From Labor to Intermediates: Firm Growth, Input Substitution, and Monopsony

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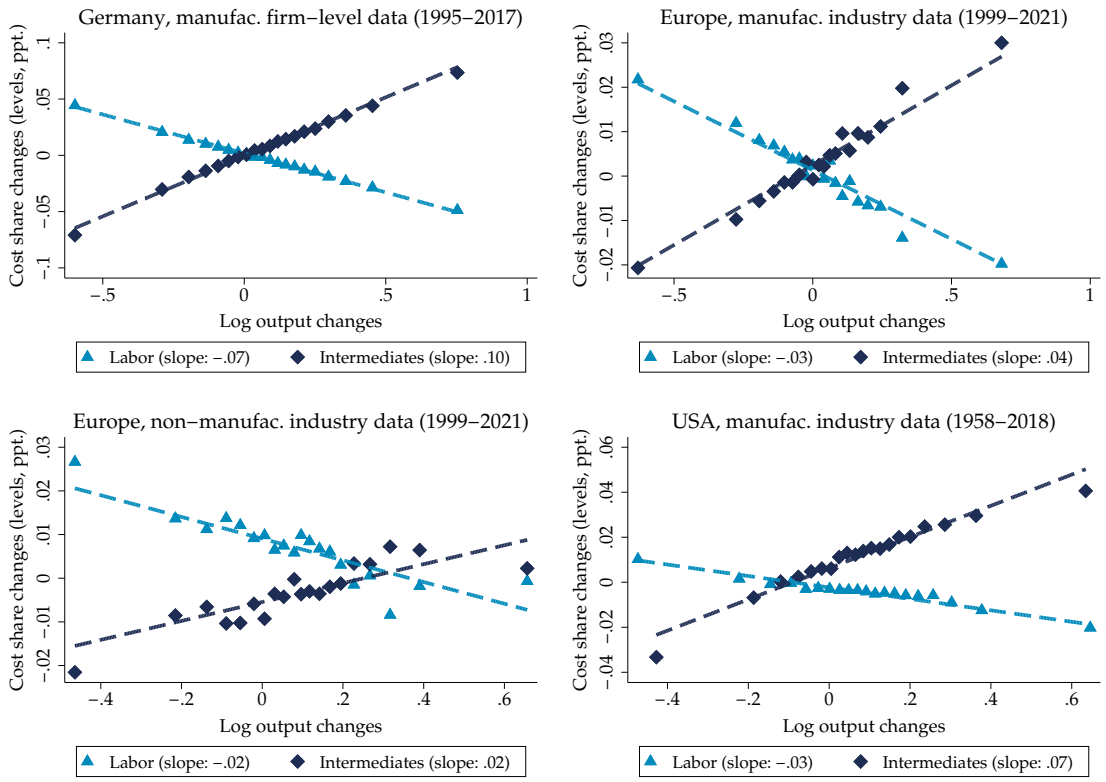
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Abstract

We document and dissect a new fact about firm growth: the shift from labor to intermediate inputs. This shift holds across the board: for input quantities, cost and output shares, and output elasticities. These patterns reject the benchmark of input-price-taking firms with Cobb Douglas production. We establish the facts using German firm-level micro data and OLS and IV regressions using export shocks, and in micro-aggregated industry data for 20 European countries (and, for cost shares, the US). A parsimonious model of firm production fully rationalizes the regularities, resting on two features: (i) an elasticity of substitution between intermediates and labor that exceeds one, and (ii) an increasing shadow price of labor, due to monopsony over labor (or labor adjustment costs). As firms grow, labor becomes expensive, incentivizing firms to reoptimize their input mix from labor to intermediates. Due to substitutability, the inputs' output elasticities shift likewise, with cost and output shares following suit. The shift in output elasticity accounts for most of the decline in the labor share in growing firms and industries (with a smaller role for markdowns and markups).

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Figure 1: From Labor to Intermediates: Change of Cost Shares for Labor and Intermediates (Levels, ppt.) against Output (Revenue) Growth, 4-Year Changes, Residualized by (Industry-)Year Fixed Effects



Notes: This figure is a binned scatter plot depicting the changes of labor and intermediate cost shares in levels (i.e., percentage points) against output growth of output (i.e., changes in log revenue). All panels depict differences at the 4-year horizon. Panel (a) does so for German micro firm data, and here all three variables are residualized by industry-year (4-digit) fixed effects. Panel (b) does so for European industry panel data in manufacturing for 2-digit NACE rev. 2 industries from CompNet after year fixed effects; Panel (c) does so for non-manufacturing industries. Panel (d) does so for the US using the NBER CES Manufacturing Database and 6-digit NAICS industries, with industry fixed effects. Panels (b)-(d) impute capital costs (to obtain total costs) as fraction 0.08 of the capital stock. Further details on the datasets, samples and empirical specifications are relegated to the main part of the paper.

1 Introduction

How do firms grow? Standard Cobb Douglas production and competitive input markets predict that firms simply scale up inputs proportionately, with constant cost shares. Our paper uncovers and makes sense of clear departures from this benchmark—one of which Figure 1 illustrates: as firms (and entire industries) grow, they shift away from labor to intermediates. They do so across the board: in terms of input quantities, output and cost shares (see the figure), and output elasticities. To rationalize this new set of facts, we offer a *parsimonious* model of firm growth, relying on two factors: (i) a labor-intermediates substitution elasticity above one, and (ii) an increasing shadow price of labor (most likely due to monopsony). Our paper connects those two features to draw their joint implications for firm growth.

We study firm growth in German micro data of manufacturing firms. We draw on OLS regressions as well as an IV strategy using export demand shocks as a firm growth shifter unrelated to factor-biased

price or technological changes. We also establish our patterns in nonparametric scatter plots of raw firm-level data. Additionally, we confirm our main results in industry level for 20 European countries using harmonized manufacturing data from the Competitiveness Research Network (CompNet)—and for a subset of outcomes also for US manufacturing industries. We show the patterns at various horizons, from 1- to 10-year changes, and across size classes of firms. We also do not find evidence for non-homotheticities, motivating our parsimonious production perspective on input substitutability. The German micro data includes firm-specific output prices, so that we address the price biases in output elasticity estimation pointed out in De Loecker et al. (2016), Bond et al. (2021), De Ridder et al. (2022)). As a complementary robustness check, we also infer output elasticities from cost shares.

The shifts from labor to intermediates in quantities and the reduction in output elasticities of labor accounts for *about half* of the negative effect of firm growth on the labor share. (Labor share changes equal labor output elasticity changes minus changes in markups and markdowns.) Hence, our framework provides a novel, technological explanation for the negative association between the labor share and firm size, complementing existing approaches that focus on large firms' product or labor market power. Importantly, unlike the existing literature on cross-sectional firm *size* gradients and concentration (e.g., Autor et al. (2020) and De Loecker et al. (2020)), we focus on *firm growth* in panel data.[1] While we focus on idiosyncratic firm growth and intermediate-labor substitution, our findings resonate work by Mertens (2022), who documents a trend decline in labor output elasticities for Germany, linking firm-level and aggregate time series.

A parsimonious model of firm growth can account for the entire set of findings. It rests on two features: substitutability between intermediates and labor, and an increasing shadow price of labor relative to intermediates. The combination of the features is needed to account for the findings—and hence the empirical findings support the model we propose, rejecting alternatives such as Cobb Douglas (unit substitability) or perfect input markets.

First, we model input substitutability in CES production function drawing on labor and intermediates (and capital potentially in an outer nest). Substitution elasticities above one imply that as the intermediate-labor input ratio increases, their output elasticities shift from labor to intermediates—consistent with our firm growth facts. Quantitatively, our firm growth regressions identify substitution elasticities well above one, ranging from 1.8 to 2.7 (OLS) and from 3.8 to 4.2 (IV). Our paper situates these values in a systematic meta analysis of related existing estimates.[2]

Second, to account for *why* growing firms choose to shift the input mix from labor to capital, our model features an increasing shadow price of labor relative to intermediate input costs. A natural source is a finitely elastic firm-specific labor supply curve, i.e., firms' monopsony power over labor inputs. (Alternatively, adjustment costs may play a role, at least in the short run.) In fact, for a given

[1] We find much smaller (but qualitatively similar) cross-sectional differences in output elasticities by firm *size* than by firm *growth*, perhaps due to firm-specific permanent factors shaping input intensities and output elasticities or due to returns to scale differences, which our main specifications of within-firm changes net out.

[2] Existing estimates of the intermediate-labor substitution elasticity vary and are difficult to compare as they depend on disparate identification strategies and production model assumptions. Atalay (2017), Oberfield and Raval (2021), and Boehm et al. (2019) estimate substitution elasticities between intermediates and a capital-labor bundle of 0.8 to 0.9, 0.6 to 1.0, and around 0, respectively. More consistent with our approach, Huneeus et al. (2022) and Chan (2023) estimate substitution elasticities between labor and intermediates of 1.6 and 1.5 to 4.0, respectively. We note that our estimates are derived from within-firm changes.

substitution elasticity, our input mix estimates identify the elasticity of the labor price to the firm (the inverse labor supply elasticity). We offer a range of implied elasticities across our specifications and wage measurement, and again situate them in a meta-analysis of existing estimates (e.g., Sokolova and Sorensen, 2021). We also find that supply elasticities are higher in the long run and for our IV estimates, and when we net out markdown shifts or use direct (average) wage estimates.[3]

As one aggregate implication, our firm-level findings point to declining labor shares as by-products of growth. Indeed, we confirm the industry-level analog of the negative association between output growth and labor shares for 20 European countries in harmonized manufacturing data from the Competitiveness Research Network (CompNet). Importantly, the declining output elasticity can fully account for this pattern, while markup or wage markdown growth cannot. Finally, we show that, in Europe, on average, *only growing* industries experienced a decline in labor shares. Consistent with substitution mechanism, these industries see much stronger declines in labor output elasticities. (Outside of our mechanism, they also see weaker increases in markups, and stronger declines in wage markdowns.) For the US, too, labor shares declined much more strongly in growing than in shrinking industries—consistent with a latent negative labor output elasticity-growth relationship. These patterns suggests that the declining output elasticity in output growth is the key factor behind recent labor share declines in European and US industries—so that this last section of our paper also generalizes and provides a micro-founded explanation for the aggregate time series based account for Germany in Mertens (2022).

We caveat that our focus is on idiosyncratic firm (and industry) growth, whereas drawing aggregate labor share implications from intermediate-labor substitution would necessitate input-output network analysis or an open economy perspective. Globalization, especially China's accession to the WTO, appears to have driven offshoring of labor-intensive tasks in many advanced countries (Elsby et al. (2013)).

Additional related literature. Broadly, our study complements work showing that estimates of the substitution elasticity between production factors are inconsistent with a Cobb-Douglas production model (e.g., Chirinko et al., 2011, Raval, 2019). Our paper also adds to existing studies that document factor substitution in response to firm-specific shocks and trends—where the existing literature has largely focused on capital-labor substitution. For example, Acemoglu and Restrepo (2019), Acemoglu and Restrepo (2020), Dauth et al. (2021), and Deng et al. (2023) study substitution of labor with robots.[4] Similarly, Hubmer and Restrepo (2021) use Compustat data and show that capital-output elasticities have increased in the largest Compustat firms in the most recent years, consistent with automation. Karabarbounis and Neiman (2014) discuss the role of declining capital prices in the global labor share decline. Huneeus et al. (2022) and Dhyne et al. (2022) study labor input and wage adjustments in

[3]We are not aware of comparable estimates for the firm-specific supply elasticities for intermediates. We assume them to be supplied elastically in our quantitative interpretation. Bilal and Lhuillier, 2022 makes an analogous assumption for a labor-only model involving labor service purchases (such as temp work agencies), although our intermediates are largely made of goods rather than services. For temp agency workers as one facet of outsourcing, Drenik et al., 2023 show that wage premia partially extend to outsourced labor. Huneeus et al., 2022 focus on intermediate input price variation from production networks as a driver of cross-sectional outsourcing differentials.

[4]Other studies explore the conditions under which relative factor prices and relative factor supply induce firms to direct their innovation toward more or less labor-intensive production processes (e.g., Hicks, 1932, Zeira, 1998, Acemoglu, 2002; 2003, Rubens, 2022), or due to non-homotheticities Lashkari et al. (2023).

context of demand shocks in production networks, but do not focus on changes in output elasticities. Dhyne et al. (2022) include a model of overhead headquarter labor units (see also Autor et al., 2020 for a discussion of the role of overhead labor costs). Huneeus et al. (2022) show that firms' monopsony power in labor markets can rationalize why growing firms substitute toward non-labor inputs. In an aggregate analysis, Castro-Vincenzi and Kleinman (2024) use aggregate data and study how rising material prices may lower labor shares if labor and materials are *complements*. We focus on firm-level mechanisms and firm growth, where in our simple account, substitutability and an increasing firm-specific labor price appear active instead. Finally, by focusing on input mix and production function dynamics accompanying firm growth, our study complements existing studies of firm growth using one-input (labor) models and measures of firm size (Sterk et al. (2021)), and we leave firm growth life cycles for future research.

Outline. The paper is organized as follows. Section 2 provides our formal model and derives predictions. Section 3 describes the firm-level data, sample and production function estimation. Section 4 uses German firm-level data to establish the shift from labor to intermediates. Section 5 interprets the results quantitatively as identifying our parameters of interest, and discuss alternative explanations for our findings. Section 6 provides a production model that connects labor output elasticity dynamics and other factors to changes in labor shares. Section 7 present our European and US industry-level results for labor share dynamics. Section 8 concludes.

2 Theory

This section provides a simple framework for firm growth and its effects on factor intensities, output elasticities, and cost and output shares. Section 2.1 defines key concepts in general terms. Section 2.2 then derives our simple production model that we use in Section 2.3 to formulate our predictions about firm growth.

2.1 Definitions

Consider a production function of firm i in period t that transforms labor (L_{it}), intermediates (M_{it}), and capital (K_{it}) into output (Q_{it}):

$$Q_{it} = F_{it}(L_{it}, K_{it}, M_{it}, \Omega_{it}). \tag{1}$$

Ω_{it} captures firm productivity. At this stage, we do not impose a specific form for productivity. In our estimation, we will delve into more specific productivity processes, including factor-biased and Hicks-neutral productivity.

Output elasticity. The output elasticity of a production factor, $X = \{L, M, K\}$ is:

$$\theta_{it}^{X} = \frac{\partial Q_{it}}{\partial X_{it}} \frac{X_{it}}{Q_{it}}. \tag{2}$$

We also define returns to scale as $RTS_{it} = \theta_{it}^{L} + \theta_{it}^{M} + \theta_{it}^{K}$.

Elasticity of substitution. The elasticity of substitution between two inputs is the ratio of the percent change in their marginal product ratio to the percent change in their input quantity ratio (Hicks, 1932). We express this in logarithms, focusing on labor-intermediate substitution, our focus:

$$\frac{1}{\sigma_{it}} = \frac{\Delta \ln(\frac{\partial Q_{it}}{\partial M_{it}}) - \Delta \ln(\frac{\partial Q_{it}}{\partial L_{it}})}{\Delta \ln(L_{it}) - \Delta \ln(M_{it})}. \tag{3}$$

Relationship between output and substitution elasticities. Inserting Equation (2) into Equation (3) highlights the following connection between output and substitution elasticities:

$$\frac{\sigma_{it} - 1}{\sigma_{it}} = \frac{\Delta \ln(\theta_{it}^{L}) - \Delta \ln(\theta_{it}^{M})}{\Delta \ln(L_{it}) - \Delta \ln(M_{it})}. \tag{4}$$

Hence, the effect of input mix shifts on relative output elasticities is shaped by the elasticity of substitution. If labor and intermediates are substitutes ($\sigma_{it} > 1$), an decrease in the labor-intermediate ratio leads to an decrease in the output elasticity of labor relative to that of intermediates (see also next section). We will use Equation (4) in our empirical analysis to back out substitution elasticities.

2.2 Firm Optimization

Ultimately, we focus on a nested homothetic CES production model. This specification will ensure tractability and is motivated by our subsequent findings that a constant substitution elasticity can account for the full set of our findings. Specifically, we employ a nested CES function, which reflects the main findings that (i) changes in labor and intermediate inputs are our dominant empirical finding, and (ii) capital accounts only for a small share in output for the average firm (i.e., the average share of labor in value-added is 0.80 in our firm-level data; the intermediate share is about 0.65). This specific model is presented in Section 5.

Functional form of the production function. A Cobb-Douglas production function is incapable of modeling changes in output elasticities or cost shares with firm size or growth. We therefore rely on a homothetic nested CES production function to guide our discussion and our interpretation of the subsequent empirical results:

$$Q_{it} = \Omega_{it} \left(\Lambda_{it}^{LM} \left(\alpha_i^{L} L_{it}^{\frac{\sigma-1}{\sigma}} + \alpha_i^{M} M_{it}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}\frac{\kappa-1}{\kappa}} + \Lambda_{it}^{K} K_{it}^{\frac{\kappa-1}{\kappa}} \right)^{\frac{\kappa}{\kappa-1}}, \tag{5}$$

where α_i^{L}, α_i^{M}, Λ_{it}^{LM}, and Λ_{it}^{K} are distribution parameters. σ is the substitution elasticity between labor and intermediates. κ denotes the substitution elasticity between capital and the second input aggregate consisting of labor and intermediate inputs. We use this specification to illustrate our arguments as our focus is on the substitution patterns between labor and intermediates (a similar setup could account for analogous capital effects). *Focusing on labor-intermediate substitution, our derivations below would go through without parametric assumptions (i.e., permitting a general σ_{it} defined as in Equation* (3)*.)*

Cost minimization. We use cost minimization to study the input and production function dynamics during firm growth. Cost minimization, taking a given output level Q_{it} as given, conveniently introduces firm size and growth as quasi-parameters and permit us to cast our reduced form empirical regression equations as structural equations.

We account for imperfect product market competition and input market frictions, such as monopsony power and adjustment costs, creating wedges between the marginal revenue products of production factors and their unit costs. Appendix B.4 contains the derivations and details.

Cost minimization implies that firms minimize costs for *given* level of output. We permit input markets are imperfect (wedges from adjustment costs or monopsony). This gives a FOC for input $X = \{L, K, M\}$:

$$P_{it}^X \cdot \underbrace{\left(1 + \frac{\partial P_{it}^X}{\partial X_{it}} \frac{X_{it}}{P_{it}^X} + \frac{\partial \chi^X}{\partial X_{it}}\right)}_{\gamma_{it}^X} = \lambda_{it} \frac{\partial Q_{it}}{\partial X_{it}}, \tag{6}$$

where P^X is price for input X (e.g., wage for L), χ^X is an adjustment cost function, λ_{it} is marginal cost, γ_{it}^X is the input price wedge, i.e., $P^X \gamma_{it}^X$ is the input *shadow price*.

Equation (6) can be written explicitly as the following input demand equations:

$$L_{it} = \frac{P_{it} Q_{it}}{\mu_{it} \gamma_{it}^L P_{it}^L} \theta_{it}^L \tag{7}$$

$$M_{it} = \frac{P_{it} Q_{it}}{\mu_{it} \gamma_{it}^M P_{it}^M} \theta_{it}^M. \tag{8}$$

Here, $\mu_{it} = \frac{P_{it}}{lambda_{it}}$ is the price over marginal cost markup. $\gamma_{it}^M = \frac{MRPM_{it}}{P_{it}^M}$ and $\gamma_{it}^L = \frac{MRPL_{it}}{P_{it}^L}$ are wedges between marginal revenue products and input costs for intermediates and labor, respectively. These wedges can, for instance, result from monopsony power or adjustment costs, as in Equation (6).

Alternatively, relative input demand can be expressed as the standard FOCs applied to CES to emphasize the role of σ in the effect of prices on input quantities:

$$\frac{L_{it}}{M_{it}} = \left(\frac{\alpha_i^L}{\alpha_i^M}\right)^{\sigma} \left(\frac{\gamma_{it}^L P_{it}^L}{\gamma_{it}^M P_{it}^M}\right)^{-\sigma}. \tag{9}$$

Monopsony. To think through input price gradients and put structure on the input wedge γ^X, we specify an inverse supply function for input X—capturing monopsony or quasi-static adjustment costs:

$$P_{it}^X = a_{it}^X X_{it}^{\varepsilon^X}. \tag{10}$$

Input demand and firm growth. The features above pin down input demand as a function of input Q_{it}, parameters, and marginal cost:

$$X_{it} = \left(\frac{\lambda_{it} \alpha_i^X}{(1+\varepsilon^X) a_{it}^X}\right)^{\frac{1}{\varepsilon^X + \frac{1}{\sigma}}} {Q_{it}}^{\frac{1}{\varepsilon^X \sigma + 1}}. \tag{11}$$

Hence, the response of input ratios (and thus output elasticities and cost and output shares) to an increase in output (i.e., firm growth) depends on $\frac{1}{\varepsilon^L\sigma+1} - \frac{1}{\varepsilon^M\sigma+1}$ (reflecting price-ratio gradients and substitution):

$$\Rightarrow \frac{L_{it}}{M_{it}} = \varrho_{it} Q_{it}^{\frac{1}{\varepsilon^L\sigma+1}-\frac{1}{\varepsilon^M\sigma+1}}, \tag{12}$$

$$\Leftrightarrow \Delta \ln \frac{L_{it}}{M_{it}} = \left(\frac{\sigma}{\varepsilon^L\sigma+1} - \frac{\sigma}{\varepsilon^M\sigma+1}\right)\Delta \ln \rho_{it} + \left(\frac{1}{\varepsilon^L\sigma+1} - \frac{1}{\varepsilon^M\sigma+1}\right)\Delta \ln Q_{it}, \tag{13}$$

where ϱ_{it} captures the *scale-independent* baseline price ratio effects and substitution effects that are unrelated to firm growth. (We sidestep potential scale-dependence of λ_{it} for the exposition below.)

Output elasticities and firm growth. With CES—away from the unit elasticity of substitution of Cobb Douglas—the following relationship emerges for the relationship between output elasticities and input mix:

$$\frac{\theta^L_{it}}{\theta^M_{it}} = \frac{\alpha^L_i}{\alpha^M_i}\left(\frac{L_{it}}{M_{it}}\right)^{\frac{\sigma-1}{\sigma}} \tag{14}$$

$$\Rightarrow \Delta \ln \frac{\theta^L_{it}}{\theta^M_{it}} = \frac{\sigma-1}{\sigma}\Delta \ln \frac{L_{it}}{M_{it}}. \tag{15}$$

This equation spotlights the pass-through of relative input quantity shifts into relative output elasticities. Specifically, if labor and intermediate inputs are substitutes (consistent with our empirical evidence below), a decrease in the labor-intermediates ratio passes through into the output elasticities. Hence, σ's value relative to one determines how the input mix effects in Equation (13) in firm growth pass through into output elasticities.

Cost and output shares and firm growth. As previously discussed, changing output elasticities have several important implications. In our analysis, we highlight the immediate implications for labor shares. Appendix B.4 shows that using the first order conditions from our production model, e.g., for labor, Equation (7), pins down an input X's cost-share as:

$$CS^X_{it} = \frac{P^X_{it}L_{it}}{P^X_{it}L_{it} + P^M_{it}M_{it} + P^K_{it}K_{it}} = \frac{\frac{\theta^L_{it}}{\gamma^L_{it}}}{\frac{\theta^L_{it}}{\gamma^L_{it}} + \frac{\theta^M_{it}}{\gamma^M_{it}} + \frac{\theta^K_{it}}{\gamma^K_{it}}}. \tag{16}$$

Similarly, by reformulating the first-order condition for input X (for labor, Equation (7)), we define the an input's output share as a function of markups, wage markdowns (i.e., the wedges between the input's marginal revenue products and its price), and output elasticities (Appendix B.4):

$$OS^X_{it} = \frac{P^X_{it}X_{it}}{P_{it}Q_{it}} = \frac{\theta^X_{it}}{\mu_{it}\gamma^X_{it}}. \tag{17}$$

Thus, a decrease in the labor output elasticity will lead to a reduction in both the cost share and output share of labor.[5]

Moreover, *relative* cost and output shares inherit the shift in output elasticities, minus adjustments for input wedges changes:

$$\Delta \ln \frac{OS_{it}^L}{OS_{it}^M} = \Delta \ln \frac{CS_{it}^L}{CS_{it}^M} = \Delta \ln \frac{\theta_{it}^L}{\theta_{it}^M} - \Delta \ln \frac{\gamma_{it}^L}{\gamma_{it}^M}. \tag{18}$$

2.3 Overview: Predictions for Firm Growth

Table 1 summarizes our predictions for various assumption about labor-intermediate substitution elasticities and relative input supply elasticities. This section's discussion remains qualitative. Our quantitative interpretation is presented in Section 5, where we back out the implied values for σ and input supply elasticities ϵ^X identified by our empirical estimates in Section 4—along with a systematic meta-analysis of existing estimates for both parameters in Figures 5 and 6.

Potential cases. We use a constant returns to scale (CRS) Cobb-Douglas production function with perfect markets as a benchmark to fix ideas (CES with $\sigma = 1$), and consider departures into complementary and substitutable territories ($\sigma < 1$ and $\sigma > 1$). We differentiate three cases for relative firm-specific input supply elasticities: labor as more or less elastic than intermediates ($(\varepsilon^L)^{-1} > (\varepsilon^M)^{-1}$ and $(\varepsilon^L)^{-1} < (\varepsilon^M)^{-1}$), and equally elastic ($(\varepsilon^L)^{-1} = (\varepsilon^M)^{-1}$). Table 1 highlights in the shaded region the combination of parameters implied by our empirical evidence: substitutability and labor being less elastically supplied to the firm than intermediates. In the table and in our discussion below, we hold the input wedges as constant and assume returns to scale, as in our structural equations in changes in Section 2.2 above. Guided by the formal derivations from Section 2.2 above, we now discuss specific cases in Table 1. (As noted below, our ceteris paribus analysis here focuses on shifts in Q_{it} and sidesteps shifts in γ^X ratios accompanying firm growth, which would tend to strengthen the predictions.)

Rejected by evidence: no (relative) monopsony. We first consider the benchmark of competitive input prices—and, equivalently, if inputs are *equally* inelastically supplied (the middle row of the table). Then, no matter the substitution elasticity σ, input ratios, output elasticities, cost shares and output shares are constant as firms grow.

Rejected by evidence: Cobb Douglas and monopsony. Second, consider the top row, i.e., the case of the shadow price of labor being increasing in firm-specific labor demand. With Cobb Douglas ($\sigma = 1$), firms substitute from labor to intermediates, which in this case does not affect output elasticities—and

[5]We add two remarks. First, we can express θ_{it}^L in terms of returns to scale ($RTS_{it} = \theta_{it}^L + \theta_{it}^M + \theta_{it}^K$) and the labor output elasticity relative to other output elasticities, $\theta_{it}^L = \frac{\theta_{it}^L}{RTS_{it}} RTS_{it}$, which separates returns to scale from the relative technological importance of labor vis-à-vis other production factors. We will utilize this in our empirical analysis. Second, under a value-added production function, the value added labor output elasticity can be approximated as $\theta_{it}^L \tau_{it}$, where τ_{it} is the ratio of sales to value added. As the value added labor share equals the labor share in sales multiplied by τ_{it}, we can extend our conclusions regarding the impact of changes in labor output elasticities on the sales labor share to the value added labor share.

Table 1: Growth predictions for different substitution and supply elasticities (ceteris paribus)

	$\sigma < 1$	$\sigma = 1$	$\sigma > 1$	
L **less** elastic than M $(\varepsilon^L)^{-1} < (\varepsilon^M)^{-1}$	$\downarrow$	$\downarrow$	$\downarrow$	
	$\uparrow$	$=$	$\downarrow$	
	$\uparrow$	$=$	$\downarrow$	
	$\uparrow$	$=$	$\downarrow$	
L **as** elastic as M $(\varepsilon^L)^{-1} = (\varepsilon^M)^{-1}$	$=$	$=$	$=$	$\Delta \ln L_{it}/M_{it}$
	$=$	$=$	$=$	$\Delta \ln \theta^L_{it}/\theta^M_{it}$
	$=$	$=$	$=$	$\Delta \ln CS^L_{it}/CS^M_{it}$
	$=$	$=$	$=$	$\Delta \ln OS^L_{it}/OS^M_{it}$
L **more** elastic than M $(\varepsilon^L)^{-1} > (\varepsilon^M)^{-1}$	$\uparrow$	$\uparrow$	$\uparrow$	
	$\downarrow$	$=$	$\uparrow$	
	$\downarrow$	$=$	$\uparrow$	
	$\downarrow$	$=$	$\uparrow$	

Notes: Ceteris paribus means constant returns to scale and non-changing market imperfections with output growth ($\Delta \ln Q_{it}$. The pink area denotes the corner of the parameter space that is consistent with our findings.

hence not cost and output shares. That is, even though labor grows more expensive, the quantity substitution from labor exactly offsets the price increase as to leave the labor cost share constant.

Rejected by evidence: complements and monopsony. Away from Cobb Douglas, the input quantity shift from labor to intermediates from the increasing price of labor does affect output elasticities and hence labor shares. Consider first the case of labor and intermediates as complements ($\sigma < 1$). Then, the shift in input quantities pushes down the output elasticity of labor relative to intermediates. And, firms' reduction in the labor-intermediate ratio is less than one to one with the input price ratio increase, leading cost and output shares of labor to *increase*.

Case supported by evidence: substitutes and monopsony. Departing from Cobb Douglas into substitutability ($\sigma > 1$) implies the opposite: the decline in the labor/intermediate input quantity ratio passes through into declines in output elasticity ratios and cost and output shares of labor. Labor becomes expensive, and firms substitute away from it more than one to one. This case is the one that our evidence points to.

Rejected by evidence and implausible: relative monopsony over intermediates. For completeness, we also include the bottom row, which depicts predictions for the case in which intermediate inputs are less elastically supplied or flexible than labor. The opposite sign for quantities emerges across the board, i.e., irrespective of σ, with moreoever output elasticities and labor shares increasing for $\sigma > 1$.

The role of input wedges γ^X. As noted, the above predictions are ceteris paribus with respect to input wedges γ^L/γ^M. We will measure those wedges and their firm growth dynamics separately in Section 6. Generally, we find that the wedges will *augment* the predictions, in that the labor wedge tends to increase in firm size and employment, rather than fall. However, an important consequence is that in that case, even the Cobb Douglas case may exhibit declines in the labor share in costs and output, but would predict constant output elasticities. We formally discuss this case in Section 6,

but additionally note that we will have direct estimates on output elasticities already in our baseline analysis, using production function methods described below.

3 Firm-level Data

We now describe the German firm-level data, sample, and summarize the estimation of output elasticities.

Production data. The firm-product-level panel data for Germany's manufacturing sector covers the period 1995-2017. The data is collected and supplied by the German Statistical Offices.[6] Variables include, among others, sales, employment, investment, intermediate input costs, wage bills, depreciation, and, most importantly, *product quantities and prices at a ten-digit product classification*.[7] The data cover 40% of firms with at least 20 employees and consists of a rotating panel that is drawn anew every 4-5 years. Labor is defined as the number of employees on the 30th September. All other variables pertain to the full calendar year. We clean and prepare the data following Mertens (2022) and provide more details on data preparation, capital stock construction, variable definitions, and summary statistics in Online Appendix D.[8]

Supplementary trade data. To provide *causal* evidence on the relationship between output growth and our variables of interest, we merge bilateral trade flows from the United Nations Comtrade Database on the firm-product-year level to the German micro data. The database contains bilateral trade flows in sales between any country at the country-year-product level from 1995 to 2017. We translate product codes in both datasets into the PRODCOM2002 classification using official concordance tables following the code of Bräuer et al. (2023). As described below, we will apply the IV-approach in Hummels et al. (2014) to instrument output changes with foreign export demand to study how output elasticities change in response to exogenous output changes.

Production function estimation To allow for time-varying and firm-specific output elasticities, we rely on the following translog production function, where lower case letters denote logs:

$$\begin{aligned} q_{it} = \beta_l l_{it} + \beta_k k_{it} + \beta_m m_{it} + \beta_{l2} l_{it}^2 + \beta_{k2} k_{it}^2 + \beta_{m2} m_{it}^2 + \\ \beta_{lk} l_{it} k_{it} + \beta_{lm} l_{it} m_{it} + \beta_{km} k_{it} m_{it} + \beta_{lkm} l_{it} k_{it} m_{it} + \omega_{it} + \epsilon_{it}, \end{aligned} \tag{19}$$

where ϵ_{it} is an i.i.d. error term.

The production function is estimated for each NACE Rev. 1.1 two-digit industry separately. We detail our production function estimation approach in Appendix E. Most importantly, we apply a correction for price biases (De Loecker et al., 2016, Bond et al., 2021) by constructing a firm-specific output price index from our firm-product-level data as in Eslava et al. (2004) and additionally follow Mertens (2022)

[6]Data source: RDC of the Federal Statistical Office and Statistical Offices of the Federal States, DOI: 10.21242/42131.2017.00.03.1.1.0, 10.21242/42221.2018.00.01.1.1.0, and 10.21242/42111.2018.00.01.1.1.0.

[7]Examples of products are "Tin sheets and tapes, thicker than 0.2mm" or "Workwear - long trousers for men, cotton".

[8]The dataset has been used in various studies, e.g., Mertens (2020), 2022, 2023, Mertens and Müller (2022), Haelbig et al. (2023), Mertens et al. (2022), Bräuer et al. (2023), and Bighelli (2023).

to include an input price control function (in a firm-level adaption of the method in De Loecker et al. (2016), using information on firms' output prices and market shares).[9]

Summary statistics and sample. We provide summary statistics of our sample in Appendix Table A.1.

4 Firm-Level Results: Reduced Form and Descriptive Analysis

We now study firm growth and associated dynamics of input use, output elasticities, and cost and output shares in the micro data. This section presents the results descriptives, while Section 5 will interpret the reduced form moments structurally and explain how they identify the implied substitution elasticities and input supply elasticities.

4.1 OLS Regressions

Strategy. For each firm i in year t and firm-level outcome X_{it}, we estimate an OLS regressions in log differences of h years of the form:

$$\Delta^h \ln(X_{it}) = \rho^h \Delta^h \ln Y_{it} + \upsilon_{jt} + \epsilon_{it}. \tag{20}$$

Y_{it} is deflated sales, and υ_{jt} captures industry (j) times year (t) fixed effects. The difference specification accounts for unobserved constant firm characteristics.[10] ϵ_{it} is an error term. We cluster standard errors at the firm level.

The coefficient of interest is ρ. It captures the percent effect (comovement) of a one percent change in firms' output on firm-level outcome x_{it} (input quantities, cost shares, output shares, output elasticities, and, later on, markups and markdowns). We examine time differences h spanning from one year to ten years.

Results. Table 2 presents OLS regression results for various outcomes, with for one- to ten-year changes across panels. As companion exhibits, Figure 2 visualizes key firm-level relationships from Table 2 using binned scatter plots for 1-, 4- and 10-year changes, residualizing both variables by industry-year fixed effects as in Equation (20)—generally revealing clearly linear relationships that support the linear regression specification. Since OLS coefficients are precisely estimated, we restrict our discussion to point estimates.

[9] This approach relies on the positive correlation between output and input prices (e.g., due to high quality outputs requiring high quality inputs). To account for the dependence of input decisions on productivity, we utilize a control function approach similar to Wooldridge (2009).

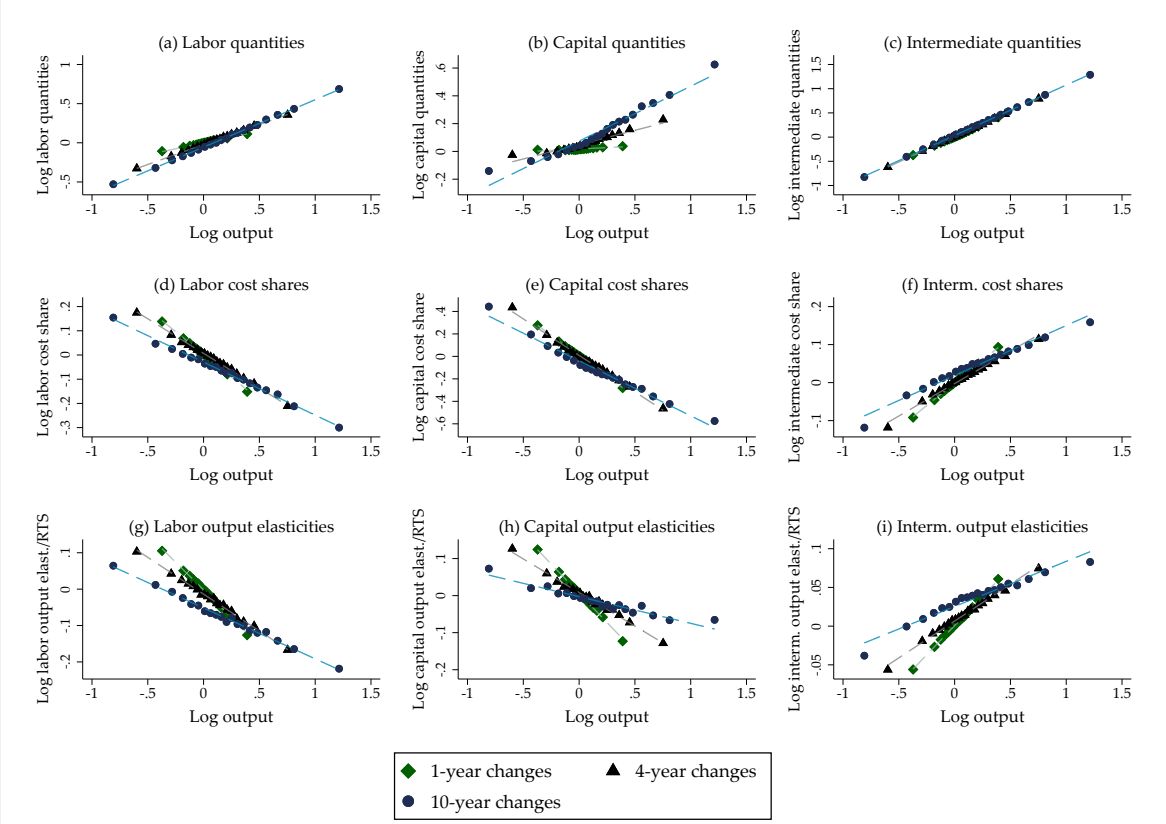
[10] In unreported robustness checks, we also ran specifications with firm effects to take out firm-specific trends. This specification leads to higher coefficient estimates, including at longer horizons, which is plausibly consistent with the remaining variation capturing more transitory fluctuations around the firm-specific trends. We have also experimented and found robust results with deflating sales by firm-specific rather than industry deflators (which drop out due to the inclusion of industry-year fixed effects).

Table 2: Firm-level adjustments in response to firm growth. OLS regressions.

	$\ln(L_{it})$	$\ln(K_{it})$	$\ln(M_{it})$	$\ln(\frac{P^L_{it}L_{it}}{C_{it}})$	$\ln(\frac{P^K_{it}K_{it}}{C_{it}})$	$\ln(\frac{P^M_{it}M_{it}}{C_{it}})$	$\ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\ln(\frac{\theta^K_{it}}{RTS_{it}})$	$\ln(\frac{\theta^M_{it}}{RTS_{it}})$	$\ln(RTS_{it})$
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output growth	0.298***	0.0405***	1.025***	-0.376***	-0.728***	0.238***	-0.304***	-0.315***	0.151***	0.0062***
	(0.0040)	(0.0022)	(0.0033)	(0.00393)	(0.0033)	(0.0022)	(0.004)	(0.0063)	(0.0015)	(0.0003)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950
R^2	0.215	0.043	0.741	0.343	0.556	0.340	0.230	0.137	0.325	0.053
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output growth	0.510***	0.206***	1.045***	-0.286***	-0.649***	0.171***	-0.202***	-0.181***	0.0951***	0.0167***
	(0.0064)	(0.0067)	(0.0045)	(0.0062)	(0.0071)	(0.0034)	(0.0060)	(0.0074)	(0.0025)	(0.0006)
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492
R^2	0.471	0.144	0.860	0.348	0.476	0.355	0.220	0.170	0.268	0.163
Panel C: 10-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output growth	0.606***	0.395***	1.041***	-0.219***	-0.490***	0.131***	-0.140***	-0.0720***	0.0582***	0.0278***
	(0.0066)	(0.0082)	(0.0043)	(0.0061)	(0.0083)	(0.0033)	(0.0053)	(0.0069)	(0.0024)	(0.0007)
Observations	49,915	49,915	49,915	49,915	49,915	49,915	49,915	49,915	49,915	49,915
N of firms	10,595	10,595	10,595	10,595	10,595	10,595	10,595	10,595	10,595	10,595
R^2	0.594	0.256	0.901	0.296	0.329	0.303	0.188	0.149	0.200	0.268
Panel D: levels, cross-section	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output growth	0.741***	0.957***	1.062***	-0.128***	-0.0405***	0.0649***	-0.0345***	0.133***	-0.0116***	0.0498***
	(0.0024)	(0.0033)	(0.0010)	(0.00192)	(0.0031)	(0.0009)	(0.0016)	(0.0022)	(0.0007)	(0.0004)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950
R^2	0.895	0.827	0.983	0.483	0.225	0.413	0.512	0.406	0.417	0.836

Notes: The table reports OLS regressions from estimating the specification in Equation (20). The dependent variables in Columns (1)-(10) are the logs of labor, capital, intermediates, labor cost shares, capital cost shares, intermediate cost shares, labor output elasticities over returns to scale, capital output elasticities over returns to scale, intermediate input output elasticities over returns to scale, and returns to scale, respectively. Panels A-C project changes in those dependent variables on changes in log output for 1-, 4- , and 10-year differences. Panel D runs a cross-sectional specification that regresses levels of the dependent variables on output levels. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm-level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Figure 2: The adjustment of production functions to output growth by horizon: output elasticities and cost shares



Notes: The figure reports binned scatter plots from estimating the specification in Equation (20) with OLS for various differences. The upper three graphs project changes in the logs of labor, capital, and intermediate output elasticities over returns to scale against log output changes, from left to right respectively. The bottom three graphs project changes in the logs of labor, capital, and intermediate cost shares against log output changes, from left to right respectively. All panels report results that are residualized by industry-year fixed effects. German firm-level data.

Input quantities. Table 2 Columns (1)-(3) and Figure 2 Panels (a)-(c) report effects on input use.

In contrast to the baseline model of Cobb Douglas and constant input prices, inputs shift in systematically different ways: intermediates respond most, and labor responds less so; finally, capital adjusts the least. The small standard errors permit us to reject the hypothesis of proportionate (i.e., equal) input growth that would emerge under Cobb Douglas and constant (or equally shifting) (shadow) input prices.

Cost shares. One possibility is that divergent input prices explain the shift towards materials and away from labor, while production can be well described by Cobb Douglas. One prediction of this view is that cost shares would be stable, as firms reduce a given input's quantity inversely proportionately to input price increases. (Adjustment costs or markdowns would not be captured by monetary costs, an issue we dissect below in Section 6.)

Table 2 Columns (4)-(6) and Figure 2 Panels (d)-(f) report effects on cost shares, i.e., input expenditures divided by total costs (C_{it}), for labor, capital, and intermediate inputs, i.e., $\frac{P^L_{it}L_{it}}{C_{it}}$, $\frac{P^K_{it}K_{it}}{C_{it}}$, and

$\frac{P_{it}^M M_{it}}{C_{it}}$.[11]

The data reveal a striking shift towards intermediate costs, away from labor (and capital). In other words, firms outsource more of their production process, with value added over total value of shipments declining.

Complementing the cost share analysis in logs and reiterating the introductory Figure 1, we also run the specification in level (ppt.) changes, showing that intermediate cost shares absorb the decrease in the labor share, with less of a shift in capital (Appendix Tables A.4 and A.5). (We dissect the analogous effects on relative input income shares below in Section 6.)

Output elasticities. The prediction of constant input cost shares under Cobb Douglas derives from the absence of shifts in mark-downs as well as constant output elasticities. We now investigate this channel. We find a strong negative association between output growth and changes in the labor output elasticity (Column (7)). At the one-year horizon, the coefficient is -0.30 (SE 0.0064). That is, a 10 percent change in output reduces the labor output elasticity by 3%. Overall, the shift away from labor is accompanied by a strong shift toward intermediates (Column (9) of Table 2).

Complementing this perspective in logs, Appendix Tables A.4 and A.5 reproduce our results without taking logs for dependent variables. Level (percentage point) changes in output elasticities of labor and capital sum up to changes in intermediate input output elasticities for firm growth (changes), with most of the action being between labor and intermediates.

We normalize the output elasticities by the returns to scale parameter to account for changing returns to scale, but find similar results without this adjustment for changes (but not in the cross section). To illustrate this relationship, Column (10) reports on a specification with returns to scale as a dependent variable. Appendix Tables A.2 and A.3 reproduce our results without dividing output elasticities by returns to scale.

Interestingly, we also see percent reduction in capital shares (Column (8)). Note, however, that the coefficients capture percent changes and that the capital share is smaller than the labor share and the intermediate input share. Column (10) shows that returns to scale are almost unchanged in the short run and only change more significantly over longer horizons, implying that changes in intermediate output elasticities fully compensate changes in labor and capital output elasticities for firm growth horizons (but not necessarily in the cross section).

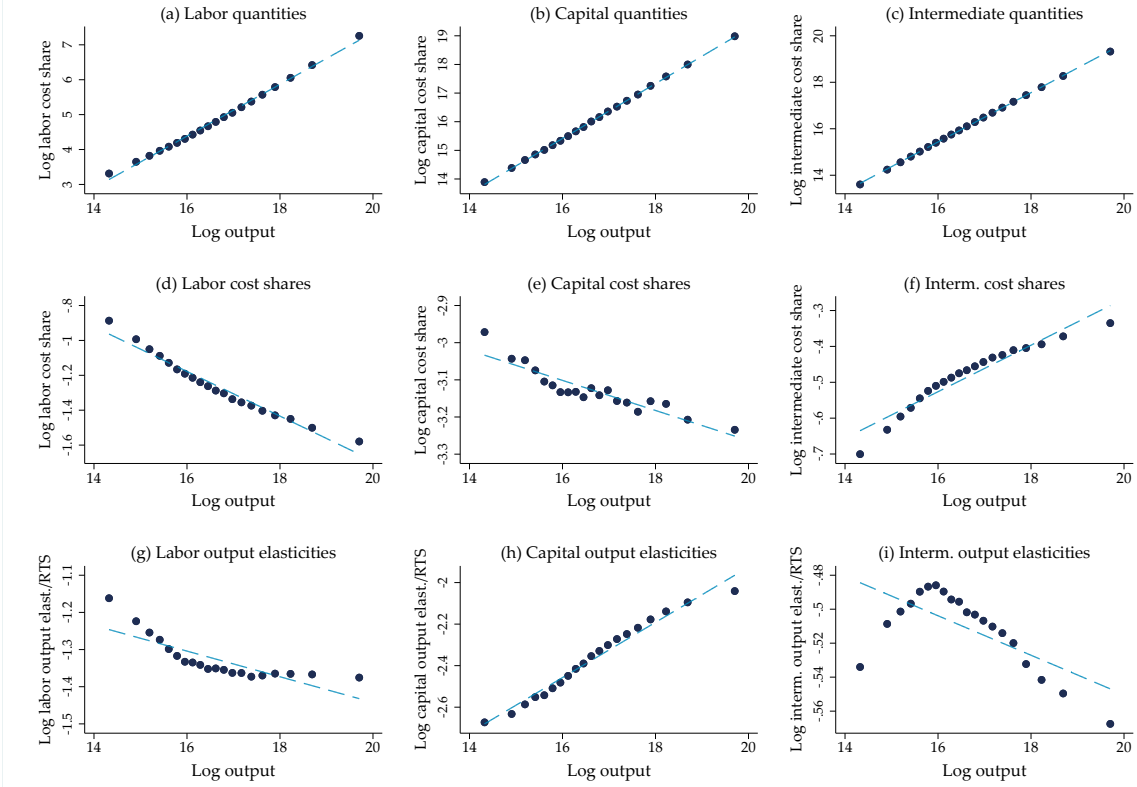
Effects across horizons. Comparing horizons across the panels highlights interesting dynamics in this pattern: the labor-output super-elasticity drops to -0.14 and -0.20 in the medium run, in 4 and 10 year horizons, respectively. Our paper focuses on within-firm changes, but our analysis includes, for completeness, a discussion of cross-sectional perspectives (bottom panel of the tables, and Figure 3, which paints a less clear picture likely due to input prices or other (permanent) heterogeneities.[12]

[11] We approximate capital costs as 8 percent of the capital stock; for the logged specifications, that homogeneous multiplicative factor does not identify the coefficient.

[12] In the cross-sectional regression (which reflects the long run but also permanent firm heterogeneity), the effect falls to -0.03, again precisely estimated. Turning to the cross-sectional scatterplot in Figure 3, reveals a slightly concave pattern, consistent with large shorter-run elasticities not extending to cross-sectional variation in firm size, which is right-skewed. The figure also clarifies that some of this behavior appears to be driven by the returns to scale-size gradient, confirmed also in Column (10) of Table 2, which plays a larger role in the cross section than even in 10-year changes. This underscores why

As we explore in Section 5, the evidence for horizon-dependency could be interpreted as a result of short-run labor adjustment costs that shape firms' production modes, or monopsony. But overall, we will find quite consistent implied values for the substitution elasticity σ across horizons, juxtaposing input mix and output elasticity/cost share gradients in firm size.

Figure 3: The cross-sectional relationships: output elasticities and cost shares



Notes: The figure reports binned scatter plots estimating the specification in Equation (20) in levels with OLS. The upper three graphs project the logs of labor, capital, and intermediate output elasticities over returns to scale against log output, from left to right respectively. The bottom three graphs project the logs of labor, capital, and intermediate cost shares against log output, from left to right respectively. All panels report results that are residualized by industry-year fixed effects. German firm-level data.

Heterogeneity: Types of intermediates. The strong response of intermediates and substitution from labor may partially reflect the fact that intermediate inputs also include intermediate services, unfinished products, and similar items (see Appendix D, and particularly summary statistics Appendix Table A.1 for average cost shares by type of intermediate input). Appendix Table A.9 reports OLS regressions for subsets of intermediates' cost shares, with results going into the expected direction (e.g., strong effects for materials and services, but less so for rent or repairs).

Heterogeneity: Size heterogeneity. Table A.8 reproduces our regressions in 1-year changes splitting firms into five size quintiles. Across all size groups, results align closely with our main results (Table 2).

it is important to account for variation in returns to scale in our regressions.

Heterogeneity: Industries. In Appendix Figure A.1, we report key results by industry and no clear pattern of heterogeneity.

4.2 Causal Effects: IV Strategy using Export Demand Shocks

We now use an instrumental variable strategy that draws on product demand shocks as variation in firm growth that is plausibly unrelated to shifts in production function parameters (output elasticities) or input price shocks as sources of firm growth. This step is to trace out the dynamics of firm growth corresponding to our structural equation in Section 2, where we appealed to shifts in firm output while holding those confounding factors constant.

Across outcomes, IV coefficients are similar to the OLS counterparts. This result is consistent with a view in which OLS unconditional firm growth (after industry-year fixed effects) largely reflects input-unbiased sources of growth such as shifters in product demand, TFP, or input price shifts across the board.

Strategy. To address such endogeneity concerns, we implement an instrumental variables (IV) approach. We follow an established literature using trade-shocks as exogenous shifter (see Autor et al., 2016 for a review). In particular, we follow Hummels et al. (2014) and instrument changes in firms' output with changes in world export demand (excluding Germany). We first compute the total exports for each product, g, from a country group, n, to the world:

$$EX_{gt} = \sum_{c} ex_{gct}^{n \to world}, \tag{21}$$

where n contains Australia, Norway, Sweden, Singpore, New Zealand, Great Britain, Canada, Japan, and the US. c denotes the country. Our country selection follows the strategy in Dauth et al. (2014), with the exception that we additionally include the US (results are robust when excluding the US and using the same countries as in Dauth et al. (2014)). There are two reasons for this selection. First, we choose other industrialized countries as their economies and thus specialization in types of export goods is more plausibly similar to Germany's. Second, this selection of farther-away countries and excluding Germany's direct neighbors and members of its EUR currency union helps mitigate potential endogeneity concerns arising from unobserved shocks that might be correlated between Germany and other nations.

Equation (21) is a product-level measure. To compute firm-specific export demand shocks, we calculate weighted averages of product-level trade flows, using the sales shares of products within firms' product portfolios as weights:

$$INS_{it} = \sum_{g} s_{git=0} \ln EX_{gt}, \tag{22}$$

where s_{git} denotes the firm-specific sales weight of product g. To limit anticipatory effects, we fix the weights for each firm to its first year of observations. Having constructed these firm-specific instruments, we instrument $\Delta \ln Y_{it}$ in Equation (20) with ΔINS_{it}, i.e., we use export demand shocks to instrument output changes.

Importantly, our set of German manufacturing firms consists of exporters, with nearly 80% of firms exporting in a given year. Our sales shares are unconditional of exporting and include domestic sales as well (as we do not observe good-specific export shares by firm).

We restrict our IV analysis to 1- and 4-year differences as we lose power for a first stage (a low F-statistic) for 10-year changes.

Finally, while standard errors somewhat widen for the IV strategy compared to the OLS regressions, the IV estimates remain generally statistically significant at 1% levels. As with the OLS design, we therefore focus our discussion of IV effects on point estimates below.

First stage. Table 3 is analogous to Table 2 and reports the corresponding IV results. Column (1) shows the first stage coefficient from projecting output growth on our instrument. We find a highly statistically significant positive association. The F-statistic is 102.6 for one-year changes and 48.57 for four-year changes. (As mentioned above, we lose a strong first-stage at the 10-year horizon, and hence end at the four-year one in the IV analysis.) We visualize the first stage regressions in Figure 4 Panel (a).

Results: IV estimates. Column (2) of Table 3 reports the second stage IV results, where we instrument output growth with export demand shocks. Across outcomes, IV coefficients are similar to the OLS counterparts. For instance, the output elasticity of labor declines by 3.3% (2.4%) in response to a 10 percent growth in output when considering one-year (four-year) changes.

Results: Reduced form estimates. In Appendix Table A.6, we additionally report reduced form estimates (i.e., we directly regress the dependent variables from Table 3 on the instrument). Results are fully consistent with our IV regressions and similar for one and four-year changes. We visualize results of these regression for key variables in Figure 4 Panels (b)-(j). For instance, the coefficients for labor output elasticities (divided by returns to scale) are approximately -0.015 under both time horizons. The corresponding intermediate input output elasticity coefficients are both 0.005, and labor cost share coefficients are -0.02 and -0.017, for one- and your-year differences, respectively.[13]

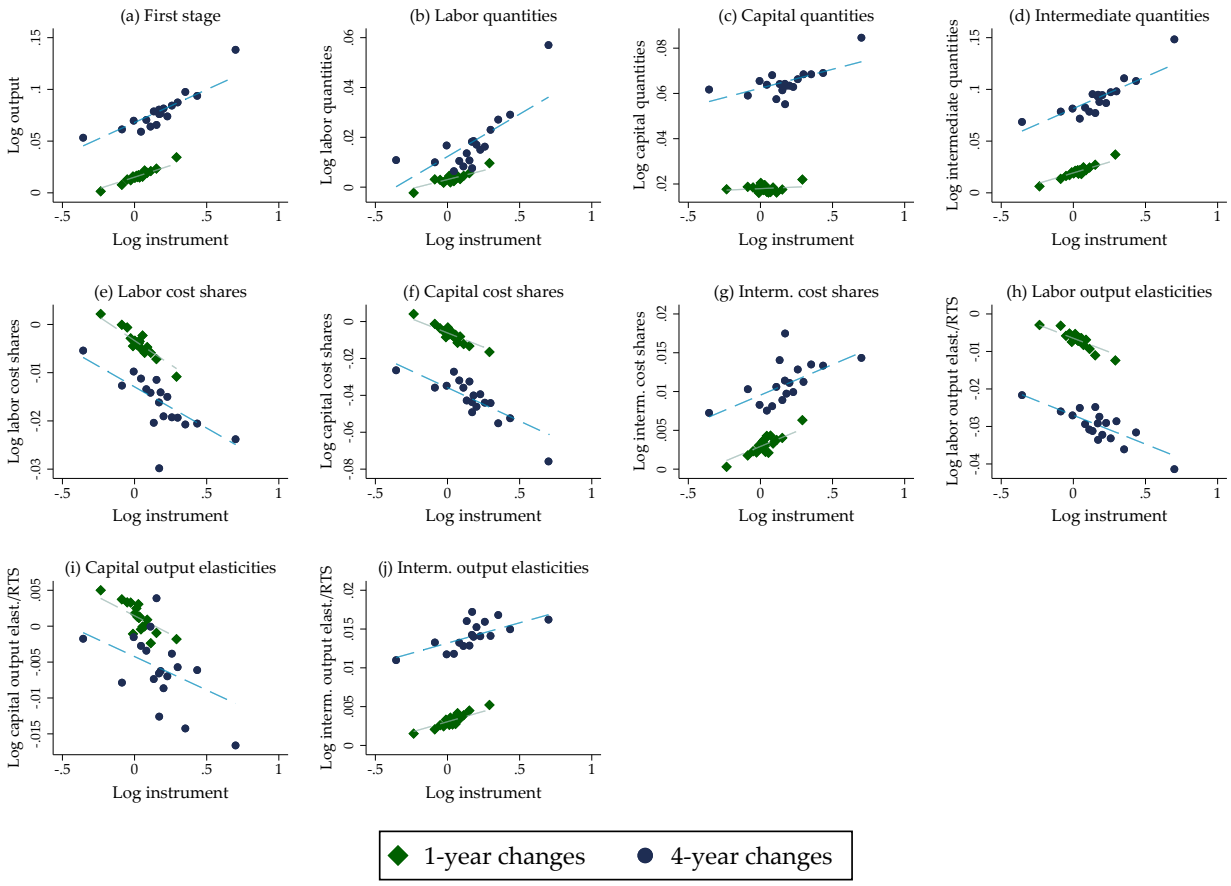
[13]In Appendix Figure A.4 we provide additional visualizations for labor output elasticities without dividing by RTS, capital output elasticities divided by RTS, and labor shares.

Table 3: Firm-level adjustments in response to firm growth. IV regressions.

	1st stage	$\ln(L_{it})$	$\ln(K_{it})$	$\ln(M_{it})$	$\ln(\frac{P^L_{it}L_{it}}{C_{it}})$	$\ln(\frac{P^K_{it}K_{it}}{C_{it}})$	$\ln(\frac{P^M_{it}M_{it}}{C_{it}})$	$\ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\ln(\frac{\theta^K_{it}}{RTS_{it}})$	$\ln(\frac{\theta^M_{it}}{RTS_{it}})$	$\ln(RTS_{it})$
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Export demand shock	0.0451***										
	(0.0045)										
Log output growth		0.324***	0.0612	0.929***	-0.444***	-0.684***	0.171***	-0.327***	-0.229**	0.119***	0.0082
		(0.0543)	(0.0461)	(0.0508)	(0.0521)	(0.0565)	(0.0292)	(0.0623)	(0.0980)	(0.0198)	(0.0053)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950
First-stage F-Statistic		102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6
R^2	0.205	0.214	0.042	0.736	0.336	0.554	0.321	0.229	0.132	0.314	0.052
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Export demand shock	0.0635***										
	(0.0091)										
Log output growth		0.536***	0.262***	0.963***	-0.269***	-0.570***	0.124***	-0.242***	-0.147	0.0823***	0.0222***
		(0.0737)	(0.0973)	(0.0542)	(0.0664)	(0.0969)	(0.0369)	(0.0706)	(0.0938)	(0.0268)	(0.0075)
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492
First-stage F-Statistic		48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57
R^2	0.183	0.470	0.139	0.855	0.347	0.471	0.339	0.217	0.169	0.266	0.156

Notes: The table reports IV regressions from estimating the specification in Equation (20) using foreign demand shocks as instruments (Equation (22)). Column (1) reports the first-stage regression results. The dependent variables in Columns (2)-(11) are the logs of labor, capital, intermediates, labor cost shares, capital cost shares, intermediate cost shares, labor output elasticities over returns to scale, capital output elasticities over returns to scale, intermediate input output elasticities over returns to scale, and returns to scale, respectively. All regressions project changes in those dependent variables on changes in log output for 1- and 4-year differences. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm-level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Figure 4: First stage and reduced form results, one- and four-year changes



Notes: The figure reports binned scatter plots for firm-level first stage and reduced form regression. The first stage projects log output on the instrument, whereas the reduced form regressions do so for the logs of labor output elasticities divided by returns to scale, intermediate output elasticities divided by returns to scale, and labor cost shares. Regressions are in one-year and four-year changes. All panels report results that are residualized by industry-year fixed effects. German firm-level data.

5 Quantitative and Structural Interpretation

We now interpret our reduced form results structurally and quantitatively through the lens of the CES production function and input price functions presented in Section 2 and summarized in Table 1. Then, we discuss alternative accounts.

Table 4 summarizes the identified parameters and the mapping from reduced-form empirical moments using the identification arguments from Section 2 sharpened below, for the substitution elasticity between labor and intermediates and the (relative) firm-specific labor supply elasticity. Figures 5 and 6 summarize existing estimates of the two key parameters, along with the values our study implies. Throughout, we focus on *firm growth* estimates (and do not apply a similar analysis to our additional results on cross-sectional firm size gradients).

Table 4: Implied substitution elasticities, changes in input cost ratios, and firm-specific labor supply elasticities

	OLS				IV			
	σ	$\Delta \ln\left(\frac{L}{M}\right)$	$\Delta \ln\left(\frac{P^L \gamma^L_{it}}{P^M \gamma^M}\right)$	$\widetilde{\epsilon}^L = 1/\varepsilon^L$	σ	$\Delta \ln\left(\frac{L}{M}\right)$	$\Delta \ln\left(\frac{P^L \gamma^L}{P^M_{it} \gamma^M_{it}}\right)$	$\widetilde{\epsilon}^L = 1/\varepsilon^L$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1-year diff.	2.673	-0.727	0.272	1.10	3.805	-0.605	0.159	2.04
4-year diff.	2.259	-0.535	0.236	2.16	4.158	-0.427	0.103	5.20
10-year diff.	1.837	-0.435	0.237	2.56				

Notes: The table reports substitution elasticities (Columns (1) and (5)), changes in input factor ratios (Columns (2) and (6)), changes in effective factor price ratios (Columns (3) and (7)), and implied labor supply elasticities, assuming perfectly elastic intermediate input supply (Columns (4) and (8)), based on our OLS (Columns (1)-(4)) and IV (Columns (5)-(8)) regressions from Tables 2 and 3 that project logged output elasticities over returns to scale and logged input factors on logged output in differences.

5.1 Identification of Substitution Elasticity

Identification argument. Our identification of the substitution elasticity rests on the production function property in Equation (5) for the relative output elasticities given by the following expressions, recapping Equations (23) and (24):

$$\frac{\theta^L_{it}}{\theta^M_{it}} = \frac{\alpha^L_i}{\alpha^M_i} \left(\frac{L_{it}}{M_{it}}\right)^{\frac{\sigma-1}{\sigma}} \tag{23}$$

$$\Rightarrow \Delta \ln \frac{\theta^L_{it}}{\theta^M_{it}} = \frac{\sigma-1}{\sigma} \Delta \ln \frac{L_{it}}{M_{it}}. \tag{24}$$

If labor and intermediates are substitutes ($\sigma > 1$), an increase in the intermediates-labor mix leads to a relative decline in the labor output elasticity, with a increase in intermediates' output elasticity. This is exactly what we find in the data: Table 2, Columns (1) and (3) show an intermediates elasticity of about 1.0 to output growth but only about 0.30 for labor (IV results in Table 3 are almost identical).

Taking log-differences in Equation (24) eliminates constant firm-heterogeneity captured by $\frac{\alpha^L_i}{\alpha^M_i}$ and recovers an expression for the substitution elasticity, σ, as identified from the co-movement of output elasticities and input quantities:

$$\frac{\sigma-1}{\sigma} = \frac{\Delta \ln(\theta^L_{it}) - \Delta \ln(\theta^M_{it})}{\Delta \ln(L_{it}) - \Delta \ln(M_{it})} \tag{25}$$

$$\Leftrightarrow \sigma = \frac{1}{1 - \frac{\Delta \ln(\theta^L_{it}) - \Delta \ln(\theta^M_{it})}{\Delta \ln(L_{it}) - \Delta \ln(M_{it})}} \tag{26}$$

Ingredients: reduced form regression estimates. Specifically, we use within-firm changes (Δ) *conditional on firm growth*, i.e., feed into the expressions the estimated OLS and IV coefficients estimated in the regression of a given outcome variable on firm growth. (Compared to the OLS estimates, the IV counterparts can be viewed as additionally purging the changes of potential confounders such as input-biased shifters in prices or production function parameters that may have underlied some of the OLS variation in firm growth.) That is, concretely, for each right-hand side component $\Delta \ln x$ in Equation (26), we plug in the corresponding regression coefficient for that outcome x estimated in

Section 4, separately by time horizon and for OLS or IV:

$$\widehat{\sigma} = \frac{1}{1 - \frac{\rho_{\Delta \ln(\theta^L),\Delta \ln(Y)} - \rho_{\Delta \ln(\theta^M),\Delta \ln(Y)}}{\rho_{\Delta \ln(L),\Delta \ln(Y)} - \rho_{\Delta \ln(M),\Delta \ln(Y)}}}, \tag{27}$$

where the various ρ correspond to the regression coefficients from Equation (20), for various horizons and outcome variables, for OLS and the IV analogs.

Hence, our difference-based estimates of changes in output elasticities and input factors in response to firm growth, derived from Tables 2 and 3, identify values for the substitution elasticity, σ. We focus on point estimates, given the precisely estimated coefficients particularly in the OLS regressions, and do not respond formal confidence intervals around the implied parameters (although for IV effects, implied values and wider confidence intervals of the coefficients may merit another approach).

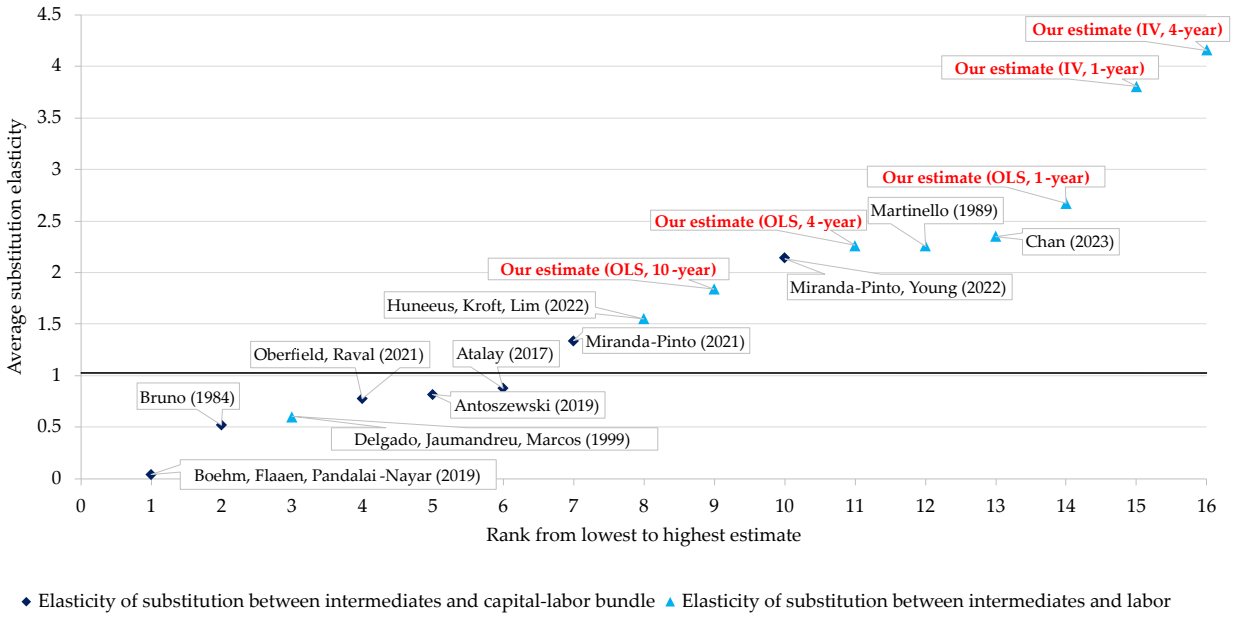
Implied parameter values. Table 4 Columns (1) and (4) report the implied values of σ based on our OLS and IV estimates. We find that labor and intermediates are indeed substitutes, with σ exceeding unity in all specifications. The OLS estimates range from 2.67 in the short run to 1.84 in the longer run. IV estimates of σ are around 4 for 1- and 4-year changes (we do not have IV estimates for 10-year changes). These substitution elasticity estimates above one provide a coherent explanation for the decline in relative output elasticities following the reduction in the labor-intermediate quantity ratio.

The relatively high substitution elasticities may also reflect the fact that intermediate inputs also include intermediate services, unfinished products, and similar items (see Appendix D, and particularly summary statistics Appendix Table A.1 for average cost shares by type of intermediate input).[14] Appendix Table A.9 reports OLS regressions for subsets of intermediates' cost shares, as discussed in Section 4, with results going into the expected directions.

Meta-analysis of existing parameter estimates. Figure 5 summarizes existing estimates of σ, along with the values our study implies and that we reported in Table 4. Quantitatively, our firm growth regressions identify substitution elasticities well above one, ranging from 1.8 to 2.7 (OLS) and from 3.8 to 4.2 (IV). The meta-analysis situates these values in a systematic meta analysis of related existing estimates. Existing estimates of the intermediate-labor substitution elasticity vary and are difficult to compare as they depend on disparate identification strategies and production model assumptions. Atalay (2017), Oberfield and Raval (2021), and Boehm et al. (2019) estimate substitution elasticities between intermediates and a capital-labor bundle of 0.8 to 0.9, 0.6 to 1.0, and around 0, respectively. More consistent with our approach, Huneeus et al. (2022) and Chan (2023) estimate substitution elasticities between labor and intermediates of 1.6 and 1.5 to 4.0, respectively. We note that our estimates are derived from within-firm changes, and focus on intermediate-labor substitution.

[14] As an aggregate perspective, consistent with labor and intermediates being substitutes, manufacturing intermediate to labor expenditure ratios increased while intermediate input to labor price ratios declined. See Appendix Figure A.5 for evidence on Germany and the US.

Figure 5: Meta analysis and quantitative interpretation: Elasticity of substitution vs. literature



Notes: The figure presents estimates from the literature as well as from our paper for the value of the labor-intermediate elasticity of substitution, σ.

5.2 Identification of (Relative) Firm-Specific Labor Supply Elasticity (Monopsony)

Why do firms change their input mix (which which follow the output elasticity effects) as they grow? Firms' cost minimization provides a natural answer, as it ties firms' optimal input mixes to input prices. We now trace out the implied input price ratio, and translate it into an implied labor supply elasticity to the firm.

Step 1: identifying the implied input price ratio change-firm growth gradient. Using our estimates of σ from above, we can infer the implied (average) change in the shadow price ratio (specifically, the change that is caused by firm growth in our empirical specification), using the CES structure from first first-order conditions for labor and intermediates:

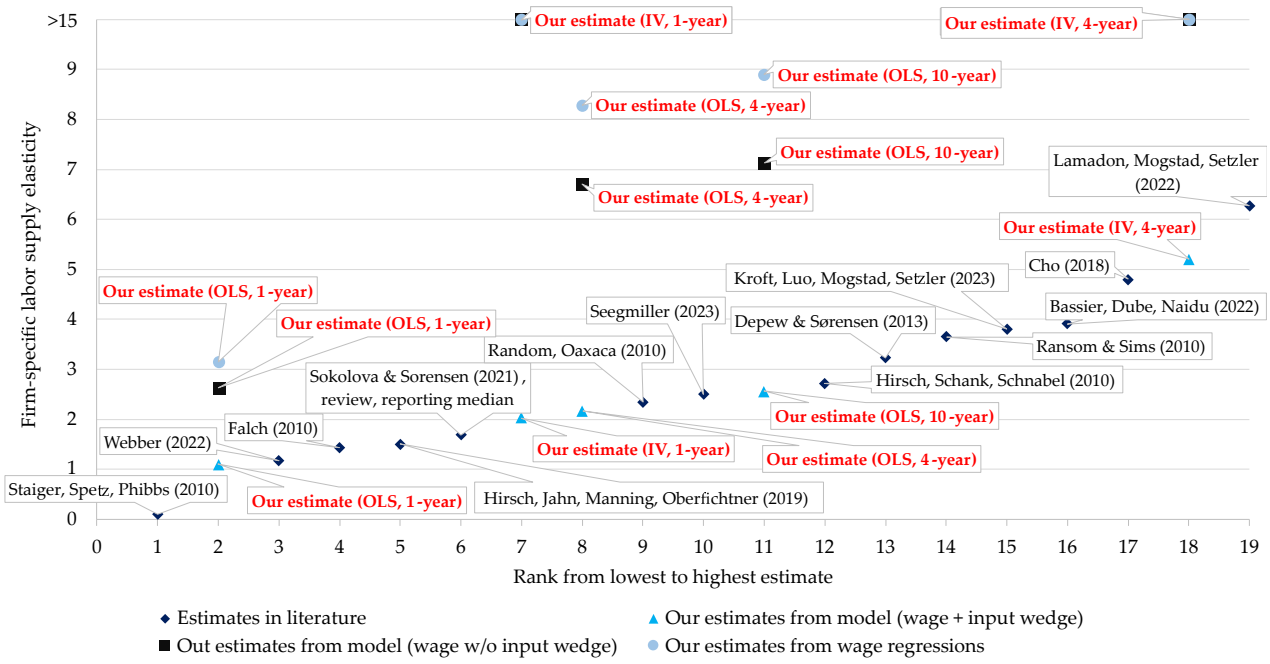
$$\Delta \ln\left(\frac{P_{it}^{L}\gamma_{it}^{L}}{P_{it}^{M}\gamma_{it}^{M}}\right) = \frac{\Delta \ln(L_{it}) - \Delta \ln(M_{it})}{-\sigma}. \tag{28}$$

Again, we feed in regression coefficients in the spirit of Equation (27) to back out the the effect of firm growth on the within-firm change in the log input (shadow) price ratio. That is, we back out the within-firm growth of the input shadow price ratio of labor vs. intermediates in response to firm growth—a regression coefficient that we have not estimated directly but argue is implied by σ and the estimated input quantity coefficients due to cost minimization:

$$\widehat{\rho}_{\Delta \ln(\frac{P^{L}\gamma^{L}}{P^{M}\gamma^{M}}),\Delta \ln(Y)} = \frac{\rho_{\Delta \ln(L),\Delta \ln(Y)} - \rho_{\Delta \ln(M),\Delta \ln(Y)}}{-\widehat{\sigma}}. \tag{29}$$

We report results for $\widehat{\rho}_{\Delta \ln(\frac{P^L \gamma^L}{P^M \gamma^M}),\Delta \ln(Y)}$ in Table 4 Columns (3) and (6).[15] OLS estimates indicate changes in the log price ratio of 0.27 in the short run and 0.23 in the long run. IV estimates are 0.16 and 0.10 for short- and longer-run changes, respectively. Our findings thus suggest that, as firms grow, effective wage costs increase relative to intermediate input costs. This incentivizes firms to substitute labor with intermediates and to change their modes of production with respect to input mix, as reflected in their output elasticities. To the best of our knowledge, we are the first to combine estimates of output elasticities with a standard production model to measure unobserved factor price changes.

Figure 6: Meta analysis and quantitative interpretation: Labor supply elasticity vs. literature



Notes: The figure presents estimates from the literature as well as from our paper for the value of the firm-specific labor supply curve, in our model the inverse of ϵ^w.

Potential sources of the relative increase in labor prices: finitely elastic supply curves (monopsony), rent sharing, and adjustment costs. What forces cause relative wage costs to rise when firms grow and move their inputs from labor to intermediate inputs? One natural explanation is that the firm-specific labor supply elasticity is low (compared to its intermediate input counterpart), raising wages as firms employ more workers. Similarly, larger labor market shares of larger firms may raise the supply elasticity and hence the markdown (Berger et al., 2022). This monopsony-based explanation is our preferred explanation, and we develop it further next, backing out the firm-specific labor supply curve elasticity.

Alternative mechanisms to monopsony through a finite firm-specific labor supply elasticity include rent sharing, whereby the shocks that increase firms' desired size may also increase wages (including marginal ones). We develop such a model in and show similar results for the production function and

[15] In Section 6, we also measure changes in $\frac{\gamma^L_{it}}{\gamma^M_{it}} = \gamma_{it}$ Hence, we can also recover the input price ratio net of input frictions.

input demand outcomes of the firm using a Nash bargaining framework Appendix C.

Lastly, rather than by affecting unit prices, the effective labor costs are also shaped by relative market frictions such as (convex) adjustment costs that may plausibly be more pronounced for labor than intermediate inputs. The fact that the price effects remain strong at the 10 year horizon suggests that such short-run wedges may not play the primary role in the implied labor price increase. Indeed, Table 4 shows that short-run changes in input price ratios are somewhat higher than longer-run changes. Our findings can thus be interpreted as direct evidence of a lower short- than longer-run labor supply elasticity (see below) or as indicating dynamic adjustment costs.

Step 2: Identifying the firm-specific factor supply elasticities. To identify the firm-specific labor supply curve, we make two key assumptions. First, intermediate inputs are perfectly elastically supplied.[16] Second, we assume that the markdown γ^L is stable in firm growth (but can permit a baseline wedge)—an assumption we relax later. Those two assumptions imply that the input shadow price gradient is solely due to wage increases. With that assumption in fact, we can infer the implied firm-specific labor supply elasticities from our estimated input price changes and the employment effects as follows:

$$\widetilde{\varepsilon}^L = \frac{\Delta \ln(L_{it})}{\Delta \ln\left(\frac{P^L_{it}\gamma^L_{it}}{P^M_{it}\gamma^M_{it}}\right)} = \frac{\Delta \ln(L_{it})}{\Delta \ln(P^L_{it})}, \tag{30}$$

which we identify on the basis of the following coefficients:

$$\widetilde{\widehat{\epsilon}}^L = \frac{\rho_{\Delta \ln(L),\Delta \ln(Y)}}{\widehat{\rho}_{\Delta \ln(\frac{P^L\gamma^L}{P^M\gamma^M}),\Delta \ln(Y)}}. \tag{31}$$

That is, we identify the labor supply elasticity by dividing the changes in labor quantities from Tables 2 and 3 by the changes in input price ratios from Table 4 as constructed above. *We emphasize that this parameter $\widetilde{\epsilon}^L$ is the* inverse *of the supply elasticity ε^L in Section 2, which denoted the effect of labor quantities on the price of labor.*

Results: implied firm-specific labor supply elasticities. We report the corresponding estimates for labor supply elasticities, ϵ^L, in Table 4. The values range from 1.10 to 2.56 for our OLS and from 2.04 to 5.20 for our IV results. Hence, these values substantiate and quantify the monopsony motivation for firms to substitute from labor to intermediates in the form of a portable statistic of firm-specific labor supply elasticities. We next show robustness checks and then situate these estimates in a meta-analysis.

Robustness: permitting markdowns and direct wage estimates. The baseline estimates of the labor supply elasticity are with respect to the shadow price of labor—which is inferred and under the

[16] We are not aware of comparable estimates for the firm-specific supply elasticities for intermediates. We assume them to be supplied elastically in our quantitative interpretation. Bilal and Lhuillier, 2022 makes an analogous assumption for a labor-only model involving labor service purchases (such as temp work agencies), although our intermediates are largely made of goods rather than services. For temp agency workers as one facet of outsourcing, Drenik et al., 2023 show that wage premia partially extend to outsourced labor. Huneeus et al., 2022 focus on intermediate input price variation from production networks as a driver of cross-sectional outsourcing differentials.

assumption of no shift in markdowns. We now calculate firm-specific labor supply elasticities under two alternative assumptions. We report the resulting values in Figure 6, discussed below.

First, we also report results that subtract the measured markdown effect on γ^L/Γ^M in Section 6. Since we find that markdowns tend to *increase* in firm size and growth, this calculation pushes up the labor supply elasticities in our study.

Second, we also measure wages directly, using the average wage measure we can construct in the firm-level data. Particularly for the OLS strategy, average wages increase in firm growth (consistent with monopsony), so that we have a direct estimate of the wage that draws in labor supply (and attribute the rest of the implied input price-firm growth gradient to the wedges γ^X). These additional wage effects are reported in Appendix Table A.12. As a result, the implied firm-specific labor supply elasticities are also higher. (For IV, since the wage effect point estimate is close to zero, these values imply large elasticities, but the wider coindidence intervals for the wages accommodate also elasticities more consistent with the literature.) Overall, we caveat that average wages are subject to composition bias (and we have not been able to merge on matched employer-employee data or data on worker skills to provide a cleaner wage estimate).

Meta-analysis of existing parameter estimates. Figure 6 summarizes existing estimates of σ, along with the values our study implies, including the additional estimates from the robustness checks discussed above (while estimates are ranked by size, for our own estimates, we provide their respective robustness checks vertically stacked). Overall, these estimates are well in line with existing findings (Sokolova and Sorensen, 2021) and again suggest a lower short- than long-run labor supply elasticity. The range of implied elasticities across our specifications and wage measurements and assumptions fall well into the range of existing estimates (e.g., Sokolova and Sorensen, 2021). The figure also illustrates that we find that supply elasticities that are higher in the long run and for our IV estimates, and when we net out markdown shifts or use direct (average) wage estimates.

5.3 Further Robustness Checks

Heterogeneity by Firm Size. Table A.8 reproduces our regressions in 1-year changes splitting firms into five size quintiles. Across all size groups, results align closely with our main results (Table 2). Table 5 further shows that the implied substitution elasticities and labor supply elasticities by size group are relatively stable and consistent with our pooled data regressions.

Using Cost Shares to Calculate Output Elasticities. In Appendix Table A.7, we calculate the substitution elasticities using cost shares as a direct measure of output elasticities, rather than production function estimation. We find higher substitution elasticities (and correspondingly lower price ratio changes, and higher labor supply elasticities). The identification assumptions are spelled out in Section 6, but naturally arise by setting $\gamma^X = 0$.

Changes vs. cross-section. We focused on a specification in changes, which netted out fixed, unobserved firm-specific factors, which are captured by $\frac{\alpha_i^L}{\alpha_i^M}$ in our model.[17] Large firms have a higher intermediate/labor ratio, without an associated increase in their output elasticity of labor that our production function seems to suggest. Since our focus is on firm growth, we do not include our detailed discussion of how to rationalize those facts with permanent heterogeneity in the paper.

5.4 Alternative Mechanisms

Automation. Recent work has highlighted capital-based technological change as one factor that reduces the importance of labor to firms (Acemoglu and Restrepo, 2020, Hubmer and Restrepo, 2021). Such technological change could explain a decline in labor's output elasticity with firm growth. However, rather than an increase in capital output elasticities, we document that those decline as well. Instead intermediate inputs gain in importance. Therefore, while automation might be an important process shaping changes in firms' production and in terms of aggregate labor market trends, it appears not to rationalize our findings in the context of idiosyncratic firm growth.[18] Additionally, note that our causality runs from plausibly input-neutral output shifters to changes in output elasticities.

Fixed costs and size dependence. Our data do not allow us to differentiate between fixed and flexible labor inputs. If firms' production involves fixed labor costs that scale up less than proportionately with flexible labor, labor output elasticities decline as firms grow. Effectively, fixed costs break the homotheticity of a production function (Dhyne et al., 2022, Savagar and Kariel, 2024). We expect fixed costs to be particularly relevant for small firms. However, when we split our sample into size quintiles, we find almost identical labor-output super-elasticity estimates (Appendix Table A.8). Therefore, fixed labor costs are unlikely to explain the diminishing output elasticity of labor. However, we find that declines in capital output elasticities and increases in intermediate inputs are somewhat moderated at larger firms. This result is consistent with large initial capital fixed costs in firms' production—something that can be particularly relevant in manufacturing.

Biased technological change. One concern could be that our output elasticities are mismeasured as we impose Hicks neutrality when estimating production functions. Yet, we do not believe that measurement error, and specifically factor-biased technological shocks, can account for our findings for two reasons.

First, our results on labor cost shares and labor output elasticities exhibit quantitatively very similar patterns—providing insights into output elasticities under very different assumptions.[19] Our estimation of output elasticities relies on Hicks neutrality and a specific model of firm behavior but allows for input adjustment frictions, input market power, and increasing/decreasing returns to scale.

[17] Indeed, applying cross-sectional estimates to our model yields unrealistic values and suggest that a 10% larger firm pays a 58% higher wage relative to intermediate unit costs, which is far beyond the 1-2.7% reported in Table 4.

[18] Evidence on the aggregate employment effects of automation is mixed. While Acemoglu and Restrepo (2020) find that robots reduced employment and wages in the US, Graetz and Michaels (2018) conclude that robots did not significantly reduce aggregate labor demand in their cross-country analysis of mostly European countries.

[19] We acknowledge that we find less consistent results for capital in the *cross-sectional regressions* (as noted above), perhaps due to confounders in the cross-sectional regressions or from difficulties in approximating capital costs. In particular, we impose homogeneous and fixed capital unit costs across all firms.

Table 5: Implied substitution elasticities, changes in input cost ratios, and firm-specific labor supply elasticities: by size quintiles for OLS 4-year changes

	OLS			
	σ	$\Delta \ln \left(\frac{L_{it}}{M_{it}} \right)$	$\Delta \ln \left(\frac{P_{it}^L \gamma_{it}^L}{P_{it}^M \gamma_{it}^M} \right)$	$\widetilde{\epsilon}^L = 1/\varepsilon^L$
	(1)	(2)	(3)	(4)
Overall	2.259	-0.535	0.236	2.16
1st quintile	2.007	-0.546	0.272	1.86
2nd quintile	2.236	-0.550	0.246	1.97
3rd quintile	2.186	-0.540	0.247	1.99
4th quintile	2.327	-0.519	0.223	2.37
5th quintile	2.277	-0.526	0.231	2.29

Notes: The table reports substitution elasticities (Column (1)), changes in input factor ratios (Column (2)), changes in effective factor price ratios (Column (3)), and implied labor supply elasticities assuming perfectly elastic intermediate input supply (Column (4)) based on our OLS regressions from Tables 2 that project logged output elasticities over returns to scale and logged input factors on logged output in differences. Size quintiles are computed by year and industry. Size is measured by sales.

Cost shares, on the other hand, measure output elasticities under perfect input markets and constant returns to scale but allow for non-Hicks-neutral productivity processes and do not impose constant parameters of the underlying production function (see Appendix F).

Second, we also instrument for firm growth using an instrumental variable approach based on foreign export demand shocks. These shocks are arguably orthogonal to firm-specific factor-biased technological shocks that affect output elasticities and increase output. Instead, our results focus on the causality that runs from firm growth to changes in output elasticities rather than from changes in technology to changes in output.

Non-homothetic CES. Our analysis focused on a homothetic CES production function. Lashkari et al. (2023) show that IT-technologies become more efficient at larger scale, which causes firms to operate on a higher IT intensity. Perhaps an analogous argument could hold for intermediates rationalizing our results: If intermediate inputs become more efficient at larger scale, firms' relative labor output elasticities decline, moving into intermediate inputs. For their focus on IT inputs, Lashkari et al. (2023) model this process using a non-homothetic CES production function. An adaptation of their framework to the CES model above and intermediates instead (and a further refinement we make to purely isolate scale-dependence of output elasticities) yields the following non-homothetic production function:

$$Q = Q(.) = A \left(\Lambda_{it}^{LM} \left(\alpha_{it}^L L^{\frac{\sigma-1}{\sigma}} + \alpha_{it}^M \left(\frac{M}{Q^\eta} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}\frac{\kappa-1}{\kappa}} + \Lambda_{it}^K K^{\frac{\kappa-1}{\kappa}} \right)^{\frac{\kappa}{\kappa-1}}, \tag{32}$$

where $\eta > -1$ is a firm-size scaling parameter that delivers the non-homoetheticity. If $\eta = 0$, Equation (32) collapses to Equation (5). Under the non-homothetic CES, the ratio of output elasticities is:

$$\frac{\theta^L}{\theta^M} = \frac{\alpha_{it}^L}{\alpha_{it}^M} \left(\frac{L}{M} \right)^{\frac{\sigma-1}{\sigma}} Q^{\eta(\frac{\sigma-1}{\sigma})}, \tag{33}$$

Compared to the homothetic production function, the ratio of output elasticities now features an additional *direct* effect of firm size on the output elasticity ratio. In particular, if labor and intermediates are substitutes, growing firms experience a decrease (increase) in their labor output elasticity relative to their intermediate input output elasticity if $\eta < 0$ ($\eta > 0$).

Equation (33) suggests a horse race as a straightforward way of testing the relevance of the non-homotheticity versus the input factor ratio in regulating changes in output elasticities. Specifically, we regress log changes in the labor output elasticity on log changes in the input factor ratio *and in the same regression* on log changes in output. We report results from this exercise in Appendix Tables A.10 and A.11 and find that, once we control for the input factor ratio, there is no statistically significant negative effect of log output changes on the log labor output elasticity changes. This result suggests that, in our case, non-homotheticity in output is irrelevant in explaining output elasticity changes with firm growth.[20] Instead, the discussed changes in the input factor ratio and relative input prices are key in understanding why growing firms move toward less labor- and more intermediate-intensive modes of production, and also offer a natural dynamic explanation for the increased relevance in the short run. In addition, we reiterate the fact that across firm size groups, we find similar results (discussed above).

6 Implications for Firm-Level Labor Shares

One immediate implication of the negative effect of firm growth on the output elasticity of labor is the negative effect of firm growth on the firm-level labor share. We now extend the basic production function framework from Section 2 to formally derive this result, and use the implied structural equation to guide our further empirical analyses.

6.1 Theoretical Framework and Identification

The labor share. Firms operate with the following cost function:

$$C_{it} = P_{it}^{L}(L_{it})L_{it} + P_{it}^{K}K_{it} + P_{it}^{M}M_{it}, \tag{34}$$

where the input prices P^X denote unit costs for labor, capital, and intermediates. We allow wages to depend on labor quantities, such that firms can exercise monopsony power (Appendix C shows that a setting where workers have bargaining power delivers essentially the same results). The first order condition for labor yields the following expression for the labor share (see Appendix B for the derivation):

$$LS_{it} = \frac{P_{it}^{L}L_{it}}{P_{it}Q_{it}} = \frac{\theta_{it}^{L}}{\mu_{it}\gamma_{it}}. \tag{35}$$

P_{it} denotes the firm's output price. LS_{it} is the labor share in sales. $\mu_{it} = \frac{P_{it}}{MC_{it}}$ is the markup of the output price over marginal cost (MC_{it}). $\gamma_{it} = \frac{MRPL_{it}}{P_{it}^{L}}$ is the wage markdown, with $MRPL_{it}$ being the marginal revenue product of labor.

[20] In some specifications, we even find a very small positive effect of output growth on the labor output elasticity once we control for the input ratio, i.e., even a reversal of the sign.

Log decomposition. Importantly, the rightmost side of Equation (35) provides a full statistical decomposition of the firm-level labor share, and is log-additive:

$$\ln(LS_{it}) = \ln(\theta^L_{it}) - \ln(\mu_{it}) - \ln(\gamma^L_{it}). \tag{36}$$

Hence, the model demonstrates a direct effect of θ^L_{it} on the labor share (for a given markup and markdown).

Identifying price markups and wage markdowns. To separate the three determinants of the labor share, we construct measures of markups, markdowns, and labor output elasticities from firm-level micro-data. We permit all these variables to be *firm- and time-specific*.

We derive output price markups and wage markdowns using the production approach of Hall et al. (1986) and De Loecker and Warzynski (2012). Assuming that intermediate inputs are a flexible input and that firms' take intermediate input prices as given—such that $\gamma^M_{it} = 1$—, markups, μ_{it}, can be derived from the first order condition for intermediates within our framework (see Appendix B):

$$\mu_{it} = \frac{P_{it}}{MC_{it}} = \theta^M_{it} \frac{P_{it}Q_{it}}{P^M_{it}M_{it}}, \tag{37}$$

where $\theta^M_{it} = \frac{\partial \ln Q_{it}}{\partial \ln M_{it}}$ is the output elasticity of intermediate inputs.

Following recent work (e.g., Dobbelaere and Mairesse, 2013, Caselli et al., 2021, Yeh et al., 2022), we can also define an expression for firms' wage markdowns, γ^L_{it}, by combining the first order conditions for intermediates and labor (see Appendix B):

$$\gamma^L_{it} = \frac{MRPL_{it}}{P^L_{it}} = \frac{\theta^L_{it}}{\theta^M_{it}} \frac{P^M_{it}M_{it}}{P^L_{it}L_{it}}. \tag{38}$$

Importantly, we acknowledge that Equation (38) also captures any biased technological differences between firms that are not captured by output elasticities (but they will be captured by cost shares as discussed above).

We also reiterate that this expression is effectively a measure of relative input cost markdowns for labor vs. intermediate inputs ($\gamma_{it} = \frac{\gamma^L_{it}}{\gamma^M_{it}}$). (Assuming exogenous intermediate input prices and flexible intermediate inputs imposes $\gamma^M_{it} = 1$.)

Finally, we also define and study the labor wedge reflecting the combined distortion from product and labor market power, namely:

$$\mu_{it}\gamma^L_{it} = \theta^L_{it} \frac{P_{it}Q_{it}}{P^L_{it}L_{it}}. \tag{39}$$

6.2 Results

Strategy. To gauge effects on firm-level labor shares, we estimate regression model Equation (20) but replace the outcome variable θ^L_{it} with labor shares, markups, markdowns, and the labor wedge. All outcome variables are logged and in differences, which allows us to quantify the part of the elasticity of the labor share to output growth that is accounted for by the diminishing output elasticity of labor

as described in Equation (36).

Throughout, we focus on the sales labor share because this is the relevant labor share definition that corresponds to the output elasticity of labor and our firm-level production function framework. However, we also include the labor share in value added as a robustness check.[21]

Again, we present OLS and IV results, and, given the important horizon dependencies we have uncovered in the previous section, we estimate this effect for 1-, 4-, and 10-year differences and additionally include the cross-sectional specification in levels.

Regression results. Tables 6 (OLS) and 7 (IV) are organized as their counterparts, Tables 2 and 3, showing the OLS and IV effects of firm output on labor shares, markups, markdowns, the labor wedge, and the value added labor share for various time horizons. We additionally add our previous estimates on labor output elasticities to facilitate the analysis. As indicated in Equation (36), the coefficients on markups, wage markdowns, and output elasticities should *add up* to fully account for the relationship between firm output and the labor share.

(While this adding up property holds up well for our regressions in differences, it does less so for our cross-sectional regression. This is because our industry-year fixed effects absorb part of the cross-sectional variation in markups, output elasticities, and markdowns—less relevant for the regression in changes, because there we solely use *changes* for identifying coefficients.)

Scatterplots. Figures 7 and 8 are the analogs of Figures 2 and 3, visualizing the variation underlying the regression estimates.

Declining labor shares. We find a strong negative effect of firm output on labor shares—which again shrinks as we widen the time horizons, ranging, in the OLS regressions, from -0.581 to -0.303 for the specifications in differences and to a much smaller but precisely estimated -0.032 in the cross-sectional regressions. Results for the value-added labor share as well as IV estimates are similar.

Half to a third of the effect: the diminishing output elasticity of labor. In the short and medium run, up to the 4-year differences, *half* of this negative effect of output growth on labor shares is driven by the diminishing output elasticity of labor. This result holds for OLS and IV regressions. For 10-year differences, the diminishing output elasticity still accounts for a third of the effect (as changing returns to scale become increasingly important over longer time horizons).

(In the cross-section, differences in labor output elasticities net of returns to scale still contribute to lower labor shares among larger firms, but the effect is much smaller (perhaps due the aforementioned confounders).)

The remainder: conventional market power effects. The remaining half of the short- to medium-run within-firm effect of firm growth on labor share changes is explained by markups and wage

[21]Note that under a value added production function, we can approximate the value added output elasticity as $\theta^L_{it}\tau_{it}$, where τ_{it} is the ratio of sales to value added. As the value added labor share equals the labor share in sales times τ_{it}, we can transfer our conclusions on the relative importance of changes in the output elasticity of labor for the sales labor share also to the value added labor share. See Appendix Figures A.2 and A.3 for the underlying empirical relationships.

markdowns increasing in firm size. As we control for industry fixed effects, output growth is effectively reflecting an increase in firms' market shares, and we expect a firm's market power to increase with its market share (as in Atkeson and Burstein, 2008 for product markets and Berger et al., 2022 for labor markets).

Over longer time horizons, wage markdowns become increasingly important in explaining the effect of firm size on labor shares. This becomes particularly clear in the cross-sectional regressions. The long-run importance of market power is consistent with Autor et al. (2020), who, however, did not investigate changes in output elasticities (and who focus on the labor wedge as their markup measure, whereas we differentiate between output price markups and wage markdowns).

Summary. We conclude that over the short to medium run, the majority of the negative effect of output growth on *firm-level* labor shares results from the diminishing output elasticity of labor. Over longer time horizons, labor market power becomes more important.

Table 6: Market power, output elasticity, and labor share adjustments in response to firm growth. OLS regressions.

	$\ln(LS_{it})$	$\ln(\mu_{it})$	$\ln(\gamma_{it})$	$\ln(\mu_{it}\gamma_{it})$	$\ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\ln(\theta^L_{it})$	$\ln(\frac{P^L_{it}L_{it}}{VA_{it}})$
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output growth	-0.581***	0.124***	0.159***	0.283***	-0.304***	-0.298***	-0.561***
	(0.0040)	(0.0025)	(0.0048)	(0.0044)	(0.005)	(0.0046)	(0.0076)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950
R^2	0.534	0.096	0.115	0.191	0.230	0.215	0.190
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output growth	-0.406***	0.0608***	0.160***	0.220***	-0.202***	-0.185***	-0.334***
	(0.0061)	(0.0032)	(0.0065)	(0.0055)	(0.0060)	(0.0063)	(0.0081)
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492
R^2	0.461	0.126	0.193	0.239	0.220	0.201	0.188
Panel C: 10-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output growth	-0.303***	0.0383***	0.152***	0.191***	-0.140***	-0.112***	-0.231***
	(0.0062)	(0.0030)	(0.0060)	(0.0048)	(0.0053)	(0.0057)	(0.0068)
Observations	49,915	49,915	49,915	49,915	49,915	49,915	49,915
N of firms	10,595	10,595	10,595	10,595	10,595	10,595	10,595
R^2	0.378	0.125	0.213	0.253	0.188	0.161	0.178
Panel D: levels, cross-section	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output growth	-0.0321***	-0.0231***	0.170***	0.147***	-0.0345***	0.0153***	-0.0994***
	(0.00133)	(0.0006)	(0.00179)	(0.00157)	(0.0016)	(0.0017)	(0.0018)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950
R^2	0.136	0.188	0.499	0.553	0.512	0.531	0.356

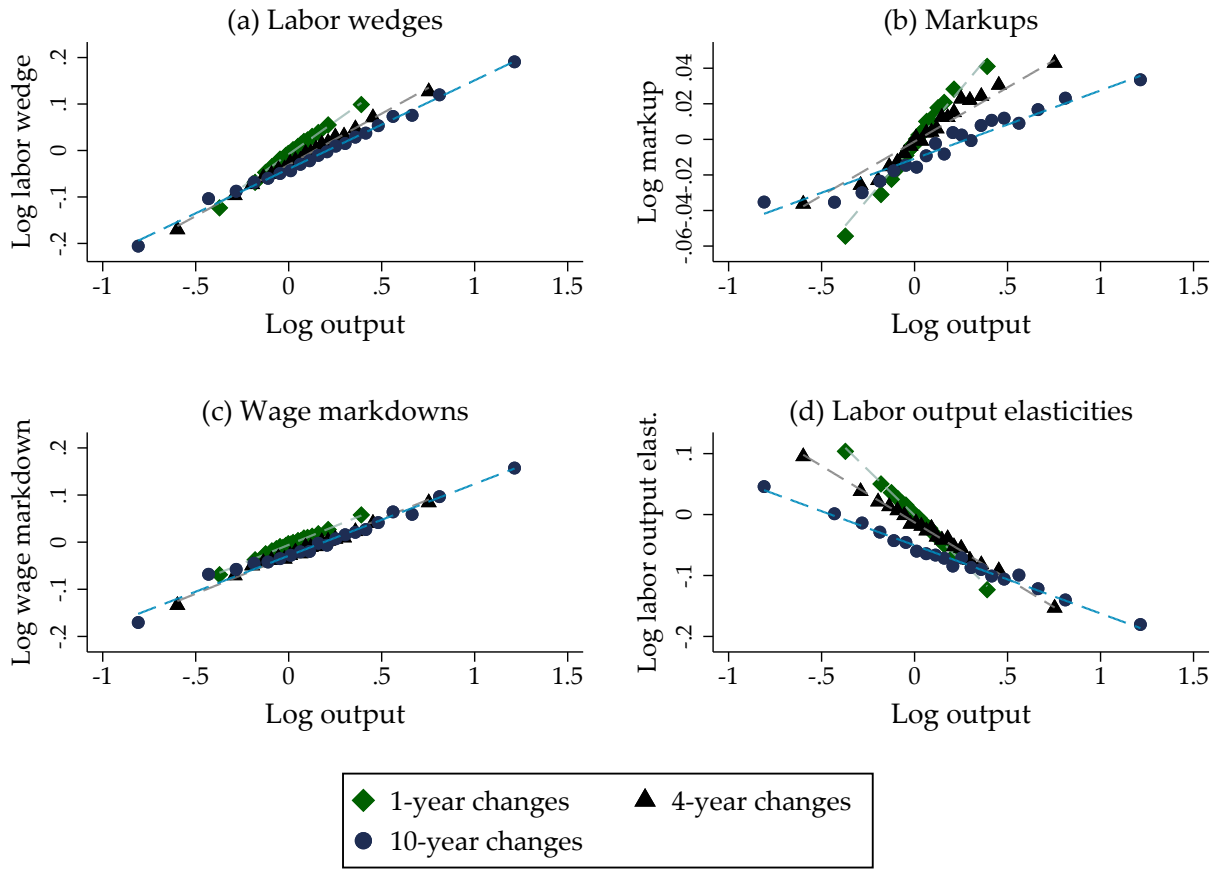
Notes: The table reports OLS regressions estimating the specification in Equation (20). The dependent variables in Columns (1)-(8) are the logs of the labor share in sales, markups, wage markdowns, labor wedge, labor output elasticity divided by returns to scale, labor output elasticity, and labor share in value added, respectively. Panels A-C project changes in those dependent variables on changes in log output for 1-, 4- , and 10-year differences. Panel D runs a cross-sectional specification that regresses levels of the dependent variables on output levels. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm-level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Table 7: Market power, output elasticity, and labor share adjustments in response to firm growth. IV regressions.

	1st stage	$\ln(LS_{it})$	$\ln(\mu_{it})$	$\ln(\gamma_{it})$	$\ln(\mu_{it}\gamma_{it})$	$\ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\ln(\theta^L_{it})$	$\ln(\frac{P^L_{it}L_{it}}{VA_{it}})$
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Export demand shock	0.0451***							
	(0.0045)							
Log output growth		-0.674***	0.186***	0.169**	0.355***	-0.327***	-0.318***	-0.842***
		(0.0521)	(0.0418)	(0.0759)	(0.0697)	(0.0623)	(0.0645)	(0.120)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950
First-stage F-Statistic		102.6	102.6	102.6	102.6	102.6	102.6	102.6
R^2	0.205	0.524	0.084	0.115	0.185	0.229	0.215	0.160
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Export demand shock	0.0635***							
	(0.0091)							
Log output growth		-0.443***	0.154***	0.0688	0.223***	-0.242***	-0.220***	-0.603***
		(0.0726)	(0.0444)	(0.0800)	(0.0713)	(0.0706)	(0.0742)	(0.119)
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492
First-stage F-Statistic		48.57	48.57	48.57	48.57	48.57	48.57	48.57
R^2	0.183	0.459	0.072	0.180	0.239	0.217	0.199	0.135

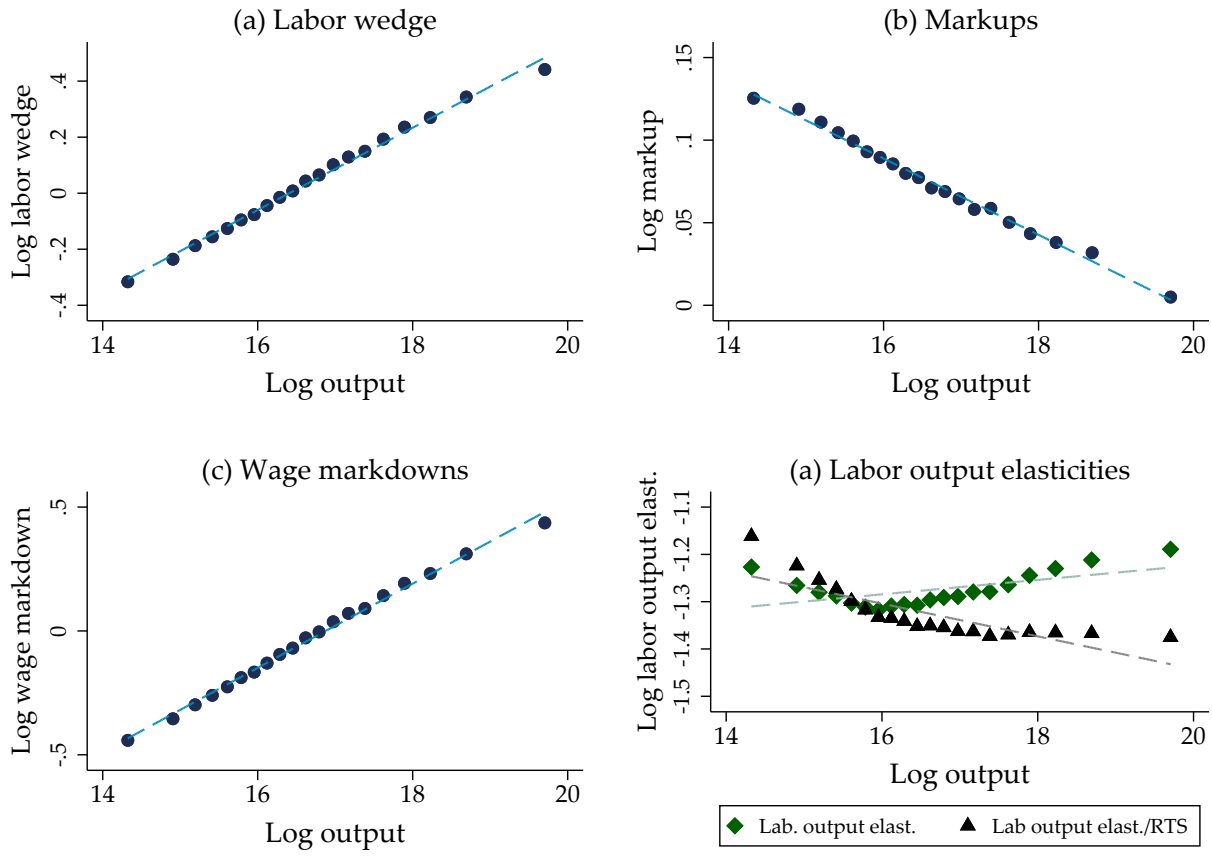
Notes: The table reports IV regressions from estimating the specification in Equation (20) using foreign demand shocks as instruments (Equation (22)). Column (1) reports the first-stage regression results. The dependent variables in Columns (1)-(5) are the logs of the labor share in sales, markups, wage markdowns, labor wedge, labor output elasticity divided by returns to scale, labor output elasticity, and labor share in value added, respectively. All regressions project changes in those dependent variables on changes in log output for 1- and 4-year differences. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm-level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Figure 7: The adjustment of market power to output growth by horizon: labor wedge, markup, markdown



Notes: The figure reports binned scatter plots estimating the specification in Equation (20) with OLS for various differences. The graphs project changes in the logs of labor wedges, markups, wage markdowns, and labor output elasticities (not divided by returns to scale) against log output changes. All panels report results that are residualized by industry-year fixed effects. German firm-level data.

Figure 8: The cross-sectional relationship between market power and firm size: labor wedge, markup, markdown



Notes: The figure reports binned scatter plots estimating the specification in Equation (20) in levels with OLS. The graphs project the logs of labor wedges, markups, wage markdowns against, and labor output elasticities (divided and not divided by RTS) against log output. All panels report results that are residualized by industry-year fixed effects. German firm-level data.

7 Aggregate Implications: Industry-Level Dynamics in 20 European Countries and the United States

We close our paper by drawing implications for aggregate industry dynamics. To do so, we access micro-aggregated data for 20 European countries and for the United States (for labor share dynamics), to show that our firm-level results carry over to industry aggregates. Our firm-level patterns suggests that the declining output elasticity in output growth may be factor behind recent labor share declines in European and US industries—so that this last section of our paper also generalizes and provides a micro-founded explanation for the aggregate time series based account for Germany in Mertens (2022).[22]

[22]That is, while we focus on idiosyncratic firm growth and intermediate-labor substitution, our findings resonate work by Mertens (2022), who documents a trend decline in labor output elasticities for Germany, linking firm-level and aggregate time series.

7.1 German Stepping Stone: Industry-Level Aggregation of Firm-Level Micro Data

As a stepping stone, we first present the industry aggregation of the firm-level results in Germany (four-digit NACE Rev. 1.1 industries), where we have access to both firm-level micro data and industry aggregates. For the 20 countries in Europe, we exclusively have industry-level data, albeit the data is derived from production function estimation results in the underlying micro data.

In the following, we run regressions as in Equation (20) but at the industry level, taking weighted averages of firm-level variables as described below and controlling for year fixed effects.

Output elasticities of labor. We construct the aggregate output elasticity as a sales-weighted average of firm-level output elasticities by industry-year cells. We then regress the change of the logged value of this weighted average on the log changes in total industry output.

The resulting industry level effect will differ from its firm-level counterpart as it additionally captures composition effects between firms of different sizes and labor intensities, including entry and exit.[23] Besides compositional effects, weighting choices, Jensen's inequality, intra-industry trade, and market-level shifts in labor demand and supply curves may detach macro and micro effects.

Labor shares, markups, markdowns. We define the industry-level labor share as total wages divided by total sales, which is equivalent to calculating a sales-weighted average of firm-level labor shares.

Again, we face aggregation issues. For markups and wage markdowns, the appropriate aggregation weight depends the assumptions on the production function and underlying model. We use intermediate input cost weights for aggregating markups, and wage costs for aggregating wage markdowns, which are the denominators of Equations (37) and (38), respectively. Our results are qualitatively robust to sales-weights (not reported).

Another important aspect when studying the industry level regards the decomposition in Equation (36), which only holds at the firm level, i.e., changes in the logs of firms' output elasticities, markups, and wage markdowns do not sum up to changes in log labor labor shares unless firms hold no market power.[24]

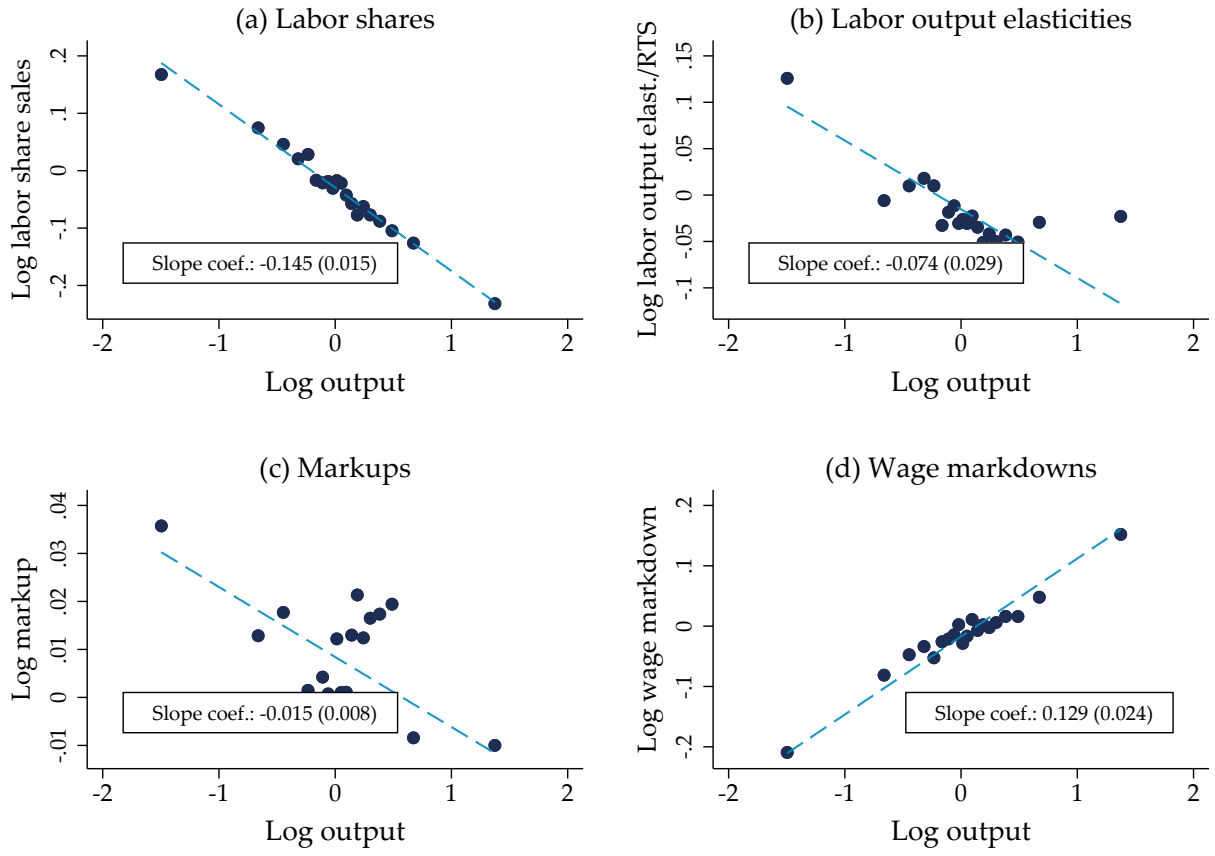
Finally, note again that Jensen's inequality applies when aggregating log labor shares as in Equation (36).

[23] The change in the aggregate output elasticity can be decomposed as: $\Delta\bar{\theta}^{L}_{Cont} + cov(s_{i,Cont}, \theta^{L}_{i,Cont}) + Net - entry$, where $Cont$ indicates the continuing firms (Melitz and Polanec, 2015). The first term is the unweighted mean of continuing firms. The second term is the covariance between firms' market shares and labor-output elasticities, which captures firm heterogeneity among continuing firms. The last term captures the contribution of entry and exit.

[24] To understand this, note that if there is no market power in output and labor markets, the aggregate labor share and labor output elasticity perfectly align: $\sum_n s_{it} LS_{it} = \sum_n s_{it}\theta^{L}_{it}$, where s_{it} is the sales-weight. If firms have market power, the right-hand side of this equation becomes $\sum_n \frac{s_{it}\theta^{L}_{it}}{\mu_{it}\gamma_{it}}$, such that the pass-through of a one-percent change in the output elasticity of labor to a change in the labor share at each individual firm is shaped by firms' markups and wage markdowns. Under imperfect competition, percentage changes in the aggregate output elasticity of labor only translate into equally large percentage changes in the labor share if market shares and product and labor market power remain constant at all firms and firm-level labor-output elasticities change by exactly the same amount at every firm. Nevertheless, qualitatively, it remains true that the partial effect of a change in the aggregate labor-output elasticity on the labor share, holding fixed firms' market power, is positive. Equally, the partial effect (ceteris paribus) of changes in aggregate markups and wage markdowns on the labor share is negative. Importantly, "holding fixed" refers here not only to the aggregate level, but also to the firm distribution.

Results. Figure 9 reports our results that replicate the previous firm-level scatterplots at the aggregate industry level (for 4-year differences). At the German industry level, there is a clear, essentially linear relationship that qualitatively mirrors the German micro firm-level relationships: output elasticities of labor and labor shares decline with output growth. Moreover, while markups decrease in industry output growth, wage markdowns increase. This suggests that also at the industry level in the German micro data, changes in output elasticities and wage markdowns are key in explaining labor share dynamics. Finally, a comparison with the 4-year OLS firm-level results makes clear that the analogous industry-level estimates differ quantitatively, likely for the reasons discussed above.

Figure 9: German micro data aggregated to industry level: 4-year changes (OLS)



Notes: The figure reports binned scatter plots (OLS) from projecting industry-level changes in log labor shares, labor output elasticities over returns to scale, markups, and wage markdowns against industry-level log output changes (4-year differences). All panels report results that are residualized by year fixed effects. German firm-level data, four-digit NACE Rev. 1.1 industries.

7.2 International Industry Panel Data: CompNet Data

We now move to our main industry-level analyses for 20 European countries and the US.

CompNet data. We use the CompNet data (9th vintage), which is a micro-aggregated database for 22 European countries.[25] The data are collected and provided by the Competitiveness Research Network (henceforth, CompNet).[26] We use the country-industry-level data (NACE Rev. 2, two-digit industries), which is the most detailed aggregation level available. The data contain, among other features, country-industry-level information on firms' sales, inputs, expenditures, markups, wage markdowns, and output elasticities that are relevant for our analysis. As there is no comparable data available for the US, we focus our analysis on Europe, but provide a limited set of results for the US based on the NBER-CES Manufacturing Industry Database for the years 1958-2018. The CompNet data covers the years 1999-2021, and we focus on manufacturing (NACE Rev. 2 two-digit industries 10-33 (manufacturing))— where we can draw on production function estimation, compare to our German micro data, and from where in most developed countries the decline in the aggregate labor stems (Dao et al., 2019). Yearly coverage varies across countries as shown in Appendix Table A.13. We focus on the data containing firms with at least 20 employees as this is available for more countries and is consistent with our German micro data. To ensure representativeness, the data is weighted by firm population weights.[27] For further details on the data, we refer to CompNet's User Guide (CompNet, 2023). Importantly, because the CompNet data only provides aggregated information, we cannot access the underlying firm-level data for individual countries.

Production Function Estimation. The CompNet data provides information on industry-level of markups, wage markdowns, and labor output elasticities using various types of estimation approaches. We rely on measures derived from industry-specific translog specifications that are estimated using the control function approach of Ackerberg et al. (2015). The log production function in the CompNet data is specified as:

$$q_{it} = \beta_l l_{it} + \beta_k k_{it} + \beta_m m_{it} + \beta_{l2} l_{it}^2 + \beta_{k2} k_{it}^2 + \beta_{m2} m_{it}^2 + \beta_{lk} l_{it} k_{it} + \beta_{lm} l_{it} m_{it} + \beta_{km} k_{it} m_{it} + \omega_{it}, \quad (40)$$

where lower case letters denote logs.[28] As there is no firm-specific price information observed in the firm-level data underlying the CompNet data, output, q_{it}, is measured as deflated sales using industry-specific deflators. To account for firm-specific price variation within industries, the CompNet estimation routine additionally controls for firms' market shares when estimating the production function.[29] Labor is defined as number of employees, whereas intermediates and capital are defined

[25] We drop Malta due to insufficient observations for output elaticities and drop the UK as its data is not available at the industry level.

[26] CompNet sources its data from highly representative administrative firm-level records located within European national statistical institutes and central banks. The CompNet team distributes harmonized data collection protocols (i.e., Stata codes) across the data providers and invests significant efforts in harmonizing the input data to maximize comparability across countries. The protocols compute micro-aggregated results that comply with the disclosure rules of each individual data provider. From these results, the CompNet team constructs the CompNet database. The data is aggregated at various levels.

[27] Our results hold for the subset of countries for which we have data based on firms of all size classes.

[28] See Ackerberg et al. (2015) for details on this estimation strategy. In a nutshell, the estimator controls for unobserved productivity (ω_{it}) using a control function containing intermediate inputs, labor, and capital. The approach assumes a simple firm decision model where intermediate inputs and labor are flexible and intermediate input demand depends on capital, labor, and productivity. Under certain restrictions, one can invert the intermediate input demand function to approximate productivity as a function of intermediates, labor, and capital.

[29] This approach follows De Loecker et al. (2020). Under a Cournot model, market shares perfectly capture markup variation. Therefore, market shares can help absorbing some of the unobserved price variation.

as deflated intermediate input expenditures and deflated capital stocks (book values). Markups, wage markdowns, and output elasticities are computed as described in Equation (37), Equation (38), and Equation (2). Industry-level values are constructed as weighted means, using sales weights for output elasticities, labor cost weights for markdowns, and intermediate input cost weights for markups (i.e., the denominators of the markup and markdown expressions).[30]

Summary statistics and sample. To construct a harmonized sample, we only keep country-industry pairs for which we observe labor shares, output elasticities, markups, and wage markdowns.[31] We provide summary statistics of key variables by country from the country-industry-level data in Appendix Table A.13. The table also reports the yearly coverage for each country in CompNet.

7.3 Industry Results for 20 European Countries

Cross-country-industry scatterplots. Figure 10 displays 4-year changes in industry-level logs of output elasticities of labor against 4-year changes in industry-level logged output across 20 European countries in CompNet. The figure shows binned scatter plots that absorb country-year and industry-year fixed effects.[32]

As for the German firm-level data, the European CompNet data reveal a strong negative relationship between the labor output elasticity and industry growth that is well described by a linear function. We obtain a precisely estimated and economically sizeable coefficient of -0.048. This means that as an industry grows, compared to others, by 10%, its output elasticity of output shrinks by about 0.5%.

In addition, the labor share shows a strong negative relationship with output growth as well, echoing the previous firm-level results.

Whereas markups are negatively related to industry growth, the relationship for wage markdowns is statistically insignificant. The latter differs from our previous results for the German micro data. Appendix Figures A.6, A.7, A.8, and A.9 reports country-specific scatterplots. The most striking results from these country-specific figures are that (i) the negative relationship between labor shares and industry output growth is found in every country in our data (including the US), and (ii) that the industry-level diminishing output elasticity of labor is also found in almost all countries (see also Figure 11 below).[33]

Labor share implications. Figure 10 suggests that the diminishing output elasticity of labor is a key driver of the negative relationship between labor shares and industry growth, because markups and wage markdowns do not increase with industry output. Hence, holding everything else fixed, only the changes in the labor output elasticity can cause labor shares to decline with output growth at the

[30] Results are robust to using sales weights for all variables (unreported).

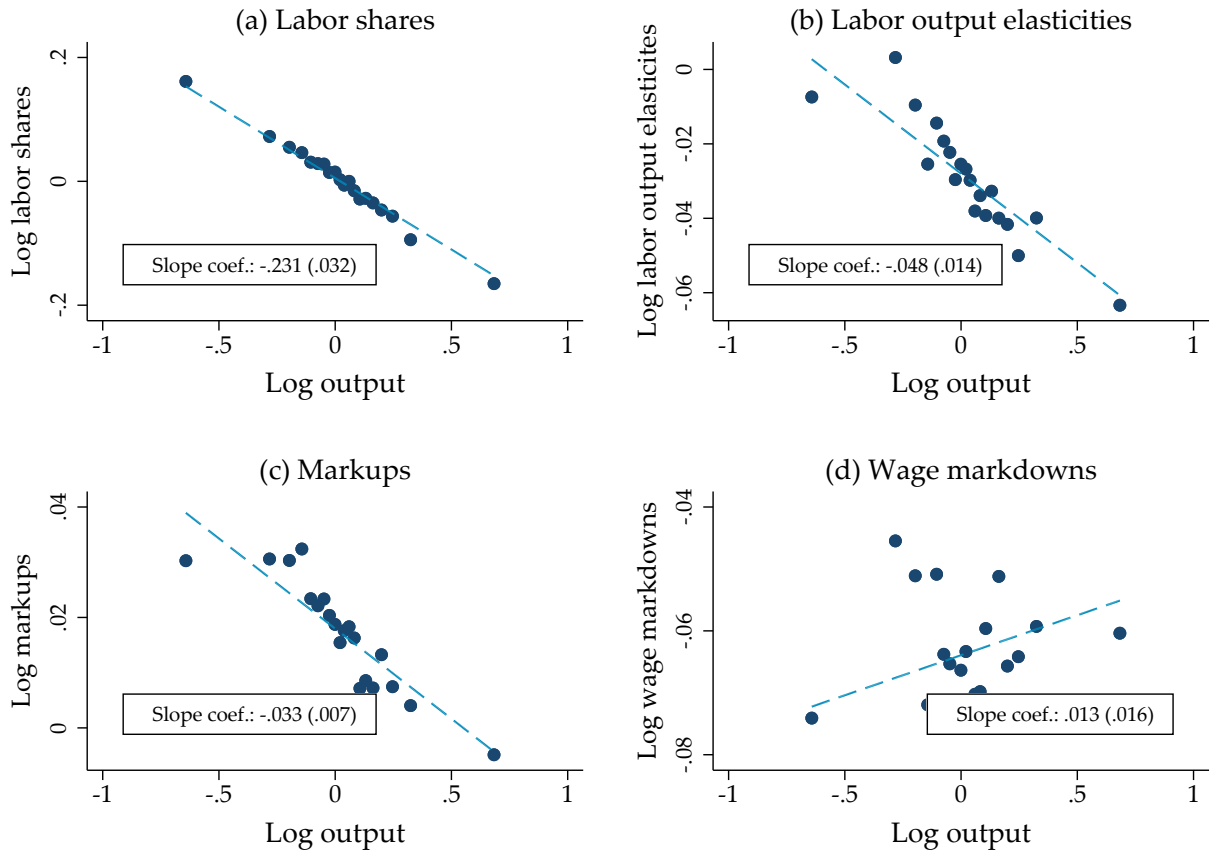
[31] Some country-industry-year cells report missing values in the CompNet data because the number of firms in the underlying micro data was too small either for passing country-specific disclosure rules, or for estimating the production functions.

[32] Results additionally including country-industry fixed effects are similar.

[33] Figure A.10 also shows that results for value added labor shares are similar.

industry level. We provide more evidence on the connection between labor shares and labor output elasticities below.

Figure 10: Changes in labor share in sales and its components on changes in output, European industry-level results



Notes: This figure reports results from projecting industry-level 4-year changes in log labor costs over sales (Panel (a)), labor output elasticities (Panel (b)), markups (Panel (c)), and wage markdowns (Panel (d)) on 4-year changes in log output. We pool all countries and control for country-year and industry-year fixed effects. CompNet data for firms with at least 20 employees, two-digit NACE Rev.2 industries.

Regression coefficients. Table 8 provides regression estimates underlying the previous scatter plots by horizon (1 year to 8 years; we cannot extend the horizon as many countries feature shorter time spans). Consistent with industry growth being a mix of firm-level growth episodes, we do not find any obvious evidence for horizon-dependence at the industry level for any outcome.

Figure 11 complements the table by showing *country-specific* regression coefficients for 4-year differences (including the labor share coefficient for the US). Point estimates suggest that the diminishing output elasticity of labor is present in almost all countries individually. Yet, standard errors are often large, presumably because of the small number of industries that we can study (the CompNet data divides manufacturing in only 23 industries).

Output elasticities. Finally, Figure 12 plots movements of output elasticities for all inputs against industry output growth (4-year differences), akin to our previous firm-level analysis. Again, we find

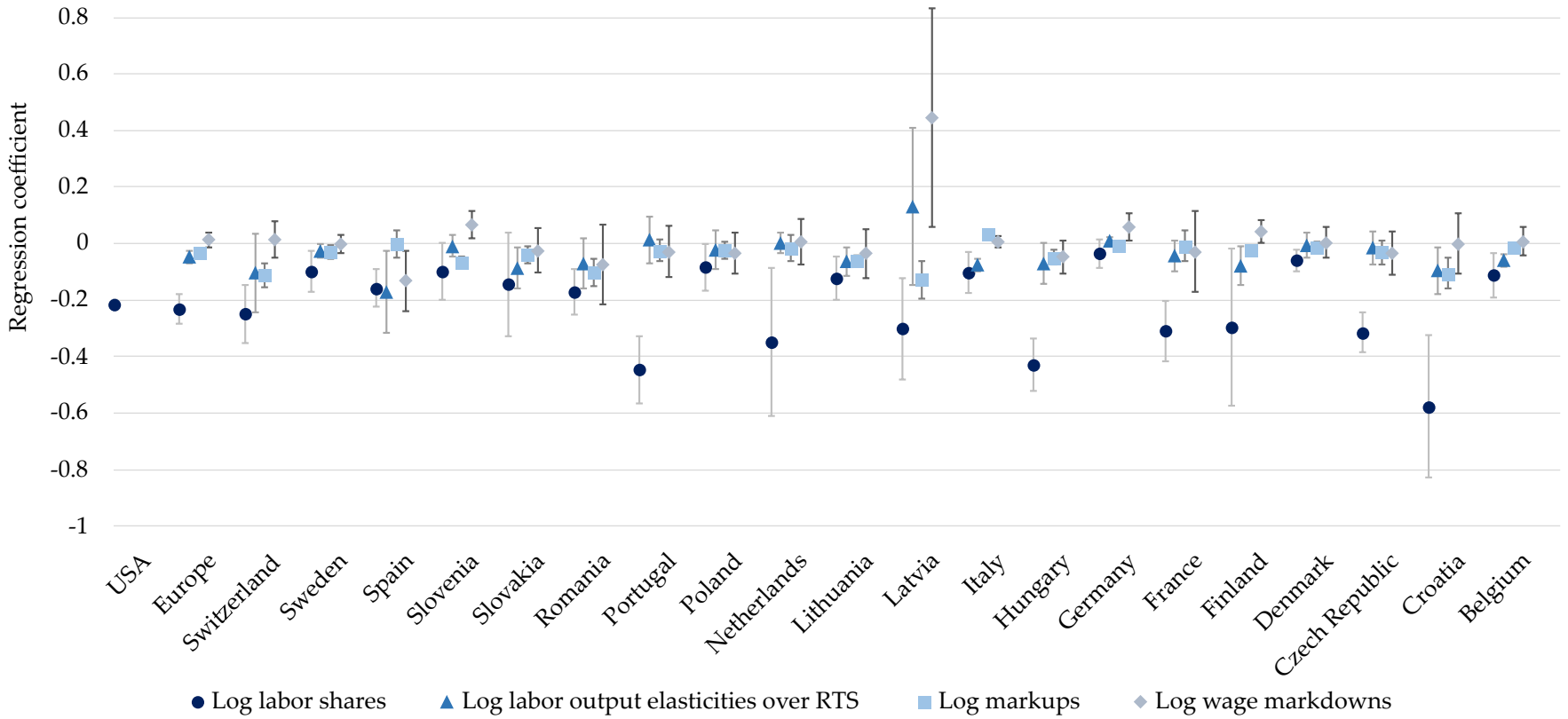
Table 8: Changes in labor share in sales and its components on changes in output, European industry-level results

	$\ln(LS_{it})$	$\ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\ln(\theta^L_{it})$	$\ln(\mu_{it})$	$\ln(\gamma_{it})$
Panel A: 1-year differences	(1)	(2)	(3)	(4)	(5)
Output growth	-0.369***	-0.057***	-0.0552***	-0.034***	0.013
	(0.0689)	(0.0129)	(0.0133)	(0.0082)	(0.0134)
Industry-year FE	yes	yes	yes	yes	yes
Country-year FE	yes	yes	yes	yes	yes
Observations	5,352	5,352	5,352	5,352	5,352
R^2	0.557	0.215	0.212	0.250	0.324
Panel B: 4-year differences	(1)	(2)	(3)	(4)	
Output growth	-0.231***	-0.048***	-0.0441***	-0.033***	0.0129
	(0.0323)	(0.0141)	(0.0148)	(0.0068)	(0.0159)
Industry-year FE	yes	yes	yes	yes	yes
Country-industry FE	yes	yes	yes	yes	yes
Observations	4,313	4,313	4,313	4,313	4,313
R^2	0.540	0.245	0.237	0.333	0.433
Panel C: 8-year differences	(1)	(2)	(3)	(4)	
Output growth	-0.218***	-0.0546***	-0.0467**	-0.034**	0.0341*
	(0.0326)	(0.0166)	(0.0182)	(0.010)	(0.0205)
Industry-year FE	yes	yes	yes	yes	yes
Country-year FE	yes	yes	yes	yes	yes
Observations	2,976	2,976	2,976	2,976	2,976
R^2	0.512	0.302	0.285	0.388	0.416

Notes: The table reports coefficients from projecting changes in log industry-level labor shares in sales, markups, wage markdowns, labor output elasticities divided by returns to scales, and labor output elasticities on changes in log output. CompNet data. Firms with at least 20 employees, two-digit NACE Rev.2 industries.

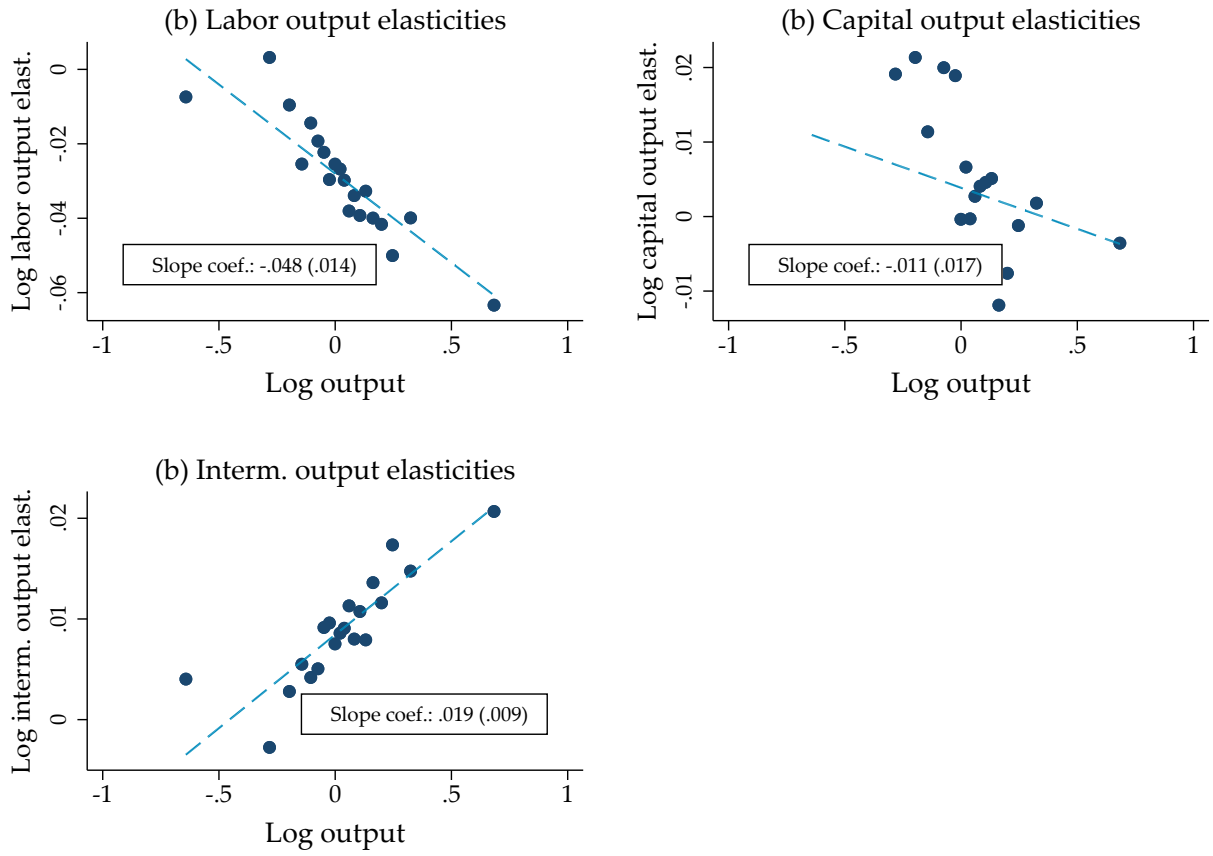
strong empirical relationships, implying an adjustment of the production factor mix away from labor and toward intermediate inputs. Changes in industry-level capital output elasticities are statistically insignificant, indicating that, at the industry level, capital-intermediate substitution is less relevant than at the firm level. Appendix Figure A.11 shows corresponding results for cost shares, which confirm the declining labor intensity of production and do not rely on parametric assumptions.

Figure 11: CompNet 4-year changes (OLS), industry level: Labor shares, labor output elasticities, markups, wage markdowns



Notes: The figure reports regression coefficients from regressing 4-year industry-level changes in the logs of labor shares in sales, labor output elasticities over returns to scale, markups, and wage markdowns on 4-year industry-level changes in log output (OLS). All country-specific coefficients result from country-level regressions that absorb year fixed effects; the Europe-wide coefficient is from a pooled regression that includes country-year fixed effects and industry-year fixed effects. CompNet data for firms with at least 20 employees, two-digit NACE Rev. 2 industries.

Figure 12: European industry level 4-year changes in output elasticities (OLS)



Notes: The figure reports binned scatter plots (OLS) from projecting 4-year industry-level changes in the logs of labor, capital, and intermediate output elasticities divided by returns to scale against industry-level log output changes (4-year differences). We pool all countries and control for country-year and industry-year fixed effects. CompNet data. Firms with at least 20 employees, two-digit NACE Rev.2 industries.

7.4 Further Implications for Labor Share Dynamics

Through the lens of our simple framework, the labor share declines when firms' markups or wage markdowns *increase* or when the output elasticity of labor *declines*. Consequently, our results above imply that the declining output elasticity drives much of the negative relationship between industry labor shares and industry output growth. The slope for the output elasticity of labor is about a quarter the magnitude of the labor share effect at the industry level.

Industry split. To further scrutinize the role of the diminishing output elasticity of labor, we split our data into growing and shrinking industries and study how industry-level labor shares, output elasticities, markups, and wage markdowns have changed over time for growing and shrinking industries. As yearly coverage varies between countries, we first calculate the log changes in each of these variables for each country-industry pair between the first and last year in the data, i.e., we pool differences for country-industry pairs of different lengths (see Table A.13 for the country-specific yearly coverage). Subsequently, we take averages of these changes across all country-industry-pairs.

Results. Figure 13 reports the results from this exercise. Panel (a) reports European results based on the CompNet data. Panel (b) reports labor share changes for the US for three 20-year periods, 1958-1978, 1978-1998, and 1998-2018. As we do not have production function estimates for the US, we only report labor shares in Panel (b).

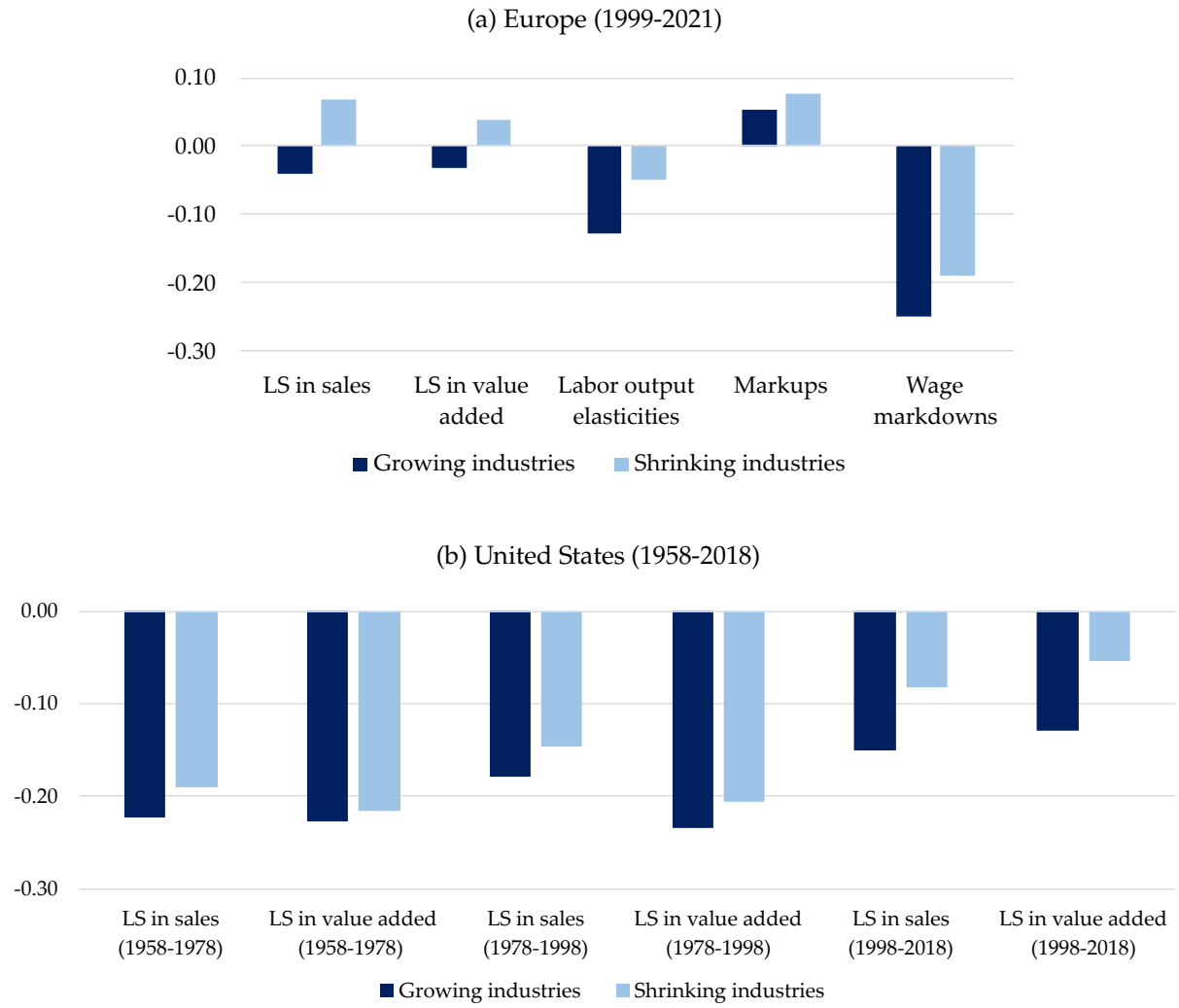
Looking at the European results (Panel (a)), a first striking result is that labor shares decreased in the group of growing industries, but not in the other group. These are also the industries that experienced particularly strong declines in labor output elasticities due to the diminishing output elasticity of labor. Markups and wage markdowns show patterns that should increase labor shares in growing industries relative to shrinking industries, i.e., markups grow less and wage markdowns decline more in growing industries. Through the lens of our framework, the diminishing output elasticity of labor is therefore the only factor that can explain why labor shares fall in growing but not in shrinking industries. This suggests a pivotal role for the decline in output elasticities of labor in shaping labor share dynamics across Europe.

Turning to the US (Panel (b)), we observe a pronounced decline in labor shares, particularly within expanding industries. The difference between growing and shrinking industries is particularly strong in the 1998-2018 period. Overall, US-results are consistent with our European results, which suggests that the diminishing output elasticity of labor may also play a significant role in driving changes in US labor shares.[34] While this evidence is consistent with the European results carrying over to the US, the industry-level data does not allow us to directly investigate the output elasticity of labor.

Robustness. We replicate our analysis splitting industries into above and below median growth rates in Appendix Figures A.12 and A.13 and find similar results. For CompNet, we define splits once based on country medians and once based on European medians, which makes almost no difference. Finally, for the US, we also report results for the entire time span from 1958-2018 in Appendix Figure A.14 and find again consistent results.

[34]The decline of labor shares in shrinking industries in the US could indicate that, in the US, market power plays a more relevant role in contributing to declining labor shares than in Europe. See De Loecker et al., 2020 and Autor et al., 2020 for a discussion on the role of market power for the declining US labor share.

Figure 13: Changes in labor shares, output elasticities, markups, and wage markdowns in growing and shrinking industries



Notes: The panels report average changes in the logs of labor shares, labor output elasticities, markups, and wage markdowns across country-industry pairs for growing and shrinking industries. Changes are first computed for each pair and then average across all pairs. Time spans differ across country-industry pairs and not all industries are available for all years within a country. Europe: CompNet data and NACE Rev. 2 two-digit industries. US: NBER-CES Manufacturing Industry Database and NAICS 6-digit industries. There are 155 shrinking and 197 growing country-industry pairs in the CompNet data. For the US data, we report three periods. We observe 9 shrinking and 325 growing industries for 1958-1978, 72 shrinking and 289 growing industries for 1978-1998, and 203 shrinking and 161 growing industries for 1998-2018 in the US data. In' a robustness test, we replicate these figures using median growth rates as splitting criteria (Appendix Figures A.12 and A.13).

8 Conclusion

We have documented and dissected a new fact about firm growth: the declining importance of labor and the increasing importance of intermediate inputs in firms' production. As labor and intermediates act as substitutes, this shift in the input mix reduces (increases) the output elasticity of labor (intermediates); as a result, the labor (intermediates) share falls (increases) in growing firms. We establish these patterns using firm-level micro data and OLS and IV regressions in Germany, as

well as in micro-aggregated industry data for 20 European countries. We rationalize the facts in a production function framework that rests on (i) an elasticity of substitution between intermediates and labor that exceeds one, and (ii) an increasing shadow price of labor (e.g., due to monopsony or adjustment costs).

The findings may have additional implications. For instance, many existing estimates of factor misallocation and productivity rely on Cobb-Douglas production functions that assume constant output elasticities. Moreover, our results imply that generally, any shocks that affect output growth will change output elasticities, input mixes and cost shares, with large magnitudes for horizons as long as 10 years. Ignoring this regularity may confound the analysis of firm-level effects from productivity shocks, trade, competition, or subsidies on a wide class of related outcomes in the short and medium runs.

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Appendix of:
From Labor to Intermediates:
Firm Growth, Input Substitution, and Monopsony

Matthias Mertens and Benjamin Schoefer

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A Appendix Tables and Figures

Table A.1: Summary statistics of the German manufacturing sample

	Mean (1)	**p25** (2)	**Median** (3)	**p75** (4)	**St.Dev.** (5)	**Obs.** (6)
Number of employees	366.63	53	109	272	2559.55	183,813
Real wage (1995 values)	34,162	25,972	33,823	41,485	11,492	183,813
Labor share (sales)	0.30	0.21	0.29	0.38	0.12	183,813
Labor share (value added)	0.80	0.63	0.76	0.88	2.89	183,813
Labor cost share	0.31	0.22	0.30	0.39	0.13	183,813
Capital cost share	0.06	0.03	0.05	0.07	0.04	183,813
Intermediates cost share	0.63	0.54	0.64	0.73	0.14	183,813
Materials, energy, ext. components cost share	0.41	0.29	0.41	0.53	0.17	183,813
Merchandise cost share	0.05	0.00	0.00	0.05	0.10	183,813
Subcontracted work by other companies cost share	0.03	0.00	0.00	0.03	0.06	183,813
Repairs, maintenance, installations cost share	0.02	0.01	0.03	0.04	0.02	183,813
Rents, leases, leasing cost share	0.03	0.01	0.02	0.04	0.03	183,813
Other intermediates cost share	0.10	0.05	0.08	0.12	0.06	183,813
Markup	1.09	0.97	1.05	1.17	0.20	183,813
Wage markdown	1.08	0.71	0.96	1.32	0.55	183,813
Output elasticity of labor	0.31	0.23	0.31	0.38	0.11	183,813
Output elasticity of capital	0.12	0.08	0.11	0.15	0.06	183,813
Output elasticity of intermediates	0.64	0.57	0.64	0.71	0.10	183,813
Returns to scale	1.06	0.98	1.05	1.13	0.12	183,813

Notes: This table presents summary statistics for selected variables from the German manufacturing sector firm-level data. Columns (1)-(5) show the mean, 25^{th} percentile, median, 75^{th}, and standard deviation, respectively. Column (6) reports the number of non-missing observations. German micro-data.

Table A.2: Firm-level adjustments in output elasticities in response to firm growth, without dividing by returns to scale. OLS regressions.

	$\ln(\theta_{it}^L)$	$\ln(\theta_{it}^K)$	$\ln(\theta_{it}^M)$	θ_{it}^L	θ_{it}^K	θ_{it}^M
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)
Log output growth	-0.298***	-0.309***	0.157***	-0.0723***	-0.0203***	0.0991***
	(0.0046)	(0.0064)	(0.00129)	(0.0008)	(0.0004)	(0.0007)
Observations	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950
R^2	0.215	0.132	0.397	0.278	0.183	0.425
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)
Log output growth	-0.185***	-0.164***	0.112***	-0.0420***	-0.0124***	0.0729***
	(0.0063)	(0.0077)	(0.0021)	(0.0014)	(0.0007)	(0.0013)
Observations	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492
R^2	0.201	0.160	0.369	0.220	0.154	0.388
Panel C: 10-year diff.	(1)	(2)	(3)	(4)	(5)	(6)
Log output growth	-0.112***	-0.0441***	0.0860***	-0.0236***	-0.0018***	0.0559***
	(0.00569)	(0.0073)	(0.0021)	(0.00137)	(0.0007)	(0.0013)
Observations	49,915	49,915	49,915	49,915	49,915	49,915
N of firms	10,595	10,595	10,595	10,595	10,595	10,595
R^2	0.161	0.141	0.322	0.174	0.142	0.334
Panel D: levels, cross-section	(1)	(2)	(3)	(4)	(5)	(6)
Log output growth	0.0153***	0.183***	0.0382***	0.0092***	0.0207***	0.0242***
	(0.0017)	(0.00243)	(0.0007)	(0.0004)	(0.0003)	(0.0004)
Observations	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950
R^2	0.531	0.451	0.476	0.533	0.580	0.481

Notes: The table reports OLS regressions from estimating the specification in Equation (20). The dependent variables in Columns (1)-(6) are the logged values and not logged values of labor, capital, and intermediate input output elasticities, respectively. Panel A-C project changes in those dependent variables on changes in log output for 1-, 4- , and 10-year differences. Panel D runs a cross-sectional specification that regresses levels of the dependent variables on output levels. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm-level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Table A.3: Firm-level adjustments in output elasticities in response to firm growth, without dividing by returns to scale. IV regressions.

	1st stage	$\ln(\theta_{it}^L)$	$\ln(\theta_{it}^K)$	$\ln(\theta_{it}^M)$	θ_{it}^L	θ_{it}^K	θ_{it}^M
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Export demand shock	0.0451***						
	(0.0045)						
Log output growth		-0.318***	-0.220**	0.127***	-0.0672***	-0.0135***	0.0914***
		(0.0645)	(0.0995)	(0.0170)	(0.0113)	(0.00455)	(0.0102)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950
First-stage F-Statistic		102.6	102.6	102.6	102.6	102.6	102.6
R^2	0.205	0.215	0.127	0.387	0.277	0.171	0.424
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Export demand shock	0.0635***						
	(0.0091)						
Log output growth		-0.220***	-0.125	0.105***	-0.0445***	-0.0005	0.0739***
		(0.0742)	(0.0973)	(0.0227)	(0.0153)	(0.0076)	(0.0143)
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492
First-stage F-Statistic		48.57	48.57	48.57	48.57	48.57	48.57
R^2	0.183	0.199	0.158	0.368	0.220	0.121	0.388

Notes: The table reports IV regressions from estimating the specification in Equation (20) using foreign demand shocks as instruments (Equation (22)). Column (1) reports the first-stage regression results. The dependent variables in Columns (2)-(7) are the logged values and not logged values of labor, capital, and intermediate input output elasticities, respectively. All regressions project changes in those dependent variables on changes in log output for 1- and 4-year differences. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm-level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Table A.4: Firm-level adjustments in response to firm growth, without taking logs. OLS regressions.

	$\frac{w_{it}L_{it}}{C_{it}}$	$\frac{r_{it}K_{it}}{C_{it}}$	$\frac{z_{it}M_{it}}{C_{it}}$	$\frac{\theta_{it}^{L}}{RTS_{it}}$	$\frac{\theta_{it}^{K}}{RTS_{it}}$	$\frac{\theta_{it}^{M}}{RTS_{it}}$	RTS_{it}
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output growth	-0.103***	-0.0380***	0.141***	-0.0710***	-0.0201***	0.0911***	0.00647***
	(0.0011)	(0.0003)	(0.0011)	(0.0007)	(0.0003)	(0.0009)	(0.0004)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950
R^2	0.296	0.447	0.406	0.312	0.224	0.335	0.052
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output growth	-0.0692***	-0.0366***	0.106***	-0.0438***	-0.0136***	0.0574***	0.0185***
	(0.00157)	(0.0006)	(0.00179)	(0.00108)	(0.000525)	(0.0014)	(0.0008)
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492
R^2	0.303	0.419	0.413	0.266	0.191	0.277	0.157
Panel C: 10-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output growth	-0.0535***	-0.0275***	0.0810***	-0.0305***	-0.0051***	0.0356***	0.0304***
	(0.0016)	(0.0006)	(0.0018)	(0.0011)	(0.0005)	(0.0014)	(0.0008)
Observations	49,915	49,915	49,915	49,915	49,915	49,915	49,915
N of firms	10,595	10,595	10,595	10,595	10,595	10,595	10,595
R^2	0.259	0.318	0.347	0.222	0.155	0.201	0.262
Panel D: levels, cross-section	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output growth	-0.0357***	-0.0028***	0.0385***	-0.0070***	0.0131***	-0.0061***	0.0541***
	(0.0005)	(0.0002)	(0.0005)	(0.0003)	(0.0002)	(0.0004)	(0.0005)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950
R^2	0.468	0.253	0.440	0.522	0.527	0.423	0.828

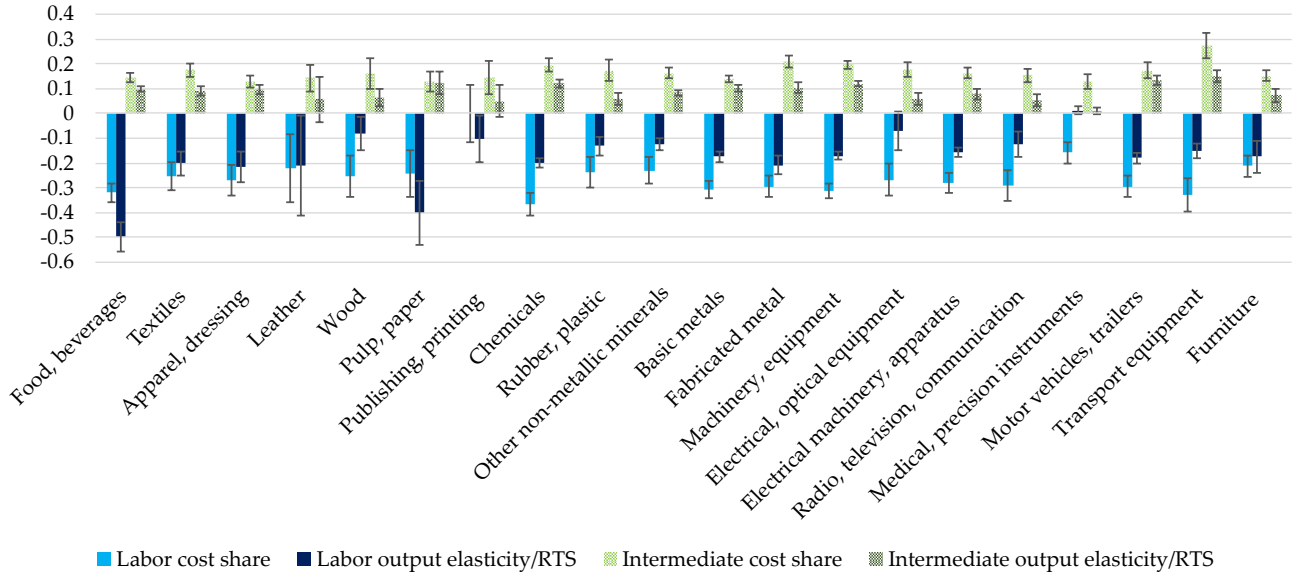
Notes: The table reports OLS regressions from estimating the specification in Equation (20) without taking logs for the dependent variable. The dependent variables in Columns (1)-(7) are labor cost shares, capital cost shares, intermediate cost shares, labor output elasticities over returns to scale, capital output elasticities over returns to scale, intermediate input output elasticities over returns to scale, and returns to scale, respectively. Panel A-C project changes in those dependent variables on changes in log output for 1-, 4- , and 10-year differences. Panel D runs a cross-sectional specification that regresses levels of the dependent variables on output levels. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm-level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Table A.5: Firm-level adjustments in response to firm growth, without taking logs. IV regressions.

	1st stage	$\frac{w_{it}L_{it}}{C_{it}}$	$\frac{r_{it}K_{it}}{C_{it}}$	$\frac{z_{it}M_{it}}{C_{it}}$	$\frac{\theta_{it}^{L}}{RTS_{it}}$	$\frac{\theta_{it}^{K}}{RTS_{it}}$	$\frac{\theta_{it}^{M}}{RTS_{it}}$	RTS_{it}
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Export demand shock	0.0451***							
	(0.0045)							
Log output growth		-0.0933***	-0.0283***	0.122***	-0.0626***	-0.0133***	0.0759***	0.0107*
		(0.0144)	(0.0042)	(0.0153)	(0.0098)	(0.0039)	(0.0116)	(0.00575)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950
First-stage F-Statistic		102.6	102.6	102.6	102.6	102.6	102.6	102.6
R^2	0.205	0.294	0.427	0.400	0.309	0.208	0.329	0.049
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Export demand shock	0.0635***							
	(0.0091)							
Log output growth		-0.0555***	-0.0280***	0.0836***	-0.0465***	-0.0070	0.0536***	0.0289***
		(0.0177)	(0.0062)	(0.0201)	(0.0127)	(0.0058)	(0.0157)	(0.00876)
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492
First-stage F-Statistic		48.57	48.57	48.57	48.57	48.57	48.57	48.57
R^2	0.183	0.297	0.403	0.402	0.266	0.175	0.276	0.139

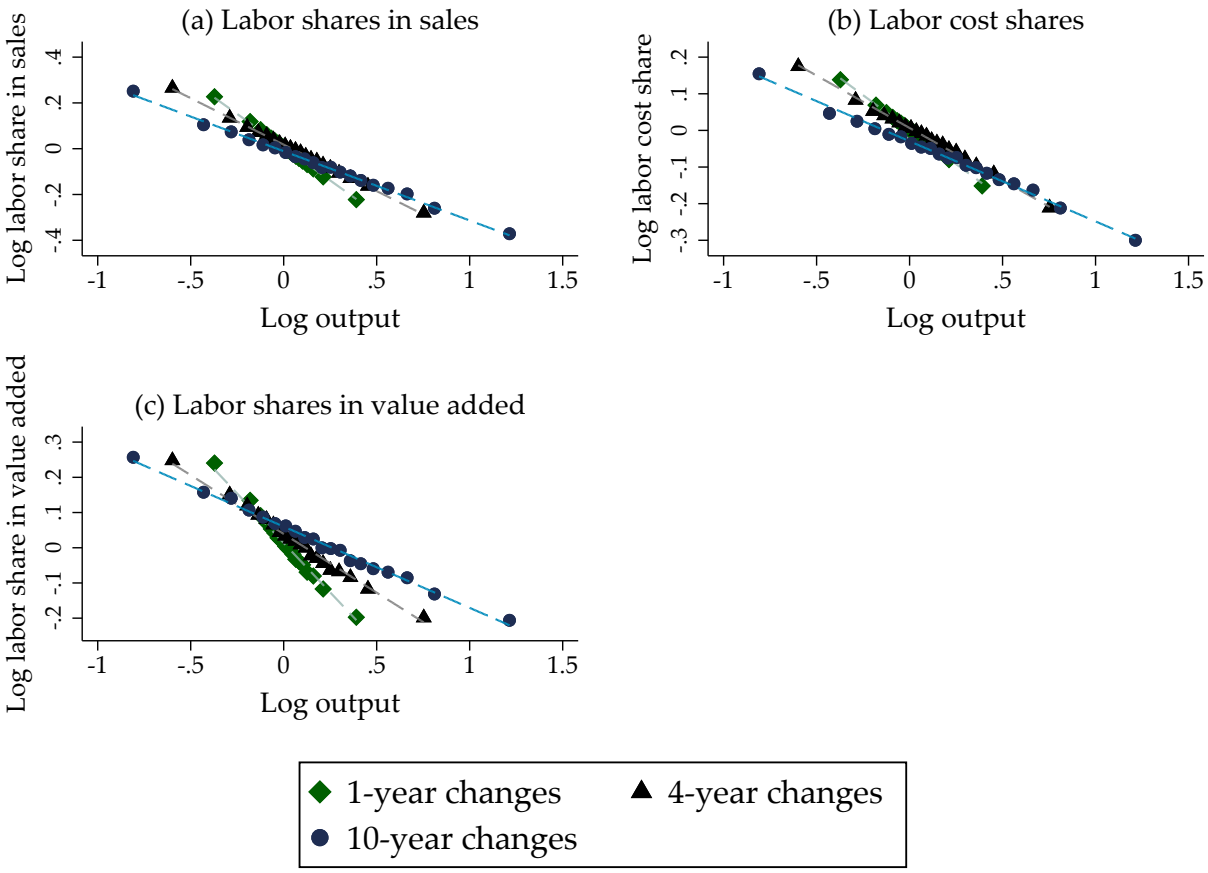
Notes: The table reports IV regressions from estimating the specification in Equation (20) using foreign demand shocks as instruments (Equation (22)). Column(1) reports the first-stage regression results. The dependent variables in Columns (2)-(8) are labor cost shares, capital cost shares, intermediate cost shares, labor output elasticities over returns to scale, capital output elasticities over returns to scale, intermediate input output elasticities over returns to scale, and returns to scale, respectively. All regressions project changes in those dependent variables on changes in log output for 1- and 4-year differences. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm-level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Figure A.1: Firm-level adjustments in labor and intermediate input output elasticities and cost shares by industries, 4-year changes, OLS



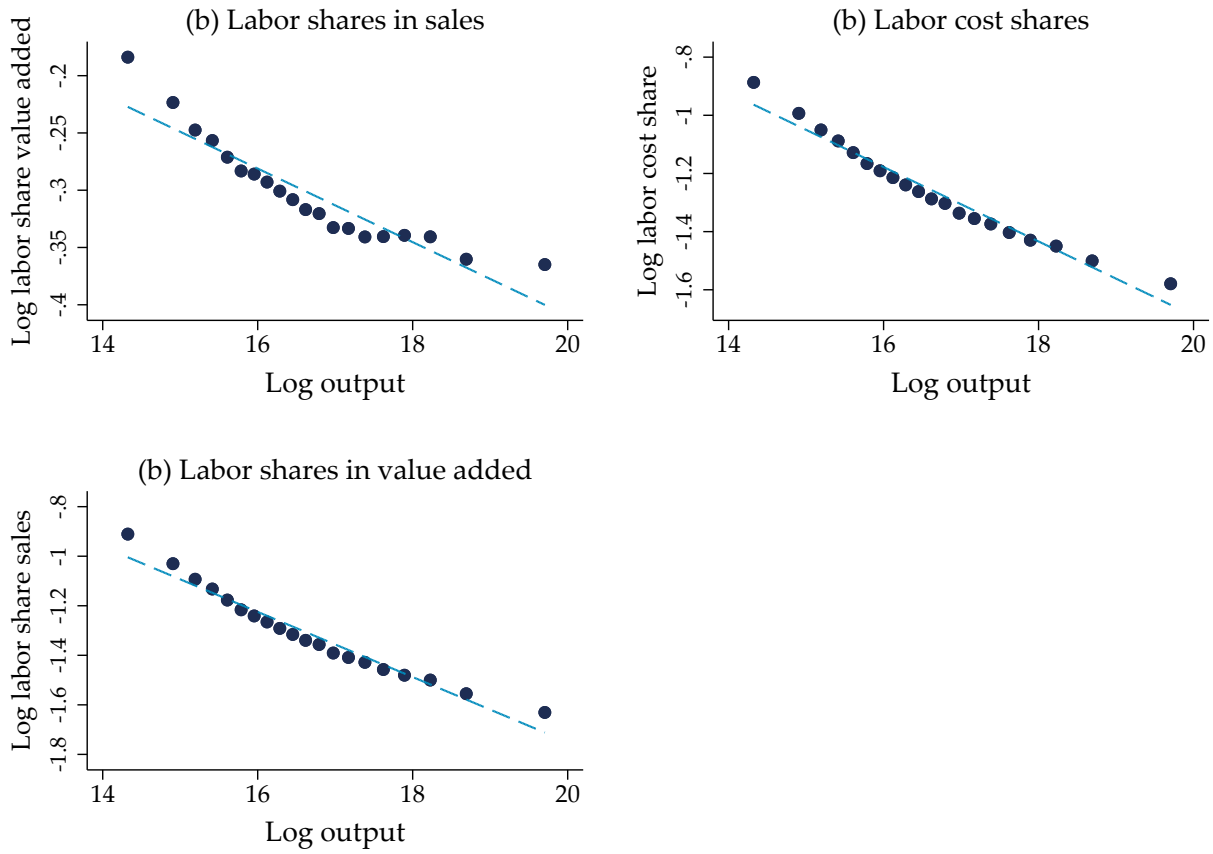
Notes: The figure reports OLS regressions from estimating Equation (20) in 4-year changes by two-digit NACE manufacturing industries. The dependent variables are logged labor and intermediate cost shares and logged labor and intermediate output elasticities divided by returns to scale. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm-level. The error bars indicate 95% confidence intervals. Results for 1- and 10-year changes are similar. German firm-level data.

Figure A.2: German micro-data firm level: Labor share sales and value added - changes



Notes: The figure reports binned scatter plots from estimating the specification in Equation (20) with OLS for various differences. The graphs project changes in the logs of labor shares in sales, costs, and value added against log output changes. All panels report results that are residualized by industry-year fixed effects. German firm-level data.

Figure A.3: German micro-data firm level: Labor share sales and value added - cross section



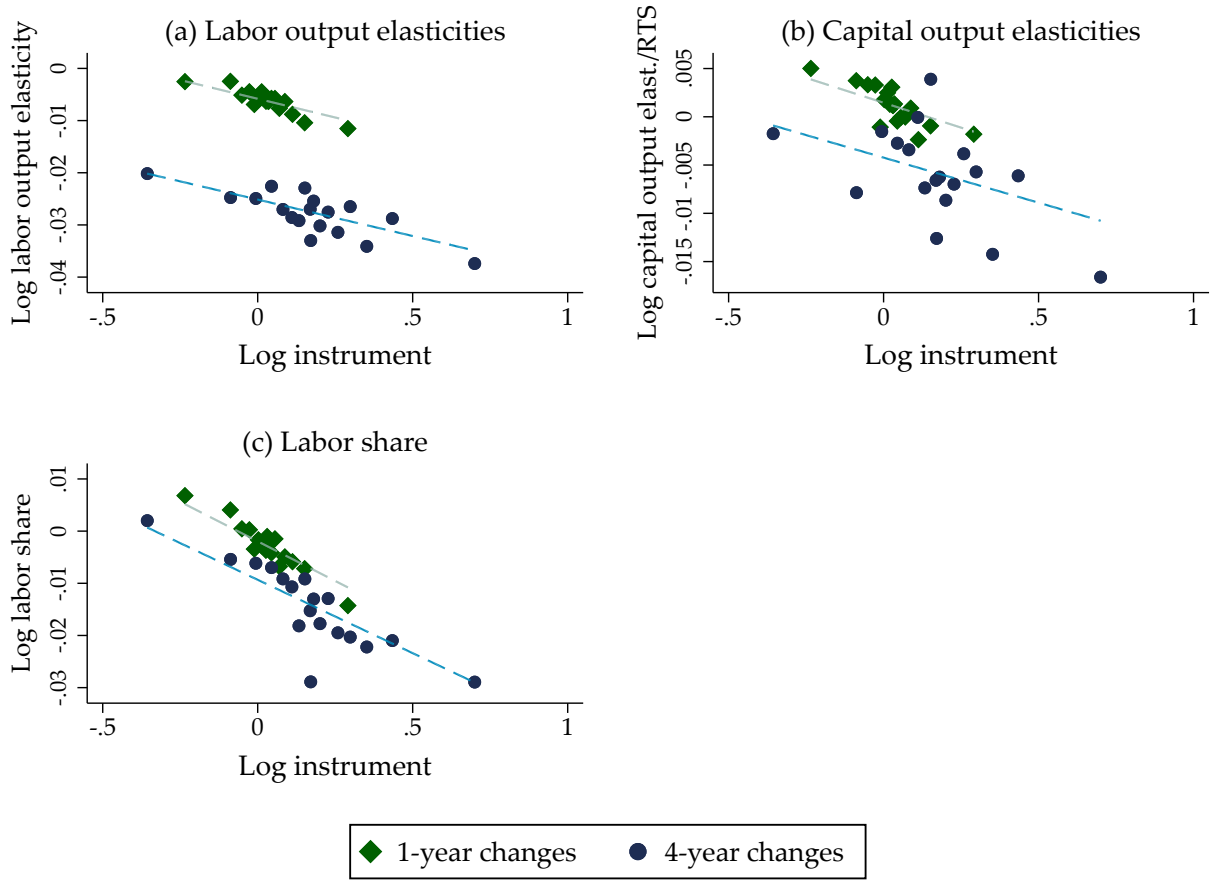
Notes: The figure reports binned scatter plots from estimating the specification in Equation (20) with OLS in levels (cross section). The graphs project the logs of labor shares in sales, costs, and value added against log output. All panels report results that are residualized by industry-year fixed effects. German firm-level data.

Table A.6: IV: Reduced form regression.

	$\ln(L_{it})$	$\ln(K_{it})$	$\ln(M_{it})$	$\ln(\frac{P^L_{it}L_{it}}{C_{it}})$	$\ln(\frac{P^K_{it}K_{it}}{C_{it}})$	$\ln(\frac{P^M_{it}M_{it}}{C_{it}})$	$\ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\ln(\frac{\theta^K_{it}}{RTS_{it}})$	$\ln(\frac{\theta^M_{it}}{RTS_{it}})$	$\ln(RTS_{it})$
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Export demand shock	0.0146***	0.0028	0.0419***	-0.0200***	-0.0308***	0.0077***	-0.0147***	-0.0092*	0.0054***	0.0004
	(0.0027)	(0.0021)	(0.0049)	(0.0028)	(0.0041)	(0.0016)	(0.0031)	(0.0048)	(0.0011)	(0.0002)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950
R^2	0.064	0.039	0.142	0.118	0.160	0.109	0.104	0.258	0.095	0.045
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Export demand shock	0.0341***	0.0167**	0.0612***	-0.0171***	-0.0362***	0.0079***	-0.0154***	-0.0093	0.0052***	0.0014***
	(0.00729)	(0.00689)	(0.0101)	(0.0046)	(0.0080)	(0.0026)	(0.0045)	(0.0061)	(0.0018)	(0.0005)
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492
R^2	0.608	0.084	0.155	0.175	0.136	0.145	0.140	0.130	0.133	0.102

Notes: The table reports results for firm-level first stage and reduced form regression corresponding to our IV regressions from estimating the specification in Equation (20) using foreign demand shocks as instruments (Equation (22)). The first stage projects log output on the instrument (Column (1)), whereas the reduced form regressions project the logs of labor, capital, intermediates, labor cost shares, capital cost shares, intermediate cost shares, labor output elasticities over returns to scale, capital output elasticities over returns to scale, intermediate input output elasticities over returns to scale, and returns to scale on the instrument (columns (2)-(10), respectively). All regressions are in 1- and 4-year differences. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm-level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

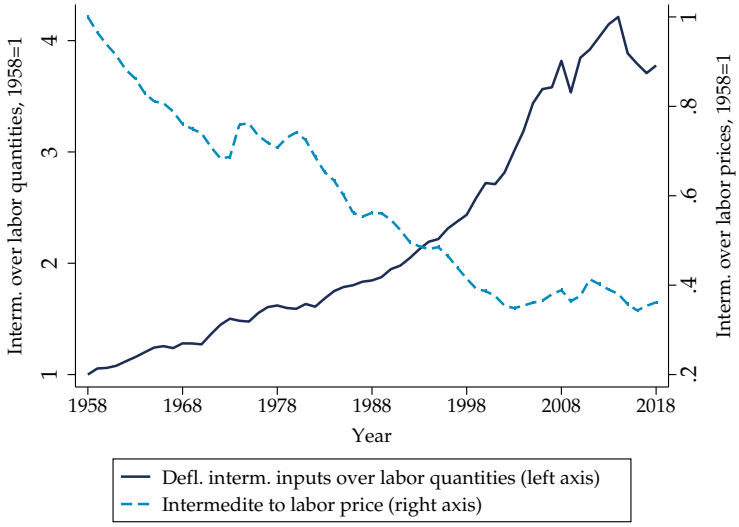
Figure A.4: Reduced form results for additional variables, one- and four-year changes



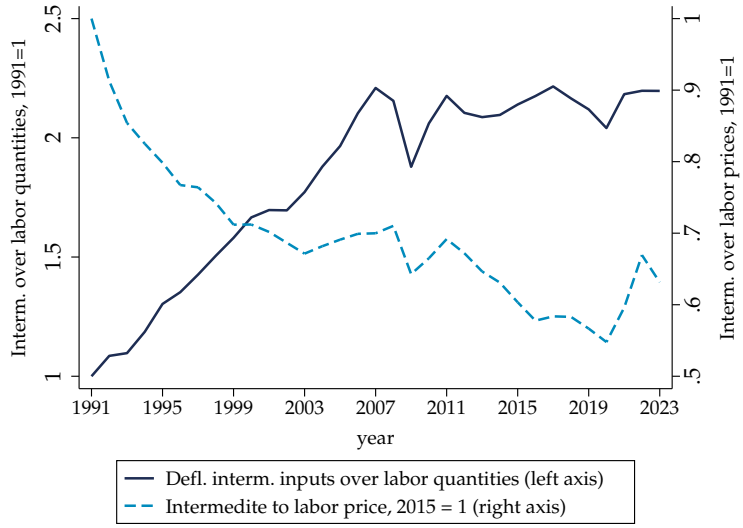
Notes: The figure reports binned scatter plots for firm-level reduced form regression. The reduced form regressions project the logs of labor output elasticities, capital output elasticities divided by returns to scale, and labor shares on the instrument. Regressions are in one-year and four-year changes. All panels report results that are residualized by industry-year fixed effects. The data are German firm-level data.

Figure A.5: Changes in aggregate intermediate to labor price and quantities

(a) Germany, entire manufacturing sector



(b) USA, manufacturing sector, 6-digit NAICS industries



Notes: The figures report changes in prices for labor (wages) and intermediate inputs (price index) and the ratio of deflated intermediate inputs to labor quantities for Germany (Panel (a)) and the US (Panel (b)). Values are normalized to unity in the first year. German data refers to the aggregate manufacturing sector. US data refers to averages across 6-digit NAICS industries. Data from the Federal Statistical Office of Germany and the NBER-CES Manufacturing Industry Database.

Table A.7: Implied substitution elasticities, changes in input cost ratios, and firm-specific labor supply elasticities: based on cost shares as output elasticity measures

	OLS			IV		
	σ	$\Delta \ln \left(\frac{w_{it}\gamma_{it}^{L}}{z_{it}\gamma_{it}^{M}} \right)$	ϵ_{it}^{L}	σ	$\Delta \ln \left(\frac{w_{it}\gamma_{it}^{L}}{z_{it}\gamma_{it}^{M}} \right)$	ϵ_{it}^{L}
	(1)	(2)	(3)	(4)	(5)	(6)
1-year diff.	6.434	0.113	2.64	2.28 - inf.	0.00 - 0.35	0.62 - inf.
4-year diff.	6.859	0.078	6.54	12.559	0.034	15.765
10-year diff.	5.118	0.085	7.13			

Notes: The table reports substitution elasticities (Columns (1) and (4)), changes in effective factor price ratios (Columns (2) and (5)), and implied labor supply elasticities assuming perfectly elastic intermediate input supply (Columns (3) and (6)) based on our OLS (Columns (1)-(3)) and IV (Columns (4)-(6)) regressions from Tables 2 and 3 that project logged output elasticities over returns to scale and logged input factors on logged output in differences. As IV point estimates suggest a substitution elasticity of infinity for 1-year changes, we report intervals using 95% confidence intervals from all point estimates entering the computation for this specification. This yields larger intervals than directly computing confidence intervals for substitution elasticites and other values.

Table A.8: Firm-level adjustments in output elasticities in response to firm growth: effects by size quintiles (4-year changes).

	$\ln(L_{it})$	$\ln(K_{it})$	$\ln(M_{it})$	$\ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\ln(\frac{\theta^K_{it}}{RTS_{it}})$	$\ln(\frac{\theta^M_{it}}{RTS_{it}})$	$\ln(\theta^{RTS}_{it})$
Panel A: 1st size quintile	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output growth	0.505***	0.259***	1.051***	-0.179***	-0.183***	0.095***	0.024***
	(0.011)	(0.016)	(0.009)	(0.010)	(0.017)	(0.005)	(0.001)
Observations	14,984	14,984	14,984	14,984	14,984	14,984	14,984
N of firms	4,408	4,408	4,408	4,408	4,408	4,408	4,408
R^2	0.558	0.290	0.862	0.337	0.255	0.351	0.357
Panel B: 2nd size quintile	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output growth	0.484***	0.217***	1.034***	-0.206***	-0.175***	0.098***	0.018***
	(0.012)	(0.015)	(0.009)	(0.011)	(0.014)	(0.004)	(0.001)
Observations	13,276	13,276	13,276	13,276	13,276	13,276	13,276
N of firms	4,508	4,508	4,508	4,508	4,508	4,508	4,508
R^2	0.540	0.268	0.872	0.383	0.295	0.384	0.325
Panel C: 3rd size quintile	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output growth	0.492***	0.208***	1.032***	-0.200***	-0.164***	0.093***	0.017***
	(0.012)	(0.013)	(0.010)	(0.011)	(0.013)	(0.005)	(0.001)
Observations	13,211	13,211	13,211	13,211	13,211	13,211	13,211
N of firms	4,265	4,265	4,265	4,265	4,265	4,265	4,265
R^2	0.546	0.295	0.877	0.353	0.355	0.394	0.312
Panel D: 4th size quintile	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output growth	0.528***	0.178***	1.047***	-0.202***	-0.169***	0.094***	0.014***
	(0.015)	(0.012)	(0.009)	(0.013)	(0.010)	(0.005)	(0.001)
Observations	13,276	13,276	13,276	13,276	13,276	13,276	13,276
N of firms	3,532	3,532	3,532	3,532	3,532	3,532	3,532
R^2	0.576	0.287	0.905	0.384	0.417	0.427	0.288
Panel E: 5th size quintile	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output growth	0.529***	0.150***	1.055***	-0.196***	-0.206***	0.099***	0.008***
	(0.016)	(0.012)	(0.012)	(0.015)	(0.021)	(0.006)	(0.002)
Observations	11,648	11,648	11,648	11,648	11,648	11,648	11,648
N of firms	2,115	2,115	2,115	2,115	2,115	2,115	2,115
R^2	0.596	0.292	0.910	0.363	0.359	0.456	0.310

Notes: The table reports on OLS regressions from estimating the specification in Equation (20) for 4-year differences. The dependent variables in Columns (1)-(7) are the logged values of labor, capital, and intermediates, the logged values of labor, capital, and intermediate input output elasticities divided by returns to scale, respectively. Panels A-E report results for the 1st, 2nd, 3rd , 4th, and 5th quintile of the firm-level output distribution. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm-level. Size quintiles are computed by year and industry. Size is measured by sales. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Table A.9: Firm-level adjustments in intermediate cost shares by intermediate type (OLS).

	$\ln(E_{it}^{CS})$	$\ln(Merch_{it}^{CS})$	$\ln(Sub_{it}^{CS})$	$\ln(Rep_{it}^{CS})$	$\ln(Rent_{it}^{CS})$	$\ln(Other_{it}^{CS})$
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)
Log output growth	0.297***	0.408***	0.388***	-0.104***	-0.417***	-0.107***
	(0.001)	(0.029)	(0.022)	(0.011)	(0.011)	(0.008)
Observations	183,807	83,975	96,062	176,845	175,391	183,813
N of firms	29,950	15,277	19,842	29,598	29,340	29,950
R^2	0.108	0.069	0.061	0.029	0.055	0.030
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)
Log output growth	0.187***	0.429***	0.218***	-0.113***	-0.281***	-0.054***
	(0.001)	(0.044)	(0.035)	(0.014)	(0.018)	(0.011)
Observations	70,933	36,495	35,962	68,859	67,827	70,936
N of firms	11,492	6,096	6,737	11,366	11,149	11,492
R^2	0.128	0.117	0.115	0.072	0.079	0.065
Panel B: 10-year diff.	(1)	(2)	(3)	(4)	(5)	(6)
Log output growth	0.137***	0.217***	0.148***	-0.072***	-0.218***	0.011
	(0.008)	(0.051)	(0.038)	(0.015)	(0.021)	(0.012)
Observations	48,950	21,102	22,376	46,996	46,143	48,953
N of firms	10,381	4,542	5,743	10,189	9,996	10,381
R^2	0.130	0.126	0.120	0.074	0.080	0.068

Notes: The table reports on OLS regressions from estimating the specification in Equation (20) for 1-year (Panel A), 4-year (Panel B), and 10-year differences. The dependent variables in Columns (1)-(6) are the logged values of raw materials, energy, and external components cost shares, merchandise cost shares, subcontracted works performed by other companies cost shares, repairs, maintenance, and installation cost shares, rents, leases, and leasing cost shares, other intermediate inputs cost shares, respectively. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm-level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Table A.10: Testing the role of non-homotheticity in the production function, OLS results

	$\ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\ln(\theta^L_{it})$	$\ln(\theta^L_{it})$	$\ln(\theta^L_{it})$	$\ln(\frac{\theta^L_{it}}{\theta^M_{it}})$	$\ln(\frac{\theta^L_{it}}{\theta^M_{it}})$	$\ln(\frac{\theta^L_{it}}{\theta^M_{it}})$
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Log output growth	-0.304***		0.006*	-0.298***		0.0338***	-0.455***		0.0206***
	(0.004)		(0.003)	(0.005)		(0.004)	(0.00549)		(0.00390)
Log labor-interm. input ratio		0.424***	0.427***		0.440***	0.456***		0.644***	0.655***
		(0.004)	(0.004)		(0.004)	(0.004)		(0.00373)	(0.00396)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950
R^2	0.230	0.466	0.466	0.215	0.471	0.472	0.276	0.618	0.618
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Log output growth	-0.202***		0.027***	-0.185***		0.061***	-0.297***		0.0512***
	(0.006)		(0.005)	(0.006)		(0.005)	(0.00785)		(0.00555)
Log labor-interm. input ratio		0.412***	0.427***		0.427***	0.461***		0.622***	0.651***
		(0.006)	(0.007)		(0.006)	(0.007)		(0.00656)	(0.00726)
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492
R^2	0.220	0.462	0.463	0.201	0.463	0.468	0.245	0.595	0.597
Panel C: 10-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Log output growth	-0.139***		0.048***	-0.111***		0.093***	-0.197***		0.0911***
	(0.005)		(0.004)	(0.006)		(0.005)	(0.00742)		(0.00522)
Log labor-interm. input ratio		0.402***	0.430***		0.415***	0.470***		0.610***	0.664***
		(0.006)	(0.007)		(0.006)	(0.007)		(0.00651)	(0.00729)
Observations	48,953	48,953	48,953	48,953	48,953	48,953	48,953	48,953	48,953
N of firms	10,381	10,381	10,381	10,381	10,381	10,381	10,381	10,381	10,381
R^2	0.189	0.492	0.497	0.163	0.485	0.503	0.199	0.612	0.623

Notes: The table reports OLS regressions from estimating the specification in Equation (20) with additionally controlling for the log of the labor-intermediate input ratio. The dependent variable in Columns (1)-(3) is the log of the labor output elasticity divided by returns to scale. The dependent variable in Columns (4)-(6) is the log of the labor output elasticity. The dependent variable in Columns (7)-(9) is the log of the labor output elasticity divided by the intermediate input output elasticity. Panels A-C project changes in those dependent variables on changes in log output and log labor-intermediate input ratios for 1-, 4- , and 10-year differences. Standard errors are clustered at the firm-level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Table A.11: Testing the role of non-homotheticity in the production function, IV results

	$\ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\ln(\frac{\theta^L_{it}}{RTS_{it}})$	$\ln(\theta^L_{it})$	$\ln(\theta^L_{it})$	$\ln(\theta^L_{it})$	$\ln(\frac{\theta^L_{it}}{\theta^M_{it}})$	$\ln(\frac{\theta^L_{it}}{\theta^M_{it}})$	$\ln(\frac{\theta^L_{it}}{\theta^M_{it}})$
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Log output growth	-0.327***		-0.100	-0.318***		-0.074	-0.445***		-0.0792
	(0.062)		(0.076)	(0.065)		(0.077)	(0.0753)		(0.0815)
Log labor-interm. input ratio		0.424***	0.374***		0.440***	0.403***		0.644***	0.605***
		(0.004)	(0.038)		(0.004)	(0.039)		(0.00373)	(0.0406)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813
First-stage F-value	102.6		90.01	102.6		90.01	102.6		90.01
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950
R^2	0.229	0.466	0.456	0.215	0.471	0.462	0.276	0.618	0.612
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Log output growth	-0.242***		-0.087	-0.220***		-0.049	-0.324***		-0.0772
	(0.071)		(0.075)	(0.074)		(0.078)	(0.0911)		(0.0857)
Log labor-interm. input ratio		0.412***	0.364***		0.427***	0.399***		0.622***	0.579***
		(0.006)	(0.042)		(0.006)	(0.044)		(0.00656)	(0.0482)
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492
First-stage F-value	48.57		39	48.57		39	48.57		39
R^2	0.217	0.462	0.445	0.199	0.463	0.452	0.244	0.595	0.583

Notes: The table reports IV regressions from estimating the specification in Equation (20) using foreign demand shocks as instruments (Equation (22)) and with additionally controlling for the log of the labor-intermediate input ratio. The dependent variable in Columns (1)-(3) is the log of the labor output elasticity divided by returns to scale. The dependent variable in Columns (4)-(6) is the log of the labor output elasticity. The dependent variable in Columns (7)-(9) is the log of the labor output elasticity divided by the intermediate input output elasticity. Panels A-B project changes in those dependent variables on changes in log output and log labor-intermediate input ratios for 1-, and 4-year differences. Standard errors are clustered at the firm-level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Table A.12: Effect of firm growth on average wages. German micro data. (OLS)

	OLS	IV
	Real wage	Real wage
Panel A: 1-year changes	(1)	(2)
Log output growth	0.095***	-0.024
	(0.003)	(0.0562)
Observations	183,813	183,813
N of firms	29,950	29,950
First-stage F-Statistic		102.6
R^2	0.062	0.035
Panel B: 4-year changes	(1)	(2)
Log output growth	0.062***	0.031
	(0.003)	(0.046)
Observations	70,936	70,936
N of firms	29,950	29,950
First-stage F-Statistic		48.57
R^2	0.126	0.122
Panel C: 10-year changes	(1)	
Log output growth	0.068***	
	(0.003)	
Observations	48,953	
N of firms	10,381	
First-stage F-Statistic		
R^2	0.156	

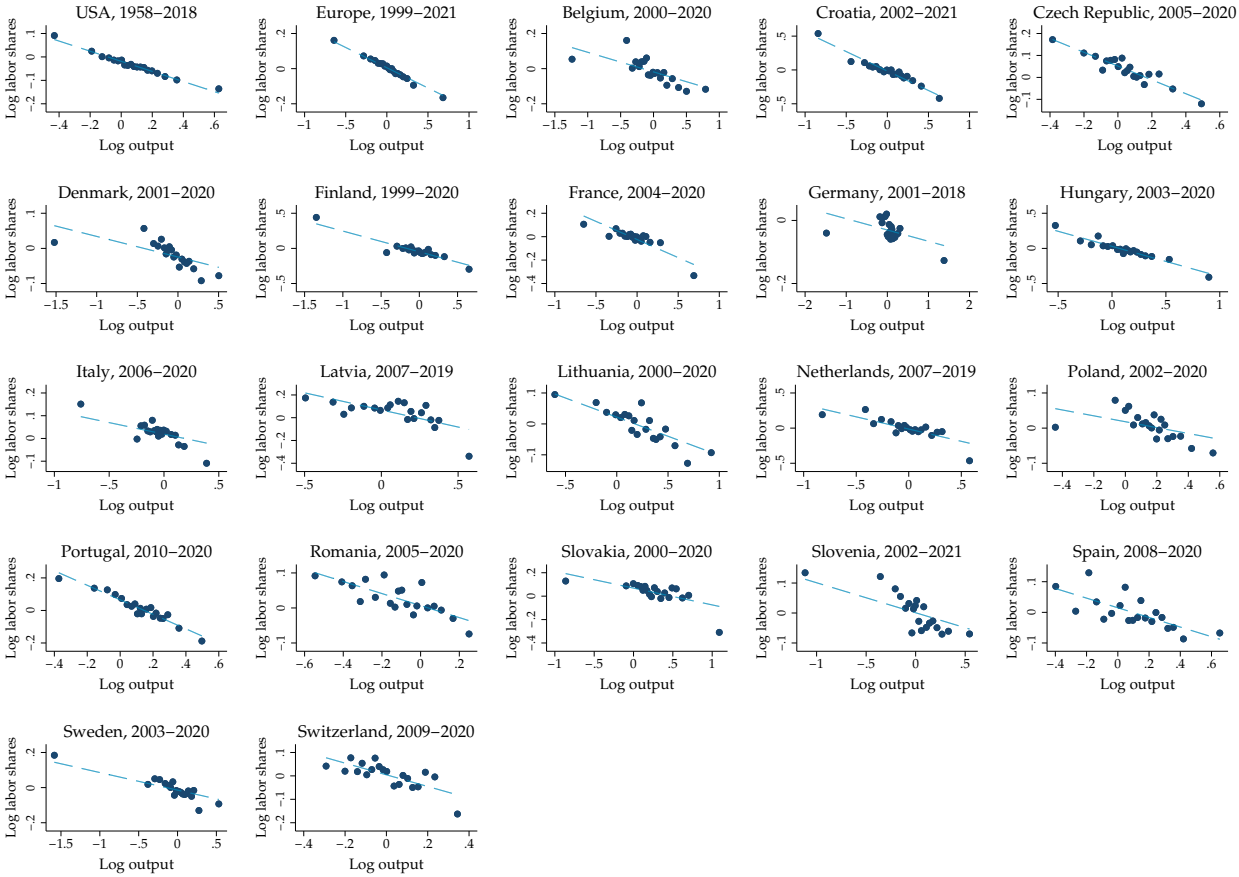
Notes: The table reports OLS and IV regressions from estimating the specification in Equation (20) for 1-, 4- and 10-year differences. The dependent variable is the logged real wage. Panels A-C report results for 1-, 4- and 10-year differences, respectively All regressions control for industry-year fixed effects. Standard errors are clustered at the firm-level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Table A.13: CompNet summary statistics, averages of industry averages (preliminary)

Country	Lab. share (sales)	Lab. share (value added)	Lab. cost share	Lab. output elast.	Markup	Wage Markdown
Belgium (2000-2020)	0.17	0.41	0.21	0.31	1.24	1.26
Croatia (2002-2021)	0.19	0.40	0.25	0.28	1.32	1.05
Czech Republic (2005-2020)	0.16	0.59	0.17	0.18	1.19	0.75
Denmark (2001-2020)	0.25	0.72	0.26	0.26	1.19	0.80
Finland (1999-2020)	0.21	0.71	0.22	0.21	1.15	0.84
France (2004-2020)	0.21	0.39	0.31	0.38	1.51	1.37
Germany (2001-2018)	0.22	0.74	0.21	0.36	1.12	1.55
Hungary (2003-2020)	0.17	0.61	0.17	0.19	1.12	0.86
Italy (2006-2020)	0.17	0.63	0.17	0.17	1.19	0.77
Latvia (2007-2019)	0.19	0.53	0.21	0.24	1.27	1.02
Lithuania (2000-2020)	0.19	0.66	0.20	0.20	1.15	1.14
Netherlands (2007-2019)	0.21	0.69	0.22	0.22	1.11	0.84
Poland (2002-2020)	0.16	0.54	0.17	0.17	1.18	0.79
Portugal (2010-2020)	0.19	0.62	0.20	0.25	1.12	1.09
Romania (2005-2020)	0.17	0.62	0.17	0.20	1.13	0.91
Slovakia (2000-2020)	0.16	0.50	0.18	0.19	1.14	1.09
Slovenia (2002-2021)	0.19	0.63	0.21	0.20	1.12	0.83
Spain (2008-2020)	0.17	0.64	0.19	0.21	1.17	0.96
Sweden (2003-2020)	0.22	0.38	0.32	0.41	1.32	1.51
Switzerland (2009-2020)	0.26	0.70	0.28	0.25	1.18	0.79

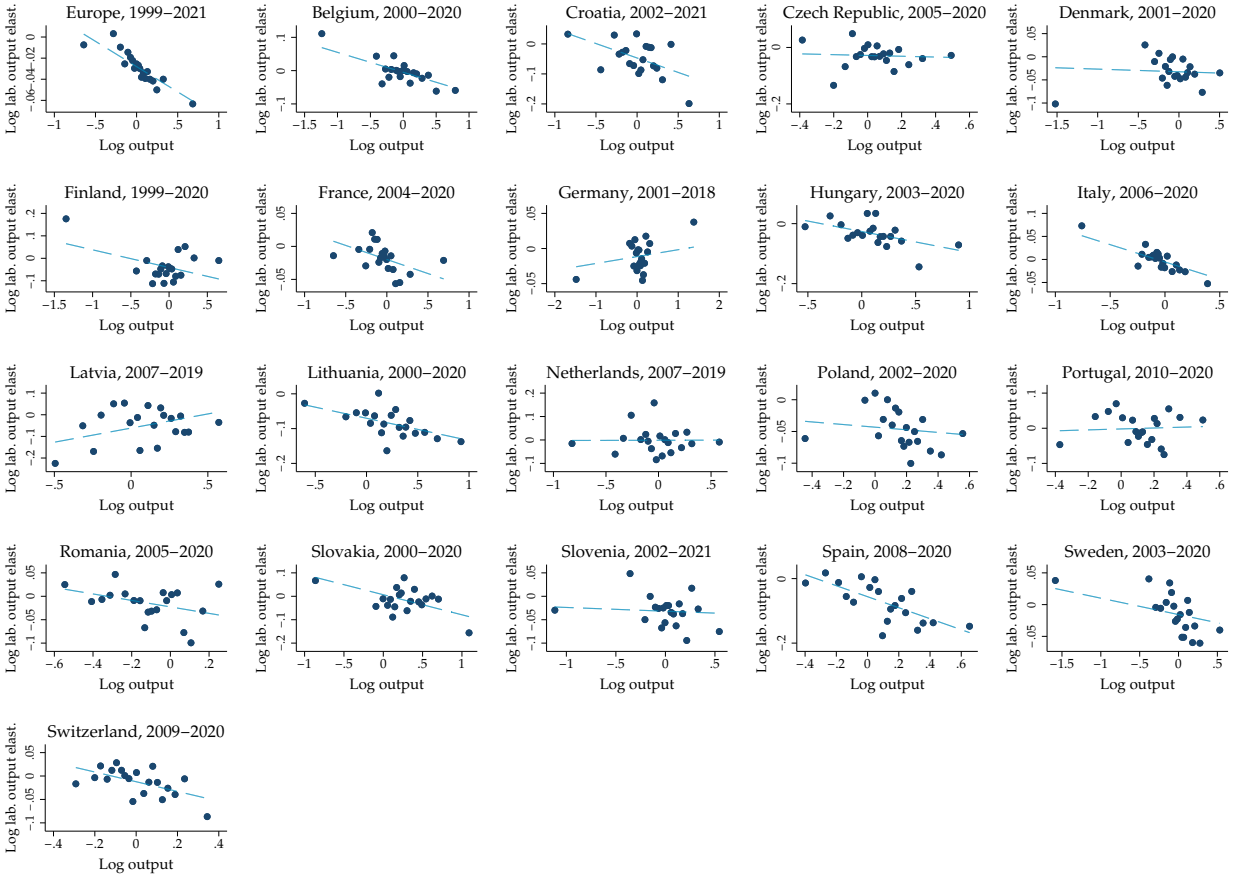
Notes: The table reports country-level averages of industry-level averages for labor shares (in sales and value added), labor output elasticities, markups, and wage mardkdowns. Industry-level aggregates for labor shares are computed from firms' total labor costs, total sales, and total value addded. Industry-level averages for labor output elasticities are sales-weighted aggregates of firm-level values. Industry-level average markups are derived as intermediate-input-cost-weighted averages of firm-level values. Industry-level average wage markdowns are derived as labor-cost-weighted averages of firm-level values. CompNet data. Firms with at least 20 employees.

Figure A.6: Log labor shares in sales changes against log output changes, industry-level, 4-year changes



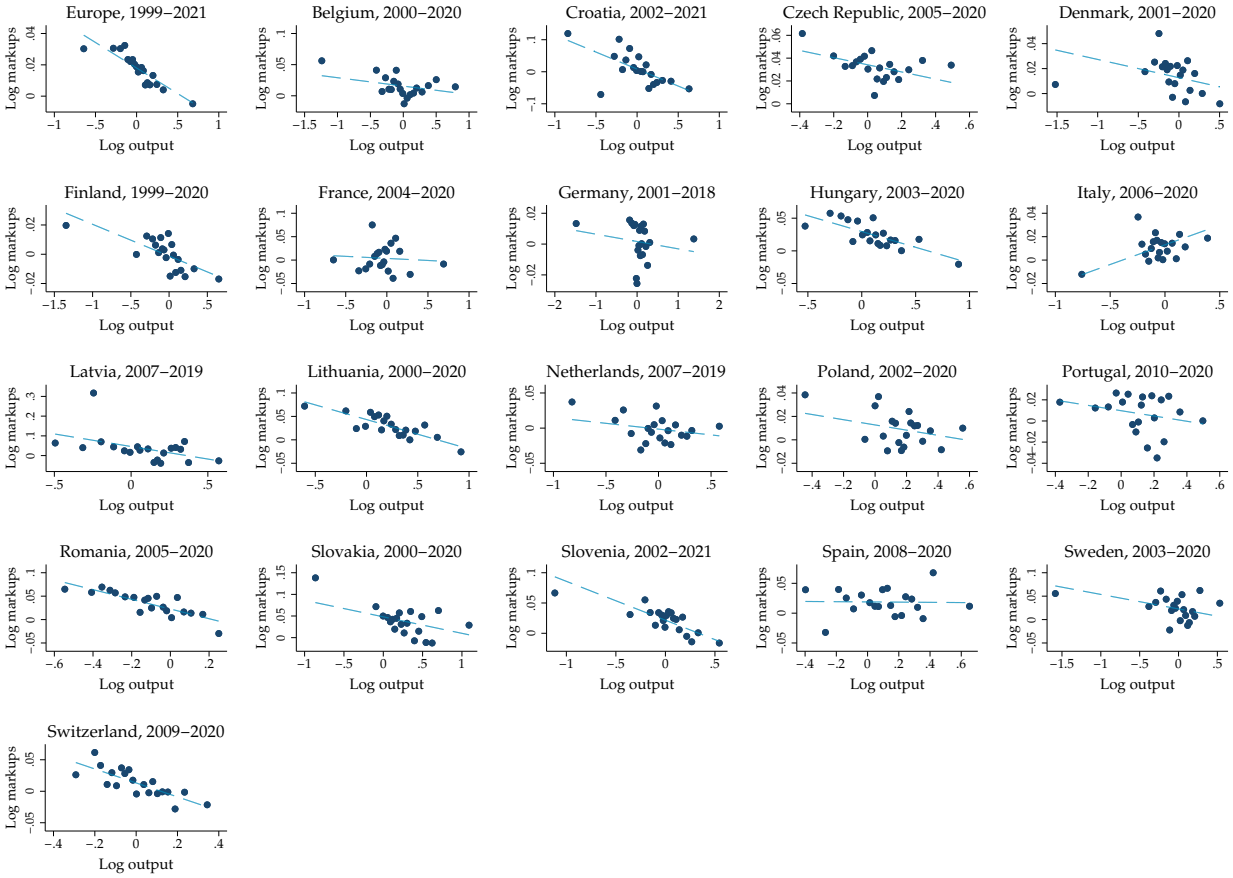
Notes: This figure reports results from separate country-level regressions (except for the "Europe" panel) that project industry-level 4-year changes in log labor costs over sales on 4-year changes in log output while controlling for year fixed effects. 2-digit manufacturing NACE Rev. 2 industries for European countries, 6-digit 2012 NAICS manufacturing industries for the US. CompNet data for Europe (firms with at least 20 employees). NBER-CES Manufacturing Industry Database for the US.

Figure A.7: Log labor output elasticity over returns to scale changes against log output changes, industry-level, 4-year changes



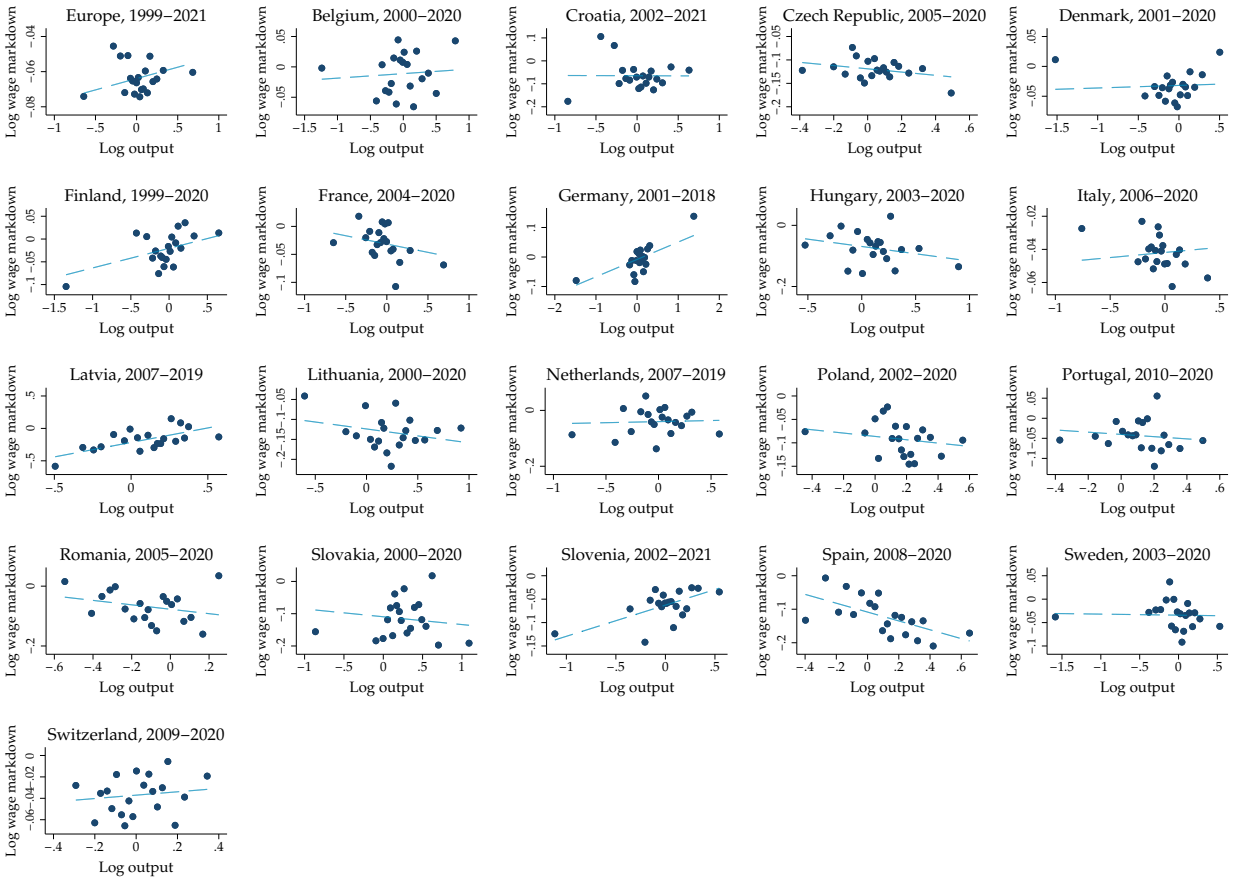
Notes: This figure reports results from separate country-level regressions (except for the "Europe" panel) that project industry-level 4-year changes in log labor output elasticity over returns to scale on 4-year changes in log output while controlling for year fixed effects. For the panel on Europe, we pool all countries and include industry-year fixed effects and country-year fixed effects. 2-digit manufacturing NACE Rev. 2 industries for European countries, 6-digit 2012 NAICS manufacturing industries for the US. CompNet data, firms with at least 20 employees.

Figure A.8: Log markup changes against log output changes, industry-level, 4-year changes



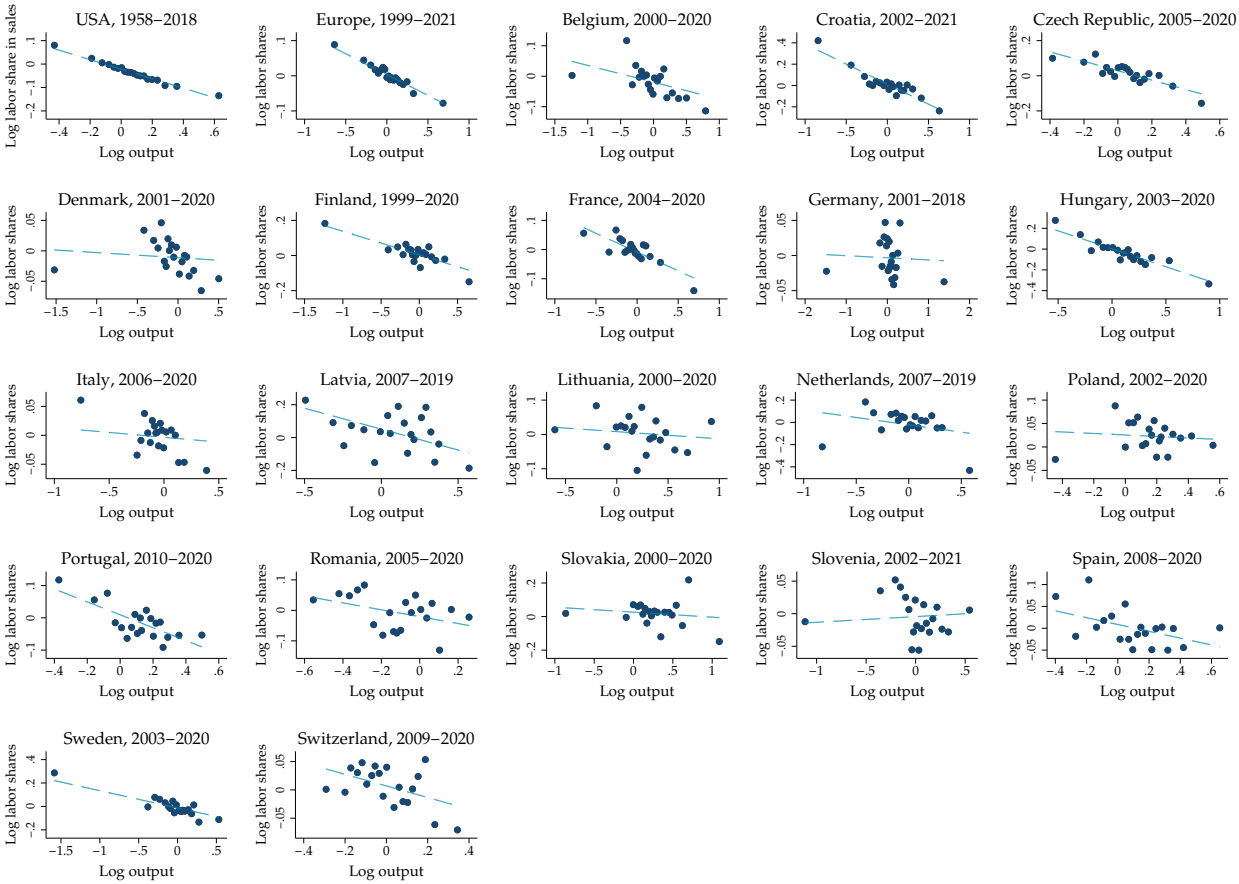
Notes: This figure reports results from separate country-level regressions (except for the "Europe" panel) that project industry-level 4-year changes in log markups on 4-year changes in log output while controlling for year fixed effects. For the panel on Europe, we pool all countries and include industry-year fixed effects and country-year fixed effects. 2-digit manufacturing NACE Rev. 2 industries for European countries, 6-digit 2012 NAICS manufacturing industries for the US. CompNet data, firms with at least 20 employees.

Figure A.9: Log wage markdown changes against log output changes, industry-level, 4-year changes



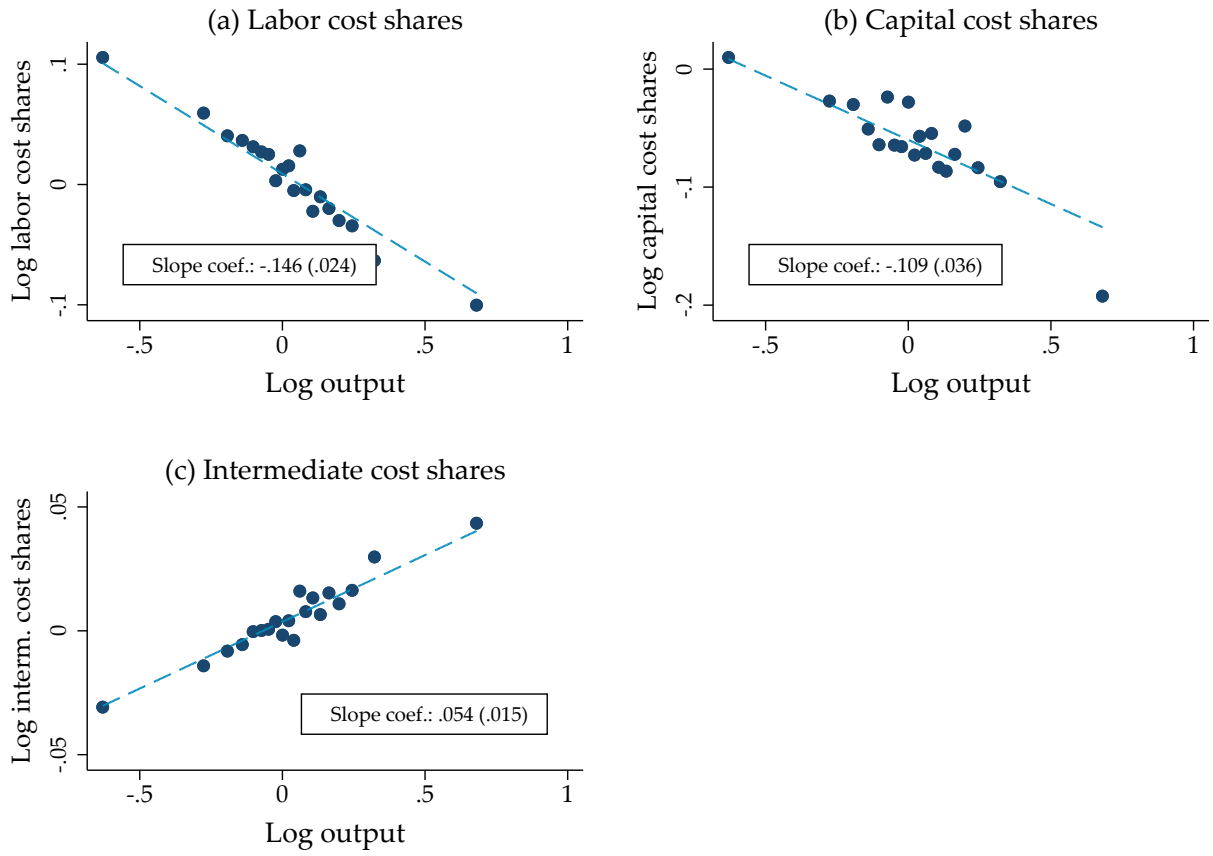
Notes: This figure reports results from separate country-level regressions (except for the "Europe" panel) that project industry-level 4-year changes in log wage markdowns on 4-year changes in log output while controlling for year fixed effects. For the panel on Europe, we pool all countries and include industry-year fixed effects and country-year fixed effects. 2-digit manufacturing NACE Rev. 2 industries for European countries, 6-digit 2012 NAICS manufacturing industries for the US. CompNet data, firms with at least 20 employees.

Figure A.10: Log labor shares in value added changes against log output changes, industry-level, 4-year change



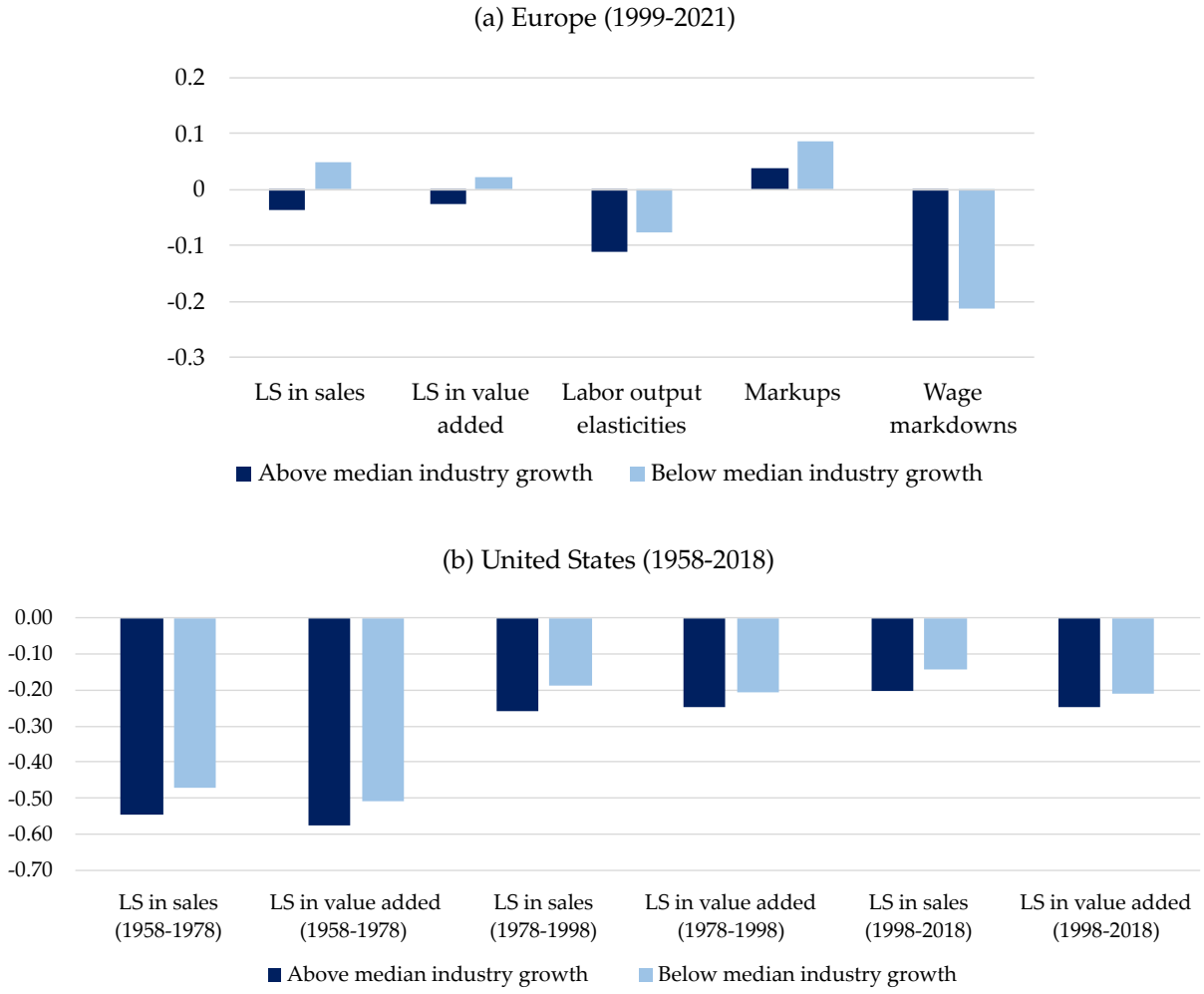
Notes: This figure reports results from separate country-level regressions (except for the "Europe" panel) that project industry-level 4-year changes in log labor costs over value added on 4-year changes in log output while controlling for year fixed effects. For the panel on Europe, we pool all countries and include industry-year fixed effects and country-year fixed effects. 2-digit manufacturing NACE Rev. 2 industries for European countries, 6-digit 2012 NAICS manufacturing industries for the US. CompNet data for Europe (firms with at least 20 employees). NBER-CES Manufacturing Industry Database for the US.

Figure A.11: European industry level 4-year changes in cost share (OLS)



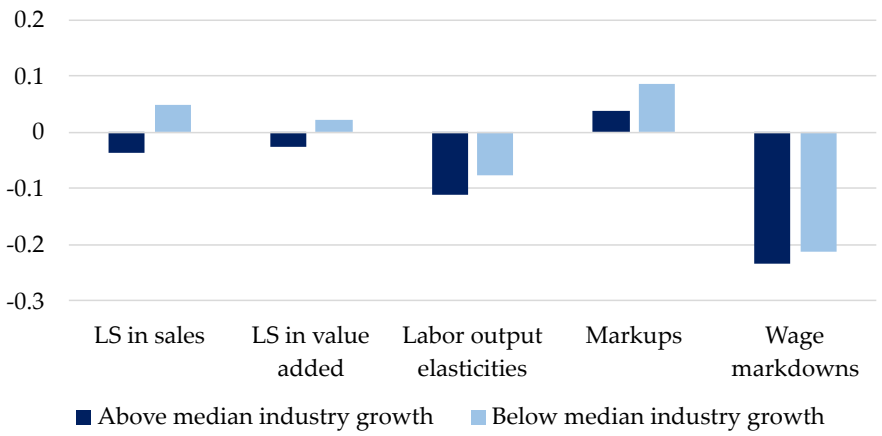
Notes: The figure reports binned scatter plots (OLS) from projecting 4-year industry-level changes in the logs of labor, capital, and intermediate cost shares against industry-level log output changes (4-year differences). We pool all countries and control for country-year and industry-year fixed effects. CompNet data. Firms with at least 20 employees, two-digit NACE Rev.2 industries.

Figure A.12: Changes in labor shares, output elasticities, markups, and wage markdowns in industries with above and below country-median growth rates



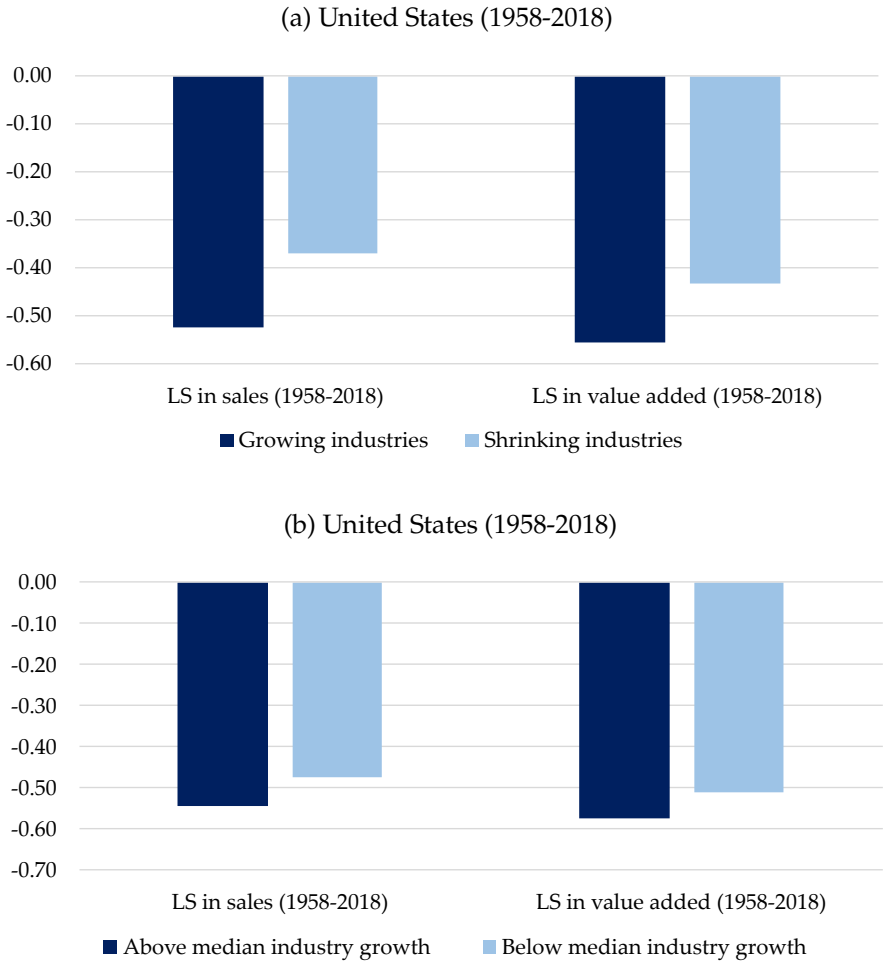
Notes: The figures report average changes in the logs of labor shares, labor output elasticities, markups, and wage markdowns across country-industry pairs for industries with growth rates above and below country-median growth rates. Changes are first computed for each pair and then average across all pairs. Time spans differ across country-industry pairs and not all industries are available for all years within a country. Europe: CompNet data and NACE Rev. 2 two-digit industries. US: NBER-CES Manufacturing Industry Database and NAICS 6-digit industries.

Figure A.13: Europe: changes in labor shares, output elasticities, markups, and wage markdowns in industries with above and below European-median growth rates



Notes: The figure reports average changes in the logs of labor shares, labor output elasticities, markups, and wage markdowns across country-industry pairs for industries with growth rates above and below European-median growth rates. Changes are first computed for each pair and then average across all pairs. Time spans differ across country-industry pairs and not all industries are available for all years within a country. CompNet data and NACE Rev. 2 two-digit industries.

Figure A.14: US: changes in labor shares in growing and shrinking industries and in industries with above and below country-median growth rates, long-run changes



Notes: The figures report average changes in the logs of labor shares across US industries for growing and shrinking industries and for industries with growth rates above and below country-median growth rates. Changes are first computed for each industry over the entire time span and then average across all pairs. NBER-CES Manufacturing Industry Database and NAICS 6-digit industries. There are 36 shrinking and 326 growing industries over the 1958-2018 time span.

B Theory Appendix

B.1 Derivation of Equation (4)

Equation (3) is defined as in Hicks (1932):

$$\frac{1}{\sigma_{it}} = \frac{\Delta \ln(\frac{\partial Q_{it}}{\partial M_{it}}) - \Delta \ln(\frac{\partial Q_{it}}{\partial L_{it}})}{\Delta \ln(L_{it}) - \Delta \ln(M_{it})}. \tag{A.1}$$

Eliminating differences (and thus adding a constant, α_i) and multiplying with -1 yields:

$$\frac{1}{\sigma_{it}} \ln\left(\frac{L_{it}}{M_{it}}\right) + \ln \alpha_i = \ln\left(\frac{\frac{\partial Q_{it}}{\partial L_{it}}}{\frac{\partial Q_{it}}{\partial M_{it}}}\right) \tag{A.2}$$

Eliminating logs yields:

$$\left(\frac{L_{it}}{M_{it}}\right)^{\frac{1}{\sigma_{it}}} \alpha_i = \frac{\frac{\partial Q_{it}}{\partial L_{it}}}{\frac{\partial Q_{it}}{\partial M_{it}}} \tag{A.3}$$

Multiplying with $\frac{\frac{L_{it}}{Q_{it}}}{\frac{M_{it}}{Q_{it}}}$ yields eq. (4) from the main text:

$$\frac{\theta_{it}^L}{\theta_{it}^M} = \alpha_i \left(\frac{L_{it}}{M_{it}}\right)^{\frac{\sigma_{it}-1}{\sigma_{it}}}. \tag{A.4}$$

B.2 CES Production Function

We can derive expression eq. (4) also from a CES production function (where σ is constant). To see this, consider the following CES production function:

$$Q_{it} = \Omega_{it}\left(\Lambda_i^{LM}\left(\alpha_i^L L_{it}^{\frac{\sigma-1}{\sigma}} + \alpha_i^M M_{it}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}\frac{\kappa-1}{\kappa}} + \Lambda_i^K K_{it}^{\frac{\kappa-1}{\kappa}}\right)^{\frac{\kappa}{\kappa-1}}, \tag{A.5}$$

where L_{it}, M_{it}, and K_{it} are labor, intermediates, and capital used in the production of output, Q. A is total factor productivity. Λ_i^{LM}, Λ_i^K, α_i^L, and α_i^M are distribution parameters. σ is the substitution elaticity between labor and intemediates. κ denotes the substitution elasticity between capital and the second input aggregate consisting of labor and intermediate inputs. This specification allows us to conveniently study substitution patterns between labor and intermediates, which is the focus of our study. Taking partial derivatives with respect to labor and intermediates yields:

$$\frac{\frac{\partial Q_{it}}{\partial L_{it}}}{\frac{\partial Q_{it}}{\partial M_{it}}} = \frac{\Omega_{it}\Lambda_i\alpha_i^L L_{it}^{\frac{-1}{\sigma}}\left(\alpha_i^L L_{it}^{\frac{\sigma-1}{\sigma}} + \alpha_i^M M_{it}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}\frac{\kappa-1}{\kappa}-1} Q^{\frac{-1}{\kappa-1}}}{\Omega_{it}\Lambda_i\alpha_i^M M_{it}^{\frac{-1}{\sigma}}\left(\alpha_i^L L_{it}^{\frac{\sigma-1}{\sigma}} + \alpha_i^M M_{it}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}\frac{\kappa-1}{\kappa}-1} Q^{\frac{-1}{\kappa-1}}} = \frac{\alpha_i^L}{\alpha_i^M}\left(\frac{L_{it}}{M_{it}}\right)^{\frac{1}{\sigma_{it}}}. \tag{A.6}$$

Multiplying with $\frac{\frac{L_{it}}{Q_{it}}}{\frac{M_{it}}{Q_{it}}}$ and defining $\alpha_i = \frac{\alpha_i^L}{\alpha_i^M}$ again yields eq. (4) of the main text.

B.3 Substitution Elasticity under Cobb-Douglas

Consider the Cobb-Douglas production function $Q_{it} = L_{it}^{\theta^L} K_{it}^{\theta^K} L_{it}^{\theta^M} \Omega_{it}$. Taking partial derivatives with respect to labor and intermediates implies:

$$\frac{\frac{\partial Q_{it}}{\partial L_{it}}}{\frac{\partial Q_{it}}{\partial M_{it}}} = \frac{\theta^L \frac{Q_{it}}{L_{it}}}{\theta^M \frac{Q_{it}}{M_{it}}}. \tag{A.7}$$

Setting the CES constants from eq. (A.6) to the Cobb-Douglas output elasticities ($\alpha_i^L = \theta^L$ and $\alpha_i^M = \theta^M$) yields the Cobb-Douglas equivalent of the CES equation eq. (A.6):

$$\frac{\frac{\partial Q_{it}}{\partial L_{it}}}{\frac{\partial Q_{it}}{\partial M_{it}}} = \frac{\alpha_i^L}{\alpha_i^M} \left(\frac{L_{it}}{M_{it}} \right)^{-1}. \tag{A.8}$$

Comparison with Equations A.6 and A.3 reveals that these equations can only be equalized if $\sigma = 1$. Therefore, the substitution elasticity between inputs equals unity in the Cobb-Douglas case.

B.4 Cost Minimization and Labor Shares in Costs and Sales

We assume that firms minimize cost. For illustration, we now allow that firms have monopsony in each input market and that each input faces adjustment costs. In the main text, we will assume flexible and competitive intermediate input markets for some parts of the analysis as standard in the related literature. Using the general production function from Section 2, we write firms' Langrangian as:

$$\begin{aligned} \mathcal{L} = P_{it}^L(L_{it}) L_{it} + \chi^L(L_{it}) P_{it}^L + P^M(M_{it}) M_{it} + \chi^M(M_{it}) P_{it}^M \\ + P^K(K_{it}) K_{it} + \chi^K(K_{it}) P_{it}^K - \lambda_{it}(Q_{it} - Q_{it}(.)). \end{aligned} \tag{A.9}$$

where, for simplicity, we model input adjustment costs through convex functions $\chi^X(X_{it})$ with $X = \{L, M, K\}$ and write the optimization in a quasi-static way as in Bond et al. (2021). One interpretation of this setting is that each input is associated with a baseline quantity, $\bar{X_{it}}$, and that firms incur adjustment costs when choosing an input quantity that differs from $\bar{X_{it}}$. In a dynamic version of the optimization problem, $\bar{X_{it}}$ could reflect input choices from previous periods. The first order conditions for each production input are:

$$P_{it}^X \underbrace{\left(1 + \frac{\partial P_{it}^X}{\partial X_{it}} \frac{X_{it}}{P_{it}^X} + \frac{\partial \chi^X}{\partial L_{it}} \right)}_{\gamma_{it}^X} = \lambda_{it} \frac{\partial Q_{it}}{\partial X_{it}}, \tag{A.10}$$

where γ_{it}^X is a measure of input market imperfections. $\frac{\partial P_{it}^X}{\partial X_{it}} \frac{X_{it}}{P_{it}^X}$ is the invers supply elasticity. Noting that $\lambda_{it} = \frac{P_{it}}{\mu_{it}}$ equals output prices over markups (i.e., marginal costs) and expanding both sides of eq. (A.10) with $\frac{X_{it}}{Q_{it}}$ yields an expression for eq. (7) and eq. (8) from the main text:

$$X_{it} = \frac{P_{it} Q_{it}}{\mu_{it} \gamma_{it}^X P_{it}^X} \theta_{it}^X, \tag{A.11}$$

where we used the definition of the output elasticity: $\theta_{it}^X = \frac{\partial Q_{it}}{\partial X_{it}} \frac{X_{it}}{Q_{it}}$. Rearranging the first order conditions yields expressions for input expenditures:

$$P_{it}^X X_{it} = P_{it} Q_{it} \frac{\theta_{it}^X}{\mu_{it} \gamma_{it}^X}, \tag{A.12}$$

Combining the cost expressions for each input with each other yields for the labor input:

$$\frac{P_{it}^L L_{it}}{P_{it}^L L_{it} + P_{it}^M M_{it} + P_{it}^K K_{it}} = \frac{\frac{\theta_{it}^L}{\gamma_{it}^L}}{\frac{\theta_{it}^L}{\gamma_{it}^L} + \frac{\theta_{it}^M}{\gamma_{it}^M} + \frac{\theta_{it}^K}{\gamma_{it}^K}}, \tag{A.13}$$

which equals eq. (16) from the main text. The right-hand side of Equation (F.9) is the cost share for the labor input. Under constant returns to scale ($\theta_{it}^L + \theta_{it}^M + \theta_{it}^K = 1$) and flexible and competitive input markets ($\gamma_{it}^L = \gamma_{it}^M = \gamma_{it}^K = 1$), this cost share equals the labor output elasticity. The same holds for other inputs. Finally, note that in this entire derivation, we did not made any assumptions on firms' production functions. This means, that cost shares also equal output elasticities in presence of non-Hicks-neutral productivity processes as long as we impose constant returns to scal and competitive and flexible input markets.

Finally, note that rearranging eq. (A.12) yields the expression for the labor share in output (eq. (18) from the main text):

$$LS_{it} = \frac{P_{it}^L L_{it}}{P_{it} Q_{it}} = \frac{\theta_{it}^L}{\mu_{it} \gamma_{it}^L}. \tag{A.14}$$

B.5 Markups and Wage Markdowns

We now show how we derive the equations for firms' markups and wage markdowns in terms of observables and output elasticities. Consider again the cost minimization setting from Appendix B.4. Following the literature, we assume that intermediates are flexible and that intermediate input prices are exogenous to firms. Strictly speaking, for our firm-level analysis in changes, it is even sufficient that flexibility and market power in intermediate input markets remain constant over time (our results provide suggestive evidence for that). This implies the following first order conditions:

$$P_{it}^M = \frac{P_{it}}{\mu_{it}} \frac{\partial Q_{it}}{\partial M_{it}}, \tag{A.15}$$

$$P_{it}^L \gamma_{it}^L = \frac{P_{it}}{\mu_{it}} \frac{\partial Q_{it}}{\partial M_{it}}. \tag{A.16}$$

Rearranging eq. (A.15) yields an expression for the markup

$$\mu_{it} = \frac{P_{it}}{MC_{it}} = \theta_{it}^M \frac{P_{it} Q_{it}}{P_{it}^M M_{it}}. \tag{A.17}$$

Combining eq. (A.16) and eq. (A.15) recovers and expression for the wage markdown:

$$\gamma_{it}^L = \frac{MRPL_{it}}{P_{it}^L} = \frac{\theta_{it}^L}{\theta_{it}^M} \frac{P_{it}^M M_{it}}{P_{it}^L L_{it}}, \tag{A.18}$$

where $MRPL_{it}$ is the marginal revenue product of labor.

C Alternative Labor Market Specification: Rent-Sharing.

For tractability, the main text focuses on a case where firms exert monopsony power. However, a large literature has highlighted that also workers possess labor market power, which can equally drive a wedge between wages and marginal revenue products (e.g., Dobbelaere and Mairesse, 2013, Treuren, 2022, Caselli et al., 2021, Mertens, 2022). We now derive a standard rent-sharing model and show that allowing for worker-side labor market power does not affect any of our derivations and conclusions. Consider that employees maximize utility:

$$U(w_{it}, L_{it}) = P_{it}^L L_{it} + (\overline{L}_{it} - L_{it})\overline{P}_{it}^L. \tag{C.1}$$

$\overline{P}_{it}^L \leq P_{it}^L$ is the reservation wage. $\overline{L}_{it}$ is the competitive employment level. Firms produce output using the production function of the main text, $Q_{it} = Q_{it}(.) = F(L_{it}, K_{it}, M_{it}, \Omega_{it})$. Firms and workers bargain over wages and employment and solve the following Nash-bargaining problem:

$$\max_{P_{it}^L, L_{it}, M_{it}, K_{it}} (\zeta_{it} \ln(L_{it}(w_{it} - \overline{L}_{it}^L)) + (1 - \zeta_{it}) \ln(P_{it}Q_{it} - P_{it}^L L_{it} - P_{it}^M M_{it} - P_{it}^K K_{it}), \tag{C.2}$$

where $\zeta_{it} \in [0, 1]$ denotes worker's bargaining power.[35] Note that in the event of a breakdown of negotiations, workers receive the reservation wage as outside options, whereas a firm's outside option is the zero-profit outcome. The latter is not essential for our conclusions, but simplifies derivations and follows the literature (e.g., Blanchflower et al., 1996). The first order condition with respect to intermediate inputs yields the same markup expression as in the main text:

$$P_{it}^M = \frac{P_{it}}{\mu_{it}} \frac{\partial Q_{it}}{\partial M_{it}} \rightarrow \mu_{it} = \theta_{it}^M \frac{P_{it}Q_{it}}{P_{it}^M M_{it}}. \tag{C.3}$$

The first order condition with respect to labor implies:

$$P_{it}^L \left(1 - \frac{\zeta_{it}}{1 - \zeta_{it}} \frac{P_{it}Q_{it} - P_{it}^L L_{it} - P_{it}^M M_{it} - P_{it}^K K_{it}}{P_{it}^L L_{it}}\right) = MRPL_{it}, \tag{C.4}$$

where $MRPL_{it}$ denotes the marginal revenue product of labor. Combining Equation (C.3) with Equation (C.4) yields the same equation for the wage markdown as in the main text:

$$\gamma_{it} = \left(1 - \frac{\zeta_{it}}{1 - \zeta_{it}} \frac{P_{it}Q_{it} - P_{it}^L L_{it} - P_{it}^M M_{it} - P_{it}^K K_{it}}{P_{it}^L L_{it}}\right) = \frac{\theta_{it}^L}{\theta_{it}^M} \frac{P_{it}^M M_{it}}{P_{it}^L L_{it}}, \tag{C.5}$$

where we used $MRPL_{it} = \frac{P_{it}}{\mu_{it}} \frac{\partial Q_{it}}{\partial L_{it}}$. Note that in this setting, $0 \leq \gamma_{it} \leq 1$, which helps explaining why the data features many firms where wages exceed marginal revenue products of labor.[36] Finally, combining Equation (C.5) with Equation (C.3) yields the expression for the labor share from the main text:

$$LS = \frac{P_{it}^L L_{it}}{P_{it}Q_{it}} = \frac{\theta_{it}^L}{\mu_{it}\gamma_{it}}. \tag{C.6}$$

[35] We follow the literature and write the rent-sharing model in static terms.

[36] Mertens (2023) generalizes this setting to a case where firms have monopsony power over some workers while they bargain with other workers over rents. This more general model rationalizes why we simultaneously observe firms with wages above and below the marginal revenue product of labor. Even in such a more complex setting, all our derivations and conclusions remain valid.

D Further Details on the German Firm-Product Level data

Table D.1: Variable definition in the German microdata.

Variable	Definition
L_{it}	Labor in headcounts.
W_{it}	Firm wage (firm average), defined as gross salary before taxes (including mandatory social costs) + "other social expenses" (including expenditures for company outings, advanced training, and similar costs) divided by the number of employees.
K_{it}	Capital derived by a perpetual inventory method as described below.
M_{it}	Deflated total intermediate input expenditures, defined as expenditures for raw materials, energy, intermediate services, goods for resale, renting, repairs, and contracted work conducted by other firms.
E_{it}	Deflated expenditures for raw, auxiliary, and operating materials and energy inputs (includes external product components). E_{it} is part of M_{it}.
$Merch_{it}$	Deflated expenditures for merchandise. $Merch_{it}$ is part of M_{it}.
Sub_{it}	Deflated expenditures for subcontracted work performed by other companies. Sub_{it} is part of M_{it}.
Rep_{it}	Deflated expenditures for repairs, maintenance, installation, and assembly. Rep_{it} is part of M_{it}.
$Rent_{it}$	Deflated expenditures for rent, leases, leasing. Rep_{it} is part of M_{it}.
$Other_{it}$	Deflated expenditures for Other intermediate costs (insurance, postage, transport, etc.). $Other_{it}$ is part of M_{it}.
$z_{it}M_{it}$	Nominal values of total intermediate input expenditures.
$P_{it}Q_{it}$	Nominal total revenue, defined as total gross output, including, among others, sales from own products, sales from intermediate goods, revenue from offered services, and revenue from commissions/brokerage.
Q_{it}	Quasi-quantity measure of physical output, i.e., $P_{it}Q_{it}$ deflated by a firm-specific price index (denoted by PI_{it}).
PI_{it}	Firm-specific Törnqvist price index, derived as in Eslava et al., 2004. See online Appendix E.1 for its construction.
P_{igt}	Price of a product g.
$share_{igt}$	Revenue share of a product g in total firm revenue.
ms_{it}	Weighted average of firms' product market shares in terms of revenues. The weights are the sales of each product in firms' total product market sales.
G_{it}	Headquarter location of the firm. 90% of firms in our sample are single-plant firms.
D_{it}	A four-digit industry indicator variable. The industry of each firm is defined as the industry in which the firm generates most of its sales.
Exp_{it}	Dummy-variable being one, if firms generate export market sales.
$NumP_{it}$	The number of products a firm produces.

Notes: The table list all variables and variable definitions used in the paper. Nominal values are deflated by a 2-digit industry-level deflator for intermediate inputs and which is supplied by the federal statistical office of Germany.

Data access. The data can be accessed at the "Research Data Centres" of the Federal Statistical Office of Germany and the Statistical Offices of the German Länder. Data request can be made at: https://www.forschungsdatenzentrum.de/en/request. The statistics we used are: "AFiD-Modul Produkte", "AFiD-Panel Industriebetriebe", "AFiD-Panel Industrieunternehmen", "Investitionserhebung im Bereich Verarbeitendes Gewerbe, Bergbau und Gewinnung von Steinen und Erden", "Panel der Kostenstrukturerhebung im Bereich Verarbeitendes Gewerbe, Bergbau und Gewinnung von Steinen und Erden". The data are combined by the statistical offices and provided as a merged dataset.

Variable definitions. Table D.1 presents an overview on the variable definitions of all variables used in this article. This includes variables used in other sections of the Appendix.

Outlier cleaning. We clean the data from the top and bottom two percent outliers with respect to value-added over revenue and revenue over labor, capital, intermediate input expenditures, and labor costs. We drop quantity and price information for products displaying a price deviation from the average price in the top and bottom one percent tails. We also drop the sectors 16 (tobacco), 23 (mineral oil and coke), and 37 (recycling) as the observation count is insufficient to derive estimates of firms' production functions in these industries.

Capital stock estimation. As capital stocks are not directly observed, we calculate a time series of capital stocks for every firm using the perpetual inventory method of Bräuer et al. (2023):

$$K_{it} = K_{it-1}(1 - \alpha_{jt-1}) + I_{it-1}, \tag{D.1}$$

where K_{it}, α_{jt} and I_{it-1} denote firm i's capital stock, the depreciation rate of capital, and investment. Investment captures firms' total investment in buildings, equipment, machines, and other investment goods. Nominal values are deflated by a two-digit industry-level deflator supplied by the German statistical office.

We derive the industry- and year-specific depreciation rate from official information on the expected lifetime of capital goods (supplied by the statistical offices). To do so, we formulate the lifetime of a capital good LT as a function of its depreciation rate:

$$LT = \alpha \int_0^\infty (1 - \alpha)^t t dt. \tag{D.2}$$

Using partial integration gives:

$$LT = \alpha \left[\frac{(1 - \alpha)^t}{\ln(1 - \alpha)} t \right]_0^\infty - \alpha \int_0^\infty \frac{(1 - \alpha)^t}{\ln(1 - \alpha)} dt, \tag{D.3}$$

where the first term on the right-hand side equals zero because $0 < \alpha < 1$. Integrating the remaining expression yields:

$$LT = \frac{\alpha}{\ln(1 - \alpha) * \ln(1 - \alpha)}, \tag{D.4}$$

which we can numerically solve for α. As the lifetime of capital goods is separately given for years and capital good types (buildings and equipment), we derive a depreciation rate for each year and capital good type separately. To derive a single industry-specific depreciation rate, we weight the depreciation rates for buildings and equipment respectively with the industry-level share of building capital in total capital and equipment capital in total capital (this information is supplied by the statistical offices). For the practical implementation, we assume that the depreciation rate of a firm's whole capital stock equals the depreciation rate of newly purchased capital.

The initial capital stock for the perpetual inventory method is derived from reported tax depreciation. We do not use the reported tax depreciation when calculating capital stock series as tax depreciation may vary due to state-induced tax incentives and might therefore not reliably reflect the true amount of depreciated capital. Given that firms likely tend to report too high depreciation due to such tax incentives, our first capital values within a capital series are likely overestimated. However, over time, observed investment decisions gradually receive a larger weight in estimated capital stocks, mitigating the impact of the first capital stock. Given that we estimate very reasonable output elasticities (see Table A.1), we are confident that our capital variables reliably reflect firms' true capital stocks.[37]

[37] As firms likely tend to overstate their capital depreciation, our capital stocks are likely a closer approximation of the true capital stock used in firms' production processes than capital measures based on book values.

Deriving a time-consistent industry classification. During our long time-series, the NACE classification of industries (and thus firms into industries) changed twice. Once in 2002 and once in 2008. Because our estimation of the production function requires a time-consistent industry classification at the firm level (as we allow for industry-specific production functions), it is crucial to recover a time-consistent NACE industry classification. Recovering such a time-consistent industry classification from official concordance tables is, however, problematic as they contain many ambiguous sector reclassifications. To address this issue, we follow the procedure described in Mertens (2022) and use information on firms' product mix to classify firms into NACE Rev 1.1 industries based on their main production activities. This procedure exploits that the first four digits of the ten-digit GP product classification reported in the German data are identical to the NACE classification (i.e., they indicate the industry of the product). Applying this method demands a consistent reclassification of all products into the GP2002 scheme (which corresponds to the NACE Rev 1.1 scheme). Reclassifying products is, due to the granularity of the ten-digit classification, less ambiguous than reclassifying industries. In the few ambiguous cases, we can follow the firms' product mix over the reclassification periods and unambiguously reclassify most products (i.e., we observe what firms produce before and after reclassification years). Having constructed a time-consistent product-industry classification according to the GP2002 scheme, we attribute every firm to the NACE Rev 1.1 industry in which it generates most of its revenue. When comparing our classification with the one of the statistical offices for the years 2002-2008 (years in which industries are already reported in NACE Rev 1.1), we find that our two-digit and four-digit classification of firms into industries matches the classification of the statistical offices in 95% and 86% of all cases, respectively.

E Production Function Estimation in the German Data

We follow Mertens (2022) in estimating the production function. This approach yields firm- and time-specific output elasticities by assuming the following translog production function (throughout, lower case letter denote logs):

$$q_{it} = \boldsymbol{\phi}'_{it}\boldsymbol{\beta} + \omega_{it} + \epsilon_{it}\,. \tag{E.1}$$

q_{it} denotes the log of produced quantities and $\boldsymbol{\phi}'_{it}$ captures the production inputs capital (k_{it}), labor (l_{it}), and intermediates (k_{it}) and its interactions. The production function is specified in logs as

$$\begin{aligned} q_{it} =& \beta_l l_{it} + \beta_m m_{it} + \beta_k k_{it} + \beta_{ll} l_{it}^2 + \beta_{mm} m_{it}^2 + \beta_{kk} k_{it}^2 \\ &+ \beta_{lk} l_{it} k_{it} + \beta_{lm} l_{it} m_{it} + \beta_{km} k_{it} m_{it} + \beta_{lkm} l_{it} k_{it} m_{it} + \omega_{it} + \epsilon_{it}. \end{aligned} \tag{E.2}$$

The output elasticity of labor is

$$\frac{\partial q_{it}}{\partial l_{it}} = \beta_l + 2\beta_{ll} l_{it} + \beta_{lm} m_{it} + \beta_{lk} k_{it} + \beta_{lkm} k_{it} m_{it}\,. \tag{E.3}$$

ϵ_{it} is an i.i.d. error term and ω_{it} denotes Hicks-neutral productivity and follows a Markov process. ω_{it} is unobserved to the econometrician, yet firms know ω_{it} before making input decisions for flexible inputs (intermediates in our case). We assume that only firms' input decision for intermediates depends on productivity shocks. Labor and capital do not respond to contemporary productivity shocks. However, our results are similar when allowing labor to respond to productivity innovations. In fact, the CompNet routine for the production function estimation models labor as flexible and we find consistent results in both datasets.
There are three issues preventing us from estimating the production function (E.1) using OLS.

(1) We need to estimate a physical production model to recover the relevant output elasticities. Although we observe product quantities, quantities cannot be aggregated across the various products of multi-product firms. Relying on the standard practice to apply sector-specific output deflators does not solve this issue if output prices vary within industries.

(2) We do not observe firm-specific input prices for capital and intermediate inputs. If input prices are correlated with input decisions and output levels, an endogeneity issue arises.

(3) The fact that productivity is unobserved and that firms' flexible input decisions depend on productivity shocks creates another endogeneity problem.

We now discuss how we solve these three identification problems.

E.1 Solving (1) by Deriving a Firm-specific Output Price Index

As we cannot aggregate output quantities across different products of a firm (a common problem), we follow Eslava et al. (2004) and construct a firm-specific price index from observed output prices. We use this price index to purge observed firm revenue from price variation by deflating firm revenues with this price index.[38] We construct firm-specific Törnqvist price indices for each firm's composite revenue from its various products in the following way:

$$PI_{it} = \prod_{g=1}^{n} \frac{p_{igt}}{p_{igt-1}}^{1/2(share_{igt}+share_{igt-1})} PI_{it-1}\,. \tag{E.4}$$

[38]This approach has also been applied in various other studies, such as Smeets and Warzynski (2013).

PI_{it} is the price index, p_{igt} is the price of good g, and $share_{igt}$ is the share of this good in total product market sales of firm i in period t. The growth of the index value is the product of the individual products' price growths, weighted with the average sales share of that product over the current and the last year. The first year available in the data is the base year (i.e., $PI_{it=1995} = 100$). If firms enter after 1995, we follow Eslava et al. (2004) and use an industry average of the computed firm price indices as a starting value. Similarly, we impute missing product price growth information in other cases with an average of product price changes within the same industry.[39] After deflating firm revenue with this price index, we have a quasi-quantity measure of output, for which, with slightly abusing notation, we keep using q_{it}.[40]

E.2 Solving (2) by Accounting for Unobserved Input Price Variation

While the recent literature stresses the so-called "output-price bias" when estimating production functions, previous work has also highlighted that unobserved input prices introduce another identification problem. To control for input price variation across firms, we use a firm-level analog of De Loecker et al. (2016) and define a price-control function from firm-product-level output price information that we add to the production function (E.1):

$$q_{it} = \boldsymbol{\phi}_{it}'\boldsymbol{\beta} + B_{it}((pi_{it}, ms_{it}, G_{it}, D_{it}) \times \boldsymbol{\phi}_{it}^{c}) + \omega_{it} + \epsilon_{it}\,. \tag{E.5}$$

$B_{it}(.) = B_{it}((pi_{it}, ms_{it}, G_{it}, D_{it}) \times \boldsymbol{\phi}_{it}^{c})$ is the price control function consisting of our logged firm-specific output price index (pi_{it}), a logged sales-weighted average of firms' product market sales shares (ms_{it}), a headquarter location dummy (G_{it}) and a four-digit industry dummy (D_{it}). $\boldsymbol{\phi}_{it}^{c} = [1; \boldsymbol{\phi}_{it}]$, where $\boldsymbol{\phi}_{it}$ includes the production function input terms as specified in (E.2). These are either in monetary terms and deflated by an industry-level deflator (capital and intermediates) or already reported in quantities (labor). The constant entering $\boldsymbol{\phi}_{it}^{c}$ highlights that elements of $B(.)$ enter the price control function linearly and interacted with $\boldsymbol{\phi}_{it}$ (a consequence of the translog production function). The idea behind the price-control function $B(.)$ is that output prices, product market shares, firm location, and firms' industry affiliation are informative about firms' input prices. Particularly, we assume that product prices and market shares contain information about product quality and that producing high-quality products requires expensive high-quality inputs. As De Loecker et al. (2016) discuss, this motivates the addition of a control function containing output price and market share information to the right-hand side of the production function to control for unobserved input price variation emerging from input quality differences across firms. We also include location and four-digit industry dummies into $B(.)$ to further absorb the remaining differences in local and four-digit industry-specific input prices.

Conditional on elements in $B(.)$, we assume that there are no remaining input price differences across firms. Although restrictive, this assumption is more general than the ones employed in most other studies estimating production functions without having access to firm-specific price data and which implicitly assume that firms face identical input and output prices within industries.

A notable difference between the original approach of De Loecker et al. (2016) and our version is that they estimate product-level production functions, whereas we transfer their framework to the firm level. For that, we use firm-product-specific sales shares in firms' total product market sales

[39] For roughly 30% of all product observations in the data, firms do not have to report quantities as the statistical office views them as not being meaningful.

[40] Note that, as discussed in Bond et al. (2021), using an output price index does not fully purge firm-specific price variation. There remains a base year difference in prices. Yet, using a firm-specific price index follows the usual practice of using price indices to deflate nominal values, we are thus following the best practice. Moreover, it is the only available approach when pooling multi- and single-product firms. Estimating the production function separately by single-plant firms requires other strong assumptions like perfect input divisibility of all inputs across all products. Finally, our results are also robust to using cost-share approaches to estimate the production function, which requires other assumptions (constant returns to scale, competitive input markets, and the absence of adjustment costs).

to aggregate firm-product-level information to the firm-level. This implicitly assumes that i) such firm aggregates of product quality increase in firm aggregates of product prices and input quality, ii) firm-level input costs for inputs entering as deflated expenditures increase in firm-level input quality, and iii) product price elasticities are equal across the various products of a firm. These or even stricter assumptions are always implicitly invoked when estimating firm-level production functions.

Finally, note that even if some of the above assumptions do not hold, including the price control function is still preferable to omitting it. This is because the price control function can nevertheless absorb some of the unobserved price variations and does not require that input prices vary between firms with respect to all elements of $B_{it}(.)$. The estimation can regularly result in coefficients implying that there is no price variation at all. The attractiveness of a price control function lies in its agnostic view about the existence and degree of input price variation.

E.3 Solving (3) by Controlling for Unobserved Productivity

To address the dependence of firms' intermediate input decision on unobserved productivity, we follow Olley and Pakes (1996) and Levinsohn and Petrin (2003) and employ a control function approach. We base our control function on firms' consumption of energy and materials, which we denote by e_{it} and which are components of total intermediate inputs. Inverting the demand function for e_{it} defines an expression for productivity:

$$\omega_{it} \equiv g_{it}(.) = g_{it}(e_{it}, k_{it}, l_{it}, \mathbf{\Gamma}_{it}). \tag{E.6}$$

$\mathbf{\Gamma}_{it}$ captures state variables of the firm, that in addition to k_{it} and l_{it} affect firms demand for e_{it}. Ideally, $\mathbf{\Gamma}_{it}$ should include a wide set of variables affecting productivity and demand for e_{it}. We include dummy variables for export (EX_{it}) activities, the log of the number of products a firm produces ($NumP_{it}$), and the average wage a firm pays (W_{it}) into $\mathbf{\Gamma}_{it}$. The latter absorbs unobserved quality and price differences that shift input demand for e_{it}. Remember that productivity follows a first-order Markov process. Firms can shift this Markov process as described in Doraszelski and Jaumandreu (2013) and De Loecker (2013), giving rise to the following law of motion for productivity: $\omega_{it} = h_{it}(\omega_{it-1}, \mathbf{T}_{it-1}) + \xi_{it} = h_{it}(.) + \xi_{it}$, where ξ_{it} denotes the innovation in productivity and $\mathbf{T}_{it} = (EX_{it}, NumP_{it})$ reflects that we allow for learning effects from export market participation and (dis)economies of scope through adding and dropping products to influence firm productivity.[41] Plugging (E.6) and the law of motion for productivity into (E.5) gives

$$q_{it} = \boldsymbol{\phi}_{it}'\boldsymbol{\beta} + B_{it}(.) + h_{it}(.) + \epsilon_{it} + \xi_{it}\,, \tag{E.7}$$

which constitutes the basis of our estimation.

E.4 Identifying Moments

We estimate Equation (E.7) separately by two-digit NACE Rev. 1.1 industries using a one-step estimator as in Wooldridge (2009).[42] Our estimator uses lagged values of flexible inputs (i.e., intermediates) as instruments for their contemporary values to address the dependence of firms' flexible input decisions on realizations of ξ_{it}. Similarly, we use lagged values of terms including firms' market share and output

[41] Doraszelski and Jaumandreu (2013) also highlight the role of R&D investment in shifting firms' productivity process. We would also like to add this information to the productivity model but do not observe R&D expenditures for the early years in our data.

[42] We approximate $h_{it}(.)$ by a third-order polynomial in all of its elements, except for the variables in $\mathbf{\Gamma}_{it}$. Those we add linearly. $B_{it}(.)$ is approximated by a flexible polynomial where we interact the output price index with elements in $\boldsymbol{\phi}_{it}$ and add the vector of market shares, the output price index, and the location and industry dummies linearly. Interacting further elements of $B_{it}(.)$ with $\boldsymbol{\phi}_{it}$ creates too many parameters to be estimated. This implementation is similar to De Loecker et al. (2016).

price index as instruments for their contemporary values as we consider these to be flexible variables.[43] We define identifying moments jointly on ϵ_{it} and ξ_{it}:

$$E[(\epsilon_{it} + \xi_{it})\mathbf{Y}_{it}] = 0. \tag{E.8}$$

$\mathbf{Y}_{it}$ includes lagged interactions of intermediate inputs with labor and capital, contemporary interactions of labor and capital, contemporary location and industry dummies, the lagged output price index, lagged market shares, lagged elements of $h_{it}(.)$, and lagged interactions of the output price index with production inputs. Formally this implies

$$\mathbf{Y}'_{it} = (J_{it}(.), A_{it-1}(.), T_{it-1}(.), \Psi_{it}(.), \boldsymbol{\nu}_{it-1}) \ , \tag{E.9}$$

where we defined:

- $J_{it}(.) = (l_{it},\, k_{it},\, l_{it}^2,\, k_{it}^2,\, l_{it}k_{it},\, G_{it},\, D_{it})$,
- $A_{it}(.) = (m_{it},\, m_{it}^2,\, l_{it}m_{it},\, k_{it}m_{it},\, l_{it}k_{it}m_{it},\, ms_{it},\, \pi_{it})$,
- $T_{it}(.) = \big((l_{it},\, k_{it},\, l_{it}^2,\, k_{it}^2,\, l_{it}k_{it},\, m_{it},\, m_{it}^2,\, l_{it}m_{it},\, k_{it}m_{it},\, l_{it}k_{it}m_{it}) \times \pi_{it}\big)$,
- $\Psi_{it}(.) = \sum_{n=0}^{3} \sum_{w=0}^{3-b} \sum_{h=0}^{3-n-b} l_{it-1}^n\, k_{it-1}^b\, e_{it-1}^h$, and
- $\boldsymbol{\nu}_{it-1} = (Exp_{it-1},\, NumP_{it-1},\, W_{it-1})$,
- with W_{it} denoting the average wage a firm pays.

[43] This also addresses any simultaneity concerns with respect to the price variables entering the right-hand side of our estimation.

F Cost Shares, Technology, and Adjustment Costs

We now discuss how input adjustment costs shape firms' input mix and cost shares (but would not directly affect our estimates of output elasticities).

F.1 Cost Shares with Flexible Inputs

As argued in the main text, cost shares measure firms' output elasticities and thus technology under constant returns to scale, perfect competition in input markets, and perfectly flexible input adjustments. To see this, consider a general production function (notation follows the main text):

$$Q_{it} = F(L_{it}, K_{it}, M_{it}, \Omega_{it}). \tag{F.1}$$

Firms' minimize costs. The Lagrangian writes:

$$\mathcal{L} = w_{it}L_{it} + z_{it}M_{it} + r_{it}K_{it} - \lambda_{it}(Q_{it} - Q_{it}(.)). \tag{F.2}$$

The first order conditions for labor, intermediates, and capital are:

$$w_{it} = \lambda_{it}\frac{\partial Q_{it}}{\partial M_{it}}, \tag{F.3}$$

$$z_{it} = \lambda_{it}\frac{\partial Q_{it}}{\partial L_{it}}, \tag{F.4}$$

$$r_{it} = \lambda_{it}\frac{\partial Q_{it}}{\partial K_{it}}, \tag{F.5}$$

where λ_{it} is the Lagrangian parameter (the shadow value of producing one more unit of output). It thus holds that: $\lambda_{it} = MC_{it} = \frac{P_{it}}{\mu_{it}}$, with MC_{it}, P_{it}, and μ_{it} denoting marginal costs, the output price, and the markup. Rearranging the first order conditions yields three expressions for input expenditures:

$$w_{it}L_{it} = P_{it}Q_{it}\frac{\theta^L_{it}}{\mu_{it}}, \tag{F.6}$$

$$z_{it}M_{it} = P_{it}Q_{it}\frac{\theta^M_{it}}{\mu_{it}}, \tag{F.7}$$

$$r_{it}K_{it} = P_{it}Q_{it}\frac{\theta^K_{it}}{\mu_{it}}, \tag{F.8}$$

where we used the definition of the output elasticity of input $X = \{L, K, M\}$ as $\theta^X_{it} = \frac{\partial Q_{it}}{\partial X_{it}}\frac{X_{it}}{Q_{it}}$.

Combining these cost expressions with each other yields for the labor input:

$$\frac{w_{it}L_{it}}{w_{it}L_{it} + z_{it}M_{it} + r_{it}K_{it}} = \frac{\theta^L_{it}}{\theta^L_{it} + \theta^M_{it} + \theta^K_{it}}. \tag{F.9}$$

The right-hand side of Equation (F.9) is the cost share for the labor input. Under constant returns to scale ($\theta^L_{it} + \theta^M_{it} + \theta^K_{it} = 1$), this cost share equals the labor output elasticity. The same holds for other inputs. Finally, note that in this entire derivation, we did not made any assumptions on firms' production functions. This derivation holds under any general production function as long as $\theta^X_{it} = \frac{\partial Q_{it}}{\partial X_{it}}\frac{X_{it}}{Q_{it}}$ is true. This means, that cost shares also equal output elasticities in presence of non-Hicks-neutral productivity processes as long as we impose constant returns to scale, competitive input markets, and flexible input adjustments.

F.2 Cost Shares with Input Adjustment Costs

Consider now that inputs face adjustment costs. We use the same setup as in Appendix F.1 but allow for labor adjustments costs. We ignore adjustments costs in other inputs for simplicity, but the following arguments can be extended to other inputs. As will become clear, adjustment costs in only one input will bias the output elasticities for all inputs if measured via cost shares.

If firms face labor adjustment costs, the optimization problems becomes dynamic. In our case with only labor adjustment costs, the first order conditions for materials and capital are unchanged. However, the condition for labor includes a new term that reflects the wedge between the input price and the shadow price of the input under adjustment costs:

$$w_{it}L_{it}\left(1+\frac{\partial\chi_{it}(L_{it},L_{it-1})}{\partial L_{it}}\right)=P_{it}Q_{it}\frac{\theta_{it}^{L}}{\mu_{it}}, \tag{F.10}$$

where $w_{it}L_{it}\frac{\partial\chi_{it}(L_{it},L_{it-1})}{\partial L_{it}}$ captures firms' adjustment costs as a share of firms' wage bill, which we write as a function of past and present labor.[44] Following the derivations of Appendix F.1, the labor cost share is now given by:

$$\frac{w_{it}L_{it}}{w_{it}L_{it}+z_{it}M_{it}+r_{it}K_{it}}=\frac{\frac{\theta_{it}^{L}}{1+\frac{\partial\chi_{it}(L_{it},L_{it-1})}{\partial L_{it}}}}{\frac{\theta_{it}^{L}}{1+\frac{\partial\chi_{it}(L_{it},L_{it-1})}{\partial L_{it}}}+\theta_{it}^{M}+\theta_{it}^{K}}, \tag{F.11}$$

which even under constant returns to scale is not equal to the output elasticity of labor. Note that, as discussed in the main text, the estimation of the input output elasticities using our parametric production function estimation routine is not *biased* by labor adjustment costs. However, as described in Appendix G, adjustment costs might directly *affect* output elasticities in certain production models through their effect on the relative factor input mix.

[44] The functional form of χ_{it} can differ, depending on the underlying adjustment cost model. Specifying the precise functional form of χ_{it} is not essential for our argument.

G CES Production Function and Changing Input Prices

Consider the following nested CES production function:

$$Q_{it} = Q(.) = A_{it}\left(\Lambda_{it}^{LM}\left(\alpha_i^L L_{it}^{\frac{\sigma-1}{\sigma}} + \alpha_i^M M_{it}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}\frac{\kappa-1}{\kappa}} + \Lambda_{it}^K K_{it}^{\frac{\kappa}{\kappa-1}}\right)^{\frac{\kappa-1}{\kappa}}, \tag{G.1}$$

where L_{it}, M_{it}, and K_{it} are labor, intermediates, and capital used in the production of output, Q. A is total factor productivity. Λ_{it}^{LM}, Λ_{it}^K, α_i^L, and α_i^M are distribution parameters. σ is the substitution elaticity between labor and intemediates. κ denotes the substitution elasticity between capital and the second input aggregate consisting of labor and intermediate inputs. The specification in Equation (G.1) allows us to explicitly study substitution patters between labor and intermediates, which is the focus of our study.

Firms minimize cost. We consider that input markets are imperfect, such that firms face adjustment costs or have monopsony power. The first order conditions of firms' optimization problem can then be written as:

$$w_{it}\gamma_{it}^L = \lambda_{it}\frac{\partial Q_{it}}{\partial L_{it}}, \tag{G.2}$$

$$z_{it}\gamma_{it}^M = \lambda_{it}\frac{\partial Q_{it}}{\partial M_{it}}, \tag{G.3}$$

where γ_{it}^L and γ_{it}^M capture input market distortions. For instance, if firms have monopsony power in labor and intermediate markets, we would have $\gamma_{it}^L = \left(1 + \frac{1}{\epsilon_{it}^L}\right)$ and $\gamma_{it}^M = \left(1 + \frac{1}{\epsilon_{it}^M}\right)$, where ϵ_{it}^L and ϵ_{it}^M denote the labor and intermediate input supply elasticity. Alternatively, γ_{it}^L and γ_{it}^M can have a different functional form and capture input adjustment costs (see Appendix F.2).

$\lambda_{it} = MC_{it} = \frac{P_{it}}{\mu_{it}}$, with MC_{it}, P_{it}, and μ_{it} being the marginal cost, output price, and markup. Multiplying the FOC of labor with $\frac{L_{it}}{Q_{it}}$ and the FOC for intermediates with $\frac{L_{it}}{Q_{it}}$ yields two equations for the optimal input decision of a firm:

$$L_{it} = \frac{P_{it}}{\mu_{it}\gamma_{it}^L w_{it}}\theta_{it}^L Q_{it}, \tag{G.4}$$

$$M_{it} = \frac{P_{it}}{\mu_{it}\gamma_{it}^M z_{it}}\theta_{it}^M Q_{it}, \tag{G.5}$$

where $\theta_{it}^L = \frac{\partial Q_{it}}{\partial L_{it}}\frac{L_{it}}{Q_{it}}$ and $\theta_{it}^M = \frac{\partial Q_{it}}{\partial M_{it}}\frac{M_{it}}{Q_{it}}$ are the output elasticities of labor and intermediates, respectively.

Using the production function, we find that:

$$\frac{\frac{\partial Q_{it}}{\partial L_{it}}}{\frac{\partial Q_{it}}{\partial M_{it}}} = \left(\frac{L_{it}}{M_{it}}\right)^{\frac{-1}{\sigma}}, \tag{G.6}$$

which, after multiplying with $\frac{L_{it}}{Q_{it}}/\frac{M_{it}}{Q_{it}}$, yields:

$$\frac{\theta_{it}^L}{\theta_{it}^M} = \left(\frac{L_{it}}{M_{it}}\right)^{\frac{\sigma-1}{\sigma}}. \tag{G.7}$$

Using Equations G.4 and G.5, we can write this expression also in terms of input prices and input

market distortions:

$$\frac{\theta_{it}^{L}}{\theta_{it}^{M}} = \left(\frac{w_{it}\gamma_{it}}{z_{it}}\right)^{1-\sigma}, \tag{G.8}$$

where we defined $\gamma_{it} = \frac{\gamma_{it}^{L}}{\gamma_{it}^{M}}$.[45] Equations G.7 and G.8 show that, under a Cobb-Douglas technology ($\sigma = 1$), relative input quantities and factor prices do not affect output elasticities. If intermediates and labor are substitutes ($\sigma > 1$), an increase in relative intermediate inputs (due to changes in relative factor prices) will reduce the labor output elasticity relative to the intermediate input output elasticity. Consequently, a factor cost ratio, $\frac{w_{it}}{z_{it}}$, increasing with firm growth, e.g., because the slope of the labor supply curve is steeper than the slope of the intermediate input supply curve, can rationalize the decline in the labor output elasticity and the increase in the intermediate input output elasticity in response to firm growth that we document. This is, for instance, consistent with i) firms having more monopsony power in labor than in intermediate input markets, or ii) firms facing higher adjustment costs with respect to labor.

[45] This is consistent with our empirical measure of γ_{it} in the main text that measures labor market power relative to intermediate input market power. The literature typically assumes exogenous intermediate input prices and perfectly flexible intermediate input markets (e.g., Yeh et al., 2022), in order to interpret γ_{it} as labor market power or a measure of labor market distortions. However, it actually measures relative input market distortions on labor compared to intermediate input markets.

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