Politics of Non-Full Democracies: Fraud, Post-Election Boycotts, and Protests

Mariam Malashkhia* Mate Malashkhia†

Abstract

Non-full democracies are often characterized by electoral manipulation and contested results. Despite the growing literature on elections and mass protests in such regimes, the strategies of opposition parties remain understudied. To address this gap, we analyze the actions of opposition parties following an election. In a context where opposition parties face significant constraints in reaching the public, the decision to boycott the results serves as a costly signal. By risking detention and violence, and forfeiting seats in the Parliament, the opposition party signals its ability and commitment to challenge the incumbent. Thus, citizen's expected net gain from regime change is higher when it follows a boycott than when it does not. Additionally, the information signaled by the boycott encourages more people to protest as the expectations about the challenger rise.

Our paper also gives insights into the political dynamics in presidential versus parliamentary elections. We show that the total loss of power after losing a presidential election makes government turnover through elections less likely in presidential systems. Moreover, while presidential elections tend to see more frequent boycotts, the scale of protests following these elections is typically smaller. This is because a boycott after a presidential election sends a weaker signal about the challenger's competence to voters, as the opposition is not giving up the benefits of holding office.

Introduction

Almost all countries hold elections, but the extent to which they are free and fair varies significantly. Elections in non-full democracies are typically marred by ballot stuffing, vote buying, intimidation of voters, and other techniques of electoral manipulation. Consequently, it is not uncommon for election results to be contested by opposition parties or rejected by the general public. In the aftermath of these elections, opposition parties

*PhD candidate of Political Economics at Stanford GSB. Contact information: 736 Escondido Road, Stanford, CA, 94305. 585-958-4403. mariami@stanford.edu. Corresponding Author

†PhD candidate of Economics at Washington University in St. Louis. Contact information: Seigle Hall Room 377, Fraternity Way, St. Louis, MO, 63105. 314-399-4649. m.malashkhia@wust.edu

may boycott, or popular protests may erupt, potentially leading to the toppling of the regime. However, the notion that incumbents always win by rigging elections and that protests inevitably follow oversimplifies the political dynamics in non-full democracies. Not all incumbents maintain power through fraudulent elections – some may choose to concede defeat and step down. Additionally, in some instances, both opposition parties and citizens may accept the election results, even when there is evidence of fraud.

This paper explores the interactions between the incumbent, the opposition, and the voters in non-full democracies, providing insights into the factors that lead to different political outcomes. By examining why some incumbents are overthrown while others step down or maintain power, and why some fraudulent elections provoke protests and boycotts while others do not, we offer a more nuanced understanding of electoral politics in these regimes.

A key aspect of the model is that, after observing the opposition party's decision to boycott following a proportional parliamentary election, voters then decide whether to protest. This sequence, combined with the high costs of boycotting – such as forfeiting parliamentary seats and facing risks of detention and violence – underscores how the opposition's decision can serve as a strong signal of their competence to voters. In non-democratic regimes, where opposition parties are significantly constrained in reaching the public (Cunha, Schuler, and Williamson 2022), the decision to boycott becomes a valuable signal – demonstrating opposition's capacity to challenge the regime.

The boycott, by conveying new, positive information about the challenger's competence, explains our finding that citizen's expected net gain from regime change is higher when it follows a boycott than when it does not. Additionally, the information signaled by the boycott encourages more people to protest as the expectations about the challenger rise.

Given the significance of the boycott decision, we explore what factors influence this choice. In particular, carrots (e.g., increasing the benefit from having seats in the Parliament) and sticks (i.e., increasing the challenger's cost from boycotting), can decrease the likelihood of a boycott. Finally, refraining from excessive electoral fraud can similarly reduce the likelihood of a boycott.

However, our analysis suggests that the incumbent is unlikely to refrain from fraud. For instance, a losing incumbent might rig the election and declare victory. Moreover, the incumbent might engage in fraud even if they could win the election without it. Finally, the incumbent may choose a positive level of fraud that ultimately results in losing the election. In such a case, fraud can enhance the incumbent's benefits once they transition into opposition.

The total loss of power after losing a presidential election makes government turnover through elections less likely in presidential systems compared to parliamentary ones. Moreover, while presidential elections tend to see more frequent boycotts, the scale of protests following these elections is typically smaller. This

is because a boycott after a presidential election sends a weaker signal about the challenger's competence to voters, as the opposition is not giving up the benefits of holding office.

In non-full democracies, it is not uncommon for the opposition to receive international support and aid in support of a boycott. However, excessive aid can sometimes do more harm than good. Too much aid can undermine the boycott's strategic signaling value, reducing the likelihood of large-scale protests sufficient to unseat the incumbent. When aid is appropriately calibrated, however, it can increase the chances of the incumbent being removed from office.

This paper contributes to the growing literature on elections and mass protests in non-full democracies. The presence of elections in such regimes is puzzling, yet most countries still choose to hold them. Various explanations have been proposed to account for this phenomenon. For instance, elections can help the incumbent identify supporters and opponents (Magaloni 2006; Brownlee 2007) or provide valuable information about the competence of party members (Blaydes 2010). Additionally, elections can reinforce agreements on power-sharing or the distribution of rents among elites (Gandhi and Przeworski 2006, 2007; Boix and Svolik 2013). Furthermore, the decision to hold elections can influence citizens' beliefs about the overall sentiment toward the incumbent (Little et al. 2012). Thus, elections can either increase or decrease the likelihood of rebelling and the chances of a successful revolution (Little et al. 2012).[1] Yet, a major feature of elections in non-full democracies is electoral manipulation. The losing incumbent may choose to commit fraud to secure victory. In addition, an incumbent can manipulate the election to receive more spoils from holding office or create an image of invincibility (Magaloni 2006). However, widespread fraud undermines the signaling power of the election and may serve as a coordination mechanism to drive rebellion (Tucker 2007; Little, Tucker, and LaGatta 2015). Thus, winning an election does not guarantee holding onto power. The incumbent may still be vulnerable to protests, which may arise for reasons other than fraud (De Mesquita 2010; Edmond 2013; Persson and Tabellini 2009; Shadmehr and Bernhardt 2011; Barbera and Jackson 2019). An important yet understudied aspect of politics in non-full democracies is the strategy of opposition parties. Opposition parties play a crucial role in mobilizing protests and challenging the regime. Our paper seeks to fill this gap in the literature by highlighting an important tool available to the opposition in non-democratic regimes – post-election boycotts. Because the opposition's ability to communicate with voters is limited, boycotting becomes a crucial mechanism for conveying the challenger party's ability to voters.

Given the prevalence of elections, we take their occurrence as a given. Thus, unlike other work, elections in our model do not serve as a public signal of the incumbent's strength. Another modeling choice we make, similar to Egorov and Sonin (2021), but different from the global games approach, is assuming that instead of individuals having private information about the incumbent, they receive private benefits from the regime.

1. See Gandhi and Lust-Okar (2009) and Gehlbach, Sonin, and Svolik (2016) for a review.

As a result, citizens's hold homogenous beliefs about the incumbent's type. Finally, unlike most existing studies on post-election protests, we focus on proportional parliamentary elections rather than presidential ones.

Model Setup

Consider a two-period model involving two political parties: the Incumbent (Party D) and the Challenger (Party C), competing for high-office seats (e.g., parliament) in a non-full democracy. Additionally, there is a continuum of citizens who can replace party D either through elections or protests following the election. We assume that the incumbent party decides the level of electoral fraud to commit. If the challenger wins the election (despite potential fraud committed by the incumbent), the game ends. However, if the incumbent wins (potentially through fraud), the challenger must decide whether to boycott the results. Regardless of the challenger's decision to boycott, each citizen chooses whether to protest, and the overall proportion of protesters determines the likelihood of the protest's success. There is no discounting, and each party's payoff from winning is proportional to an exogenous payoff from holding office, the party's competence/ability, and the vote share they receive in the election. Finally, even if a party loses the election, it still receives payoffs (such as seats in parliament or salaries), which are proportional to its vote share and ability. Having given a general overview of the model, we will proceed by getting into particularities of it.

We assume that the abilities of the challenger (a_C) and the incumbent (a_D) are drawn from a normal distribution with mean a_0 and variance σ_a^2. Both parties know their own abilities and citizens observe the ability of the party in power, as well as their own economic well-being, which includes a noise component δ_i. This contrasts with the framework of Egorov and Sonin (2021), where individuals observe only their own economic well-being, and cannot directly observe incumbent party's ability. Thus, in our model, by the end of the first period individuals hold homogeneous beliefs about the dictator's ability; however, their economic well-being is heterogeneous. The payoff for individual i when party $P \in \{D, C\}$ is in office during period $t \in \{1, 2\}$ is given by

$$r_i^P = a_P + \delta_i^P$$

Where δ_i^P is independent and follows a normal distribution with mean 0 and variance σ_δ^2. Therefore, if the party D remains in office in the second period, individual i's payoff remains unchanged (Egorov and Sonin 2021). Hence, this assumption highlights the inherent conflict of interest among citizens.[2]

After the election, in which citizens are assumed to vote sincerely, the incumbent receives an actual vote

2. Note that citizens do not gain any additional information about the challenger's competence by the end of first period.

share τ. The incumbent party then determines the level of fraud (x) to commit. Consequently, after fraud, the incumbent's vote share becomes $\tau + x$. However, fraud is costly, entailing a cost of $k(x)$. Similar to Little et al. (2012), we assume that $k(0) = 0, k'(0) = 0, k'(x) \geq 0$ (the cost of fraud increases as the level of fraud increases), and $k''(x) > 0$. Moreover, we assume that as the level of fraud approaches 1, the marginal cost of fraud approaches infinity. This implies that falsifying first vote incurs no cost, but as fraud increases, the cost increases at an increasing rate (Little et al. 2012). This assumption captures the idea that the cost of ballot stuffing or other methods of falsifying election results will escalate as the level of fraud increases. This is because a larger network of people will need to be compensated both for committing fraudulent activities and for ensuring their silence.

If the challenger wins the election, the game ends. The challenger receives a payoff $e^{A+a_C}(1-(\tau+x))$, and the incumbent receives $e^{M_D+a_D}(\tau+x) - k(x)$ where $(\tau + x) \leq \tilde{\tau}$. Where, A denotes the exogenous payoff from winning the election, M_D denotes the net payoff the incumbent party receives when they lose the election but still retain seats in parliament. M_D might be less than M_C (the net payoff for the challenger from, for example, holding seats in parliament after losing the election), as the payoff to the dictator party upon losing the election may be smaller than that of the challenger party, since the incumbent was already in government.

If the incumbent wins the election ($\tau + x \geq \tilde{\tau}$), where $\tilde{\tau}$ is the threshold required to win (with 0.5 being the most natural case), the challenger must then decide whether to boycott the results.

If the challenger chooses not to boycott and the subsequent protest fails, the challenger's payoff is given by $e^{M_C+a_C}(1-(\tau+x))$. Where M_C represents the net payoff from, for example, holding seats in parliament after losing the election. The challenger's payoff is proportional to the vote share that the party receives and the party's ability. Because we assume the payoff takes the form $e^{M_C+a_C}(1-(\tau+x))$ and as a_C is normally distributed, it follows that $e^{M_C+a_C}$ is log-normally distributed with mean $a_0 + M_C$ and variance σ_a^2. This ensures that the payoff is always non-negative. For the incumbent, the payoff in case of a failed protest is $e^{A+a_D}(\tau+x) - k(x)$. Thus, incumbent's payoff is also proportional to its vote share and ability. We make the intuitive assumption that $A > M_C$. However, if the protest after a no boycott is successful, the challenger's payoff becomes zero, while the incumbent's payoff equals $-k(x)$, as the incumbent has already incurred the cost of fraud.

The payoff for citizen i in the case of no boycott is as follows: if party D remains in power (i.e., the protest fails), citizen i will receive the same payoff as in the first period, $r_i^D = a_D + \delta_i^D$. If the regime changes (i.e., the protest is successful), citizen i will receive an expected payoff of $E(a_V) = a_0$. This implies that, upon a successful protest, a new group (denoted as V) will take power, with an ability that is also normally

distributed with mean a_0 and variance σ_a^2.[3]

Thus, the expected net payoff from regime change for the individual i in the case of no boycott is given by $b_i^V = a_0 - a_D - \delta_i^D = b^V + \delta_i$ where $\delta_i = -\delta_i^D$ and $b^V = a_0 - a_D$. Beyond the payoff from regime change, there are costs and benefits associated with protesting.[4] In particular, if an individual i joins the protest and it succeeds, she gets an additional payoff - αb_i^V. If the protest fails, she receives γb_i^V, with $\alpha > \gamma > 0$. This reflects the "warm glow" utility from protesting against a regime the citizen dislikes, which is proportional to the expected net payoff from regime change (for more details, see Egorov and Sonin (2021), p.1669). However, protesting is costly. If an individual joins the protest, she incurs a disutility of $c > 0$. Conversely, if i decides not to protest, there will be no extra costs and benefits. Therefore, an individual with a negative net expected payoff from regime change (i.e., someone who prefers the current regime) will never protest. Additionally, an individual with $b_i^V < c/\alpha$ will never protest, while someone with $b_i^V > c/\gamma$ will always protest.

Next, we consider the case where the challenger decides to boycott. The boycott is costly for the challenger party, with a cost equal to f. The cost reflects the notion that in non-democratic regimes, boycotting comes with significant risks. For example, members of the challenger party may face arrests, violence, and threats during the boycott. Therefore, in the case of a boycott, if the protest is successful, the challenger receives a maximum payoff of $e^{A+a_C} - f$, while the incumbent receives $-k(x)$. Conversely, if the protest fails, the challenger incurs a loss of f and the incumbent receives the maximum payoff of $e^{A+a_D} - k(x)$. While the payoff for citizen i is as follows: if party D remains in power, citizen i receives the same payoff $r_i^D = a_D + \delta_i^D$. If the regime changes, citizen i will receive the expected payoff of $E(a_C|Boycott)$. To specify the expected payoff from the regime change following a boycott, we assume that the challenger party follows a threshold rule: they will boycott if and only if $a_C \geq \hat{a}_C$. That is, given the costs associated with boycotting, only a sufficiently competent challenger will boycott. Therefore, the decision to boycott sends a signal to voters about the challenger party's competence. In non-full democracies, where challenger parties have limited channels to demonstrate their type, boycotting, thus, becomes a valuable strategy for signaling their ability to voters.

With this assumption,

$$E(a_c|Boycott) = E(a_c|a_C \geq \hat{a}_C) = a_0 + \frac{\phi\left(\frac{\hat{a}_C - a_0}{\sigma_a}\right)}{1 - \Phi\left(\frac{\hat{a}_C - a_0}{\sigma_a}\right)}\sigma_a = a_C^{New}$$

3. We might also consider a scenario where V is drawn from a normal distribution with a lower mean and higher variance, reflecting the greater uncertainty regarding who will come to power, as citizens likely have some information about the challenger before the election. However, it should be noted that in this case, the probability of protest in the event of no boycott would be even lower.

4. The payoff from protesting is same as in Egorov and Sonin (2021).

Where $\phi()$ and $\Phi()$ are PDF and CDF of standard normal distribution, respectively.

Thus, the expected net payoff from the regime change for individual i is given by $b_i^{CB} = a_C^{New} - a_D - \delta_i^D = b^{CB} + \delta_i$ where $\delta_i = -\delta_i^D$ and $b^{CB} = a_C^{New} - a_D$. Similar to the case of a protest without a boycott, if individual i joins the protest and it succeeds, she receives an extra benefit - αb_i^{CB}; if the protest fails, she receives γb_i^{CB}. However, protesting is costly, and an individual who protests incurs a disutility of c. If the individual does not protest, her payoff is 0 in both cases. Additionally, an individual with $b_i^{CB} < \frac{c}{\alpha}$ will never protest, while someone with $b_i^{CB} > \frac{c}{\gamma}$ will always protest.

Finally, as in Egorov and Sonin (2021) and Persson and Tabellini (2009), we assume that the probability of the incumbent leaving office after protests, π, is equal to the share of citizens who protest.

To summarize, the game has the following time structure:

1. The abilities of the incumbent (a_D) and the challenger (a_C) are realized.

2. The incumbent and the challenger learn their own abilities.

3. Each citizen observes her economic well-being in the first period (r_i^D), learns the competence of the incumbent, and everyone forms the same expectation about the challenger's ability.

4. Each citizen votes sincerely for her preferred candidate, and the votes are counted.

5. The incumbent decides the level of fraud (x) to commit. If the incumbent's total vote share after fraud is less than $\tilde{\tau}$, the challenger takes office in the second period. In this case, the game ends: the challenger receives a payoff $e^{A+a_C}(1-(\tau+x))$, the incumbent receives $e^{M_D+a_D}(\tau+x) - k(x)$ where $(\tau+x) \leq \tilde{\tau}$, and each citizen receives her second period utility r_i^C.

6. If the incumbent's total vote share (after possible fraud) is at least $\tilde{\tau}$, the challenger decides whether to boycott the outcome. The boycott decision follows a threshold rule: the challenger boycotts if and only if $a_C \geq \hat{a}_C$.

7. Regardless of the challenger's decision, each citizen decides whether to protest. The probability π of the incumbent leaving office is equal to the proportion of citizens who protest. If the protest after a boycott is successful, the challenger comes to power. If the protest after no boycott is successful, a new group (denoted as V) takes power.

8. Each citizen i receives her second period utility r_i^P where $P \in \{D, C, V\}$:

    - If the protest after a boycott succeeds, the challenger receives a maximum payoff of $e^{A+a_C} - f$, while the incumbent incurs a loss of $k(x)$, and the citizen i receives a payoff r_i^C.

- If the protest fails but there was boycott, the challenger incurs a loss of f, and the incumbent receives a maximum payoff of $e^{A+a_D} - k(x)$, while citizen i receives payoff r_i^D.
- If the protest without a boycott succeeds, the challenger receives 0, the incumbent incurs a loss of $k(x)$ and citizen i receives payoff r_i^V.
- If the protest without a boycott fails, the challenger receives $e^{M_C+a_C}(1-(\tau+x))$, and the incumbent receives payoff of $e^{A+a_D}(\tau+x) - k(x)$, while citizen i receives a payoff r_i^D.

We are interested in Perfect-Bayesian equilibrium in pure strategies where citizens vote sincerely and the challenger's boycott decision follows the threshold rule. The analysis of the model is organized as follows. We first discuss the protest stage, followed by the boycott stage and the fraud stage. Finally, we explore several implications of the model, including the differing dynamics in parliamentary versus presidential elections, the impact of fixed versus variable benefits received by the challenger upon losing, and the role of international support.

Protest Stage

In this subsection, we first establish that the expected net gain from the regime change for the citizen i when the challenger party boycotts (b_i^{CB}) is higher than if there were no boycott (b_i^V).

Lemma 1. *The expected net gain from regime change for the citizen i when the challenger party boycotts (b_i^{CB}) is greater than if there were no boycott (b_i^V). Moreover, as the threshold for the challenger's boycott decision ($\hat{a}_C$) increases, the expected net gain from regime change for citizen i when the party boycotts (b_i^{CB}) also increases, i.e., $\frac{\partial b_i^{CB}}{\partial \hat{a}_C} > 0$.*

Specifically, when the boycott threshold is very low ($\hat{a}_C \to -\infty$), the expected net gain from the regime change for the citizen i when the party boycotts converges to the expected gain from the regime change when there is no boycott ($b_i^{CB} \to b_i^V$). In this case, the boycott decision does not provide citizens with any new information about the challenger's ability.

Conversely, when the boycott threshold is very high ($\hat{a}_C \to \infty$), the expected net gain from the regime change for the citizen i when the party boycotts approaches infinity ($b_i^{CB} \to \infty$). In this scenario, the decision to boycott conveys very strong information about the challenger's ability.

Proof. In the Appendix.

□

The intuition behind the Lemma 1 is that the challenger's decision to boycott signals new, positive

information about its competence. Hence, the expected net gain for citizens from regime change is higher if it happened after the boycott rather than without it.

The information conveyed by the decision to boycott also influences the size of the protest. In particular, if the variance of the individual's shock towards a party is sufficiently large (Assumption 1), the challenger's decision to boycott, leads to an increase in the size of the protest (Lemma 2).

Assumption 1. *The variance of the individual's shock towards a party is sufficiently large. In particular,*

$$\sigma_\delta > \frac{1}{2\sqrt{2ln2}} \frac{c(\alpha - \gamma)}{\gamma^2}$$

Lemma 2. *Suppose that citizen i protests if and only if $b_i \in R$ where R (set of protesters) satisfies the following: if $b_i = x > \frac{c}{\gamma}$ then $x \in R$, if $b_i = x < \frac{c}{\alpha}$ then $x \notin R$. The value of b_i is b^V in the case of no boycott and b^{CB} in the case of a boycott. Then, the size of the protest in the case of a boycott, π^B, will be greater than in the case of no boycott, π^{NB}: $\pi^B > \pi^{NB}$.*

Moreover, as the threshold for the boycott decision, $\hat{a}_C$, increases, the size of the protest in the case of a boycott will also increase. Specifically, when the boycott threshold is very low $\hat{a}_C \to -\infty$, the size of the protest in the case of a boycott converges to the size of the protest in the case of no boycott $\pi^B \to \pi^{NB}$.

Conversely, when the threshold for boycott is very high $\hat{a}_C \to \infty$, the size of the protest in the case of a boycott approaches 1, meaning that every individual will protest ($\pi^B \to 1$).

Proof. In the Appendix. □

An interesting implication arises from Lemma 2. A voter who initially supported the incumbent might reconsider and decide to protest after observing the challenger's decision to boycott. This shift occurs because the costly decision to boycott conveys new, positive information about the challenger's competence

Having discussed how the protest size is affected by the decision to boycott, we now show that the set of protesters takes the threshold form. Specifically, Lemma 3 demonstrates that, similar to the findings in Egorov and Sonin (2021), there exists a unique equilibrium protest threshold for boycott and no boycott scenarios. This threshold determines whether an individual will protest: the individual will protest if and only if b_i is no less than the threshold. Furthermore, Lemma 3 provides comparative static results, showing how equilibrium thresholds respond to changes in the underlying parameters.

Lemma 3. *The set of protesters in both the boycott and no-boycott scenarios can be represented as $R^B = [z^B, \infty)$ and $R^{NB} = [z^{NB}, \infty)$, respectively. Specifically, in the case of boycott, an individual i will protest if and only if $b_i^{CB} \geq z^B$. In the case of no boycott, an individual i will protest if and only if $b_i^V \geq z^{NB}$.*

Moreover, the protest threshold in the case of no boycott is higher than in the case of a boycott, i.e., $z^{NB} > z^{B}$. *Additionally, when the boycott threshold is very low* ($\hat{a}_C \to -\infty$), *the protest threshold in the case of a boycott converges to that of the no-boycott case, i.e.,* $z^{B} \to z^{NB}$. *Conversely, when the boycott threshold is very high* ($\hat{a}_C \to \infty$), *the protest threshold in the case of a boycott converges to* $\frac{c}{\alpha}$.

Furthermore, the protest threshold z^B *decreases as the boycott threshold* $\hat{a}_C$ *increases:* $\frac{\partial z^B}{\partial \hat{a}_C} < 0$.

Finally, the comparative statics imply that, for both boycott and no boycott scenarios: $\frac{\partial z}{\partial c} > 0, \frac{\partial z}{\partial \alpha} < 0, \frac{\partial z}{\partial \gamma} < 0, \frac{\partial z}{\partial b} < 0$

Proof. In the Appendix.

□

Boycott Stage

In the Boycott stage, after losing the election – potentially due to fraud committed by the incumbent – the challenger decides whether to boycott the results. The boycott decision follows a threshold rule: the challenger will boycott if and only if $a_C \geq \hat{a}_C$. This implies that the challenger with the ability $a_C = \hat{a}_C$ is indifferent between boycotting and not boycotting.

As a reminder, if the challenger decides to boycott and the subsequent protest is successful, the challenger receives the maximum payoff, $e^{A+a_C} - f$. Conversely, if the protest fails, the challenger incurs a loss of f. In the case of no boycott, if the protest fails, the challenger's payoff is $e^{M_C+a_C}(1-(\tau+x))$, where $1-(\tau+x)$ represents the total vote share the challenger receives after potential fraud by the incumbent party. However, if the protest succeeds after a no-boycott decision, the challenger's payoff is zero.

Moreover, because we assume sincere voting, players can determine the level of fraud used by the incumbent. In particular, as $\tau = \Phi\left(\frac{-b^V}{\sigma_\delta}\right)$ then after observing election result $\tau + x$, players can determine the level of fraud $x = \tau + x - \Phi\left(\frac{-b^V}{\sigma_\delta}\right)$.

We assume that the challenger values winning significantly more than losing. This intuitive assumption is crucial because it ensures that the challenger is sufficiently motivated to take risks, such as boycotting, in pursuit of victory.

Assumption 2. *The exogenous payoff for the challenger (A) from winning the election is significantly higher than the exogenous payoff from losing the election,* M_C. *Specifically:*

$$A - M_C > ln\left(\frac{\Phi\left(\frac{\frac{c}{\gamma}-b^V}{\sigma_\delta}\right)}{1-\Phi\left(\frac{\frac{c}{\gamma}-b^V}{\sigma_\delta}\right)}\right)$$

With this assumption, we show that there is a unique equilibrium threshold for boycotting.

Proposition 1. *There exists a unique equilibrium threshold ($\hat{a}_C$) such that the challenger with ability, $\hat{a}_C$, is indifferent between boycotting and not boycotting the election. Specifically, if the challenger's ability a_C is greater than $\hat{a}_C$, the challenger will choose to boycott after the election. Conversely, if a_C is lower than $\hat{a}_C$, the challenger will not boycott. The equilibrium threshold $\hat{a}_C$ is determined by the solution to the following equation:*

$$\pi^B(\hat{a}_C)e^{A+\hat{a}_C} - f - (1-\pi^{NB})e^{M_C+\hat{a}_C}(1-(\tau+x)) = 0$$

The bounds for $\hat{a}_C$ are given by:

$$\left(ln\left(\frac{f}{e^A-(1-\pi^{NB})e^{M_C}(1-(\tau+x))}\right), ln\left(\frac{f}{\pi^{NB}e^A-(1-\pi^{NB})e^{M_C}(1-(\tau+x))}\right)\right)$$

Proof. In the Appendix. □

Because in the model, the boycott decision conveys strong information about the challenger's type, the challenger party may choose to boycott, even if the incumbent did not commit fraud. Specifically, if $\tau \geq \hat{\tau}$, the bounds for the boycott threshold in the absence of fraud are given by:

$$\left(ln\left(\frac{f}{e^A-(1-\pi^{NB})e^{M_C}(1-\tau)}\right), ln\left(\frac{f}{\pi^{NB}e^A-(1-\pi^{NB})e^{M_C}(1-\tau)}\right)\right)$$

When π^{NB} is at its lowest, the lower bound for the boycott threshold is also smallest. However, we assume that the boycott threshold in the absence of fraud ($\hat{a}_C^{LB}(0)$) is non-negative, where $\hat{a}_C^{LB}(0)$ is defined as:

$$\hat{a}_C^{LB}(0) = ln\left(\frac{f}{e^A-\Phi\left(\frac{\frac{c}{\gamma}-b^V}{\sigma_\delta}\right)e^{M_C}(1-\tau)}\right) \geq 0$$

This is equivalent to the Assumption 3.

Assumption 3. *The cost of boycotting is sufficiently high to ensure that only challengers with positive competence ($\hat{a}_C^{LB}(0) \geq 0$) choose to boycott when there is no fraud. Specifically, we assume the following:*

$$f \geq e^A - \Phi\left(\frac{\frac{c}{\gamma}-b^V}{\sigma_\delta}\right)e^{M_C}(1-\tau)$$

This condition essentially states that, in the absence of fraud, a challenger who chooses to boycott must be of sufficiently high competence.

Comparative Statics: Boycott threshold

As highlighted in Lemma 3, the boycott threshold has important implications, particularly in influencing the size of the protest. Hence, in this subsection, we apply the implicit function theorem to derive the comparative statics for the boycott threshold.

1. The effect of increasing boycott cost (f):

$$\frac{\partial \hat{a}_C}{\partial f} = -\frac{\frac{\partial F}{\partial f}}{\frac{\partial F}{\partial \hat{a}_C}} = \frac{1}{\frac{\partial F}{\partial \hat{a}_C}} > 0$$

   Where $F = \pi^B(\hat{a}_C)e^{A+\hat{a}_C} - f - (1-\pi^{NB})e^{M_C+\hat{a}_C}(1-(\tau+x))$.

   Thus, an increase in the cost of boycotting raises the boycott threshold.

2. The effect of increasing the exogenous payoff from winning (A):

$$\frac{\partial \hat{a}_C}{\partial f} = -\frac{\frac{\partial F}{\partial A}}{\frac{\partial F}{\partial \hat{a}_C}} = -\frac{\pi^B(\hat{a}_C)e^{A+\hat{a}_C}}{\frac{\partial F}{\partial \hat{a}_C}} < 0$$

   Therefore, when the exogenous payoff from winning increases, the boycott threshold decreases.

3. The effect of increasing exogenous payoff from losing (M_C):

$$\frac{\partial \hat{a}_C}{\partial M_C} = -\frac{\frac{\partial F}{\partial M_C}}{\frac{\partial F}{\partial \hat{a}_C}} = \frac{(1-\pi^{NB})e^{M_C+\hat{a}_C}(1-(\tau+x))}{\frac{\partial F}{\partial \hat{a}_C}} > 0$$

   Therefore, an increase in the exogenous payoff from holding seats in Parliament in the event of losing leads to a higher boycott threshold.

4. The effect of increasing fraud (x):

$$\begin{aligned}\frac{\partial \hat{a}_C}{\partial x} &= -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial \hat{a}_C}} \\ &= \frac{(1-\pi^{NB})e^{M_C+\hat{a}_C}}{e^{A+\hat{a}_C}\left(\frac{d\pi^B(\hat{a}_C)}{d\hat{a}_C} + \pi^B(\hat{a}_C)\right) - (1-\pi^{NB})e^{M_C+\hat{a}_C}(1-(\tau+x))} \\ &= \frac{1}{(1-(\tau+x)) - \frac{e^{A-M_C)}}{(1-\pi^{NB})}\left(\frac{d\pi^B(\hat{a}_C)}{d\hat{a}_C} + \pi^B(\hat{a}_C)\right)} < 0\end{aligned}$$

   Thus, as the fraud level increases, the boycott threshold decreases.

Moreover, we derive the lower bound of $\frac{\partial \hat{a}_C}{\partial x}$. We can see that $\frac{\partial \hat{a}_C}{\partial x}$ is an increasing function of $\frac{e^{A-M_C}}{(1-\pi^{NB})}\left(\frac{d\pi^B(\hat{a}_C)}{d\hat{a}_C}+\pi^B(\hat{a}_C)\right)$, which goes to 0 when $\frac{e^{A-M_C}}{(1-\pi^{NB})}\left(\frac{d\pi^B(\hat{a}_C)}{d\hat{a}_C}+\pi^B(\hat{a}_C)\right)$ goes to infinity. Thus, $\frac{\partial \hat{a}_C}{\partial x}$ is lowest when $\frac{e^{A-M_C}}{(1-\pi^{NB})}\left(\frac{d\pi^B(\hat{a}_C)}{d\hat{a}_C}+\pi^B(\hat{a}_C)\right)$ is lowest. But because $\frac{e^{A-M_C)}}{(1-\pi^{NB})}\left(\frac{d\pi^B(\hat{a}_C)}{d\hat{a}_C}+\pi^B(\hat{a}_C)\right) > 1$, then, $\frac{\partial \hat{a}_C}{\partial x} > -\frac{1}{\tau+x}$.

Thus,

$$-\frac{1}{\tau+x} < \frac{\partial \hat{a}_C}{\partial x} < 0$$

5. The effect of a change in the probability of protest success:

   - Change in the probability of protest success in the case of a boycott (π^B):

   $$\frac{\partial \hat{a}_C}{\partial \pi^B(\hat{a}_C)} = -\frac{\frac{\partial F}{\partial \pi^B(\hat{a}_C)}}{\frac{\partial F}{\partial \hat{a}_C}} = -\frac{e^{A+\hat{a}_C}}{\frac{\partial F}{\partial \hat{a}_C}} < 0$$

   This indicates that an increase in the probability of protest success during a boycott results in a decrease in the boycott threshold.

   - Change in the probability of protest success in the case of no boycott (π^{NB}):

   $$\frac{\partial \hat{a}_C}{\partial \pi^{NB}} = -\frac{\frac{\partial F}{\partial \pi^{NB}}}{\frac{\partial F}{\partial \hat{a}_C}} = -\frac{e^{M_C+\hat{a}_C}(1-(\tau+x))}{\frac{\partial F}{\partial \hat{a}_C}} < 0$$

   Thus, similarly, an increase in the probability of protest success when no boycott occurs leads to a decrease in the boycott threshold.

6. The effect of a change in protest parameters (cost of protesting (c), benefits from participation in successful (α) and unsuccessful (γ) protests) and the perception of the challenger (b):

   By Lemma 3 we know:

   $$\frac{\partial z}{\partial c} > 0, \frac{\partial z}{\partial \alpha} < 0, \frac{\partial z}{\partial \gamma} < 0, \frac{\partial z}{\partial b} < 0$$

   Thus, when the cost of protesting (c) increases, both the probabilities of a successful protest in the case of a boycott ($\pi^B(\hat{a}_C)$) and no boycott (π^{NB}) decrease. Conversely, when benefits from participating in successful (α) and unsuccessful (γ) protests increase, both $\pi^B(\hat{a}_C)$ and π^{NB} increase. Consequently, an increase in c raises the boycott threshold, while increase in α and γ leads to a decrease in the boycott threshold.

   Now, consider the case where the general attitude toward the challenger, ($b = a_0 - a_D$), improves. This has two effects. First, as shown in Lemma 3, if everything else remains constant, a more favorable public perception of the challenger before the election results in a larger share of protesters, thereby

lowering the boycott threshold. However, as b increases, the vote share the challenger receives in the event of a loss also rises. This second effect raises the boycott threshold. [5] Thus, the overall effect on the boycott threshold depends on which of these two effects dominates.

Summary of the Comparative Static results discussed above: When parameters that directly influence the challenger's decision to boycott change, the following comparative static results hold:

$$\frac{\partial \hat{a}_C}{\partial f} > 0, \frac{\partial \hat{a}_C}{\partial A} < 0, \frac{\partial \hat{a}_C}{\partial M_C} > 0, \frac{\partial \hat{a}_C}{\partial x} < 0$$

Changes in protest parameters, which affect the decision to boycott through changing the probability of protest success, yield the following results:

$$\frac{\partial \hat{a}_C}{\partial \pi^B(\hat{a}_C)} < 0, \frac{\partial \hat{a}_C}{\partial \pi^{NB}} < 0, \frac{\partial \hat{a}_C}{\partial c} > 0, \frac{\partial \hat{a}_C}{\partial \alpha} < 0, \frac{\partial \hat{a}_C}{\partial \gamma} < 0$$

According to Lemma 2, an increase in the boycott threshold leads to a higher share of protests in the case of a boycott. Thus, we obtain that

$$\frac{\partial \pi^B(\hat{a}_C)}{\partial f} > 0, \frac{\partial \pi^B(\hat{a}_C)}{\partial A} < 0, \frac{\partial \pi^B(\hat{a}_C)}{\partial M_C} > 0, \frac{\partial \pi^B(\hat{a}_C)}{\partial x} < 0$$

Incumbent Decision: Fraud Level

We now turn to the incumbent party's decision. If the incumbent loses the election, the payoff is given by $e^{M_D+a_D}(\tau + x) - k(x)$. Because the incumbent is uncertain about the challenger's ability, they form an expectation over the challenger's type. If the incumbent wins the election, there is a boycott with probability $a_C \geq \hat{a}_C$ and no boycott with probability $a_C < \hat{a}_C$.

The probability of a no-boycott can be expressed as:

$$Pr(a_C < \hat{a}_C) = Pr\left(\frac{a_C - a_0}{\sigma_\alpha} < \frac{\hat{a}_C - a_0}{\sigma_\alpha}\right) = Pr\left(Z < \frac{\hat{a}_C - a_0}{\sigma_\alpha}\right) = \Phi\left(\frac{\hat{a}_C - a_0}{\sigma_\alpha}\right)$$

In the event of a boycott, if the protest is successful, the incumbent receives a payoff of $-k(x)$. If the protest fails, the incumbent receives the maximum payoff of $e^{A+a_D} - k(x)$. Conversely, if there is no boycott and the protest succeeds, the incumbent again faces the cost $-k(x)$, while if the protest fails, the payoff is $e^{A+a_D}(\tau + x) - k(x)$.

5. To see this note under sincere voting, we find that: $\frac{d(1-\tau-x)}{db} = \frac{d}{db}\left(1 - \Phi\left(\frac{-b}{\sigma_\delta}\right) - x\right) = \frac{1}{\sigma_\delta}\phi\left(\frac{-b}{\sigma_\delta}\right) > 0$.

Thus, the expected payoff from winning the election is:

$$\Phi\left(\frac{\hat{a}_C - a_0}{\sigma_\alpha}\right)(-\pi^{NB}k(x) + (1-\pi^{NB})(e^{A+a_D}(\tau+x) - k(x))$$
$$+\left(1-\Phi\left(\frac{\hat{a}_C - a_0}{\sigma_\alpha}\right)\right)(-\pi^{B}(\hat{a}_C)k(x) + (1-\pi^{B}(\hat{a}_C)(e^{A+a_D} - k(x))$$

Simplifying further:

$$e^{A+a_D}\left(\Phi\left(\frac{\hat{a}_C - a_0}{\sigma_\alpha}\right)(\tau+x)(1-\pi^{NB}) + \left(1-\Phi\left(\frac{\hat{a}_C - a_0}{\sigma_\alpha}\right)\right)(1-\pi^{B}(\hat{a}_C))\right) - k(x)$$

To refine the incumbent's decision making, we make the following assumptions.

Assumption 4. *The variance of abilities, σ_a, is sufficiently large:*

$$\sigma_a > \max\left(\{\frac{\phi(1)}{1-\Phi(1)} \approx 1.5, \alpha_0\}\right)$$

Assumption 5. *At $1-\tau$, the marginal cost of fraud is sufficiently high, making it not optimal for the incumbent to win the election with 100% of the vote. Specifically, we assume the following:*

$$k'(1-\tau) > e^{M_C+a_D}\Phi\left(\frac{\frac{c}{\gamma} - b^V}{\sigma_\delta}\right)\left(\Phi\left(\frac{\hat{a}_C^{UB}(1-\tau) - a_0}{\sigma_a}\right)(1+e^{A-M_C}) - 1\right)$$

Where

$$\hat{a}_C^{UB}(1-\tau) = ln(f) - A - ln\left(1-\Phi\left(\frac{\frac{c}{\gamma} - b^V}{\sigma_\delta}\right)\right)$$

Assumption 6. *If the incumbent loses the election without fraud (i.e., $\tau < \tilde{\tau}$), then the inverse marginal cost at the maximum payoff from losing the election is lower than the minimum fraud level required to win:*

$$k'^{-1}(e^{M_D+a_D}) < \tilde{\tau} - \tau^6$$

Under these assumptions, Proposition 2 establishes that the incumbent will commit a positive level of fraud in equilibrium. Moreover, the proposition shows that it is possible for the incumbent to lose the election, yet still engage in fraud.

Proposition 2. *If the incumbent is winning the election without fraud (i.e., $\tau \geq \tilde{\tau}$), there exists a level of fraud that maximizes the incumbent's payoff. Moreover, under assumptions that the variance of abilities (σ_a) is sufficiently large (Assumption 4) and $\hat{a}_C^{LB}(0) \geq 0$ (Assumption 3), the incumbent will engage in a*

6. Note: the inverse of $k'(x)$ exists since $k''(x) > 0$.

positive level of fraud even if winning the election without it.

Furthermore, if at $1-\tau$, the marginal cost of fraud is sufficiently high, so that it is not optimal for the incumbent to achieve a 100% election victory (Assumption 5), the equilibrium fraud level satisfies the following first-order condition:

$$e^{A+a_D}\left(\left(\frac{d\hat{a}_C(x)}{dx}\times\frac{1}{\sigma_a}\phi\left(\frac{\hat{a}_C(x)-a_0}{\sigma_a}\right)\left[(\tau+x)(1-\pi^{NB})-(1-\pi^B(\hat{a}_C(x)))\right]\right)\right.$$
$$\left.+\Phi\left(\frac{\hat{a}_C(x)-a_0}{\sigma_a}\right)(1-\pi^{NB})-\left(1-\Phi\left(\frac{\hat{a}_C(x)-a_0}{\sigma_a}\right)\right)\frac{d\pi^B(\hat{a}_C(x))}{d\hat{a}_C}\frac{d\hat{a}_C(x)}{dx}\right)=k'(x)$$

In the case where the incumbent is losing the election without fraud (i.e., $\tau<\tilde{\tau}$) and under the assumption that $k'^{-1}(e^{M_D+a_D})<\tilde{\tau}-\tau$ (Assumption 6), there will still be a positive level of fraud in equilibrium. Furthermore, the incumbent may choose a level of fraud that leads to losing the election. The optimal fraud level, x^L, in this losing case is given by:

$$0<x^L=k'^{-1}(e^{M_D+a_D})<\tilde{\tau}-\tau$$

The optimal fraud level when the incumbent is losing without fraud but can win with fraud is either equal to the minimum level of fraud required to win the election ($\tilde{\tau}-\tau$), or it is the solution to the first-order condition described above.

Proof. In the Appendix.

□

Sincere Voting

As mentioned previously, we consider sincere voting. The assumption that guarantees sincere voting is that the citizen i receives a very high additional utility from voting sincerely. Then, since δ_i^P is distributed according $N(0,\sigma_\delta^2)$, the vote share for the incumbent party without fraud is:

$$\tau=Pr(\delta_i^P>a_0-a_D)=1-\Phi\left(\frac{a_0-a_D}{\sigma_\delta}\right)=\Phi\left(\frac{a_D-a_0}{\sigma_\delta}\right)=\Phi\left(\frac{-b^V}{\sigma_\delta}\right)$$

Thus, the incumbent party wins the election without fraud if and only if:

$$\tau=\Phi\left(\frac{-b^V}{\sigma_\delta}\right)\geq\tilde{\tau}$$

Discussion

Parliamentary vs Presidential Elections

In this section, we compare parliamentary and presidential elections in non-full democracies, emphasizing their fundamental differences. Our analysis indicates that government turnover through elections is more likely in parliamentary systems than in presidential ones. Furthermore, while presidential elections often result in more frequent boycotts, the scale of protests following these elections is generally smaller compared to those following parliamentary elections.

The key distinctions between presidential and parliamentary elections are captured in Proposition 3.

Proposition 3. *The boycott threshold in a presidential election is significantly lower than in a parliamentary election, leading to more frequent boycotts after presidential elections. However, the size of protests following a parliamentary election tends to be larger than that after presidential elections. Furthermore, following a presidential election the challenger's decision to boycott does not depend on the level of fraud in the election, but rather on the cost of boycotting, the exogenous payoff from winning the election, and the probability of a successful protest in the event of a boycott.*

It is also less likely that the incumbent loses in a presidential election than in a parliamentary election. The optimal level of fraud in presidential elections is either zero or equal to $\tilde{\tau} - \tau$, meaning that when fraud is used, the post-fraud vote share will be close to the required winning threshold. This implies that, conditional on winning, the vote share after fraud is higher in parliamentary elections compared to presidential elections. Additionally, in presidential systems, a government change is more likely to occur through protests rather than through elections.

Proof. In the appendix. □

Fixed Benefits for the Challenger

We next turn to discussing a case, where instead of receiving a benefit proportional to its vote share, the challenger receives a fixed benefit denoted by L_C. The challenger will be indifferent between boycotting and not boycotting when:

$$\pi^B(\hat{a}_C)e^{A+\hat{a}_C} - f = L_C$$

When benefits shift from proportional to fixed, the effect is similar to the change observed when transitioning from parliamentary to presidential elections. However, the shift to fixed benefits increases the cost of boycotting to $f + L_C$, resulting in a higher boycott threshold. As a result, in case of fixed benefits, the

threshold for boycotting will be higher, and a larger share of protesters will participate in the case of a boycott compared to a presidential election.

International Support

We next analyze the scenario when the international community seeks to influence the outcome by offering additional benefits (B) to the challenger in the event of a boycott. These benefits are proportional to the vote share the challenger receives. The challenger will be indifferent between boycotting and not boycotting when the following condition is satisfied:

$$\pi^B(\hat{a}_C)e^{A+\hat{a}_C} + B(1-\tau) - f - (1-\pi^{NB})e^{M_C+\hat{a}_C}(1-(\tau+x)) = 0$$

This scenario effectively reduces the cost of boycotting for the challenger. As a result, the boycott threshold will be lower, meaning that weaker challengers are more likely to boycott. Additionally, the size of the protest in the event of a boycott will be smaller.

If the international community provides excessive aid to the challenger, it could backfire.[7] Specifically, if the benefits are large enough, the boycott threshold could become negative infinity, meaning that every challenger would choose to boycott. Consider the scenario where the benefits are sufficiently large so that:

$$B \geq \frac{1}{(1-\tau)}\left[f + (1-\pi^{NB})e^{M_C+\hat{a}_C}(1-(\tau+x))\right]$$

In this case, the benefit from boycotting will always be positive because $\pi^B(\hat{a}_C)e^{A+\hat{a}_C} > 0$. Therefore, all challengers will boycott, which drives the boycott threshold to negative infinity. The probability of the incumbent being removed from office will then be:

$$\Phi\left(\frac{\hat{a}_C - a_0}{\sigma_a}\right)\pi^{NB} + \left(1 - \Phi\left(\frac{\hat{a}_C - a_0}{\sigma_a}\right)\right)\pi^B(\hat{a}_C) = \pi^{NB}$$

This means that the probability of the incumbent leaving office will revert to the same level as if no aid was provided to the challenger, defeating the purpose of international support. Excessive aid results in boycott losing its strategic value, and hence the following protest might not be large enough to remove the incumbent.

However, when the aid is not excessive, it becomes possible to increase the probability that the incumbent

7. Note that the threshold for excessive aid (i.e., one that encourages all challenger's to boycott) is lower in presidential than in the parliamentary system. In the case of a presidential election, if the international community provides benefits that are greater than or equal to the cost of boycotting, f, the boycott threshold will indeed drop to negative infinity, causing all challengers to boycott.

leaves office. The key lies in balancing the aid such that it incentivizes boycotting, but still ensures that only sufficiently strong challengers choose to boycott. This approach keeps the quality of the challenger high enough to result in a large protest size if a boycott occurs.

Conclusion

In non-democratic regimes, opposition parties face significant constraints in reaching the public (Cunha, Schuler, and Williamson 2022). As a result, a decision to boycott following (a proportional parliamentary) election becomes one of the few tools available for communicating with voters. The costly nature of boycotting allows the challenger to convey new, positive information about its competence. Consequently, a boycott encourages greater public participation in protests as citizens' expected net gain from regime change is higher when it follows a boycott than when it does not.

A major pillar of non-full democracies is electoral manipulation. Our analysis suggests that fraud may be used to inflate the incumbent's support or to help a losing incumbent secure victory. A further interesting result in our model is that, in a parliamentary election, the incumbent might choose a level of fraud that leads to losing the election.

The paper offers a range of empirical implications that could be explored in future research. For instance, the model implies that the government turnovers through elections are more likely in parliamentary systems than in presidential ones. Furthermore, boycotts are more frequent following presidential elections but are accompanied by smaller protests. Finally, too much foreign aid can decrease the chance of the incumbent being removed from office.

Another promising direction for future research would be to expand the model by incorporating multiple challengers. Additionally, integrating both pre and post-election periods could provide deeper insights into the electoral processes in non-full democracies. Finally, it would be valuable to explore the effects of broadening the incumbent's strategy set by allowing them to influence the challenger's costs and benefits associated with boycotting.

Appendix for Politics of Non-Full Democracies: Fraud, Post-Election Boycotts, and Protests

Mariam Malashkhia* Mate Malashkhia†

Lemma 1. *The expected net gain from regime change for the citizen i when the challenger party boycotts (b_i^{CB}) is greater than if there was no boycott (b_i^V). Moreover, as the threshold for the challenger's boycott decision ($\hat{a}_C$) increases, the expected net gain from regime change for citizen i when the party boycotts (b_i^{CB}) also increases, i.e., $\frac{\partial b_i^{CB}}{\partial \hat{a}_C} > 0$.*

Specifically, when the boycott threshold is very low ($\hat{a}_C \to -\infty$), the expected net gain from the regime change for the citizen i when the party boycotts converges to the expected gain from the regime change when there is no boycott ($b_i^{CB} \to b_i^V$). In this case, the boycott decision does not provide citizens with any new information about the challenger's ability.

Conversely, when the boycott threshold is very high ($\hat{a}_C \to \infty$), the expected net gain from the regime change for the citizen i when the party boycotts approaches infinity ($b_i^{CB} \to \infty$). In this scenario, the decision to boycott conveys very strong information about the challenger's ability.

Proof. Because $a_C^{New} > a_0$, it follows that $b^{CB} = a_C^{New} - a_D > b^V = a_0 - a_D$. Therefore, $b_i^{CB} = b^{CB} + \delta_i > b_i^V = b^V + \delta_i$, which implies that the expected net gain from regime change for the citizen i is greater when the party boycotts than when there is no boycott.

Furthermore, since the Mills Ratio is strictly decreasing over the entire real line, the inverse Mills ratio

*PhD candidate of Political Economics at Stanford GSB. Contact information: 736 Escondido Road, Stanford, CA, 94305. 585-958-4403. mariami@stanford.edu. Corresponding Author

†PhD candidate of Economics at Washington University in St. Louis. Contact information: Seigle Hall Room 377, Fraternity Way, St. Louis, MO, 63105. 314-399-4649. m.malashkhia@wust.edu

is strictly increasing everywhere. Thus, as $\hat{a}_C$ increases, a_C^{New} also increases. Therefore,

$$\frac{\partial b_i^{CB}}{\partial \hat{a}_C} = \frac{\partial b^{CB}}{\partial \hat{a}_C} = \frac{\partial a_C^{New}}{\partial \hat{a}_C} = \frac{\partial}{\partial \hat{a}_C} \frac{\phi(\frac{\hat{a}_C - a_0}{\sigma_a})}{1 - \Phi(\frac{\hat{a}_C - a_0}{\sigma_a})} \sigma_a =$$

$$= \frac{\sigma_a \left[1 - \Phi(\frac{\hat{a}_C - a_0}{\sigma_a}) \frac{\partial}{\partial \hat{a}_C} \phi(\frac{\hat{a}_C - a_0}{\sigma_a}) - \phi(\frac{\hat{a}_C - a_0}{\sigma_a}) \big[- \frac{1}{\sigma_a} \phi(\frac{\hat{a}_C - a_0}{\sigma_a}) \big] \right]}{\Big(1 - \Phi(\frac{\hat{a}_C - a_0}{\sigma_a})\Big)^2} =$$

$$= \frac{\Big[(1 - \Phi(\frac{\hat{a}_C - a_0}{\sigma_a}))(-(\frac{\hat{a}_C - a_0}{\sigma_a}))\phi(\frac{\hat{a}_C - a_0}{\sigma_a}) + \phi^2(\frac{\hat{a}_C - a_0}{\sigma_a})\Big]}{\Big(1 - \Phi(\frac{\hat{a}_C - a_0}{\sigma_a})\Big)^2} > 0$$

The equation above is the derivative of the Inverse Mills Ratio which is positive.

Moreover, as the cutoff $\hat{a}_C$ approaches negative infinity (i.e., almost every challenger boycotts), a_C^{New} converges to a_0 (because the inverse Mills ratio approaches 0). Thus, $b^{CB} \to b^V$ and $b_i^{CB} \to b_i^V$.

Conversely, as $\hat{a}_C$ approaches positive infinity (i.e., almost no challenger boycotts), a_C^{New} also approaches infinity (because the inverse Mills ratio approaches infinity). Thus, $b^{CB} \to \infty$ and $b_i^{CB} \to \infty$.

□

Lemma 2. *Suppose that citizen i protests if and only if $b_i \in R$ where R (set of protesters) satisfies the following: if $b_i = x > \frac{c}{\gamma}$ then $x \in R$, if $b_i = x < \frac{c}{\alpha}$ then $x \notin R$. The value of b_i is b^V in the case of no boycott and b^{CB} in the case of a boycott. Then, the size of the protest in the case of a boycott, π^B, will be greater than in the case of no boycott, π^{NB}: $\pi^B > \pi^{NB}$.*

Moreover, as the threshold for the boycott decision, $\hat{a}_C$, increases, the size of the protest in the case of a boycott will also increase. Specifically, when the boycott threshold is very low $\hat{a}_C \to -\infty$, the size of the protest in the case of a boycott converges to the size of the protest in the case of no boycott $\pi^B \to \pi^{NB}$.

Conversely, when the threshold for boycott is very high $\hat{a}_C \to \infty$, the size of the protest in the case of a boycott approaches 1, meaning that every individual will protest ($\pi^B \to 1$).

Proof. First, note that the probability that $b_i = b + \delta_j$ is equal to x is given by:

$$Pr(b + \delta_j = x) = \frac{d}{dx}\Phi\left(\frac{x-b}{\sigma_\delta}\right) = \frac{1}{\sigma_\delta}\phi\left(\frac{x-b}{\sigma_\delta}\right)$$

The size of the protest in the case of no boycott (π^{NB}) and in the case of a boycott (π^B) can be expressed as:

$$\pi^{NB} = Pr(b^V + \delta_j \in R) = \int_{x\in R} \frac{1}{\sigma_\delta}\phi\left(\frac{x-b^V}{\sigma_\delta}\right)dx = 1 - \Phi\left(\frac{\frac{c}{\gamma} - b^V}{\sigma_\delta}\right) + \frac{1}{\sigma_\delta}\int_{x\in R, \frac{c}{\alpha}<x<\frac{c}{\gamma}} \phi\left(\frac{x-b^V}{\sigma_\delta}\right)dx$$

$$\pi^{B} = Pr(b^{CB} + \delta_j \in R) = \int_{x\in R} \frac{1}{\sigma_\delta}\phi\left(\frac{x-b^{CB}}{\sigma_\delta}\right)dx = 1 - \Phi\left(\frac{\frac{c}{\gamma} - b^{CB}}{\sigma_\delta}\right) + \frac{1}{\sigma_\delta}\int_{x\in R, \frac{c}{\alpha}<x<\frac{c}{\gamma}} \phi\left(\frac{x-b^{CB}}{\sigma_\delta}\right)dx$$

By applying Lemma A2 from Egorov and Sonin (2021), which shows that the probability of protest success increases in b (when b is fixed and known), and noting that $b^{CB} > b^V$ (by Lemma 1), we conclude that $\pi^B > \pi^{NB}$.

Furthermore, as the threshold for the boycott decision ($\hat{a}_C$) increases, the only term that changes is b^{CB}, which also increases. Since all other terms remain unchanged, the size of the protest in the case of a boycott (π^B) will increase as $\hat{a}_C$ increases.

Now, consider the limiting cases:

1. $\hat{a}_C \to -\infty$. As shown in Lemma 1, when $\hat{a}_C \to -\infty$, $b^{CB} \to b^V$, leading to $\pi^B \to \pi^{NB}$. This means that as the boycott threshold becomes very low, the size of the protest in the case of a boycott converges

to the size of the protest in the case of no boycott.

2. $\hat{a}_C \to \infty$. As shown in Lemma 1, when $\hat{a}_C \to \infty$, $b^{CB} \to \infty$. Consequently, $\pi^B \to 1$. This implies that as the boycott threshold becomes very high, the size of the protest in the case of a boycott approaches 1, meaning that every individual will protest.

□

Lemma 3. *The set of protesters in both the boycott and no-boycott scenarios can be represented as* $R^B = [z^B, \infty)$ *and* $R^{NB} = [z^{NB}, \infty)$*, respectively. Specifically, in the case of boycott, an individual* i *will protest if and only if* $b_i^{CB} \geq z^B$*. In the case of no boycott, an individual* i *will protest if and only if* $b_i^V \geq z^{NB}$*.*

Moreover, the protest threshold in the case of no boycott is higher than in the case of a boycott, i.e., $z^{NB} > z^B$*. Additionally, when the boycott threshold is very low (*$\hat{a}_C \to -\infty$*), the protest threshold in the case of a boycott converges to that of the no-boycott case, i.e.,* $z^B \to z^{NB}$*. Conversely, when the boycott threshold is very high (*$\hat{a}_C \to \infty$*) the protest threshold in the case of a boycott converges to* $\frac{c}{\alpha}$*.*

Furthermore, the protest threshold z^B *decreases as the boycott threshold* $\hat{a}_C$ *increases:* $\frac{\partial z^B}{\partial \hat{a}_C} < 0$*.*

Finally, the comparative statics imply that, for both boycott and no boycott scenarios: $\frac{\partial z}{\partial c} > 0, \frac{\partial z}{\partial \alpha} < 0, \frac{\partial z}{\partial \gamma} < 0, \frac{\partial z}{\partial b} < 0$

Proof. First, note that all individuals use the same probabilities to calculate the expected payoff of protesting, denoted as π^B in the case of a boycott and π^{NB} in the case of no boycott. For a given π^B, the expected payoff from protesting in the case of a boycott for individual i is given by:

$$\pi^B \alpha b_i^{CB} + (1 - \pi^B)\gamma b_i^{CB} - c + \pi^B a_C^{New} + (1 - \pi^B)(a_D + \delta_i^D)$$

The payoff from not protesting is:

$$\pi^B a_C^{New} + (1 - \pi^B)(a_D + \delta_i^D)$$

Thus, the decision to protest in the case of a boycott reduces to

$$\pi^B \alpha b_i^{CB} + (1 - \pi^B)\gamma b_i^{CB} - c \geq 0$$

This expression is increasing in b_i^{CB}, implying that the cutoff z^B must satisfy:

$$z^B = \frac{c}{(\alpha - \gamma)\pi^B + \gamma}$$

Similarly, in the case of no boycott, the decision to protest is:

$$\pi^{NB} \alpha b_i^V + (1 - \pi^{NB})\gamma b_i^V - c \geq 0$$

This expression is also increasing in b_i^V, implying that the cutoff z^{NB} must satisfy:

$$z^{NB} = \frac{c}{(\alpha - \gamma)\pi^{NB} + \gamma}$$

Thus, the set of protesters is $[z^B, \infty)$ in the case of a boycott and $[z^{NB}, \infty)$ in the case of no boycott.

Next, since $\pi^B > \pi^{NB}$ by Lemma 2, it follows that $z^B < z^{NB}$. Therefore, the protest threshold in the case of no boycott is higher than in the case of a boycott. Moreover, according to Lemma 2 when $\hat{a}_C \to -\infty$, $\pi^B \to \pi^{NB}$, and thus $z^B \to z^{NB}$. When $\hat{a}_C \to \infty$, $\pi^B \to 1$, and thus $z^B \to \frac{c}{\gamma}$.

The existence of such thresholds follows from Proposition 1 in Egorov and Sonin (2021). Specifically, by defining the function $Q^B(\tilde{z})$ such that it equals zero at z^B, we have:

$$Q^B(\tilde{z}^B) = \tilde{z}^B - \frac{c}{(\alpha-\gamma)\pi^B + \gamma} = \tilde{z}^B - \frac{c}{(\alpha-\gamma)\left(1 - \Phi\left(\frac{\tilde{z}^B - b^{CB}}{\sigma_\delta}\right)\right) + \gamma}$$

Similarly, defining $Q^{NB}(\tilde{z}^{NB})$ such that it equals zero at z^{NB}, we have:

$$Q^{NB}(\tilde{z}^{NB}) = \tilde{z}^{NB} - \frac{c}{(\alpha-\gamma)\pi^{NB} + \gamma} = \tilde{z}^{NB} - \frac{c}{(\alpha-\gamma)\left(1 - \Phi\left(\frac{\tilde{z}^{NB} - b^{V}}{\sigma_\delta}\right)\right) + \gamma}$$

In both cases, the limit as $\tilde{z}$ approaches negative infinity is negative infinity, and as $\tilde{z}$ approaches positive infinity, it is positive infinity. As shown iEgorov and Sonin (2021), under the assumption that the variance of the individual shock σ_δ is sufficiently large, the derivative is positive, making the function increasing everywhere:

$$\frac{dQ(\tilde{z})}{d\tilde{z}} = 1 - \frac{\left(c(\alpha-\gamma)\phi\left(\frac{\tilde{z}^{NB} - b^{V}}{\sigma_\delta}\right)\right)}{\sigma_\delta\left((\alpha-\gamma)(1 - \Phi\left(\frac{\tilde{z}^{NB} - b^{V}}{\sigma_\delta}\right)) + \gamma\right)^2} > 0$$

Thus, there is a unique solution to the equation. Furthermore, note the following:

$$\frac{dQ^B}{d\hat{a}_C} = \frac{dQ^B}{d\hat{a}_C}\left(\tilde{z}^B - \frac{c}{(\alpha-\gamma)\left(1 - \Phi\left(\frac{\tilde{z}^B - b^{CB}}{\sigma_\delta}\right)\right) + \gamma}\right) =$$

$$\frac{c(\alpha-\gamma)}{\left((\alpha-\gamma)\left(1 - \Phi\left(\frac{\tilde{z}^B - b^{CB}}{\sigma_\delta}\right)\right) + \gamma\right)^2}\frac{\partial}{\partial\hat{a}_C}\left(-\Phi\left(\frac{\tilde{z}^B - b^{CB}}{\sigma_\delta}\right)\right) =$$

$$\frac{c(\alpha-\gamma)}{\sigma_\delta\left((\alpha-\gamma)\left(1 - \Phi\left(\frac{\tilde{z}^B - b^{CB}}{\sigma_\delta}\right)\right) + \gamma\right)^2}\phi\left(\frac{\tilde{z}^B - b^{CB}}{\sigma_\delta}\right)\frac{\partial b^{CB}}{\partial\hat{a}_C} > 0$$

The last term is positive by Lemma 1. Since, $\frac{dQ(z)}{dz} > 0$, the Implicit Function Theorem implies:

$$\frac{\partial\tilde{z}^B}{\partial\hat{a}_C} = -\frac{\frac{\partial Q^B}{\partial\hat{a}_C}}{\frac{\partial Q^B}{\partial\tilde{z}^B}} < 0$$

Thus, when the boycott threshold increases, the protest threshold decreases. For other comparative statics,

note that:

$$\frac{\partial Q}{\partial c} < 0, \frac{\partial Q}{\partial \alpha} > 0, \frac{\partial Q}{\partial \gamma} > 0, \frac{\partial Q}{\partial b} > 0$$

Therefore as derived in Egorov and Sonin (2021),

$$\frac{\partial z}{\partial c} > 0, \frac{\partial z}{\partial \alpha} < 0, \frac{\partial z}{\partial \gamma} < 0, \frac{\partial z}{\partial b} < 0$$

□

Proposition 1. *There exists a unique equilibrium threshold ($\hat{a}_C$) such that the challenger with ability, $\hat{a}_C$, is indifferent between boycotting and not boycotting the election. Specifically, if the challenger's ability a_C is greater than $\hat{a}_C$, the challenger will choose to boycott after the election. Conversely, if a_C is lower than $\hat{a}_C$, the challenger will not boycott. The equilibrium threshold $\hat{a}_C$ is determined by the solution to the following equation:*

$$\pi^B(\hat{a}_C)e^{A+\hat{a}_C} - f - (1-\pi^{NB})e^{M_C+\hat{a}_C}(1-(\tau+x)) = 0$$

The bounds for $\hat{a}_C$ are given by:

$$\left(ln\left(\frac{f}{e^A-(1-\pi^{NB})e^{M_C}(1-(\tau+x))}\right), ln\left(\frac{f}{\pi^{NB}e^A-(1-\pi^{NB})e^{M_C}(1-(\tau+x))}\right)\right)$$

Proof. The indifference condition implies that:

$$\pi^B(\hat{a}_C)e^{A+\hat{a}_C} - f - (1-\pi^{NB})e^{M_C+\hat{a}_C}(1-(\tau+x)) = 0$$

Let us define the function $J(\tilde{a}_C)$ such that it equals zero at $\hat{a}_C$:

$$J(\tilde{a}_C) = \pi^B(\tilde{a}_C)e^{A+\tilde{a}_C} - f - (1-\pi^{NB})e^{M_C+\tilde{a}_C}(1-(\tau+x))$$

The limits of $J(\tilde{a}_C)$ are the following:

$$\lim_{\hat{a}_C\to-\infty} J(\tilde{a}_C) = \lim_{\hat{a}_C\to-\infty} \pi^B(\tilde{a}_C)e^{A+\tilde{a}_C} - f - (1-\pi^{NB})e^{M_C+\tilde{a}_C}(1-(\tau+x)) = -f$$

And:

$$\begin{aligned}\lim_{\hat{a}_C\to\infty} J(\tilde{a}_C) = \lim_{\hat{a}_C\to\infty} \pi^B(\tilde{a}_C)e^{A+\tilde{a}_C} - f - (1-\pi^{NB})e^{M_C+\tilde{a}_C}(1-(\tau+x)) = \\ \lim_{\hat{a}_C\to\infty} e^{A+\tilde{a}_C} - f - (1-\pi^{NB})e^{M_C+\tilde{a}_C}(1-(\tau+x)) \geq \\ \lim_{\tilde{a}_C\to\infty} e^{M_C+\tilde{a}_C}(1-(1-\pi^{NB})(1-(\tau+x)) - f = \infty\end{aligned}$$

This holds because $A > M_C$ and the term in brackets is necessarily positive.

Next, we will show that $J(\tilde{a}_C)$ is an increasing function of $\tilde{a}_C$.

$$\frac{dJ(\tilde{a}_C)}{d\tilde{a}_C} = \frac{d}{d\tilde{a}_C}\pi^B(\tilde{a}_C)e^{A+\tilde{a}_C} - f - (1-\pi^{NB})e^{M_C+\tilde{a}_C}(1-(\tau+x)) =$$
$$e^{A+\tilde{a}_C}\left(\frac{d\pi^B(\tilde{a}_C)}{d\tilde{a}_C} + \pi^B(\tilde{a}_C)\right) - (1-\pi^{NB})e^{M_C+\tilde{a}_C}(1-(\tau+x)) > 0$$
$$\Leftrightarrow \frac{e^{A+\tilde{a}_C}}{e^{M_C+\tilde{a}_C}} > \frac{(1-\pi^{NB})(1-(\tau+x))}{\frac{d\pi^B(\tilde{a}_C)}{d\tilde{a}_C} + \pi^B(\tilde{a}_C)}$$
$$\Leftrightarrow e^{A-M_C} > \frac{(1-\pi^{NB})(1-(\tau+x))}{\frac{d\pi^B(\tilde{a}_C)}{d\tilde{a}_C} + \pi^B(\tilde{a}_C)}$$
$$\Leftrightarrow A - M_C > ln\left(\frac{(1-\pi^{NB})(1-(\tau+x))}{\frac{d\pi^B(\tilde{a}_C)}{d\tilde{a}_C} + \pi^B(\tilde{a}_C)}\right)$$

Given that $(1-(\tau+x)) < 1$, and by Lemma 2, $\frac{d\pi^B(\tilde{a}_C)}{d\tilde{a}_C} > 0$ and $\pi^B(\tilde{a}_C) > \pi^{NB}$ it follows that

$$ln\left(\frac{(1-\pi^{NB})(1-(\tau+x))}{\frac{d\pi^B(\tilde{a}_C)}{d\tilde{a}_C} + \pi^B(\tilde{a}_C)}\right) < ln\left(\frac{(1-\pi^{NB})}{\frac{d\pi^B(\tilde{a}_C)}{d\tilde{a}_C} + \pi^B(\tilde{a}_C)}\right) < ln\left(\frac{1-\pi^{NB}}{\pi^B(\tilde{a}_C)}\right) < ln\left(\frac{1-\pi^{NB}}{\pi^{NB}}\right)$$

Because $\frac{1-\pi^{NB}}{\pi^{NB}}$ decreases as π^{NB} increases within the interval $[0,1]$, it reaches its maximum value when π^{NB} is at its minimum. Given that there are always some citizens who protest, this sets a lower bound for π^{NB} at $1-\Phi\left(\frac{\frac{c}{\gamma}-b^V}{\sigma_\delta}\right)$. Consequently,

$$ln\left(\frac{1-\pi^{NB}}{\pi^{NB}}\right) < ln\left(\frac{\Phi\left(\frac{\frac{c}{\gamma}-b^V}{\sigma_\delta}\right)}{1-\Phi\left(\frac{\frac{c}{\gamma}-b^V}{\sigma_\delta}\right)}\right)$$

Therefore,

$$ln\left(\frac{(1-\pi^{NB})(1-(\tau+x))}{\frac{d\pi^B(\tilde{a}_C)}{d\tilde{a}_C} + \pi^B(\tilde{a}_C)}\right) < ln\left(\frac{\Phi\left(\frac{\frac{c}{\gamma}-b^V}{\sigma_\delta}\right)}{1-\Phi\left(\frac{\frac{c}{\gamma}-b^V}{\sigma_\delta}\right)}\right)$$

Given the assumption that $A - M_C > ln\left(\frac{\Phi\left(\frac{\frac{c}{\gamma}-b^V}{\sigma_\delta}\right)}{1-\Phi\left(\frac{\frac{c}{\gamma}-b^V}{\sigma_\delta}\right)}\right)$ (Assumption 2), it follows that $A - M_C > ln\left(\frac{(1-\pi^{NB})(1-(\tau+x))}{\frac{d\pi^B(\tilde{a}_C)}{d\tilde{a}_C}+\pi^B(\tilde{a}_C)}\right)$. Consequently, this inequality confirms that the function $J(\tilde{a}_C)$ increases in $\tilde{a}_C$.

Thus, because $\lim_{\hat{a}_C\to-\infty} J(\tilde{a}_C) = -f$ and $\lim_{\hat{a}_C\to\infty} J(\tilde{a}_C) = \infty$ and because the function is increasing, there exists a unique threshold $\hat{a}_C$ that solves the equation:

$$\hat{a}_C = ln\left(\frac{f}{\pi^B(\tilde{a}_C)e^A - (1-\pi^{NB})e^{M_C}(1-(\tau+x))}\right)$$

Furthermore, the value of $\hat{a}_C$ is highest when $\pi^B(\hat{a}_C)$ is at its lowest (at π^{NB}) and is lowest when $\pi^B(\hat{a}_C)$ equals 1. Therefore, the bounds for $\hat{a}_C$ are:

$$\left(ln\left(\frac{f}{e^A-(1-\pi^{NB})e^{M_C}(1-(\tau+x))}\right), ln\left(\frac{f}{\pi^{NB}e^A-(1-\pi^{NB})e^{M_C}(1-(\tau+x))}\right)\right)$$

Next, consider a challenger whose ability (a_C) is lower than the threshold $\hat{a}_C$ (i.e., $a_C < \hat{a}_C$). In equilibrium, such a challenger should not boycott. We will next demonstrate that such a challenger has no profitable deviation.

The expected payoff for the challenger, if she choose to boycott, is $\pi^B(\hat{a}_C)e^{A+\hat{a}_C} - f$. Conversely, if she chooses not to boycott, her payoff is $(1-\pi^{NB})e^{M_C+\hat{a}_C}(1-(\tau+x))$. If the challenger deviates by boycotting when her ability is below the threshold, citizens will believe that the challenger's ability exceeds $\hat{a}_C$. Thus, the share of protesters will be $\pi_B(\hat{a}_C)$.

For there to be no profitable deviation, the following inequality must hold:

$$\pi^B(\hat{a}_C)e^{A+a_C} - f < (1-\pi^{NB})e^{M_C+a_C}(1-(\tau+x)) \leftrightarrow$$
$$\pi^B(\hat{a}_C)e^{A+a_C} - f - (1-\pi^{NB})e^{M_C+a_C}(1-(\tau+x)) < 0$$

This condition is always true because the function is strictly increasing in a_C and equals zero when $a_C = \hat{a}_C$. To see this note that:

$$\frac{d}{da_C}\pi^B(\hat{a}_C)e^{A+a_C} - f - (1-\pi^{NB})e^{M_C+a_C}(1-(\tau+x))$$
$$\Leftrightarrow \pi^B(\hat{a}_C)e^{A+a_C} - (1-\pi^{NB})e^{M_C+a_C}(1-(\tau+x)) > 0$$
$$\Leftrightarrow A - M_C > ln\left(\frac{(1-\pi^{NB})(1-(\tau+x)}{\pi^B(\hat{a}_C)}\right)$$

Since this condition is true by assumption, it follows that if $a_C < \hat{a}_C$, the function is negative, confirming that there is no incentive to deviate from the non-boycotting strategy.

By the same reasoning, a challenger whose ability exceeds the threshold is better off boycotting than not boycotting. □

Proposition 2. *If the incumbent is winning the election without fraud (i.e., $\tau \geq \tilde{\tau}$), there exists a level of fraud that maximizes the incumbent's payoff. Moreover, under assumptions that the variance of abilities (σ_a) is sufficiently large (Assumption 4) and $\hat{a}_C^{LB}(0) \geq 0$ (Assumption 3), the incumbent will engage in a positive level of fraud even if winning the election without it.*

Furthermore, if at $1-\tau$, the marginal cost of fraud is sufficiently high, so that it is not optimal for the incumbent to achieve a 100% election victory (Assumption 5), the equilibrium fraud level satisfies the following first-order condition:

$$e^{A+a_D}\left(\left(\frac{d\hat{a}_C(x)}{dx} \times \frac{1}{\sigma_a}\phi\left(\frac{\hat{a}_C(x)-a_0}{\sigma_a}\right)\left[(\tau+x)(1-\pi^{NB}) - (1-\pi^B(\hat{a}_C(x)))\right]\right)\right.$$
$$\left.+\Phi\left(\frac{\hat{a}_C(x)-a_0}{\sigma_a}\right)(1-\pi^{NB}) - \left(1-\Phi\left(\frac{\hat{a}_C(x)-a_0}{\sigma_a}\right)\right)\frac{d\pi^B(\hat{a}_C(x))}{d\hat{a}_C}\frac{d\hat{a}_C(x)}{dx}\right) = k'(x)$$

In the case where the incumbent is losing the election without fraud (i.e., $\tau < \tilde{\tau}$) and under the assumption that $k'^{-1}(e^{M_D+a_D}) < \tilde{\tau}-\tau$ (Assumption 6), there will still be a positive level of fraud in equilibrium. Furthermore, the incumbent may choose a level of fraud that leads to losing the election. The optimal fraud level, x^L, in this losing case is given by:

$$0 < x^L = k'^{-1}(e^{M_D+a_D}) < \tilde{\tau}-\tau$$

The optimal fraud level when the incumbent is losing without fraud but can win with fraud is either equal to the minimum level of fraud required to win the election ($\tilde{\tau}-\tau$), or it is the solution to the first-order condition described above.

Proof. We begin by considering the case where the incumbent party is winning the election even without fraud.

The level of fraud chosen by the incumbent must satisfy two conditions: (1) the total vote share obtained by the incumbent must be less than or equal to 1, and (2) the fraud level must be nonnegative. Therefore, the level of fraud to choose lies in the interval $[0, 1-\tau]$. We define the expected payoff as a function of fraud $H(x) : [0, 1-\tau] \to \mathbb{R}$ as follows

$$H(x) = e^{A+a_D}\left(\Phi\left(\frac{\hat{a}_C(x)-a_0}{\sigma_a}\right)(\tau+x)(1-\pi^{NB}) + \left(1-\Phi\left(\frac{\hat{a}_C(x)-a_0}{\sigma_a}\right)\right)(1-\pi^B(\hat{a}_C(x)))\right) - k(x)$$

Since $H(x)$ is continuous over the closed and bounded interval $[0, 1-\tau]$, by the extreme value theorem, $H(x)$ attains both its maximum and minimum values on this interval. This guarantees that there exists a level of fraud that maximizes the incumbent's payoff.

To find critical points, we differentiate $H(x)$ with respect to x, noting that $\hat{a}_C(x)$ is also a function of x:

$$\frac{d}{dx}\left(e^{A+a_D}\left(\Phi\left(\frac{\hat{a}_C(x)-a_0}{\sigma_a}\right)(\tau+x)(1-\pi^{NB})+\left(1-\Phi\left(\frac{\hat{a}_C(x)-a_0}{\sigma_a}\right)\right)(1-\pi^B(\hat{a}_C(x)))\right)-k(x)\right)=0$$

$$\Leftrightarrow e^{A+a_D}\left(\left(\frac{d\hat{a}_C(x)}{dx}\times\frac{1}{\sigma_a}\phi\left(\frac{\hat{a}_C(x)-a_0}{\sigma_a}\right)[(\tau+x)(1-\pi^{NB})-(1-\pi^B(\hat{a}_C(x)))]\right)+\Phi\left(\frac{\hat{a}_C(x)-a_0}{\sigma_a}\right)(1-\pi^{NB})\right.$$

$$\left.-\left(1-\Phi\left(\frac{\hat{a}_C(x)-a_0}{\sigma_a}\right)\right)\frac{d\pi^B(\hat{a}_c(x))}{d\hat{a}_c}\frac{d\hat{a}_c(x)}{dx}\right)=k'(x)$$

Because $\frac{d\hat{a}_C}{dx}<0$ and $\frac{d\pi^B(x)}{d\hat{a}_C}>0$, the second and third terms are always positive. The only negative term is

$$\frac{d\hat{a}_C(x)}{dx}\frac{1}{\sigma_a}\phi\left(\frac{\hat{a}_C(x)-a_0}{\sigma_a}\right)(\tau+x)(1-\pi^{NB})$$

Let us now consider the derivative when $x=0$ then $k'(x)=0$. Given that all the other terms are positive and $\frac{\partial\hat{a}_C}{\partial x}>-\frac{1}{(\tau+x)}$, a sufficient condition for the derivative to be positive at $x=0$ is

$$e^{A+a_D}\left(-\frac{1}{\sigma_a}\phi\left(\frac{\hat{a}_C(0)-a_0}{\sigma_a}\right)\left[(1-\pi^{NB})-\frac{1-\pi^B(\hat{a}_C(0))}{\tau}\right]+\Phi\left(\frac{\hat{a}_C(0)-a_0}{\sigma_a}\right)(1-\pi^{NB})\right)>0$$

Equivalently,

$$\Phi\left(\frac{\hat{a}_C(0)-a_0}{\sigma_a}\right)(1-\pi^{NB})>\frac{1}{\sigma_a}\phi\left(\frac{\hat{a}_C(0)-a_0}{\sigma_a}\right)\left[(1-\pi^{NB})-\frac{1-\pi^B(\hat{a}_C(0))}{\tau}\right]$$

Which implies

$$\begin{aligned}\sigma_a &> \frac{\phi\left(\frac{\hat{a}_C(0)-a_0}{\sigma_a}\right)\left[(1-\pi^{NB})-\frac{1-\pi^B(\hat{a}_C(0))}{\tau}\right]}{\Phi\left(\frac{\hat{a}_C(0)-a_0}{\sigma_a}\right)(1-\pi^{NB})}\\ &=\frac{\phi\left(\frac{\hat{a}_C(0)-a_0}{\sigma_a}\right)}{\Phi\left(\frac{\hat{a}_C(0)-a_0}{\sigma_a}\right)}-\frac{\phi\left(\frac{\hat{a}_C(0)-a_0}{\sigma_a}\right)\frac{1-\pi^B(\hat{a}_C(0))}{\tau}}{\Phi\left(\frac{\hat{a}_C(0)-a_0}{\sigma_a}\right)(1-\pi^{NB})}\\ &=\frac{\phi\left(\frac{a_0-\hat{a}_C(0)}{\sigma_a}\right)}{1-\Phi\left(\frac{a_0-\hat{a}_C(0)}{\sigma_a}\right)}-\frac{\phi\left(\frac{a_0-\hat{a}_C(0)}{\sigma_a}\right)(1-\pi^B(\hat{a}_C(0)))}{1-\Phi\left(\frac{a_0-\hat{a}_C(0)}{\sigma_a}\right)(1-\pi^{NB})\tau}\\ &=\frac{\phi\left(\frac{a_0-\hat{a}_C(0)}{\sigma_a}\right)}{1-\Phi\left(\frac{a_0-\hat{a}_C(0)}{\sigma_a}\right)}\left(1-\frac{1-\pi^B(\hat{a}_C(0))}{(1-\pi^{NB})\tau}\right)\end{aligned}$$

First, consider the case where $\hat{a}_C(0)-a_0>0$. As σ_a increases, $\frac{a_0-\hat{a}_C(0)}{\sigma_a}$ increases (becomes less negative). Because the inverse Mills ratio is an increasing function in its argument, $\frac{\phi\left(\frac{a_0-\hat{a}_C(0)}{\sigma_a}\right)}{1-\Phi\left(\frac{a_0-\hat{a}_C(0)}{\sigma_a}\right)}$ also increases. This

ratio reaches its maximum when σ_a approaches infinity, where it converges to $\frac{\phi(0)}{1-\Phi(0)} = \frac{2}{\sqrt{2\pi}}$. Moreover, the right-hand side of the inequality will be largest when $\frac{1-\pi^B(\hat{a}_C(0)))}{(1-\pi^{NB})\tau}$ is smallest. Consequently, for any $\sigma_a > 0$, the following holds:

$$\frac{\phi\left(\frac{a_0-\hat{a}_C(0)}{\sigma_a}\right)}{1-\phi\left(\frac{a_0-\hat{a}_C(0)}{\sigma_a}\right)}\left(1-\frac{1-\pi^B(\hat{a}_C(0)))}{(1-\pi^{NB})\tau}\right) < \frac{\phi\left(\frac{a_0-\hat{a}_C(0)}{\sigma_a}\right)}{1-\Phi\left(\frac{a_0-\hat{a}_C(0)}{\sigma_a}\right)} < \frac{\phi(0)}{1-\Phi(0)} = \frac{2}{\sqrt{2\pi}} \approx 0.8$$

Further, if $\hat{a}_C(1-\tau) - a_0 = 0$, we have $\frac{\phi\left(\frac{a_0-\hat{a}_C(0)}{\sigma_a}\right)}{1-\phi\left(\frac{a_0-\hat{a}_C(0)}{\sigma_a}\right)} = \frac{\phi(0)}{1-\Phi(0)} = \frac{2}{\sqrt{2\pi}}$. Thus, if $\hat{a}_C(0) - a_0 \geq 0$, the assumption on σ_a implies that

$$\sigma_a > \frac{\phi(1)}{1-\Phi(1)} > \frac{\phi(0)}{1-\Phi(0)} > \frac{\phi\left(\frac{a_0-\hat{a}_C(0)}{\sigma_a}\right)}{1-\Phi\left(\frac{a_0-\hat{a}_C(0)}{\sigma_a}\right)}\left(1-\frac{1-\pi^B(\hat{a}_C(0)))}{(1-\pi^{NB})\tau}\right)$$

Thus, the derivative of the payoff function at a 0 fraud level is positive, which implies that the payoff of the incumbent is increasing in fraud.

Now, if $a_0 - \hat{a}_C(0) > 0$, the inverse Mills ratio decreases as σ_a increases. Thus, if $\sigma_a > a_0 - \hat{a}_C(0)$ we obtain that

$$\frac{\phi\left(\frac{a_0-\hat{a}_C(0)}{\sigma_a}\right)}{1-\Phi\left(\frac{a_0-\hat{a}_C(0)}{\sigma_a}\right)} < \frac{\phi\left(\frac{a_0-\hat{a}_C(0)}{a_0-\hat{a}_C(0)}\right)}{1-\Phi\left(\frac{a_0-\hat{a}_C(0)}{a_0-\hat{a}_C(0)}\right)} = \frac{\phi(1)}{1-\Phi(1)}$$

Therefore, since $\sigma_a > \frac{\phi(1)}{1-\Phi(1)}$ then the condition is satisfied.

However, note that if $a_0 - \hat{a}_C(0) > 0$ becomes very large (i.e. when $\hat{a}_C(0)$ is very small) then the condition might not hold. In particular, as $a_0 - \hat{a}_C(0)$ approaches infinity, $1 - \pi^B(\hat{a}_C(0))$ tends to 0 and we obtain

$$\frac{\phi\left(\frac{a_0-\hat{a}_C(0)}{\sigma_a}\right)}{1-\Phi\left(\frac{a_0-\hat{a}_C(0)}{\sigma_a}\right)}\left(1-\frac{1-\pi^B(\hat{a}_C(0))}{(1-\pi^{NB})\tau}\right) = \frac{\phi\left(\frac{a_0-\hat{a}_C(0)}{\sigma_a}\right)}{1-\Phi\left(\frac{a_0-\hat{a}_C(0)}{\sigma_a}\right)} \to \infty$$

Thus, the assumption that $\hat{a}_C^{LB}(0)$ is positive (Assumption 3) prevents this case from occurring. Given that $\hat{a}_C^{LB}(0)$ is positive, we have $a_0 - \hat{a}_C^{LB}(0) \leq a_0$. Then, because $\sigma_a > \max\left\{\frac{\phi(1)}{1-\Phi(1)}, a_0\right\}$, this implies that the derivative is necessarily positive, which implies that the payoff increases in fraud.

Now let us consider the derivative at $x = 1 - \tau$.

$$e^{A+a_D}\left(\frac{d\hat{a}_C(1-\tau)}{dx} \times \frac{1}{\sigma_a}\phi\left(\frac{\hat{a}_C(1-\tau)-a_0)}{\sigma_a}\right)[\pi^B(\hat{a}_C(1-\tau)) - \pi^{NB}]\right) + \Phi\left(\frac{\hat{a}_C(1-\tau)-a_0)}{\sigma_a}\right)(1-\pi^{NB})$$
$$-\left(1-\Phi\left(\frac{\hat{a}_C(1-\tau)-a_0)}{\sigma_a}\right)\frac{d\pi^B(\hat{a}_C(1-\tau))}{d\hat{a}_C}\frac{d\hat{a}_C(1-\tau)}{dx}\right) - k'(1-\tau)$$

We are interested in finding the upper bound for this derivative. First, note that the first term is always

negative, so it will be largest at 0. Also, observe:

$$-\frac{d\hat{a}_C(1-\tau)}{dx} = \frac{1}{\frac{e^{A-M_C}}{1-\pi^{NB}}\left(\frac{d\pi^B(\hat{a}_C)}{d\hat{a}_C} + \pi^B(\hat{a}_C)\right)} < \frac{1-\pi^{NB}}{e^{A-M_C}\frac{d\pi^B(\hat{a}_C)}{d\hat{a}_C}}$$

Thus,

$$-\frac{d\pi^B(\hat{a}_C(1-\tau))}{d\hat{a}_C}\frac{d\hat{a}_C(1-\tau)}{dx} < \frac{1-\pi^{NB}}{e^{A-M_C}}$$

The derivative is therefore less than:

$$\begin{aligned} &e^{A+a_D}(1-\pi^{NB})\left(\Phi\left(\frac{\hat{a}_C(1-\tau)-a_0)}{\sigma_a}\right) - \left(1-\Phi\left(\frac{\hat{a}_C(1-\tau)-a_0)}{\sigma_a}\right)\right)\frac{1}{e^{A-M_C}}\Bigg) - k'(1-\tau) \\ &= e^{A+a_D}(1-\pi^{NB})\left(\Phi\left(\frac{\hat{a}_C(1-\tau)-a_0)}{\sigma_a}\right)\left(1+\frac{1}{e^{A-M_C}}\right) - \frac{1}{e^{A-M_C}}\right) - k'(1-\tau) \end{aligned}$$

Since $\Phi\left(\frac{\hat{a}_C(1-\tau)-a_0)}{\sigma_a}\right)$ is increasing in $\hat{a}_C(1-\tau)$, the function reaches it highest value when $\hat{a}_C(1-\tau)$ is at its upper bound. The bounds for $\hat{a}_C(1-\tau)$ are given by:

$$\left(ln\left(\frac{f}{e^A}\right), ln\left(\frac{f}{\pi^{NB}e^A}\right)\right) = (lnf - A, lnf - A - ln\pi^{NB}) \tag{1}$$

Thus, the upper bound is $lnf - A - ln\pi^{NB}$, which is largest when π^{NB} is lowest. The lowest value of π^{NB} occurs when $\pi^{NB} = 1 - \Phi\left(\frac{\frac{c}{\gamma}-b^V}{\sigma_\delta}\right)$, meaning only always-protesters are protesting. Thus, the upper bound, denoted by $\hat{a}_C^{UB}(1-\tau)$, is given by:

$$\hat{a}_C^{UB}(1-\tau) = lnf - A - ln\left(1-\Phi\left(\frac{\frac{c}{\gamma}-b^V}{\sigma_\delta}\right)\right)$$

Therefore, the derivative at $x = 1-\tau$ is less than:

$$\begin{aligned} &e^{A+a_D}(1-\pi^{NB})\left(\Phi\left(\frac{\hat{a}_C^{UB}(1-\tau)-a_0)}{\sigma_a}\right)\left(1+\frac{1}{e^{A-M_C}}\right) - \frac{1}{e^{A-M_C}}\right) - k'(1-\tau) \\ &= e^{M_C+a_D}(1-\pi^{NB})\left(\Phi\left(\frac{\hat{a}_C^{UB}(1-\tau)-a_0)}{\sigma_a}\right)(1+e^{A-M_C}) - 1\right) - k'(1-\tau) \end{aligned}$$

Moreover, $(1-\pi^{NB})$ is largest when π^{NB} is lowest. Thus, the derivative is lower than

$$e^{M_C+a_D}\Phi\left(\frac{\frac{c}{\gamma}-b^V}{\sigma_\delta}\right)\left(\Phi\left(\frac{\hat{a}_C^{UB}(1-\tau)-a_0)}{\sigma_a}\right)(1+e^{A-M_C}) - 1\right) - k'(1-\tau)$$

Therefore, the sufficient condition for derivative at $x = 1 - \tau$ to be negative is that

$$k'(1-\tau) > e^{M_C + a_D} \Phi\left(\frac{\frac{c}{\gamma} - b^V}{\sigma_\delta}\right)\left(\Phi\left(\frac{\hat{a}_C^{UB}(1-\tau) - a_0)}{\sigma_a}\right)(1 + e^{A - M_C}) - 1\right)$$

This is assumed in the Assumption 6. Hence, if the marginal cost of fraud at $(1-\tau)$ is sufficiently high, the derivative at $(1-\tau)$ is necessarily negative.

Thus, under these assumptions, the derivative when the incumbent party does not commit fraud is positive and when the total vote share reaches 100% due to fraud, the derivative is negative. This means that the optimal fraud level lies in the interior of the interval $[0, 1-\tau]$ and satisfies the following first-order condition:

$$e^{A+a_D}\left(\left(\frac{d\hat{a}_C(x)}{dx} \times \frac{1}{\sigma_a}\phi\left(\frac{\hat{a}_C(x) - a_0)}{\sigma_a}\right)[(\tau + x)(1 - \pi^{NB}) - (1 - \pi^B(\hat{a}_C(x))]\right) + \Phi\left(\frac{\hat{a}_C(x) - a_0)}{\sigma_a}\right)(1 - \pi^{NB})$$
$$- \left(1 - \Phi\left(\frac{\hat{a}_C(x) - a_0)}{\sigma_a}\right)\right)\frac{d\pi^B(\hat{a}_C(x))}{d\hat{a}_C}\frac{d\hat{a}_C(x)}{dx}\Bigg) = k'(x)$$

This equation represents the equilibrium condition that determines the optimal level of fraud. The solution to this first-order condition gives the incumbent's optimal strategy regarding how much fraud to commit in order to maximize their expected payoff, while taking into account the impact of fraud on the likelihood of protest and the reaction of the challenger party.

Now let us consider the case where the incumbent party is losing the election without fraud. The payoff for the incumbent party in this scenario is the following:

$$\begin{cases} e^{M_D + a_D}(\tau + x) - k(x) \quad if \quad \tau + x < \tilde{\tau} \\ e^{A + a_D}\left(\Phi\left(\frac{\hat{a}_C(x) - a_0}{\sigma_a}\right)(\tau + x)(1 - \pi^{NB}) + \left(1 - \Phi\left(\frac{\hat{a}_C(x) - a_0}{\sigma_a}\right)\right)(1 - \pi^B(\hat{a}_C(x)))\right) - k(x) \quad if \quad \tau + x \geq \tilde{\tau} \end{cases}$$

Thus, the payoff function is not continuous at $\tilde{\tau}$. To find the optimal level of fraud, first consider the derivative when $\tau + x < \tilde{\tau}$:

$$\frac{d}{dx} e^{M_D + a_D}(\tau + x) - k(x) = e^{M_D + a_D} - k'(x)$$

Since $k'(0) = 0$, when the fraud level is 0, the derivative is positive, implying that no fraud cannot be an equilibrium. Moreover, the first-order condition implies that

$$e^{M_D + a_D} = k'(x) \Leftrightarrow x^L = k'^{-1}(e^{M_D + a_D})$$

Thus, if $k'^{-1}(e^{M_D + a_D}) < \tilde{\tau} - \tau$ and it is optimal for the incumbent to lose the election, the optimal

level of fraud will be $x^L = k'^{-1}(e^{M_D+a_D})$. This is a global maximum because the second derivative equals $-k''(x) < 0$.

Further, note that $x = 1 - \tau$ cannot be an equilibrium by the argument discussed above. Therefore, if the payoff from losing the election at the equilibrium fraud level x^L, $(e^{M_D+a_D})(\tau + k'^{-1}(e^{M_D+a_D}) - k(k'^{-1}(e^{M_D+a_D})))$ exceeds the payoff from winning the election at the equilibrium fraud level (at $(\tilde{\tau} - \tau)$ with payoff $e^{A+a_D}\left(\Phi\left(\frac{\hat{a}_C(\tilde{\tau}-\tau)-a_0}{\sigma_\alpha}\right)\tilde{\tau}(1-\pi^{NB}) + \left(1-\Phi\left(\frac{\hat{a}_C(\tilde{\tau}-\tau)-a_0}{\sigma_\alpha}\right)\right)(1-\pi^B(\hat{a}_C(\tilde{\tau}-\tau)))\right) - k(\tilde{\tau}-\tau)$ and at the solution to the first-order condition), then it is optimal for the incumbent to commit a positive level of fraud, but still lose the election. If the reverse holds, it is optimal for the incumbent to win the election with fraud.

□

Proposition 3. *The boycott threshold in a presidential election is significantly lower compared to that in a parliamentary election. As a result, boycotts are more frequent after presidential elections. However, the size of protests following a parliamentary election tends to be larger than that after presidential elections. Furthermore, following a presidential election the challenger's decision to boycott does not depend on the level of fraud in the election, but rather on the cost of boycotting, the exogenous payoff from winning the election, and the probability of a successful boycott in the event of a protest.*

It is also less likely that the incumbent loses in a presidential election than in a parliamentary election. The optimal level of fraud in presidential elections is either zero or equal to $\tilde{\tau} - \tau$, meaning that when fraud is used, the post-fraud vote share will be close to the required winning threshold. This implies that, conditional on winning, the vote share after fraud is higher in parliamentary elections compared to presidential elections. Additionally, in presidential systems, a government change is more likely to occur due to protests rather than through elections.

Proof. In presidential elections, the challenger's payoff from losing is zero. Thus, the challenger will be indifferent between boycotting and not boycotting when:

$$\pi^B(\hat{a}_C)e^{A+\hat{a}_C} - f = 0$$

Equivalently,

$$\hat{a}_C = ln\left(\frac{f}{\pi^B(\hat{a}_C)}\right) - A$$

Thus,

$$\hat{a}_C \in (lnf - A, lnf - A - ln\pi^{NB})$$

This shows that the boycott threshold is lower in presidential elections than in parliamentary ones, which leads to more frequent boycotts in the former. However, the size of protests following a parliamentary election is larger than after a presidential election because $\frac{\partial \pi^B(\hat{a}_C)}{\partial \hat{a}_C} < 0$.

Importantly, the boycott decision in presidential elections does not depend on the level of fraud (because x does not appear in the equation). It depends only on the cost of boycotting, the exogenous payoff from winning the election, and the probability of a successful boycott in the case of a protest.

Additionally, in the equilibrium, the incumbent cannot lose a presidential election with a positive level of fraud. If the incumbent loses the election, their payoff is $-k(x)$, while losing without fraud results in a payoff of zero. Therefore, losing with fraud is not an equilibrium strategy.

In contrast, if the incumbent is winning the election without fraud, the payoff is

$$e^{A+a_D}\left(\Phi\left(\frac{\hat{a}_C - a_0}{\sigma_a}\right)(1-\pi^{NB}) + \left(1-\Phi\left(\frac{\hat{a}_C - a_0}{\sigma_a}\right)\right)(1-\pi^{B}(\hat{a}_C(x)))\right) - k(x)$$

In this scenario, using fraud is irrational because it increases the cost without changing the benefit. Consequently, fraud is only optimal in a presidential election when the incumbent is losing without fraud but can win with it. In this case, the optimal level of fraud is $x = \tilde{\tau} - \tau$.

Given:

$$e^{A+a_D}\left(\Phi\left(\frac{\hat{a}_C - a_0}{\sigma_a}\right)(1-\pi^{NB}) + \left(1-\Phi\left(\frac{\hat{a}_C - a_0}{\sigma_a}\right)\right)(1-\pi^{B}(\hat{a}_C(x)))\right) > k(\tilde{\tau} - \tau)$$

It becomes optimal for the incumbent to use fraud to win the election. □