Monetary-Based Asset Pricing: A Mixed-Frequency Structural Approach

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Abstract

We integrate a high-frequency monetary event study into a mixed-frequency macro-finance model and structural estimation. The model and estimation allow for jumps at Fed announcements in investor beliefs, providing granular detail on why markets react to central bank communications. We find that the reasons involve a mix of revisions in investor beliefs about the economic state and/or future regime change in the conduct of monetary policy, and subjective reassessments of financial market risk. However, the structural estimation also finds that much of the causal impact of monetary policy on markets occurs outside of tight windows around policy announcements.

JEL: E7, E27, E32, E17, G4.

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1 Introduction

In early February of 2003, then Governor Ben Bernanke of the Federal Reserve Board gave a speech pressing the virtues and proven successes of "constrained discretion," his shorthand for the monetary policy framework of the United States since Volcker that arguably continues to this day. In it, the central bank first establishes a strong commitment to low and stable inflation and, only once this condition is met, does it implement descretionary short-term policies aimed at stabilizing real activity. Properly implemented, Bernanke argued, such a policy regime achieves not only price stability but output stability around its full-employment potential.

Yet with the successes of constrained discretion firmly in place, how should we interpret the outsize volatility of financial markets in reaction to the actions and announcements of central banks? Through what channels, if any, might such a framework been a driver of financial market volatility and market reactions to central bank news? A central challenge to addressing these questions is that workhorse New Keynesian models typically used to study how monetary policy impacts the economy are silent both on regime changes in policy conduct, and on what markets actually learn from central bank communications.

Our contribution to addressing these challenges is to integrate a high-frequency monetary event study into a mixed-frequency macro-finance model and structural estimation. We examine Federal Reserve (Fed) communications alongside both high- and lower-frequency data through the lens of a structural equilibrium asset pricing model with New Keynesian style macroeconomic dynamics, using dozens of series ranging from minutely financial market data to biannual survey forecast data in our structural estimation. The model and estimation allow us to infer how investor beliefs about the latent state of the economy, the perceived sources of economic risk, and the future conduct of monetary policy jump in response to Fed communications. The general approach can be applied in a wide variety of other structural and semi-structural settings, whenever a granular understanding of financial market responses to almost any type of news is desired.

In this paper, we apply the approach to a two-agent asset pricing model with New Keynesian style macroeconomic dynamics in which the two agents have heterogeneous beliefs, as in Bianchi, Lettau, and Ludvigson (2022). One agent is a representative "investor" who is forward-looking, reacts swiftly to news, and earns income solely from investments in the stock market and a one-period nominal bond. Macroeconomic dynamics are specified by a set of equations similar to those in New Keynesian models, and can be thought of as driven by a representative "household/worker" that supplies labor and has access to the nominal bond but holds no stock market wealth. Unlike investors, the household/worker forms inflation expectations using adaptive learning rules

and (under our estimated parameter values) commensurately responds only slowly to signals indicating a shift in the central bank's inflation objectives, consistent with survey evidence on how households actually arrive at their beliefs (Malmendier and Nagel (2016), Coibion, Gorodnichenko, Kumar, and Pedemonte (2020)). Beyond bringing sensible empirical realism to the model's household expectations process, our estimates show how highly inertial adaptive learning by households play a crucial role in equity markets, even when they own no equity themselves. We discuss this further below.

An important feature of our model is that the conduct of monetary policy is not static over time, but is instead subject to infrequent nonrecurrent regime shifts, or "structural breaks," that take the form of shifts in the parameters of a nominal interest rate rule. Such regime changes in what we refer to as the *conduct* of monetary policy give rise to endogenously long-lasting changes in real interest rates in the model and are conceptually distinct from those generated by the monetary policy *shock*, an innovation in the nominal rate that is uncorrelated with inflation, economic growth, and shifts in the policy rule parameters. To interpret how these breaks in policy conduct affect the stock market, we explicitly model investor beliefs about future regime change. Investors in the model are presumed to closely monitor central bank communications, continuously updating their beliefs about the probability of transitioning out of the current policy regime into a perceived "Alternative regime" that they predict will come next. Investors are aware that they may change their minds subsequently about the likelihood of near-term monetary regime change, and take that into account when forming expectations.

A Fed announcement in our model is bonafide news shock to which investors may react by revising their nowcasts and forecasts of the current and future economic state, their beliefs about the future conduct of monetary policy, and their perceptions of financial market risk. To ensure that model expectations evolve in a manner that closely aligns with observed expectations, we map the theoretical implications for these beliefs into data on numerous forward-looking variables, including professional forecaster surveys and financial market indicators from spot and futures markets, estimating all parameters and latent states.

Our main empirical results may be summarized as follows. First, we find that the way in which monetary policy affects the stock market is state contingent. In policy regimes for which the Fed was focused on establishing a strong and credible commitment to low and stable inflation, as in the "Great Moderation" regime that began roughly with Volcker, policy affects the stock market mainly through a real interest rate channel. For this channel, the slow, adaptive learning about inflation by households plays a crucial role. In moving from the prior "Great Inflation" regime to the Great Moderation regime, slow adaptive learning by households requires large and persistent shifts in policy conduct to re-anchor household inflation expectations toward lower levels. These shifts show up

as large and persistent changes in real interest rates that affect the stock market.

First, the structural estimation implies that investors seldom learn only about conventional monetary policy shocks from central bank announcements. Instead, jumps in financial market variables are typically the result of a mix of factors, including announcement-driven revisions in investor beliefs about the in the composition of primitive economic shocks hitting the economy and/or about the probability of near-term monetary regime change.

For example, on January 3, 2001 the Fed surprised markets by reducing its target for the federal funds rate by 50 basis points, causing the stock market to vault 4.2% in the 20 minutes following the announcement. Yet our estimates imply that the perception of a surprisingly accommodative monetary policy shock played only a small role in the stock market surge. Instead, the market jumped upward because, all else equal, the announcement caused investors to lower their perception of what financial market liquidity premia would be, and increase their perception of aggregate demand and the corporate earnings share of output. On April 18, 2001, the market leapt 2.5% after the Greenspan Fed again surprised with another 50 basis point reduction in the funds rate. In this case, the big driver of the stock market was a jump upward in the perceived probability that the conduct of monetary policy going forward would more aggressively protect against the downside risks that affect stocks. The results for this event are new to the literature and illustrate an important channel of monetary transmission to markets, namely the role of Fed communications in altering investor beliefs about future Fed policy to contain economic risks, thereby immediately impacting subjective risk premia.

Our second main finding is that fluctuating beliefs about the conduct of future monetary policy generate significant market volatility throughout the sample and that most of the variation in these beliefs occurs at times that are not close to a policy announcement. An obvious explanation for this result is that most Fed announcements are not immediately associated with a change in the policy stance, but instead provide "forward guidance" in the form of a data-dependent sketch of what could trigger a change in the conduct of policy down the road. These results underscore the challenges with relying solely on high-frequency event studies for quantifying the channels of monetary transmission to markets and the real economy.

Finally, our results indicate that investor beliefs about a future monetary policy regime change are especially important for the stock market because of their role in shaping perceptions of equity market risk. We find that the S&P 500 would have been 50% higher than it was in February of 2020 had investors counterfactually believed that the Fed was very likely to shift in the next year to a policy rule that featured greater activism to stabilize economic volatility, thereby lowering the quantity of risk in the stock market.

***A growing academic literature has offered a myriad of competing explanations for why financial markets react strongly to the actions and announcements of central banks. A classic view is that surprise central bank announcements proxy for shocks to a nominal interest rate rule of the type emphasized by Taylor (1993), which have short-run affects on the real economy in a manner consistent with canonical New Keynesian models (e.g., Christiano, Eichenbaum, and Evans (2005)). More recently, other hypotheses have emerged, including the effects such announcements have on financial market risk premia, the information they impart about the state of the economy (the "Fed information effect"), or the role they play in revising the public's understanding of the central bank's reaction function and objectives.

As the mushrooming debate over how to interpret this evidence indicates, many questions about the interplay between markets and monetary policy remain unanswered. In this paper we consider three of them. First, theories focused on a single channel of monetary transmission are useful for elucidating its marginal effects, but may reveal only part of the overall picture. To what extent are several competing explanations or others entirely playing a role simultaneously? Second, monetary policy communications cover a range of topics from interest rate policy, to forward guidance, to quantitative interventions, to the macroeconomic outlook. How do these varied communications affect market participants' perceptions of the primitive economic sources of risk hitting the economy in real time? Third, high frequency events studies only capture the causal effects of the surprise component of a policy announcement, a lower bound on its overall impact. How much of the causal influence of shifting monetary policy occurs outside of tight windows around Fed communications, effects that are by construction impossible to observe from high-frequency event studies?***

The research in this paper connects with a large and growing body of evidence that the values of long-term financial assets and expected return premia respond sharply to the announcements of central banks.[1] A classic assumption of this literature is that high-frequency financial market reactions to Fed announcements proxy for conventional monetary policy "shocks," i.e., innovations in a Taylor (1993)-type nominal interest rate rule. By contrast, Jarocinski and Karadi (2020), Cieslak and Schrimpf (2019) and Hillenbrand (2021) argue that some of the fluctuations are likely driven by the revelation of information by the Fed, a "Fed information effect" channel emphasized in earlier work by Romer and Romer (2000), Campbell, Evans, Fisher, and Justiniano (2012), Melosi (2017), and Nakamura and Steinsson (2018). Related, Cieslak and Pang (2021) identify monetary, growth, and risk premium shocks from Fed news using sign-

[1] See Cochrane and Piazzesi (2002), Piazzesi (2005), Bernanke and Kuttner (2005), Krishnamurthy and Vissing-Jorgensen (2011), Hanson and Stein (2015), Gertler and Karadi (2015), Gilchrist, López-Salido, and Zakrajšek (2015), Brooks, Katz, and Lustig (2018), Kekre and Lenel (2021), and Pflueger and Rinaldi (2020).

restricted VARs. Bauer and Swanson (2023) instead argue that markets are surprised by the Fed's response to recent economic events, while Bauer, Pflueger, and Sundaram (2022) use survey data to estimate perceived policy rules, finding that they are subject to substantial time-variation. The mixed-frequency structural approach proposed in this paper extends these literatures by integrating a high-frequency event study into a structural model and estimation. We also add to this literature by providing evidence that expected return premia vary, in part, because the perceived quantity of stock market risk fluctuates with beliefs about future monetary policy conduct.

Our work relates to a theoretical literature focused on the implications of monetary policy for asset prices going back to Piazzesi (2005). Kekre and Lenel (2021) and Pflueger and Rinaldi (2020) develop carefully calibrated theoretical models that imply stock market return premia vary in response to a monetary policy shock. These theories use different mechanisms but are all silent on the possible role of Fed announcement information effects or of changing policy rules in driving market fluctuations, features that are at the heart of our analysis.

The two-agent structural model of this paper builds on Bianchi, Lettau, and Ludvigson (2022) (BLL hereafter), who focus on the low frequency implications for asset valuations of changes in the conduct of monetary policy. The mixed-frequency structural approach of this paper offers a significant methodological advancement over BLL and (to the best of our knowledge) the extant literature, by developing a methodology to exploit large datasets of relevant information at different frequencies, by integrating an event study into a structural model, and by explicitly modeling revisions in investor beliefs about future monetary policy in the minutes surrounding Fed announcements as well as at lower frequencies. Moreover, unlike BLL and the extant literature, we model regime changes in the conduct of monetary policy as nonrecurrent regimes, i.e., structural breaks, a more plausible specification given that new policy regimes are never expected to be *identically* equal to old ones. We show how forward looking variables, such as survey expectations and asset prices, can be used both to estimate the market's perceived probability of a near-term policy regime change, and to extract beliefs about the nature of future policy regimes.

In contemporaneous work, Caballero and Simsek (2022) also study a two-agent, "two-speed" economy with investors and households similar in spirit to our framework, in which the Fed directly controls aggregate asset prices in an attempt to steer the household spending. This differs from our study in that it is a purely theoretical investigation that studies asset pricing at an abstract level by thinking of the risky asset price as a broad-based financial conditions index while allowing for two-way feedback between financial conditions and the economy. Our work is an empirical compliment that studies the impact of the Fed on markets by integrating a high-frequency monetary event

study into a mixed-frequency asset pricing model and structural estimation, specifically modeling the risky asset as the stock market. In future work, we plan to extend our structural estimation to allow for simultaneous feedback between markets, the Fed, and the economy.

Finally, our mixed-frequency structural approach connects with a pre-existing reduced-form forecasting/nowcasting literature using mixed-frequency data in state space models with the objective of augmenting lower frequency prediction models with more timely high-frequency data (e.g., Giannone, Reichlin, and Small (2008), Ghysels and Wright (2009), Schorfheide and Song (2015)). Our use of mixed-frequency data is designed for a very different purpose, namely as way of integrating a high-frequency event study into a structural model and estimation for the purpose of modeling and measuring news shocks. We use high-frequency, forward-looking data available *within* the decision interval to infer revisions in the intraperiod beliefs of investors about the economic state to be realized at the *end* of the decision interval. This allows us to treat Fed announcements as bonafide news shocks (as perceived by investors) rather than as ultra high frequency macro shocks that happen to occur around Fed communications.

The rest of this paper is organized as follows. The next section presents preliminary empirical evidence that we use to pin down the timing of monetary regime changes in our sample. Section 3 describes the mixed-frequency structural macro-finance model and equilibrium solution. Section 4 describes the structural estimation, while Section 5 presents our empirical findings from the structural estimation. Section 6 concludes. A large amount of additional material on the model, estimation, and data has been placed in an Online Appendix.

2 Preliminary Evidence

In the structural model of the next section, investors form beliefs about future regime change in the conduct of monetary policy. We therefore begin by presenting preliminary evidence suggestive of infrequent, sizable shifts in the conduct of monetary policy over our the course of our sample.

To that end, Figure 1, panel (a) plots the real federal funds rate, r_t, measured for the purposes of this plot in real terms as the nominal rate minus a four quarter moving average of inflation, while panel (b) plots the difference between this rate and an estimate of the neutral rate of interest, r_t^*, from Laubach and Williams (2003), effectively a low-frequency component of the real funds rate. We refer to th spread between r_t and r_t^* as the *monetary policy spread*, and denote its time t value as mps_t. The mps_t may be considered a crude measures of the stance of monetary policy, i.e., whether monetary policy is accommodative or restrictive.

We allow for the possibility of infrequent regime changes in the means of r_t and mps_t, governed by a discrete valued latent state variable, ξ_t^P that is presumed to follow a N_P-state *nonrecurrent* regime-switching Markov process, i.e., structural breaks. That is, there is no expectation that regime shifts in the means of either variable must move to a new regime that is *identically* equal to one in the past (mathematically a probability zero event), though it could be quite similar. The specification here is more general and more plausible than recurrent regime switching, since the estimation is free to choose parameter values across regimes that are arbitrarily close to those that have occurred in the past, without restricting them to be identically equal.[2]

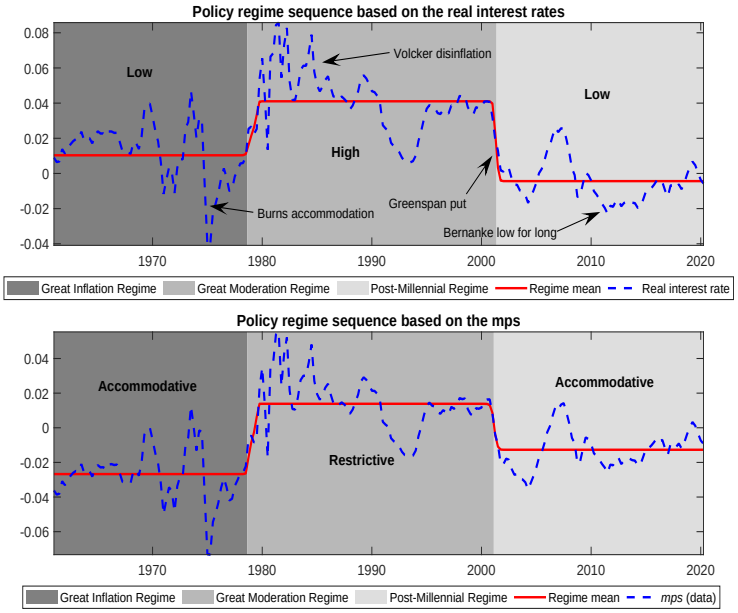
Figure 1 reports the results for the case of two structural breaks ($N_P = 3$) estimated separately for r_t and mps_t, with the estimated regime subperiods reported in the figure notes. We identify decades-long breaks in both variables, consistent with an earlier literature documenting that monetary regime changes are infrequent (Clarida, Gali, and Gertler (2000); Lubik and Schorfheide (2004); Sims and Zha (2006); Bianchi (2013)). Importantly, regardless of whether we measure breaks in the mean of r_t or mps_t, the break dates are identical and thus so are the regime subperiods. This shows that the r_t^* measure–while useful to get a sense of the persistence of swings in r_t around that low frequency component–has no influence on the estimated break dates.

The first estimated subperiod spans 1961:Q1 to 1978:Q3, a time period in which r_t was low and mps_t was persistently negative. This "Great Inflation" regime coincides with a run up in inflation and with two oil shocks in the 1970s that were arguably exacerbated by a Fed that failed to react sufficiently proactively. A second, "Great Moderation," regime begins in 1978:Q4, when a structural break drove upward jumps in both r_t and mps_t. This period of more restrictive monetary policy lasted through 2001:Q3 and covers the Volcker disinflation and moderation in economic volatility that followed. The third, "Post Millennial," regime spans 2001:Q4 to 2020:Q1 and represents a new prolonged period of low real interest rates. The beginning of this regime follows shortly after the inception of public narratives on the "Greenspan Put," the perceived attempt of Chair Greenspan to prop up securities markets in the wake of the IT bust, a recession, and the aftermath of 9/11, by lowering interest rates. This low rate subperiod continues with the explicit forward guidance "low-for-long" policies under Chair Bernanke that repeatedly promised over several years to keep interest rates at ultra low levels for an extended period of time. Below we refer to the Great Inflation, the Great Moderation and the Post Millennial regimes in abbreviated terms as the GI, GM, and PM regimes.

In the next section we formally assess the extent to which estimated monetary policy rules actually shifted across these identified regime subperiods associated with large, low frequency movements in the real federal funds rate. To accomplish this, we set the

[2]Details of this procedure are provided in Appendix C of the Online Appendix.

Figure 1: Breaks in Monetary Policy



Notes: Monetary policy spread $mps_t \equiv FFR_t - \text{Expected Inflation}_t - r_t^*$. r^* is from Laubach and Williams (2003). The blue (dashed) line represents the data. The red (solid) line is the estimated regime mean of each series. Great Inflation Regime: 1961:Q1-1978:Q3. Great Moderation Regime: 1978:Q4-2001:Q3. Post-Millennial Regime: 2001:Q4-2020:Q1. The sample spans 1961:Q1-2020:Q1.

break dates for regime changes in the policy rule to coincide with the regime sequence for ξ_t^P displayed in Figure 1. We use Bayesian model comparison of different estimated structural models to decide on the appropriate number N_P of policy regimes, and find $N_P = 3$ works well. With this, our structural estimation spans three different policy regimes across the Great Inflation, the Great Moderation, and the Post Millennial subperiods shown in Figure 1.

The purpose of estimating break dates in this way is that it allows us to build a structural model to fit these empirical observations, rather than establishing evidence about the sequence of monetary regimes that would be contingent on the many details of the structural model. It should be emphasized, however, that the preliminary evidence of this section is used only to set the *timing* of policy regime changes in the structural model. In particular, all regime-dependent parameters of the policy rule are freely estimated under symmetric priors, so are treated as equally likely to increase or decrease across the regime subperiods for ξ_t^P, if they change at all.

3 Mixed-Frequency Macro-Finance Model

This section presents a two-agent dynamic asset pricing model of monetary policy transmission. Risky asset prices are determined by the behavior of a forward-looking repre-

sentative investor who reacts swiftly to news and forms beliefs about future monetary policy. Households/workers supply labor, invest only in the bond, and form expectations using adaptive learning rules that predominate in aggregate inflation and output growth expectations. As in BLL, it is through such heterogeneity in beliefs that regime changes in the conduct of monetary policy have large and prolonged effects on real interest rates, despite the forward-looking, non-inertial nature of market participant expectations. We work with a risk-adjusted loglinear approximation to the model that can be solved analytically, in which all random variables are conditionally lognormally distributed.

Let the "decision" interval t of both agents be monthly and let lowercase variables denote log variables, e.g., $\ln(D_t) = d_t$. For investors, this means that they receive payout and can only observe the economic state S_t at the end of each month. However, as explained below, they nevertheless price assets continuously and update expectations in the wake of Fed announcements.

Asset Pricing Block Assets are priced by a representative investor who consumes per-capita aggregate shareholder payout, D_t, and earns all income from trade in two assets: a one-period nominal risk-free bond and a stock market. The investor's intertemporal marginal rate of substitution in consumption is the stochastic discount factor (SDF) and its logarithm takes the form:

$$m_{t+1} = \ln\left(\beta_p\right) + \vartheta_{pt} - \sigma_p\left(\Delta d_{t+1}\right). \tag{1}$$

where σ_p is a relative risk aversion coefficient and $\ln\left[\beta_p \exp\left(\vartheta_{pt}\right)\right]$ is a subjective time discount factor that varies over time with the patience shifter ϑ_{pt}. Individual investors take ϑ_{pt} as given, driven by the market as a whole.[3] A time-varying specification for the subjective time-discount factor is essential for ensuring that, in equilibrium, investors are willing to hold the nominal bond at the interest rate set by the central bank's policy rule, specified below.

Aggregate payout is a time-varying share K_t of real output Y_t, implying $D_t = K_t Y_t$ or in logs $d_t - \ln(Y_t) = k_t$. Since in the model all earnings are paid out to shareholders, we refer to K_t simply as the *earnings share* hereafter. Variation in k_t, follows an exogenous primitive process:

$$k_t - \overline{k} = (1-\rho_k)\,\lambda_{k,\Delta y}\Delta y_t + \rho_k\left(k_{t-1} - \overline{k}\right) + \sigma_k \varepsilon_{k,t}.$$

[3]This specification for ϑ_{pt} is a generalization of those considered in previous work (e.g., Campbell and Cochrane (1999) and Lettau and Wachter (2007)) where the preference shifter is taken as an exogenous process that is the same for each shareholder. Combining (1) and (3) below, we see that $\vartheta_{p,t}$ is implicitly defined as

$$\vartheta_t^p = -\left[i_t - \mathbb{E}_t^b\left[\pi_{t+1}\right]\right] + \mathbb{E}_t^b\left[\sigma_p \Delta d_{t+1}\right] - .5\mathbb{V}_t^b\left[-\sigma_p \Delta d_{p,t+1} - \pi_{t+1}\right] - lp_t - \ln\left(\beta_p\right).$$

Thus k_t varies with economic growth and an independent i.i.d. shock $\varepsilon_{kt} \sim N(0,1)$.

The first-order-condition for optimal holdings of the one-period nominal risk-free bond with a face value equal to one nominal unit is

$$LP_t^{-1} Q_t = \mathbb{E}_t^b \left[M_{t+1} \Pi_{t+1}^{-1} \right], \tag{2}$$

where Q_t is the nominal bond price, $\mathbb{E}_t^b$ denotes the subjective expectations of the investor, and $\Pi_{t+1} = P_{t+1}/P_t$ is the gross rate of general price inflation. Investors' subjective beliefs, indicated with a "b" superscript, play a central role in asset pricing and are discussed in detail below. Investors have a time-varying preference for nominal risk-free assets over equity, accounted for by $LP_t > 1$, implying that Q_t is higher than it would be absent these benefits, i.e., when $LP_t = 1$.

Taking logs of (2) and using the properties of conditional lognormality delivers the real interest rate as perceived by the investor:

$$i_t - \mathbb{E}_t^b [\pi_{t+1}] = -\mathbb{E}_t^b [m_{t+1}] - .5 \mathbb{V}_t^b [m_{t+1} - \pi_{t+1}] - lp_t \tag{3}$$

where $i_t = -\ln(Q_t)$, $\pi_{t+1} \equiv \ln(\Pi_{t+1})$ is net inflation, $\mathbb{V}_t^b[\cdot]$ is the conditional variance under the subjective beliefs of the investor, and $lp_t \equiv \ln(LP_t) > 0$. Variation in lp_t follows an AR(1) process

$$lp_t - \overline{lp} = \rho_{lp} \left(lp_{t-1} - \overline{lp} \right) + \sigma_{lp} \varepsilon_{lp,t}$$

subject to an i.i.d. shock $\varepsilon_{lp,t} \sim N(0,1)$.

Let P_t^D denote total value of market equity, i.e., price per share times shares outstanding. Optimal shareholder consumption obeys the following log Euler equation:

$$\begin{aligned} pd_t &= \kappa_{pd,0} + \mathbb{E}_t^b [m_{t+1} + \Delta d_{t+1} + \kappa_{pd,1} pd_{t+1}] + \\ &\quad + .5 \mathbb{V}_t^b [m_{t+1} + \Delta d_{t+1} + \kappa_{pd,1} pd_{t+1}], \end{aligned}$$

where $pd_t \equiv \ln\left(P_t^D / D_t\right)$. The log equity return $r_{t+1}^D \equiv \ln\left(P_{t+1}^D + D_{t+1}\right) - \ln\left(P_t^D\right)$ obeys the following approximate identity (Campbell and Shiller (1989)):

$$r_{t+1}^D = \kappa_{pd,0} + \kappa_{pd,1} pd_{t+1} - pd_t + \Delta d_{t+1},$$

where $\kappa_{pd,1} = \exp(\overline{pd})/(1+\exp(\overline{pd}))$, and $\kappa_{pd,0} = \log\left(\exp(\overline{pd}) + 1\right) - \kappa_{pd,1} \overline{pd}$. Combining the above, the log equity premium as perceived by the investor is:

$$\underbrace{\mathbb{E}_t^b \left[r_{t+1}^D \right] - \left(i_t - \mathbb{E}_t^b [\pi_{t+1}] \right)}_{\text{subj. equity premium}} = \underbrace{\left[\begin{array}{c} -.5 \mathbb{V}_t^b \left[r_{t+1}^D \right] - \mathbb{COV}_t^b \left[m_{t+1}, r_{t+1}^D \right] \\ +.5 \mathbb{V}_t^b [\pi_{t+1}] - \mathbb{COV}_t^b [m_{t+1}, \pi_{t+1}] \end{array} \right]}_{\text{subjective risk premium}} + \underbrace{lp_t}_{\text{liquidity Premium}}, \tag{4}$$

where $\mathbb{COV}_t^b[\cdot]$ is the investor's subjective conditional covariance.

The equity premium has two components, a subjective risk premium is attributable to the agent's subjective perception of risk, and a "liquidity premium" lp_t that represents a time-varying preference for risk-free nominal debt over equity. The subjective risk premium varies endogenously in the model with fluctuations in investor beliefs about the conduct of future monetary policy, as explained below. The liquidity premium captures all sources of time-variation in the equity premium other than those attributable to subjective beliefs about the monetary policy rule. These could include variation in the liquidity and safety attributes of nominal risk-free assets (e.g., Krishnamurthy and Vissing-Jorgensen (2012)), variation in risk aversion, flights to quality, or jumps in sentiment.

Macro Dynamics Macroeconomic dynamics are described by a set of equations similar to prototypical New Keynesian models, but with two distinctive features: adaptive learning, and regime changes in the conduct of monetary policy. Strictly speaking, we consider equations (5) through (7) below as equilibrium dynamics and not a microfounded structural model. We consider an equilibrium in which bonds are in zero-net-supply in both the macro and asset pricing blocks and thus there is no trade between the investor and households.[4]

Let $\ln(A_t/A_{t-1}) \equiv g_t$ represent the stochastic trend growth of the economy, which follows an AR(1) process $g_t = g + \rho_g(g_{t-1} - g) + \sigma_g \varepsilon_{g,t}$, $\varepsilon_{g,t} \sim N(0,1)$. Log of detrended output in the model is defined as $\ln(Y_t/A_t)$. Let variables with tildes, e.g., $\widetilde{y}_t = \ln(Y_t/A_t)$, denote detrended variables. Thus $\widetilde{y}_t > 0$ (< 0) when y_t is above (below) potential output, so $\widetilde{y}_t \neq 0$ can be interpreted as a New Keynesian output gap. In keeping with New Keynesian models, we write most equations in the macro block in terms of detrended real variables.

Macroeconomic dynamics satisfy a loglinear Euler or "IS" equation that is a function of household consumption $(1 - K_t)Y_t$:[5]

$$\widetilde{y}_t = \mathbb{E}_t^m(\widetilde{y}_{t+1}) - \sigma[i_t - \mathbb{E}_t^m(\pi_{t+1}) - \overline{r}] + f_t \tag{5}$$

where $\mathbb{E}_t^m(\cdot)$ is the expectation under the subjective beliefs of the macro agent, $\overline{r}$ is the steady state real interest rate, and f_t is a demand shock that also absorbs any variation in the macro agent's consumption attributable to movements in the labor share, $\ln(1 - K_t)$.

[4]Models with trade are computationally slow to solve and would present a significant challenge to estimation; hence we leave this to future research. However, an empirically plausible version of our model with trade may not imply appreciably different aggregate dynamics. For example, Chang, Chen, and Schorfheide (2021) provide econometric evidence that spillovers between aggregate and distributional dynamics in heterogeneous agent models are generally small.

[5]We assume that the Euler equation (5) holds under nonrational expectations. Honkapohja, Mitra, and Evans (2013) provide microfoundations for such Euler equations with nonrational beliefs.

The demand shock follows an AR(1) process $f_t = \rho_f f_{t-1} + \sigma_f \varepsilon_f$, $\varepsilon_f \sim N(0,1)$. The coefficient σ in (5) is a positive parameter.

Inflation dynamics are described by the following equation, which takes the form of a New Keynesian Phillips curve:

$$\begin{aligned} \pi_t - \overline{\pi}_t &= \beta\left(1-\lambda_{\pi,1}-\lambda_{\pi,2}\right)\mathbb{E}_t^m\left[\pi_{t+1}-\overline{\pi}_t\right] + \beta\lambda_{\pi,1}\left[\pi_{t-1}-\overline{\pi}_t\right] \\ &\quad +\beta\lambda_{\pi,2}\left[\pi_{t-2}-\overline{\pi}_t\right] + \kappa_0\widetilde{y}_t + \kappa_1\widetilde{y}_{t-1} + \sigma_\mu\varepsilon_{\mu,t} \end{aligned} \tag{6}$$

where $\overline{\pi}_t$ denotes the household's perceived trend inflation rate (specified below) and $\varepsilon_{\mu,t} \sim N(0,1)$ is a markup shock.[6] Lags beyond the current values of variables are used to capture persistent inflation dynamics. The coefficients β, λ_{π_1}, λ_{π_2}, κ_0, and κ_1 are positive parameters.

The central bank obeys the following nominal interest rate rule subject to nonrecurrent regime changes in its parameters:

$$\begin{aligned} i_t - \left(\overline{r} + \pi^T_{\xi^p_t}\right) &= \left(1-\rho_{i,\xi^p_t}-\rho_{i_2,\xi^p_t}\right)\left[\psi_{\pi,\xi^p_t}\widehat{\pi}_{t,t-3} + \psi_{\Delta y,\xi^p_t}\left(4\widehat{\Delta y}_{t,t-3}\right)\right] \\ &\quad +\rho_{i_1,\xi^p_t}\left[i_{t-1}-\left(\overline{r}+\pi^T_{\xi^p_t}\right)\right] + \rho_{i_2,\xi^p_t}\left[i_{t-2}-\left(\overline{r}+\pi^T_{\xi^p_t}\right)\right] + \sigma_i\varepsilon_i. \end{aligned} \tag{7}$$

The central bank is presumed to react to quarterly data (at monthly frequency) given that it is unlikely to react to the more volatile monthly variation in growth and inflation. Thus $\widehat{\pi}_{t,t-3} \equiv \sum_{l=0}^{2}\left(\pi_{t-l} - \pi^T_{\xi^p_t}\right)$ is quarterly inflation in deviations from the implicit time t target $\pi^T_{\xi^p_t}$, $4\Delta y_{t,t-3} \equiv 4\sum_{l=0}^{2}\left(\Delta y_t - g\right)$ is annualized quarterly output growth in deviations from steady-state growth g, and $\varepsilon_{i,t} \sim N(0,1)$ is an i.i.d. monetary policy shock. Lags of the left-hand-side variable appear in the rule to capture the observed smoothness in adjustments to the central bank's target interest rate.

The interest rate policy rule allows for nonrecurrent regime changes in the conduct of monetary policy driven by ξ^p_t, which indexes changes in the parameters of (7). The parameter $\pi^T_{\xi^P_t}$ plays the role of an *implicit* time-t inflation target. In particular, this time-varying parameter may deviate from the central bank's stated long-term inflation objective when it is actively trying to move inflation back toward that objective. The activism coefficients ψ_{π,ξ^P_t}, and $\psi_{\Delta y,\xi^P_t}$ govern how strongly the central bank responds to deviation from the implicit target and to economic growth and are also subject to regime shifts, as are the autocorrelation coefficients ρ_{i,ξ^P_t} and ρ_{i_2,ξ^p_t}. We treat shifts in the policy rule parameters as exogenous and latent parameters to be estimated.[7] These coefficients vary with ξ^P_t and the identified regime sequence for $r_{\xi^P_t}$ from Figure 1. It

[6]This equation can be micro-founded by assuming that managers of firms are workers who form expectations as households/workers do rather than as shareholders do, consistent with evidence that the discount rates managers use when making investment and employment decisions are different from those observed in financial markets (Gormsen and Huber (2022)), and with evidence that those expectationsn do not appear rational ()Gennaioli, Ma, and Shleifer (2016).

[7]This approach side-steps the need to take a stand on why the Fed changes its policy rule, an

is important to emphasize, however, that we freely estimate the policy rule parameters under symmetric priors, so they could in principle show no shift across regimes.

We assume that households form expectations about inflation using an adaptive algorithm on the autoregressive process, $\pi_t = \alpha + \phi\pi_{t-1} + \eta_t$, where the agent must learn about α. Each period, agents update a belief α_t^m about α. Define *perceived trend inflation* to be the $\lim_{h\to\infty} \mathbb{E}_t^m [\pi_{t+h}]$ and denote it by $\overline{\pi}_t$. Given the presumed autoregressive process, it can be shown that $\overline{\pi}_t = (1-\phi)^{-1} \alpha_t^m$ and that $\mathbb{E}_t^m [\pi_{t+1}] = (1-\phi)\overline{\pi}_t + \phi\pi_t$.

We allow the evolution of beliefs about α_t^m and $\overline{\pi}_t$ to potentially reflect both adaptive learning as well as a signal about the central bank's inflation target that could reflect the opinion of experts (as in Malmendier and Nagel (2016)) or a credible central bank announcement. For the adaptive learning component, we follow evidence in Malmendier and Nagel (2016) that the University of Michigan Survey of Consumers (SOC) mean inflation forecast is well described by a constant gain learning algorithm. Combining these yields updating rules for α_t^m and $\overline{\pi}_t$:

$$\alpha_t^m = \left(1-\gamma^T\right)\left[\alpha_{t-1}^m + \gamma\left(\pi_t - \phi\pi_{t-1} - \alpha_{t-1}^m\right)\right] + \gamma^T\left[(1-\phi)\,\pi_{\xi_t}^T\right] \quad (8)$$

$$\overline{\pi}_t = \left(1-\gamma^T\right)\left[\overline{\pi}_{t-1} + \gamma(1-\phi)^{-1}\left(\pi_t - \phi\pi_{t-1} - (1-\phi)\,\overline{\pi}_{t-1}\right)\right] + \gamma^T\pi_{\xi_t}^T, \quad (9)$$

where γ is the constant gain parameter that governs how much last period's beliefs α_{t-1}^m and $\overline{\pi}_{t-1}$ are updated given new information, π_t. The second term in square brackets captures the effect of the signal about the implicit inflation target $\pi_{\xi_t}^T$. The parameter γ^T controls the informativeness of the inflation target signal. If $\gamma^T = 1$, the signal is perfectly informative and the household's belief about trend inflation is the same as the implicit target. If $\gamma^T = 0$, the signal is completely uninformative and the agent's belief about trend inflation depends only on the adaptive learning algorithm. A weight of $\gamma^T < 1$ could arise either because the target is imperfectly observed, or because central bank announcements about the target are not viewed as fully informative or credible. Small values for γ^T are indicative of slow learning and low central bank credibility, since in that case the macro agent continues to base inflation expectations mostly on a backward looking rule even when there has been a shift in the inflation target.

Finally, expectations about detrended output follow a simple backward looking rule:

$$\mathbb{E}_t^m\left(\widetilde{y}_{t+1}\right) = \varrho_1\widetilde{y}_{t-1} + \varrho_2\widetilde{y}_{t-2} + \varrho_3\widetilde{y}_{t-3}. \quad (10)$$

Investors take the above dynamics into account when forming expectations but they must form beliefs about the future conduct of monetary policy.

important consideration given that the reasons for such changes would be difficult if not impossible to credibly identify as a function of past historical data, due to the degree of discretion the Fed has in interpreting its dual mandate and the likelihood that distinct regimes are the result of a gradual learning process interacting with the bespoke perspectives of different central bank leaders across time.

Investor Beliefs About Future Monetary Policy To model the uncertainty investors face about the conduct of monetary policy, we assume that they expect the monetary policy rule to be subject to infrequent, nonrecurrent regime changes. We further assume that investors can accurately estimate the policy rule currently in place. This latter assumption, we argue, is a reasonable approximation of the role of beliefs in stock market dynamics in the context of infrequent regime changes, for two reasons. First, plausible uncertainty solely about the current rule–holding fixed beliefs about future monetary policy–is unlikely to be important for long duration assets such as the stock market. This is because what matters for heavily forward-looking assets is not where the policy rule is *today*, but where it will be for the foreseeable future. In our specification, investors continuously update their beliefs about the probability of moving to a new policy regime as soon as the beginning of the next month, so any reasonable additional uncertainty about where the rule is currently is relatively unimportant. Appendix K of the Online Appendix, shows that results for the stock market shown below are virtually unchanged if investors update their understanding of the current rule after the announcement–even by sizable amounts.This contrasts with the large effects found for changing beliefs about when and where policy will settle for the foreseeable future. Second, these assumptions are consistent with evidence that investors closely monitor Fed communications, combined with the observed practice of the Fed to clearly telegraph an intentional change in the stance of policy but to be comparatively vague about how long that change will last and what will come afterwards. Investors closely scrutinize Fed communications, not because they are most interested in learning what the central bank is doing today, but rather because they understand they must contemplate a future in which the central bank could operate differently from the one today, or any that has come before.

To model these circumstances, we assume that, for each time t policy rule regime indexed by ξ_t^P, investors hold in their minds a perceived "Alternative policy rule" indexed by ξ_t^A that they believe will come next, whenever the current policy regime ends:

$$\begin{aligned} i_t - \left(\overline{r} + \pi^T_{\xi_t^A}\right) &= \left(1 - \rho_{i,\xi_t^A} - \rho_{i_2,\xi_t^A}\right)\left[\psi_{\pi,\xi_t^A}\widehat{\pi}_{t,t-3} + \psi_{\Delta y,\xi_t^A}\left(4\widehat{\Delta y}_{t,t-3}\right)\right] \\ &\quad + \rho_{i_1,\xi_t^A}\left[i_{t-1} - \left(\overline{r} + \pi^T_{\xi_t^A}\right)\right] + \rho_{i_2,\xi_t^A}\left[i_{t-2} - \left(\overline{r} + \pi^T_{\xi_t^A}\right)\right] + \sigma_i\varepsilon_i, \end{aligned} \tag{11}$$

Investors do not have perfect foresight. When the current policy regime ends, the new policy regime that replaces it will never be exactly as previously imagined by the investor. When a regime ends, investors update their understanding of the new current policy rule and proceed to contemplate a new perceived Alternative for the next rule.

Investors in the model form beliefs not only about what the next policy rule ξ_t^A will look like, they also continuously assess the likelihood of switching to ξ_t^A by the beginning of next period. Specifically, for each ξ_t^P, investors have beliefs about the probability of

remaining in ξ_t^P versus changing to ξ_t^A next month, but do not consider anything after that. This latter aspect of the specification is a form of bounded rationality that is arguably plausible in the context of infrequent regime changes. In the nonrecurrent regime setup of the model, this implies that the pondered Alternative is treated as an absorbing state as of time t, since the probability of returning to any previous rule must be zero by definition.

We formalize these ideas with a *belief regime* sequence governed by a discrete-valued variable $\xi_t^b \in \{1, 2, ...B, B+1\}$ with $B+1$ states. The regimes $\xi_t^b = 1, 2, ...B$ represent a grid of beliefs taking the form of perceived probabilities that the current policy rule will still be in place next period. The regime $\xi_t^b = B+1$ is a belief regime capturing the perceived probability of staying in the Alternative regime once it is reached. We order these so that belief regime $\xi_t^b = 1$ is the lowest perceived probability that the current policy rule will remain in place and $\xi_t^b = B$ is the highest.

The perceived regimes are modeled with a transition matrix taking the form:

$$\mathbf{H}^b = \begin{bmatrix} p_{b1}p_s & p_{b2}p_{\Delta 1|2} & \cdots & p_{bB}p_{\Delta 1|B} & 0 \\ p_{b1}p_{\Delta 2|1} & p_{b2}p_s & & p_{bB}p_{\Delta 2|B} & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ p_{b1}p_{\Delta B|1} & & & p_{bB}p_s & 0 \\ 1-p_{b1} & 1-p_{b2} & \cdots & 1-p_{bB} & p_{B+1,B+1}=1 \end{bmatrix}, \tag{12}$$

where $\mathbf{H}_{ij}^b \equiv p\left(\xi_t^b = i | \xi_{t-1}^b = j\right)$ and $\sum_{i \neq j} p_{\Delta i|j} = 1 - p_s$. In the above, p_{b1} is the subjective probability of remaining in the current policy rule under belief 1. For example, $p_{b1} = 0.05$ implies that investors believe there is a 5% chance that the current policy rule will still be in place next period. The non-zero off diagonal elements in the upper left $B \times B$ submatrix allow for the possibility that investors might receive subsequent information that could change their beliefs, and take that into account when forming expectations. The parameter, p_s is the probability investors assign to not changing their minds, i.e., to having the same beliefs tomorrow as today. The parameter $p_{\Delta i|j}$ is the probability that agents assign to changing to belief i tomorrow as a result of new information, conditional on having belief j today. Thus $p_{bj}p_s$ measures the subjective probability of being in belief j tomorrow, conditional on having belief j today, while $p_{bj}p_{\Delta i|j}$ is the subjective probability of being in belief i tomorrow conditional on having belief j today. Finally, $1-p_{bi}$ is the probability of having belief i today but exiting to the Alternative regime tomorrow. The parameter $p_{B+1,B+1}$ is the perceived probability of remaining in the Alternative regime conditional on having moved there. With perceived nonrecurrent regimes and our bounded rationality assumption, this probability is unity by definition. The model of beliefs therefore takes the form of a reducible Markov chain, implying that investors believe with probability 1 that they will eventually transition out of the current policy rule to the perceived Alternative rule.

Define the *overall policy regime* $\xi_t^{P,A} = \left\{\xi_t^P, \xi_t^A\right\}$ as characterized by the time t policy regime ξ_t^P and its associated time t perceived Alternative policy rule ξ_t^A. Thus with $N_p = 3$ true policy regimes over the course of the sample, there are also 3 perceived Alternative regimes over the same time span.

Equilibrium An equilibrium is defined as a set of prices (bond prices, stock prices), macro quantities (inflation, output growth, inflation expectations), laws of motion, and investor beliefs such that the equations in the asset pricing block are satisfied, the equations in the macro block are satisfied, with investor beliefs about monetary policy characterized by the perceived Alternative policy rule (11) and the perceived belief regime sequence described above with transition matrix (12).

Model Solution To solve the model we use the algorithm of Farmer, Waggoner, and Zha (2011) applied to solve the system of model equations that must hold in equilibrium. Appendix J of the Online Appendix explains the approximation used to preserve lognormality of the entire system, following Bansal and Zhou (2002) and Bianchi, Kung, and Tirskikh (2018). The solution of the model takes the form of a Markov-switching vector autoregression (MS-VAR) in the state vector

$$S_t = \left[S_t^M, m_t, pd_t, k_t, lp_t, \mathbb{E}_t^b\left(m_{t+1}\right), \mathbb{E}_t^b\left(pd_{t+1}\right)\right],$$

where $S_t^M \equiv \left[\widetilde{y}_t, g_t, \pi_t, i_t, \overline{\pi}_t, f_t\right]'$, with

$$S_t = \underbrace{C\left(\theta_{\xi_t^{P,A}}, \xi_t^b, \mathbf{H}^b\right)}_{\text{level}} + \underbrace{T(\theta_{\xi_t^{P,A}}, \xi_t^b, \mathbf{H}^b)S_{t-1}}_{\text{propagation}} + \underbrace{R(\theta_{\xi_t^{P,A}}, \xi_t^b, \mathbf{H}^b)Q\varepsilon_t}_{\text{amplification}}, \tag{13}$$

where $C\left(\cdot\right)$, $T\left(\cdot\right)$, and $R\left(\cdot\right)$ are matrices whose elements depend on primitive parameters, $\varepsilon_t = (\varepsilon_{f,t}, \varepsilon_{i,t}, \varepsilon_{g,t}, \varepsilon_{\mu,t}, \varepsilon_{k,t}, \varepsilon_{lp,t})$ is the vector of primitive Gaussian shocks, and $\theta_{\xi_t^{P,A}}$ is a vector of parameters that include the time-varying parameters of the current policy regime ξ_t^P, and the time-varying parameters of each associated Alternative regime ξ_t^A.

To solve the model we use the assumption that investors condition on the economic state S_t once it is observed at the end of each month. With this assumption, investor expectations in the presence of nonrecurrent regime switching and the perceived Alternative rule maybe be computed for any variable, as explained in Appendix G of the Online Appendix.

Equation (13) shows that the realized policy regime ξ_t^P (along with the associated Alternative regime ξ_t^A) and investor beliefs ξ_t^b about the probability of a shift in the policy rule amplify and propagate shocks in three distinct ways. First, there are "level" effects, captured by the coefficients $C\left(\theta_{\xi_t^{P,A}}, \xi_t^b, \mathbf{H}^b\right)$, that affect the economy absent shocks. These are driven by changes in the central bank's objectives such as the inflation

target, as well as by the perceived risk of the stock market given by the risk-premium terms in (4). Second, there are "propagation" effects governed by the matrix coefficient $T(\theta_{\xi_t^{P,A}}, \xi_t^b, \mathbf{H}^b)$ that determine how today's economic state is related to tomorrow's. Third, there are "amplification" effects governed by the matrix coefficient $R(\theta_{\xi_t^{P,A}}, \xi_t^b, \mathbf{H}^b)$ that generate endogenous heteroskedasticity of the primitive Gaussian shocks.

This heteroskedasticity implies that perceived quantity of risk in the stock market varies endogenously with the expected future conduct of monetary policy. Indeed, it is only through $R(\theta_{\xi_t^{P,A}}, \xi_t^b, \mathbf{H}^b)$ that the subjective risk premium in (4) varies. In turn, $R(\theta_{\xi_t^{P,A}}, \xi_t^b, \mathbf{H}^b)$ varies only with (i) realized regime changes ξ_t^P in the conduct of monetary policy–each of which are associated with a distinct perceived Alternative regime ξ_t^A–and (ii) time-varying beliefs ξ_t^b about the probability of switching to ξ_t^A by next period. This shows that the perceived quantity of risk varies only with changes in the current rule and beliefs about the future rule. It is especially sensitive to the activism coefficient in the perceived Alternative rule, $\psi_{\Delta y,\xi_t^A}$ governing investor beliefs about how strongly future monetary policy will react to fluctuations in economic growth. The greater $\psi_{\Delta y,\xi_t^A}$ is relative to $\psi_{\Delta y,\xi_t^P}$, the more agents perceive that future central bank policy will do more to proactively limit economic volatility and thus the systematic risks that affect stocks.

Investor Information and Updating Let $\mathbb{I}_t$ denote the time t information set of investors, which includes the current policy regime ξ_t^P, their perceived Alternative regime ξ_t^A, their beliefs about monetary policy ξ_t^b, and additional data available at mixed frequencies that we don't explicitly specify. Investors can observe the economic state S_t only at the end of each month, but the price assets continuously and their beliefs may exhibit jumps in response to Fed communications. With S_t observed only at the end of the month, any Fed news that the investor attends to *within* a month results in the updating of a *nowcast* of S_t, which we assume they produce by filtering a potentially extensive database of timely, high-frequency information in $\mathbb{I}_t$. This database is unobserved by the econometrician, but our estimation won't rely on it as explained below.

Investors use $\mathbb{I}_t$ in two ways. First, given a baseline monthly decision interval, they update their previous nowcasts and subjective expectations once S_t is observed at the end of every t. Second, investors allocate attention to updating nowcasts of S_t and beliefs ξ_t^b about future monetary policy at specific times *within* a month when the central bank releases information. This higher-frequency attentiveness to Fed news echoes real-world "Fed watching" and is the mechanism through which the model accommodates swift market reactions to surprise central bank announcements, driving jumps in investor perceptions of stock market risk $\mathbb{COV}_t^b\left[m_{t+1}, r_{t+1}^D\right]$.

4 Structural Estimation

Define $\xi_t \equiv \left(\xi_t^P, \xi_t^A, \xi_t^b\right)$ to be the collection of belief and policy regime indicators. The system of estimable equations may be written in state-space form by combining the state equations (13) with an observation equation taking the form

$$\begin{aligned} X_t &= D_{\xi_t,t} + Z_{\xi_t,t}\left[S_t', \widetilde{y}_{t-1}\right]' + U_t v_t \\ v_t &\sim N\left(0, I\right), \end{aligned} \tag{14}$$

where X_t denotes a vector of data, v_t is a vector of observation errors, U_t is a diagonal matrix with the standard deviations of the observation errors on the main diagonal, and $D_{\xi_t,t}$, and $Z_{\xi_t,t}$ are parameters mapping the model counterparts of X_t into the latent discrete- and continuous-valued state variables ξ_t and S_t, respectively, in the model. The matrices $Z_{\xi_t,t}$, U_t, and the vector $D_{\xi_t,t}$ depend on t independently of ξ_t because some of our observable series are not available at all frequencies and/or over the full sample. As a result, the state-space estimation uses different measurement equations to include series when they are available, and exclude them when they are missing.

We estimate the state-space representation with Bayesian methods using a modified version of Kim's (Kim (1994)) basic filter and approximation to the likelihood for Markov-switching state space models, and a random-walk metropolis Hastings MCMC algorithm to characterize uncertainty. The parameters of the monetary policy rule are estimated under symmetric priors, while the priors on the other parameters are standard and specified to be loosely informative except where there are strong restrictions dictated by theory, e.g., risk aversion must be non-negative. A complete description of the priors is provided in Appendix A of the Online Appendix.

Mixed-Frequency Filtering Algorithm The filtering algorithm described in this section is used to infer real-time jumps in investor beliefs in response to news events and refers to the state space equations (13) and (14). We provide a short description of the algorithm in this section, with full detail provided in Appendix I of the Online Appendix. To further facilitate interpretation, in Appendix H we explain the procedure for a simple state space representation with a single state variable and no regime switching.

The algorithm uses mixed-frequency data but differs from common reduced-form settings in which high-frequency data are used primarily to augment prediction models with more timely information, an objective typically accomplished by specifying the state/transition equations at the highest frequency of data used. In this case, the objects to be filtered are high-frequency estimates of the state space and, correspondingly, high-frequency structural shocks. Our mixed-frequency algorithm is designed for a very

different purpose, namely as way of integrating a high-frequency event study into a structural estimation for the purpose of measuring market reactions to news shocks. In our setting, the state/transition equation is part of the structural model and needs to correspond to the monthly frequency over which investors observe S_t and consume payout. However, agents price assets continuously and thus update their *beliefs* about the economic state in tight windows around FOMC news. Note that it would be not be appropriate to interpret high-frequency revisions in expectations as attributable–in general–to actual macroeconomic shocks occurring over a 30 minute window containing a Fed communication. A Fed announcement can be considered a set of signals about different variables with varying degrees of precision. An FOMC statement announcing a change in the target federal funds rate, which the Fed directly controls, is a signal with infinite precision that removes all uncertainty about what the end-of-month funds rate will be. In this special case, the belief update coincides with a true change in the interest rate. For all other variables in S_t, this will not be true. An announcement about the macroeconomic outlook is a signal with considerable noise, but in any case there's little reason to think that true macro shocks systematically happen over the narrow window that the fed happens to be speaking. What *can* plausibly change over the course of an announcement are the beliefs of investors about the current and future economic state.

To model these ideas, the filter is designed to measure within-month belief updates (i.e., nowcasts) reflecting investors' changing perceptions of the current (end-of-month) economic state attributable to Fed news. This is accomplished by mapping jumps in high-frequency, forward-looking data from markets and surveys into the model's implications for expectations, a procedure that allows the econometrician to infer investor belief updates around Fed news without having to take a stand on the unobservable nowcasting models and information sets of investors. Because the model implies the economic state is fully revealed at the end of each month, within-month nowcasts of S_t are then supplanted by their observed values the end of each t.

The algorithm may be summarized as follows. Suppose the econometrician has information up through the end of month $t-1$ and new high-frequency information arrives at $t-1+\delta_h$. Here $\delta_h \in (0,1)$ represents the number of time units that have passed during month t up to $t-1+\delta_h$. For example, δ_h could correspond to the number of time units that have passed when we are at 10 minutes before or 20 minutes after an FOMC announcement. Let X^{δ_h} denote the subset of X available at high frequency around Fed news, and let $X^{t-1} = (X_{t-1}, X_{t-2}, ...)$ denote all observations in the sample up through $t-1$. We use the suffix $(t \backslash t-1+\delta_h)$ to denote filtered objects related to investor nowcasts of the state at time t, conditional on their information at $t-1+\delta_h$. We use "|" to refer to conditioning that is with respect to the econometrician's information set. Thus, $S_{(t\backslash t-1+\delta_h)|t-1}$ refers to the econometrician's time $t-1$ filtered estimate of the

investor's nowcast of S_t conditional on whatever information investors had at $t-1+\delta_h$. The algorithm involves iterating on the following steps:

(i) Kalman Filter: Conditional on $\xi^b_{t-1} = j$ and $\xi^b_t = i$ run the Kalman filter for $i, j = 1, 2, ..., B$ to produce $S^{(i,j)}_{(t\backslash t-1+\delta_h)|t-1}$ and its mean squared error $P^{(i,j)}_{(t\backslash t-1+\delta_h)|t-1}$. At $t-1+\delta_h$, compute updated conditional forecast errors $e^{(i,j)}_{(t\backslash t-1+\delta_h)|t-1+\delta_h,t-1} = X^{\delta_h}_{t-1+\delta_h} - D_i - Z_i \left[S^{(i,j)'}_{(t\backslash t-1+\delta_h)|t-1}, \widetilde{y}_{t-1}\right]'$ using the series X^{δ_h} available at $t-1+\delta_h$. Fixing $S^{(i,j)}_{(t\backslash t-1+\delta_h)|t-1}$ and $P^{(i,j)}_{(t\backslash t-1+\delta_h)|t-1}$ from $t-1$, use $e^{(i,j)}_{(t\backslash t-1+\delta_h)|t-1+\delta_h,t-1}$ to re-run the filter and update $S^{(i,j)}_{(t\backslash t-1+\delta_h)|t-1+\delta_h}$ and $P^{(i,j)}_{(t\backslash t-1+\delta_h)|t-1+\delta_h}$.

(ii) Hamilton Filter: With $e^{(i,j)}_{(t\backslash t-1+\delta_h)|t-1+\delta_h,t-1}$ in hand, re-run the Hamilton filter to estimate new regime probabilities $\Pr\left(\xi^b_t, \xi^b_{t-1}|X_{t-1+\delta_h}, X^{t-1}\right)$, $\Pr\left(\xi^b_t|X_{t-1+\delta_h}, X^{t-1}\right)$ for $i, j = 1, 2, ..., B$.

(iii) Approximations: Collapse the $B\times B$ values of $S^{(i,j)}_{(t\backslash t-1+\delta_h)|t-1+\delta_h}$ and $P^{(i,j)}_{(t\backslash t-1+\delta_h)|t-1+\delta_h}$ into B values $S^{(j)}_{(t\backslash t-1+\delta_h)|t-1+\delta_h}$ and $P^{(j)}_{(t\backslash t-1+\delta_h)|t-1+\delta_h}$ using Kim's (Kim (1994)) approximation.

(iv) Store or Iterate: If $t-1+\delta_h = t$ iterate forward by setting $t-1 = t$ and return to step (i). Otherwise store the updates $S^{(j)}_{(t\backslash t-1+\delta_h)|t-1+\delta_h}$, $P^{(j)}_{(t\backslash t-1+\delta_h)|t-1+\delta_h}$, $\Pr\left(\xi^b_t, \xi^b_{t-1}|X_{t-1+\delta_h}, X^{t-1}\right)$, and $\Pr\left(\xi^b_t|X_{t-1+\delta_h}, X^{t-1}\right)$ and return to step (i) at the next intramonth time unit $\delta_k > \delta_h$, keeping $t-1$ fixed.

Two points about this algorithm bear noting. First, the filter is rerun at least twice, once immediately before and once after an FOMC announcement. In general, the filter can be rerun as frequently as desired within a month, even as transition dynamics are still specified across months. It is therefore straightforward to handle news events that are spaced non-uniformly over the sampling interval, as when the number of FOMC meetings during a month varies over the sample.

Second, the entire perceived state vector S_t can be reestimated at high-frequency within a month, provided that a subset of data are available in tight windows around announcements. Thus we can infer revisions to e.g., investor nowcasts of aggregate demand or of the earnings share from the information encoded in more timely financial market observations, even if data on output, earnings, inflation, etc., are only available once per month.

Data and Measurement Our full dataset spans January 1961 through February 2020. The sample of Fed news consists of 220 Federal Open Market Committee (FOMC) press releases covering February 4th, 1994 to January 29th, 2020. Observations on most

series are available monthly. For quarterly GDP growth we interpolate to monthly frequency using the method in Stock and Watson (2010). An explicit description of the mapping between our observables and model counterparts and complete description of each data series and sources is given in Appendix I of the Online Appendix.

We use high-frequency pre- and post-FOMC observations on the following variables: daily survey expectations of inflation, GDP growth, and S&P 500 earnings from Bloomberg (BBG), daily observations on the 20-year Baa credit spread with the 20-year Treasury bond rate (Baa spread hereafter), minutely observations on four distinct federal funds futures (FFF) contract rates with different expiries, and minutely observations on the S&P 500 market value. These high-frequency data serve two purposes. First, they allow us to measure the affect of Fed news on investor beliefs and perceptions in tight windows around announcements. Second, the timely information contained in these forward-looking series allow us to account for economic news that pre-dated the FOMC announcement but arrived after the latest observations on stale monthly survey data (Bauer and Swanson (2023)). By conditioning on close-range, pre- and post-announcement observations for inflation, GDP growth, and earnings expectations, as well as credit spreads (the day before and day after), interest rate futures, and the stock market (10 minutes before and 20 minutes after), we ensure that our estimated post-announcement jumps in nowcasts cannot be readily attributed to stale economic news.

At lower frequencies, we use the household-level Survey of Consumers (SOC) from the University of Michigan to discipline household expectations and three additional professional forecaster surveys from Bluechip (BC), Survey of Professional Forecasters (SPF) and Livingston (LIV) to discipline investor expectations. We measure investor expectations at multiple horizons using the four different professional surveys and treat each of these as a noisy signal on the true underlying investor expectations process.

A number of series are used because they have obvious model counterparts. Data for Gross Domestic Product (GDP) growth and inflation are mapped into the model implications for output growth and inflation; data on the current effective federal funds rate (FFR) are mapped into the model's implications for the current nominal interest rate; data on the FFF market and the BC survey measure of the expected FFR 12 months-ahead are mapped into the model's implications for investor expectations of the FFR.[8] The inclusion of data on long-dated FFF contracts and survey forecasts of the funds rate a year or more out are especially helpful for identifying the parameters of the

[8]In principle, fed funds futures market rates may contain a risk premium that varies over time. If such variation exists, it is absorbed in the estimation by the observation error for these equations. In practice, risk premia variation in fed funds futures is known to be small when that variation is measured over the short 30-minute windows surrounding FOMC announcements that we analyze (Piazzesi and Swanson (2008)).

Alternative policy rule, since investors' longer-term forecasts are dominated by where they believe future policy will be and not by the rule currently in place.

We discipline the earnings share of output K_t with observations on the ratio of S&P 500 earnings to GDP. We account for the fact that earnings in the data differs from the payout shareholders actually receive by mapping the theoretical concept for k_t into its respective data series allowing for observation error in the relevant observation equation.

Finally, data on the Baa spread are mapped into the model's implications for the liquidity premium, lp_t, a catchall for many factors outside of the model that could effect the subjective equity premium, including changes in the perceived liquidity and safety attributes of Treasuries, default risk, flights to quality, and/or sentiment. We use the Baa spread as an observable likely to be correlated with many of these factors, but our measurement equation allows for both a constant and a slope coefficient on the Baa spread along with observation error, in order to soak up variation in this latent variable that may not move identically with the spread.

Estimating Beliefs We take the parameters p_{bi} in $\mathbf{H}^b$ from a discretized beta distribution, estimating its mean and variance as additional parameters of the structural estimation. The parameters $p_{\Delta i|j}$ are specified as $(1-p_s)\left(\rho_b^{|i-j-1|}/\sum_{i\neq j}\rho_b^{|i-j-1|}\right)$, where p_s and $\rho_b<1$ are estimated parameters and $|i-j-1|$ measures the distance between beliefs j and i, for $i\neq j\in(1,2,...,B)$. This creates a decaying function that makes the probability of moving to contiguous beliefs more likely than jumping to very different beliefs.

Let T be the sample size used in the estimation and let the vector of observations as of time t be denoted by X_t. Let $\Pr\left(\xi_t^b=i|X_T;\boldsymbol{\theta}\right)\equiv\pi_{t|T}^i$ denote the probability that $\xi_t^b=i$, for $i=1,2,...,B+1$, based on information that can be extracted from the whole sample and knowledge of the parameters $\boldsymbol{\theta}$, while $\pi_{t|T}$ is a $(B+1)\times1$ vector containing the elements $\left\{\pi_{t|T}^i\right\}_{i=1}^{B+1}$. We refer to these as the smoothed regime probabilities. The time t perceived probability of exiting the current policy rule, i.e., of transitioning in the next period to the Alternative policy regime ξ_t^A, is given by $\overline{P}_t^{bE}\equiv\sum_{i=1}^{B}\pi_{t|T}^i(1-p_{bi})$. The time t perceived probability of exiting the current policy rule and transitioning in h periods to ξ_t^A is $\overline{P}_{t+h,t}^{bE}=\mathbf{1}'_{B+1}\left(\mathbf{H}^b\right)^h\pi_{t|T}$, where $\mathbf{1}'_{B+1}$ is an indicator vector with 1 in the $(B+1)$th position and zeros elsewhere. We use these estimated regime probabilities to compute the most likely belief regime at each point in time and track how it changes around Fed announcements and the whole sample. In the applied estimation, we set $B=11$.

5 Estimation Results

This section presents results from the structural estimation based on the modal values of the posterior distribution for the parameters. The estimated credible sets indicate that the parameters are tightly identified and we report other moments of the posterior in Table A.1 of the Online Appendix. In the estimation, we allow for observation errors on all variables except for inflation, GDP growth, the FFR, and the SP500-lagged GDP ratio. The estimated model-implied series track their empirical counterparts closely, as shown in Figure A.1 of the Online Appendix.

Parameter and Latent State Estimates Table 1 reports the posterior modes for the policy rule parameters $\pi^T_{\xi^P_t}$, ψ_{π,ξ^P_t}, $\psi_{\Delta y,\xi^P_t}$ and ρ_{i,ξ^P_t}, where we use symmetric priors. The results imply that the regime subperiods reported in Figure 1 are associated with quantitatively large changes in the estimated policy rule, as well as in the associated Alternative policy rules that we estimate investors perceived would come next. The Great Inflation (GI) regime (1961:Q1-1978:Q3) is characterized by a high implicit

Table 1: **Taylor Rule Parameters**

	Great Inflation Regime		Great Moderation Regime		Post-Millennial Regime	
	Realized	Alternative	Realized	Alternative	Realized	Alternative
π^T_ξ	12.3763	3.7021	2.2366	0.7033	2.5032	0.0600
ψ_π	1.8617	0.6817	1.9532	2.7249	0.9163	0.8103
ψ_y	1.0214	0.4541	0.1015	0.6472	0.0550	0.5499
ψ_π/ψ_y	1.8227	1.5012	19.2433	4.2103	16.6600	1.4735
$x = \rho_{i,1} + \rho_{i,2}$	0.9955	0.9818	0.9868	0.5680	0.9958	0.8922

Notes: Posterior mode values of the parameters for the current and Alternative policy rules. Great Inflation Regime: 1961:Q1-1978:Q3. Great Moderation Regime: 1978:Q4-2001:Q3. Post-Millennial Regime: 2001:Q4-2020:Q1. The estimation sample spans 1961:Q1-2020:Q1.

inflation target and a moderate level of inflation activism (ψ_{π,ξ^P_t}), consistent with previous research arguing that the Fed accommodated high inflation during this period. The perceived Alternative policy rule for this subperiod has a much lower inflation target, but features less activism against both inflation and output growth, with inflation stabilization perceived as the main objective. The anticipation of a lower inflation target is in fact a defining feature of the subsequent Great Moderation (GM) regime that began in 1978:Q4. The GM also featured a stronger emphasis on inflation stabilization than the GI regime but little activism on economic growth. Moving to the Post-Millennial (PM) regime, we find that policy rule parameters then shifted back toward slightly more accommodative values with a higher implicit inflation target, but with far less activism on inflation and comparably low activism on output growth .

The estimated perceived Alternative policy rules of each regime show how investors expected policy to change in the future. In the GM regime, investors evidently expected the next rule to have an inflation target that was even lower than what was actively in place at the time, along with greater activism in stabilizing both inflation and economic growth. In the PM period investors expected an inflation target that was lower still, but with a greater emphasis on output growth stabilization relative to inflation stabilization compared to the realized rule during the PM period. Thus both the GM and PM periods are characterized by expectations that the next policy rule would be both more hawkish and more active on output growth than the realized rules of those periods. Since more activism on output growth is indicative of more aggressive action to stabilize the real economy, these features of the perceived Alternative rules are closely related to perceived risk in the stock market, as discussed below.

A comment is in order about the estimated magnitudes for $\pi^T_{\xi^P_t}$ shown in Table 1. Although this parameter plays the role of an "inflation target" in the interest rate rule, unlike traditional New Keynesian models with a time invariant inflation target, $\pi^T_{\xi_t}$ is appropriately interpreted as an implicit time t target rather than an explicit long-run objective. To understand why, consider the PM period as an example. The structural estimation implies that, to achieve the observed average CPI inflation of roughly 1.96% over this period, $\pi^T_{\xi^P_t}$ needed to be 2.5%, well above what officially became in 2012 the explicitly stated long-run inflation objective of 2%. Forward guidance "low-for-long" interest rate policies and quantitative easing, two tools that were employed at the zero-lower-bound (ZLB), are channels that manifest in the model as a higher values for $\pi^T_{\xi^P_t}$, since with $\gamma^T > 0$ these tools generate higher perceived trend inflation by households even as nominal interest rates remain unchanged at the ZLB (equation (9)). Likewise, the high value for $\pi^T_{\xi^P_t}$ in the GI regime represents an implicit central bank accommodation of the high inflation of the 1970s that is difficult to explain otherwise.

Table 2 presents estimation results for key model parameters other than those of the policy rule.[9] The estimates imply a very high level of inertia in household inflation expectations. The constant gain parameter γ controlling the speed with which households update beliefs about inflation with new information on inflation is estimated to be low ($\gamma = 0.0001$). Furthermore, the parameter γ^T controlling the speed with which households' perceived trend inflation is influenced by shifts in $\pi^T_{\xi^P_t}$ is also estimated to be quite small, though non-zero ($\gamma^T = 0.006$), consistent with survey evidence that households are innattentive to monetary policy communications, even around major policy events extensively covered by media (Coibion, Gorodnichenko, Kumar, and Pedemonte (2020)). Taken together, these findings imply that households revise their beliefs about

[9]The model has a large number of additional auxiliary parameters that are used to map observables into their model counterparts. To conserve space, estimates of these parameters are reported in the Online Appendix.

trend inflation only very slowly over time, both in response to changes in the implicit inflation target and with past inflation realizations.

Table 2: **Other Key Parameters**

Parameter	Mode	Parameter	Mode	Parameter	Mode	Parameter	Mode
σ	0.1051	γ^T	0.0056	σ_f	6.4146	σ_{lp}	0.2035
β	0.7552	σ_p	6.8668	σ_i	0.0352	σ_g	1.4568
ϕ	0.7477	β_p	0.9964	σ_μ	0.1302		
γ	0.0001	p_s	0.9410	σ_k	6.3330		

Notes: Posterior mode values of the parameters named in the row. The sample spans 1961:Q1-2020:Q1.

We estimate a moderate level of risk aversion for the investor ($\sigma_P = 6.9$). In terms of the magnitude of the primitive economic shocks, monthly demand shocks are estimated to be the largest quantitatively ($\sigma_f = 6.4$), compared to "supply side" shocks to trend growth ($\sigma_g = 1.46$) or the markup shock ($\sigma_\mu = 0.13$). Finally, the parameter p_s is estimated to be 0.94, indicating that investors maintain very firmly held beliefs, rarely contemplating the possibility that they may change their minds about the likelihood of moving to the next policy rule on the basis of new information.

Before leaving this section we report the model implications for basic asset pricing moments. Table 3 shows that the model based moments for the log stock return, real interest rate, and earnings growth, based on the modal parameter and latent state estimates, match their data counterparts closely.

Table 3: **Asset Pricing Moments**

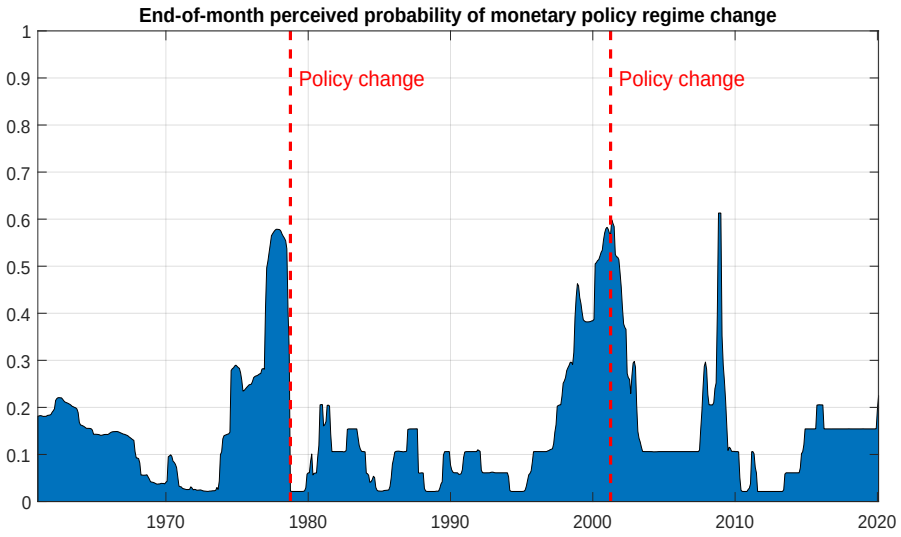
Moments	Model		Data	
	Mean	StD	Mean	StD
Log Excess Return	7.20	14.93	7.42	14.85
Real Interest Rate	1.65	2.48	1.72	2.53
Log Real Earning Growth	2.62	25.06	1.96	17.24

Notes: Annualized monthly statistics (means are multiplied by 12 and standard deviations by $\sqrt{12}$) and reported in units of percent. Excess returns are the log difference in the SP500 market capitalization minus FFR. Real interest rate is FFR minus the average of the average of the one-year ahead forecasts of inflation from the BC, SPF, SOC, and Livingston surveys. SP500 Earnings is deflated using the GDP deflator and divided by population. The sample is 1961:M1 - 2020:M2.

Investor Beliefs About Monetary Policy Over the Sample Figure 2 plots the estimated perceived probability that investors assign to being in a new policy rule regime in one year's time. Specifically, the figure reports the end-of-the-month value for $\overline{P}^{bE}_{t+12,t} \equiv \pi^{B+1}_{t+h,t|T} = \mathbf{1}'_{B+1}\left(\mathbf{H}^b\right)^{12}\pi_{t|T}$, where $\mathbf{1}'_{B+1}$ is an indicator vector with 1 in the $(B+1)$th position and zeros elsewhere and $\pi_{t|T}$ is the vector of smoothed time t belief

regime probabilities. The vertical lines mark the timing of the two realized policy regime changes in our sample.

Figure 2: Perceived Probability of Monetary Policy Regime Change



Notes: Estimated end-of-month perceived probability that investors assign to exiting the current monetary policy rule within one year. The sample spans 1961:M1-2020:M2.

Figure 2 shows that the perceived probability of a policy rule regime change fluctuates strongly over the sample and typically increases before a realized policy change, suggesting that financial markets have some ability to anticipate realized shifts in the conduct of policy even though they cannot perfectly predict what the next policy rule will look like. The perceived probability of a policy rule change also spikes upward sharply in the financial crisis when no actual change occurred subsequently, though this "mistake" is short-lasting. One interpretation of this brief spike is that investors may have initially believed that the Fed could shift to a policy rule with more aggressive stabilization of economic growth, but soon realized that the severity of the crisis and the reality of the ZLB would constrain their ability to do so.

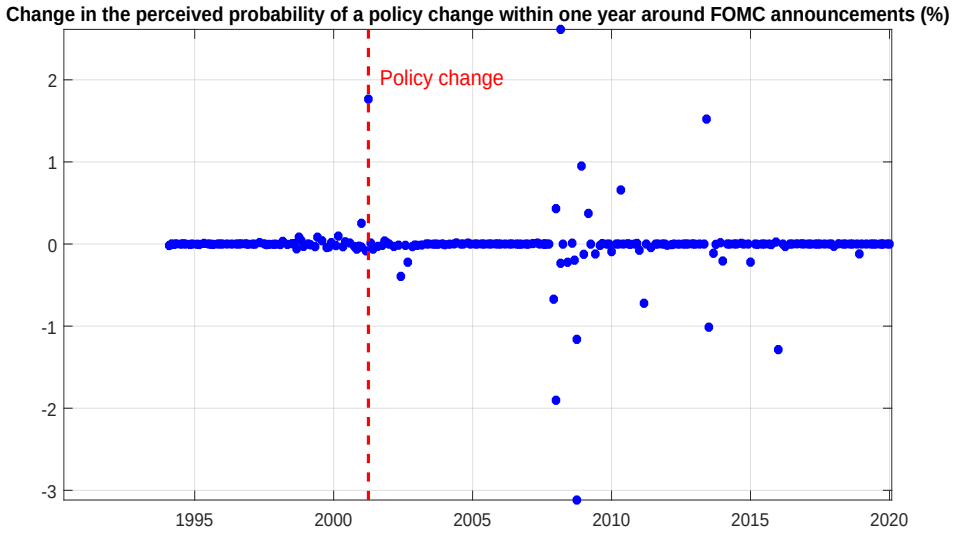
An important feature of the findings displayed in Figure 2 is that investor beliefs about the probability of a regime change in the Fed's policy rule continuously evolve *out*side of tight windows surrounding policy announcements. Indeed, most of the variation in investor beliefs about the future conduct of monetary policy occurs at times over the sample that are not close temporally to an FOMC announcement, indicating that the causal effect of central bank policy on investor beliefs and therefore on markets is substantially more far reaching than what can be observed from market reactions in tight windows surrounding Fed communications.[10] An obvious explanation for this result is that most Fed announcements are not immediately associated with a change in the rule.

[10]Brooks, Katz, and Lustig (2018) report a related finding for the Treasury market with evidence of persistent post-FOMC announcement drift in longer term yields.

Instead, what they mainly provide is a form of forward guidance on the factors that are likely to trigger a change in the policy stance down the road. As new data become available in between Fed communications, investor beliefs about future monetary policy are shaped by what was previously communicated, having consequences for markets even if current policy is unchanged. Because high frequency event studies surrounding Fed communications only capture the causal effects of the surprise component of any announcement, they are by construction incapable of accommodating these additional channels of influence outside of tight windows around events. The estimates portrayed in Figure 2 are key inputs into our estimated overall causal impact of the Fed on markets over the sample, discussed below in Section 5.

To underscore this point, Figure 3 shows the *change* in the estimated perceived probability of a monetary policy regime change within the next year in tight windows around every FOMC announcement in our sample. We see that most FOMC announcements result in little if any change in the perceived probability of a regime change in monetary policy, again implying that financial markets do not learn about the possibility of policy regime change only from the surprise component of a policy announcement. Naturally, many FOMC announcements carry little news of any kind, consistent with the majority of points lining up along the horizontal line at zero and the idea that significant changes in the policy rule are infrequent.

Figure 3: Change in the probability of a policy switch around FOMC announcements



Notes: Pre-/post- FOMC announcement log changes (10 minutes before/20 minutes after) in the probability that financial markets assign to a switch in the monetary policy rule occurring within one year. The full sample has 220 announcements spanning February 4th, 1994 to February 28th, 2020. The sample reported in the figure is 1993:M1-2020:M2.

Nevertheless, we find that some announcements are associated with sizable changes

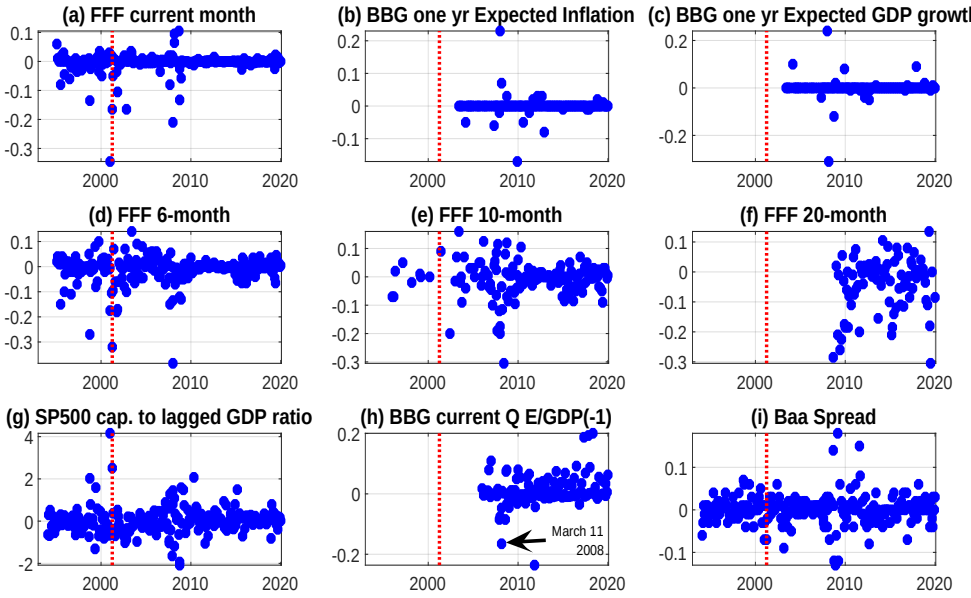
in the perceived probability of exiting the current policy regime. The largest decline in this perceived probability occurred on October 29, 2008 when the FOMC announced a 50 basis point reduction in the fed funds rate target and the perceived probability of a regime change in the next year declined by more than 2% in the 30 minutes surrounding the FOMC press release. The largest increase in the perceived probability of a policy regime change occurs on March 11, 2008 when the Federal Reserve annouced an expansion of securities lending in a new facility, the Term Securities Lending Facility (TSLF). In this case the perceived probability of policy regime change increased more than 2%. We discuss these differing reactions below.

High-Frequency Analysis To study why markets sometimes react strongly to Fed announcements, we investigate what happens in tight windows around FOMC press releases. In our analysis the pre-FOMC value is always either 10 minutes before or the day before the FOMC press release time, depending on data availability (daily versus minutely), and the post-FOMC value is either 20 minutes after or the day after the release. Figure 4 displays the log change in pre-/post- FOMC announcement values of variables we measure at high frequency, for each FOMC announcement in our sample. Some announcements are associated with declines in the stock market within 30 minutes surrounding the FOMC press release that exceed 2% in absolute terms or increases above 4%. Many announcements also produce large jumps in other financial market variables such as FFF rates and the Baa spread.

The mixed-frequency structural approach developed in this paper allows us to investigate a variety of possible explanations for these large market reactions. Consider an FOMC announcement in month t. As above, let $\delta_h \in (0,1)$ represent the number of time units that have passed during month t up to some particular point $t-1+\delta_h$. Let $S^i_{(t\backslash t-1+\delta_h)|t-1+\delta_h}$ denote a filtered estimate of investors' perceived time t economic state based on their information up to time $t-1+\delta_h$, conditional on $\xi^b_t = i$. We use the filtering algorithm described above along with high-frequency, forward-looking data on investor expectations and financial markets to obtain estimates of the pre- and post-FOMC announcement nowcasts $S^i_{(t\backslash t-1+\delta_h)|t-1+\delta_h}$, and the associated filtered belief regime probabilities $\pi^i_{t|t-1+\delta_h} \equiv \Pr\left(\xi^b_t = i | X_{t-1+\delta_h}, X^{t-1}\right)$, where δ_h assumes distinct values d_{pre} and d_{post} that denote the times right before and right after the FOMC meeting. These *pre* and *post* differences represent our estimates of the market's revised nowcasts for S and beliefs about the future conduct of monetary policy that are attributable to the FOMC announcement.

Figure 5 displays the percent changes in pre-/post- announcement nowcasts of different elements of S_t for every FOMC announcement in our sample. The figure shows that some FOMC announcements led to frequent and large changes in investor perceptions

Figure 4: HF Changes in Prices and Expectations



Notes: Log change in the observed variables in a short time-window around FOMC meetings. For all but panels (b) and (c), this corresponds to a change measured from 10 minutes before to 20 minutes after an FOMC statement is released. For panels (b) and (c), this corresponds to one day before to one day after the FOMC statement is released. The full sample has 220 FOMC announcements spanning February 4th, 1994 to February 28th, 2020. The sample reported in the figure is 1993:M1-2020:M2.

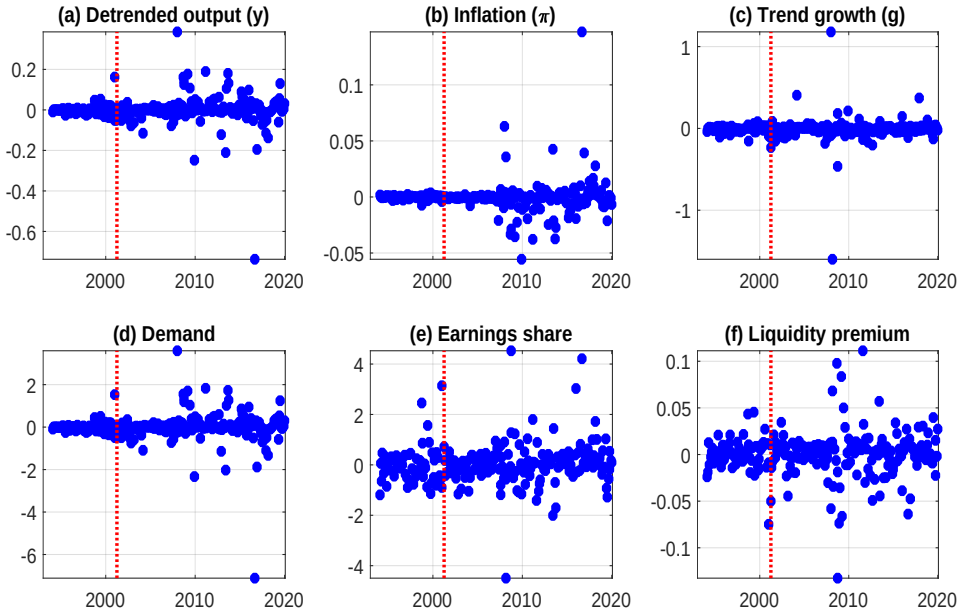
about trend growth g_t, detrended output, $\widetilde{y}_t$, inflation, current demand f_t, the earnings share k_t, and the liquidity premium lp_t. This implies that some announcements cause investors to significantly revise their beliefs about the state of the economy and its core driving forces.

To make further progress of our understanding of what markets learn from FOMC announcements, we use estimates of $S^i_{(t\backslash t-1+\delta_h)|t-1+\delta_h}$ and the belief regimes $\pi^i_{t|t-1+\delta_h}$ in the minutes and days surrounding an FOMC meeting to observe changes in the *perceived shocks* $\varepsilon^i_{(t\backslash t-1+\delta_h)|t-1+\delta_h}$ that investors must have discerned in order to explain revisions in $S^i_{(t\backslash t-1+\delta_h)|t-1+\delta_h}$ and $\pi^i_{t|t-1+\delta_h}$. To do so consider the model solution applied to the intramonth nowcasting updates:

$$\begin{aligned} S^i_{(t\backslash t-1+\delta_h)|t-1+\delta_h} &= C\left(\theta_{\xi_t^{P,A}}, \xi^b_t = i, \mathbf{H}^b\right) + T(\theta_{\xi_t^{P,A}}, \xi^b_t = i, \mathbf{H}^b) S^j_{t-1} \\ +R(\theta_{\xi_t^{P,A}}, \xi^b_t &= i, \mathbf{H}^b) Q \varepsilon^i_{(t\backslash t-1+\delta_h)|t-1+\delta_h}, \end{aligned} \tag{15}$$

where $\varepsilon^i_{(t\backslash t-1+\delta_h)|t-1+\delta_h}$ denotes the perceived Gaussian shocks estimated on the basis of data available at time $t-1+\delta_h$, conditional on being in belief regime $\xi^b_t = i$. Given estimates of $S^i_{(t\backslash t-1+\delta_h)|t-1+\delta_h}$, $C(\cdot)$, $T(\cdot)$, $R(\cdot)$, Q, and S^j_{t-1} using the most likely belief regime j at $t-1$, invert (15) to solve for $\varepsilon^i_{(t\backslash t-1+\delta_h)|t-1+\delta_h}$. The contribution of one

Figure 5: HF Changes in State Variables



Notes: Estimated changes in the perceived state of the economy from 10 minutes before to 20 minutes after an FOMC press release. The full sample has 220 FOMC announcements spanning February 4th, 1994 to February 28th, 2020. The sample reported in the figure is 1993:M1-2020:M2.

particular perceived shock k is to variation in $S^{i}_{(t\backslash t-1+\delta_h)|t-1+\delta_h}$ is given by:

$$S^{\cdot,k}_{(t\backslash t-1+\delta_h)|t-1+\delta_h} = \sum_{i=1}^{B} \pi^{i}_{t|t-1+\delta_h} R(\theta_{\xi_t^{P,A}}, \xi_t^b = i, \mathbf{H}^b) Q \varepsilon^{i,k}_{(t\backslash t-1+\delta_h)|t-1+\delta_h} \tag{16}$$

where $\varepsilon^{i,k}_{(t\backslash t-1+\delta_h)|t-1+\delta_h}$ is a vector constructed by setting each element of $\varepsilon^{i}_{(t\backslash t-1+\delta_h)|t-1+\delta_h}$ to zero other than the kth. The contribution of the belief regime is the remaining part:

$$S^{\cdot,b}_{(t\backslash t-1+\delta_h)|t-1+\delta_h} = \sum_{i=1}^{B} \pi^{i}_{t|t-1+\delta_h} \left[C\left(\theta_{\xi_t^{P,A}}, \xi_t^b = i, \mathbf{H}^b\right) + T(\theta_{\xi_t^{P,A}}, \xi_t^b = i, \mathbf{H}^b) S^{j}_{t-1} \right]. \tag{17}$$

Finally, the contribution of revisions in perceived shocks and belief regimes to jumps in observed variables X_t is computed by taking the difference between the post- and pre-announcement values of $S^{\cdot,k}_{(t\backslash t-1+\delta_h)|t-1+\delta_h}$ and $S^{\cdot,b}_{(t\backslash t-1+\delta_h)|t-1+\delta_h}$ and linking them back to X_t using the mapping (14). We refer to these as shock decompositions.

Figure 6 reports shock decompositions for the FOMC announcements that generated the 10 most quantitatively important absolute changes in the stock market. The largest of these occurred on January 3, 2001 when the Fed met off-cycle to lower the target funds rate by 50 basis points, driving the S&P 500 surge 4.2% over the 30 minutes surrounding the news. The long red bar in panel (c) shows that the news led to investors to perceive a large accommodative monetary policy shock. Yet panel (d) of Figure 6 shows that the main driver of the market's jump was not the surprise decline in the funds rate *per se*, but instead an upward revision in the nowcast for the corporate earnings share,

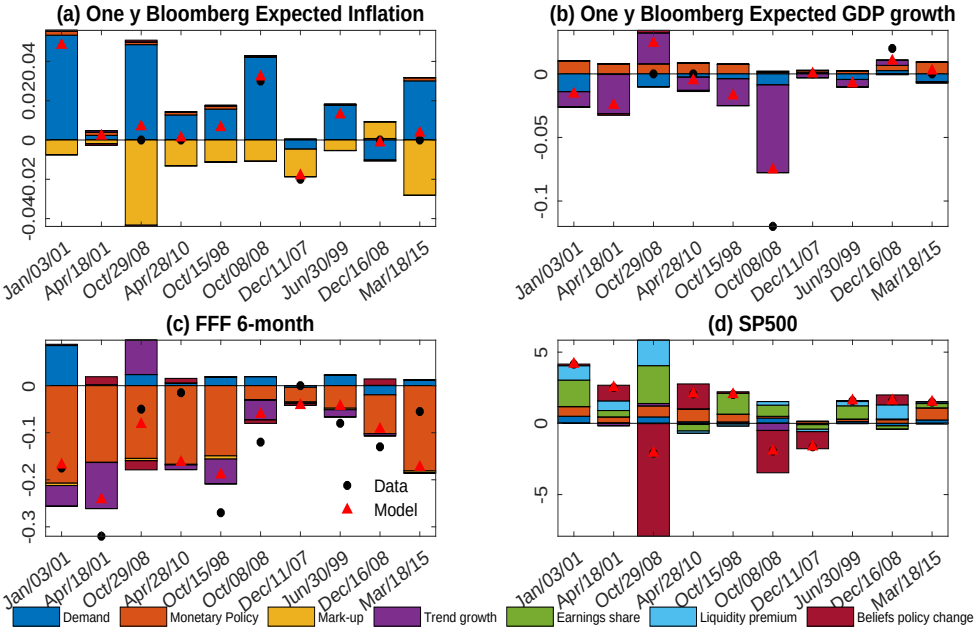
and a downward revision in the nowcast of the liquidity premium component of the equity premium. These empirical results occur for two reasons. First, the liquidity premium falls because the actual BAA spread fell surrounding the news. Second, the nowcast of the earnings-to-lagged GDP share rises because the actual BBG nowcast of current quarter earnings rose sharply surrounding the news. To understand this latter result, note that our linearized structural model captures the first order, direct, effects of each channel. Although second order effects are not explicitly modeled, they can still be accounted for by the empirical procedure given observed movements in data. The surprise cut in the FFR on Jan 3, 2001 provides an example. This action–widely interpreted as unexpected insurance in response to the dot-com bust–leads to an estimated upward revision in both the actual earnings-to-lagged-GDP ratio and in investor nowcasts of this variable. These upward revisions are related to estimated second-order effects of the surprise cut in rates, rather than to the direct impact of the actual cut or to changing beliefs about future policy regimes.

An alternative interpretation is that the January, 2001 announcement caused investors to update their assessment of the parameters of the current policy rule. This is not a feature of our baseline model, but we allow for it in an alternative version of the model and report the results in Appendix K of the Online Appendix, which shows that the results for the stock market shown in Figure 6 are virtually unchanged if investors update their understanding of the current rule after the announcement–even by sizable amounts. The reason for this is that what matters for the long duration stock market is not where the policy rule is today, but where it is likely to settle for the foreseeable future. Beliefs about the latter are already captured in our baseline model by the investor's continuous belief updating about the probability of moving to the Alternative rule, so allowing for additional updating about the parameters of the current rule changes little about the longer-term outlook of relevance for the stock market.

The second and third most important FOMC events for the stock market were those on April 18, 2001 and October 29, 2008, respectively, when the market increased 2.5% and declined 2%, respectively, in the 30 minutes surrounding those press releases. For these and many subsequent events, investor beliefs about the probability of near-term monetary policy regime change played the largest quantitative role in the market's jump. We discuss the channels through which beliefs affect markets in the next section.

Discount Rate or Cash Flow Effects? In principle, the actions and announcements of central banks can affect financial markets through either discount rate or cash flow effects, or both. To study these different channels, we decompose price-lagged output ratio into components of the representative investor's subjective expectations. Start

Figure 6: Top Ten FOMC: SP500



Notes: The figure reports shock decomposition for the 10 most relevant FOMC announcements based on changes in the SP500-lagged GDP ratio. The sample is 1961:M1-2020:M2.

with

$$\frac{P_t^D}{Y_{t-1}} = \frac{P_t^D}{D_t}\frac{D_t}{Y_t}\frac{Y_t}{Y_{t-1}}$$

or in logs

$$pgdp_t = pd_t + k_t + \Delta y_t, \tag{18}$$

where $pgdp_t \equiv \ln\left(P_t^D/Y_{t-1}\right)$ and $pd_t \equiv \ln\left(P_t^D/D_t\right)$. Let r_t^{ex} denote the log return r_t^D in excess of the log real interest rate, rir_t. Decompose pd_t as in Campbell and Shiller (1989) into the sum of three forward-looking terms:

$$pd_t = \frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + pdv_t\left(\Delta d\right) - pdv_t\left(r^{ex}\right) - pdv_t\left(rir\right) \tag{19}$$

where $pdv_t\left(x\right) \equiv \sum_{h=0}^{\infty}\beta_p^h\mathbb{E}_t^b\left[x_{t+1+h}\right]$, $rir_{t+1} \equiv \left(i_{t+1} - \mathbb{E}_t^b\left[\pi_{t+1}\right]\right)$ are computed under the subjective expectations of the investor $\mathbb{E}_t^b\left[\cdot\right]$. Subjectively expected return premia $pdv_t\left(r^{ex}\right)$ are driven in the model by three factors: (i), realized regime change in monetary policy ξ_t^P, (ii) changing investor beliefs about the probability of future regime change ξ_t^b, and (iii) the liquidity premium lp_t. Subjectively expected real interest rates $pdv_t\left(rir\right)$ depend these factors, as well as on expectations about inflation and output growth that enter the monetary policy rule.

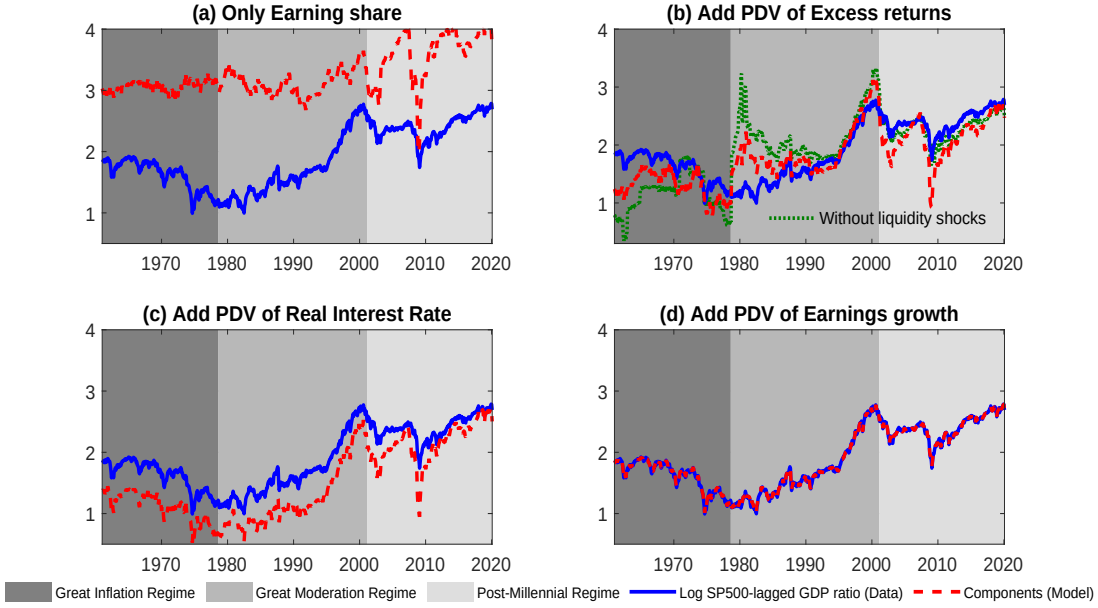
Substituting (19) into (18), we can decompose $pgdp_t$ into four components:

$$pgdp_t = \underbrace{ey_t}_{\text{earning share}} + \underbrace{pdv_t\left(\Delta d\right)}_{\text{earnings}} - \underbrace{pdv_t\left(r^{ex}\right)}_{\text{premia}} - \underbrace{pdv_t\left(rir\right)}_{\text{real int rate}}, \tag{20}$$

where $ey_t \equiv \frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + k_t + \Delta y_t$ is the earnings-to-lagged output ratio, or "earnings share" for brevity.

Figure 7 decomposes historical variation in $pgdp_t$ into the estimated components of (20). The solid (blue) line in each panel plots the data for $pgdp_t$, measured as the S&P 500-lagged GDP ratio. The red lines in panels (a)-(d) successively cumulate the right hand side components in (20) so that they add to the observed $pgdp_t$ as we move from panel (a) to panel (d).

Figure 7: SP500-to-GDP decomposition



Notes: Decomposition of the log SP500-to-lagged GDP ratio $pgdp$. The blue (solid) line represents the data. The dashed (red) lines represent component in the model, decomposed as $pgdp_t = ey_t + pdv_t(\Delta d) - pdv_t(r^{ex}) - pdv_t(rir)$, where $pdv_t(x) \equiv \sum_{h=0}^{\infty} \beta_p^h \mathbb{E}_t^b [x_{t+1+h}]$. Panel (a) plots $pgdp_t$ along with ey_t. Panel (b) plots $pgdp_t$ with $ey_t - pdv_t(r^{ex})$. Panel (c) plots $pgdp_t$ with $ey_t - pdv_t(r^{ex}) - pdv_t(rir)$. Panel (d) plots $pgdp_t$ in the data along with $ey_t + pdv_t(\Delta d) - pdv_t(r^{ex}) - pdv_t(rir)$. Great Inflation Regime: 1961:Q1-1978:Q3. Great Moderation Regime: 1978:Q4-2001:Q3. Post-Millennial Regime: 2001:Q4-2020:Q1. The sample spans 1961:M1 - 2020:M2

Panel (a) of Figure 7 shows that ey_t alone plays little role in fluctuations in $pgdp_t$ up to about the year 2000, but it declines sharply in the financial crisis of 2008/09 contributing to the sharp drop in the stock market during the crisis and subsequently boosting the market thereafter, echoing previous findings on the role of the earnings share in Greenwald, Lettau, and Ludvigson (2019).

A comparison of panels (a) and (b) shows how the picture changes when we add (the negative of) subjectively expected return premia $-pdv_t(r^{ex})$ to ey_t. The green line in panel (b) plots a counterfactual in which we turn off the liquidity premium shocks lp_t, implying that–within a policy regime–the only factor driving fluctuations

in $pdv_t(r^{ex})$ are changing investor beliefs about the probability of a regime change. Outside of a few episodes, we see that the green counterfactual line is quite close to the baseline estimate, implying that much of the variation in the estimated subjective return premium is driven by beliefs about future policy regime shifts, rather than by fluctuations in the liquidity premium. The exception to this occurs in the years after the switch to the GM regime, where, absent liquidity shocks, the market would have been substantially higher. Looking at the end of the GM regime, panel (b) shows that lower subjective return premia drove a surge in the market because investors perceived a greater likelihood that the central bank would move to a policy rule more focused on stabilizing the real economy. This can be understood from the results reported in from Figure 2, which shows the sharp rise in the perceived probability of regime change at the end of the GM period, in conjunction with the parameter estimates of the perceived Alternative rule that investors expected to come next from Table 1. These shifts in beliefs about future policy drove down the perceived quantity of risk in the stock market and drove up valuations.

Panel (c) of Figure 7 adds $-pdv_t(rir)$ to $ey_t - pdv_t(r^{ex})$, so that the differences between panels (b) and (c) isolates the role of subjectively expected real interest rates in stock market fluctuations. Expectations of persistently low future real rates helped support the stock market in the GI regime from 1961:Q1-1978:Q3, but by contrast, expectations of persistently higher real rates pulled down the market with the shift to a hawkish policy rule during the Volcker disinflation. Comparing panels (b) and (c) we see that expectations of persistently higher future real interest rates largely explain the low stock market valuations between 1978:Q3 to about 1990. Taken together, these results imply that the Volcker disinflation and the Great Moderation that followed set the stage for the high valuations in 1990s, by reducing expected volatility and lowering subjective return premia. Initially, however, the switch into the GM regime dragged the market down through the shift to a more hawkish policy rule with persistently higher real interest rates.

Finally, panel (d) of Figure 7 adds $pdv_t(\Delta d)$ to $ey_t - pdv_t(r^{ex}) - pdv_t(rir)$. Expected future cash flow growth plays a small role in these stock market fluctuations.

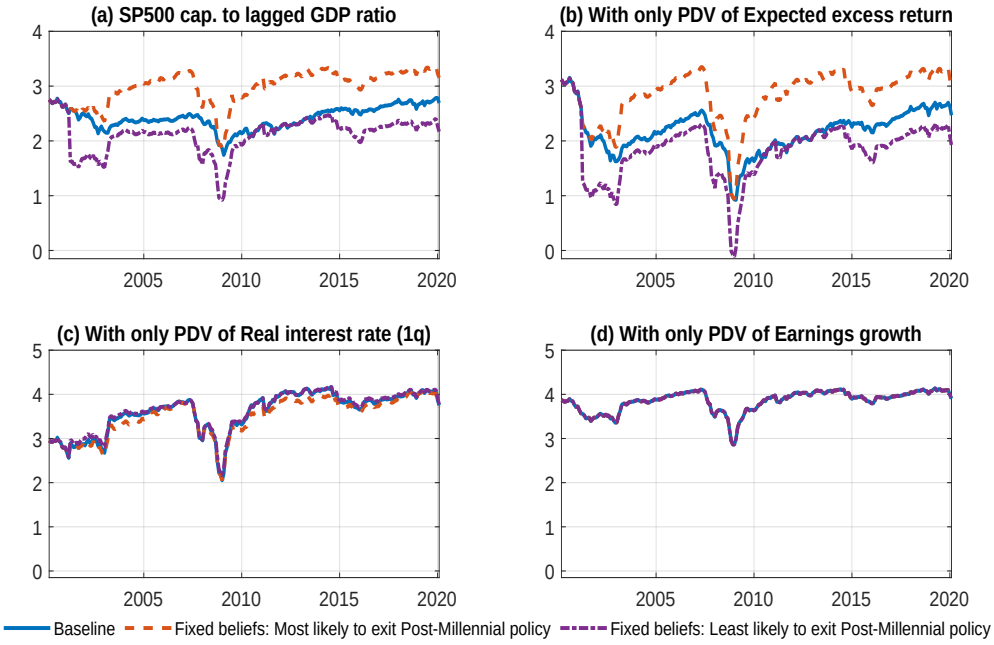
Figure 8 exhibits a counterfactual for the PM period with a slightly different decomposition of $pgdp_t$, this time adding only one of the $pdv(\cdot)$ terms in (20) at a time to ey_t. We use the notation

$$pgdp_{r^{ex},t} \equiv ey_t - pdv_t(r^{ex}); \quad pgdp_{rir,t} \equiv ey_t - pdv_t(rir); \quad pgdp_{\Delta d,t} \equiv ey_t + pdv_t(\Delta d).$$

The solid (blue) line in each panel of Figure 8 plots our baseline estimate of the component series named in the subpanel. For panel (a), which plots $pgdp_t$, our baseline model estimate and the data series coincide by construction. Panels (b)-(c) plot the components

$pgdp_{r^{ex},t}$, $pgdp_{rir,t}$, and $pgdp_{\Delta d,t}$, respectively. The red/dashed (purple/dashed-dotted) line in each panel plots a counterfactual in which the belief regime with the highest (lowest) perceived probability of exiting the policy rule was always in place.[11]

Figure 8: Counterfactual simulations: The Post-Millennial period



Notes: Counterfactual for the post-Millennial period. The red/dashed (purple/dashed-dotted) line plots a counterfactual in which the belief regime with the highest (lowest) perceived probability of exiting the policy rule was always in place. Panel (a) plots the model implications for $pgdp_t$. Panel (b) plots $pgdp_{r^{ex},t}$. Panel (c) plots $pgdp_{rir,t}$. Panel (d) plots $pgdp_{\Delta d,t}$. The sample for the counterfactual spans 2000:M3 to 2020:M2.

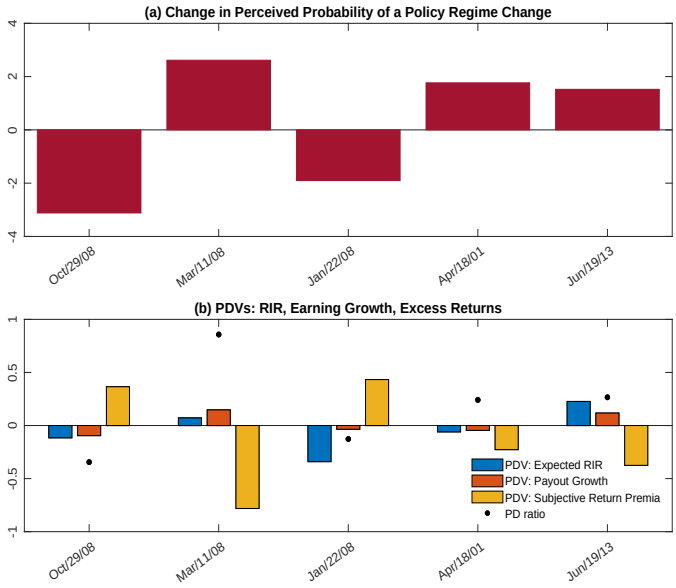
Figure 8 conveys two main findings. First, it shows that investor beliefs about the conduct of future monetary policy play an outsized role in stock market fluctuations. This can be observed from the quantitatively large gap between the red and purple lines in panel (a). Had investors counterfactually maintained the belief throughout the PM period that the central bank was very likely to exit the PM policy rule, the stock market would have been much higher than it actually was over most of this period. Second, panels (b)-(d) show that the reason for this large discrepancy has to do with the affect of these beliefs on investors' subjectively expected future return premia, rather than their effect on subjectively expected real rates or payout growth. This can be observed by noting that the red/blue line discrepancy is largest for $pgdp_{r^{ex},t}$ in panel (b), small for $pgdp_{rir,t}$ in panel (c), and non-existent for $pgdp_{\Delta d,t}$ in panel (d). Appendix K of the

[11] The $(B+1)\times 1$ vector $\pi_{t|T}$ collects the estimated probabilities $P\left(\xi_t^b = i|X_T;\boldsymbol{\theta}\right) \equiv \pi_{t|T}^i$ that $\xi_t^b = i$, for $i = 1, 2,...,B+1$. The red-dashed (purple dashed-dotted) counterfactual replaces $\pi_{t|T}$ with a vector that has 1 as the first (Bth) element and zeros elsewhere.

Online Appendix shows an alternative counterfactual in which investors had different beliefs about the *current* policy rule, while keeping fixed their beliefs about the the conduct of *future* policy. In contrast the large effects found for different beliefs about future policy shown in panel (a) of Figure 8, the Appendix figure shows that variation in beliefs about the current rule have virtually no affect on the market's level or evolution. This happens because what matters for the heavily forward-looking stock market is beliefs about Fed policies that extend well into the future, not those relevant only for the present or very immediate near-term.

Figure 9 examines cash flow versus discount rate effects at high frequency around FOMC announcements. The figure decomposes the announcement-related jumps in pd_t into fluctuations driven by the $pdv_t(\cdot)$ components on the right-hand-side of (19) for the 5 most relevant FOMC announcements sorted on the basis of jumps in the estimated perceived probability of a regime change in the conduct of monetary policy over the next year. Panel (a) of Figure 9 shows how the perceived probabilities of a regime change shifted in the 30 minute windows surrounding each FOMC announcement, while panel (b) shows the decomposition of the jump in pd_t into its $pdv_t(\cdot)$ components.

Figure 9: Jumps in risk perceptions, short rates, and earnings expectations



Notes: Panel (a) shows the pre-/post-FOMC announcement change (10 minutes before/20 minutes after) in the perceived probability of a monetary policy regime change occurring within one year. Panel (b) decomposes the jump in the log price-payout ratio $pd = pdv_t(\Delta d) - pdv_t(r^{ex}) - pdv_t(rir)$ into movements in the subjective equity risk premia $pdv_t(r^{ex})$ (yellow bar), subjective expected real interest rates $pdv_t(RIR)$ (blue bar), and subjective expected payout growth $pdv_t(\Delta d)$ (red bar). PD ratio is $pdv_t(\Delta d) - pdv_t(r^{ex}) - pdv_t(rir)$. The sample is 1961:M1-2020:M2.

The FOMC announcement of October 29, 2008, is associated with the largest absolute decline in the perceived probability of monetary regime change, as shown in panel (a).

In this case, the perceived probability that the central bank would soon transition to an Alternative policy rule capable of more actively stabilizing the real economy falls sharply, resulting in a large jump down in subjective risk premia and *pd*. Less stabilization of economic growth increases expected volatility and with it the perceived quantity of risk in the stock market, driving down the price-dividend ratio. This event suggests that the announcement fueled worries the Fed might soon re-approach the ZLB with limited capacity for economic stabilization.

The March 11, 2008 announcement that the Federal Reserve annouced an expansion of securities lending via the creation of a new facility, the Term Securities Lending Facility (TSLF) is the event associated with largest increase in the perceived probability of exiting the policy rule over the next 12 months, and is the mirror image of the October 29, 2008 event. The stock market rose surrounding this announcement, and Panel (b) shows that the most important contributor to this rise was a decline in subjective return premia driven by a lower perceived quantity of risk in the stock market. Evidently, the announcement of new lending facilities to support the economy amid a financial crisis crisis triggered an increase in the perceived probability of shifting to a new monetary policy regime characterized by more aggressive stabilization of economic growth. The "Fed put" flavor (Cieslak and Vissing-Jorgensen (2021)) of this event shows how Fed news can move markets by altering beliefs about future policies to limit downside risk, immediately changing risk premia.

Still, Figure 4 shows that the March 11, 2008 announcement nevertheless created a deterioration in nowcasts about the current state of the real economy. This is especially evident from the behavior of the BBG current quarter S&P earnings nowcast, which–as separately demarcated in Figure 4–fell 17% surrounding the announcement. But it is also true for the BBG expectations of GDP growth. This suggests that news of yet another unprecedented emergency lending facility was received as bad news for the broader economy even as the announcement led perceived stock market risk to decline. In this sense, the March 11, 2008 event speaks to the "information effects" of Fed announcements, emphasized in Romer and Romer (2000), Campbell, Evans, Fisher, and Justiniano (2012), and Nakamura and Steinsson (2018).

6 Conclusion

We integrate a high-frequency monetary event study into a mixed-frequency macro-finance model and structural estimation. The approach allows for jumps at Fed announcements in investor beliefs, providing granular detail on why markets react to central bank announcements. We also provide a methodology for modeling expectations in the presence of structural breaks, and show how forward-looking data can be used to infer what agents expect from the next policy regime. The overall approach can be used

in a variety of other settings to provide a richer understanding of the role of news shocks of any kind in driving financial market volatility.

The heightened responsiveness of financial markets to central bank communications raises an important question: What are the underlying drivers of this phenomenon? We find that the reasons involve a mix of factors, including revisions in investor beliefs about the latent state of the economy, uncertainty over the future conduct of monetary policy, and subjective reassessments of risk in the stock market. These dynamics stem from three primary sources. First, beliefs about the conduct of future policy react to Fed news even if current policy is unchanged, affecting the perceived quantity of risk in the stock market. Second, realized shifts in the central bank policy rule over the sample have had a persistent influence on short rates, affecting valuations. Third, some announcements are associated with sizable shifts in investor perceptions of the economic state, altering the composition of perceived shocks investors believe will hit the economy. Yet approach developed here also permits us to estimate the effects of monetary policy over an extended sample, not merely in tight windows around Fed announcements. Doing so, we find that beliefs about the future conduct of policy continuously evolve over time, implying that announcement effects alone understate the impact of central banks on markets.

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Online Appendix

A Priors, Posterior, and Smoothed Series

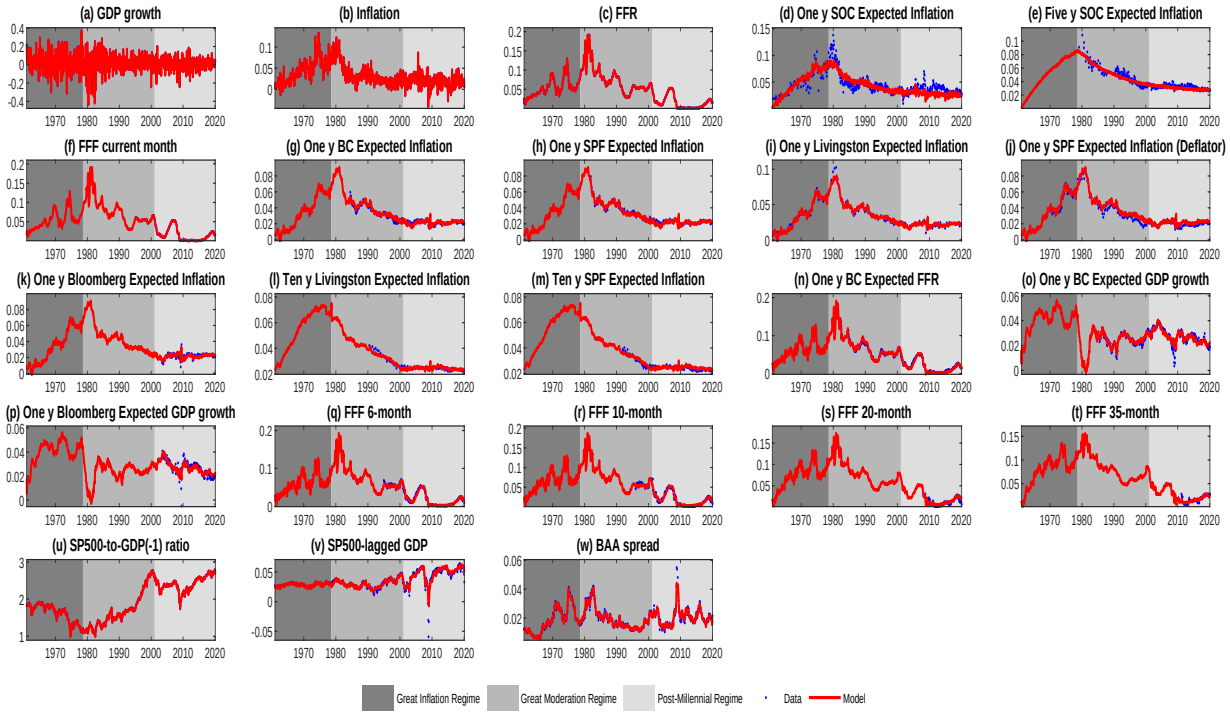
Table A.1 describes the posterior (left-hand-side of the table) and prior (right-hand-side of the table) distributions for the parameters of the model. In the column "Type," N stands for Normal, G stands for Gamma, IG stands for Inverse Gamma, and B stands for Beta distribution, respectively. For all prior distributions, we report the mean and the standard deviation. The priors for all parameters are diffuse and centered around values typically found in the literature. We choose symmetric priors for the parameters of the realized and alternative policy rules. For the posterior, we report the mode and 90% credible sets.

Table A.1: **Parameters**

	Posterior			Prior				Posterior			Prior		
	Mode	5%	95%	Mean	Std	Type		Mode	5%	95%	Mean	Std	Type
π_{ξ}^{T}	12.5335	12.2776	12.7404	5	2.5	G	β	0.7566	0.7502	0.7612	0.8	0.1	B
$\rho_{i,1}$	1.2911	1.2768	1.3286	0.5	0.25	N	κ_1	0.0043	0.0042	0.0043	0.1	0.05	G
$\psi_{\pi,1}$	1.8866	1.8425	1.9429	0.5	0.25	N	γ	0.0002	0.0002	0.0002	0.05	0.02	B
$\rho_{i,2}+\rho_{i,1}$	0.9954	0.995	0.9958	0.5	0.2	B	ρ_g	0.1332	0.1322	0.1343	0.5	0.2	B
$\psi_{\Delta y}$	1.0113	0.9969	1.0342	2	1	G	κ_0	0.0047	0.0047	0.0048	0.1	0.05	G
π_{ξ}^{T}	3.393	3.3787	3.4945	5	2.5	G	ρ_f	0.542	0.5384	0.5466	0.5	0.2	B
$\rho_{i,1}$	0.3597	0.3482	0.3648	0.5	0.25	N	ϕ	0.751	0.7444	0.7592	0.5	0.2	B
$\psi_{\pi,1}$	0.6893	0.6822	0.7196	0.5	0.25	N	$\bar{r}$	0	0	0	0.0017	0.0008	G
$\rho_{i,2}+\rho_{i,1}$	0.9804	0.9758	0.9818	0.5	0.2	B	γ^T	0.0056	0.0055	0.0057	0.2	0.1	B
$\psi_{\Delta y}$	0.4488	0.4328	0.4612	2	1	G	$exp(\bar{k})$	0.0345	0.0342	0.0347	0.04	0.02	B
π_{ξ}^{T}	2.2249	2.198	2.2694	5	2.5	G	σ_p	6.868	6.8268	6.9741	4	2	G
$\rho_{i,1}$	1.1878	1.154	1.2017	0.5	0.25	N	β_p	0.9964	0.9963	0.9968	0.95	0.025	B
$\psi_{\pi,1}$	2.0546	1.9692	2.074	0.5	0.25	N	$\bar{lp}$	0.0015	0.0015	0.0016	0.0033	0.0017	N
$\rho_{i,2}+\rho_{i,1}$	0.985	0.9833	0.9866	0.5	0.2	B	$\lambda_{\pi,1}$	0.3127	0.3111	0.3162	0.5	0.2	B
$\psi_{\Delta y}$	0.117	0.1136	0.1208	2	1	G	$\lambda_{\pi,2}$	0.3056	0.302	0.3075	0.5	0.2	B
π_{ξ}^{T}	0.7463	0.7305	0.7705	5	2.5	G	ρ_k	0.9967	0.9966	0.9972	0.8	0.1	B
$\rho_{i,1}$	0.5372	0.5261	0.5477	0.5	0.25	N	ρ_{lp}	0.923	0.9162	0.9259	0.5	0.2	B
$\psi_{\pi,1}$	2.7719	2.7013	2.8571	0.5	0.25	N	ϱ_2	0.2418	0.2397	0.2434	0	1	N
$\rho_{i,2}+\rho_{i,1}$	0.9608	0.9478	0.9652	0.5	0.2	B	ϱ_3	0.1594	0.1584	0.161	0	1	N
$\psi_{\Delta y}$	0.652	0.6334	0.6538	2	1	G	$\lambda_{k,\Delta y}$	57.1559	56.3137	57.4958	20	10	G
π_{ξ}^{T}	2.4961	2.4702	2.5226	5	2.5	G	p_s	0.9409	0.9357	0.9426	0.9	0.08	B
$\rho_{i,1}$	1.3435	1.2988	1.3645	0.5	0.25	N	ρ_H	0.3026	0.3005	0.305	0.15	0.1	B
$\psi_{\pi,1}$	0.9189	0.8852	0.9445	0.5	0.25	N	mean beta bel	0.962	0.9612	0.963	0.6	0.25	B
$\rho_{i,2}+\rho_{i,1}$	0.9956	0.995	0.9961	0.5	0.2	B	std beta bel	0.0358	0.0353	0.0361	0.15	0.05	B
$\psi_{\Delta y}$	0.071	0.0695	0.0726	2	1	G	int BAA	0.0196	0.0195	0.0199	0.02	0.01	N
π_{ξ}^{T}	0.0608	0.0582	0.0606	5	2.5	N	scale BAA	0.9233	0.9161	0.9327	2	1	G
$\rho_{i,1}$	0.5183	0.5068	0.5309	0.5	0.25	N	σ_f	6.495	6.2551	6.9863	5	5	IG
$\psi_{\pi,1}$	0.8102	0.8032	0.8379	0.5	0.25	N	σ_i	0.0353	0.0341	0.0366	0.0167	0.0167	IG
$\rho_{i,2}+\rho_{i,1}$	0.8885	0.8765	0.8985	0.5	0.2	B	σ_μ	0.1308	0.125	0.1376	1	1	IG
$\psi_{\Delta y}$	0.5625	0.5498	0.5685	2	1	G	σ_k	6.3224	6.1467	6.403	0.1	0.05	IG
σ	0.1099	0.1087	0.1109	2	1	G	σ_{lp}	0.2059	0.1953	0.2155	0.0083	0.0083	IG
ϱ_1	0.543	0.5406	0.5453	1	1	N	σ_g	1.4543	1.4077	1.5003	1	1	IG

Notes: The table describes the posterior and prior distributions for the parameters of the model. In the column "Type", N stands for Normal, G stands for Gamma, IG stands for Inverse Gamma, and B stands for Beta distribution, respectively. For all prior distributions, we report the mean and the standard deviation. For the posterior, we report the mode and 90% credible sets.

Figure A.1: Smoothed Series



Notes: The figure displays the model-implied series (red, solid line) and the actual series (blue dotted line). The model-implied series are based on smoothed estimates $S_{t|T}$ of S_t, and exploit the mapping to observables in (14) using the modal parameter estimates. The difference between the model-implied series and the observed counterpart is attributable to observation error. We allow for observation errors on all variables except for GDP growth, inflation, the FFR, and the SP500 capitalization to GDP ratio. Great Inflation Regime: 1961:Q1-1978:Q3. Great Moderation Regime: 1978:Q4-2001:Q3. Post-Millennial Regime: 2001:Q4-2020:Q1. The sample is 1961:M1-2020:M2.

B Data

Real GDP

The real Gross Domestic Product is obtained from the US Bureau of Economic Analysis. It is in billions of chained 2012 dollars, quarterly frequency, seasonally adjusted, and at annual rate. The source is from Bureau of Economic Analysis (BEA code: A191RX). The sample spans 1959:Q1 to 2021:Q2. The series was interpolated to monthly frequency using the method in Stock and Watson (2010). The quarterly series was downloaded on August 20th, 2021.

GDP price deflator

The Gross Domestic Product: implicit price deflator is obtained from the US Bureau of Economic Analysis. Index base is 2012=100, quarterly frequency, and seasonally

adjusted. The source is from Bureau of Economic Analysis (BEA code: A191RD). The sample spans 1959:Q1 to 2021:Q2. The series was interpolated to monthly frequency using the method in Stock and Watson (2010). The quarterly series was downloaded on August 20th, 2021.

Federal funds rate (FFR)

The Effective Federal Funds Rate is obtained from the Board of Governors of the Federal Reserve System. It is in percentage points, quarterly frequency, and not seasonally adjusted. The sample spans 1960:02 to 2021:06. The series was downloaded on August 20th, 2021

SP500 and SP500 futures

For our high-frequency analysis, we use tick-by-tick data on SP500 index obtained from tickdata.com. The series was downloaded on September 22th, 2021 from `https://www.tickdata.com/`.We create the minutely data using the close price within each minute. Within trading hours, we construct minutely S&P 500 market capitalization by multiplying the S&P 500 index by the previous month's S&P 500 Divisor. (The index is the market capitalization of the 500 companies covered by the index divided by the S&P 500 divisor, roughly the number of shares outsanding across all companies.) The S&P 500 divisor is available at the URL: `https://ycharts.com/indicators/sp_500_divisor`. We supplement SP500 index using SP500 futures for events that occur in off-market hours. We use the current-quarter contract futures. We purchased the SP500 futures from CME group at URL: https://datamine.cmegroup.com/. Our sample spans January 2nd 1986 to September 17th, 2021. The SP500 futures data were downloaded on October 6, 2021.

SP500 Earnings and Market Capitalization

We use S&P 500 earnings divided by GDP as a noisy signal on K_t in the structural estimation. To map into a monthly estimation, we ideally would use monthly earnings data. Instead, we have quarterly S&P 500 earnings per share (EPS) data that starts in 1988:Q2 from S&P Global `https://www.spglobal.com/spdji/en/documents/additional-material/sp-500-eps-est.xlsx`, which we linearly interpolate (and divide it by 3) to monthly observations. To extend our sample backward, we use monthly (interpolated) EPS data on the S&P 500 from Robert Shillers data depository at URL:`http://www.econ.yale.edu/~shiller/data/ie_data.xls`. These are monthly EPS data computed from the S&P four-quarter trailing totals (and divide it by 12) and we use data for this series that span 1959:01-1988:03. (There is no quarterly EPS data prior to 1988:03. Observation error in the structural estimation absorbs fluctuations attributable to splicing

together an annual moving average with an annualized quarterly sequence of observations.) Splicing the two series together results in a monthly earnings per share series spanning 1959:01 to 2021:06. In the S&P Global dataset there is one observation in 2008:Q4 with a negative EPS, which appears in 2008:12 after interpolation. To deal with this in earnings growth rate calculations we replace the 2008:12 observation with the Shiller four quarter total trailing EPS observation in 2008:12. To convert EPS to total earnings, we multiply EPS by the monthly S&P 500 Divisor available at URL:`https://ycharts.com/indicators/sp_500_divisor`. For a monthly stock market value series, we use the S&P 500 market capitalization, obtained as the end-of-month series from Ycharts.com available at `https://ycharts.com/indicators/sp\_500\_market\_cap`. All series span the periods 1959:01 to 2021:06 and were downloaded on December 22nd, 2021.

Baa Spread, 20-yr T-bond, Long-term US government securities

We obtained daily Moody's Baa Corporate Bond Yield from FRED (series ID: DBAA) at URL: `https://fred.stlouisfed.org/series/BAA`, US Treasury securities at 20-year constant maturity from FRED (series ID: DGS20) at URL: `https://fred.stlouisfed.org/series/DGS20`, and long-term US government securities from FRED (series ID: LTGOVTBD) at URL: `https://fred.stlouisfed.org/series/LTGOVTBD`. The sample for Baa spans the periods 1986:01 to 2021:06. To construct the long term bond yields, we use LTGOVTBD before 2000 (1959:01 to 1999:12) and use DGS20 after 2000 (2000:01 to 2021:06). The Baa spread is the difference between the Moody's Corporate bond yield and the 20-year US government yield. The excess bond premium is obtained at URL: `https://www.federalreserve.gov/econres/notes/feds-notes/ebp_csv.csv`. All series were downloaded on Feb 21, 2022.

Bloomberg Consensus Inflation, GDP forecasts, and Earnings Nowcasts

We obtain the Bloomberg (BBG) US GDP (id: ECGDUS) and inflation (id: ECPIUS) consensus mean forecast from the Bloomberg Terminal available on a daily basis up to a few days before the release of GDP and inflation data. The Bloomberg (BBG) US consensus forecasts are updated daily (except for weekends and holidays) and reports daily quarter-over-quarter real GDP growth and CPI forecasts from 2003:Q1 to 2021Q2. These forecasts provide more high-frequency information on the professional outlook for economic indicators. Both forecast series were downloaded on October 21, 2021.

We obtain daily nowcasts of S&P 500 earnings per share (EPS) from BBG. The survey respondents are equity strategists that are asked to provide nowcasts of earnings per share (EPS) for the constituents of the S&P 500. For each S&P 500 constituent, BBG provides the mean nowcast across survey respondents as well as a bottom-up aggregate

nowcast of EPS for the S&P 500 by aggregating the EPS nowcasts across the S&P 500 constituents. We construct a mean respondent nowcast for the level of S&P 500 earnings by multiplying this aggregate with the S&P 500 index divisor. (The index is the market capitalization of the 500 companies covered by the index divided by the S&P 500 divisor, roughly the number of shares outsanding across all companies.) The S&P 500 divisor is available at the URL: `https://ycharts.com/indicators/sp_500_divisor`. These nowcasts are available daily for the current standard financial quarter (Jan-Mar, Apr-June, Jul-Sep, Oct-Dec). For example, the observation for July 10, 2024 (which falls under the standard financial quarter of 2024:Q3) would contain nowcasts for 2024:Q3 EPS for the S&P 500. Bloomberg does not require respondents to submit their nowcasts on a specific timeline or frequency. Instead, respondents voluntarily decide how often to update their nowcasts. To ensure that consensus nowcasts are not heavily influenced by outdated information, Bloomberg excludes stale nowcasts submitted before the most recent earnings announcement date. The data was downloaded from the Bloomberg terminal on July 24, 2024, using the Earnings & Estimates (EE) function on the S&P 500 index (SPX Index). The aggregated consensus nowcasts are available daily, except weekends and holidays, spanning the period from Jan 2, 2006 to July 24, 2024. The divisor series series span the periods 1959:01 to 2021:06 and was downloaded on December 22nd, 2021.

Livingston Survey Inflation Forecast

We obtained the Livingston Survey mean 1-year and 10-year CPI inflation forecast from the Federal Reserve Bank of Philadelphia, URL: `https://www.philadelphiafed.org/surveys-and-data/real-time-data-research/livingston-historical-data`. Our sample spans 1947:06 to 2021:06. The forecast series were downloaded on September 20, 2021.

Michigan Survey of Consumers Inflation Forecasts

We construct MS forecasts of annual inflation of respondents answering at time t. Each month, the SOC contains approximately 50 core questions, and a minimum of 500 interviews are conducted by telephone over the course of the entire month, each month. We use two questions from the monthly survey for which the time series begins in January 1978.

1. Annual CPI inflation: To get a point forecast, we combine the information in the survey responses to questions A12 and A12b.

    - Question A12 asks (emphasis in original): *During the next 12 months, do you think that prices in general will go up, or go down, or stay where they are*

now?

- A12b asks (emphasis in original): *By about what percent do you expect prices to go (up/down) on the average, during the next 12 months?*

2. Long-run CPI inflation: To get a point forecast, we combine the information in the survey responses to questions A13 and A13b.

   - Question A13 asks (emphasis in original): *What about the outlook for prices over the next 5 to 10 years? Do you think prices will be higher, about the same, or lower, 5 to 10 years from now?*
   - A13b asks (emphasis in original): *By about what percent per year do you expect prices to go (up/down) on the average, during the next 5 to 10 years?*

All series were downloaded on September 17th, 2021.

Bluechip Inflation and GDP Forecasts

We obtain Blue Chip expectation data from Blue Chip Financial Forecasts from Wolters Kluwer. The surveys are conducted each month by sending out surveys to forecasters in around 50 financial firms such as Bank of America, Goldman Sachs & Co., Swiss Re, Loomis, Sayles & Company, and J.P. Morgan Chase. The participants are surveyed around the 25th of each month and the results published a few days later on the 1st of the following month. The forecasters are asked to forecast the average of the level of U.S. interest rates over a particular calendar quarter, e.g. the federal funds rate and the set of H.15 Constant Maturity Treasuries (CMT) of the following maturities: 3-month, 6-month, 1-year, 2-year, 5-year and 10-year, and the quarter over quarter percentage changes in Real GDP, the GDP Price Index and the Consumer Price Index, beginning with the current quarter and extending 4 to 5 quarters into the future.

In this study, we look at a subset of the forecasted variables. Specifically, we use the Blue Chip micro data on individual forecasts of the quarter-over-quarter (Q/Q) percentage change in the Real GDP, the GDP Price Index and the CPI, and convert to quarterly observations as explained below.

1. CPI inflation: We use quarter-over-quarter percentage change in the consumer price index, which is defined as

   "Forecasts for the quarter-over-quarter percentage change in the CPI (consumer prices for all urban consumers). Seasonally adjusted, annual rate."

   Quarterly and annual CPI inflation are constructed the same way as for PGDP inflation, except CPI replaces PGDP.

2. For real GDP growth, We use quarter-over-quarter percentage change in the Real GDP, which is defined as

   "Forecasts for the quarter-over-quarter percentage change in the level of chain-weighted real GDP. Seasonally adjusted, annual rate. Prior to 1992, Q/Q % change (SAAR) in real GNP."

The surveys are conducted right before the publication of the newsletter. Each issue is always dated the 1st of the month and the actual survey conducted over a two-day period almost always between 24th and 28th of the month. The major exception is the January issue when the survey is conducted a few days earlier to avoid conflict with the Christmas holiday. Therefore, we assume that the end of the last month (equivalently beginning of current month) is when the forecast is made. For example, for the report in 2008 Feb, we assume that the forecast is made on Feb 1, 2008. We obtained Blue Chip Financial Forecasts from Wolters Kluver in several stages starting in 2017 and with the last update purchased in June of 2022 and received on June 22, 2022. URL:https://law-store.wolterskluwer.com/s/product/blue-chip-financial-forecast-print/ 01tG000000LuDUCIA3.

Survey of Professional Forecasters (SPF)

The SPF is conducted each quarter by sending out surveys to professional forecasters, defined as forecasters. The number of surveys sent varies over time, but recent waves sent around 50 surveys each quarter according to officials at the Federal Reserve Bank of Philadelphia. Only forecasters with sufficient academic training and experience as macroeconomic forecasters are eligible to participate. Over the course of our sample, the number of respondents ranges from a minimum of 9, to a maximum of 83, and the mean number of respondents is 37. The surveys are sent out at the end of the first month of each quarter, and they are collected in the second or third week of the middle month of each quarter. Each survey asks respondents to provide nowcasts and quarterly forecasts from one to four quarters ahead for a variety of variables. Specifically, we use the SPF micro data on individual forecasts of the price level, long-run inflation, and real GDP.[1] Below we provide the exact definitions of these variables as well as our method for constructing nowcasts and forecasts of quarterly and annual inflation for each respondent.[2]

The following variables are used on either the right- or left-hand-sides of forecasting models:

[1]Individual forecasts for all variables can be downloaded at https://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/historical-data/individual-forecasts.

[2]The SPF documentation file can be found at https://www.philadelphiafed.org/-/media/research-and-data/real-time-center/survey-of-professional-forecasters/spf-documentation.pdf?la=en.

1. Quarterly and annual inflation (1968:Q4 - present): We use survey responses for the level of the GDP price index (PGDP), defined as

   "Forecasts for the quarterly and annual level of the chain-weighted GDP price index. Seasonally adjusted, index, base year varies. 1992-1995, GDP implicit deflator. Prior to 1992, GNP implicit deflator. Annual forecasts are for the annual average of the quarterly levels."

   Since advance BEA estimates of these variables for the current quarter are unavailable at the time SPF respondents turn in their forecasts, four quarter-ahead inflation and GDP growth forecasts are constructed by dividing the forecasted level by the survey respondent-type's nowcast. Let $\mathbb{F}_t^{(i)}[P_{t+h}]$ be forecaster i's prediction of PGDP h quarters ahead and $\mathbb{N}_t^{(i)}[P_t]$ be forecaster i's nowcast of PGDP for the current quarter. Annualized inflation forecasts for forecaster i are

   $$\mathbb{F}_t^{(i)}[\pi_{t+h,t}] = (400/h) \times \ln\left(\frac{\mathbb{F}_t^{(i)}[P_{t+h}]}{\mathbb{N}_t^{(i)}[P_t]}\right),$$

   where $h = 1$ for quarterly inflation and $h = 4$ for annual inflation. Similarly, we construct quarterly and annual nowcasts of inflation as

   $$\mathbb{N}_t^{(i)}[\pi_{t,t-h}] = (400/h) \times \ln\left(\frac{\mathbb{N}_t^{(i)}[P_t]}{P_{t-h}}\right),$$

   where $h = 1$ for quarterly inflation and $h = 4$ for annual inflation, and where P_{t-1} is the BEA's advance estimate of PGDP in the previous quarter observed by the respondent in time t, and P_{t-4} is the BEA's most accurate estimate of PGDP four quarters back. After computing inflation for each survey respondent, we calculate the 5th through the 95th percentiles as well as the average, variance, and skewness of inflation forecasts across respondents.

2. Long-run inflation (1991:Q4 - present): We use survey responses for 10-year-ahead CPI inflation (CPI10), which is defined as

   "Forecasts for the annual average rate of headline CPI inflation over the next 10 years. Seasonally adjusted, annualized percentage points. The "next 10 years" includes the year in which we conducted the survey and the following nine years. Conceptually, the calculation of inflation is one that runs from the fourth quarter of the year before the survey to the fourth quarter of the year that is ten years beyond the survey year, representing a total of 40 quarters or 10 years. The fourth-quarter level is the quarterly average of the underlying monthly levels."

   Only the median response is provided for CPI10, and it is already reported as an inflation rate, so we do not make any adjustments and cannot compute other moments or percentiles.

3. Real GDP growth (1968:Q4 - present): We use the level of real GDP (RGDP), which is defined as

   *"Forecasts for the quarterly and annual level of chain-weighted real GDP. Seasonally adjusted, annual rate, base year varies. 1992-1995, fixed-weighted real GDP. Prior to 1992, fixed-weighted real GNP. Annual forecasts are for the annual average of the quarterly levels. Prior to 1981:Q3, RGDP is computed by using the formula NGDP / PGDP * 100."*

Source: Federal Reserve Bank of Philadelphia.All series were downloaded on September 17th, 2021.

Fed Funds Futures

We use tick-by-tick data on Fed funds futures (FFF) and Eurodollar futures obtained from the CME Group. Our sample spans January 3, 1995 to June 2, 2020. FFF contracts settle based on the average federal funds rate that prevails over a given calendar month. Fed funds futures are priced at $100 - f_t^{(n)}$, where $f_t^{(n)}$ is the time-t contracted federal funds futures market rate that investors lock in. Contracts are monthly and expire at month-end, with maturities ranging up to 60 months. For the buyer of the futures contract, the amount of $\left(f_t^{(n)} - r_{t+n}\right) \times \$D$, where r_{t+n} is the ex post realized value of the federal funds rate for month $t+n$ calculated as the average of the daily Fed funds rates in month $t+n$, and \$D is a dollar "deposit", represents the payoff of a zero-cost portfolio.

Contracts are cleaned following communication with the CME Group. First, trades with zero volume, which indicate a canceled order, are excluded. Floor trades, which do not require a volume on record, are included. Next, trades with a recorded expiry (in YYMM format) of 9900 indicate bad data and are excluded (Only 1390 trades, or less than 0.01% of the raw Fed funds data, have contract delivery dates of 9900). For trades time stamped to the same second, we and keep the trade with the lowest sequence number, corresponding to the first trade that second.

Fed funds futures trade prices were quoted in different units prior to August 2008. To standardize units across our sample, we start by noting that Fed funds futures are priced to the average effective Fed funds rate realized in the contract month. And in our sample, we expect a reasonable effective Fed funds rate to correspond to prices in the 90 to 100 range. As such, we rescale prices to be less than 100 in the pre-August 2008 subsample.[3] After rescaling, a small number of trades still appear to have prices that are far away from the effective Fed funds rates at both trade day and contract expiry, along with trades in the immediate transactions. The CME Group could not explain this data

[3]For trades with prices significantly greater than 100, we repeatedly divide by 10 until prices are in the range of 90 to 100. We exclude all trades otherwise.

issue, so following Bianchi, Kind, and Kung (2019) and others in the high frequency equity literature, we apply an additional filter to exclude trades with such non-sensible prices. Specifically, for each maturity contract, we only keep trades where

$$|p_t - \overline{p}_t(k,\delta)| < 3\sigma_t(k,\delta) + \gamma,$$

where p_t denotes the trade price (where t corresponds to a second), and $\overline{p}_t(k,\delta)$ and $\sigma_t(k,\delta)$ denote the average price and standard deviation, respectively, centered with $k/2$ observations on each side of t excluding $\delta k/2$ trades with highest price and excluding $\delta k/2$ trades with lowest price. Finally, γ is a positive constant to account for the cases where prices are constant within the window. Our main specification uses $k = 30$, $\delta = 0.05$ and $\gamma = 0.4$, and alternative parameters produce similar results.

C Structural Breaks as Nonrecurrent Regime-Switching

Let T be the sample size used in the estimation and let the vector of observations as of time t be denoted $z_{r,t}$. The sequence $\xi_t^P = \{\xi_1^P, ..., \xi_T^P\}$ of regimes in place at each point is unobservable and needs to be inferred jointly with the other parameters of the model. We use the Hamilton filter (Hamilton (1994)) to estimate the smoothed regime probabilities $P\left(\xi_t^P = i | z_{r,T}; \boldsymbol{\theta}_r\right)$, where $i = 1, ..., N_P$. We then use these regime probabilities to estimate the most likely historical regime sequence ξ_t^P over our sample as described in the next subsection.

The specifications to be estimated are

$$z_{r,t} = r_{\xi_t^P} + \epsilon_t^r, \quad z_{r,t} = \{r_t, mps_t\}$$

where $\epsilon_t^r \sim N(0, \sigma_r^2)$, and $r_{\xi_t^P}$ is a time-varying intercept governed by a discrete valued latent state variable, ξ_t^P, that follows a N_P-state nonrecurrent regime-switching Markov with transition matrix $\mathbf{H}$. Bayesian methods with flat priors are used estimate the parameters $\boldsymbol{\theta}_r = \left(r_{\xi_t^P}, \sigma_r^2, vec(\mathbf{H})'\right)'$ over the period 1961:Q1-2020:Q1 and to estimate the most likely historical regime sequence for ξ_t^P over that sample.

To capture the phenomenon of nonrecurrent regimes, we suppose that ξ_t^P follows a Markov-switching process in which new regimes can arise but do not repeat exactly as before. This is modeled by specifying the transition matrix over nonrecurrent states, or "structural breaks." If the historical sample has N_P nonrecurrent regimes (implying

$N_P - 1$ structural breaks), the transition matrix for the Markov process takes the form

$$\mathbf{H} = \begin{bmatrix} p_{11} & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ 1-p_{11} & p_{22} & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & 1-p_{22} & p_{33} & 0 & \cdots & \cdots & \vdots \\ \vdots & 0 & 1-p_{33} & \ddots & & & \\ \vdots & \vdots & 0 & \vdots & \ddots & & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & p_{N_P,N_P} & 0 \\ 0 & \cdots & \cdots & \cdots & 0 & 1-p_{N_P,N_P} & 1 \end{bmatrix}, \tag{A.1}$$

where $\mathbf{H}_{ij} \equiv p\left(\xi_t^P = i | \xi_{t-1}^P = j\right)$. For example, if there were $N_P = 2$ nonrecurrent regimes in the sample, we would have

$$\mathbf{H} = \begin{bmatrix} p_{11} & 0 \\ 1-p_{11} & 1 \end{bmatrix}.$$

The above process implies that, if you are currently in regime 1, you will remain there next period with probability p_{11} or exit to regime 2 with probability $1 - p_{11}$. Upon exiting to regime 2, since there are only two regimes in the sample and the probability p_{12} of returning exactly to the previous regime 1 is zero, $p_{22} = 1$.

D Most Likely Regime Sequence

For regime switches in the mean of mps_t where the specification that is estimated is

$$mps_t = r_{\xi_t^P} + \epsilon_t^r,$$

$\epsilon_t^r \sim N(0, \sigma_r^2)$, and $r_{\xi_t^P}$ is an intercept governed by a discrete valued latent state variable, ξ_t^P, that is presumed to follow a N_P-state nonrecurrent regime-switching Markov with transition matrix $\mathbf{H}$. The vector $\boldsymbol{\theta}_r = \left(r_{\xi_t^P}, \sigma_r^2, vec(\mathbf{H})'\right)'$ denotes the set of parameters to be estimated. The most likely regime sequence is the regime sequence $\xi^{P,T} = \{\hat{\xi}_1^P, ..., \hat{\xi}_T^P\}$ that is most likely to have occurred, given the estimated posterior mode parameter values for $\boldsymbol{\theta}_r$. This sequence is computed as follows.

Let $P\left(\xi_t^P = i | z_{t-1}; \boldsymbol{\theta}_r\right) \equiv \pi_{t|t-1}^i$. First, run Hamilton's filter to get the vector of filtered regime probabilities $\pi_{t|t}$, $t = 1, 2, ..., T$. The Hamilton filter can be expressed iteratively as

$$\begin{aligned} \pi_{t|t} &= \frac{\pi_{t|t-1} \odot \eta_t}{\mathbf{1}'\left(\pi_{t|t-1} \odot \eta_t\right)} \\ \pi_{t+1|t} &= \mathbf{H}\pi_{t|t} \end{aligned}$$

where the symbol $\odot$ denotes element by element multiplication, η_t is a vector whose j-th element contains the conditional density $p(mps_t | \xi_t^P = j; \boldsymbol{\theta}_r)$, i.e.,

$$\eta_{j,t} = \frac{1}{\sqrt{2\pi}\sigma_r} \exp\left\{\frac{-(mps_t - r_j)^2}{2\sigma_r^2}\right\},$$

and where **1** is a vector with all elements equal to 1. The final term, $\pi_{T|T}$ is returned with the final step of the filtering algorithm. Then, a recursive algorithm can be implemented to derive the other smoothed probabilities:

$$\pi_{t|T} = \pi_{t|t} \odot \left[\mathbf{H}' \left(\pi_{t+1|T} \left(\div\right) \pi_{t+1|t}\right)\right]$$

where $(\div)$ denotes element by element division. To choose the regime sequence most likely to have occurred given our parameter estimates, consider the recursion in the next to last period $t = T - 1$:

$$\pi_{T-1|T} = \pi_{T-1|T-1} \odot \left[\mathbf{H}' \left(\pi_{T|T} \left(\div\right) \pi_{T|T-1}\right)\right].$$

Suppose we have $N_p = 3$ regimes. We first take $\pi_{T|T}$ from the Hamilton filter and choose the regime that is associated with the largest probability, i.e., if $\pi_{T|T} = (.8, .1, .1)$, where the first element corresponds to the probability of regime 1, we select $\hat{\xi}_T^P = 1$, indicating that we are in regime 1 in period T. We now update $\pi_{T|T} = (1, 0, 0)$ and plug into the right-hand-side above along with the estimated filtered probabilities for $\pi_{T-1|T-1}$, $\pi_{T|T-1}$ and estimated transition matrix $\mathbf{H}$ to get $\pi_{T-1|T}$ on the left-hand-side. Now we repeat the same procedure by choosing the regime for $T-1$ that has the largest probability at $T-1$, e.g., if $\pi_{T-1|T} = (.2, .7, .1)$ we select $\hat{\xi}_{T-1} = 2$, indicating that we are in regime 2 in period $T-1$, we then update to $\pi_{T-1|T} = (0, 1, 0)$, which is used again on the right-hand-side now

$$\pi_{T-2|T} = \pi_{T-2|T-2} \odot \left[\mathbf{H}' \left(\pi_{T-1|T} \left(\div\right) \pi_{T-1|T-2}\right)\right].$$

We proceed in this manner until we have a most likely regime sequence $\xi^{P,T}$ for the entire sample $t = 1, 2, ..., T$. Two aspects of this procedure are worth noting. First, it fails if the updated probabilities are exactly $(.333, .333, .333)$. Mathematically this is virtually a zero probability event. Second, note that this procedure allows us to choose the most likely regime sequence by using the recursive formula above to update the filtered probabilities sequentially working backwards from $t = T$ to $t = 1$. This allows us to take into account the time dependence in the regime sequence as dictated by the transition probabilities.

Follow the same procedure to obtain the most likely belief regime sequence ξ_t^b, where the structural model is described by B^2 conditional densities

$$f\left(X_{t-1+\delta_h} | \xi_{t-1}^b = j, \xi_t^b = i, X^{t-1}\right) = (2\pi)^{-N_X/2} |f_{t|t-1+\delta_h}^{(i,j)}|^{-1/2} \exp\left\{-\frac{1}{2} e_{t|t-1+\delta_h,t-1}^{(i,j)\prime} f_{t|t-1+\delta_h,t-1}^{(i,j)} e_{t|t-1+\delta_h,t-1}^{(i,j)}\right\}.$$

Define ξ_t^* describe a B^2-state Markov chain incorporating all the (i, j) combinations above and recast $f(\cdot)$ as B^2 densities $\eta_t = f\left(X_{t-1+\delta_h} | \xi_t^* = i, X^{t-1}\right)$ to use in the computation of $\pi_{t|t}$.

E Price-Output Decompositions

Mapping from price to output (measured as GDP_t) is

$$\begin{aligned}\frac{P_t}{GDP_{t-1}} &= \frac{P_t}{D_t}\frac{D_t}{GDP_t}\frac{GDP_t}{GDP_{t-1}} \\ pgdp_t &= pd_t + k_t + \widetilde{y}_t + g_t - \widetilde{y}_{t-1}\end{aligned}$$

Below we decompose pd_t to write:

$$\begin{aligned}pgdp_t &= \underbrace{\frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + k_t + y_t + g_t - \widetilde{y}_{t-1}}_{\text{earning share component}} + \underbrace{pdv_t\left(\Delta d\right)}_{\text{earnings}} - \underbrace{pdv_t\left(r^{ex}\right)}_{\text{premia}} - \underbrace{pdv_t\left(rir\right)}_{\text{RIR}} \\ pgdp_{r^{ex},t} &= \underbrace{\frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + k_t + \widetilde{y}_t + g_t - \widetilde{y}_{t-1}}_{\text{earning share component}} - \underbrace{pdv_t\left(r^{ex}\right)}_{\text{premia}} \\ pgdp_{rir,t} &= \underbrace{\frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + k_t + \widetilde{y}_t + g_t - \widetilde{y}_{t-1}}_{\text{earning share component}} - \underbrace{pdv_t\left(rir\right)}_{\text{RIR}} \\ pgdp_{\Delta d,t} &= \underbrace{\frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + k_t + \widetilde{y}_t + g_t - \widetilde{y}_{t-1}}_{\text{earning share component}} + \underbrace{pdv_t\left(\Delta d\right)}_{\text{earnings}}\end{aligned}$$

where

$$\begin{aligned}pd_t &= \kappa_{pd,0} + \mathbb{E}_t^b\left[m_{t+1} + \Delta d_{t+1} + \kappa_{pd,1} pd_{t+1}\right] + \\ &\quad + .5\mathbb{V}_t^b\left[m_{t+1} + \Delta d_{t+1} + \kappa_{pd,1} pd_{t+1}\right].\end{aligned}$$

The solution approximates around the balanced growth path with $\frac{D_{t+1}}{D_t} = G$, where G is the gross growth rate of the economy. The Euler equation under the balanced growth path is

$$\begin{aligned}1 &= \left[M_{t+1}\left(\frac{P_{t+1}/D_{t+1}+1}{P_t/D_t}\right)\frac{D_{t+1}}{D_t}\right] \\ &= \left[\beta_p\left(\frac{D_{t+1}}{D_t}\right)^{-\sigma_p}\left(\frac{P_{t+1}/D_{t+1}+1}{P_t/D_t}\right)\frac{D_{t+1}}{D_t}\right] \\ &= \left[\underbrace{\beta_p G^{1-\sigma_p}}_{\widetilde{\beta}_p}\left(\frac{P/D+1}{P/D}\right)\right] => \\ \frac{1}{\widetilde{\beta}_p} &= \left(\frac{P/D+1}{P/D}\right) => \\ P/D &= \frac{\widetilde{\beta}_p}{1-\widetilde{\beta}_p}.\end{aligned}$$

Denote the log steady state price-payout ratio as $\ln(P/D) = \overline{pd}$, thus we have

$$\overline{pd} = \ln\left(\frac{\widetilde{\beta}_p}{1-\widetilde{\beta}_p}\right).$$

$$\begin{aligned}
\kappa_{pd,1} &= \exp(\overline{pd})/(1+\exp(\overline{pd})) = \frac{\widetilde{\beta}_p}{1-\widetilde{\beta}_p}\left[1+\frac{\widetilde{\beta}_p}{1-\widetilde{\beta}_p}\right]^{-1} = \widetilde{\beta}_p \\
\kappa_{pd,0} &= \ln(\exp(\overline{pd})+1) - \kappa_{pd,1}pd = \ln\left(\frac{1}{1-\widetilde{\beta}_p}\right) - \widetilde{\beta}_p \ln\frac{\widetilde{\beta}_p}{1-\widetilde{\beta}_p} \\
&= -\widetilde{\beta}_p \ln\widetilde{\beta}_p - \left(1-\widetilde{\beta}_p\right)\ln\left(1-\widetilde{\beta}_p\right)
\end{aligned}$$

The log return obeys the following approximate identity (Campbell and Shiller (1989)):

$$r^D_{t+1} = \kappa_{pd,0} + \kappa_{pd,1}pd_{t+1} - pd_t + \Delta d_{t+1},$$

where $\kappa_{pd,1} = \exp(\overline{pd})/(1+\exp(\overline{pd}))$, and $\kappa_{pd,0} = \log\left(\exp(\overline{pd})+1\right) - \kappa_{pd,1}\overline{pd}$. Combining all of the above, the log equity premium is

$$\underbrace{\mathbb{E}^b_t\left[r^D_{t+1}\right] - \left(i_t - \mathbb{E}^b_t\left[\pi_{t+1}\right]\right)}_{\text{Equity Premium}} = \underbrace{\left[\begin{array}{c} -.5\mathbb{V}^b_t\left[r^D_{t+1}\right] - \mathbb{COV}^b_t\left[m_{t+1}, r^D_{t+1}\right] \\ +.5\mathbb{V}^b_t\left[\pi_{t+1}\right] - \mathbb{COV}^b_t\left[m_{t+1}, \pi_{t+1}\right] \end{array}\right]}_{\text{Risk Premium}} + \underbrace{\overline{lp}_t}_{\text{Liquidity Premium}},$$

Then

$$\begin{aligned}
pd_t &= \kappa_{pd,0} + \mathbb{E}^b_t\left[\Delta d_{t+1} - r^D_{t+1} + \kappa_{pd,1}pd_{t+1}\right] \\
pd_t &= \kappa_{pd,0} + \mathbb{E}^b_t\left[\Delta d_{t+1} - \left(r^{ex}_{t+1} - rir_{t+1}\right) + \kappa_{pd,1}pd_{t+1}\right]
\end{aligned}$$

where $\mathbb{E}^b_t\left[r^{ex}_{t+1}\right] = \mathbb{E}^b_t\left[r^D_{t+1}\right] - rir_{t+1}$, where $rir_{t+1} \equiv \left(i_{t+1} - \mathbb{E}^b_t\left[\pi_{t+1}\right]\right)$.

Solving forward:

$$\begin{aligned}
pd_t &= \kappa_{pd,0} + \mathbb{E}^b_t\left[\Delta d_{t+1} - r^{ex}_{t+1} - rir_{t+1}\right] + \\
&\quad + \kappa_{pd,1}\mathbb{E}^b_t\left[\kappa_{pd,0} + \mathbb{E}^b_t\left[\Delta d_{t+2} - r^{ex}_{t+2} - rir_{t+1} + \kappa_{pd,1}pd_{t+2}\right]\right]
\end{aligned}$$

Thus:

$$pd_t = \frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + \left(1_{\Delta d} - 1_{\mathbb{E}(r^{ex})} - 1_{rir}\right)\sum_{h=0}^{\infty}\kappa^h_{pd,1}\mathbb{E}^b_t\left[S_{t+1+h}\right]$$

where 1_x is a vector of all zeros except for a 1 in the xth position. This can be written as:

$$pd_t = \frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + pdv_t\left(\Delta d\right) - pdv_t\left(r^{ex}\right) - pdv_t\left(rir\right)$$

Using the solution:

$$pd_t = \frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + \left(1_{\Delta d} - 1_{\mathbb{E}(r^{ex})} - 1_{r^b}\right)\left(\mathbf{I} - \kappa_{pd,1}T_{\xi_t}\right)^{-1}\left[T_{\xi_t}S_t + \left(\mathbf{I} - \kappa_{pd,1}\right)^{-1}C_{\xi_t}\right].$$

Thus, we can decompose movements in the pd_t into those attributable to expected dividends, equity premia, and expected real interest rates:

$$pgdp_t = \underbrace{\frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + k_t + y_t + g_t - y_{t-1}}_{\text{earning share component}} + \underbrace{pdv_t\left(\Delta d\right)}_{\text{earnings}} - \underbrace{pdv_t\left(r^{ex}\right)}_{\text{premia}} - \underbrace{pdv_t\left(rir\right)}_{\text{RIR}}.$$

F Solution and Estimation Details

This appendix presents details on the solution and estimation. An overview of the steps are as follows.

1. We first solve the macro block set of equations involving a set of macro state variables $S_t^M \equiv [\widetilde{y}_t, g_t, \pi_t, i_t, \overline{\pi}_t, f_t]'$. The MS-VAR solution consists of a system of equations taking the form

$$S_t^M = C_M\left(\theta_{\xi_t^P}\right) + T_M(\theta_{\xi_t^P})S_{t-1}^M + R_M(\theta_{\xi_t^P})Q_M\varepsilon_t^M,$$

   where $\varepsilon_t^M = (\varepsilon_{f,t}, \varepsilon_{i,t}, \varepsilon_{g,t}, \varepsilon_{\mu,t})$. Since this block involves no forward-looking variables and only depends on the pre-determined policy regimes, this block can be solved analytically. See Bianchi, Lettau, and Ludvigson (2022).

2. Use the solution for S_t^M based on the current realized policy regime ξ_t^P and then resolve the model based on the Alternative regime, i.e., obtain

$$S_t^M = C_M\left(\theta_{\xi_t^A}\right) + T_M(\theta_{\xi_t^A})S_{t-1}^M + R_M(\theta_{\xi_t^A})Q_M\varepsilon_t^M.$$

   Store the two solutions. S_t^M under ξ_t^P is mapped into the observed current macro variables in our observation equation.

3. To identify the parameters of the Alternative policy rule, the perceived transition matrix $\mathbf{H}^b$ and belief regime probabilities governing moving to the Alternative rule, we use:

   (a) Measures of expectations from professional forecast surveys and futures markets. Given the perceived transition matrix of the investor $\mathbf{H}^b$, use it to compute investor expectations for future macro variables that take into account the perceived probability of transitioning to the Alternative rule in the future. See the section below on "Computing Expectations with Regime Switching and Alternative Policy Rule." These give us investor expectations of the macro block variables used in our observation equation.

(b) Stock prices. The asset pricing block of equations involves conditional subjective variance terms that are affected by Markov-switching random variables in the model. The subsection "Risk Adjustment with Lognormal Approximation," below explains the approximation used to preserve lognormality of the entire system. This part uses the approach in Bianchi, Kung, and Tirskikh (2018) who in turn build on Bansal and Zhou (2002) and is combined with the algorithm of Farmer, Waggoner, and Zha (2011) to solve the overall system of model equations, where investors form expectations taking into account the probability of regime change in the future. The state variables for the full system are

$$S_t = \left[S_t^M, m_t, pd_t, k_t, lp_t, \mathbb{E}_t^b\left(m_{t+1}\right), \mathbb{E}_t^b\left(pd_{t+1}\right)\right].$$

This leaves us with the MS-VAR solution consists of a system of equations taking the form

$$S_t = C\left(\theta_{\xi_t^P}, \xi_t^b, \mathbf{H}^b\right) + T(\theta_{\xi_t^P}, \xi_t^b, \mathbf{H}^b)S_{t-1} + R(\theta_{\xi_t^P}, \xi_t^b, \mathbf{H}^b)Q\varepsilon_t,$$

where $\varepsilon_t = (\varepsilon_{f,t}, \varepsilon_{i,t}, \varepsilon_{g,t}, \varepsilon_{\mu,t}, \varepsilon_{k,t}, \varepsilon_{lp,t})$. Since pd_t depends the risk adjustment and $\mathbb{E}_t^b\left(pd_{t+1}\right)$, its value is also informative about the parameters of the Alternative rule, $\mathbf{H}^b$ and belief regime probabilities. Unlike the formulas that are required to relate data on expectations to future macro variables in step (a), the formulas governing these relationships are solved numerically using the solution algorithm described above.

4. We estimate the model by combining the solution above with an observation equation that includes macro, asset pricing, and survey expectation variables. See the subsection "Estimation" below.

G Computing Expectations with Regime Switching and Alternative Policy Rule

In what follows, we explain how to use expectations to infer what alternative regimes agents have in mind. Expectations about inflation, FFR, and GDP growth depend on the regime currently in place, the alternative regime, and the probability of moving to such regime. This note is based on "Methods for measuring expectations and uncertainty" in Bianchi (2016). That paper explains how to computed expected values in presence of regime changes. In the models described above, for each policy rule in place, agents would have different beliefs about alternative future policy rules. This would lead to changes in expected values for the endogenous variables of the model.

Consider a MS model:

$$S_t = C_{\xi_t} + T_{\xi_t} S_{t-1} + R_{\xi_t} Q \varepsilon_t \tag{A.2}$$

where $\xi_t = \left\{\xi_t^P, \xi_t^b\right\}$ controls the policy regime ξ_t^P controls the policy rule currently in place and the alternative policy rule, while the belief regime ξ_t^b controls agents' beliefs about the possibility of moving to the alternative policy rule.

Let n be the number of variables in S_t. Let $m = B + 1$ be the number of Markov-switching states and define

$$\xi_t = i \equiv \left\{\xi_t^P, \xi_t^b = i\right\}, \quad i = 1, ..., B+1.$$

Define the $mn \times 1$ column vector q_t as:

$$\underset{mn\times 1}{q_t} = \left[q_t^{1\prime}, ..., q_t^{m\prime}\right]'$$

where the individual $n \times 1$ vectors $q_t^i = \mathbb{E}_0\left(S_t 1_{\xi_t=i}\right) \equiv \mathbb{E}\left(S_t 1_{\xi_t=i} | \mathbb{I}_0\right)$ and $1_{\xi_t=i}$ is an indicator variable that is one when belief regime i is in place and zero otherwise. Note that:

$$q_t^i = \mathbb{E}_0\left(S_t 1_{\xi_t=i}\right) = \mathbb{E}_0\left(S_t | \xi_t = i\right) \pi_t^i$$

where $\pi_t^i = P_0\left(\xi_t = i\right) = P\left(\xi_t = i | \mathbb{I}_0\right)$. Therefore we can express $\mu_t = \mathbb{E}_0\left(S_t\right)$ as:

$$\mu_t = \mathbb{E}_0\left(S_t\right) = \textstyle\sum_{i=1}^m q_t^i = w q_t$$

where the matrix $\underset{n\times mn}{w} = [I_n, ..., I_n]$ is obtained placing side by side m n-dimensional identity matrices. Then the following proposition holds:

PROPOSITION 1: *Consider a Markov-switching model whose law of motion can be described by (A.2) and define* $q_t^i = \mathbb{E}_0\left(S_t 1_{\xi_t=i}\right)$ *for* $i = 1...m$. *Then* $q_t^j = C_j \pi_t^j + \sum_{i=1}^m T_j q_{t-1}^i p_{ji}$.

It is then straightforward to compute expectations conditional on the information available at a particular point in time. Suppose we are interested in $\mu_{t+s|t} \equiv \mathbb{E}_t^b\left(S_{t+s}\right)$, i.e. the expected value for the vector S_{t+s} conditional on the information set available at time t. If we define:

$$q_{t+s|t} = \left[q_{t+s|t}^{1\prime}, ..., q_{t+s|t}^{m\prime}\right]'$$

where $q_{t+s|t}^i = \mathbb{E}_t^b\left(S_{t+s} 1_{\xi_t=i}\right) = \mathbb{E}_t^b\left(S_{t+s} | \xi_t = i\right) \pi_{t+s|t}^i$, where $\pi_{t+s|t}^i \equiv P\left(\xi_{t+s} = i | \mathbb{I}_t\right)$, we have

$$\mu_{t+s|t} = \mathbb{E}_t^b\left(S_{t+s}\right) = w q_{t+s|t}, \tag{A.3}$$

where for $s \geq 1$, $q_{t+s|t}$ evolves as:

$$q_{t+s|t} = C\pi_{t+s|t} + \Omega q_{t+s-1|t} \tag{A.4}$$

$$\pi_{t+s|t} = \mathbf{H}^b \pi_{t+s-1|t} \tag{A.5}$$

with $\pi_{t+s|t} = \left[\pi^1_{t+s|t}, ..., \pi^m_{t+s|t}\right]'$, $\Omega = bdiag\left(T_1, ..., T_m\right)\left(\mathbf{H}^b \otimes I_n\right)$, and $\underset{mn\times m}{C} = bdiag\left(C_1, ..., C_m\right)$, where e.g., C_1 is the $n \times 1$ vector of constants in regime 1, $\otimes$ represents the Kronecker product and *bdiag* is a matrix operator that takes a sequence of matrices and use them to construct a block diagonal matrix.

The formulas above are used to compute expectations conditional on each belief regime ξ^b_t and policy rule regime ξ^P_t. For each composite regime $\xi_t = \left\{\xi^P_t, \xi^b_t\right\}$, we can obtain a forecast for each of the variables of the model. For example, conditional on ξ^P_t and $\xi^b_t = j$ in place we have

$$q_{t,\xi_t=j} = e_j \otimes S_t$$

where e_j is a variable that has elements equal to zero except for the one in position ξ^b_t. For example, with $B = 5$ belief regimes and $\xi^b_t = 3$ we have

$$q_{t,\xi_t=3} = \left[\mathbf{0}', \mathbf{0}', S_t', \mathbf{0}', \mathbf{0}', \mathbf{0}'\right]'.$$

where 0 and S_t are column vectors with n rows. We have $B + 1$ subvectors in $q_{t,\xi_t=j}$ to take into account the alternative policy mix. The fact that all subvectors are zero except for the one corresponding to the belief regime $b = 3$ reflects the assumption that agents can observe the current state S_t and, by definition, their own beliefs (while the econometrician cannot observe any of the two and she uses macro data and survey expectations to estimate both S_t and agents' beliefs).

Thus, suppose we want to compute the expected value for a variable x over the next year under the assumption that agents' beliefs are $\xi^b_t = j$. With monthly data, we have:

$$\begin{aligned}
\mathbb{E}^b_t\left(x_{t,t+s}|\xi_t = j\right) &= \textstyle\sum_{s=1}^{12} \mathbb{E}^b_t\left(x_{t+s}|\xi_t = j\right) \\
&= e_x \textstyle\sum_{s=1}^{12} \mu_{t+s|t,\xi_t=j} \\
&= e_x w \textstyle\sum_{s=1}^{12} q_{t+s|t,\xi_t=j}
\end{aligned}$$

where for $s \geq 1$, $q_{t+s|t}$ evolves as:

$$q_{t+s|t,\xi_t=j} = C\pi_{t+s|t} + \Omega q_{t+s-1|t,\xi_t=j} \tag{A.6}$$

$$\pi_{t+s|t,\xi_t=j} = \mathbf{H}^b \pi_{t+s-1|t,\xi_t=j} \tag{A.7}$$

with $\pi_{t+s|t} = \left[\pi^1_{t+s|t}, ..., \pi^m_{t+s|t}\right]'$, $\Omega = bdiag\left(T_1, ..., T_m\right)\left(\mathbf{H}^b \otimes I_n\right)$, and $\underset{mn\times m}{C} = bdiag\left(C_1, ..., C_m\right)$, where e.g., C_1 is the $n \times 1$ vector of constants in regime 1, $\otimes$ represents the Kronecker product and *bdiag* is a matrix operator that takes a sequence of matrices and use them to construct a block diagonal matrix. The recursive algorithm is initialized with $\pi_{t|t,\xi_t=j} = 1_{\xi_t=j}$ and $q_{t,\xi_t=j} = e_j \otimes S_t$.

The formulas (A.6) and (A.7) can be written in a more compact form. If we define $\widetilde{q}_{t|t} = [q'_{t|t}, \pi'_{t|t}]'$, with $\pi_{t|t}$ a vector with elements $\pi^i_{t|t} \equiv P\left(\xi_t = i|\mathbb{I}_t\right)$ we can compute the conditional expectations in one step:

$$\mu_{t+s|t} = \mathbb{E}^b_t\left(S_{t+s}\right) = \widetilde{w}\widetilde{\Omega}^s \widetilde{q}_{t|t} \tag{A.8}$$

where $\widetilde{w} = [w, 0_{n\times m}]$. The formula above can be used to compute the expected value from the point of view of the agent of the model with beliefs $\xi_t = j$:

$$\mathbb{E}_t^b\left(x_{t+s}|\xi_t = j\right) = e_x\mu_{t+s|t,\xi_t=j} = e_x\widetilde{w}\widetilde{\Omega}^s\widetilde{q}_{t|t,\xi_t^b=j} = \underbrace{e_x w\widetilde{\Omega}^s_{\{1,nm\},\{n(j-1)+1,nj\}}}_{Z_{\xi_t,x_{t+s}}}\underbrace{S_t}_{(n\times 1)} + \underbrace{e_x w\widetilde{\Omega}^s_{\{1,nm\},nm+j}}_{D_{\xi_t,x_{t+s}}} \tag{A.9}$$

where $D_{\xi_t,x_{t+s}}$ is a scalar, $Z_{\xi_t,x_{t+s}}$ is an $(1\times n)$ vector, $\widetilde{\Omega}^s_{\{1,nm\},\{n(j-1)+1,nj\}}$ is the submatrix obtained taking the first nm rows and the columns from $n(j-1)+1$ to nj of $\widetilde{\Omega}^s$, while $\widetilde{\Omega}^s_{\{1,nm\},nm+j}$ is the submatrix obtained taking the first nm rows and the $nm+j$ column of $\widetilde{\Omega}^s$. Thus, we have that conditional on one belief regime and a policy rule regime, we can map the current state of the economy S_t into the expected value reported in the survey. The matrix algebra in (A.9) returns the same results of the recursion in (A.6) and (A.7).

To see what the formulas above do, consider a simple example with $B = 2$ and we are currently in belief regime $b = 2$:

$$\begin{aligned}
\mathbb{E}_t^b\left(x_{t+s}|\xi_t = 2\right) &= e_x\widetilde{w}\widetilde{\Omega}^s\widetilde{q}_{t|t,\xi_t=2} = e_x\widetilde{w}\widetilde{\Omega}^s\begin{bmatrix}\mathbf{0}_{n\times 1}\\ S_t\\ \mathbf{0}_{n\times 1}\\ 0\\ 1\\ 0\end{bmatrix}\\
&= e_x\widetilde{w}\begin{bmatrix}\widetilde{\Omega}^s_{11} & \widetilde{\Omega}^s_{12} & \widetilde{\Omega}^s_{13} & \widetilde{\Omega}^s_{14} & \widetilde{\Omega}^s_{15} & \widetilde{\Omega}^s_{16}\\ \widetilde{\Omega}^s_{21} & \widetilde{\Omega}^s_{22} & \widetilde{\Omega}^s_{23} & \widetilde{\Omega}^s_{24} & \widetilde{\Omega}^s_{25} & \widetilde{\Omega}^s_{26}\\ \widetilde{\Omega}^s_{31} & \widetilde{\Omega}^s_{32} & \widetilde{\Omega}^s_{33} & \widetilde{\Omega}^s_{34} & \widetilde{\Omega}^s_{35} & \widetilde{\Omega}^s_{36}\\ & & & \widetilde{\Omega}^s_{44} & \widetilde{\Omega}^s_{45} & \widetilde{\Omega}^s_{46}\\ & & & \widetilde{\Omega}^s_{54} & \widetilde{\Omega}^s_{55} & \widetilde{\Omega}^s_{56}\\ & & & \widetilde{\Omega}^s_{64} & \widetilde{\Omega}^s_{65} & \widetilde{\Omega}^s_{66}\end{bmatrix}\begin{bmatrix}\mathbf{0}_{n\times 1}\\ S_t\\ \mathbf{0}_{n\times 1}\\ 0\\ 1\\ 0\end{bmatrix}\\
&= e_x\widetilde{w}\begin{bmatrix}\widetilde{\Omega}^s_{12}S_t + \widetilde{\Omega}^s_{15}\\ \widetilde{\Omega}^s_{22}S_t + \widetilde{\Omega}^s_{25}\\ \widetilde{\Omega}^s_{32}S_t + \widetilde{\Omega}^s_{35}\\ \widetilde{\Omega}^s_{44}\\ \widetilde{\Omega}^s_{54}\\ \widetilde{\Omega}^s_{64}\end{bmatrix}\\
&= e_x\left(\widetilde{\Omega}^s_{12} + \widetilde{\Omega}^s_{22} + \widetilde{\Omega}^s_{32}\right)S_t + e_x\left(\widetilde{\Omega}^s_{15} + \widetilde{\Omega}^s_{25} + \widetilde{\Omega}^s_{35}\right)
\end{aligned}$$

Finally, suppose we are interested in the forecast $E_t^b\left(x_{t,t+s}|\xi_t^b = j, \xi_t^p\right)$:

$$\mathbb{E}_t^b\left(x_{t,t+s}|\xi_t = j\right) = \underbrace{\left[e_x\sum_{s=1}^{12} w\widetilde{\Omega}^s_{\{1,nm\},\{n(j-1)+1,nj\}}\right]}_{Z_{\xi_t,x_{t,t+s}}}\underbrace{S_t}_{(n\times 1)} + \underbrace{e_x\sum_{s=1}^{12} w\widetilde{\Omega}^s_{\{1,nm\},nm+j}}_{D_{\xi_t,x_{t,t+s}}} \tag{A.10}$$

Thus, we can include $Z_{\xi_t,x_{t,t+s}}$ as a row in Z_{ξ_t} and $D_{\xi_t,x_{t,t+s}}$ as a row in D_{ξ_t} in the mapping from the model to the observables described in (A.13). Note that the matrix Z and vector D are now regime dependent.

For GDP growth, we are interested in the average growth over a certain horizon. Our state vector contains $\widetilde{y}_t$. Thus, we can use the following approach:

$$\begin{aligned}\mathbb{E}_t^b\left[\left(gdp_{t+h}-gdp_t\right)h^{-1}|\xi_t=j\right] &= \mathbb{E}_t^b\left[\left(\widetilde{y}_{t+h}-\widetilde{y}_t+\textstyle\sum_{s=1}^h\widehat{g}_t+hg\right)h^{-1}|\xi_t=j\right]\\ &= h^{-1}\mathbb{E}_t^b\left[\widetilde{y}_{t+h}|\xi_t=j\right]-h^{-1}\widetilde{y}_t+g+h^{-1}\textstyle\sum_{s=1}^h\widehat{g}_t\end{aligned}$$

where g is the average growth rate in the economy and $\widetilde{y}_t$ is GDP in deviations from the trend. With deterministic growth we have $gdp_{t+h}-gdp_t-hg\equiv\widetilde{y}_{t+h}-\widetilde{y}_t$. We then have

$$\begin{aligned}&\mathbb{E}_t^b\left[\left(gdp_{t+h}-gdp_t\right)h^{-1}|\xi_t=j\right]\\ =\;& h^{-1}\mathbb{E}_t^b\left[\widetilde{y}_{t+h}|\xi_t=j\right]-h^{-1}\widetilde{y}_t+g\\ =\;& h^{-1}\left[\underbrace{e_{\widetilde{y}}w\widetilde{\Omega}^s_{\{1,nm\},\{n(j-1)+1,nj\}}}_{Z_{\xi_t,\widetilde{y}_{t+s}}}\underbrace{S_t}_{(n\times1)}+\underbrace{e_{\widetilde{y}}w\widetilde{\Omega}^s_{\{1,nm\},nm+j}}_{D_{\xi_t,\widetilde{y}_{t+s}}}-e_{\widetilde{y}}S_t\right]+g\\ =\;& h^{-1}\underbrace{\left[e_{\widetilde{y}}w\widetilde{\Omega}^s_{\{1,nm\},\{n(j-1)+1,nj\}}-e_{\widetilde{y}}\right]}_{Z_{\xi_t,\widetilde{y}_{t+s}-\widetilde{y}_t}}\underbrace{S_t}_{(n\times1)}+h^{-1}\underbrace{e_{\widetilde{y}}w\widetilde{\Omega}^s_{\{1,nm\},nm+j}}_{D_{\xi_t,\widetilde{y}_{t+s}}}+g\end{aligned}$$

The expected values for the endogenous variables depend on the perceived transition matrix H^b and the properties of the alternative regime. The latter can be seen by recalling that the regime $\xi_t=B+1$ applies to the perceived Alternative regime. Thus, data from survey expectations and futures markets provide information about the perceived probability of moving across belief regimes as well as the parameters of the Alternative regime.

H Simplified Example of Mixed-Frequency Filtering Algorithm

In this appendix, we present a simplified example to understand how to interpret the mixed frequency filtering that we implement in our analysis of FOMC announcements.

Let t denote a month. Let d_h denote the number of time units that have passed within a month when we have reached a particular point in time, and let nd denote the total number of time units in the month. Then $0\leq d_h/nd\leq 1$, and the intramonth time period is denoted $t-1+\delta_h$ with $\delta_h\equiv d_h/nd\in[0,1]$. For example, if the time unit is in days and we are at the beginning of the 11th day in a 31 day month, then $\delta_h\equiv 10/31=0.3226$.

Consider a simple state space model in which there is a single state for inflation, which follows an autoregressive process with no regime changes. The process is specified

at monthly frequency:

$$\pi_t = \rho\pi_{t-1} + \sigma_\pi \varepsilon_{\pi,t} \tag{A.11}$$

Investors are always asked to predict inflation for a particular time period, for example next month $(t+1)$, even when surveyed intramonth $(t-1+\delta_h < t)$. Furthermore, agents know the data generating process (A.11) and understand that the process is specified at monthly frequency. At the end of month t, we assume that the data are fully revealed and, accordingly, inflation expectations for the next month $(t+1)$ reflect the realized value of inflation at the end of the current month (t):

$$\mathbb{E}_t\left[\pi_{t+1}\right] = \rho\pi_t.$$

Given the realized value of π_t, the above forecast will in general differ from the two-month-ahead forecast of the same object π_{t+1} as of the end of $t-1$:

$$\mathbb{E}_{t-1}\left[\pi_{t+1}\right] = \rho^2\pi_{t-1}.$$

Now suppose we are within month t at day $t-1+\delta_h$, where $\delta_h < 1$. At that time, investors are presumed to have a belief or *nowcast* of what inflation will be at the end of t. We assume that such nowcasts are formed from extensive information sets that are by definition unobserved by the econometrician. Denote the investor's nowcast of π_t as of at time $t-1+\delta_h$ as $\pi_{t\backslash t-1+\delta}$, where we use the symbol "\" to indicate that the conditioning is with respect to the agent's unobserved information set. (Given that investors observe π_t at the end of the month, $\pi_{t\backslash t} \equiv \pi_t$). Using the nowcast $\pi_{t\backslash t-1+\delta}$, investors can update their forecast of π_{t+1} relative to what it was at the end of $t-1$ using

$$\mathbb{E}_{t-1+\delta_h}\left[\pi_{t+1}\right] = \rho\pi_{t\backslash t-1+\delta} = \rho^2\pi_{t-1} + \rho\sigma_\pi\varepsilon_{\pi,t\backslash t-1+\delta}.$$

This shows that the nowcast implicitly depends on the *perceived shock* $\varepsilon_{\pi,t\backslash t-1+\delta}$, which denotes the agent's belief about what $\varepsilon_{\pi,t}$ will be revealed to be at the end of the month t, given any new information received between $t-1$ and $t-1+\delta_h$. We do not attempt to model how investors obtain these nowcasts or what information is summarized by $\varepsilon_{\pi,t\backslash t-1+\delta}$, a task that would be difficult if not impossible given the large amount of information processed by financial markets and professional forecasters.

Instead, we develop a filtering algorithm to allow an econometrician to *infer* revisions in investor expectations/nowcasts induced by FOMC announcements. To do this, the econometrician must have access to high frequency forward-looking data from markets or surveys summarizing investor expectations. Suppose for this example that we observe a daily inflation expectations measure that proxies for investors evolving inflation expectations, where the expectation as of time $t-1+\delta_h$ is denoted $\mathbb{E}_{t-1+\delta_h}\left[\pi_{t+1}\right]^o$. We use superscipt o to denote the observed value of variables in the data.

More formally, we have the following state space representation of this simple model.

1. At the end of month t, both inflation and inflation expectations are observed. The observation equation is:

$$\underbrace{\begin{bmatrix} \pi_t^o \\ \mathbb{E}_t\left[\pi_{t+1}\right]^o \end{bmatrix}}_{\equiv X_t} = \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{\equiv D_t} + \underbrace{\begin{bmatrix} 1 \\ \rho \end{bmatrix}}_{\equiv Z_t} \pi_t + \underbrace{\begin{bmatrix} \sigma_{u,\pi} & 0 \\ 0 & \sigma_{u,\mathbb{E}[\pi]} \end{bmatrix}}_{\equiv U_t} \begin{bmatrix} u_{\pi,t} \\ u_{\mathbb{E}[\pi],t} \end{bmatrix}$$

where we allow for observation errors to avoid stochastic singularity because we have one stochastic process (model inflation process) mapped into two observables (inflation and expected inflation). The transition equation is given by (A.11).

2. Intramonth, only the high frequency inflation expectations are obseved. We can use them to filter out what agents think inflation will be over the current month. As explained above, this is equivalent to filtering out what the perceived shock will be at the end of the current month:

$$\underbrace{\begin{bmatrix} - \\ \mathbb{E}_{t-1+\delta_h}\left[\pi_{t+1}\right]^o \end{bmatrix}}_{\equiv X_{t-1+\delta_h}} = \underbrace{\begin{bmatrix} - \\ 0 \end{bmatrix}}_{\equiv D_{t-1+\delta_h}} + \underbrace{\begin{bmatrix} - \\ \rho \end{bmatrix}}_{\equiv Z_{t-1+\delta_h}} \pi_{t\backslash t-1+\delta} + \underbrace{\begin{bmatrix} - & - \\ - & \sigma_{u,\mathbb{E}[\pi],t} \end{bmatrix}}_{\equiv U_{t-1+\delta_h}} \begin{bmatrix} u_{\pi,t-1+\delta_h} \\ u_{\mathbb{E}[\pi],t-1+\delta_h} \end{bmatrix}$$

where

$$\pi_{t\backslash t-1+\delta} = \rho\pi_{t-1} + \sigma_\pi \varepsilon_{\pi,t\backslash t-1+\delta} \tag{A.12}$$

This provides a specific interpretation of our filtering results. We do not interpret high frequency revisions in expectations around FOMC announcements as stemming from macroeconomic shocks occurring over a 30 minute window. Instead, we interpret these as revisions in investor beliefs about what the shocks for the month will turn out to be. In some cases the information revealed during an FOMC announcement will completely remove any uncertainty about what a variable will be at the end of the month, such as when the announcent is about a change in the target federal funds rate. For other variables, the announcement will merely constitute a noisy signal.

The idea is to use the filter to infer investor nowcasts at high frequency around news events, without having to take a stand on the unobservable nowcasting model and information set investors use to obtain and update these nowcasts. In what follows, we use the suffix $(t\backslash t-1+\delta_h)$ to denote filtered objects related to investor nowcasts which are implicitly based on the agent's latent information set. We use the symbol "|" to refer to conditioning in the filter that is with respect to the econometrician's information set.

Steps in the high frequency filtering:

1. Suppose the econometrician has information up through month $t-1$ and new high frequency data arrives at $t-1+\delta_h$. Compute one step-ahead nowcast estimates:

$$\begin{aligned} \pi_{(t\backslash t-1+\delta_h)|t-1} &= \rho\pi_{t-1|t-1} \\ P_{(t\backslash t-1+\delta_h)|t-1} &= \rho^2 P_{t-1|t-1} + \sigma_\pi^2 \end{aligned}$$

2. Compute forecast error given high frequency information at $t-1+\delta_h$:

$$\begin{aligned} e_{(t\backslash t-1+\delta_h)|t-1+\delta_h,t-1} &= X_{t-1+\delta_h} - D_{t-1+\delta_h} - Z_{t-1+\delta_h}\pi_{(t\backslash t-1+\delta_h)|t-1} \\ f_{(t\backslash t-1+\delta_h)|t-1+\delta_h,t-1} &= Z_{t-1+\delta_h}P_{(t\backslash t-1+\delta_h)|t-1}Z'_{t-1+\delta_h} + U^2_{t-1+\delta_h} \end{aligned}$$

3. Update estimates of nowcast and its variance

$$\begin{aligned} \pi_{(t\backslash t-1+\delta_h)|t-1+\delta_h} &= \pi_{(t\backslash t-1+\delta_h)|t-1} + P_{(t\backslash t-1+\delta_h)|t-1}Z'_{t-1+\delta_h}\left(f_{(t\backslash t-1+\delta_h)|t-1+\delta_h,t-1}\right)^{-1} e_{(t\backslash t-1+\delta_h)|t-1+\delta_h,t-1} \\ P_{(t\backslash t-1+\delta_h)|t-1+\delta_h} &= P_{(t\backslash t-1+\delta_h)|t-1} - P_{(t\backslash t-1+\delta_h)|t-1}Z'_{t-1+\delta_h}\left(f_{(t\backslash t-1+\delta_h)|t-1+\delta_h,t-1}\right)^{-1} Z_{t-1+\delta_h}P_{(t\backslash t-1+\delta_h)|t-1} \end{aligned}$$

4. Repeat the above procedure the day before and the after an FOMC announcement to infer revisions in nowcasts due to Fed news.

I Estimation and Filtering

The solution of the model takes the form of a Markov-switching vector autoregression (MS-VAR) in the state vector $S_t = \left[S_t^M, m_t, pd_t, k_t, lp_t, \mathbb{E}_t^b\left(m_{t+1}\right), \mathbb{E}_t^b\left(pd_{t+1}\right)\right]$. Here, S_t^M is a vector of macro block state variables given by $S_t^M \equiv [\widetilde{y}_t, g_t, \pi_t, i_t, \overline{\pi}_t, f_t]'$. The asset pricing block of equations involves conditional subjective variance terms that are affected by Markov-switching random variables in the model. The subsection "Risk Adjustment with Lognormal Approximation," below, explains the approximation used to preserve lognormality of the entire system.

The model solution in state space form is

$$\begin{aligned} X_t &= D_{\xi_t,t} + Z_{\xi_t,t}\left[S'_t, \widetilde{y}_{t-1}\right]' + U_t v_t \\ S_t &= C\left(\theta_{\xi_t^P}, \xi_t^b, \mathbf{H}^b\right) + T(\theta_{\xi_t^P}, \xi_t^b, \mathbf{H}^b)S_{t-1} + R(\theta_{\xi_t^P}, \xi_t^b, \mathbf{H}^b)Q\varepsilon_t \\ Q &= diag\left(\sigma_{\varepsilon_1}, ..., \sigma_{\varepsilon_G}\right),\ \varepsilon_t \sim N\left(0, I\right) \\ U &= diag\left(\sigma_1, ..., \sigma_X\right),\ v_t \sim N\left(0, I\right) \\ \xi_t^P &= 1...N_P,\ \xi_t^b = 1, ...B+1, \mathbf{H}_{ij}^b = p\left(\xi_t^b = i|\xi_{t-1}^b = j\right). \end{aligned}$$

where X_t is a $N_X \times 1$ vector of data, v_t are a vector of observation errors, U_t is a diagonal matrix with the standard deviations of the observation errors on the main diagonal, and $D_{\xi_t,t}$, and $Z_{\xi_t,t}$ are parameters mapping the model counterparts of X_t into the latent discrete- and continuous-valued state variables ξ_t and S_t, respectively, in the model. The vector X_t of observables is explained below. Note that the parameters $D_{\xi_t,t}$, $Z_{\xi_t,t}$, and U_t vary with t independently of ξ_t because not all variables are observed at each data sampling period. To reduce computation time, we calibrate rather than estimate the parameters in $U = diag\left(\sigma_1, ..., \sigma_X\right)$ such that the variance of the observation error

is 0.05 times the sample variance of the corresponding variable in X. In addition, some of the parameters in the system are dependent on the current policy rule and the associated Alternative rule, ξ_t^P, and the unobserved, discrete-valued $(B+1)$-state Markov-switching variable ξ_t^b $\left(\xi_t^b = 1, 2, ..., B+1\right)$ with perceived transition probabilities

$$\mathbf{H}^b = \begin{bmatrix} p_{b1}p_s & p_{b2}p_{\Delta 1|2} & \cdots & p_{bB}p_{\Delta 1|B} & 0 \\ p_{b1}p_{\Delta 2|1} & p_{b2}p_s & & p_{bB}p_{\Delta 2|B} & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ p_{b1}p_{\Delta B|1} & & & p_{bB}p_s & 0 \\ 1-p_{b1} & 1-p_{b2} & \cdots & 1-p_{bB} & p_{B+1,B+1}=1 \end{bmatrix},$$

where $H_{ij}^b \equiv p\left(\xi_t^b = i|\xi_{t-1}^b = j\right)$, and $\sum_{i\neq j} p_{\Delta i|j} = 1 - p_s$. We take the parameters p_{bi} from a discretized estimated beta distribution, where the mean and variance of the beta distribution are estimated. We specify the probability of transitioning to belief i tomorrow, conditional on having belief j today, while remaining in the same policy regime, as $p_{\Delta i|j} \equiv (1-p_s)\left(\rho_b^{|i-j-1|}/\sum_{i\neq j}\rho_b^{|i-j-1|}\right)$, where p_s and $\rho_b < 1$ are parameters to be estimated and $|i-j-1|$ measures the distance between beliefs j and i, for $i \neq j \in (1, 2, ..., B)$. This creates a decaying function that makes the probability of moving to contiguous beliefs more likely than jumping to very different beliefs. For computational reasons, we also eliminate very unlikely transitions ($p_{\Delta i|j} < 0.0001$) by setting their probabilities to zero.

We use the following notation:

$$\begin{aligned} C_{\xi_t^P,i} &= C\left(\theta_{\xi_t^P}, \xi_t^b = i\right),\ T_{\xi_t^P,j} = T\left(\theta_{\xi_t^P}, \xi_t^b = i\right),\ R_{\xi_t^P,j} = R\left(\theta_{\xi_t^P}, \xi_t^b = i\right) \\ D_{i,t} &= D_{\xi_t|\xi_t^b=i}, \quad Z_{i,t} = Z_{\xi_t|\xi_t^b=i}. \end{aligned}$$

Kim's Approximation to the Likelihood and Filtering We use Kim's (Kim (1994)) basic filter and approximation to the likelihood.

First note that, from the econometricians viewpoint, investors are only ever observed in the first B regimes, since the perceived Alternative is never actually realized. For this reason the filtering algorithm for the latent belief regimes involves only the upper $B \times B$ submatrix of H^b, rescaled so that the elements sum to unity. Even though the filtering loops over just B states rather than $B+1$, this is done conditional on the parameters for the full $(B+1)\times(B+1)$ transition matrix, which is estimated from all the data by combining the likelihood with the priors, as described below.

The sample is divided into N_P policy regime subperiods indexed by ξ_t^P. Denote the last observation of each regime subperiod of the sample $T_1, ..., T_{N_P}$. The algorithm for the basic filter is described as follows.

Initiate values $\widetilde{S}_{0|0}$, $P_{0|0}$, for the Kalman filter and $\Pr\left(\xi_0^b\right) = \pi_0$ for the Hamilton filter and initialize $L(\theta) = 0$. Denote $X^{t-1} \equiv \{X_1, ..., X_{t-1}\}$ and $\xi^{PT} = \left\{\xi_1^P, ..., \xi_T^P\right\}$.

As explained for the simplified example above, in the mixed-frequency estimation we use high frequency, forward-looking intramonth data to infer updates to investor nowcasts of the state space that will be revealed at the end of the month. Our "final" estimates of the state space are obtained using a more complete set of data available at the end of each month. Let t denote a month. Let d_h denote the number of time units that have passed within a month when we have reached a particular point in time, and let nd denote the total number of time units in the month. Then $0 \leq d_h/nd \leq 1$, and the intramonth time period is denoted $t-1+\delta_h$ with $\delta_h \equiv d_h/nd \in [0,1]$. For example, δ_{100} could denote the point within the month that is exactly 10 minutes before an FOMC meeting during the month, while δ_{130} could denote point in the month 20 minutes after the same meeting. Intra-month observations used just prior to an FOMC meeting will typically include the daily BBG consensus forecasts and Baa credit spread from the day before the meeting, and the 10-minutes before FFF, ED and stock market data. Intermonth observations for the point of the month right after the FOMC meeting will typically include the daily BBG consensus forecasts and Baa spread from the day after the meeting, and the 20-minutes after FFF, ED and stock market data.

Suppose we are within month t at day $t-1+\delta_h$, where $\delta_h < 1$. Investors are presumed to have a belief or *nowcast* of what S will be at the end of t. We assume that such nowcasts are formed from extensive information sets that are by definition unobserved by the econometrician. The filtering algorithm below is designed to allow the econometrician to infer investor nowcasts at any time $t-1+\delta_h$ by using the structural model combined with high frequency forward-looking data, without taking a stand on the investor's unobservable information sets and nowcasting model. We use the suffix $(t\backslash t-1+\delta_h)$ to denote filtered objects related to investor nowcasts which are implicitly based on the agent's latent information set, i.e., $S_{(t\backslash t-1+\delta_h)}$ refers to the investor's nowcast of S_t based on information through $t-1+\delta_h$. Given that investors observe S_t at the end of t, $S_{t\backslash t} \equiv S_t$. We use the symbol "|" to refer to conditioning in the filter that is with respect to the econometrician's information set.

- For $t=1$ to T_1 and $\theta_{\xi_t^P}$ relevant when $\xi_t^P = 1$:

1. Suppose the econometrician has information up through month $t-1$ and new high frequency data arrives at $t-1+\delta_h$. Conditional on $\xi_{t-1}^b = j$ and $\xi_t^b = i$ run the Kalman filter given below for $i,j = 1,2,...,B$ to update estimates of the latent state:

$$
\begin{aligned}
S^{(i,j)}_{(t\backslash t-1+\delta_h)|t-1} &= C_{\xi_t^P,i} + T_{\xi_t^P,i} S^j_{t-1|t-1} \\
P^{(i,j)}_{(t\backslash t-1+\delta_h)|t-1} &= T_{\xi_t^P,i} P^j_{t-1|t-1} T'_{\xi_t^P,i} + R_{\xi_t^P,i} Q^2 R'_{\xi_t^P,i} \text{ with } Q^2 \equiv QQ' \\
e^{(i,j)}_{(t\backslash t-1+\delta_h)|t-1+\delta_h,t-1} &= X_{t-1+\delta_h} - D_{i,t-1+\delta_h} - Z_{i,t-1+\delta_h}\left[S^{(i,j)'}_{(t\backslash t-1+\delta_h)|t-1}, \widetilde{y}_{t-1}\right] \\
f^{(i,j)}_{(t\backslash t-1+\delta_h)|t-1+\delta_h,t-1} &= Z_{i,t-1+\delta_h} P^{(i,j)}_{(t\backslash t-1+\delta_h)|t-1} Z'_{i,t-1+\delta_h} + U^2_{t-1+\delta_h} \quad \text{with } U^2_{t-1+\delta_h} \equiv U_{t-1+\delta_h} U'_{t-1+\delta_h} \\
S^{(i,j)}_{(t\backslash t-1+\delta_h)|t-1+\delta_h} &= S^{(i,j)}_{(t\backslash t-1+\delta_h)|t-1} + P^{(i,j)}_{(t\backslash t-1+\delta_h)|t-1} Z'_{i,t-1+\delta_h} \left(f^{(i,j)}_{(t\backslash t-1+\delta_h)|t-1+\delta_h,t-1}\right)^{-1} e^{(i,j)}_{(t\backslash t-1+\delta_h)|t-1+\delta_h,t-1} \\
P^{(i,j)}_{(t\backslash t-1+\delta_h)|t-1+\delta_h} &= P^{(i,j)}_{(t\backslash t-1+\delta_h)|t-1} - P^{(i,j)}_{(t\backslash t-1+\delta_h)|t-1} Z'_{i,t-1+\delta_h} \left(f^{(i,j)}_{(t\backslash t-1+\delta_h)|t-1+\delta_h,t-1}\right)^{-1} Z_{i,t-1+\delta_h} P^{(i,j)}_{(t\backslash t-1+\delta_h)|t-1}
\end{aligned}
$$

2. Run the Hamilton filter to calculate new regime probabilities $\Pr\left(\xi_t^b, \xi_{t-1}^b | X_{t-1+\delta_h}, X^{t-1}\right)$ and $\Pr\left(\xi_t^b | X_{t-1+\delta_h}, X^{t-1}\right)$, for $i, j = 1, 2, ..., B$

$$\begin{aligned}
\Pr\left(\xi_t^b, \xi_{t-1}^b | X^{t-1}\right) &= \Pr\left(\xi_t^b | \xi_{t-1}^b\right) \Pr\left(\xi_{t-1}^b | X^{t-1}\right) \\
\ell\left(X_{t-1+\delta_h} | X^{t-1}\right) &= \sum_{j=1}^{B} \sum_{i=1}^{B} f\left(X_{t-1+\delta_h} | \xi_{t-1}^b = j, \xi_t^b = i, X^{t-1}\right) \\
&\quad \Pr\left[\xi_{t-1}^b = j, \xi_t^b = i | X^{t-1}\right] \\
f\left(X_{t-1+\delta_h} | \xi_{t-1}^b = j, \xi_t^b = i, X^{t-1}\right) &= (2\pi)^{-N_X/2} \left| f_{t|t-1+\delta_h}^{(i,j)} \right|^{-1/2} \\
&\quad \exp\left\{ -\frac{1}{2} e_{(t\backslash t-1+\delta_h)|t-1+\delta_h, t-1}^{(i,j)\prime} f_{(t\backslash t-1+\delta_h)|t-1+\delta_h, t-1}^{(i,j)} e_{(t\backslash t-1+\delta_h)|t-1+\delta_h, t-1}^{(i,j)} \right\} \\
\mathcal{L}(\theta) &= \mathcal{L}(\theta) + \ln\left(\ell\left(X_{t-1+\delta_h} | X^{t-1}\right)\right) \\
\Pr\left(\xi_t^b, \xi_{t-1}^b | X_{t-1+\delta_h}, X^{t-1}\right) &= \frac{f\left(X_{t-1+\delta_h} | \xi_t^b, \xi_{t-1}^b, X^{t-1}\right) \Pr\left(\xi_t^b, \xi_{t-1}^b | X^{t-1}\right)}{\ell\left(X_{t-1+\delta_h} | X^{t-1}\right)} \\
\Pr\left(\xi_t^b | X_{t-1+\delta_h}, X^{t-1}\right) &= \sum_{j=1}^{B} \Pr\left(\xi_t^b, \xi_{t-1}^b = j | X_{t-1+\delta_h}, X^{t-1}\right)
\end{aligned}$$

3. Using $\Pr\left(\xi_t^b, \xi_{t-1}^b | X_{t-1+\delta_h}, X^{t-1}\right)$ and $\Pr\left(\xi_t^b | X_{t-1+\delta_h}, X^{t-1}\right)$, collapse the $B \times B$ values of $S_{t|t-1+\delta_h}^{(i,j)}$ and $P_{t|t-1+\delta_h}^{(i,j)}$ into B values represented by $S_{(t\backslash t-1+\delta_h)|t-1+\delta_h}^{i}$ and $P_{(t\backslash t-1+\delta_h)|t-1+\delta_h}^{i}$:

$$S_{(t\backslash t-1+\delta_h)|t-1+\delta_h}^{i} = \frac{\sum_{j=1}^{B} \Pr\left[\xi_{t-1}^b = j, \xi_t^b = i | X_{t-1+\delta_h}, X^{t-1}\right] S_{(t\backslash t-1+\delta_h)|t-1+\delta_h}^{(i,j)}}{\Pr\left[\xi_t^b = i | X_{t-1+\delta_h}, X^{t-1}\right]}$$

$$P_{(t\backslash t-1+\delta_h)|t-1+\delta_h}^{i} = \frac{\sum_{j=1}^{B} \Pr\left[\xi_{t-1}^b = j, \xi_t^b = i | X_{t-1+\delta_h}, X^{t-1}\right] \begin{pmatrix} P_{t|t-1+\delta_h}^{(i,j)} + \left(S_{(t\backslash t-1+\delta_h)t-1+\delta_h}^{i} - S_{(t\backslash t-1+\delta_h)|t-1+\delta_h}^{(i,j)}\right) \\ \left(S_{(t\backslash t-1+\delta_h)|t-1+\delta_h}^{i} - S_{(t\backslash t-1+\delta_h)|t-1+\delta_h}^{(i,j)}\right)' \end{pmatrix}}{\Pr\left[\xi_t^b = i | X_{t-1+\delta_h}, X^{t-1}\right]}$$

4. If $t - 1 + \delta_h = t$, move to the next period by setting $t - 1 = t$ and returning to step 1

5. else, store the updated $S_{(t\backslash t-1+\delta_h)|t-1+\delta_h}^{(j)}$, $P_{(t\backslash t-1+\delta_h)|t-1+\delta_h}^{(j)}$, $\Pr\left(\xi_t^b, \xi_{t-1}^b | X_{t-1+\delta_h}, X^{t-1}\right)$, and $\Pr\left(\xi_t^b | X_{t-1+\delta_h}, X^{t-1}\right)$, move to the next intramonth time unit $\delta_k > \delta_h$, and repeat steps 1-5 keeping $t - 1$ fixed.

- At $t = T_1 + 1$ use $\theta_{\xi_t^P}$ relevant when $\xi_t^P = 2$, set $t - 1 = t$, and repeat steps 1-5
- At $t = T_2 + 1$ use $\theta_{\xi_t^P}$ relevant when $\xi_t^P = 3$, set $t - 1 = t$, and repeat steps 1-5
- $\vdots$
- At $t = T_{N_P - 1} + 1$ use $\theta_{\xi_t^P}$ relevant when $\xi_t^P = N_P$, set $t - 1 = t$ and repeat steps 1-5

- At $t = T_N = T$ stop. Obtain $L(\theta) = \sum_{t=1}^{T} \sum_{\delta_h \in (0,1)} \ln\left(\ell\left(X_{t-1+\delta_h} | X^{t-1}\right)\right)$.

The algorithm above is described in general terms; in principle the intramonth loop could be repeated at every instant within a month for which we have new data. Since we have only a subset of data intramonth, we vary the dimension of the vector of observables $X_{t-1+\delta_h}$ as a function of time $t-1+\delta_h$. In application, we repeat steps 1-5 only at certain minutes or days pre- and post-FOMC meeting. We initialize the algorithm with guesses for the Markov-switching parameters that vary across regime subperiods (only the policy rule parameters), while the fixed-coefficient parameters have guessed values that are identical across regime subperiods. These guesses are used to evaluate the posterior by combining the likelihood $L(\theta)$ with the priors. We continue guessing parameters and evaluating the posterior in this manner, until we find parameter values that maximize the posterior. With the posterior mode in hand, we evaluate the entire posterior distribution, as described below.

Observation Equation The mapping from the variables of the model to the observables in the data can be written using matrix algebra to obtain the observation equation $X_t = D_{\xi_t,t} + Z_{\xi_t,t}\left[S_t', \widetilde{y}_{t-1}\right]' + U_t v_t$. Denote $\widehat{g}_t \equiv g_t - g$, and $\widehat{lp}_t = lp_t - lp$. Using the definition of stochastically detrended output, we have $\widetilde{y}_t = \ln\left(Y_t/A_t\right)$, $\Delta\ln\left(A_t\right) \equiv g_t = g + \rho_g\left(g_{t-1} - g\right) + \sigma_g \varepsilon_{g,t} \Rightarrow \widetilde{y}_t - \widetilde{y}_{t-1} = \Delta\ln\left(Y_t\right) - g_t \Rightarrow \Delta\ln\left(Y_t\right) = \widetilde{y}_t - \widetilde{y}_{t-1} + g_t = \widetilde{y}_t - \widetilde{y}_{t-1} + \widehat{g}_t + g$. Annualizing the monthly growth rates to get annualized GDP growth we have $\Delta\ln\left(GDP_t\right) \equiv 12\Delta\ln\left(Y_t\right) = 12g + 12\left(\widetilde{y}_t + \widehat{g}_t - \widetilde{y}_{t-1}\right)$. For quarterly GDP growth we interpolate to monthly frequency using the method in Stock and Watson (2010). For our other quarterly variables (SPF survey measures) and our biannual Liv survey, we drop these from the observation vector in the months for which they aren't available. The observation equation when all variables in X_t are available takes the form:

$$\begin{bmatrix} \Delta \ln\left(GDP_t\right) \\ Inflation \\ FFR \\ SOC\left(Inflation\right)_{12m} \\ SOC\left(Inflation\right)_{60m} \\ f_t^{(0)} \\ BC\left(Inflation\right)_{12m} \\ SPF\left(Inflation\right)_{12m} \\ Liv\left(Inflation\right)_{12m} \\ SPF\left(GDPDInfl\right)_{12m} \\ BBG\left(Inflation\right)_{12m} \\ Liv\left(Inflation\right)_{120m} \\ SPF\left(Inflation\right)_{120m} \\ BC\left(FFR\right)_{12m} \\ BC\left(\Delta GDP\right)_{12m} \\ BBG\left(\Delta GDP\right)_{12m} \\ SPF\left(\Delta GDP\right)_{12m} \\ f_t^{(n)} \\ Baa_t \\ pgdp_t \\ EGDP_{t,t-1} \end{bmatrix} = \begin{bmatrix} 12g \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ D_{\xi_t,\pi_{t,t+12}} \\ D_{\xi_t,\pi_{t,t+12}} \\ D_{\xi_t,\pi_{t,t+12}} \\ D_{\xi_t,\pi_{t,t+12}} \\ D_{\xi_t,\pi_{t,t+12}} \\ D_{\xi_t,\pi_{t,t+120}} \\ D_{\xi_t,\pi_{t,t+120}} \\ D_{\xi_t,i_{t,t+12}} \\ D_{\xi_t,y_{t+s}} \\ D_{\xi_t,y_{t+s}} \\ D_{\xi_t,y_{t+s}} \\ D_{\xi_t,i_{t+n}} \\ C_{Baa} \\ \ln\left(K\right)+g \\ K\exp\left(g\right) \end{bmatrix} + \begin{bmatrix} 12\left(\widetilde{y}_t+\widehat{g}_t-\widetilde{y}_{t-1}\right) \\ 12\pi_t \\ 12i_t \\ \left[h+\left(h-1\right)\phi+\left(h-2\right)\phi^2+...+\phi^{11}\right]\left(1-\phi\right)\overline{\pi}_t+\Sigma_{j=1}^{12}\phi^j\pi_t \\ \left[h+\left(h-1\right)\phi+\left(h-2\right)\phi^2+...+\phi^{59}\right]\left(1-\phi\right)\overline{\pi}_t+\Sigma_{j=1}^{60}\phi^j\pi_t \\ 12i_t \\ Z_{\xi_t,\pi_{t,t+12}}S_t \\ Z_{\xi_t,\pi_{t,t+12}}S_t \\ Z_{\xi_t,\pi_{t,t+12}}S_t \\ Z_{\xi_t,\pi_{t,t+12}}S_t \\ Z_{\xi_t,\pi_{t,t+12}}S_t \\ Z_{\xi_t,\pi_{t,t+120}}S_t \\ Z_{\xi_t,\pi_{t,t+120}}S_t \\ Z_{\xi_t,i_{t,t+12}},S_t \\ Z_{\xi_t,y_{t+s}-y_t}S_t \\ Z_{\xi_t,y_{t+s}-y_t}S_t \\ Z_{\xi_t,y_{t+s}-y_t}S_t \\ Z_{\xi_t,i_{t+n}}S_t \\ B\widehat{lp}_t \\ k_t-k+pd_t+\widehat{g}_t+\widetilde{y}_t-\widetilde{y}_{t-1} \\ K\exp\left(g\right)\left[k_t-k+\widehat{g}_t+\widetilde{y}_t-\widetilde{y}_{t-1}\right] \end{bmatrix} \tag{A.13}$$
$$+U_t v_t$$

where we have used the fact that expectations for the macro agent in the model is:

$$\begin{aligned} \mathbb{E}_t^m\left[\pi_{t,t+h}\right] &= \left[h+\left(h-1\right)\phi+\left(h-2\right)\phi^2+...+\phi^{h-1}\right]\alpha_t^m+\left[\phi+\phi^2+...+\phi^h\right]\pi_t \\ &= \left[h+\left(h-1\right)\phi+\left(h-2\right)\phi^2+...+\phi^{h-1}\right]\left(1-\phi\right)\overline{\pi}_t+\left[\phi+\phi^2+...+\phi^h\right]\pi_t \end{aligned}$$

The term $Inflation$ in the above stands for CPI inflation; $GDPDInfl$ refers to GDP deflator inflation. The variable $f_t^{(n)}$ refers to the time-t contracted federal funds futures market rate. Here we use $n=\{6,10,20,35\}$. The variable $pgdp$ is the log of the SP500 capitalization-to-lagged GDP ratio, i.e., $\ln\left(P_t/GDP_{t-1}\right)$; $EGDP_{t,t-1}$ is the level of the SP500 earnings-to-lagged GDP ratio; taking a first order Taylor approximation of $EGDP_{t,t-1}$ around the log earnings-output ratio, we have $EGDP_{t,t-1}\approx K\exp\left(g\right)\left[1+k_t-k+\widehat{g}_t+\widetilde{y}_t-\widetilde{y}_{t-1}\right]$, where K is the steady state level of $EGDP_t=\exp\left(k\right)$. To obtain high-frequency information on $EGDP_{t,t-1}$, we use the BBG earnings nowcasts divided by one-month lagged real-time GDP. For announcements that occur during the time frame for which these nowcasts are available (Jan 2, 2006 to July 24, 2024), we use the pre- and post- announcement BBG earnings nowcast-to-lagged GDP ratio. For announcements that pre-date this time frame, we use the end-of-month earnings-lagged GDP ratio allowing for observation error as a noisy proxy for these nowcasts. Baa_t is the Baa spread described above, where C_{Baa} and B are parameters. To allow for the fact that the true convenience yield is only a function of Baa_t, we add a constant C_{Baa} to our model-implied convenience yield lp_t and scale it by the parameter B to be estimated. Unless otherwise indicated, all survey expectations are 12 month-ahead forecasts in annualized units.

The above uses multiple measures of observables on a single variable, e.g., investor expectations of inflation 12 months ahead are measured by four different surveys (BC, SPF, LIV, and BBG). In the filtering algorithm above, these provide four noisy signals on the same latent variable.

Computing the Posterior

The likelihood is computed with the Kim's approximation to the likelihood, as explained above, and then combined with a prior distribution for the parameters to obtain the posterior. A block algorithm is used to find the posterior mode as a first step. Draws from the posterior are obtained using a standard Metropolis-Hastings algorithm initialized around the posterior mode. Here are the key steps of the Metropolis-Hastings algorithm:

- Step 1: Draw a new set of parameters from the proposal distribution: $\vartheta \sim N\left(\theta_{n-1}, c\overline{\Sigma}\right)$
- Step 2: Compute $\alpha\left(\theta^m; \vartheta\right) = \min\left\{p\left(\vartheta\right)/p\left(\theta^{m-1}\right), 1\right\}$ where $p\left(\theta\right)$ is the posterior evaluated at θ.
- Step 3: Accept the new parameter and set $\theta^m = \vartheta$ if $u < \alpha\left(\theta^m; \vartheta\right)$ where $u \sim U\left([0,1]\right)$, otherwise set $\theta^m = \theta^{m-1}$
- Step 4: If $m \geq n^{sim}$, stop. Otherwise, go back to step 1

The matrix $\overline{\Sigma}$ corresponds to the inverse of the Hessian computed at the posterior mode $\overline{\theta}$. The parameter c is set to obtain an acceptance rate of around 30%. We use four chains of $540,000$ draws each (1 of every 200 draws is saved). The four chains combined are used to form an estimate of the posterior distribution from which we make draws. Convergence is checked by using the Brooks-Gelman-Rubin potential reduction scale factor using within and between variances based on the four multiple chains used in the paper.

J Risk Adjustment with Lognormal Approximation

The asset pricing block of equations involves conditional subjective variance terms that are affected by Markov-switching random variables in the model. We extend the approach in Bansal and Zhou (2002) of approximating a model with Markov-switching random variables using a risk-adjustment while maintaining conditional log-normality. Consider the forward looking relation for the price-payout ratio:

$$\begin{aligned} P_t^D &= \mathbb{E}_t^b\left[M_{t+1}\left(P_{t+1}^D + D_{t+1}\right)\right] \\ \frac{P_t^D}{D_t} &= \mathbb{E}_t^b\left[M_{t+1}\frac{D_{t+1}}{D_t}\left(\frac{P_{t+1}^D}{D_{t+1}} + 1\right)\right]. \end{aligned}$$

Taking logs on both sides, we get:

$$pd_t = \log\left[\mathbb{E}_t^b\left[\exp\left(m_{t+1} + \Delta d_{t+1} + \kappa_{pd,0} + \kappa_{pd,1} pd_{t+1}\right)\right]\right].$$

Applying the approximation implied by conditional log-normality we have:

$$\begin{aligned} pd_t &= \kappa_0 + \mathbb{E}_t^b\left[m_{t+1} + \Delta d_{t+1} + \kappa_{pd,1} pd_{t+1}\right] + \\ &\quad + .5\mathbb{V}_t^b\left[m_{t+1} + \Delta d_{t+1} + \kappa_{pd,1} pd_{t+1}\right]. \end{aligned}$$

To implement the solution, we follow Bansal and Zhou (2002) and approximate the conditional variance as the weighted average of the objective variance across regimes, conditional on ξ_t. Using the simpler notation of the state equation,

$$S_t = C_{\xi_t} + T_{\xi_t} S_{t-1} + R_{\xi_t} Q \varepsilon_t,$$

the approximation takes the form

$$\mathbb{V}_t^b\left[x_{t+1}\right] \approx e_x' \mathbb{E}_t^b\left[R_{\xi_{t+1}} Q Q' R_{\xi_{t+1}}'\right] e_x \tag{A.14}$$

where e_x is a vector used to extract the desired linear combination of the variables in S_t. This approximation maintains conditional log-normality of the entire system. In the solution, C_{ξ_t} depends on the risk adjustment term $V_t^b\left[m_{t+1} + \Delta d_{t+1} + \kappa_{pd,1} pd_{t+1}\right]$ which depends on R_{ξ_t}. Conditional on the risk adjustment term, the numerical solution delivers the appropriate coefficients, C_{ξ_t}, T_{ξ_t}, and R_{ξ_t}. To solve this fixed point problem, we employ the iterative approach of Bianchi, Kung, and Tirskikh (2018). Specifically, we solve the model and get S_t for an initial guess on the risk adjustment V_t^b, denoted $V_t^{b(0)}$. Given the approximation (A.14) the term $V_t^b\left[m_{t+1} + \Delta d_{t+1} + \kappa_{pd,1} pd_{t+1}\right]$ only depends on ξ_t. For each policy regime ξ_t^P our initial guess $V_t^{b(0)}$ is therefore one value of V_t^b for each of the belief regimes ξ_t^b. The initial solution based on the initial guess $V_t^{b(0)}$ gives an initial value for R_{ξ_t}, denoted $R_{\xi_t}^{(0)}$. So far we have not used (A.14). Then, given $R_{\xi_t}^{(0)}$, we use (A.14) to get an updated $V_t^{b(1)} \approx e_x E_t^b\left[R_{\xi_{t+1}}^{(0)} Q Q' R_{\xi_{t+1}}^{(0)\prime}\right] e_x$. Given the updated risk adjustment $V_t^{b(1)}$ we resolve the model for S_t one more time, and verify that the new $R_{\xi_{t+1}}$ is the same as the one obtained before, i.e., the same as $R_{\xi_{t+1}}^{(0)}$. Note that, although $V_t^b\left[x_{t+1}\right]$ depends on $R_{\xi_{t+1}}$ only (it does not depend on C_{ξ_t} due to the approximation (A.14)), $R_{\xi_{t+1}}$ does not depend on V_t^b. Thus, we can stop here.

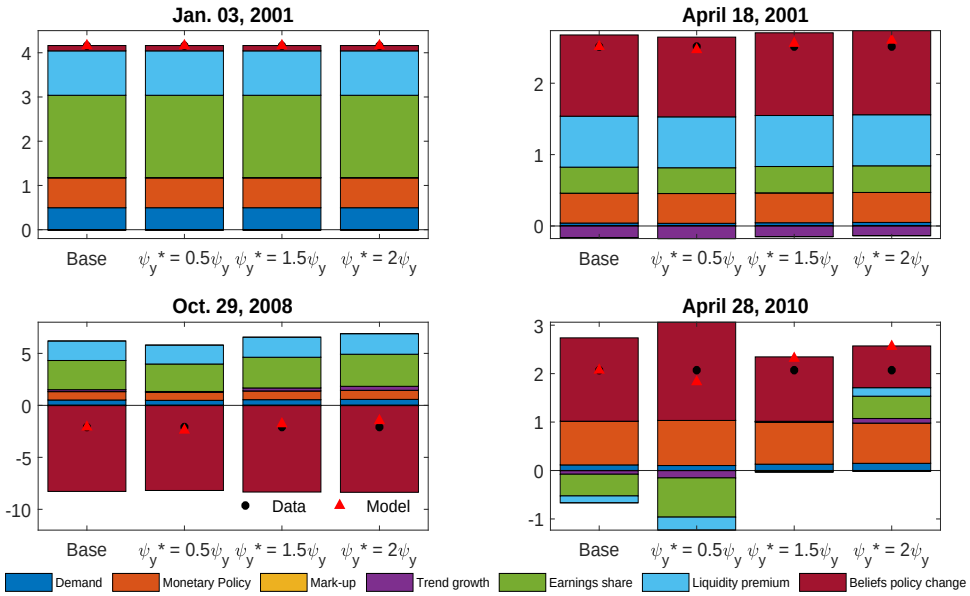
K Allowing for Belief Uncertainty About the Current Policy Rule

This Appendix shows what would happen in our results if beliefs about the current rule–holding constant beliefs/uncertainty about future policy regimes–had differed from

the true estimated rule, as in our baseline estimates. First, we redo the shock decompositions for the stock market assuming that investors change their belief about the current rule after a Fed announcement. Figure A.2 shows a decompsition that is analogous to that in Figure 6 for the top four FOMC announcements in terms of absolute jumps in the market. Each panel plots the shock decomposition for one announcement. The first bar labeled "base" shows the results for our baseline model, which repeats information from Figure (6). The next three bars show what would happen if investors had–in constrast to our baseline model–updated beliefs about the current policy rule in the wake of the announcement. Specifically, we assume they update their belief about the activism coefficient on output growth, ψ_y as a result of the announcement. In these cases investors' pre-announcement belief is equal to the true ψ_y, but the post-announcement belief changes to some $\psi_y^* \neq \psi_y$. The plot shows different cases where $\psi_y^* = \left\{0.5\psi_y, 1.5\psi_y, 2\psi_y\right\}$. The red triangles show the jump in the market implied by each specification, while the black dot shows the jump in the data. For the baseline model results shown in the first bars, the black dots and red triangles coincide. For the other specifications, the differences show the result of allowing investors to update their assessment of the current policy rule as a result of the announcement. We can see that the differences are negligble: the difference between the baseline model (black dot) and the red triangles in each case are small. Moreover, the relative contribution of different perceived shocks is virtually unchanged from the baseline case. In particular, changing perceptions about the economic state and/or beliefs about future regime change remain important contributors to the market's jump in all cases.

Next, we show how the model's implication for historical variation in the stock market would have changed if we allow for different beliefs about the parameters of the current policy rule. Panel (a) of A.3 repeats the results for the historical variation in the log stock market-lagged GDP ratio implied by our baseline model and displayed in in panel (a) of Figure 8 for the post-millennial period. The blue line shows $pgdp_t$, (the log stock market-to-last months GDP ratio) from our baseline model which coincides with true data series. The red/dashed (purple/dashed-dotted) line in each panel plots a counterfactual in which the belief regime with the highest (lowest) perceived probability of exiting the policy rule was always in place. Repeating the information from 8, we see that investor beliefs about the conduct of *future* monetary policy play an outsized role in stock market fluctuations, as can be observed from the quantitatively large gap between the red and purple lines in panel (a). Panel (b) of A.3 plots two different counterfactual series, in which investors believed that the activism coefficient ψ_y on output growth in the policy rule had been double (half) the true estimated value over the post-millennial period. Panel (b) shows that a substantial range of different beliefs about the *current* policy rule has negligle effects on the historical variation in the stock market. By contrast, panel (a) shows that differing beliefs about the probability of

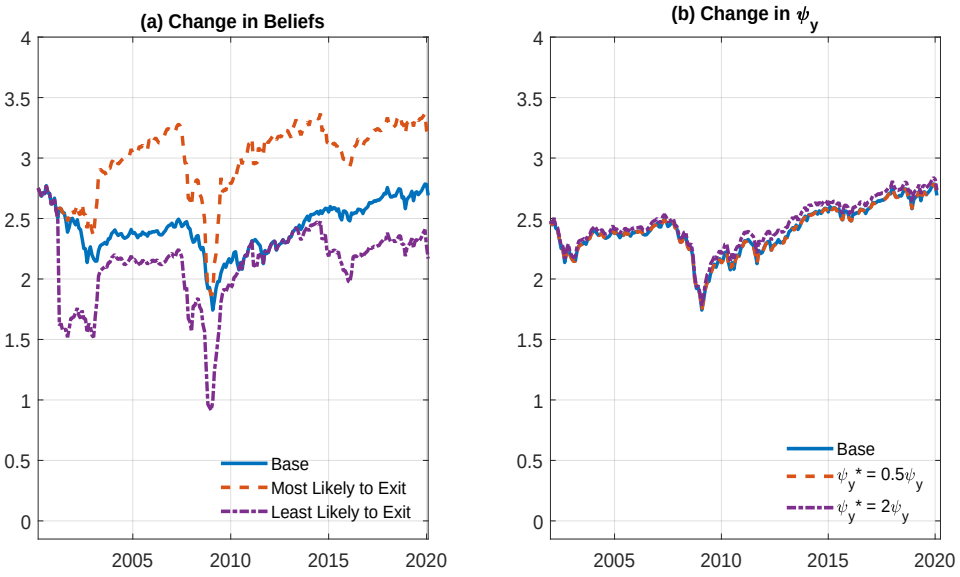
Figure A.2: Effects of Post-FOMC Updates to Beliefs About Current Rule



Notes: The figure reports a decomposition for the 4 most relevant FOMC announcements based on changes in the SP500-lagged GDP ratio. The first bar on the left gives our baseline estimate where investors know the true reaction coefficient on output growth in the current regime is ψ_y. The next three bars show what would have happened if instead investors had updated their belief about ψ_y to ψ_y^* *after* the FOMC meeting. The sample is 1961:M1-2020:M2.

switching to a new policy rule that is likely to be in place for an extended period of time has large effects.

Figure A.3: Effects of Beliefs About Future Rule vs Beliefs About Current Rule



Notes: The blue line in each panel plots the log of the S&P 500 market capitalization-lagged GDP ratio. In panel (a) this coincides with the historical variation implied by the baseline model. The red/dashed (purple/dashed-dotted) line in panel (a) plots a counterfactual S&P 500 to GDP ratio in which the belief regime with the highest (lowest) perceived probability of exiting the policy rule was always in place. The red/dashed (purple/dashed-dotted) line in panel (b) plots a counterfactual S&P 500 to GDP ratio in which the investor had counterfactually believed the activism coefficient on output growth in the policy rule had been double (half) the true estimated value. The sample for the counterfactual spans 2000:M3 to 2020:M2.