# Aging, Diseases, and the Great Dispersion of Wealth Distribution

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**Abstract**

This paper studies how households' cumulative health shocks affect their health investment, health insurance premium, and wealth. We develop a rich lifecycle model of endogenous health capital, insurance premium and wealth, with a diffusion income process and Poisson health shocks. In this model, health shocks affect a household's wealth through three channels. It increases medical spending, lowers productivity and thus income, and increases the probability of death. Theoretical results show that since health increases life expectancy, it becomes a luxury good relative to consumption. When income rises, health generates larger marginal utility relative to consumption. As a result, rich people invest more in health, live longer, and save more. This implies that vital health shocks may facilitate top concentration in wealth distribution. With the CDC data for the top nine death-causing diseases we estimate vital disease processes and calibrate the model. The counterfactual exercise shows that effects of vital health shocks indeed help explain wealth inequality.

Keywords: Wealth Distribution, Health and Income Shocks, Health Capital Evolution, Health Insurance, Endogenous Life Expectancy.

JEL Classification Numbers: D91, E20, I10.

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# 1 Introduction

Since Piketty's celebrated book (2014), much attention has been paid to the drastic widening of top income and wealth inequalities. While conventional factors inclusive of stochastic survival, warm-glow bequest, wage income shock, and entrepreneurial shock fail to explain such widening in a systematic manner, recent research considers two layers of extreme distributions (e.g. in superstar models) to better match the observation. While so much effort has been put to relying on entrepreneurship and monetary wealth, one may wonder whether another dimension of human wealth, health, may play an important role in explaining old age wealth inequalities. In the present paper, we intend to address this still largely open issue in a quantitative lifecycle framework.

Our model setting has three important features that differentiate ours from early contributions by De Nardi, French and Jones (2010) and De Nardi, Porapakkarm and Pashchenko (2017). First, we model the dynamic process of health capital evolution, depending crucially on age-dependent health shocks and out-of-pocket medical expenses. Second, we allow for mandatory and self-paid health insurance against random health shocks with actuarially fair premia depending on expected rates of arrivals. Third, life expectancy is endogenous, depending on ones initially inherited health capital and the dynamic path of health capital evolution. Thus, while current health status helps future health standing, an individual with higher monetary and human wealth are better insured and more capable of paying large out-of-pocket medical expenses when facing a harmful health shock. Health shocks will in turn affect one's future health stock, survival probability, saving decision and wealth accumulation over the entire life course.

We note that unhealthy people have significantly higher medical spending and lower income than healthy people, due to their lower labor supply and lower earnings conditional on working. Among prime age men with a high school degree, participation among the healthy is over 90% while participation among the unhealthy is only about 70%. Conditional on participation the healthy cohort earns on average 28% more than the unhealthy. Moreover, unhealthy people's wealth share tend to be lower than healthy people's. The gap in wealth by health starts at young people and becomes very large by retirement age. Among 65-year-old men with a high school degree, the median wealth of the healthy is almost twice that of the unhealthy; $\$230,000$ for the former versus $\$120,000$ for the latter. This suggests that the accumulated effects of bad health is important.

In this paper, we develop a rich lifecycle model of endogenous health capital, insurance premium and wealth, with a diffusion income process and Poisson health shocks. In this model, health shocks affect a household's wealth through three channel, by raising out-of-pocket medical spending, lowering productivity and thus income, and increasing the probability of death. Theoretical results show that since health increases life expectancy, it becomes a luxury good relative to consumption. When income rises, health generates larger marginal utility relative to consumption. As a result, rich people invest more in health,

live longer, and save more. This implies that vital health shocks may facilitate top concentration in wealth distribution.

With medical data from the Centers for Disease Control and Prevention (CDC) and economic data from the Health and Retirement Study (HRS), we calibrate the model and perform counterfactual exercises. Our preliminary results show that our baseline model fits quite well in predicting of the share of wealth for the 5%. Our counterfactual analysis suggests that vital health shocks are indeed important for wealth inequalities, by accounting for more than 40% of the model predicted wealth inequalities at the 90 percentile or higher.

# 2 Background and Evidence

In this section, we explore the relationship between wealth, health care expenditure and life expectancy using data from the Health and Retirement Study (HRS). We use RAND HRS Longitudinal File 2014 which derives data from all waves of HRS from 1992 to 2014 and contains useful information such as wealth, medical expenditure, and death year (if one died) for individuals above age 50 .

## 2.1 Lifecycle wealth distribution

First, we look into wealth distribution by age group. We use household net worth data from wave 4 (year 1998) to wave 9 (year 2008). [1] We take average age of household members if the household consists of a couple and construct seven age groups (50-59, 60-69, 70-79, 80-84, 85-89, 90-94, and 95+). Note that we separate older groups (above 80) into 5-year age groups rather than 10-year groups because large death trimming for the very old may bias the result.

Table 1 reports average wealth ratios of different wealth percentiles to the median within each age group from the six waves. While wealth ratios are relatively stable for those at relatively young ages, they increase sharply when people approach very advanced ages. For example, top wealth concentration (99-50 wealth ratio) increases substantially for those above 85, and the 50-20 ratio starts to surge when people enter their 80 s.

Table 1. average wealth ratio for given age groups (waves 4-9)

| age/wealth(%) | $99-50$ | $95-50$ | $90-50$ | $80-50$ | $50-20$ |
|---|---|---|---|---|---|
| $50-59$ | 28.06 | 10.30 | 6.24 | 3.58 | 9.89 |
| $60-69$ | 29.95 | 9.97 | 6.19 | 3.48 | 7.29 |
| $70-79$ | 24.62 | 9.00 | 5.87 | 3.27 | 7.19 |
| $80-84$ | 22.55 | 10.37 | 6.33 | 3.45 | 12.29 |
| $85-89$ | 29.14 | 11.65 | 6.92 | 3.87 | 44.28 |
| $90-94$ | 47.54 | 17.95 | 10.87 | 5.06 | 395.87 |
| 95+ | 286.00 | 48.78 | 28.02 | 12.75 | 286.13 |

[01] Asset data is severely underreported in wave 2 (De Nardi et al., 2010) and is unavailable in wave 3 , and the data pattern of wealth distribution after wave 9 could be heavily influenced by the 2008 financial crisis.

One might wonder what the within-cohort wealth ratios may look like when people age. To see this, we use an unbalanced panel data and fix the cohorts as in wave 4 , and then examine how the wealth ratios within cohort may change over the next ten years. Table 2(a)-(d) show the results from wave 4 (year 1998), 6 (year 2002), 7 (year 2004), and 9 (year 2008) for all the 5 -year cohort groups. Consistent with that in Table 1, we note that all the wealth ratios are relatively stable at relatively young ages but tend to rise when the cohort get older. For instance, the 99-50 ratio of cohort 7 (aged 80-84 in 1998) rises continuously from 25.64 in 1998 to 36.10 in 2008, while the 50-20 ratio starts to rise for a much younger cohort, beginning for cohort 4 (aged 65-69 in 1998) from 5.27 in 1998 to 8.94 in ten years; the same ratio for cohort 7 even rises from 16.20 in 1998 to 40.50 in 2002, and up to 112.55 in 2008 .

We interpret these results regarding rising wealth inequality for the elderly from two perspectives. First, the rich earn much more capital income than the poor at the old ages, magnifying wealth inequality. Second and which is more relevant to our paper, as the poor invest less in health than the rich throughout their life, their health condition deteriorates faster and they would thus expect a shorter life; therefore, they start to dissave at a younger age, running out of wealth earlier than their rich counterpart in the same cohort. Moreover, as medical expenditure rises sharply at advanced ages (to be shown later), the poor are much more financially constrained when facing medical shocks and their wealth can be dried out much faster than the rich. We also note that our interpretation applies to the documentation in Saez and Zucman (2016) that the elderly hold a larger fraction of wealth among the top wealth group than among the general population. Since the rich invest more in health, stay healthier and live longer than the poor, they constitute a larger share of the old group. In other words, the wealthy group consists more of old people than the general population does, leading to a larger share of wealth held by the elderly.

Table 2(a). wealth ratios for 9 cohorts in wave 4 (1998)

| cohort(age)/wealth(%) | $99-50$ | $95-50$ | $90-50$ | $80-50$ | $50-20$ |
|---|---|---|---|---|---|
| $1(50-54)$ | 22.95 | 9.50 | 5.76 | 2.95 | 6.18 |
| $2(55-59)$ | 29.89 | 9.97 | 6.10 | 3.37 | 5.94 |
| $3(60-64)$ | 32.64 | 9.19 | 5.84 | 3.25 | 10.61 |
| $4(65-69)$ | 27.80 | 9.46 | 5.96 | 3.15 | 5.27 |
| $5(70-74)$ | 20.68 | 8.41 | 5.32 | 2.94 | 4.58 |
| $6(75-79)$ | 26.57 | 8.45 | 5.63 | 3.29 | 7.78 |
| $7(80-84)$ | 25.64 | 10.31 | 5.86 | 3.26 | 16.20 |
| $8(85-89)$ | 37.80 | 12.84 | 7.46 | 4.15 | 51.59 |
| 9(90+) | 55.90 | 19.15 | 11.34 | 6.02 | 92.72 |

Table 2(b). wealth ratios for 9 cohorts in wave 6 (2002)

| cohort(age)/wealth(%) | $99-50$ | $95-50$ | $90-50$ | $80-50$ | $50-20$ |
|---|---|---|---|---|---|
| $1(54-58)$ | 21.43 | 9.01 | 5.78 | 3.42 | 5.16 |
| $2(59-63)$ | 27.31 | 9.71 | 6.11 | 3.54 | 4.68 |
| $3(64-68)$ | 24.47 | 8.82 | 5.93 | 3.30 | 9.63 |
| $4(69-73)$ | 24.21 | 8.88 | 5.66 | 3.44 | 5.44 |
| $5(74-78)$ | 18.52 | 8.65 | 5.37 | 3.16 | 5.61 |
| $6(79-83)$ | 22.31 | 9.75 | 6.76 | 3.61 | 10.56 |
| $7(84-88)$ | 29.14 | 11.33 | 6.13 | 3.37 | 40.50 |
| $8(89-93)$ | 37.09 | 13.97 | 10.59 | 4.98 | 106.50 |
| 9(94+) | 46.11 | 23.20 | 9.31 | 4.56 | 166.67 |

Table 2(c). wealth ratios for 9 cohorts in wave 7 (2004)

| cohort(age)/wealth(%) | $99-50$ | $95-50$ | $90-50$ | $80-50$ | $50-20$ |
|---|---|---|---|---|---|
| $1(56-60)$ | 22.78 | 9.46 | 5.93 | 3.62 | 6.06 |
| $2(61-65)$ | 31.76 | 9.42 | 5.65 | 3.44 | 5.87 |
| $3(66-70)$ | 34.39 | 9.28 | 5.85 | 3.29 | 8.80 |
| $4(71-75)$ | 22.42 | 8.88 | 5.92 | 3.31 | 7.50 |
| $5(76-80)$ | 23.06 | 9.04 | 5.75 | 3.10 | 6.40 |
| $6(81-85)$ | 24.98 | 10.67 | 6.81 | 3.89 | 14.56 |
| $7(86-90)$ | 33.44 | 14.16 | 7.75 | 4.10 | 50.00 |
| $8(91-95)$ | 59.65 | 18.42 | 13.95 | 5.99 | 400.00 |
| 9(96+) | 171.57 | 99.22 | 78.43 | 22.16 | 510.00 |

Table 2(d). wealth ratios for 9 cohorts in wave 9 (2008)

| cohort(age)/wealth(%) | $99-50$ | $95-50$ | $90-50$ | $80-50$ | $50-20$ |
|---|---|---|---|---|---|
| $1(60-64)$ | 24.03 | 10.14 | 6.23 | 3.45 | 5.75 |
| $2(65-69)$ | 25.49 | 9.42 | 6.04 | 3.39 | 6.60 |
| $3(70-74)$ | 28.84 | 9.17 | 5.91 | 3.33 | 10.75 |
| $4(75-79)$ | 25.82 | 9.97 | 6.02 | 3.35 | 8.94 |
| $5(80-84)$ | 25.95 | 10.93 | 6.22 | 3.39 | 11.93 |
| $6(85-89)$ | 24.64 | 11.58 | 7.10 | 3.88 | 21.70 |
| $7(90-94)$ | 36.10 | 17.50 | 10.67 | 5.47 | 112.55 |
| $8(95-99)$ | 86.35 | 25.92 | 17.69 | 7.13 | 1300.00 |
| 9(100+) | 198.81 | 198.81 | 198.81 | 32.92 | 120.00 |

# 3 2.2 Wealth and medical expense

To investigate the relationship between wealth and medical expenses, we use individual-level medical expenditure data from wave 3 to 12 of HRS for each age-wealth group. The medical expenditure includes all out-of-pocket expenses and health insurance premium. We use 5-year interval age groups (i.e., $50-54, 55-59, \ldots$) and divide the sample into seven wealth groups (100-99, 99-95, 95-80, 80-60, 60-40, 40-20, and 20-0 percentiles) based on household wealth distribution in wave 4. All expenses are in 2014 US dollars.

We first compute average medical expense for each age-wealth group and show results in Figure 1 (top panel). Then in order to capture individuals' medical expenses when facing health shocks, we also plot the 95 and 99 percentiles medical expense, respectively, for each age-wealth group in Figure 1 (bottom panel). On average, for almost all ages above 50, wealthier people spend more on medical care than the less wealthy do. The divergence becomes more remarkable at more advanced ages (e.g., above 85), when the rich's medical spending rises sharply while the poor's spending does not change significantly from that in earlier life. Turning to medical expenditure when facing health shocks (measured by 95 and 99 percentiles of medical expense, respectively, of each group), we find that the pattern is qualitatively similar, albeit with a much higher level of medical expenditure for all groups. In addition and different from the average spending, there is a noticeable increase of medical expenditure of the bottom 40 percent wealth groups when facing health shocks at advanced ages - from age 80 for the 95 percentile and from 75 for the 99 percentile, respectively, although the amount of spending is still far less than that of the top wealth groups.

### 3.1 Wealth and life expectancy

Next, we explore the relationship between wealth and life years using the survival data in HRS. To do so, we compute the survival probability at each age for each wealth group (again, we use the same seven wealth groups as for medical expense based on wealth distribution in wave 4), and then impute life years for each wealth group while matching the average life expectancy of the US population.

Figure 2 plots the average survival probability of seven wealth groups and of the whole sample above 50 (conditional on survival at 50). With the exception of those within the top 5 percent wealth group, the wealthier has a higher average survival probability than the poorer at all ages.

To impute life expectancy, we assume that life begins at age 20 and that death follows a Poisson process. We first estimate the average arrival rate of death of the whole population so that the distance between the average

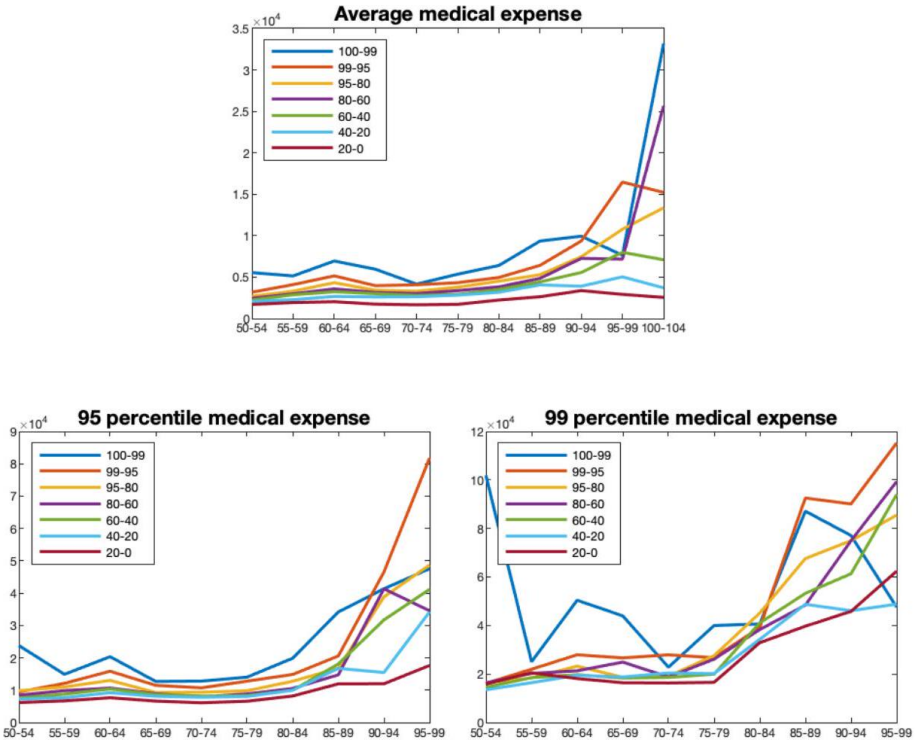



Figure 1: Lifecycle medical expense by wealth group

survival probability over ages 50-90 imputed from this arrival rate and that in data is minimized. [2] Meanwhile, since the HRS data is old-age biased as it only selects people above 50, a hypothetical conditional age (upon which an HRS respondent survives) is picked jointly with the death arrival rate to match the average life expectancy of the US population which is 78 . We then estimate the death arrival rate of each wealth group so that the ratio of survival probability of each group to that of the whole population over ages

[2] I trim the old age at 90 because the sample size becomes very small above 90 and there can be severe under-report of death beyond 90 (i.e., for all wealth groups, the survival probability changes very little above 90 which is unrealistic).

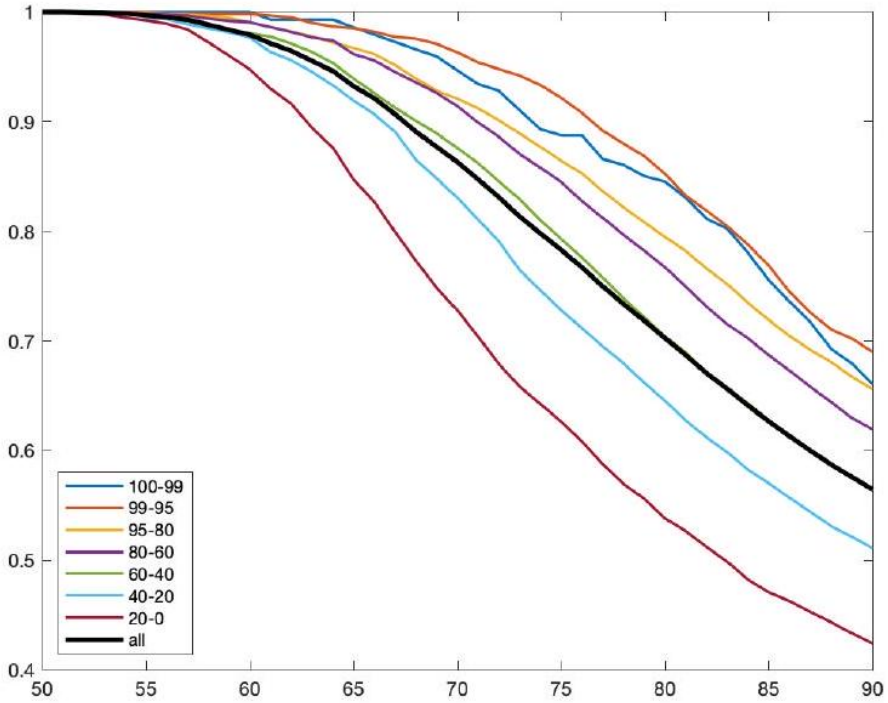



Figure 2: Lifecycle survival probability by wealth group

50-90 imputed from the death arrival rates equalizes that from the data. Finally, we compute life expectancy for each wealth group and for the whole population using the estimated arrival rates of death. Table 3 reports the results.

Table 3. Life expectancy by wealth groups

| wealth groups | $100-99$ | $99-95$ | $95-80$ | $80-60$ | $60-40$ | $40-20$ | $20-0$ | all |
|---|---|---|---|---|---|---|---|---|
| life years | 84.6 | 85.5 | 82.6 | 81.3 | 78.4 | 75.4 | 70.2 | 77.7 |

Overall, the wealthier may expect to live a longer life than the less wealthy (except for those within the top 5 percent wealth group). While the middle wealth group (60-40) have a life expectancy close to the average of the US population, the top 5 percent can expect to live 7 years (or 10 percent) longer than the average and the bottom 20 percent can only expect to live about 7 years (or 10 percent) shorter. Comparing top 5 percent with bottom 20 percent, the latter have a life expectancy about 17 percent lower than the former. [3]

In summary, our data shows that within-cohort wealth inequality rises when the population ages, and that the wealthier spend more on health care on average

[03] Our estimates are roughly comparable to that documented by Cutler et al. (2006) that Americans in the bottom 5 percent of the income distribution in 1980 had a life expectancy at all ages about 25 percent lower than the that of those in the top 5 percent of the income distribution.

and when facing health shocks, and expect longer years of life than the less wealthy. Our analysis suggests that these facts are inter-related. As the rich value health more relatively to normal consumption than the poor and as they are less financially constrained, they invest more in health and expect to live longer; thus, they accumulate wealth toward more advanced ages than the poor. This, together with a higher capital income for the rich in the old ages, elevates wealth inequality for the elderly.

# 4 A Benchmark Lifecycle Model

Time is continuous. An individual's life starts at $t_0$ and retires at $R$ and ends at $T$ which is endogenous. Throughout $[t_0, T]$, she values consumption $c$ and health capital $h$, and derives utility from leaving a bequest. The lifetime utility is assumed to take the following form (note that there is a **warm-glow** utility term):

$$V(a_0, h_0, \epsilon_0, \nu_0, t_0, \delta_0) = \max_{c,x,s} \mathbb{E} \int_{t_0}^{T} e^{-\rho(t-t_0)} [\ln(c(t)) + \beta \ln(h(t))] dt + e^{-\rho(T-t_0)} q_0 \ln(1 + \frac{a(T)}{q_1}) \tag{1}$$

where the state variables are wealth $a$, health capital $h$, the random error in the income process $\epsilon$, insurance premium $\nu$, time $t$, and disease indicator $\delta$ (indicating whether the agent is currently hit by the disease shock). Their evolution equations are as follows.

Let the out-of-pocket medical spending $\bar{M}(n(t))$ be an increasing function of health deterioration. Upon paying a health insurance premium $\nu(t)$ under standard private healthcare insurance in the benchmark case, an individual is qualified to have the insurance to cover the medical cost accrued up to the total amount of health investment $x(t)$. That is, when out-of-pocket medical spending exceeds the necessary health investment based on standard medical practice, the insurance only covers the amount of health investment $x(t)$. Letting $\kappa(\nu)$ denote the insurance coverage rate, the amount of medical cost covered by the insurance is then equal to $\kappa(\nu(t)) \cdot \min\{x(t), \bar{M}(n(t))\}$. An individual's asset accumulation over the life course is therefore governed by:

$$da = \{ra + y - c - s - x + \kappa(\nu) min[x, \bar{M}(n(t))]\delta\} dt \tag{2}$$

where $r$ is net rate of wealth return, $s$ is the inflow of insurance premium, $y$ is the labor income, $x$ is health expenditure, $\kappa(\nu)$ is the percentage of health expenditure covered by the insurance company ($1 - \kappa(\nu)$ is the co-insurance rate), $\bar{M}$ is the maximum disease-related health expenditure (The amount of health expenditure exceeding $\bar{M}$ is not considered for insurance coverage. Asset can be accumulated when wage and asset incomes exceed insurance premium and net medical spending after insurance coverage. In the benchmark setting we assume the rate of returns on asset $r$ is time-invariant but may vary across agents

depending on demographics (for example, better educated may earn higher return). Note that besides disease-related medical expenses, health expenditure may also covers costs of leisure, nutritional supplements, and health-enhancing activities etc.).

By allowing for an income shock $\epsilon(t)$, the log-income process can be specified as

$$\begin{cases} \ln(y(t)) = \ln(\theta) + \ln(w(t)) + \ln[h(t) + \omega(t)] + \epsilon(t), & \text{if } t < R \\ y(t) = \frac{1}{2} y(R), & \text{if } t \geq R \end{cases} \tag{3}$$

where $\theta$ measures demographic heterogeneity and $\omega$ measures on-the-job learning effect with

$$\omega(t) = \xi(e^{b_1(t-t_0)+b_2(t-t_0)^2} - 1)$$

and the Wiener process:

$$d\epsilon(t) = \eta dt + \sigma dW(t).$$

and $w(t)$ is taken as 1 for now.

To specify the evolution of health capital, we first note that while health can be improved by health investment, it does so at a diminishing rate, i.e., the rate of health capital evolution is decreasing in health investment per unit of health capital $x/h$ (cf. Chen, Wang and Yao 2017) and the marginal return decreases with age (cf. Hall and Jones 2007). Moreover, the rate of health deterioration depends negatively on the current health status $h(t)$. Therefore we specify the health evolution process as follows :

$$dh = (\phi x^{\alpha} h^{1-\alpha} - \psi \underline{\delta} h^{\mu} - \psi h^{\mu} \delta) dt. \tag{4}$$

where the detrimental effect of health shocks on one's health stock is measured by a constant scaling parameter $\psi$ under which the health deteriorating rate $\psi \delta h(t)^{\mu-1}$ is already increasing with age. $\mu \in (0, 1)$ and $\alpha \in (0, 1-\mu)$ measure the returns to scale of current health stock and health investment to incremental health capital, respectively. To counter health deterioration, an individual may invest in health with $x(t)$, but its effect on the health stock faces diminishing return not only in its amount ($\alpha \in (0, 1-\mu)$) but also over the individual's age - the latter is captured by the age-dependent scaling parameter $\phi$:

$$\phi(t) = \phi_0(B_1 - e^{\upsilon_1 t}).$$

The rest two terms on the RHS are natural and disease-related deterioration on health capital, where the natural health deterioration follows:

$$\underline{\delta}(t) = \delta_0(B_2 + e^{\upsilon_2 t}).$$

Turning now to disease-induced health shocks, we denote the Poisson arrival rate and intensity of disease $i$ as $\tilde{q}_i$ and $d_i$, respectively. To simplify the Poisson

processes of diseases, we normalize the measurement of the severity of disease in such a way that every disease hit the individual with the same intensity but severer diseases are associated with higher Poisson arrival rates. That is, we can denote an average disease intensity as $\bar{d}$ to rewrite

$$\tilde{q}_i d_i = \lambda_i \bar{d}$$

where $\lambda_i$ measures the relative arrival rate of disease $i$ adjusted by its disease intensity. In other words, under this normalization, every disease affects the health capital by the same amount but severer diseases are more likely to hit. In this way disease shocks can be conveniently represented by a single Poisson process, as does natural deterioration included health shocks:

$$d\delta = -\delta dt + dN_t \tag{5}$$

where $dN_t = \sum_i dN_t^i$, and $dN_t^i$ is the Poisson process for disease $i$. The arrival rate of the aggregate Poisson process is then $\lambda = \sum_i \lambda_i$.

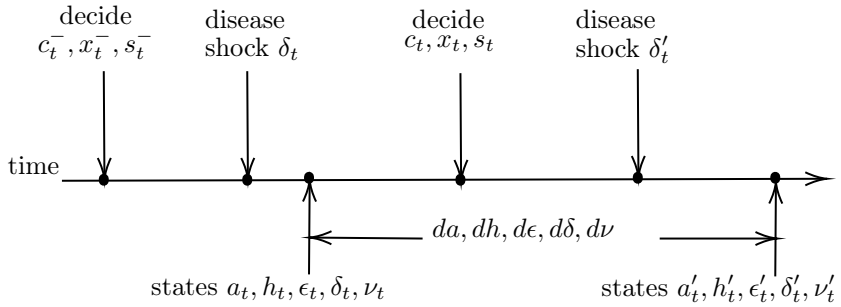
Insurance premium process is specified as

$$d\nu = -\nu dt + s dt \tag{6}$$

which means that insurance premium depreciates at rate of 100% and at the same time is increased by the premium inflow $s$ paid by the individual.

The agent leaves the economy at time $T$. $T$ is the period when health capital reaches the minimum requirement $\underline{h}$ for the first time.

## 4.1 Timeline




Note that at time $t$, the disease-related medical expenditure is covered with the insurance associated with premium $\nu_t$. The agent chooses premium inflow $s_t$ which only affects the future insurance premium $\nu_t'$. In other words, in this model, agents decide their insurance plan before observing the realization of the disease shock which affects the time $t$ health capital.

## 4.2 Hamilton-Jacobi-Bellman equation

Since the income process of the agent in this model is not continuous, we cannot directly optimize the agent's lifetime utility over $[t_0, T]$ with the Hamilton-Jacoby-Bellman (HJB) approach. To bypass the discontinuity in the income process, we divide the problem into two and solve them backwards. We firstly maximize the agent's utility after retirement (from $t = R + 1$ to $t = T$ ). This problem can be safely solved because there is no discontinuity in $y$ over this period. Denote the maximized value by $v_R$, and it is a function of the state variables at $t = R + 1$. We then maximize the agent's utility from $t = t_0$ to $t = R$ with the terminal condition that the agent's terminal utility equals $v_R$. Again this problem can be safely solved because there is no discontinuity in y over this period. Note that both problems have very similar HJB equations. The only difference is that $y$ in the first problem is a constant, $\ln(0.5) + \ln(y(R))$.In this way, the HJB of the first problem can be regarded as a special case of that in the second problem. Therefore, we only solve the second problem here.

With the value function $v(a, h, \epsilon, t, \nu, \delta)$, the HJB is

$$\rho v(a,h,\epsilon,t,\nu,\delta) = \max_{c,x,s} \quad u(c,h) + v_a\frac{da}{dt} + v_h\frac{dh}{dt} + \eta V_\epsilon + \frac{\sigma^2}{2}V_{\epsilon\epsilon} + v_t + v_\nu(-\nu + s) \\ - v_\delta\delta + \lambda\big[v(\delta+1) - v(\delta)\big]$$

It is straightforward to derive FOC w.r.t. $c$ and $x$:

$$1/c = v_a$$

$$v_a\big[1 - \kappa(\nu_t)\cdot\delta\cdot(1 - I_{x_t \geq \bar{M}})\big] = v_h\phi\alpha x_t^{\alpha-1}h_t^{\mu} \tag{7}$$

The FOC for $s$ is

$$v_a = v_\nu \tag{8}$$

Due to diminishing marginal utility of $a$ and $\nu$, this means that if $v_\nu > v_a$ the agent increases $s$ and if $v_\nu < v_a$ the agent reduces $s$. Intuitively, $s$ is determined by balancing the marginal benefit and marginal cost of the future insurance premium $(\nu_{t'})$.

In numerical solution we determine $s$ such that

$$\mathbb{E}v_a^{t+1} = \mathbb{E}v_\nu^{t+1}$$

or

$$\mathbb{E}v_a\Big[a_{t+1}(s_t), h_{t+1}, \epsilon_{t+1}, \nu_{t+1}(s_t), t{+}1, \delta_{t+1}\Big] = \mathbb{E}v_\nu\Big[a_{t+1}(s_t), h_{t+1}, \epsilon_{t+1}, \nu_{t+1}(s_t), t{+}1, \delta_{t+1}\Big] \tag{9}$$

where

$$a_{t+1}(s_t) = a_t + ra_t + y_t - c_t - x_t - s_t + \kappa(\nu_t)min[x_t, m_t]$$

$$\nu_{t+1}(s_t) = s_t$$

# 5 Numerical Solution: Algorithm

Achdou et al. (2022) develops the implicit finite difference method with the upwind scheme to numerically solve a heterogeneous agent economy in a simple lifecycle model with an exogenous life expectancy. We extend this algorithm to solve our model.

Boundary conditions: We assume there exists a natural upper limit of life span, $\bar{T}$. Agents leave the economy at $\bar{T}$ even if her health capital does not reach $\underline{h}$ yet. We also assume reflective upper and lower bounds for $\epsilon$ and $a$: $\bar{\epsilon}$, $\underline{\epsilon}$, $\bar{a}$, $\underline{a}$. This gives rise to the following state-constraint boundary conditions (as in Achdou et al (2022)):

$$0 = V_\epsilon(h, a, \bar{\epsilon}, t) = V_\epsilon(h, a, \underline{\epsilon}, t), \tag{10}$$

$$V_a \geq u'[r\underline{a} + y - x(1 - \kappa(\nu) \cdot \min\{1, \frac{\bar{M}}{x}\}) - \nu] \tag{11}$$

$$V_a \geq u'[r\bar{a} + y - x(1 - \kappa(\nu) \cdot \min\{1, \frac{\bar{M}}{x}\}) - \nu] \tag{12}$$

Space discretization: discretize $(\ln a, \epsilon, h, \nu, \delta, t)$ as $a_i, i = 1, ..., I$ with step size $\Delta a$; $\epsilon_j, j = 1, ..., J$ with step size $\Delta\epsilon$; $h_k, k = 1, ..., K$ with step size $\Delta h$; $t^n, n = 1, ..., N$ with period length $\Delta t$; $\nu_m, m = 1, ..., M$ with step size $\Delta\nu$; $t^n, n = 1, ..., N$ with period length $\Delta t$; $\delta_d$ as simply 0 or 1; $t^n, n = 1, ..., N$ with period length $\Delta t$; and use notation $v^n_{ijkmd} = V(a_i, \epsilon_j, h_k, \nu_m, \delta_d, t^n)$. We also discretize the retirement income into $y^R_l, l = 1, ..., L$ with step size $\Delta y$;

FOC discretization: value function derivatives are evaluated in period $t + 1$ and control variables are in period $t$, as in Achdou et al(2022).

HJB discretization: HJB equation is discretized as

$$\begin{aligned}
\rho v^n_{ijkmd} = \ln c^n_{ijkmd} + \beta \ln h^n_k &+ \partial^F_a v^n_{ijkmd}\big[\mu^{FF;n+1}_{a;ijkmd} I^{FF;n+1}_{ijkmd} + \mu^{FB;n+1}_{a;ijkmd} I^{FB;n+1}_{ijkmd}\big] \\
&+ \partial^B_a v^n_{ijkmd}\big[\mu^{BF;n+1}_{a;ijkmd} I^{BF;n+1}_{ijkmd} + \mu^{BB;n+1}_{a;ijkmd} I^{BB;n+1}_{ijkmd}\big] \\
&+ \partial^F_h v^n_{ijkmd}\big[\mu^{FF;n+1}_{h;ijkmd} I^{FF;n+1}_{ijkmd} + \mu^{BF;n+1}_{h;ijkmd} I^{BF;n+1}_{ijkmd}\big] \\
&+ \partial^B_h v^n_{ijkmd}\big[\mu^{FB;n+1}_{h;ijkmd} I^{FB;n+1}_{ijkmd} + \mu^{BB;n+1}_{h;ijkmd} I^{BB;n+1}_{ijkmd}\big] \\
&+ \partial^F_\nu v^n_{ijkmd}\big[\mu^{FF;n+1}_{\nu;ijkmd} I^{FF;n+1}_{ijkmd} + \mu^{BF;n+1}_{\nu;ijkmd} I^{BF;n+1}_{ijkmd}\big] \\
&+ \partial^B_\nu v^n_{ijkmd}\big[\mu^{FB;n+1}_{\nu;ijkmd} I^{FB;n+1}_{ijkmd} + \mu^{BB;n+1}_{\nu;ijkmd} I^{BB;n+1}_{ijkmd}\big] \\
&+ \eta\big[\partial^F_\epsilon v^n_{ijkmd}\big] + \frac{\sigma^2}{2}\partial_{\epsilon\epsilon} v^n_{ijkmd} \\
&+ (\lambda - \delta_d)\big[v^n_{ijkm}(\delta_d = 1) - v^n_{ijkm}(\delta_d = 0)\big] + \frac{v^{n+1}_{ijkm} - v^n_{ijkm}}{\Delta t}
\end{aligned}$$

where $F$ denotes the forward difference approximation and $B$ denotes the backward difference approximation; more precisely, for example, for $a$

$$\partial_a^F v_{ijkmd}^n = \frac{v_{i+1,jkmd}^n - v_{ijkmd}^n}{\Delta a},$$

$$\partial_a^B v_{ijkmd}^n = \frac{v_{ijkmd}^n - v_{i-1,jkmd}^n}{\Delta a},$$

for $\epsilon$

$$\partial_\epsilon^F v_{ijkmd}^n = \frac{v_{i,j+1,kmd}^n - v_{ijkmd}^n}{\Delta \epsilon},$$

$$\partial_\epsilon^B v_{ijkmd}^n = \frac{v_{ijkmd}^n - v_{i,j-1,kmd}^n}{\Delta \epsilon},$$

and for $h$

$$\partial_h^F v_{ijkmd}^n = \frac{v_{ij,k+1,md}^n - v_{ijkmd}^n}{\Delta h},$$

$$\partial_h^B v_{ijkmd}^n = \frac{v_{ijkmd}^n - v_{ij,k-1,md}^n}{\Delta h},$$

and

$$\partial_{\epsilon\epsilon} v_{ijkmd}^n = \frac{v_{i,j+1,kmd}^n - 2v_{ijkmd}^n + v_{i,j-1,kmd}^n}{(\Delta \epsilon)^2};$$

and $\mu_{a;ijkmd}^{FB;n+1}$ denotes the drift term for the state variable $a$ computed using the forward difference in the $a$ dimension and the backward difference in the $h$ dimension (note that drift terms depend on control variables, and the superscript $n+1$ indicates that control variables are computed with value function derivatives in period $t+1$), e.g.

$$\mu_{a;ijkmd}^{FB;n+1} = ra + y - c - x(1 - \kappa(\nu) \cdot \min\{1, \frac{\bar{M}}{x}\}) - \nu$$

where $c$, $x$, and $\nu$ are computed from FOCs as functions of $\partial_a^F v_{ijkmd}^{n+1}$ and $\partial_h^B v_{ijkmd}^{n+1}$ defined above; similarly

$$\mu_{h;ijkmd}^{BF;n+1} = \phi(n) x^\alpha h_k^{1-\alpha} - \left[\psi \underline{\delta}(n) + \psi \delta_d\right] h_k^\mu$$

where $c$, $x$, and $\nu$ are again computed from FOCs as functions of $\partial_a^B v_{ijkmd}^{n+1}$ and $\partial_h^F v_{ijkmd}^{n+1}$ defined above; and the indicator functions pick the combination of forward/backward differences consistent with the upwind scheme,

$$I_{ijkmd}^{FF;n+1} = \begin{cases} 1 & \text{if } \mu_{a;ijkmd}^{FF;n+1} > 0 \text{ and } \mu_{h;ijkmd}^{FF;n+1} > 0 \\ 0 & \text{otherwise} \end{cases};$$

$$I_{ijkmd}^{FB;n+1} = \begin{cases} 1 & \text{if } \mu_{a;ijkmd}^{FB;n+1} > 0 \text{ and } \mu_{h;ijkmd}^{FB;n+1} \leq 0 \\ 0 & \text{otherwise} \end{cases};$$

$$I_{ijkmd}^{BF;n+1} = \begin{cases} 1 & \text{if } \mu_{a;ijkmd}^{BF;n+1} \leq 0 \text{ and } \mu_{h;ijkmd}^{BF;n+1} > 0 \\ 0 & \text{otherwise} \end{cases};$$

$$I_{ijkmd}^{BB;n+1} = \begin{cases} 1 & \text{if } \mu_{a;ijkmd}^{BB;n+1} \leq 0 \text{ and } \mu_{h;ijkmd}^{BB;n+1} \leq 0 \\ 0 & \text{otherwise} \end{cases}.$$

Collecting terms on the discretized HJB, we have a system of difference equations

$$\rho \mathbf{v}^n = \mathbf{u}(\mathbf{v}^{n+1}) + A(\mathbf{v}^{n+1})\mathbf{v}^n + \frac{\mathbf{v}^{n+1} - \mathbf{v}^n}{\Delta t} \tag{13}$$

We solve the model backwards along $t$, starting with period $\bar{T}$ with the terminal condition

$$V(a,t) = e^{-\rho(t-t_0)} q_0 \ln(1 + \frac{a}{q_1}).$$

For each period $t = n+1$, firstly we solve for optimal $c$, $x$ and $s$ from the FOCs. Then compute the matrix $A(\mathbf{v}^{n+1})$ in the discretized HJB equation (13). And lastly, solve for the value function in the previous period $\mathbf{v}^n$. For $t > R$, since $y^R$ is not yet determined we add $y^R$ as an auxiliary state variable, and the value function is solved for each $y_l^R$ as well. At $t = R$ we compute the exact $y^R$ and then match it with the corresponding $y_l^R$.

# 6 Estimation and Calibration

Parameters are either borrowed from other researches, directly estimated from data, or calibrated in the model to match real-life targets. Datasets include HRS, SCF, MEPS, PSID, Disease-Based Price Index (from BLS), Health Care Satellite Account (from BEA), CDC data

Parameters directly estimated from data:

- On-the-job learning Mincerian equation:$\zeta = 127.26$, $b_1 = 0.0451$, $b_2 = -0.00089$. Calibrate to match lifetime income profile, Guvenen 2015.
- Diffusion income process: $\eta = 0.005$, $\sigma = 0.1037$. Control age and demographic heterogeneity to compute income residual and then fit into the Wiener process.
- Demographic heterogeneity in income: $\theta$. Allow heterogeneity in gender, race, education. Estimate from PSID labor income data.
- $\underline{h} = 26.61$: assume annual mortality rate to be inverse to health capital and death a Poisson process, and match life expectancy.
- Poisson intensity of disease process: $\lambda = 0.0572$. Estimate from incidence rates and death rates of 9 leading death-causing diseases. We firstly compute the weighted average death rate of these vital diseases and use the

relative death rates (relative to the weighted average death rate) as weights to compute the weighted average incidence rate, which is then converted to the Poisson intensity.

| Diseases | Incidence | Death |
|---|---|---|
| Heart attack | 0.31% | 62.42% |
| Cancers | 0.45% | 8.61% |
| Chronic lower resp. | 2.80% | 0.75% |
| Stroke | 0.33% | 12.36% |
| Alzheimer | 0.20% | 1.12% |
| Diabetes | 0.62% | 0.25% |
| Pneumonia | 0.42% | 3.87% |
| Kidney | 0.05% | 5.27% |
| Septicemia | 0.71% | 1.61% |

Table 1: Incidence rates and death rates of the top nine death-causing diseases

- Maximum disease-related medical expense: $\bar{M} = 9072$. Estimate from DBPI and HCSA. Similar to the estimation of $\lambda$.

| Diseases | TC($,1999) | DBPI(2004) | TC($,2004) |
|---|---|---|---|
| Heart attack | 10250 | 1.145 | 11735 |
| Cancers | 7174 | 1.047 | 7512 |
| Chronic lower resp. | 430 | 1.570 | 675 |
| Stroke | 5372 | 1.495 | 8032 |
| Alzheimer | 1022 | 1.127 | 1152 |
| Diabetes | 1424 | 0.9334 | 3217 |
| Pneumonia | 2244 | 1.434 | 3217 |
| Kidney | 1558 | 2.410 | 3754 |
| Septicemia | 541 | 1.918 | 1038 |

Table 2: Treatment costs of the top nine death-causing diseases

- Insurance coverage function: $\kappa(\nu)$. Estimate from MEPS data.

$\rho$ is set to 0.98. The bequest parameter $q_1$ is set to 11.6 as in De Nardi 2004. $r$ is set to 0.03.

The model calibrate the following parameters $\beta$ (health preference), $q_0$ (bequest), $\phi_0$, $B_1$, $\upsilon_1$ (health investment efficacy), $\underline{\delta}_0$, $B_2$, $\upsilon_2$ (health natural deterioration), $\alpha$ (return to scale in health investment), $\mu$ (health curvature), and $\psi$ (magnitude of disease), to match the following targets: the consumption-income ratio, the out-of-pocket medical expense rate, the OOP-income ratio, the life expectancy distribution ($0-60\%$, $60-90\%$, $90-99\%$, top $1\%$), and the OOP expense distribution ($0-60\%$, $60-90\%$, $90-99\%$, top $1\%$).

# 7 Quantitative Results

## 7.1 Targets Matching

Overall speaking, the majority of targets are well matched. However the benchmark model with the calibrated parameters appear to be somewhat too harsh on the poor while motivate the rich to invest too much on their health. The over-investment of the rich on health drives up their out-of-pocket expense and the overall out-of-pocket expense as well. This allows the rich to live longer than the data shows while the poor's life expectancy is slightly lower than reality.

| Target | $c/y$ | $cov.rate$ | $oop/y$ |
|---|---|---|---|
| Data | 0.42-0.61 | 0.75 | 0.075 |
| Calibration | 0.45 | 0.72 | 0.091 |

Table 3: Target matching results for ratios.

| wealth bins | $0-60\%$ | $60-90\%$ | $90-99\%$ | top $1\%$ |
|---|---|---|---|---|
| Data | 75.7 | 81.0 | 83.2 | 83.9 |
| Calibration | 72.1 | 76.5 | 85.2 | 87.0 |

Table 4: Target matching results for the life expectancy distribution.

| Wealth bins | $60-90\%$ | $90-99\%$ | top $1\%$ |
|---|---|---|---|
| Age 86-90 | 1.25 | 1.69 | 3.03 |
| Calibration | 1.25 | 1.82 | 3.40 |
| Age 56-60 | 1.29 | 1.80 | 2.26 |
| Calibration | 1.33 | 2.07 | 4.16 |

Table 5: Target matching results for the out-of-pocket expense distribution (over the value of the $0-60\%$ wealth group).

## 7.2 Wealth Distribution

The simulation result matches the data well except for the very wealthy and old groups. The model generates severe wealth inequality but tends to underestimate wealth shares of the top $1\%$ while overestimate those of the $0-60\%$ $60-90\%$ and $90-99\%$ groups. This kind of tendency exacerbates with age. The model over-motivates the rich to invest in their health which generates too much out-of-pocket expenses and in turn reduces their wealth. This may be because of the model does not incorporate public healthcare programs such Medicaid and Medicare which may greatly reduce out-of-pocket health expenditures for the elderly.

| wealth \ age (in percentage) | data 61-69 | model 61-69 | data 71-79 | model 71-79 | data 81-89 | model 81-89 |
|---|---|---|---|---|---|---|
| $0-60\%$ | 8.2 | 4.9 | 10.2 | 12.6 | 11.1 | 26.3 |
| $60-90\%$ | 25.7 | 33.4 | 26.0 | 31.4 | 26.7 | 37 |
| $90-99\%$ | 37.7 | 43.7 | 34.8 | 35.1 | 41.0 | 27.5 |
| top 1% | 28.4 | 18.0 | 29 | 15 | 29.8 | 9.2 |

Table 6: Comparison of simulated wealth distribution with data on various age cohorts and wealth groups

### 7.3 Wealth distribution: Counterfactual analysis

We conduct the counterfactual analysis by replacing the Poisson disease process with a constant health deterioration ($\delta = \lambda$). The result shows that the model with the income process and demographic heterogeneity alone cannot generate severe wealth inequality, especially the wealth shares of the rich.

| wealth \ age (in percentage) | CF 61-69 | model 61-69 | CF 71-79 | model 71-79 | CF 81-89 | model 81-89 |
|---|---|---|---|---|---|---|
| $0-60\%$ | 2.1 | 4.9 | 10.3 | 12.6 | 33.6 | 26.3 |
| $60-90\%$ | 52 | 33.4 | 47.1 | 31.4 | 46.5 | 37 |
| $90-99\%$ | 38.5 | 43.7 | 36.4 | 35.1 | 17.8 | 27.5 |
| top 1% | 7.4 | 18.0 | 6.2 | 15 | 2.1 | 9.2 |

Table 7: Comparison of scounterfactual wealth distribution with benchmark results on various age cohorts and wealth groups

## 8 A Lifecycle Model with Public Insurance

This section incorporates Medicare in the benchmark model. [TO BE UPDATED]

## 9 Concluding Remarks

In this paper, we have constructed a rich lifecycle model of endogenous health capital, insurance premium and wealth accumulation, with a diffusion income process and Poisson health shocks. We have established that rich people tend to invest more in health, live longer, and save more. As a consequence, vital health shocks have been found to facilitate top concentration in wealth distribution. We have then calibrated the model to fit the US medical and economic data. While we have shown our baseline model to fit well in predicting of the share of

wealth for the 5%, we have also confirmed the quantitative importance of vital health shocks for top wealth inequalities.

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