Prices and mergers in a general model of multi-sided markets*

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Abstract

We develop a general oligopoly model of multi-sided platform markets, in which end-users decide endogenously which sides and platforms to join in any combination of single- and multi-homing, and participation potentially limited to one side. Heterogeneous end-user valuations are drawn from a general density function. We derive formally uniqueness of the equilibrium number of end-users and novel switching effects that distort optimal platform pricing. This can lead to positive cross-price elasticities under positive network externalities, markups exceeding the Lerner index, and the ruling out of "cross-subsidization". To analyze multi-sided platform mergers, we provide a framework that unifies seemingly contradictory results in the previous literature. Given observable pre-merger variables, our model is the first to predict post-merger upward pricing pressure of mergers among (un)related platforms based on the switching effects. We show that while optimal pricing is determined by the nature of end-users' side choices, their platform choices are crucial for merger analysis.

Keywords: Multi-sided markets; Heterogeneous end-users; Endogenous side choice; Mergers of platforms; Digital platforms.

JEL Classification Numbers: D43, G34, L11, L13, L22, L86.

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1 Introduction

Multi-sided markets are ubiquitous. The remarkable rise in information technologies enables end-users to participate in multiple segments of these markets. Prime examples of multi-sided markets include Amazon and eBay, which enable end-users to buy and sell goods, and Airbnb, where guests, private hosts, and lodgings interact (Farronato and Fradkin, 2022). As digitization progresses, end-users enjoy even more immediate and flexible platform choices and side-switching options, influenced by market forces. For example, apps such as Uber and Bolt allow car owners to switch between driving passengers, delivering food, or renting out their cars with a tap of their fingertips; Rover and Wag allow end-users to either have their dogs taken for a walk or to walk someone else's dog; and platforms such as Piclo (UK) and Drift (USA) are leading a pioneering trend that allows households to buy or sell energy based on their needs and on demand and supply conditions (IRENA, 2020). In this paper, we build a novel tractable general model of multi-sided markets that captures these features of digital platforms and use this model to study optimal pricing and mergers.

In several dimensions, our model generalizes recent seminal contributions to this literature by introducing, for a richer set of scenarios, *both* endogenous side and platform choices of heterogeneous end-users whose valuations are drawn from a general density function. For example, Tan and Zhou (2021) study price competition and market entry with the assumption that end-users cannot choose sides (*fixed sides*) on single-homing platforms. Under full coverage and symmetry, prices increase to the detriment of end-users as additional platforms enter. Jullien and Pavan (2019) show that, under imperfect information, end-users' beliefs about participation affect demand elasticities and prices when the preferences of end-users are correlated with their beliefs.[1] The authors consider two-sided platforms with fixed sides, also limiting the role of end-users' choices.

Instead, we study optimal pricing in a model in which end-users choose the side(s) that they join. A quick Google search for "Rideshare vs. Food Delivery" reveals numerous tutorials advising Uber and Bolt car owners on the optimal choice between these two sides based on observable factors, illustrating the increasing prevalence of endogenous side decisions. We incorporate these decisions in a model that accommodates any combination of both single- and multi-homing plat-

[1] Despite playing a significant role in certain classic examples of two sided-markets—such as video games or operating systems—that require undertaking substantial sunk cost investments, beliefs are not especially relevant for many digital platforms in which end-users can easily appear on one side/platform or another.

forms, obtaining results that markedly depart from those of previous contributions—e.g., positive cross-price elasticities under positive externalities, markups exceeding the Lerner index, no cross-subsidization. These results reveal the importance of considering the different sides of platforms (e.g., ordering food, courier services, ride-hailing, or car rental in the case of Uber and Bolt) rather than entire platforms, as a fundamental unit of analysis in multi-sided markets, as we demonstrate that end-users' side choices crucially affect optimal pricing. In addition, we show that the nature of platform choices (single- vs. multi-homing) is key in determining the effects of platform mergers, to which we apply our theoretical framework after establishing it.

As Rysman (2009) discussed, the implications of mergers for end-users are very different in multi-sided markets than in other markets.[2] Based on this premise, we provide a novel and unifying framework that can accommodate mergers between platforms in a broad range of multi-sided markets and that predicts post-merger prices based on observable parameters before the merger occurs. Considering the potential overlap of some side(s) between two or more platforms (e.g., the side of peer-to-peer rental cars between Uber and Turo), our merger analysis sheds light on the harmful, though usually perceived as innocuous, effects of mergers of *seemingly unrelated* platforms—exemplified by Ola Cabs' takeover of Foodpanda, a food delivery app whose couriers often deliver by car—and other *conglomerate mergers*. For a variety of previous empirical studies on platform mergers that focus on particular industries (Fan, 2013; Song, 2021) and which yield mixed results, our general model predicts in which direction the corresponding post-merger prices will move based on observable pre-merger variables. Our model thus provides a general theoretical prediction toolkit for antitrust authorities and regulators dealing with platform mergers.

In our general model, we derive a unique subgame perfect equilibrium number of end-users and characterize the optimal pricing of platforms. Specifically, our analysis considers *(i)* that end-users choose whether they join no platform or one platform (single-homing), or even several platforms (multi-homing), in which case some of the end-users may still endogenously choose to single-home;[3] *(ii)* that end-users can participate on either one side or multiple sides at once, wherein some of the end-users may opt for single side participation; and *(iii)* that cross-group externalities can

[2]Nocke and Whinston (2013) provide a framework for the antitrust authority's optimal merger approval policy, while Evans and Schmalensee (2013) show why traditional antitrust analysis fails in the case of multi-sided platforms.

[3]Previous authors frequently focus their analysis on multi-homing platforms (Caillaud and Jullien, 2003; Rochet and Tirole, 2003; Armstrong, 2006). Moreover, most of the literature, except for Gao (2018) and Choi and Zennyo (2019), assumes fixed sides, ignoring side choices of end-users.

be either positive or, conversely, negative (*congestion effect*). We assume throughout that for the end-users, platform-specific idiosyncratic valuations for joining each side[4] are drawn from a general, well-behaved density function. This general density function allows us to accommodate end-users' valuation profiles that place more weight on likely side-switching decisions (e.g., the choice of a car owner between renting out her car or providing rides in the Uber case) while placing less weight on more implausible ones (e.g., the choice between providing rides or taking a ride). These valuation profiles are then key in determining participation decisions.

First, we let end-users decide to join any side on any multi-homing platform based on *participation constraints* (PC): an end-user joins a side on a platform if the sum of the valuation of joining it and the cross-group externality, net of the price (also called fee), exceeds the outside option. Using PCs, we obtain closed-form expressions for the demand for sides, from which we then derive price elasticities. In line with the intuition of previous authors, we show formally that the own-price elasticity is negative; that is, an increase in the fee on one side reduces participation on it. However, the effect on all other sides, measured by the cross-price elasticity, depends on the network effect: it is negative under positive cross-group externalities, and it becomes positive under congestion effects. This ambiguity is also reflected in the characterization of the Lerner index, which, in the former case, is dragged down by the markup charged to end-users on other sides—capturing the *opportunity cost*[5] in terms of lost revenue—while increasing in the latter case.

We then consider that participation is limited to only one side on certain platforms, such as Uber and Bolt, where a car owner faces the decision whether to rent out her vehicle, use it to provide rides, or engage in food delivery. In this case, endogenous side choices are further determined by *incentive compatibility constraints* (ICC): an end-user joins a side if the utility gained is greater than the outside option (PC) and greater than the utility obtained by joining any other mutually exclusive side (ICC). The latter yields a novel *switching-side effect* that we characterize formally, by which an increase in the fee on one side induces some end-users to switch to another side. Thus, increasing the fee has two (potentially countervailing) effects: it negatively (under positive cross-group externalities) or positively (under congestion) affects participation on the other sides

[4]This approach is commonly used to capture end-user heterogeneity (Armstrong, 2006; Correia-da Silva et al., 2019; Jullien and Pavan, 2019; Tan and Zhou, 2021). Other authors alternatively (or additionally) introduce heterogeneity in cross-group network externalities (Rochet and Tirole, 2003, 2006; Weyl, 2010).

[5]Rochet and Tirole (2006) coined this term. Unlike them, we capture not only the markup lost from participation on other sides but also the resulting network effect on the side where the price increased.

through the network effect and also increases participation on the other sides due to the switching-side effect, making the sign of the cross-price elasticity (ex-ante) ambiguous. This critically affects platform pricing as, contrary to previous results, the markup is greater than the Lerner index if the switching-side effect dominates the network effect, even under positive externalities. This stands in stark contrast to previous authors who show that additional participation and higher platform profits following an increase in the fee on one side can occur only under congestion.

Finally, with some subtle yet important particularities, our results prove robust if we assume that a subset of platforms are single-homing; i.e., end-users are allowed to join only one platform on each side. This occurs, for example, if car owners on Uber or Bolt are barred from renting out their cars on Turo or Getaround and/or from providing rides through Lyft. This case is micro-founded by introducing additional ICCs, which give rise to *switching-platform effects*.[6] We derive formally these effects, show how they interact with both the network effect and the switching-side effect discussed above, and obtain the unique participation equilibrium and optimal platform pricing. We remark that our results hold for any combination of single- and multi-homing platforms with any number of mutually/non-mutually exclusive sides. Additionally, they extend to no double-counting of network externalities and to end-users' random demand, as formally shown in Appendix B.

Two of the key features of our model (end-user endogenous platform and side choices) were previously considered only separately in more specific settings. For example, in Bakos and Halaburda (2020), Jeitschko and Tremblay (2020), and Teh et al. (2023), end-users who belong to one of two (fixed) sides simply decide which platform(s) to join. Jeitschko and Tremblay (2020) and Teh et al. (2023) derive an equilibrium in which some end-users single-home, while others multi-home. Bakos and Halaburda (2020) study two platforms competing à la Hotelling to show that, under full coverage and multi-homing, the classic result of cross-subsidization breaks down. Gao (2018) studies bundling in a monopoly platform that allows end-users to choose sides and offers a discount to those who join both sides. Choi and Zennyo (2019) model instead a two-sided platform duopoly on a Hotelling line in which end-users choose a platform and a side. However, these authors restrict to single-homing with mutually exclusive sides and full coverage. With the exception of Gao (2018), none of these authors considers congestion effects.

Our approach also contrasts with pioneer contributions to this literature (Caillaud and Jullien,

[6] Chandra and Collard-Wexler (2009) and Affeldt et al. (2013) identify intuitively these effects.

2003; Rochet and Tirole, 2003, 2006; Armstrong, 2006; Hagiu, 2006; Weyl, 2010), which rely on the trade-off between the fees incurred and the benefit obtained by joining the platform to capture end-user demand. As in any standard market model, participation decreases as own-side fees increase and increases with the number of end-users on the other side. These models capture well the characteristics of the classic examples of two-sided markets (e.g., video games, shopping malls, credit cards), considering that *(i)* end-users—at least on one side—are unlikely to be widely heterogeneous (game developers, retailers, advertising agencies), and *(ii)* end-users on each side appear as clearly differentiated groups (players/developers, card holders/merchants, readers/advertisers) with little or no relevant possibility of inter-group participation choices and side overlap. Instead, our model captures a wide range of easy-to-access digital platforms that allow heterogeneous end-users, whose valuations are drawn from a general density function, to actively choose and switch between the side(s) of the platform(s) they join. While these switching options may seem convenient for end-users, we show that they enable platforms to increase their markups.

Another contribution of this paper is to provide a novel general framework for evaluating the impact of mergers between (possibly unrelated) multi-sided platforms. This framework unifies the disparate results of previous authors who studied mergers in two-sided markets—Correia-da Silva et al. (2019) offer an overview of this literature. Departing from the platforms' optimal pricing and the corresponding unique participation equilibrium we characterize, we build on the work of Affeldt et al. (2013). These authors adapt Farrell and Shapiro (2010)'s Upward Pricing Pressure (UPP) measure of mergers to two-sided markets and demonstrate that the incentives to increase prices depend on the diverted sales that the merged entity recaptures. The value of these diverted sales is characterized formally by *diversion ratios* (ratios of own- and cross-price elasticities). We extend Affeldt et al. (2013)'s analysis to multi-sided markets and,[7] based on our micro-founded model, also offer additional and more general insights into the direction and strength of diversion ratios in a broader set of cases (single-/multi-homing, mutually exclusive sides, congestion effects, etc.). Since these ratios can be measured and estimated (Conlon and Mortimer, 2021), we can predict precisely the UPP of platform mergers in all these cases before the merger occurs.

Theoretically, Chandra and Collard-Wexler (2009) and Correia-da Silva et al. (2019) show that a merger has ambiguous effects on prices, depending on cross-subsidization. We not only identify

[7]Cosnita-Langlais et al. (2021) study cross-side effects in the UPP of a merger of two two-sided platforms.

formally the conditions under which cross-subsidization takes place but also quantify the strength of the UPP based on the different switching effects that we identify formally. From an empirical point of view, Fan (2013) finds by using data on newspapers that ad rates and subscription prices post-merger tend to move in opposite directions. Along the same lines, Song (2021) uses data from magazines to estimate a model of a two-sided market and finds that post-merger prices might go either up or down, with prices on different sides usually moving in opposite directions. Using diversion ratios, we identify the conditions under which these results hold, and we present intuition regarding the forces and effects that yield them. Bearing in mind the well-acknowledged features of media markets, our analysis explicitly considers congestion effects (Anderson and Peitz, 2020).

Most papers on platform mergers impose the assumptions that best suit the specific markets (e.g., newspapers, radio) on which they focus. Instead, we not only provide a general toolkit for mergers of platforms offering similar services, but we also apply it to mergers of unrelated platforms that are connected through one side; for example, a food-delivery platform and a ride-hailing platform (which clearly offer different services) both allow drivers to choose to deliver food or to give rides to passengers. Since end-users' side choices are at the heart of our analysis, this framework permits us to unmask the impact of digital conglomerate mergers and other agreements—perceived as innocuous by antitrust authorities—on prices, such as the mergers of Ola Cabs and Foodpanda, Uber and JUMP, Airbnb and HotelTonight, and the partnership agreement between Sixt and Tier.

The rest of the paper is as follows. We set up the model in Section 2. In Section 3, we provide the system of equations that characterize market demand for each of the cases we study. Section 4 proves the existence and uniqueness of participation equilibria for each of the cases, characterizes the equilibria and optimal pricing, and discusses certain features of the equilibria and optimal pricing. In Section 5, we study platform mergers, while Section 6 concludes. All proofs are in Appendix A, and some extensions are in Appendix B.

2 Model setup

We consider an oligopoly with a set of m platforms, denoted by $\mathcal{M}$ and indexed by $q \in \mathcal{M} := \{1, ..., m\}$. Each platform q enables interaction between end-users on $i \in \mathcal{D}^q := \{1, ..., d^q\}$ different sides, each capturing a different good or service offered or requested—e.g., in the Uber and Bolt

cases, passengers requesting car rides, car owners renting out cars, pedestrians booking e-scooters, and consumers ordering food represent different sides of these platforms. We assume that $d^q \geq 2$ for all $q \in \mathcal{M}$. The super-index q reflects that the number of sides need not be identical across platforms. However, for simplicity, we drop this super-index from now on.[8] A unit-measure continuum of end-users choose to join side i on any of the m platforms or to join no platform at all—sometimes we allow end-users to join multiple sides on multiple platforms. Slightly abusing notation, we denote by N_i^q the set of end-users that join side i on platform q.

Each end-user obtains an idiosyncratic surplus upon joining side i on platform q, denoted by v_i^q. This surplus parameter, also called *gross membership benefit*, is independent of the number of end-users on q's other sides and, therefore, of the market size or the number of transactions performed. Let $\mathbf{v} := \left(v_1^1, ..., v_d^m\right) \in \mathbb{R}_+^{m \times d}$ be the set of possible combinations of surplus parameters. Without loss of generality, we let $\mathbf{v} \in [0,1]^{m \times d}$, since only the relative relationship between the parameters is relevant. We assume that end-users are heterogeneous in v_i^q for all i and q, whose realization is drawn independently from a continuous, strictly increasing joint cumulative distribution function $F(\mathbf{v})$, with an associated joint density function $f(\mathbf{v})$, which has full and bounded support. This general formulation of end-users' valuations allows us to capture distribution profiles that align with the likely marginal decisions of end-users between different sides (e.g., a car owner deciding between providing rides or delivering food in Uber), while placing little weight on more implausible side decisions at the margin (e.g., an Uber user deciding between providing rides or ordering food).

We denote the *cross-group (network) externalities* by $\phi_i^q(\mathbf{N}_{-i}^q)$, which captures how end-users on platform q's side i are affected by participation $\mathbf{N}_{-i}^q$ on all sides other than i on q; that is, $\phi_i^q(\cdot)$ is a continuously differentiable, strictly monotone mapping, $\phi_i^q : [0,1]^{d-1} \to \mathbb{R}$, with $\phi_i^q(\mathbf{0}) = 0$ (normalization). There are *positive cross-group externalities* on platform q if end-users on side i enjoy meeting additional end-users on other sides—for example, the presence of additional buyers is beneficial for sellers on e-commerce websites. Conversely, there are *congestion effects* if those on side i do not enjoy meeting additional end-users on other sides—for example, readers dislike additional advertising on media outlets.[9] While we do not impose a specific functional form on $\phi_i^q(\cdot)$, we assume, in line with previous authors, that the aggregate impact of an (arbitrarily) small

[8] If one side is present on one platform but not on others, this poses no problem for our analysis.

[9] With few exceptions, such as Tan and Zhou (2021), previous authors assume linear effects in $\mathbf{N}_{-i}^q$; i.e., $\phi_i^q(\cdot) = \sum_j \alpha_{ij}^q N_j^q$ for some $\alpha_{ij}^q \in \mathbb{R}$, for all $i, j \in \mathcal{D}$, $i \neq j$, and for all $q \in \mathcal{M}$.

change in $\mathbf{N}_{-i}^q$ on $\phi_i^q(\cdot)$ is bounded.

Assumption 1. *(Bounded feedback loop)* $\sum_{j \in \mathcal{D} \setminus \{i\}} \left| \frac{\partial \phi_i^q(\mathbf{N}_{-i}^q)}{\partial N_j^q} \right| \in (0,1)$ *for all* $i \in \mathcal{D}$ *and* $q \in \mathcal{M}$.

Assumption 1 ensures that changes in participation on one side of the platform do not trigger disproportionately larger changes in participation on the other sides. If this assumption is not satisfied, then the "feedback loop" stemming from a change in market participation on one side generates increasingly amplified effects on participation on other sides, ultimately resulting in corner solutions where end-users either join only one side or none (outcomes that lack meaningful analytical insights).

Finally, we assume that end-users who join side i on platform q pay a fixed membership or *participation fee*, usually called price and denoted by p_i^q.[10] We let $\mathbf{p}^q := (p_1^q, ..., p_d^q) \in \mathbb{R}^d$ be the vector of prices charged by platform $q \in \mathcal{M}$, and we denote by $\mathbf{p} := (\mathbf{p}^1, ..., \mathbf{p}^m) \in \mathbb{R}^{m \times d}$ the vector of prices charged by all platforms.

The timing in our model is as follows. First, nature chooses end-users' idiosyncratic surplus parameters $\mathbf{v}$, and the platforms choose prices $\mathbf{p}$. Second, end-users learn the surplus parameters and prices and decide which side(s) on which platform(s) to join. Finally, end-users access the corresponding side(s), and payoffs are realized.

Given $\mathbf{v}$, $\mathbf{p}$, $\mathbf{N}_{-i}^q$, and $\phi_i^q(\cdot)$, an end-user who joins side i on platform q obtains a utility of

$$u_i^q := v_i^q + \phi_i^q(\mathbf{N}_{-i}^q) - p_i^q. \tag{1}$$

If platforms are multi-homing, (1) implies *double-counting* of cross-group effects, as end-users on different sides realize the network externality every time they meet on different platforms. This is the case, for example, when buyers benefit from meeting the same seller on multiple e-commerce websites, which occurs if the buyers are interested in a seller's reviews on different websites (Bakos and Halaburda, 2020). However, in other cases, only the first interaction matters. For example, advertisers may value the first impression of an ad on a viewer, while subsequent impressions on other platforms are not valued (Ambrus et al., 2016; Anderson et al., 2018). In Appendix B, we

[10] End-users pay no per-interaction fees. For monopoly platforms, Armstrong (2006) shows that lump-sum fees are equivalent to per-transaction fees, as discussed by Rochet and Tirole (2003). In our model, Bajo-Buenestado and Kinateder (2019) arrive at a similar conclusion, which extends to a combination of both fee types given the same transaction count.

extend our model to no double-counting of cross-group externalities and show that we capture the key results in this literature (incremental pricing; lower or no cross-subsidization).

Unlike end-users from the existing literature, end-users in our model decide endogenously not only whether to join a platform[11] but also which side(s) to join. In particular, to join side i on platform q, the participation constraint PC_i^q requires that the utility of an end-user is larger than her outside option (normalized to 0), i.e., $u_i^q \geq 0$. Moreover, we consider platforms where, for various reasons (examples of which are provided below), simultaneous participation on multiple sides is unfeasible (mutually exclusive sides). In such cases, we require additionally that the utility of joining one side is larger than that of joining any other; i.e., to participate on side i on platform q, the incentive compatibility constraint ICC_{ij}^q requires that $u_i^q \geq u_j^q$ for all $j \neq i$. We consider these features in two standard contexts: multi-homing and single-homing platforms.

3 Characterization of the demand for sides

In this section, we characterize end-user demand for each side and platform for the different cases we consider. The baseline case constitutes a micro-founded version of the framework frequently employed by previous authors. Subsequently, we introduce modifications for specific sides and platforms due to various constraints, inspired by particularly notable examples. Importantly, our analysis considers that sides and platforms may exhibit any combination of these distinct cases. As shown below, this highlights the importance of examining (subsets of) sides rather than entire platforms as the fundamental unit of analysis.

Baseline case. First, we assume that end-users can join any side on any platform, i.e., their participation decisions across different sides and platforms are independent. Consequently, each platform solves its own profit maximization problem (Belleflamme and Peitz, 2019b). Leaving aside the endogenous choices, this baseline scenario resembles the classic framework examined in the previous literature,[12] whose intuitive conclusions we show formally and which serves as the building block for our subsequent analysis. Examples of this setup include e-commerce platforms such as

[11] Previous authors, such as Bakos and Halaburda (2020), Choi and Zennyo (2019), and Tan and Zhou (2021), assume full coverage. In our equilibrium analysis, whether there is full or less than full coverage arises endogenously.

[12] For example, Armstrong (2006) and Rochet and Tirole (2003, 2006) assume that end-users, assigned to a fixed side on a platform, determine independently their participation on the corresponding side.

eBay, Facebook Marketplace, and Taobao, where end-users simultaneously engage in selling, buying, and advertising across multiple platforms without facing any exclusivity constraints. Similarly, classified ads websites such as Craigslist, Recycler, and Oodle follow this setup, as they enable end-users to choose independently their participation across various sides on the platforms.

Formally, given the realization of gross membership benefits $\mathbf{v}$ and the vector of prices $\mathbf{p}$, an end-user joins side $i \in \mathcal{D}$ on platform $q \in \mathcal{M}$ if, and only if,

$$(PC_i^q) \quad v_i^q + \phi_i^q(\mathbf{N}_{-i}^q) - p_i^q \quad \geq \quad 0.$$

From PC_i^q, the demand for side i on platform q can be obtained as follows:

$$N_i^q(\mathbf{N}_{-i}^q, \mathbf{p}^q) = pr\big(v_i^q \geq p_i^q - \phi_i^q(\mathbf{N}_{-i}^q)\big). \tag{2}$$

Let us denote by $f_i^q(v_i^q)$ the marginal density function of the valuation of side i on platform q, which need not follow the same marginal density as that of any other side on any platform. Then, the number of end-users joining side i on platform q can be written as

$$N_i^q(\cdot) = \int_{p_i^q - \phi_i^q(\mathbf{N}_{-i}^q)}^{1} f_i^q(v_i^q) dv_i^q,$$

or, equivalently,

$$N_i^q(\cdot) = 1 - F_i^q\big(p_i^q - \phi_i^q(\mathbf{N}_{-i}^q)\big), \tag{3}$$

where $F_i^q(\cdot)$ is the corresponding marginal distribution function of v_i^q. □

Mutually exclusive sides. We now modify the baseline case and consider that a subset of sides (possibly all), denoted as $\mathcal{D}' \subseteq \mathcal{D}$ with $|\mathcal{D}'| \geq 2$, are mutually exclusive.[13] In this context, end-users cannot be simultaneously on sides i and j on any platform, for all $i, j \in \mathcal{D}'$, $i \neq j$, but rather have to choose between one side or another. This exclusivity between sides, which can arise, for example, from physical constraints or economic (opportunity cost-related) reasons, is commonly observed in prominent digital peer-to-peer platforms.

For example, in app-based transportation companies such as Uber and Bolt, an end-user can

[13] Our model can also accommodate a platform-specific set of mutually exclusive sides. In this case, we would introduce a super-index q in the set $\mathcal{D}'$. For expositional clarity, we omit this super-index.

either request a ride or rent someone's vehicle to complete her trip. This decision may be influenced by market conditions and the end-user's opportunity cost. However, performing both activities concurrently is physically unfeasible. Likewise, in addition to renting out their vehicles, vehicle owners can offer rides to passengers or engage in food delivery services. These side choices are guided by similar trade-offs based on market conditions and opportunity cost. Again, undertaking these activities simultaneously is physically impractical. This also occurs in dog boarding and walking service platforms such as Rover and Wag: an end-user might use both platforms to walk dogs but cannot plausibly have her dog taken for a walk while walking someone else's dog. The same concept applies to many other platforms that require the physical presence of an end-user who either provides or seeks a service.

Formally, given certain mutually exclusive sides, say, i and j, end-users do not just decide whether to join platform q but also which of the sides (if any) to join. We capture this feature by introducing ICCs; that is, given $\mathbf{v}$ and $\mathbf{p}$, an end-user joins side $i \in \mathcal{D}' \subseteq \mathcal{D}$ on platform $q \in \mathcal{M}$ if, and only if, PC_i^q defined above holds and

$$(ICC_{ij}^q) \quad v_i^q + \phi_i^q(\mathbf{N}_{-i}^q) - p_i^q \quad \geq \quad v_j^q + \phi_j^q(\mathbf{N}_{-j}^q) - p_j^q,$$

also holds for all $j \in \mathcal{D}' \setminus \{i\}$, that is, if an end-user is better off by joining side i than side j on platform q.

Together these conditions yield the number of end-users on platform q's side i:

$$N_i^q(\cdot) = pr\left(v_i^q + \phi_i^q(\mathbf{N}_{-i}^q) - p_i^q \geq \max\left\{\max_{j \in \mathcal{D}' \setminus \{i\}}\left\{v_j^q + \phi_j^q(\mathbf{N}_{-j}^q) - p_j^q\right\}, 0\right\}\right). \tag{4}$$

Given $f_i^q(v_i^q)$ and considering that the valuations of different mutually exclusive sides are independent, the demand for side i is characterized by

$$N_i^q(\cdot) = \int_{p_i^q - \phi_i^q(\mathbf{N}_{-i}^q)}^{1} f_i^q(v_i^q) \prod_{j \in \mathcal{D}' \setminus \{i\}} \int_0^{v_i^q - \phi_j^q(\mathbf{N}_{-j}^q) + p_j^q - p_i^q + \phi_i^q(\mathbf{N}_{-i}^q)} f_j^q(v_j^q) dv_j^q dv_i^q,$$

or, equivalently,

$$N_i^q(\cdot) = \int_{p_i^q - \phi_i^q(\cdot)}^{1} f_i^q(v_i^q) \prod_{j \in \mathcal{D}' \setminus \{i\}} F_j^q\big(v_i^q - \phi_j^q(\mathbf{N}_{-j}^q) + p_j^q - p_i^q + \phi_i^q(\mathbf{N}_{-i}^q)\big) dv_i^q, \tag{5}$$

where $F_j^q\big(v_i^q - \phi_j^q(\mathbf{N}_{-j}^q) + p_j^q - p_i^q + \phi_i^q(\mathbf{N}_{-i}^q)\big) = \int_0^{v_i^q - \phi_j^q(\mathbf{N}_{-j}^q) + p_j^q - p_i^q + \phi_i^q(\mathbf{N}_{-i}^q)} f_j^q(v_j^q) dv_j^q$ is the marginal cumulative distribution function of v_j^q. □

Single-homing platforms. Departing again from the baseline case, we now assume instead that a subset of platforms (possibly all), denoted by $\mathcal{M}' \subseteq \mathcal{M}$ with $|\mathcal{M}'| \geq 2$, are single-homing.[14] In this scenario, an end-user cannot simultaneously join any side i on platforms q and r, for all $q, r \in \mathcal{M}'$, $q \neq r$. As a result, she must choose for each side whether to join it on one platform or another (or not to join it at all). This exclusivity between platforms can arise due to reasons such as legal (contractual), technical, or cost-related constraints.

For example, peer-to-peer car rental platforms such as Turo explicitly prohibit car owners in the US and Australia from listing their vehicles on multiple platforms simultaneously. Similarly, certain food delivery apps incorporate exclusivity clauses into their contracts, preventing a restaurant from delivering through multiple platforms. In several countries, this exclusivity extends to couriers, who are restricted from working for various apps.[15] For content development, platform choices also hinge on technical constraints related to differing programming languages; for example, Android developers must employ a C/C++-based environment, whereas iOS developers use a Java-based ecosystem (Shekhar, 2021). Another case in point is wireless technology standards, such as IEEE 802.11 or 3GPP (Farrell and Simcoe, 2012). Both chip manufacturers and portable device companies must adhere to one standard since developing products compatible with multiple standards would be extremely costly. Therefore, in all these cases, the decisions of end-users to participate on side i on different platforms are no longer independent.

Formally, given $\mathbf{v}$ and $\mathbf{p}$, an end-user joins side $i \in \mathcal{D}$ on platform $q \in \mathcal{M}' \subseteq \mathcal{M}$ if, and only if,

[14] Our model can accommodate the case in which the single-homing constraint applies to a subset of sides. This would require introducing a sub-index d in the set $\mathcal{M}'$, omitted for the sake of expositional clarity.

[15] In practice, the 2021 Rider Act in Spain converted riders into workers of a specific platform. Prior to this regulation, riders were considered contractors and had the flexibility to work for multiple platforms.

PC_i^q defined above holds and

$$(ICC_i^{qr}) \quad v_i^q + \phi_i^q(\mathbf{N}_{-i}^q) - p_i^q \;\geq\; v_i^r + \phi_i^r(\mathbf{N}_{-i}^r) - p_i^r,$$

also holds for all $r \in \mathcal{M}' \setminus \{q\}$, where $\mathcal{M}' \subseteq \mathcal{M}$ is the subset of single-homing platforms. The condition ICC_i^{qr} arises due to single-homing: participating on side i on platform q precludes an end-user from joining side i on any other single-homing platform $r \neq q$.

Putting these conditions together yields the number of end-users on platform q's side i:

$$N_i^q(\cdot) = pr\left(v_i^q + \phi_i^q(\mathbf{N}_{-i}^q) - p_i^q \geq \max\left\{\max_{r\in\mathcal{M}'\setminus\{q\}}\left\{v_i^r + \phi_i^r(\mathbf{N}_{-i}^r) - p_i^r\right\}, 0\right\}\right).$$

Thus, given $f_i^q(v_i^q)$, the number of end-users on platform q's side i is characterized by

$$N_i^q(\cdot) = \int_{p_i^q - \phi_i^q(\mathbf{N}_{-i}^q)}^{1} f_i^q(v_i^q) \prod_{r\in\mathcal{M}'\setminus\{q\}} \int_0^{v_i^q - \phi_i^r(\mathbf{N}_{-i}^r) + p_i^r - p_i^q + \phi_i^q(\mathbf{N}_{-i}^q)} f_i^r(v_i^r) dv_i^r dv_i^q,$$

or, equivalently, since $F_i^r\big(v_i^q - \phi_i^r(\mathbf{N}_{-i}^r) + p_i^r - p_i^q + \phi_i^q(\mathbf{N}_{-i}^q)\big) = \int_0^{v_i^q - \phi_i^r(\mathbf{N}_{-i}^r) + p_i^r - p_i^q + \phi_i^q(\mathbf{N}_{-i}^q)} f_i^r(v_i^r) dv_i^r$,

$$N_i^q(\cdot) = \int_{p_i^q - \phi_i^q(\cdot)}^{1} f_i^q(v_i^q) \prod_{r\in\mathcal{M}'\setminus\{q\}} F_i^r\big(v_i^q - \phi_i^r(\mathbf{N}_{-i}^r) + p_i^r - p_i^q + \phi_i^q(\mathbf{N}_{-i}^q)\big) dv_i^q.$$

□

Single-homing platforms with mutually exclusive sides. Finally, we consider the most restrictive case in which some or all platforms are single-homing and, simultaneously, some or all sides are mutually exclusive. This case is exemplified by several of the previously mentioned transportation companies in jurisdictions that require contractual exclusivity for drivers: e.g., in certain EU countries, the drivers have the status of employees.[16] In this case, drivers providing rides on one platform can neither do so on other platforms (single-homing) nor can they engage in renting out their cars (mutually exclusive sides). Another example are the increasingly popular platform-based energy utilities, such as Piclo (UK), Powerledger (Australia), and Drift (US) (IRENA, 2020; Baake et al., 2023). Upon joining one of the utilities—securing electricity from two utilities is unworkable—households equipped with solar panels and domestic batteries can decide

[16] Belleflamme and Peitz (2019a) note that third-party apps conveniently display offers from various companies to drivers on a single screen. However, in certain countries, exclusivity policies restrict such practices.

whether to buy or sell electricity based on their needs (e.g., appliances, electric vehicles charge) and demand-supply conditions. However, engaging in both selling and buying energy simultaneously is unfeasible.

Formally, this double exclusivity (within and across certain platforms) requires the introduction of additional ICCs. More precisely, given $\mathbf{v}$ and $\mathbf{p}$, an end-user joins side $i \in \mathcal{D}'$ on platform $q \in \mathcal{M}'$ if, and only if, PC_i^q, ICC_{ij}^q, and ICC_i^{qr} defined above hold and

$$(ICC_{ij}^{qr}) \quad v_i^q + \phi_i^q(\mathbf{N}_{-i}^q) - p_i^q \quad \geq \quad v_j^r + \phi_j^r(\mathbf{N}_{-j}^r) - p_j^r,$$

also holds for all $j \in \mathcal{D}' \subseteq \mathcal{D}$, $i \neq j$, and for all $r \in \mathcal{M}' \subseteq \mathcal{M}$, $q \neq r$. This latter constraint implies that an end-user is better off by joining platform q's side i than platform r's side j. Together these conditions yield the number of end-users on platform q's side i:

$$N_i^q(\cdot) = pr\left(v_i^q + \phi_i^q(\mathbf{N}_{-i}^q) - p_i^q \geq \max\left\{\max_{j\in\mathcal{D}',\, r\in\mathcal{M}'}\left\{v_j^r + \phi_j^r(\mathbf{N}_{-j}^r) - p_j^r\right\}, 0\right\}\right).$$

Using the marginal density functions of the surplus parameters and their corresponding marginal distribution functions, we rewrite participation on platform q's side i as follows:

$$N_i^q(\cdot) = \int_{p_i^q - \phi_i^q(\mathbf{N}_{-i}^q)}^{1} f_i^q(v_i^q) \prod_{r\in\mathcal{M}'\setminus\{q\}} F_i^r\left(v_i^q - \phi_i^r(\cdot) + p_i^r - p_i^q + \phi_i^q(\cdot)\right)$$
$$\prod_{j\in\mathcal{D}'\setminus\{i\}} F_j^q\left(v_i^q - \phi_j^q(\cdot) + p_j^q - p_i^q + \phi_i^q(\cdot)\right) \prod_{j\in\mathcal{D}'\setminus\{i\}} \prod_{r\in\mathcal{M}'\setminus\{q\}} F_j^r\left(v_i^q - \phi_j^r(\cdot) + p_j^r - p_i^q + \phi_i^q(\cdot)\right) dv_i^q.$$

□

By capturing the number of end-users on each side and platform while considering the exclusivity that might exist across and within platforms, we can build a system of $m \times d$ equations that captures the number of end-users on all sides and platforms, denoted as $\mathbf{N}(\cdot)$. Importantly, and as explained above, this system can accommodate any combination of all the distinct cases examined in this section, where all, some, or none of the sides are mutually exclusive and in which some, all, or none of the platforms are single-homing.

4 Participation equilibrium and optimal pricing

We solve our model backwards to characterize a subgame perfect equilibrium and begin with the second stage, where end-users decide which side(s) and platform(s) to join, given any $\mathbf{p}$. We prove formally the existence and uniqueness of the participation equilibrium (PE), and then characterize the non-empty solutions in which there are end-users on all sides and platforms. Finally, we study the platforms' optimal pricing and derive explicit price elasticity formulas, discussing their implications tailored to the four scenarios introduced in Section 3.

4.1 Participation equilibrium: existence, uniqueness, and characterization

In Proposition 1, the proof of which can be found in Appendix A, we show the existence and uniqueness of a PE, $\mathbf{N}^*(\mathbf{p})$, given $\mathbf{p}$. This result holds if Assumption 1 and an additional standard condition related to the maximum change in N_i^q resulting from a small change in N_j^q are satisfied.

Proposition 1. *Given* $\mathbf{p}$*, there is a unique PE (participation equilibrium),* $\mathbf{N}^*(\mathbf{p})$*, if Assumption 1 and the following condition hold:*

$$\beta := \underbrace{\max_{\substack{i\in\mathcal{D}\\ q\in\mathcal{M}}} \sup_{\mathbf{N}^q\in[0,1]^d} \sum_{j\in\mathcal{D}} \left|\frac{\partial N_i^q(\cdot)}{\partial N_j^q}\right|}_{\beta_1=\text{condition on dispersion of } v_i\text{'s}} \times \underbrace{\max_{\substack{i\in\mathcal{D}\\ q\in\mathcal{M}}} \left(\sum_{j\in\mathcal{D}\setminus\{i\}} \sup_{\mathbf{N}_{-i}^q\in[0,1]^{d-1}} \left|\frac{\partial \phi_i^q(\mathbf{N}_{-i}^q)}{\partial N_j^q}\right| \right)}_{\beta_2=\text{condition on externalities}} < 1.$$

If all sides are mutually exclusive, Proposition 1 holds under Assumption 1 alone. However, if some sides are non-mutually exclusive, the additional condition $\beta_1 < \beta_2^{-1}$ must be satisfied. Intuitively, β_1 bounds the maximum change in N_i^q resulting from a small change in N_j^q, among all possible $\mathbf{N}^q$ vectors. The absolute value of the derivative of N_i^q with respect to N_j^q is a marginal density $f_{ij}^q(\cdot) \in [0,1]$. In the case where some sides are non-mutually exclusive, then the sum of these marginal densities for all $j \in \mathcal{D}$ may exceed 1 since each end-user can join multiple sides on the same platform. By contrast, when all sides are mutually exclusive, end-users can join only one side, ensuring that this sum remains below 1; consequently, $\beta_1 < 1$ is trivially satisfied. Condition $\beta_2 < 1$, which bounds the feedback loop of cross-group externalities, holds by Assumption 1. Therefore, in cases where all sides are mutually exclusive, $\beta < 1$ follows directly from Assumption 1, while if some sides are non-mutually exclusive, we additionally require $\beta_1 < \beta_2^{-1}$.[17]

[17]Note that $\beta < 1$ is a sufficient condition, also fulfilled at non-empty corner solutions, in which all end-users join

Next, we provide necessary and sufficient conditions for the existence of a non-empty PE, which is an equilibrium with at least one end-user on each side and platform. A non-empty PE requires binding PCs, i.e., that the price, net of the cross-group externalities, must be below 1 for all sides and platforms. If this condition is not satisfied for a specific side and platform, no end-user would join it—as end-user valuations are at most 1. Under multi-homing, if all sides are non-mutually exclusive, this condition stands as both necessary and sufficient for the existence of a non-empty PE. In all other cases, this condition together with binding ICCs, which prevent a particular side and/or platform from consistently yielding the highest utility in comparison to others, are both necessary and sufficient.

First, when some sides are mutually exclusive, the ICCs across all mutually exclusive sides for all platforms must be binding. Second, if all sides are non-mutually exclusive but some platforms are single-homing, then the ICCs must be binding across all sides for each single-homing platform. Lastly, if there are single-homing platforms with some mutually exclusive sides, both sets of ICCs must be met simultaneously, and furthermore, the ICCs of all mutually exclusive sides across each single-homing platform must also be binding. These conditions are provided formally in Proposition 2, the proof of which is in Appendix A.

Proposition 2. *Given* $\mathbf{p}$*, there exists a non-empty PE,* $\mathbf{N}^*(\mathbf{p})$*, if, and only if,*

a) (PC_i^q) $\left[\phi_i^q(\cdot) - p_i^q\right] \geq -1$ *holds for all* $i \in \mathcal{D}$ *and all* $q \in \mathcal{M}$ *in the baseline case;*

b) (PC_i^q) *and* (ICC_{ij}^q) $\left[\phi_i^q(\cdot) - p_i^q\right] - \left[\phi_j^q(\cdot) - p_j^q\right] \in [-1, 1]$ *hold for all* $i, j \in \mathcal{D}'$*,* $i \neq j$*, and all* $q \in \mathcal{M}$*, in case sides* $i, j \in \mathcal{D}' \subseteq \mathcal{D}$ *are mutually exclusive;*

c) (PC_i^q) *and* (ICC_i^{qr}) $\left[\phi_i^q(\cdot) - p_i^q\right] - \left[\phi_i^r(\cdot) - p_i^r\right] \in [-1, 1]$ *hold for all* $i \in \mathcal{D}$ *and all* $q, r \in \mathcal{M}'$*,* $q \neq r$*, in case platforms* $q, r \in \mathcal{M}' \subseteq \mathcal{M}$ *are single-homing;*

d) (PC_i^q)*,* (ICC_{ij}^q)*,* (ICC_i^{qr}) *and* (ICC_{ij}^{qr}) $\left[\phi_i^q(\cdot) - p_i^q\right] - \left[\phi_j^r(\cdot) - p_j^r\right] \in [-1, 1]$ *hold for all* $i, j \in \mathcal{D}'$*,* $i \neq j$*, and all* $q, r \in \mathcal{M}'$*,* $q \neq r$*, in case platforms* $q, r \in \mathcal{M}' \subseteq \mathcal{M}$ *are single-homing and sides* $i, j \in \mathcal{D}' \subseteq \mathcal{D}$ *are mutually exclusive.*

Throughout the rest of the paper, we focus our analysis on the set of non-empty PE as characterized in Proposition 2. Such an equilibrium is unique, provided the conditions outlined in

at least one side. Tan and Zhou (2021) impose a very similar condition to ensure uniqueness under single-homing and fixed sides.

Proposition 1 are satisfied. We also emphasize that non-empty equilibria should not be confused with solutions involving full market coverage, which is an assumption made frequently by previous authors. In our framework, it can be easily shown that having at least one PC slack for all end-users is sufficient for $\mathbf{N}(\cdot)$ to yield full market coverage.

4.2 Platform optimal pricing

In the first stage, platform q maximizes its profit π^q by choosing prices for all sides. Assuming a cost per end-user of $c_i^q > 0$ on side i, platform q's profit maximization problem is given by

$$\max_{\{p_1^q,\cdots,p_d^q\}} \quad \pi^q := \sum_{i\in\mathcal{D}} (p_i^q - c_i^q) N_i^q. \tag{6}$$

At an interior solution, the first-order-condition (FOC) of (6), that characterizes platform q's optimal price for side i, is given by

$$N_i^q + (p_i^q - c_i^q)\frac{\partial N_i^q}{\partial p_i^q} + \sum_{j\in\mathcal{D}\setminus\{i\}} (p_j^q - c_j^q)\frac{\partial N_j^q}{\partial p_i^q} = 0, \tag{7}$$

where N_i^q and its derivatives are evaluated at optimal prices. By the extreme value theorem, given the continuity of the profit function, generically there exists a vector of optimal prices that solves (7) for all $i \in \mathcal{D}$ and all $q \in \mathcal{M}$.[18] The left-hand-side (LHS) of (7) is the usual FOC with respect to p_i^q for any profit-maximizing firm, augmented by $\sum_{j\in\mathcal{D}\setminus\{i\}}(p_j^q - c_j^q)\frac{\partial N_j^q}{\partial p_i^q}$. This sum captures the fact that increasing p_i^q also affects participation on all other sides through different cross-group effects, formally identified below.

When (7) is rearranged, optimal pricing can be rewritten in the familiar Lerner index notation:

$$\frac{p_i^q - c_i^q}{p_i^q} = \frac{1}{|\varepsilon_i^q|} + \sum_{j\in\mathcal{D}\setminus\{i\}} \lambda_{ij}^q \frac{(p_j^q - c_j^q)}{p_i^q}, \tag{8}$$

where $\varepsilon_i^q := \frac{\partial N_i^q}{\partial p_i^q}\frac{p_i^q}{N_i^q}$ is the own-price elasticity of demand for platform q's side i, and $\lambda_{ij}^q := \frac{\partial N_j^q}{\partial p_i^q} \Big/ \left|\frac{\partial N_i^q}{\partial p_i^q}\right|$ is the *diversion ratio*:[19] it reflects the change in the number of end-users on platform q's

[18] The optimal price vector is unique under additional standard assumptions, such as log-concavity of $f_i^q(\cdot)$. This assumption is sufficient for uniqueness and is made commonly. In a related context, it is made by Zhou (2017), Choi et al. (2018), and (for some results) Tan and Zhou (2021).

[19] Due to Shapiro (1996), this term is widely used in the anti-trust literature (Conlon and Mortimer, 2021).

side j relative to the change on side $i \neq j$ when p_i^q increases. Hence, the diversion ratio captures the extent to which platform q internalizes the different effects of a price change on the number of end-users on other sides. To quantify these effects, in the following subsection, we characterize own- and cross-price elasticities for the different combinations of the scenarios considered. In the rest of the paper, without loss of generality, we assume that platforms charge positive prices,[20] which, however, does not preclude platforms from pricing below marginal cost (cross-subsidizing), if they find that doing so is optimal.

4.3 Price elasticities and implications for optimal platform pricing

4.3.1 Baseline case

First, we consider the baseline case in which end-users can join any side on any platform. Given the marginal density function of valuations of a given side i on platform q, denoted as $f_i^q(\cdot)$, the derivative of N_i^q with respect to p_i^q and p_j^q can be formally obtained from the expressions that characterize the number of end-users in (3):

$$\frac{\partial N_i^q}{\partial p_i^q} = \underbrace{- f_i^q\big(p_i^q - \phi_i^q(\mathbf{N}_{-i}^q)\big)}_{\text{price effect}} + \underbrace{f_i^q\big(p_i^q - \phi_i^q(\mathbf{N}_{-i}^q)\big)\frac{\partial \phi_i^q(\mathbf{N}_{-i}^q)}{\partial p_i^q}}_{\text{network effect}}, \tag{9}$$

$$\frac{\partial N_i^q}{\partial p_j^q} = \underbrace{f_i^q\big(p_i^q - \phi_i^q(\mathbf{N}_{-i}^q)\big)\frac{\partial \phi_i^q(\mathbf{N}_{-i}^q)}{\partial p_j^q}}_{\text{network effect}}. \tag{10}$$

The own-price derivative (9) can be decomposed into a *price effect*, which arises from the negative impact of p_i^q on PC_i^q, and a *network effect*. Under positive cross-group externalities, the decrease in N_i^q due to the negative price effect reduces participation on all sides except side i, which further decreases N_i^q. Under congestion, the effect is opposite in sign, and N_i^q increases. In the cross-price derivative of N_i^q with respect to p_j^q (10), there is only a network effect. In Lemma 1, we formally show that, given a non-empty interior PE with $N_i^q \in (0,1)$ for all i and q,[21] the sign of (10)

[20] This restriction simplifies the interpretation of the own- and cross-price elasticities and the matrix of relative prices defined below in Section 5. Our analysis extends to the case in which platforms charge a strictly negative price on some side(s).

[21] If all end-users join a side on a platform (i.e., $N_i^q = 1$ for some i and q), a small change in p_i^q (or p_j^q) does not affect participation on that side and platform (i.e., $N_i^q = 1$ still holds). Consequently, own- and cross-price derivatives are zero. These non-empty corner equilibria cannot occur at a non-empty PE in all other cases considered below

depends on the nature of the network effect: it is negative under positive cross-group externalities and positive under congestion. We also show that, despite the ambiguity in the sign of the network effect, an increase in p_i^q always decreases N_i^q—the price effect dominates the network effect under Assumption 1. From these results, the signs of the own- and the cross-price elasticities then follow.

Lemma 1. *Consider that end-users can join any side on any platform. Given a non-empty interior PE,* $\mathbf{N}^*(\mathbf{p})$*, the own-price elasticity* $\varepsilon_i^q := \frac{\partial N_i^q}{\partial p_i^q}\frac{p_i^q}{N_i^q} < 0$*, the cross-price elasticity* $\varepsilon_{ij}^q := \frac{\partial N_i^q}{\partial p_j^q}\frac{p_j^q}{N_i^q} < 0$ *under positive cross-group externalities, and* $\varepsilon_{ij}^q > 0$ *under congestion.*

Lemma 1's proof can be found in Appendix A. It implies that λ_{ij}^q is negative under positive cross-group externalities and positive under congestion. Therefore, the optimal pricing rule captures several results that are in line with previous studies. First, if cross-group externalities are positive, the optimal markup on one side is adjusted downward by the opportunity cost in terms of the revenue lost from all other sides, resulting in a markup lower than the usual Lerner index. Second, this optimal pricing rule may involve *cross-subsidization*; that is, for some parameters, platform q finds it optimal to set a price below marginal cost on one side, resulting in a strictly positive profit on another side. For example, if $|\varepsilon_i^q|$ is sufficiently large, we have $p_i^q - c_i^q < 0$, requiring $p_j^q - c_j^q > 0$ for some side j.[22]

4.3.2 Mutually exclusive sides

We now consider that a subset of sides $\mathcal{D}' \subseteq \mathcal{D}$ are mutually exclusive. That is, end-users face a choice between joining either side i or j, for all $i, j \in \mathcal{D}'$, $i \neq j$. In this case, the same price and network effects (stemming from the PCs) as those described above occur following a change in p_i^q. However, there is an additional effect through the ICCs: an increase in p_i^q induces some end-users, who are on side i but are close-to-indifferent between joining side i or j, to switch from the former to the latter. We call this the *switching-side effect* and show its existence in Proposition 3, the proof of which can be found in Appendix A.

Proposition 3. *Consider that a subset of sides* $\mathcal{D}' \subseteq \mathcal{D}$*,* $|\mathcal{D}'| \geq 2$*, on platform* q *are mutually exclusive. Given a non-empty PE,* $\mathbf{N}^*(\mathbf{p})$*, suppose that platform* q *changes prices from* $\mathbf{p}^q$ *to* $\tilde{\mathbf{p}}^q$*,*

since side i is mutually exclusive and/or platform q is single-homing.

[22] Only when we also impose that end-users' valuations are uniformly distributed does the characterization of optimal prices in our model yield those obtained, e.g., by Armstrong (2006) and Rochet and Tirole (2006).

where $\tilde{p}_k^q = p_k^q$ *for all* $k \in \mathcal{D} \setminus \{i\}$*, and* $\tilde{p}_i^q = p_i^q + \delta$ *for some* $i \in \mathcal{D}'$ *and* $\delta > 0$*. Then some end-users switch from side* i *to all other mutually exclusive sides* $j \in \mathcal{D}' \setminus \{i\}$ *on platform* q*, while no end-user changes from any side* j *to side* i*.*

Therefore, an increase in p_i^q has a twofold impact on participation on sides i and j. First, N_i^q decreases due to the price and the switching-side effects. Second, N_j^q increases due to the switching-side effect and further increases (decreases) under congestion (positive cross-group externalities). To illustrate the interplay of these effects, we provide a numerical example.

Example 1. Consider a platform with two mutually exclusive sides, and let end-users' valuations be jointly uniformly distributed; i.e., for each end-user, any $(v_i, v_j) \in [0,1]^2$ arises with equal probability. Let the cross-group externality be $\phi_i(N_j) = N_j$ and $\phi_j(N_i) = N_i$, and suppose that $p_i = p_j = .4$. Then, PC_i is given by $v_i - .4 + N_j \geq 0$, while ICC_{ij} is given by $v_i - .4 + N_j \geq v_j - .4 + N_i$. It can be easily shown that there is a unique PE with full market coverage in which half of the end-users join side i, and the other half join side j (since the cross-group externality $N_i = N_j = .5$ is larger than the price .4).

Given this PE, suppose that the platform increases p_i to .42. Then, a new PE arises with full market coverage. However, because joining side i is now relatively more expensive than joining side j, some end-users switch from side i to side j. This is captured by ICC_{ij}, which becomes $v_i - .42 + N_j \geq v_j - .4 + N_i$. To find the end-user who is indifferent between joining side i or j, we let ICC_{ij} hold with equality and find that $N_j = .51$ and $N_i = .49$; that is, the switching-side effect makes .01 of end-users on side i drop it and switch to side j.

However, $N_j = .51$ and $N_i = .49$ cannot be the new PE: if it were, this would imply that in equilibrium, $v_i = v_j$ holds, in which case, half of the end-users join each side (which is a contradiction). In fact, we also need to take into account the network effect, which prevents some end-users from leaving side i due to the additional end-users that switch to side j and, consequently, also attenuates the increase in participation on side j.

To find the new PE, we calculate the valuation for side i, denoted by $\bar{v}_i$, for an end-user with $v_j = 0$, such that she is indifferent to joining side i or j at prices $p_i = .42$ and $p_j = .4$; that is, $\bar{v}_i$ captures the higher valuation for side i that the indifferent end-user needs to join side i rather than side j, given the higher p_i. For such an indifferent end-user, the equality $\bar{v}_i - .42 + N_j = -.4 + N_i$

must hold.

Considering that the valuations are jointly uniformly distributed, N_i and N_j can be calculated from $N_i = .5(1 - \bar{v}_i)^2$, and due to full market coverage, $N_j = 1 - .5(1 - \bar{v}_i)^2$ must also hold. Replacing these conditions in the equality $\bar{v}_i - .42 + N_j = -.4 + N_i$ yields, after some manipulations, $\bar{v}_i - .02 + 1 - (1 - \bar{v}_i)^2 = 0$. Solving for $\bar{v}_i$ yields two solutions, namely, one that is larger than 1 and thus discarded and another that is $\bar{v}_i = .0067$. Hence, at the new PE, $N_i = .4933$ and $N_j = .5067$.

In summary, when p_i increases from .4 to .42 (with p_j unchanged at .4), there is a switching-side effect that decreases N_i from .5 to .49 and increases N_j from .5 to .51, and the network effect attenuates this increase in N_j to .5067 and augments N_i to .4933. □

Next, we analytically derive both the own- and the cross-price elasticity for any pair of mutually exclusive sides, say, i and j. Regarding the own-price elasticity, one might conclude that its sign is ambiguous: since N_j^q can either increase (as shown in Example 1) or decrease (as we explain below) following an increase in p_i^q, the network effect on N_i^q could be either positive or negative, potentially offsetting the price effect. However, once again, we show that the own-price elasticity is always negative. By contrast, the implications of the different effects on the cross-price elasticity are not as straightforward. Hence, the sign of the cross-price elasticity depends on the model's parameters and on the features of the PE.

More precisely, if there is full participation on platform q, such as in Example 1, an increase in p_i^q does not induce any end-user to drop the platform (i.e., PC_i^q remains slack), but some of the end-users will switch from side i to another mutually exclusive side, say, j. In this context, the cross-price elasticity is unambiguously positive—regardless of whether the cross-group externalities are positive (which attenuates the increase in N_j^q) or negative (which reinforces the increase in N_j^q). If participation is less than full (i.e., PCs are binding), the effect of this decrease in N_i^q on N_j^q depends on the nature of the cross-group externalities. Under congestion, the cross-price derivative is unambiguously positive because both the network and the switching-side effect increase participation on side j. However, if the cross-group externalities are positive, the two effects go in opposite directions, and the impact on N_j^q can be either positive or negative (depending on which effect dominates). These results are formally presented in Theorem 1, the proof of which can be found in Appendix A.

Theorem 1. *Given a subset $\mathcal{D}' \subseteq \mathcal{D}$, $|\mathcal{D}'| \geq 2$, of mutually exclusive sides on platform q, at a non-empty PE, $\mathbf{N}^*(\mathbf{p})$, $\varepsilon_i^q < 0$ for all $i \in \mathcal{D}'$. For $j \in \mathcal{D}' \setminus \{i\}$, $\varepsilon_{ij}^q > 0$ if $\sum_{i \in \mathcal{D}'} N_i^q = 1$ or if there is congestion, and ε_{ij}^q can be either positive or negative if $\sum_{i \in \mathcal{D}'} N_i^q < 1$ and cross-group externalities are positive.*

Theorem 1 captures a novel result that is worth highlighting, namely, that an increase in p_i^q can increase participation on all sides other than i, even in the presence of positive cross-group externalities (and regardless of whether market coverage is full or not). Intuitively, consistent with the previous literature, a decrease in the number of end-users on one side would typically reduce the number of end-users on all other sides through the network effect. However, in our model, an increase in p_i^q has a negative impact on side j only if the network effect dominates the switching-side effect; otherwise, N_j^q increases. Previous authors do not formally find this switching-side effect.

Finally, we examine the implications of these results for optimal platform pricing. As shown, the Lerner index (8) depends on the sign of the diversion ratio λ_{ij}^q for all $i, j \in \mathcal{D}$, $i \neq j$. If all sides are non-mutually exclusive (Section 4.3.1), the sign of λ_{ij}^q is determined solely by whether there are positive cross-group externalities or congestion. However, when sides i and j are mutually exclusive, determining the sign of λ_{ij}^q is more intricate, as follows from Theorem 1.

If the PE yields full coverage, then those end-users that drop side i following an increase in p_i^q are *diverted* to another (mutually exclusive) side j, and $\lambda_{ij}^q > 0$. Consequently, the optimal markup is greater than the usual Lerner index. The same result is obtained in the presence of congestion effects as, in this case, an increase in p_i^q also increases N_j^q. However, with positive cross-group externalities and less than full coverage, an increase in p_i^q has two opposite effects on N_j^q, and thus, the result is ambiguous. Namely, if the switching-side effect dominates the network effect, increasing p_i^q raises participation on side j (since $\lambda_{ij}^q > 0$), and hence, the optimal markup is greater than the Lerner index. Conversely, if the network effect dominates the switching-side effect, $\lambda_{ij}^q < 0$, and consequently, the optimal markup is smaller than the usual Lerner index. This ambiguity under positive cross-group externalities is stated formally in Corollary 1.

Corollary 1. *Given a subset $\mathcal{D}' \subseteq \mathcal{D}$, $|\mathcal{D}'| \geq 2$, of mutually exclusive sides on platform q, at a non-empty PE, $\mathbf{N}^*(\mathbf{p})$, the vector of optimal platform prices is characterized by the Lerner index (8), adjusted downward or upward by $\sum_{j \in \mathcal{D} \setminus \{i\}} (p_j^q - c_j^q)\lambda_{ij}^q$.*

Compared to the previous literature, the characterization of optimal prices in Corollary 1 differs in two important aspects. First, all previous authors show that optimal prices are adjusted downward (absent congestion) after a price increase, resulting in a markup lower than the usual Lerner index (e.g., Armstrong, 2006 and Rochet and Tirole, 2006). This is due to the fact that an increase in p_i^q has an opportunity cost for the platform in terms of revenue lost on side j. To the contrary, we show that for some parameters, the impact of an increase in p_i^q on side-j participation is positive, which results in optimal platform markups higher than the usual Lerner index (even with positive cross-group externalities). Second, in the latter case, we find that it is never optimal for the platform to cross-subsidize—a result that also stands in stark contrast with the conclusions of previous authors.[23]

4.3.3 Single-homing platforms

Parting again from the baseline case, in which end-users can join multiple sides within a platform, we now assume that a subset of platforms $\mathcal{M}' \subseteq \mathcal{M}$ are single-homing. This implies that each end-user faces a choice between joining side i either on platform q or r, for all $q, r \in \mathcal{M}'$, $q \neq r$, and for all $i \in \mathcal{D}$. Thus, each end-user joins side i on the platform that yields the highest utility (if any) through ICCs. This introduces an additional effect that we call *switching-platform effect*: an increase in p_i^q induces some end-users on side i on platform q, who are close-to-indifferent between joining side i on platform q or r, to switch from q to r. We now show formally the existence of this effect.

Proposition 4. *Consider that a subset of platforms $\mathcal{M}' \subseteq \mathcal{M}$, $|\mathcal{M}'| \geq 2$, are single-homing. Given a non-empty PE, $\mathbf{N}^*(\mathbf{p})$, suppose that platform $q \in \mathcal{M}'$ changes prices from $\mathbf{p}^q$ to $\tilde{\mathbf{p}}^q$, where $\tilde{p}_j^q = p_j^q$ for all $j \in \mathcal{D} \setminus \{i\}$, and $\tilde{p}_i^q = p_i^q + \delta$ for some $i \in \mathcal{D}$ and $\delta > 0$. Then, on side i, some end-users switch from platform q to all other single-homing platforms $r \in \mathcal{M}' \setminus \{q\}$, while no end-user changes from any platform r to q.*

The proof of this proposition follows closely that of Proposition 3 but employs condition ICC_i^{qr}

[23] A notable exception is Bakos and Halaburda (2020), who show under several additional assumptions that if there are multi-homing end-users on all sides, it is never optimal for the platform to cross-subsidize, as the interdependence between the different sides—captured by the network effect—falls apart. In our model, the transmission channel is slightly different: when the interdependence of sides (through the network effect) becomes less important *vis-à-vis* our novel switching-side effect, then cross-subsidization does not arise.

instead of ICC_{ij}^{q}. Importantly, the switching-platform effect identified in Proposition 4 occurs regardless of whether there are positive cross-group externalities or congestion. Hence, this effect further decreases the number of end-users on platform q's side i following an increase in p_i^q relative to the baseline case. In both cases, Lemma 1 applies regarding the sign of the own- and cross-price derivatives. However, these derivatives are "larger" in the single-homing case because the switching-platform effect amplifies the negative impact of an increase in p_i^q on platform q's participation. In any case, platform q's profit maximization problem is similar to the one in Section 4.3.1. Thus, the Lerner index charged to end-users on one side is dragged down by the markup charged to end-users on the other sides under positive cross-group externalities,[24] and the Lerner index increases in the case of congestion. Table 1 summarizes and compares all the effects that occur when platforms q and r are multi-homing, relative to the case in which they are single-homing, assuming that sides i and j are non-mutually exclusive.

Table 1: Effects of an increase in p_i^q on end-user participation for non-mutually exclusive sides i and j

	Platforms q and r are multi-homing		**Platforms q and r are single-homing**	
	Effect	**Sign**	**Effect**	**Sign**
Side N_i^q	Price effect Network effect	– – (+)	Price effect Network effect Switching-platform effect	– – (+) –
Side N_j^q	Network effect	– (+)	Network effect	– (+)
Side N_i^r	$\emptyset$		Switching-platform effect	+

Note: The signs of the different effects indicated are those under positive cross-group externalities, while the signs in parenthesis are those under congestion (indicated only for the cases in which they differ).

4.3.4 Mutually exclusive sides and single-homing platforms

Finally, we consider that a subset of platforms $\mathcal{M}' \subseteq \mathcal{M}$ are single-homing, and a subset of sides $\mathcal{D}' \subseteq \mathcal{D}$ are mutually exclusive. Therefore, each end-user is restricted to joining one of the mutually exclusive sides (say, i or j) on one of the single-homing platforms (say, q or r). In this setup, the

[24] Due to the negative impact on the markup charged, platforms might have the incentive to segment the market in order to limit the switching-platform effect (Karle et al., 2020).

switching-side and the switching-platform effects derived above occur. In addition, there exists an end-user who is close-to-indifferent between joining side i on platform q or side j on platform r—e.g., a car owner might be nearly indifferent between renting her car out on Uber or providing rides on Bolt. Thus, through ICC_{ij}^{qr}, an increase in p_i^q induces some end-users to switch from platform q's side i to platform r's side j. We call this last and novel effect *switching-side-and-platform effect*, stated formally in Proposition 5, the proof of which is analogous to that of Proposition 3 but uses ICC_{ij}^{qr} instead of ICC_{ij}^{q}.

Proposition 5. *Consider that a subset of sides $\mathcal{D}' \subseteq \mathcal{D}$, $|\mathcal{D}'| \geq 2$, are mutually exclusive and that a subset of platforms $\mathcal{M}' \subseteq \mathcal{M}$, $|\mathcal{M}'| \geq 2$, are single-homing. Given a non-empty PE, $\mathbf{N}^*(\mathbf{p})$, suppose that platform $q \in \mathcal{M}'$ changes prices from $\mathbf{p}^q$ to $\tilde{\mathbf{p}}^q$, where $\tilde{p}_k^q = p_k^q$ for all $k \in \mathcal{D} \setminus \{i\}$, and $\tilde{p}_i^q = p_i^q + \delta$ for some $i \in \mathcal{D}'$ and $\delta > 0$. Then some end-users switch from side i on q to all other mutually exclusive sides $j \in \mathcal{D}' \setminus \{i\}$ on all other single-homing platforms $r \in \mathcal{M}' \setminus \{q\}$, while no end-user changes to platform q's side i.*

If platforms q and r are multi-homing, participation on side i decreases following an increase in p_i^q, as we show in Section 4.3.2. When these platforms are single-homing, this negative impact on participation on side i is "larger" due to two additional effects: the switching-platform and the switching-side-and-platform effects. Consequently, the own-price elasticity is negative. By contrast, the effect of an increase in p_i^q on participation on another mutually exclusive side j on platform q remains ambiguous. When q and r are multi-homing, this ambiguity is driven by two opposite effects, namely, the switching-side effect—which increases participation on side j—and the network effect, which decreases it absent congestion. If these platforms are single-homing, both the switching-platform and switching-side-and-platform effects unambiguously increase participation on all mutually exclusive sides of all single-homing platforms except q. These effects are summarized in Table 2, which provides a comparison between the case in which platforms q and r are multi-homing and single-homing, respectively, provided that sides i and j are mutually exclusive.

As in Section 4.3.2, the platform's optimal markup on each side is larger or smaller than the Lerner index depending on whether there is congestion or not and, if not, on whether the switching-side effect dominates the network effect. The role of the switching-platform and switching-side-and-platform effects on prices will become more relevant, though, in the following section.

Table 2: Effects of an increase in p_i^q on end-user participation for mutually exclusive sides i and j

	Platforms q and r are multi-homing		**Platforms q and r are single-homing**	
	Effect	**Sign**	**Effect**	**Sign**
Side N_i^q	Price effect	–	Price effect	–
	Network effect	– (+)	Network effect	– (+)
	Switching-side effect	–	Switching-side effect	–
			Switching-platform effect	–
			Switching-side-and-platform effect	–
Side N_j^q	Network effect	– (+)	Network effect	– (+)
	Switching-side effect	+	Switching-side effect	+
Side N_i^r	$\emptyset$		Switching-platform effect	+
Side N_j^r	$\emptyset$		Switching-side-and-platform effect	+

Note: The signs of the different effects indicated are those under positive cross-group externalities, while the signs in parenthesis are those under congestion (indicated only for the cases in which they differ).

5 Platform merger analysis

We rely on the results derived above to study the impact of platform mergers on markups. Following the analysis by Farrell and Shapiro (2010), we quantify the UPP resulting from a merger of platforms based on diversion ratios which measure the value of diverted sales that the merged entity recaptures following a change in prices. The importance of diversion ratios to understand the consequences of mergers has gained momentum among practitioners in competition and antitrust; e.g., the 2023 merger guidelines of the US Federal Trade Commission (FTC and DOJ, 2023) use diversion ratios as key statistics to measure unilateral price effects of mergers. Diversion ratios are not only acknowledged as a powerful tool to analyze mergers, particularly in the context of non-homogeneous goods, but also as a practical one, as they can be estimated using market, scanner, or survey data—see Affeldt et al. (2013) and Conlon and Mortimer (2021). Given the deeper insights that our theoretical analysis provides into the direction of diversion ratios (based on the different effects we identify formally), we provide a general and unifying framework to predict the UPP of platform mergers in multi-sided markets. Our framework explains consistently the mixed results

obtained previously by authors in more specific contexts.[25]

Theoretically, Chandra and Collard-Wexler (2009) study the merger of two two-sided platforms that compete à la Hotelling. These authors show intuitively that the merging platforms internalize the effect of end-users who switch across platforms (on the same side, as in Section 4.3.3). Correia-da Silva et al. (2019) assume instead that two merging platforms compete à la Cournot. Both studies conclude that the merged entity increases or decreases prices depending solely on whether the platforms cross-subsidize before the merger or not. We not only identify formally the conditions under which cross-subsidization occurs but also derive the impact on post-merger prices based on the relative strength of the network effect *vis-à-vis* the switching-side effect that is captured by diversion ratios.

Using data on TV magazines, Song (2021) finds empirically that post-merger prices might either go up or down, with prices on different sides usually moving in opposite directions. Similarly, using data on US daily newspapers, Fan (2013) shows that ad rates and subscription prices post-merger tend to move in opposite directions.[26] Considering that an increase in subscription fees (in ad rates) by magazines and newspapers is implausible to induce a reader to *switch side* and become an advertiser (or vice versa), and bearing in mind that cross-group externalities are positive on the advertising side, our model rationalizes these empirical results. Indeed, our model predicts that when the network effect across different sides on the same platform dominates the switching-side effect, as is the case in the examples under consideration, post-merger prices on both sides do move in opposite directions.[27]

5.1 General case

We consider the same theoretical framework as in Section 2 but additionally assume that a subset of platforms, denoted by $\mathcal{N} \subseteq \mathcal{M}$, merge, where $\mathcal{N} = \{1, ..., n\}$ with $2 \leq n \leq m$ (as long as $m \geq 2$).

[25] We abstract from certain features of platform mergers in particular industries, such as political motives in the media industry (Anderson and McLaren, 2012), "time paying attention" by end-users in digital outlets and social networks (Prat and Valletti, 2022), or content variety in the radio industry (Sweeting, 2010).

[26] Jeziorski (2014), who studies mergers in the radio industry, documents a similar trade-off that the merged firms face between exercising market power on one side of the market (advertisers) or the other (listeners).

[27] As we show later, if the switching-side effect dominates the network effect or if there are congestion effects on both sides of a platform, then prices on both sides will unambiguously increase after the merger.

Then, the profit maximization problem for the merged firms is

$$\max_{\{p_1^1,\cdots,p_d^n\}} \quad \sum_{s\in\mathcal{N}} \pi^s = \sum_{s\in\mathcal{N}}\sum_{i\in\mathcal{D}} (p_i^s - c_i^s)N_i^s.$$

In the most general case, i.e., considering all potential across and within platform effects that may exist, the optimal price that the merged entity charges to end-users on side i of platform s at an interior solution is characterized by the following FOC:

$$\frac{(p_i^s - c_i^s)}{p_i^s} = \frac{1}{|\varepsilon_i^s|} + \sum_{j\in\mathcal{D}\setminus\{i\}} \lambda_{ij}^s \frac{(p_j^s - c_j^s)}{p_i^s} + \underbrace{\sum_{t\in\mathcal{N}\setminus\{s\}}\sum_{j\in\mathcal{D}} \lambda_{ij}^{st}\frac{(p_j^t - c_j^t)}{p_i^s}}_{\text{across-platform diversion effects}}, \tag{11}$$

where λ_{ij}^s is the diversion ratio (within platform s) defined above and $\lambda_{ij}^{st} := \frac{\partial N_j^t}{\partial p_i^s} / \left|\frac{\partial N_i^s}{\partial p_i^s}\right|$ is the diversion ratio between side j on platform t and side i on platform s for all $s,t \in \mathcal{N}$, $s \neq t$, and where the derivatives are evaluated at the optimal solution. Note that the right-hand side (RHS) of (11) adds an additional term relative to the RHS of (8), which characterizes optimal pricing before the merger. This last term in (11) appears since the merged entity internalizes the different across-platform effects which now determine the UPP of the merger.

As is customary in the literature (Affeldt et al., 2013; Miller et al., 2017; Conlon and Mortimer, 2021), we characterize the UPP on the optimal markup for each side and platform after the merger holding the prices of all sides and platforms on the RHS of (11) fixed at the optimal pre-merger level from (8). To do so, it is convenient to present a "stacked" version of (11) for all platforms that merge. For that purpose, let us denote the matrices of the *within* and *across* (pre-merger) platform diversion ratios, respectively, by

$$\Lambda^s = \begin{pmatrix} 1 & -\lambda_{12}^s & \dots & -\lambda_{1d}^s \\ -\lambda_{21}^s & 1 & \dots & -\lambda_{2d}^s \\ \vdots & & \ddots & \vdots \\ -\lambda_{d1}^s & -\lambda_{d2}^s & \dots & 1 \end{pmatrix} \qquad \text{and} \qquad \Lambda^{st} = \begin{pmatrix} -\lambda_{11}^{st} & -\lambda_{12}^{st} & \dots & -\lambda_{1d}^{st} \\ -\lambda_{21}^{st} & -\lambda_{22}^{st} & \dots & -\lambda_{2d}^{st} \\ \vdots & & \ddots & \vdots \\ -\lambda_{d1}^{st} & -\lambda_{d2}^{st} & \dots & -\lambda_{dd}^{st} \end{pmatrix},$$

and let the corresponding matrices of (pre-merger) relative optimal prices be

$$\Theta^s = \begin{pmatrix} 1 & \frac{p_2^s}{p_1^s} & \cdots & \frac{p_d^s}{p_1^s} \\ \frac{p_1^s}{p_2^s} & 1 & \cdots & \frac{p_d^s}{p_2^s} \\ \vdots & & \ddots & \vdots \\ \frac{p_1^s}{p_d^s} & \frac{p_2^s}{p_d^s} & \cdots & 1 \end{pmatrix} \qquad \text{and} \qquad \Theta^{st} = \begin{pmatrix} \frac{p_1^t}{p_1^s} & \frac{p_2^t}{p_1^s} & \cdots & \frac{p_d^t}{p_1^s} \\ \frac{p_1^t}{p_2^s} & \frac{p_2^t}{p_2^s} & \cdots & \frac{p_d^t}{p_2^s} \\ \vdots & & \ddots & \vdots \\ \frac{p_1^t}{p_d^s} & \frac{p_2^t}{p_d^s} & \cdots & \frac{p_d^t}{p_d^s} \end{pmatrix},$$

for all $s, t \in \mathcal{N}$, $s \neq t$.

Then, optimal pricing for the merged platforms is given by the following (block) matrix equation:

$$\underbrace{\left(\begin{array}{c|c|c|c} \Lambda^1 \odot \Theta^1 & \Lambda^{12} \odot \Theta^{12} & \cdots & \Lambda^{1n} \odot \Theta^{1n} \\ \hline \Lambda^{21} \odot \Theta^{21} & \Lambda^2 \odot \Theta^2 & \cdots & \Lambda^{2n} \odot \Theta^{2n} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline \Lambda^{n1} \odot \Theta^{n1} & \Lambda^{n2} \odot \Theta^{n2} & \cdots & \Lambda^n \odot \Theta^n \end{array}\right)}_{\mathcal{H}} \begin{pmatrix} \boldsymbol{\mu}^1 \\ \boldsymbol{\mu}^2 \\ \vdots \\ \boldsymbol{\mu}^n \end{pmatrix} = \begin{pmatrix} \mathcal{E}^1 \\ \mathcal{E}^2 \\ \vdots \\ \mathcal{E}^n \end{pmatrix},$$

where $\odot$ denotes the Hadamard product of two matrices; ${}^T\boldsymbol{\mu}^s := \left(\frac{p_1^s - c_1^s}{p_1^s}, \cdots, \frac{p_d^s - c_d^s}{p_d^s}\right) \in \mathbb{R}^d$ is the vector of optimal markups for each merging platform s that result from incorporating the predicted post-merger UPP; and $\mathcal{E}^s$ is the vector of pre-merger inverse own-price elasticities on platform s, where ${}^T\mathcal{E}^s := \left(\frac{1}{|\varepsilon_1^s|}, \cdots, \frac{1}{|\varepsilon_d^s|}\right) \in \mathbb{R}^d_+$. Therefore, the UPP impact of the merger of platforms on their markups depends on the diversion ratios across platforms, i.e., on Λ^{st} for all $s, t \in \mathcal{N}$, $s \neq t$.[28] We now analyze the impact of the merger for each of the possible cases considered in Section 3.

Multi-homing platforms. First, we consider the case in which all platforms are multi-homing. Then, there are no switching-platform effects, and consequently, across-platform diversion ratios are zero (regardless of whether sides are mutually exclusive or not). Therefore, the merger does not affect markups.

Proposition 6. *If all merged platforms are multi-homing, then* $\mathcal{H} = \bigoplus_{s \in \mathcal{N}} (\Lambda^s \odot \Theta^s)$*, all off-diagonal block matrices in* $\mathcal{H}$ *are null, and the merger does not change the markups of platforms relative to those in the pre-merger case.*

Under multi-homing, there is no UPP post-merger. This result follows from the *independent*

[28] Following Song (2021), we ignore any efficiency gains that arise from the merger, for example, from an increase in productivity (Braguinsky et al., 2015) or innovation (Cabral, 2021). However, we can accommodate efficiency gains along the lines of Affeldt et al. (2013).

nature of platforms (Belleflamme and Peitz, 2019b). To illustrate this, consider e-commerce platforms (e.g., Taobao, Facebook Marketplace, or eBay), on which end-users may sell and buy goods. Suppose that an end-user sells an item through several e-commerce platforms. Then, an increase in the selling fee charged by one of the platforms might induce the end-user to drop it. However, this does not induce the end-user to drop the other platforms. In other words, changes in prices do not trigger a switching of end-users across platforms, and hence, there are no across-platform diversion effects to be internalized by the merged entity.[29] Therefore, if some of the platforms merge, the optimal prices are identical to those charged by the platforms separately. Consistently, Farronato et al. (2023) find empirically that fees did not increase in the context of a multi-homing platform merger between two dog boarding/walking companies (Rover and DogVacay).

Single-homing platforms. We now consider that a subset of the merging platforms, denoted by $\mathcal{N}' \subseteq \mathcal{N}$, with $|\mathcal{N}'| \geq 2$, are single-homing. In this case, an end-user must choose whether she joins one platform or another for each of their sides. This exclusivity that single-homing imposes on end-users triggers different across-platform effects, which are internalized when they merge. As a consequence, optimal markups before and after the merger generically differ.

Suppose first that the sides of all platforms are non-mutually exclusive. In this scenario, there are only switching-platform effects across the merging single-homing platforms, and the following properties regarding matrix $\mathcal{H}$ follow immediately.

Lemma 2. *Consider that a subset of the merged platforms, $\mathcal{N}' \subseteq \mathcal{N}$, are single-homing, on which all sides are non-mutually exclusive. Then, all off-diagonal block matrices of $\mathcal{H}$ are non-positive. In particular, they are equal to $\Lambda^{st} \odot \Theta^{st} = diag\big(-\lambda_{ii}^{st}\frac{p_i^t}{p_i^s}\big)$ for all $i \in \mathcal{D}$ and all $s,t \in \mathcal{N}'$, $s \neq t$, and equal to the null matrix for all the other cases.*

Since switching-side-and-platform effects are null under non-mutually exclusive sides, all the off-diagonal elements in Λ^{st} are zero, i.e., $\lambda_{ij}^{st} = 0$ for all $i \neq j$. Thus, the post-merger UPP on side i of single-homing platform s is determined solely by the diversion ratios that capture the switching-platform effects λ_{ii}^{st} and their interaction with the markups charged by the other single-homing platforms $t \neq s$ on side i. That is, the last element on the RHS of (11) boils down to

[29] We implicitly assume that end-users have no time or budget constraints. Otherwise, price changes may induce substitution effects, and end-users may switch platforms. In this case, the single-homing framework—studied below—would apply.

$\sum_{t\in\mathcal{N}'\setminus\{s\}} \lambda_{ii}^{st}\frac{(p_i^t-c_i^t)}{p_i^s}$, which (evaluated at the optimal pre-merger prices) determines the UPP on side i of platform s. Bearing in mind that λ_{ii}^{st} is unambiguously positive for all $i \in \mathcal{D}$ and for all $s, t \in \mathcal{N}'$, $s \neq t$,[30] it follows that the post-merger markup on side i of platform s can go up or down. The direction of this change depends on the sign of $(p_i^t - c_i^t)$ for all $t \in \mathcal{N}' \setminus \{s\}$, i.e., on whether the other merging single-homing platforms charge a pre-merger price above or below marginal cost to side-i end-users. While previous studies recognize the importance of cross-subsidization in predicting post-merger prices (Chandra and Collard-Wexler, 2009), our model provides formal conditions under which cross-subsidization takes place. Moreover, with the pricing conditions we derive we can quantify the strength of post-merger price changes based on diversion ratios and switching-platform effects, which are multiplied by the corresponding markup.

In particular, if cross-group externalities are negative, then the network effect is positive, and as a consequence, optimal pre-merger prices are always above marginal cost, i.e., $(p_i^t - c_i^t) > 0$ for all $t \in \mathcal{N}'$. Hence, the merger unambiguously increases prices on all sides for all single-homing platforms. However, if cross-group externalities are positive, optimal pre-merger markups are adjusted downward, and cross-subsidization may take place; i.e., $(p_i^t - c_i^t)$ may be negative for some $t \in \mathcal{N}'$. Thus, the UPP of the merger is (ex-ante) indeterminate. For example, if end-users on a side of a single-homing platform are cross-subsidized before the merger, then the price on this side further decreases post-merger. This also implies that this platform charges a positive pre-merger markup to end-users on another side, who experience an increase in the markup after the merger. By contrast, if the platform charges positive markups on all sides before the merger, then post-merger prices unambiguously increase.

Proposition 7. *Consider that a subset of the merged platforms, $\mathcal{N}' \subseteq \mathcal{N}$, are single-homing on which all sides are non-mutually exclusive. Under congestion effects, the merger increases markups on all sides of the platforms in $\mathcal{N}'$. If cross-group externalities are positive and $\sum_{t\in\mathcal{N}'\setminus\{s\}} \lambda_{ii}^{st}\frac{(p_i^t-c_i^t)}{p_i^s} > 0$ (or conversely ≤ 0), the merger increases (conversely weakly decreases) the markup on side $i \in \mathcal{D}$ of platform $s \in \mathcal{N}'$, $s \neq t$.*

To illustrate this result, consider the merger of two single-homing platforms A and B, each with two sides. If pre-merger prices are above marginal cost on both sides of both platforms, then the

[30] This holds regardless of the sign of the cross-group externalities. As shown in Section 4.3.3, the switching-platform effect is unambiguously positive under both positive cross-group externalities and congestion effects.

merger unambiguously raises prices. However, this is not true if the platforms cross-subsidize end-users on one side (which requires the cross-group externalities to be positive), say, on side 1. In this case, the merger further decreases markups on side 1 for both platforms, while the merger raises markups on side 2 for both platforms. This result is in line with Song (2021), who shows empirically that for the newspaper industry, post-merger ad prices tend to move in the opposite direction of post-merger copy prices (consistent with the fact that readers are usually cross-subsidized and single-home). However, prices sometimes increase on both sides after the merger, which is the case if newspapers are able to charge positive markups to advertisers and readers.

Finally, we consider that on the merging single-homing platforms in $\mathcal{N}'$, a subset of sides, denoted by $\mathcal{D}' \subseteq \mathcal{D}$, with $|\mathcal{D}'| \geq 2$, are mutually exclusive. This case applies, for example, to apps such as Uber or Bolt in jurisdictions in which riders are restricted to working for a single platform. Unlike in the previous case, some of the off-diagonal elements in matrix Λ^{st} are now strictly negative due to the switching-side-and-platform effect identified in Section 4.3.4.

Lemma 3. *Consider that a subset of the merged platforms, $\mathcal{N}' \subseteq \mathcal{N}$, are single-homing on which a subset of sides, $\mathcal{D}' \subseteq \mathcal{D}$, are mutually exclusive. Then, all off-diagonal block matrices of $\mathcal{H}$ are non-positive. In particular, $\Lambda^{st} \odot \Theta^{st}$ is a non-diagonal, non-positive matrix for all $s, t \in \mathcal{N}'$, $s \neq t$, and it is equal to the null matrix for all the other cases.*

Based on Lemma 3, we unveil a surprising result. Namely, that the UPP on the markup of a merging single-homing platform $s \in \mathcal{N}'$ depends on the strength of the switching-side effects on the other merging platforms $t \in \mathcal{N}' \setminus \{s\}$, and not only on all the across-platform effects and the nature of the cross-group externalities (as in the previous case). Before exploring this result, it is important to recall that the switching-platform effect between platforms s and t occurs for all $i \in \mathcal{D}$ due to the single-homing nature of the platforms (e.g., a rider choosing between Uber and Bolt). However, the switching-side-and-platform effect between side i on platform s and side j on platform t occurs exclusively across the subset of mutually exclusive sides $i, j \in \mathcal{D}', i \neq j$ (e.g., a car owner providing rides with Bolt or renting out her car on Uber). Propositions 4 and 5 confirm that both these effects, i.e., the switching-platform effects (captured by λ_{ii}^{st} for all $i \in \mathcal{D}$) and the switching-side-and-platform effects (captured by λ_{ij}^{st} for all $i, j \in \mathcal{D}'$, $i \neq j$), are positive for all $s, t \in \mathcal{N}'$, $s \neq t$.

Equipped with these insights, we now explain how the markup charged by merging platform $s \in \mathcal{N}'$ on mutually exclusive side i evolves. If cross-group externalities are negative (congestion), both the switching-side effect between any pair of mutually exclusive sides, such as i and j, and the network effect go in the same direction, and hence, the diversion ratios across these sides on platforms $t \in \mathcal{N}' \setminus \{s\}$ (captured by λ_{ij}^t) are unambiguously positive. In such instances, optimal pre-merger prices on platform $t \in \mathcal{N}' \setminus \{s\}$ always exceed marginal cost. This also occurs if cross-group externalities are positive and the switching-side effect between all pairs of mutually exclusive sides dominates the network effect on platform $t \in \mathcal{N}' \setminus \{s\}$. Consequently, in both cases, the merger unambiguously increases markups on all mutually exclusive sides of platform s. Conversely, when cross-group externalities are positive and the network effect dominates the switching-side effect for a pair of mutually exclusive sides, such as i and j, on platform $t \in \mathcal{N}' \setminus \{s\}$, λ_{ij}^t turns negative. Then, platform t might find it optimal to cross-subsidize (albeit not necessarily) some end-users. In this case, the UPP of the merger on platform s is ambiguous and depends on the model's parameters.

Proposition 8. *Consider that a subset of the merged platforms, $\mathcal{N}' \subseteq \mathcal{N}$, are single-homing on which a subset of sides, $\mathcal{D}' \subseteq \mathcal{D}$, are mutually exclusive. If $\lambda_{ij}^t \geq 0$ for all $i, j \in \mathcal{D}$, $i \neq j$, then the merger increases markups on all mutually exclusive sides of platform $s \in \mathcal{N}'$, $s \neq t$. Otherwise, if $\sum_{t \in \mathcal{N}' \setminus \{s\}} \left(\lambda_{ii}^{st} \frac{(p_i^t - c_i^t)}{p_i^s} + \sum_{j \in \mathcal{D}' \setminus \{i\}} \lambda_{ij}^{st} \frac{(p_j^t - c_j^t)}{p_i^s} \right) > 0$ (or conversely ≤ 0), then the merger increases (conversely weakly decreases) the markup on side $i \in \mathcal{D}'$ of platform $s \in \mathcal{N}'$, $s \neq t$.*

Intuitively, Proposition 8 documents that the effects on platform t—specifically, the strength of the switching-side effect relative to the network effect—determine whether the markups on platform s increase or decrease post-merger. The across-platform effects between platforms s and t only further reinforce this (positive or negative) impact on platform s' markups.

By combining these findings with the optimal platform pricing results presented in Section 4, we can derive policy-relevant implications of platform mergers for welfare—defined as the sum of end-user surplus (given by the valuation of those who join any side on any platform, net of the network effect and price paid) and the profits of platforms. These implications are relevant for mergers involving single-homing platforms which, as shown above, internalize the different across-

platform effects leading to post-merger price changes.[31] Therefore, the overall impact on welfare hinges on *(i)* the magnitude of the across-platform switching effects (how many end-users switch platforms) which increase the profits of merged platforms at the expense of end-user surplus, and on *(ii)* the magnitude of the own-price effect (how many end-users drop/join the platforms) which affects end-user participation and, therefore, surplus depending on the sign of the markups charged by the platforms.

First, if there are congestion effects or if there are positive cross-group externalities and the switching-side effect dominates the network effect, cross-subsidization does not occur. Consequently, there is a strictly positive UPP post-merger and platforms obtain an additional profit (via the internalized across-platform switching effects) at the expense of end-user surplus. However, the net impact on welfare is negative since some end-users drop some side on some platform due to higher post-merger prices.[32] Conversely, when there are positive cross-group externalities and the network effect dominates the switching-side effect, cross-subsidization on a side may occur. In such instances, the merger leads to lower prices on the cross-subsidized sides—thereby increasing participation—while the prices on other sides rise, causing decreased participation, resulting in an ambiguous effect on end-user surplus (Song, 2021). Consequently and since the post-merger profits of platforms increase (otherwise, they would not change prices), the impact on welfare is (ex-ante) ambiguous in this case.[33]

In summary, our general framework accommodates the analysis of mergers involving multi-sided platforms, regardless of whether some of their sides are mutually exclusive or not. The presence of non-zero diversion ratios in Λ^{st} plays a pivotal role in shaping post-merger prices and welfare—in particular, if platforms are single-homing (diagonal elements) and sides are mutually exclusive (off-diagonal elements). In the following subsection, we show how this framework also provides a useful workhorse for analyzing the merger of platforms that offer "unrelated" services but that are connected through some sides.

[31] These implications are also relevant for multi-homing platforms in which end-users effectively single-home (due to budget or time constraints). In this case, price changes may also induce across-platform effects, and consequently, it would be more appropriate to apply the single-homing framework.

[32] This result is consistent with the literature on platform mergers involving substitutes, such as Nevo (2000), which is a framework that closely resembles the cases discussed here.

[33] Our analysis abstracts from other issues, such as regressive effects arising when one side of the market is "excessively subsidized" at the expense of others (Sarin, 2020) and the potential for platforms to implement "own-rules" that enhance welfare (Johnson et al., 2023) that lie beyond the scope of this paper.

5.2 Special case: merger of "unrelated" platforms

For now, we have referred mainly to previous studies of mergers among multi-sided platforms within the same industry (e.g., newspapers, radio stations) or to platforms that offer similar services (e.g., dog walking). However, our main results above highlight that the relevant unit of analysis is not a platform but a side. Consequently, we finally show that our framework also provides a useful toolkit to analyze mergers of platforms that offer different services (potentially in different industries)—usually called *conglomerate mergers*—but in which there are across-platform effects between some sides of the merging platforms.[34]

For example, this is the case if a peer-to-peer ride-sharing platform merges with another platform that offers food delivery services. These platforms not only offer unrelated services but also belong to very different sectors. However, they are connected through one of their sides if food couriers deliver orders from restaurants to customers by car: in this case, the courier might choose either to deliver food or to drive passengers (i.e., there is a switching-side-and-platform effect between the two platforms on the drivers'/couriers' side). A case in point is provided by Ola Cabs' acquisition of the food delivery platform Foodpanda—whose drivers often deliver food by car—in December 2017.

Another example is the acquisition of HotelTonight—a platform that connects travelers and hotels for very last-minute bookings—by Airbnb in March 2019. Again, although these platforms offer different services (vacation rentals vs. hotel rooms), travelers who search for apartments using Airbnb might switch and check for hotels at HotelTonight's website (in particular, if Airbnb users are last-minute travelers). Hence, there is again a switching-platform effect that the acquirer firm internalizes. A similar internalization of across-platform effects can be expected in other mergers of platforms, such as that of Lyft and Motivate (a US bicycle sharing platform) and that of Uber and JUMP (a scooter sharing system), as users might choose either to receive a ride by car or to complete their trips by finding a nearby bike or scooter to use. In all these cases, whether the merger has harmful effects depends on whether end-users behave as single-homers or multi-homers before the merger as it follows from the above results.

[34] Even though conglomerate mergers are becoming increasingly popular in (digital) platform markets, thus far previous studies discuss only the potentially harmful effects of conglomerate mergers in other (standard) markets (Garcia and de Azevedo, 2019).

Formally, in all these examples, end-users on some sides, such as $i, j \in \mathcal{D}$, on each platform are unrelated, which implies that $\frac{\partial N_i^s}{\partial p_j^t} = 0$ for all $s, t \in \mathcal{N}$, $s \neq t$ (e.g., riders are not directly affected by a change in the fee charged to those who order food, and those who order food are not affected by a change in the fee charged for a ride), which in turn implies that $\lambda_{ij}^{st} = 0$. However, the cross-price derivative will be different from 0 for end-users on some other sides, such as $k, l \in \mathcal{D}$, through which platforms s and t are related (e.g., Uber riders might choose to drive a JUMP scooter instead of booking an Uber service if fees charged by the latter service are relatively high). In these cases, $\lambda_{kl}^{st} > 0$, and consequently, $\mathcal{H}$ is a diagonal matrix in which some of the diagonal elements are equal to 0 (for the unrelated sides), while others are strictly lower than 0 (for the related sides).

Suppose that platforms s and t merge, which are unrelated except through sides k and l. Then, the optimal price charged by merging platform s on side k is characterized by[35]

$$\frac{(p_k^s - c_k^s)}{p_k^s} = \frac{1}{|\varepsilon_k^s|} + \sum_{i \in \mathcal{D} \setminus \{k\}} \lambda_{ki}^s \frac{(p_i^s - c_i^s)}{p_k^s} + \underbrace{\lambda_{kl}^{st} \frac{(p_l^t - c_l^t)}{p_k^s}}_{\text{across-platform diversion effects}} . \tag{12}$$

Therefore, if the price-cost markup charged by platform t on side l is strictly positive (i.e., in the absence of cross-subsidization), the merger is beneficial for the merging platforms (to the detriment of end-users), as it increases side k's markup by the across-platform diversion effects indicated in (12). This occurs because these platforms internalize the switching-side-and-platform effect on the sides through which they are related (i.e., sides k and l in the example).[36] This result holds even though platforms s and t offer different services, and therefore, its consequences might potentially receive less attention from anti-trust authorities—e.g., in countries that investigate mergers only if the market share of the combined entity is above a certain threshold (Motta and Peitz, 2021; Nocke and Whinston, 2022). Indeed, antitrust and competition authorities did not actively investigate as potentially harmful some of the mergers and acquisitions mentioned above, such as that of Foodpanda and Ola Cabs or that of HotelTonight and Airbnb.

[35] This result easily generalizes to more than two related sides and platforms. In this case, the post-merger effect in (12) should include the sum for all other platforms t whose sides relate to those on platform s.

[36] Argentesi et al. (2021) note that most acquisitions by Amazon, Facebook, or Google target companies offering a wide range of products and services that are complementary to those supplied (i.e., those facilitating the switching effects).

6 Conclusions

This paper presents a novel general model of multi-sided markets in which heterogeneous end-users, whose valuations are drawn from a general density function, endogenously choose both which side and which platform they join. These decisions are considered with the assumption that participation on one side of a platform sometimes precludes participation on another side and that platforms can be either single- or multi-homing.

From the primitives of the model, we derive the demand of the different sides for various scenarios. For each scenario, we find a unique participation equilibrium and then derive the optimal prices of platforms, from which closed-form expressions for own- and cross-price elasticities arise. Using the closed-form expressions, we formally characterize a set of novel switching effects, by which an increase in the fee on one side of a platform induces some end-users to switch to different sides and/or platforms. Optimal prices differ mainly along the dimension of whether the sides of platforms are mutually exclusive or not.

Then, we analyze the consequences of multi-sided platform mergers. We show that for end-users, mergers of multi-homing platforms are innocuous, as they do not affect optimal pricing, while mergers of single-homing platforms do affect optimal pricing (and there is a nuanced difference depending on whether the platform sides are mutually exclusive or not). Thus, we provide a unifying framework to analyze platform mergers, which allows us to explain reliably the mixed results obtained by previous authors.

Both dimensions—single- vs. multi-homing and mutually exclusive sides or otherwise—are important in our framework. The first determines whether mergers between multi-sided platforms have harmful effects for end-users through post-merger price increases, and the second whether optimal markups of platforms can be larger than the Lerner index.

The tractability of the general theoretical model we provide will be useful for applied and future empirical research, in particular, to study antitrust issues. We believe, moreover, that the model will be an appealing and convenient starting point for authors interested in more specific applications that can be obtained by modifying our relatively general assumptions.

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Appendix A. Proofs

Proof of Proposition 1. Let $\mathbf{p}$ be given. Define set $N^{m+1} := \{v_i^q \mid pr\big(v_i^q + \phi_i^q(\mathbf{N}_{-i}^q) - p_i^q < 0\big)$ for all $i \in \mathcal{D}$ and all $q \in \mathcal{M}\}$, i.e., any end-user in set N^{m+1} joins no platform at all since her valuation net of price and cross-group externality is less than zero. Then, let $\tilde{N} := (\mathbf{N}^1, ..., \mathbf{N}^m, N^{m+1})$, where $\mathbf{N}^q := (N_1^q, ..., N_d^q)$ for all $q \in \mathcal{M}$, and define the mapping $\psi : \tilde{N} \to \tilde{N}$, such that $\psi : [0,1]^{m\times d+1} \to [0,1]^{m\times d+1}$. By Brouwer's fixed point theorem, ψ has a fixed point since it is a continuous function from a compact and convex set to itself. Denote such a fixed point by $\mathbf{N}^*(\mathbf{p})$. This shows that a PE, $\mathbf{N}^*(\mathbf{p})$, exists.

To show uniqueness of the PE,[37] suppose that Assumption 1 holds and note that the L^∞ norm on some vector $\mathbf{X}$ is defined as $\|X\|_\infty = \max_{i\in\mathcal{D},q\in\mathcal{M}} |x_i^q|$. Then, $\|\mathbf{N} - \mathbf{N}'\|_\infty$ denotes the infinity norm of

[37] Our proof of uniqueness follows a similar line of argument as Tan and Zhou (2021). Yet, they consider only single-homing and mutually exclusive (fixed) sides and also assume full coverage. For more details about the mathematical arguments we use, any advanced book on mathematical analysis is helpful, such as Dieudonné (1964).

the distance between vectors $\mathbf{N}$ and $\mathbf{N}'$. We show uniqueness by using the contraction mapping theorem, and thus, need to show that for all $\mathbf{N}, \mathbf{N}'$,

$$\|\tilde{N}(\mathbf{N}) - \tilde{N}(\mathbf{N}')\|_\infty \leq \beta \|\mathbf{N} - \mathbf{N}'\|_\infty. \tag{A.1}$$

Let

$$\beta := \max_{\substack{i\in\mathcal{D}\\ q\in\mathcal{M}}} \sup_{\mathbf{N}^q\in[0,1]^d} \sum_{j\in\mathcal{D}} \Big|\frac{\partial N_i^q(\cdot)}{\partial N_j^q}\Big| \times \max_{\substack{i\in\mathcal{D}\\ q\in\mathcal{M}}} \Big(\sum_{j\in\mathcal{D}\setminus\{i\}} \sup_{\mathbf{N}_{-i}^q\in[0,1]^{d-1}} \Big|\frac{\partial \phi_i^q(\mathbf{N}_{-i}^q)}{\partial N_j^q}\Big|\Big) < 1.$$

To show that (A.1) holds, we use the following result which is an immediate consequence of the mean value theorem: for any continuously differentiable function $g(a_1, ..., a_n)$ on a convex domain $E \subset \mathbf{R}^n$, and for any vectors $\mathbf{a} = (a_1, ..., a_n)$ and $\mathbf{b} = (b_1, ..., b_n)$,

$$|g(\mathbf{a}) - g(\mathbf{b})| \leq \Big(\max_i |a_i - b_i|\Big)\Big(\sup_{\mathbf{e}\in E} \sum_{j=1}^n \Big|\frac{\partial g(\mathbf{e})}{\partial a_j}\Big|\Big),$$

where $\max_i |a_i - b_i|$ is L^∞ norm of the vector $\mathbf{a} - \mathbf{b}$, and $\sup_{\mathbf{e}\in E} \sum_{j=1}^n \left|\frac{\partial g(\mathbf{e})}{\partial a_j}\right| \leq \sum_{j=1}^n \sup_{\mathbf{e}\in E} \left|\frac{\partial g(\mathbf{e})}{\partial a_j}\right|$, which is the supremum of the L^1 norm of the gradient ∇g over E.

Applying these results to any i and q and all $\mathbf{N}, \mathbf{N}'$, yields $|N_i^q(\phi_i^q(\mathbf{N}_{-i}^q)) - N_i^q(\phi_i^q(\mathbf{N}_{-i}^{q'}))| \leq$

$$\leq \max_{i\in\mathcal{D}} \Big|\phi_i^q(\mathbf{N}_{-i}^q) - \phi_i^q(\mathbf{N}_{-i}^{q'})\Big| \times \max_{i\in\mathcal{D}} \sup_{\mathbf{N}^q\in[0,1]^d} \sum_{j\in\mathcal{D}} \Big|\frac{\partial N_i^q(\cdot)}{\partial N_j^q}\Big| \leq$$

$$\max_{i\in\mathcal{D}} \Big(\sum_{j\in\mathcal{D}\setminus\{i\}} \sup_{\mathbf{N}_{-i}^q\in[0,1]^{d-1}} \Big|\frac{\partial \phi_i^q(\mathbf{N}_{-i}^q)}{\partial N_j^q}\Big|\Big) \|\mathbf{N}^q - \mathbf{N}^{q'}\|_\infty \Big(\max_{i\in\mathcal{D}} \sup_{\mathbf{N}^q\in[0,1]^d} \sum_{j\in\mathcal{D}} \Big|\frac{\partial N_i^q(\cdot)}{\partial N_j^q}\Big|\Big)$$

$$\leq \beta \|\mathbf{N} - \mathbf{N}'\|_\infty,$$

which holds for any $\mathbf{N}$, $\mathbf{N}'$, any $i \in \mathcal{D}$, and $q \in \mathcal{M}$. □

Proof of Proposition 2. Given $\mathbf{p}$, to show that there is a non-empty PE, $\mathbf{N}^*(\mathbf{p})$, consider first baseline case a). Suppose *ad absurdum* that $[\phi_i^q(\cdot) - p_i^q] < -1$ for some $i \in \mathcal{D}$ and $q \in \mathcal{M}$, i.e., PC_i^q is not fulfilled. Then even for an end-user with the highest valuation $v_i^q = 1$, $v_i^q + \phi_i^q(\cdot) - p_i^q < 0$, and she does not join side i on platform q. Thus, the PE is empty, a contradiction. Therefore, PC is necessary for the PE to be non-empty. Suppose next that $\mathbf{N}^*(\mathbf{p}) \neq \emptyset$, i.e., there is some end-user on each side i on each platform q. Since $v_i^q = 1$ is the highest valuation for any side on any

platform, this implies that $\left[\phi_i^q(\cdot) - p_i^q\right] \geq -1$ for all $i \in \mathcal{D}$ and all $q \in \mathcal{M}$, and so PC is sufficient for the PE to be non-empty.

Consider next case b) of multi-homing with sides $i, j \in \mathcal{D}' \subseteq \mathcal{D}$, $i \neq j$, being mutually exclusive. From case a), it follows that $\mathbf{N}^*(\mathbf{p}) = \emptyset$, when PC_i^q is not fulfilled. Suppose next *ad absurdum* that $[\phi_i^q(\cdot) - p_i^q] - [\phi_j^q(\cdot) - p_j^q] \notin [-1, 1]$ for $i, j \in \mathcal{D}'$, $i \neq j$ and $q \in \mathcal{M}$. If $[\phi_i^q(\cdot) - p_i^q] - [\phi_j^q(\cdot) - p_j^q] < -1$, then an end-user with $(v_i^q = 1, v_j^q = 0)$ does not join side i on q, and if $[\phi_i^q(\cdot) - p_i^q] - [\phi_j^q(\cdot) - p_j^q] > 1$, then an end-user with $(v_i^q = 0, v_j^q = 1)$ does not join side j on q. Both end-users exist by full support. Hence, the PE is empty, a contradiction. Therefore PC_i^q and ICC_{ij}^q are necessary. Suppose next that $\mathbf{N}^*(\mathbf{p}) \neq \emptyset$. Then it follows from case a) that $[\phi_i^q(\cdot) - p_i^q] \geq -1$ for all i and q and, by full support, the existence of the two end-users just found implies that $[\phi_i^q(\cdot) - p_i^q] - [\phi_j^q(\cdot) - p_j^q] \in [-1, 1]$ holds for $i, j \in \mathcal{D}'$, $i \neq j$, and $q \in \mathcal{M}$. Therefore, PC_i^q and ICC_{ij}^q are sufficient for the PE to be non-empty.

Consider next case c) of non-mutually exclusive sides with single-homing platforms $q, r \in \mathcal{M}' \subseteq \mathcal{M}$, $q \neq r$. From case a), it follows that $\mathbf{N}^*(\mathbf{p}) = \emptyset$, when PC_i^q is not fulfilled. Suppose next *ad absurdum* that $[\phi_i^q(\cdot) - p_i^q] - [\phi_i^r(\cdot) - p_i^r] \notin [-1, 1]$ for $i \in \mathcal{D}$ and for $q, r \in \mathcal{M}'$, $q \neq r$. If $[\phi_i^q(\cdot) - p_i^q] - [\phi_i^r(\cdot) - p_i^r] < -1$, then an end-user with $(v_i^q = 1, v_i^r = 0)$ does not join side i on q, and if $[\phi_i^q(\cdot) - p_i^q] - [\phi_i^r(\cdot) - p_i^r] > 1$, then an end-user with $(v_i^q = 0, v_i^r = 1)$ does not join side i on r. Both end-users exist by full support. Hence, the PE is empty, a contradiction. Therefore PC_i^q and ICC_i^{qr} are necessary. Suppose next that $\mathbf{N}^*(\mathbf{p}) \neq \emptyset$. Then it follows from case a) that $[\phi_i^q(\cdot) - p_i^q] \geq -1$ for all i and q and, by full support, the existence of the two end-users just found implies that $[\phi_i^q(\cdot) - p_i^q] - [\phi_i^r(\cdot) - p_i^r] \in [-1, 1]$ for $i \in \mathcal{D}$ and for $q, r \in \mathcal{M}'$, $q \neq r$. Therefore, PC_i^q and ICC_i^{qr} are sufficient for the PE to be non-empty.

Finally, consider case d) of mutually exclusive sides $i, j \in \mathcal{D}' \subseteq \mathcal{D}$, $i \neq j$ with single-homing platforms $q, r \in \mathcal{M}' \subseteq \mathcal{M}$, $q \neq r$. From above, it follows that $\mathbf{N}^*(\mathbf{p}) = \emptyset$, when PC_i^q, ICC_{ij}^q or ICC_i^{qr} is not fulfilled. Suppose next *ad absurdum* that $[\phi_i^q(\cdot) - p_i^q] - [\phi_j^r(\cdot) - p_j^r] \notin [-1, 1]$ for $i, j \in \mathcal{D}'$, $i \neq j$, and $q, r \in \mathcal{M}'$, $q \neq r$. If $[\phi_i^q(\cdot) - p_i^q] - [\phi_j^r(\cdot) - p_j^r] < -1$, then an end-user with $(v_i^q = 1, v_j^r = 0)$ does not join side i on q, and if $[\phi_i^q(\cdot) - p_i^q] - [\phi_j^r(\cdot) - p_j^r] > 1$, then an end-user with $(v_i^q = 0, v_j^r = 1)$ does not join side j on r. Both end-users exist by full support. Hence, the PE is empty, a contradiction. Therefore PC_i^q, ICC_{ij}^q, ICC_i^{qr} and ICC_{ij}^{qr} are necessary. Suppose next that $\mathbf{N}^*(\mathbf{p}) \neq \emptyset$. Then it follows from above that PC_i^q, ICC_{ij}^q and ICC_i^{qr} hold and, by full support,

the existence of the two end-users just found implies that $\left[\phi_i^q(\cdot) - p_i^q\right] - \left[\phi_j^r(\cdot) - p_j^r\right] \in [-1, 1]$ for $i, j \in \mathcal{D}'$, $i \neq j$, and $q, r \in \mathcal{M}'$, $q \neq r$. Therefore, PC_i^q, ICC_{ij}^q, ICC_i^{qr} and ICC_{ij}^{qr} are sufficient for the PE to be non-empty. □

Proof of Lemma 1. To ease notation, we drop the platform superscript q. Given the price effect in (9), $\Delta N_i^{PE} := -f_i(\cdot) < 0$, we solve for the sign of the cross-price derivative (10):

$$\frac{\partial N_j}{\partial p_i} = \frac{\partial N_j}{\partial N_i}\frac{\partial N_i}{\partial p_i} = \frac{\partial(1 - F_j(p_j - \phi_j(\mathbf{N}_{\text{-}j})))}{\partial N_i}\Delta N_i^{PE} =$$

$$= \frac{\partial(1 - F_j(p_j - \phi_j(\mathbf{N}_{\text{-}j})))}{\partial \phi_j(\mathbf{N}_{\text{-}j})} \sum_{i \neq j} \frac{\partial \phi_j(\mathbf{N}_{\text{-}j})}{\partial N_i}\Delta N_i^{PE}, \tag{A.2}$$

where we just apply the chain rule. Note that $\frac{\partial(1-F_j(p_j-\phi_j(\mathbf{N}_{\text{-}j})))}{\partial \phi_j(\mathbf{N}_{\text{-}j})} = f_j(p_j - \phi_j(\mathbf{N}_{\text{-}j})) \in (0, 1)$, and $\sum_{i \neq j} \left|\frac{\partial \phi_j(\mathbf{N}_{\text{-}j})}{\partial N_i}\right| \in (0, 1)$ by Assumption 1. It thus follows that $\frac{\partial N_j}{\partial p_i} \in (-\Delta N_i^{PE}, \Delta N_i^{PE})$, that is, the absolute value of the cross-price derivative is smaller than that of the price effect in the own-price derivative. Finally, the sign of $\frac{\partial N_j}{\partial p_i}$ is determined by the sign of $\sum_{i \neq j} \frac{\partial \phi_j(\mathbf{N}_{\text{-}j})}{\partial N_i}$, i.e., $\frac{\partial N_j}{\partial p_i} < 0$ under positive cross-group externalities and $\frac{\partial N_j}{\partial p_i} > 0$ under congestion effects.

Given that $\sum_{i \neq j} \frac{\partial \phi_j(\mathbf{N}_{\text{-}j})}{\partial N_i}\Delta N_i^{PE} = \sum_{i \neq j} \frac{\partial \phi_j(\mathbf{N}_{\text{-}j})}{\partial N_i}\frac{\partial N_i}{\partial p_i} = \frac{\partial \phi_j(\mathbf{N}_{\text{-}j})}{\partial p_i}$, rewriting (A.2) yields (10) as required:

$$\frac{\partial N_j}{\partial p_i} = f_j(p_j - \phi_j(\mathbf{N}_{\text{-}j}))\frac{\partial \phi_j(\mathbf{N}_{\text{-}j})}{\partial p_i}.$$

Now we show formally that $\frac{\partial N_i}{\partial p_i} < 0$. From (9) we obtain

$$\frac{\partial N_i}{\partial p_i} = -f_i(\cdot) + f_i(\cdot)\frac{\partial \phi_i(\mathbf{N}_{\text{-}i})}{\partial p_i} = f_i(\cdot)\Big(-1 + \sum_{j \neq i} \frac{\partial \phi_i(\mathbf{N}_{\text{-}i})}{\partial N_j}\frac{\partial N_j}{\partial p_i}\Big).$$

By Assumption 1, $\sum_{j \neq i} \left|\frac{\partial \phi_i(\mathbf{N}_{\text{-}i})}{\partial N_j}\right| \in (0, 1)$, and as just shown, $\left|\frac{\partial N_j}{\partial p_i}\right| < \left|\Delta N_i^{PE}\right| = f_i(\cdot) \in (0, 1)$. Therefore, $\sum_{j \neq i} \frac{\partial \phi_i(\mathbf{N}_{\text{-}i})}{\partial N_j}\frac{\partial N_j}{\partial p_i} \in (-1, 1)$, and thus the magnitude of the price effect (the first term) in (9) is strictly larger than that of the network effect which implies that $\frac{\partial N_i}{\partial p_i} < 0$, and the result follows. □

Proof of Proposition 3. Given $\mathbf{p}$ and $\mathbf{N}^*(\mathbf{p})$, consider end-user #1 with valuations $(\bar{v}_i^q, \bar{v}_j^q)$ who joins side $i \in \mathcal{D}'$ on platform q since she would benefit less from joining side $j \in \mathcal{D}' \setminus \{i\}$ on platform

q, i.e., for her $\bar{v}_i^q + \phi_i^q(\mathbf{N}_{-i}^q) - p_i^q > \bar{v}_j^q + \phi_j^q(\mathbf{N}_{-j}^q) - p_j^q > 0$. Rewriting this yields

$$\bar{v}_j^q - \bar{v}_i^q < \phi_i^q(\mathbf{N}_{-i}^q) - p_i^q + p_j^q - \phi_j^q(\mathbf{N}_{-j}^q). \tag{A.3}$$

Next consider end-user #2 with valuations $(\tilde{v}_i^q, \tilde{v}_j^q)$ who joins side j on platform q since she would benefit less from joining side i on platform q, i.e., for her $\tilde{v}_j^q + \phi_j^q(\mathbf{N}_{-j}^q) - p_j^q > \tilde{v}_i^q + \phi_i^q(\mathbf{N}_{-i}^q) - p_i^q > 0$. Rewriting this yields

$$\tilde{v}_j^q - \tilde{v}_i^q > \phi_i^q(\mathbf{N}_{-i}^q) - p_i^q + p_j^q - \phi_j^q(\mathbf{N}_{-j}^q). \tag{A.4}$$

Both end-users exist by full support. Combining (A.3) and (A.4) yields:

$$\tilde{v}_j^q - \tilde{v}_i^q > \phi_i^q(\mathbf{N}_{-i}^q) - p_i^q + p_j^q - \phi_j^q(\mathbf{N}_{-j}^q) > \bar{v}_j^q - \bar{v}_i^q. \tag{A.5}$$

Given $\mathbf{p}^q$ and $\tilde{\mathbf{p}}^q$ such that $\tilde{p}_k^q = p_k^q$ for all $k \in \mathcal{D} \setminus \{i\}$, and $\tilde{p}_i^q = p_i^q + \delta$ for some $i \in \mathcal{D}'$, where $\delta > 0$, we want to show that if end-user #1 switches to side j, then end-user #2 does not switch to side i. Therefore, we state the new ICCs for both end-users. For end-user #1, this is $\bar{v}_j^q + \phi_j^q(\mathbf{N}_{-j}^q) - p_j^q > \bar{v}_i^q + \phi_i^q(\mathbf{N}_{-i}^q) - \tilde{p}_i^q$. Rewriting this yields

$$\bar{v}_j^q - \bar{v}_i^q > \phi_i^q(\mathbf{N}_{-i}^q) - \tilde{p}_i^q + p_j^q - \phi_j^q(\mathbf{N}_{-j}^q). \tag{A.6}$$

Next consider end-user #2 and suppose *ad absurdum* that she switches to side i on platform q. Then for her it holds that $\tilde{v}_i^q + \phi_i^q(\mathbf{N}_{-i}^q) - \tilde{p}_i^q > \tilde{v}_j^q + \phi_j^q(\mathbf{N}_{-j}^q) - p_j^q$. Rewriting yields

$$\tilde{v}_j^q - \tilde{v}_i^q < \phi_i^q(\mathbf{N}_{-i}^q) - \tilde{p}_i^q + p_j^q - \phi_j^q(\mathbf{N}_{-j}^q). \tag{A.7}$$

Analogously as before, combining (A.6) and (A.7) yields:

$$\bar{v}_j^q - \bar{v}_i^q > \phi_i^q(\mathbf{N}_{-i}^q) - \tilde{p}_i^q + p_j^q - \phi_j^q(\mathbf{N}_{-j}^q) > \tilde{v}_j^q - \tilde{v}_i^q. \tag{A.8}$$

Clearly this contradicts (A.5) which yields $\tilde{v}_j^q - \tilde{v}_i^q > \bar{v}_j^q - \bar{v}_i^q$. Therefore, we have shown that in case of switching (following a price change in a given PE), this always takes place from the side on which the price increases to another mutually exclusive side. End-users from the side on which the

price did not change never switch to the side on which it increases. □

Proof of Theorem 1. Consider first full coverage, i.e., that $\sum_{i\in\mathcal{D}'} N_i^q = 1$. Without loss of generality pick side $i \in \mathcal{D}'$, and for ease of notation, drop superscript q. Given our assumption of full support, from ICC_{ij} it follows that for some end-user on side $i \in \mathcal{D}'$, $v_i = v_j + \phi_j(\cdot) - p_j + p_i - \phi_i(\cdot)$, for all $j \in \mathcal{D}' \setminus \{i\}$. Then, as p_i increases, any such end-user is strictly better off by joining side j instead. Since $\sum_{i\in\mathcal{D}'} N_i = 1$ and by Proposition 3, it follows that N_i decreases while N_j increases for all $j \in \mathcal{D}' \setminus \{i\}$ after an increase in p_i. Therefore, $\varepsilon_i < 0$ and $\varepsilon_{ij} > 0$ for all $j \in \mathcal{D}' \setminus \{i\}$.

Suppose next that $\sum_{i\in\mathcal{D}'} N_i < 1$. Given the own-price elasticity $\varepsilon_i := \frac{\partial N_i}{\partial p_i}\frac{p_i}{N_i}$, we show next that $\frac{\partial N_i}{\partial p_i} < 0$. The number of side-$i$ end-users in equilibrium is given by (5). Taking the derivative with respect to p_i yields

$$\frac{\partial N_i}{\partial p_i} = \overbrace{\Big(-1 + \frac{\partial \phi_i(\cdot)}{\partial p_i}\Big) f_i(p_i - \phi_i(\mathbf{N}_{\text{-}i})) \prod_{j\in\mathcal{D}'\setminus\{i\}} F_j\left(p_j - \phi_j(\mathbf{N}_{\text{-}j})\right)}^{\text{price and network effect from } PC_i \Rightarrow \text{ end-users that drop the platform if } p_i \text{ increases}} + \underbrace{\int_{p_i-\phi_i(\mathbf{N}_{\text{-}i})}^{1} f_i(v_i) \frac{\partial}{\partial p_i} \prod_{j\in\mathcal{D}'\setminus\{i\}} F_j\left(v_i - \phi_j(\mathbf{N}_{\text{-}j}) + p_j - p_i + \phi_i(\mathbf{N}_{\text{-}i})\right) dv_i}_{\text{price and network effect from } ICC_{ij} \text{ for all } j\in\mathcal{D}'\setminus\{i\} \Rightarrow \text{ end-users that switch away from side } i \text{ if } p_i \text{ increases}}, \tag{A.9}$$

Now, we solve the derivative in the second line of (A.9):

$$\tfrac{\partial}{\partial p_i}\textstyle\prod_{j\in\mathcal{D}'\setminus\{i\}} F_j\left(v_i - \phi_j(\mathbf{N}_{\text{-}j}) + p_j - p_i + \phi_i(\mathbf{N}_{\text{-}i})\right) = \sum_{j\in\mathcal{D}'\setminus\{i\}} \tfrac{\partial F_j(\cdot)}{\partial p_i} \prod_{k\in\mathcal{D}'\setminus\{i,j\}} F_k(\cdot) =$$

$$= \textstyle\sum_{j\in\mathcal{D}'\setminus\{i\}} \big(-\tfrac{\partial \phi_j(\cdot)}{\partial p_i} - 1 + \tfrac{\partial \phi_i(\cdot)}{\partial p_i}\big) f_j(\cdot) \prod_{k\in\mathcal{D}'\setminus\{i,j\}} F_k(\cdot).$$

Let $C := \int_{p_i-\phi_i(\mathbf{N}_{\text{-}i})}^{1} f_i(v_i) \sum_{j\in\mathcal{D}'\setminus\{i\}} (-\frac{\partial \phi_j(\cdot)}{\partial p_i}) f_j(\cdot) \prod_{k\in\mathcal{D}'\setminus\{i,j\}} F_k(\cdot) dv_i$, which captures the network effect that follows from the switching-side effect (i.e., the network impact generated by those that switch) on all other sides $k \in \mathcal{D}' \setminus \{i,j\}$. By Assumption 1, since the cross-group externalities are bounded, it cannot be larger than the switching-side effect itself. Formally, $\frac{\partial \phi_j(\cdot)}{\partial p_i} = \sum_{k\in\mathcal{D}'\setminus\{j\}} \frac{\partial \phi_j(\cdot)}{\partial N_k}\frac{\partial N_k}{\partial p_i}$ and by Assumption 1, $\sum_{k\in\mathcal{D}'\setminus\{j\}} \frac{\partial \phi_j(\cdot)}{\partial N_k} \in (-1,1)$. If this expression as well as $\sum_{k\in\mathcal{D}'\setminus\{i,j\}} \frac{\partial N_k}{\partial p_i}$ have the same sign, then $\frac{\partial \phi_j(\cdot)}{\partial p_i} > 0$ and $C < 0$. Otherwise, $\frac{\partial \phi_j(\cdot)}{\partial p_i} < 0$ and $C > 0$. We now show that this implies that $\frac{\partial N_i}{\partial p_i} < 0$. Since $\sum_{k\in\mathcal{D}'\setminus\{i,j\}} \frac{\partial N_k}{\partial p_i} = \sum_{k\in\mathcal{D}'\setminus\{i,j\}} \frac{\partial N_k}{\partial N_i}\frac{\partial N_i}{\partial p_i}$, given positive cross-group externalities $\sum_{k\in\mathcal{D}'\setminus\{j\}} \frac{\partial \phi_j(\cdot)}{\partial N_k} \in (0,1)$ and $\sum_{k\in\mathcal{D}'\setminus\{i,j\}} \frac{\partial N_k}{\partial N_i} > 0$, so for

$\frac{\partial \phi_j(\cdot)}{\partial p_i} < 0$ to hold requires that $\frac{\partial N_i}{\partial p_i} < 0$; while, under congestion, $\sum_{k \in \mathcal{D}' \setminus \{j\}} \frac{\partial \phi_j(\cdot)}{\partial N_k} \in (-1, 0)$ and $\sum_{k \in \mathcal{D}' \setminus \{i,j\}} \frac{\partial N_k}{\partial N_i} < 0$, and for $\frac{\partial \phi_j(\cdot)}{\partial p_i} < 0$ to hold requires that $\frac{\partial N_i}{\partial p_i} < 0$. In both cases, the own-price elasticity needs to be negative since otherwise a contradiction arises.

Let $A := f_i\big(p_i - \phi_i(\mathbf{N}_{\text{-}i})\big) \prod_{j \in \mathcal{D}' \setminus \{i\}} F_j\big(p_j - \phi_j(\mathbf{N}_{\text{-}j})\big)$, and note that $A \in (0,1)$ is a measure of end-users that drop side i due to the shift in PC_i. Then, replacing A and the derivative into (A.9) yields

$$\frac{\partial N_i}{\partial p_i} = \big(-1 + \tfrac{\partial \phi_i(\cdot)}{\partial p_i}\big)A + \int_{p_i - \phi_i(\mathbf{N}_{\text{-}i})}^{1} f_i(v_i) \textstyle\sum_{j \in \mathcal{D}' \setminus \{i\}} \big(-\tfrac{\partial \phi_j(\cdot)}{\partial p_i} - 1 + \tfrac{\partial \phi_i(\cdot)}{\partial p_i}\big) f_j(\cdot) \prod_{k \in \mathcal{D}' \setminus \{i,j\}} F_k(\cdot) dv_i.$$

By the Fubini/Tonelli theorem, the previous expression can be rewritten as follows

$$\frac{\partial N_i}{\partial p_i} = \big(-1 + \tfrac{\partial \phi_i(\cdot)}{\partial p_i}\big)A + \textstyle\sum_{j \in \mathcal{D}' \setminus \{i\}} \big(-\tfrac{\partial \phi_j(\cdot)}{\partial p_i} - 1 + \tfrac{\partial \phi_i(\cdot)}{\partial p_i}\big) \int_{p_i - \phi_i(\mathbf{N}_{\text{-}i})}^{1} f_i(v_i) f_j(\cdot) \prod_{k \in \mathcal{D}' \setminus \{i,j\}} F_k(\cdot) dv_i.$$

Let $B := \sum_{j \in \mathcal{D}' \setminus \{i\}} \int_{p_i - \phi_i(\mathbf{N}_{\text{-}i})}^{1} f_i(v_i) f_j(\cdot) \prod_{k \in \mathcal{D}' \setminus \{i,j\}} F_k(\cdot) dv_i$, and note that $B \in (0,1)$ is a measure of end-users that drop side i due to the shift in ICC_{ij} for all $j \in \mathcal{D}' \setminus \{i\}$. Then,

$$\frac{\partial N_i}{\partial p_i} = \Bigg(\underbrace{-1}_{\text{price effect } \Delta N_i^{PE}} + \underbrace{\frac{\partial \phi_i(\cdot)}{\partial p_i}}_{\text{network effect}} \Bigg)(A + B) + C. \tag{A.10}$$

Since sides are mutually exclusive and the total measure of end-users is 1, it follows that $(A+B) \in (0,1)$, since A arises from PC_i, while B arises from ICC_{ij} for all $j \in \mathcal{D}' \setminus \{i\}$, i.e., A and B measure different end-users that drop side $i \in \mathcal{D}'$. Hence, $\frac{\partial N_i}{\partial p_i} < 0$ as long as $\frac{\partial \phi_i(\cdot)}{\partial p_i} \in (-1, 1)$, which we show next. Note that $\frac{\partial \phi_i(\cdot)}{\partial p_i} = \sum_{j \in \mathcal{D}' \setminus \{i\}} \frac{\partial \phi_i(\cdot)}{\partial N_j} \frac{\partial N_j}{\partial N_i} \frac{\partial N_i}{\partial p_i}$, with $\frac{\partial N_i}{\partial p_i} = \Delta N_i^{PE} := -(A+B) \in (-1, 0)$, i.e., this is the drop due to the price effect which triggers all other changes. Hence, $\frac{\partial \phi_i(\cdot)}{\partial p_i} = \sum_{j \in \mathcal{D}' \setminus \{i\}} \frac{\partial \phi_i(\cdot)}{\partial N_j} \frac{\partial N_j}{\partial N_i} \Delta N_i^{PE}$ and $\sum_{j \in \mathcal{D}' \setminus \{i\}} \frac{\partial \phi_i(\cdot)}{\partial N_j} \frac{\partial N_j}{\partial N_i} \leq \sum_{j \in \mathcal{D}' \setminus \{i\}} \frac{\partial \phi_i(\cdot)}{\partial N_j} \sum_{i \in \mathcal{D}' \setminus \{j\}} \frac{\partial N_j}{\partial N_i}$, which is in $(-1, 1)$ by Assumption 1 and since sides are mutually exclusive, respectively. Finally, since $C < 0$ or if $C > 0$, then $\frac{\partial N_i}{\partial p_i} < 0$, and $\varepsilon_i < 0$ holds.

Before showing the sign of $\frac{\partial N_j}{\partial p_i}$, we remark that the change in p_i affects N_j for all $j \in \mathcal{D}' \setminus \{i\}$ and the changes in N_k for $k \in \mathcal{D}' \setminus \{i, j\}$ have feedback effects on N_j as well which, however, are bounded; i.e., their sum is smaller in magnitude than the effect of p_i on N_j, given Assumption 1 and since sides are mutually exclusive—each end-user joins at most one side. Hence, to show the

sign of $\frac{\partial N_j}{\partial p_i}$, we abstract from these higher order effects whereby changes in N_k for $k \in \mathcal{D}' \setminus \{i,j\}$ (due to the change in p_i) lead again to changes in N_j.

The number of side-j end-users in a PE is given by (5). Taking the derivative w.r.t. p_i yields

$$\frac{\partial N_j}{\partial p_i} = \overbrace{\frac{\partial \phi_j(\cdot)}{\partial p_i} f_j(v_j) \prod_{k \in \mathcal{D}' \setminus \{j\}} F_k\big(v_j - \phi_k(\mathbf{N}_{\text{-}k}) + p_k - p_j + \phi_j(\mathbf{N}_{\text{-}j})\big)}^{\text{network effect from } PC_j} + \underbrace{\int_{p_j - \phi_j(\mathbf{N}_{\text{-}j})}^{1} f_j(v_j) \frac{\partial}{\partial p_i} \prod_{k \in \mathcal{D}' \setminus \{j\}} F_k\big(v_j - \phi_k(\mathbf{N}_{\text{-}k}) + p_k - p_j + \phi_j(\mathbf{N}_{\text{-}j})\big) dv_j}_{\text{network effect from } ICC_{jk} \text{ for all } k \in \mathcal{D}' \setminus \{j\}}. \tag{A.11}$$

Solving for the partial derivative in the second line of (A.11) yields

$$\tfrac{\partial}{\partial p_i} \textstyle\prod_{k \in \mathcal{D}' \setminus \{j\}} F_k\big(v_j - \phi_k(\mathbf{N}_{\text{-}k}) + p_k - p_j + \phi_j(\mathbf{N}_{\text{-}j})\big) = \sum_{k \in \mathcal{D}' \setminus \{j\}} \tfrac{\partial F_k(\cdot)}{\partial p_i} \prod_{l \in \mathcal{D}' \setminus \{k,j\}} F_l(\cdot)$$

Focusing on the effect with respect to i and, as discussed above, abstracting from higher order effects through ICC_{jk} for $k \in \mathcal{D}' \setminus \{i,j\}$, we obtain the following expression,

$$\tfrac{\partial F_i(\cdot)}{\partial p_i} \textstyle\prod_{k \in \mathcal{D}' \setminus \{i,j\}} F_k(\cdot) = \big(1 - \tfrac{\partial \phi_i(\cdot)}{\partial p_i} + \tfrac{\partial \phi_j(\cdot)}{\partial p_i}\big) f_i(\cdot) \textstyle\prod_{k \in \mathcal{D}' \setminus \{i,j\}} F_k(\cdot).$$

Let $X := f_j(v_j) \prod_{k \in \mathcal{D}' \setminus \{j\}} F_k(g(v_j - \phi_k(\mathbf{N}_{\text{-}k}) + p_k - p_j + \phi_j(\mathbf{N}_{\text{-}j}))$ and note that $X \in (0,1)$. Then, replacing X and the partial derivative in (A.11), and applying the Fubini/Tonelli theorem yields

$$\tfrac{\partial N_j}{\partial p_i} = \tfrac{\partial \phi_j(\cdot)}{\partial p_i} X + \Big(1 - \tfrac{\partial \phi_i(\cdot)}{\partial p_i} + \tfrac{\partial \phi_j(\cdot)}{\partial p_i}\Big) \textstyle\int_{p_j - \phi_j(\mathbf{N}_{\text{-}j})}^{1} f_j(v_j) f_i(\cdot) \prod_{k \in \mathcal{D}' \setminus \{i,j\}} F_k(\cdot) dv_j.$$

Now replace $Y := \int_{p_j - \phi_j(\mathbf{N}_{\text{-}j})}^{1} f_j(v_j) f_i(\cdot) \prod_{k \in \mathcal{D}' \setminus \{i,j\}} F_k(\cdot) dv_j$, note that $Y \in (0,1)$, and simplify:

$$\tfrac{\partial N_j}{\partial p_i} = \tfrac{\partial \phi_j(\cdot)}{\partial p_i}(X + Y) + \Big(1 - \tfrac{\partial \phi_i(\cdot)}{\partial p_i}\Big) Y. \tag{A.12}$$

The second term on the RHS of (A.12) captures end-users switching from side i to side j after p_i increases. This term is positive since $\frac{\partial \phi_i(\cdot)}{\partial p_i} \in (-1,1)$, as shown before in this proof. In the first term on the RHS of (A.12), $(X+Y) \in (0,1)$ since sides are mutually exclusive and the network effect $\frac{\partial \phi_j(\cdot)}{\partial p_i} \in (-1,1)$, as also shown before: $\frac{\partial \phi_j(\cdot)}{\partial p_i} < 0$ if side i has positive cross-group externalities

on side j, and $\frac{\partial \phi_j(\cdot)}{\partial p_i} > 0$ if there is congestion. Adding the two terms on the RHS of (A.12), under congestion, the cross-price derivative is strictly positive, while under positive cross-group externalities, the negative network effect is compensated by the positive switching-side effect, and the cross-price derivative is positive if $\big(1 - \frac{\partial \phi_i(\cdot)}{\partial p_i}\big)Y > \big|\frac{\partial \phi_j(\cdot)}{\partial p_i}\big|(X+Y)$, or $\big(1 - \frac{\partial \phi_i(\cdot)}{\partial p_i} - \big|\frac{\partial \phi_j(\cdot)}{\partial p_i}\big|\big)Y > \big|\frac{\partial \phi_j(\cdot)}{\partial p_i}\big|X$, and negative otherwise. □

Proof of Proposition 6. If all platforms are multi-homing, there are neither switching-platform nor switching-side-and-platform effects; that is, $\lambda_{ij}^{st} = 0$ for all $i, j \in \mathcal{D}$ and for all $s, t \in \mathcal{N}$, $s \neq t$. Hence, Λ^{st} is a null matrix, for all $s, t \in \mathcal{N}$, $s \neq t$. Therefore, markups are given by $(\Lambda^s \odot \Theta^s)\boldsymbol{\mu}^s = \mathcal{E}^s$ for all $s \in \mathcal{N}$. This is identical to the pre-merger markups. □

Proof of Lemma 2. If a subset of the merged platforms, $\mathcal{N}' \subseteq \mathcal{N}$, are single-homing with all sides being non-mutually exclusive, then there are switching-platform effects (Proposition 4), but no switching-side-and-platform effects; that is, for all $s, t \in \mathcal{N}'$, $s \neq t$, it holds, *(i)* $\lambda_{ii}^{st} > 0$ for all $i \in \mathcal{D}$; while *(ii)* $\lambda_{ij}^{st} = 0$ for all $j \in \mathcal{D}$, $i \neq j$. Therefore, $\Lambda^{st} \odot \Theta^{st}$ is a diagonal matrix. This matrix is non-positive since $\lambda_{ii}^{st} > 0$, as follows from Proposition 4. For all other platforms $s, t \in \mathcal{N} \setminus \mathcal{N}'$ (i.e., those that are not single-homing), there are neither switching-platform nor switching-side-and-platform effects; that is, $\lambda_{ij}^{st} = 0$ for all $i, j \in \mathcal{D}$ and for all $s, t \in \mathcal{N} \setminus \mathcal{N}'$, $s \neq t$ and, hence, the corresponding Λ^{st} matrices are null. □

Proof of Proposition 7. By Lemma 2, $\lambda_{ij}^{st} = 0$ for all $i, j \in \mathcal{D}$, $i \neq j$, and for all $s, t \in \mathcal{N}'$, $s \neq t$, and by Proposition 4, $\lambda_{ii}^{st} > 0$ for all $i \in \mathcal{D}$ and for all $s, t \in \mathcal{N}'$, $s \neq t$. After incorporating all optimal pre-merger prices in the RHS of (11), this expression becomes

$$\frac{(p_i^s - c_i^s)}{p_i^s} = \frac{1}{|\varepsilon_i^s|} + \sum_{j \in \mathcal{D} \setminus \{i\}} \lambda_{ij}^s \frac{(p_j^s - c_j^s)}{p_i^s} + \sum_{t \in \mathcal{N}' \setminus \{s\}} \lambda_{ii}^{st} \frac{(p_i^t - c_i^t)}{p_i^s}. \tag{A.13}$$

Relative to the pre-merger case given by (8), post-merger the last term on the RHS of (A.13) appears additionally. Hence, the sign of this last term determines whether the markup of platform $s \in \mathcal{N}'$ on side $i \in \mathcal{D}$ increases or decreases post-merger. If $(p_i^t - c_i^t) > 0$ for all $i \in \mathcal{D}$ and for all $t \in \mathcal{N}'$, $s \neq t$ (which unambiguously occurs if there are congestion effects), then this additional term is unambiguously positive. However, if $(p_i^t - c_i^t) < 0$ for some $i \in \mathcal{D}$ and $t \in \mathcal{N}'$, $s \neq t$ (which only may occur if there are positive cross-group externalities), then this additional term can be either positive or negative. □

Proof of Lemma 3. If a subset of the merged platforms, $\mathcal{N}' \subseteq \mathcal{N}$, are single-homing on which a subset of sides $\mathcal{D}' \subseteq \mathcal{D}$ are mutually exclusive, then there are switching-platform effects (by Proposition 4) and switching-side-and-platform effects (by Proposition 5); that is, for all $s, t \in \mathcal{N}'$, $s \neq t$, it holds, *(i)* $\lambda_{ii}^{st} > 0$ for all $i \in \mathcal{D}$, and *(ii)* $\lambda_{ij}^{st} > 0$ for all $i, j \in \mathcal{D}'$, $i \neq j$. Therefore, $\Lambda^{st} \odot \Theta^{st}$ is non-positive for all $s, t \in \mathcal{N}'$, $s \neq t$. Moreover, since $\lambda_{ij}^{st} > 0$ for all $i, j \in \mathcal{D}'$, $i \neq j$, and for all $s, t \in \mathcal{N}'$, $s \neq t$, some of the off-diagonal elements in Λ^{st} are strictly negative and, consequently, $\Lambda^{st} \odot \Theta^{st}$ is non-diagonal. Finally, for all other platforms that are not single-homing $s, t \in \mathcal{N} \setminus \mathcal{N}'$, there are neither switching-platform nor switching-side-and-platform effects; that is, $\lambda_{ij}^{st} = 0$ for all $i, j \in \mathcal{D}$ and for all $s, t \in \mathcal{N} \setminus \mathcal{N}'$, $s \neq t$, and consequently, the corresponding Λ^{st} matrices are null. □

Proof of Proposition 8. From the proof of Lemma 3, it follows that for all $s, t \in \mathcal{N}'$, $s \neq t$, *(i)* $\lambda_{ii}^{st} > 0$ for all $i \in \mathcal{D}$, and *(ii)* $\lambda_{ij}^{st} > 0$ for all $i, j \in \mathcal{D}'$, $i \neq j$. Then, after incorporating all pre-merger optimal prices in the RHS of (11), this expression becomes

$$\frac{(p_i^s - c_i^s)}{p_i^s} = \frac{1}{|\varepsilon_i^s|} + \sum_{j \in \mathcal{D} \setminus \{i\}} \lambda_{ij}^s \frac{(p_j^s - c_j^s)}{p_i^s} + \sum_{t \in \mathcal{N}' \setminus \{s\}} \Big(\lambda_{ii}^{st} \frac{(p_i^t - c_i^t)}{p_i^s} + \sum_{j \in \mathcal{D}' \setminus \{i\}} \lambda_{ij}^{st} \frac{(p_j^t - c_j^t)}{p_i^s} \Big). \tag{A.14}$$

Therefore, relative to the pre-merger case given by (8), the last term on the RHS of (A.14) (i.e., the whole summation in parenthesis) additionally appears post-merger. Hence, the sign of this last term determines whether the markup of platform $s \in \mathcal{N}'$ on side $i \in \mathcal{D}'$ increases or decreases after the merger. If $(p_i^t - c_i^t) > 0$ for all $i \in \mathcal{D}$ and for all $t \in \mathcal{N}'$, $t \neq s$ (which unambiguously occurs if $\lambda_{ij}^t \geq 0$ for all $i, j \in \mathcal{D}$, $i \neq j$), then this additional term is unambiguously positive. However, if $(p_i^t - c_i^t) < 0$ for some $i \in \mathcal{D}$ and $t \in \mathcal{N}'$, $t \neq s$ (which may occur only in all other cases not included in the previous condition), then this additional term can be either positive or negative. □

Appendix B: Model extensions (for online publication)

B.1 Partial or no double-counting with multi-homing platforms

In Section 3, we characterize the demand for sides assuming that end-users on one side, say, side i, value equally all those on another side, say, side j, on the same platform. However, as discussed by some authors (Bakos and Halaburda, 2020), this assumption might not always be appropriate in the context of multi-homing platforms. For example, buyers on e-commerce platforms benefit from viewing a certain product on one platform, but viewing the same product on another one does not yield them extra value. Other authors note that this also applies to media markets: for advertisers, media end-users that can be reached through a single outlet are much more valuable than those that can be reached through multiple ones (Ambrus et al., 2016; Anderson et al., 2018). Thus, for the sake of completeness, in this appendix, we extend our model to accommodate this alternative assumption.

Recall that, for multi-homing platforms and non-mutually exclusive sides, the demand function for side i on platform q is given by (2), while (4) characterizes the demand for that side and platform under mutually exclusive sides. Note that (4) is similar to (2) but augmented by a term that captures the ICC on the RHS. In our analysis, we primarily focus on the former, although similar conclusions apply to the latter. By the law of total probability, the previous expression can be decomposed as

$$N_i^q(\cdot) = \underbrace{pr\big(v_i^q \geq p_i^q - \phi_i^q(\cdot) \cap v_i^r < p_i^r - \phi_i^r(\cdot)\big)}_{\text{single-homing end-users } (\hat{N}_i^q)} + \underbrace{\big(v_i^q \geq p_i^q - \phi_i^q(\cdot) \cap v_i^r \geq p_i^r - \phi_i^r(\cdot)\big)}_{\text{multi-homing end-users } (\bar{N}_i^q)}$$

for all $i \in \mathcal{D}$ and all $q, r \in \mathcal{M}, q \neq r$. That is, the total number of end-users on side i on platform q is equal to the sum of two groups: end-users who join side i on platform q but do not join side i on any other platform r (single-homers), and those who also join side i on some other platform r (multi-homers). Let us denote the former set of end-users by $\hat{N}_i^q$ and the latter one by $\bar{N}_i^q$.

Under the partial or no double-counting assumption, end-users value fully the presence of those on other sides on the same platform who single-home, giving little (or no) value to those who multi-home. The network effect in this case is expressed as $\phi_i^q(\hat{\mathbf{N}}_{-i}^q + \boldsymbol{\sigma}_{-i}^q \cdot \bar{\mathbf{N}}_{-i}^q)$, where $\hat{\mathbf{N}}_{-i}^q$ (resp., $\bar{\mathbf{N}}_{-i}^q$) is the vector of single-homing (resp., multi-homing) end-users on all sides other than i on platform

q. The vector $\boldsymbol{\sigma}_{-i}^{q}$ contains $(d-1)$ parameters in the interval $[0,1)$, where each parameter, such as σ_{j}^{q}, can be interpreted as either the probability with which end-users on side i interact with multi-homing end-users on side j on platform q for the first time or as a parameter that captures the lower cross-group network externality associated with multi-homing end-users on side j. This setup also accommodates the extreme case in which end-users on other sides shared with more than one platform have no market value, i.e., $\sigma_{j}^{q}=0$, as proposed by Anderson et al. (2018).

In this alternative case, the derivatives of N_{i}^{q} with respect to p_{i}^{q} and p_{j}^{q} differ slightly from those obtained in Sections 4.3.1 and 4.3.2. These derivatives, which can be obtained formally using the modified version of (3) that incorporates the new network effect indicated above, are as follows:

$$\frac{\partial N_{i}^{q}}{\partial p_{i}^{q}}=-\underbrace{f_{i}^{q}\big(p_{i}^{q}-\phi_{i}^{q}(\mathbf{N}_{-i}^{q})\big)}_{\text{price effect}}+\underbrace{f_{i}^{q}\big(p_{i}^{q}-\phi_{i}^{q}(\mathbf{N}_{-i}^{q})\big)\left[\frac{\partial\phi_{i}^{q}(\hat{\mathbf{N}}_{-i}^{q})}{\partial p_{i}^{q}}+\boldsymbol{\sigma}_{-i}^{q}\cdot\frac{\partial\phi_{i}^{q}(\bar{\mathbf{N}}_{-i}^{q})}{\partial p_{i}^{q}}\right]}_{\text{network effect}}, \tag{B.1}$$

$$\frac{\partial N_{i}^{q}}{\partial p_{j}^{q}}=\underbrace{f_{i}^{q}\big(p_{i}^{q}-\phi_{i}^{q}(\mathbf{N}_{-i}^{q})\big)\left[\frac{\partial\phi_{i}^{q}(\hat{\mathbf{N}}_{-i}^{q})}{\partial p_{j}^{q}}+\boldsymbol{\sigma}_{-i}^{q}\cdot\frac{\partial\phi_{i}^{q}(\bar{\mathbf{N}}_{-i}^{q})}{\partial p_{j}^{q}}\right]}_{\text{network effect}}. \tag{B.2}$$

These derivatives differ from (9) and (10) in Section 4.3.1 only in the network effect, which now depends not only on the total number of end-users on the other sides of platform q but also on the composition of these end-users (i.e., on how many of them are single-homers and multi-homers, respectively). The impact that this new expression characterizing the network effect has on our analysis depends on the type of market solution we encounter.

In case all end-users multi-home, (B.2) boils down to $\frac{\partial N_{i}^{q}}{\partial p_{j}^{q}}=f_{i}^{q}\big(p_{i}^{q}-\phi_{i}^{q}(\mathbf{N}_{-i}^{q})\big)\left[\boldsymbol{\sigma}_{-i}^{q}\cdot\frac{\partial\phi_{i}^{q}(\bar{\mathbf{N}}_{-i}^{q})}{\partial p_{j}^{q}}\right]$. In this case, the network effect plays a less prominent role relative to that obtained in Sections 4.3.1 and 4.3.2, as its magnitude is significantly reduced by $\boldsymbol{\sigma}_{-i}^{q}$. In fact, in the limiting case in which $\boldsymbol{\sigma}_{-i}^{q}\to\mathbf{0}$, the network effect vanishes. These results have different implications (but similar conclusions) in the two cases that we consider in Sections 4.3.1 and 4.3.2.

First, in the case of platforms with non-mutually exclusive sides, as discussed in Section 4.3.1, if the network effect vanishes, then the cross-price derivative becomes small. In this scenario, optimal pricing, as characterized by (7), resembles closely the usual Lerner pricing formula for a standard monopolist, as the third element on the LHS of (7) converges to zero; that is, optimal

pricing depends exclusively on the value associated with single-homing end-users. Therefore, as noted in Bakos and Halaburda (2020), cross-subsidization—where different sides of a platform are interdependent through the network effect—does not occur. Importantly, since the cross-price derivative is relatively close to zero, so is λ_{ij}^{q}.

Second, if platforms' sides are mutually exclusive and cross-group externalities are positive, as discussed in Section 4.3.2, increasing p_i^q has two countervailing effects on side j. Namely, it decreases participation on that side (due to the network effect), but it also increases it (due to the switching-side effect). In the limiting case where $\boldsymbol{\sigma}_{-i}^{q} \to \mathbf{0}$, the network effect vanishes, and the switching-side effect is more likely to dominate the network effect. In this scenario, the cross-price derivative will more likely be positive (and so is λ_{ij}^{q}), resulting in an optimal markup greater than the usual Lerner index, even in the absence of congestion. This optimal pricing rule depends mostly on the value associated to single-homing end-users and, to a lesser degree, on the marginal value associated to multi-homers—given by the extent to which some of the single-homing end-users switch sides in response to a marginal change in price. This observation is consistent with the so-called "principle of incremental pricing" (Anderson et al., 2018). Obviously, cross-subsidization cannot occur in this case.

Finally, in case there is full market participation with both single-homing and multi-homing end-users, and also in case there is less than full participation—implying the coexistence of both types of end-users due to the full support and continuity of the distribution of valuations—the outcomes are similar. These cases represent intermediate cases of the previous ones, wherein the network effect is attenuated as the presence of multi-homers increases. It becomes akin to the results in Sections 4.3.1 and 4.3.2 as the number of multi-homing end-users decreases.

B.2 Random demand

In our main analysis, every end-user on the platform is "always" interacting with the end-users on the other sides. However, there are cases where an end-user who joins the platform is not always *active*, which is they are not always interacting with end-users on the other sides. Consider, for example, an Uber driver. She does not offer her services 24/7, but rather decides when to be active; sometimes she may decide to earn money as an Uber driver and sometimes she may enjoy her free time.

In order to capture this, suppose that an end-user who joins side i on platform q is expected to be active on side i with probability $\theta_i^q \in (0,1)$. With the complementary probability $1-\theta_i^q$ she is not (even though she joined that side and paid the corresponding fee p_i^q). This probability can be interpreted as the ratio at which end-users are usually active or as the quantity below 1 that they exchange on the platform. While intuitively both cases have slightly different interpretations, formally they are equivalent. Correspondingly, an end-user is active on platform q's side i with probability $\theta_i^q \in (0,1)$. These ex-ante probabilities are resolved before an end-user interacts on the platform. In this case, an end-user that joins side i on platform q obtains a utility of

$$u_i^q := \theta_i^q \left[v_i^q + \phi_i^q(\boldsymbol{\theta}_{-i}^q \cdot \mathbf{N}_{-i}^q) \right] - p_i^q, \tag{B.3}$$

where $\boldsymbol{\theta}_{-i}^q$ is the vector of probabilities of being active on all sides other than i on platform q.
The utility a representative end-user obtains upon joining side i is first multiplied by the probability with which she is expected to be active on side i on platform q. If she is active, then, as before, she obtains v_i^q in addition to the network effect multiplied by the expected number of end-users she encounters on the other sides. Finally, the corresponding fee to join a side is paid for sure by the end-user (even if she interacts less than all the time or trades less than one unit of the good or service on the platform).
Now we introduce three changes of variables that will simplify our analysis:

1. Define a new random variable for any end-user i: $\check{v}_i^q = \theta_i^q v_i^q$.
2. Define a new random variable $\check{\mathbf{N}}_{-i}^q = \boldsymbol{\theta}_{-i}^q \cdot \mathbf{N}_{-i}^q$.
3. Finally, define a new random variable $\check{\phi}_i^q(\check{\mathbf{N}}_{-i}^q) = \theta_i^q \left[\phi_i^q(\boldsymbol{\theta}_{-i}^q \cdot \mathbf{N}_{-i}^q) \right]$; since end-users' demand is random, their expected cross-group externality is also a random variable $\check{\phi}_i^q(\cdot)$.

Therefore, the expression that captures the utility for an end-user becomes

$$u_i^q := \check{v}_i^q + \check{\phi}_i^q(\check{\mathbf{N}}_{-i}^q) - p_i^q. \tag{B.4}$$

Under the multi-homing assumption, if platforms' sides are non-mutually exclusive, then the par-

ticipation constraint that yields sides' demand is as follows

$$(PC_i^q) \quad \check{v}_i^q + \check{\phi}_i^q(\check{\mathbf{N}}_{-i}^q) - p_i^q \quad \geq \quad 0,$$

and, if platform q's sides i and j are mutually exclusive for $i, j \in \mathcal{D}' \subseteq \mathcal{D}$, then the demands are characterized additionally by the following incentive compatibility constraint

$$(ICC_{ij}^q) \quad \check{v}_i^q + \check{\phi}_i^q(\check{\mathbf{N}}_{-i}^q) - p_i^q \quad \geq \quad \check{v}_j^q + \check{\phi}_j^q(\check{\mathbf{N}}_{-j}^q) - p_j^q.$$

These conditions resemble the corresponding ones provided in Section 3, and similar ICCs can be obtained for the case in which platforms are single-homing. Therefore, all results obtained above hold here as well after applying the change of variables, although all variables are "smaller" since they are random.