Central Advantage:

A geographic measure of partisan fairness in redistricting

Jeffrey T. Barton* Jon X. Eguia †

July 23, 2024

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Abstract

We introduce a measure of partisan advantage in redistricting that takes into account a state's political geography. We define a party's geographic baseline seat-share as the proportion of the state's inhabitants for which the party would win a district centered around the inhabitant's location of residence. If the geographically ideal district for each voter is a compact district with this voter at its center, a party's geographic baseline is the share of inhabitants for whom the party wins their ideal district. A party's "Central Advantage" is then the difference between the seats the party wins, and its geographic baseline number of seats. We compute the Central Advantage for each legislative and congressional 2024 district map for all 50 states using precinct-level statewide elections results from 2016 to 2020. We observe that Democrats' geographic baseline increases with district size, and that Democrats are underrepresented in nationwide seat aggregates: the GOP has a Central Advantage of about twenty-eight U.S. congressional seats, and over a hundred and fifty state legislative seats.

Keywords: Redistricting, gerrymandering, political geography, partisan advantage, geographic baseline.

*Hampshire College, email: jtbNS@hampshire.edu.

†Michigan State University, email: eguia@msu.edu.

1 Introduction

In an electoral system in which the candidate with most votes in each district gains office, and other candidates do not, the number of seats that each political party obtains is not necessarily proportional to the party's aggregate number of votes statewide. Factors such as the geographic distribution of votes for each party across the state and the boundaries of the state's electoral district map, are crucial as well. Normative measures of partisan fairness in redistricting that rely solely on statewide vote-shares, such as proportionality or the Efficiency Gap[1] omit any consideration of political geography. And yet, as argued by the United States Supreme Court, local political geography matters as well: "*to translate statewide support into seats in the legislature"* is "*a norm that does not exists in our electoral system.*"[2]

In an idealized local vision of community representation by a geographic district, each voter is in the same district as all her closest neighbors in her own community. Individuals whose enacted district has a different partisan composition than this hypothetical district composed of their nearest neighbors suffer "political dislocation" (DeFord, Eubanks and Rodden 2022). For each state in the U.S., and for each political party, including Democrats, Republicans, and any third party, we ask: which share of residents would be represented by this party, if they were not politically dislocated? In other words: what is the share of residents for which the party would win a hypothetical district drawn with the resident at its center surrounded by her nearest neighbors? This share, multiplied by the number of districts in the state, is our geographic baseline of representation for the party. If we restrict attention only to votes for either of the two main parties, this geographic baseline is the share of residents whose closest geographic neighbors vote majoritarily for the party.

We define a measure of partisan advantage with respect to this geographic benchmark as

[1] Computed as "*seat margin minus twice the vote margin*" by Stephanopoulus and McGhee (2015), p. 853.

[2] *Rucho v. Common Cause*, 139 S. Ct. 2484, 2499 (2019).

the difference between the number of enacted districts in which a party wins most votes, and the geographic baseline number of seats for the party. Because this baseline is constructed by imagining each resident at the geographic center of her own hypothetical district, we refer to the baseline as the "central geographic baseline" and to its associated measure of partisan advantage as the "Central Advantage."

The central geographic baseline, and with it the Central Advantage, depend not only on the number of votes cast for each party and the geographic distribution of these votes, but also on the number of districts —and therefore on their population size. Holding all individual votes constant, as we increase the population size of the hypothetical district of closest neighbors around a given voter, the political composition of such hypothetical district changes, and its partisan majority may switch from one party to another. Therefore, the central geographic baseline is a function of district size and we can quantify how different district sizes relatively favor or disfavor a party.

The idea of defining a partisan advantage for each party, for a given election result, and for a given map, as the difference between the number of the map's districts won by the party at that election, and a baseline number obtained from the distribution of vote tallies across all precincts, is well-established. Measures of partisan advantage that calculate the deviation from a baseline number of seats include those using baselines that are linear with slope one (proportionality) or slope two (Stephanopoulos and McGhee 2015), or quadratic (Pegden, Proaccia and Yu 2017; Barton 2022), or cubic (Goedert 2014) in statewide vote-share; or a baseline proportional to the population of jurisdictions in which the party is the one to win most votes, as in the Jurisdictional Advantage (Eguia 2022). Closely related as well are "outlier tests" that define a range of acceptable values obtained from the distribution of the number of seats that the party would obtain over a large collection —an ensemble— of computationally generated maps (see Chen and Rodden 2013 for a seminal contribution, and DeFord, Duchin and Solomon 2021 for a discussion of the ensemble approach). Among these measures, the Jurisdictional Advantage and the outlier test also take into account political

geography.

The Jurisdictional Advantage is best applicable in environments in which the population size of any county is not much more than that of one district, so that the county map can be used to aggregate vote by county to compute the jurisdictional baseline. If counties comprise multiple districts as in Arizona, California, or Texas in congressional elections, or in elections to the state legislature in most states, the Jurisdictional Advantage relies on a more delicate partition of counties into subcounty jurisdictions. Whereas, the Central Advantage is uniquely defined, without any degree of researcher freedom in the definition of the geographic unit of comparison to define the baseline, and with a consistent operationalization of the definition any map with districts of any size in any state. Results are therefore more comparable across states, and easier to interpret.

The Central Advantage's unique definition, with zero algorithmic parameters used to compute it, is also a relative strength compared to the outlier test approach. The properties of the ensemble of maps used for comparison depend on the algorithm used to generate the ensemble. Competing algorithmic approaches exist in the literature, and each approach features multiple degrees of freedom in the choice of algorithmic parameters within each approach, both on the determination of which maps are admissible for consideration, and on the probability of selection among admissible maps. With the same underlying data, different ensembles can lead to conflicting outlier test results, a conflict of difficult interpretation to outsiders to this literature. Whereas, the Central Advantage is a unique measure of easier computation and unambiguous interpretation.

The definition of the Central Advantage is rooted in DeFord, Eubanks and Rodden's (2022) idea that the ideal unit of representation for a voter is the community formed by her nearest neighbors. With this notion, they define a voter's Political Dislocation as the deviation in the distribution of partisan support between this ideal district and the voter's actual district. Such Political Dislocation averaged at the district level captures to what extent a district is representative of the distribution of the set of nearest neighbors for each

voter in the district. This measure is therefore best designed to quantify the harm to local representation that arises if a district fails to capture a community of voters defined by their geographic proximity (DeFord, Eubanks and Rodden 2022, pp 407-408).

On the other hand, Political Dislocation does not directly speak to the partisan composition of a state's legislature or congressional delegation. In contrast, the Central Advantage is solely a statewide measure of partisan (un)fairness quantified in number of seats.

We illustrate the difference between the Political Dislocation and the Central Advantage by an abstract example. Consider a state with two congressional districts, and assume the distribution of voters is such that for half of the voters in the state, the half of her state peers geographically closest to the voter lean Democratic 51%-49%, whereas for the other half of the voters, their geographically closest neighbors lean Republican 55%-45%. Political Dislocation and Central Advantage are both zero with a map A in which Democrats and Republicans each win one district by their respective margins. Consider a map B in which Republicans win both districts 52%-48%. Political Dislocation is very small (0.03), but the change in the partisan composition is relatively very large, as one of the two districts flips; the Central Advantage notices this difference, reporting a Central Advantage to the GOP of +1.00 seats. Whereas, consider map C that carefully sorts voters into two landslides, one 96%-4% for Democrats, and another 100%-0% for Republicans. Political Dislocation is huge, but there is no effect on the partisan composition of the delegation, so the Central Advantage is zero. Partisan Dislocation and Central Advantage measure different threats to fair representation, and can used as complementary tools to quantify partisan gerrymandering.

The Central Advance has a practical advantage over the other measures of partisan advantage in number of seats that are grounded on political geography: it is easy to compute. Calculating the central geographic baseline avoids the algorithmic complexities of ensemble methods, and the need for partitioning counties into sub-county juristictions in the Jurisdictional Advantage. With a small adaptation of the code that DeFord, Eubanks and Rodden (2022) made public, we can easily compute the Central Advantage for a partition of any state

into any number of districts that are larger than voting precincts. We demonstrate this ease of computation by providing results for districts of the size of the lower (House) and upper (Senate) chamber and congressional districts in each state, and for districts of any size, with election data from 2016, 2018 and 2020. Further, we conduct two sensitivity analysis, one with respect to the size of the districts, providing results for any district size from $10,000$ to $1,000,000$ in increments of $10,000$, and another with respect to election results, providing results with a 2% or a 4% swing away from the actual 2016-2020 results.

We highlight our main findings. First, if the population size of each district is large, the central geographic baseline assigns more seats to the Democratic party than baselines derived solely on statewide vote-share would suggest. Second, the Democratic seat-share according to the baseline is monotonically increasing in district size. Third, the Democratic party seats under enacted maps falls far short of this baseline, so the GOP enjoys a large Central Advantage in seats in state legislatures and in Congress. And fourth, the measure of Central Advantage in congressional seats bears a close correlation to the measure of Jurisdictional Advantage.

On the comparison to non-geographic baselines, using election data from 2016-2020 statewide elections in which Democrats obtained an average 51.77% of the two-party vote-share, the central geographic baseline assigns 57.7% of congressional seats to Democrats, more than proportionality (51.8%), the efficiency gap or the quadratic rule (53.5%) or the cubic rule (55.3%). On the positive relation between district size and the Democratic central geographic baseline, we compute a sensitivy analysis on the effect of district size, and we find that with districts of size 10k residents, and the same 51.77% vote-share, the central geographic baseline seat-share for Democrats would be 54.8%, with the Democratic seat-share increasing gradually with the population size of the district from 10k to $760k$ as in the average congressional district.[3]

[3]Also, across the 39 states that have single-member districts in both their lower and upper state chamber, with 51.74% of the vote, and weighing each seat by its population, the central geographic baseline assigns to Democrats 55.6% of the lower chamber seats, which represent smaller districts, and 57.5% of the seats in

The Democratic party baseline seat-share increases with district size because Democrats perform disproportionally well in large cities, and since population density is higher in cities, for rural voters who live just outside the city, most of their closest neighbors are in the city and vote Democratic. Even if these city-adjacent rural voters vote Republican, their geographic community defined by closest proximity (not by interests, culture or other traits) is mostly urban and votes Democrat; since other rural voters who may be culturally closer are spread out over vast distances, whereas the city and its Democratic voters are nearby, it follows that to avoid political dislocation in the precise sense of DeFord, Eubanks and Rodden (2022), these city-adjacent Republican voters must be in a Democratic district. As district size grows, this halo of thinly populated Republican areas that fall under the influence of the city expands outwards, encompassing a large share of the state's population, so the Democratic central geographic baseline accordingly increases.

The Central Advantage for the GOP (i.e. the Democratic central geographic baseline and the number of districts in the enacted maps won by Democrats) averages twenty-eight seats in Congress over the 2016-2020 elections in our sample, and to 156 seats in state legislatures across the 39 states with single-member districts in each of two state legislative chambers. This GOP advantage amounts to over 6% of the seats in the U.S. House of Representatives, over 5% of population-weighted seats in state legislative upper chambers, and over 2.5% of seats in state legislative lower upper chambers. Half of the GOP's Central Advantage in congressional seats concentrates in just three states: Texas (+6 seats), Florida (+4 seats) and North Carolina (+4 seats). These three states are the only ones with a Central Advantage above three seats for either party in Congress, and notably, in all three the GOP exerted partisan control over the drawing of the enacted map. Only two other congressional maps provide a Central Advantage of more than two seats: Ohio (+2 seats), drawn by and favoring the GOP; and Illinois (-2 seats), drawn by and favoring Democrats. State legislature results are less comparable given the variance in the population size that each seat represents, but

the upper chamber, which represent larger districts.

Texas (+14 state legislature seats) and North Carolina (+13 seats) are the two states with largest Central Advantage, both for the GOP, with again Ohio (+11 seats) and Florida (+9 seats) not too far behind.

With regard to a comparison to an existing measure of partisan advantage over a geographic baseline, despite the conceptual difference between the Central Advantage focus on geometric distance alone, and the Jurisdictional Advantage's reliance on jurisdictional geography, the two measures correlate strongly. Jurisdictional Advantage values accross states are only publicly available for congressional seats; Comparing their results for congressional seats, the Pearsson correlation coefficient across all 50 states is +0.87. Their Spearman rank-order correlation is +0.74. Under both measures, Texas and Florida are the only two states to provide more than 4 seats of partisan advantage, North Carolina is the only other with more than 3 seats of advantage; and on the other side, Illinois is the only one to provide at least approximately 2 to Democrats.

In the next section, we detail our methodology to compute the measure of Central Advantage. Thereafter, we present results for congressional seats in each state, and for state legislature seats in each state legislature with single-member districts.

2 Methodology

In this section we detail the method to compute the Central Advantage for each legislative body and each state. We first describe the data, then the algorithm to compute the central geographic baseline for each election result, and from these, the computation of the Central Advantage.

2.1 Data

We use precinct-level data from 2016-2020 statewide election results for the offices of U.S. President, U.S. Senator, and State Governor, in each state. The list of elections used in each

state, driven by the availability of precinct-level data, is in Table 6 in the Appendix.

For each election, the unit of observation is the electoral precinct, or precinct-equivalent geographic area.[4] For each election and each precinct, the relevant data we use comprises: the precinct's geographic boundary and population, and the precinct's vote tallies for the Democratic candidate and for the Republican candidate at that election.

We obtained the precinct boundaries were obtained directly from the 2020 U.S. Census with the exception of Maine's, which was provided to us by DRA. We obtain all our population and election data from DRA.[5]

2.2 Computation of the central geographic baseline

The input necessary to compute the central geographic baseline is: a precinct map of a state S, population and result in one election for each precinct, and a number d of electoral districts. Let $P_1, P_2, \ldots, P_K$ be the list of precincts in state S, containing corresponding populations $p_1, p_2, \ldots, p_K$, numbers of Democratic voters $d_1^e, d_2^e, \ldots, d_K^e$, numbers of Republican voters $r_1^e, r_2^e, \ldots, r_K^e$, and total two-party voters $v_1^e, v_2^e, \ldots, v_K^e$ in election e, where $d_k^e + r_k^e = v_k^e$ for each precinct $k = 1, \ldots, K$, for the election data from election e used to compute the baseline.

With this data as input, we proceed as follows.

1. Determine each precinct's centroid, c_k. Since the precise location of voters within a precinct is unknown, we assume they are all located at their precinct's centroid c_k.[6]

[4] In the case of West Virginia, Oregon, Hawaii, and California, which do not report precincts to the Census Bureau, we use Census block group-level results. In the case of Maine, which reports precincts for only a portion of the state, we use the combination of precincts and other geographic units as provided by Dave's Redistricting App (DRA). Hereafter, we refer to all these minimal units of vote aggregation indistinctly as "precincts."

[5] DRA obtained its population data from the 2020 U.S. Census; it compiled its precinct-level election data from its data partners listed at: https://davesredistricting.org/maps#aboutdata, and it aggregated this data to 2020 precincts to publish the 2020-precinct-level results that we use.

[6] Here we depart from the more complex algorithm by DeFord, Eubanks and Rodden (2022), who assume each voter's location is a random variable uniformly distributed over the entire precinct, and they then draw a sample of voters. Our centroid is the expectation of their voters' location; their method introduces

2. With m districts and total state population $p_S = \sum_{k=1}^{k} p_k$, the population per district, denoted n, is $n = \frac{p_S}{m}$. For each precinct k, we construct its own neighborhood in S of size n, denoted $N_k(n)$, as follows:

   (a) Order all precincts in S by the Euclidean distances from their centroids to centroid c_k of precinct P_k, from smallest to largest.

   (b) Add population to neighborhood $N_k(n)$, starting with residents in the precinct nearest to P_k until its population is exhausted, then from the precinct second-nearest, and then continuing adding population sequentially from the precincts that are next in the order of proximity to P_k, until the population of the neighborhood equals n, at which point, the process ends. Notice that, typically, when the process ends, several precincts have been completely added to the neighborhood, and from one, only a fraction of its residents have been added.

3. Determine the winning party for neighborhood $N_k(n)$ by summing each party's total vote tallies, $D_k(n))$ for Democrats, and $R_k(n)$ for Republicans, in all precincts fully included in the neighborhood, plus the fraction of the vote tallies corresponding to the fraction of the precinct that has been only fractionally added to the neighborhood. If the fractionally added precinct is P_f and the fraction added is λ, then

$$D_k^e(n) = \sum_{P_j \subset N_k(n)} d_j^e + \lambda d_f^e, \text{ and } R_k^e(n) = \sum_{P_j \subset N_k(n)} r_j^e + \lambda r_f^e.$$

4. We determine each party's *central geographic baseline* number of seats for the election by crediting each party with the proportion of the seats equal to the proportion of the state's population whose neighborhood it wins. In the event of a tie in a neighborhood of a precinct, each party is credited with half of the precinct's population. The baseline

uncertainty, and thus it generates stochastic results.

for each party is then, respectively

$$GB^e_D(n) = m\frac{\sum_{D^e_k(n)>R^e_k(n)} p_k + \sum_{D^e_k(n)=R^e_k(n)} 0.5p_k}{p_S}$$

and

$$GB^e_R(n) = m\frac{\sum_{D^e_k(n)<R^e_k(n)} p_k + \sum_{D^e_k(n)=R^e_k(n)} 0.5p_k}{p_S}.$$

5. For each party $J \in \{D, R\}$, and for a given map of the state into districts of size n, we define the *Central Advantage* for party J, for election e, denoted $CA^e_J(n)$, as the difference between the number of districts that the party wins with the vote tallies in election e, and its central geographic baseline $GB^e_J(n)$.

6. For a given neighborhood size n, once we have the central geographic baseline $GB^e_J(n)$ for each party $J \in \{D, R\}$, and for each election e in the dataset for state S, we average the geographic baselines over all elections to arrive at the state's overall geographic baseline $GB_J(n)$ for party J and districts of size n. Similarly, we average the Central Advantage $CA^e_J(n)$ across every election e in the data set, to arrive at our main measure: the Central Advantage for Party J for districts of size n, denoted $CA_J(n)$.

We illustrate this algorith in Figure **??**, showing one particular precinct contributes to the central geographic benchmark in North Carolina, computed for districts of congressional size (745,671 residents), using election data from the 2020 U.S. presidential election. The first map (top left) shows Guilford County precinct GR [6], at county's southeast corner, with 3,511 residents according to the 2020 U.S. Census, and 400 Democratic and 1,735 Republican votes in the 2020 U.S. Presidential election. The second (top right), third (middle left) and fourth (middle right) panels show the neighborhood of size 745,671 around precinct GR [6], with the colors in the fourth panel capturing the vote margin in each precinct.[7] The fifth

[7]Notice that the shape of the neighborhood of each precinct depends on its location. The neighborhood

panel shows that the aggregate vote for the entire neighborhood is a Democratic majority. To avoid political dislocation, the 3,511 residents in precinct GR [6] must be in a Democratic district, so they are added to the Democratic party tally of residents for the geographic baseline, as reflected in the blue coloring in the sixth panel (bottom right).

Proceeding in an analogous manner with each precinct in the state, we end up coloring every precinct in the sixth panel blue (if their neighborhood is Democratic) or red (if its neighborhood is Republican); adding up the population in all the precincts colored blue in panel six, and dividing by the total population of North Carolina (10,439,388 residents) we obtain the share of residents with Democratic neighborhoods (0.53); multiplying by the number of congressional districts in the state (14), we obtain the Democratic central geographic baseline for this election data, for congressional seats in North Carolina, namely: 7.4 seats. Since Democrats would win only 4 of the districts in the 2024 North Carolina congressional map, this GOP-drawn map, using the 2020 U.S. Presidential election data, features a GOP Central Advantage of 3.4 seats.

Repeating the exercise with other election data, we obtain similar results that average to a central geographic baseline of 7.7 seats for Democrats and 6.3 seats for Republicans, and a Central Advantage for the GOP of 3.8 seats. In the next section, we present and discuss results for each state.

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of a precinct in the interior of a state, as in Figure 1, is roughly circular. Large neighborhoods near state borders reflect the state's geography and appear as the intersection of a circle with the state's boundary.

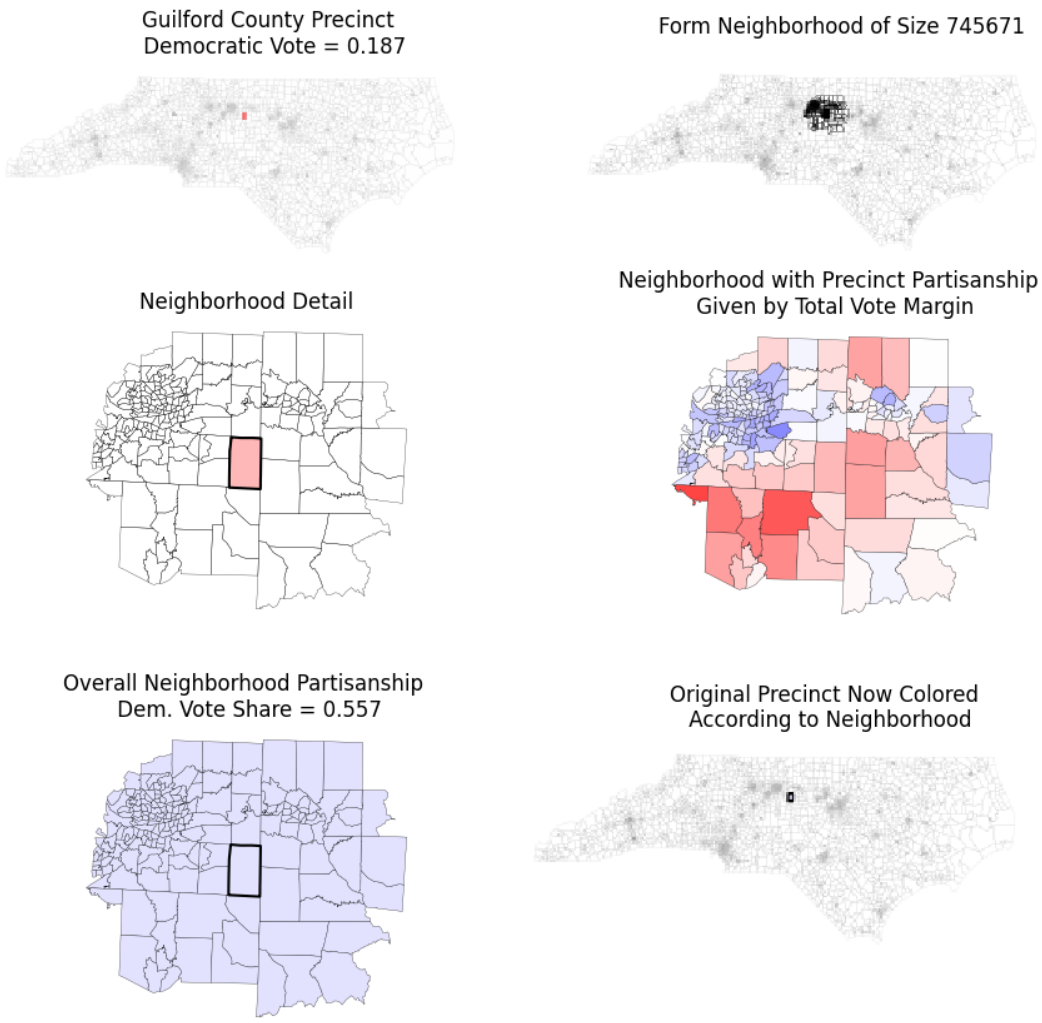


Figure 1: Contribution of Precinct GR[6] (Guilford County, NC) to the geographic baseline.

3 Results

We first present results for U.S. congressional district maps, and for state legislative district maps. We compute the central geographic baseline with the precinct voting tallies in each election in our data set. We also compute the number of each map's districts that a party would win with such election tallies, for each map that was a state's official congressional or legislative map as of February 24, 2024; the difference between this number and the baseline is the Central Advantage. For Congressional maps, we compare the Central Advantage to the

Jurisdictional Advantage value[8], and for all maps, we also compare it to Cubic rule baseline (i.e. a seat-share ratio that is the cube of the ratio of vote-shares of the two parties), which, albeit of questionable normative interpretation, is an acceptable fit for the historic of seat outcomes as a function of statewide vote-share (Goedert 2014).

We present aggregate results nationwide, results for each individual state, and results partially aggregated by the partisan control of map drawing, distinguising Democratic control, Republican control, or no party control in maps drawn by independent or bipartisan commissions, or with substantial Court intervention.[9]

We extend these results with sensitivity analysis with respect to district size, creating a counterfactual geographic baseline for districts of any size from 10k to 1M in multiples of 10k; and with respect to changes in the electoral environment, computing counterfactual results for proportional swings of 2% and 4% in favor of either party.

3.1 U.S. Congressional Elections

Democratic candidates obtained an average 51.8% vote-share across the elections in our dataset. Given its geographic distribution, this vote-share was good for a Democratic central geographic baseline of 251 seats in the U.S. House of Representatives. Democrats only won 223 U.S. House districts on average across the elections in our dataset,[10] so results show an aggregate GOP Central Advantage of 28 seats. Table 1 summarizes the results, both in absolute terms, and as a share of the 435 seats in the House of Representatives, with the jurisdictional and cubic law baselines in additional rows for comparison.

[8]Publicly available online at the Partisan Advantage Tracker.

[9]The degree of partisan control or Court intervention in the drawing of the maps is a continuum, and any asignment of this continuous variable to three discrete categories is debatable at best. A reader who prefers a different classification of specific maps can easily aggregate our individual state results to obtain results by partisan control of map-drawing with her preferred classification.

[10]We first compute the average number of districts in each state, and then we sum these up to obtain a nationwide total.

	Dem.	Rep.	Dem. %	Rep. %
Avg. 2024 map seats	222.55	212.45	51.2%	48.8%
Geographic baseline	251.03	184.00	57.7%	42.3%
Central Advantage	-28.48	28.48	-6.5%	6.5%
Jurisdic. baseline	241.07	193.93	55.4%	44.6%
Cubic law baseline	236.47	198.53	54.4%	45.6%

Table 1: Geographic baseline and Central Advantage in congressional seats, nationwide.

The states with greatest Central Advantage in congressional seats are Texas, Florida, North Carolina and Ohio for the GOP, and Illinois for the Democrats. These five states are the only ones with a Central Advantage of more than two seats, and the four Republican ones contribute more than half of the 28 states of the aggregate GOP Central Advantage nationwide. We highlight these results in Table 2, as an excerpt of the complete results for each state in Table 7 and Table 8 in the Appendix. Values represent the advantage for the GOP; negative values reflect a Democratic advantage.

State	Central Adv.	Jurisdict. Adv.
TX	6.11	4.06
FL	4.30	5.11
NC	3.75	3.01
OH	2.38	1.49
IL	-2.35	-1.92

Table 2: GOP Central Advantage in congressional seats, selected states.

The Central Advantage values are aligned with the map's Jurisdictional Advantage. Not only are the maps of Texas, Florida and North Carolina the most favorable to the GOP (and the only ones to give an advantage to either party) and the Illinois map the most favorable to Democrats according to both measures, further, if we look at the values across

all fifty states, the Pearson correlation between these two geographich measures of partisan advantage is +0.87.[11].

The five maps with greatest Central Advantage for either party share a characteristic in common: they were drawn under full partisan control by the party advantaged by the map. In fact, if we extend the analysis of party control as an explanatory variable of Central Advantage, we find that, even though congressional maps were drawn under GOP partisan control in only 18 states with only 181 districts among them, these maps contribute 26 out of the 28 seats of aggregate GOP Central Advantage nationwide.[12] Maps drawn under Democratic control generate cntain 75 districts in total, but generate only the two seats of Democratic Central Advantage in Illinois. Courts, bipartisan and non-partisan commissions, or state governments with split partisan control drew the maps of 173 districts across 18 states, and generated only 4 seats of Central Advantage for the GOP.

We illustrate these findings in Figure 2 and Figure 3. As shown in Figure 3 maps drawn without partisan control generate a number of Democratic seats that falls just below the Democratic central geographic baseline, generating a Central Advantage for the GOP of about 2% of the seats.[13] Maps drawn under Democratic partisan control (Figure 2, bottom), with the outlying exception of Illinois, closely track the central geographic baseline, generating no Central Advantage; whereas, maps drawn under GOP control (and Illinois mirroring them) track a different line that gives Democrats only about 70% of the seats they would obtain according to the central geographic baseline, generating a large GOP Central Advantage that amounts to over 14% of the seats.[14]

[11]Their Spearman rank-order correlation is +0.74.

[12]Congressional maps drawn were drawn under GOP in AR, FL, GA, IA, IN, KS, KY, LA, MO, MS, NC, OH, OK, SC, TN, TX, UT and WV; and by Democrats in IL, MA, MD, NM, NV, NY, OR, and RI.

[13]California, with its almost 45 seats in both the baseline and the enacted map, is off the chart; if we zoomed out, it would appear almost on the line of equality.

[14]If one were to interpret the outcomes of non-partisan redistricting as "fair", then from that premise one would conclude that the central geographic baseline and maps drawn under Democratic control outside Illinois are only slightly biased in favor of Democrats, whereas the set of all eighteen GOP-drawn maps (and Illinois for Democrats) are on average six times more biased toward partisan interests.

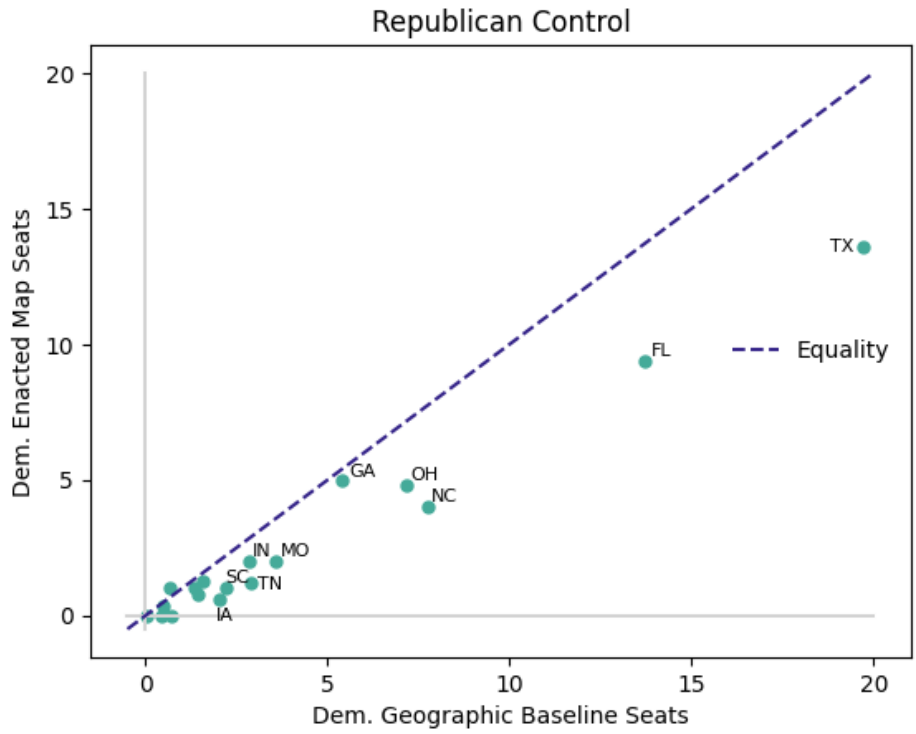
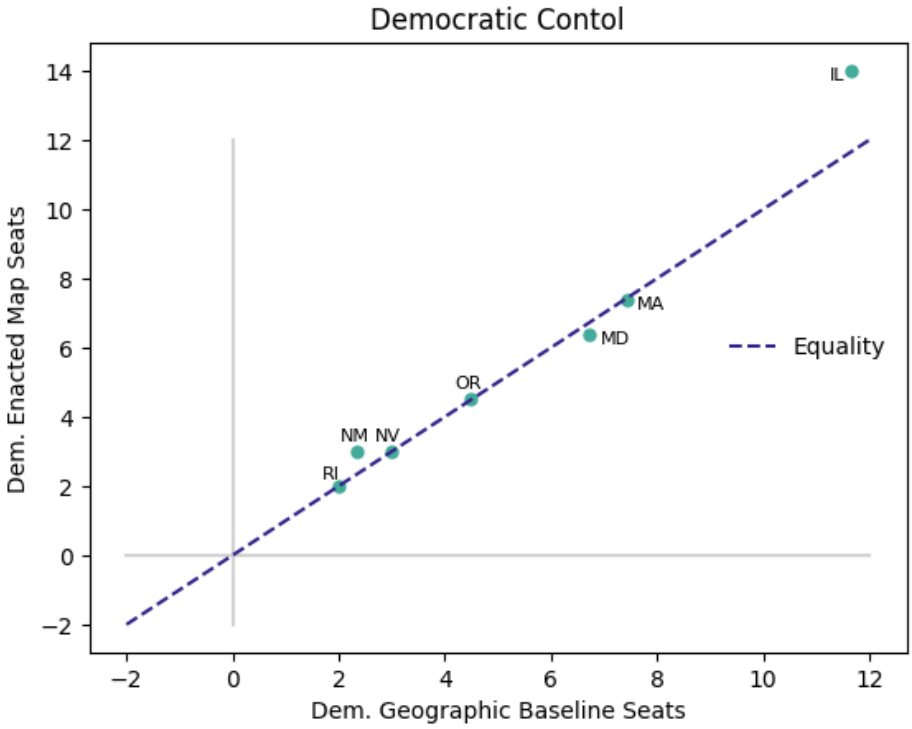


Figure 2: Central Advantage with GOP (above) and Dem. (below) partisan control.



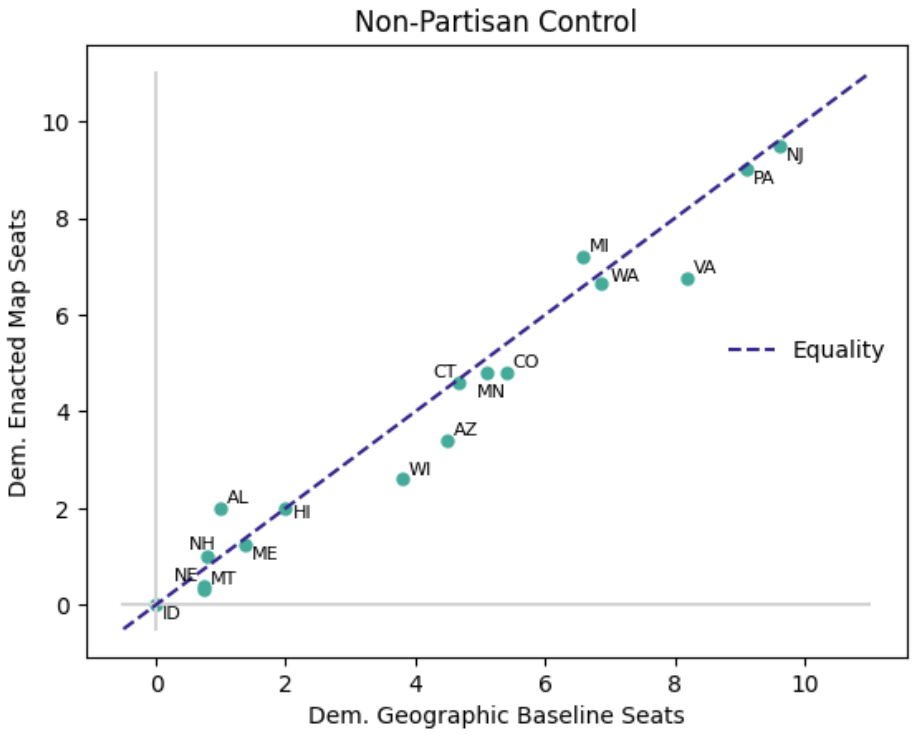


Figure 3: Central Advantage in maps without partisan control.

3.2 State legislative elections

We next analyze results for districts of population size equal to those of districts for the state's legislative lower champer and the state's legislative upper chamber, in each state with partisan representatives elected in single-member districts. This excludes the non-partisan state assembly in Nebraska; the upper chambers with multi-member districts in Vermont and West Virginia, and the lower chambers with multi-member districts in Arizona, Idaho, Maryland, New Hampshire, New Jersey, North Dakota, South Dakota, Vermont, and Washington.

In Table 3 and Table 4. Because (unlike in Congress) the number of residents in each seat varies wildly across states for state legislative seats, the percentages in the tables capture the share of population represented by each party, and not their number of seats. Tables 7 – 12 in the Appendix provide detailed results for each individual state.

	Dem.	Rep.	Dem. %	Rep. %
Avg. 2024 map seats	853.0	1,007.0	52.4%	47.6%
Geographic baseline	923.1	936.9	57.8%	42.2%
Central Advantage	-69.9	69.9	-5.4%	5.4%
Cubic law baseline	880.5	979.5	54.7%	45.3%

Table 3: Geographic baseline and Central Advantage in state upper chamber seats.

	Dem.	Rep.	Dem. %	Rep. %
Avg. 2024 map seats	1,976.4	2,273.7	52.7%	47.3%
Geographic baseline	2,062.5	2.187.5	55.4%	44.6%
Central Advantage	-86.1	86.1	-2.7%	2.7%
Cubic law baseline	1,978.2	2,271.8	53.9%	46.1%

Table 4: Geographic baseline and Central Advantage in state lower chamber seats.

The main finding is that the GOP also has a Central Advantage in state legislative seats, albeit a smaller one as a share of total seats available than in Congress, particularly for the lower chambers, where the advantage amounts to less than 3% of the seats. Since, to our knowledge, no computation of any alternative geographic baseline is publicly available for state legislature seats, we provide the cubic law number of seats as an alternative, non-geographic baseline. The central geographic baseline assigns more seats than the cubic law baseline.

A reason for this discrepancy between the central geographic baseline and the cubic rule (or, more generally, between the geographic baseline and any baseline that is a function solely of statewide vote-share) is that the cubic rule does not account for the geographic distribution of the non-voting population. Democratic-voting areas contain more residents who do not vote (including non-citizens), so for the same vote-share, Democrats represent more residents. Since seats are apportioned by number of represented residents, not just

voters, for the same vote-shares, since Democratic representatives represent more residents, there should be more of them. Any baseline that relies solely on statewide vote-share is by construction unable to detect this effect; whereas, the central geographic baseline, accounts for it.[15] Indeed, if we nullify the effect of the non-voting population, in a counterfactual in which instead of using Census data for precinct population, we use number of voters as the precinct's population, in such counterfactual that assumes away the effect of the non-voting population, the geographic baseline converges toward the cubic rule: it assigns to Democrats seats in upper chambers representing 56.3% of the population, down from 57.8% with a correct population tally; and seats in lower chambers representing 53.5% of the population (down from 55.4% with a correct population tally).

Among individual states, Texas, North Carolina and Ohio, all favoring the GOP, are the only three to feature a Central Advantage of more than 4 seats and more than 12% of their state's upper chamber seats; and once again Texas, North Carolina and Ohio, this time together with Florida, Virginia and Iowa, are among the only six to feature a Central Advantage of more than 4 seats and more than 5% of their state's lower chamber seats, always favoring the GOP. These seat advantages are consequential: for at least one election result, the GOP's Central Advantage is critical for the party to gain a majority in each of the two chambers in Florida, North Carolina and Texas; without this advantage, with such election results, the GOP would hold only a minority of seats in each chamber, and Democrats would have full control of the legislature.

Classifying all maps according to the partisan control of the map-drawing process, we find that as in the case of congressional maps, those drawn with limited partisan control, through Court-appointed experts, non-partisan or bipartisan commissions, or split governments, generate a Central Advantage for the GOP of approximately 2% of the districts

[15]The right to representation of non-voters (including non-citizens) also runs contrary to partisan symmetry as a legal ideal in the U.S.: in a democracy in which representation is by geographic districts of equal population of residents, for equal vote-shares, geographic regions with more residents are entitled to more representation, so seat outcomes ought to be asymmetric.

they drew; the ones drawn by Democrats exhibit closer to zero Central Advantage (with no Illinois exception in state legislative maps), whereas maps drawn under GOP control favor Republicans, but not as much as in the case of congressional maps, and not much more so than maps of non-partisan creation.[16] Table 5 summarizes these results.[17]

	Upper chambers			Lower chambers		
Control	Rep.	Other	Dem.	Rep.	Other	Dem.
Number of districts	759	803	298	1,988	1,546	716
Dem. 2024 map seats	193.0	444.7	215.4	604.1	843.0	529.2
Dem. geographic baseline	227.5	474.9	220.7	652.3	870.4	539.9
GOP Central Advantage	34.5	30.2	5.3	48.2	27.4	10.7

Table 5: Central Advantage in state legislatures, by partisan control.

3.3 Sensitivity analysis

We analyze the comparative static on the central geographic baseline with respect to changes in the district size, and changes in voting patterns.

First we compute the geographic baseline for neighborhood sizes from $10,000$ to $1,000,000$ in 10K increments. To provide better intuition, all results in this section are expressed in terms of the number of seats in the U.S. House of Representatives that is equivalent to the share of Democratic districts according to the baseline. For each neighborhood size, we take each state's share of residents for whom their neighborhood of closest neighbors votes Democratic, we multiply this share by the state's number of U.S. seats according to

[16]The state legislative maps of Louisiana, Mississippi and South Carolina, all drawn under Republican partisan control, concede a 2% Central Advantage to Democrats. We conjecture that efforts in these states to comply with protections for minority communities under the Voting Rights Act might explain the creation of more districts in which Democrats win a majority of votes, than those that would arise if the only goal were to eliminate political dislocation.

[17]The states we classify as GOP control are: AL, AR, FL, GA, IA (lower chamber), IN, KS, KY, LA, MS, NC, ND (upper chamber), NH (upper chamber), OK, SC, SD (upper chamber), TN, TX, UT, WV (lower chamber) and WY. Under Democratic control: DE, IL, MA, NM, NV, NY, OR and RI. All others in our sample we classify as "Other".

the 2020 apportionment, and we sum the results. This produces an aggregate baseline this is approximately state population based but includes the effect of the current apportionment method in the U.S, which slightly favors small states. The result is Figure 4 displays the U.S. aggregate geographic baselines for Democrats and Republicans in terms of U.S. Congressional seats. For reference, we include three benchmarks on the figure:

1. The party's *proportional seat share*. This is the seat share a party would win if in each state and each relevant election it were awarded its share of U.S. congressional seats proportional to the party's statewide vote.

2. The party's *state share*. This is the seat share a party would win if for each state and each relevant election it were allocated all of the U.S. congressional seats for the state if it won a majority of the vote in an election.

3. The party's *jurisdictional benchmark share*. This is the seat share a party would win if for each state and each relevant election it were allocated seats equal to the state's jurisdictional benchmark share.

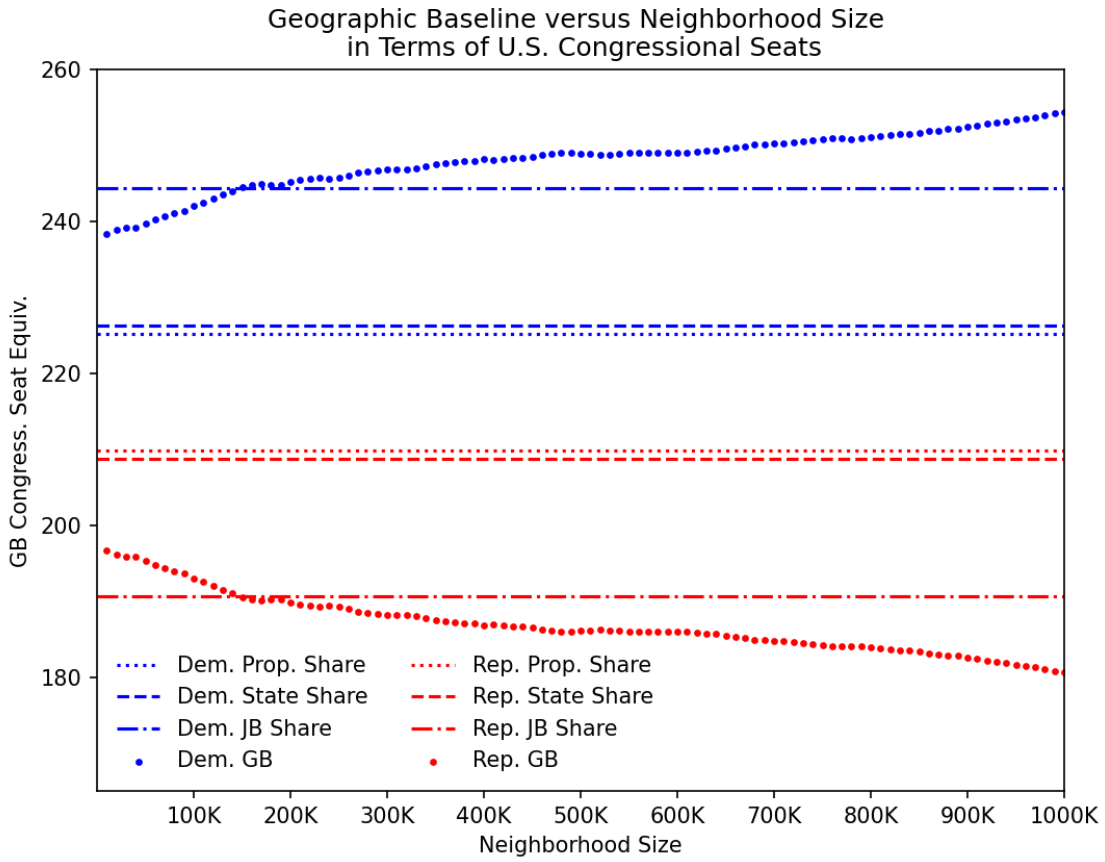


Figure 4: U.S. aggregate geographic baseline versus neighborhood size.

We see that the geographic baseline becomes more favorable for Democrats as the neighborhood size increases. An implication of the graph is that increases in the number of U.S. congressional representatives would put Republicans in a more advantageous geographic position.

As we discuss in the introduction, the explanation for this comparative static is that as neighborhood size increases, the "nearest neighborhood" of residents in rural areas adjacent to cities ceases to be rural (and Republican voter) and becomes majority urban (and Democratic voting), so for larger neighborhood sizes —implicitly, for a larger size of the notion of "local community"— there are fewer voters whose community is rural, and more whose community is urban, so minimizing political dislocation at larger scales requires more rep-

resentation of urban political preferences than at minimizing political dislocation at smaller scales. Accordingly, the Democratic central geographic baseline grows in district size.

We also analyze the sensitivity of the central geographic baseline to changes in vote-shares. We first note that in cross-sectional data, the geographic baseline as a function of vote-share, with a fixed district size, and variation in vote-share across different states tracks the cubic rule seat baseline reasonably well. We illustrate this relation in Figure 5, for a common district size of $60,000$, which is approximately the size of the average lower chamber district across states.

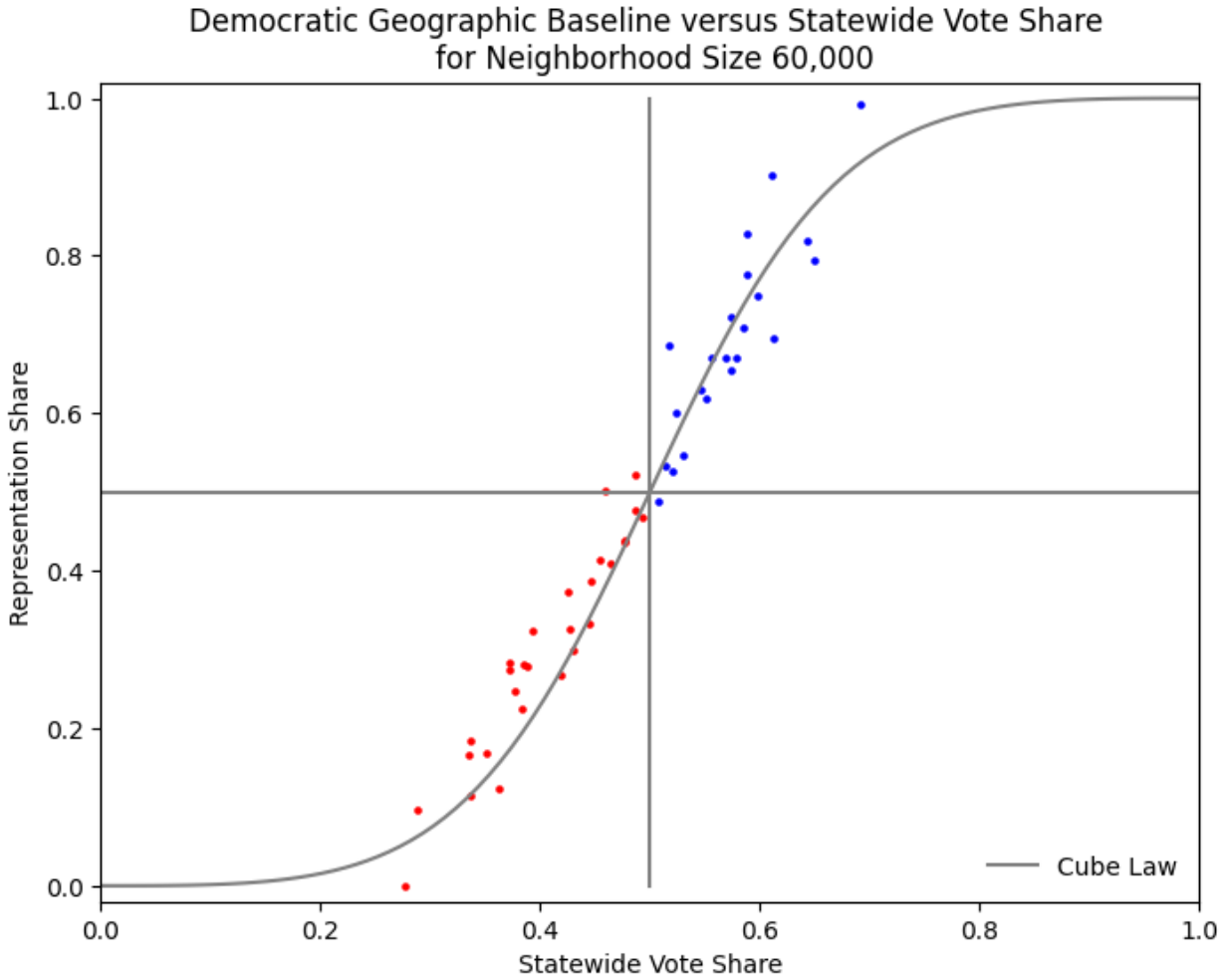


Figure 5: Geographic baseline as a function of Democratic vote-share across states.

We next study the effect of changes in vote-share across states over the aggregate central

geographic baseline. We estimate the "swing ratio" or "responsiveness" of the baseline, defined as the change in seat-share according to the baseline divided by the change in vote-share (Tufte 1973). We find a value of responsiveness of the central geographic benchmark ranging from approximately 2 if districts are as small as the typical state house district, to approximately 2.5 for larger districts the size of a congressional district.[18]

In this computation, we assume a proportional vote swing such that each party's vote total in every precinct is multiplied by the same factor, where each party's factor is chosen so that the total number of votes across parties stays constant, and the overall effect is a vote swing of exactly the specified size statewide. Figure 6 (top) shows the responsiveness of the central geographic baseline, for any district size, given vote-share swings of 2% and 4% in favor or against Democrats. Given such responsiveness, the bottom graph of Figure 6 extends Figure 4 to track the Democratic central geographic baseline at each district size given 2% or 4% vote swings.

[18]This compares to a responsiveness of the the jurisdictional baseline of about 2.2 (Eguia 2022), and a theoretical responsiveness of up to 3 of the cubic rule near equal vote-share, and 2 for the Efficiency Gap.

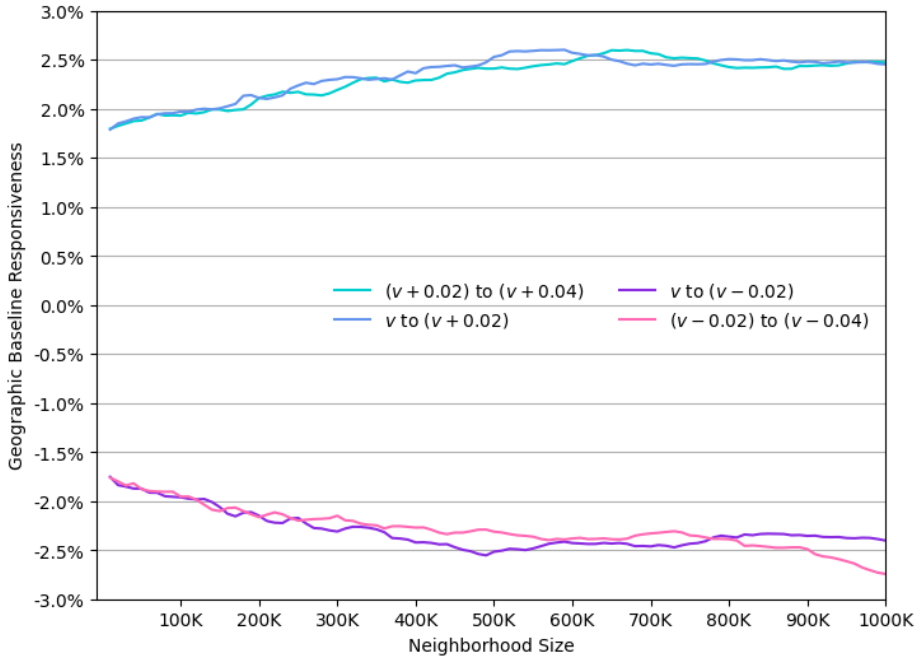
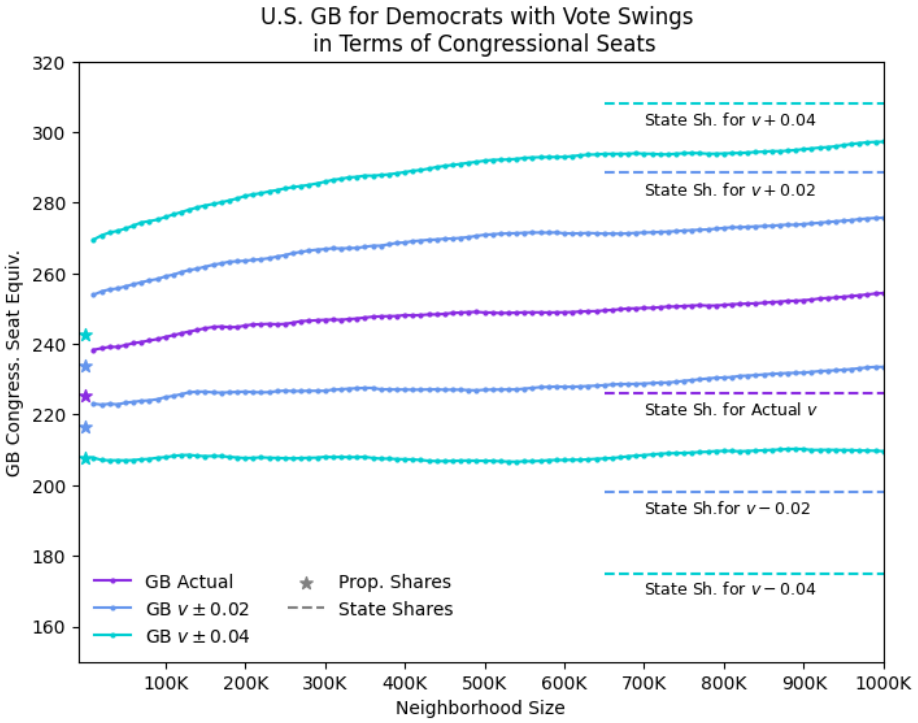


Figure 6: Responsiveness of the central geographic baseline to changes in vote-share (above), and the baseline as a function of changes in vote-share and district size (below).



3.4 Results for Individual States

We provide additional graphs to illustrate the results for individual states. We include four maps, coloring each precinct according to the proportion of elections in which each party won the precinct's ideal neighborhood, from more intense purple if Democrats win all elections, to more intense orange if Republicans win all elections. The first map uses neighborhoods of size smaller than the smallest precinct; the second, neighborhoods of size equal to the state's average state house district; the third, equal to the state's average state senate district, and the fourth, to the U.S. congressional district. For each state, we also provide a graph that shows the geographic baseline representation proportion for the minority party for neighborhood sizes ranging from $10,000$ to $1,000,000$ in $10K$ increments, as well as those corresponding to the average district sizes for the state's house, senate, and U.S. Congressional districts. For comparison we include the actual map average seat proportions Democrats earned for each body and the average jurisdictional benchmark (JB) for the state's U.S. Congressional seats. Finally, as a reference line we include the minority party's average statewide vote proportion.

We include here as an example the map of North Carolina. North Carolina's 2024 congressional map is its fourth in a decade, after the 2012-14 one was struck down as a racial gerrymander, its 2016-18 one was struck down as a partisan gerrymander, and a change in jurisprudence by the NC Supreme Court led the NC legislature to replace its remedial 2022 map with one for 2024 that the Court's previous jurisprudence would have deemed unconstitutional. We provide analogous information for selected states in Appendix B, online.

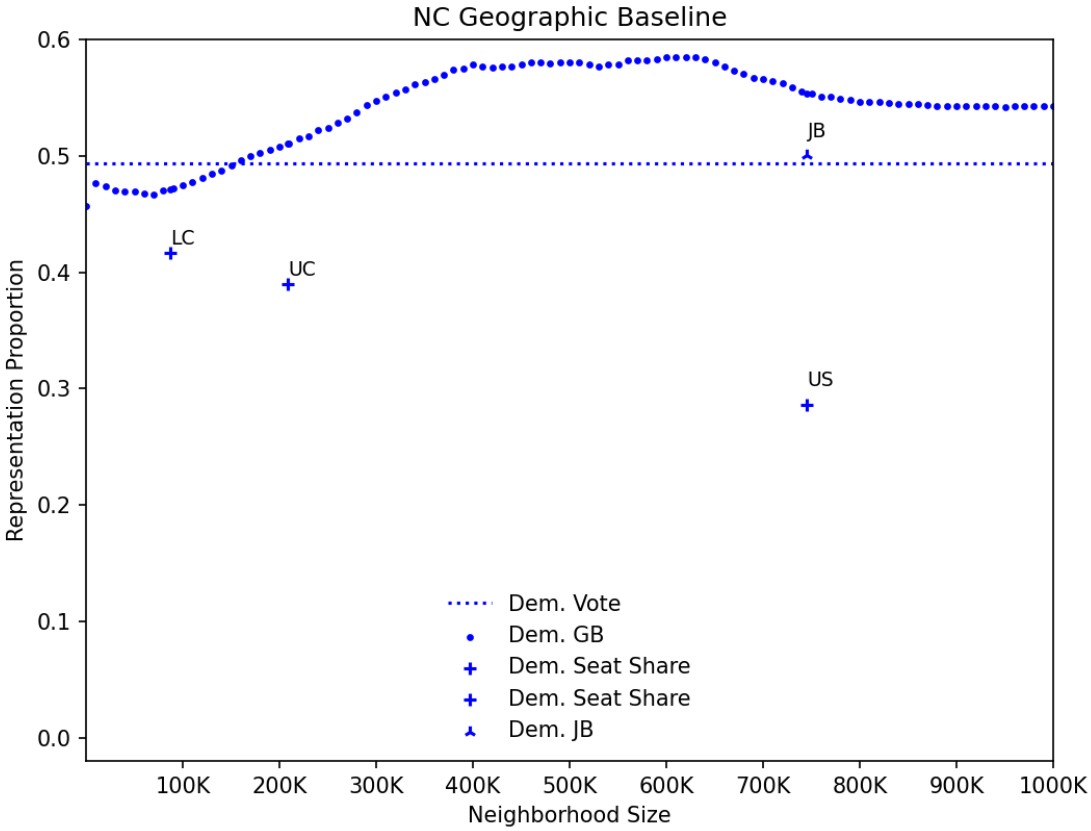


Figure 7: Geographic baseline for the minority party along with actual seat shares for the state's lower and upper chambers and for its U.S. congressional seats.

While North Carolina Democrats gain a majority of votes only in two out of six elections in our dataset, for large enough districts (at or above the size of the state senate districts), in four out of six of these elections, a majority of voters have a Democratic-majority neighborhood around them, and on average, Democrats obtain a central geographic baseline with more than half the seats (Figure 7). The color maps illustrate why: as long as neighborhoods are small (as in the two maps at the top), the Democratic-voting area around Halifax County to the Northwest of the state, and the Democratic-voting cluster in the Research Triangle (Raleigh, Durham and Chapel Hill) appear as two distinct Democratic areas, separated by Republican neighborhoods in between. Whereas, if neighborhoods are large (as in the bottom right map), the more heavily populated Democratic cluster sweeps these Repub-

lican areas in between, creating a larger area of connected Democratic neighborhoods, while to the South, a Democratic neighborhood similarly grows around Charlotte. These large Democratic neighborhoods would only be overwhelmed by the slight majority of dispersed Republican voters —collapsing the Democratic central geographic baseline— if votes are aggregated into a single district, which the majority party necessarily wins.

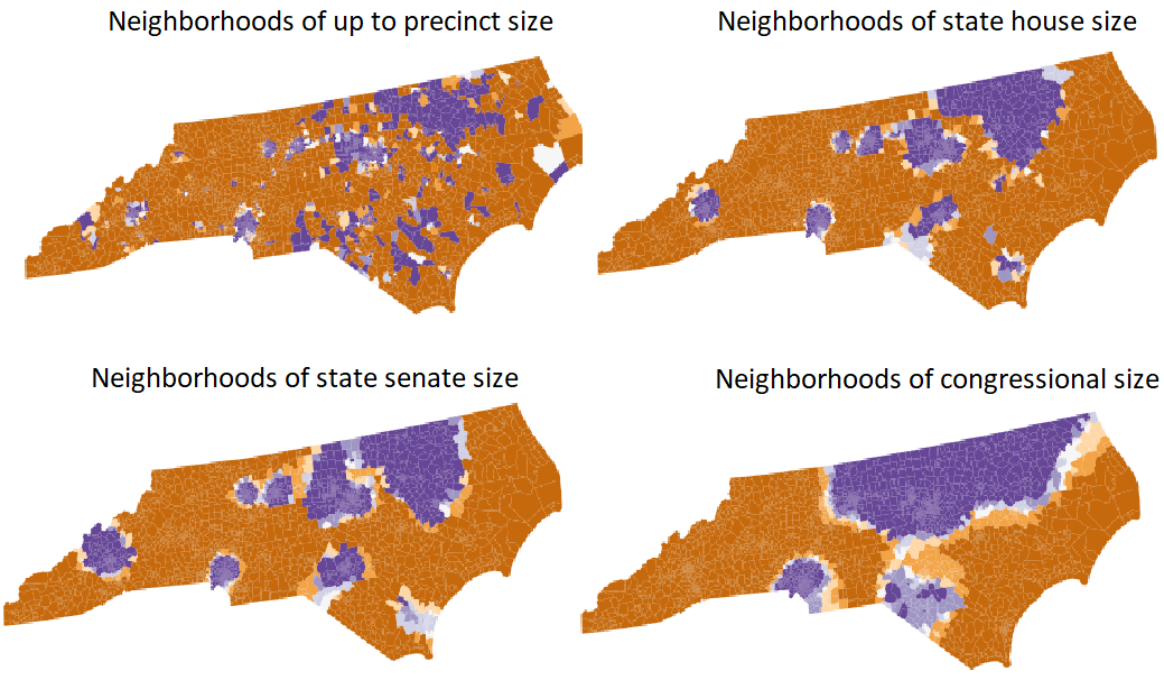


Figure 8: Probabilities of precinct neighborhoods being represented by a party for precincts, state house, state senate, and U.S. Congress.

We provide figures and maps analogous to those of Figure 7 and Figure 8 for each other state in an Online Appendix.

4 Discussion

The Central Advantage is a measure of partisan advantage in redistricting, measured in the number of seats that a party obtains above a geographic baseline underpinned by the concept of "political dislocation" (DeFord, Eubanks and Rodden 2022). To avoid political

dislocation, a voter must be represented by the party that most of her closest neighbors vote for, in a theoretical neighborhood with the voter at the center, and with the same population as a district. For a map to attain a perfect score of zero Central Advantage, each party should gain a seat share equal to the share of voters it would represent if no voter were politically dislocated. A party with more seats than that baseline has a positive Central Advantage.

This notion of partisan advantage is easy to compute for any redistricting map with any number of districts of any population size larger than the size of an individual precinct. It takes into account political geography, the distribution of votes for each party across the state, and the distribution of the non-voting population. Its normative appeal as a notion of partisan fairness derives from the idea that electoral districts are meant to represent local communities of voters defined by their close geographic proximity to each other, so that, in principle, each voter would be ideally represented by the majority preference of other voters around him.[19]

We demonstrate the ease of computation of the Central Advantage by providing results for congressional, state senate, and state house in each state and chamber that elects partisan representatives in single-member districts. We find that the central geographic baseline is more favorable (i.e. it assigns more seats) to Democrats than baselines that rely solely on statewide vote-share. We also show that the representation of the non-voting population, and the disproportionate geographic distribution of this population, which is by construction not considered by measures that rely solely on statewide vote share, accounts for much of this difference. We conjecture that the Central Advantage correlates much more closely with other measures of partisan advantage that do take into account political geography (such as

[19]Kromkowski (2002) discusses the evolution of this notion of local representation by geographically defined units of representation, from Colonial America under British rule, to the early decades of the newly independent Republic of United States. This geographic notion of representation is sharply in contrast to notions of representation by affinity, for instance in shared social class (nobility, clergy, and everyone else) in the États Généraux in the Ancien Régime in pre-revolutionary France; income bracket in the Centuriate Assembly in Republican Rome; ethnicity (Bosniak, Serb, or Croat) in the three-member Presidency of Bosnia; or ideology (Unionist, Nationalist, or neither) in the Northern Ireland Assembly and its two-member First Minister.

the jurisdictional advantage, or the outlier test using computational ensemble methods) and do show this closer correlation in the case of congressional districts, but we are not aware of comprehensive and publicly available results for other geographic measures of partisan fairness in the composition of state legislatures.

While an established measure of partisan advantage can be used to evaluate the partisan fairness of redistricting maps, in this paper we are introducing a new measure, and when introducing a new measure, we can reverse the exercise, using the results and our priors about which maps are fair to assess the face validity of the new measure. Following this reasoning, two findings on the congressional maps support our interpretation of the Central Advantage as a valid measure of partisan fairness. First, the five congressional maps with the largest Central Advantage are from states that are notorious for their partisan redistricting practices: Texas, Florida, North Carolina and Ohio, under Republican control, and Illinois under Democratic control. Second, Courts, non-partisan and bipartisan commissions, and legislatures under split control adopt maps with minimal Central Advantage. Hence, regardless of its other normative merits, at the very least we can conclude that the central geographic baseline number of seats is observationally close to the seat outcome generated by maps drawn without partisan control.

We interpret the Central Advantage as a valid measure of partisan fairness, one that quantifies the map's deviation from an ideal seat outcome based solely on the geographic distribution of voters and residents, under the ideal that citizens should be represented by a party that carries the majority preference of their nearest neighbors, which also underpins the notion of political dislocation (DeFord, Eubanks and Rodden). The representation of communities of voters defined by proximity in space is only one of several traditional redistricting goals. Communities whose limits are also determined by considerations such as ethnicity or language as spelled out under the Voting Rights Act; by affinity of cultural, economic or historic ties that lead to a "community of interests", or by jurisdictional boundaries such as county, city and township limits also seek representation. For this reason, we

see the Central Advantage not as a replacement, but rather as a complement to be used in conjunction with other measures of partisan advantage.

Under the notion of an ideally fair district map as one in which each voter is represented by the party preferred by the voter's local community, while defining the local community by geographic proximity leads to the Central Advantage as a measure of partisan fairness, other measures are useful too to take into account the political geography of communities that are defined not only by proximity, but also by their cultural or institutional attributes. Evaluating a map according to multiple measures, both those that take into account different aspects of political geography, and those that do not take geography into account at all, can lead to a more comprehensive understanding of the map's overall fairness.

5 Appendix

State	US Pres.	US Sen.	Governor	State	US Pres	US Sen	Governor
AK	16, 20	16, 20	18				
AL	16, 20	16, 20	18	MS	16, 20	18, 20	
AR	16, 20	16, 20	18	MT	16, 20	18, 20	16, 20
AZ	16, 20	16, 20	18	NC	16, 20	16, 20	16, 20
				ND	16, 20	16, 18	16, 20
CA	16, 20		18	NE	16, 20	18, 20	18
CO	16, 20	16, 20	18	NH	16, 20	16, 20	16, 18, 20
CT	16, 20	16, 18	18	NJ	16, 20	18, 20	
DE	16, 20	18, 20	16, 20				
FL	16, 20	16, 18	18	NM	16, 20	18, 20	18
GA	16, 20	16, 20	18	NV	16, 20	16, 18	18
HI	16, 20	16, 18	18	NY	16, 20	16, 18	18
IA	16, 20	16, 20	18	OH	16, 20	16, 18	18
ID	16, 20	16, 20	18	OK	16, 20	16, 20	18
IL	16, 20	16, 20	18	OR	16, 20	16, 20	16, 18
IN	16, 20	16, 18	16, 20	PA	16, 20	16, 18	18
KS	16, 20	16, 20	18	RI	16, 20	18, 20	18
KY	16, 20	16, 20		SC	16, 20	16, 20	18
				SD	16, 20	16, 20	18
LA	16, 20	16, 20		TN	16, 20	18, 20	18
MA	16, 20	18, 20	18	TX	16, 20	18, 20	18
MD	16, 20	16, 18	18	UT	16, 20	16, 18	16, 20
ME	16, 20	20	18	VA	16, 20	18, 20	
				VT	16, 20	16	16, 18, 20
MI	16, 20	18, 20	18	WA	16, 20	16, 18	16, 20
MN	16, 20	18, 20	18	WI	16, 20	16, 18	18
MO	16, 20	16, 18	16, 20	WV	16, 20	18, 20	16, 20
				WY	16, 20	18, 20	18

Table 6: 2016, 2018 and 2020 elections included in our analysis, per state. For states with only one U.S. House district, a party's jurisdictional benchmark is the fraction of elections in which the party has the majority of the statewide vote.

5.1 U.S. Congressional seat results

Table 7: Democratic congressional geographic baseline, by state.

State	U.S. seats	Average. Democratic vote-share	Democratic Actual Map seats	Democratic Geographic baseline	Democratic Jurisdict. baseline	Democratic Cube Law baseline
Total	435	51.8%	222.55	251.03	241.07	236.44
AL	7	37.8%	2.0	1.00	1.57	1.30
AK	1	39.4%	0.0	0.00	0.00	0.27
AZ	9	47.8%	3.4	4.49	3.59	3.93
AR	4	35.1%	0.0	0.04	0.75	0.55
CA	52	64.3%	44.7	44.73	42.92	44.32
CO	8	54.6%	4.8	5.39	5.09	5.07
CT	5	58.8%	4.6	4.67	4.61	3.66
DE	1	59.7%	1.0	1.00	1.00	0.76
FL	28	48.7%	9.4	13.70	14.51	12.90
GA	14	47.0%	5.0	5.38	6.94	6.08
HI	2	69.1%	2.0	2.00	2.00	1.82
ID	2	33.7%	0.0	0.00	0.18	0.25
IL	17	58.%5	14.0	11.65	12.08	12.54
IN	9	42.8%	2.0	2.84	2.90	2.73
IA	4	44.6%	0.6	2.02	1.40	1.40
KS	4	42.5%	0.8	1.45	1.27	1.22
KY	6	38.4%	1.3	1.55	1.55	1.21
LA	6	38.5%	1.0	1.32	1.46	1.19
ME	2	51.4%	1.3	1.38	1.02	1.09
MD	8	61.3%	6.4	6.73	5.34	6.10
MA	9	58.8%	7.4	7.43	7.09	6.44
MI	13	52.1%	7.2	6.57	7.28	7.30
MN	8	55.1%	4.8	5.10	4.99	5.14

State	U.S. seats	Average. Democratic vote-share	Democratic Actual Map seats	Democratic Geographic baseline	Democratic Jurisdict. baseline	Democratic Cube Law baseline
MS	4	41.9%	1.0	0.65	1.12	1.10
MO	8	44.4%	2.0	3.58	3.00	2.75
MT	2	45.4%	0.3	0.74	0.80	0.75
NE	3	37.2%	0.4	0.73	1.11	0.56
NV	4	51.7%	3.0	2.99	3.01	2.21
NH	2	48.7%	1.0	0.80	0.90	0.95
NJ	12	57.4%	9.5	9.61	9.03	8.50
NM	3	56.9%	3.0	2.33	2.12	2.07
NY	26	65.0%	21.8	22.78	21.56	22.22
NC	14	49.3%	4.0	7.75	7.01	6.72
ND	1	28.9%	0.0	0.00	0.00	0.10
OH	15	46.4%	4.8	7.18	6.29	6.02
OK	5	33.7%	0.0	0.44	0.30	0.68
OR	6	57.3%	4.5	4.48	3.77	4.21
PA	17	53.0%	9.0	9.10	9.22	9.92
RI	2	61.1%	2.0	2.00	1.88	1.58
SC	7	43.0%	1.0	2.21	1.59	2.14
SD	1	36.3%	0.0	0.00	0.00	0.18
TN	9	38.9%	1.2	2.89	2.17	1.90
TX	38	45.9%	13.6	19.71	17.66	14.45
UT	4	33.5%	0.0	0.71	0.54	0.49
VT	1	52.4%	0.5	0.50	0.50	0.56
VA	11	55.5%	6.8	8.17	7.26	7.25
WA	10	57.9%	6.7	6.86	7.26	7.20
WV	2	37.2%	0.3	0.49	0.00	0.48
WI	8	50.8%	2.6	3.80	3.43	4.19
WY	1	27.8%	0.0	0.00	0.00	0.06

Table 8: GOP Central Advantage for all 50 states (negative values favor Democrats).

State	U.S. seats	Avg. Dem. vote	GOP Central Advantage	GOP Jurisd. Advantage	GOP seats −Cubic rule
Total	435	51.8%	28.48	18.52	14.11
AL	7	37.8%	-1.00	-0.43	-0.70
AK	1	39.4%	0.00	0.00	0.27
AZ	9	47.8%	1.09	0.19	0.53
AR	4	35.1%	0.04	0.75	0.55
CA	52	64.3%	0.09	-1.75	-0.35
CO	8	54.6%	0.59	0.29	0.27
CT	5	58.8%	0.07	0.01	-0.94
DE	1	59.7%	0.00	0.00	-0.24
FL	28	48.7%	4.30	5.11	3.50
GA	14	47.7%	0.38	1.94	1.08
HI	2	69.1%	0.00	0.00	-0.18
ID	2	33.7%	0.00	0.18	0.25
IL	17	58.5%	-2.35	-1.92	-1.46
IN	9	42.8%	0.84	0.90	0.73
IA	4	44.6%	1.42	0.80	0.80
KS	4	42.5%	0.65	0.47	0.42
KY	6	38.4%	0.30	0.30	-0.04
LA	6	38.5%	0.32	0.46	0.19
ME	2	51.4%	0.13	-0.23	-0.16
MD	8	61.3%	0.33	-1.06	-0.30
MA	9	58.8%	0.03	0.19	-0.96
MI	13	52.1%	-0.63	0.08	0.10
MN	8	55.1%	0.30	0.19	0.34

State	U.S. seats	Avg. Dem. vote	GOP Central Advantage	GOP Jurisd. Advantage	GOP seats −Cubic rule
MS	4	41.9%	-0.35	0.12	0.10
MO	8	41.9%	1.58	1.00	0.75
MT	2	45.4%	0.41	0.47	0.42
NE	3	37.2%	0.33	0.71	0.16
NV	4	51.7%	-0.01	0.01	-0.79
NH	2	48.7%	-0.20	-0.10	-0.05
NJ	12	57.4%	0.11	-0.47	-1.00
NM	3	56.9%	-0.67	-0.88	-0.93
NY	26	65.0%	1.03	-0.24	0.42
NC	14	49.3%	3.75	3.01	2.72
ND	1	28.9%	0.00	0.00	0.10
OH	15	46.4%	2.38	1.49	1.22
OK	5	33.7%	0.44	0.30	0.68
OR	6	57.3%	-0.02	-0.73	-0.29
PA	17	53.0%	0.10	0.22	0.92
RI	2	61.1%	0.00	-0.12	-0.42
SC	7	43.0%	1.21	0.59	1.14
SD	1	36.3%	0.00	0.00	0.18
TN	9	38.9%	1.69	0.97	0.70
TX	38	45.9%	6.11	4.06	0.85
UT	4	33.5%	0.71	0.54	0.49
VT	1	52.4%	0.00	0.00	0.06
VA	11	55.5%	1.42	0.51	0.50
WA	10	57.9%	0.19	0.59	0.53
WV	2	37.2%	0.15	-0.33	0.15
WI	8	50.8%	1.20	0.83	1.59
WY	1	27.8%	0.00	0.00	0.06

5.2 Upper Chamber Results

Table 9: Democratic central geographic baseline in state legislative upper chambers.

State	Upper Chamber seats	Average. Democratic vote-share	Democratic Actual Map seats	Democratic Geographic baseline	Democratic Cube Law baseline
Total	1860	51.8%	853.05	923.10	880.54
AL	35	37.8%	8.20	9.76	6.49
AK	20	39.4%	6.20	6.40	5.32
AZ	30	47.8%	13.20	13.26	13.09
AR	35	35.1%	5.80	5.73	4.84
CA	40	64.3%	33.00	34.80	34.09
CO	35	54.6%	22.80	22.78	22.17
CT	36	58.8%	29.60	30.18	26.37
DE	21	59.7%	15.50	15.69	16.03
FL	40	48.7%	17.20	19.53	18.43
GA	56	47.7%	22.20	25.25	24.33
HI	25	69.1%	24.80	24.79	22.70
ID	35	33.7%	4.60	4.32	4.29
IL	59	58.5%	42.80	41.97	43.51
IN	50	42.8%	10.83	16.10	15.15
IA	50	44.6%	15.20	19.47	17.49
KS	40	42.5%	11.80	14.98	12.20
KY	38	38.4%	6.00	8.69	7.68
LA	39	38.5%	10.75	9.68	7.74
ME	35	51.4%	18.25	18.70	18.99
MD	47	61.3%	31.00	33.09	35.81
MA	40	58.8%	31.40	32.00	28.61
MI	38	52.1%	20.60	20.90	21.34
MN	67	55.1%	38.60	41.62	43.02

State	Upper Chamber seats	Average. Democratic vote-share	Democratic Actual Map seats	Democratic Geographic baseline	Democratic Cube Law baseline
MS	52	41.9%	15.00	14.25	14.32
MO	34	44.4%	10.00	11.73	11.67
MT	50	45.4%	21.00	20.63	18.72
NV	21	51.7%	14.40	15.51	11.58
NH	24	48.7%	9.43	12.42	11.39
NJ	40	57.4%	28.00	30.15	28.33
NM	42	56.9%	28.80	28.26	28.93
NY	63	65.0%	50.80	52.88	53.83
NC	50	49.3%	19.50	25.53	23.99
ND	47	28.9%	5.17	5.07	4.71
OH	33	46.4%	10.80	15.62	13.24
OK	48	33.7%	6.80	8.47	6.49
OR	30	57.3%	19.67	20.94	21.04
PA	50	53.0%	26.40	28.86	29.16
RI	38	61.1%	31.80	33.23	30.01
SC	46	43.0%	15.40	13.64	14.09
SD	35	36.3%	4.20	4.81	6.38
TN	33	38.9%	6.00	9.16	6.97
TX	31	45.9%	12.00	16.42	11.78
UT	29	33.5%	4.50	4.73	3.55
VA	40	55.5%	24.75	28.06	26.38
WA	49	57.9%	30.50	33.92	35.28
WI	33	50.8%	15.60	15.81	17.29
WY	31	27.8%	2.20	3.28	1.73

Table 10: GOP Central Advantage in state legislative upper chambers.

State	Chamber seats	Avg. Dem. vote	GOP Central Advantage	GOP seats −Cubic rule
Total	1869	51.8%	70.05	27.50
AL	35	37.8%	1.56	-1.71
AK	20	39.4%	0.20	-0.88
AZ	30	47.8%	0.06	-0.11
AR	35	35.1%	-0.07	-0.96
CA	40	64.3%	1.80	1.09
CO	35	54.6%	-0.02	-0.63
CT	36	58.8%	0.58	-3.23
DE	21	59.7%	0.19	0.53
FL	40	48.7%	2.33	1.23
GA	56	47.7%	3.05	2.13
HI	25	69.1%	-0.01	-2.10
ID	35	33.7%	-0.28	-0.31
IL	59	58.5%	-0.83	0.71
IN	50	42.8%	5.27	4.32
IA	50	44.6%	4.27	2.29
KS	40	42.5%	3.18	0.40
KY	38	38.4%	2.69	1.68
LA	39	38.5%	-1.07	-3.01
ME	35	51.4%	0.45	0.74
MD	47	61.3%	2.09	4.81
MA	40	58.8%	0.60	-2.79
MI	38	52.1%	0.30	0.74
MN	67	55.1%	3.02	4.42

State	Chamber seats	Avg. Dem. vote	GOP Central Advantage	GOP seats −Cubic rule
MS	52	41.9%	-0.75	-0.68
MO	34	44.4%	1.74	1.67
MT	50	45.4%	-0.37	-2.28
NV	21	51.7%	1.11	-2.82
NH	24	48.7%	2.99	1.96
NJ	40	57.4%	2.15	0.33
NM	42	56.9%	-0.54	0.13
NY	63	65.0%	2.08	3.03
NC	50	49.3%	6.03	4.49
ND	47	28.9%	-0.10	-0.46
OH	33	46.4%	4.82	2.44
OK	48	33.7%	1.67	-0.31
OR	30	57.3%	1.28	1.37
PA	50	53.0%	2.46	2.76
RI	38	61.1%	1.43	-1.79
SC	46	43.0%	-1.76	-1.31
SD	35	36.3%	0.61	2.18
TN	33	38.9%	3.16	0.97
TX	31	45.9%	4.42	-0.22
UT	29	33.5%	0.23	-0.95
VA	40	55.5%	3.31	1.63
WA	49	57.9%	3.44	4.78
WI	33	50.8%	0.21	1.69
WY	31	27.8%	1.08	-0.47

5.3 Lower Chamber Results

Table 11: Democratic central geographic baseline in state legislative lower chambers.

State	Lower Chamber seats	Average. Democratic vote-share	Democratic Actual Map seats	Democratic Geographic baseline	Democratic Cube Law baseline
Total	4,250	51.8%	1,976.35	2,062.54	1,978.22
AL	105	37.8%	28.40	25.61	19.48
AK	40	39.4%	12.40	12.31	10.64
AR	100	35.1%	16.40	17.42	13.83
CA	80	64.3%	66.00	66.74	68.19
CO	65	54.6%	43.20	41.58	41.18
CT	151	58.8%	117.80	122.99	110.62
DE	41	59.7%	29.50	30.32	31.30
FL	120	48.7%	50.80	57.48	55.28
GA	180	47.7%	78.60	78.43	78.20
HI	51	69.1%	50.00	50.24	46.31
IL	118	58.5%	84.60	84.83	87.02
IN	100	42.8%	28.67	32.82	30.30
IA	100	44.6%	32.60	39.96	34.98
KS	125	42.5%	40.80	46.67	38.11
KY	100	38.4%	18.25	22.81	20.20
LA	105	38.5%	31.00	29.35	20.85
ME	151	51.4%	69.75	75.16	81.94
MA	160	58.8%	120.20	123.57	114.42
MI	110	52.1%	56.20	56.98	61.77

State	Lower Chamber seats	Average. Democratic vote-share	Democratic Actual Map seats	Democratic Geographic baseline	Democratic Cube Law baseline
MN	134	55.1%	79.60	82.85	86.04
MS	122	41.9%	43.50	41.29	33.61
MO	163	44.4%	53.33	53.29	55.94
MT	100	45.4%	43.67	38.72	37.45
NV	42	51.7%	28.40	29.01	23.16
NM	70	56.9%	48.80	46.79	48.22
NY	150	65.0%	116.60	121.57	128.16
NC	120	49.3%	50.00	56.52	57.58
OH	99	46.4%	36.20	41.99	39.71
OK	101	33..7%	16.60	19.53	13.66
OR	60	57.3%	38.50	39.55	42.07
PA	203	53.0%	107.00	111.30	118.40
RI	75	61.1%	62.60	64.26	59.23
SC	124	43.0%	39.80	36.17	37.97
TN	99	38.9%	23.60	27.11	20.90
TX	150	45.9%	64.40	74.32	57.02
UT	75	33.5%	10.83	12.73	9.18
VA	100	55.5%	60.25	67.85	65.95
WV	100	37.0%2	24.50	27.71	24.03
WI	99	50.8%	47.60	48.38	51.88
WY	62	27.8%	5.40	6.33	3.46

Table 12: GOP Central Advantage in state legislative lower chambers.

State	Chamber seats	Avg. Dem. vote	GOP Central Advantage	GOP seats −Cubic rule
Total	4,250	51.8%	86.19	1.87
AL	105	37.8%	-2.79	-8.92
AK	40	39.4%	-0.09	-1.76
AR	100	35.1%	1.02	-2.57
CA	80	64.3%	0.74	2.19
CO	65	54.6%	-1.62	-2.02
CT	151	58.8%	5.19	-7.18
DE	41	59.7%	0.82	1.80
FL	120	48.7%	6.68	4.48
GA	180	47.7%	-0.17	-0.40
HI	51	69.1%	0.24	-3.69
IL	118	58.5%	0.23	2.42
IN	100	42.8%	4.16	1.63
IA	100	44.6%	7.36	2.38
KS	125	42.5%	5.87	-2.69
KY	100	38.4%	4.56	1.95
LA	105	38.5%	-1.65	-10.15
ME	151	51.4%	5.41	12.19
MA	160	58.8%	3.37	-5.78
MI	110	52.1%	0.78	5.57

State	Chamber seats	Avg. Dem. vote	GOP Central Advantage	GOP seats −Cubic rule
MN	134	55.1%	3.25	6.44
MS	122	41.9%	-2.21	-9.89
MO	163	44.4%	-0.04	2.60
MT	100	45.4%	-4.94	-6.22
NV	42	51.7%	0.61	-5.24
NM	70	56.9%	-2.01	-0.58
NY	150	65.0%	4.97	11.56
NC	120	49.3%	6.52	7.58
OH	99	46.4%	5.79	3.51
OK	101	33.7%	2.93	-2.94
OR	60	57.3%	1.05	3.57
PA	203	53.0%	4.30	11.40
RI	75	61.1%	1.66	-3.37
SC	124	43.0%	-3.63	-1.83
TN	99	38.9%	3.51	-2.70
TX	150	45.9%	9.92	-7.38
UT	75	33.5%	1.90	-1.65
VA	100	55.5%	7.60	5.70
WV	100	37.2%	3.21	-0.47
WI	99	50.8%	0.78	4.28
WY	62	27.8%	0.93	-1.94

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6 Appendix B: Results for each State

6.1 Arizona

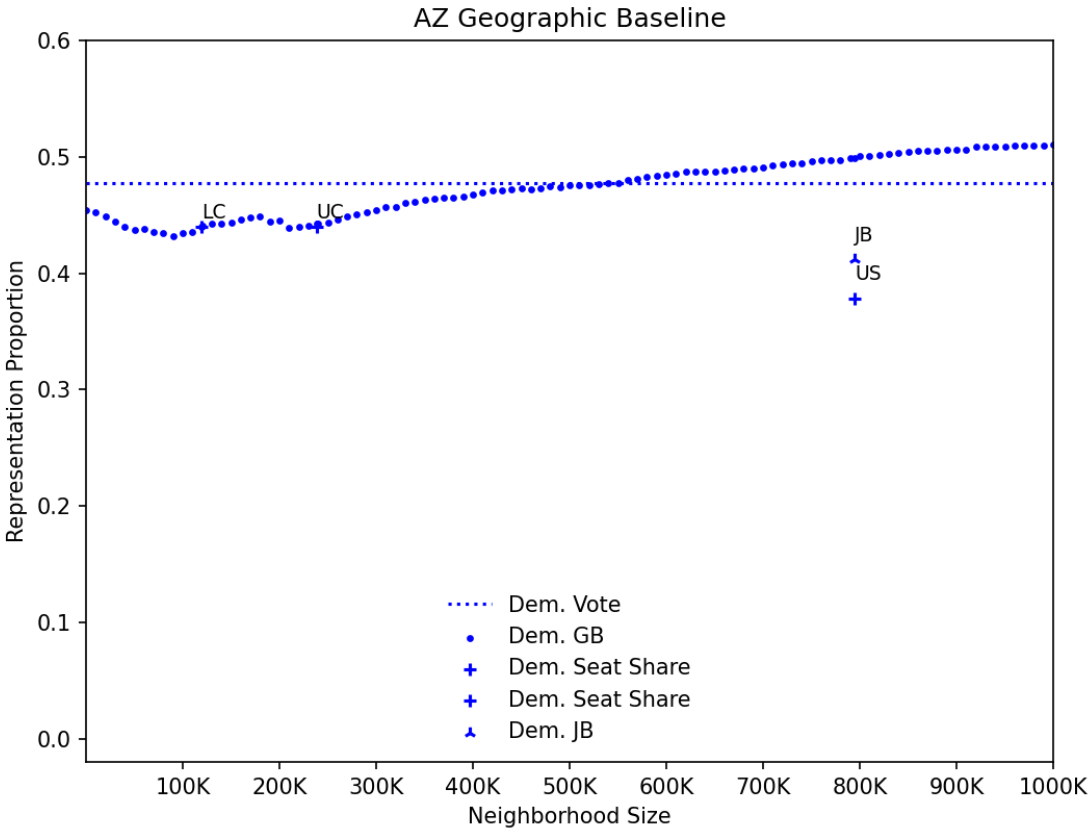
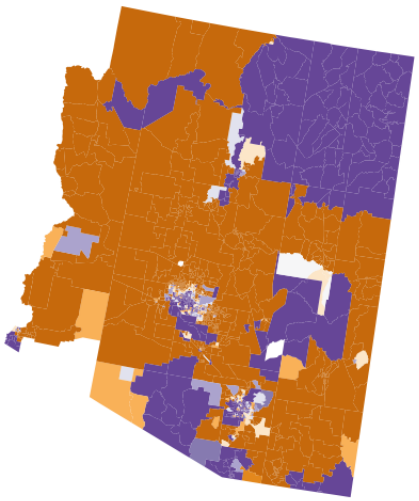


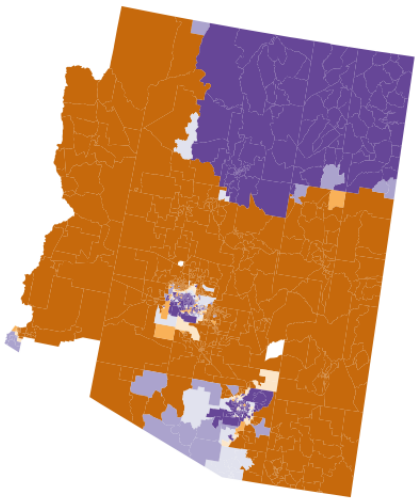
Figure 9: Geographic baseline for the minority party along with actual seat shares for the state's lower and upper chambers and for its U.S. congressional seats.

AZ Precinct Neighborhood Partisan Probabilities

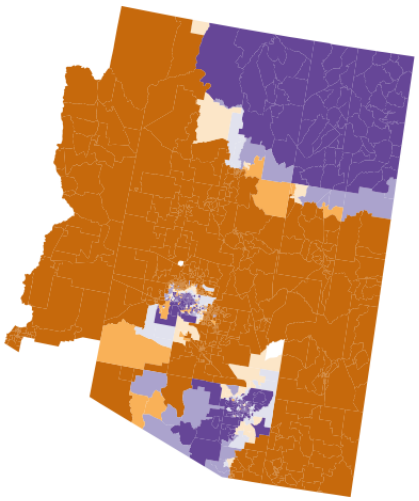
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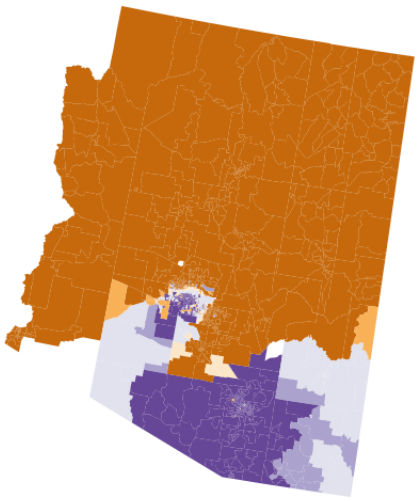
State House, GB = 0.440



State Senate, GB = 0.442



U.S. Congress, GB = 0.499



6.2 California

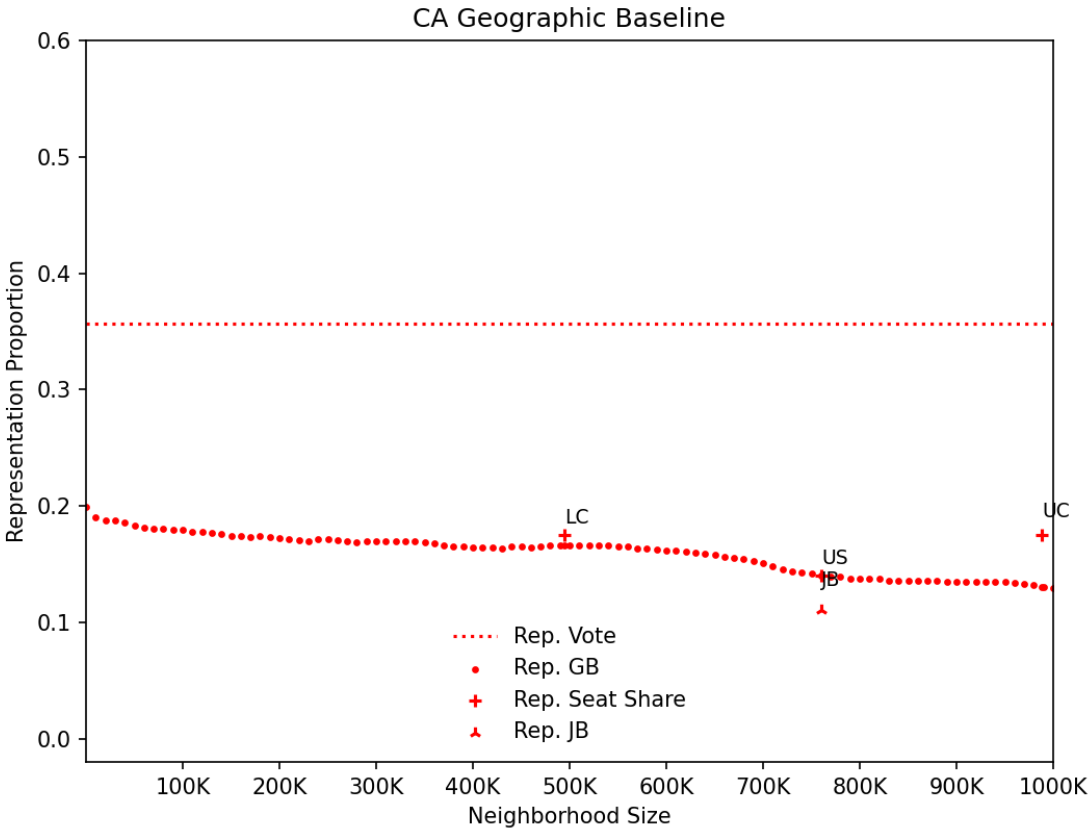


Figure 11: Geographic baseline for the minority party along with actual seat shares for the state's lower and upper chambers and for its U.S. congressional seats.

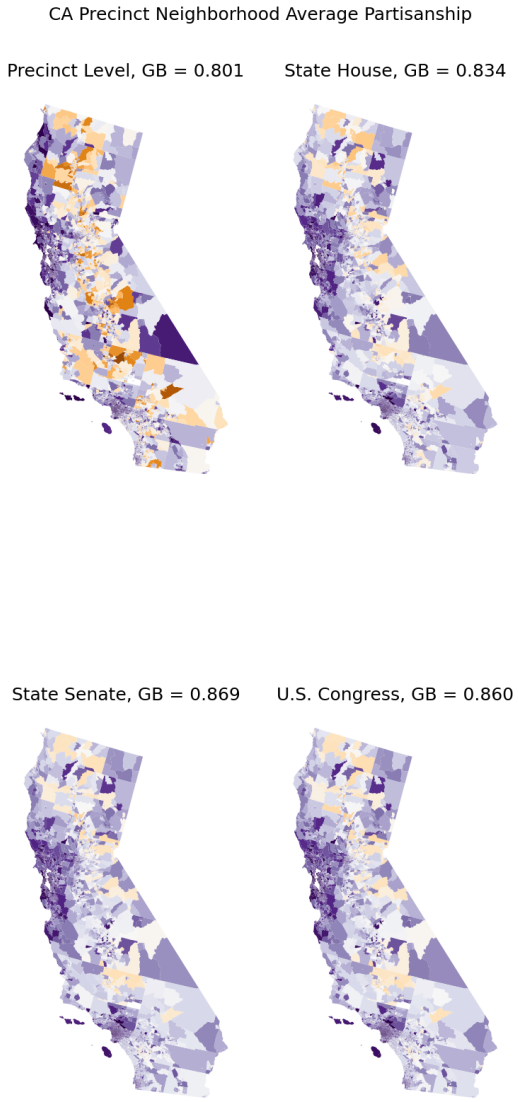


Figure 12: Territories induced by the geographic baseline for precincts, state house, state senate, and U.S. Congress.

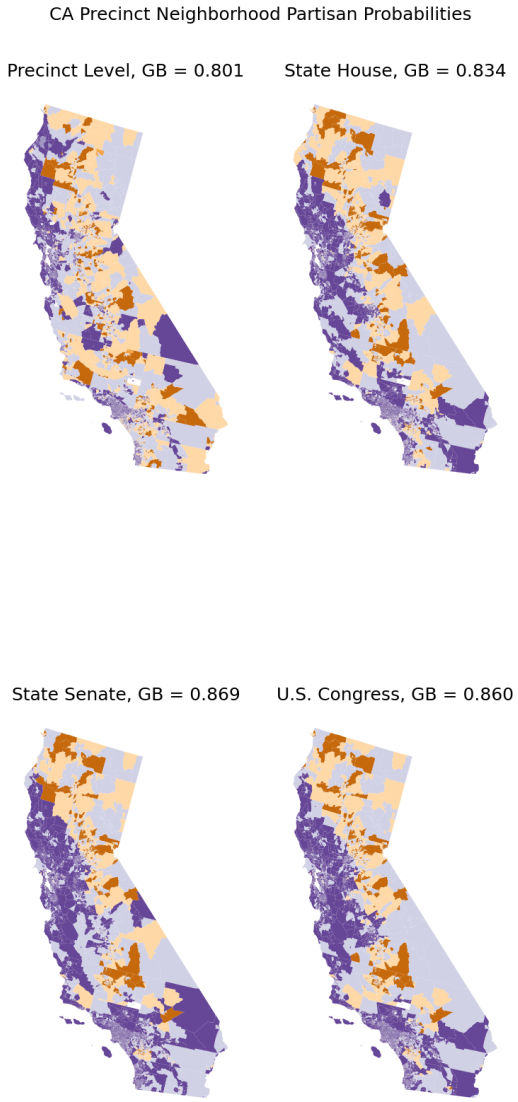


Figure 13: Territories induced by the geographic baseline for precincts, state house, state senate, and U.S. Congress.

6.3 Florida

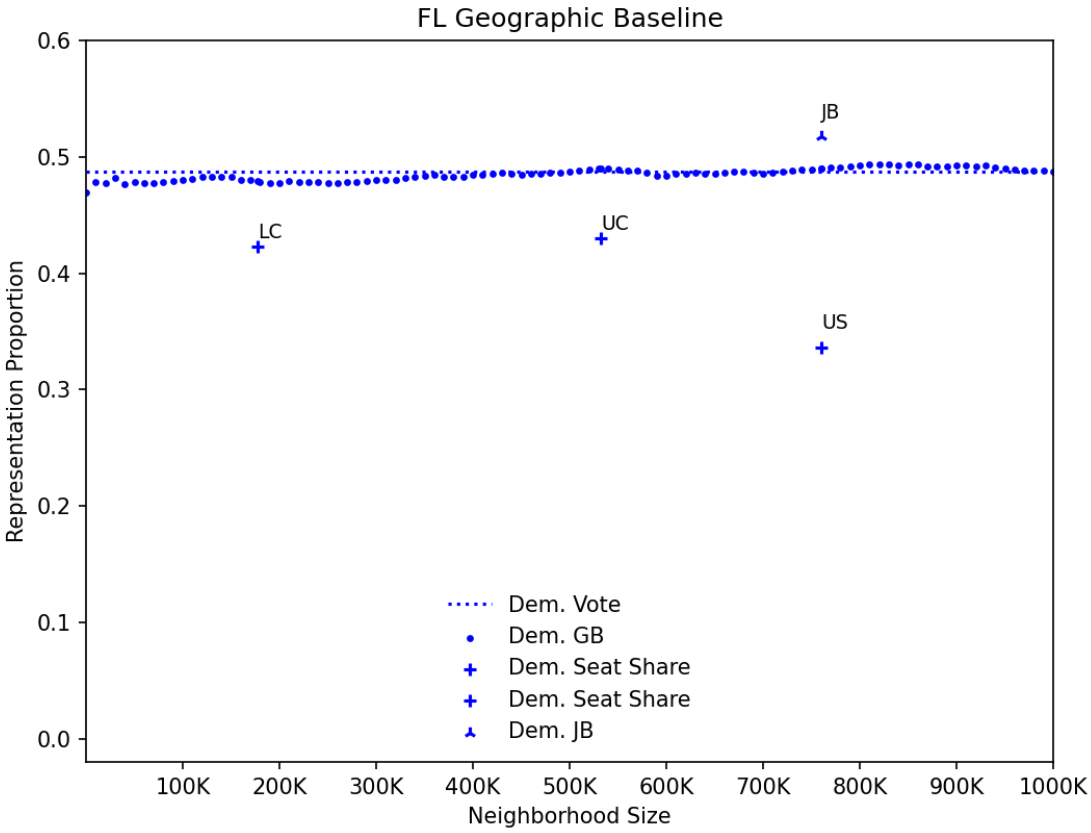


Figure 14: Geographic baseline for the minority party along with actual seat shares for the state's lower and upper chambers and for its U.S. congressional seats.

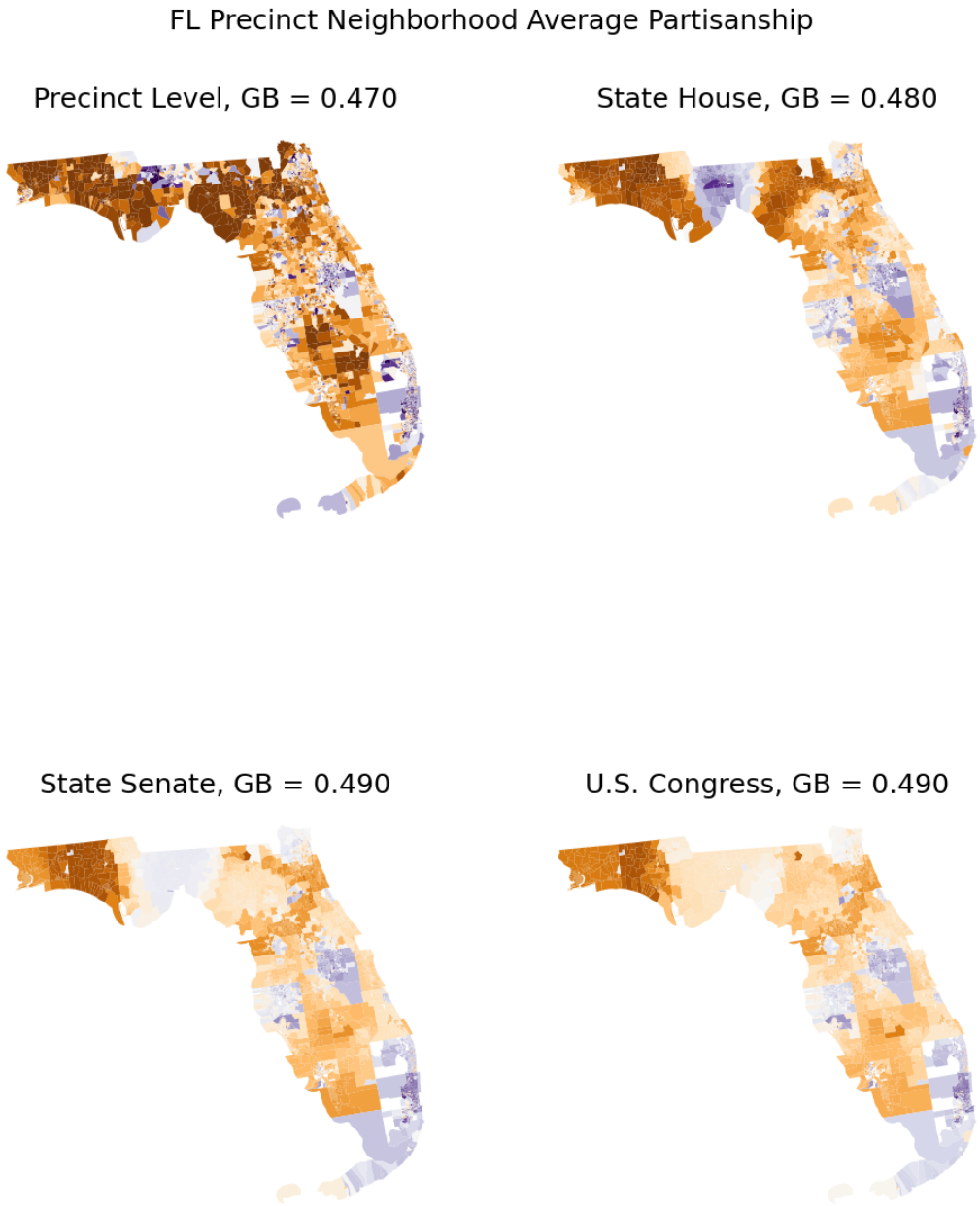


Figure 15: Territories induced by the geographic baseline for precincts, state house, state senate, and U.S. Congress.

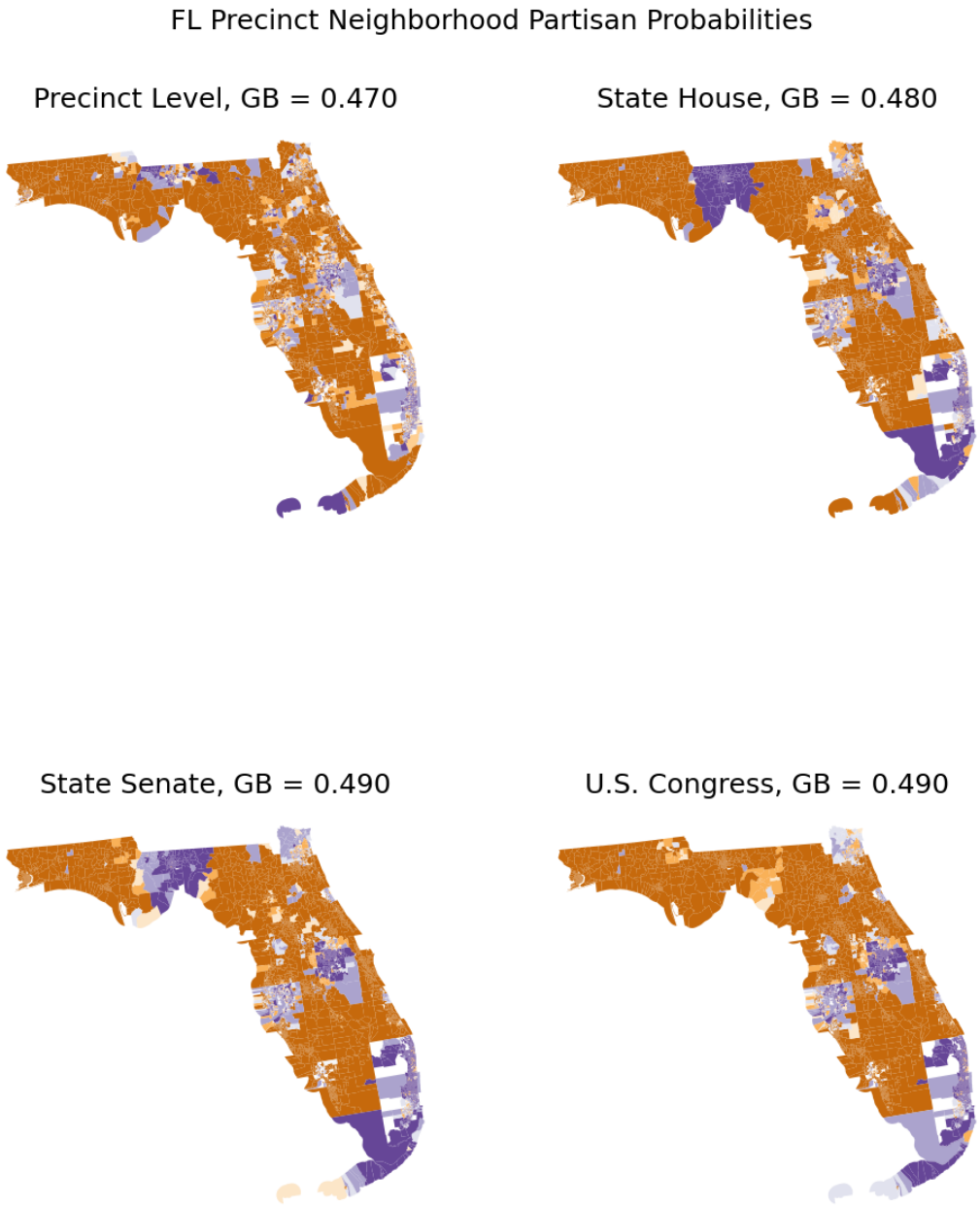


Figure 16: Territories induced by the geographic baseline for precincts, state house, state senate, and U.S. Congress.

6.4 Georgia

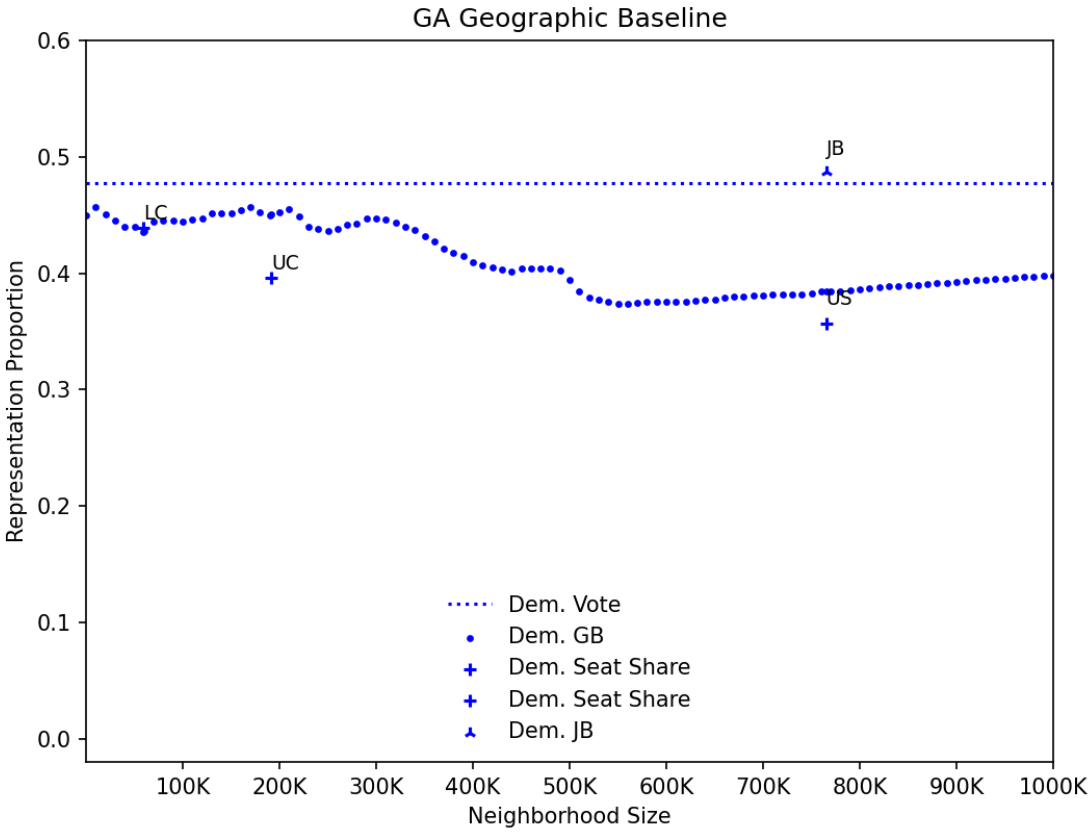
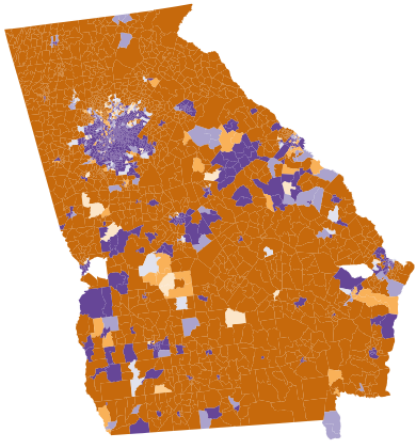


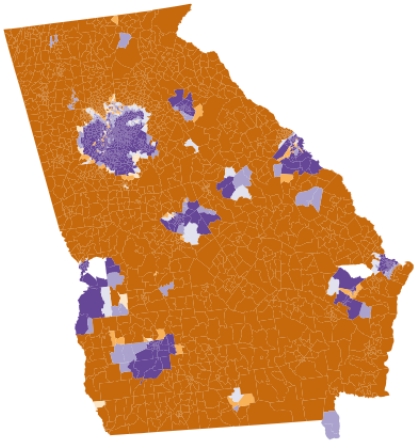
Figure 17: Geographic baseline for the minority party along with actual seat shares for the state's lower and upper chambers and for its U.S. congressional seats.

GA Precinct Neighborhood Partisan Probabilities

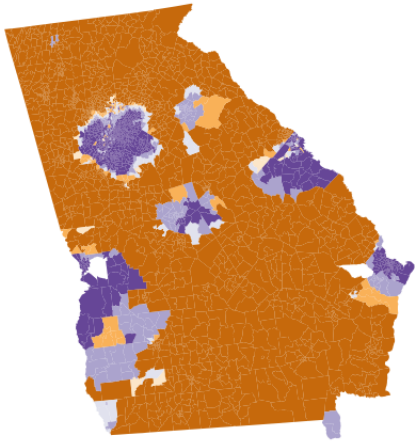
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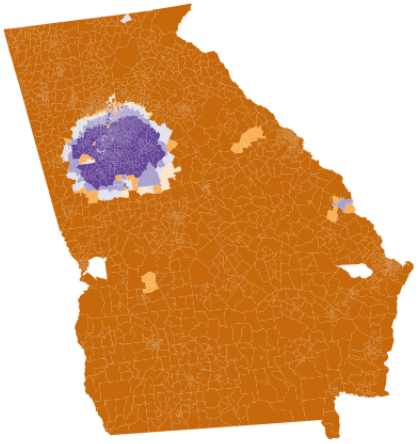
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State Senate, GB = 0.451



U.S. Congress, GB = 0.384



6.5 Massachusetts

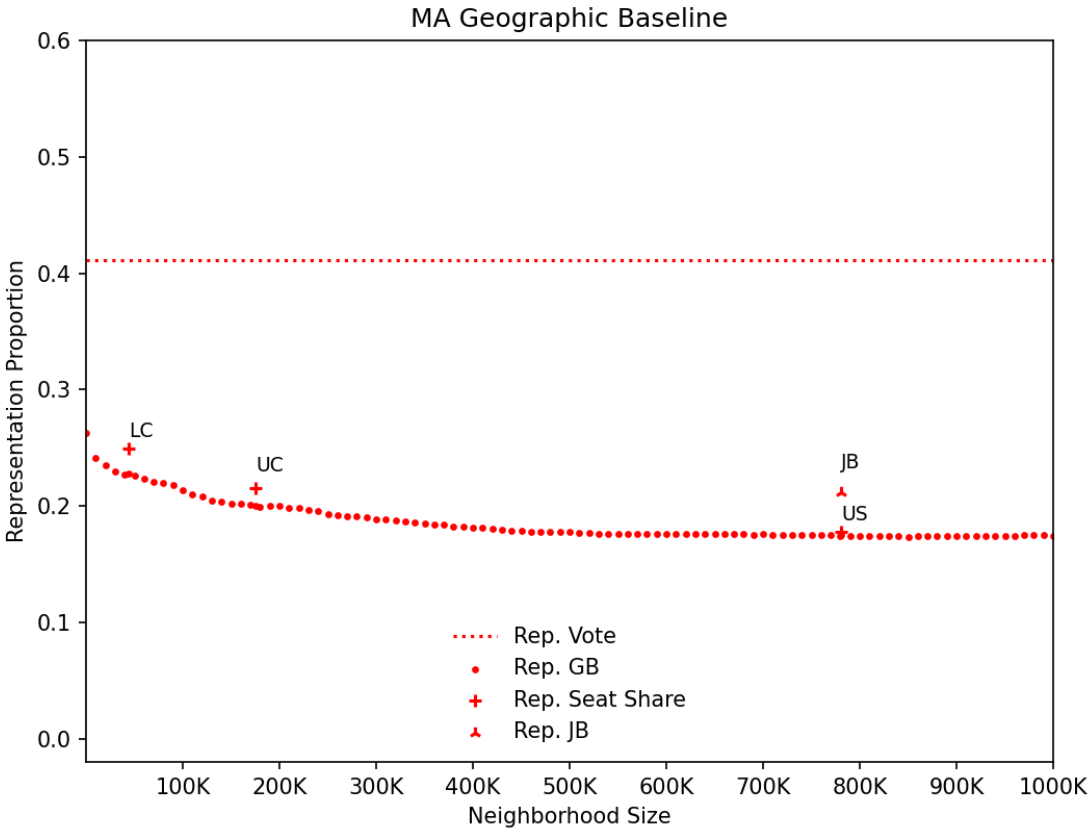


Figure 19: Geographic baseline for the minority party along with actual seat shares for the state's lower and upper chambers and for its U.S. congressional seats.

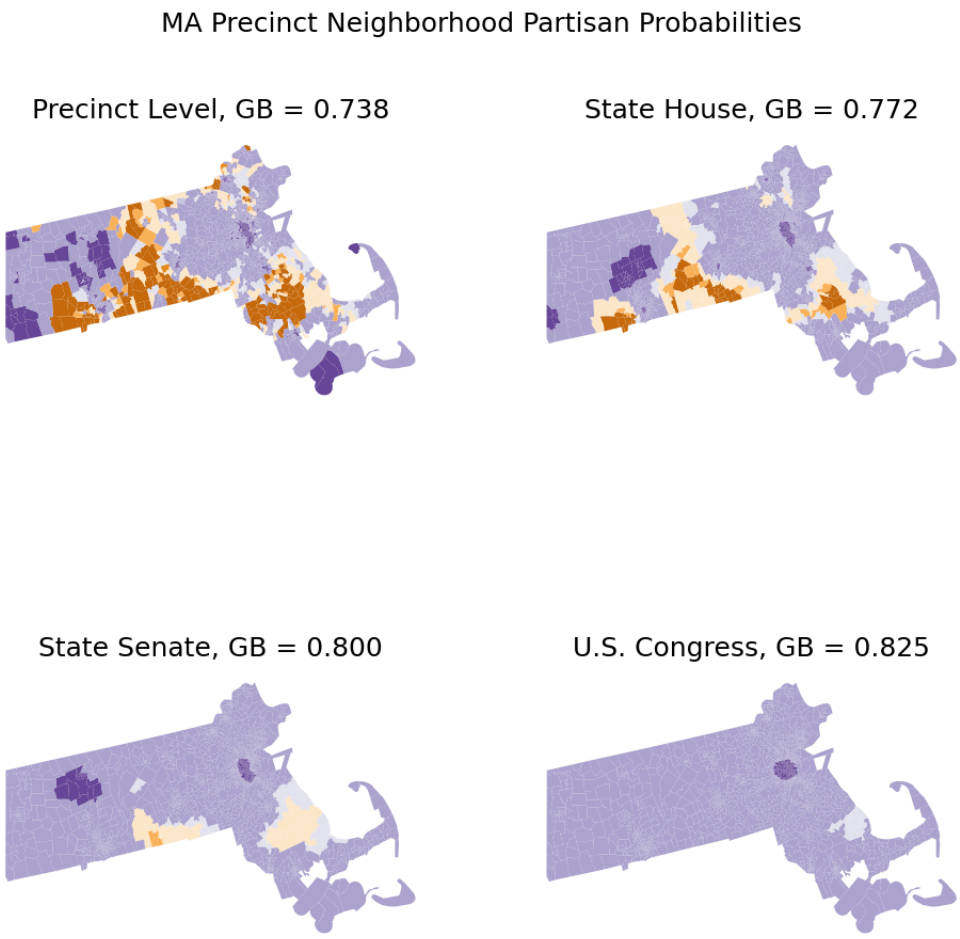


Figure 20: Territories induced by the geographic baseline for precincts, state house, state senate, and U.S. Congress.

6.6 Michigan

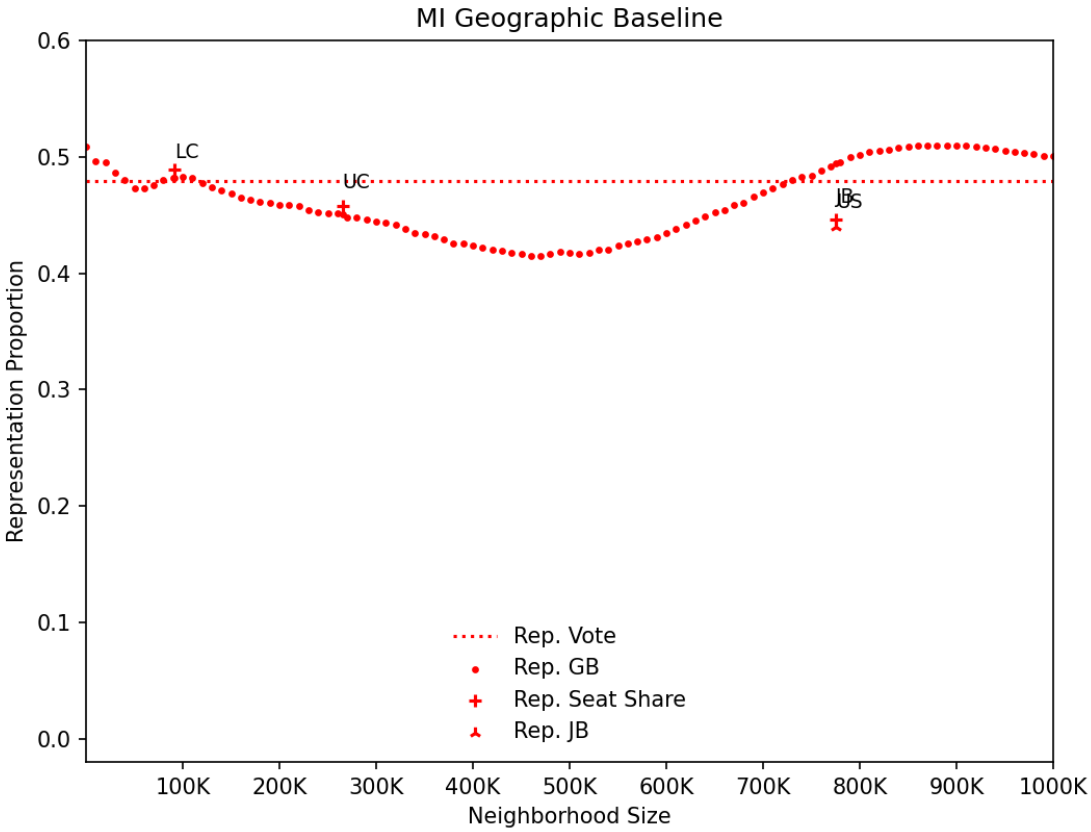


Figure 21: Geographic baseline for the minority party along with actual seat shares for the state's lower and upper chambers and for its U.S. congressional seats.

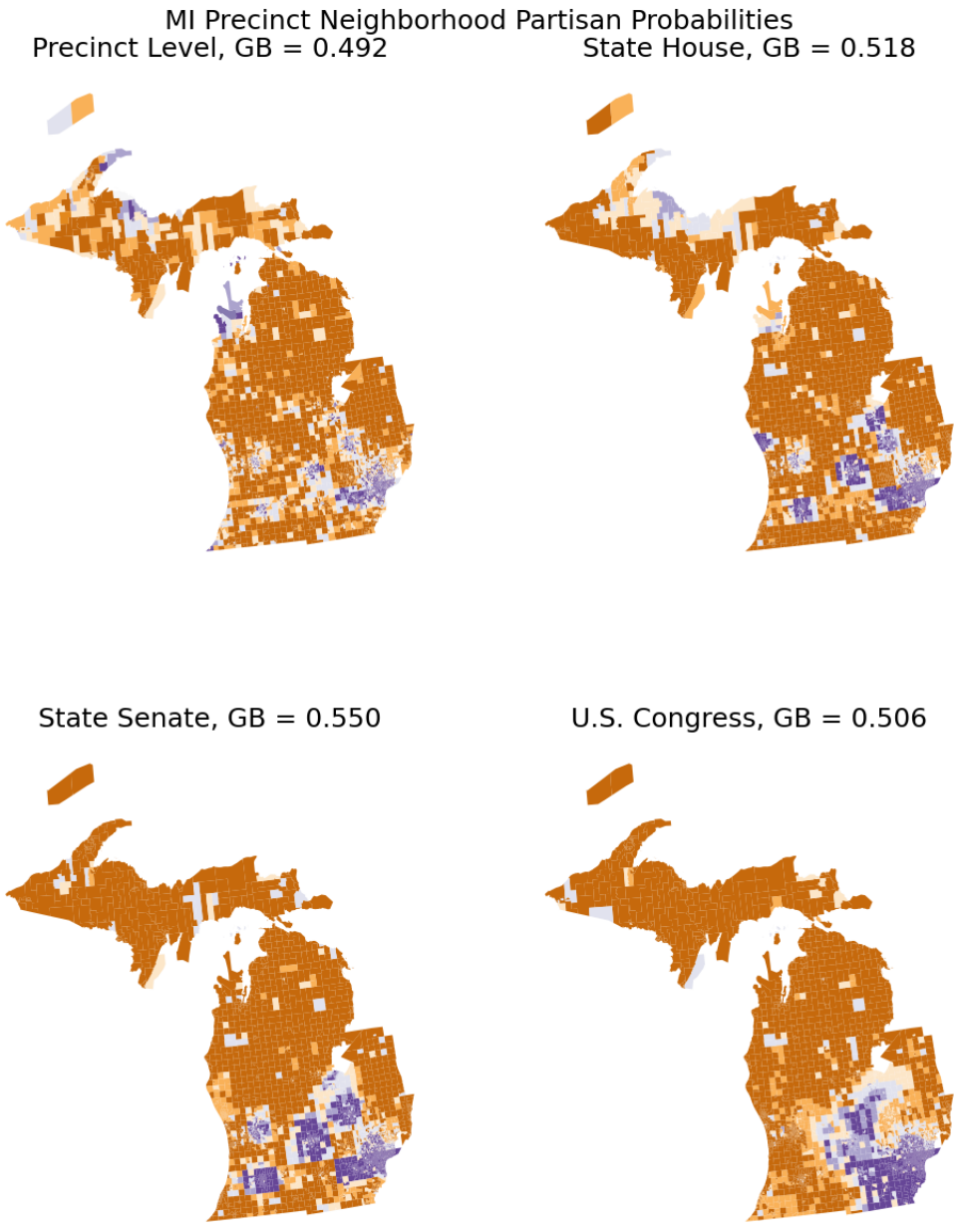


Figure 22: Territories induced by the geographic baseline for precincts, state house, state senate, and U.S. Congress.

6.7 Nevada

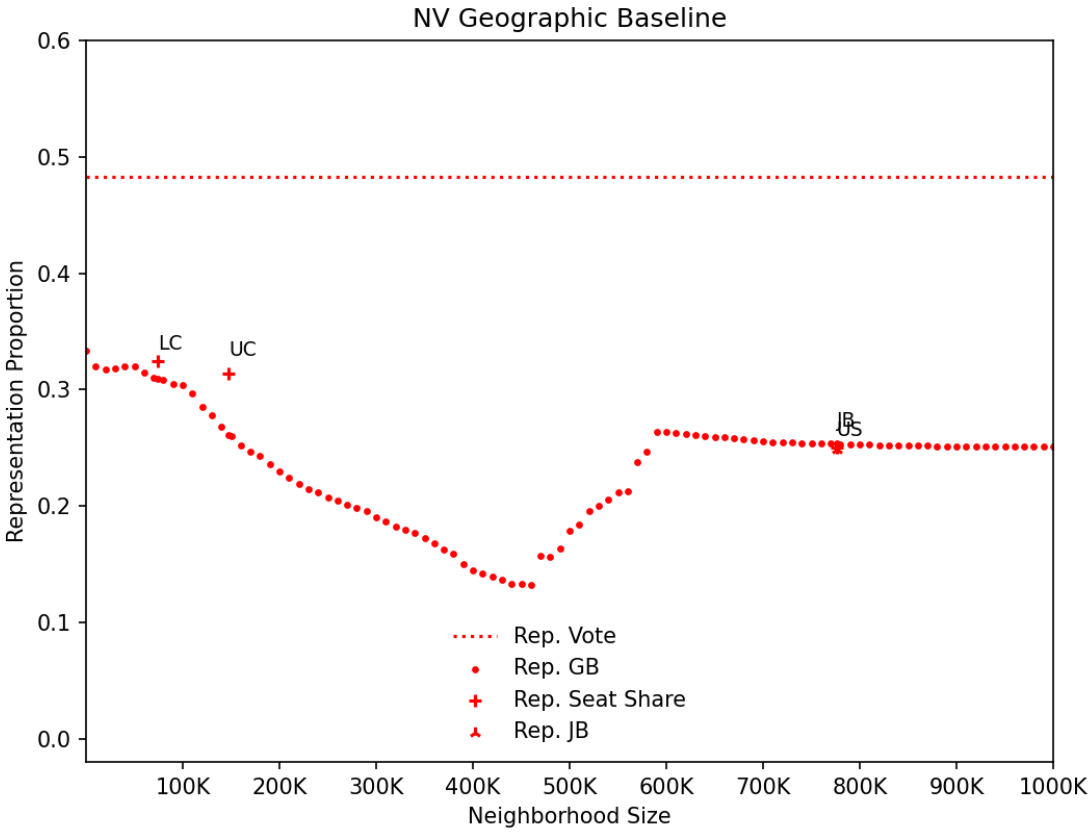


Figure 23: Geographic baseline for the minority party along with actual seat shares for the state's lower and upper chambers and for its U.S. congressional seats.

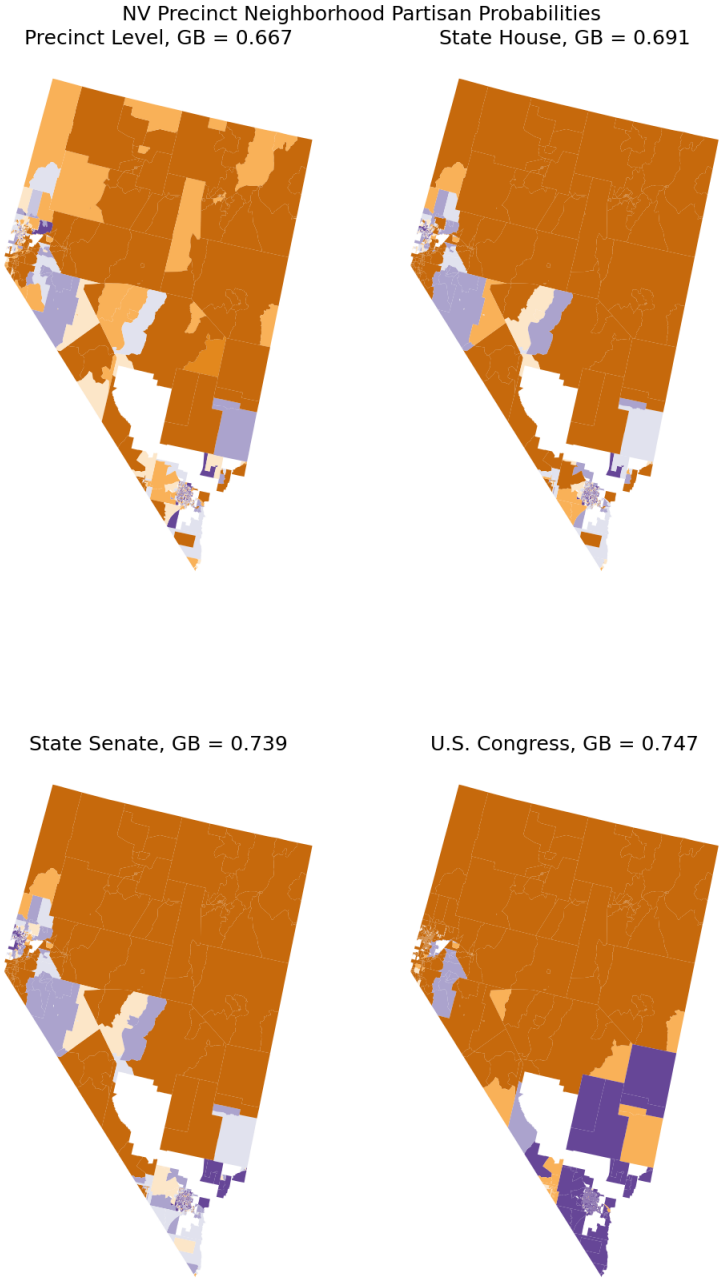


Figure 24: Territories induced by the geographic baseline for precincts, state house, state senate, and U.S. Congress.

6.8 Ohio

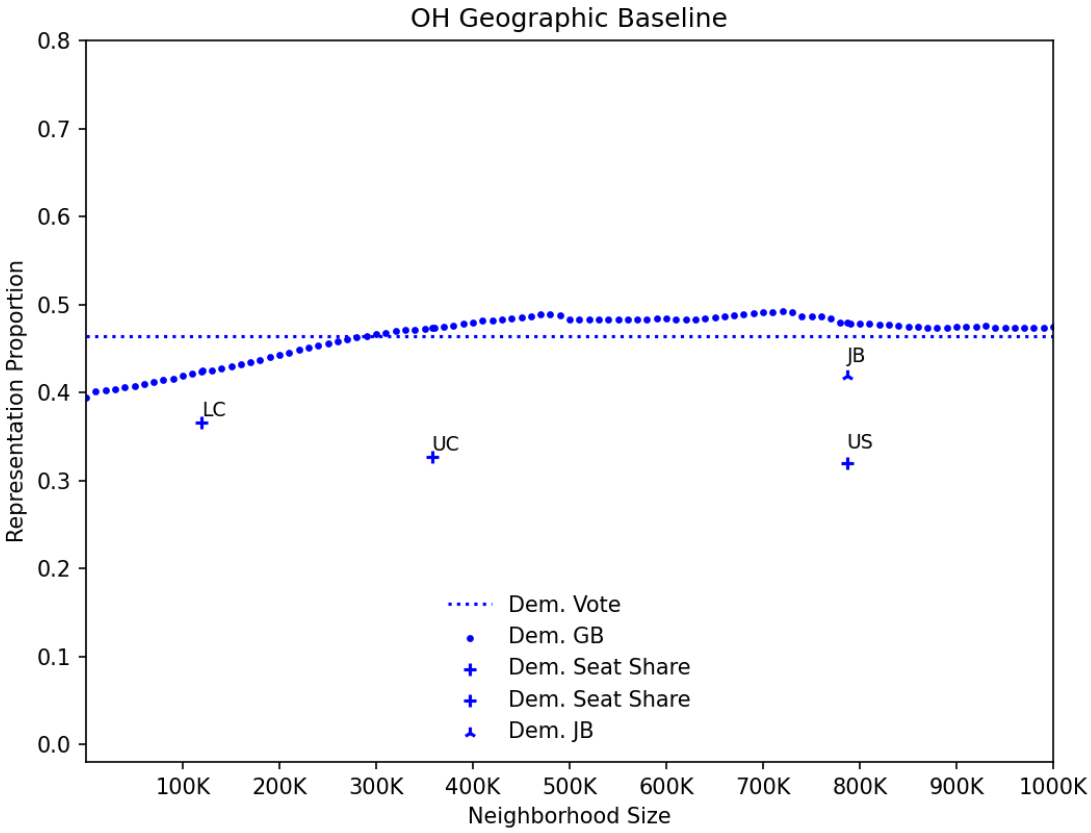


Figure 25: Geographic baseline for the minority party along with actual seat shares for the state's lower and upper chambers and for its U.S. congressional seats.

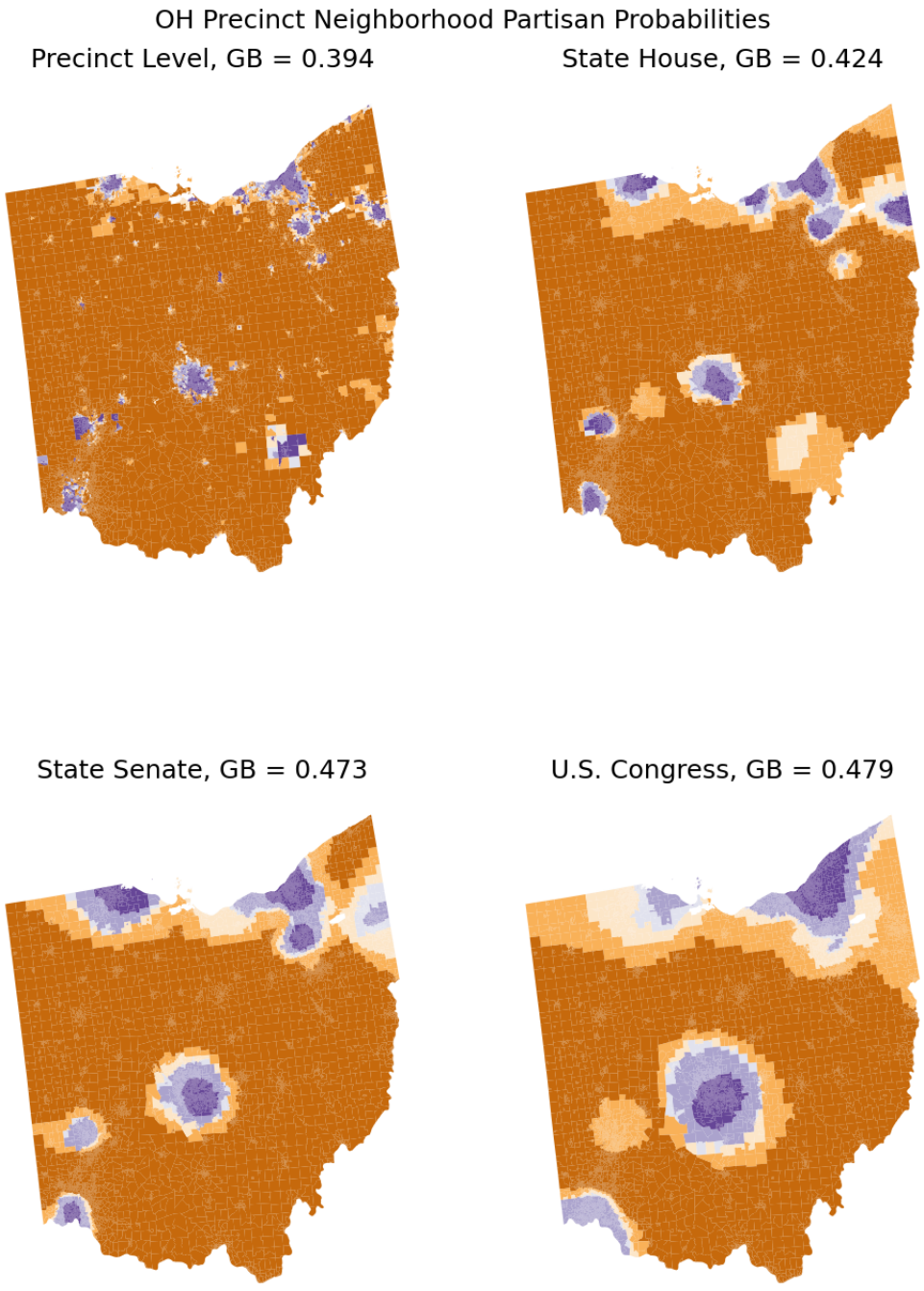


Figure 26: Territories induced by the geographic baseline for precincts, state house, state senate, and U.S. Congress.

6.9 Pennsylvania

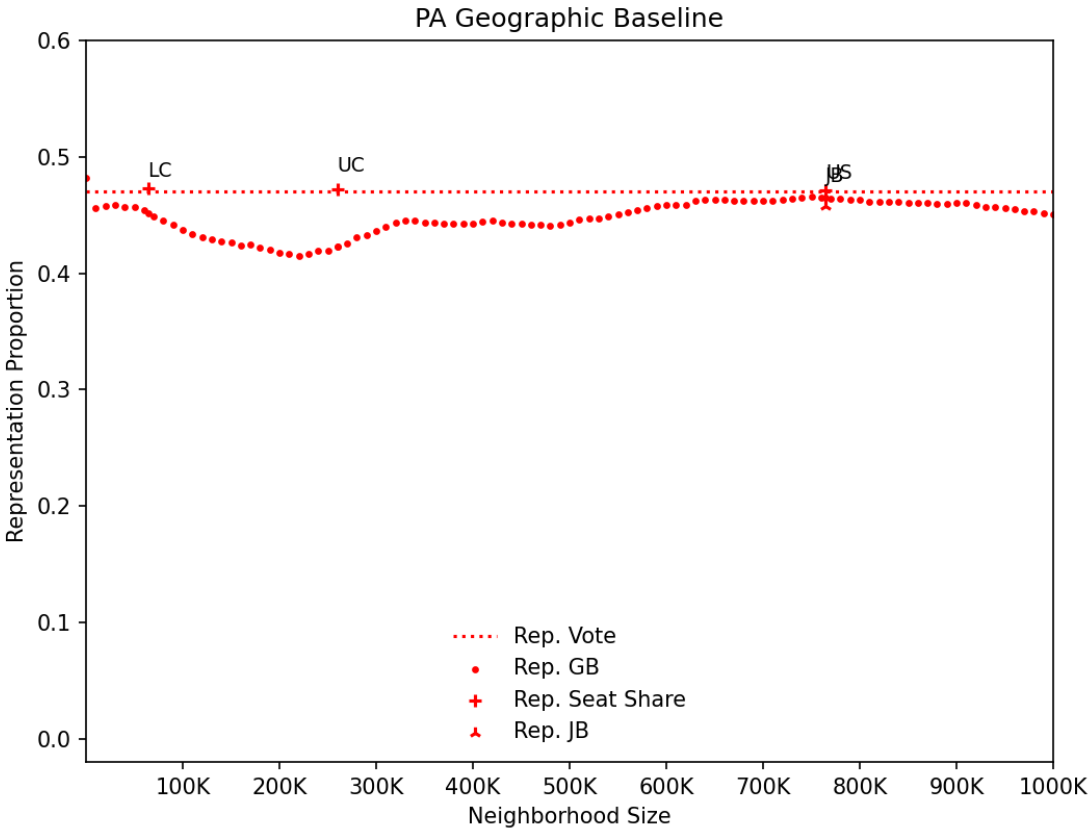


Figure 27: Geographic baseline for the minority party along with actual seat shares for the state's lower and upper chambers and for its U.S. congressional seats.

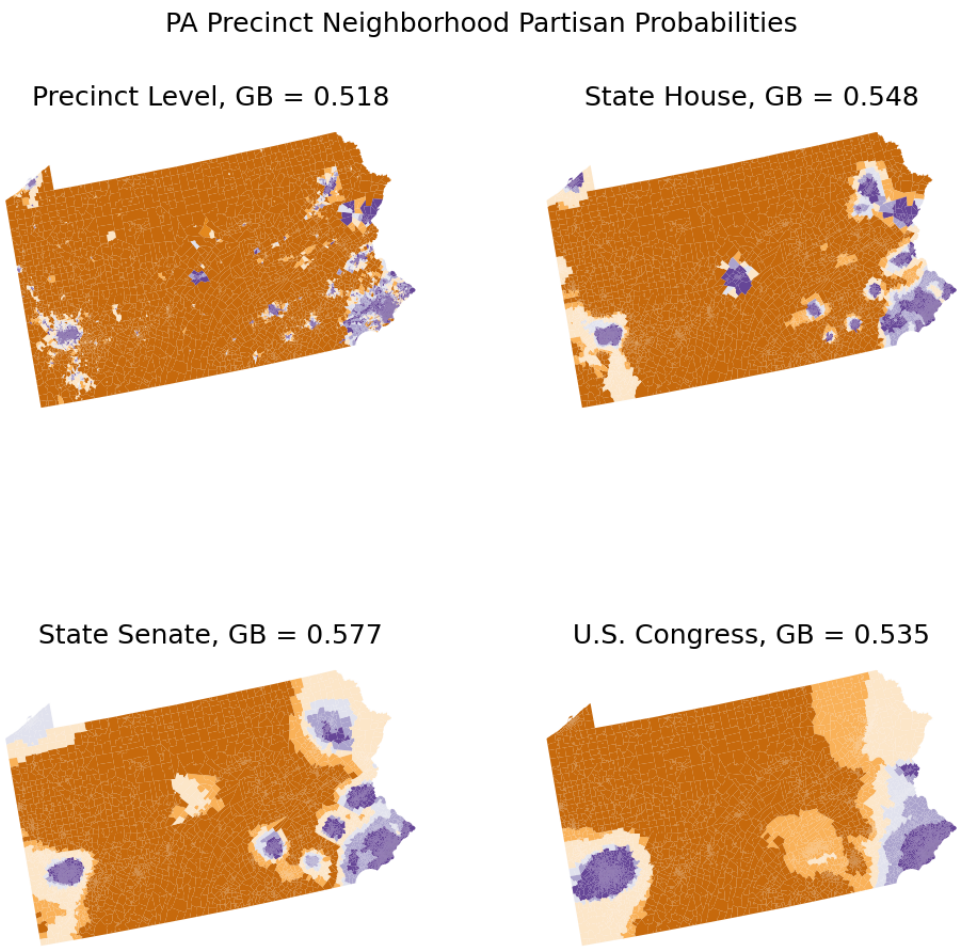


Figure 28: Territories induced by the geographic baseline for precincts, state house, state senate, and U.S. Congress.

6.10 Texas

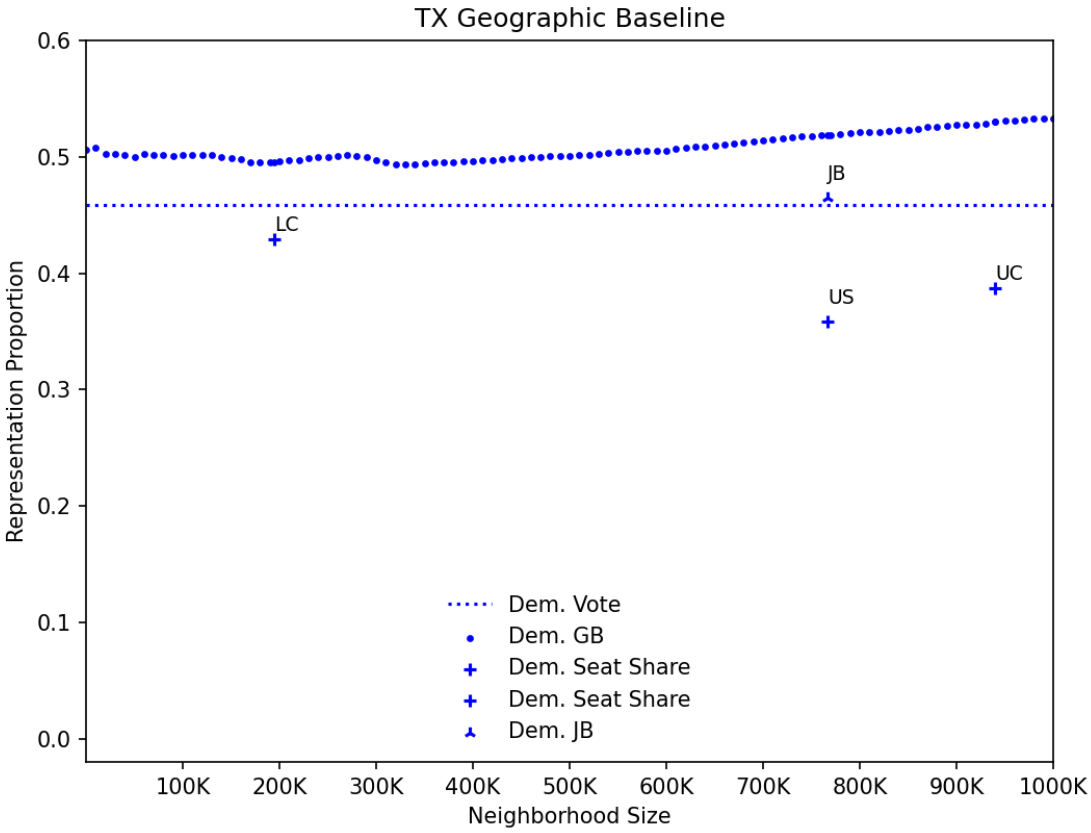


Figure 29: Geographic baseline for the minority party along with actual seat shares for the state's lower and upper chambers and for its U.S. congressional seats.

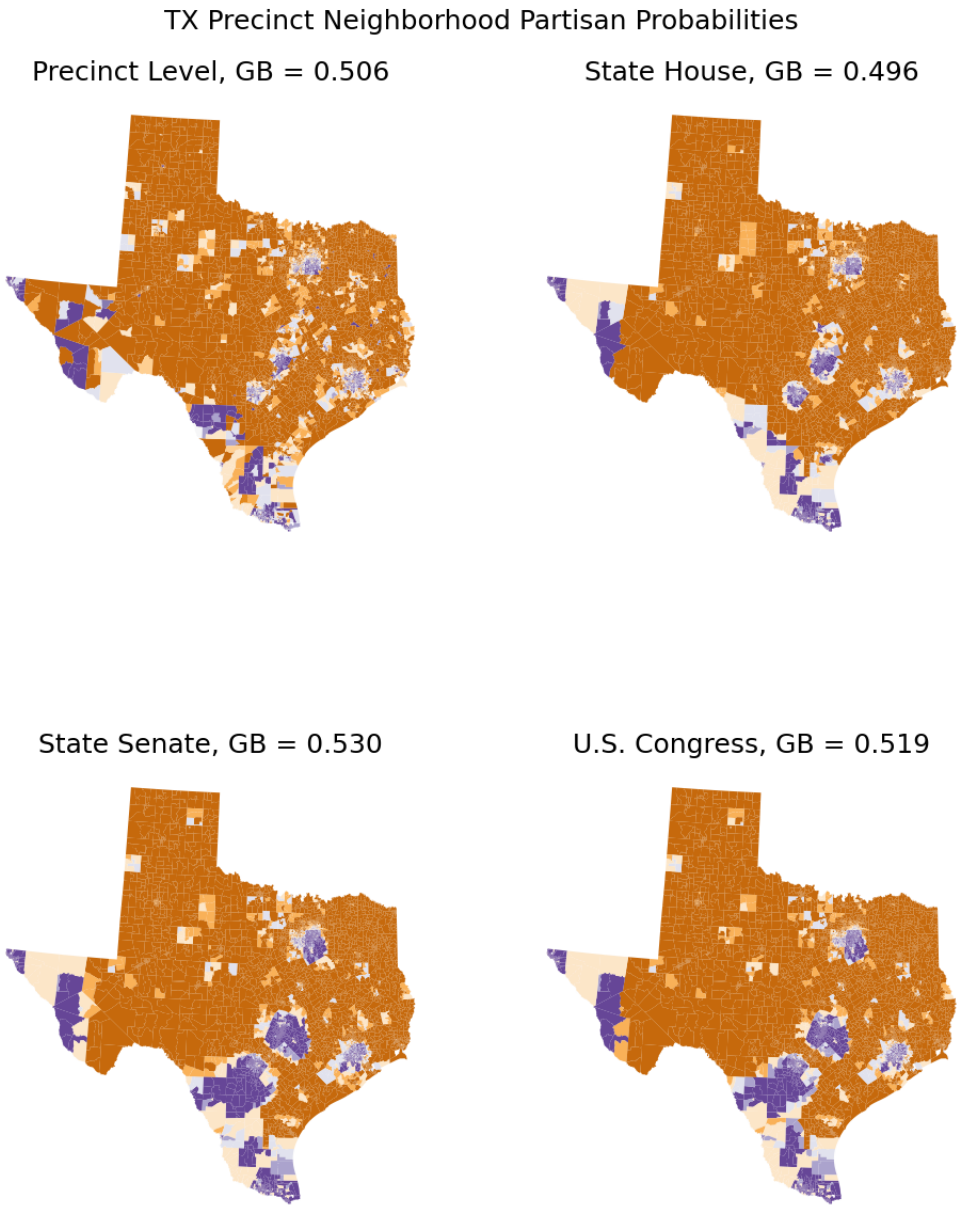


Figure 30: Territories induced by the geographic baseline for precincts, state house, state senate, and U.S. Congress.

6.11 Virginia

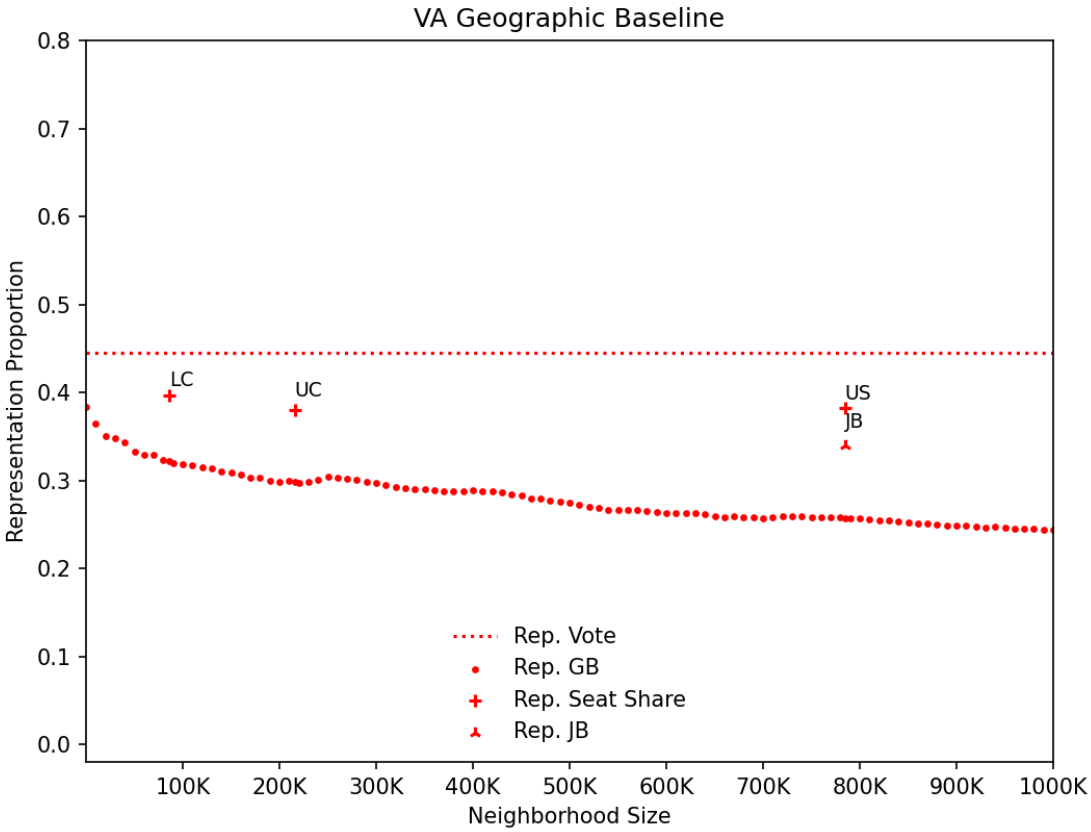


Figure 31: Geographic baseline for the minority party along with actual seat shares for the state's lower and upper chambers and for its U.S. congressional seats.

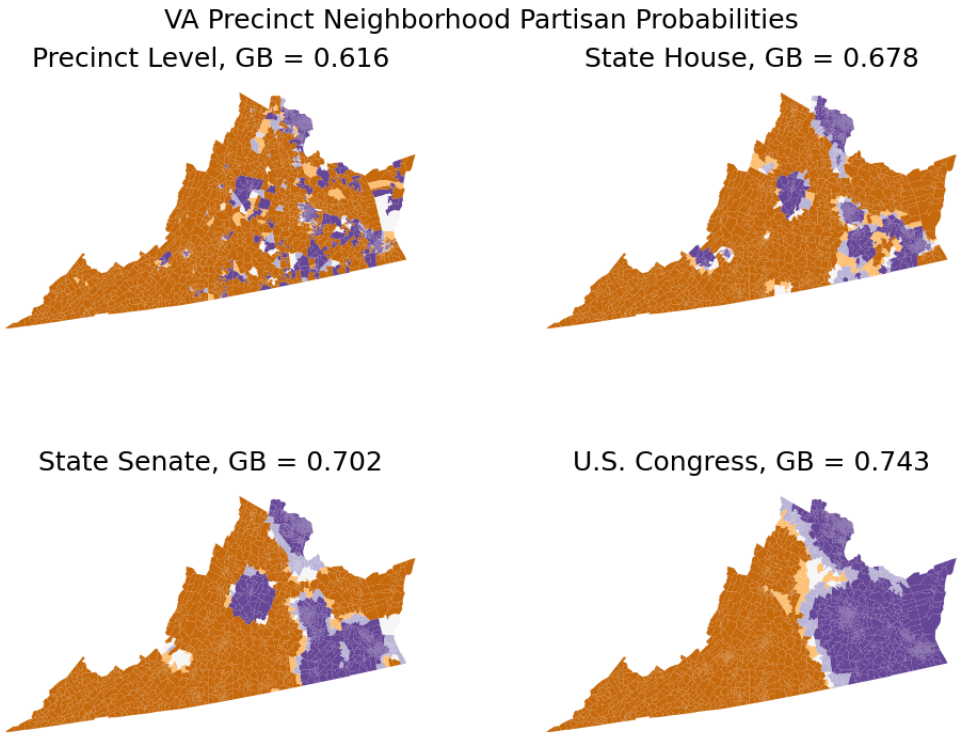


Figure 32: Territories induced by the geographic baseline for precincts, state house, state senate, and U.S. Congress.

6.12 Wisconsin

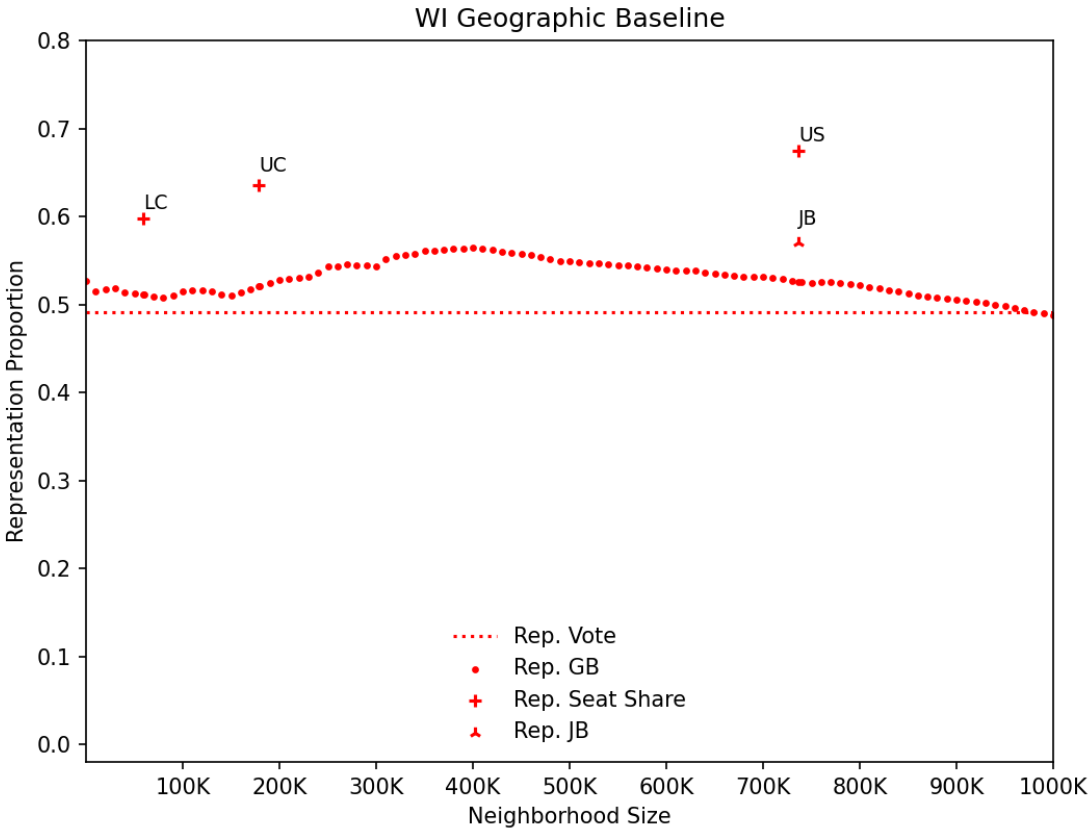


Figure 33: Geographic baseline for the minority party along with actual seat shares for the state's lower and upper chambers and for its U.S. congressional seats.

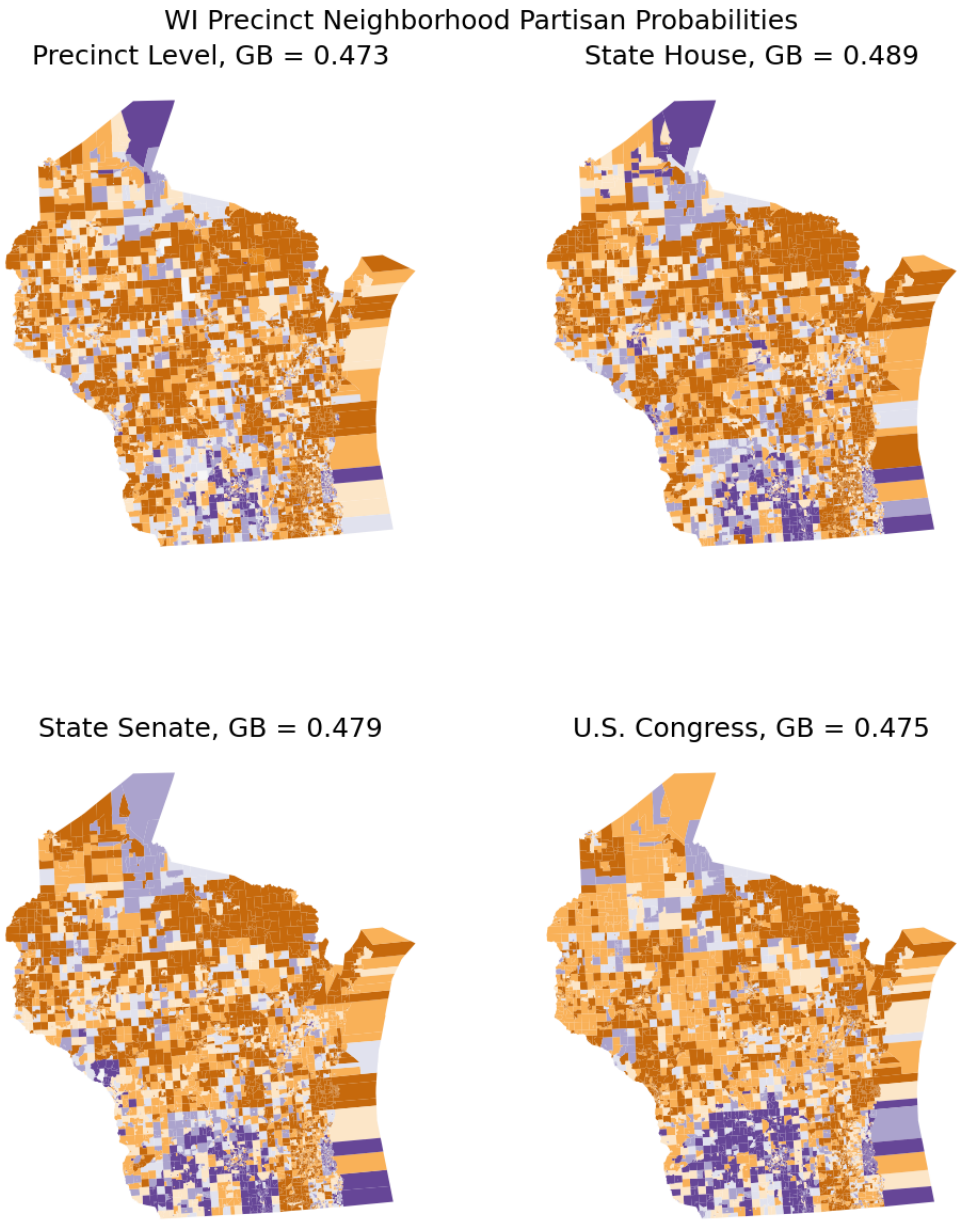


Figure 34: Territories induced by the geographic baseline for precincts, state house, state senate, and U.S. Congress.