Retail Investment under Aggregate Fluctuations

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Abstract

Although local market conditions are normally considered to be the key drivers of forward-looking and strategic retail chain expansion (or contraction), broader aggregate states and their movements (i.e., aggregate fluctuations) might also be of relevance; however, the extent to which these aggregate states actually impact retail investment decisions remains unknown. This paper develops a new framework for modeling, identifying, and estimating dynamic investment games that accommodates for aggregate states. In particular, firms in our model make strategic investment decisions based on beliefs they have about the evolving aggregate state. With this novel model framework, we present a set of identification results, as well as a likelihood-based estimation methodology. We then conduct an empirical case study about fast food industry dynamics. Analysis of estimated models reveals that while aggregate states do indeed impact retail investment, these effects are heterogeneous, as some of the retail chains are noticeably more/less sensitive to the aggregate state. Furthermore, the estimated model demonstrates a connection between heightened volatility in the aggregate fluctuations and dampened retail investment. Finally, the relationship between volatility of the aggregate state and investment has the potential to further reinforce the market presence of already dominant firms.

Keywords: Aggregate Fluctuations; Firm Investment; Industry Dynamics; Learning Models; Market Presence; Retail Competition and Strategy; Partially Observable Stochastic Games

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1 Introduction

Retail investment decisions are driven by a number of market factors. Broadly speaking, these factors can be categorized based by their degree of regional specificity. At one end of the spectrum, highly *localized* factors may include proxies for market size and competitive conditions, while at the other end of the spectrum, are *aggregate* states (and their movements, i.e., aggregate fluctuations) that potentially impact an entire industry as a whole. One may interpret the aggregate state of the economy as a firm payoff-relevant index that parsimoniously captures observable macroeconomic factors. For example, aggregate state can be a strong state (akin to economic prosperity), or a weak state (akin to economic downturn): from the demand/revenue perspective, we know that household price elasticities might be sensitive to aggregate states (Beck and Lein 2020, Gordon et al. 2013, Kryvtsov and Vincent 2021, Van Heerde et al. 2013), while from the operating cost perspective, firm inventory decisions (Bils and Kahn 2000) and access to credit (Gaiotti 2013) may also be impacted by these aggregate states. Ultimately, market structure (Etro and Colciago 2010, Hall et al. 1986) and firm profit margins (Machin and Van Reenen 1993) might fluctuate with aggregate state. Consequently, retailers, and more generally, firms may condition their corporate-level investment strategies around their subjective (and possibly heterogeneous) beliefs about the aggregate states (Andre et al. 2022).[1]

Although the existing state-of-the-art for analyzing retail investment decisions is well-equipped for incorporating information about localized market factors, we have less clarity about how broader aggregate states shape a firm's overall investment strategies across *all* local markets. To address this shortcoming, we develop a novel structural modeling framework for analyzing retail chain investment decisions under the presence of evolving aggregate state.

The introduction of aggregate states creates distinct challenges for structural modeling and estimation. In particular, while macroeconomic data is often widely available, important to the investment decisions is *the way* that firms translate this macroeconomic information to the aggregate state, which in turn impacts firms' investment payoffs. For instance, a firm might anticipate a higher payoff for more active investment if it deems a strong economic condition by evaluation of relevant macroeconomic data. We denote the mapping from macroeconomic data to aggregate state by the *model uncertainty* encountered by firms.[2] In

[1] For example, marketing budgets and investments set by corporate executives might adapt to macroeconomic conditions (Peers et al. 2017, Srinivasan et al. 2005, Steenkamp and Fang 2011).

[2] This type of uncertainty is commonly used in studying macroeconomic policy (Levin et al. 2003), and is conceptually different from the notion of *measurement error* in the observed macroeconomic data that firms use in their

fact, getting an accurate assessment of how aggregate states impact firm outcomes based on observable information is a non-trivial task for firms, financial investors, and monetary policy-makers (Bai and Ng 2006, Bomfim 2001, Orphanides 2003, Angeletos and La'o 2010). Thus, firm's data-driven Bayesian belief of aggregate state may play an essential role in investment decisions.[3] Moreover, research has shown that disagreements among firms about aggregate state can exist, even when conditioned on the same economic information, because the firms' beliefs of aggregate state might be based on their own subjective models (Andre et al. 2022, Angeletos and La'o 2010, Coibion et al. 2018, Molavi 2019, Weber et al. 2023). To this end, we allow firms to have their subjective (i.e., heterogeneous) beliefs. Additionally, firms' beliefs about the overall economy evolve over time as information is continually updated. In summary, consideration of broader aggregate conditions introduces an unobserved state that impacts firm decisions across all local markets. Therefore, we develop a novel estimable model framework for analyzing dynamic games with time-varying aggregate factors that are *unobservable to both firms and researcher*. Our framework leverages the innovations in artificial intelligence and control theory about partially observable stochastic games (POSGs), which offers a new paradigm for studying dynamic discrete choice models. By setting up the retail investment decision as a POSG, we can be more flexible in how we interpret each firm's information structure, by partitioning them into public observations (e.g.,, local market factors, GDP/inflation) and evolving subjective beliefs of time-varying and unobserved aggregate state. Our model is an extension of dynamic Bayesian games if one considers firms' initial beliefs (on aggregate state) as their "types". We provide formal theoretical results about non-parametric identification for both aggregate state dynamics and payoff functions. In particular, rank conditions are established for which the aggregate state dynamics associated with macroeconomic indicators can indeed be identified.

Our new framework can support policy-relevant insights. In particular, macro-finance policy might have an interest in investigating the resilience of certain retail sectors to weak aggregate state (associated with recession). As recessionary periods often involve discussions about government-induced stimulus packages and targeted transfers (Oh and Reis 2012, Taylor 2014), knowing which sectors are likely to face diminished firm-level investment can help government prioritize firms/industries to help. Also relevant for macroeconomic policy is the role of uncertainty on firm investment. Our framework can be used to establish connections between volatility in aggregate states and retail investment activity. As retailers

decision making. For example, Chi (2023) documents the impact of out-dated (i.e., noisy) census data about local market conditions on retail investment outcomes (i.e., establishment failure and exit).

[3]Note here that the notion of beliefs is different from the existing literature about belief evolution under strategic uncertainty (Aguirregabiria and Magesan 2020), as firms in our theoretical environment do not form beliefs about whether or not their competitors are using strategies that are in equilibrium.

employ a significant number of workers, their investment decisions have direct implications on employment dynamics. As uncertainty itself sometimes stems from policy itself (Baker et al. 2016, Gulen and Ion 2016), understanding the potential "chilling" effect of uncertainty on retail investment can help the government calibrate potential (long-run) costs associated with unstable policies. Methodologically, our framework provides an important point of departure from how investment decisions are normally studied in macro-finance (Bond and Van Reenen 2007). In particular, as our focus is on oligopoly competition, forward-looking investment decisions in our setting are made strategically (i.e., one firm's decision has a direct impact on a competitor's decision and outcomes).

By developing a framework that accommodates for strategic interaction between competing firms, our research also yields antitrust implications. In particular, our framework can be used to determine whether weak aggregate state has uniform or asymmetric effects across all relevant firms within a retail sector. If, for example, some firms are more resilient, periods of recession might allow the most resilient firms to increase investments and grow aggressively, thereby leading to more concentrated markets.[4] Under the backdrop of rising mark-ups (Gutiérrez and Philippon 2017, De Loecker et al. 2020), the extent to which a weak aggregate state is conducive to market consolidation might be particularly relevant for policy discussions. Taken together, our framework aims to offer researchers a way to explore investment heterogeneity between firms (and between industries on a case-by-case basis), which remains an important shortcoming in the macro-market power literature (Syverson 2019).

To demonstrate proof-of-concept for our new framework using real-world data, we conduct a comprehensive empirical case study about the Canadian fast food industry from 1970 to 2004. This analysis is based on the data originally collected and used in Blevins et al. (2018), Fang and Yang (2023a), Igami and Yang (2016), Yang (2020), supplemented with additional information about fundamental macroeconomic indicators, including GDP growth rate and inflation rate during the same time period. This empirical setting fits particularly well with our theoretical framework, as the local market investment decisions are often influenced by the country's head office (e.g., McDonald's Canadian headquarters in Toronto), which leads to the consideration for aggregate condition.[5] With this data, we demonstrate

[4] In macroeconomic research, this phenomena has also been referred to as the scarring effect, in that less productive firms are allocated fewer resources. However, an important cost of this scarring effect is that firms with potential (in the longer-run) to be high-performing might not survive past infancy (Ouyang 2009). More generally, the impact of aggregate fluctuations on employment, productivity and sales may be asymmetric between firms (Salgado et al. 2019).

[5] In general, a chain's national headquarters makes decisions about overall expansion decisions. See, for example, "McDonald's Plans 'Fastest Period Of Growth In Brand's History' With 10,000 New Stores By 2027" (*Forbes*, December 7, 2023).

that our proposed framework (considering both observable local market information as well as beliefs about the unobserved aggregate state) outperforms existing methods (that incorrectly treat the unobserved aggregate state as observable) with respect to predictive power (Appendix D). Our estimated model also has the power to infer the underlying dynamics of aggregate state, as well as the extent to which these states are correlated with observed macroeconomic proxies like GDP and inflation. This linkage (i.e., model uncertainty) between unobserved aggregate state and observed factors has yet to be captured in existing frameworks for studying dynamic games. In addition to serving as a proof-of-concept for our methodological framework, this case study aims to uncover substantive insights about firm resilience during macroeconomic downturns, as fast food has been widely posited to be "recession-proof." We show that the estimates of base profit are positive (and close to each other) across various aggregate conditions for the fast-food chains in Canada. A few key insights emerge from this analysis. First, estimated firm profitability in general appears to be resilient to macroeconomic downturns, especially so for McDonald's. Second, outlet growth does however, slow down slightly (but still net positive) in the midst of these downturns. Furthermore, we show that volatility in the aggregate fluctuations deters investment, which suggests that one may view retail expansion under the real options framework.[6] Finally, we illustrate the effect that volatility potentially has on market presence. In a hypothetical scenario in which McDonald's faces a competitor that is more sensitive to aggregate fluctuations, volatility further solidifies McDonald's market leadership in terms of its number and share of establishments. Moreover, with estimates from real data, we observe similar qualitative patterns as in the hypothetical scenario. Together, these findings highlight the importance of acknowledging strategic competition when studying firm investment in oligopolistic markets.

Methodologically, our research explores a novel type of time-varying heterogeneity that is unobservable to *both* firms and researcher (e.g., aggregate state), which extends the scope of existing work on the structural estimation of dynamic discrete choice models with unobserved states or serial correlation[7] (Aguirregabiria and Mira 2007, Arcidiacono and Miller 2011, Berry and Compiani 2023, Blevins et al. 2018, Blevins 2016, Hu and Shum 2013, Hu et al. 2017, Hwang 2020, Kalouptsidi et al. 2021, Kasahara and Shimotsu 2009, Luo et al. 2022, Norets 2009), whereas these past innovations have made important contributions towards the factors that are *observed by firms but not researcher* (e.g., unobserved profitability of

[6] In a real options framework (Myers 1977), partial irreversibility of investments that involve high sunk costs leads firms to be more cautious during times of uncertainty in order to maintain the option to "wait and see" how the economy unfolds (Bloom et al. 2007). Consequently, pauses in investment and hiring could lead to decreases in firm productivity (Bloom 2009).

[7] For a summary, we refer the reader to Abbring (2010).

certain geographic markets). Central to incorporating such "unobserved-to-both" state is a new model paradigm (i.e., partially observable stochastic games), which requires a different approach to the identification and estimation methods. This type of heterogeneity might be particularly relevant when incorporating aggregate factors into a retailer's investment decisions, as these factors, captured by model uncertainty and likely permeating throughout the decision process via broader corporate-level sentiment and its subsequent directives, are difficult to identify by firms *ex ante*.

We also contribute to the literature about firm learning, as our framework involves firm continuously updating subjective beliefs about aggregate states. Learning models are commonly studied using structural estimation, and there are new efforts to establish identification results for these class of models (Bunting et al. 2024).[8] In particular, they have been useful for studying individual (Crawford and Shum 2005, Erdem and Keane 1996, Fang 2022, Zhang 2010) and firm (Aguirregabiria and Jeon 2020, Ching 2010, Hitsch 2006, Huang 2019, Huang et al. 2019, 2022, Jovanovic 1982, Kellogg 2011, Shen 2014, Yang 2020) decisions.[9] We depart from this literature around Bayesian learning in two important avenues. First, the existing literature on firm learning is primarily focused on the unobserved variable that is fixed, while, in contrast, the unobserved state (i.e., aggregate state) faced by firms in our framework evolves dynamically. Second, our framework allows for learning signals (e.g., macroeconomic data) that impact the beliefs for *all* cross-sectional observations (i.e., investment decisions for each geographical market). This feature makes our framework particularly useful for studying investment decisions in the retail sector that can be impacted by broader aggregate conditions.

To accommodate for uncertainty faced by firms, our framework is closely related to the literature about POSGs.[10] POSGs are particularly useful in modeling scenarios in which the information received by agent is corrupted by noise (Isler and Karnad 2008, Isler et al. 2005, Chung et al. 2011). Under the context of retail investment, model uncertainty implies that the aggregate conditions are only partially observable to firms through proxies of macroeconomic factors. The control and optimization community has largely focused on computing optimal equilibrium policies for all agents given the POSG model's primitives (e.g., unobserved state transition probabilities, utility functions), which is computationally

[8] We refer the reader to Ching et al. (2013) and Ching et al. (2017) for an extensive surveys about empirical applications using such models.

[9] This work is also related to research about investment under uncertainty, largely built around the seminal work of Dixit and Pindyck (1994). More recent efforts have focused on empirical documentation of firm investment dynamics under uncertainty (Bloom 2009, Bloom et al. 2007, Chi 2023, Collard-Wexler 2013, Handley and Li 2020, Kumar et al. 2023).

[10] Our methodology can be viewed as a generalization of Chang et al. (2023), which considers estimating dynamic single agent models (e.g., optimal machine replacement) with unobserved states.

challenging (Bernstein et al. 2005). In light of this high complexity, solving POSGs with special structures is the most practical. For example, POSGs with cooperative agents where all agents share the same utility function (Hansen et al. 2004, Dibangoye et al. 2014, Aras and Dutech 2010), and zero-sum games (Chatterjee et al. 2013) are both active research areas. Our research contributes to this area of work as our focus is on the "inverse" estimation of the model primitives given observed behaviors for a specially structured game, as opposed to computation/optimization alone.

2 The Model

In this section, we present a dynamic firm investment model that incorporates forward-looking and strategic behavior. To accommodate for aggregate states, we introduce subjective beliefs held by firms that relate to these broader economic conditions. We begin this section by providing a high-level overview of the key components in our model framework.

2.1 Overview

Our model will capture firm investment decisions that are conditional to aggregate and local market factors. The underlying aggregate state is not fully known (i.e., unobserved) to the firm (and researcher), which we denote by s_t^h. The firm, however, can form a belief about this unobserved state, and a belief of other firms' beliefs using observable macroeconomic data, such as GDP and inflation (i.e., z_t). In particular, this observable information about the aggregate states is incorporated into the firm's ever-evolving private belief state $h_t^i \triangleq \{x_t^i, \mathbb{P}(x_t^{-i}|h_t, x_0^i, \mathbb{P}(x_0^{-i}))\}$ (unknown to researcher, but known to firms), where x_t^i is firm i's subjective belief, and abusing notation h_t is the publicly observable aggregate related information (e.g., past GDP readings) up to time t, and $\mathbb{P}(x_t^{-i}|h_t, x_0^i, \mathbb{P}(x_0^{-i}))$ is firm i's belief about other firms' beliefs x_t^{-i}, and x_0^i, x_0^{-i} are the initial beliefs of firm i, and of the firms except i, respectively.

Firms private belief state $h_t^i, i = 1, 2, ...$ impacts their investment decision a_t^i (e.g., open/close a store). Local market factors s^o are a culmination of the realized market structure (i.e., how many stores each retail firm has active) and market size proxies (i.e., population, income, property value). It can be easily seen that past investment decisions have implications on the evolution of local market factors via the realized market structure. The schematic representation of our model is summarized in Figure 1.

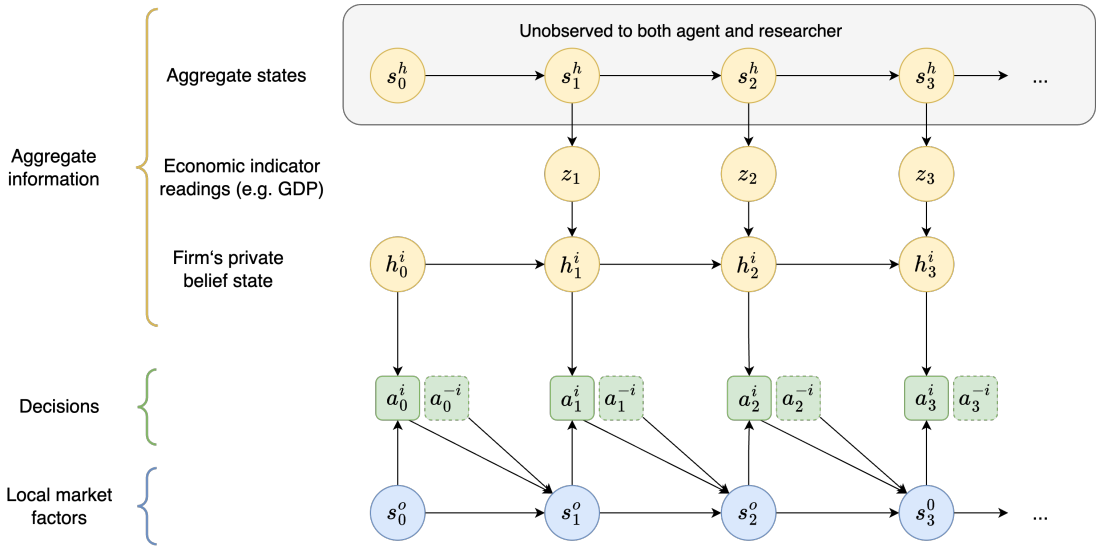


Figure 1: Schematic representation of the relevant local market factors, aggregate information and firm investment decisions.

2.2 Environment

There are M independent geographical markets, indexed by $m \in \{1, 2, ..., M\}$; therefore, without loss of generality, the following description focuses on a given market. At time $t = 0, 1, 2, ...$, each firm $i \in \{1, \ldots, I\}$ operates $n_t^i \in \{0, 1, \ldots, N\}$ active stores in the given market where N is a uniform upper bound on the number of stores in any market. Let $n_t^{-i} = \sum_{j \neq i} n_t^j$ and define the market structure by $n_t = \{n_t^i\}_{i=1}^I$. All firms observe a profile of local market size factors $s_t^{ds} \in S^{ds} = \mathbb{R}^{|S^{ds}|}$ (e.g., population, income, property value), so we define the observable local market factors by $s_t^o = (n_t, s_t^{ds}) \in S^o$.

Each firm independently makes its own investment decision by selecting a discrete action $a_t^i \in A = \{-1, 0, +1\}$, which respectively correspond to exit (or downsizing by closing one store), status quo, and entry (or expansion by adding one store).[11] This investment decision can be interpreted as an extensive margin increase (or decrease) in production units. Thus, the number of stores follow a simple first order dynamics $n_{t+1}^i = n_t^i + a_t^i$.[12] The local market size factors s^{ds} are assumed to exogenously evolve according to a first-order Markov process with transition probability matrix $\mathbf{F}$, whose (i, j) element is $\mathbf{F}_{i,j} = \mathbb{P}(s_{t+1}^{ds} = j | s_t^{ds} = i)$.

In addition to local market factors, we also allow for a broader aggregate state that impacts all local markets. This underlying and time-varying state, due to model uncertainty,

[11] We note that it is possible that, at a time t, firm i opens more than one store in a market (i.e., $a_t^i > 1$), or closes more than one store (i.e., $a_t^i < -1$). However, these situations are extremely rare in the dataset (less than 0.01%). Hence, we denote by $a_t^i = 1$ for any actions $a_t^i \geq 1$; similarly, $a_t^i = -1$ for any actions $a_t^i \leq -1$.

[12] This additive structure is not required; we use the additive structure in favor of application context. Our model permits general function that maps n_t^i, a_t^i to n_{t+1}^i if necessary.

is not perfectly known (i.e., unobserved) to the firms, and they will ultimately form beliefs about this factor. The aggregate state is denoted by $s_t^h \in S^h = \{0, 1\}$, where $s_t^h = 0$ corresponds to a weak state and $s_t^h = 1$ corresponds to a strong state. The movement of aggregate states (e.g., from strong to weak) creates fluctuations, and the movement of the state is a Markov process with transition probabilities given by matrix $\mathbf{P} \in \mathbb{R}_+^{|S^h| \times |S^h|}$:

$$\mathbf{P} = \begin{array}{c} \\ \textit{weak } (0) \\ \textit{strong } (1) \end{array} \begin{array}{c} \begin{array}{cc} \textit{weak } (0) & \textit{strong } (1) \end{array} \\ \begin{pmatrix} \eta_0 & 1-\eta_0 \\ 1-\eta_1 & \eta_1 \end{pmatrix} \end{array}, \tag{1}$$

where the parameters $\eta_0, \eta_1 \in [0, 1]$ and $\eta = \{\eta_0, \eta_1\}$ describe the persistence of aggregate conditions.

Generally speaking, aggregate fluctuations are characterized by upswings and downturns of commonly used macroeconomic variables (e.g., GDP, inflation) $z \in Z$. We thus model the macroeconomic observations via $z_t = z_t^{gdp}$ or $z_t = z_t^{inf}$, where

$$z_t^{gdp} \in \{\textit{negative growth } (0), \textit{positive growth } (1)\}$$

represents GDP growth rate and

$$z_t^{inf} \in \{\textit{high inflation rate } (0), \textit{mild inflation rate } (1)\}$$

is the inflation rate. We note that theoretically one can combine both GDP and inflation observations as the observation z, we choose not to do so to avoid multicollinearity issue, as the correlation between GDP and inflation is relatively strong (-0.56).

Model uncertainty prevents firm from mapping observation to aggregate state with certainty.[13] Therefore, with these observed macroeconomic indicators, we can construct the following observation probability matrices $\mathbf{O} \in \mathbb{R}_+^{|S^h| \times |Z|}$ for GDP and inflation, respectively:

$$\mathbf{O}^{gdp} = \begin{array}{c} \\ \textit{weak (0)} \\ \textit{strong (1)} \end{array} \begin{array}{c} \begin{array}{cc} \textit{negative} & \textit{positive} \\ \textit{growth (0)} & \textit{growth (1)} \end{array} \\ \begin{pmatrix} 1-\omega^{gdp} & \omega^{gdp} \\ \omega^{gdp} & 1-\omega^{gdp} \end{pmatrix} \end{array},$$

[13] Model uncertainty aligns with existing work has shown that noises exist in the observation of inflation and GDP (Orphanides 2003, 2001).

$$\mathbf{O}^{inf} = \begin{array}{c} \\ weak\ (0) \\ strong\ (1) \end{array} \begin{array}{c} \begin{array}{cc} high\ inflation & mild\ inflation \end{array} \\ \begin{pmatrix} rate\ (0) & rate\ (1) \\ 1-\omega^{inf} & \omega^{inf} \\ \omega^{inf} & 1-\omega^{inf} \end{pmatrix} \end{array},$$

where $\omega^{gdp}, \omega^{inf} \in [0,1]$ are the probabilities of uncertainty shock in the economic data. We see here that the observation probability matrix $\mathbf{O}$ (with ω) has a better observation quality than $\mathbf{O}'$ (with ω') if $\omega \leq \omega'$.[14] To proceed with our discussion, we make the following rank assumption on the observation probability matrix $\mathbf{O}$:[15]

Assumption 1. The observation matrix $\mathbf{O}$ is of full row rank.

Assumption 1 ensures that there is sufficient variation in observations for different aggregate states in that the observation probability distribution of any state is not simply a linear combination of other states' counterpart distributions. This data variation prerequisite is related to the notion of cointegration in time series analysis (Hamilton 2020). Having full rank would be similar to the observed aggregate factors as not being cointegrated.

Let $s_t = (s_t^o, s_t^h)$ denote the payoff-relevant state, and $a_t = (a_t^1, ..., a_t^I)$ denote the collection of all firms' actions. Thus, given the conditional independence assumption, the payoff relevant states dynamics can be summarized by:

$$\mathbb{P}(z_{t+1}, s_{t+1} | z_t, s_t, a_t) = \mathbb{P}_{\mathbf{F}}(s_{t+1}^o | s_t^o, a_t) \mathbf{P}_{s_t^h, s_{t+1}^h} \mathbf{O}_{s_{t+1}^h, z_{t+1}}, \tag{2}$$

where $\mathbf{P}$ is a matrix of size $|S^h| \times |S^h|$, whose s_t^h-th row and s_{t+1}^h-th column is $\mathbf{P}_{s_t^h, s_{t+1}^h} \triangleq \mathbb{P}(s_{t+1}^h | s_t^h)$; and where $\mathbf{O}$ is a matrix of size $|S^h| \times |Z|$, whose s_t^h-th row and z_t-th column is $\mathbf{O}_{s_t^h, z_t} \triangleq \mathbb{P}(z_t | s_t^h)$.

Remark 1 (On the independent structure in Eq.(2))**.** We assume that a few firms' investment decisions do not broadly affect the aggregate economy. Hence, the aggregate state transition probability is independent of firms' decisions. Moreover, local market size factors are assumed to be independent of unobserved aggregate state, as we calculate the correlation between aggregate state and local market size factors is not strong.[16]

[14] Note that the symmetric structure of $\mathbf{O}^{inf}$ and $\mathbf{O}^{inf}$ only serves the purpose of illustration. Symmetry is not a requirement for our theoretical findings. Moreover, in the numerical analysis one can demonstrate that our identification results continue to hold even with non-symmetry of $\mathbf{O}$.

[15] This rank assumption is the only one we need to making about $\mathbf{O}$.

[16] We use the strength level defined in https://sphweb.bumc.bu.edu/otlt/MPH-Modules/PH717-QuantCore/PH717-Module9-Correlation-Regression/PH717-Module9-Correlation-Regression4.html

In order to proceed, we place more structure on the per-period payoff function.[17] Here, we generalize the per-period payoff function for firm i from Igami and Yang (2016) so as to explicitly model the retail investment decisions under unobserved aggregate state as follows:

$$\begin{aligned}\tilde{r}^i(s_t^h, s_t^o, a_t^i, \epsilon_t^i) &= r^i(s_t^h, s_t^o, a_t^i) + \epsilon_t^i(a_t^i) \\ &= n_t^i \cdot \left(\theta^i_{base,s_t^h} + \theta^i_{own} n_t^i + \theta^i_{rival} n_t^{-i} + \theta^i_{ds} s_t^{ds}\right) - \mathbb{1}_{\{a_t^i>0\}}\theta^i_{entry} + \epsilon_t^i(a_t^i), \end{aligned} \quad (3)$$

where the first equation is built on the assumption of additively separable payoff perturbation,[18] and $\theta_1^i = \{\theta^i_{base,s^h}, \theta^i_{own}, \theta^i_{rival}, \theta^i_{ds}, \theta^i_{entry}\}$ stand for the parameters of base profit depending on the aggregate state, competition with stores of the same brand (own competition), competition with stores of rival brands (rival competition), local market size factor and (net) entry cost, respectively,[19] and $\epsilon^i \in \mathcal{E}^i$ is private information to firm i conditional on firm i's action, along the lines of Seim (2006). The payoff function of firm i, r^i, does not directly depend on other firms' current actions; rather, r^i is related to the past actions of other firms through the market structure (n^i, n^{-i}). For notational simplicity, we denote $\theta_1 = \{r^i\}_{i=1}^I, \theta_2 = \{\mathbf{F}, \mathbf{O}, \mathbf{P}\}$ and $\theta = \{\theta_1, \theta_2\}$.

Assumption 2. The random variables $\{\epsilon_t^i(a_t^i) : a_t^i \in A\}_{t\geq 0}$ are independent with identical distribution $\mu(\cdot)$.

Assumption 2 implies that payoff shocks ϵ_t do not affect aggregate state dynamics. In addition, although this information about private information abstracts away from the permanent market-specific heterogeneity that is commonly known to all firms (but not to the researcher), we note that in practice, market-specific heterogeneity can still be partially controlled for when estimating the dynamic game (see Section 4).[20]

2.3 Subjective Firm Belief

Let $x_0^i \in X$ denote the prior distribution of firm i's belief about the aggregate state s^h where X is the probability simplex on $\mathbb{R}^{|S^h|}$. Let $h_t = (z_t, ..., z_1)$ be the publicly received aggregate state related information up to and including time t. Let x_t^i denote firm i's subjective belief as $x_t^i \triangleq \mathbb{P}(s^h | h_t, x_0^i)$.

[17] We note that although the payoff function is parametrized for illustration, the identification of payoff function (Section 3) is essentially non-parametric.

[18] Additive separability allows us to be agnostic as to the exact channel to which aggregate states impact the retail investment outcomes. That is, these states can manifest through per-store revenue and/or operational costs. If revenue/cost data were available, more structure could be introduced into this mechanism.

[19] Here, we do not include the sunk cost of exit in the payoff function as by convention we normalize the sunk cost to be zero for identification issue; see related discussion in Aguirregabiria and Suzuki (2014).

[20] Integrating market permanent heterogeneity into our framework is an interesting future research topic.

Let us also denote by $\sigma^i_\theta(z_{t+1}, x^i_t)$ the firm i's subjective probability of observing new economic indicator z_{t+1},[21] given current Bayesian belief x^i_t, i.e.:

$$\sigma^i_\theta(z_{t+1}, x^i_t) \triangleq \sum_{\tilde{s}^h \in S^h} \sum_{s^h \in S^h} x^i_t(s^h) \mathbf{P}_{s^h, \tilde{s}^h} \mathbf{O}_{\tilde{s}^h, z_{t+1}},$$

where $\mathbf{P}, \mathbf{O}$ are defined in Eq.(2).

The computation of belief update can thus be recursively written as follows:

$$x^i_{t+1,\theta}(s^h) \triangleq \mathbb{P}(s^h | h_t, x^i_0) = \frac{\sum_{s^h_t} x^i_t(s^h_t) \mathbf{P}_{s^h_t, s^h} \mathbf{O}_{s^h, z_{t+1}}}{\sigma^i_\theta(z_{t+1}, x^i_t)}, \tag{4}$$

for all $s^h \in S^h$ whenever $\sigma^i_\theta(z_{t+1}, x^i_t) > 0$. As the updated belief x^i_{t+1} depends only on z_{t+1}, x^i_t, and structural parameters θ, for the ease of notation we shall re-write Eq.(4) as $x^i_{t+1,\theta} = \lambda^i_\theta(z_{t+1}, x^i_t)$ for the rest of the paper.[22] In the following analysis, we sometimes omit θ in $x_{t,\theta}$ for salience of notation.

2.4 Markov Perfect Equilibrium with Incomplete Information on Firms' Initial Beliefs

The previous section describes firm's own belief and its update. However, beliefs from other firms can also affect strategic investment decisions of the company. In this section, we consider the case where a firm knows its own belief, but not others' beliefs. Further, not knowing others' beliefs is amount to not knowing others initial beliefs, as with the initial belief x^j_0 for firm j, firm i can calculate the belief of j at any time given publicly observable history h_t with Eq.(4). Therefore, firm i's information of other firms' beliefs is incomplete. Let $x^i_0 \in \Delta^{|S^h|}$ denote the initial belief for firm i, and let the joint distribution of firms' initial beliefs $\mathbb{P}(x^{1:I}_0)$ be *common knowledge*, where $x^{1:I}_0 \triangleq \{x^i_0\}^I_{i=1}$. Firm i's belief about other firms' initial beliefs can be obtained using the Bayes rule $\mathbb{P}(x^{-i}_0 | x^i_0) = \frac{\mathbb{P}(x^{1:I}_0)}{\mathbb{P}(x^i_0)}$ where $x^{-i}_0 \triangleq (x^1_0, ..., x^{i-1}_0, x^{i+1}_0, ...x^I_0)$ denotes the collection of firms' initial beliefs except firm i.

Let us denote by $\mathbb{P}(x^{-i}_t | h_t, x^i_0, \mathbb{P}(x^{-i}_0))$, the updated distribution of firm i's belief about other firms' beliefs. Recall that $h_t = (z_1, ..., z_t)$ is the history of aggregate state relevant observations. Abusing notation, let us define the *private belief state* $h^i_t \triangleq \{x^i_t, \mathbb{P}(x^{-i}_t | h_t, x^i_0, \mathbb{P}(x^{-i}_0))\}$ as the tuple composed of firm i's updated private belief about the aggregate state, firm i's up-

[21] We note that the dependence of σ^i_θ on structural parameter θ is only through θ_2, i.e., $\sigma^i_\theta = \sigma^i_{\theta_2}$. We drop the subscript of θ for salience of notation.

[22] By induction, one can see $x_t, t > 0$ is a function of θ. We further note that the dependence of $x^i_{t,\theta}$ on structural parameter θ is only through θ_2, i.e., $x^i_{t,\theta} = x^i_{t,\theta_2}$.

dated distribution of other firms' beliefs.[23] At $t=0$, $h_0^i = \{x_0^i, \mathbb{P}(x_0^{-i}|x_0^i)\}$. Correspondingly, the collection of private belief states except firm i is $h_t^{-i} = (h_t^1, ..., h_t^{i-1}, h_t^{i+1}, ..., h_t^I)$.

A stationary and deterministic policy for the firm i is a map $\delta^i : (\mathcal{H}^i \times \mathcal{E}^i \times S^o) \to A$, where $\mathcal{H}^i$ is the set of feasible private belief states h_t^i. From other firms' perspective, firm i's payoff perturbation ϵ^i is private. Therefore, define a stationary and probabilistic policy $\pi^i : (\mathcal{H}^i \times S^o) \to \Delta^{|A|}$, where $\Delta^{|A|}$ is the probability simplex in $\mathbb{R}^{|A|}$, such that $\pi^i(a_t^i|h_t^i, s_t^o)$ is the conditional choice probability (CCP) that marginalizes firm i's private payoff shock ϵ^i. In other words, CCP is a strategy that other firms believe that firm i will play if they knew that firm i's initial belief is x_0^i:

$$\pi^i(a^i|h_t^i, s_t^o) \triangleq \int \mathbb{1}\{\delta^i(h_t^i, \epsilon_t^i, s_t^o) = a^i\} d\mu(\epsilon_t^i), \tag{5}$$

where $\mathbb{1}$ is an indicator function. We further define $\pi^{-i}(a_t^{-i}|h_t^{-i}, s_t^o) \triangleq \prod_{j \neq i} \pi^j(a_t^j|h_t^j, s_t^o)$ be the probability for all but except firm i's actions, where $a_t^{-i} = (a_t^1, ..., a_t^{i-1}, a_t^{i+1}, ..., a_t^I)$ is the collection of actions except the firm i.

Given π^{-i}, θ, firm i considers the optimization problem:

$$V_{\theta,\pi^{-i}}(h_t^i, \epsilon_t^i, s_t^o) \triangleq \sup_{\delta^i} \mathbb{E}\left[\sum_{\tau \geq t} \beta^{\tau - t} \tilde{r}^i(s_\tau^h, s_\tau^o, \delta^i, \epsilon_\tau^i) | h_t^i, \epsilon_t^i, s_t^o, \pi^{-i}, \theta\right], \tag{6}$$

where $\beta \in (0,1)$ is the discount factor and the expectation is taken with respect to the joint probability of actions $\{a_t^{-i}\}_{t \geq \tau}$ induced by probabilistic policies π^{-i}, other firms' private belief state distributions $\{h_t^{-i}\}_{t \geq \tau}$, macroeconomic indicators $\{z_t\}_{t > \tau}$, local market size proxies $\{s_t^o\}_{t > \tau}$, and future random shocks $\{\epsilon_t^i\}_{t > \tau}$.

Problem (6) admits a unique fixed point solution $V^*_{\theta,\pi^{-i}}$ to the Bellman equation, such that:

$$\begin{aligned} V^*_{\theta,\pi^{-i}}(h_t^i, \epsilon_t^i, s_t^o) &= \max_{a_t^i \in A} \left\{ r(x_t^i, s_t^o, a_t^i) + \epsilon_t^i(a_t^i) + \beta \mathbb{E}[V^*_{\theta,\pi^{-i}}(h_{t+1}^i, s_{t+1}^o, \epsilon_{t+1}^i)] \right\} \\ &= \max_{a_t^i \in A} \{ Q^*_{\theta,\pi^{-i}}(h_t^i, s_t^o, a_t^i) + \epsilon_t^i(a_t^i) \} \end{aligned} \tag{7}$$

where $r(x_t^i, s_t^o, a_t^i) \triangleq \sum_{s^h} x_t^i(s^h) r^i(s^h, s_t^o, a_t^i)$, and $h_{t+1}^i = \{x_{t+1}^i, \mathbb{P}(x_{t+1}^{-i}|h_{t+1}, x_0^i, \mathbb{P}(x_0^{-i}))\}$ can be computed recursively with h_t^i and z_{t+1}. The expectation is taken with respect to (z_{t+1}, s_{t+1}^o) with probability distribution $\mathbb{P}_\theta(z_{t+1}, s_{t+1}^o|x_t^i, s_t^o, a_t) = \sigma_\theta^i(z_{t+1}, x_t^i)\mathbb{P}_\theta(s_{t+1}^o|s_t^o, a_t)$; h_t^{-i} with $\mathbb{P}(x_0^{-i}|x_0^i)$[24]; a_t^{-i} with the probabilistic policy $\pi^{-i}(a_t^{-i}|h_t^{-i}, s_t^o)$; ϵ_{t+1}^i with μ. Moreover, Q^* is

[23] Note that $h_t^i, \mathbb{P}(x_t^{-i}|h_t, x_0^i, \mathbb{P}(x_0^{-i})), t > 0$ is a function of model primitives θ. We omit θ for simplicity of exposition.

[24] For $j \neq i$, if x_0^j is known, give observable history h_t, firm i can calculate j's private belief state at time t,

the state-action value function without considering ϵ_t^i such that

$$Q^*_{\theta,\pi^{-i}}(h_t^i, s_t^o, a_t^i) \triangleq r(x_t^i, s_t^o, a_t^i) + \beta \mathbb{E}[V^*_{\theta,\pi^{-i}}(h_{t+1}^i, \epsilon_{t+1}^i, s_{t+1}^o)]. \tag{8}$$

The related proof of fixed point theorem can be found in Appendix A.1.

Further, given that $\{\epsilon^i\}_{i=1}^I$ are i.i.d. with Gumbel distribution. Let $a_t^i = a^i, h_t^i = h^i, s_t^o = s^o$. For fixed θ, there exists a conditional choice probability (CCP) combination $\{\pi_\theta^i\}_{i=1}^I$ that is a Markov Perfect Equilibrium (MPE) and satisfies the following condition:

$$\pi_\theta^i(a^i|h^i, s^o) \triangleq \frac{\exp Q^*_{\pi_\theta^{-i}}(h^i, s^o, a^i)}{\sum_{\tilde{a}^i \in A} \exp Q^*_{\pi_\theta^{-i}}(h^i, s^o, \tilde{a}^i)} \tag{9}$$

for all $i \in \{1, 2, ..., I\}, h^i \in \mathcal{H}^i, s^o \in S^o, a^i \in A$. The related proof can be found in Appendix A.2. We denote by π_θ^i the MPE policy, and by $Q^*_{\pi_\theta^{-i}}$ the fixed point value function for the equilibrium policy.

Remark 2 (Relationship with Perfect Bayes-Nash Equilibrium)**.** The above MPE equilibrium is the (subgame Perfect) Bayes-Nash Equilibrium for the associated dynamic game where each subgame corresponds to a joint collection (for all firms) of private belief states.

2.5 Markov Perfect Equilibrium with Complete Information on Firms' Initial Beliefs

For identification (to be discussed in the next section) and tractability consideration, in the rest of the paper, we simplify the model proposed in Section 2.4 by assuming the collection of initial beliefs $x_0^{1:I}$ are known to all firms, i.e., each firm knows other firms' initial beliefs. Hence, the information on initial beliefs is complete for each firm. To this end, $h_0^i = h_0^j = x_0^{1:I}, i \neq j$. At any time $t > 0, i = 1, 2, ..., I$, x_t^i can be computed recursively with x_0^i and the belief update function λ_θ^i. Hence, $h_t^i = h_t^j = x_t^{1:I}$. As a result, Eq.(8) becomes

$$Q^*_{\theta,\pi^{-i}}(\mathbf{x}_t, s_t^o, a_t^i) \triangleq r(x_t^i, s_t^o, a_t^i) + \beta \mathbb{E}[V^*_{\theta,\pi^{-i}}(\mathbf{x}_{t+1}, \epsilon_{t+1}^i, s_{t+1}^o)], \tag{10}$$

where $\mathbf{x}_t \triangleq x_t^{1:I}$ and $x_{t+1}^i = \lambda_\theta^i(z_{t+1}, x_t^i), i = 1, ..., I$. The expectation is taken with respect to a_t^{-i} from $\pi^{-i}(a_t^{-i}|\mathbf{x}_t, s_t^o)$, z_{t+1}, s_{t+1}^o and ϵ_{t+1}^i. The closed-form MPE in Eq.(9) becomes

$$\pi_\theta^i(a^i|\mathbf{x}, s^o) \triangleq \frac{\exp Q^*_{\pi_\theta^{-i}}(\mathbf{x}, s^o, a^i)}{\sum_{\tilde{a}^i \in A} \exp Q^*_{\pi_\theta^{-i}}(\mathbf{x}, s^o, \tilde{a}^i)}. \tag{11}$$

h_t^j. However, x_0^j is unobserved by firm i, only the distribution $\mathbb{P}(x_0^{-i}|x_0^i)$ is known by firm i. Hence, from firm i's perspective, random variable $h_t^{-i} = \{h_t^j\}_{j \neq i}$ depends on $x_0^{-i} = \{x_0^j\}_{j \neq i}$ through $\mathbb{P}(x_0^{-i}|x_0^i)$.

3 Identification and Estimation

3.1 Identification

In this section, we study the non-parametric identification of the aggregate state dynamics θ_2 (Section 3.1.1) and the payoff function θ_1 (Section 3.1.2-3.1.3). Following Section 2.5, we will focus on the case where initial beliefs are known to firms. In particular, for the identification of θ_1, we first discuss the case where firms have heterogeneous beliefs such that x^i might be different from x^j (Section 3.1.2), and then we discuss the long panel case where firms' beliefs will converge ($x^i = x^j, j \neq i$) (Section 3.1.3).

3.1.1 Local and Aggregate Factors

The state that relates to the local market size factor s_t^{ds} is directly observable, so its dynamics $\mathbf{F}$ can be directly identified from the data. In what follows, we will focus on the identification of unobserved aggregate state dynamics $\mathbf{P}, \mathbf{O}$.

As firms' beliefs about the aggregate state s^h and its corresponding observed macroeconomic signals z evolve independently of the observable state s^o and firms' actions a, for the purpose of identifying the observation probability $\mathbf{O}$ and transition probability $\mathbf{P}$ we only consider the partial dataset $\{z_t\}$ and a true prior of aggregate state ρ (a vector of size S^h) such that $\rho(s^h) = \mathbb{P}(s_0^h = s^h)$. Note that ρ is the distribution from which the initial aggregate state s_0^h is sampled from, and ρ might be different from any firm's initial belief x_0^i.

Consider the joint probability of $2k+1$ periods of observations $z_{1:2k+1} \triangleq \{z_t\}_{t=1}^{2k+1}$, which can be factorized into three parts conditioned on s_{k+1}^h: the observations prior to time $k+1$, the observation at $k+1$, and the observation after $k+1$.[25] That is,

$$\mathbb{P}(z_{1:2k+1}|\rho) = \sum_{s_{k+1}^h} \mathbb{P}(z_{1:k}, s_{k+1}^h|\rho)\mathbb{P}(z_{k+1}|s_{k+1}^h)\mathbb{P}(z_{k+2:2k+1}|s_{k+1}^h), \tag{12}$$

where the left hand side is the statistics from data, while the right hand side is built through model primitives.

Factorization has been used for establishing non-parametric identification in past research, such as Kasahara and Shimotsu (2009). Loosely speaking, the identification arguments used in factorization-based approaches all share similar qualities. In particular, identification often rests on rich variation between periods t and $t+1$, t and $t+2$, t and

[25] The factorization is ensured by the property of hidden Markov models. Namely, observation z_t only depends on s_t^h and $\{s_t^h\}_{t\geq 0}$ is a Markov process (Allman et al. 2009).

$t+3$, and so on.[26] There are some important differences though. In non-parametric identification of permanent heterogeneity (i.e., cross-sectional), key sources of variation are more reliant on dynamics of the action (i.e., entry/exit) between periods, as well as sufficiently rich support for the exogenous market size factors. In our case, however, the non-parametric identification of unobserved aggregate state dynamics, temporal observation of macroeconomic factors becomes particularly important; support conditions alone (i.e., range of values GDP or inflation can potentially take) for the macroeconomic factors is unlikely to be sufficient. That said, nonparametric identification of hidden aggregate state dynamics have milder conditions than permanent heterogeneity, as rank conditions are less dependent on rich multiperiod investment decisions.[27] As a brief aside, we note that joint identification of both aggregate state dynamics *and* permanent heterogeneity is an interesting future topic; such joint identification is out of the scope of the current paper.[28]

We now rewrite Eq.(12) in tensor form, which uses the following matrices. Matrix $\mathbf{A}(k)$ of $|S^h|$ rows and $|Z|^k$ columns contains the probabilities for different realizations of $z_{1:k}$. That is, the probability $\mathbb{P}(z_{1:k}, s^h_{k+1}|\rho)$ resides in the matrix $\mathbf{A}(k)$ with row index s^h_{k+1} and column index $Z(...Z(Zz_1 + z_2) + ... + z_{k-1}) + z_k$.[29]

Example. Under the context of our specific model about retail investment, we have $|S^h| = |Z| = 2, k = 2$, such that

$$\mathbf{A}(2) \triangleq \begin{pmatrix} p^1_{1,1} & p^1_{1,2} & p^1_{2,1} & p^1_{2,2} \\ p^2_{1,1} & p^2_{1,2} & p^2_{2,1} & p^2_{2,2} \end{pmatrix},$$

where $p^s_{i,j} \triangleq \mathbb{P}(z_1 = i, z_2 = j, s^h_3 = s|\rho)$. The probability of realization $(s^h = 1, z_1 = 1, z_2 = 0)$ occupies $\mathbf{A}_{1,2}(2)$ (i.e., the second row and third column of $\mathbf{A}(2)$). Similarly, we define matrix $\mathbf{B}(k)$ of size $|S^h| \times |Z|^k$ to hold the probabilities $\mathbb{P}(z_{k+2:2k+1}|s^h_{k+1})$. For a realization of $z_{k+2:2k+1}, s^h_{k+1}$, the probability $\mathbb{P}(z_{k+2:2k+1}|s^h_{k+1})$ is an element of $\mathbf{B}(k)$ with coordinate $(s^h_{k+1}, Z(...Z(Zz_{k+2} + z_{k+3}) + ... + z_{2k}) + z_{2k+1})$.

Recall that $\mathbf{O} \in \mathbb{R}_+^{|S^h|\times|Z|}$ is the observation probability matrix, and Eq.(12) can be

[26] Finite dependence (Arcidiacono and Miller 2011, 2019) may alleviate the burden of observing such variation in the data when identifying non-parametrically unobserved permanent heterogeneity.

[27] Variation of this sort may be hard to achieve with investment decisions, that often exhibit sparseness.

[28] This joint identification may require the availability of candidate instrumental variables (Berry and Compiani 2023). For example, past population or GDP might potentially serve as valid exclusion restrictions, as these proxies might be correlated with the current market structure, but could in principle be uncorrelated with the current unobserved shocks (conditional on other observed market factors). Alternatively, observation of policy changes may also serve as candidate instrumental variables.

[29] Note that the first row/column index of matrices $\mathbf{A}(k)$ and $\mathbf{B}(k)$ (to be defined later) is 0.

written in tensor form as:

$$\mathbf{T}_{i,j,l}(k) = \sum_{r=1}^{|S^h|} \mathbf{A}_{r,i}(k)\mathbf{O}_{r,j}\mathbf{B}_{r,l}(k) \tag{13}$$

where $\mathbf{A}_{r,i}(k), \mathbf{O}_{r,j}, \mathbf{B}_{r,l}(k)$ respectively denote the row r and column i, j, l of the following matrices $\mathbf{A}(k), \mathbf{O}, \mathbf{B}(k)$, and $\mathbf{T}(k)$ is a three-way tensor. Therefore, the identification of observation probabilities requires rank conditions, which we describe below:

Definition 1 (Kruskal (1977))**.** The Kruskal rank of a matrix $\mathbf{A}$, or $K(\mathbf{A})$, is defined as the largest number K such that *any* K rows in $\mathbf{A}$ are linearly independent.

Having set up the relevant probabilities in the firm investment problem, we are now ready to present the identification of observation probabilities, which serves as our paper's key theoretical result:

Theorem 1. Assume the cardinality of each spaces S^h, Z is known, denoted by $|S^h|, |Z|$. The observation probability matrix $\mathbf{O}$ is non-parametrically identified up to simultaneous row permutation from $2k+1$ periods of consecutive observations (i.e., $z_{1:2k+1}$), provided that k satisfies:

$$K(\mathbf{A}(k)) + K(\mathbf{B}(k)) + K(\mathbf{O}) \geq 2|S^h| + 2. \tag{14}$$

Example (continued). Theorem 1 suggests that with our model specification described in Section 2 where $|S^h| = |Z| = 2$, with 3 consecutive observations (i.e., $k = 1$), if the corresponding matrices $\mathbf{A}(1)$ of size (2, 2) containing $\mathbb{P}(z_1, s_2^h|\rho)$, matrix $\mathbf{B}(1)$ of size (2, 2) containing $\mathbb{P}(z_3|s_2^h)$, and observation probability matrix $\mathbf{O}$ of size (2, 2) are all full rank, then $\mathbf{O}$ is identified as $K(\mathbf{A}(1)) + K(\mathbf{B}(1)) + K(\mathbf{O}) = 6 \geq 2|S^h| + 2 = 6$.

With the identification of the observation probabilities, in the following theorem, we show that the transition probabilities of aggregate state are identified with two consecutive periods of data (i.e., $\{z_t, z_{t+1}\}$). We note that this condition is less stringent than the panel length requirements for identification of permanent heterogeneity, which requires $T \geq 5$ as a necessary condition (Kasahara and Shimotsu 2009).

Theorem 2. Assume the cardinalities of the spaces S^h, Z are known, denoted by $|S^h|, |Z|$. The observation probability matrix $\mathbf{O}$ is known (or readily identified). The transition probability matrix $\mathbf{P}$ of the unobserved aggregate state is identified with two consecutive periods of data (i.e., $\{z_t, z_{t+1}\}$).

In summary, we show the identification of observation and transition probabilities, so the identified dynamics can be summarized as $\theta_2^* = \{\mathbf{F}, \mathbf{P}, \mathbf{O}\}$.

3.1.2 Payoff Function with Heterogeneous Beliefs

Following Section 2.5, we assume each firm knows others' initial beliefs. Therefore, upon observing h_t, any firm i knows x_t^{-i} as x_t^j can be calculated with x_0^j and Eq.(4). Practically, these initial beliefs can be determined by domain knowledge (such as interviewing a firm's manager). We note, however, this requirement of the common knowledge of initial beliefs can be relaxed with long panel as those beliefs will eventually converge, which is to be discussed in Section 3.1.3.

We denote firms' initial beliefs by $\mathbf{x}_0 = x_0^{1:I}$, the true pay-off function by $\theta_1^* = \{r^{*,i}\}_{i=1}^I$ and $\theta^* = \{\theta_1^*, \theta_2^*\}$. For firm i, θ^* should "match" data by

$$\forall (h_t, a_t^i), \quad \underbrace{\hat{\mathbb{P}}(a_t^i | h_t, \mathbf{x}_0)}_{\text{Data}} = \underbrace{\pi_{\theta^*}^i(a_t^i | \mathbf{x}_{t,\theta^*}, s_t^o)}_{\text{Model}},$$

where $\mathbf{x}_{t,\theta^*} = x_{t,\theta^*}^{1:I}$, and $\hat{\mathbb{P}}(a_t^i | h_t, \mathbf{x}_0^{1:I})$ is a function of dataset.

The definitions of observational equivalence and identification from Magnac and Thesmar (2002) can be generalized for our model as follows. Let Θ_1, Θ_2 be the set of payoff functions and model dynamics, respectively, $\Theta = \Theta_1 \times \Theta_2$, and let $\overset{o}{\iff}$ denote observational equivalence. Then $\forall \theta = \{\theta_1, \theta_2^*\} \in \Theta, \theta' = \{\theta_1', \theta_2^*\} \in \Theta, \theta \overset{o}{\iff} \theta'$ if and only if $\pi_\theta^i(a_t^i | \mathbf{x}_{t,\theta}, s_t^o) = \pi_{\theta'}^i(a_t^i | \mathbf{x}_{t,\theta'}, s_t^o), \forall i = 1, ..., I$. The model is identified if $\forall \theta, \theta' \in \Theta, \theta \overset{o}{\iff} \theta'$ implies $\theta = \theta'$.

As the aggregate state dynamics is identified (i.e., θ_2^*), we can transform any sample path $\{(z_t)_{t=1}^T, (s_t^o)_{t=0}^T, (a_t)_{t=0}^{T-1}, \mathbf{x}_0\}$ to a corresponding belief process $\{(s_t^o)_{t=0}^T, (a_t)_{t=0}^{T-1}, (\mathbf{x}_{t,\theta^*})_{t=1}^T, \mathbf{x}_0\}$ where $(\mathbf{x}_{t,\theta^*})_{t=1}^T$ can be recursively calculated via Eq.(4) and $\mathbf{x}_0$. Hence, although aggregate state is unobserved, the belief of aggregate state is "observed". Consequently, our model with unobserved aggregate state is transformed to the classic dynamic games with observable state. Thus, MPE policies $\pi_{\theta^*}^i(a_t^i | \mathbf{x}_{t,\theta^*}, s_t^o)$ can also be identified from the data $\hat{\mathbb{P}}(a_t^i | h_t, \mathbf{x}_0)$.

Theorem 3. Assume the following are known: the discount factor β, the random perturbation distribution μ, aggregate state dynamics $\mathbf{O}, \mathbf{P}, \mathbf{F}, \mathbf{x}_0$, and firm i's payoff function $r^i(s^o, s^h, a^{i,0}), \forall i, \forall s^o \in S^o, s^h \in S^h$ when the reference action $a^{i,0}$ is taken. The payoff function $\{r^i\}_{i=1}^I$ is identified.

The related proof can be found in Appendix A.5.

3.1.3 Payoff Function with Common Beliefs (for Long Panel)

In this section, we begin by showing firms' heterogeneous beliefs will eventually converge in the long run. As a result, the knowledge of firms' initial beliefs is not necessary for the identification of payoff functions: one may use the data after the initial τ periods (i.e., $h_t, t \geq \tau$) for the (approximate) identification purpose. As τ increases, the belief will converge, so will $\pi^i_{\theta^*}$.

We now show that when firms have different initial beliefs, the individual belief process of each firm will eventually be approximately equivalent to a single common belief process, provided that there are a sufficiently long history of observations. To demonstrate this theoretical feature, we first define the metric function $\mathcal{D} : X \times X \to \mathbb{R}_+$ as

$$\mathcal{D}(x, x') \triangleq \max\{d(x, x'), d(x', x)\}, \tag{15}$$

where

$$d(x, x') \triangleq 1 - \min\left\{\frac{x(s^h)}{x'(s^h)} : s^h \in S^h, x'(s^h) > 0\right\}, \forall x, x' \in X.$$

Proposition 1. *$\forall \epsilon > 0$, for all $x^i_0, x^j_0 \in X$ there exists $T_\epsilon < \infty$ such that $\mathcal{D}(x^i_{t,\theta}, x^j_{t,\theta}) < \epsilon$ for all $t > T_\epsilon$, $i, j \in \{1, \ldots, I\}$.*

Proposition 1 demonstrates that when the decision making process continues for sufficiently long periods, the belief of each firm will ultimately converge to a common belief irrespective of (potentially different) initial beliefs $\{x^i_0\}^I_{i=1}$ across firms $i = 1, ..., I$.[30] This notion of converged beliefs shares some similarities with the commonly used assumption about the nature of permanent heterogeneity (i.e., time-invariant market-specific unobserved factors), in that most studies often assume that the market-specific heterogeneity impacts all firms in the same manner. An important difference though, in the type of heterogeneity we study, is that the symmetry is derived theoretically (i.e., Bayesian belief updating), as opposed to being imposed as a functional form assumption.

We demonstrate Proposition 1 via a numerical example to illustrate the rate of convergence of heterogeneous beliefs. We set prior of unobserved aggregate state to be $\rho = [0.5, 0.5]$, and two heterogeneous beliefs are $[1, 0], [0, 1]$. Note that such setting of heterogeneous belief is the extreme case where two firms do not share any belief in common. Aggregate state

[30] Proposition 1 is consistent with the finding from control and optimization research (Munie et al. 2009, Davis et al. 2021, Munie and Shoham 2010, Vlasceanu et al. 2020) that demonstrate that individual beliefs on a given product/service will converge after observing a long history of the product reviews.

transition probability and $\mathbf{P}$ observation probability $\mathbf{O}$ are:

$$\mathbf{P} = \begin{pmatrix} 0.8 & 0.2 \\ 0.1 & 0.9 \end{pmatrix}, \quad \mathbf{O} = \begin{pmatrix} 1-\omega & \omega \\ \omega & 1-\omega \end{pmatrix},$$

where $\omega = 0.1, 0.15, 0.2, 0.25$ denotes different level of noises in observation. The larger ω is, the noisier aggregate state is correlated to observations.[31] The convergence of belief is plotted in Figure 2. As we can see, even with extreme heterogeneity ($x_0 = [1, 0]$ vs. $x_0 = [0, 1]$) the difference between beliefs quickly vanishes, usually within 7 periods of data. If the initial beliefs are reasonably close, one can expect beliefs to be well converged within 5 periods. Moreover, with noisier observations (i.e., greater ω), it takes longer time for heterogeneous beliefs to converge.

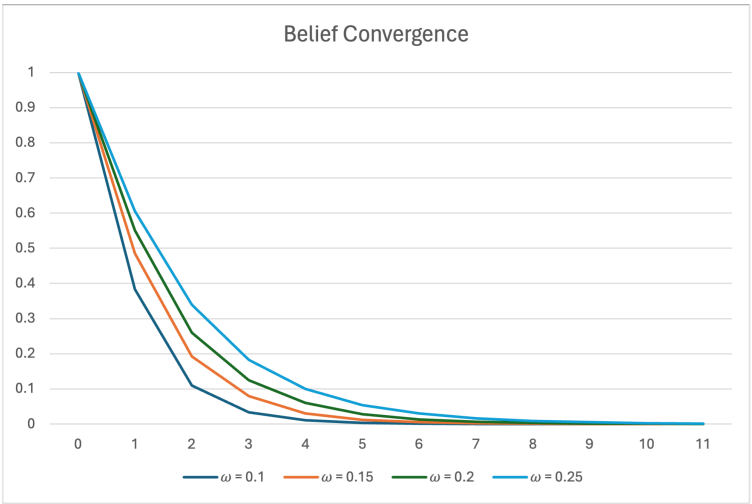


Figure 2: The convergence of heterogeneous beliefs. Vertical axis is the L_2 norm measuring the difference between two beliefs, horizontal axis shows the length of data.

In light of Proposition 1 and the associated numerical experiment, we consider that all firms hold identical beliefs about the underlying aggregate state at all times. Consequently, the superscript i in terms σ^i, and λ^i can be dropped, and $x_t^i = x_t^j = x_t, t \geq 0$ denotes the common belief of firms at any time t. As a result, $\mathbf{x}_t$ in Eq.(10) and Eq.(11) can be represented by the common belief x_t.

Remark 3. We acknowledge that treating firms share identical belief serves an approximation of the original problem where there is heterogeneity in firms' beliefs. For example, when firms' beliefs are not identical but arbitrarily close, the MPE characterized in Eq.(11) is

[31] We note that the values for transition and observation probabilities are derived from economic data we use in this paper to justify their appropriateness.

an *approximate* equilibrium, i.e., the strategies identified are approximately best responses. However, given the fast convergence of beliefs (as shown in the numerical example), the benefit of this approximation is clear: it facilitates a simpler representation of the firm's decision so that a firm need not track its competitors' (potentially different) beliefs, which in turn helps alleviate the computational burden of finding optimal policies.

In view of belief convergence, we have the following Corollary drived from Theorem 3:

Corollary 1. For long panel, assume the following are known: the discount factor β, the random perturbation distribution μ, aggregate state dynamics $\mathbf{O}, \mathbf{P}, \mathbf{F}$, and firm i's payoff function $r^i(s^o, s^h, a^{i,0}), \forall i, \forall s^o \in S^o, s^h \in S^h$ when the reference action $a^{i,0}$ is taken. The payoff function $\{r^i\}_{i=1}^I$ is approximately identified.

The proof of this corollary directly follows from Theorem 3 and Proposition 1.

3.2 Estimation

In this section, we aim to estimate our model primitives with long panel data; hence we consider a common belief for firms. We assume the data of M sample paths have been generated by only one MPE and are given as $\{(s^o_{m,t})_{t=0}^T, (a_{m,t})_{t=0}^{T-1}, (z_{m,t})_{t=1}^T\}_{m=1}^M$. Our aim here is to estimate the primitives of the proposed model, including the payoff function θ_1 and the dynamics θ_2, from M observed data sequences.

We consider a non-parametric setting for local market size factor dynamics $\mathbf{F}$ and aggregate state dynamics $\mathbf{P}, \mathbf{O}$, and a parametric setting of payoff function $\{r^i\}_{i=1}^I$. Specifically, payoff function is parametrized by $\theta_1^i = \{\theta^i_{base,s^h}, \theta^i_{own}, \theta^i_{rival}, \theta^i_{ds}, \theta^i_{entry}\}$; dynamics is $\theta_2 = \{\mathbf{F}, \eta, \omega\}$. Consequently, we have $\sigma_\theta = \sigma_{\theta_2}, x_{m,t,\theta} = x_{m,t,\theta_2}$. With this notation in place, the log-likelihood function $\mathcal{L}(\theta)$ of the data sequences can be written as

$$\begin{aligned}\mathcal{L}(\theta) &= \log \prod_{m=1}^{M} \mathbb{P}_\theta(\text{data sequence } m) \\ &= \sum_{m=1}^{M}\sum_{t=0}^{T-1} \big(\log \sigma_{\theta_2}(z_{m,t+1}, x_{m,t,\theta_2}) + \log \mathbb{P}_{\theta_2}(s^o_{m,t+1}|s^o_{m,t}, a_{m,t})\big) \\ &\quad + \sum_{m=1}^{M}\sum_{t=0}^{T-1}\sum_{i=1}^{I} \log \pi^i_\theta(a^i_{m,t}|x_{m,t,\theta_2}, s^o_{m,t}).\end{aligned} \tag{16}$$

With this likelihood function, we adopt the nested fixed-point algorithm for estimation. We first search for the value of $\hat{\theta}_2$, for example, by maximizing the log-likelihood of the unobserved (i.e., aggregate) and observable state dynamics (i.e., the first two terms on the right-hand side of Eq.(16)). Note that once $\hat{\theta}_2$ is obtained, all $\{x_{m,t,\hat{\theta}_2}\}_{m=1}^M$ are determined.

Therefore, θ_1 can be established by solving the following likelihood optimization

$$\max_{\theta_1} \sum_{m=1}^{M} \sum_{t=0}^{T-1} \sum_{i=1}^{I} \log \pi^i_{\theta_1,\hat{\theta}_2}(a^i_{m,t}|x_{m,t,\hat{\theta}_2}, s^o_{m,t}).$$

To assess the validity of our proposed approach, we test our estimation methodology with a synthetically generated data set (for which the ground-truth model parameters are known). The test confirms that our estimation approach correctly and precisely recovers the underlying model primitives. Details about the test are provided in Appendix B.

4 Empirical Application

We now present a comprehensive empirical case study about fast food industry investments. This case study serves two main purposes. First, it demonstrates how our framework can be applied to an actual setting involving strategic retail investment under aggregate fluctuations. Second, this case study aims to uncover substantive insights about the extent to which fast food chain investments are sensitive to the aggregate fluctuation (and its corresponding volatility). This sensitivity ultimately has implications on retail investment's resilience to macroeconomic recessions/expansions as well as uncertainty. Moreover, if this sensitivity differs across retail chains, market structure can potentially be impacted.

4.1 Industry Background and Data

We analyze the market entry and exit decisions of five hamburger retail chains in Canada in the time frame from 1970 to 2004 during which four economic recessions occurred according to *C.D. Howe Institute Business Cycle Council*.[32] We utilize data from Igami and Yang (2016), which has information about the entry and exit of hamburger shops across seven major cities of Canada. The five major chains include A&W, Burger King, Harvey's, McDonald's, and Wendy's. A geographical market was defined on the basis of a cluster of stores located within a 0.5-mile radius of each other between 1970 and 2004. This dataset contains 400 geographical markets and it was used to analyze entry and exit decisions of hamburger chains and other relevant economic strategies (e.g., cannibalization and/or preemption). The entry and exit decision data provides a required piece of information for our analysis. For each market and year, information about population, income, and property value are used as proxies for local market size conditions.

[32]See the Council's 2012 report on its website: https://www.cdhowe.org/council/business-cycle-council.

To analyze the impact of aggregate fluctuations on retail investment decisions, we expand the dataset of Igami and Yang (2016) by gathering macroeconomic indicators of Canada during the same time period (1970-2004). Two commonly used and reported economic indicators, quarterly real gross domestic product (GDP) growth rate and inflation rate of Canada, are collected from *Federal Reserve* economic data.[33] The Howe Council provides the relevant information about Canada's economy, since the time when GDP data became available. The Howe Council examined recessions from three perspectives (i.e., duration, amplitude, and scope), and it collected multiple data sources across different industry sectors, and identified an authoritative set of business cycle dates. Their analysis in 2012 on historical chronology of Canadian recessions reveals that there were four recessions from 1970 to 2004: 1974Q4–1975Q1, 1979Q4–1980Q2, 1981Q2–1982Q4, and 1990Q1–1992Q2.

Lastly, the work of Igami and Yang (2016) demonstrated that there were sources of permanent market-specific heterogeneity in the data. Their analysis using the factorization results from Kasahara and Shimotsu (2009) revealed that there were at least three types of markets. Two results of the market classification were presented. The authors first deterministically assigned the geographical markets into three types according to market fixed-effect estimates. They further developed an advanced algorithm to assign probabilities to unobservable market types, and with these market-type probabilities, estimated a dynamic game of retail investment. While the two approaches are dramatically different, the two distributions tend to broadly agree.[34] In our paper, we use the deterministic results from the first method, which allow us to directly partition the data to three subsets for high-, middle-, and low-type markets, respectively. The use of probabilistic labeling for each market would require *joint* consideration of *both* permanent market-specific heterogeneity (only unobservable to the researcher) and hidden aggregate state (unobservable to *both* firms and the researcher), which is beyond the scope of our current analysis. We further note that for our analysis, we only focus on the analysis of the middle type of market. Other market types (low and high types of market) share similar results; hence their data and analysis are omitted in our discussions about the main findings.[35]

To examine the descriptive relationship between retail investment and the aggregate state of Canada, we first depict Canada's real GDP and the number of stores of each of the five burger chains from 1970 to 2004. No surprisingly, Figure 3 shows that the burger chains were more likely to grow in strong aggregate state. However, during a recession, especially a prolonged one around 1991, some burger chains tended to close stores, and even if some

[33] See the website of Economic Research from *Federal Reserve Bank of St. Louis*: `https://fred.stlouisfed.org/`.
[34] See the related comparison discussion in Igami and Yang (2016).
[35] Details about these alternative specifications are available upon request.

brands were still opening new stores, the speed (slope) was reduced.

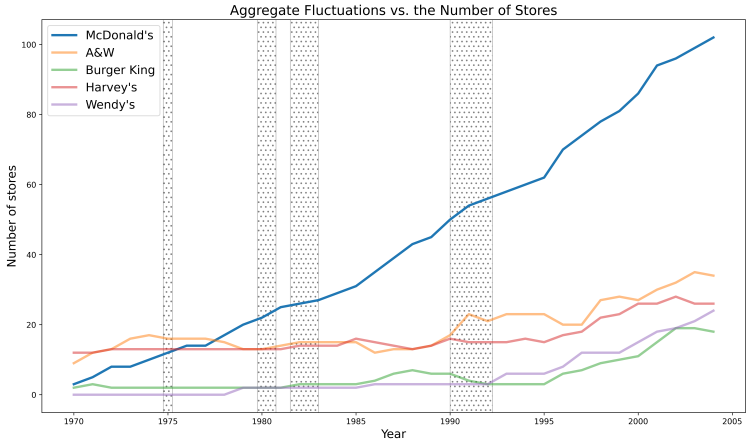


Figure 3: The aggregate fluctuations and Canada fast-food market. The four dotted areas are recession time periods defined by the *C.D. Howe Institute Business Cycle Council*.

Next, we examine the correlation between firms' decisions and related economic factors via a reduced-form ordered probit regression model. We use n_t^i, n_t^{-i} to denote market structure, z_t to denote economic indicator, the GDP (z_t^{gdp}) or the inflation rate (z_t^{inf}),[36] and the local market size factor terms s_t^{pop}, s_t^{pro}, and s_t^{inc} stand for the market population, property value, and income, respectively. Define the reduced-form regression model of entry and exit decisions by

$$a_t^i = \begin{cases} \text{close an existing store,} & y_t^i \leq c_1; \\ \text{status quo,} & c_1 < y_t^i \leq c_2; \\ \text{open a new store,} & y_t^i > c_2. \end{cases} \tag{17}$$

$$y_t^i = \gamma_{own} n_t^i + \gamma_{rival} n_t^{-i} + \gamma_{pop} s_t^{pop} + \gamma_{pro} s_t^{pro} + \gamma_{inc} s_t^{inc} + \gamma_{inf} z_t^{inf} (\gamma_{gdp} z_t^{gdp}) + v_t^i,$$

where c_1 and c_2 are cutoff values, v_t follows the i.i.d standard normal distribution, and γs are the related coefficients of competition with same-brand stores (γ_{own}), competition with rival-brand stores (γ_{rival}), market population (γ_{pop}), market property value (γ_{pro}), market income (γ_{inc}), the real GDP (γ_{gdp}), and the inflation rate (γ_{inf}).

The regression results in Table 1 show that retail investment decisions are correlated with economic indicators. The sign of coefficients indicates the growth of the real GDP is positively correlated with entry ($\hat{\gamma}_{dgp} > 0$), and the correlation between entry and high

[36] Specifically, the one-quarter-lag inflation rate are selected as the observation z_t^{inf}, because the peak effect on inflation rate is usually achieved after the implementation of monetary policies (Batini and Nelson 2001, Fama 1990). For example, z_t^{inf} with t =1970Q1 uses the inflation rate reading at the time 1970Q2.

inflation is negative ($\hat{\gamma}_{inf} < 0$). The likelihood of entry is negatively correlated with own and rival store competition, consistent with Igami and Yang (2016).

Model Parameters	Decision to Enter/Exit/Unchanged	
	GDP Model	Inflation Model
Own-store Competition (γ_{own})	-0.9285**** (0.0662)	-0.8480****(0.0640)
Rival-store Competition (γ_{rival})	-0.3827****(0.0363)	-0.3178****(0.0350)
Population (γ_{pop})	0.0870****(0.0187)	0.1143****(0.0185)
Property Value (γ_{pro})	0.0705***(0.0238)	0.0974****(0.0227)
Income (γ_{inc})	0.4082***(0.1243)	0.5703****(0.1170)
Real GDP (γ_{gdp})	0.2489****(0.0356)	-
Inflation Rate (γ_{inf})	-	-0.0654***(0.0233)
Cutoff 1 (c_1)	-2.7267(0.0907)	-3.1700(0.0946)
Cutoff 2 (c_2)	1.8674(0.0106)	1.8580(0.0101)
Number of Observations	93800	93800
Pseudo R^2	0.0729	0.0630

Table 1: Ordered probit regressions on entry/exit decisions with quarterly real GDP (inflation rate), population, income, and property value. The standard deviation error are in the parentheses. The symbols ****, ***, **, and * represent the significance level within 0.1%, 1%, 5%, and 10%, respectively.

In summary, this descriptive analysis of the data provides suggestive evidence that retail investment decisions are correlated with publicly observable macroeconomic indicators. However, such indicators may not offer a complete characterization about the underlying unobserved aggregate state.

In what follows, we use the model framework developed in early sections, which allows firms to base their decisions upon Bayesian beliefs about the aggregate state which evolves following a Markov process. As we shall show below such model reveals whether an industry is robust to changes in the macroeconomy. Our empirical analysis will focus on competition between the dominant firm (i.e., McDonald's) and a fringe of smaller firms (i.e. A&W, Wendy's, Burger King, Harvey's), similar to how Igami and Yang (2016) studied the industry. Those smaller firms are assumed to be homogeneous with the same strategies and pay-off structures. In the rest of the analysis we will differentiate the aforementioned five companies by McDonald's and others. Moreover, the local market size factors (including population, income, and property value) are discretized into quartiles.[37]

[37] One could argue that property value might serve as relevant information about broader aggregate conditions.

4.2 Estimates of Aggregate State Dynamics

The estimates of aggregate dynamics are shown in Table 2, where the state transition probabilities $(\eta_0^{gdp}, \eta_1^{gdp}, \eta_0^{inf}, \eta_1^{inf})$ are estimated with the GDP indicator (Table 2a), and with inflation indicator (Table 2b).[38] Despite using different indicators, the implied aggregate dynamics are similar. First, a state's self transition probability is materially larger than the probability of transitioning to a different state. Taking GDP results as an example (inflation indicator shows similar results), it is suggested by $\eta_0^{gdp} > 1 - \eta_0^{gdp} (\eta_1^{gdp} > 1 - \eta_1^{gdp})$ that the aggregate state tends to stay in a weak (strong) condition for the next quarter if it is experiencing a weak (or strong) condition in the current period, implying persistence of economy. Second, Canada's economy experienced more growth than contraction as $\eta_0^{gdp} < \eta_1^{gdp}$ during the chosen time frame (from 1970 to 2004). Based on our estimates, in the long term, the proportion of strong aggregate state constitutes 85.7% of historical data, significantly higher than the weak (14.3%). Our estimation of economic dynamics aligns with published data from the Council (Table 2c).

	$s_{t+1}^h = 0$	$s_{t+1}^h = 1$
$s_t^h = 0$	$\eta_0^{gdp} = 0.8499$	0.1501
$s_t^h = 1$	0.0251	$\eta_1^{gdp} = 0.9749$

(a) Transition probability matrix with GDP indicator.

	$s_{t+1}^h = 0$	$s_{t+1}^h = 1$
$s_t^h = 0$	$\eta_0^{inf} = 0.8501$	0.1499
$s_t^h = 1$	0.0251	$\eta_1^{inf} = 0.9749$

(b) Transition probability matrix with inflation indicator.

	$s_{t+1}^h = 0$	$s_{t+1}^h = 1$
$s_t^h = 0$	0.8007	0.1993
$s_t^h = 1$	0.0288	0.9712

(c) Transition probability matrix summarized from C.D. Howe Institute Business Cycle Council report.

Table 2: Uobserved aggregate state transition probability **P**. Note that $s_t^h = 0 (= 1)$ represents a weak (strong) state.

In view of model uncertainty, the accuracy of GDP (inflation) indicator is captured by observation probability $\omega^{gdp} (\omega^{inf})$, whose estimates are shown in Table 3. The result shows that neither of the two indicators is perfect and each indicator comes with some level of uncertainty; hence there is always a positive probability that the actual observation of

For our analysis, we do not incorporate property values into firm beliefs about the aggregate conditions, as past research has viewed property values as factors that impact *consumer*, not firm, sentiment (Chahrour and Gaballo 2021).

[38] The transition probabilities of local market size factors are also fitted to data and can be found in Appendix E.

the indicators fails to match the "true" aggregate state, corroborating the need of using a partially observable model. Furthermore, we observe that GDP indicator performs better in describing the aggregate state as it has a smaller estimated false probability of 0.0655, compared to that of the inflation indicator being 0.1619. These patterns are consistent with widely held view that GDP is a primary economic indicator for its wider coverage of the whole economy as compared to the inflation indicator.

	$z_t^{gdp}=0$	$z_t^{gdp}=1$
$s_t^h=0$	0.9345	$\omega^{gdp}=0.0655$
$s_t^h=1$	0.0655	0.9345

(a) GDP indicator: $z_t^{gdp}=0(=1)$ indicates a negative (positive) growth of GDP.

	$z_t^{inf}=0$	$z_t^{inf}=1$
$s_t^h=0$	0.8381	$\omega^{inf}=0.1619$
$s_t^h=1$	0.1619	0.8381

(b) Inflation indicator: $z_t^{inf}=0(=1)$ indicates a high (mild) inflation.

Table 3: Observation probability matrix $\mathbf{O}$, where $s_t^h=0(=1)$ represents a weak (strong) state.

4.3 Estimates from Payoff Functions

Anecdotes indicate that while economic downturns may severely hurt many industries, its impact on the fast-food industry appears limited (Creswell 2022, Ritchie 2010, Rivas 2022). Motivated by these observations, our estimation results aim to explore these conjectures more comprehensively in a quantitative manner (Table 4).

Our results suggest that the apparent resilience to weak aggregate conditions is a manifestation of the positive base profits under both aggregate conditions, and the base profit under weak state is close to that under strong state ($\theta_{base,s} \simeq \theta_{base,w} \geq 0$) for all fast-food chains.[39] In addition, the fast-food industry appears to perform well during strong economy, as shown in the second row of Table 4, where the slightly increased base profit $\theta_{base,s}$ under strong condition compared to the base profit $\theta_{base,w}$ under weak condition encapsulates an improved profitability of the fast-food industry in an economic upturn.

As the true aggregate state is not directly observable, firms might base their investment decisions on their beliefs about the aggregate state. Our estimation is consistent with this assertion, as reported in Figure 5. First, from the figure on the left, we find that periods with a high belief of weak state are largely aligned with the four reported economic recessions,

[39] A positive weak state base profit implies a positive margin profit under recession, making new entries viable from profitability perspective. Besides, the closeness between base profits under two aggregate states shrinks the profitability gap among aggregate conditions.

Model Parameters	GDP Model		Inflation Model	
	MCD	Others	MCD	Others
Base Profit in Weak State ($\theta_{base,w}$)	1.9211	1.4748	1.5632	1.3848
	(0.0008)	(0.0011)	(0.0032)	(0.0017)
Base Profit in Strong State ($\theta_{base,s}$)	2.015	1.563	1.8483	1.7215
	(0.0030)	(0.0024)	(0.0035)	(0.0017)
Own-store (θ_{own})	-0.4746	-0.4414	-0.4789	-0.4825
	(0.0026)	(0.0040)	(0.0031)	(0.0048)
Rival-store (θ_{rival})	-0.0666	-0.1172	-0.0604	-0.138
	(0.0054)	(0.0064)	(0.0066)	(0.0070)
Population (θ_{pop})	0.061	0.0134	0.0652	0.0061
	(0.0069)	(0.0046)	(0.0084)	(0.0049)
Property Value (θ_{pro})	0.0569	0.0326	0.0628	0.0245
	(0.0042)	(0.0049)	(0.0058)	(0.0048)
Income (θ_{inc})	0.0628	0.0379	0.0621	0.0278
	(0.0076)	(0.0040)	(0.0094)	(0.0045)
Net Entry Sunk Cost (θ_{entry})	12.7440	10.3903	10.9158	10.3253
	(0.0004)	(0.0002)	(0.0003)	(0.0006)

Table 4: Estimation results of payoff function on Canada's fast-food market (standard deviation in parenthesis).

where the dotted areas are the economic recessions reported by the Council and the shaded areas indicate a high belief of weak state (over 80%).

Interestingly, we observe that highly-believed weak state periods do not always fully cover the identified economic recessions by the Howe Council's report; see Figure 4. For example in 1980 and 1990-1992 recessions, the belief of a weak state occurs only near the ending and the beginning of the two Council reported recessions, respectively, and the believed weak condition duration is shorter than the recession duration for general markets, which may indicate that fast-food industry's capability to survive (lastly affected than general market as in 1980 recession) and to thrive (faster-than-average recovery as in 1990-1992) during weak economic condition.[40]

Notably, the left figure (of Figure 5) *empirically* finds that the probability of new entry decreases as the belief of weak state increases: during the extended believed weak state (shaded areas), the fast-food chains' entry decreases (though still net positive). The right

[40]It may be worthwhile to point out that the recession occurred in 1980, which was originally included in 2012 version of the Council's reports, but was later removed in its 2021 version. Our model calculating only a very short period of highly believed weak state in 1980 provides a possible account for such removal.

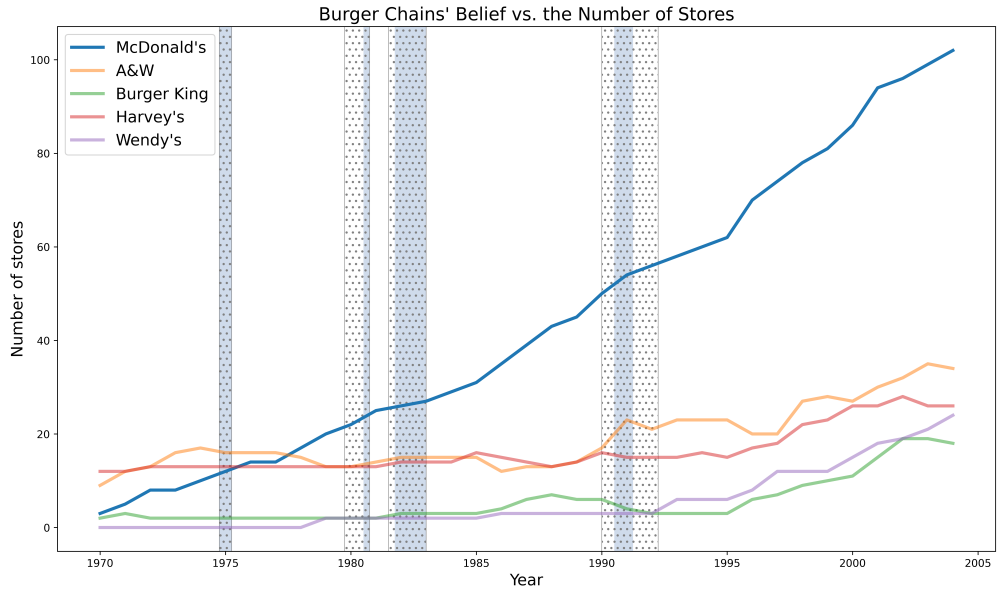


Figure 4: Aggregate fluctuations vs. fast-food chains' belief evolution. The dotted areas are recession time periods defined by the *C.D. Howe Institute Business Cycle Council*. The shaded areas are the time periods when fast-food chains' belief of weak state is above 80%.

figure from our estimated policy *quantitatively* reflects this trend: an increase in the belief of weak state is accompanied by a decrease of the likelihood for a fast-food chain to open new stores. Moreover, while the trend is the same for both McDonald's and others, McDonald's has a notably higher probability of entry across the whole spectrum of belief, attesting to its higher growth rate.

While fast-food chains' investment decisions are generally aligned with their beliefs about the aggregate conditions, we find that within fast-food industry, different firms may adopt diverse investment strategies in response to aggregate fluctuations. In particular, our estimated model (Table 4) captures McDonald's advantage with base profits: McDonald's base profit θ_{base}^{mcd} is notably larger than other fast-food chains θ_{base}^{others} under various aggregate conditions. A larger base profit, *ceteris paribus*, increases the marginal profitability for new stores and thus encourages more market entries, corroborating the empirical data of McDonald's faster growth as in Figure 4.

Moreover, our model finds that in most cases, estimates of own-brand store competition cost θ_{own} and between-brands competition cost θ_{rival} are negative across the board, indicating business stealing effects from competition (Table 4, row 3-4).[41] Moreover, our estimation of

[41] Notably, our results hint that McDonald's is less sensitive to the competition compared to its rivals. To see this, taking GDP model for an example, McDonald's has a lower rival cost (less negative) $\theta_{rival}^{mcd} = -0.0774$ than that for other stores $\theta_{rival}^{others} = -0.1044$. This observation confirms McDonald's leading position and brand recognition in the fast-food market.

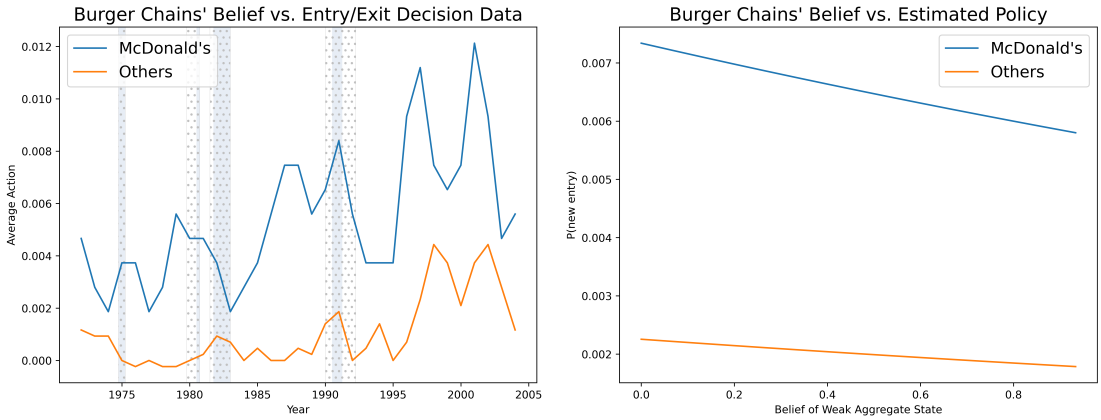


Figure 5: Canada fast-food chains' entry/exit decisions are affected by the belief of aggregate states. Left figure: (8-quarter) Moving average of action (*exit*(-1), *status quo*(0), *entry*(1)). The dotted areas are recession periods reported by *C.D. Howe Institute Business Cycle Council*. The shaded areas are the time periods when fast-food chains' belief of weak state is above 80%. Right figure: The estimated policy in terms of the probability of new entry against the belief of weak state.

net entry sunk costs is consistent with Igami and Yang (2016), suggesting that McDonald's incurs a higher investment cost to open a new store compared to its competitors (Love 1995). Yet, such larger sunk cost can potentially be offset by the significantly its higher base profit.

Finally, our results show that demographic variables appear to positively affect all fast-food chain investment decisions, implying that a larger population, higher income, and higher property value all facilitate a fast-food chain's expansion, in general.

4.4 Estimated Model Validation and Comparison

To validate our model, we conduct the following tests: (1) We test the prediction power of the estimated model, results show that our model can closely track firms' investment decisions in the dataset. More details can be found in Appendix C. (2) We compare our model with a benchmark model (e.g., Igami and Yang (2016)), where aggregate state is considered observed: we use macroeconomic indicator reading as if it were the true aggregate state. For example, by observing a positive GDP growth (i.e., $z^{gdp} = 1$) we set aggregate condition to be a strong state (i.e., $s^h = 1$). We show that the benchmark model yields an average 13.64% larger model prediction error than our model when tested on the dataset. Related information on benchmark model formulation, estimation, and comparison are in Appendix D.

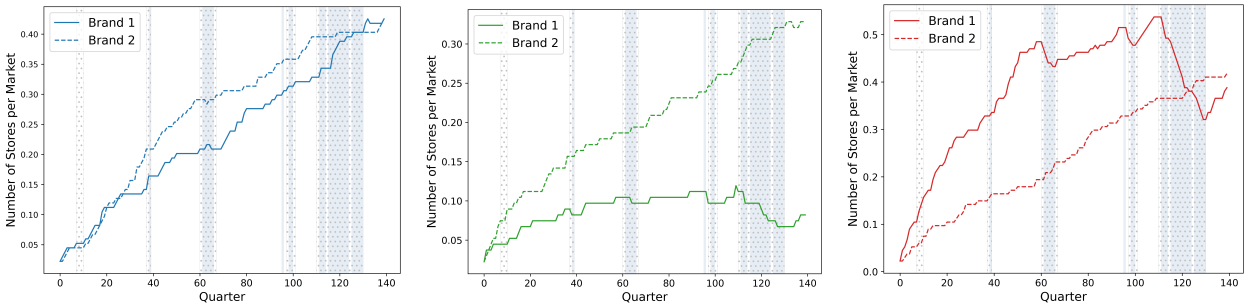
4.5 Implications of Aggregate Fluctuations

For our simulation analysis with the estimated model, we employ a two-firm model with GDP economic indicator and its corresponding dynamics (Table 2-3), as well as the estimated local market size factor dynamics in Appendix E. Before proceeding, we do note an important caveat: although the estimated model involves competition between five chains, our simulations will involve two firms. We model a firm (i.e., brand 1) with McDonald's estimates, and vary certain parts of brand 1's model across hypothetical settings. The other firm's (i.e., brand 2's) model is fixed (to McDonald's estimates), serving as a controlled reference. Here, two cases are considered. In case 1, we address only the effects of base profit under weak aggregate state (i.e., $\theta_{base,w}$). To this end, the brand 1's weak state base profit $\theta^1_{base,w}$ is designed to decrease by 60% proportionally to McDonald's recession base profit estimate, emulating brand 1's decisions under extreme cases such as a major recession.

In reality, the proportion of the strong aggregate state is significantly longer than the weak state on average, as shown in Table 2. Hence, the base profit under strong state may be critical in understanding a firm's strategic entries/exits. In case 2, we introduce the effects of base profit in a strong state (i.e., $\theta_{base,s}$). To this end, we set brand 1's strong state base profit $\theta^1_{base,s}$ to increase by +15% while its weak state base profit is fixed to 20% of McDonald's counterpart. In addition to the above two cases, we also provide a baseline case for comparison, in which both brand 1 & 2 use McDonald's estimates as their payoff functions.

We initialize the local market size factor s^o from the 1970Q1 data in the Canada fast-food dataset and perform forward simulations. One simulation instance of payoff relevant states (i.e., observed and unobserved states) is generated using the estimated dynamics for 140 quarters (35 years) for all 134 geographical markets. In order to generate the simulated firms' investment decisions, we obtain equilibrium retail investment decisions under different scenarios via the value function iteration.

The results of a simulation instance are summarized in Figure 6. This specific instance contains multiple weak aggregate state periods with various lengths to emulate the actual economic conditions. Starting with the baseline case where brand 1 and brand 2 share the same pay-off function as McDonald's (Figure 6a), we do not find significant long-term discrepancy between blue dashed and solid lines, verifying the comparable outcome of the two brands with identical payoff function. In case 1 (Figure 6b), facing profit declines under weak state, brand 1 (green solid line) fails to expand consistently, which hands over the market share to its rival brand 2 (green dashed line) especially during weak condition, and brand 2 enjoys relatively stable growth in the long run. When comparing the green solid



(a) Baseline case: both brands' payoff functions are from McDonald's estimates.

(b) Case 1: brand 1 has lower recession base profit.

(c) Case 2: brand 1 has lower (higher) weak (strong) state base profit.

Figure 6: Simulation with two-firm models with different $\theta_{base,w}$ and $\theta_{base,s}$ for brand 1. y-axis is the average number of stores per market. The dashed lines are generated by brand 2s. The dotted areas are for the simulated periods with weak aggregate state, while the shaded areas for the periods with a belief of weak stateover 80%.

line, with the fixed competitor's payoff function, the brand with lower recession base profit leads to slower growth rate.

The market structure of the two brands described in Case 2 is shown in Figure 6c (red lines). The business expansion (in terms of store number) of brand 1 (red solid line) with large strong-state base profit and small weak-state base profit is volatile. The brand 1 tends to open more stores as compared to the corresponding brand 2 during a strong economy, but many of these establishments fail to survive a weakened economy. In the long run, the brand 1 may not necessarily be able to maintain a strong position in the competition with the corresponding brand 2, which depends on how aggregate condition unfolds. Prolonged recessions (or even a single one) may leave the brand 1 with a permanent decrease of their market share in the longer run (i.e., the decreasing red solid line during the elongated weak state around 120th quarter).

The aforementioned experiment only shows how business sensitivity affects firm investment and/or market share *within one experiment instance*. To account for the stochastic nature of the evolving state, in the following set of experiments, we repeat the previous experiment with Monte Carlo simulations. Further, we consider a two-firm competition in a hypothetical weaker macroeconomy (i.e., η_0 trends larger and/or η_1 trends smaller), compared to the benchmark economic dynamics estimated from Canada economy data from 1970 to 2004. To this end, we modify the aggregate state transition probabilities with inflation indicator to $\eta_0 = 0.925$, whose original estimate is 0.85. Note that $\eta_1 = 0.975$ is fixed to its estimate. Such modification creates a weaker aggregate dynamics with 25% long-term weak state, compared to the benchmark economy (estimated from Canada's data) with 14% long

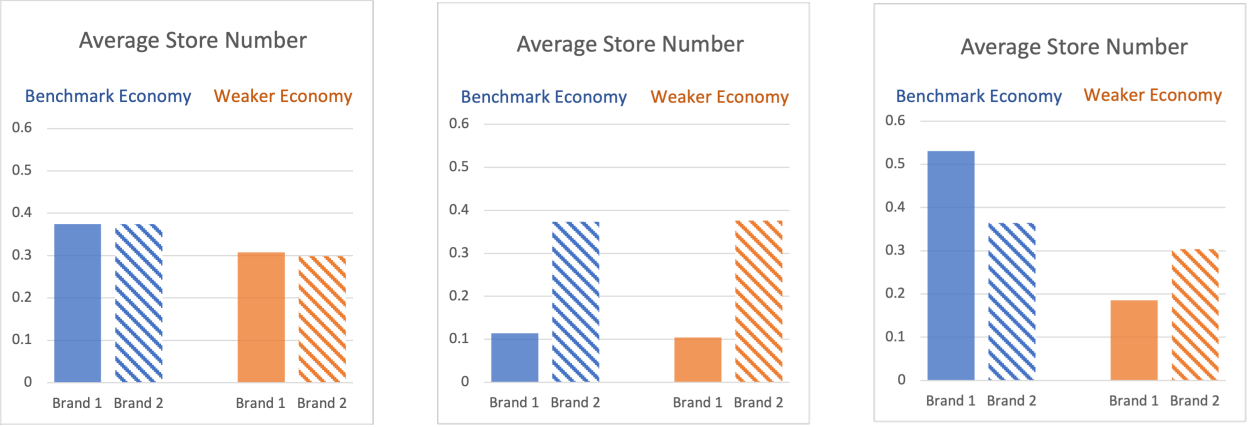
term weak state.

Under the benchmark economy with transition probability estimates in Table 2, and under the weaker economy, Monte Carlo simulations are conducted 50 times, where each trial includes a 140-quarter horizon on all 134 markets for all aforementioned three cases (i.e., two firm with identical McDonald's payoff function, firm 1 with smaller weak state base profit, firm 1 with smaller weak state profit and larger strong state base profit). The initialization of the local market size factors s^o follows from our earlier discussion.

We summarize the averaged change of store numbers for all three cases in different macroeconomies in Figure 7. For comparison purposes, we normalize the number of stores by subtracting the average number of stores (over 134 markets) at time $t = 0$ from that number at time $t = 139$, and then average the resulting difference over 50 simulations. In the reference case (Figure 7a), brand 1 & 2 have similar store number growth in both economies, although they perform consistently worse in the weaker economy (orange group) compared to the benchmark economy (blue group). Figure 7b shows that the brand 1 with reduced weak state base profit is always outperformed by the corresponding brand 2 in both economies and the discrepancy can be even larger in the weaker economy. Figure 7c indicates that a weak economy has a significant impact on the firm that is more sensitive to weak states (case in point, brand 1 with a low weak state base profit and a high strong state base profit). In the benchmark economy (the blue group), brand 1 has 42% ($= \frac{0.53-0.36}{0.36} \times 100\%$) *more* stores on average than the corresponding brand 2. However, this competitive advantage dissipates as the economic landscape changes. In the weaker economy, brand 1 has only 60% of the average number of stores relative to the corresponding brand 2, which experiences a relatively minor decrease in the weaker economy. The results show that merely experiencing an increase in strong state base profit may not suffice for a firm to sustain its market share in weaker economy.

Unlike typical macro-finance models of firm investment under uncertainty, our framework allows for strategic interactions between competitors.[42] This capability thus allows us to investigate the effect of aggregate state and its volatility on market structure dynamics. Here, we consider the following two-firm game, where the aggregate state transition probabilities (i.e., η_0, η_1) vary while the long-term proportion of recession and expansion is fixed to 25% and 75%, respectively. By changing values for the pair (η_0, η_1), one may induce different levels of aggregate volatility. For instance, smaller η_0 represents less persistence in weak state: with fixed long-term proportions, smaller η_0, implying smaller η_1, leads to more aggregate state transitions, and hence, higher aggregate volatility. The payoff functions

[42] Some exceptions include Fang and Yang (2023b) and Grenadier (1996), where they demonstrate the phenomenon of overinvestment due to strategic concerns (e.g., preemptive-type motivations).



(a) Both brands' payoff functions are from McDonald's estimates.

(b) Brand 1 has smaller weak state base profit.

(c) Brand 1 has smaller (larger) weak (strong) state base profit.

Figure 7: The change of average store numbers for different pairs of two-firm competition under benchmark and weaker economic environments. Blue bars are associated with benchmark economy (estimated from Canada economic data). Orange bars represent a weaker economy. Solid bars are generated by brand 1, while the hatched bars by brand 2.

of firm 1 and firm 2 are derived from McDonald's estimates: firm 1 has reduced weak-state base profit ($\theta^1_{base,w} = 0.2\theta^{mcd}_{base,w}$) and increased strong-state base profit ($\theta^1_{base,s} = 1.15\theta^{mcd}_{base,s}$); other than the base profits, firm 1 has the estimated McDonald's payoff function. Firm 2 has the estimated McDonald's payoff function. With these primitives of the model, we proceed with the Monte Carlo simulation for all 134 geographical markets over a 35-year horizon, and we denote by n^1, n^2 the store numbers for firm 1 and firm 2 at the end of the 35th year. The market share (in terms of establishment counts) for firm 1 can be then calculated as $\frac{n^1}{n^1+n^2} \times 100\%$.

The results are summarized in Figure 8. From the figure on the left, higher volatility leads to a decline in the average store number (at the end of the 35-year simulation horizon) of both firms, consistent with our earlier finding about the volatility-induced chilling effects on firm investment. The new insight that emerges from this simulation is that high volatility appears to have more impact on the firms with high growth (during strong state) and low weak state resistance (e.g., firm 1) than resilient firms (e.g., firm 2). In other words, McDonald's would gain market presence over a hypothetical competitor that is more sensitive to expansionary and recessionary aggregate conditions. Ultimately, uncertainty has the double effects of dampening firm investments as well as reinforcing the resilient firm's market presence.

Our last set of simulations aim to compare the sensitivity of investment decisions by McDonald's versus its competitors. We replicate the analysis in the previous simulation, where the main difference is that firm 1's estimated payoff function is based on the estimates

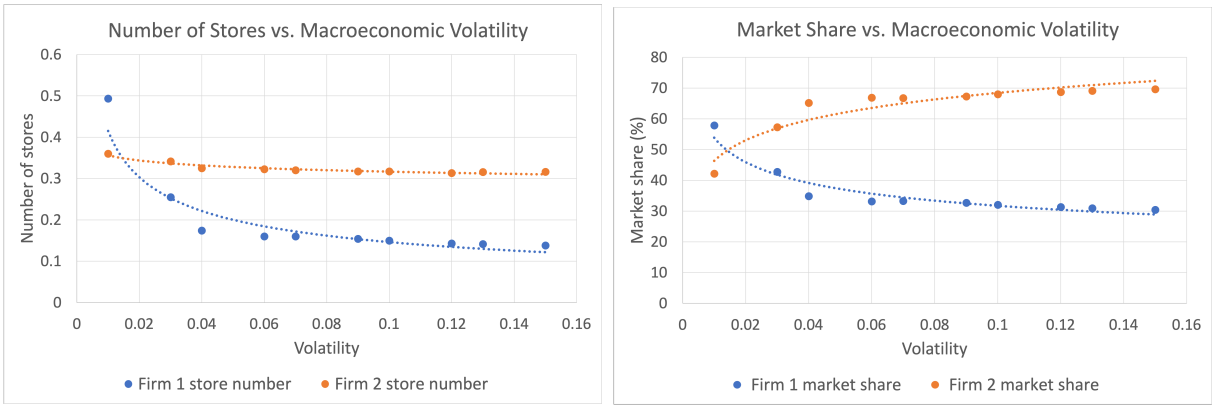


Figure 8: The effects of aggregate volatility on firm's investment. x-axis represents volatility measured by $\mathbb{P}(\text{weak state}|\text{strong state})(=1-\eta_1)$. Left figure describes the relationship between volatility and average store numbers while the right figure shows the impact of volatility on market share.

of the other chains. As we consider a two-firm model, one may view this exercise as capturing the fast food investment dynamics for local markets where McDonald's faces competition from only one of the chains (which is certainly the case in many geographic markets).

The simulation results from this comparison between McDonald's and the other chain are presented in Figure 9. These plots show that an increase in volatility is associated with a slight increase in McDonald's share of outlets (i.e., 84% to 86%). This finding suggests that uncertainty associated with aggregate fluctuations might have subtle benefits to McDonald's, with respect to market presence.

Summary. We end this section by summarizing the key results from our counterfactual simulation analysis. First, we show that dampened sensitivity (i.e., resilience) to aggregate fluctuations contributes to sustained retail investment and business expansion. Second, we show that low (high) volatility in the aggregate conditions can induce increased (decreased) retail investment, consistent with insights about investment under uncertainty from macro-finance. Finally, we highlight a plausible scenario in which McDonald's, the market leader, sustains its market presence even in high volatile aggregate conditions.

5 Conclusion

We present a novel framework to study the effects of aggregate fluctuation on firm investment in retail. Non-parametric identification of our model uncover situations where (1) firms' beliefs are heterogeneous, and (2) firms have a common belief with long the panel data. We apply our framework to develop an empirical case study about the Canadian fast-

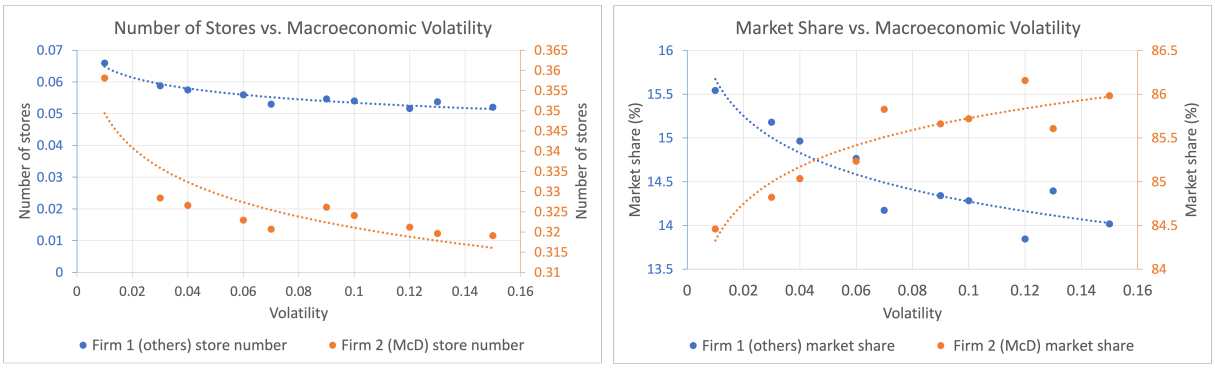


Figure 9: The effects of aggregate volatility on firm's investment. x-axis represents volatility measured by $\mathbb{P}(\text{weak state}|\text{strong state})(= 1 - \eta_1)$. Left figure describes the relationship between volatility and average store numbers while the right figure shows the impact of volatility on market share. Firm 2 uses McDonald's estimates, while firm 1 uses others' estimates.

food market. Analysis of this industry's dynamics highlights that the fast-food chains are resilient to economic downturn, and especially so for McDonald's. Moreover, we highlight the connection between dampened investment and aggregate volatility, which suggests that retailer investment strategies conform with the traditional understanding about firm investment under uncertainty (Dixit and Pindyck 1994). We anticipate that our framework can be extended to address settings of strategic firm investment, where some state variables are unobserved to both firms and researcher.

Our current framework fits most closely with dynamic oligopolistic competition (i.e., retail sectors with a few dominant firms). It might be interesting to incorporate innovations from POSGs to non-MPE models with many firms, such as partially oblivious equilibrium models of competition (Benkard et al. 2015). By extending our results to this setting, future research can explore the extent to which aggregate fluctuation disproportionately benefits (or hurts) fringe versus dominant firms. For example, Chi (2023) shows that large and small retailer investment outcomes may be impacted differently to variability in market information quality.

As our framework allows for firm beliefs to transcend across geographic markets, future work can aim to incorporate retail network effects into the investment decision process under aggregate fluctuations, as investment decisions in one market might impact the cost (or returns) for other markets. For example, retailer investments might be driven in part by the benefits from density and scale (Ellickson et al. 2013, Ergin et al. 2022, Holmes 2011, Jia 2008); but these scale efficiencies may at the same time be hindered (or accelerated) by broader aggregate conditions (e.g., cost of capital, customer sentiment).

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Online Appendix (Not for Publication)

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A Proofs

A.1 Fixed Point of Bellman Operator

Assume $\mathcal{B}$ is the Banach space consisting of bounded and Borel measurable functions Q for firm $i : (\mathcal{H}^i \times S^o \times A) \to \mathbb{R}$ with supremum norm $||\cdot||$. Let $a_t = a, s_t^o = s^o, h_t^i = h^i, a_{t+1} = \tilde{a}, s_{t+1}^o = \tilde{s}^o, h_{t+1}^i = \tilde{h}^i, z_{t+1} = \tilde{z}, \epsilon_{t+1} = \tilde{\epsilon}$. We define the Bellman operator $H_{\theta,\pi^{-i}} : \mathcal{B} \to \mathcal{B}$ as:

$$[H_{\theta,\pi^{-i}}Q](h^i, s^o, a^i) = r(x^i, s^o, a^i) + \beta\mathbb{E}[V_{\theta,\pi^{-i}}(\tilde{h}^{-i}, \tilde{s}^o, \tilde{\epsilon})], \tag{18}$$

where the expectation is taken with respect to $\tilde{z}, \tilde{s}^o, h^{-i}, a^{-i}$, and $\tilde{\epsilon}^i$.

The argument used in the proof is similar to Theorem 3.2 in Rust (1994). Assume the following regularity conditions:

R.1 (Bounded Upper Semicontinuous) The payoff function $r^i \in \mathcal{B}$. For each $a^i \in A$, $r^i(x^i, s^o, a^i)$ is upper semicontinuous in $S^o \times X \times A$ with bounded expectation and

$$\int \max_{a^i}\{r^i(x^i, s^o, a^i) + \epsilon^i(a^i)\}d\mu(\epsilon^i) < \infty.$$

R.2 (Weakly Continuous) The stochastic kernels $\{\mathbb{P}_\theta(\tilde{s}^o|s^o, a)\}_{\tilde{s}^o \in S^o}$ and $\{\sigma_\theta(z, x^i)\}_{z \in Z}$ are weakly continuous in $S^o \times A$ and X, respectively.

R.3 (Bounded Expectation) For each $Q \in \mathcal{B}$, $\mathbb{E}_{\theta,\pi^{-i}}Q \in \mathcal{B}$, where

$$[\mathbb{E}_{\theta,\pi^{-i}}Q](h^i, s^o, a^i) \triangleq \mathbb{E}_{h^{-i},a^{-i},\tilde{z},\tilde{s}^o,\tilde{\epsilon}^i}[V_{\theta,\pi^{-i}}(\tilde{h}^i, \tilde{s}^o, \tilde{\epsilon}^i)].$$

Under these regularity conditions, $H_{\theta,\pi^{-i}}$ is well defined. Thus, $\forall i \in \{1, ..., I\}, \forall Q, Q' \in \mathcal{B}$, Define the supremum norm

$$||Q - Q'|| \triangleq \max_{h^i,s^o,a^i} |Q(h^i, s^o, a^i) - Q'(h^i, s^o, a^i)|.$$

For any $s^o \in S^o, h^i \in \mathcal{H}^i, a^i \in A$, we have:

$$
\begin{aligned}
&|H_{\theta,\pi^{-i}}Q(h^i,s^o,a^i) - H_{\theta,\pi^{-i}}Q'(h^i,s^o,a^i)| \\
&\leq \beta \mathbb{E}_{h^{-i},a^{-i},\tilde{z},\tilde{s}^o,\tilde{\epsilon}^i}\left[\int \Big|\max_{\tilde{a}^i\in A}\{Q(\tilde{h}^i,\tilde{s}^o,\tilde{a}^i)+\epsilon^i(\tilde{a}^i)\} - \max_{\tilde{a}^i\in A}\{Q'(\tilde{h}^i,\tilde{s}^o,\tilde{a}^i)+\epsilon^i(\tilde{a}^i)\}\Big| d\mu(\epsilon^i)\right] \\
&\leq \beta \mathbb{E}_{h^{-i},a^{-i},\tilde{z},\tilde{s}^o,\tilde{\epsilon}^i}\left[\int \max_{\tilde{a}^i\in A}\Big|Q(\tilde{h}^i,\tilde{s}^o,\tilde{a}^i) - Q'(\tilde{h}^i,\tilde{s}^o,\tilde{a}^i)\Big| d\mu(\epsilon^i)\right] \\
&\leq \beta \mathbb{E}_{h^{-i},a^{-i},\tilde{z},\tilde{s}^o,\tilde{\epsilon}^i}||Q-Q'|| \leq \beta||Q-Q'||.
\end{aligned}
$$

Hence

$$
||H_{\theta,\pi^{-i}}Q - H_{\theta,\pi^{-i}}Q'|| \triangleq \max_{h^i,s^o,a^i}|H_{\theta,\pi^{-i}}Q(h^i,s^o,a^i) - H_{\theta,\pi^{-i}}Q'(h^i,s^o,a^i)| \leq \beta||Q-Q'||.
$$

Then for $i = 1, 2, ..., I$, $H_{\theta,\pi^{-i}}$ is a contracting operator and there exists an unique fixed point $Q^*_{\theta,\pi^{-i}}$ such that $H_{\theta,\pi^{-i}}Q^*_{\theta,\pi^{-i}} = Q^*_{\theta,\pi^{-i}}$.

A.2 Proof of Existence of Markov Perfect Equilibrium

Let $Q \in \mathcal{B}$, and define the best response strategy as:

$$
BR^i_Q(a^i|h^i,s^o) \triangleq \mathbb{P}\big(a^i \in \arg\max_{\tilde{a}^i\in A}\{Q(h^i,s^o,\tilde{a}^i)+\epsilon^i(\tilde{a}^i)\}\big)
$$

for all $i \in \{1,\ldots,I\}$. We introduce the definition of Markov Perfect Equilibrium, along the lines of Ericson and Pakes (1995):

Definition 2. The CCP combination $\{\pi^i\}_{i=1}^I$ is a Markov Perfect Equilibrium (MPE) if and only if for $Q^*_{\theta,\pi^{-i}}$ satisfying Eq. (8),

$$
\pi^i(a^i|h^i,s^o) = BR^i_{Q^*_{\theta,\pi^{-i}}}(a^i|h^i,s^o) \tag{19}
$$

for all $a^i \in A, h^i \in \mathcal{H}^i, s^o \in S^o, i \in \{1.\ldots,I\}$.

The argument used in the proof is similar to Theorem 1 in McKelvey and Palfrey (1995). Let $Q = \{Q^i\}_{i=1}^I$, where $Q^i \in \mathcal{B}$, $\pi = \{\pi^i\}_{i=1}^I$ and $BR_Q = \{BR^i_Q\}_{i=1}^I$. Let Δ_A denote the

simplex on $\mathbb{R}^{|A|}$. Consider the map $g : \mathcal{B}^I \times \Delta_A^I \mapsto \mathcal{B}^I \times \Delta_A^I$ defined as follows:

$$g(Q, \pi) = (H_{\theta,\pi} Q, BR_{H_{\theta,\pi}Q}).$$

Since g is continuous and the domain is compact and convex, by Brouwer's fixed point theorem, there exists a fixed point such that $Q^*_{\pi_\theta} = H_{\theta,\pi_\theta} Q^*_{\pi_\theta}, \pi_\theta = BR_{H_{\theta,\pi_\theta}Q^*_{\pi_\theta}}$. Such fixed point satisfies Eq.(9) if ϵ^i is i.i.d with Gumbel distribution. To see this, for $i = 1, 2, ..., I$,

$$\begin{aligned}
\pi^i_\theta(a^i|s^o, h^i) &\triangleq \frac{\partial}{\partial Q^*_{\pi_\theta^{-i}}(h^i, s^o, a^i)} \mathbb{E}_{\epsilon^i}[V(h^i, s^o, \epsilon^i)] \\
&= \frac{\partial}{\partial Q^*_{\pi_\theta^{-i}}(h^i, s^o, a^i)} \log\Big(\sum_{\tilde{a}^i \in A} \exp Q^*_{\pi_\theta^{-i}}(h^i, s^o, \tilde{a}^i)\Big) \\
&= \frac{\exp Q^*_{\pi_\theta^{-i}}(h^i, s^o, a^i)}{\sum_{\tilde{a}^i \in A} \exp Q^*_{\pi_\theta^{-i}}(h^i, s^o, \tilde{a}^i)},
\end{aligned}$$

where the closed-form solution $\mathbb{E}_{\epsilon^i}[V(h^i, s^o, h^i)] = \log\Big(\sum_{\tilde{a}^i \in A} \exp Q^*_{\pi_\theta^{-i}}(h^i, s^o, \tilde{a}^i)\Big)$ follows from the distributional assumption of ϵ^i.

A.3 Proof of Theorem 1

Condition (14) implies the *Canonical Polyadic Decomposition* (CPD) in Eq.(13) is essentially unique (Kolda and Bader 2009, Kruskal 1977). Note that the left hand side of Eq.(13) is directly available from data and the prior of unobserved state ρ. Thus, the observation probability matrix $\mathbf{O}$ (on the right hand side of Eq.(13)) is uniquely determined up to row permutation and rescaling, provided Eq.(14) is satisfied. Moreover, $\mathbf{O}$ is a stochastic matrix, i.e., each row of $\mathbf{O}$ sums up to 1. Hence, $\mathbf{O}$ is identified up to row permutation (row rescaling ruled out).

A.4 Proof of Theorem 2

Denote two consecutive periods of observations z_t, z_{t+1} by $z, \tilde{z}$, respectively. We define a square matrix $\mathbf{S}_{z,\tilde{z}}$, whose (i, j) element is the probability $\mathbb{P}(z = i, \tilde{z} = j)$. Assuming $s^h, \tilde{s}^h$

denote respectively the aggregate state at time $t, t+1$, we define another square matrix $\mathbf{S}_{s^h,\tilde{s}^h}$, whose (i,j) element is the probability $\mathbb{P}(s^h = i, \tilde{s}^h = j)$. We have

$$\mathbb{P}(z = i, \tilde{z} = j) = \sum_{s^h} \sum_{\tilde{s}^h} \mathbb{P}(z = i|s^h)\mathbb{P}(s^h, \tilde{s}^h)\mathbb{P}(\tilde{z} = j|\tilde{s}^h).$$

In matrix form, the above equation can be written as

$$\mathbf{S}_{z,\tilde{z}} = \mathbf{O}^\top \mathbf{S}_{s^h,\tilde{s}^h} \mathbf{O}. \tag{20}$$

Recall the observation probability matrix $\mathbf{O} \in \mathbb{R}_+^{|S^h| \times |Z|}$. Assuming $\mathbf{O}$ is full row rank, we consider the following left and right inverse matrices: the right inverse of $\mathbf{O}$ is $\mathbf{O}^\top(\mathbf{O}\mathbf{O}^\top)^{-\mathbf{1}}$ and the left inverse of $\mathbf{O}^\top$ is $(\mathbf{O}\mathbf{O}^\top)^{-1}\mathbf{O}$. Eq.(20) becomes

$$\mathbf{S}_{s^h,\tilde{s}^h} = [(\mathbf{O}\mathbf{O}^\top)^{-1}\mathbf{O}]\mathbf{S}_{z,\tilde{z}}[\mathbf{O}^\top(\mathbf{O}\mathbf{O}^\top)^{-\mathbf{1}}].$$

As $\mathbf{S}_{z,\tilde{z}}$ is obtainable from data and $\mathbf{O}$ has already been identified, $\mathbf{S}_{s^h,\tilde{s}^h}$, or the joint distribution of s_t^h, s_{t+1}^h can be derived. Consequently, the unobserved aggregate state transition probabilities can be calculated from $\mathbf{S}_{s^h,\tilde{s}^h}$. To see this, for $s^h, \tilde{s}^h \in S^h$:

$$\mathbb{P}(\tilde{s}^h|s^h) = \frac{\mathbb{P}(s^h, \tilde{s}^h)}{\sum_{\tilde{s}^h} \mathbb{P}(s^h, \tilde{s}^h)}.$$

A.5 Proof of Theorem 3

Assuming the dynamics $\theta_2 = \{\mathbf{F}, \mathbf{P}, \mathbf{O}\}$ is identified and firms' initial beliefs $\mathbf{x}_0$ are known, we could use Eq.(4) to transform each sample path and initial beliefs to a corresponding belief process

$$\{(s_t^o)_{t=0}^T, (a_t)_{t=0}^{T-1}, (\mathbf{x}_{t,\theta_2})_{t=1}^T, \mathbf{x}_0\}.$$

As $\{\mathbf{x}_{t,\theta_2}, t > 0\}$ can be recursively calculated, hence, fully "observed". Thus, the MPE policy for firm i $\pi_\theta^i(a_t^i|\mathbf{x}_{t,\theta_2}, s_t^o)$ can also be identified from the data $\hat{\mathbb{P}}(a_t^i|h_t, \mathbf{x}_0)$.

Treating belief as the state and since $|h_t| < \infty$, Proposition 1 in Hotz and Miller (1993) and Magnac and Thesmar (2002) show that there is a one-to-one mapping q, only depend-

ing on the distribution μ of payoff perturbation, which maps the choice probability set $\{\pi_\theta^i(a^i|s^o,\mathbf{x})\}_{a^i\in A}$ to the set of the difference in (fixed point) state-action value function $\{Q^*_{\pi_\theta^{-i}}(s^o,\mathbf{x},a^i) - Q^*_{\pi_\theta^{-i}}(s^o,\mathbf{x},a^{i,0})\}_{a^i\in A}$, for firm i,

$$Q^*_{\pi_\theta^{-i}}(s^o,\mathbf{x},a^i) - Q^*_{\pi_\theta^{-i}}(s^o,\mathbf{x},a^{i,0}) = q_{a^i}(\pi_\theta^i(a^i|s^o,\mathbf{x});\mu), \tag{21}$$

where $q_0(\cdot)=0$, and $q=(q_0,...,q_{|A|-1})$ and π_θ^i is the equilibrium strategy for firm i. Thus, if we know $Q^*_{\pi_\theta^{-i}}(s^o,\mathbf{x},a^{i,0})$, we can recover $Q^*_{\pi_\theta^{-i}}(s^o,\mathbf{x},a^i), \forall a^i\in A\backslash a^{i,0}$. Note that $r(s^o,x^i,a^i) = \sum_{s^h} x^i(s^h) r^i(s^o,s^h,a^i)$, then we have:

$$\begin{aligned}
Q^*_{\pi_\theta^{-i}}(s^o,\mathbf{x},a^i) &= r^i(s^o,x^i,a^i) + \beta\mathbb{E}_{a^{-i}|s^o,\mathbf{x}}\Big\{\mathbb{E}_{\tilde{s}^o,\tilde{z}|s^o,x^i,a}\Big\{\mathbb{E}_{\tilde{\epsilon}}\max_{\tilde{a}^i\in A}\big\{Q^*_{\pi_\theta^{-i}}(\tilde{s}^o,\tilde{\mathbf{x}},\tilde{a}^i)+\tilde{\epsilon}(\tilde{a}^i)\big\}\Big\}\Big\} \\
&= r^i(s^o,x^i,a^i) + \beta\mathbb{E}_{a^{-i}|s^o,\mathbf{x}}\Big\{\mathbb{E}_{\tilde{s}^o,\tilde{z}|s^o,x^i,a}\Big\{\mathbb{E}_{\tilde{\epsilon}}\max_{\tilde{a}^i\in A}\big\{Q^*_{\pi_\theta^{-i}}(\tilde{s}^o,\tilde{\mathbf{x}},\tilde{a}^i)+\tilde{\epsilon}(\tilde{a}^i)\big\} \\
&\quad - Q^*_{\pi_\theta^{-i}}(\tilde{s}^o,\tilde{\mathbf{x}},a^{i,0})\Big\}\Big\} + \beta\mathbb{E}_{a^{-i}|s^o,\mathbf{x}}\Big\{\mathbb{E}_{\tilde{s}^o,\tilde{z}|s^o,x^i,a}\big\{Q^*_{\pi_\theta^{-i}}(\tilde{s}^o,\tilde{\mathbf{x}},a^{i,0})\big\}\Big\},
\end{aligned}$$

where $\tilde{x}^i = \lambda_\theta(\tilde{z},x^i)$ and $\tilde{x}^{-i} = \lambda_\theta^{-i}(\tilde{z},x^{-i}), \tilde{\mathbf{x}} = (\tilde{x}^i,\tilde{x}^{-i})$.

Because of the mapping q in Eq.(21), and that all game dynamics, μ, and equilibrium strategies $\{\pi^i(a^i|s^o,\mathbf{x})\}_{i=1}^I$ are known (i.e., identifiable from data), the quantity

$$C = \beta\mathbb{E}_{a^{-i}|s^o,\mathbf{x}}\Big\{\mathbb{E}_{\tilde{s}^o,\tilde{z}|s^o,x^i,a}\Big\{\mathbb{E}_{\tilde{\epsilon}}\max_{\tilde{a}^i\in A}\big\{Q^*_{\pi_\theta^{-i}}(\tilde{s}^o,\tilde{\mathbf{x}},\tilde{a}^i)+\tilde{\epsilon}(\tilde{a}^i)\big\} - Q^*_{\pi_\theta^{-i}}(\tilde{s}^o,\tilde{\mathbf{x}},a^{i,0})\Big\}\Big\}$$

is identifiable. Without loss of generality, under the assumption of $r^i(s^o,s^h,a^{i,0})=0, \forall s^o\in S^o, s^h\in S^h$, we have

$$Q^*_{\pi_\theta^{-i}}(s^o,\mathbf{x},a^{i,0}) = C + \beta\mathbb{E}_{a^{-i}|s^o,\mathbf{x}}\Big\{\mathbb{E}_{\tilde{s}^o,\tilde{z}|s^o,x^i,a}\big\{Q^*_{\pi_\theta^{-i}}(\tilde{s}^o,\tilde{\mathbf{x}},a^{i,0})\big\}\Big\} \tag{22}$$

with only unknown $Q^*_{\pi_\theta^{-i}}(s^o,\mathbf{x},a^{i,0})$. It is easy to see there is a unique solution to Eq.(22) due to the contraction mapping theorem. Consequently, all $Q^*_{\pi_\theta^{-i}}(s^o,\mathbf{x},a^i), a^i\in A$ can be recovered by Eq.(21). Therefore,

$$r^i(s^o,x^i,a^i) = Q^*_{\pi_\theta^{-i}}(s^o,\mathbf{x},a^i) - C - \beta\mathbb{E}_{a^{-i}|s^o,\mathbf{x}}\Big\{\mathbb{E}_{\tilde{s}^o,\tilde{z}|s^o,x^i,a}\big\{Q^*_{\pi_\theta^{-i}}(\tilde{s}^o,\tilde{\mathbf{x}},a^{i,0})\big\}\Big\},$$

we can identify $r^i(x^i, s^o, a^i), \forall x^i \in X$.

A.6 Proof of Proposition 1

Let $\tilde{\mathbf{P}}_\theta(z) \in \mathbb{R}_+^{|S^h|\times|S^h|}$ be a probability matrix, whose (i,j) element is $\mathbb{P}_\theta(z_{t+1} = z|s^h_{t+1} = j)\mathbb{P}_\theta(s^h_{t+1} = j|s^h_t = i)$. Take the notation of *contraction coefficient* (coefficient of ergodicity) γ for $\tilde{\mathbf{P}}_\theta(z)$ and λ^τ in White and Scherer (1994),

$$\gamma(\tilde{\mathbf{P}}_\theta(z)) = \max_{i,j}\{\mathcal{D}(\lambda_\theta(z, e_i), \lambda_\theta(z, e_j)) : i, j \in S^h\}, \tag{23}$$
$$\lambda^\tau_\theta(z_{1:\tau}, x_0) = \lambda_\theta(z_\tau, \lambda_\theta(...\lambda_\theta(z_1, x_0))),$$

where $e_i \in X : e_i(i) = 1$, $z_{1:\tau} = \{z_1, \ldots, z_\tau\}, x_0 \in X$.

According to Section 2.3 in White and Scherer (1994), $\gamma(\tilde{\mathbf{P}}_\theta(z)) < 1, \forall z \in Z$ and

$$\mathcal{D}(\lambda^\tau_\theta(z_{1:\tau}, x^i_0), \lambda^\tau_\theta(z_{1:\tau}, x^j_0)) \leq \Big(\max_{z\in Z}\gamma(\tilde{\mathbf{P}}_\theta(z))\Big)^\tau, \forall x^i_0, x^j_0 \in X.$$

Note that $x^i_{\tau,\theta_2} = \lambda^\tau_{\theta_2}(z_{1:\tau}, x^i_0)$. Hence, $\forall x^i_0, x^j_0 \in X$,

$$\lim_{\tau\to\infty}\mathcal{D}(x^i_{\tau,\theta}, x^j_{\tau,\theta}) = \lim_{\tau\to\infty}\mathcal{D}(\lambda^\tau_\theta(z_{1:\tau}, x^i_0), \lambda^\tau_\theta(z_{1:\tau}, x^j_0)) = 0.$$

B Estimation with Long Synthetic Dataset

In this section, we test the estimation of our model on a synthetic dataset. We consider a two-firm model with the aggregate state dynamics parametrized by $\theta_2 = \{\eta_0, \eta_1, \omega\}$. Note that for testing purposes (i.e., estimating aggregate state dynamics and payoff functions), we do not include market size factors. The payoff structure is defined by $\theta_1 = \{\theta^1_1, \theta^2_1\}$, where for firm i, $\theta^i_1 = \{\theta^i_{base,w}, \theta^i_{base,s}, \theta^i_{own}, \theta^i_{rival}, \theta^i_{entry}\}, i = 1, 2$. The numerical settings for $\theta = \{\theta_1, \theta_2\}$ can be found in the Table A1.

For the given θ, we randomly generate panel data sequences with length $T = 100$. Each data sequence contains: a sequence of observable state $(s^o_t)^T_{t=0}$ (in this case, $s^o_t = (n^i_t, n^{-i}_t), \forall i$), macroeconomic indicator observation sequence $(z_t)^T_{t=1}$ and investment decision sequence for

each firm $(a_t^i)_{t=0}^{T-1}, i = 1, 2$. We consider the case where the initial belief is shared, and the case where the firms' initial beliefs are different.

	θ_2				θ_1^i				
	η_0	η_1	ω		$\theta^i_{base,w}$	$\theta^i_{base,s}$	θ^i_{own}	θ^i_{rival}	θ^i_{entry}
Shared x_0	0.93	0.99	0.05	$i = 1$	4	13.5	-2	-1	3
					(4.000)	(13.500)	(-2.000)	(-0.999)	(3.002)
	(0.930)	(0.990)	(0.050)	$i = 2$	3	12	-1	-2	2.5
					(2.999)	(12.000)	(-1.002)	(-2.001)	(2.505)
Different x_0	0.93	0.99	0.05	$i = 1$	4	13.5	-2	-1	3
					(3.999)	(13.499)	(-2.003)	(-1.001)	(3.003)
	(0.930)	(0.990)	(0.050)	$i = 2$	3	12	-1	-2	2.5
					(2.997)	(11.996)	(-1.010)	(-2.009)	(2.509)

Table A1: The estimation of our model with synthetic dataset. The ground truth of the model primitives θ_1, θ_2 are in the corresponding columns. The estimates of θ_1, θ_2 are in parenthesis.

We use the simulated data to perform estimation. The results are summarized in the Table A1, which show that the estimates of the model primitives are close to their true values, attesting to the effectiveness our estimation methodology.

C Model Predictive Power

We test the model's predictive power by simulations. We use our estimated model to perform rolling 8-quarter predictions from 1970 to 2004, and then we compare our model predictions with empirical data.

For a brand in a geographical market, we forward simulate the brand's number of stores at period $t+\tau$, based on the brand's history up to time t. For simulation purposes, we set $\tau = 8$ (8 quarters). The simulation is performed for each brand in every geographical market, with both GDP and inflation indicators. The results of the first numerical experiment are shown in Figure A1. As can be seen from the figure, our model can closely track the number of stores for all the firms in the Canada fast-food dataset, under different economic indicators.

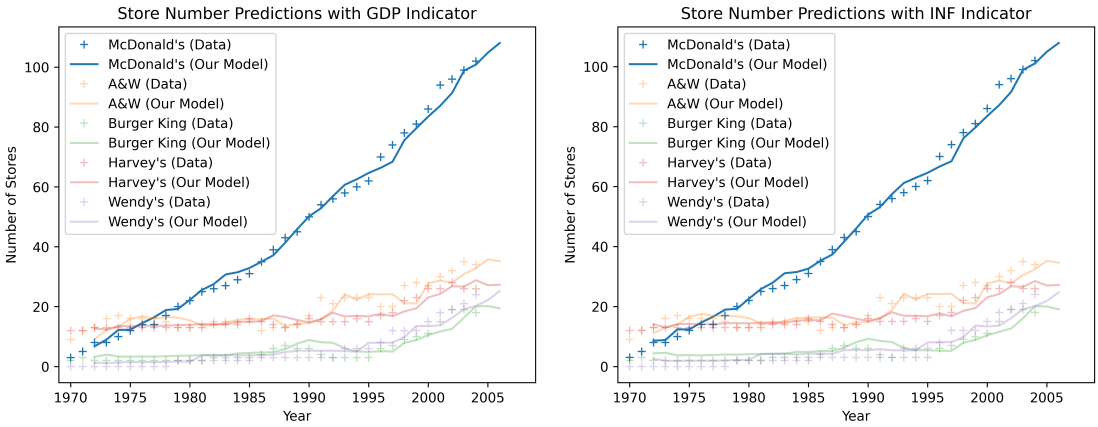


Figure A1: Simulation of store counts for the five burger chains. Left (Right) figure uses the estimated model with GDP (inflation) indicator. Crosses and lines represent our model's 8-quarter-ahead prediction of the chains' store number and real data, respectively.

D Benchmark (Observable State) Model

D.1 Benchmark Model Formulation and Estimation

We provide results from the benchmark model as proposed in Igami and Yang (2016), Blevins et al. (2018) to model strategic market entry under aggregate fluctuations. In this approach, the observation of economic indicators (e.g., GDP or inflation) is considered the "true" aggregate state, i.e., $s^h = z$. Hence, aggregate state is observable. For example, $z = 0$ suggests a weak aggregate state $s^h = 0$. Therefore, we have different aggregate state transition probabilities depending on which observation is used. The state transition probability matrices of aggregate state under different indicators are:

$$\mathbf{P}(s^h_{t+1}|s^h_t) = \begin{array}{c} \\ \textit{weak state}(0) \\ \textit{strong state}(1) \end{array} \begin{array}{c} \begin{array}{cc} \textit{weak state}(0) & \textit{strong state}(1) \end{array} \\ \begin{pmatrix} \eta_0^{gdp} & 1-\eta_0^{gdp} \\ 1-\eta_1^{gdp} & \eta_1^{gdp} \end{pmatrix} \end{array}, \text{or}$$

$$\mathbf{P}(s^h_{t+1}|s^h_t) = \begin{array}{c} \\ weak\ state(0) \\ strong\ state(1) \end{array} \begin{array}{c} \begin{array}{cc} weak\ state(0) & strong\ state(1) \end{array} \\ \begin{pmatrix} \eta_0^{inf} & 1-\eta_0^{inf} \\ 1-\eta_1^{inf} & \eta_1^{inf} \end{pmatrix} \end{array}.$$

The estimation results of aggregate state transitions under various indicators are summarized in Table A2. The estimation results of payoff function under various indicators are summarized in Table A3.

	GDP Model	Inflation Model
η_0	$\eta_0^{gdp} = 0.4783$	$\eta_0^{inf} = 0.7143$
η_1	$\eta_1^{gdp} = 0.8966$	$\eta_1^{inf} = 0.7101$

Table A2: State transition probabilities with observable aggregate state (i.e., $s^h = z$).

Model Parameters	GDP Model		Inflation Model	
	MCD	Others	MCD	Others
Base Profit in Weak State ($\theta^i_{base,w}$)	1.9210	1.1432	1.5085	1.3922
	(0.0003)	(0.0004)	(0.0013)	(0.0015)
Base Profit in Strong State ($\theta^i_{base,s}$)	1.9927	1.6911	1.9604	1.8855
	(0.0020)	(0.0021)	(0.0025)	(0.0025)
Own-store (θ^i_{own})	-0.4830	-0.4629	-0.4830	-0.4815
	(0.0027)	(0.0044)	(0.0027)	(0.0048)
Rival-store (θ^i_{rival})	-0.0773	-0.1092	-0.0825	-0.1170
	(0.0032)	(0.0051)	(0.0040)	(0.0047)
Population (θ^i_{pop})	0.0448	0.0184	0.0446	0.0158
	(0.0055)	(0.0049)	(0.0060)	(0.0046)
Property Value (θ^i_{pro})	0.0453	0.0381	0.0482	0.0359
	(0.0042)	(0.0039)	(0.0040)	(0.0043)
Income (θ^i_{inc})	0.0437	0.0413	0.0446	0.0385
	(0.0068)	(0.0041)	(0.0063)	(0.0038)
Net Entry Sunk Cost (θ^i_{entry})	12.7331	10.3952	10.6245	10.3633
	(0.0002)	(0.0004)	(0.0004)	(0.0005)

Table A3: Estimation results of benchmark model on Canada fast-food dataset (standard deviation in parenthesis).

D.2 Comparison with the Proposed (Unobserved State) Model

We compare our estimated model with the estimated benchmark model by simulation, aiming to reveal the importance of modeling unobserved aggregate state and the consequences of neglecting such unobserved feature by using the benchmark model. We perform simulations with both models. For illustration purposes, we consider inflation indicator for both models.[43]

For our model, we define the average decision (over 500 simulation instances) of brand i in market m at time t as $\bar{a}_{i,m,t}$. We further denote $a_{i,m,t}$ as the actual action implemented by brand i in market m at time t, which is directly obtained from the dataset. The average error (over different geographical market) between model prediction and data is defined by $e_{i,t} \triangleq \frac{1}{M}\sum_m(\bar{a}_{i,m,t} - a_{i,m,t})^2$, where M is the total number of geographical markets. The total action error is the sum of action errors for all brands, denoted by $e_t = \sum_i e_{i,t}$. We further define $\tilde{e}_t = \sum_{\tau=1}^{t} e_\tau$ to quantify the cumulative error up to time t. For benchmark model, we define the error term $\bar{a}^{\text{Ben}}_{i,m,t}, e^{\text{Ben}}_{i,t}, e^{\text{Ben}}_t, \tilde{e}^{\text{Ben}}_t$ as the counterparts of the error terms in our model.

We find that although for a short prediction horizon, the difference of the cumulative decision prediction error between our model and the benchmark model is almost negligible, as the prediction horizon increases, our model starts to show higher prediction accuracy compared to the benchmark model; see Figure A2. By the end of year 2004, the benchmark model's cumulative error is on average 13.64% higher than our model, the maximum of which being 23.48% occurring at 2004Q4.

E Transition Probability of Local Market Size Factors

In Table A4, we include the local market size factors' transition probability matrix $\mathbf{F}$, fitted to Canada fast-food dataset.

[43] GDP indicator reveals similar results, thus omitted. Details are available upon request.

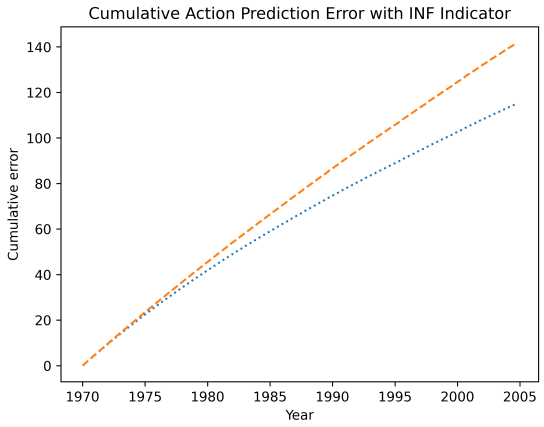


Figure A2: Cumulative decision prediction error comparison between the our model (blue) and the benchmark model (orange).

s^{pop}	0	1	2	3
0	0.9949	0.0051	0.0000	0.0000
1	0.0000	0.9883	0.0117	0.0000
2	0.0000	0.0000	0.9898	0.0102
3	0.0000	0.0000	0.0003	0.9997

s^{pro}	0	1	2	3
0	0.9828	0.0172	0.0000	0.0000
1	0.0002	0.9787	0.0211	0.0000
2	0.0000	0.0016	0.9733	0.0251
3	0.0000	0.0000	0.0033	0.9967

s^{inc}	0	1	2	3
0	0.9773	0.0227	0.0000	0.0000
1	0.0020	0.9662	0.0318	0.0000
2	0.0000	0.0042	0.9759	0.0199
3	0.0000	0.0000	0.0042	0.9958

Table A4: Estimates of transition probability matrix **F** of local market size factors population s^{pop}, property value s^{pro} and income s^{inc}.