Global Hegemony and Exorbitant Privilege *

Carolin E. Pflueger Pierre Yared

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Abstract

We present a dynamic two-country model in which military spending, geopolitical risk, and government bond prices are jointly determined. The model is consistent with three empirical facts: hegemons have a funding advantage, this advantage rises with geopolitical tensions, and war losers suffer from higher debt devaluation than victors. Even though higher debt capacity increases the military and financial advantage of the exogenously stronger country, it also gives rise to equilibrium multiplicity and the possibility that the weaker country overwhelms the stronger country with support from financial markets. For intermediate debt capacity, transitional dynamics exhibit *geopolitical hysteresis*, with dominance determined by initial conditions, unless war is realized and induces a hegemonic transition. For high debt capacity, transitional dynamics exhibit *geopolitical fragility*, where bond market expectations drive unpredictable transitions in dominance, and hegemonic transitions occur even in the absence of war.

Keywords: International Conflicts, Financial Globalization, International Financial Markets, National Security and War
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For war you need three things: 1. Money. 2. Money. 3. Money

Raimondo Montecuccoli, *Memorie della Guerra* (ca. 1661)

1 Introduction

A government's ability to borrow to finance military spending has historically played a critical role in military success. During the Napoleonic Wars, for example, Great Britain's credibility with lenders allowed for significant debt-financing and delivered a military advantage over France, leading to Great Britain's ultimate victory (Bordo and White (1991), Sargent and Velde (1995)). That victory paved the way for Great Britain to take over as the global hegemon in the nineteenth century (Ferguson (2008)).

Much like Great Britain in the nineteenth century, the U.S. today serves as the global hegemon, and it also benefits from an "exorbitant privilege" in its debt financing since it can borrow in international markets at preferential interest rates. As China grows economically and militarily, an open question concerns the conditions under which the U.S.'s hegemonic status and funding privilege can be preserved in the face of a more intense geopolitical rivalry.

In this paper, we examine the link between government financing and geopolitical rivalry in an environment with globalized debt markets. We ask the following three questions: How does the presence of global debt markets impact the military balance between countries? How does the military balance affect global debt markets? And how do hegemonic transitions take place?

In the first part of the paper, we document three empirical observations to motivate our theoretical exploration. First, using evidence on historical government borrowing rates around the time of hegemonic transitions, we document that global hegemons have historically borrowed at lower interest rates than other countries. Second, we document that the interest spread for different countries relative to the global hegemon rises when geopolitical tensions increase. Third, we present evidence that in the aftermath of realized geopolitical conflict, defeated countries experience greater inflation and debt devaluation relative to victorious countries.

In the second part of the paper, we construct a simple dynamic two-country model which is consistent with these three facts in order to address our motivating questions. In the model, military strength is a function of endogenous accumulated military spending, which we refer to as military capital, and exogenous factors (such as geography or technology). We allow one country to have an exogenous advantage over the other, but otherwise assume symmetry across countries. Countries decide how much to spend on defense versus other

public goods, how much to borrow to finance total spending, and whether to default. As is common in the sovereign debt literature, we model the cost of default as an exogenous proportional endowment cost, which parameterizes a country's debt capacity. War occurs exogenously with some positive probability in every period, and the likelihood of victory is increasing in a country's relative military strength. For realism and to ensure consistency with our third motivating empirical fact, we assume that military defeat results in the complete destruction of a country's endowment. This implies that a defeated country must default, and this default risk is in turn reflected in the country's financing costs.

Our first main result is that if the debt capacity of the two countries is low, there is a unique steady state conditional on remaining in a state of peace, with the exogenously stronger country winning (in expectation) if war takes place. The weaker country is defeated (in expectation) and therefore faces a higher default risk than the stronger country. Thus, and in line with our first motivating empirical fact, the stronger country benefits from a lower funding cost. Moreover, and in line with our second motivating empirical fact, an increase in the likelihood of war due to higher geopolitical tensions raises the interest spread between the two countries, since greater war risk raises the defeat and default risk for the weaker country relative to the stronger country.

An important feature of this steady state is the complementarity between military success and funding advantage. As default costs and hence debt capacity increase for both countries, the funding advantage of the stronger country also increases, translating into greater relative military spending and a larger probability of victory. Analogously, as the exogenous military advantage of the stronger country increases, so does its funding advantage, which amplifies the military advantage by facilitating an increase in endogenous military spending.

Our second main result is that if the debt capacity of the two countries is intermediate or high, there are multiple steady states conditional on remaining in a state of peace. While the steady state in which the stronger country wins the war is preserved, a second steady state emerges in which the weaker country wins. This second steady state is sustained by bond market participants' anticipation that the exogenously weaker country will invest enough in the military to overwhelm the stronger country's exogenous military advantage. That belief underpins a lower funding cost for the exogenously weaker country which is now less likely to default; this in turn supports a larger military investment by the weaker country that is facilitated by the country's debt capacity. Other factors beyond debt capacity also support multiplicity of steady states. Multiplicity is more likely if the exogenous difference in military capability across the two countries is small so that the weaker country can more feasibly dominate the stronger country. This is also more likely if the depreciation rate on military capital stock is low, so that the exogenously weaker country can build up enough

capacity over time to overwhelm the stronger country. In addition, a higher likelihood of war or a higher war risk premium enable multiplicity since these increase investors' desire for bonds issued by an expected victor.

Our final result considers the dynamic evolution of an expected victor's military and funding advantage. Conditional on remaining in a state of peace, there always exists a convergent monotonic path towards the steady state, where the military and funding advantage for an expected victor jointly increase over time. Our main result is that the uniqueness of this monotonic path depends on debt capacity. If debt capacity is low or intermediate, then the path is unique. An implication of this result is a form of *geopolitical hysteresis* that emerges for intermediate debt capacity: with multiple steady states, the initial relative level of military power determines which country will ultimately dominate militarily and financially. In this circumstance, the realization of war, which alters the balance of power, is required for a hegemonic transition to take place, with countries converging to a different steady state. In contrast, if debt capacity is high, then the convergent path is not unique, and initial conditions do not necessarily predict the ultimate steady state. In this case of *geopolitical fragility*, non-monotonic dynamics emerge, and a prospective victor's military and funding advantage can be ephemeral. Thus, hegemonic transitions can occur even in the absence of war. Geopolitical fragility is more likely the higher the likelihood of war and the higher the investor war risk premium. Under such conditions, market expectations within the zone of fragility can change suddenly, altering the funding and military advantages that the previously dominant country enjoyed. The two countries can subsequently transition out of the zone of fragility and converge towards a new steady state, with the military and financial dominance of the new hegemon being solidified over time.

A key insight from the second and third results of our model is that factors that enhance the complementarity between geopolitical and financial dominance—higher debt capacity, higher war probability, and higher war risk premium—also entail costs for the stronger country. They introduce steady-state multiplicity with the possibility that the stronger country is overwhelmed by the weaker country. They also introduce fragility in the sense that the equilibrium may no longer exhibit monotonic convergence. To investigate this issue further, we consider an extension of our environment that introduces asymmetric debt capacity between the two countries. We find that financial development that raises the debt capacity of the weaker country or impairs debt capacity of the stronger country also raises the likelihood of multiplicity. This extension provides a useful lens for interpreting the ultimate dominance of Great Britain over France during the Napoleonic Wars. It also implies that any successful attempt to internationalize China's currency and raise China's debt capacity relative to the U.S. is likely to matter for U.S. national security. The same is true to for any U.S. policy

decisions–such as a technical default on U.S. Treasury bonds due to a failure to raise the debt ceiling–that erode its own debt capacity.

This paper builds on the literature studying rationalist models of war going back to the work of Fearon (1995) and Powell (1993, 1999).[1] Relative to this literature, we abstract away from decisions to go to war, and instead focus on the role of international debt markets, which endogenizes the resource constraint of a country choosing armaments and introduces equilibrium multiplicity and fragility.[2]

Our paper is complementary with the recent growing literature that explores the effects of geopolitical risk and global hegemony on globalization, building on the landmark contributions of Hirschman (1945, 1958).[3] The focus of this growing literature is on trade volumes, trade networks, and trade policy, whereas our focus is on the interaction between global debt markets and military spending.

This paper is also contributes to the political economy literature on government debt. This literature emphasizes the role that political incentives play in driving government debt dynamics.[4] Most related is the work that considers these incentives and coordination in open economies with multiple countries (e.g., Chari and Kehoe (2007)), Azzimonti, de Francisco and Quadrini (2014), Aguiar et al. (2015), Halac and Yared (2018)). We contribute to this literature by considering how defense spending and the prospect of military conflict influences bond prices and government policies.

Finally, there is a large literature that has studied the "exorbitant privilege" of the United States, referring to its ability to borrow large amounts in its own currency at relatively low rates (Eichengreen (2011), Gourinchas and Rey (2022), Atkeson, Heathcote and Perri (2022)). More specifically, we relate to work studying the rise and fall of bond market dominance and its determinants. The prior literature has studied the link from bond market dominance to trade invoicing (Gopinath and Stein, 2021), market depth and liquidity (He, Krishnamurthy and Milbradt (2019), Coppola, Krishnamurthy and Xu (2023)), market power in the issuance of safe debt (Farhi and Maggiori, 2018), risk premia due to market

[1] Among others, see also Skaperdas (1992), Baliga and Sjöström (2004), Acemoglu et al. (2012), Acemoglu and Wolitzky (2011), Padró i Miquel and Yared (2012), and König et al. (2017).

[2] Gennaioli and Voth (2015) also endogenize the resource constraint of governments by considering the decision to engage in state capacity building, but they do not consider issues of debt markets, multiplicity, or hegemonic transitions.

[3] For example, see Maggi (2016), Becko and O'Connor (2024), Broner et al. (2024), Clayton, Maggiori and Schreger (2023), Liu and Yang (2024), Fernández-Villaverde, Mineyama and Song (2024), and Thoenig (2024).

[4] There is a large literature dating back to Persson and Svensson (1989) and Alesina and Tabellini (1990) that considers these issues, and a more recent literature on the pricing of political uncertainty in financial markets (Pastor and Veronesi (2012), Pastor and Veronesi (2020), Kelly, Pastor and Veronesi (2016), Koijen, Philipson and Uhlig (2016)). See Alesina and Passalacqua (2016)and Yared (2019) for surveys of the literature.

size and inflation credibility (Hassan (2013), Du, Pflueger and Schreger (2020)), and fiscal position (Chen et al., 2022). The connection between bond market dominance and military strength has received relatively less attention, despite the evidence that wars represent the majority of disasters priced in asset markets (Barro (2006), Barro and Ursua (2008)) and have often coincided with switches in bond market dominance (Eichengreen and Flandreau (2009), Chen et al. (2022)). We contribute to this literature by theoretically modeling the link and complementarity between military and financial dominance in a framework that elucidates issues of multiplicity, fragility, and hegemonic transitions.[5]

2 Motivating Empirical Facts

This section provides empirical evidence for three key features of our model. First, the expected winner from military conflict enjoys lower borrowing costs; second, this funding advantage increases with the ex-ante probability of a military conflict; and third, the winners from wars choose to devalue their debts by less than the losers.

2.1 Funding Advantage of Hegemon

Military hegemons have historically experienced a lower cost of funding, and this fact can be illustrated by considering the interest rates faced by governments around the time of hegemonic transitions. Such a transition occurred from Great Britain to the U.S. in the period spanning World War I and World War II, a period during which the British Empire weakened significantly, and the financial center of the world moved from London to New York. Figure 1 Panel A compares the 10-year government bond yield for Great Britain relative to the U.S. Prior to World War I, Britain was able to borrow at cheaper rates than the U.S. This pattern reversed after World War I, and the U.S. was able to borrow at the same rates or lower rates as Britain. After World War II, the U.S.'s funding advantage was solidified with lower interest rates relative to Britain.[6]

Another military and financial transition occurred at the end of the eighteenth century

[5] Our work also relates to the literature on multiplicity in sovereign default models dating back to Cole and Kehoe (2000). As in this literature, here self-fulfilling investor expectations change default risk. However, in our framework self-fulfilling expectations come at the expense of another borrower and occur through endogenous changes in geopolitical risk.

[6] Schmelzing (2020) and Ilzetzki, Reinhart and Rogoff (2022) have noted that on the eve of World War I, German imperial bonds experienced a brief 5-year period in which they dominated those of Great Britain. This means that Germany may have ascended to the role of global hegemon during the great naval arms race with Britain prior to its defeat in the calamity of World War I. This is consistent with our model in which the global hegemon is the country that is expected to be victorious while still potentially losing in the event of war.

from the Netherlands to the United Kingdom. Prior to this period, the Netherlands was a dominant global trading power, as exemplified by the Dutch East India company. The company maintained armies, built military structures, formed alliances, and conducted military campaigns to an extent unmatched by any other European power (Taylor (2018)). The Dutch military was also one of the fiercest in the world. The Dutch navy posed a formidable challenge to the British throughout the 16th, 17th and 18th centuries and was responsible for the last armed invasion of Great Britain during the Glorious Revolution of 1688 (e.g., Parker (2004), Schama (1988), Rodger (2006)). After the Dutch defeat in the Anglo-Dutch war of 1780, invasion by Napoleonic armies in 1795, and the bankruptcy of the Dutch East India company in 1796, the financial center of the world moved from Amsterdam to London. Figure 1 Panel B compares the borrowing rates of the Dutch and British government during that time period.[7] While the Dutch had lower borrowing rates before the 1800s, bond yields rose quickly after the Anglo-Dutch war of 1780 and particularly the Napoleonic invasion. After that period, the Dutch lost their funding advantage to the British.

2.2 Geopolitical Risk and Funding Advantage

We now show that the funding advantage of the hegemon rises with the risk of military conflict. Figure 2 plots the difference between the average developed country three-year government bond yields and the U.S. three-year government bond yields against the U.S. geopolitical risk index from Caldara and Iacoviello (2022). We see that when geopolitical risk is high, the spread—capturing the U.S. funding advantage—rises. This relationship is strong with a correlation coefficient of 45 percent.

Figure 3 considers the behavior of Ukrainian and Russian bond prices (which are inversely related to yields) around the Russian invasion of Ukraine on February 24, 2022. Both of these prices relative to the price of U.S. bonds fall precipitously during the invasion, again showing that the funding advantage of the U.S. is rising during this period of rising geopolitical tension.

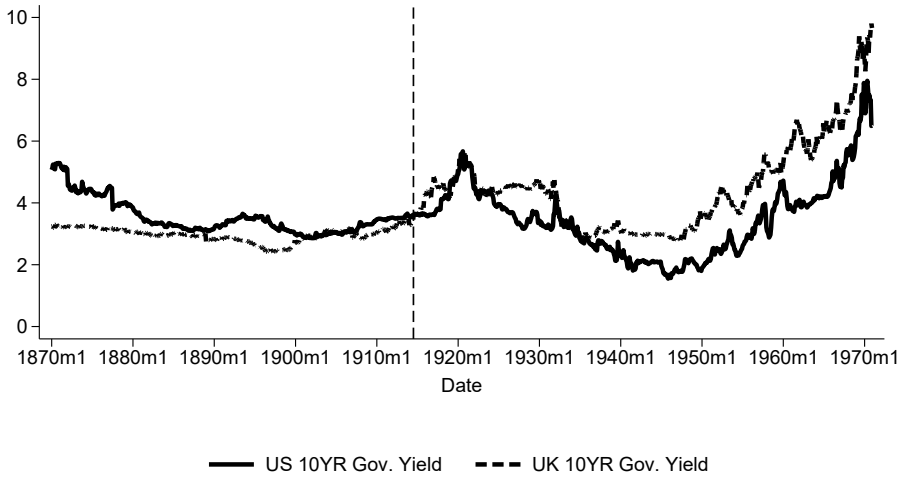
Our findings in Figures 2 and 3 are consistent with many other observations in the literature. They are related to the historical analysis of Barro (2006) who shows that U.S. borrowing costs generally fall during wars. Using a different geopolitical conflict index than we do here, Hirshleifer, Mai and Pukthuanthong (2023) provide complementary recent evidence that U.S. bond risk premia decline with global conflict. Similar patterns are found by Rigobon and Sack (2005) who study the period around the second Iraq war.[8]

[7] This figure is similar to Figure 11 of Chen et al. (2022) who also analyze transitions in financial dominance but focus on fiscal spending rather than the interaction with military dominance.

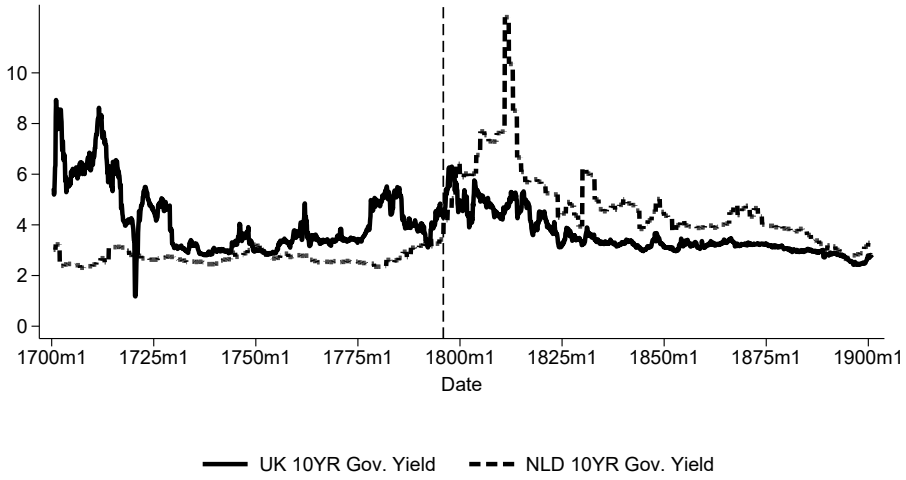
[8] These findings are consistent with asset pricing models admitting a time-varying probability of a disaster

Figure 1: Changing Military Dominance and Bond Yields in History

Panel A: U.S./U.K. Bond Yields around World War I



Panel B: U.K./Netherlands Bond Yields around Napoleonic Wars

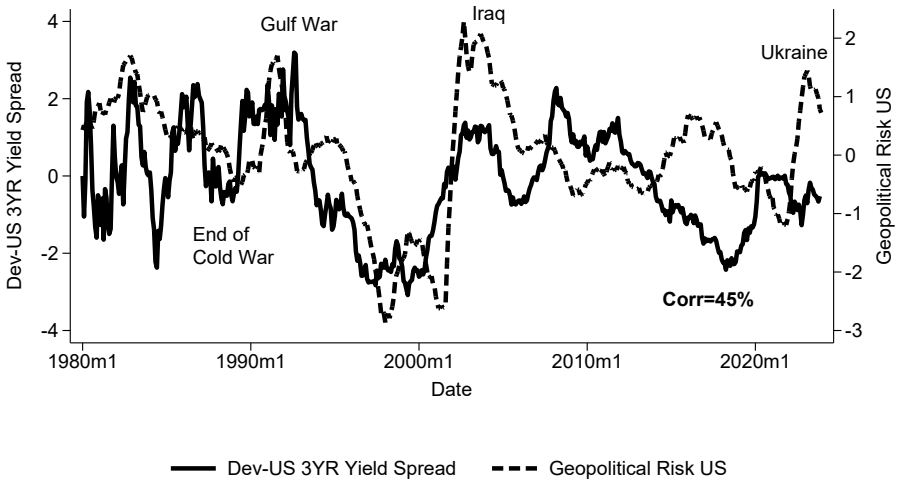


Panel A plots the 10-year government bond yields for the U.S. and the U.K. from January 1870 until December 1970 (end of Bretton Woods). A vertical line indicates the start of World War I (July 1914). Panel B plots the 10-year government bond yields for the U.K. and Netherlands from 1700 (when U.K. yields become available) until 1900. The vertical line indicates 1795, when the Netherlands was invaded by Napoleon. U.S. and U.K. bond yields are from Global Financial Data. Netherlands bond yields are from Korevaar, Francke and Eichholtz (2020). The Netherlands yield for the 1700s uses data for the Province of Holland.

(Gabaix (2012), Wachter (2013)). In such frameworks, government bond yields are driven down by a negative risk premium if they admit little risk of default through inflation in a disaster, but they are driven up by a positive risk premium if inflation is expected to jump up conditional on a disaster.

These empirical findings are also consistent with previous work on the relative volatility of bond returns during the British Empire. Ferguson (2006), for example, shows that British bond prices barely fluctuated in response to geopolitical events in the second half of the nineteenth century, whereas the bond prices of all other European economies tended to drop during periods of rising geopolitical tensions.

Figure 2: Developed Country-U.S. Borrowing Costs vs. Geopolitical Risk



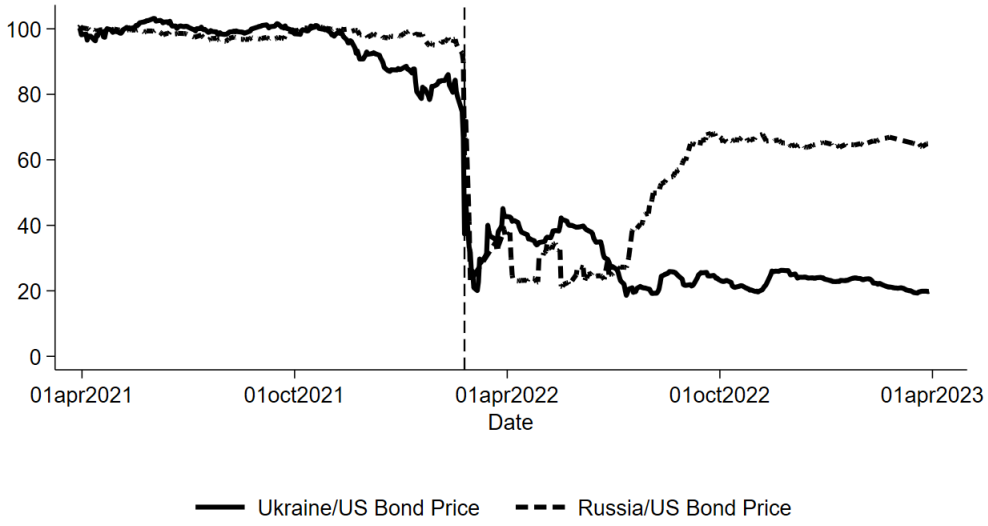
This figure plots the spread between an equal-weighted average three-year government yield from six developed countries (Australia, France, Germany, Italy, Japan, U.K.) minus the three-year U.S. government bond yield against the historical geopolitical risk index for the U.S (in annualized %). (GPHRC USA) from Caldara and Iacoviello (2022). The yield spread is stochastically de-trended by subtracting a 10-year moving average, with all yield data from Global Financial Data. The German three-year government bond yield is extended with Bloomberg data after 2021. We take the log of the geopolitical risk index and a 12-month moving average to reduce the effect of outliers. The sample is monthly and runs from from January 1980 to November 2023. Geopolitical risk is standardized to have a mean of zero and standard deviation 1.

2.3 Post-War Devaluations

Our final empirical pattern is that countries that are defeated in war devalue their debts by more than those that are victorious. Table 1 shows the average inflation rates for the U.S. and defeated countries for the ten years following World Wars I and II. Inflation in the U.S. was significantly lower than that in the defeated countries.[9]

[9]The German hyperinflation of the 1920s following World War I has been well documented. See Brunnermeier et al. (2023) for an analysis. Table 1 does not cover inflation in all defeated countries involved in World War I or World War II. Excluding countries with data gaps, such as the Ottoman Empire after World War I and Hungary after World War II, is conservative in that these countries generally experienced high inflation, with Hungary experiencing the highest inflation rate ever recorded in 1946.

Figure 3: Government Bond Prices around Ukraine Invasion



This figure plots scaled daily bond prices from April 2021 until April 2023 for a Russian and a Ukrainian bond, both of which are denominated in U.S. Dollars. Both bond prices are divided by the price of a U.S. bond price of matching maturity. We use Bloomberg tickers UKRAIN 7 $\frac{3}{4}$ 09/01/2029, RUSSIA 7 $\frac{1}{2}$ 03/31/2030, and T 1 5/8 08/15/29 Gov. A vertical line indicates the surprise invasion of Ukraine by Russia on February 24, 2022.

This pattern is mirrored by the experience of wars in the nineteenth century, a period during which many countries experienced significant debasement, currency depreciation, and inflation, whereas Great Britain–the hegemon at the time–did not.[10] In particular, Austria, Russia, and Turkey experienced a cumulative decline in silver content of 50 percent or more during the 19th century, most of it realized during the Napoleonic wars of 1799-1815 (Reinhart and Rogoff (2009)). By contrast, Great Britain experienced a cumulative decline in silver content of only 6 percent over the 19th century.[11]

3 Two-Period Example

Our complete model involves two countries in an infinite horizon framework that make decisions over military and public goods spending, borrowing, and default in every period.

[10]This evidence is complemented by that in Federle et al. (2024) who find that the realization of conflict, even in a neighboring country, has a detrimental effect on GDP.

[11]See Reinhart and Rogoff (2009), Table 11.2. The samples for Germany and Portugal in that table are significantly shorter, so we exclude them from our discussion. Even the U.S. experienced one significant episode of default through inflation during the 19th century, with inflation peaking at 24 percent in 1864 after the Civil War.

Table 1: Inflation and Geopolitical Conflict in the 20th Century

Avg. 10-Year Inflation Rate	post-WWI	post-WWII
U.S.	1.68	4.20
Avg. Defeated Countries	3.84E+08	33.04

This table reports 10-year post-war inflation rates in an annualized percent for the U.S. vs. an equal-weighted average of defeated countries. For the 10 years post World War I (1919-1928), the included defeated countries are Austria and Germany. For the 10 years post World War II (1946-1955), the included defeated countries are Austria, Germany, Italy, and Japan. U.S. monthly CPI inflation data is from Robert Shiller's website. All other CPI inflation data is from Global Financial Data.

To facilitate the discussion of equilibrium uniqueness and multiplicity, we begin by presenting a simple two-period example.

3.1 Environment

There are two time periods $t = 0, 1$. There are two countries $i = 1, 2$, each of mass 1, and risk neutral global investors with deep pockets. For simplicity, we assume the two countries are identical in every dimension other than exogenous military technology.[12]

The date 0 government budget constraint of country i is

$$g_{i0} + m_i = \tau + q_i b_i,$$

where $g_{i0} \geq 0$ is non-military government spending, m_i is military spending, $\tau > 0$ is an exogenous tax revenue, q_i is the global bond price for country i bonds, and $b_i \geq 0$ is the level of borrowing. The date 1 government budget constraint of country i under peace is

$$g_{i1} = \tau \left(1 - \chi d_i\right) - b_i \left(1 - d_i\right),$$

where $g_{i1} \geq 0$, $d_i = \{0, 1\}$ is country i's default decision, and $\chi \in (0, 1]$ is a default cost which is proportional to the tax revenue of the government. The parameter χ can be interpreted as a reduced form representation of a country's debt capacity resulting from the institutional frameworks that restrain sovereigns from defaulting on their debt. The post-Glorious Revolution constraints on British monarchs are an example of such an institutional framework (e.g., North and Weingast (1989)), as these fostered the development of the British government bond market.

[12]While we discuss our model in the context of two countries that are adversarial to one another (e.g., U.S. and China in the present), an equivalent interpretation of our model involves two countries that are allied with one another (e.g., U.S. and Great Britain in the world wars), where the crucial force in the model is that the militarily more dominant country is more likely to repay its debt once the war is over.

The date 1 budget constraint under war if the country wins is identical to that under peace. However, if the country loses the war, the tax endowment is destroyed and the budget constraint becomes

$$g_{i1} = -b_i (1 - d_i).$$

Note that this implies that if country i loses the war, then it defaults on its debt since default is cost-less. This feature is consistent with the third empirical observation that losers from wars devalue their debt by more than winners.[13]

The probability of war is exogenous and equal to $\phi \in (0, 1)$. Country i's probability of winning the war is $w_i (m_i, m_{-i})$ which is increasing in m_i and decreasing in m_{-i}. Throughout our paper, we consider the difference contest function of Hirshleifer (1989):

$$w_1 (m_1, m_2) = \frac{exp (A + m_1)}{exp (A + m_1) + exp (m_2)} \equiv F (A + m_1 - m_2). \quad (1)$$

This implies that the probability of winning for country 2 equals $w_2 (m_2, m_1) = 1 - F (A + m_1 - m_2)$. The parameter $A \geq 0$ denotes the exogenous military advantage of country 1 due to factors such as geography or technology. The case with $A = 0$ corresponds to the situation where both countries have equal chances of dominating in a military conflict under symmetric military spending.

The preferences of country i are

$$\mathbb{E} \{g_{i0} + \kappa g_{i1} - \lambda\}.$$

κ takes a value of 1 under peace and a value of $\theta > 1$ under war, capturing the fact that marginal utility of resources under war is higher.[14] We refer to the parameter θ as the war premium. λ takes a value of 0 under peace or war with victory or a value of $\gamma > 0$ under war with defeat.[15]

International investors have preferences

$$\mathbb{E} \{c_0 + \kappa c_1\},$$

where c_t represents the consumption of investors.

[13] In the two-period model, default is necessary for feasibility of non-negative government spending. In the infinite horizon model of the next section, default is potentially not necessary for feasibility, but it is strictly optimal.

[14] Our results for the two-period model can easily be solved and are robust in an environment with concave preferences, but this nonlinearity would make the fully dynamic model non-tractable.

[15] Without loss of generality, we can easily introduce a cost of war with victory which is smaller than the cost of war with defeat.

The order of events is as follows. At date 0, both countries simultaneously choose g_{i0} and b_i, financial markets open and clear, and countries choose m_i to satisfy the government budget constraint.[16]

3.2 Equilibrium

We solve the program by backward induction. Under war with defeat at date 1, country i defaults on any outstanding debt. Under war with victory or under peace, it repays its debt only if $b_i \leq \tau\chi$. Thus, $\tau\chi$ captures the maximal amount of debt a country can borrow. For any $b_i \in (0, \tau\chi]$, bond prices satisfy

$$q_i\left(m_i, m_{-i}\right) = 1 - \phi + \phi\theta w_i\left(m_i, m_{-i}\right). \tag{2}$$

Observe that the bond price depends on the probability of war ϕ, the war premium θ, and the likelihood of victory $w_i\left(m_i, m_{-i}\right)$.[17]

Substituting the bond pricing equation into the country's budget constraint and into its preferences, country i's optimization program, assuming that it chooses $b_i > 0$, can thus be written as

$$\max_{m_i, b_i \in (0,\tau\chi]} \left\{ \begin{array}{c} \tau + \left(1 - \phi + \phi\theta w_i\left(m_i, m_{-i}\right)\right) b_i - m_i \\ + \left(1 - \phi + \phi\theta w_i\left(m_i, m_{-i}\right)\right)(\tau - b_i) \\ -\phi\theta\left(1 - w_i\left(m_i, m_{-i}\right)\right)\gamma \end{array} \right\} \tag{3}$$

subject to

$$\tau + \left(1 - \phi + \phi\theta w_i\left(m_i, m_{-i}\right)\right) b_i \geq m_i, \tag{4}$$

where the last constraint corresponds to the non-negativity constraint on non-military government spending at date 0. Observe that the terms in the objective function involving debt cancel out. This is because the government and international lenders have the same preferences, and the revenue raised from bond issuance at date 0 equals the expected payment to international lenders at date 1. Thus, if the non-negativity constraint on non-military spending does not bind for a country, the equilibrium choice of debt is irrelevant and independent of military spending, which is a reflection of Ricardian Equivalence (Barro (1974)). To ensure an interaction between military spending and debt and thereby break Ricardian Equivalence, we assume that the cost of war γ is sufficiently large that debt capacity constrains military

[16] The staggered timing of the choices of g_{i0} (which must be non-negative) and m_i is necessary to ensure that the government budget constraint is satisfied off the equilibrium path.

[17] Our model thus delivers a positive average risk premium on risky government bonds, which is consistent with long-run evidence on bond risk premia documented by Meyer, Reinhart and Trebesch (2022).

spending.[18]

Assumption 1. $\gamma \to \infty$.

Under this assumption, the cost of losing a war is sufficiently severe that both countries choose zero spending on public goods at date 0.[19] Since constraint (4) binds for both countries, it follows that $b_1 = b_2 = \tau\chi$. Since (4) holds with equality for both countries, it follows that these can be combined, taking into account the functional form for the contest function (1) and bond pricing equation (2):

$$\underbrace{m_1 - m_2}_{\text{Gap in military expenditure}} = \tau\chi \times (q_1 - q_2) = \underbrace{\tau\chi \times \phi\theta\,(2F(A + m_1 - m_2) - 1)}_{\text{Gap in bond market revenue}}. \tag{5}$$

Equation (5) states that in equilibrium, the gap in military expenditure between countries is proportional to the gap in bond market revenue, which is determined by the gap in bond prices multiplied by debt capacity χ and the tax endowment τ.

3.3 Uniqueness and Multiplicity

We now examine what condition (5) implies for the uniqueness or multiplicity of equilibria. We use m_i^* to denote the equilibrium values for country i's military investment and q_i^* for the equilibrium bond price. In this two-period model, we define equilibrium stability in the sense of Fudenberg and Tirole (1991, p. 24). They consider an equilibrium to be stable if it is the result of an iterative tatonnement adjustment process in the neighborhood of a Nash equilibrium which converges back to the same equilibrium. An analogous definition holds here if we consider the bond market to be a third player along with the two countries.

Proposition 1 *There exists a debt capacity threshold $\chi' > 0$ that is a positive function of the exogenous advantage A and a negative function of the war probability ϕ and the war premium θ that satisfies the following properties:*

i) If $\chi < \chi'$, then the equilibrium is unique. If country 1 does not have an exogenous military advantage, i.e. $A = 0$, then the equilibrium is symmetric with $w_1(m_1^*, m_2^*) = \frac{1}{2}$. If country 1 has an exogenous military advantage, i.e. $A > 0$, then the equilibrium is asymmetric with country 1 dominating militarily and financially: $w_1(m_1^*, m_2^*) > \frac{1}{2}$, $m_1^* > m_2^*$, and $q_1^* > q_2^*$.

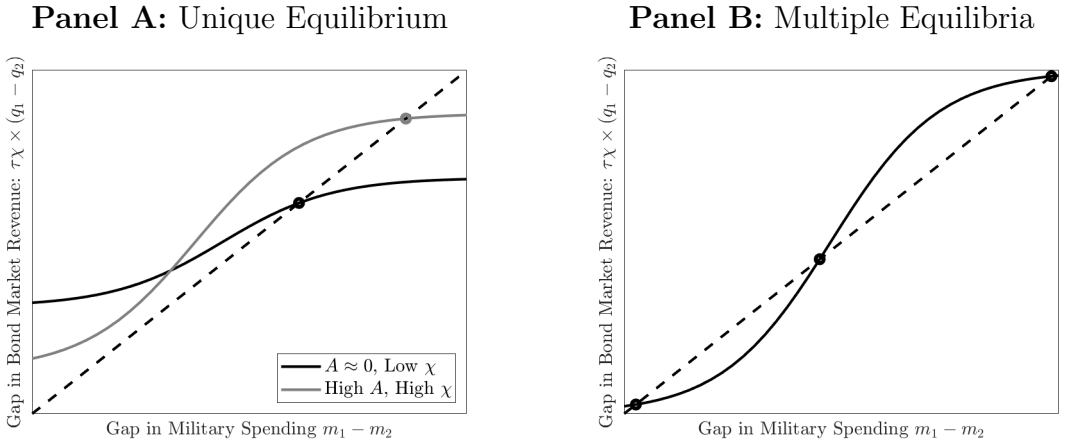
[18]There is a finite threshold for γ above which our results hold; we take this value to infinity to reduce notation.

[19]Note that one can interpret the level of equilibrium government spending, which is zero in our model, to reflect necessary or pre-committed public goods.

ii) If $\chi > \chi'$, then there are two stable equilibria and one unstable equilibrium. In one stable equilibrium, country 1 dominates militarily and financially: $w_1(m_1^*, m_2^*) > \frac{1}{2}$, $m_1^* > m_2^*$, and $q_1^* > q_2^*$. In the other, country 2 dominates militarily and financially: $w_1(m_1^*, m_2^*) < \frac{1}{2}$, $m_1^* < m_2^*$, and $q_1^* < q_2^*$.

Figure 4 Panel A illustrates two examples in which the equilibrium is unique and the condition of Proposition 1 i) is satisfied. It depicts equation (5), which shows that the difference in military spending between countries 1 and 2 (shown on the x-axis) must match the difference in their revenue from international bond markets (shown on the y-axis). This revenue difference is influenced by the relative probability of winning the war, which in turn is a function of how much each country spends on its military. The dotted line corresponds to the 45 degree line and the black and gray solid lines correspond to the gap in bond market revenue as a function of the gap in military capacity in two different cases. In each case, an equilibrium corresponds to the intersection of the solid line and the 45 degree line.

Figure 4: Equilibria in Two-Period Model



This figure plots the equilibrium condition (5) for different cases. A solid line depicts the gap in bond market revenue. An equilibrium is defined by the intersection of the solid line and the dotted 45 degree line. Panel A depicts two examples of unique equilibria and Panel B depicts an example of multiple equilibria. In Panel A, the solid black line represents the gap in bond market revenue under a low exogenous military advantage (i.e. $A \approx 0$). Debt capacity is sufficiently low that the slope of the curve is always less than one. The solid gray line represents the gap in bond market revenue under a high exogenous military advantage $A >> 0$ and high debt capacity. The maximum slope of this curve exceeds one, but the exogenous military advantage A is large enough to ensure equilibrium uniqueness. Panel B depicts the gap in bond market revenue under a small exogenous military advantage (i.e. $A \approx 0$) and high debt capacity $\chi > \chi'$, so that the maximum slope of this curve exceeds one and there are multiple equilibria.

The solid black line corresponds to the case for which the military advantage for country 1 is small with $A \approx 0$. Debt capacity is sufficiently small that the slope of the revenue gap curve is below 1 for all values of $m_1 - m_2$. In this case, the bond market revenue gap

increases less than one-for-one with the gap in military expenditures. As a result, there is only one point of intersection between the solid and dotted lines, and the equilibrium is unique. Country 1 invests relatively more in the military, wins the war in expectation, faces lower funding costs, and raises more revenue in the bond market.

The solid gray line corresponds to the case for which there is a sizable military advantage for country 1 so that $A >> 0$, and while debt capacity satisfies $\chi < \chi'$, it is still sufficiently high that the slope of the bond market revenue gap curve exceeds 1 for some values of $m_1 - m_2$. Because debt capacity is sufficiently large, relative bond market revenue depends more than one for one on relative military spending for some range of relative military spending. However, despite this, there is only one point of intersection between the solid and dotted lines because the military advantage (which shifts the solid line) is sufficiently large.

Figure 4 Panel B illustrates an example of multiplicity in the case where the military advantage for country 1 is small with $A \approx 0$. Recall that the value of χ' is an increasing function of A, meaning that a small value for A facilitates equilibrium multiplicity. Because $\chi > \chi'$, the slope of the revenue gap curve exceeds 1 for some values of $m_1 - m_2$, similar to the second case in Panel A. In contrast to that case, the exogenous advantage for country 1 in Panel B is not large, so the bond market revenue curve is lower than the gray line in Panel A, leading to the emergence of multiple equilibria. Intuitively, even though there exists an equilibrium where the exogenously stronger country wins the war in expectation and benefits from a funding advantage, there is another equilibrium where the exogenously weaker country can overwhelm the stronger country with a sufficiently large military investment. Bond markets rationally anticipate a higher probability of victory for the weaker country in this equilibrium, thus providing it with a funding advantage that reinforces its military advantage.[20]

Observe that this result regarding multiplicity holds even if there is no exogenous military advantage and $A = 0$. In that case, there are two stable equilibria where either country 1 or country 2 dominates, despite the fact that neither country has an exogenous military advantage.

The examples of uniqueness and multiplicity clarify that our model is consistent with the first and second empirical facts discussed in the previous section. The country that wins the war in expectation also benefits from a funding advantage since it faces lower borrowing costs. Moreover, this funding advantage rises in the probability of war, holding relative

[20] Our multiplicity result does not rely on our choice of a difference contest function which convex-concave. We have numerically verified that this multiplicity also arises with ratio contest functions which are concave. See Hirshleifer (1989) for a discussion of contest functions.

military spending fixed. This is clear from the bond market pricing equation (2) that shows that the difference between bond prices is rising in the war probability ϕ when military spending is held constant. The following corollary expands on these ideas by examining the complementarity between military success and funding advantage.

Corollary 1 *Consider the equilibrium comparative statics in the neighborhood of the stable equilibrium in which country i dominates. Country i's probability of winning the war $w_i(m_i^*, m_{-i}^*)$ and funding advantage $q_i^* - q_{-i}^*$ both increase in debt capacity χ, in the war probability ϕ, the war premium θ, and in country i's exogenous military advantage.*

The victorious country's military and financial dominance are amplified by several features. For example, as debt capacity increases, so does the victor's ability to borrow and invest in military capacity. This ability in turn increases the probability of winning the war, which reduces that country's likelihood of default. Thus, at higher debt capacity, international investors are even more willing to hold the victorious country's bonds, which increases their price and reduces the relative funding costs of the victorious country.

An analogous logic holds if the probability of war or the war premium increase. In both cases, the victorious country's funding advantage rises holding relative military spending fixed, since investors are even more willing to hold that country's bonds. In response, the victorious country invests relatively more in the military while the defeated country invests relatively less, thus amplifying the victorious country's military and financial dominance.[21]

A related observation involves the feedback loop from military advantage to funding advantage. Consider an increase in the exogenous advantage A starting from an equilibrium in which the exogenously stronger country is expected to win a war. Holding relative military spending fixed, an increase in the exogenous military advantage increases the funding advantage of the stronger country since it increases its likelihood of victory and reduces its likelihood of default. In response, the stronger country invests relatively more in the military, which further boosts its geopolitical and funding advantage.

We conclude our analysis of the two-period model with one final observation relating to the fact that the cutoff χ' is a negative function of the war probability ϕ and the war premium θ, as stated in Proposition 1. Combined with Corollary 1, this means that financial market factors that enhance the complementarity between military and financial dominance for the exogenously stronger country–such as higher debt capacity, war probability, and war premium–also invite the possibility of equilibrium multiplicity, with the weaker country

[21] This means that if financial markets put a lot of weight on the war state, as they would in a model of rare disasters (Barro (2006)), then the feedback mechanism from military investment to bond market revenue is strengthened.

successfully using financial markets to overwhelm the stronger country. In this sense, military strength derived from the ability to borrow in financial markets is inherently different from exogenous military advantage, which, while also enhancing the complementarity between military and financial dominance, reduces the potential for equilibrium multiplicity.

4 Infinite Horizon Analysis

We now examine our results in an infinite horizon environment in which peace or war can occur in every date. We begin by showing that the insights regarding equilibrium multiplicity in the two-period model translate to a dynamic environment, in which equilibrium multiplicity now corresponds to the multiplicity of steady states conditional on the two countries remaining at peace. Moreover, we evaluate transition dynamics around those steady states and examine the conditions under which those dynamics are uniquely determined. We use this characterization to delineate conditions under which a hegemonic transition from one peaceful steady state to another is possible.

4.1 Environment

There are periods $t = 0, 1, ...$ and three potential states of the world: peace, war with victory for country 1, or war with defeat for country 1.

The date t government budget constraint of country i under peace is

$$g_{it} + m_{it} = \tau (1 - \chi d_{it}) + (1 - \delta) m_{it-1} + q_{it} b_{it} - b_{it-1} (1 - d_{it}) .$$

As in the two period model, $g_{it} \geq 0$ is non-military government spending, $\tau > 0$ is an exogenous tax revenue, q_{it} is the global bond price for country i bonds, $b_{it} \geq 0$ is the level of borrowing, $d_{it} = \{0, 1\}$ is country i's default decision, and $\chi \in (0, 1]$ is a default cost as a share of tax revenue. Note that in contrast to the two-period model, m_{it}, which we refer to as military capital, is accumulated over time, with a depreciation rate of $\delta \in (0, 1)$, so that $m_{it} - (1 - \delta) m_{it-1}$ is the period t level of military spending.

The government budget constraint under war with victory is

$$g_{it} + m_{it} = \tau (1 - \chi d_{it}) + q_{it} b_{it} - b_{it-1} (1 - d_{it}) ,$$

where we have assumed that military capital is destroyed during war. The budget constraint under war with defeat is

$$g_{it} + m_{it} = q_{it} b_{it} - b_{it-1} (1 - d_{it}) ,$$

where again military capital is destroyed in war. As in the two-period model, if the country loses the war, its tax endowment is also destroyed.

The probability of war in every date is exogenous and equal to $\phi \in (0,1)$. Country i's probability of winning the war at $t+1$ is $w_i(m_{it}, m_{-it})$, with $w_1(m_{1t}, m_{2t}) = F(A + m_{1t} - m_{2t})$ and $w_2(m_{2t}, m_{1t}) = 1 - F(A + m_{1t} - m_{2t})$ as defined in the contest function in (1).

There are overlapping generations of governments that make decisions at date t, where the date t government has preferences represented by:[22]

$$\mathbb{E}\{g_{it} + \kappa_{t+1} g_{it+1} - \lambda_{t+1}\}.$$

κ_{t+1} takes a value of 1 under peace at $t+1$ and a value of $\theta > 1$ under war, with θ corresponding to the war premium as in the two-period model. λ_{t+1} takes a value of 0 under peace or war with victory at $t+1$ or a value of $\gamma > 0$ under war with defeat. This parameter captures the cost of losing a war.

International investors at date t have preferences

$$\mathbb{E}\{c_t + \kappa_{t+1} c_{t+1}\},$$

where c_t represents the consumption of investors.

At any date t, both countries simultaneously choose $\{g_{it}, b_{it}, d_{it}\}$, financial markets open and clear, and countries choose m_{it} to satisfy the government budget constraint.

4.2 Equilibrium

Because each government solves a two-period problem, we can solve the program by backward induction. Under war with defeat at date t, country i defaults on any outstanding debt by the same logic as in the two-period model. Moreover, under war with victory or under peace, it repays its debt only if $b_{it} \leq \tau\chi$, and therefore $\tau\chi$ captures the maximal amount of debt a country can borrow. For any $b_{it} \in (0, \tau\chi]$, bond prices satisfy the analog of (2):

$$q_{it}(m_{it}, m_{-it}) = 1 - \phi + \phi\theta w_i(m_{it}, m_{-it}). \tag{6}$$

Now consider the optimization problem of a government at date t. By the same logic as in the two-period model, under Assumption 1 that states that $\gamma \to \infty$, it follows that $g_{it} = 0$ and $b_{it} = \tau\chi$ for all t. The dynamic budget constraint under peace at t can thus be rewritten

[22] If instead governments were infinitely lived, then additional strategic considerations involving coordination across countries would need to be taken into account, which would lead to even more equilibrium multiplicity than what we highlight in this simpler case.

as

$$m_{it} = \tau + (1-\delta)\, m_{it-1} - (\phi - \phi\theta w_i\,(m_{it}, m_{-it}))\,\tau\chi. \tag{7}$$

The budget constraints for countries 1 and 2 can be combined, taking into account the functional form for the contest function, and defining the gap in military capital $\mu_t \equiv m_{1t} - m_{2t}$ to yield:

$$\underbrace{\mu_t - (1-\delta)\,\mu_{t-1}}_{\text{Gap in military expenditure}} = \underbrace{\tau\chi \times \phi\theta\,(2F(A+\mu_t) - 1)}_{\text{Gap in bond market revenue}}. \tag{8}$$

Equation (8) is analogous to equation (5) since it states that in equilibrium, the gap in military expenditure between countries is proportional to the gap in bond market revenue. Relative to the two-period model, that gap in military expenditure is a function of the gap in military capital at t as well as a $t-1$.

We can write the equivalent of condition (8) under war with victory for country 1 as

$$\mu_t = \tau\,(1-\chi) + \tau\chi \times \phi\theta\,(2F(A+\mu_t) - 1), \tag{9}$$

with an analogous condition that would hold under war with defeat for country 1:

$$\mu_t = -\tau\,(1-\chi) + \tau\chi \times \phi\theta\,(2F(A+\mu_t) - 1). \tag{10}$$

During peace at t, condition (8) defines a correspondence from μ_{t-1} to μ_t which may or may not be one-to-one. During war at t with victory for country 1, condition (9) defines a value for μ_t that is independent of history. Condition (10) defines an analogous value for μ_t during war at t with defeat for country 1. Together, these three equations characterize equilibrium dynamics for μ_t.

4.3 Uniqueness and Multiplicity

We define a steady state μ_{ss} as the level of μ_t to which the equilibrium converges after an infinite sequence of peaceful states. We focus our attention on monotonically stable steady states, that is, a steady state value of μ_{ss} for which the transition dynamics under peace defined by equation (8) admit a monotonic transition path for μ_t in the neighborhood of μ_{ss}.

Proposition 2 *There exists a debt capacity threshold χ' that is a positive function of the exogenous advantage A and the depreciation rate δ and a negative function of the war probability ϕ and the war premium θ that satisfies the following properties:*

i) If $\chi < \chi'$, then the steady state is unique and is monotonically stable. If country 1 does not have an exogenous military advantage, i.e. $A = 0$, then the steady state is symmetric with $w_1(m^*_{1ss}, m^*_{2ss}) = \frac{1}{2}$. If country 1 has an exogenous military advantage, i.e. $A > 0$, then the steady state is asymmetric with country 1 dominating militarily and financially: $w_1(m^*_{1ss}, m^*_{2ss}) > \frac{1}{2}$, $m^*_{1ss} > m^*_{2ss}$, and $q^*_{1ss} > q^*_{2ss}$.

ii) If $\chi > \chi'$, then there are two monotonically stable steady states and one monotonically unstable steady state. In one monotonically stable steady state, country 1 dominates militarily and financially: $w_1(m^*_{1ss}, m^*_{2ss}) > \frac{1}{2}$, $m^*_{1ss} > m^*_{2ss}$, and $q^*_{1ss} > q^*_{2ss}$. In the other, country 2 dominates militarily and financially: $w_1(m^*_{1ss}, m^*_{2ss}) < \frac{1}{2}$, $m^*_{1ss} < m^*_{2ss}$, and $q^*_{1ss} < q^*_{2ss}$.

Figure 5 Panel A illustrates an example with a unique steady state. The x-axis depicts the value of μ_{t-1} and the y-axis depicts the value of μ_t. The solid line shows the correspondence defined by condition (8), and the dotted line is the 45 degree line. The intersection of these two lines is a steady state, and in this example, there is a single intersection and a unique equilibrium. The logic for this result is similar to the two-period case. Because debt capacity is below χ', the exogenously weaker country is unable to use financial markets to its advantage to overwhelm the stronger country, and the exogenously stronger country has a geopolitical and funding advantage that complement each other.

Figure 5 Panels B and C illustrate two examples with multiple steady states. In both examples, the correspondence defined by condition (8) intersects the 45 degree line three times, with each intersection depicting a steady state. The middle steady state is not monotonically stable, but the steady states to the right and to the left of the middle steady state are monotonically stable.[23] Note that in the example in Panel B, the correspondence from μ_{t-1} to μ_t is one-to-one, whereas this is not the case in Panel C. We will return to this difference in the next subsection in our discussion of transition dynamics.

In each of the examples depicted in Panels B and C, debt capacity is sufficiently large that the exogenously weaker country is able to overwhelm the stronger country by borrowing in financial markets. As in the two-period model, this multiplicity requires a higher probability of war, a higher war premium, and a smaller exogenous advantage for country 1. In addition, Proposition 2 shows that a lower depreciation rate for military capital increases the scope for multiple steady states by making it more feasible for the exogenously weaker country

[23] In both panels, the transition path diverges away from the middle steady state. There are also examples where the slope of the correspondence at the middle steady state is between 0 and -1, and convergent transitions paths around the middle steady state exist; however, such paths features oscillations and are thus not monotonic.

to accumulate military capital over time, and thereby overwhelm the stronger country in steady state.

While not formally repeated here, the comparative statics for equilibria in the two-period model in Corollary 1 also translate directly to steady states of the dynamic model. Hence, in a steady state where country i dominates, the steady-state funding advantage increases in the debt capacity χ, the war probability ϕ, the war premium θ, and country i's exogenous advantage. As before, a stronger ability to tap international bond markets or a stronger exogenous military advantage amplifies the dominant country's funding and military advantage.

4.4 Transition Dynamics

Each panel in Figure 5 depicts a monotonic transition path in the neighborhood of the monotonically stable steady state where the exogenously stronger country dominates. The slope of the mapping from μ_{t-1} to μ_t at the intersection defining the such a steady state is between 0 and 1; thus, a convergent transition path around the steady state exists. It is clear that in the case of a unique steady state in Panel A, such a transition path is globally unique: starting from any value of μ_0, multiple consecutive periods of peace cause μ_t to converge towards the steady-state value μ_{ss}. For instance, if $\mu_0 < \mu_{ss}$, then μ_t rises over time along with the military and financial advantage of country 1.

We now turn to transition dynamics in the more complicated cases with multiple steady states.

Proposition 3 *Suppose that $\chi > \chi'$ so that there are two monotonically stable steady states. Then there exists a debt capacity threshold χ'' that is a negative function of the war probability ϕ and the war premium θ that satisfies the following properties:*

i) ***(geopolitical hysteresis)*** *If $\chi < \chi''$, then there does not exist any non-monotonic peaceful transition path to a monotonically stable steady state, and initial conditions determine the steady state.*

ii) ***(geopolitical fragility)*** *If $\chi > \chi''$, then there exists a non-monotonic peaceful transition path to each of the monotonically stable steady states, and the same initial conditions can lead to different steady states.*

Figure 5 Panel B depicts an example of *geopolitical hysteresis* with $\chi' < \chi \leq \chi''$. Since the correspondence from μ_{t-1} to μ_t is one-to-one, the transition path under consecutive peaceful periods is uniquely determined starting from any initial condition μ_0. If μ_0 is sufficiently high, then the equilibrium converges to the steady state in which country 1 dominates militarily

and financially. The opposite occurs if μ_0 is sufficiently low. As such, the relative levels of military and financial dominance at a point in time determine the eventual identity of the dominant country after multiple periods of peace.

Figure 5 Panel C depicts a situation of *geopolitical fragility* with $\chi > \chi'' > \chi'$. While a monotonic path towards each of the monotonically stable steady state exists, there also exists non-monotonic paths that begin within the zone of geopolitical fragility. This is a zone where where the starting gap in military capacity μ_{t-1} is neither too large nor to small. Within this zone, the correspondence from μ_{t-1} to μ_t is one-to-many; there can be multiple peaceful transition paths that begin from some initial condition μ_0 that lead to either steady state. In this example, the relative military and financial dominance at a point in time need not determine the eventual identity of the dominant country after multiple periods of peace.

Within the zone of fragility, the different possible equilibrium paths arise because of the interaction between financial markets and military capacity. Forward-looking rational investors link the "exorbitant privilege" of the victorious country to its military dominance. Market expectations can quickly change, causing the erosion of a country's previous financial and military dominance. Once the equilibrium transitions out of the zone of fragility, it settles on a convergence path towards a new steady state, with the new prospective victor establishing greater and greater military and financial dominance over time.

Higher debt capacity matters for this analysis, because it makes geopolitical fragility more likely than hysteresis. The intuition is that when debt capacity is high, bond market coordination around a potential victor has a much greater effect on countries' ability to raise financing, amplifying the effect on relative military strength. Thus, those same factors that make steady-state multiplicity more likely–high debt capacity, high war probability, and high war premium–also make geopolitical fragility more likely.

Using this proposition, we now investigate conditions under which a hegemonic transition can occur. We define a hegemonic transition as a transition path–which may include the realization of war–that begins in one monotonically stable steady state but eventually converges to the other monotonically stable steady state.

Corollary 2 *Suppose that $\chi > \chi'$ so that there are two monotonically stable steady states.*

i) If $\chi < \chi''$, then there does not exist a hegemonic transition path that avoids war.

ii) If $\chi > \chi''$, then there exists a hegemonic transition path that avoids war if the depreciation rate δ is sufficiently high.

The logic for the first part of the corollary stems from the first part of Proposition 3. Any path that avoids war is predetermined by initial conditions μ_0, thus implying that a

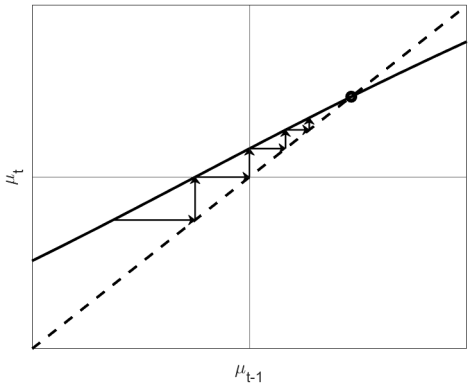
hegemonic transition in the absence of war is not possible. To see how war could lead to a hegemonic transition in this case, consider the example in Figure 6 Panel A where we have depicted a situation in which the military advantage of country 1 is low with $A \approx 0$. Suppose that at date 0, the equilibrium begins in the steady state in which country 1 dominates. Suppose that war takes place at date 1, and suppose that country 1 loses the war. At date 1, military capital is destroyed for both countries, and the tax revenue of country 1 is also destroyed, with μ_1 now satisfying condition (10). Given that $A \approx 0$, it follows that $\mu_1 < 0$, so that country 2 now dominates militarily and financially. Moreover, it follows that any peaceful transition path necessarily converges to the steady state in which country 2 is the hegemon. This example illustrates a situation in which a hegemonic transition is made possible by the realization of war and defeat of the exogenously stronger country, similar to the historical transitions depicted in Figure 1.

The second part of the corollary shows that if debt capacity is sufficiently high, a second type of hegemonic transition is possible, originating from financial markets and without the realization of war. It directly follows from the second part of Proposition 3. To see how a hegemonic transition can occur in peacetime, consider the example in Figure 6 Panel B under small exogenous advantage $A \approx 0$. Observe that in this example, the zone of fragility is sufficiently wide that it encompasses both steady states. Suppose that at date 0, the equilibrium begins in the steady state in which country 1 dominates. While there exists a path under peace where both countries remain in this steady state, there also exists a path under peace that converges monotonically to the steady state in which country 2 is the hegemon, as depicted Figure 6. Thus, in contrast to the previous case, a hegemonic transition need not require war. This is because country 2's debt capacity is sufficiently large that country 2 can fund itself in international markets to achieve eventual dominance along some transition path. Of course, the second steady state in this example is also in the zone of fragility, which means that transitions toward either steady may occur repeatedly in the absence of war.

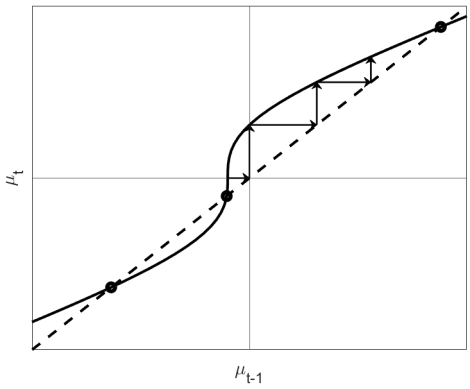
Observe that this type of transition occurs if the depreciation of military capital δ is sufficiently high to ensure that the steady states are within the zone of fragility. If depreciation is sufficiently high, then the disadvantaged country can more rapidly accumulate military capital to overwhelm the prospective victor, which is why the steady state is fragile in such an environment. This means that if the depreciation rate of military technology in the future rises due to shifts away from expensive fighter planes towards cheaper replaceable drones, for example, then geopolitical fragility will be more likely.

Figure 5: Steady States and Convergence in Dynamic Model

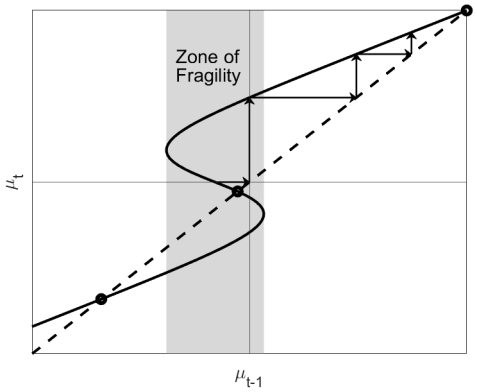
Panel A: Uniqueness



Panel B: Hysteresis



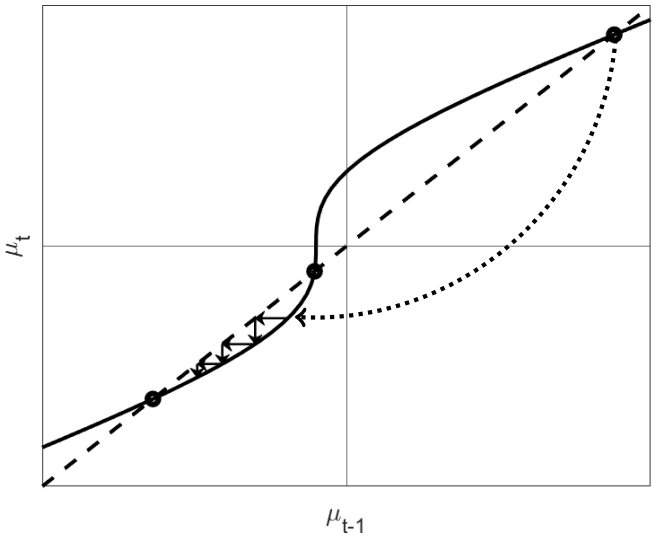
Panel C: Fragility



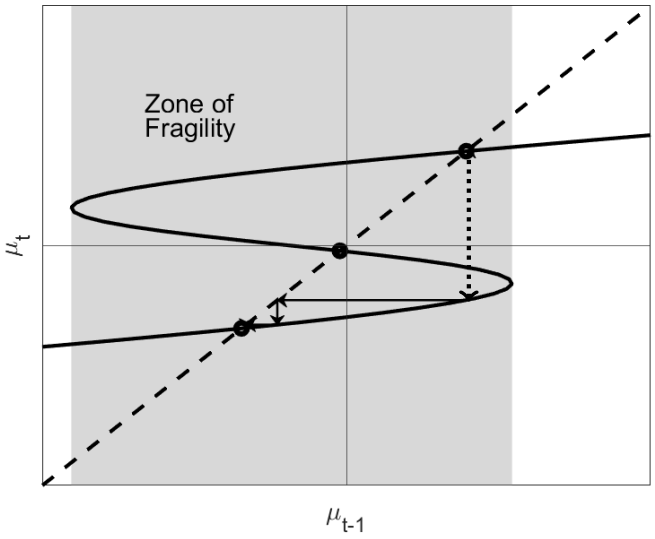
This figure plots the correspondence from μ_{t-1} to μ_t under peace in (8) for three different cases. A steady state is defined by the intersection of the curve and the dotted 45 degree line. Panel A depicts a case for which $\chi \leq \chi'$ and there is a unique monotonically stable steady state. Panels B and C depict cases for which $\chi > \chi'$ and there are two monotonically stable steady states. Panel B depicts the case of geopolitical hysteresis with $\chi < \chi''$. Panel C depicts the case of geopolitical fragility with $\chi > \chi''$. The arrows in each figure depict a monotonic convergent transition path towards a steady state.

Figure 6: Hegemonic Transitions

Panel A: War with Defeat



Panel B: Peaceful Transition



This figure plots the correspondence from μ_{t-1} to μ_t under peace in (8) for two different cases. A steady state is defined by the intersection of the curve and the dotted 45 degree line. Panel A depicts a case for which $\chi'' > \chi > \chi'$ and there are two monotonically stable steady state with geopolitical hysteresis. The arrows depict a hegemonic transition during war with defeat for country 1 starting from the steady state in which country 1 dominates. Panels B depicts a case for which $\chi > \chi'' > \chi'$ and there are two monotonically stable steady states with geopolitical fragility. The arrows depict a hegemonic transition under peace starting from the steady state in which country 1 dominates.

5 Extension: Asymmetric Debt Capacity

We now consider an extension of our model to a situation in which the two countries have asymmetric debt capacity. This extension is motivated by the observation that Great Britain prevailed during the Napoleonic Wars in large part because of its higher debt capacity. It is also motivated by the question of what China's efforts to develop its international bond markets, such as studied in Clayton et al. (2022), would imply for its geopolitical rivalry with the U.S. To facilitate exposition, we let $A = 0$ so that the only source of asymmetry between the two countries is their relative debt capacity, and we let χ_i correspond to country i's debt capacity. We let $\chi_1 \geq \chi_2$ so that country 1 has weakly larger debt capacity.

Country i's budget constraint taking into account that $g_{it} = 0$ and $b_{it} = \tau\chi_i$ can be represented analogously to equation (7) as

$$m_{it} = \tau + (1-\delta)\, m_{it-1} - (\phi - \phi\theta w_i\,(m_{it}, m_{-it}))\,\tau\chi_i. \tag{11}$$

Combining this equation for countries 1 and 2 yields

$$\mu_t - (1-\delta)\,\mu_{t-1} = \left(-\phi + \frac{\phi\theta}{2}\right)\tau\,(\chi_1 - \chi_2) + \tau\frac{\chi_1+\chi_2}{2} \times \phi\theta(2F(\mu_t) - 1). \tag{12}$$

This equation shows that relatively higher debt capacity $\chi_1 - \chi_2$ shifts country 1's relative military investment up as long as the war premium θ is sufficiently high, acting similarly to an exogenous military advantage in the previous section. The intuition is that if the probabilities of winning are relatively similar, a relatively higher debt capacity increases the gap in bond market revenue proportionally.[24] On the other hand, the gap in the probability of winning $2F(\mu_t) - 1$ affects military investment through the average debt capacity $\frac{\chi_1+\chi_2}{2}$, because higher debt capacity for either country increases the spillover from the probability of winning to the gap in bond market revenue.

We now evaluate this equilibrium condition and consider the circumstances under which there is a unique steady state or there are multiple steady states. We consider the case where country 1's debt capacity is sufficiently high and weakly exceeds that of country 2, and where the war premium is sufficiently high such that $\theta > 2$. Moreover, for simplicity, we consider comparative statics for the difference in debt capacity $\chi_1 - \chi_2$ holding total debt capacity $\chi_1 + \chi_2$ fixed.

[24] The condition on the war premium θ ensures that the average bond price exceeds one, so a country with an even probability of winning raises more funding from new bonds issued than the cost of repaying last period's bonds.

Proposition 4 *Suppose that* $\chi_1 \geq \chi_2$, $\chi_1 > \frac{4}{\tau\phi\theta}$. *For any given value of global debt capacity* $\chi_1 + \chi_2$ *there exists a debt capacity difference threshold* η *with the following properties:*

i) If $\chi_1 - \chi_2 > \eta$, *then the steady state is unique and is monotonically stable. The steady state is asymmetric with country 1 dominating militarily and financially:* $w_1(m^*_{1ss}, m^*_{2ss}) > \frac{1}{2}$, $m^*_{1ss} > m^*_{2ss}$, *and* $q^*_{1ss} > q^*_{2ss}$.

ii) If $\chi_1 - \chi_2 < \eta$, *then there are two monotonically stable steady states and one monotonically unstable steady state. In one monotonically stable steady state, country* 1 *dominates militarily and financially:* $w_1(m^*_{1ss}, m^*_{2ss}) > \frac{1}{2}$, $m^*_{1ss} > m^*_{2ss}$, *and* $q^*_{1ss} > q^*_{2ss}$. *In the other, country 2 dominates militarily and financially:* $w_1(m^*_{1ss}, m^*_{2ss}) < \frac{1}{2}$, $m^*_{1ss} < m^*_{2ss}$, *and* $q^*_{1ss} < q^*_{2ss}$.

This proposition states that even if there is no exogenous military advantage for country 1, country 1 will dominate country 2 militarily if its debt capacity is sufficiently larger than country 2's debt capacity.[25] The logic for this result emerges from the complementarity between military and financial power that we have highlighted in previous sections: Greater debt capacity supports a larger military buildup, which generates a funding advantage that further supports the military. This ability to dominate country 2 is eroded if either country 2 increases its debt capacity or alternatively if country 1's own debt capacity is eroded. In both of these circumstances, country 2's ability to raise sufficient funds to overwhelm country 1 are enhanced and multiple steady states can emerge.[26]

6 Conclusion

We have presented a simple dynamic model in which geopolitics and international bond markets interact. The model is consistent with the three empirical facts we have highlighted: hegemons have a funding advantage, this advantage rises with geopolitical tensions, and war losers suffer from higher devaluation than victors.

A key insight of our model is that financial market factors such as greater debt capacity and a higher war premium can facilitate a hegemon's rise to power because of the complementarity between military and financial dominance. However, this complementarity also gives rise to equilibrium multiplicity as well a geopolitical fragility, whereby even the exogenously stronger country can lose its hegemonic status if the weaker country is supported by

[25] A corollary to this result is that, with a sufficiently high debt capacity, country 1 can dominate country 2 even if it suffers an exogenous disadvantage with $A < 0$.

[26] Whether the case of multiplicity involves hysteresis or fragility depends on the average of χ_1 and χ_2. If that sum is sufficiently small, then the case of multiplicity admits hysteresis. Otherwise, it admits fragility.

financial markets. This insight can explain Great Britain's rise to power at the beginning of the nineteenth century. And it highlights the risk that the U.S. faces today if China's debt capacity rises through the internationalization of its currency or if the U.S.'s debt capacity becomes imperiled by government policies that erode it. Our model also shows that while hegemonic transitions can occur in the aftermath of wars, peaceful bond market led transitions from one military and financial hegemon to another are also possible when both countries have very high debt capacities.

Our framework leaves three interesting directions for future work. First, we have abstracted away from any time-varying or endogenous geopolitical risk. An interesting and empirically relevant avenue for future research can consider how this risk itself responds to armament and indebtedness across countries. This would lead to some richer comparative statics and predictions than those considered here, since equilibria could transition between cases of uniqueness and multiplicity or hysteresis and fragility over time. Second, and relatedly, such theoretical extensions would allow for a quantitative analysis that measures the strength of the feedback between military and financial dominance and that allows for counterfactual scenario analysis given current global geoeconomic conditions. Finally, we have also abstracted away from the strategic decision to go to war and how those interact with the level of armament and debt. An interesting question for future work would consider these strategic interactions and what they would imply for financial markets.

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Appendix for Online Publication

A Proofs

A.1 Proof of Proposition 1

We define

$$\chi'(A,\chi,\phi,\tau,\theta) \;=\; \frac{\tilde{h}^{-1}(A)}{\phi\tau\theta}, \tag{A.1}$$

where $\tilde{h}^{-1}$ is the inverse function of

$$\tilde{h}: \qquad [2,\infty)\mapsto[0,\infty), \tag{A.2}$$

$$\tilde{h}(x) \;=\; -2\tanh^{-1}\left(\sqrt{1-\frac{2}{x}}\right)+x\sqrt{1-\frac{2}{x}}. \tag{A.3}$$

Here $\tanh:(\infty,\infty)\mapsto(-1,1)$ is the hyperbolic tangent function. Differentiating $\tilde{h}$ gives

$$\frac{d\tilde{h}(x)}{dx} \;=\; \sqrt{1-\frac{2}{x}}\;\forall x\in[2,\infty)\,. \tag{A.4}$$

Here, we used that $\frac{d}{dz}\tanh(z)=1-\tanh^2(z)$. This implies that $\tilde{h}$ is a strictly increasing function $[2,\infty)\mapsto[0,\infty)$, so its inverse $\tilde{h}^{-1}\,[0,\infty)\mapsto[2,\infty)$ exists and is also strictly increasing. We therefore have that $\chi'(A,\chi,\phi,\tau,\theta)\geq\frac{2}{\phi\tau\theta}$, $\frac{d\chi'}{dA}>0$, $\frac{d\chi'}{d\phi}<0$, $\frac{d\chi'}{d\tau}<0$, and $\frac{d\chi'}{d\theta}<0$.

Now suppose that $\chi<\chi'$, i.e. we are in case i). Rearranging equation (5) and defining the effective difference in military strength $z=A+m_1-m_2$, any equilibrium must be a zero of the function

$$h(z) \;=\; -A+z-\chi\phi\tau\theta\,(2F(z)-1) = -A+z-\chi\phi\tau\theta\tanh\left(\frac{z}{2}\right), \tag{A.5}$$

The derivative of h with respect to z is

$$\frac{dh(z)}{dz} \;=\; 1-\frac{\chi\phi\tau\theta}{2}\left(1-\tanh^2\left(\frac{z}{2}\right)\right) \tag{A.6}$$

Since $-1<\tanh(z)<1$ and $\tanh(z)\underset{z\to\pm\infty}{\to}\pm1$, it is clear that $h(z)$ goes from $-\infty$ to ∞. There are two sub-cases to case i). If χ is sufficiently low that $\chi\leq\chi'(0,\chi,\phi,\tau,\theta)=\frac{2}{\phi\tau\theta}$, then $0<\frac{dh(z)}{dz}<1$ almost everywhere and h is strictly increasing, showing that the equilibrium is unique.

Next, consider the sub-case where $\chi<\chi'$ but $\chi>\frac{2}{\phi\tau\theta}$. Local maxima and minima are

roots of $\frac{dh}{dz}(z) = 0$

$$z_{\pm} \quad = \quad \pm 2 \tanh^{-1}\left(\sqrt{1 - \frac{2}{\chi\phi\tau\theta}}\right). \tag{A.7}$$

Hence, there exist exactly two local extrema. Because h goes from $-\infty$ to ∞, the lower root, z_-, is a local maximum and the upper root, z^+, is a local minimum. Substituting for z_- and using that $\tanh(-z) = -\tanh(z)$ the height of the local maximum is

$$h(z_-) \quad = \quad -A + \underbrace{\left(-2\tanh^{-1}\left(\sqrt{1 - \frac{2}{\chi\phi\tau\theta}}\right) + \chi\phi\tau\theta\sqrt{1 - \frac{2}{\chi\phi\tau\theta}}\right)}_{\tilde{h}(\chi\phi\tau\theta)}. \tag{A.8}$$

Because $\chi < \chi'$ the local maximum is negative, implying that h crosses zero only once and the equilibrium is unique.

If in addition $A = 0$ then $h(0) = 0$, implying that the unique equilibrium is given by $z^* = 0$. Conversely, if $A > 0$, then $h(A) < 0$, and since $h \underset{z\to\infty}{\to} \infty$ the unique zero satisfies $z^* > A$, implying that $w_1(m_1^*, m_2^*) > \frac{1}{2}$, $m_1^* > m_2^*$ and $q_1^* > q_2^*$.

Now suppose that $\chi > \chi'$, i.e. we are in case ii). Equation (A.8) then implies that $h(z_-) > 0$. Because country 1 is assumed to have a weak exogenous military advantage, i.e. $A \geq 0$, we have $h(0) \leq 0$. With $z_- < 0$, h going from minus to plus infinity, and h having exactly one local maximum at z_- and one local minimum at z_+ it follows that h crosses zero exactly three times, and there are three equilibria.

Label the equilibria $z^{1,*} < z^{2,*} < z^{3,*}$. Since z_- is a local maximum and z_+ is a local minimum of h, we have that $z^{1,*} < -z_- < 0$ and $z^{3,*} > z_+ > 0$ implying that $w_1\left(m_1^*(z^{1,*}), m_2^*(z^{1,*})\right) = \frac{1}{2}\tanh\left(\frac{z^{1,*}}{2}\right) + \frac{1}{2} < \frac{1}{2}$ and $w_1\left(m_1^*(z^{3,*}), m_2^*(z^{3,*})\right) = \frac{1}{2}\tanh\left(\frac{z^{3,*}}{2}\right) + \frac{1}{2} > \frac{1}{2}$. With $w_2(m_1^*, m_2^*) = 1 - w_1(m_1^*, m_2^*)$ and the bond pricing equation (2), the remaining inequalities follow.

Employing an asymptotic stability concept, where the bond market sets prices in alternation with governments choosing military investment, an equilibrium is stable if and only if the slope of the bond revenue function at the equilibrium is below one in absolute value. Because $\tanh(\frac{z^{1,*}}{2}) < \tanh(\frac{z_-}{2}) < \tanh(\frac{z^{2,*}}{2}) < \tanh(\frac{z_+}{2}) < \tanh(\frac{z^{3,*}}{2})$, and $z_\pm$ are zeros of (A.6) and tanh is between zero and one, it follows that $\frac{\chi\phi\tau\theta}{2}\left(1 - \tanh^2\left(\frac{z^{1,*}+A}{2}\right)\right) < 1$, $\frac{\chi\phi\tau\theta}{2}\left(1 - \tanh^2\left(\frac{z^{3,*}+A}{2}\right)\right) < 1$ and $\frac{\chi\phi\tau\theta}{2}\left(1 - \tanh^2(\frac{z^{2,*}+A}{2})\right) > 1$, i.e. the slope of the bond revenue function with respect to $m_1 - m_2$ is greater than one at $z^{2,*}$, but less than one at $z^{1,*}$ and $z^{3,*}$. ■

A.2 Proof of Corollary 1

Suppose that country 1 is dominant in equilibrium, meaning that $w_1(m_1^*, m_2^*) > \frac{1}{2}$. Further assume that the equilibrium is stable. We start by proving the statement for θ. Note that $\frac{dw_1^*}{dz^*} > 0$ from equation (1) and $\frac{d(q_1^* - q_2^*)}{dw_1^*} > 0$ from equation (2), so it is sufficient to prove that $\frac{dz^*}{d\theta} > 0$. We totally differentiate the equilibrium condition $h(z^*) = 0$ with respect to θ and

use the implicit function theorem:

$$\frac{dz^*}{d\theta} = \frac{\chi\phi\tau\tanh\left(\frac{z^*}{2}\right)}{1-\frac{1}{2}\chi\phi\tau\theta\left(1-\tanh^2\left(\frac{z^*}{2}\right)\right)}. \tag{A.9}$$

The denominator in this expression is positive because we are in a stable equilibrium and the numerator is positive because country 1 is dominant, implying that $\tanh\left(\frac{z^*}{2}\right) = 2\left(w_1(m_1^*, m_2^*) - 1\right) > 0$. If country 2 is dominant in equilibrium, the sign of the numerator in (A.9) is negative, and $\frac{dz^*}{d\theta} < 0$, implying $\frac{dw_2^*}{dz^*} > 0$ and $\frac{d(q_2^* - q_1^*)}{dz^*} > 0$. The proofs for χ and ϕ are analogous.

Finally, totally differentiating the equilibrium condition $h(z^*)$ with respect to the exogenous military advantage A gives

$$\frac{dz^*}{dA} = \frac{1}{1-\frac{1}{2}\chi\phi\tau\theta\left(1-\tanh^2\left(\frac{z^*}{2}\right)\right)} > 0, \tag{A.10}$$

Because the military advantage of country 2 equals $-A$, this proves the comparative static with respect to the exogenous military advantage for both countries. ■

A.3 Proof of Proposition 2

Define the threshold

$$\chi'\left(A, \chi, \phi, \tau, \theta, \delta\right) = \frac{\delta\tilde{h}^{-1}\left(A\right)}{\tau\phi\theta}, \tag{A.11}$$

where the function $\tilde{h}\left[0, \infty\right) \mapsto \left[2, \infty\right)$ is defined as in Appendix Section A.1. Because $\tilde{h}$ is strictly increasing, it is clear that $\chi' \geq \frac{2\delta}{\phi\tau\theta}$, $\frac{d\chi'}{dA} > 0$, $\frac{d\chi'}{d\phi} < 0$, $\frac{d\chi'}{d\tau} < 0$, $\frac{d\chi'}{d\theta} < 0$, $\frac{d\chi'}{d\delta} > 0$.

With an abuse of notation, a steady state is a zero of the function

$$h(z) = \delta(z - A) - \tau\chi\phi\theta\tanh\left(\frac{z}{2}\right). \tag{A.12}$$

The remainder of the proof for cases i) and ii) is analogous to Appendix Section A.1. ■

A.4 Proof of Proposition 3

Define the threshold

$$\chi'' = \frac{2}{\tau\phi\theta}. \tag{A.13}$$

The threshold χ'' has the following properties: $\frac{d\chi''}{d\phi} < 0$, $\frac{d\chi''}{d\theta} < 0$, $\frac{d\chi''}{d\tau} < 0$.

Suppose we are in case i), i.e. $\chi < \chi''$. Define again $z_t = \mu_t + A$ as the effective relative

military capacity of country 1. The equilibrium mapping (8) can be written as

$$z_{t-1} = H(z_t), \tag{A.14}$$

$$H(z_t) \equiv \frac{z_t - \tau\chi\phi\theta \tanh\left(\frac{z_t}{2}\right) - \delta A}{1-\delta}. \tag{A.15}$$

Taking the derivative

$$\frac{dH(z)}{dz} = \frac{1 - \frac{\tau\chi\phi\theta}{2}\left(1 - \tanh^2\left(\frac{z}{2}\right)\right)}{1-\delta} \tag{A.16}$$

Because $\chi < \chi''$ and tanh is bounded between -1 and 1, it follows that H is strictly increasing and hence one-to-one. As a result, any transition path must be monotonic and the steady-state is uniquely determined by the initial conditions.

Now, suppose we are in case ii). In that case, $\frac{dH(z)}{dz}(z) < 0$ for all $z \in (\underline{z}, \bar{z})$, where

$$\underline{z} = -2\tanh^{-1}\left(\sqrt{1 - \frac{2}{\chi\phi\tau\theta}}\right) \tag{A.17}$$

$$\bar{z} = 2\tanh^{-1}\left(\sqrt{1 - \frac{2}{\chi\phi\tau\theta}}\right). \tag{A.18}$$

For any $z_{t-1} \in (H(\underline{z}), H(\bar{z}))$ there exist multiple z_t such that (A.14) is satisfied, and we call this interval the zone of fragility. Hence, there exists at least one z_{t-1} such that z_t could take multiple values.

Next, we show that there exists a z_{t-1} such that z_t can take multiple values and $z_t - z_{t-1}$ can take either sign. To prove this, we show that the middle steady state lies in the zone of fragility. The local extrema of $h(z)$, defined in equation (A.12), are given by

$$z_{\pm} = \pm 2\tanh^{-1}\left(\sqrt{1 - \frac{2\delta}{\chi\phi\tau\theta}}\right). \tag{A.19}$$

Because the middle steady state, $z^{2,*}$, satisfies $z_- < z^{2,*} < z_+$ and $\delta < 1$ it follows that $\underline{z} < z^{2,*} < \bar{z}$ and $H(\bar{z}) > H(z^{2,*}) = z^{2,*} > H(\underline{z})$. This proves that the middle steady state lies in the zone of fragility.

We next show that there exists a non-monotonic transition path within an arbitrarily small neighborhood of $z^{2,*}$. We know that there exists $\epsilon > 0$ such that $(z^{2,*} - \epsilon, z^{2,*} + \epsilon)$ lies entirely in the zone of fragility. Since H intersects the 45 degree line at $z^{2,*}$ it follows that there exists a z_{t-1} within this neighborhood such that $z_t - z^{2,*}$ and $z_t - z_{t-1}$ can take either sign. Hence, there exists at least one non-monotonic transition path and transition paths need not be unique.

Since $h'(z^{2,*}) < 0$ we know that $\frac{H(z^{2,*})}{dz} < 1$. If $-1 < \frac{H(z^{2,*})}{dz} < 1$, the middle steady state $z^{2,*}$ is unstable. If z_t jumps below $z^{2,*}$, there exists a convergence path to $z^{3,*}$ and for $z_t < z^{2,*}$ there exists a convergence path to $z^{1,*}$.

If $\frac{H(z^{2,*})}{dz} < -1$, the middle steady state is stable but transition paths converging to it are non-monotonic. Because H^{-1} is downward-sloping at $z^{2,*}$, from within a neighborhood of

$z^{2,*}$, it is hence possible to jump to three different values for z_t, one of which is on the upper portion $(\bar{z}, \infty)$ and one of which is on the lower portion $(-\infty, \underline{z})$ of the curved line in Figure 5, Panel C. If $z_t \in (\bar{z}, \infty)$ there exists a transition path leading to $z^{3,*}$, while if $z_t \in (\infty, \underline{z})$ there exists a transition path to $z^{1,*}$. This proves that there exists a non-monotonic transition path to each of the monotonically stable steady states. ■

A.5 Proof of Corollary 2

Corollary 2 follows from Proposition 3. Only the statement about discount rates in ii) requires an additional proof. Because we want to take the limit as $\delta \to 1$ while ensuring that $\chi > \chi'$ and $\chi > \chi''$ throughout, assume that $\chi > \frac{\tilde{h}^{-1}(A)}{\tau\phi\theta} \geq \chi''$. By continuity, as $\delta \to 1$ the zeros of h defined in (A.12) converge to those of $h^{\delta=1}$. This shows that $z^{1,*}$ and $z^{3,*}$ are bounded as $\delta \to 1$. Conversely, the region of fragility equals

$$(H(\underline{z}), H(\bar{z})) \quad = \quad \left(-\frac{\delta A}{1-\delta} - \frac{\tilde{h}(\chi\phi\tau\theta)}{1-\delta}, -\frac{\delta A}{1-\delta} + \frac{\tilde{h}(\chi\phi\tau\theta)}{1-\delta} \right), \tag{A.20}$$

whose left bound goes to $-\infty$ and the right bound goes to ∞. This shows that for δ sufficiently close to one, all three steady states lie within the region of fragility. ■

A.6 Proofs of Proposition 4

A steady state is a zero of the function

$$h(z) \quad = \quad \delta z - \left(-\phi + \frac{\phi\theta}{2} \right) \tau \left(\chi_1 - \chi_2 \right) - \phi\theta\tau \frac{\chi_1 + \chi_2}{2} \tanh\left(\frac{z}{2} \right). \tag{A.21}$$

By analogy to Proposition 2 there exists a unique steady state with country 1 dominating if

$$\tilde{h}\left(\frac{\frac{\chi_1+\chi_2}{2}\tau\phi\theta}{\delta} \right) < \frac{1}{\delta}\left(-\phi + \frac{\phi\theta}{2} \right) \tau \left(\chi_1 - \chi_2 \right), \tag{A.22}$$

where $\tilde{h} : [2, \infty) \mapsto [0, \infty)$ is the same strictly increasing function as before. The condition $\chi_1 > \frac{4}{\tau\phi\theta}$ ensures that h has a local maximum and a local minimum for any value of χ_2 and that $\tilde{h}$ is defined on its argument.

Define the threshold η

$$\eta \quad = \quad \frac{\tilde{h}\left(\frac{\frac{\chi_1+\chi_2}{2}\tau\phi\theta}{\delta} \right)}{\frac{1}{\delta}\left(-\phi + \frac{\phi\theta}{2} \right)\tau}. \tag{A.23}$$

By continuity of $\tilde{h}$, it then follows that the steady state is unique if $\chi_1 - \chi_2 > \eta$, but if $\chi_1 - \chi_2 < \eta$ there are two monotonically stable and one not monotonically stable steady state. The remainder of the proof for cases i) and ii) is analogous to Appendix Section A.1. ■