Screening Using a Menu of Contracts in Imperfectly Competitive Markets with Adverse Selection

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Abstract

I develop a model of screening with menus under imperfect competition. The model has a unique equilibrium in pure strategies, which I characterise in closed form. I use it to analyse a contractual externality and build a sufficient statistic to test for its empirical relevance. I also use the model to study credit market policies in the presence of screening. Contrary to conventional wisdom, increasing the Federal Reserve rate or decreasing competition can increase lending. I provide an empirical application in the context of consumer credit and show that, due to the externality, the menus contain too many maturity options.

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1 Introduction

In many markets, firms use menus of contracts to make their customers reveal their private information through their choices (i.e., screening). Examples include the insurance market (Handel, Hendel, and Whinston 2015; Einav, Finkelstein, and Tebaldi 2019); the mortgage market (Taburet, Polo, and Vo 2024); the market for credit cards (Nelson 2018); and the housing rental market (Humphries, Nelson, Nguyen, Dijk, and Waldinger 2024, Calder-Wang and Kim 2024).

Because asymmetric information is not directly observable, applied studies rely on structural models (see Einav, Finkelstein, and Mahoney 2021 for a survey). However, existing frameworks abstract from screening with menus. This largely stems from screening models' equilibrium prediction and comparative statics being fragile. Theoretical models only consider extreme market structures (e.g., Stiglitz 1977, Rothschild and Stiglitz 1976, Azevedo and Gottlieb 2017) and face issues of equilibrium non existence. As such, their predictions heavily depend on the choice of market structure or equilibrium refinement used to solve the model (e.g., Riley 1979, Wilson 1980).

This paper bridges theory and empirical research on adverse selection. I incorporate imperfect competition in an otherwise standard screening model à la Rothschild and Stiglitz (1976). Following the empirical industrial organization literature, I model competition using a discrete choice approach, as McFadden (1981). This approach has two benefits. First, it gives the model convenient analytical properties and yields demand functions that nest various degrees of competition, from perfect competition to monopoly. Second, it makes the model empirically tractable.

I use the framework to analyze theoretically how incentives to screen and welfare vary with competition or policies affecting firms' marginal costs. I also discuss the identification and estimation of model parameters for a situation in which an econometrician has access to data on the menus of contracts and choices for one firm (e.g., Hertzberg, Liberman, and Paravisini 2018); or when they only observe the contracts chosen for the full market (e.g., credit register data, as in Crawford, Pavanini, and Schivardi 2018). Finally, I provide a working example in the context of consumer credit using data from Hertzberg, Liberman, and Paravisini (2018).

Giving firms incomplete market power has three main effects on the analysis. First, it solves the equilibrium non existence problem. The model delivers a unique pure strategy Nash equi-

librium, which I characterize in closed form.[1] The closed-form characterization is important for empirical applications because it allows me to derive sharp moment conditions and renders the exclusion restrictions transparent.

Second, the competition between firms creates a contractual externality, for which I derive a sufficient statistic. This externality results from the fact that when lenders screen borrowers using a menu, they change the composition of their competitors' borrowers. The equilibria can thus be inefficient in the second-best sense, so an informationally constrained social planner can create a menu that makes both firms and their clients better off. The existence of the externality does not depend on the use of a particular equilibrium refinement.

Third, I show that policies that affect competition or the cost of originating a contract have unintended consequences due to the presence of competition and screening. For instance, in the context of credit markets, increasing capital requirements, increasing the Federal Reserve rate, or decreasing competition can increase lending.

To be close to the empirical application, I present my model in the context of credit markets. However, the framework is general enough to capture labour markets, insurance markets or other financial markets.[2] I provide micro-foundations for the functional forms used and develop a unifying framework for screening in credit markets.

In the baseline model, a contract comprises a loan size and an interest rate. Borrowers have quasilinear utility over the present value of the loan. They have private information about their willingness to pay (WTP) for each dollar borrowed, their demand elasticity, and their default probabilities. Their WTP is positively correlated with their default probabilities. In that context, screening is achieved by setting the interest rate above the low-default borrower's WTP for larger loans. That way, high-default borrowers self-select into higher-rate contracts. Screening can, therefore, restore perfect information pricing. However, it also requires that low-default borrowers get a smaller loan than high-default borrowers, which is not necessarily what would happen in the first best.

To model borrowers' choice of lender, I assume that part of borrowers' utility contains a borrower-bank-specific random shock following a continuous probability distribution and entering the utility additively (as in, for instance, McFadden 1981). This shock can be interpreted

1. To derive this result, I need to impose a capacity constraint on firms as Azevedo and Gottlieb (2017), but only when the market is close to perfect competition.

2. A key requirement is that contracts are effectively exclusive; see, for instance, Attar, Mariotti, and Salanié 2011 for non-exclusive contracts under perfect competition. Search costs may prevent borrowers from getting multiple contracts in practice, which allows firms to screen.

as borrowers' heterogeneous preferences over lenders' characteristics (e.g., distance from the closest branch as in Hoteling 1929), search costs or as borrowers being imperfectly informed about the contracts offered by each bank (Varian 1980). I consider the situation in which the random shock is uncorrelated with borrowers' preferences for contract characteristics, so that banks cannot use their menus to sort borrowers on their random shock realization. When the shocks follow an extreme value distribution, its variance parameterizes the product demand elasticity. In the limit case in which the variance of the shock tends to infinity, each lender behaves like a monopolist. When the variance tends to zero, borrowers' demand elasticity becomes infinite, as in the perfect competition case. I allow the variance of the shock to be correlated with the willingness to pay. This provides additional price discrimination incentives and renders the model more realistic.

Unlike Rothschild and Stiglitz (1976), a pure strategy equilibrium always exists in my model. In Rothild and Stiglitz, the equilibrium does not exist when the relative number of low-default borrowers is high enough. I show that in my model, deviations that break the equilibrium in Rothild and Stiglitz attract relatively too many high-default borrowers to be profitable. This is because those deviations are more valuable to high-default borrowers, and lenders cannot attract the whole market under imperfect competition. In the spirit of Einav, Finkelstein, and Cullen (2010), I provide a graphical intuition to illustrate why having an imperfectly elastic demand restores the pure strategy equilibrium.[3]

I show that the equilibrium is constrained inefficient, so there is room for policy intervention in this market. The intuition is the following. Maintaining customers' incentives to self-select is costly, because it may require distorting contracts relative to the first best. When the distortions are too high, it is more efficient to pool borrowers instead. Yet, if pooling contracts are offered and competition is high, a competitor can take advantage of a pooling contract by introducing a product that steals the most profitable customers only (cream skimming). Cream skimming can be ex-post inefficient because lenders do not internalize how their screening strategies (e.g., cream skimming) change the types of borrowers who select competitors' products—and thus the cost of lending via those products. The reasoning is the same as in Rothschild and Stiglitz

3. When competition is large enough, I must add a capacity constraint to the model. Formally, I assume that lenders cannot serve more than half of the total market. This can be interpreted as an extreme case of a lending cost that is increasing in the number of customers. This assumption is needed because the lender objective function becomes convex when markets are competitive. Indeed, deviations from screening attract bad borrowers in high proportion first, but once most of the bad borrowers have been attracted, the good borrowers become easier to attract relative to the bad borrowers. With this condition, the model equilibrium under perfect competition is the same as the one using Azevedo and Gottlieb (2017) equilibrium concept.

(1976). However, contrary to Rotchild and Stiglitz, I can compute the equilibrium and characterize the size of this friction. I show that the equilibrium is inefficient when competition is high, and the marginal total surplus per dollar lent in the market for low-default borrowers is higher than the difference between borrowers' willingness to pay (i.e., the information rent in Rotchild and Stiglitz).

When the equilibrium is constrained inefficient, decreasing competition or increasing the marginal cost of lending can be a Pareto improvement. It lowers incentives to implement possibly inefficient cream-skimming deviations, and thus restores lenders' ability to pool. A lower level of competition can be beneficial to low-default borrowers, because it lowers their credit constraint at the cost of a higher markup. It is also beneficial to high-default borrowers as competition can lower the interest rate of their contracts due to pooling. Lenders are also better off because market power allows them to extract more surplus from borrowers. The same result holds when the policy increases lenders' cost of originating low loan-size contracts and decreases the cost of originating high loan-size contracts, but only if competition is low enough. Examples of such policies are capital requirements in credit markets.

To accommodate a wide variety of empirical setups, I extend the model to allow for endogenous default (moral hazard, burden of payment), maturity, loan size, interest rate collateral, and fees. I also develop a guide for how to bring the model to the data using either proprietary data on the menu of contracts and choices for one firm or credit register data. Those two cases are the most common in practice (See, for instance, Hertzberg, Liberman, and Paravisini (2018) and Crawford, Pavanini, and Schivardi (2018) for the credit market or Handel and Kolstad (2015) for the insurance market).

The identification and estimation of the model parameters rely on using equilibrium conditions linking price to quantities and contract shares. It enables me to recover the marginal cost, the information rent, and the demand semi-elasticity as a constant and a slope of linear regression. My approach does not require estimating default elasticities, nor does it require data on menus of contracts offered by other lenders to recover demand elasticities. This is convenient because those elasticities are difficult to estimate under adverse selection, given that borrowers who chose different contracts are not comparable.

To illustrate the approach, I provide an application using data from an online lending platform (Lending Club) as in Hertzberg, Liberman, and Paravisini (2018).[4] I use the estimates to

4. Although contracts are not formally exclusive in this market, the fact that Hertzberg, Liberman, and Paravisini (2018) finds that screening with menus happens in practice shows that using Rothschild and Stiglitz (1976)

decompose the equilibrium interest rate into a break-even price, a perfect information markup and an information premium or discount. The information premium divided by the perfect information markup of the contract designed for high-default borrowers is a sufficient statistic for the existence of the contractual externality.

Interpreting the regression error term through the lens of the theoretical model, I show that the independent variables need to be uncorrelated with unobserved demand heterogeneity. As a result, I use the variation coming from the borrower's observable risk category. The identifying assumption is that the average search costs of borrowers who chose high-maturity contracts are uncorrelated with the borrower risk category assigned by the lender.

As in Hertzberg, Liberman, and Paravisini (2018), I find that high-default borrowers self-select into higher-maturity loans. Using the structural model, I can also characterize the distortions caused by adverse selection and imperfect competition. I show that prices are close to the first best case, but maturities are not. Borrowers who choose high-maturity contracts are less price elastic but get an asymmetric information discount that mitigates the competition channel. The information rent is equal to 15 percent of the loan's present value. Using sufficient statistics, I show that both borrowers and lenders would be better off if lenders could cross-subsidize customers in order to increase the maturity of their lower-maturity contracts while maintaining incentives to self-select. The threat of cream skimming prevents this situation from emerging.

Literature Review—The paper contributes to the large theoretical literature on screening. The vast majority focuses on perfect competition and discuss equilibrium existence and refinements (Rothschild and Stiglitz 1976, Riley 1979, Wilson 1980, Farinha Luz 2017, Azevedo and Gottlieb 2017). Recent papers have studied imperfect competition (Lester, Shourideh, Venkateswaran, and Zetlin-Jones 2019, Chade and Swinkels 2021). In contrast to those papers, I focus on financial markets and on developing an empirically tractable framework.

The paper also contributes to the empirical literature on asymmetric information. A first strand of the literature developed reduced form approaches to test for the presence of screening. They do so by comparing the default probability of borrowers choosing different contracts or comparing borrowing that chose the same contract but were differently self-selected (Hertzberg, Liberman, and Paravisini 2018, Karlan and Zinman 2009, Chiappori and Salanie 2000). A second strand of the literature used structural models or sufficient statistics approaches to assess the cost of asymmetric information (Einav, Finkelstein, and Levin 2010, Handel, Kolstad, and Spin-

framework (exclusive contracts) instead of Attar, Mariotti, and Salanié (2011) (non-exclusive contracts) is a valid approach in this context.

newijn 2019, Crawford, Pavanini, and Schivardi 2018, Yannelis and Zhang 2021). I contribute to the literature by developing a model with an endogenous menu of contracts. My framework allows us to estimate the contract term distortions that stem from screening. The closest paper is Handel, Hendel, and Whinston (2015), which uses Rothschild and Stiglitz (1976) in the context of health insurance. In contrast, my paper focuses on credit markets, and the empirical predictions do not rely on the use of equilibrium refinement or the assumption that the market is perfectly competitive, which may not hold in credit markets.

Appendix B provides an extensive discussion of the literature.

The rest of the paper is structured as follows. Section B provides a literature review. I describe the model setup in section 2. Then, in section 3, I provide intuition about the incentives to screen, I prove the existence and uniqueness of the equilibrium and solve for the model in closed form in section 4. In section 5, I provide comparative statics and discuss the potential implications of various widely used policy interventions. Section 6 present the empirical application. Section 7 concludes.

2 Model Set up

This section presents the baseline model with two periods, two borrower types, and two contract characteristics (interest rate and loan size).

Subsection C and appendices D and E present a micro-foundation for the functional form used for contract features such as loan size, interest rate, maturity, collateral or fees.

To cater to a wide variety of empirical setups, Appendix F extends the model to include an arbitrary number of periods, endogenous default probabilities (i.e., moral hazards and burden of payment), borrower types (with restrictions on preferences), and contract terms (including maturity, collateral, or fees).

2.1 General considerations

Economic Agents: The economy is populated by two groups of agents: borrowers and lenders. I also refer to the second group as banks. There is a finite number of banks $\mathbf{B} > 1$ indexed by $b \in [\![1, \mathbf{B}]\!]$. Borrowers have heterogeneous characteristics (age, income, risk aversion..). This heterogeneity implies that borrowers have different default probabilities and demand elasticities. There are two types of borrowers indexed by $i \in \{G, B\}$. The number of type i borrowers

is denoted n_i. I denote j a given borrower. Banks offer menus of contracts. I index the contract offered by bank b and designed to be chosen by borrower i by the subscript ib or by c to simplify the notation.

Timing: There are two periods. At the beginning of the first period, each borrower makes a decision to enter or not the credit market. There is no entry cost, borrowers choose to participate if the utility they get from borrowing is higher than the one of not borrowing. Conditional on participation, a borrower chooses one loan contract from one lender.

Loans mature in the second period, and borrowers may default on their loans.

Information structure: There is asymmetric information in the economy: Lenders do not perfectly observe borrowers' type (i.e. their preference and their default probabilities).[5] Whenever it is profitable and feasible, lenders use a menu of contracts to make observational equivalent borrowers reveal their type.

2.2 Borrowers

In this section, I model borrowers' decision to participate in the credit market as well as their choice of loan contract and repayment behaviour. I provide a micro-foundation of the demand system in Appendix C. All parameters defined in this section are part of borrower i's information set at the time they choose their contract and bank.

2.2.1 Choice of contract and bank

Utility: The utility of type i borrowers when borrowing an amount L_c in period 1 via contract c that requires repayment of (R_c) at maturity is specified as

$$u_i(L_c, R_c) := \alpha_i F(L_c) - \theta_i R_c,\ WTP_i := \frac{\alpha_i}{\theta_i} > 0 \tag{1}$$

$WTP_i F'(\cdot)$ is the borrower's willingness to pay for each extra dollar borrowed. Without loss of generality, I assume that $WTP_G < WTP_B$. A high WTP captures that the borrower derives a high utility level from housing, or less disutility from having a high interest rate.

5. In practice, lenders can observe some characteristics such as income, job or credit score. The model and the empirical application consider borrowers that are observational equivalent based on observable characteristics.

Heterogeneous WTP coefficients are necessary for lenders to be able to screen using loan size L and the face value of the debt R. Screening is achieved by offering a contract whose pricing for an extra loan unit is in between the two borrowers' willingness to pay. That way, larger loans are only purchased by high WTP borrowers.

θ_i is borrower i survival probability. This interpretation of the coefficient θ_i suggests that the loan is non-recourse and uncolateralized. This is one way, among others, to rationalize adverse selection in the model. The key requirement is that WTP_i are correlated with default probabilities, so θ_i can be relaxed to be a function of default probabilities.

Allowing α_i to be heterogeneous allows more flexibility in the empirical application, and can be normalized to 1 for the theoretical analysis.

I provide a micro-foundation for this utility form in Appendix C. The model can be extended to feature a finite number of contract characteristics X as long as they enter linearly in the utility.

Choice of bank b contract c: Borrower i chooses the bank that offers them the best deal. Formally:

$$\max_{\{c\}}\{u_i(L_c, R_c) + \sigma_i^{-1}\epsilon_{jb}\},\ \epsilon_{jb}\ iid,\ EV,\ \text{independent of type} \tag{2}$$

$\sigma_i^{-1}\epsilon_{jb}$ is the main departure from the classic principal-agent model. ϵ_{jb} is a borrower-bank-specific shock. σ drives the demand elasticity (i.e., competition) and can be interpreted as the distance between the borrower and the closest bank branch—as in, for instance, Hoteling (1929)—or more generally about heterogeneous preferences over bank characteristics (brand name, customer services, existing relationship, etc) or switching costs. The assumption that ϵ_{jb} draws are type-independent ensures that lenders cannot screen on borrower-specific draws.

It is convenient to write equation (2) as a set of inequalities that need to be satisfied for the borrower to choose bank b:

$$(PC_{ijb}):\ u_i(L_{ib}, R_{ib}) \geqslant u_i(L_{cd}, R_{cd}) + \sigma_i^{-1}[\epsilon_{jb} - \epsilon_{jd}], \forall cd \in \{G, B\} \times [\![1, \mathbf{B}]\!], \forall j \tag{3}$$

Equation (3) renders the interpretation of $\sigma_i^{-1}[\epsilon_{jb} - \epsilon_{jd}]$ as switching cost more salient. σ_i^{-1} parametrizes the importance of those switching costs. When σ tends to infinity, borrowers only care about the contract features the banks offer (i.e perfect competition). When σ^{-1} tends to 0,

each bank behaves like a monopolist with their borrowers.[6]

Participation constraint (PC): Borrower i accepts a contract if it provides a higher level of utility than what would get if they did not take any loan. Formally, borrower i accepts the loan if:

$$(PC_i): \ u_i(L_c, R_c) \geqslant \bar{V}_i = \bar{V} \in \mathbb{R}^+ \tag{4}$$

The fact that ϵ_{ib} is not present here is in favour of the interpretation of ϵ_{ib} being a sunk cost that has to be paid to switch banks. This is done for simplicity of the exposition so that each borrower type has the same participation constraint. Using a nested logit approach, we could use the condition $E[u_i(L_c, R_c) + \sigma_i^{-1}\epsilon_{ib}]) = ln(\sum_c u_i(L_c, R_c)) \geqslant \bar{V}_i$. This participation constraint will not be important for the vast majority of the results as it will be non-binding.

Survival probability: The market is adversely selected:

$$Adverse\ selection: \ Corr(\theta_i, WTP_i) < 0 \tag{5}$$

Equation (5) states that borrowers with high willingness to pay $WTP_B > WTP_G$ are more likely to default $\theta_B < \theta_G$. The classic justification is that high-default borrowers are less sensitive to interest rate changes, since they do not expect to repay the full face value of the debt. This is the case when, for instance, $\alpha_G = \alpha_B$.

2.3 Lenders

Lenders do not observe individual borrowers' type $(WTP_i, \theta_i, \sigma_i)$, but they know the population distribution. They can use the revelation principle to design a menu of contracts to make borrower reveal their private information through their choices. I assume that ϵ_{ib} is independent of other borrower parameters $(WTP_i, \sigma_i, \theta_i)$, so banks cannot screen borrowers on their ϵ_{ib} draw. The model is thus the classic principal-agent model, but with demand elasticities that are not infinite or null.

6. when $(\varepsilon_{i,j})_j$ are not all equal

Formally, a lender b designs a menu of contracts to maximize its expected profits subject to some incentive compatibility and participation constraints:

$$\max_{\{(L_{ib},R_{ib})\in\mathcal{F}\}} \sum_{i\in\{G,B\}} \overbrace{E[\sum_{i=1}^{n_j} \mathbf{1}_{\{PC_{ijb}\ satisfied\}}\mathbf{1}_{\{P_i\ satisfied\}}]}^{Demand} \overbrace{\overbrace{NPV_i}}^{Expected\ profit\ on\ loan} \tag{6}$$

$$s.t.\ (IC_{ib}):\ u_i(L_{ib},R_{ib}) \geqslant u_i(L_{jb},R_{jb})\ \forall i,j \in \{G,B\}$$

$$(P_{ib}):\ u_i(L_{ib},R_{ib}) \geqslant \bar{V}$$

$$(PC_{ijb}):\ u_i(L_{ib},R_{ib}) \geqslant u_j(L_{cd},R_{cd}) + \sigma_i^{-1}[\epsilon_{jb} - \epsilon_{jd}],\ \forall ijcd$$

The (IC_{ib}) constraints ensure that type i borrowers chose the contract designed for them.

The first participation constraint (P_{ib}) implies that type i borrowers enter (or do not exit) the credit market.

When the constraint (PC_{ijb}) is satisfied, borrowers j of type i choose lender b instead of its competitors. Those sets of constraints are the main departure from the classic model, and they allow me to include imperfect competition.

For problem (6) to be well defined when the function F is linear, I assume that contracts' characteristics are bounded. Formally, lenders need to offer contracts within the following set $\mathcal{F} := \{(L_c,R_c) : L_c \in [0,\bar{L}],\ R \geqslant 0\}$ with $\bar{L} > 0$. $\bar{L}$ is the maximum house size available in the market. Alternatively, this constraint can be written as a maximum Loan-to-Value constraint to capture existing regulations. I use a constraint on L rather than a constraint on the borrower's second-period income (and thus R) so that the house size is fixed in the first best and independent of the competition level.

2.3.1 Parametrization and simplification of lenders' problem

Problem (6) is difficult to solve, because it involves a set of constraints that may or not be binding. Lemma 1 allows to replace them and the term $E[\sum_{j=1}^{n_i} \mathbf{1}_{\{PC_{jb}\ satisfied\}}]$ by a continuous demand function.

Lemma 1: PC_{ijb} ***constraints.*** *When the shocks ϵ_{jb} are iid and extreme value distributed conditional on borrower type, the participation constraints (PC_{ijb}) can be removed from problem*

(6) *and the term* $E[\sum_{j=1}^{n_i} \mathbf{1}_{\{PC_{jb}\ satisfied\}}]$ *has a closed form expression denoted by* $N_i^b(u_i(L_{ib}, R_{ib}))$. *The expected number of borrowers that come to lender b is:*

$$N_i^b(u_i(L_{ib}, R_{ib})) := E[\sum_{j=1}^{n_i} \mathbf{1}_{\{PC_{ijb}\ satisfied\}}] \tag{7}$$

$$= \overbrace{n_i}^{market\ size} \overbrace{\frac{exp(\sigma_i u_i(L_{ib}, R_{ib}))}{\sum_{x \in \mathbf{B}} exp(\sigma_i u_i(L_{ix}, R_{ix}))}}^{Probability\ i\ chooses\ b} \tag{8}$$

$N_i^b(u_i(L_{ib}, R_{ib}))$ is a continuous demand function that flexibly captures different degrees of competition (captured by the demand semi-elasticity $-\frac{\partial_u N_i^b}{N_i^b} = \sigma_i(1 - N_i^b)$). $\sigma_i \to \infty$ captures the perfect competition case. In that case, if the contract from lender b (L_{ib}, R_{ib}) is epsilon better than that of its competitors, it attracts the full market (i.e., $\frac{exp(\sigma_i u_i(L_{ib}, R_{ib}))}{\sum_{x \in B} exp(\sigma_i u_i(L_{ix}, R_{ix}))} \to 1$). $\sigma_i \to 0$, then each lender behaves as a monopoly. The market share of lender b is $\frac{n_i}{B}$ no matter what contract it offers.

Net Present Value of Lending: Lender b's net present value of lending to borrower i via a contract ib is denoted NPV_{ib}:

$$NPV_{ib} := \theta_i R_{ib} - mcL_{ib} \tag{9}$$

Where mc is the marginal cost of lending via contract ib. $\theta_i R_{ib}$ is the present value of the loan. The use of collateral as in the mortgage mico-foundation presented in the previous section would give us the additional term capturing the amount recovered upon default $(1 - \theta_i)min\{\gamma H, R_{ib}\}$, where γ is the value of the house upon default. For risk discrimination to be relevant, I consider the case in which the price of the house upon default γ is low enough so that the collateral is not enough to repay the face value of the debt. This is in line with surveys on collateral recoup rates (e.g., Davydenko and Franks 2008 for corporate loans). Without loss of generality, I assume $\gamma = 0$ for notational simplicity.

Assumptions A1: In order to obtain closed-form solutions, I assume that F is linear on $[0, \bar{L}]$ and equal to $F(L) = L$.[7]

7. As shown in the next section, as can use non linear pricing, this assumptions does not affect the economic channels driving perfect information contracts. The assumption does not affect the imperfect information case: the

To make the use of menus relevant, I consider the situation in which both market segments have positive NPV $\alpha_i - mc > 0 \; \forall i$ and $\theta_i > 0$. If only the low WTP borrowers has positive NPV, the model will be similar to that of Akerlof (1978), in which only one contract is offered. Similarly, if only the high WTP borrower has a positive NPV, then banks exclude the low WTP borrowers using one contract.

3 Optimal Menu Design

To gain intuition, I first analyze how each lender set contract terms under under perfect information.

Then I focus on the imperfect information case. Using the continuous demand function, I propose a decomposition of the economic forces at play in screening models without relying on numerical methods or equilibrium refinements.

3.1 Contracts when borrower's type is observable

The first-order conditions of problem (6) without the incentive compatibility constraint (IC_B) yield:

Proposition 1: Banks' contract under perfect information

Each bank uses product characteristics (L_i) to maximize the surplus of lending ($S_i(L_i) := (\alpha_i - mc)L_i$), then uses the interest rate to split the surplus between itself and the borrowers.

Dropping the b index for clarity of notation, the optimal contract (L_i^{PI}, R_i^{PI}) is:

Characteristics:

$$L_i^{PI} := \bar{L} \tag{10}$$

linearity assumption affects the cost of credit rationing, which is also flexibly captured by the α_i parameters.

Pricing:

$$R_i^{PI} := \begin{cases} \overbrace{\frac{mc_i L_i^{PI}}{\theta_i}}^{fair\ price} + \overbrace{\frac{N_i}{-\partial_R N_i}}^{\text{“}markup\text{”}} & \text{if } u_i \geqslant \bar{V}_i \\ WTP_i L_i^{PI} - \frac{\bar{V}_i}{\theta_i} & \text{Otherwise} \end{cases} \tag{11}$$

Utility:

$$u_i = \begin{cases} \overbrace{S_i(L_i^{PI})}^{Lending\ surplus} - \overbrace{\frac{N_i}{-\partial_R N_i}}^{\text{“}markup\text{”}} & \text{if } u_i \geqslant \bar{V}_i \\ \bar{V}_i & \text{Otherwise} \end{cases} \tag{12}$$

with $S_i(L_i) := (\alpha_i - mc)L_i$ being the surplus generated by the lending activity.

Equation (10) states that the optimal contract maximizes the social surplus of lending. $\bar{L}$ is the threshold above which lending generates negative NPV ($\alpha_i - mc > 0,\ \forall i$).

In a symmetric equilibrium, the right-hand side of equation (11) does not depend on rates, because the markup $\frac{N_i}{\partial_R N_i}$ simplifies to $\frac{1}{\sigma_i(1-B^{-1})\theta_i}$ when the demand takes a logit form.

The interest rate expression captures the following economic channels of the equation (11). The upper-right-hand side of equation (11) captures the classic extensive and intensive margin channels driving the interest rate level. The first term ($\frac{mcL_i^{PI}}{\theta_i}$) is the fair price. That is the price at which banks break even. The second term is a “markup”. It is equal to the inverse price semi-elasticity. The numerator of the “markup” captures the intensive margin: By increasing R, banks earn more on each borrower. The denominator ($\partial_R N_i$) captures the impact of the extensive margin on pricing: By increasing R, the bank loses customers.

Impact of additive separability: The assumption that the utility of loan size and interest rate are additively separable implies that, absent asymmetric information, imperfect competition does not distort loan size away from its first best value. The reason is that, under perfect information, banks set the contract characteristics that maximise the lending surplus and use interest rates to split the surplus between borrowers and lenders. This separability assumption may not be valid when the loan is used to buy a good that is a complement to consumption. I do not analyze this case in the model as I want to focus on the distortions related to the contractual externality only. An empirical structural model should, however, consider how this assump-

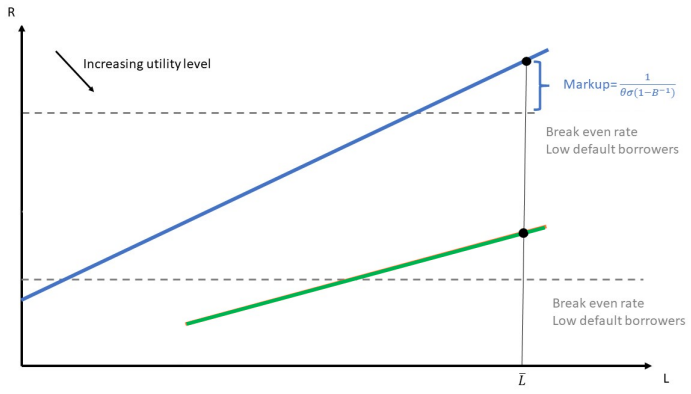


Figure 1: Equilibrium contracts under perfect information

tion impacts the counterfactual simulations. The linearity of F does not impact the economic intuition behind Proposition 1. Without the linearity assumption the $L_i^{PI} = F'^{-1}(mc)$.

3.2 Contracts under imperfect information

Now, let us focus on the perfect information case and how it impacts banks' contracts.

3.2.1 Intuition

As shown in the previous section, given assumption A1 stated in section 2.3: (i) borrowers are offered the same loan size in the first-best, but (ii) banks would like to price them differently due to borrowers' different default probability or price elasticity. Figure (1) summarizes the model equilibrium under perfect information. I plot borrower indifference curves (blue for B borrowers and green for G borrowers) and lenders' break-even conditions for each market segment. To render the graph more legible, I consider the situation in which lenders have to pay a fixed cost of originating contracts (i.e., $mcL = f \in \mathbf{R}^+$).

However, the perfect information contracts in best contracts are not incentive-compatible. If they were both offered in a menu, all borrowers would choose the cheaper product (see Figure 2).

Thus, under imperfect information, banks must distort the first best contracts to maintain borrowers' incentives to self-select. To do so, lenders can use two tools: interest rates (via cross-subsidies) and loan size (via credit rationing). Figures 3 and 4 give a visual representation of how incentives are maintained using each of these margins.

The next subsections analyse which market segment/contract characteristic lender distort.

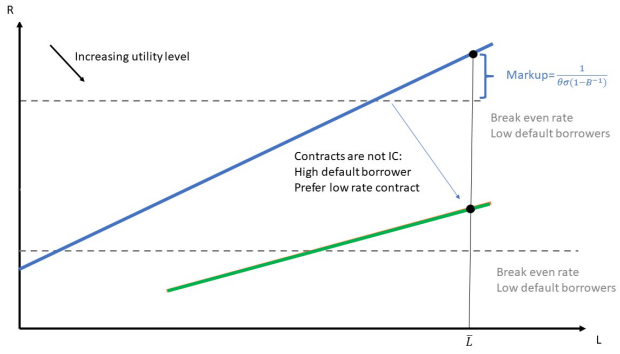


Figure 2: First best contracts are not incentive compatible

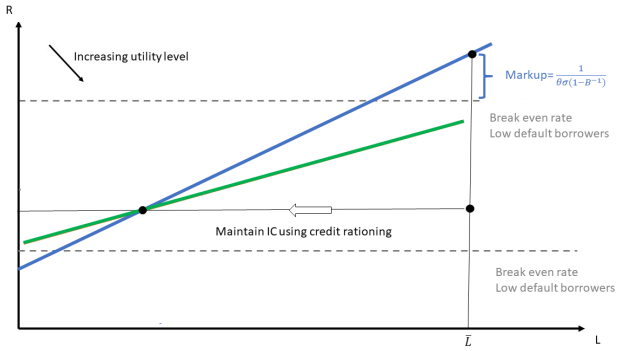


Figure 3: Tool 1 to maintain IC: Use credit rationing

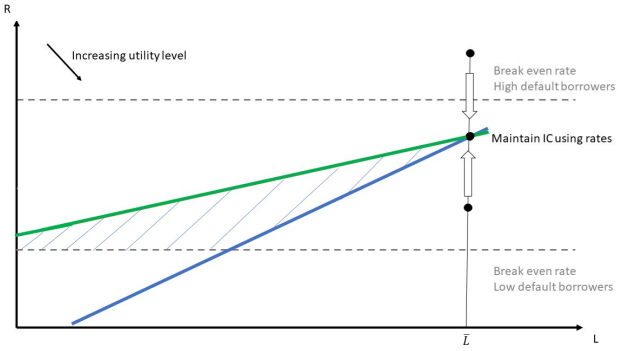


Figure 4: Tool 2 to maintain IC: Use interest rates

I formally show that high competition forces lenders to set prices close to their perfect information values and maintain incentives to self-select using loan size. Otherwise, a competitor can offer a contract that steals only the most profitable customers by offering a contract in the dashed region of Figure 3. I also show that low competition allows the use of cross-subsidies and that lenders choose to distort the contracts of the market segment that generates the lower surplus.

3.2.2 Formal analysis

As in the textbook principal-agent model, the system of IC can be simplified. The simplifications are summarized in the following Lemmas 1, 2 and 3.

Lemma 1: At least one IC is binding. *The IC constraint of B borrower is always binding. When competition is high enough, no participation constraints are binding.*

Proof: Appendix H.

Solving for the problem 6 using Lemma 1, we get:

Lemma 2: No distortion at the top. Credit rationing at the bottom.

(i) The loan size of B borrowers is equal to its first best value.

(ii) The loan size of G borrowers is lower than the first best. The credit rationing is proportional to the interest rate spread. Formally:

$$L_B = \bar{L} \tag{13}$$

$$(IC_B) : \bar{L} - L_G = \frac{R_B^{II} - R_G^{II}}{WTP_B} \in [0, \bar{L}] \tag{14}$$

Proof: Appendix H

The intuition behind those results can be grasped from Figures 3 and 4. The no distortion at the top results from the fact that lowering the B borrower's loan size does not help with self-selection: It renders the B borrower worse off and the G contract even more attractive. Credit rationing works because B borrowers are less willing to pay for loan size. Equation (14) is the B borrower's incentive compatibility. It states that screening works when the average cost of one extra dollar above L^{PI} is priced at the B borrower's willingness to pay. That way, only B borrowers find the extra amount of borrowing attractive.

Lemma 3: Interest rate distortions.

Relative to the first best, the interest rate of B borrowers is lower, and the one of the G

borrowers is higher. The pricing takes the form:

$$R_i := \begin{cases} \overbrace{\frac{mcL_i}{\theta_i}}^{fair\ price} + \overbrace{\frac{N_i}{-\partial_R N_i}}^{\text{“markup”}}[1 + \overbrace{I_i}^{Asymmetric\ information\ discount/premium}] & \text{if } u_i \geqslant \bar{V}_i \\ WTP_i \cdot L_i - \bar{V}_i & \text{Otherwise} \end{cases} \tag{15}$$

$I_G > 0, I_B < 0$

Proof: (i) Use Lemmas 2 and 1. (ii) solve the lender problem using the Lagrangian.

Equation (15) states that the price can be written as in the perfect information case (fair price and markup) with an extra additive term (I_i). I call this extra term the asymmetric information discount or premium. I_i is positive for G borrowers (i.e., premium) and negative for B borrowers (i.e., discount).

Absent I_i, the spread between rates would be higher, which implies that banks would have to distort L_G more intensively. I_i thus provides information about the incentives to screen: An increase in $|I_B|$ lowers the B market segment profits compared to the perfect information case but allows for an increase in G market segment profits by relaxing the incentive compatibility constraint. An increase in I_G increases G interest rate distortion but lowers the credit rationing level.

The formula captures channels that are absent from perfect competition and monopoly models. Under perfect competition, only the fair price $\frac{mcL_i^I}{\theta_i}$ is present. Under monopoly, the participation constraint is binding for one borrower, and the price of the other borrower is (partially) driven by this outside option. The markup $(\frac{N_i}{\partial_R N_i})$ and the asymmetric information premium or discount (I_i) are thus specific to the imperfect competition case and endogenously determined. Analysing (I_G, I_B) thus allows us to study how various changes in the economic environment affect incentives to use both interest rates and loan size distortion to screen.

The interest rate formula (15) is a key novelty of the paper. I show in the next section that a pure strategy equilibrium exists so that using equation (15) is valid. Previous papers do not use a continuous demand system and thus cannot derive an equation such as this one and rely on numerical methods to interpret the results. The interest rate formula is used as a key moment condition to recover model parameters in the empirical analysis. Having a pure strategy equi-

librium is a good depiction of strategies observed in practice and simplifies the estimation and identification.

Proposition 2: Banks' incentives to screen.

Banks have incentives to screen (i.e., to use lower cross-subsidies) when the surplus generated by lending in the G market segment is low and when screening is costly. The asymmetric information premium and discount are given by:

$$I_B := -\frac{n_G}{n_B}\frac{\overbrace{\alpha_G - mc}^{Benefit\ of\ pooling:\ More\ surplus}}{\underbrace{WTP_B - WTP_G}_{Cost\ of\ pooling:\ Information\ rent}} > 0 \tag{16}$$

$$-I_G := \frac{\alpha_G - mc}{WTP_B - WTP_G} > 0 \tag{17}$$

I show in the next section (section 4) that an interior solution (i.e., $L_G^{II} \in [0, \bar{L}]$ and $u_i \geqslant \bar{V}$) in which the participation constraints are not binding exists when competition is high enough. To gain intuition about incentives to screen, I focus on the interior solution in the subsequent paragraphs.

Equation (16) captures the following trade-off. By lowering high default borrower interest rates above their perfect information value, lenders are able to relax the incentive compatibility constraint and extract the surplus generated ($\alpha_G - mc$ per dollar lent). The benefit is proportional to the number of low-default borrowers they have (N_G). However, to be able to lend one dollar more and maintain incentives, lenders have to provide a discount of $WTP_B - WTP_G$. This can be seen using the incentive compatibility constraint of the B borrower in terms of utility (IC_B : $u_B = u_G + (WTP_B - WTP_G)L_G$). $(WTP_B - WTP_G)L_G$ is referred to as the information rent in the literature. If a lender wants to provide utility u_G to low-default borrowers, lenders must design a B contract that gives them an extra amount of utility proportional to the differences in willingness to pay ($(WTP_B - WTP_G)L_G$).

Equation (17) captures the following trade-off. Increasing the interest rate allows lending more and up to ($\alpha_G - mc$ per dollar lent). However, to maintain incentives, there is a limit to how much of the surplus they can extract. This is captured by $\frac{1}{(WTP_B - WTP_G)}$.

4 Equilibrium: Existence and Uniqueness

In this section, I discuss equilibrium existence and characterize the equilibrium contracts in closed form. My model features a unique pure strategy equilibrium. I provide intuition about the underlying mechanism and contrast my approach to the literature.

Non-existence result in the literature: In the textbook perfect competition model of screening (i.e., Rothschild and Stiglitz 1976), pooling contracts cannot be offered. This is because of cream-skimming: A lender can take advantage of its competitors pooling contracts by introducing a new contract with a smaller loan size and pricing it so that it only steals the most profitable borrowers (See Figure 5). Yet, offering screening contracts cannot be an equilibrium when the number of low-default borrowers is large enough: A pooling deviation exists because low-default borrowers prefer to be pooled; since that allows them to get a lower credit constraint at the cost of a small interest rate increase (See Figure 6).

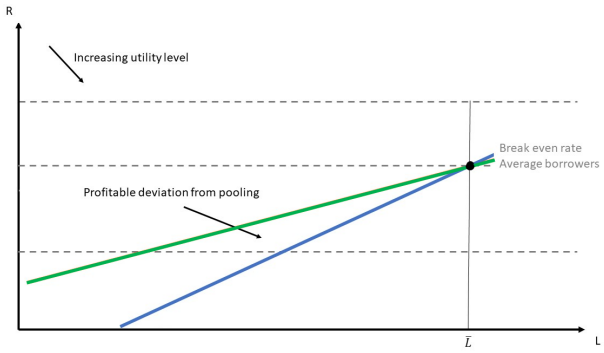


Figure 5: Perfect competition case: Deviation from pooling always exists

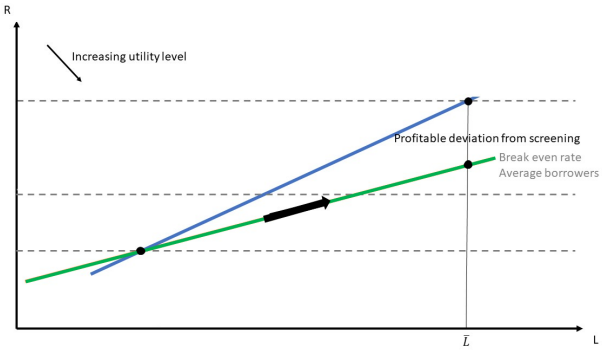


Figure 6: Perfect competition case: Deviation from screening always when the share of good borrower is high

To overcome this non-existence result, one can use other equilibrium concepts as in, for instance, Riley (1979), Bisin and Gottardi (2006), or Wilson (1980). The two first equilibrium concepts restore the existence of the screening equilibrium, while the third one restores the pooling equilibrium. Another way to overcome this issue is to change the modelling. For instance, Guerrieri, Shimer, and Wright (2010) assume that the principal can match at most one borrower. Finally, allowing for mixed strategies also solves the non-existence problem (see Dasgupta and Maskin (1986) for proof that a mixed strategy equilibrium exists, and Lester, Shourideh, Venkateswaran, and Zetlin-Jones (2019) or Farinha Luz (2017) for a numerical so-

lution to those mixed strategies). Mixed strategies are computationally demanding, so they are restricted to stylized models that cannot flexibly capture important channels in an empirical application. Furthermore, mixed strategies are not observed in most set-ups.

In my model: I show that imperfect competition restores the existence of a pure strategy Nash equilibrium. I derive the conditions for the equilibrium to exist. When demand elasticities are large, I use a capacity constraint as in Azevedo and Gottlieb (2017). The following proposition states the conditions under which the capacity constraint is needed and characterizes the equilibrium.

Definition 1: Capacity constraint

A lender has a capacity constraint if they cannot serve more than a certain number of customers $\bar{N}$. I denote by k_i the maximum share of customers that the firm can serve if they only serve type i borrowers (i.e., $\{k_i : k_i \cdot n_i = \bar{N}\}$). I denote $k = \max_{i \in \{G,B\}} k_i$.

For simplicity of notation, we now assume that the price elasticity of both borrowers is the same ($\sigma := \sigma_G(1 - \frac{n_G}{B}) = \sigma_B(1 - \frac{n_B}{B})$). The equilibrium existence result does not rely on this assumption. However, it allows a simple characterization of when the capacity constraint is required.

Proposition 3: Equilibrium existence, uniqueness and characterization

Existence and uniqueness: *There exists a unique pure strategy equilibrium as long as the demand elasticity σ is lower than $\bar{\sigma} := \frac{1}{[WTP_B - mc]\bar{L} - 1}$ or as long as lenders have a capacity constraint $k < \frac{1}{2}$.*

Characterization: *The equilibrium is symmetric and is given by replacing N_i with $\frac{n_i}{\mathbf{B}}$ and $\frac{N_i}{\partial_{R_i} N_i}$ with $\frac{1}{\sigma_i(1 - \frac{n_i}{\mathbf{B}})\theta_i}$ in the formulas (15) and (16).*

Proof: See Appendix I.

In my model, deviations from screening are not profitable when the demand elasticities are

low enough, since they attract too many high-default borrowers to be profitable. The reason is that lenders cannot attract all borrowers and that the deviation is relatively more valuable to high default borrowers. Deviations from screening (i.e., using cross-subsidies) are more valuable to high-default borrowers because they require increasing the G contract loan size and interest rate, which is relatively more valuable and less costly to the high-default borrowers. A heuristic derivation of this argument is given in Appendix J.

A visual representation of this argument is provided in Figure 6. The most profitable perfect competition pooling deviation implies moving along the G borrower indifference curve. This deviation makes the G borrower indifferent and the B borrower better off. Under perfect competition, the deviation attracts the full market segment. The break-even rate is thus below the pooling contract deviation when there are enough low-default borrowers. Under imperfect competition, those deviations attract too many high default borrowers to be profitable. This is as if the break-even condition for the representative borrowers endogenously moves above the pooling contract.

All in all, the equilibrium under imperfect competition features just enough cross-subsidy so that any pooling deviations are not profitable as they attract a relative number of types of agents so that the existence condition in Rothschild and Stiglitz (1976) is satisfied. Deviations using cream-skimming strategies (as in the deviation from pooling case) are also unprofitable because they attract too few borrowers from other banks and make you lose too many high default borrowers for which you make profits on.[8]

When competition is high enough, pooling deviations become profitable because lenders can attract the full market segment. Therefore, the argument that pooling deviations are relatively more valuable to high default borrowers is not valid anymore. To prevent this, I introduce a capacity constraint.

Using Lemma 2 3 and 4 and Proposition 3, we can derive the level of credit rationing. Given (14), the level of credit rationing is proportional to the interest rate spread. Formally:

Lemma 4: Equilibrium level of credit rationing (interior solution)*: Abusing notation I use*

8. Similarly, when the equilibrium is such that once makes negative profits on the B market segment, the deviation from pooling do not deter enough B borrower from moving to another lender.

σ_G and σ_B as a shortcut for $\sigma_G(1-\frac{n_G}{\mathbf{B}})$ and $\sigma_B(1-\frac{n_B}{\mathbf{B}})$.

$$L_G = \overbrace{\frac{1}{S_{BG}}[S_B\bar{L}}^{perfect\ competition\ distortion\ level} + \underbrace{(\frac{1}{\sigma_G\theta_G}-\frac{1}{\sigma_B\theta_B})}_{competition\ channel} + \overbrace{\frac{I_G}{\sigma_G\theta_G}+\frac{|I_B|}{\sigma_B\theta_B})}^{cross-subsidy\ channel} \tag{18}$$

$$S_{BG} := WTP_B - \frac{mc}{\theta_G} \tag{19}$$

$$S_i := WTP_i - \frac{mc}{\theta_i},\ i \in \{G,B\} \tag{20}$$

Equation (73) captures the three channels that drive credit rationing. I use the notation $WTP_i = \frac{\alpha_i}{\theta_i}$ to distinguish the elements in the formula that depend on borrowers' preferences from the one that comes from the supply side.

The first part of the formula (i.e., $\frac{1}{\alpha_B - mc\frac{\theta_B}{\theta_G}}(\alpha_B - mc)\bar{L}$) captures the level of distortion absent any markup or cross-subsidy (i.e., the perfect competition credit rationing level). $(\alpha_B - mc)\bar{L}$ is the utility of B borrowers under perfect competition when the contract is priced at marginal costs ($R_B = \frac{mc}{\theta_B}$). $\alpha_B - mc\frac{\theta_B}{\theta_G}$ is the utility derived from B borrowers per dollar lent when choosing the contract designed for G borrowers under perfect competition ($R_G = \frac{mc}{\theta_G}$). Loan size $L_G = \frac{(\alpha_B - mc)\bar{L}}{\alpha_B - mc\frac{\theta_B}{\theta_G}}$ renders the B borrower indifferent between the two contacts when each contract is priced at the break-even price.

The second term captures the distortion level once we add the perfect information markups (i.e., the inverse interest rate elasticities). $(\frac{1}{\sigma_G\theta_G} - \frac{1}{\sigma_B\theta_B})$ is the difference between the perfect information markups of the G and B contracts when the competition level is (σ_G, σ_B). A higher spread makes the G contract less attractive for B borrowers which relaxes the credit constraint.

The last term captures how much cross-subsidies are used to relax the credit rationing.

5 Analysis of the Equilibrium Contracts

In this section, I provide a positive and normative analysis of the equilibrium contracts.

5.1 Positive analysis of the equilibrium contracts

This section provides comparative statics with respect to competition (σ) and changes in the marginal costs of lending (mc) of a given contract type. I allow for the marginal cost to depend on contract characteristics. Appendix M details the formal analysis of this extension.

I focus on an interior solution (i.e., $L_G < \bar{L}$ and the participation constraint is not binding) as changes in L_G provide intuition about the different forces behind pooling and screening incentives.

In appendix N, I provide additional comparative statics with respect to adverse selection ($\frac{\theta_G}{\theta_B}$), and changes in prices of the good bought with the loan (α) — also referred to as house prices.

Result 1: Competition. *Let us denote $\Delta\sigma := \frac{\sigma_G}{\sigma_B}$ and look and changes in competition levels ($\sigma = \sigma_B$), holding the ratio $\Delta\sigma$ constant.*

(i) Lender screen when competition is high enough (i.e., $\sigma \in [\sigma^*, \infty)$).

(ii) When competition decreases ($\sigma \downarrow$), the amount of credit rationing and the interest rate spread decreases as long as the cross-subsidy effect described in Lemma 4 dominates that of the competition (i.e., $MU + CS > 0$). The switching point between screening and pooling (σ^*) is defined by:

$$\frac{1}{\sigma^*} := \frac{\bar{L}[S_{BG} - S_G]}{MU + CS} \tag{21}$$

$$MU := \frac{1}{\theta_B} - \frac{1}{\Delta\sigma\theta_G} < 0 \tag{22}$$

$$CS := (\frac{I_G}{\Delta\sigma\theta_G} + |I_B|) > 0 \tag{23}$$

The intuition behind result 1 (i) is presented in Section 4. When the market is competitive, lenders have to price each borrower close to their perfect information prices; otherwise, they can lose their most profitable customers to a competitor.

Result 1 (ii) derives from the following considerations. Let us consider for simplicity that demand elasticities decrease proportionally (i.e., σ_B decreases holding $\Delta\sigma$ constant). Under perfect information, banks can increase rates by approximately $\frac{1}{\theta_i} d\sigma_B^{-1}$ as demand elasticities decreases. However, under imperfect information, it is optimal for lenders to increase the rate relatively less for B borrowers. By doing so, they make the G contract less attractive to B borrowers, which allows lenders to lend more to G borrowers. Thus as competition increases, the spread between interest decrease rates decreases. When competition decreases relatively more in the G market segment, it creates and additional channel that goes in the opposite direction.

Equation (21) states that lenders tend to pool more often ($\sigma^* \uparrow$) when there are more profits to be made in the G market segment and when asymmetric information discounts and premiums

are effective in dealing with the distortions. Those channels are captured by the CS term in the denominator. The term $MU < 0$ captures the spread between perfect information markups. When $|MU|$ is high, the perfect competition contract is less incentive-compatible, making it more likely to screen. The numerator captures how large the distortion would be absent any cross-subsidies or markups. It is a function of the spread between defaults and the loan size. The higher those elements, the more cross-subsidies are required to maintain pooling. High levels of cross-subsidies are only feasible when competition is low.

Overall, this first result goes against the standard argument that lowering competition leads to lower loan sizes due to monopoly distortions.

I summarize Result 1 in Figure 7, where I plot the various types of equilibrium that arise when competition and the number of borrowers variy. To focus on risk discrimination, the figure is plotted for the case in which $\frac{\sigma_B}{\sigma_G} = 1$ and varies the level of product elasticity (σ) while keeping the ratio of product elasticities constant. Figure 8 highlights that the capacity constraint has no impact on the equilibrium once the competition is low enough.

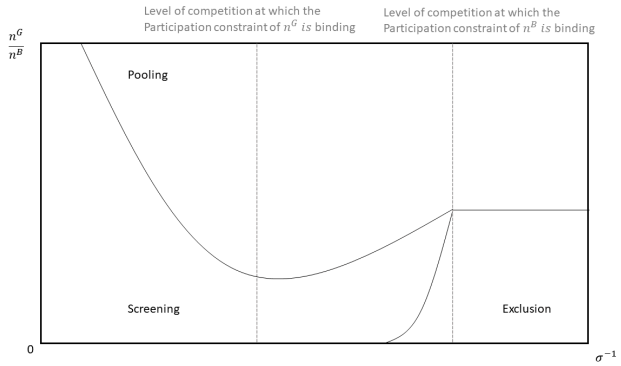


Figure 7: Equilibrium regions with the capacity constraint

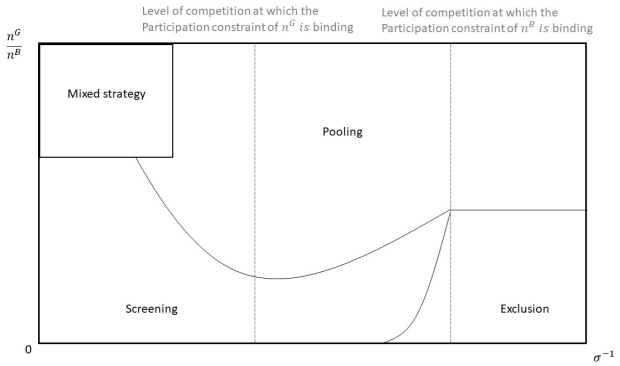


Figure 8: Equilibrium regions without the capacity constraint

Result 2: Marginal costs (monetary policy, risk weights).

(i) Increasing the marginal cost of lending via for instance capital requirements of low loan contracts decreases the credit rationing level and interest rate spread under high competition but increases it it under low competition (i.e., $\sigma < \sigma_{mc_G} := \frac{CS[\frac{S_{BG}}{S_G} - 1] - MU}{PC}$).

(ii) Increasing the marginal cost of lending of high loan contracts increases the amount of credit rationing.

Proof: See Appendix K

To analyse result 2, I decompose the effect of marginal cost into a perfect competition and an imperfect competition channel. For intuition, let us consider a situation in which the marginal cost of the G contract increases.

Under perfect competition, the cost has to be passed through interest rates. As a result, the rate of the G contracts increases, which makes it less attractive to B borrowers. As a result, the incentive compatibility constraint is relaxed and lenders can increase the G loan size.[9] Under imperfect competition, the increase in the G marginal cost decreases the potential social surplus generated by the G market segment relative to the B market segment. This creates incentives to distort the G contracts more heavily in order to extract more surplus from the B market segment.

This comparative static is particularly relevant for credit markets. Indeed, banking policies often affect the marginal cost of lending but are not analyzed through the lens of screening models. Depending on the contract characteristics used to screen, different policies can be analysed with my model. For instance, in markets in which lenders screen with maturity (see Hertzberg, Liberman, and Paravisini 2018 for empirical evidence), quantitative easing (QE) is a pertinent policy. This is because maturity plays the same role as interest rate in the model, and that QE affect marginal costs of originating contracts with different maturity. In markets where lenders screen with Loan-to-Values (See Taburet, Polo, and Vo 2024 for empirical evidence, then capital requirements are the relevant policy. LTV plays the same role as loan size in the model, and capital requirements are often LTV-specific.

I summarize Result 2 in Figure 9.

5.2 Normative Analysis: Screening Externality and Pareto Improvement

In this subsection, I show that a contractual externality may cause excessive screening. This implies that there is room for policy interventions. I derive a sufficient statistic to test for its presence in the data. This externality is present even in the region in which the capacity constraint is irrelevant (see figure 8), which renders it a robust feature of screening models.

9. Increasing the marginal cost of B contracts has the opposite effect, as it tightens the incentive compatibility constraints. When both marginal costs increase, the second effect dominates as interest rates increase relatively more in the B contract because default probabilities scale them up.

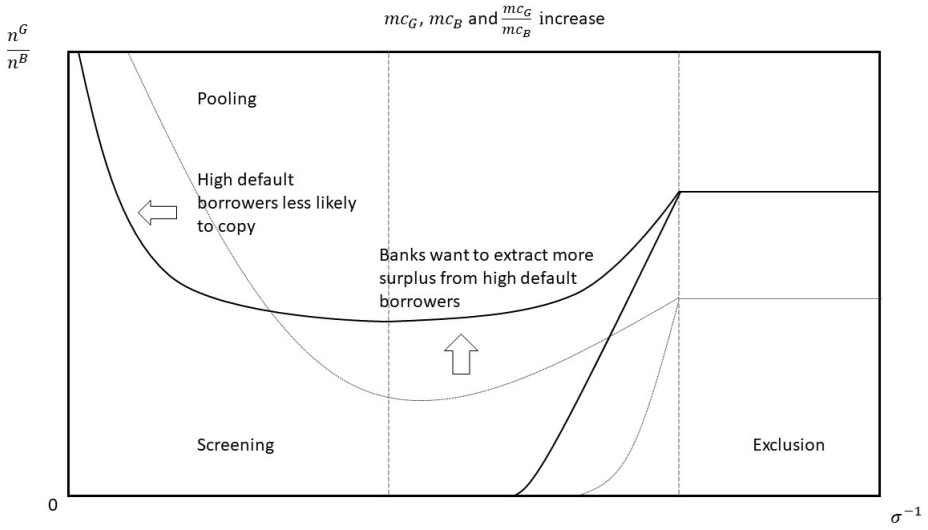


Figure 9: Equilibrium regions when marginal costs increase

Contractual externality: To obtain a lower bound on the contractual externality, I show that there exists a menu of contracts that makes all borrowers and lenders better off. I do so by solving for the optional contract of an informationally constrained social planner. Formally, the social planner maximizes lenders' profits subject to (i) the same incentives compatibility constraints as lenders and (ii) providing at least as much utility to borrowers as the market contracts for a given competition level (denoted (u_B^*, u_G^*)). That is, the problem is the same as (6) but with constant market shares (n_i) and the participation constraints being equation to u_i^* instead of $\bar{V}$. The constant market shares assumption shuts down the contractual externality. The social planner's problem can be written:

$$\max_{\{(L_B,L_G)\in[0,\bar{L}]^2\}} \sum_i \overbrace{n_i}^{Market\ size} \overbrace{[(WTP_i - \frac{mc}{\theta_i})L_i - u_i^*]}^{profits\ on\ type\ i\ market\ segment}$$

$$s.t.\ (IC_B) : u_B^* = u_G^* + (WTP_B - WTP_G)L_G$$

Using the first-order conditions, I derive the following Proposition:

Proposition 4: Sufficient Statistic for the Contractual Externality.
Banks should pool borrowers together at $\bar{L}$ if and only if:

$$\frac{WTP_G - \frac{mc}{\theta_G}}{(WTP_B - WTP_G)} \frac{n_G}{n_B} > 1 \tag{24}$$

However, except under monopoly, the competitive equilibrium switching point differs from the social planner's one (24). For instance, according to Result 2, when both participation constraints are not binding, banks pool borrowers if and only if: $\sigma \in [0, \sigma^*]$.

Proposition 4 shows the conditions under which the equilibrium is inefficient in the second-best sense.

The friction emerges for the following reasons. Maintaining customers' incentives to self-select is costly since it may require distorting contracts relative to the first best. When the distortions are too high, it is more efficient to pool borrowers instead. Yet, if pooling contracts are offered and competition is high, a competitor can take advantage of a pooling contract by introducing a product that steals only the most profitable customers (cream-skimming). The friction arises because lenders do not internalize how their screening strategies (e.g., cream-skimming) change the types of borrowers selecting competitors' products — and thus the cost of lending via those products.

This friction exists only when pooling is a Pareto improvement over screening so that cream-skimming deviations are ex-post inefficient. Equation (24) captures the condition for the latter to be true. The numerator is the benefit of increasing L_G, and the denominator represents the costs. Increasing the loan size allows the social planner to generate more profits $(WTP_i - \frac{mc}{\theta_i})$ per G borrowers. However, to maintain incentives to self-select, lenders have to decrease the interest rate on the B market segment, which leads to a loss of $(WTP_B - WTP_G)$ per B borrower. In this model, the utilitarian welfare function—used in, for instance, Lester, Shourideh, Venkateswaran, and Zetlin-Jones (2019)—is equivalent to decreasing the level of credit rationing (i.e., considering only the numerator of equation (24)).

Equation (24) can be used to test for the existence of the contractual externality. We can use a revealed preference approach to recover borrowers' willingness to pay and relative contracts' shares to obtain the relative numbers of borrowers. The marginal cost can be recovered either using accounting data or by estimating it using interest rate first-order conditions as is standard in the industrial organisation literature.

Effect of policy interventions: In Appendix O, I show that decreasing competition can lead to a Pareto improvement. A similar result is obtained by changing marginal costs. The intuition is that it lowers' incentives to implement cream-skimming deviations. The policy can render G borrowers better off by lowering their credit constraint at the cost of increasing the markup. B Borrowers get a higher information rent but also a higher markup. Finally, a decrease in competition allows lenders to extract more surplus from their customers. Figure 10 illustrates the set of parameters for which decreasing competition is a Pareto improvement. This is the case when cream-skimming occurs and is inefficient. That is, when the number of high borrowers is relatively large and competition is high.

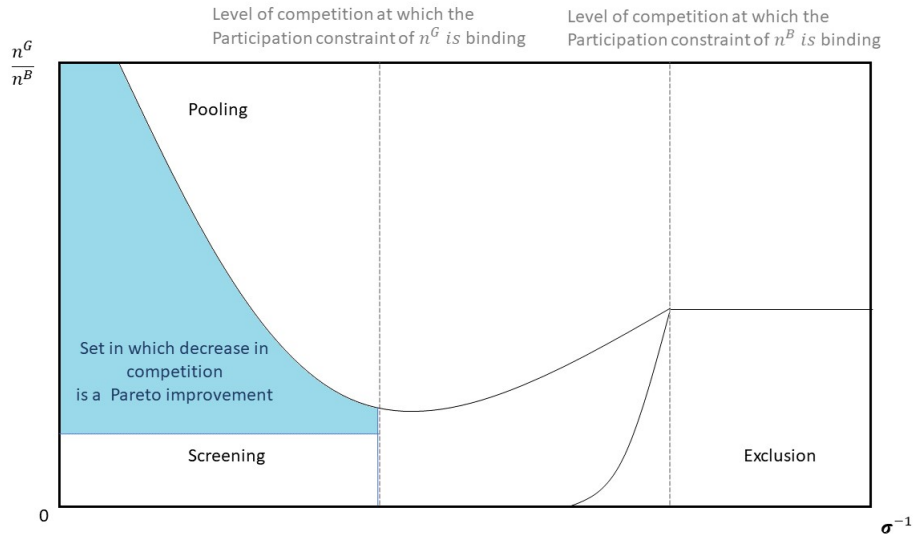


Figure 10: Pareto set

6 Empirical Application

This section illustrates how to bring the model to the data.

The identification and estimation in a general credit market set-up are discussed in length in Appendix P. I discuss the identification and estimation of model parameters for types of datasets commonly used in banking or insurance. In the first scenario, I consider that the econometrician has data on menus and customer choices but only for an individual firm. This is the most common scenario in which menus are observable (see, for instance, Hertzberg, Liberman, and Paravisini (2018) for the credit market or Tebaldi (2024) for the insurance market). I refer to this as the proprietary data case. Second, I analyse the second situation in which the econometrician

has data on multiple firms but can only see the equilibrium contracts chosen by each customer. This is the data constraint faced when using credit register data (see, for instance, Crawford, Pavanini, and Schivardi 2018).

The Appendix P also summarizes the empirical facts that guided my modelling assumptions and develops extensions that render the model applicable to a wide range of empirical setups. In particular, I discuss how to deal with maturity and the fact that default probabilities depend on contract terms in the main text, since these are directly related to the empirical application in Section 6. I defer to Appendix F the extensions such as multiple borrower types and contract characteristics.

6.1 Data and Setting

I use data from a key paper on screening in credit markets: Hertzberg, Liberman, and Paravisini (2018).

The data concern an online lending company, Lending Club (henceforth LC) in 2013. This is an ideal institutional setting for studying screening with menus of contracts. The dataset contains all information observed by lenders at the time of origination. The menu offered, borrowers' choices, and defaults are also observable.

LC is an online lending platform that offers unsecured loans for amounts between $1,000 and $35,000. LC was the largest online lending platform in the U.S. in 2013. It originated $4.4B in consumer loans across 45 states. Prosper Marketplace, its nearest rival, originated $1.6B in the same year.

The borrowing process is the following. Prior to the borrower's selection of a loan amount or maturity, LC assigns them to one of 25 risk categories using a proprietary credit risk assessment algorithm. The algorithm uses the hard information in a borrower's credit report (e.g., FICO score, outstanding debt, repayment status) and income. Conditional on the risk category, borrowers are offered the same menu. They can choose how much to borrow and a maturity of 36 or 60 months. The interest rate only depends on the risk category, not on the loan size or maturity. For low loan size amounts, only the low-maturity contract is available. I observe the risk category, the menu available to the borrower, and their choice.

Model setup: Hertzberg, Liberman, and Paravisini (2018) show that LC can screen borrowers with maturity. To capture the LC set up, I thus allow lenders to choose monthly payments,

maturity and loan size. I use a flexible model that allows for moral hazard and adverse selection: Default probabilities depend on borrower characteristics and monthly payments. I also allow for an arbitrary number of types.[10] Maturity plays the same role as the interest rate in the baseline model. I derive the pricing equation by taking the first-order conditions with respect to maturity. Appendix R derives sufficient conditions for the model to match menu patterns (i.e., fixed interest rates conditional on the risk category and a continuum of loan size offered).

6.2 Identification and Estimation

I apply the approach discussed in Section P.1. Given the variation in menus generated by risk categories, I can use only step 1 and a generalized version of step 3 to recover key model parameters. Nonetheless, I present the results of step 2 at the end of the section to show how they can be used to derive information about the distribution of willingness to pay and that the results are plausible.

Default Probability Estimates (Step 1): I start by calculating the average default probability on long and short-maturity conditional on the borrower's investment grade and the monthly payment-to-income ratio (Denoted PI_i). I run the following regression:

$$1 - Default_i = \delta_i^{gt} + \beta \frac{Montly\ Payment_i}{Income_i} + \alpha \mathbf{1}_{\{T_i=60\ Month\}} + e_i \tag{25}$$

$Default_i$ is a dummy equal to 1 if the loan is reported as charged off after the loan should have been completely repaid. δ_i^{gt} are investment grade-time fixed effects. The superscript g indexes the grade and t the time. $\frac{Montly\ Payment_i}{Income_i}$ is the monthly payment to yearly income ratio. I normalize the standard deviation to one. α is the coefficient of interest. This coefficient captures whether high-default borrowers self-select into different loans.

I define θ_i as the monthly survival probability. Formally, denoting λ_i the hazard rate:

$$\theta_i := e^{-\lambda_i} \text{ where } \lambda_i := -\frac{log(E[1 - Default_i | g_i, T_i, \frac{Montly\ Payment_i}{Income_i}])}{T_i} \tag{26}$$

Marginal costs, Demand elasticities, asymmetric information premium and discounts

10. Appendix (S) shows that the equilibrium conditions of the model with an arbitrary number of borrower types but with LC choosing to offer two optimally, and the optimality conditions with two borrower types are identical.

(Generalized Step 3):

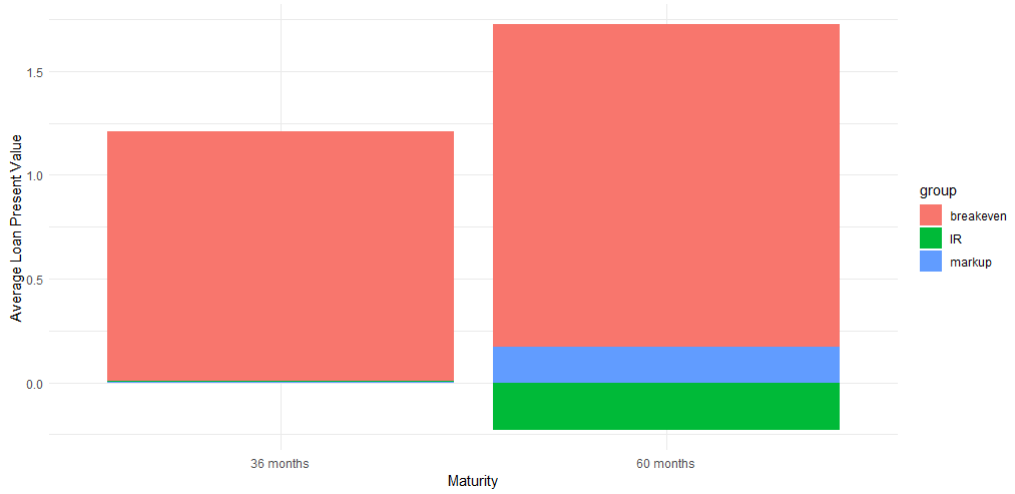


Figure 11: Decomposition of the present value of the average loan (normalized by loan size)

I use the optimal pricing condition to recover marginal costs, the asymmetric information premium and discounts and the demand elasticity. Instead of inverting the equation as suggested in section (P), I leverage variation in the relative market share of the contracts for each investment grade. I do not observe the marginal cost.

In practice, I run the following two regressions. For the high-maturity contract, regress the present value of the loan on loan size, a constant, and the relative contract shares ($\frac{n_{36}}{n_{60}}$, where 36 stands for the low maturity contract). For the high-maturity contract, I regress the present value of the loan on the loan size and a constant. I parameterize the principal of the loan R as a linear approximation of the continuous time annuity formula ($\rho R := M[1 - e^{\rho T}] \approx \rho m T$ where m is the monthly payment, and T is the maturity so that $\theta M \rho T$ is the approximate loan present value). All coefficients on the right-hand side should thus be interpreted as divided by ρ. This implies that the equation identifies the relative value of each component of the pricing equation. Formally, I run the two regressions for each month:

$$\overbrace{E[\theta_i|g_i, T_i = 60]R_i}^{Present\ Value} = \overbrace{mc_{T_i}\text{Ł}_i}^{Break\ even} + \overbrace{\sigma_{T_i}}^{\text{“}mark\ up\text{”}} + \overbrace{\beta\frac{n_{36g}}{n_{60g}}}^{Information\ rent:\ -\sigma_{T_i}\frac{n_{36g}}{n_{60g}}I_g} + e_i \tag{27}$$

$$\underbrace{E[\theta_i|g_i, T_i = 36]R_i}_{Present\ value} = \underbrace{mc_{T_i}\text{Ł}_i}_{Break\ even} + \underbrace{\alpha}_{Total\ markup:\ \sigma_{T_i}(1+I_g)} + e_i \tag{28}$$

$(mc_{T_i}, \sigma_{T_i}, \beta, \alpha)$ are the parameters to estimate. I interpret them through the lens of the

structural model. The error term e_i captures the heterogeneity in demand elasticity $\sigma_i - \sigma$ and information rent $I_g\sigma_i - I\sigma$ where $I_g := \frac{\alpha_{36g}-mc}{WTP_{60g}-WTP_{36g}}$. I allow for the marginal cost to vary at contract-month level, so it is unlikely that the error term contains heterogeneity in costs.

The coefficient β is the information rent $\sigma_{60}I$. I recover $\hat{I}$ as $\frac{\hat{\beta}}{\hat{\sigma}_{60}}$. The coefficient α captures the perfect information markup and the asymmetric information premium $\sigma_{36}[1 + I]$. I recover $\hat{\sigma}_{36}$ as $\frac{\hat{\alpha}}{1+\hat{I}}$.

Equations (88) and (28) are identity regressions. Thus the independent variable impact should not be interpreted as causal. However, allowing for error terms to arise from borrower unobservable heterogeneity implies the following exclusion restrictions.

The variation in relative market shares across investment grades allows us to identify β. The identifying assumption is that the relative market share variations ($\frac{n_{36g}}{n_{60g}}$) are uncorrelated with preference heterogeneity $WTP_{60g} - WTP_{36g}$ and uncorrelated with the average demand elasticity σ_B of borrowers who choose 60-month maturity contracts. Since the investment grade goal is to predict default (i.e., the borrower's cost) and not demand elasticities, this assumption is plausible.

The variation in loan size within a group allows for recovery of the marginal cost mc. Conditional on the market segment, the loan size variation should be uncorrelated with deviation from the average demand elasticities σ_{T_i} (i.e., search costs). To mitigate the endogeneity concern, I use a granular instrument approach. That is, I instrument loan size using the residual from a loan size regression while controlling for contract characteristics and observable borrowers' characteristics. The underlying assumption is that search cost heterogeneity is uncorrelated with residual demand. The results are very similar with and without the instrument.

The linearity of the model allows use of the full set of tools developed in the reduced-form literature, such as difference in difference. This is a main advantage of this model compared with a theory-oriented framework such that of Lester, Shourideh, Venkateswaran, and Zetlin-Jones (2019). The simplicity of the model also makes the exclusion restrictions transparent, as the error term has a structural interpretation. A caveat of this approach is that results depend on the interpretation of the coefficient of the linear regression. Given that markup and adverse selection are not directly observable, imposing some structure is, however, necessary.

6.3 Results

Table 1 reports the results of the default regressions. The average default probability is 15 percent. Borrowers who choose high-maturity contracts are α 4 per cent more likely to see their loan charged off. The implied average hazard rate is $4 \cdot 10^{-3}$, and that of borrowers with high-maturity contracts is $2.7 \cdot 10^{-3}$. The hazard rate allows accounting for the fact that with constant default probabilities, longer loans are more likely to be charged off.

Results of the present value regressions are reported in Tables 2 and 3. All coefficients are significant and have the sign predicted by the theory. Figure (11) provides a visual decomposition of the present value of the loan. All else equal, high-maturity contracts are more expensive to originate (the marginal cost is 1.5 per dollar lent instead of 1.2). Borrowers who self-select into low-maturity contracts are less likely to default (from step 1). They are also very elastic, so they do not pay any markups. This is consistent with the fact that search costs are very low in the online lending market. Borrowers who self-select into high-maturity contracts are more likely to default (from step 1) and are less price elastic. Their perfect information markup would be 0.2 dollars per dollar borrowed. However, because of adverse selection they receive an information rent that cancels this markup.

Overall, due to screening, prices are close to the first best case. However, low-default borrowers get a lower maturity compared to what they should get under perfect information. To understand whether there is too much screening, I use the social planner's framework described in Section 5.2.

Competition: Sufficient Statistic. Using Proposition 4, I find that: $\frac{n_{36g}}{n_{60g}} I = 1.32 > E[\theta_i | g_i, T_i = 60] = 0.99$. This result implies that there is excessive screening. Due to the contractual externality, low-default borrowers obtain a lower maturity than what they should get under perfect information but also lower maturity than what they should get in the second-best scenario (informationally constrained social planner). Decreasing competition or increasing the marginal cost of lending via low-maturity contracts can thus lead to a Pareto improvement. The exact cost of the distortion requires extrapolating preferences away from their local estimates. Using the linear model assumption, low-default borrowers would prefer to go all the way to 60-month contracts.

Distribution of willingness to pay (Step 2): To test whether the inequality constraint as-

sumption delivers reasonable results, I implement step 2. I use equation (85) to recover the distribution of borrowers' willingness to pay. For each borrower, I calculate the implied loan size that would give them the same monthly payment if they were to change maturity. I consider only borrowers for whom the alternative combination of loan size and maturity is feasible, given the menus offered by LC.

For a given investment grade, the inequality allows me to recover one lower bound for the willingness to pay of high-maturity borrowers. I use interest rate variation across borrower investment grades to recover the shape of the distribution of willingness to pay. The identifying assumption is that the distribution of loan size marginal utility ($\alpha \sim G$) does not change with investment grades.

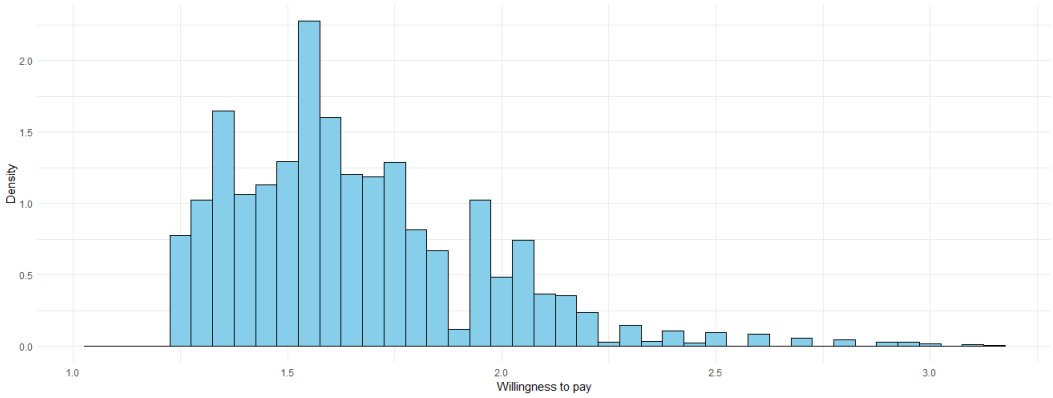


Figure 12: Borrowers' willingness to pay (all market segment pooled)

Figure (12) report the distribution of willingness to pay. The average willingness to pay is 1.6. Interpreting the value in terms of a two-period model, the results imply that for each dollar borrowed in period 0, a borrower is willing to pay up to 1.5 dollars in period 2.

7 Conclusion

This paper bridges theory and empirics. I develop a new structural model to study contract term distortions that stem from adverse selection. I analyse the theoretical properties of the model and discuss identification of its parameters.

The key innovation is to use a discrete choice approach to model imperfect competition. It makes the model both well-behaved and empirically tractable. The model can accommodate various empirical situations by allowing for moral hazard and adverse selection, multiple

borrower types and contract terms.

The paper carefully analyses the equilibrium existence, which is one of the central theoretical issues of screening models. I show that the equilibrium always exists and characterize it in closed form. I use my closed for characterization to analyse a contractual externality that arises from screening and derive novel implications for monetary policy and capital requirement.

The model is empirically tractable, which allows for discussion of exclusion restrictions and provides a structural interpretation of a linear regression of prices on quantity. I propose a research design that is robust to adverse selection and does not rely on estimating borrowers' choice set and default elasticities, which is problematic in the context of adverse selection.

The tractability of the model also paves the way for studying a wide set of counterfactuals with endogenous menus of contracts. This is crucial since empirical works on asymmetric rely on counterfactual analysis as changes in information are difficult to measure. Existing models with endogenous menus, such as Wollmann (2018), use numerical methods that are computationally demanding and not robust to the initial conditions.

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A Tables

Table 1: Survival Probabilities

	Loan is fully paid at maturity
Term: 60 months	-4
	(0.0118)
Monthly payment to income ratio	-2
	(0.01445)
Subgrade fixed effect	Yes
Average number of defaults at maturity	15

Table 2: Present value regression (60-month maturity)

	Present Value
Intercept	$4e^3$
	$(1.5e^2)$
Loan Size	1.5
	$(5.5e^{-3})$
Market share ratio	-9.3 e^3
	$(1.3e^2)$
Average present value	34,438
R^2	0.98

Table 3: Present value regression (36-month maturity)

	Present Value
Intercept	82
	(25.4)
Loan Size	1.2
	$(1.5e^{-3})$
Average present value	16,940
R^2	0.98

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B Literature Review

There is a growing empirical literature on adverse selection. A first strand of the literature developed tests for adverse selection using reduced-form approaches (Chiappori and Salanié 2002, Karlan and Zinman 2009, Hertzberg, Liberman, and Paravisini 2018). Following Einav, Finkelstein, and Cullen 2010, a second strand of the literature developed models to estimate the cost of adverse selection based on Akerlof (1978). In particular, papers in credit markets have focused on the case in which lenders can only choose interest rates (e.g., Crawford, Pavanini, and Schivardi 2018, Nelson 2018, Darmouni 2020), or in which lenders can pay a fixed cost to learn about a borrower's type (Yannelis and Zhang 2021). My paper differs from previous work by looking at a situation in which lenders can screen their customers by designing their menu of contracts.[11]

My paper also contributes to the theoretical literature on adverse selection and on the role of contract terms and prices as screening devices. The vast majority of the literature assumes either perfect competition (Rothschild and Stiglitz 1976) or monopoly (à la Stiglitz 1977). Recent examples of perfect competition models are those of Azevedo and Gottlieb (2017) for a model solution based on an equilibrium refinement and Guerrieri, Shimer, and Wright (2010) for a pure strategy characterization based on the assumption that the principal can match with at most one agent. The use of refinements emerged because the mixed strategy equilibrium is difficult to compute even in stylized frameworks (see Farinha Luz (2017) for the first numerical characterization in a Rothschild and Stiglitz (1976) model).

My paper is closer to that of Chade and Swinkels (2021) and Lester, Shourideh, Venkateswaran, and Zetlin-Jones (2019), who use a framework with imperfect competition. Chade and Swinkels (2021) focus on a situation in which the principal has heterogeneous costs of producing a good of a given quality and uses the menu to screen agents with heterogeneous willingness to pay. In contrast, this paper focuses on a situation that is closer to the banking market, in which the cost heterogeneity is on the agent, not the principal.

Lester, Shourideh, Venkateswaran, and Zetlin-Jones (2019) is closer to my framework. It

11. The closest paper is by Taburet, Polo, and Vo (2024), who use a structural model of screening for default probability. I make different modelling assumptions to analyse the theoretical properties of the model and develop a tractable model that allows studying a wider range of counterfactual experiments. I also propose a different research design that can be applied in situations in which only data on an individual firm are available.

uses a search model à la Burdett and Judd (1983) to model imperfect competition in an otherwise standard screening model in goods of different quality (lemon market). In contrast, my paper focuses on credit markets and aim to develop and empirical framework. As such, the modelling assumptions, the analysis and the comparative statics are different. I use a discrete choice approach to develop a model that is empirically tractable and discuss the identification of the model parameters. The theoretical analysis relies on a continuous demand system instead of a discrete one. This implies that I can use first-order conditions to analyze the model properties and deliver closed-form solutions in pure strategy.[12] I also use the model to derive different sets of comparative statistics relevant to credit markets and focus on the contractual externality.

C Possible micro-foundation of borrowers' indirect utility

This section provides a micro foundation for the indirect utility function u. I use a mortgage market micro-foundation. Alternatively, one can assume that lending is used for consumption in the first period and get rid of the housing modelling. Appendix D presents micro-foundations for a model with maturity, collateral and fees.

Borrowers have income I in period 1. They can get a loan (L_c) to invest in a house of size (H_1) that yields utility $F(H_1)$ in each period as long as they do not sell it. They can also consume (C_1), from which they derive utility ($u(C_1)$). The function F and u are increasing and concave with $F(0) = u(0) = 0$. Borrowers discount period 2 utilities with the discount factor δ. In the second period, borrowers' income is either W with probability (θ) or 0 with probability ($1 - \theta$). Borrowers use their income W to consume (C_2) and to repay the loan (R_c). When their income is equal to zero, they fire sell the house and get (γH) to repay the loan R_c. The borrower's maximization problem is:

12. Papers such as Lester, Shourideh, Venkateswaran, and Zetlin-Jones (2019) rely on a mixed strategy numerical characterization. Their model has to be solved in pure strategy because of their modelling of demand. By modelling borrowers as some being infinitely price elastic while others being completely inelastic, Lester, Shourideh, Venkateswaran, and Zetlin-Jones (2019) end is a situation similar to Rothschild and Stiglitz (1976) in which both pooling deviations attract the same proportion of good and bad borrowers as in the full market. As discussed in the introduction, this creates, in some situations, incentives to deviation from both pooling and screening, preventing the model from being solved using pure strategies.

$$\max_{\{C\}} u(C_1) + F(H_1) + \delta\theta \overbrace{\left[u(C_2) + F(H_1)\right]}^{utility\ when\ not\ defaulting} + \delta(1-\theta) \overbrace{\left[u(max\{\gamma H_1 - R, 0\})\right]}^{utility\ when\ defaulting} \quad (29)$$

$$s.t.\ C_1 + H_1 = L_c + I \quad (30)$$

$$C_2 = W - R_c \quad (31)$$

Assuming, for simplicity of notation, that fire selling the house in the second period is costly ($\gamma = 0$)[13] and that borrowers prefer to invest in the house rather than consuming in the first period (i.e. $u'(0) < [1 + WTP(1-d)F'(\bar{L})]$, with $\bar{L}$ being the maximum loan size available), but that they prefer consuming rather than getting a new house in the second period (i.e., $u'(W) > F'(0)$) the indirect utility can then be written:

$$u_i = [\delta\theta + 1]F(L_c + I) - \delta\theta u(W - R_c) \quad (32)$$

The expression implies that high default borrowers are more likely to have a high willingness to pay for the loan, since they do not expect to repay the full face value of the debt (captured by $\frac{1}{\delta\theta}$). When the utility of consumption is linear, the same indirect utility as in equation (1) is obtained (up to a normalization of the R coefficient). This assumption is consistent with Hertzberg, Liberman, and Paravisini (2018) empirical finding that self-selection in the consumer lending market seems to be driven by private information on the income process rather than risk aversion.

D Extension: Loan-to-value fees, Maturity, and Collateral

The loan size L and the repayment R can also be interpreted as LTV and maturity, respectively. The following paragraph presents a formal argument for the LTV and its maturity.

Loan-to-value and fees: The model presented in Section C can be extended by allowing

13. In that particular case, borrowers would be better off keeping the house (and lender would be no worse off). Selling the house upon default can be, however rationalized by the use of the house as collateral to prevent borrower from filling for default even when income is equal to W (see appendix G).

borrowers to have income (A) in the first period of the contract to allow for down payments (D) and fees (f). The utility thus becomes

$$u_i = [\delta\theta + 1]F(L + D) + u(A - D - f) - \delta\theta u(W - R) \tag{33}$$

In that case, we get that high-default borrowers are less willing to put their own wealth into their houses. This justifies the fact that, under screening, high-LTV loans will be selected by borrowers with unobservably high default probabilities.

When the marginal cost of lending mc is greater than 1, the optimal contract is such that the lender requires much down payment as possible (i.e., $D = A$). Loan size L in equation (1) can thus also be interpreted as loan-to-value.

Maturity: The full model with maturity is presented in Appendix E. This section focuses solely on the main equation. When the default rate is constant in each period, the borrower's value function can be written as

$$u_i = \frac{F(L + D) - \theta C[1 - e^{-\rho T}]}{\rho} - D - f. \tag{34}$$

$\frac{F(L)}{\rho}$ is the present value of the loan. $\frac{C}{\rho}[1 - e^{-\rho T}]$ is the present value of the debt for borrowers. ρ is the discount rate. θ is the per-period survival probability. T is the maturity. $\theta C := \theta r + (1 - \theta)[\gamma K + \Delta]$ is the expected per period loan cost for borrowers, with r as the monthly payment, γK the collateral value, and Δ as a cost of defaulting beyond the loss of collateral.

The supply side is identical to the one presented in the following section but set in continuous time. Lenders offer differentiated loan products and maximize their expected profits. I show in Appendix E that it is optimal for the lender to set very high monthly payments (infinite) and minimise the contract's maturity so that the borrower's probability of paying the non monetary cost is minimal. If there is a limit on how high the monthly payment can be, for instance, repayment cannot be higher than disposable monthly income, it is optimal to set the monthly payment at the limit and extract surplus with the longer maturity.

Collateral: Appendix G states the conditions required for collateralized debt to arise as the optimal contract. There are two critical assumptions. The first states that lenders cannot observe the cash flow (which is costly to do so as in Townsend 1979), so borrowers can lie about their income. The second assumption is that the bank can use collateral and seize it upon default if this is more efficient than spending the verification cost. For the collateral to be seized only upon default, the bank must value it less than the borrower. This assumption is adapted from Lacker (2001) to fit mortgage markets (see Appendix G). Papoutsi, Paravisini, Rappoport, and Taburet (2024) show that empirically, this assumption holds in the market for corporate loans.

E Model with Maturity

E.1 Borrower

A borrower derives utility $F(z)$ during the time interval Δt. With probability $(1-\theta\Delta t)$ the borrower can repay the monthly loan payment rL during that period. With probability $(1-\theta)\Delta t$, the borrower defaults and loses collateral cK.

Denoting $V(t)$ the value function of the borrower at time t when he did not default and $\tilde{V}(t)$ when he defaults, denoting the discount factor by ρ, the value function is

$$V(t) = F(z)\Delta t + \frac{\Delta t\lambda(t)}{1+\rho\Delta t}\{[\tilde{V}(t+\Delta t) - cK]\} + \frac{1-\lambda(t)\Delta t}{1+\rho\Delta t}[V(t+\Delta t) - rL]. \quad (35)$$

Assuming constant default probability, constant hazard rate λ and outside option, taking the limit when $\Delta t \to 0$ and using the boundary condition $V(T) = \bar{V}(L) = V(T) = F(L)\int_0^\infty e^{-\rho t}dt = \frac{1}{\rho}F(L)$ and using the discount factor ρ for the period below the maturity, we get for $t < T$:

$$V = \frac{F(L)}{\rho}e^{-\rho T} + \frac{F(L)-R}{\rho}[1-e^{-\rho T}] \quad (36)$$

$\frac{F(L)}{\rho}$ is the present value of lending. $e^{-\rho T}$ is the probability of survival until maturity times the discount rate. $R := \theta rL + \theta[cK + \Delta V]$ is the expected per period cost of the loan for borrowers.

I parameterize $\tilde{V} = V - \delta$ so that ΔV is a constant equal to δ. This implies that lenders

expect next-period value functions $\tilde{V}(t+\Delta t)$ and $V(t+\Delta t)$ to satisfy an equilibrium condition $\tilde{V} = V - \delta$. This can be interpreted as borrowers who renegotiate the debt upon default and get their previous value function minus some amount δ.

E.2 Lender

Each lender maximizes expected profits:

$$\max_{\{r,L,K,T\}} \Phi(V(t))\{\frac{(1-\lambda)rL + \lambda cK}{\rho}[1 - e^{-\rho(T-t)}] - mcL\} \tag{37}$$

Using equation (34) to express the problem in terms of promised utility yields

$$\max_{\{V,K,T,L\}} \Phi(V)\{\frac{F(L)}{\rho} - V - \lambda\frac{\Delta V}{\rho}[1 - e^{-\rho(T-t)}] - mcL\} \tag{38}$$

So, lenders want to minimize borrowers' default to prevent borrowers from paying the cost of defaulting ΔV. With exogenous default, lenders want to set maturity to zero and monthly payment to infinity.

If there is a limit to how high the monthly payment can be (for instance, if monthly payment rL cannot be above monthly income W), then the lender sets rL to W and extracts surplus using the interest rate rather than maturities.

F Extensions: Many Borrowers, Many Contract Features

The model can be extended to feature an arbitrary number of characteristics X, as long as they enter linearly:[14]

$$\beta'(L, X) - \theta R \tag{39}$$

14. Interaction terms are possible, since $\beta_1 L + \beta_2 L \cdot C = \beta_1 L + \beta_2 X$.

where R is the present value of the loan and X a set of characteristics such as fees or financial advice, and β is a vector of preferences.

Using the fact that $R = WTP(L, X) - u$, the lender problem is thus

$$\max_{\{u_i, X_i, L_i\}} \sum_i N_i(\theta_i u_i)[\beta'(L_i, X_i) - \theta_i u_i - mc(X, L)] \tag{40}$$
$$s.t.\ u_i \geqslant u_j + (WTP_i - WTP_j)'(X_j, L_j)$$

F.1 Many borrower types, unidimensional case

Let us consider x borrower types. I denote them with the index $i \in \mathcal{N}^*$, with $WTP_i > WTP_j,\ \forall i < j$. The incentive compatibility constraints can be written as a set of IC and a monotonicity constraint:

$$(ICs):\ u_1 \geqslant u_2 + \Delta WTP_1 L_2 \geqslant u_3 + \Delta WTP_2 L_3 ... \geqslant u_x + \Delta WTP_{x-1} L_x \tag{41}$$
$$(Ms): L_i \geqslant L_j, \forall i > j \tag{42}$$

$\Delta WTP_i := WTP_i - WTP_{i+1}$

Let us focus on a situation in which competition is high so that we have an interior solution (i.e., $L_i < \bar{L},\ \forall i > 1$). At equilibrium, the optimal contract is

$$L_1 = \bar{L} \tag{43}$$
$$L_{i-1} - L_i = \frac{R_{i-1} - R_i}{WTP_{i-1}} \in [0, \bar{L}],\ i > 1 \tag{44}$$
$$R_i = \frac{mcL_i}{\theta_i} + \frac{1}{\sigma_i \theta_i}\{[1 + \frac{1}{n_i}\frac{\alpha_i - mc}{\Delta WTP_i} 1_{i>1}] - [\frac{1}{n_{i-1}}\frac{\alpha_{i+1} - mc}{\Delta WTP_{i+1}}] 1_{i<x}\} \tag{45}$$

F.2 Many borrower types, multidimensional case

Let us denote X_i a vector of non-interest-rate characteristics of the contract designed for borrower i. The incentive compatibility constraints are

$$(ICs):\ u_1 \geqslant u_2 + \Delta WTP_1(L_2, X_2) \geqslant u_3 + \Delta WTP_2(L_3, X_3)... \geqslant u_x + \Delta WTP_{x-1}(L_x, X_x) \tag{46}$$

$$(Ms): WTP_{ij}L_i, X_i) \geqslant WTP_{ij}(L_j, X_j), \forall i > j \tag{47}$$

$\Delta WTP_{ij} := WTP_i - WTP_j$

The difficulty comes from having to guess and track which ICC is binding. The simplest case is when the ordering of borrowers and willingness to pay is the same along each characteristic X. This is a natural case when the only source of unobservable heterogeneity comes from default probabilities. In that case, the lender distorts the characteristics that have the lowest shadow cost.

An alternative solution is to assume that lenders maximize the worst-case scenario, as in Carroll (2017). In that case, the optimum for the principal is to screen along each contract term separately.

Appendix G discusses the necessary conditions for collateralized debt to be the optimal contract.

F.3 Endogeneous default probabilities

The model can be extended to allow for endogenous default probabilities.

The analysis stays exactly the same, and closed form solution are available when survival probabilities have the form:

$$\theta_i = \delta_i + \frac{\beta_i}{R_i} \tag{48}$$

Where δ_i and β_i are borrower-specific parameters. β_i drives moral hazard or burden of payment and δ_i is the adverse selection. R_i is the monthly payment.

G Conditions for Collateralized Debt to Be the Optimal Contract

The assumptions are:

ASSUMPTION 1: *Ex post private information.* In this paper, I model this assuming that banks cannot observe the second period cash flow (W) of the borrower,[15] borrowers can thus lie about their income and hide it from the bank. The bank can spend some amount to verify it.

ASSUMPTION 2: *The house can be used as collateral (i.e., housing is observable). Using collateral to deal with ex post private information is less costly than verifying cash flows. The borrower values the house more than the bank.* This assumption ensures that in the optimal mortgage contract, the bank asks for cash instead of housing when possible. In this paper, the reason the borrower prefers the house more than the bank is that borrower values the house more than its selling price and the bank has a utility over cash rather than house.

At t=2, borrowers privately observe their income realization ($\tilde{W}$) and choose whether to file for default or not. Borrowers default when they cannot repay (i.e., $\delta H + W - R < 0$) or when it's better for them to strategically default (i.e., $H + u(W - R) < u(K + W)$ or $u(\delta H - R + W) < u(K + W)$. K is the amount the bank gives back (or asks for) after seizing the house and selling it. K thus has to be lower than or equal to $\delta H - R$ so that all inequalities are satisfied. In my model, since borrowers value cash more than upon default than banks ($\alpha > 1$), the constraint is binding $K = \delta H - R$. Notice that the bank wants to prevent strategic default when the borrower is in negative equity ($\delta H < R$), so it punishes the the borrower by seizing more than the house (i.e. $K = \delta H - R$ can be negative).

H Lemmas

Proofs of Lemmas 1 and 2: The two ICCs can be written $(WTP_B - WTP_G)'L_B \geqslant u_B - u_G \geqslant (WTP_B - WTP_G)'L_G$. or $IC_B: \ (WTP_B - WTP_G)'L_B \geqslant u_B - u_G$ and $MC: \ L_B \geqslant L_G$. When $L_B = L_G$, as in the FB, this implies that $\bar{u}_B - u_G = (WTP_B - WTP_G)'\bar{L}$, which is

15. Or it is costly to do so.

the case when $R_G = R_B$. This implies that both IC constraints are binding. If $L_B > L_G$, $(WTP_B - WTP_G)'L_B \geqslant u_B - u_G$ must be binding.

Assuming that the constraint (MC) is not binding and solving for problem 40, we notice that (IC_B) does not depend on L_B. Thus $L_B = \bar{L}$.

Intuitively, the self-selection problem comes from high-default (high-WTP) borrowers who want to pretend to be low-default (low-WTP) borrowers to get a cheaper loan. If the bank offers a contract with distorted B loan size ($L_B < \bar{L}$), there is a Pareto improvement that leads to either more profits in the B market segment or relaxes the IC constraint. For instance, increasing L_B up to $\bar{L}$ and increasing the interest rate by $WTP_B(L_B - \bar{L})$ increases bank profits without affecting the incentive compatibility constraints.

The fact that the participation constraint is not binding is shown by solving for the perfect competition model to get the equilibrium conditions from Proposition 3 and then using comparative statics by varying the competition level.

I Existence and Uniqueness

Let us denote borrower utility by

$$u(L, R) := WTPL - \theta R \Longleftrightarrow \theta R = WTPL - u. \tag{49}$$

We use the following change of variable:

$$u := WTPL - \theta R \Longleftrightarrow \theta R = WTPL - u. \tag{50}$$

The bank's objective function for a particular market is thus

$$\Pi(L, u) := \Phi(u)\pi(L, u) \tag{51}$$

$$\pi(L, u) := (WTP - mc)L - u. \tag{52}$$

For the lender's maximization problem to be well behaved, the Hessian matrix of $\Pi(L, u)$ needs to be negative definite. Using Sylverster's criterion, we need to show that the leading principal minors of the Hessian matrix of $-\Pi(L, u)$ have positive determinants:

The conditions to be satisfied are

$$\pi\{(1-2\phi)\} < \frac{2}{\sigma} \; and \;\; -\left[(\Phi_u \pi + \Phi\pi')\Phi(WTP - mc)\right]^2 < 0. \tag{53}$$

The second condition is always satisfied.

We separate the case in which competition is low enough (a) from the one in which competition is high (b). If (b) is satisfied, we do not need the assumption on (a) for the Nash equilibrium to exist.

(a) **Condition when competition is low enough:** The left-hand side is bounded above by $\pi\{(1-2\phi)\} < \{[WTP - mc]\bar{L} - u\}(1-2\phi) < \{[WTP - mc]\bar{L} - \bar{u}\} < \{[WTP - mc]\}\bar{L}$. For the last inequality, we use the fact that utilities cannot be negative because of a participation constraint $(i.e., u \geqslant 0)$.

The inequality implies that there exists a level of competition $\sigma > \bar{\sigma} := \frac{1}{[WTP-mc]\bar{L}-1}$ such that the Nash equilibrium exists and is unique.

Condition when competition is high:

Because of the participation constraint, individual profits are bounded by $c := [WTP - mc]\bar{L}$. Then we have:

$$\partial_u \Pi(u) < \partial_u \Phi c \implies \pi < c + \frac{1}{\sigma(1-\phi)}. \tag{54}$$

If there is a capacity constraint k, then

$$\pi < c + \frac{1}{\sigma(1-k)}. \tag{55}$$

The market share is bounded by one, so that $((1-2\phi) < 1)$.

We thus have $\pi\{(1-2\phi)\} < \frac{1}{\sigma(1-k)} + c < \frac{2}{\sigma}$.

Hence, k should be lower than $\frac{1-c\sigma}{2-c\sigma}$; this is decreasing for σ large enough, so we have $k < \frac{1}{2}$.

J Intuition: Why the Capacity Constraint Works

Small deviations from screening to pooling attract a high proportion of high-default borrowers ($\bar{n}$). Indeed, starting from the screening candidate, we get

$$\frac{\Delta\bar{u}}{\Delta u}\cdot\frac{\Phi(\bar{u})(1-\Phi(\bar{u}))}{\Phi(u)(1-\Phi(u))} \tag{56}$$

$$\implies \frac{u^n-u^o+(\frac{\bar{W}TP}{\bar{\theta}}-\frac{WTP}{\theta})L}{u^n-u^o}\cdot\frac{\bar{n}}{n}\frac{(1-\bar{n})}{(1-n)} \tag{57}$$

$$\implies \frac{\bar{n}}{n}<\frac{u^n-u^o+(\frac{\bar{W}TP}{\bar{\theta}}-\frac{WTP}{\theta})L}{u^n-u^o}\cdot\frac{\bar{n}}{n}<\frac{u^n-u^o+\overbrace{(\frac{\bar{W}TP}{\bar{\theta}}-\frac{WTP}{\theta})}^{>0}L}{u^n-u^o}\cdot\frac{\bar{n}}{n}\frac{(1-\bar{n})}{(1-n)} \tag{58}$$

Local deviations are not profitable, but large ones are. The capacity constraint limits the ability to do non-local deviations and thus restores pure strategy equilibrium existence.

Condition for screening inefficient:

$$\frac{n_G}{n_B}IR_B:=\frac{n_G\theta_G}{n_B\theta_B}\frac{WTP_G-\frac{mc}{\theta_G}}{WTP_B-WTP_G}>1 \tag{59}$$

Equilibrium L_G amount:

$$L_G=\delta[\bar{L}+\frac{1}{\sigma}\tilde{\delta}\frac{1}{WTP_B-WTP_G}(1+\frac{\theta_G n_G}{\theta_B n_B})]=\delta[\bar{L}+\frac{1}{\sigma}\frac{\tilde{\delta}}{WTP_B-WTP_G}+\frac{IR}{\sigma[WTP_B-\frac{mc}{\theta_B}]})]<\bar{L}\text{ for } \tag{60}$$

with: $\delta:=\frac{WTP_B-\frac{mc}{\theta_B}}{WTP_B-\frac{mc}{\theta_G}}$ is the strength of the AS problem. $\tilde{\delta}:=\frac{WTP_G-\frac{mc}{\theta_G}}{WTP_B-\frac{mc}{\theta_B}}$ is the relative profit measure.

When $\sigma=\frac{2}{A_B\bar{L}}$, $IR=1^+$:

$$L_G=\delta[1+\frac{\tilde{\delta}}{WTP_B-WTP_G}\frac{A_B}{2}+\frac{1^+}{2})]<\bar{L}\text{ For } WTP_B-WTP_G\text{ large enough or } A_B\text{ low enough (this i} \tag{61}$$

Given the condition for the function to be well behaved and screening to be inefficient, there exists a zone in which banks screen. We can thus analyse the interior solution to understand

the screening property trade-off. The level of inefficiency should be low enough so that the bound on the degree of competition is low enough that banks to not have enough freedom to pool borrowers.

K Proof of Results

Result 2: (i)

$$\partial_{\theta_G^{-1}} L_G < 0 \Longleftrightarrow mcL_G < \frac{1}{\sigma}[-\partial_{\theta_G^{-1}} CS - \partial_{\theta_G^{-1}} MC] \tag{62}$$

$$\Longleftrightarrow mc\sigma \frac{(WTP_B - \frac{mc}{\theta_B})\bar{L}}{WTP_B - \frac{mc}{\theta_G}} < [\underbrace{-\partial CS}_{cross-subsidy\ channel} - \overbrace{mc\frac{CS}{WTP_B - \frac{mc}{\theta_G}}}^{perfect\ competition\ channel} - \underbrace{\partial MU}_{competition\ channel} - \overbrace{mc\frac{MU}{WTP_B}}^{perfect\ competiti} \tag{63}$$

$-\partial MU - mc\frac{MU}{WTP_B - \frac{mc}{\theta_G}}$ is negative. $-\partial CS$ is positive $-mc\frac{CS}{WTP_B - \frac{mc}{\theta_G}}$ is negative, so the difference can be positive or negative. It is negative when the no-cross-subsidy effect is stronger than the cross-subsidy effect.

To focus on the cross-subsidy channel, let us consider that borrowers have the same elasticity (i.e., $\sigma := \sigma_B\theta_B = \sigma_G\theta_G$), so the competition channel is absent. In that case, we have

$$\frac{\partial CS}{CS} = -\frac{WTP_B}{WTP_B - WTP_G} \tag{64}$$

$$\Longrightarrow \frac{\partial CS}{CS} - \frac{mc}{WTP_B - \frac{mc}{\theta_G}} \geqslant 0 \Longleftrightarrow \frac{1}{1 + \theta_G^{-1} - \frac{WTP_G}{WTP_B}} \geqslant \frac{mc}{WTP_B} \tag{65}$$

This implies that a decrease in θ_G has a positive impact when competition is low. It has a negative impact when competition is high if $\frac{WTP_B}{1+\theta_G^{-1} - \frac{WTP_G}{WTP_B}} \geqslant mc$.

The right-hand side captures the perfect competition channel. When the marginal costs are high, changes in survival probabilities have a high impact on the perfect competition spread between rates. When WTP_B is high, the high-default borrower is less likely to copy the low-default contract, so the perfect competition channel is relatively less important.

The left-hand side captures the strength of the cross-subsidy channel. When the spread between preferences is high ($\frac{WTP_G}{WTP_B}$) or the survival probability of the low-default borrower is low (θ_G), the cross-subsidy channel is low. This renders the channel less likely to dominate the

marginal cost effect.

(ii)

$$\partial_{\theta_B} L_G > 0 \Longleftrightarrow 0 < \frac{mc}{\theta_G} L_G + \frac{1}{\sigma}\{\partial_{\theta_B} CS + \partial_{\theta_B} MC\} \tag{66}$$

We have: $\partial_{\theta_B^{-1}}(\theta_B MC) > 0$ $\partial_{\theta_B^{-1}}(\theta_B CS) < 0$. To focus on the cross-subsidy channel, let us consider that borrowers have the same elasticity (i.e., $\sigma := \sigma_B \theta_B = \sigma_G \theta_G$), so the competition channel is absent. In that case, we have

$$\frac{\partial_{\theta_B} CS}{CS} = -\frac{WTP_B}{WTP_B - WTP_G}\frac{1}{\theta_B} \tag{67}$$

$$\Longrightarrow -\frac{\partial_{\theta_B} CS}{CS} - \frac{mc}{\theta_G}\frac{1}{WTP_B - \frac{mc}{\theta_G}} > 0 \Longleftrightarrow \frac{WTP_B}{1 + \frac{1}{\theta_B} - \frac{WTP_G}{WTP_B}}\frac{\theta_G}{\theta_B} \geqslant \frac{mc}{WTP_B} \tag{68}$$

Result 3: (i) $\partial_\alpha L_G > 0 \Longleftrightarrow L_G < \bar{L} + \underbrace{\frac{mc}{\alpha^2}\frac{1}{\sigma}\frac{\alpha}{WTP_B - WTP_G}[\frac{\theta_B}{\theta_G} + \frac{n_B}{n_G}]}_{>0}$

(ii) $\partial_{\alpha_G} L_G > 0 \Longleftrightarrow \frac{\alpha_G}{\theta_B} L_G < \frac{1}{\sigma} CS \implies \bar{\sigma}_\alpha := \frac{CS[\frac{mc}{\alpha_G} - 1] - MU}{PC}$, with $PC := S_B \bar{L}$

Result 4: (i) $\partial_{mc} L_G > 0 \Longleftrightarrow \frac{1}{\theta_G} L_G - \frac{1}{\theta_B}\bar{L} - \frac{1}{\theta_G}\frac{CS}{S_G} > 0$ $\sigma_{mc} := \frac{CS[\frac{S_{BG}}{S_G} - 1] - MU}{PC - \frac{\theta_G}{\theta_B}\bar{L} \cdot S_{BG}} < 0$ if $CS[\frac{S_{BG}}{S_G} - 1] - MU > 0$ with $S_{BG} := WTP_B - \frac{mc_G}{\theta_G}$ and $S_G := WTP_G - \frac{mc_G}{\theta_G}$.

(ii) $\partial_{mc_G} L_G > 0 \Longleftrightarrow \frac{1}{\theta_G} L_G - \frac{1}{\theta_G \sigma}\frac{CS}{S_G} > 0$ so $\sigma_{mc_G} := \frac{CS[\frac{S_{BG}}{S_G} - 1] - MU}{PC}$ with $S_{BG} := WTP_B - \frac{mc_G}{\theta_G}$ and $S_G := WTP_G - \frac{mc_G}{\theta_G}$.

(iii) $\partial_{mc_B} L_G < 0 \Longleftrightarrow \frac{-\frac{\bar{L}}{\theta_B}}{WTP_B - \frac{\theta_G}{mc_G}} < 0$ which always holds.

L Proof of Propositions

Proposition 4 $\partial_\delta L_G = \underbrace{\frac{\frac{mc_2}{(\theta_B + \delta_B)^2}}{WTP_B - \frac{mc_2}{\theta_G}} L_{90}}_{\text{“perfect competition” effect}} - \underbrace{IR_B \frac{1}{\theta_B + \delta_B}}_{\text{“monopoly” competition effect}}$

Proof (ii): $u_i = S(L_i) - \frac{1}{\sigma} - IR_G 1_{i=G} + IR_B 1_{i=B}$. For G borrowers, $S(L_G)$ increases when L_G increases. In the other market segment, $\partial_\delta u_B = -\partial_\delta R_B = \frac{1}{\theta_B + \delta_B}[\underbrace{\frac{mcL_{95}}{\theta_B + \delta_B}}_{fair\ price} - \underbrace{IR_B}_{Information\ rent}]$

M Model with Heterogeneous Marginal Costs

In this section, I introduce heterogeneous marginal costs to analyze the impact of marginal cost policies (i.e. capital requirements).

For simplicity, since they do not add anything to the policy analysis, I assume that the price elasticity of both borrowers is the same ($\sigma := \sigma_G(1 - \frac{n_G}{B}) = \sigma_B(1 - \frac{n_B}{B})$).

The key point of those policies is that they are designed differently for each specific product. For tractability, it is useful to model those policies as a piecewise linear function over some product characteristics. As a result, it is also convenient to introduce discontinuities in the marginal costs at the same thresholds so that the first-best products are different for each borrower. This assumption ensures that demand is continuous.

Formally, I consider a situation in which the marginal cost of lending via a contract with a characteristic X (e.g. LTV) is higher above a certain threshold $\bar{T}$:

$$mc(X) = mc_1 + 1_{X>\bar{T}}(mc_2 - mc_1) \tag{69}$$

Borrowers have preferences such that

$$WTP_G \geqslant \frac{mc_1}{\theta_G},\ WTP_G < \frac{mc_1}{\theta_G} \tag{70}$$

$$WTP_B > \frac{mc_2}{\theta_B},\ WTP_B \geqslant \frac{mc_2}{\theta_B} \tag{71}$$

For tractability, I consider that the policies are not large enough to change the above ordering.

M.0.1 Policy experiment 1: Effect of changes in capital requirements

Capital requirements are often based on loan-to-values ratios (for instance, in Basel III). For this reason, I model capital requirements based on LTV. I consider that the marginal cost now takes the form

$$mc_c = mc - \omega^l \frac{1}{LTV} 1_{LTV<L\bar{T}V} - (\omega^h - \omega^l)\frac{1}{LTV} 1_{LTV\geqslant L\bar{T}V} \tag{72}$$

with $\omega \frac{1}{LTV}$ being the capital requirements and ω capturing how the capital requirements vary with the loan leverage. A positive ω^l (or ω^h) implies that capital requirements are increasing in

LTV when LTV is below (above) a threshold.

Assumptions 5: Let us assume that the increase in capital requirements is not high enough to render the NPV of lending negative.

For simplicity of exposition, let us consider the case in which the optional contract is such that both borrowers put the same level of down payment (D) (as in D)[16] This assumption does not impact the results.

Using Proposition 2 and Lemma 4, the equilibrium distortions can be written:

$$L_G = \overbrace{\frac{1}{S_{BG}}[S_B(\bar{L}-D)}^{perfect\ competition\ distortion\ level} + \underbrace{(\frac{1}{\sigma_G\theta_G}-\frac{1}{\sigma_B\theta_B})}_{competition\ channel} + \overbrace{\frac{I_G}{\sigma_G\theta_G}+\frac{|I_B|}{\sigma_B\theta_B})}^{cross-subsidy\ channel} \tag{73}$$

$$S_{BG} := WTP_B - \frac{mc+\omega^G}{\theta_G} \tag{74}$$

$$S_i := WTP_i - \frac{mc+\omega^i}{\theta_i},\ i \in \{G,B\} \tag{75}$$

$$I_B := -\frac{n_G}{n_B}\frac{\overbrace{\alpha_G - mc - \omega^G}^{Benefit\ of\ pooling:\ More\ surplus}}{\underbrace{WTP_B - WTP_G}_{Cost\ of\ pooling:\ Information\ rent}} > 0 \tag{76}$$

$$-I_G := \frac{\alpha_G - mc - \omega^G}{WTP_B - WTP_G} > 0 \tag{77}$$

N More Comparative statics

Result 2: Default probabilities (Adverse Selection). *An increase in adverse selection, measured as the survival probability spread ($\frac{\theta_G}{\theta_B}$), has different effects depending on whether the changes come from default probability in the B or G market segment.*
(i) An increase in $\frac{\theta_G}{\theta_B}$ caused by a decrease in G default probability increases credit rationing under high competition levels. Under low competition levels, it relaxes credit rationing if the cross-subsidy channel is higher than the perfect competition channel and the competition chanel. When demand elasticities are the same (i.e., $\sigma := \sigma_B\theta_B = \sigma_G\theta_G$), so that the competi-

16. That is, $\frac{1}{\epsilon} < \frac{mc}{\theta}$ so that the optimal contract features screening on loan size and a maximum amount of deposit D.

tion channel is absent, the cross-subsidy channel dominates if and only if $\frac{1}{1+\theta_G^{-1}-\frac{WTP_G}{WTP_B}} \geqslant \frac{mc}{WTP_B}$. *The competition switching point is* $\sigma = \frac{CS(\varepsilon_{CS}-mc)}{mc \cdot L^{PC}}$, *where* L^{PC} *is the perfect competition loan size and* ε *is the cross-subsidy semi-elasticity to* θ_G^{-1}.

(ii) An increase in $\frac{\theta_G}{\theta_B}$ *caused by an increase in B default probability decreases credit rationing under high competition levels. Under low competition levels, it relaxes credit rationing if the cross-subsidy channel is higher than the perfect competition channel and the competition channel. When demand elasticities are the same (i.e.,* $\sigma := \sigma_B\theta_B = \sigma_G\theta_G$*), so that the competition channel is absent, the cross-subsidy channel dominates if and only if* $\frac{WTP_B}{1+\frac{1}{\theta_B}-\frac{WTP_G}{WTP_B}}\frac{\theta_G}{\theta_B} \geqslant \frac{mc}{WTP_B}$.

Proof: See Appendix K.

Three forces are at play. To illustrate, let us consider a situation in which the default probability of the B market segment increases.

First, under a high level of competition, the default probability increase must be fully passed through B contract interest rates. In turn, the loan size of the G contract must be lowered to maintain incentives to self-select. If borrowers' willingness to pay is an increasing function of default probabilities, an additional force pushes the credit constraint in the opposite direction. The increase in default probability increases borrowers' willingness to pay for loan size. As a result, for a given level of credit rationing, the low-default contract thus becomes relatively less attractive. This effect allows lenders to decrease the credit constraint. When borrowers' willingness to pay is $WTP_i = \frac{\alpha_i}{\theta_i}$, the latter effect dominates.

Second, the decrease in the default probability of B borrowers renders them more sensitive to interest rate changes. Thus, the perfect information markup must decrease. As a result, the spread between interest rates and credit constraints decreases. This allows lenders to lower the credit constraint level.

Third, the decrease in the default probability of B borrowers increases the potential profits in this market. This creates incentives to distort market segment G to extract more surplus from contracts designed for market B.

I summarize Result 2 in Figure 13 for the case in which the cross-subsidy channel dominates. When adverse selection decreases due to a decrease in market B default probability, the equilibrium region moves from the dotted lines to the solid lines in Figure 13.

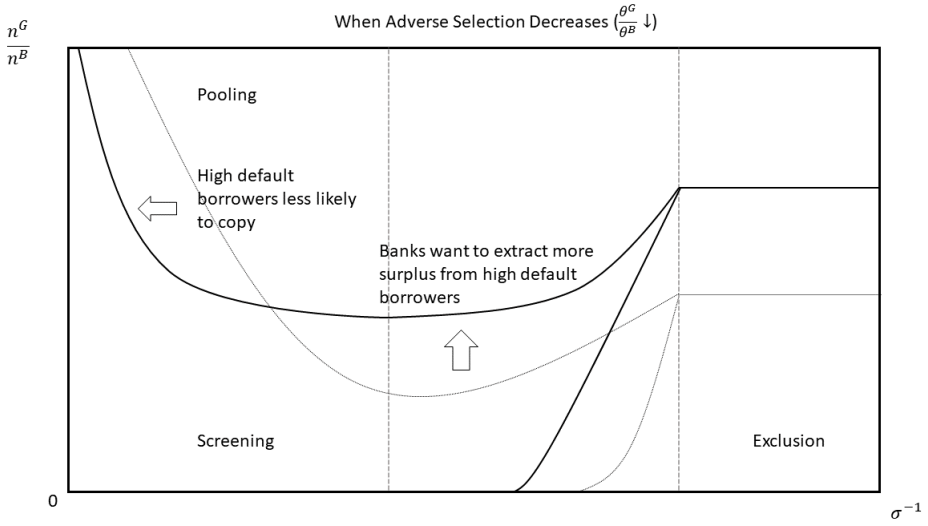


Figure 13: Equilibrium regions when adverse selection decreases

Result 3: Willingness to pay (House Prices and Help-to-Buy).

(i) A decrease in house prices (i.e., $\alpha_G = \alpha_B$ increases) decreases the interest rate and loan size spread between contracts. This effect is stronger under low levels of competition as long as the lender is not pooling (i.e., $\sigma > \bar{\sigma}$).

(ii) A government intervention to increase the value of small-size loans via a help-to-buy scheme (i.e., α_G increases) increases the credit rationing and the interest rate spread under high competition. It decreases it under low competition (i.e., $\sigma < \bar{\sigma}_\alpha$) if and only if its $CS\frac{mc}{\alpha_G} > CS + MC$. The competition threshold is then $\bar{\sigma}_\alpha := \frac{-MU - CS[1-\frac{mc}{\alpha_G}]}{PC} > 0$, where $PC := S_B\bar{L}$ denotes the utility of B borrowers when getting the perfect competition B contract.

Proof: See Appendix K

The intuition for the result (i) is the following. Decreasing house prices increase borrowers' willingness to pay for each dollar lent. This lowers credit constraints via two channels. First, the price change renders credit rationing more costly for high-default borrowers. All else constant, the G contract becomes less attractive, so the perfect competition credit rationing level decreases. Second, credit rationing becomes more costly for lenders as the G market segment generates more surplus. This creates incentives to use asymmetric information discounts and premiums more intensively to lower credit rationing.

Two channels drive the help-to-buy result. First, holding contract terms constant, the G con-

tract becomes more attractive to B borrowers. This renders the level of cross-subsidy required to completely undo credit rationing higher. Second, the help-to-buy policy renders the G market segment more profitable, which creates incentives to increase the cross-subsidy level. In the formula $CS\frac{mc}{\alpha_G} > CS + MC$, $CS + MC$ captures the first channel while $CS\frac{mc}{\alpha_G}$ captures the second channel. $\frac{mc}{\alpha_G}$ is the strength of the marginal impact of an increase in help to buy on the cross-subsidy.

O Normative Analysis

To analyze whether policy interventions have a positive impact on welfare, I use the following lemma:

***Lemma 5: Equilibrium utility and profit levels (Interior Solution)**:*

$$u_B = (\alpha_B - mc)\bar{L} - \pi_B \tag{78}$$

$$u_G = (\alpha_B - mc)L_G - \pi_G \tag{79}$$

$$\Pi_B := n_B\pi_B = n_B\frac{1}{\sigma_B\theta_B}[1 - \frac{\alpha_G - mc}{WTP_G - WTP_B}\frac{n_G}{n_B}] \tag{80}$$

$$\Pi_G := n_G\pi_G = n_G\frac{1}{\sigma_G\theta_G}[1 + \frac{\alpha_G - mc}{WTP_G - WTP_B}] \tag{81}$$

Equations (78) and (79) state that borrowers' utility is the surplus generated by the loan minus the lender's profits per borrower. Equations (80) and (80) state that the lender's profits are the markup plus or minus the asymmetric information premium or discount.

Changes in competition: I analyse how changes in competition (σ^{-1}) affect the welfare of different market participants. For the welfare results to be independent of the welfare function used, I then examine whether there exists a situation in which a decrease in competition leads to Pareto improvements.

Decreases in competition decrease the interest rate of the B contract and increase the B borrower's utility when the asymmetric information discount is higher than the pure markup effect (i.e., $\frac{\alpha_G - mc}{WTP_G - WTP_B}\frac{n_G}{n_B} > 1$).[17] This happens when the surplus in the B market segment

17. Because of the linearity assumption of the utility, the condition implies that lenders make negative profits on

is large (i.e., $\frac{\alpha_G - mc}{WTP_G - WTP_B}$ large) and when the cost of screening is low (i.e., $WTP_G - WTP_B$ low) and the relative number of G borrowers is high.

A decrease in competition increases G borrowers' rate but also relaxes the credit rationing. It increases welfare when the increase in price is compensated by the increase in loan size (i.e., $\frac{S_G}{S_{BG}}[MU + CS] > \frac{1}{\Delta\sigma\theta_G} + \frac{1}{\Delta\sigma\theta_G}[\frac{S_G}{WTP_B - WTP_G}]$). This happens when the relative number of G borrowers is high or if the G market segment is relatively more elastic.

A decrease in competition has an ambiguous effect on lenders' profits. It increases lenders' profits if incentives to provide an information rent are low enough $n_G \frac{S_G}{WTP_B - WTP_G}[\frac{1}{\Delta\sigma\theta_G} - \frac{1}{\theta_B}] + \frac{n_B}{\theta_B} + \frac{n_G}{\Delta\sigma\theta_G} > 0$. For instance, when interest rate elasticities are the same (i.e., $\sigma_G\theta_G = \sigma_B\theta_B$), a decrease in competition always increases profits. When the interest rate elasticity of the G market segment is high and the benefit of screening is large ($\frac{S_G}{WTP_B - WTP_G} > n_B$ and $\Delta\sigma$ large enough), a decrease in competition can decrease profits. This is because a decrease in competition creates incentives to distort the B market segment more to extract surplus on the G market segment. However, since the G market segment is relatively more elastic, lenders begin to compete more on this market. Overall, the latter effect dominates.

As a result, when the number of high-default-borrowers is high enough and the G market's surplus is high enough, a decrease in competition is a Pareto improvement.

Changes in the marginal cost of lending: I analyse how changes in the marginal cost of lending (mc) affect the welfare of different market participants. For welfare results to be independent of the welfare function used, I then examine whether there exists a situation in which a decrease in competition leads to Pareto improvements.

Let us denote mc_i the marginal cost of lending via i contract.

An increase in mc_G renders interest rates in the B market higher and the surplus in the G market lower, which renders the surplus in the B market relatively higher. This gives incentives to provide less information rent to B borrowers. A decrease in mc_B is passed through lower interest rates. This increases the welfare of B borrowers.

The effect of an increase in mc_G on G borrower is ambiguous. An increase in mc_G is partially passed through interest rates. It also lowers the relative surplus in the G market segment, which triggers incentives to increase the asymmetric information discount (i.e., the interest rate) so that more profits can be made in the B market segment. The rate increase also decreases

the B market segment. This is not necessarily true otherwise.

cream-skimming incentives, so lenders can increase the cross-subsidy and the amount lent in the G market segment without the threat of losing their customers. Overall, welfare can increase. mc_B changes have no impact.

Increases in mc_G positively impact profits if the interest rate elasticity is higher in the G market segment. This is because an increase in mc_G triggers incentives to provide lower information rents. mc_B changes have no impact on lenders' profits in my model, because the cost is fully passed through borrowers.

As a result, an increase in mc_G coupled with a decrease in mc_B can be a Pareto improvement policy. The extreme-case scenario is equivalent to banning contracts with lower than $\bar{L}$ loan size, which prevents cream-skimming. This is a Pareto-efficient policy when cream-skimming is ex post inefficient.

P Bringing the Model to the Data: A Guidebook

This section develops a guide for how to bring the model to the data.

I discuss the identification and estimation of model parameters for types of datasets commonly used in banking or insurance. In the first scenario, I consider that the econometrician has data on menus and customer choices but only for an individual firm. This is the most common scenario in which menus are observable (see, for instance, Hertzberg, Liberman, and Paravisini (2018) for the credit market or Tebaldi (2024) for the insurance market). I refer to this as the proprietary data case.

In Appendix P.2, I analyse the second situation in which the econometrician has data on multiple firms but can only see the equilibrium contracts chosen by each customer. This is the data constraint faced when using credit register data (see, for instance, Crawford, Pavanini, and Schivardi 2018).

This section also summarizes the empirical facts that guided my modelling assumptions and develops extensions that render the model applicable to a wide range of empirical setups. In particular, I discuss how to deal with maturity and the fact that default probabilities depend on contract terms in the main text, since these are directly related to the empirical application in Section 6. I defer to Appendix F the extensions such as multiple borrower types and contract characteristics.

P.1 First case: proprietary data on a single firm

Estimation of the model parameters is based on a three-step procedure. Step 1 recovers the default probabilities ($1-\theta$), step 2 the willingness to pay (WTP), and step 3 the marginal costs (mc) as well as the demand elasticities (σ).

Step 1: Default Probabilities ($1-\theta_i$).

To render the model more realistic, I allow for default probabilities to depend on contract terms. More specifically, I assume that borrowers default if their disposable income mp_{it} is lower than the monthly payment M_{it}. I consider that disposable income is observable by the lender at origination and that the probability of its changing over time is private information. In practice, lenders derive a good sense of this value through credit checks.

The parametrization of defaults rules out strategic default and implies that the income process is the most important source of heterogeneity (and of private information) among borrowers. The modelling is consistent with the empirical evidence in Hertzberg, Liberman, and Paravisini (2018), who show that the unobserved heterogeneity that drives borrowers' behaviour in the consumer lending market is due to private information about the income process rather than a difference in risk aversion or interest rate risk. It is also consistent with the Bank of England's NMG household survey, which documents that the main reasons for defaulting are an increase in other bills or unexpected expenses/costs (21 percent), being made redundant, unemployed or getting a wage cut (67 percent), or an increase in loan payment (10 percent).

In addition to its tractability, this modelling assumption has a convenient implication: It allows us to recover the key model parameters without having to estimate default elasticities (see Step 2). Identifying default elasticities relies on comparing borrowers who are observationally equivalent but choose different contracts. Because of the selection on unobservables, finding the right control group is problematic when lenders screen.[18]

Formally, denoting mp_{it} the random variable that captures the disposable income process of borrower i at time t, the borrower's survival probability at maturity T_i is:

$$S_i := E[\mathbf{1}_{\{\exists t<T_i : M_i<mp_{it}\}} | T_i, X_i, i \; chose \; contract \; c] \tag{82}$$

18. Alternatively, the default probability can be parametrized as $\theta = \alpha + \frac{\beta}{R}$. This approach also yields closed-form solutions.

X_i are borrowers' characteristics or economic environment variables that are observable by the lender, affect the income variance, and are legal to use in making acceptance and rejection or pricing decisions.

For each observationally equivalent borrower who chooses contract c (indexed by j $\in [\![1, n_c]\!]$), we can estimate the average survival probability ($\hat{S}_i$) using realised default at maturity:

$$\hat{S}_i := \sum_{j=1}^{n_c} \mathbf{1}_{\{j \ did \ not \ default\}} \tag{83}$$

Using a constant hazard rate model and denoting T_i the loan maturity in months yields the following constant monthly survival probability:

$$\hat{\theta}_i := e^{-\hat{\lambda}_i}, \ with \ \lambda_i : \ \hat{S}_i = e^{-\hat{\lambda}_i T_i} \tag{84}$$

The present value of a loan is then $[1 - e^{-\lambda T}]M \approx \lambda T M$. The constant hazards rate model is convenient when screening with maturity (see empirical application). In that case, λ plays the same role as θ and TM as R in the theoretical model. Constant hazard rates are consistent with assuming that the income process follows the same Bernoulli distribution in each period with values mp_i and $\delta < M_i$. δ captures the bad shock described in the Bank of England NMG household survey.

Step 2: Borrower preferences (α_i).

I use a revealed preference approach to recover borrowers' preferences. The identification leverages the idea that if a borrower chooses a contract with a loan size of \$1,000 when they had access to a contract with a loan size of \$1,100 for an interest rate increase of, say, 100 bps, it must be that their willingness to pay for loan size is lower than 100 bps. Figure (14) provides a visual representation of this argument. I plot the interest rate as a function of loan size (i.e., the pricing schedule) and the borrower indifference curve. When a continuum of contracts is offered, the optimality condition implies that the slope of the indifference curve (i.e., the willingness to pay) must be tangent to the pricing schedule at the contract chosen.

One challenge with this approach is that willingness to pay depends on borrowers' default probabilities, which are endogenous objects. This implies that the econometrician potentially

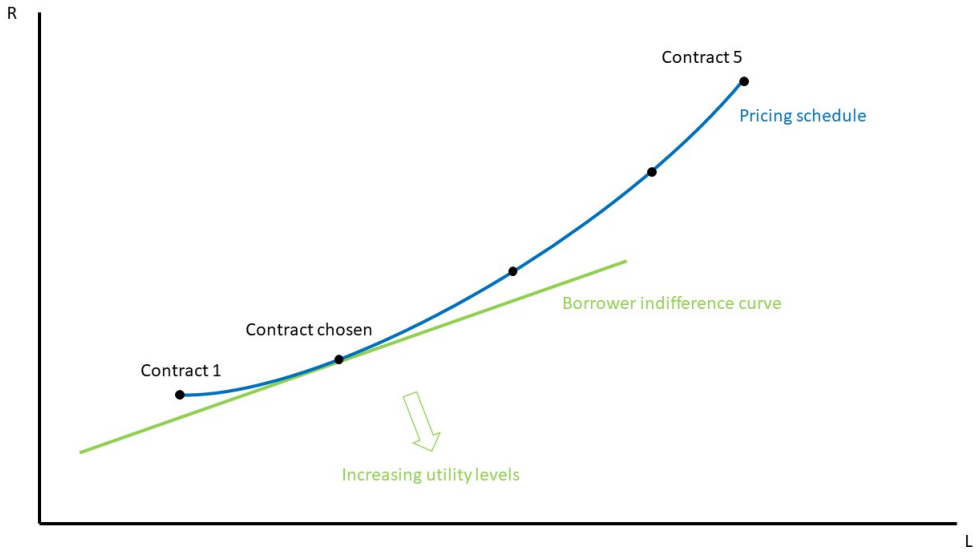


Figure 14: The slope of the pricing schedule at the contract chosen allows us to recover the borrower's willingness to pay $\frac{\alpha_i}{\theta_i}$

needs to estimate the default probability if the borrower were to choose another contract. Because of adverse selection, this is problematic.

As a solution to this problem, I propose using contracts that yield the same monthly payments (leveraging the variety of maturity and loan size options offered in practice) so that default probability is plausibly constant at the individual level (see step 1). Depending on the setting and the data available, it may be preferable to depart from this assumption. The interested reader willing to do so can refer, for instance, to Taburet, Polo, and Vo (2024).

Using contracts with similar monthly payments to recover preferences does not necessarily come at the cost of efficiency. In a model with linear utility and inefficient default (see footnote[19] and Appendix E) borrowers max out their maximum monthly payment capacity (mp_i) so using contracts with the same maturity yields are the relevant set of moments to use. Furthermore, monthly payment targeting is consistent with empirical evidence (Argyle, Nadauld, and Palmer 2020) that borrowers use a monthly payment rule when making their decisions.

Formally, borrower i chooses the contract designed for them (contract i) if its indirect utility is higher than the one they would get if they were to choose similar contracts (i-1 and i+1). Considering contracts that yield the same monthly payment and denoting G the probability distribution function of α, the indirect utility inequalities yield:

19. For instance with the following model: $\max_L \alpha L - p(rL > W)rL - (1-p)c$, $c > r\bar{W}$ and W follows a Bernoulli with its maximum value being $\bar{W}$.

$$u_i(R_i, L_i) \geqslant u_i(R_j, L_j)\ \forall j \implies a := \frac{R_i - R_{i+1}}{L_i - L_{i+1}} \leqslant \frac{\alpha_i}{\theta_i} \leqslant b := \frac{R_i - R_{i-1}}{L_i - L_{i-1}} \tag{85}$$

Relative contract market shares ($\frac{n_i}{\sum_c n_c}$) and the variables (a, b) are observable by the econometrician. Thus an estimate of G is:

$$\hat{G}(a \leqslant \frac{\alpha}{\theta} \leqslant b) = \frac{n_i}{\sum_c n_c} \tag{86}$$

Exogenous variation in a or b—i.e. variation uncorrelated with the distribution of α (i.e., the G function)—allows recovery of the full distribution non parametrically. We can use variation in contract terms coming from either cross-sectional or time series variation. For instance, assuming that the distribution of borrower type stays constant over time exogenous variation in the spread between contracts' interest rates allows recovery of the distribution of borrowers' preferences to default ratio.

Parametric assumptions on G can help with power when the number of observations is small. Moral hazard, as long as its effects are homogeneous across types, can be dealt with by considering variations in contracts other than contract i. For instance, Taburet, Polo, and Vo (2024) use variation in the contract-specific marginal cost of lending to recover preference heterogeneity in a model with moral hazard and adverse selection.

Step 3: Demand elasticities (σ_i).

I recover the demand elasticity using pricing equation (15) as a moment condition.[20] Having a pure strategy equilibrium makes the moment condition tractable and realistic, since we typically do not observe mixed strategies in practice (i.e., lenders do not change their menu each period).

The approach used in this section leverages the fact that proprietary data offer information on marginal costs or that it is common practice in banking to consider that the deposit rate is the marginal cost of lending. The empirical application in section 6 showcases how this approach can be generalized to recover the demand elasticity, marginal cost and information rent.

The estimation procedure is the following. Given an estimate of the borrower's parameter

20. In a model where default only depends on monthly payments, the pricing equation (15) can be derived using the first order conditions with respect to maturity holding monthly payment constant.

(α_i, θ_i) and conditional on observing the cost of lending (mc) (for instance, equal to the deposit rate) we can construct the asymmetric information premium and discount using the equilibrium conditions from Proposition 2 extended to an arbitrary number of contracts (see Appendix F.1):

$$\hat{\theta}_i \hat{I}_i := [\frac{1}{n_i} \frac{\alpha_i - mc}{\Delta WTP_i} 1_{i>1}] - [\frac{1}{n_{i-1}} \frac{\alpha_{i+1} - mc}{\Delta WTP_{i+1}}] 1_{i<x} \tag{87}$$

Abusing notation (I use σ_i in the empirical section instead of $\frac{1}{\sigma(1-\frac{n_i}{\mathbf{B}})}$), the demand elasticity of each group is recovered using the loan present value equilibrium conditions:

$$\theta_i R_i = \hat{m}c\mathbf{Ł}_i + \sigma_i[1 + I_i] \implies E[\hat{\sigma}|c] := R_i - \frac{\hat{m}c\mathbf{Ł}_i}{E[\hat{\theta}_i|c]}[1 + \hat{I}_i]^{-1} \tag{88}$$

By assuming that semi-elasticities are borrower-specific parameters (σ_i), I avoid estimating demand elasticities using choice data. Similar to default elasticities, demand elasticities are problematic to estimate when there is a selection of unobservables, even with credit-register data. This is because the econometrician only observes the contract chosen and thus has to estimate the contract offered by competitors using observationally equivalent borrowers.

The constant semi-elasticity with respect to the loan present value (θR) assumption can be justified in two ways. First, it arises naturally in the symmetric equilibrium model (the semi-elasticity is $\frac{1}{\sigma(1-\frac{n_i}{\mathbf{B}})}$ which are parameters in my model). Second, it can be interpreted as lenders' considering the local borrowers' semi-elasticity as an approximation when setting rates.

Alternative to Steps 2 and 3.

Instead of estimating utility parameters and demand elasticity separately, one can run a linear regression based on the pricing equation (15). Variation in the loan size allows recovery of the marginal cost (mc), and the variation in information rents identifies the demand semi-elasticity (σ). I illustrate this approach and discuss exclusion restrictions in the empirical application (Section 6).

Discussion: Heterogeneity within contracts.

With those three steps, I recover borrowers' preferences (α_i), default $(1 - \theta_i)$ and elasticity (σ) conditional on contract choice. If lenders pool borrowers, there may be substantial borrower

heterogeneity within a contract.

To learn more about this heterogeneity without having to impose parametric assumptions, we can use the fact that the pooling contract was optimal. For instance, if the econometrician observes that only one contract with loan size $\bar{L}$, and is concerned that the distribution has at least two borrower types, they can deduce that the L_G implied by equation (73) is above $\bar{L}$. Conditional on, for instance, (σ, θ), this inequality provides a bound on the value of θ within borrowers who chose this sample.

P.2 Second Case: Credit Register Data

When data on menus are unavailable, we have to predict borrower i menu using observationally equivalent borrowers. Crawford, Pavanini, and Schivardi (2018) use this approach and predict the contracts offered by competitors.

In that situation, data on lenders' marginal cost of lending are often not available. It is then common practice in industrial organisations to estimate demand elasticity (σ) using a logit model together with bank choice data and use equation (88) to recover marginal costs.

In the context of my model, we can estimate the pricing schedule for each borrower and characteristics and use it to recover the willingness to pay (WTP) for each contract characteristic (X) using step 2 of the previous section.

Using a linear utility model ($u^*_{i,b} := WTP \cdot X - PV$), we can then recover the demand elasticity using market share data or the pricing equation. For the former, we can use a logit model together with market share data. A standard parameterization of the utility is: $u^*_{i,b} + \mu_b + \sigma\varepsilon_{ib}$, with ε_{ib} iid and extreme value distributed and μ_b is a bank fixed effect. This procedure is similar to the nested logit approach with random coefficients but is more flexible and tractable. For the latter, the pricing equation can be used as a regression to recover both the demand and default elasticity (see Section Alternative to Steps 2 and 3). This approach, compared to the logit model, has the advantage whereby we do not have to take a stance on the choice set available to the borrower.

Q Borrower preference approximation

Using data on menus, I approximate the pricing schedule $(L_c, r_c, T_c)_c$, where r is the interest rate and T is the maturity using a spline function. This interpolation allows to use first-order

conditions to derive the moment equation. I discuss alternative options in the paragraph in section P.1 labelled "Heterogeneity within contracts".

Formally, I first calculate the present value of the loan $R_{i,c} = \sum_{t=1}^{T} \theta_i^t T_i$ for each contract c holding the monthly payment (T_i) constant. Then, I find the coefficients $(\gamma_1, \gamma_2, \gamma_3)$ such that $R_i(L) := \hat{\gamma}_{1,i} + \hat{\gamma}_{2,i} L + \frac{\hat{\gamma}_{3,i}}{2} L^2$ best match the data points ($R_{i,c}$).

I recover borrowers' preferences using a revealed-preference approach. For a given monthly payment and an implied pricing schedule, I have

$$u_i^* := \max_L u(L) := \alpha_i L - \theta_i R_i(L) \implies L_i = \frac{\alpha_i}{\hat{\gamma}_{3,i}\theta_i} - \frac{\hat{\gamma}_{2,i}}{\theta_i} \tag{89}$$

For each contract c, I can thus recover borrowers' average preference by inverting equation (89) and using the average default estimates:

$$E[\hat{\alpha}_i|c] = L_i \hat{\gamma}_{2,i}(1 + E[\hat{\theta}_i|c]) - \hat{\gamma_{2,i}} \tag{90}$$

Equation (90) states that the slope of the pricing schedule for the contract chosen allows us to recover the borrower's willingness to pay $\frac{\alpha_i}{\theta_i}$. Figure 5 provides a visual representation of the identification strategy. An identification threat would be the slope of the pricing schedules, which varies due to unobserved contract characteristics such as covenant. In that case, the econometrician is not identifying the willingness to pay for loan size but rather the willingness to pay for both covenant and loan size. The "Heterogeneity within contracts" paragraph discusses how to deal with this issue.

R Adapting the Model to the Empirical Setting

For the model to be able to deliver a continuum of loan sizes, I consider the following extensions. For a given risk category, I consider that there are N types of borrowers (i.e., N combinations of $(mp_i, \frac{\alpha_i}{\theta_i}, \sigma_i)$), where mp_i is the disposable income, $\frac{\alpha_i}{\theta_i}$ is the willingness to pay for one dollar, and σ_i is the demand elasticity.

Borrowers have linear utility on loan size so that, conditional on maturity, they borrow as much as possible until their monthly payment capacity (mp_i) is maxed out. For each borrower,

I can thus construct the following incentive compatibility constraint:

$$\alpha_i L_i - PV(mp_i, T_i) \geqslant \alpha_i L_j(mp_i, T_j) - PV_i(mp_i, T_j) \tag{91}$$

$L_j(mp_i, T_j)$ is the loan size that would be chosen by the borrower if they were to choose a contract with the other maturity. $PV(mp_i, T_i)$ is the present value of the loan as a function of maturity T_i.

Using the annuity formula, I can show that the heterogeneity in mp is irrelevant, because it cancels out in the incentive compatibility constraints. This feature allows us to abstract away from mp and focus only on the two contract cases. [21]

The fact that LC offers the same interest rate independent of loan size is consistent with the assumption that the distribution of maximum monthly payment mp_i is independent of the distribution of willingness to pay and $\frac{\alpha_i}{\theta_i}$ and demand elasticities σ_i conditional on risk category. This is the modelling approach I use for this application. I recover the distribution of mp using the observed distribution of monthly payments.

I am agnostic about the number of types $(\frac{\alpha_i}{\theta_i}, \sigma_i)$. Appendix (S) shows that the equilibrium conditions of the model with an arbitrary number of borrower types – but with LC choosing to offer two optimally–and the optimality conditions with two borrower types are identical.

Since I do not observe maturity above 60 months, I assume that its cost is such that longer than 60-month loans are not profitable. This assumption plays the same role as the maximum loan size $\bar{H}$ in the baseline model.

S Empirical Application Model

I consider that there are N types of borrowers (i.e., N combinations of $(mp_i, \frac{\alpha_i}{\theta_i}, \sigma_i)$), where $\bar{m}p_i$ is the maximum monthly payment the borrower can repay. For each borrower type, I construct an incentive compatibility constraint:

$$L_i - \frac{\theta_i}{\alpha_i} mp_i T_i \geqslant L_j(mp_i, M_j) - \frac{\theta_i}{\alpha_i} mp_i M_j \tag{92}$$

21. In continuous time, the principal is $L = mp \cdot [1 - \frac{e^{-r_i T_i}}{r_i}]$ and the present value is $mp \cdot e^{-\lambda_i M_j}$. λ_i is the hazard rate, so that $e^{-\lambda_i}$ is the monthly survival probability. Formally, I linearize $e^{-\lambda_i M_j}$ and use the change of variable $\tilde{L}_i := [1 - \frac{e^{-r_i T_i}}{r_i}]$.

where $L_j(mp_i, M_j)$ is the loan size that would be chosen by the borrower if they were to choose a contract with the other maturity.

$$\sum_{i=1}^{l} n_i \phi(u_i) \theta_i [R_l - \frac{mc}{\theta_i} L_h] + \sum_{i=h}^{n} n_i \phi(u_i) \theta_i [R_h - \frac{mc}{\theta_i} L_h] \tag{93}$$

$$with\ u_i := WTP_i L_i - R_i \tag{94}$$

$$\{h := l+1, l\} : u_h(L_h, R_h) := u_l(L_l, R_l) + (WTP_h - WTP_l) L_i \tag{95}$$

Using the average utility in each market segment ($\bar{u} := u_i - \Delta_i L_i$ with $\bar{\Delta}_i := (\bar{WTP} - WTP_i)$):

$$\sum_{i=1}^{l} n_i \phi(\bar{u} + \bar{\Delta}_i L_h) \theta_i [(W\bar{T}P - \frac{mc}{\theta_i}) L_h - \bar{u}] + \sum_{i=h}^{n} n_i \phi(\underline{u} + \Delta_i L_l) \theta_i [(\underline{WTP} - \frac{mc}{\theta_i}) L_h - \underline{u}] \tag{96}$$

$$\bar{u} + (WTP_h - W\bar{T}P) L_h = \underline{u} + (WTP_l - \underline{WTP}) L_l + (WTP_h - WTP_l) L_l \tag{97}$$

Replacing the constraint and taking the first-order conditions with respect to ($\bar{u}$ and $\underline{u}$), we obtain the same first-order conditions as in the two-borrower models.