motivate support for a host of other pro-labor policies. Importantly, the results below do not require a strong connection across issues: *any* non-zero correlation is enough to orient the equilibrium, so that issues are bundled just as they would be if they were *perfectly* correlated.

Observers of politics often express concern that the standard liberal-conservative divide bundles policy issues poorly. The model below lends structure to these concerns, as inefficient bundlings of issues can sometimes persist in equilibrium. With the present formulation of the model, however, inefficient equilibria are unstable and may not exist at all, making efficiency the essentially unique prediction of the model. Since the orientation of this unique equilibrium depends on the logical connections between issues and not on institutional details, it should be similar across elections, consistent with empirical evidence.[9]

2 Literature

There are two papers that study information aggregation in multidimensional common-interest settings, but neither addresses candidate positioning or unidimensionality, which are the focus of this paper. Feddersen and Pesendorfer (1997) show that conflicts of interest impede information aggregation when there are more dimensions than one. Barelli, Bhattacharya, and Siga (2022) identify conditions on the information structure that, absent any conflict, are necessary and sufficient for efficient information aggregation in arbitrary dimensions.

Social learning literature offers two accounts of multidimensional beliefs collapsing to a single dimension, but no account as to why the orientation of this direction should be consistent across elections. In Spector (2000), individuals with one of two prior beliefs about a multidimensional common-interest decision take turns reporting private signals to influence the group. Over time, beliefs converge in every direction but the one of prior disagreement, which presumably should differ from election to election.[10] In DeMarzo, Vayanos, and Zwiebel (2003), individuals update their beliefs after circulating their initial signals through a social network but fail to discount repeated information. The direction of slowest consensus can be interpreted as the left and right of politics (see also Louis, Troumpounis, and Tsakas, 2023), but depends on prior beliefs and the network structure, both of which are likely to vary across elections.[11]

The private interest election model of Schnakenberg (2016) assumes that voters do not know candidates' policy preferences, and policy platforms are non-binding. In equilibrium, a line divides the policy space in two, and a candidate's platform can only credibly communicate which side of the line her preferred policy bundle lies on. This has the familiar geometry of candidates pitted opposite one another, but since the line

[9] As further evidence that institutional features are not essential for the empirical correlation between issues, note that, historically, political factions (e.g. Tories and Whigs in England, Federalists and Anti-Federalists in the U.S.) arose before political parties were ever formalized (see https://en.wikipedia.org/wiki/Political_party). Pan and Xu (2017) find ideological structure similar to that of western democracies, even in authoritarian China. Outside the realm of politics, scientific and other non-political communities similarly divide into competing "schools of thought".

[10] With three or more priors, the cross-section of different senders' messages should be fully revealing, as in Battaglini (2002).

[11] In addition to these models, Duggan and Martinelli (2011) and Egorov (2015) show how the orientation of political conflict can be influenced by a monolithic media or by candidate messaging, respectively, but take unidimensionality as an exogenous constraint on communication.

dividing them could be oriented in any direction, it offers no reason for different elections to be oriented the same way. With enough symmetry, equilibria with different geometry exist, as well.

3 The Model

A society consists of N voters (indexed by i), where N is drawn from a Poisson distribution with mean $n \in \mathscr{N}$, and candidates A and B (indexed by j). Together, voters and candidates must choose a D-dimensional policy vector x from a compact set $X \subset R^D$ of alternatives.

At the beginning of the game, Nature designates one policy vector $z \in Z \subseteq X$ as socially optimal. Voters and candidates share a *common interest* to implement policies as close as possible to z. If policy $x \in X$ is implemented then voters and candidates each receive utility

$$u(x,z) = -\|x-z\|^2 \tag{1}$$

that decreases quadratically in the distance between x and z.[12]

Voters and candidates do not observe Nature's choice: they perceive z to be drawn from a distribution with log-concave density f.[13] For ease of exposition, normalize $E(z) = 0$. Conditional on information Ω (and suppressing terms that do not depend on the policy outcome), expected utility

$$E[u(x,z)|\Omega] = -\|x - E(z|\Omega)\|^2 \tag{2}$$

decreases quadratically in the distance between x and the updated expectation $E(z|\Omega)$ of the optimum.

Candidates each choose policy platforms $x_A, x_B \in X$ that they will implement if elected, then voters each vote for one of the two candidates. The candidate $w \in \{A, B\}$ with more votes (breaking ties with equal probability) wins the election and implements her vector of platform policies.[14] Noting that the candidate platform closest to z is socially optimal, platforms x_A and x_B partition Z into regions Z_A and Z_B where it is better to implement x_A or x_B, respectively.[15] Abusing notation, let j double as shorthand notation for the event $w = j$ of a candidate winning the election.

Before voting, voters form private beliefs about the location of the optimal policy, modeled as informative private signals $s_i \in S$. For simplicity, let $S = X$ and assume that s_i is D-dimensional normal with mean

[12] That candidates share voters' policy preferences is parsimonious in that, in reality, candidates are drawn from the pool of voters. Other policy objectives might also be overcome by candidates' desires to be viewed favorably by voters. For simplicity, candidates do not care about winning. I discuss in McMurray (2022) how candidates who want to win will polarize less—not at all, if winning matters enough—but how large elections mute office motives because voters' interest in z generates aggregate uncertainty. Beyond muting polarization, office motivation seems unlikely to alter how candidates bundle political issues.

[13] f if log-concave if $\ln f(z)$ is concave in z. Many of the most well-known distributions are log-concave (Bagnoli and Bergstrom, 2005).

[14] As in McMurray (2017b), candidates have an incentive to adjust their platforms after taking office, as they learn more about the location of z, and with common interests, voters have no reason to prevent this. Binding commitments make the analysis more tractable by limiting each candidate to one policy outcome. They also make the model more comparable to existing literature.

[15] Implicitly, the definition of these sets depends on the locations of both candidates' platforms: $Z_j(x_A, x_B) = \{z \in Z : \|x_j - z\| \leq \|x_{-j} - z\|\}$

z, truncated to domain X so that S remains compact.[16] In the voting subgame, a voting strategy $v : S \to \{A, B\}$ (from set V) specifies a vote choice for every signal vector $s \in S$.[17] If candidates adopt platforms (x_A, x_B) and a voter's peers all follow strategy $v \in V$, his best response v^{br} maximizes his expected utility. A (symmetric) Bayesian Nash equilibrium $v^* \in V$ is a voting strategy that is its own best response. In the complete game, a voting strategy $\sigma : X^2 \to V$ (from set Σ) specifies voting behavior for every pair $(x_A, x_B) \in X^2$ of candidate plaforms.

Candidates do not observe private signals, but may infer some voter information in equilibrium.[18] A candidate's platform $x_j^{br} \in X$ is a best response if it maximizes expected utility, taking her opponent's platform $x_{-j} \in X$ and the voting strategy $\sigma \in \Sigma$ as given. A perfect Bayesian equilibrum is a triple (x_A^*, x_B^*, σ^*) such that σ^* induces a Bayesian Nash equilibrium in every subgame and x_j^* is a best response to x_{-j}^* and to σ^*, for $j = A, B$.

4 Equilibrium Analysis

4.1 Voters

This section analyzes behavior in the voting subgame associated with candidate platforms (x_A, x_B), first for small and then for large elections. As Proposition 1 now states, best response voter behavior is straightforward: a voter votes A if his expectation of z lies closer to x_A than to x_B, and votes B otherwise. Naturally, this expectation depends on a voter's private signal vector s_i. As in existing voting literature, it also depends on the event P of casting a *pivotal* vote, meaning that it makes or breaks a tie. Proofs of all formal results are presented in the Appendix.

Proposition 1. *$v^{br} \in V$ is a best response to $(x_A, x_B, v) \in X^2 \times V$ if and only if $v^{br}(s) = \arg\min_{j \in \{A,B\}} \|x_j - E(z|P, s)\|$. For any pair $(x_A, x_B) \in X^2$ of candidate platforms, there exists a strategy $v^*(x_A, x_B) \in V$ that maximizes expected utility and is a Bayesian Nash equilibrium.*

Proposition 1 implies that, in any best response, voters with expectations in Z_j (after conditioning on being pivotal) vote for candidate j. Since signals are informative of z, this translates into a partition of S into regions S_A and S_B of signals that favor candidate A and B, respectively. Existence of an optimal voting

[16] This signal distribution is not essential to any of the results below. The only important feature of S is that signals are rich enough to identify the superior candidate as the number of voters grows large, no matter what policies candidates propose. Specifically, the distribution of signals must satisfy the *linear refinement* condition identified by Barelli, Bhattacharya, and Siga (2022).

[17] Mixed strategies could be allowed, but would only be used by the zero-measure set of indifferent voters.

[18] Given their career choice in policymaking and their privileged access to policy advice, it is reasonable that candidates should possess more private information than voters, not less. If voters and candidates all receive private signals, however, then each should try to infer the private information of the $N + 1$ others from their behavior. As their behavior comes to reflect inference from others in addition to their private information, everyone's inference about everyone else becomes more complicated, triggering an infinite heirarchy of beliefs about beliefs. The one-dimensional model of McMurray (2024) explores these nuances, but finds that even if candidates' signals are much better quality than voters', their inference from voters can overwhelm their own private opinions when N is large, partly justifying the omission here. Alternatively, if candidates receive private information and announce it publicly (which is credible in this common interest setting), the prior f can simply be reinterpreted as the common posterior that already takes candidates' publicized signals into account.

strategy follows from the continuity of expected utility over the compact set of strategies. That the optimal strategy constitutes a Bayesian Nash equilibrium follows from McLennan's (1998) observation that socially optimal behavior in a common interest setting is also individually optimal.

In stating the existence of a socially optimal equilibrium voting strategy, Proposition 1 leaves open the possibility of additional equilibria. However, inferior equilibria seem unlikely here: no matter how others vote, the first part of Proposition 1 states that a voter should vote informatively in response. If inferior equilibria do exist, they seem unlikely targets for coordination, so socially optimal voting seems likely to be focal. On that basis, the analysis below presumes that voters follow this strategy. Proposition 1 begins by stating the jury theorem, which holds here by familiar reasoning: as the number of voters grows large, the candidate with the superior platform almost certainly wins the election.

Proposition 2. *For any* $(x_A, x_B) \in X^2$ *and for every* n*, let* v_n^* *denote any socially optimal voting strategy. If voters follow* v_n^* *then* $\lim_{n\to\infty} \Pr(w = j|Z_j) = 1$ *for* $j = A, B$*.*

4.2 Candidates

Let σ^* denote the strategy in Σ that implements optimal voting in the subgame associated with every platform pair $(x_A, x_B) \in X^2$. This section proceeds by backward induction to analyze the platform selection incentives of candidates, first in small and then in large elections, assuming each candidate expects voters to react to her policy platform according to σ^*. To begin, Proposition 3 states that candidates, like voters, favor policies as close as possible to their expectations of the optimal policy.

Proposition 3. $x_j^{br} \in X$ *is a best response to* $x_{-j} \in X$ *and* σ^* *only if* $x_j^{br} = E(z|j)$*.*

According to Proposition 3, the expectation that a candidate uses in choosing her policy position conditions on the event $w = j$ of winning the election. This may seem odd, given that she must choose her policy stance before voting takes place. As I explain in McMurray (2022), the logic behind this result is *pivotal* reasoning analogous to voters': a candidate's platform position only influences her utility if she wins, so in equilibrium she presumes that this will have been the case. Winning from a particular position (against an opponent in a particular position) has implications about the signals that voters received, which in turn has implications about the location of the optimal policy. In equilibrium, a candidate chooses a policy that will be optimal in light of the information that she will infer if she wins from that position.[19]

As the number of voters grows large, Proposition 2 implies that the candidate whose platform is closer to z almost surely wins the election. Conditional on winning, candidate j therefore infers with certainty that the optimal policy is closer to her own platform than to her opponent's. In other words, she infers that $z \in Z_j$, as Proposition 4 now states.

[19]The proof of Proposition 3 is complicated by the fact that adjusting her platform affects a candidate's welfare directly, but also triggers a new voting subgame. However, the optimality of the voting strategy induces a kind of envelope theorem, by which voter reactions to changes in a candidate's platform have only second-order impact on utility.

Proposition 4. *If voters follow the optimal voting strategy* σ_n^* *for all* n *and* $\left(x_{A,n}^*, x_{B,n}^*, \sigma_n^*\right)$ *is a perfect Bayesian equilibrium for all* n *then* $\lim_{n\to\infty} x_{j,n}^* = E\left(z|Z_j\right)$.

One implication of Proposition 4 is that, as in the one-dimensional models of McMurray (2022, 2024), candidates' policy platforms are substantially polarized. That is, the set Z of possible locations of the optimal policy is partitioned into Z_A and Z_B, and candidates take positions at the respective means of these opposite subsets. The direction of this polarization is the subject of the analysis below.

5 Large Elections

The game between $N+2$ players is cumbersome in that every player must anticipate the behavior of each of the $N+1$ others and adjust his or her behavior accordingly. As everyone adjusts their behavior to anticipate others' information, however, the information content of their own behavior shifts, requiring others to adjust further in response. Increasing the number of voters further complicates individual incentives but simplifies behavior in the aggregate: according to Propositions 2 and 4, the candidate with the superior platform wins outright, so candidate platforms simply approach the respective means of Z_A and Z_B.

Commensurate with this aggregate simplicity, this section simplifies the model of Section 3: instead of a voting subgame, a single omniscient "voter" observes z and elects the candidate whose platform is truly superior. For ease of exposition, repurpose notation from earlier, now letting σ^* denote the behavior of this voter. As in Proposition 4, a candidate's best response x_j^{br} to x_{-j} and to σ^* is the solution to $x_j = E\left(z|Z_j\left(x_A, x_B\right)\right)$, where $E\left(z|Z_j\left(x_A, x_B\right)\right)$ is her expectation of the optimal policy, conditional on z being closer to x_j than to x_{-j} (that is, conditional on $z \in Z_j\left(x_A, x_B\right)$). A perfect Bayesian equilibrium is then a platform pair $\left(x_A^*, x_B^*\right) \in X^2$ such that x_j^* is a best response to x_{-j}^* and to σ^*.[20]

6 Equilibrium Analysis

For $d = 1, 2, ..., D$, the d^{th} *principal component vector* α_d of f is defined as the unit vector that maximizes the variance of the dot product of α_d and z, subject to being orthogonal to prior principal components $\alpha_1, \alpha_2, ..., \alpha_{d-1}$ (Pearson, 1901; Hotelling, 1933). Proposition 6 now presents the main result of this paper, which is that in any perfect Bayesian equilibrium, candidates take positions along one of the principal components of the policy space. On every principal component of f, there is a pair $\left(x_A^{*d}, x_B^{*d}\right) \in X^2$ of policy platforms that are *local best responses* to each other, meaning that neither candidate can increase her expected utility within a neighborhood of x_j^{*d}. These may or may not be Nash equilibria. The local best responses on the first principal component necessarily constitute a Nash equilibrium.

[20] Alternatively, the voting subgame could be modeled simply as a mechanical process, whereby the superior candidate simply takes office. In the resulting two-player game between candidates, $\left(x_A^*, x_B^*\right)$ is a Bayesian Nash equilibrium.

Proposition. *For $d = 1, 2, ..., D$, there exist unique $\left(x_A^{*d}, x_B^{*d}\right) \in X^2$ such that $x_j^{*d} = \gamma_{j,d}\alpha_d$ for some non-zero $\gamma_{j,d} \in R$, and such that, for $j = A, B$, x_j^{*d} is a local best response to x_{-j}^{*d}. If $(x_A^*, x_B^*) \in X^2$ is a Nash equilibrium then, for some d, $x_j^* = x_j^{*d}$ for $j = A, B$. Expected welfare associated with $\left(x_A^{*d}, x_B^{*d}\right)$ and polarization $\left\|x_j^{*d}\right\|$ both decrease with d. Moreover, $\left(x_A^{*1}, x_B^{*1}\right)$ is a Nash equilibrium.*

The logic behind Proposition 6 is that candidates seek to minimize average quadratic deviations from z, which is simply variance. The direction of maximal variance, which is the first principal component of the policy space, minimizes variance in orthogonal directions, so candidates optimally locate on this axis. By McLennan's (1998) logic, platforms that are socially optimal in a common interest setting are also individually optimal, thus constituting an equilibrium.

Reorienting the policy space in the direction of the first principal component α_1, the logic of Proposition 4 implies that candidate positions are given by $E(z|Z_A) = E(z|z_1 < \bar{x}_1)$ and $E(z|Z_B) = E(z|z_1 > \bar{x}_1)$, where $\bar{x} = \frac{x_A^{*1} + x_B^{*1}}{2}$ is the midpoint between the two platforms. Log-concavity of f ensures a unique $\left(x_A^{*1}, x_B^{*2}\right)$ on this axis for which these conditions hold, as in the one-dimensional model of McMurray (2022).

Restricting candidates to the subspace of X that is orthogonal to α_1, optimal policy positioning leads candidates to the second principal component. Naturally, welfare is lower than when candidates optimize without this constraint. Polarization also decreases, because the second principal component is less informative than the first principal component, so conditioning on winning does not make a candidate as confident as she is when winning on the first principal component. Again, log-concavity of f ensures a unique $\left(x_A^{*2}, x_B^{*2}\right)$ such that $E(z|Z_A) = E(z|z_2 < \bar{x}_2)$ and $E(z|Z_B) = E(z|z_2 > \bar{x}_2)$, and these positions are local best responses to one another. McLennan's (1998) reasoning is not sufficient to guarantee these as an equilibrium, since they are optimal only within the subspace of X orthogonal to α_1, not optimal globally. Repeating this logic yields a unique pair $\left(x_A^{*d}, x_B^{*d}\right)$ of local best responses on every principal component axis. Consistently, welfare and polarization decrease with d.[21]

6.1 Inefficient Equilibria

As noted above, the local best responses on the first principal component necessarily constitute an equilibrium since they are globally optimal, but the same is not guaranteed for the best responses on higher principal components, which are optimal only locally. This section constructs examples in which these inferior policy positions do constitute equilibria, but also presents examples in which the efficient equilibrium is the only equilibrium.

Intuitively, what determines whether higher principal components can sustain equilibria or not is the symmetry or asymmetry of Z and f: with sufficient symmetry, later principal components provide welfare

[21] That the direction of greatest polarization also generates the highest welfare is familiar from the one-dimensional analysis of McMurray (2022) but may seem jarring to readers familiar with private interest models, where policy extremes inevitably create the greatest disutility, namely for voters who prefer the opposite extreme. Polarization can be good for welfare in a common interest setting because voters do not necessarily benefit from the policies they favor: they benefit from policies as close as possible to the true z. A moderate policy avoids the disutility of a severe policy mistake, but in this model, large electorates never make such mistakes, and so benefit from policy options that are (moderate within Z_j but) substantially different from each other.

similar to earlier principal components, and unilaterally deviating from the local best responses does not improve expected utility, preserving these as equilibria. With highly asymmetric Z or f, deviating from the inferior axis to a superior axis provides greatear advantage, so the former does not support an equilibrium.

To clarify this principle, consider the example of a two-dimensional state space with uniform discrete distribution over $Z = \left\{\left((1-c)\sqrt{2}, -(c+1)\sqrt{2}\right), \left(-(c+1)\sqrt{2}, (1-c)\sqrt{2}\right), \left((1+c)\sqrt{2}, (c-1)\sqrt{2}\right), \left((c-1)\sqrt{2}, (c+1)\sqrt{2}\right)\right\}$. If $c > 1$ then the first principal component of this distribution is the major diagonal (i.e., the upward-sloping 45 degree line); the second is the minor diagonal (i.e., the downward-sloping 45-degree line). Rotating the basis to put the first principal component on the horizontal axis (and the second on the vertical axis) allows Z to be written as $Z = \{(-c,-1), (-c,1), (c,-1), (c,1)\}$.

In terms of this rotated basis, local best responses $x_A^{*1} = E(z|z_1 < 0) = (-c, 0)$ and $x_B^{*1} = E(z|z_1 > 0) = (c, 0)$ on the first principal component are also optimal globally, and so together constitute an equilibrium. Local best responses on the second principal component are $x_A^{*2} = E(z|z_2 < 0) = (0, -1)$ and $x_B^{*2} = E(z|z_2 > 0) = (0, 1)$. Whether these local best responses also constitute global best responses depends on whether either candidate can profitably deviate.

The candidates are symmetric, so without loss, consider the decision of candidate B. If she remainins at x_B^{*2}, she will win in states $(-c, 1)$ and $(c, 1)$ and A will win in states $(-c, -1)$ and $(c, -1)$, so she will receive expected utility $\frac{1}{4}\left[-(-c-0)^2 - (-1+1)^2\right] + \frac{1}{4}\left[-(-c-0)^2 - (1-1)^2\right] + \frac{1}{4}\left[-(c-0)^2 - (-1+1)^2\right] + \frac{1}{4}\left[-(c-0)^2 - (1-1)^2\right] = -c^2$. If she deviates from x_B^{*2}, her most profitable deviation is to $(c, 0)$ (or, equivalently, to $(-c, 0)$). If x_A remains at $(0, -1)$ but B deviates then A will win in states $(-c, -1)$ and $(-c, 1)$ and B will win in states $(c, -1)$ and $(c, 1)$, producing expected utility $\frac{1}{4}\left[-(-c-0)^2 - (-1+1)^2\right] + \frac{1}{4}\left[-(-c-0)^2 - (1+1)^2\right] + \frac{1}{4}\left[-(c-c)^2 - (-1-0)^2\right] + \frac{1}{4}\left[-(c-c)^2 - (1-0)^2\right] = -\frac{1}{2}c^2 - \frac{3}{2}$.

If $c < \sqrt{3}$ then this deviation is unprofitable, sustaining $\left(x_A^{*2}, x_B^{*2}\right)$ as a Nash equilibrium. This is the case where f is close to being symmetric ($c = 1$ would imply perfect symmetry). If $c > \sqrt{3}$ then the deviation is profitable, implying that x_j^{*2} is a best response only locally, not globally. This is the case where f is highly asymmetric. This example shows that local best responses on principal components beyond the first can be equilibria, but need not be.

7 Example

The following example gives clarity to the logic and the impact of Proposition 6. Let $Z = X$ be the unit disk and define f as follows (expressed both in Cartesian and polar coordinates),

$$f(z;\rho) = \frac{1}{\pi}\left(1 + \rho\frac{z_1 z_2}{\|z\|}\right) = \frac{1}{\pi}\left[1 + \rho r_z \cos(\theta_z)\sin(\theta_z)\right] \tag{3}$$

where $\|z\| = \sqrt{z_1^2 + z_2^2}$denotes the magnitude of z. For $\rho = 0$, (3) is uniform and z_1 is uncorrelated with z_2. More generally, the correlation coefficient between z_1 and z_2 is proportional to the parameter ρ. Figure 1

illustrates the case in which z_1 and z_2 are positively correlated.

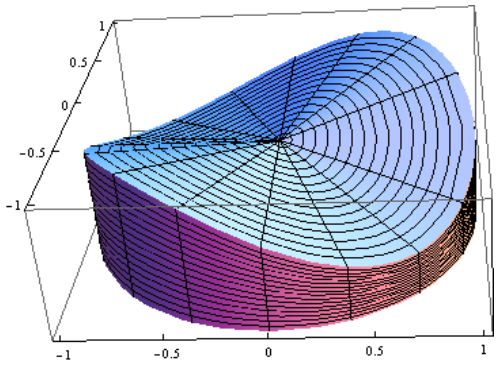


Figure 1: Parametric example density (3), with $\rho > 0$.

Consider first the case of $\rho = 0$. In that case, it is straightforward to derive an equilibrium along the horizontal axis, with $x_A^* = E(z|z_1 < 0)$ and $x_B^* = E(z|z_1 > 0)$. That is, candidates ignore issue 2 and campaign on issue 1 alone. With candidates taking equivalent (centrist) positions on issue 2, the winner is determined on the basis of issue 1 alone.[22] Rotating the policy space by any angle θ, the radial symmetry of f generates identical incentives. Thus, there are a continuum of equilibria, with A and B polarizing any direction from the origin. In particular, candidates can be in quadrants 1 and 3, with one taking positive positions on both issues and the other taking negative positions on both issues, or in quadrants 2 and 4, with both candidates taking one positive position and one negative position. (Within these categories, equilibria differ only in the extent of polarization on each issue.) In a higher-dimensional extension of this example, any configuration of policy positions could be adopted by the candidates in equilibrium.

In contrast, consider now the case of $\rho > 0$. By Proposition 6, the number of equilibria falls precipitously from infinity to one or two: there is an efficient equilibrium with A in quadrant 3 and B in quadrant 1, and there is a pair of local best responses in quadrants 2 and 4 (which may or may not be an equilibrium).

There is no longer an equilibirum in which candidates ignore issue 2 and align on the horizontal axis: if candidates polarized horizontally, the winner would be determined on the basis of issue 1 alone; winning the election would still signal to candidate B that $z_1 > 0$, vindicating her position right of the origin, but she would no longer wish to remain on the horizontal axis, reasoning that if $z_1$is positive then $z_2$is likely positive, as well, since the two issues are now correlated. Thus, she would prefer to deviate to a position in the interior of quadrant 1. Similarly, candidate A would prefer a candidate on the interior of quadrant 3. If this logic led candidates to rotate counter-clockwise, of course, the boundary between Z_A and Z_B would rotate in tandem, so that the candidates wish to rotate further. This rotation would continue until candidates reach the main diagonal.

Orthogonal to the efficient equilibrium, there are local best responses in quadrants 2 and 4, satisfying

[22]With finitely many voters, as in the model of Section 3, voters would rely wholly on s_1 and ignore s_2, so that $S_A = \{(s_1, s_2) : s_1 < 0\}$ and $S_B = \{(s_1, s_2) : s_1 > 0\}$; taking pivotal considerations into account, it can be shown in this case that $E(z_1|P, s)$ has the same sign as s_1.

$x_A^{*2} = E(z|z_1 < z_2)$ and $x_B^{*2} = E(z|z_1 > z_2)$. Upon winning, candidate B infers that z lies southeast of the origin, which coincides with the location of her platform. Now, however, she is less polarized than before.[23] The reason for this is she infers mixed messages, making her less confident: since z_1 and z_2 are positive, the inference that z is east of the origin suggests it should also be north, while the inference that z is south of the origin suggests that it should also be west. In contrast, the messages she received in the efficient equilibrium (namely, that z_1 and z_2 are both positive) were mutually reinforcing.

The positions in quadrants 2 and 4 are also less stable: if policy positions were slightly clockwise from these locations, a chain of best responses would continue to rotate further clockwise till they reach the efficient equilibrium; if policy positions were slightly counter-clockwise then best responses would rotate counter-clockwise toward the efficient equilibrium. At $\left(x_A^{*2}, x_B^{*2}\right)$, candidates are delicately balanced between rotating either direction. Neither candidate can improve utility in a neighborhood of x_j^{*2}; whether a larger deviation (such as to the main diagonal) improves utility or not determines whether or not this policy pair constitutes an equilibrium.

This example illustrates why inefficient equilibria, if they exist, are less polarized than the efficient equilibrium. It also highlights another distinction between the equilibria, which is that only the efficient equilibrium is *stable*: that is, perturbations from the efficient equilibrium trigger best responses that rotate back agian, while perturbations from inefficient equilibria trigger best responses that rotate further away. Even when inefficient equilibria exist, the efficient equilibrium seems likely to prevail, both for its stability and because efficiency makes it a more likely target for coordination.

8 Applications

When a thousand policy decisions yield $2^{1,000}$ (or over 10^{300}) policy combinations (plus continua of intermediate positions between these extremes), it seems woefully inadequate to model politics with a single dimension, ranging from liberal to conservative with moderates in between. Empirically, however, this seems to be the political world we live in. The model above offers a partial explanation for this, positing simply that underlying policy truths are correlated, so that voters who believe that a liberal policy is optimal on one issue tend to favor liberal policies on other issues, as well. Logical connections across issues are not likely to depend on location, timing, or institutional details, explaining why issues are bundled so consistently across elections.

For many policies, of course, the logical connection is sufficiently subtle that the correlation, while non-zero, is likely to be weak. The example of Section 7 is useful in that regard: equilibrium can be polarized in any direction when $\rho = 0$, but as soon as $\rho \neq 0$, the first principal component emerges as the uniquely

[23] For the parametric density (3), $\left(x_A^{*1}, x_B^{*1}\right) = \left(\begin{pmatrix} \frac{6+\rho}{9\pi}\sqrt{2} \\ \frac{6+\rho}{9\pi}\sqrt{2} \end{pmatrix}, \begin{pmatrix} \frac{6+\rho}{9\pi}\sqrt{2} \\ \frac{6+\rho}{9\pi}\sqrt{2} \end{pmatrix}\right)$ and $\left(x_A^{*2}, x_B^{*2}\right) = \left(\begin{pmatrix} -\frac{6-\rho}{9\pi}\sqrt{2} \\ \frac{6-\rho}{9\pi}\sqrt{2} \end{pmatrix}, \begin{pmatrix} \frac{6-\rho}{9\pi}\sqrt{2} \\ -\frac{6-\rho}{9\pi}\sqrt{2} \end{pmatrix}\right)$.

efficient equilibrium, and if other equilibria exist, they are unstable. In that sense, *any* positive correlation is sufficient to orient the equilibrium, so that candidates behave just as if $\rho = 1$.

Converse (1964) and Shor (2014) find that political candidates are more ideologically consistent than voters. This, too, is consistent with the analysis above: with two issues, for example, voters hold opinions in every quadrant, but candidates only ever take positions in quadrants 1 and 3. For large D, candidates adopt consistent positions on every issue, but voters form opinions in every orthant, and the fraction of voters who favor the party line on every issue tends to zero.

The results above also shed light on modern political arguments. The U.S. Libertarian party, for example, is liberal on social issues such as immigration, abortion and marriage, but conservative on economic issues such as taxes and regulation. Its website emphasizes logical consistency, arguing that while Democrats and Republicans each favor personal or economic liberty, Libertarians favor both.[24] In terms of the model above, this can be formalized as a claim that the optimal social and economic policies should be correlated, suggesting that society is stuck in an inferior equilibrium. The analysis above acknowledges that an inefficient bundling of issues is an equilibrium possibility: if somehow issues were bundled inefficiently, this bundling could be self-perpetuating.

On the other hand, the Libertarian perspective might be wrong: conservative policies might be logically unified by the underlying importance of preserving social and economic traditions, while liberal or progressive policies are unified by the importance of modernizing on both fronts. If so, the prevailing political divide matches the underlying policy correlation, and reorientation would reduce welfare. The analysis above does nothing to settle the political dispute, but lends useful structure to the question: if the direction of strongest correlation is the traditional one, the traditional bundling of issues is efficient; if the principal component is the opposite direction, moving to the axis proposed by Libertarians would enhance welfare.[25]

9 Conclusion

Left, right, and center is the widely acknowledged geometry of politics, but how or why the myriad of political issues reduce to a single dimension has remained a theoretical mystery. In the common interest setting of this paper, a candidate can't learn about one issue without also inferring information about issues that are logically related. Even if logical connections are weak, this is enough to orient political competition along the principal component axis of the underlying issue space. If logical structure pervades times and space, this may explain why the bundling of political issues is so universal, and why it is more pronounced among politicians than among ordinary citizens. Efficiency is the unique (or uniquely stable) equilibrium prediction, but the possibility of orthonogal equilibria gives structure to prevelant public concern that the standard bundling of issues might be inefficient.

[24] See www.lp.org/platform.

[25] If anything, the instability of inefficient equilibria suggests that an inferior bundling issues would be unlikely to have lodged as the status quo. On the other hand, enriching the present model might vindicate the Libertarian narrative: perhaps ρ is imperfectly observed, for example, with different voters seeing it as positive or negative.

Section 3 assumes that every issue matters equally to voters, but more generally, (1) can be generalized as follows,

$$u(x,z) = \sum_{d=1}^{D} \omega_d (x_d - z_d)^2$$

to accommodate different weight ω_d on each issue. Generalizing this way has the same impact as stretching important dimensions, further polarizing these dimensions.[26] As issue importance fluctuates across space and time, then, candidates polarize most highly on the prominent issue of the day, but the underlying bundling of issues remains largely the same.

In the analysis above, the key assumption that z_d and $z_{d'}$ are correlated seems plausible given the shared logic of many policy decisions, but similar unidimensionality could also arise if voter learning were somehow biased such that opinions s_d and $s_{d'}$ on the two issues were correlated *conditional* on z. As a simple example of this, suppose z is uniform on $Z = \{(-1,-1),(-1,1),(1,-1),(1,1)\}$ and that s_1 and s_2 are perfectly correlated, with $s = z$ whenever $z_1 = z_2$ but $s = (-1,-1)$ and $s = (1,1)$ are equally likely when $z_1 \neq z_2$. In that case, if candidates polarized on the horizontal axis then $z = (1,1)$ would result in a unanimous B victory while $z = (-1,1)$ and $z = (1,-1)$ both result in expected ties. Conditional on winning, therefore, $E(z_1|B) = \frac{1}{8}(-1) + \frac{3}{8}(1) = \frac{3}{4}$, vindicating the position right of center, but $E(z_2|B) = \frac{1}{8}(-1) + \frac{3}{8}(1) = \frac{3}{4}$ as well, so candidate B would prefer to deviate in the vertical direction (as would A, by symmetry).

That a common interest paradigm sheds light where private interest literature has not highlights the utility of this general approach to elections. Literally identical preferences are improbable, though, so generalizing this is an important direction for future work.[27] As long as preferences share a substantial common element, the results above seem likely: a voter who cares partly about the social optimum should still formulate an expectation $E(z|P,s_i)$ using his private signal and whatever he can infer from other voters, and a candidate should infer what she can from voters.[28] If z_d and $z_{d'}$ are correlated, voter opinions in the direction of polarization will reveal information about other dimensions, as well, thus rotating candidates in the direction of the principal component.

Whatever the level of common interest, future work should also enrich the information structure above. In the current framework, for example, pairs of voters could find consensus simply by sharing and combining their private signals. In large groups, public opinion should be so reliable that voters abandon minority opinions. Empirically, individuals routinely disagree with one another, and knowingly maintain unpopular opinions. In McMurray (2022) I discuss cognitive or other information barriers that might explain these features, and the same discussion applies here. A dynamic model of how opinions update between elections

[26] The necessary equilibrium condition generalizes to $\left[D^{1/2}(x_B - x_A)\right]' \left[D^{1/2}(z - \bar{x})\right] > 0$, where $D^{1/2}$ denotes the square root of the diagonal matrix with diagonal elements ω_d. This is equivalent to the original equilibrium condition after a change of basis, where the units of each dimension are scaled in proportion to $\sqrt{\omega_d}$. If one issue is nine times as important as another for utility, for example, its axis stretches so that policy distances are three times as large (with nine times the consequence for utility), leaving the correlation structure intact.

[27] Even if private interests are important empirically, a pure common interest model is useful as a theoretical benchmark against which imperfectly aligned interests can be compared.

[28] Voting that is only partly informative may polarize candidates less for a given n, but still eventually convey perfect information in the limit.

would also be useful: Krasa and Polborn (2014) present evidence, for example, that the correlation of voter attitudes across political issues has increased over time. Noted above is the possibility that voters must learn ρ in addition to determining z; changes over time in voters' understanding of the correlation structure of f might explain policy realignments that occur occasionally.

The jury theorem guarantees that the winning platform is closer to the social optimum than the losing platform is, but with platforms always on the principal component axis, welfare loss may still be substantial. This is an inherent limitation of binary decisions in a complex policy environment, but additional candidates cannot easily solve this problem, as they reduce the best candidate's ability to attract a plurality of votes.[29] If candidates can modify their policy positions after taking office, however, as in McMurray (2017b), vote shares (even of losing candidates) convey additional information that can steer the winning candidate toward the social optimum. This may be even more important in higher dimensions, as votes for Libertarian, Green party, or other minor candidates give voters a way to signal opinions off the main diagonal. With far more issues than parties, however, the ability to precisely identify z remains limited, underscoring the importance of informal political activities such as petitions, rallies, public opinion surveys, and letters to legislators, which communicate policy-specific opinions that a coarse voting mechanism cannot.[30]

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