The Diminishing Output Elasticity of Labor

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May 16, 2024
Preliminary. Please do not circulate.

Abstract

We establish a novel property of production functions and a new fact about firm growth: the *diminishing output elasticity of labor*. As a firm grows, its elasticity of output to labor falls, with an offsetting shift into materials. Using firm-level micro data and OLS and IV regressions, we estimate this "super-elasticity" to be -0.30 in the short run (one-year horizon), -0.20 to -0.14 in the medium run (4-10 years), and -0.03 in the long run (cross section). We find similar patterns for cost shares, with a reduced labor share and an increase in intermediate inputs. The diminishing output elasticity of labor provides a novel, technological explanation for the negative association between the labor share and firm size, complementing existing approaches that focus on product or labor market power. Using micro-aggregated data for 20 European countries, we show that these firm-level patterns extend to aggregate industry growth. Strikingly, only growing industries display declining labor shares, which suggests an important role for the diminishing output elasticity of labor in driving aggregate labor share dynamics.

We thank Michael Rößner and Denise Henker for their invaluable support with the data. We are grateful to all data-providing institutions, funding members of the CompNet project, and members of the CompNet team.

1 Introduction

We establish and dissect a new property of production functions: the *diminishing output elasticity of labor*, i.e., the elasticity of output to labor declines in output. This systematic relationship can be well described by a "super-elasticity" (the elasticity of the labor-output elasticity to output). In firm-level panel data, we estimate this super-elasticity to be -0.38 in the short run. The longer the horizon, the smaller this elasticity in magnitude. It falls to -0.2 to -0.14 in the medium run (4-10 years); it settles at -0.03 in the cross-section (i.e., the long run). Many standard approaches restrict the labor-output elasticity to be constant; and the studies that do permit it to vary across time or units of observations have not established systematic patterns, and specifically not the regularity we uncover.

Our primary strategy estimates firm- and time-specific labor output elasticities in firm-level data on German manufacturing firms.[1] We find similar effects of output growth on the labor-output elasticity in OLS regressions as well as when estimating *causal* effects of firm growth, by instrumenting firm-level output growth with foreign export demand shifts.

The diminishing labor-output elasticity is accompanied by a tantamount increase in the materials elasticity, rather than increases in the capital-output elasticity or decreases in returns to scale. In fact, we also find some evidence for a reduction in capital output elasticities. Hence, as firms grow, they shift their production toward intermediates rather than in-house production.

Complementing our analysis of structurally estimated production function parameters, we study firms' factor mix directly. We find similar patterns for cost shares and input quantities: firm growth is associated with lower labor shares and a shift toward intermediate inputs. Notably, this additional analysis is based on nonparametric relationships in the raw data that do not require specifying a production function.

Potential microfoundations for the diminishing output elasticity of labor include technological innovations or firms' selecting from a menu of production modes that changes as firms grow (e.g., due to fixed costs of certain technologies or changes in relative factor prices - see Sections 2 and 5). By contrast, we argue that conventional adjustment cost for labor vis-á-vis capital or intermediates cannot explain this property. In our study, we do not derive a theoretical model but focus on documenting this production function property in the data.

We then draw one immediate implication for firm-level labor shares. Basic economic theory of profit maximization pins down the (log) labor share as the output elasticity of labor minus markups and markdowns (all in logs). Hence, we can precisely decompose the labor share-firm size gradient into those components. We find that the diminishing output

[1]The dataset contains firm-specific price information and thus permits us to address the price biases in the estimation of output elasticities, discussed in De Loecker et al. (2016), Bond et al. (2021), and De Ridder et al. (2022).

elasticity of labor accounts for *about half* of the negative effect of firm growth on the labor share. Hence, the diminishing output elasticity of labor provides a novel, technological explanation for the negative association between the labor share and firm size, complementing existing approaches that focus on large firms' product or labor market power.

As one aggregation implication, our firm-level findings point to declining labor shares as by-products of growth. Indeed, we confirm the industry-level analog of the negative association between output growth and labor shares for 20 European countries in harmonized manufacturing data from the Competitiveness Research Network (CompNet) covering 1999 to 2021. Importantly, the declining output elasticity can fully account for this pattern, while markup or wage markdown growth cannot. Finally, we show that, in Europe, on average, *only growing* industries experienced a decline in labor shares. Simultaneously, these industries see much stronger declines in labor output elasticities, weaker increases in markups, and stronger declines in wage markdowns than shrinking industries.[2] These patterns suggests that the diminishing output elasticity is the key factor behind recent declines in European labor shares. Although we cannot estimate output elasticities, markups, and wage markdowns for the US, we document from publicly available US data ranging from 1958 to 2018 that labor shares declined much more significantly in growing industries than in shrinking industries. These patterns corroborate our conclusions based on European data and are consistent with a key role for the diminishing output elasticity of labor in explaining declining labor shares also in the US.

The paper is organized as follows. Section 2 reviews related work. Section 3 introduces the key concepts in a general production model. Section 4 uses German firm-level data to establish the diminishing output elasticity of labor. Section 5 reviews potential mechanisms that can rationalize our firm-level results. Section 6 discusses a production model that connects the diminishing labor output elasticity and other factors to changes in labor shares. Section 7 present our European results highlighting a key role for the diminishing output elasticity of labor in driving aggregate labor share changes. Section 8 concludes.

2 Related Literature

Most closely related to our paper is Mertens (2022), who documents a decline in labor output elasticities over time for Germany. Our work differs from this study as we focus on the systematic and causal relationship between output growth and the labor output elasticity and establish the diminishing output elasticity of labor as a regularity. We also show that this regularity is accompanied by a shift of firms' input mix toward intermediate inputs.

[2]It is not surprising that growing industries display lower markup growth and stronger declines in wage markdowns, as higher markups and wage markdowns incentivize firms to decrease their output.

Additionally, we extend this analysis to a large set of European countries, for which evidence on changes in labor output elasticities and their connection to labor shares does not exist yet.

More broadly, our study complements work showing that estimates of the substitution elasticity between production factors are inconsistent with a Cobb-Douglas production model (e.g., Chirinko et al., 2011, Raval, 2019). A new insight that we highlight is that firm- and industry-level factor elasticities systematically *change* as output grows, which is also inconsistent with a Cobb-Douglas model. Consequently, our findings also have implications for several research strands that rely on constant output elasticities as in a Cobb-Douglas model. This includes work on factor misallocation (e.g., Hsieh and Klenow, 2009, Petrin and Sivadasan, 2013) and firm-level and aggregate productivity (e.g., Brynjolfsson and Hitt, 2003, Petrin and Levinsohn, 2012,Konings and Vanormelingen, 2015, Andrews et al., 2016, Berlingieri et al., 2018,Czarnitzki et al., 2023, Berlingieri et al., 2024). Studying these and other applications is beyond the scope of this paper and left for future work.

We further relate to work studying factor substitution. Particularly relevant for us are Huneeus et al. (2021) and Dhyne et al. (2022), who study labor input and wage adjustments in context of demand shocks in production networks. Both studies do not specifically focus on changes in output elasticities, but provide insights that complement our findings. Dhyne et al. (2022) show that labor adjustments to demand shocks are weaker in the short- than in the long-run. While we confirm this finding, we highlight the role of changing production technologies in shaping firms' input mix. Huneeus et al. (2021) show that firms' monopsony power and relative factor prices can rationalize why growing firms substitute towards non-labor inputs. This mechanism could potentially serve as a micro-foundation for the declining output elasticity of labor as firms' decision about their production technology (output elasticities) is shaped by relative factor prices. In this context, also other studies explore the conditions under which relative factor prices and relative factor supply induce firms to direct their innovation toward more or less labor-intensive production processes (e.g., Hicks, 1932, Zeira, 1998, Acemoglu, 2002; 2003, Rubens, 2022).

Besides this work, there is a significant amount of studies that document factor substitution in response to firm-specific shocks. For example, Acemoglu and Restrepo (2020), Dauth et al. (2021), and Deng et al. (2023) study substitution of labor with robots. Similarly, Hubmer and Restrepo (2021) use Compustat data and show that capital-output elasticities have increased in the largest Compustat firms in the most recent years, consistent with automation. Changes in output elasticities with firm growth, such as we document, can help rationalize these existing findings from a production function perspective.

Finally, our paper relates to the large body of work that debates the mechanisms behind declining labor shares. Karabarbounis and Neiman (2014) and Lashkari et al. (2023) discuss the role of declining capital and IT prices. Elsby et al. (2013) highlight the offshoring of

domestic production. De Loecker et al. (2020) and Autor et al. (2020) emphasize rising product markups and concentration, while Mertens (2022) compares the relative importance of product and factor market power vs. changes in labor output elasticities (Autor et al., 2020 additionally discuss the role of overhead labor costs). Koh et al. (2020) and Gutiérrez and Piton (2020) argue that measurement error is key in understanding declining labor shares, and Acemoglu and Restrepo (2019) and Hubmer and Restrepo (2021) emphasize the importance of capital technologies. Our results on a key role of changing output elasticities in shaping labor share patterns in Europe support the production technology-based explanations for the recent decline in Europe's labor share rather than changes in firms' market power. This perspective is also in line with recent evidence by Harrison (2024) showing that technological explanations as proxied by R&D investment and productivity growth exert a much stronger negative effect on labor shares than proxies for market power in many countries.

3 Formal Definitions

Production function. Firm i produces output, Q_{it}, in period t using the production function:

$$Q_{it} = F(L_{it}, K_{it}, M_{it}, \Omega_{it}). \tag{1}$$

L_{it}, M_{it}, and K_{it} denote labor, intermediate, and capital inputs. Ω_{it} is Hicks-neutral total factor productivity (TFP).[3] We do not restrict the functional form of the production function and only require it to be continuous and twice differentiable.

The labor-output *super*-elasticity. The output elasticity of labor is a property of a production function given by the derivative of log output with respect to log labor:

$$\theta^L_{it} = \frac{\partial \ln Q_{it}}{\partial \ln L_{it}}. \tag{2}$$

Our empirical analysis will establish that θ^L depends on firms' output level. While we will nonparametrically trace out the relationship between θ^L_{it} and Q_{it}, we will find that it can be described well by a super-elasticity that is constant across growth magnitudes for a given time horizon (but shrinks over longer time periods):

Definition: *The labor-output super-elasticity is the elasticity of the labor-output elasticity*

[3]Hicks-netraulity is a strong but common assumption in the empirical production function literature. When estimating output elasticities, we cannot identify non-Hicks-neutral productivity without imposing perfectly flexible and competitive labor markets (see for instance Doraszelski and Jaumandreu, 2018 and Demirer, 2020). We therefore will rely on Hicks-neutrality when estimating output elasticities and complement our results with additional evidence on input cost share adjustments that confirms our key mechanisms.

to output:

$$\sigma^L = \frac{\partial \ln \theta_{it}^L}{\partial \ln Q_{it}}. \tag{3}$$

Diminishing output elasticity of labor. We define the following property:

> ***Definition:*** *A production function exhibits a diminishing output elasticity of labor if θ_{it}^L falls in output, Q_{it}, i.e., if the labor-output super-elasticity, σ^L, is negative.*

4 Firm-Level Evidence

We now establish the diminishing output elasticity of labor in rich firm-level German manufacturing data. We describe the data, the production function estimation, and the results.

4.1 Data

Production data. The firm-product-level data for Germany's manufacturing sector covers the period 1995-2017. The data is collected and supplied by the German Statistical Offices.[4] Variables include, among others, sales, employment, investment, intermediate input costs, wage bills, depreciation, and, most importantly, *product quantities and prices at a ten-digit product classification*.[5] The data cover 40% of firms with at least 20 employees and consists of a rotating panel that is drawn anew every 4-5 years. Labor is defined as the number of employees on the 30th September. All other variables pertain to the full calendar year. We clean and prepare the data following Mertens (2022) and provide more details on data preparation, capital stock construction, variable definitions, and summary statistics in online Appendix D.[6]

Supplementary trade data. To provide *causal* evidence on the relationship between output growth and our variables of interest, we merge bilateral trade flows from the United Nations Comtrade Database on the firm-product-year level to the German micro data. The database contains bilateral trade flows in sales between any country at the country-year-product level from 1995 to 2017. We translate product codes in both datasets into the PRODCOM2002 classification using official concordance tables following the Stata code of Bräuer et al. (2023). As described below, we will apply the IV-approach in Hummels et al. (2014) to instrument

[4]Data source: RDC of the Federal Statistical Office and Statistical Offices of the Federal States, DOI: 10.21242/42131.2017.00.03.1.1.0, 10.21242/42221.2018.00.01.1.1.0, and 10.21242/42111.2018.00.01.1.1.0.

[5]Examples of products are "Tin sheets and tapes, thicker than 0.2mm" or "Workwear - long trousers for men, cotton".

[6]The data has been used in various studies, e.g., Mertens, 2020; 2022; 2023, Mertens and Müller, 2022, Haelbig et al., 2023, Mertens et al., 2022, Bräuer et al., 2023, and Bighelli, 2023.

output changes with foreign export demand to study how output elasticities change in response to exogenous output changes.

4.2 Production Function Estimation

The definition of output elasticities depends on the functional form of the production function (Equation (1)). To allow for time-varying and firm-specific output elasticities, we rely on the following translog production function, where lower-case letters denote logs:

$$\begin{aligned} q_{it} = \beta_l l_{it} + \beta_k k_{it} + \beta_m m_{it} + \beta_{l2} l_{it}^2 + \beta_{k2} k_{it}^2 + \beta_{m2} m_{it}^2 + \\ \beta_{lk} l_{it} k_{it} + \beta_{lm} l_{it} m_{it} + \beta_{km} k_{it} m_{it} + \beta_{lkm} l_{it} k_{it} m_{it} + \omega_{it} + \epsilon_{it}, \end{aligned} \tag{4}$$

where ϵ_{it} is an i.i.d. error term.

We detail our production function estimation approach in online Appendix E and summarize it in the following. The literature has extensively discussed that Equation (4) cannot be directly estimated by OLS because firms' input decisions for flexible inputs are correlated with unobserved productivity (Olley and Pakes, 1996). Additionally, unobserved input and output prices introduce additional biases when estimating production functions (De Loecker et al., 2016, Bond et al., 2021). Fortunately, we can apply a correction for these price biases using our firm-product level price data. Specifically, we control for output price variation by constructing a firm-specific price index from our firm-product-level data as in Eslava et al. (2004). To control for input price variation, we apply a firm-level adaptation of the approach in De Loecker et al. (2016) and formulate an input price control function from information on firms' output prices and market shares. This relies on the positive correlation between output and input prices (e.g., due to high quality outputs requiring high quality inputs). To account for the dependence of input decisions on productivity, we utilize a control function approach similar to Wooldridge (2009). The implementation of this approach with the German data closely follows previous work by Mertens (2022). The production function is estimated for each NACE Rev. 1.1 two-digit industry separately.

In a robustness check, we also discuss results using directly measured input cost shares in total costs instead of output elasticities from an estimated production function (cost shares equal firms' output elasticities under perfect competition, constant returns to scale, and perfect input flexibility).

Summary statistics and sample. We provide summary statistics of our sample in Appendix Table A.1.

4.3 OLS Strategy: The Correlation of Firm Growth and the Labor-Output Elasticity

Strategy. For each firm i in year t, we estimate an OLS regressions in differences of h years of the form:

$$\Delta^h \ln(\theta_{it}^L) = \sigma^h \Delta^h \ln Y_{it} + \upsilon_{jt} + \epsilon_{it}. \tag{5}$$

Y_{it} is deflated sales, and υ_{jt} captures industry (j) times year (t) fixed effects.[7] ϵ_{it} is an error term. We cluster standard errors at the firm level in our panel data (with each firm being observed multiple times).

The coefficient of interest is σ. It captures the percent effect of a one percent change in firms' output on the output elasticity of labor—hence identifying the (average) labor-output "super-elasticity." The difference specification accounts for unobserved constant firm characteristics.

In additional regressions mirroring Equation (5), we also consider additional dependent variables such as cost shares, markups/markdowns, or output elasticities of other factors.

Horizons h. We consider time differences ranging from one year up to ten years and subsequently implement a cross-sectional specification (mirroring Equation (5)) but in log levels rather than differences. Thus, we estimate the super-elasticity for the short, medium, and long run. While we interpret the cross-sectional regression as capturing the long run (as we cannot track even longer within-firm changes in our panel data), we acknowledge that this approach introduces confounders (permanent heterogeneity in production function parameters). Therefore, we focus mostly on the more cleanly identified regressions in changes for our analysis.

Regression results. Table 1 presents the OLS regression results. It does so for various outcomes across columns. The panels report results for the short run (one year), the medium run (4 and 10 years), and the long run (in the form of the cross-sectional regression described above). We normalize the output elasticities by the returns to scale parameter to account for changing returns to scale, but find similar results without this adjustment for changes (but not in the cross section). To illustrate that, Column (10) runs a specification with returns to scale as a dependent variable (Appendix Tables A.2 and A.3 reproduce our results without dividing output elasticities by returns to scale).

[7] In unreported robustness checks, we also ran specifications with firm effects to take out firm-specific trends. This specification leads to higher coefficient estimates, including at longer horizons, which is plausibly consistent with the remaining variation capturing more transitory fluctuations around the firm-specific trends.

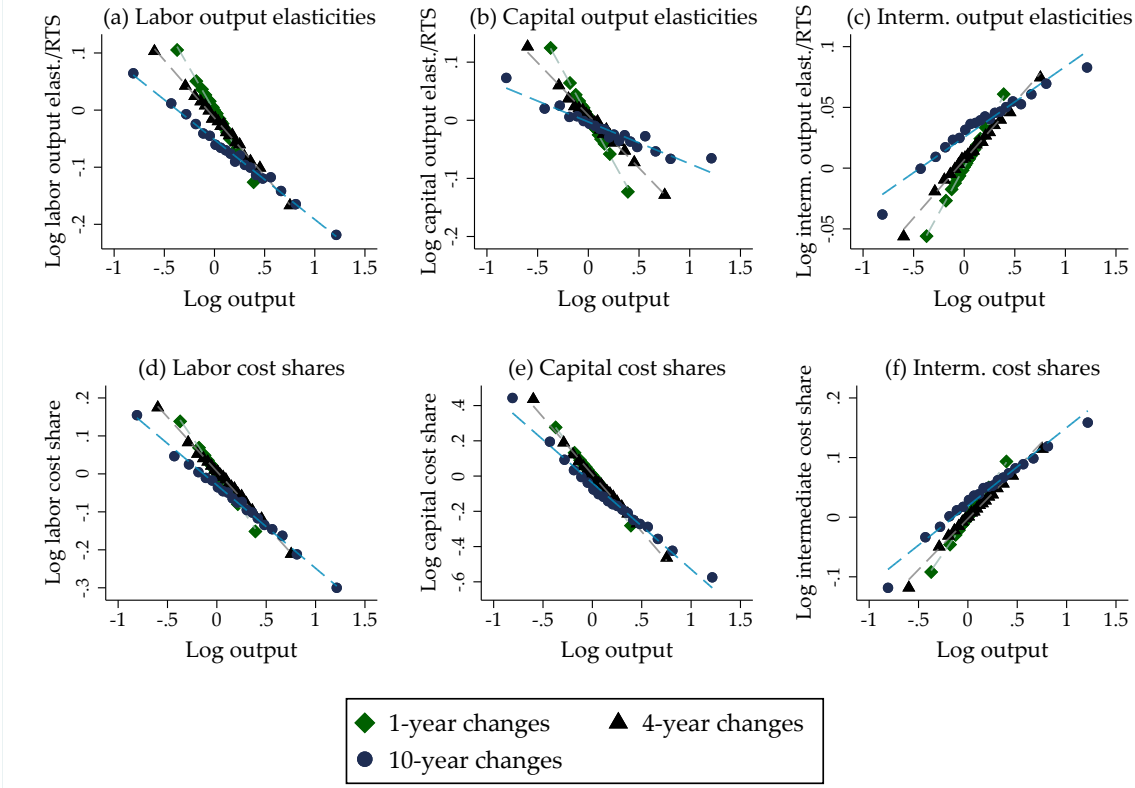
Complementary perspective: nonparametric scatterplots. As companion exhibits, Figure 1 visualizes key firm-level relationships from Table 1 as binned scatter plots. It does so for the 1, 4 and 10 year changes; Figure 2 reports binned scatter plots for the cross-sectional regression. The figures are residualized as in Equation (5).

Table 1: Firm-level adjustments in response to firm growth. OLS-regressions.

	$log(L_{it})$	$log(K_{it})$	$log(M_{it})$	$log(\frac{w_{it}L_{it}}{C_{it}})$	$log(\frac{r_{it}K_{it}}{C_{it}})$	$log(\frac{z_{it}M_{it}}{C_{it}})$	$log(\frac{\theta^L_{it}}{RTS_{it}})$	$log(\frac{\theta^K_{it}}{RTS_{it}})$	$log(\frac{\theta^M_{it}}{RTS_{it}})$	$log(RTS_{it})$
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output growth	0.298***	0.0405***	1.025***	-0.376***	-0.728***	0.238***	-0.304***	-0.315***	0.151***	0.0062***
	(0.0040)	(0.0022)	(0.0033)	(0.00393)	(0.0033)	(0.0022)	(0.005)	(0.0063)	(0.0015)	(0.0003)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950
R^2	0.215	0.043	0.741	0.343	0.556	0.340	0.230	0.137	0.325	0.053
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output growth	0.510***	0.206***	1.045***	-0.286***	-0.649***	0.171***	-0.202***	-0.181***	0.0951***	0.0167***
	(0.0064)	(0.0067)	(0.0045)	(0.0062)	(0.0071)	(0.0034)	(0.0060)	(0.0074)	(0.0025)	(0.0006)
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492
R^2	0.471	0.144	0.860	0.348	0.476	0.355	0.220	0.170	0.268	0.163
Panel C: 10-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output growth	0.606***	0.395***	1.041***	-0.219***	-0.490***	0.131***	-0.140***	-0.0720***	0.0582***	0.0278***
	(0.0066)	(0.0082)	(0.0043)	(0.0061)	(0.0083)	(0.0033)	(0.0053)	(0.0069)	(0.0024)	(0.0007)
Observations	49,915	49,915	49,915	49,915	49,915	49,915	49,915	49,915	49,915	49,915
N of firms	10,595	10,595	10,595	10,595	10,595	10,595	10,595	10,595	10,595	10,595
R^2	0.594	0.256	0.901	0.296	0.329	0.303	0.188	0.149	0.200	0.268
Panel D: levels, cross-section	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log output growth	0.741***	0.957***	1.062***	-0.128***	-0.0405***	0.0649***	-0.0345***	0.133***	-0.0116***	0.0498***
	(0.0024)	(0.0033)	(0.0010)	(0.00192)	(0.0031)	(0.0009)	(0.0016)	(0.0022)	(0.0007)	(0.0004)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950
R^2	0.895	0.827	0.983	0.483	0.225	0.413	0.512	0.406	0.417	0.836

Notes: The table reports OLS-regressions from estimating the specification in Equation (5). The dependent variables in Columns 1-10 are the logs of labor, capital, intermediates, labor cost shares, capital cost shares, intermediate cost shares, labor output elasticities over returns to scale, capital output elasticities over returns to scale, intermediate input output elasticities over returns to scale, and returns to scale, respectively. Panels A-C project changes in those dependent variables on changes in log output for 1-, 4- , and 10-year differences. Panel D runs a cross-sectional specification that regresses levels of the dependent variables on output levels. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm-level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Figure 1: The adjustment of production functions to output growth by horizon: output elasticities and cost shares



Notes: The figure reports binned scatter plots from estimating the specification in Equation (5) with OLS for various differences. The upper three graphs project changes in the logs of labor, capital, and intermediate output elasticities over returns to scale against log output changes, from left to right respectively. The bottom three graphs project changes in the logs of labor, capital, and intermediate cost shares against log output changes, from left to right respectively. All panels report results that are residualized by industry-year fixed effects. German firm-level data.

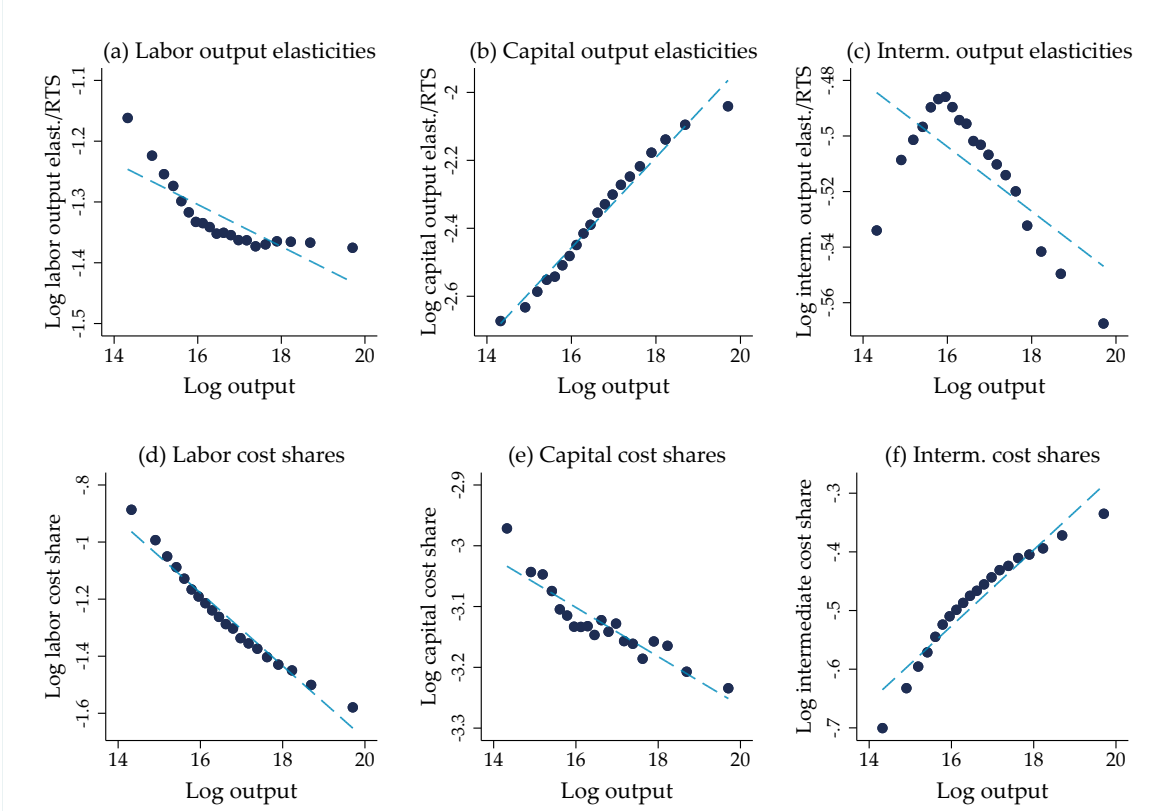
Main result: the diminishing output elasticity of labor. Across specifications, we find a strong negative association between output growth and changes in the labor output elasticity (Column (7)). At the one-year horizon, the estimate of σ equals -0.30. This effect is significantly different from zero, with a small standard error of 0.0064. That is, a 10 percent change in output reduces the labor output elasticity by 3%.

Visualizing the underlying variation, Figure 1, Panel (a) reveals a striking linear relationship between the output elasticity of labor and firm size, both again in log changes and after residualizing the two variables as in regression Equation (5). This linear relationship motivates our proposal of a constant super-elasticity σ. The scatter plots also support the linear functional form imposed in regression Equation (5) above.

Larger effects in the short run. Comparing horizons across the panels highlights interesting dynamics in this pattern: the labor-output super-elasticity drops to -0.14 and -0.20 in the

medium run, in 4 and 10 year horizons, respectively. In fact, in the long run, in the form of the cross-sectional regression, the effect falls to -0.03, again relatively precisely estimated. Turning to the cross-sectional scatterplot in Figure 2, we see a slightly concave pattern, consistent with the fact that the large shorter-run elasticities do not extend to large cross-sectional variation in firm size, which is right-skewed. The figure also makes clear that some of this behavior appears to be driven by the returns to scale-size gradient, confirmed also in Column (10) of Table 1, which plays a larger role in the cross section than even in 10-year changes. This is precisely why it is important to account for variation in returns to scale in our regressions. The strong evidence for horizon-dependency could be interpreted as a result of short-run imperfections in technological adjustments or different kinds of shocks by horizon. Section 5 further contrasts the role of input adjustments costs vs. technological adjustments in context of our results.

Figure 2: The cross-sectional relationships: output elasticities and cost shares



Notes: The figure reports binned scatter plots estimating the specification in Equation (5) in levels with OLS. The upper three graphs project the logs of labor, capital, and intermediate output elasticities over returns to scale against log output, from left to right respectively. The bottom three graphs project the logs of labor, capital, and intermediate cost shares against log output, from left to right respectively. All panels report results that are residualized by industry-year fixed effects. German firm-level data.

A shift towards materials. Overall, the shift away from labor is accompanied by a shift toward intermediates (Column (9) of Table 1). Interestingly, the effect is even accompanied

by a reduction in capital shares (Column (8)). Note, however, that the coefficients capture percent changes and that the capital share is smaller than the labor share and the intermediate input share. In this context, Column (10) shows that returns to scale are almost unchanged in the short run and only change more significantly over longer horizons, implying that changes in intermediate output elasticities fully compensate changes in labor and capital output elasticities in the short run.[8] Finally, the cross-sectional regressions show consistent patterns for the output elasticity of labor (a negative coefficient but much attenuated compared to the long run), but exhibit a positive effect for the capital elasticity. However, as mentioned above, the cross-sectional regression is a rough approximation for long-run changes that potentially confound several other factors with our main effects. For instance, for this specification, the returns to scale parameter exhibits a larger effect, dominating the output elasticity effects for labor and materials (Appendix Tables A.2 and A.3 present results without dividing output elasticities by returns to scale).

Raw input use and cost shares. Columns (1)-(6) report raw data on input use and cost shares for labor, capital, and materials. We define input cost shares as input expenditures divided by total costs (C_{it}), which is the sum of input expenditures. Labor, capital, and intermediate cost shares are denoted by $\frac{w_{it}L_{it}}{C_{it}}$, $\frac{r_{it}K_{it}}{C_{it}}$, and $\frac{z_{it}M_{it}}{C_{it}}$, respectively.[9] Consistent with a shift away from labor (and to capital), we see a large increase in intermediate use as measured by its costs share. In other words, firms outsource more of its production process, with value added over total value of shipments declining.[10] We note that for this specification, effects on capital are negative too even for the cross section, with a shift from labor (and capital) into intermediate input costs.

4.4 Causal Effects: IV Strategy using Export Demand

We now move beyond OLS relationships to identify the *causal* effect of firm growth on input intensities and production function output elasticities.

Strategy. To address endogeneity concerns, we implement an instrumental variables (IV) approach. We follow an established literature using trade-shocks as exogenous shifter (see Autor et al., 2016 for a review). In particular, we follow Hummels et al. (2014) and instrument changes in firms' output with changes in world export demand (excluding Germany). We

[8] Appendix Tables A.4 and A.5 reproduce our results without taking logs for dependent variables and indeed show that changes in output elasticities of labor and capital sum up to changes in intermediate input output elasticities in the short run.

[9] We approximate capital costs as 8 percent of the capital stock.

[10] Again, short-run changes in labor and capital cost shares add up to short-run changes in intermediate cost shares (appendix Tables A.4 and A.5).

first compute the total exports for each product, g, from a country group, n, to the world:

$$EX_{gt} = \sum_{c} ex_{gct}^{n \to world}, \tag{6}$$

where n contains Australia, Norway, Sweden, Singpore, New Zealand, Great Britain, Canada, Japan, and the US. c denotes the country. Our country selection follows Dauth et al. (2014), with the exception that we additionally include the US (results are robust when excluding the US and using the same countries as in Dauth et al. (2014)). There are two reasons for this selection. First, we choose other industrialized countries as their wages and thus specialization in types of export goods is arguably more similiar to Germany compared to developing/low-wage countries. Second, this selection helps mitigate potential endogeneity concerns arising from unobserved shocks that might be correlated between Germany and other nations. To avoid this, we exclude Germany's direct neighbors and countries sharing the same currency as Germany from the instrument set.

Equation (6) is a product-level measure. To compute firm-specific export demand shocks, we calculate weighted averages of product-level trade flows, using the sales shares of products within firms' product portfolios as weights:

$$INS_{it} = \sum_{g} s_{git=0} \ln EX_{gt}, \tag{7}$$

where s_{git} denotes the firm-specific sales weight of product g. To limit anticipatory effects, we fix the weights for each firm to its first year of observations. Having constructed these firm-specific instruments, we instrument $\Delta \ln Y_{it}$ in Equation (5) with ΔINS_{it}, i.e., we use export demand shocks to instrument output changes.

Limited horizons. As the instrument is in changes, we cannot provide cross-sectional results. Moreover, our first stage is not sufficiently strong to study 10-year differences. We therefore restrict our IV analysis to one- and four-year differences.

First stage. Table 2 reports the IV results. Column (1) shows the first stage coefficient from projecting output growth on our instrument. We find a highly statistically significant positive association. The F-statistic is 102.6 for one-year changes and 48.57 for four-year changes. We visualize the first stage regressions in Figure 3 Panel (a).

IV effects. Column (2) of Table 2 reports the 2nd stage IV results where we instrument output growth with export demand shocks. Coefficients are similar to the OLS results. The output elasticity of labor declines by 3.3% (2.4%) in response to a 10 percent growth in output when considering one-year (four-year) changes. Also for other variables, IV and OLS

estimates are closely in line. This suggests that the OLS results reflect a causal empirical regularity stemming from output and firm scale to shifts in the intensity of inputs, away from labor (and to a lesser degree capital) and toward intermediates.

Reduced form. In Appendix Table A.6, we additionally report reduced form results for one- and four-year changes corresponding to our IV-regressions; i.e., we project the dependent variables from Table 2 directly on the instrument in one- and four-year changes. Results are fully consistent with our IV-regressions and similar for one and four-year changes. We visualize results of these regression for key variables in Figure 3, Panels (b)-(d). The coefficients for labor output elasticities (divided by returns to scale) are approximately -0.015 under both time horizons. The corresponding intermediate input output elasticity coefficients are both 0.005, and labor cost share coefficients are -0.02 and -0.017, for one- and your-year differences, respectively. All results are statistically significant at the 1% level.[11]

5 Interpretation and Potential Mechanisms

We now discuss potential mechanisms behind the diminishing output elasticity of labor, before exploring implications for labor shares in the next two sections.

Measurement error. One potential concern could be that our output elasticities are mismeasured. One particularly strong assumption when estimating output elasticities is Hicks neutrality. Yet, we do not believe that measurement error can account for our findings as our results on labor cost shares and labor output elasticities exhibit *quantitatively* very similar patterns.[12] This robustness is reassuring because output elasticities and cost shares measure firms' production technology under very different assumptions. Our estimation of output elasticities relies on Hicks-neutrality and a specific model of firm behavior but allows for input adjustment frictions, input market power, and increasing/decreasing returns to scale. Cost shares, on the other hand, measure output elasticities under perfect input markets and constant returns to scale but allow for non-Hicks-neutral productivity processes and do not impose constant parameters of the underlying production function (see online Appendix F).

Input adjustment costs. One might wonder whether our results could simply be attributed to input adjustment costs. We acknowledge the importance of input adjustment costs in understanding *raw input* choices of firms: labor and capital, as relatively fixed inputs, lead

[11] In Appendix Figure A.3 we provide additional visualizations for labor output elasticities without dividing by RTS, capital output elasticities divided by RTS, and labor shares.

[12] We acknowledge that we find less consistent results for capital in the cross-sectional regressions (as noted above). This could results from confounders in the cross-sectional regressions or from difficulties in approximating capital costs. Particularly, we impose homogeneous and fixed capital unit costs across all firms.

firms to scale up intermediates in the short run when expanding, with labor and capital adjusting more slowly over time.

However, our parametric estimates of output elasticities are unaffected by such input adjustment costs. These estimates are directly derived from the production function, measuring the actual change in output following a realized shift in inputs and input mixes by marginal units.[13] As a result, output elasticities genuinely reflect shifts in firms' production technology, conditional on firms' input mix.

In fact, our analysis permits gauging the presence of input adjustment costs, by comparing changes in cost shares with estimated output elasticities. Intuitively, changes in cost shares solely reflect changes in output elasticities when inputs are perfectly flexible and have fixed prices (or markdowns move in lockstep across inputs), such that excess movement in input cost shares above and beyond output elasticities reflects adjustment costs (see online Appendix F). Indeed, our findings suggest that labor and capital face higher input adjustment costs than intermediates, because coefficients on labor and capital cost shares exhibit higher negative values than those on labor and capital output elasticities.[14]

While adjustment costs are a natural explanation given that we find smaller effects for longer time horizons (Table 1), a notion of adjustment costs would need to act on overall technological change, with firms possibly adopting different modes of production in the short and long run.

Automation or Offshoring? Recent work has highlighted capital-based technological change as one factor that reduces the importance of labor to firms (Hubmer and Restrepo, 2021, Acemoglu and Restrepo, 2020). Such technological change can therefore explain a decline in labor's output elasticity with firm growth. However, our findings indicate that capital output elasticities also decline, and instead intermediate input intensity increases. Therefore, while automation might be an important process shaping general changes in firms' production, it cannot rationalize our findings.[15]

Instead, technological change that replaces labor, and to a lesser extent, capital with intermediate inputs, seems to be more relevant in our case. This would be consistent with the rise of offshoring that Europe and particularly Germany experienced over the last decades due to China's accession to the WTO Autor et al., 2016.[16] From a production perspective, the avail-

[13]Similar to De Loecker et al. (2016), our production function estimation routine explicitly models labor and capital as dynamic inputs.

[14]As discussed, differences between cost shares and output elasticities may also arise from input market power.

[15]Evidence on the aggregate employment effects of automation is mixed. WhileAcemoglu and Restrepo (2020) find that robots reduced employment and wages in the US, Graetz and Michaels (2018) conclude that robots did not significantly reduce aggregate labor demand in their cross-country analysis of mostly European countries.

[16]Note that our intermediate input variable also contains unfinished products, products for resale, and

ability of cheap offshore production possibilities induces firms to change their production technologies toward cheaper inputs, leading to an increase in the intermediate input-output elasticity and a decline in output elasticities for factors that became relatively more expensive (i.e., labor and capital).[17]

Slope of input supply curves. The relative slope of firms' input supply curves is another factor that affects relative factor prices as firms grow. One simple micro-foundation for the diminishing labor output elasticity that is independent of macro-trends, such as declining capital prices or globalization, is that firms operate on an increasing labor supply curve, while facing a less steep (or flat) intermediate input supply curve. Such a framework has recently been discussed in Huneeus et al., 2021 (in context of a monopsony model). In this case, firms' relative costs of using labor increase with size, incentivizing firms to adopt less labor-intensive technologies. Consequently, labor output elasticities decline with firm size, while intermediate input output elasticities increase.

Functional form of the production function. Changes in firms' output elasticities with firm growth do occur in certain production models. For instance, in the context of declining IT prices, Lashkari et al. (2023) model a production technology where firms that increase output become more IT-capital intensive. As a result, their production model features a negative labor-output super-elasticity.[18]

More generally, non-homothetic production functions, such as the non-homothetic CES production function, allow for the level of output to affect the input factor mix and output elasticity of labor (Sato, 1977). Our results can thus be formalized within such a non-homothetic production function and highlight the empirical relevance of these types of production models.

intermediate services (see Appendix D).

[17]Consistent with this, Bena and Simintzi (2023) argue that "Machines could not compete with Chinese labor."

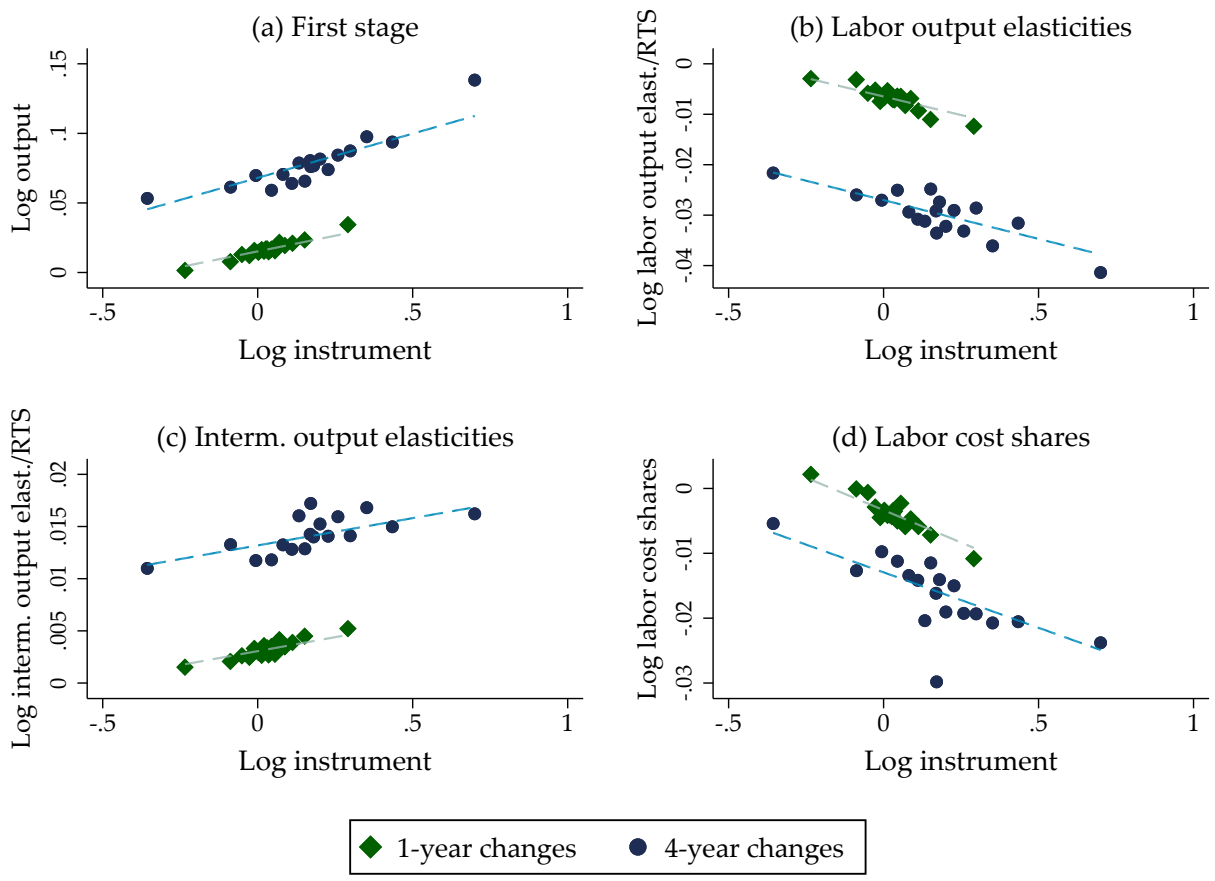
[18]Similarly, production functions that feature biased technological change can also account for a decreasing importance of labor with output growth (e.g., Doraszelski and Jaumandreu, 2018, Oberfield and Raval, 2021).

Table 2: Firm-level adjustments in response to firm growth. 2SLS IV-regressions.

	1st stage	$log(L_{it})$	$log(K_{it})$	$log(M_{it})$	$log(\frac{w_{it}L_{it}}{C_{it}})$	$log(\frac{r_{it}K_{it}}{C_{it}})$	$log(\frac{z_{it}M_{it}}{C_{it}})$	$log(\frac{\theta^L_{it}}{RTS_{it}})$	$log(\frac{\theta^K_{it}}{RTS_{it}})$	$log(\frac{\theta^M_{it}}{RTS_{it}})$	$log(RTS_{it})$
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Export demand shock	0.0451***										
	(0.0045)										
Log output growth		0.324***	0.0612	0.929***	-0.444***	-0.684***	0.171***	-0.327***	-0.229**	0.119***	0.0082
		(0.0543)	(0.0461)	(0.0508)	(0.0521)	(0.0565)	(0.0292)	(0.0623)	(0.0980)	(0.0198)	(0.0053)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950
First-stage F-value		102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6	102.6
R^2	0.205	0.214	0.042	0.736	0.336	0.554	0.321	0.229	0.132	0.314	0.052
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Export demand shock	0.0635***										
	(0.0091)										
Log output growth		0.536***	0.262***	0.963***	-0.269***	-0.570***	0.124***	-0.242***	-0.147	0.0823***	0.0222***
		(0.0737)	(0.0973)	(0.0542)	(0.0664)	(0.0969)	(0.0369)	(0.0706)	(0.0938)	(0.0268)	(0.0075)
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492
First-stage F-value		48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57
R^2	0.183	0.470	0.139	0.855	0.347	0.471	0.339	0.217	0.169	0.266	0.156

Notes: The table reports 2SLS IV-regressions from estimating the specification in Equation (5) using foreign demand shocks as instruments (Equation (7)). Column 1 reports the first-stage regression results. The dependent variables in Columns 2-11 are the logs of labor, capital, intermediates, labor cost shares, capital cost shares, intermediate cost shares, labor output elasticities over returns to scale, capital output elasticities over returns to scale, intermediate input output elasticities over returns to scale, and returns to scale, respectively. All regressions project changes in those dependent variables on changes in log output for 1- and 4-year differences. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm-level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Figure 3: First stage and reduced form results, one- and four-year changes



Notes: The figure reports binned scatter plots for firm-level first stage and reduced form regression. The first stage projects log output on the instrument, whereas the reduced form regressions project the logs of labor output elasticities divided by returns to scale, intermediate output elasticities divided by returns to scale, and labor cost shares on the instrument. Regressions are in one-year and four-year changes. All panels report results that are residualized by industry-year fixed effects. German firm-level data.

6 Implications for Firm-Level Labor Shares

One immediate implication of the diminishing output elasticity of labor is the causal negative effect of firm growth and firm size on the firm's labor share. We now extend the basic production function framework from Section 3 to formally derive this result, and use the implied structural equation to guide our further empirical analysis.

6.1 Theoretical Framework and Identification

The labor share. Firms operate with the following cost function:

$$C_{it} = w_{it}(L_{it})L_{it} + r_{it}K_{it} + z_{it}M_{it}. \tag{8}$$

w, r, and z denote unit costs for labor, capital, and intermediates. We allow wages to depend on labor quantities, such that firms can exercise monopsony power (Appendix C shows that a setting where workers have bargaining power delivers essentially the same results). The first order condition for labor yields the following expression for the labor share (see Appendix B for the derivation):

$$LS_{it} = \frac{w_{it}L_{it}}{P_{it}Q_{it}} = \frac{\theta_{it}^{L}}{\mu_{it}\gamma_{it}}. \tag{9}$$

P_{it} denotes the firm's output price. LS_{it} is the labor share in sales. $\mu_{it} = \frac{P_{it}}{MC_{it}}$ is the markup of the output price over marginal cost (MC_{it}). $\gamma_{it} = \frac{MRPL_{it}}{w_{it}}$ is the wage markdown, with $MRPL_{it}$ being the marginal revenue product of labor.

Log decomposition. Importantly, the rightmost side of Equation (9) provides a full decomposition of the firm-level labor share, and is log-additive:

$$\ln(LS_{it}) = \ln(\theta_{it}^{L}) - \ln(\mu_{it}) - \ln(\gamma_{it}). \tag{10}$$

Hence, the model demonstrates a direct *causal* effect of θ_{it}^{L} on the labor share (for a given markup and markdown).

Identifying price markups and wage markdowns. To separate the three determinants of the labor share, we construct measures of markups, markdowns, and labor output elasticities from firm-level micro-data. We permit all these variables to be *firm- and time-specific*.

We derive output price markups and wage markdowns using the production approach of Hall et al. (1986) and De Loecker and Warzynski (2012). Assuming that intermediate inputs are a flexible input and that firms' take intermediate input prices as given, markups, μ_{it}, can be derived from the first order condition for intermediates within our framework (see Appendix B):

$$\mu_{it} = \frac{P_{it}}{MC_{it}} = \theta_{it}^{M}\frac{P_{it}Q_{it}}{z_{it}M_{it}}, \tag{11}$$

where $\theta_{it}^{M} = \frac{\partial \ln Q_{it}}{\partial \ln M_{it}}$ is the output elasticity of intermediate inputs.

Following recent work (e.g., Dobbelaere and Mairesse, 2013, Caselli et al., 2021, Yeh et al., 2022), we can also define an expression for firms' wage markdowns, γ_{it}, by combining the first order conditions for intermediates and labor (see Appendix B):

$$\gamma_{it} = \frac{MRPL_{it}}{w_{it}} = \frac{\theta_{it}^{L}}{\theta_{it}^{M}}\frac{z_{it}M_{it}}{w_{it}L_{it}}. \tag{12}$$

Finally, we also define and study the labor wedge reflecting the combined distortion from

product and labor market power, namely:

$$\mu_{it}\gamma_{it} = \theta_{it}^{L}\frac{P_{it}Q_{it}}{w_{it}L_{it}}. \tag{13}$$

6.2 Results

Strategy. To gauge effects on firm-level labor shares, we estimate regression model Equation (5) but replace the outcome variable θ_{it}^{L} in Equation (5) with labor shares, markups, markdowns, and the labor wedge. All outcome variables are logged and in differences, which allows us to quantify the part of the elasticity of the labor share to output growth that is accounted for by the diminishing output elasticity of labor as described in Equation (10).

Throughout the study, we focus on the sales labor share because this is the relevant labor share definition that corresponds to the output elasticity of labor and our firm-level production function framework. However, we also include the labor share in value added as a robustness check.[19]

Again, we present OLS and IV results, and, given the important horizon dependencies we have uncovered in the previous section, we estimate this effect for 1-, 4-, and 10-year differences and additionally include the cross-sectional specification in levels.

Regression results. Tables 3 (OLS) and 4 (IV) are organized as their counterparts Tables 1 and 2, showing the OLS and IV effects of firm output on labor shares, markups, markdowns, the labor wedge, and the value added labor share for various time horizons. We additionally add our previous estimates on labor output elasticities to facilitate the analysis. As indicated in Equation (10), the coefficients on markups, wage markdowns, and output elasticities should *add up* to fully account for the relationship between firm output and the labor share. While this adding up property holds up well for our regressions in differences, it does less so for our cross-sectional regression. This is because our industry-year fixed effects absorb part of the cross-sectional variation in markups, output elasticities, and markdowns.[20]

Scatterplots. Figures 4 and 5 are the analogs of Figures 1 and 2, visualizing the variation underlying the regression estimates.

[19]Note that under a value added production function, we can approximate the value added output elasticity as $\theta_{it}^{L}\kappa_{it}$, where κ_{it} is the ratio of sales to value added. As the value added labor share equals the labor share in sales times κ_{it}, we can transfer our conclusions on the relative importance of changes in the output elasticity of labor for the sales labor share also to the value added labor share. See Appendix Figures A.1 and A.2 for the underlying empirical relationships.

[20]This is less relevant for the regression in changes, because there we solely use *changes* for identifying coefficients.

Declining labor shares. We find a strong negative effect of firm output on labor shares—which again shrinks as we widen the time horizons, ranging, in the OLS regressions, from -0.581 to -0.303 for the specifications in differences and to a much smaller but precisely estimated -0.032 in the cross-sectional regressions. Results for the value-added labor share as well as IV estimates are similar.

Half to a third of the effect: the diminishing output elasticity of labor. In the short and medium run, up to the 4-year differences, *half* of this negative effect of output growth on labor shares is driven by the diminishing output elasticity of labor. This result holds for OLS and IV regressions. For 10-year differences, the diminishing output elasticity still accounts for a third of the effect (as changing returns to scale become increasingly important over longer time horizons). In the cross-section, differences in labor output elasticities net of returns to scale still contribute to lower labor shares among larger firms, but the effect is much smaller (perhaps due the aforementioned confounders).

The remainder: conventional market power effects. The remaining half of the short- to medium-run within-firm effect of firm growth on labor share changes is explained by markups and wage markdowns increasing in firm size. This result likely reflects market power: as we control for industry fixed effects, output growth is effectively reflecting an increase in firms' market shares, and we expect a firm's market power to increase with its market share (as in Atkeson and Burstein, 2008 for product markets and Berger et al., 2022 for labor markets). Over longer time horizons, wage markdowns become increasingly important in explaining the effect of firm size on labor shares. This becomes particularly clear in the cross-sectional regressions.[21]

Summary. We thus conclude that over the short to medium run, the majority of the negative effect of output growth on *firm-level* labor shares results from the diminishing output elasticity of labor, whereas, over longer time horizons, labor market power becomes more important.

[21]The long-run importance of market power is consistent with Autor et al. (2020), who, however, did not investigate changes in output elasticities. Notably, whereas Autor et al. (2020) focus on the labor wedge as their markup measure, we differentiate between markups and wage markdowns and can thus uncover a higher importance of wage markdowns relative to product markups for explaining cross-sectional differences in labor shares (particularly over longer time horizons).

Table 3: Market power, output elasticity, and labor share adjustments in response to firm growth. OLS-regressions.

	$log(LS_{it})$	$log(\mu_{it})$	$log(\gamma_{it})$	$log(\mu_{it}\gamma_{it})$	$log(\frac{\theta^L_{it}}{RTS_{it}})$	$log(\theta^L_{it})$	$log(\frac{w_{it}L_{it}}{VA_{it}})$
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output growth	-0.581***	0.124***	0.159***	0.283***	-0.304***	-0.298***	-0.561***
	(0.0040)	(0.0025)	(0.0048)	(0.0044)	(0.005)	(0.0046)	(0.0076)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950
R^2	0.534	0.096	0.115	0.191	0.230	0.215	0.190
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output growth	-0.406***	0.0608***	0.160***	0.220***	-0.202***	-0.185***	-0.334***
	(0.0061)	(0.0032)	(0.0065)	(0.0055)	(0.0060)	(0.0063)	(0.0081)
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492
R^2	0.461	0.126	0.193	0.239	0.220	0.201	0.188
Panel C: 10-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output growth	-0.303***	0.0383***	0.152***	0.191***	-0.140***	-0.112***	-0.231***
	(0.0062)	(0.0030)	(0.0060)	(0.0048)	(0.0053)	(0.0057)	(0.0068)
Observations	49,915	49,915	49,915	49,915	49,915	49,915	49,915
N of firms	10,595	10,595	10,595	10,595	10,595	10,595	10,595
R^2	0.378	0.125	0.213	0.253	0.188	0.161	0.178
Panel D: levels, cross-section	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output growth	-0.0321***	-0.0231***	0.170***	0.147***	-0.0345***	0.0153***	-0.0994***
	(0.00133)	(0.0006)	(0.00179)	(0.00157)	(0.0016)	(0.0017)	(0.0018)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950
R^2	0.136	0.188	0.499	0.553	0.512	0.531	0.356

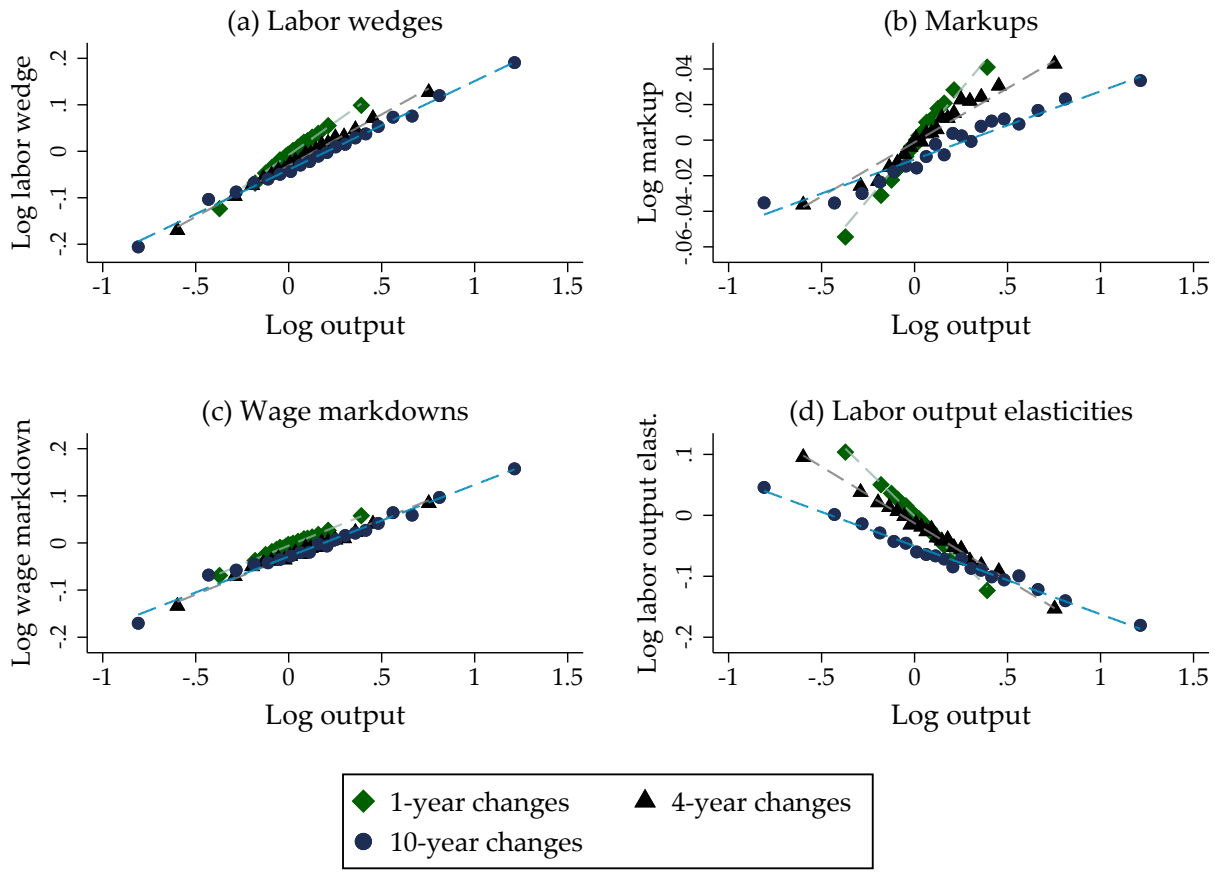
Notes: The table reports OLS-regressions estimating the specification in Equation (5). The dependent variables in Columns 1-8 are the logs of the labor share in sales, markups, wage markdowns, labor wedge, labor output elasticity divided by returns to scale, labor output elasticity, and labor share in value added, respectively. Panel a-c project changes in those dependent variables on changes in log output for 1-, 4- , and 10-year differences. Panel D runs a cross-sectional specification that regresses levels of the dependent variables on output levels. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm-level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Table 4: Market power, output elasticity, and labor share adjustments in response to firm growth. 2SLS IV-regressions.

	1st stage	$log(LS_{it})$	$log(\mu_{it})$	$log(\gamma_{it})$	$log(\mu_{it}\gamma_{it})$	$log(\frac{\theta^L_{it}}{RTS_{it}})$	$log(\theta^L_{it})$	$log(\frac{w_{it}L_{it}}{VA_{it}})$
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Export demand shock	0.0451***							
	(0.0045)							
Log output growth		-0.674***	0.186***	0.169**	0.355***	-0.327***	-0.318***	-0.842***
		(0.0521)	(0.0418)	(0.0759)	(0.0697)	(0.0623)	(0.0645)	(0.120)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950
First-stage F-value		102.6	102.6	102.6	102.6	102.6	102.6	102.6
R^2	0.205	0.524	0.084	0.115	0.185	0.229	0.215	0.160
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Export demand shock	0.0635***							
	(0.0091)							
Log output growth		-0.443***	0.154***	0.0688	0.223***	-0.242***	-0.220***	-0.603***
		(0.0726)	(0.0444)	(0.0800)	(0.0713)	(0.0706)	(0.0742)	(0.119)
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492
First-stage F-value		48.57	48.57	48.57	48.57	48.57	48.57	48.57
R^2	0.183	0.459	0.072	0.180	0.239	0.217	0.199	0.135

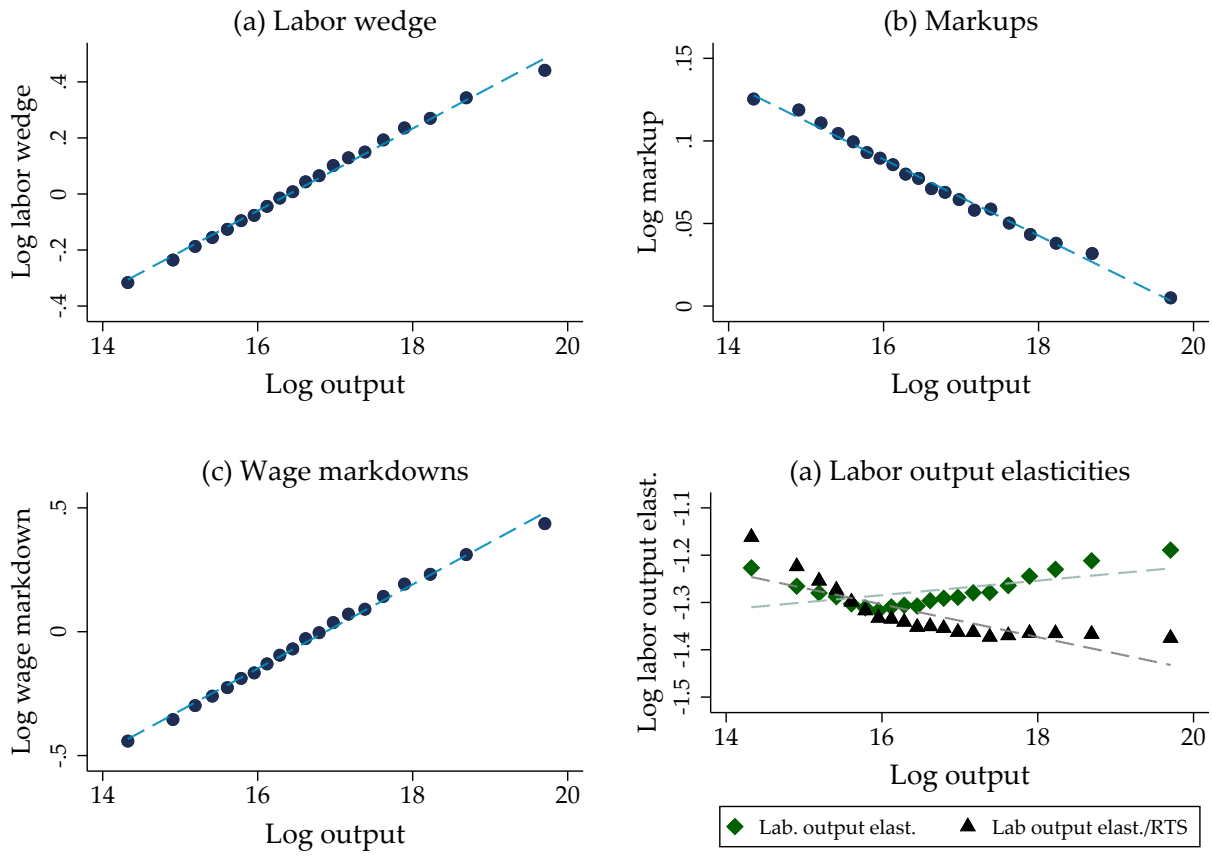
Notes: The table reports 2SLS IV-regressions from estimating the specification in Equation (5) using foreign demand shocks as instruments (Equation (7)). Column 1 reports the first-stage regression results. The dependent variables in Columns 1-5 are the logs of the labor share in sales, markups, wage markdowns, labor wedge, labor output elasticity divided by returns to scale, labor output elasticity, and labor share in value added, respectively. All regressions project changes in those dependent variables on changes in log output for 1- and 4-year differences. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm-level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Figure 4: The adjustment of market power to output growth by horizon: labor wedge, markup, markdown



Notes: The figure reports binned scatter plots estimating the specification in Equation (5) with OLS for various differences. The graphs project changes in the logs of labor wedges, markups, wage markdowns, and labor output elasticities (not divided by returns to scale) against log output changes. All panels report results that are residualized by industry-year fixed effects. German firm-level data.

Figure 5: The cross-sectional relationship between market power and firm size: labor wedge, markup, markdown



Notes: The figure reports binned scatter plots estimating the specification in Equation (5) in levels with OLS. The graphs project the logs of labor wedges, markups, wage markdowns against, and labor output elasticities (divide and not divided by RTS) against log output. All panels report results that are residualized by industry-year fixed effects. German firm-level data.

7 Aggregate Implications: Industry-Level Dynamics in 20 European Countries and the United States

We close our paper by drawing implications for aggregate industry dynamics. To do so, we access micro-aggregated data for 20 European countries and for the United States (for labor share dynamics), to show that our firm-level results carry over to industry aggregates.

7.1 German Stepping Stone: Industry-Level Aggregation of Firm-Level Micro Data

As a stepping stone, we first present the industry aggregation of the firm-level results in Germany (four-digit NACE Rev. 1.1 industries), where we have access to both firm-level

micro data and industry aggregates. We add this stepping stone using our German micro data as a bridge because for the 20 countries in Europe, we exclusively have industry-level data, albeit the data is derived from production function estimation in the underlying micro data.

In the following, we run regressions as in Equation (5) but do so at the industry level, taking weighted averages of firm-level variables as described below, and control for year fixed effects.

Output elasticities of labor. We construct the aggregate output elasticity as a sales-weighted average of firm-level output elasticities by industry-year cells. We then regress the change of the logged value of this weighted average on the log changes in total industry output.

The resulting super-elasticity at the industry level will differ from its firm-level counterpart (Equation (5)), which isolates within-firm changes. Instead, the industry aggregates will additionally capture composition effects between firms of different sizes and labor intensities, including entry and exit.[22] Besides compositional effects, other differences may detach the micro and macro estimates, such as weighting choices, Jensen's inequality, intra-industry trade, and market-level shifts in labor demand and supply curves. Nevertheless, to understand the impact of the diminishing output elasticity on aggregate labor shares, we proceed with the industry analysis and test for links between aggregate economic growth and changes in labor shares and output elasticities akin to our firm-level results.

Labor shares, markups, markdowns. We define the industry-level labor share as total wages divided by total sales, which is equivalent to calculating a sales-weighted average of firm-level labor shares.

Again, we face aggregation issues. For markups and wage markdowns, the appropriate aggregation weight depends the assumptions on the production function and underlying model. We use intermediate input cost weights for aggregating markups, and wage costs for aggregating wage markdowns, which are the denominators of Equations (11) and (12), respectively. Our results are qualitatively robust to sales-weights (not reported).

Another important aspect when studying the industry level regards the decomposition in Equation (10), which only holds at the firm level, i.e., changes in the logs of firms' output elasticities, markups, and wage markdowns do not sum up to changes in log labor labor shares unless firms hold no market power.[23]

[22]The change in the aggregate output elasticity can be decomposed as: $\Delta\bar{\theta}^{L}_{Cont} + cov(s_{i,Cont}, \theta^{L}_{i,Cont}) + Net - entry$, where $Cont$ indicates the continuing firms (Melitz and Polanec, 2015). The first term is the unweighted mean of continuing firms. The second term is the covariance between firms' market shares and labor-output elasticities, which captures firm heterogeneity among continuing firms. The last term captures the contribution of entry and exit.

[23]To understand this, note that if there is no market power in output and labor markets, the aggregate labor

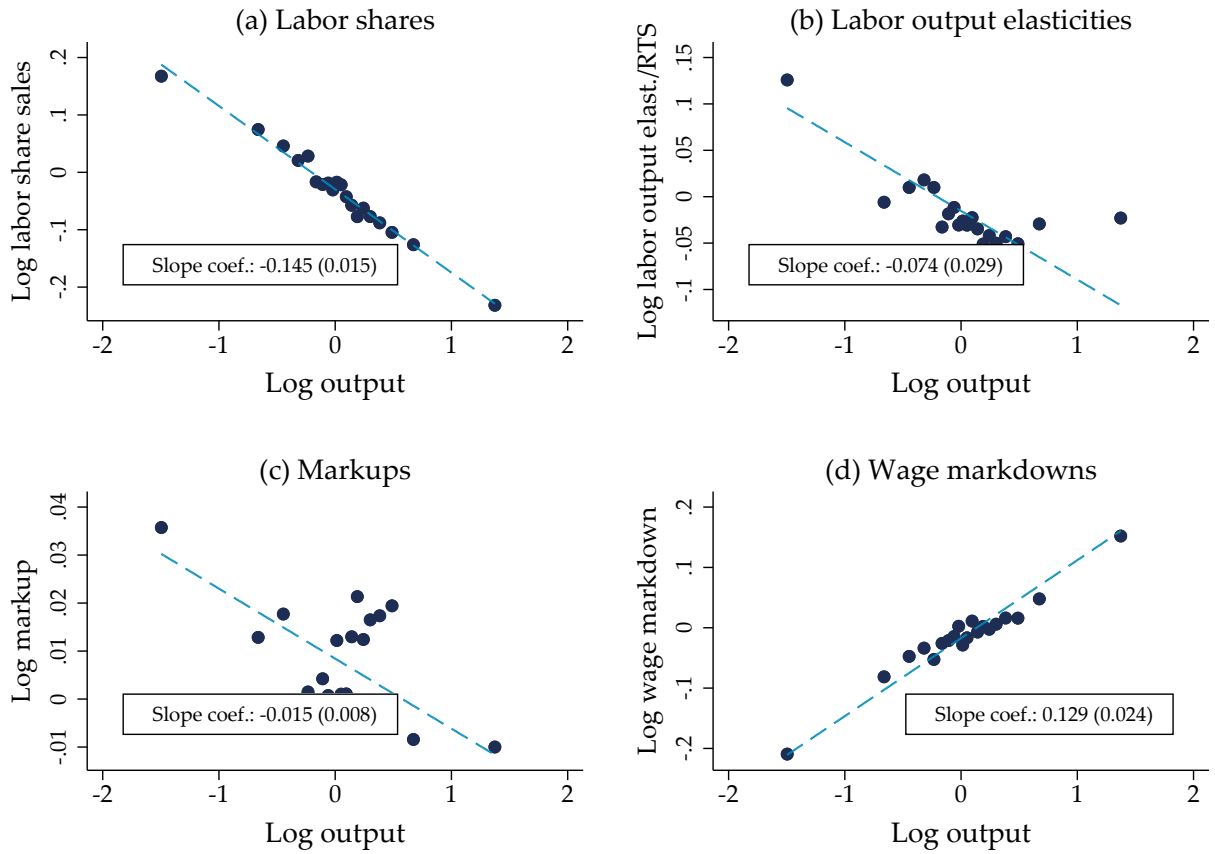
Finally, note again that Jensen's inequality applies when aggregating log labor shares as in Equation (10).

Results. Figure 6 reports our results that replicate the previous firm-level scatterplots at the aggregate industry level (at the 4-year level). At the German industry level, there is a clear, essentially linear relationship that qualitatively mirrors the German micro firm-level relationships: output elasticities of labor and labor shares decline with output growth. Moreover, while markups decrease in industry output growth, wage markdowns increase. This suggests that also at the industry level in the German micro data, changes in output elasticities and wage markdowns are key in explaining labor share dynamics. Finally, a comparison with the 4-year OLS firm-level results makes clear that the analogous industry-level estimates differ quantitatively, likely for the reasons discussed above.

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share and labor output elasticity perfectly align: $\sum_n s_{it} LS_{it} = \sum_n s_{it} \theta^L_{it}$, where s_{it} is the sales-weight. If firms have market power, the right-hand side of this equation becomes $\sum_n \frac{s_{it}\theta^L_{it}}{\mu_{it}\gamma_{it}}$, such that the pass-through of a one-percent change in the output elasticity of labor to a change in the labor share at each individual firm is shaped by firms' markups and wage markdowns. Under imperfect competition, percentage changes in the aggregate output elasticity of labor only translate into equally large percentage changes in the labor share if market shares and product and labor market power remain constant at all firms and firm-level labor-output elasticities change by exactly the same amount at every firm. Nevertheless, qualitatively, it remains true that the partial effect of a change in the aggregate labor-output elasticity on the labor share, holding fixed firms' market power, is positive. Equally, the partial effect (ceteris paribus) of changes in aggregate markups and wage markdowns on the labor share is negative. Importantly, "holding fixed" refers here not only to the aggregate level, but also to the firm distribution.

Figure 6: German micro data aggregated to industry level: 4-year changes (OLS)



Notes: The figure reports binned scatter plots (OLS) from projecting industry-level changes in log labor shares, labor output elasticities over returns to scale, markups, and wage markdowns against industry-level log output changes (4-year differences). All panels report results that are residualized by year fixed effects. German firm-level data, four-digit NACE Rev. 1.1 industries.

7.2 International Industry Panel Data: CompNet Data

We now move to our main industry-level analyses for 20 European countries and the US.

CompNet data. We use the CompNet data (9th vintage), which is a micro-aggregated database for 22 European countries.[24] The data are collected and provided by the Competitiveness Research Network (henceforth, CompNet). CompNet sources its data from highly representative administrative firm-level records located within European national statistical institutes and central banks. The CompNet team distributes harmonized data collection protocols (i.e., Stata codes) across the data providers and invests significant efforts in harmonizing the input data to maximize comparability across countries. The protocols compute micro-aggregated results that comply with the disclosure rules of each individual data provider.

[24]We drop Malta due to insufficient observations for output elaticities and the UK as data is not available at the industry level.

From these results, the CompNet team constructs the CompNet database. The data is aggregated at various levels. We use the country-industry-level data (NACE Rev. 2, two-digit industries), which is the most detailed aggregation level available.

The data contain, among other features, country-industry-level information on firms' sales, inputs, expenditures, markups, wage markdowns, and output elasticities that are relevant for our analysis. As there is no comparable data available for the US, we focus our analysis on Europe. Yet, we also provide a limited set of results for the US based on the NBER-CES Manufacturing Industry Database for the years 1958-2018. The CompNet data covers the years 1999-2021 and the NACE Rev. 2 two-digit industries 10-33 (manufacturing), 41-43 (construction), 45-47 (wholesale/retail trade and repair of motor vehicles and motorcycles), 49-53 (transportation/storage), 55-56 (accommodation/food services), 58-63 (information and communication technology), 68 (real estate), 69-75 (professional/scientific/technical activities), and 77-82 (administrative/support service activities). Yearly coverage varies across countries as shown in Appendix Table A.7. In our analysis, we focus on manufacturing, as we require estimates of firms' production functions which are difficult to define and estimate in other sectors.[25] The CompNet data comes in two versions; one being based on firms with at least 20 employees and the other featuring firms of all size classes. We focus on the data containing firms with at least 20 employees as this is available for more countries. To ensure representativeness, the data is weighted by firm population weights.[26] For further details on the data, we refer to CompNet's User Guide (CompNet, 2023).[27] Importantly, because the CompNet data only provides aggregated information, we cannot access the underlying firm-level data for the individual countries.

Production Function Estimation. The CompNet data provides information on industry-level of markups, wage markdowns, and labor output elasticities using various types of estimation approaches. We rely on measures derived from industry-specific translog specifications that are estimated using the control function approach of Ackerberg et al. (2015). The log production function in the CompNet data is specified as:

$$q_{it} = \beta_l l_{it} + \beta_k k_{it} + \beta_m m_{it} + \beta_{l2} l_{it}^2 + \beta_{k2} k_{it}^2 + \beta_{m2} m_{it}^2 + \beta_{lk} l_{it} k_{it} + \beta_{lm} l_{it} m_{it} + \beta_{km} k_{it} m_{it} + \omega_{it}, \quad (14)$$

[25] In most developed countries, declining manufacturing sector labor shares explain large parts of declining aggregate labor shares (Dao et al., 2019).

[26] Our results hold for the subset of countries for which we have data based on firms of all size classes.

[27] Among others, the CompNet data has recently been used in Autor et al. (2020), Cortes and Tschopp (2020), Gutiérrez and Piton (2020), Bighelli et al. (2023), Biondi et al. (2023), and Mertens and Mottironi (2023).

where lower case letters denote logs.[28] As there is no firm-specific price information observed in the firm-level data underlying the CompNet data, output, q_{it}, is measured as deflated sales using industry-specific deflators. To account for firm-specific price variation within industries, the CompNet estimation routine additionally controls for firms' market shares when estimating the production function.[29] Labor is defined as number of employees, whereas intermediates and capital are defined as deflated intermediate input expenditures and deflated capital stocks (book values). For further details on the production function estimation routine in the CompNet data, we refer to CompNet's User Guide (CompNet, 2023). Markups, wage markdowns, and output elasticities are computed as described in Equation (11), Equation (12), and Equation (3). Industry-level values are constructed as weighted means, using sales weights for output elasticities, labor cost weights for markdowns, and intermediate input cost weights for markups (i.e., the denominators of the markup and markdown expressions).[30]

Summary statistics and sample. To construct a harmonized sample, we only keep country-industry pairs for which we observe labor shares, output elasticities, markups, and wage markdowns.[31] We provide summary statistics of key variables by country from the country-industry-level data in Appendix Table A.7. The table also reports the yearly coverage for each country in CompNet.

7.3 Results: The Diminishing Output Elasticity of Labor in Europe

Cross-country-industry scatterplots. Figure 7 displays 4-year changes in industry-level logs of output elasticities of labor against 4-year changes in industry-level logged output across 20 European countries in CompNet. The figure shows binned scatter plots that absorb country-year and industry-year fixed effects.[32]

As for the German firm-level data, the European CompNet data reveal a strongly negatively relationship between the labor output elasticity and industry growth that is well described by a linear function—and hence again tracing out a constant super-elasticity. We

[28]See Ackerberg et al. (2015) for details on this estimation strategy. In a nutshell, the estimator controls for unobserved productivity (ω_{it}) using a control function containing intermediate inputs, labor, and capital. The approach assumes a simple firm decision model where intermediate inputs and labor are flexible and intermediate input demand depends on capital, labor, and productivity. Under certain restrictions, one can invert the intermediate input demand function to approximate productivity as a function of intermediates, labor, and capital.

[29]This approach follows De Loecker et al. (2020). Under a Cournot model, market shares perfectly capture markup variation. Therefore, market shares can help absorbing some of the unobserved output price variation.

[30]Results are robust to using sales weights for all variables (unreported).

[31]Some country-industry-year cells report missing values in the CompNet data because the number of firms in the underlying micro data was too small either for passing country-specific disclosure rules, or for estimating the production functions.

[32]Results additionally including country-industry fixed effects are similar.

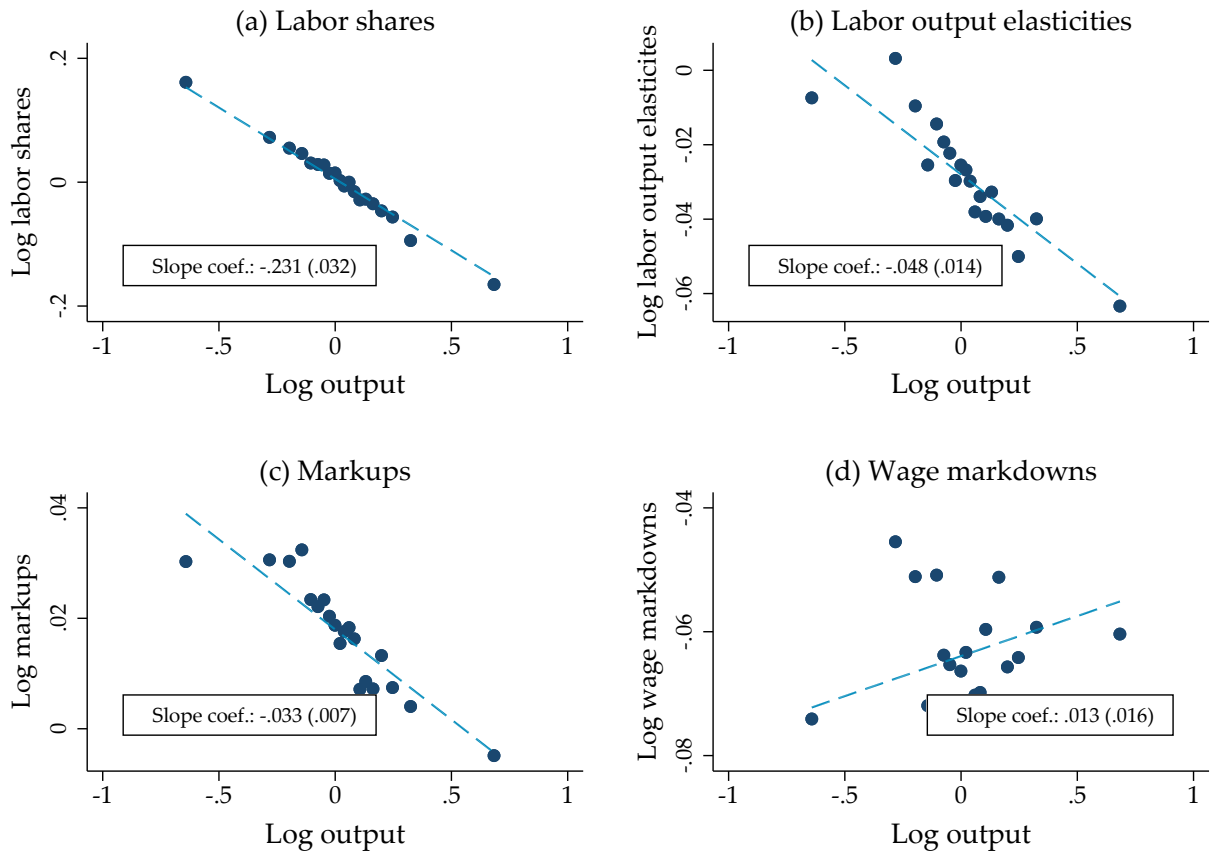
obtain a precisely estimated and economically sizeable coefficient of -0.048. This means that as an industry grows, compared to others, by 10%, its output elasticity of output shrinks by about 0.5%. Hence, the diminishing aggregate labor output elasticity is consistent with the diminishing output elasticity of labor that we established with firm-level micro data. In addition, the labor share shows a strong negative relationship with output growth as well, echoing the previous results at the firm level.

Whereas markups are negatively related to industry growth, the relationship for wage markdowns is statistically insignificant. The latter differs from our previous results for the German micro data. Appendix Figures A.4, A.5, A.6, and A.7 reports country-specific scatterplots. The most striking results from these country-specific figures are i) that the negative relationship between labor shares and industry output growth is found in every country in our data (including the US), and ii) that the industry-level diminishing output elasticity of labor is also found in almost all countries (see also Figure 8 below).[33]

Labor share implications. Figure 7 suggests that the diminishing output elasticity of labor is a key driver of the negative relationship between labor shares and industry growth, because markups and wage markdowns do not increase with industry output. Hence, holding everything else fixed, only the changes in the labor output elasticity can cause labor shares to decline with output growth at the industry level. We provide more evidence on the connection between labor shares and labor output elasticities below.

[33]Figure A.8 also shows that results for value added labor shares are similar.

Figure 7: Changes in labor share in sales and its components on changes in output, European industry-level results



Notes: This figure reports results from projecting industry-level 4-year changes in log labor costs over sales (Panel (a)), labor output elasticities (Panel (b)), markups (Panel (c)), and wage markdowns (Panel (d)) on 4-year changes in log output. We pool all countries and control for country-year and industry-year fixed effects. CompNet data for firms with at least 20 employees, two-digit NACE Rev.2 industries.

Regression coefficients. Table 5 provides regression estimates underlying the previous scatter plots by horizon (1 year to 8 years; we cannot extend the horizon as many countries feature shorter time spans). Consistent with industry growth being a mix of firm-level growth episodes, we do not find any obvious evidence for horizon-dependence at the industry level for any outcome.

Figure 8 complements the table by showing *country-specific* regression coefficients for 4-year differences (including the labor share coefficient for the US). Point estimates suggest that the diminishing output elasticity of labor is present in almost all countries individually. Yet, standard errors are often large, presumably because of the small number of industries that we can study (the CompNet data divides manufacturing in only 23 industries).

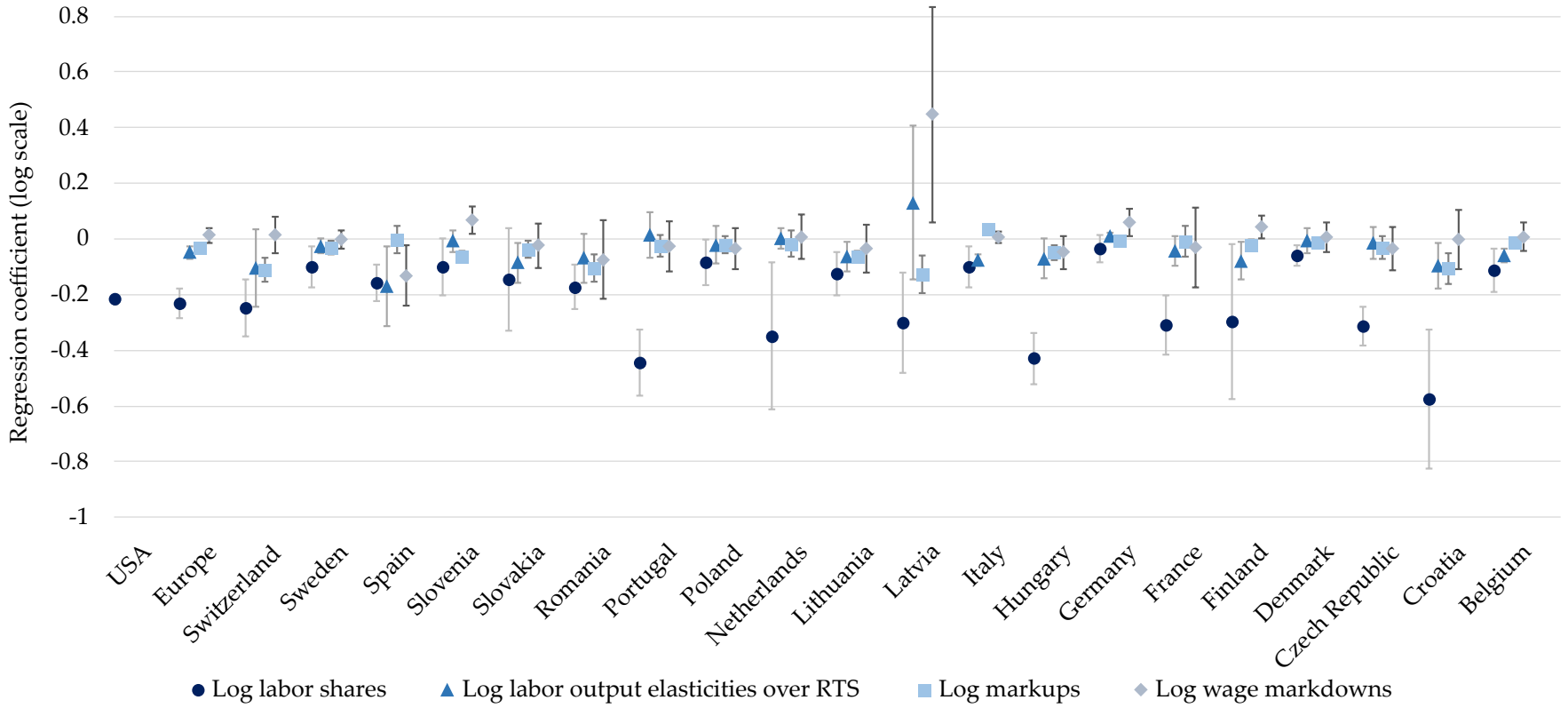
Table 5: Changes in labor share in sales and its components on changes in output, European industry-level results

	$log(LS_{it})$	$log(\frac{\theta^L_{it}}{RTS_{it}})$	$log(\theta^L_{it})$	$log(\mu_{it})$	$log(\gamma_{it})$
Panel A: 1-year differences	(1)	(2)	(3)	(4)	(5)
Output growth	-0.369***	-0.057***	-0.0552***	-0.034***	0.013
	(0.0689)	(0.0129)	(0.0133)	(0.0082)	(0.0134)
Industry-year FE	yes	yes	yes	yes	yes
Country-year FE	yes	yes	yes	yes	yes
Observations	5,352	5,352	5,352	5,352	5,352
R^2	0.557	0.215	0.212	0.250	0.324
Panel B: 4-year differences	(1)	(2)	(3)	(4)	
Output growth	-0.231***	-0.048***	-0.0441***	-0.033***	0.0129
	(0.0323)	(0.0141)	(0.0148)	(0.0068)	(0.0159)
Industry-year FE	yes	yes	yes	yes	yes
Country-industry FE	yes	yes	yes	yes	yes
Observations	4,313	4,313	4,313	4,313	4,313
R^2	0.540	0.245	0.237	0.333	0.433
Panel C: 8-year differences	(1)	(2)	(3)	(4)	
Output growth	-0.218***	-0.0546***	-0.0467**	-0.034**	0.0341*
	(0.0326)	(0.0166)	(0.0182)	(0.010)	(0.0205)
Industry-year FE	yes	yes	yes	yes	yes
Country-year FE	yes	yes	yes	yes	yes
Observations	2,976	2,976	2,976	2,976	2,976
R^2	0.512	0.302	0.285	0.388	0.416

Notes: The table reports coefficients from projecting changes in log industry-level labor shares in sales, markups, wage markdowns, labor output elasticities divided by returns to scales, and labor output elasticities on changes in log output. CompNet data. Firms with at least 20 employees, two-digit NACE Rev.2 industries.

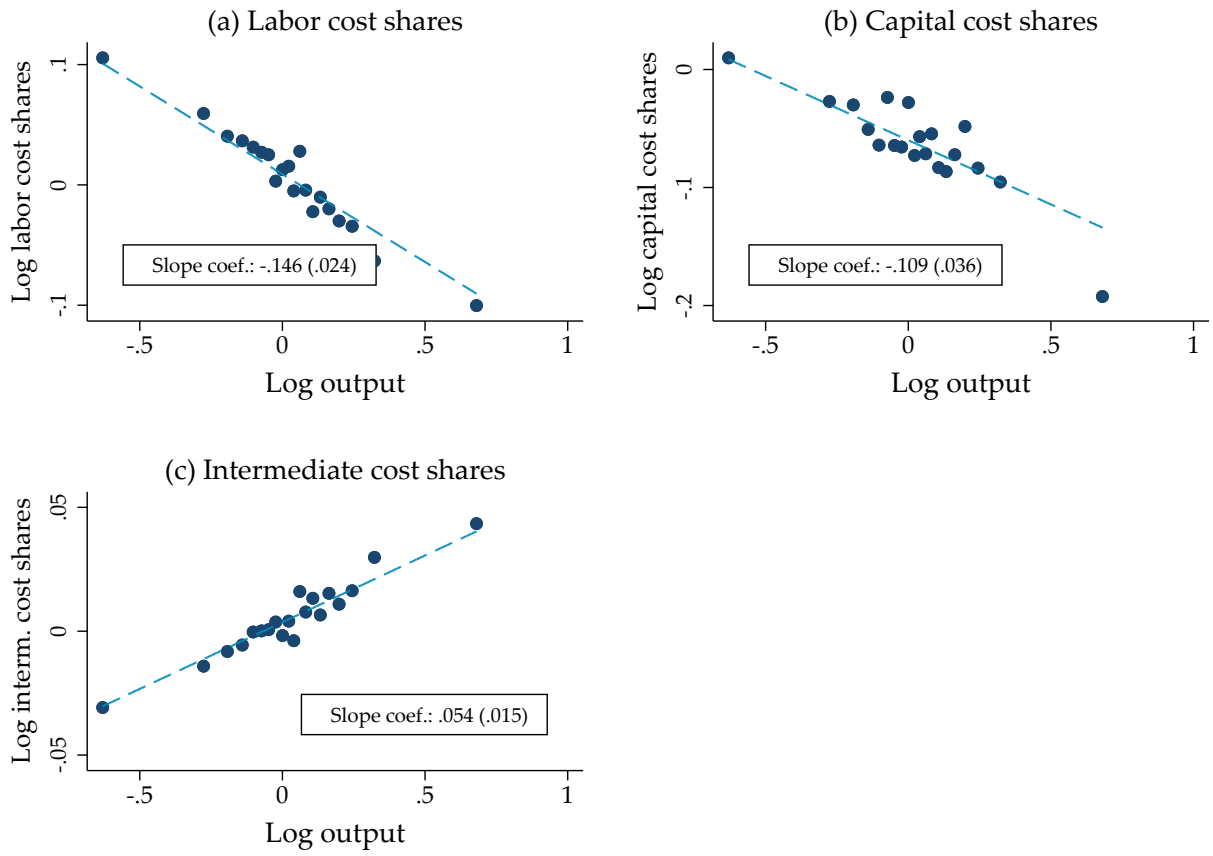
Input cost shares. Finally, Figure 9 again shows raw movements of input cost shares against industry output growth (4-year differences), akin to our previous firm-level analysis. This analysis does not rely on production function estimation or parametric assumptions. Again, we find strong empirical relationships, implying an adjustment of the production factor mix away from labor and capital and toward intermediate inputs.

Figure 8: CompNet 4-year changes (OLS), industry level: Labor shares, labor output elasticities, markups, wage markdowns



Notes: The figure reports regression coefficients from regressing 4-year industry-level changes in the logs of labor shares in sales, labor output elasticities over returns to scale, markups, and wage markdowns on 4-year industry-level changes in log output (OLS). All country-specific coefficients result from country-level regressions that absorb year fixed effects; the Europe-wide coefficient is from a pooled regression that includes country-year fixed effects and industry-year fixed effects. CompNet data for firms with at least 20 employees, two-digit NACE Rev. 2 industries.

Figure 9: European industry level 4-year changes in cost share (OLS)



Notes: The figure reports binned scatter plots (OLS) from projecting 4-year industry-level changes in the logs of labor, capital, and intermediate cost shares against industry-level log output changes (4-year differences). We pool all countries and control for country-year and industry-year fixed effects. CompNet data. Firms with at least 20 employees, two-digit NACE Rev.2 industries.

7.4 Further Implications for Labor Share Dynamics

Through the lens of our simple framework, the labor share declines when firms' markups or wage markdowns *increase* or when the output elasticity of labor *declines*. Consequently, our results above imply that the diminishing output elasticity drives much of the negative relationship between industry labor shares and industry output growth. The slope for the output elasticity of labor is about a quarter the magnitude of the labor share effect at the industry level.

Industry split. To further scrutinize the role of the diminishing output elasticity of labor, we split our data into growing and shrinking industries and study how industry-level labor shares, output elasticities, markups, and wage markdowns have changed over time for growing and shrinking industries. As yearly coverage varies between countries, we first calculate the log changes in each of these variables for each country-industry pair between the first and

last year in the data, i.e., we pool differences for country-industry pairs of different lengths (see Table A.7 for the country-specific yearly coverage). Subsequently, we take averages of these changes across all country-industry-pairs.

Results. Figure 10 reports the results from this exercise. Panel (a) reports European results based on the CompNet data. Panel (b) reports labor share changes for the US for three 20-year periods, 1958-1978, 1978-1998, and 1998-2018. As we do not have production function estimates for the US, we only report labor shares in Panel (b).

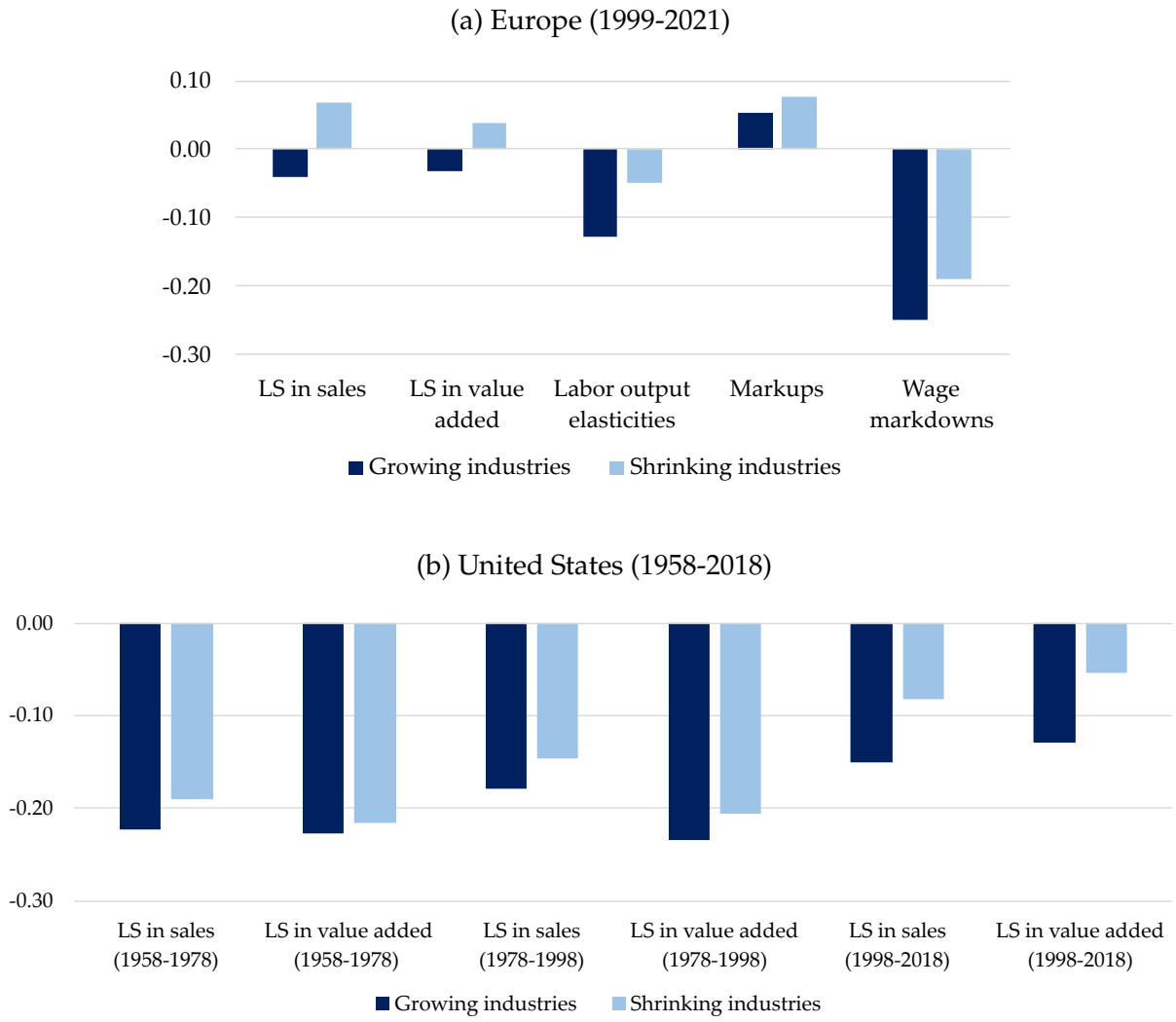
Looking at the European results (Panel (a)), a first striking result is that labor shares *only* decreased in growing industries. These are also the industries that experienced particularly strong declines in labor output elasticities due to the diminishing output elasticity of labor, exerting a downward pressure on labor shares in growing industries. Markups and wage markdowns show patterns that should increase labor shares in growing industries relative to shrinking industries, i.e., markups grow less and wage markdowns decline more in growing industries. Through the lens of our framework, the diminishing output elasticity of labor is therefore the only factor that can explain why labor shares fall in growing but not in shrinking industries. This suggests a pivotal role for the diminishing output elasticity of labor in shaping labor share dynamics in Europe.

Turning to the US (Panel (b)), we observe a pronounced decline in labor shares, particularly within expanding industries. The difference between growing and shrinking industries is particularly strong in the 1998-2018 period. Overall, US-results are consistent with our European results, which suggests that the diminishing output elasticity of labor may also play a significant role in driving changes in US labor shares.[34] While this evidence is consistent with the European results carrying over to the US, the industry-level data does not allow us to directly investigate the output elasticity of labor.

Robustness. We replicate our analysis splitting industries into above and below median growth rates in Appendix Figures A.10 and A.11 and find similar results: More strongly growing industries exhibit stronger declines in labor output elasticities and labor shares. For CompNet, we define splits once based on country medians and once based on European medians, which makes almost no difference. Finally, for the US, we also report results for the entire time span from 1958-2018 in online Appendix Figure A.12 and find again consistent results.

[34]The decline of labor shares in shrinking industries in the US could indicate that, in the US, market power plays a more relevant role in contributing to declining labor shares than in Europe. See De Loecker et al., 2020 and Autor et al., 2020 for a discussion on the role of market power for the declining US labor share.

Figure 10: Changes in labor shares, output elasticities, markups, and wage markdowns in growing and shrinking industries



Notes: The panels report average changes in the logs of labor shares, labor output elasticities, markups, and wage markdowns across country-industry pairs for growing and shrinking industries. Changes are first computed for each pair and then average across all pairs. Time spans differ across country-industry pairs and not all industries are available for all years within a country. Europe: CompNet data and NACE Rev. 2 two-digit industries. US: NBER-CES Manufacturing Industry Database and NAICS 6-digit industries. There are 155 shrinking and 197 growing country-industry pairs in the CompNet data. For the US data, we report three periods. We observe 9 shrinking and 325 growing industries for 1958-1978, 72 shrinking and 289 growing industries for 1978-1998, and 203 shrinking and 161 growing industries for 1998-2018 in the US data. In' a robustness test, we replicate these figures using median growth rates as splitting criteria (online appendix Figures A.10 and A.11).

8 Conclusion

We have established, quantified, and dissected a novel property of production functions: the diminishing output elasticity of labor. Using firm-level data for German manufacturing and OLS and IV regressions, we show that a 10% growth in output reduces the firm-level labor output elasticity by precisely estimated responses of 3% to 0.3%, depending on the

time horizon. Raw scatter plots clarify a strikingly *linear* relationship between log output elasticities of labor and firm growth, suggesting a constant super-elasticity (that is horizon-dependent). The shift away from labor translates into an increase in intermediates, at least in the manufacturing sector that we have studied in this paper. Using industry data for 20 European countries, we show that these micro-level patterns also hold at the aggregate industry level and across a large set of countries. Direct implications emerge for labor share dynamics at the micro and macro levels: the output elasticity of labor is smaller in larger firms, rationalizing about half of the labor share-size gradient in the data when applied to short- to medium run firm growth. Similarly, growing industries display declining labor output elasticities and labor shares. This *technological* explanation complements existing studies that have focused on price shifts, mismeasurement, or concentration and market power. Overall, our findings suggest that a declining labor share may be a by-product of aggregate, industry, or firm growth even in the absence of any changes in firms' market power.

While we have focused on the implications of the diminishing output elasticity for labor shares, there are many additional issues relying on production functions that this regularity may affect, which we leave for future work. For instance, many existing estimates of factor misallocation rely on Cobb-Douglas production functions; and, similarly, the measurement of productivity at the micro and macro level is often based on constant output elasticities.

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Online Appendix of:
The Diminishing Output Elasticity of Labor

Matthias Mertens and Benjamin Schoefer

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A Appendix Tables and Figures

Table A.1: Summary statistics of the German manufacturing sample

	Mean (1)	**p25** (2)	**Median** (3)	**p75** (4)	**St.Dev.** (5)	**Observations** (6)
Number of employees	366.63	53	109	272	2559.55	183,813
Labor share (sales)	0.30	0.21	0.29	0.38	0.12	183,813
Labor share (value added)	0.80	0.63	0.76	0.88	2.89	183,813
Real wage (1995 values)	34,162	25,972	33,823	41,485	11,492	183,813
Markup	1.09	0.97	1.05	1.17	0.20	183,813
Wage markdown	1.08	0.71	0.96	1.32	0.55	183,813
Output elasticity of labor	0.31	0.23	0.31	0.38	0.11	183,813
Output elasticity of capital	0.12	0.08	0.11	0.15	0.06	183,813
Output elasticity of intermediates	0.64	0.57	0.64	0.71	0.10	183,813
Returns to scale	1.06	0.98	1.05	1.13	0.12	183,813

Notes: This table presents summary statistics for selected variables from the German manufacturing sector firm-level data. Columns 1-5 show the mean, 25^{th} percentile, median, 75^{th}, and standard deviation, respectively. Column 6 reports the number of non-missing observations. German micro-data.

Table A.2: Firm-level adjustments in output elasticities in response to firm growth, without dividing by returns to scale. OLS-regressions.

	$log(\theta_{it}^L)$	$log(\theta_{it}^K)$	$log(\theta_{it}^M)$	θ_{it}^L	θ_{it}^K	θ_{it}^M
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)
Log output growth	-0.298***	-0.309***	0.157***	-0.0723***	-0.0203***	0.0991***
	(0.0046)	(0.0064)	(0.00129)	(0.0008)	(0.0004)	(0.0007)
Observations	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950
R^2	0.215	0.132	0.397	0.278	0.183	0.425
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)
Log output growth	-0.185***	-0.164***	0.112***	-0.0420***	-0.0124***	0.0729***
	(0.0063)	(0.0077)	(0.0021)	(0.0014)	(0.0007)	(0.0013)
Observations	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492
R^2	0.201	0.160	0.369	0.220	0.154	0.388
Panel C: 10-year diff.	(1)	(2)	(3)	(4)	(5)	(6)
Log output growth	-0.112***	-0.0441***	0.0860***	-0.0236***	-0.0018***	0.0559***
	(0.00569)	(0.0073)	(0.0021)	(0.00137)	(0.0007)	(0.0013)
Observations	49,915	49,915	49,915	49,915	49,915	49,915
N of firms	10,595	10,595	10,595	10,595	10,595	10,595
R^2	0.161	0.141	0.322	0.174	0.142	0.334
Panel D: levels, cross-section	(1)	(2)	(3)	(4)	(5)	(6)
Log output growth	0.0153***	0.183***	0.0382***	0.0092***	0.0207***	0.0242***
	(0.0017)	(0.00243)	(0.0007)	(0.0004)	(0.0003)	(0.0004)
Observations	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950
R^2	0.531	0.451	0.476	0.533	0.580	0.481

Notes: The table reports OLS-regressions from estimating the specification in Equation (5). The dependent variables in Columns 1-6 are the logged values and not logged values of labor, capital, and intermediate input output elasticities, respectively. Panel a-c project changes in those dependent variables on changes in log output for 1-, 4- , and 10-year differences. Panel d runs a cross-sectional specification that regresses levels of the dependent variables on output levels. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm-level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Table A.3: Firm-level adjustments in output elasticities in response to firm growth, without dividing by returns to scale. 2SLS IV-regressions.

	1st stage	$log(\theta_{it}^L)$	$log(\theta_{it}^K)$	$log(\theta_{it}^M)$	θ_{it}^L	θ_{it}^K	θ_{it}^M
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Export demand shock	0.0451***						
	(0.0045)						
Log output growth		-0.318***	-0.220**	0.127***	-0.0672***	-0.0135***	0.0914***
		(0.0645)	(0.0995)	(0.0170)	(0.0113)	(0.00455)	(0.0102)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950
First-stage F-value		102.6	102.6	102.6	102.6	102.6	102.6
R^2	0.205	0.215	0.127	0.387	0.277	0.171	0.424
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Export demand shock	0.0635***						
	(0.0091)						
Log output growth		-0.220***	-0.125	0.105***	-0.0445***	-0.0005	0.0739***
		(0.0742)	(0.0973)	(0.0227)	(0.0153)	(0.0076)	(0.0143)
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492
First-stage F-value		48.57	48.57	48.57	48.57	48.57	48.57
R^2	0.183	0.199	0.158	0.368	0.220	0.121	0.388

Notes: The table reports 2SLS IV-regressions from estimating the specification in Equation (5) using foreign demand shocks as instruments (Equation (7)). Column 1 reports the first-stage regression results. The dependent variables in Columns 2-7 are the logged values and not logged values of labor, capital, and intermediate input output elasticities, respectively. All regressions project changes in those dependent variables on changes in log output for 1- and 4-year differences. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm-level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Table A.4: Firm-level adjustments in response to firm growth, without taking logs. OLS-regressions.

	$\frac{w_{it}L_{it}}{C_{it}}$	$\frac{r_{it}K_{it}}{C_{it}}$	$\frac{z_{it}M_{it}}{C_{it}}$	$\frac{\theta^L_{it}}{RTS_{it}}$	$\frac{\theta^K_{it}}{RTS_{it}}$	$\frac{\theta^M_{it}}{RTS_{it}}$	RTS_{it}
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output growth	-0.103***	-0.0380***	0.141***	-0.0710***	-0.0201***	0.0911***	0.00647***
	(0.0011)	(0.0003)	(0.0011)	(0.0007)	(0.0003)	(0.0009)	(0.0004)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950
R^2	0.296	0.447	0.406	0.312	0.224	0.335	0.052
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output growth	-0.0692***	-0.0366***	0.106***	-0.0438***	-0.0136***	0.0574***	0.0185***
	(0.00157)	(0.0006)	(0.00179)	(0.00108)	(0.000525)	(0.0014)	(0.0008)
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492
R^2	0.303	0.419	0.413	0.266	0.191	0.277	0.157
Panel C: 10-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output growth	-0.0535***	-0.0275***	0.0810***	-0.0305***	-0.0051***	0.0356***	0.0304***
	(0.0016)	(0.0006)	(0.0018)	(0.0011)	(0.0005)	(0.0014)	(0.0008)
Observations	49,915	49,915	49,915	49,915	49,915	49,915	49,915
N of firms	10,595	10,595	10,595	10,595	10,595	10,595	10,595
R^2	0.259	0.318	0.347	0.222	0.155	0.201	0.262
Panel D: levels, cross-section	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log output growth	-0.0357***	-0.0028***	0.0385***	-0.0070***	0.0131***	-0.0061***	0.0541***
	(0.0005)	(0.0002)	(0.0005)	(0.0003)	(0.0002)	(0.0004)	(0.0005)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950
R^2	0.468	0.253	0.440	0.522	0.527	0.423	0.828

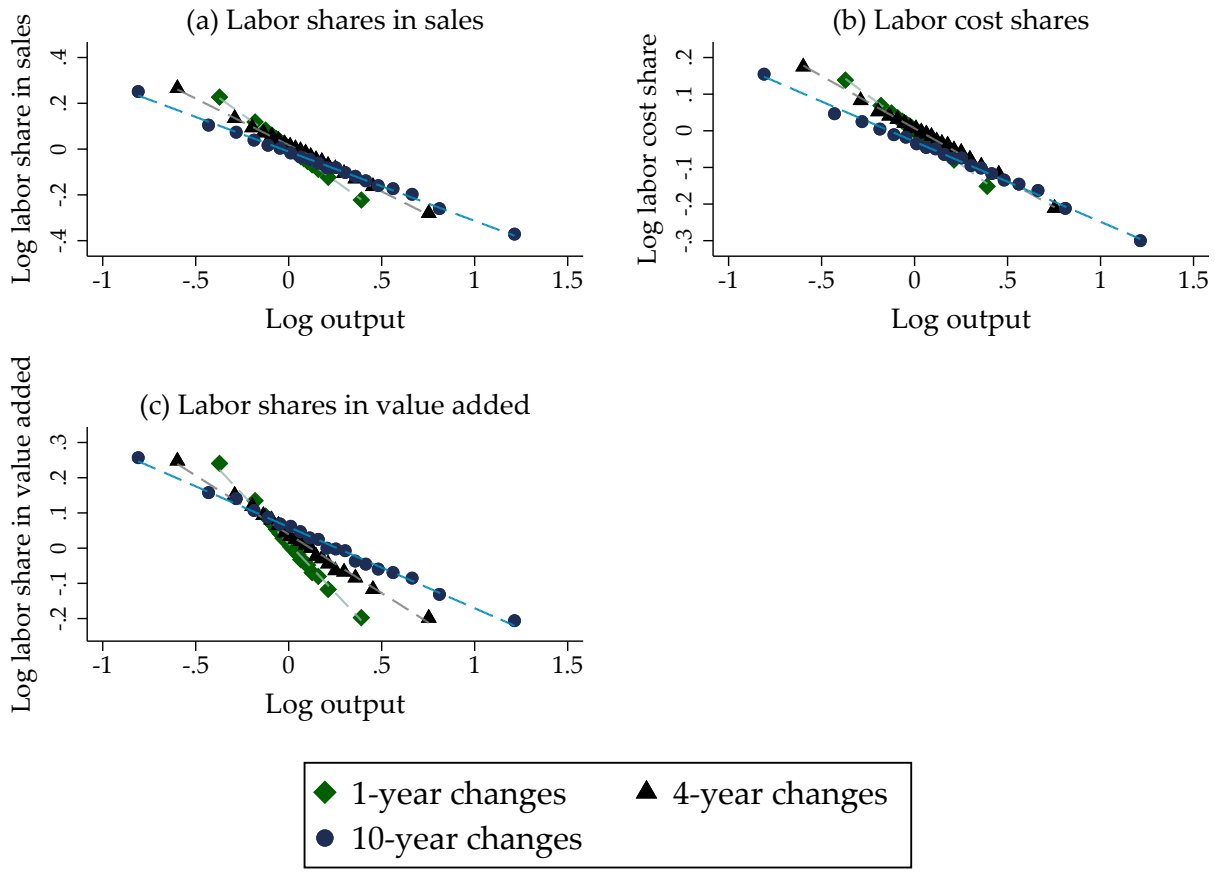
Notes: The table reports OLS-regressions from estimating the specification in Equation (5) without taking logs for the dependent variable. The dependent variables in Columns 1-7 are labor cost shares, capital cost shares, intermediate cost shares, labor output elasticities over returns to scale, capital output elasticities over returns to scale, intermediate input output elasticities over returns to scale, and returns to scale, respectively. Panel a-c project changes in those dependent variables on changes in log output for 1-, 4- , and 10-year differences. Panel d runs a cross-sectional specification that regresses levels of the dependent variables on output levels. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm-level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Table A.5: Firm-level adjustments in response to firm growth, without taking logs. 2SLS IV-regressions.

	1st stage	$\frac{w_{it}L_{it}}{C_{it}}$	$\frac{r_{it}K_{it}}{C_{it}}$	$\frac{z_{it}M_{it}}{C_{it}}$	$\frac{\theta^L_{it}}{RTS_{it}}$	$\frac{\theta^K_{it}}{RTS_{it}}$	$\frac{\theta^M_{it}}{RTS_{it}}$	RTS_{it}
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Export demand shock	0.0451***							
	(0.0045)							
Log output growth		-0.0933***	-0.0283***	0.122***	-0.0626***	-0.0133***	0.0759***	0.0107*
		(0.0144)	(0.0042)	(0.0153)	(0.0098)	(0.0039)	(0.0116)	(0.00575)
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950
First-stage F-value		102.6	102.6	102.6	102.6	102.6	102.6	102.6
R^2	0.205	0.294	0.427	0.400	0.309	0.208	0.329	0.049
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Export demand shock	0.0635***							
	(0.0091)							
Log output growth		-0.0555***	-0.0280***	0.0836***	-0.0465***	-0.0070	0.0536***	0.0289***
		(0.0177)	(0.0062)	(0.0201)	(0.0127)	(0.0058)	(0.0157)	(0.00876)
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492
First-stage F-value		48.57	48.57	48.57	48.57	48.57	48.57	48.57
R^2	0.183	0.297	0.403	0.402	0.266	0.175	0.276	0.139

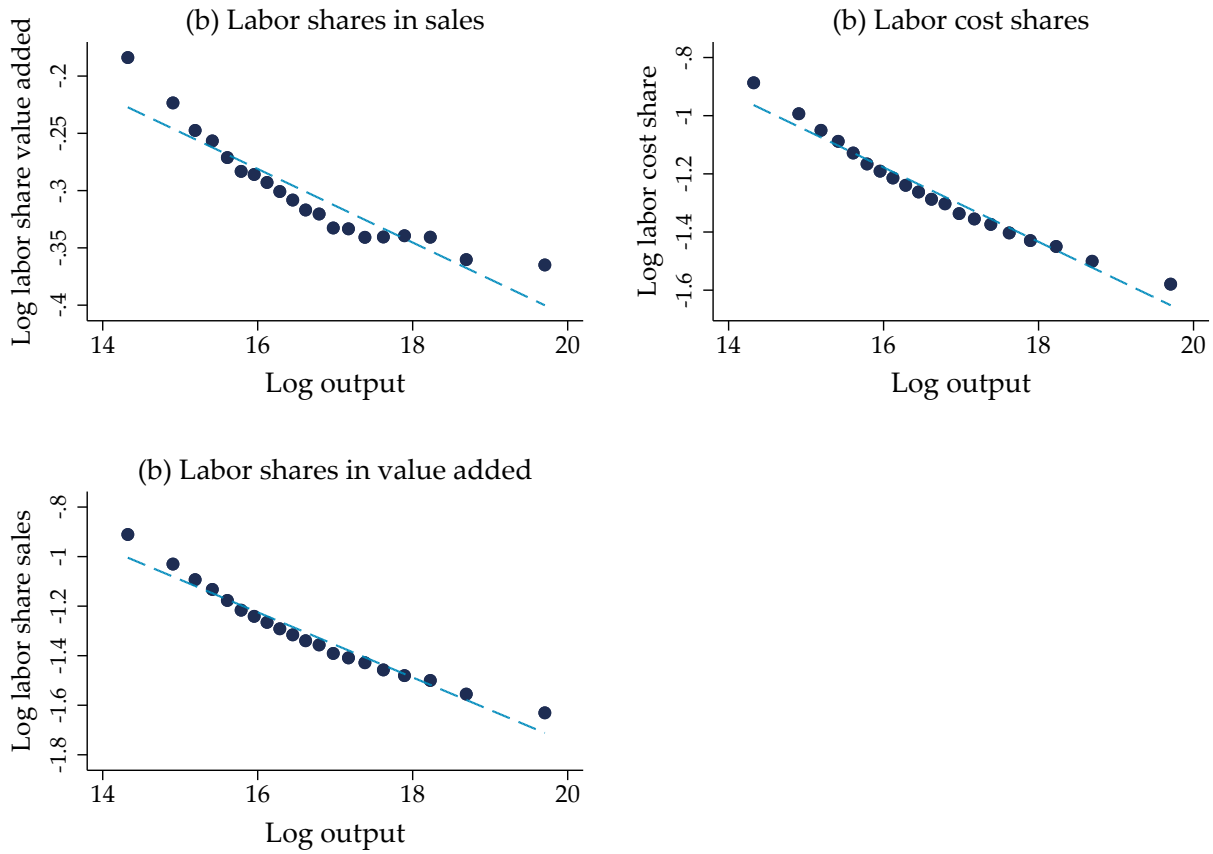
Notes: The table reports 2SLS IV-regressions from estimating the specification in Equation (5) using foreign demand shocks as instruments (Equation (7)). Column 1 reports the first-stage regression results. The dependent variables in Columns 2-8 are labor cost shares, capital cost shares, intermediate cost shares, labor output elasticities over returns to scale, capital output elasticities over returns to scale, intermediate input output elasticities over returns to scale, and returns to scale, respectively. All regressions project changes in those dependent variables on changes in log output for 1- and 4-year differences. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm-level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Figure A.1: German micro-data firm level: Labor share sales and value added - changes



Notes: The figure reports binned scatter plots from estimating the specification in Equation (5) with OLS for various differences. The graphs project changes in the logs of labor shares in sales, costs, and value added against log output changes. All panels report results that are residualized by industry-year fixed effects. German firm-level data.

Figure A.2: German micro-data firm level: Labor share sales and value added - cross section



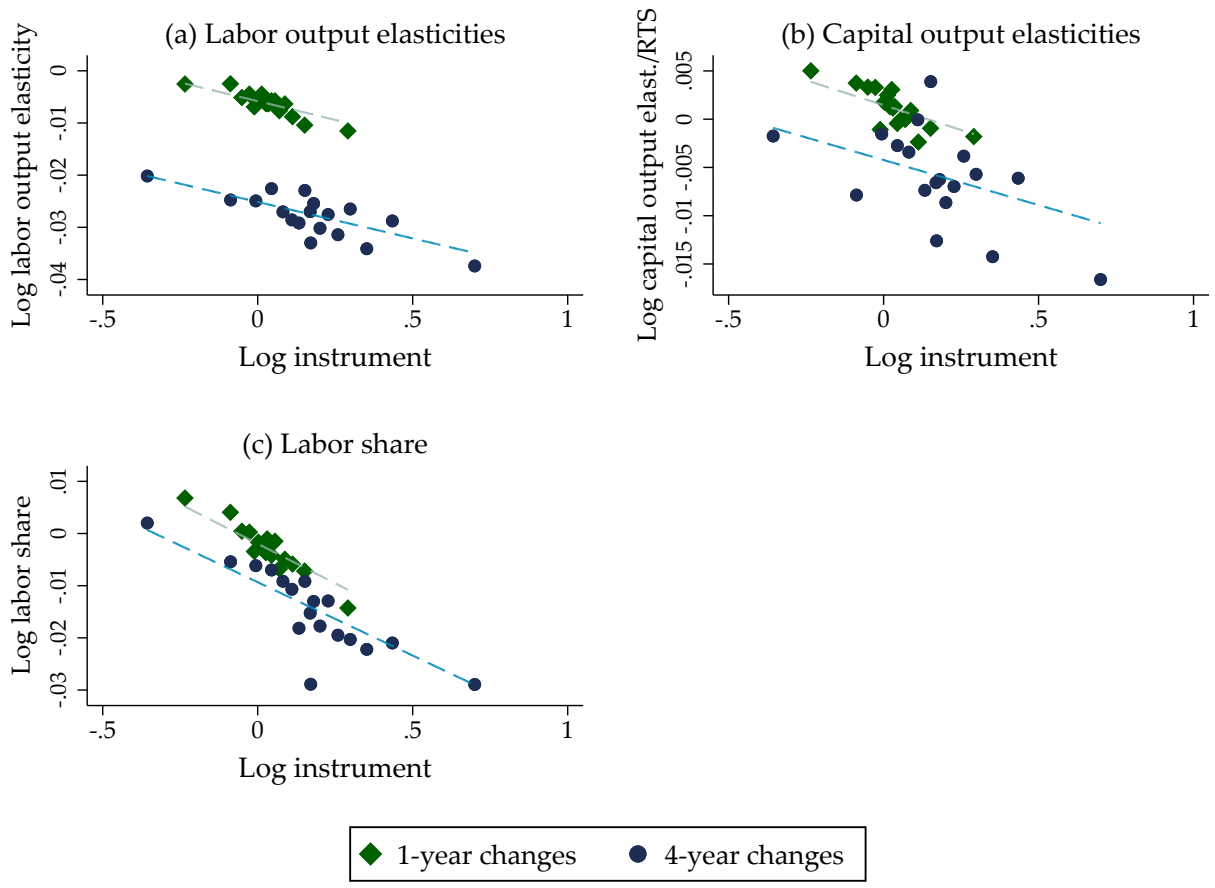
Notes: The figure reports binned scatter plots from estimating the specification in Equation (5) with OLS in levels (cross section). The graphs project the logs of labor shares in sales, costs, and value added against log output. All panels report results that are residualized by industry-year fixed effects. German firm-level data.

Table A.6: Reduced form regression.

	$log(L_{it})$	$log(K_{it})$	$log(M_{it})$	$log(\frac{w_{it}L_{it}}{C_{it}})$	$log(\frac{r_{it}K_{it}}{C_{it}})$	$log(\frac{z_{it}M_{it}}{C_{it}})$	$log(\frac{\theta^L_{it}}{RTS_{it}})$	$log(\frac{\theta^K_{it}}{RTS_{it}})$	$log(\frac{\theta^M_{it}}{RTS_{it}})$	$log(RTS_{it})$	
Panel A: 1-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	
Export demand shock	0.0146***	0.0028	0.0419***	-0.0200***	-0.0308***	0.0077***	-0.0147***	-0.0092*	0.0054***	0.0004	
	(0.0027)	(0.0021)	(0.0049)	(0.0028)	(0.0041)	(0.0016)	(0.0031)	(0.0048)	(0.0011)	(0.0002)	
Observations	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	183,813	
N of firms	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	29,950	
R^2	0.064	0.039	0.142	0.118	0.160	0.109	0.104	0.258	0.095	0.045	
Panel B: 4-year diff.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	
Export demand shock	0.0341***	0.0167**	0.0612***	-0.0171***	-0.0362***	0.0079***	-0.0154***	-0.0093	0.0052***	0.0014***	
	(0.00729)	(0.00689)	(0.0101)	(0.0046)	(0.0080)	(0.0026)	(0.0045)	(0.0061)	(0.0018)	(0.0005)	
Observations	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936	70,936
N of firms	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492	11,492
First-stage F-value		48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57	48.57
R^2	0.608	0.084	0.155	0.175	0.136	0.145	0.140	0.130	0.133	0.102	

Notes: The table reports results for firm-level first stage and reduced form regression corresponding to our 2SLS IV-regressions from estimating the specification in Equation (5) using foreign demand shocks as instruments (Equation (7)). The first stage projects log output on the instrument (Column 1), whereas the reduced form regressions project the logs of labor, capital, intermediates, labor cost shares, capital cost shares, intermediate cost shares, labor output elasticities over returns to scale, capital output elasticities over returns to scale, intermediate input output elasticities over returns to scale, and returns to scale on the instrument (columns 2-10, respectively). All regressions are in 1- and 4-year differences. All regressions control for industry-year fixed effects. Standard errors are clustered at the firm-level. Significance: 0.01***, 0.05**, 0.1*. German firm-level data.

Figure A.3: Reduced form results for additional variables, one- and four-year changes



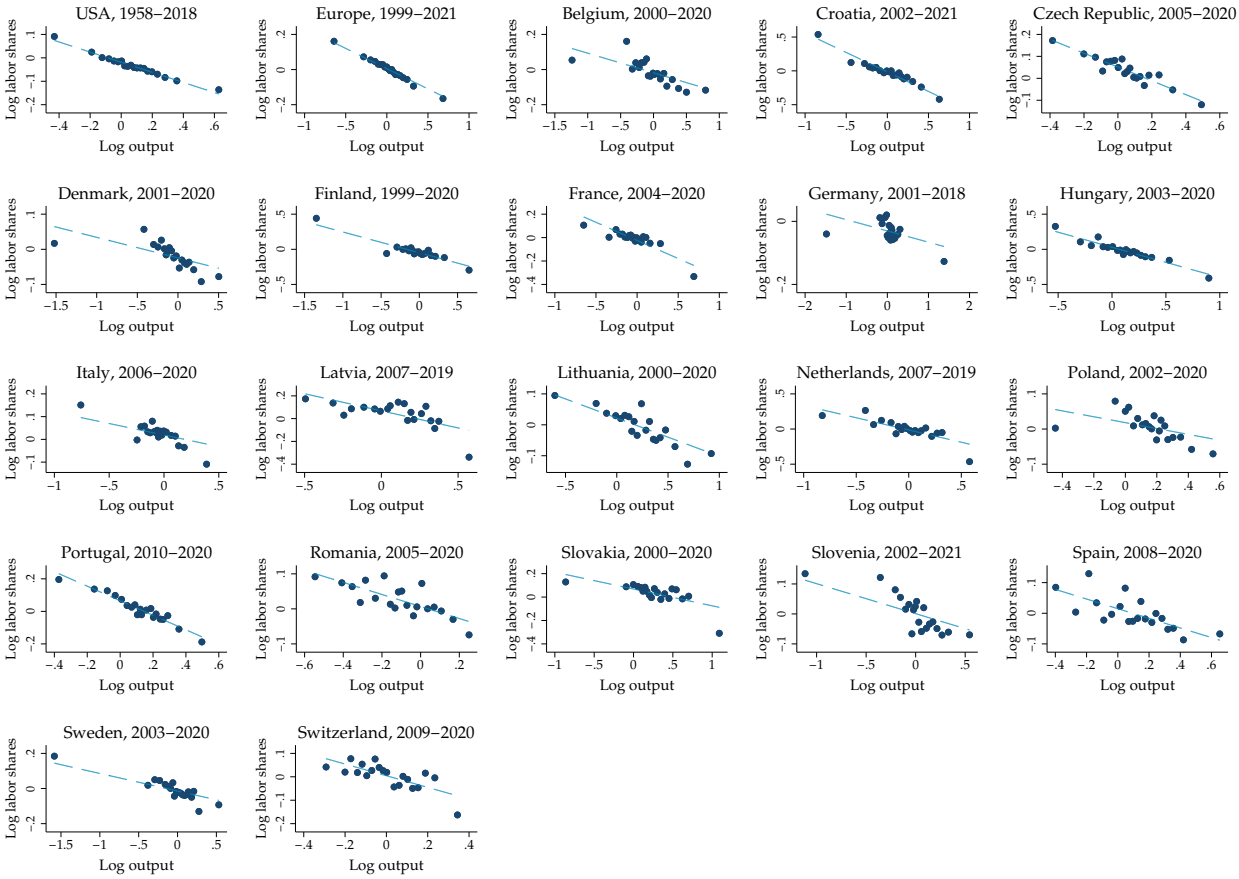
Notes: The figure reports binned scatter plots for firm-level reduced form regression. The reduced form regressions project the logs of labor output elasticities, capital output elasticities divided by returns to scale, and labor shares on the instrument. Regressions are in one-year and four-year changes. All panels report results that are residualized by industry-year fixed effects. The data are German firm-level data.

Table A.7: CompNet summary statistics, averages of industry averages (preliminary)

Country	Lab. share (sales)	Lab. share (value added)	Lab. cost share	Lab. output elast.	Markup	Wage Markdown
Belgium (2000-2020)	0.17	0.41	0.21	0.31	1.24	1.26
Croatia (2002-2021)	0.19	0.40	0.25	0.28	1.32	1.05
Czech Republic (2005-2020)	0.16	0.59	0.17	0.18	1.19	0.75
Denmark (2001-2020)	0.25	0.72	0.26	0.26	1.19	0.80
Finland (1999-2020)	0.21	0.71	0.22	0.21	1.15	0.84
France (2004-2020)	0.21	0.39	0.31	0.38	1.51	1.37
Germany (2001-2018)	0.22	0.74	0.21	0.36	1.12	1.55
Hungary (2003-2020)	0.17	0.61	0.17	0.19	1.12	0.86
Italy (2006-2020)	0.17	0.63	0.17	0.17	1.19	0.77
Latvia (2007-2019)	0.19	0.53	0.21	0.24	1.27	1.02
Lithuania (2000-2020)	0.19	0.66	0.20	0.20	1.15	1.14
Netherlands (2007-2019)	0.21	0.69	0.22	0.22	1.11	0.84
Poland (2002-2020)	0.16	0.54	0.17	0.17	1.18	0.79
Portugal (2010-2020)	0.19	0.62	0.20	0.25	1.12	1.09
Romania (2005-2020)	0.17	0.62	0.17	0.20	1.13	0.91
Slovakia (2000-2020)	0.16	0.50	0.18	0.19	1.14	1.09
Slovenia (2002-2021)	0.19	0.63	0.21	0.20	1.12	0.83
Spain (2008-2020)	0.17	0.64	0.19	0.21	1.17	0.96
Sweden (2003-2020)	0.22	0.38	0.32	0.41	1.32	1.51
Switzerland (2009-2020)	0.26	0.70	0.28	0.25	1.18	0.79

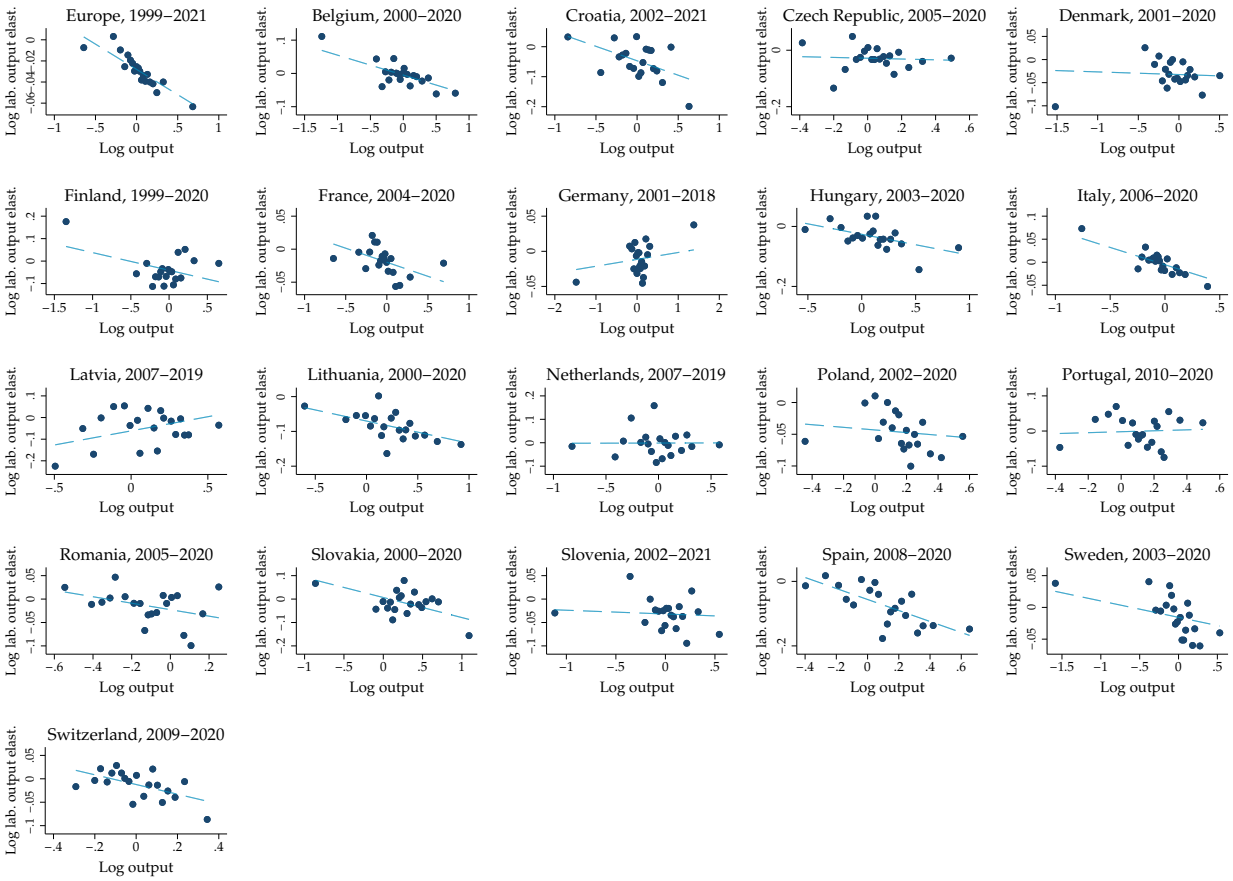
Notes: The table reports country-level averages of industry-level averages for labor shares (in sales and value added), labor output elasticities, markups, and wage mardkdowns. Industry-level aggregates for labor shares are computed from firms' total labor costs, total sales, and total value addded. Industry-level averages for labor output elasticities are sales-weighted aggregates of firm-level values. Industry-level average markups are derived as intermediate-input-cost-weighted averages of firm-level values. Industry-level average wage markdowns are derived as labor-cost-weighted averages of firm-level values. CompNet data. Firms with at least 20 employees.

Figure A.4: Log labor shares in sales changes against log output changes, industry-level, 4-year changes



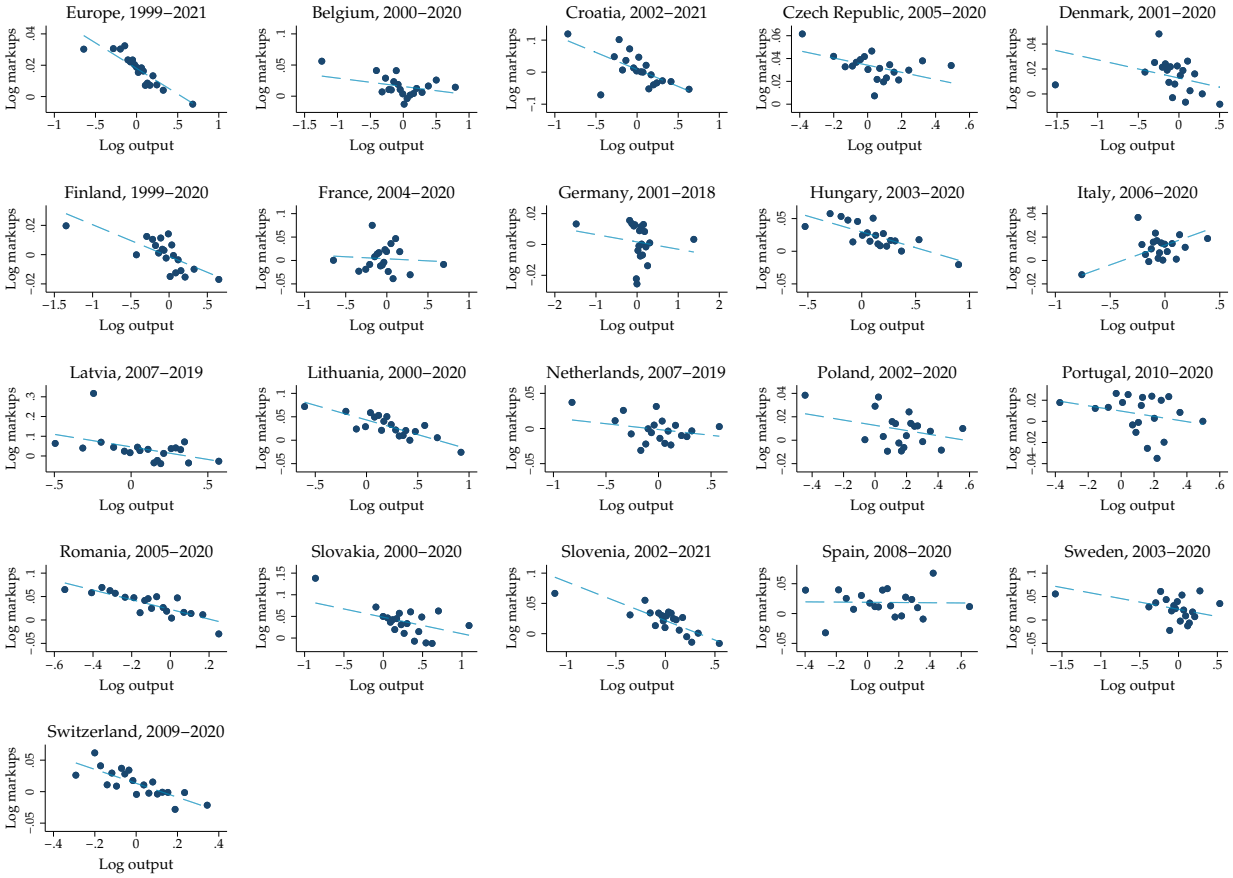
Notes: This figure reports results from separate country-level regressions (except for the "Europe" panel) that project industry-level 4-year changes in log labor costs over sales on 4-year changes in log output while controlling for year fixed effects. 2-digit manufacturing NACE Rev. 2 industries for European countries, 6-digit 2012 NAICS manufacturing industries for the US. CompNet data for Europe (firms with at least 20 employees). NBER-CES Manufacturing Industry Database for the US.

Figure A.5: Log labor output elasticity over returns to scale changes against log output changes, industry-level, 4-year changes



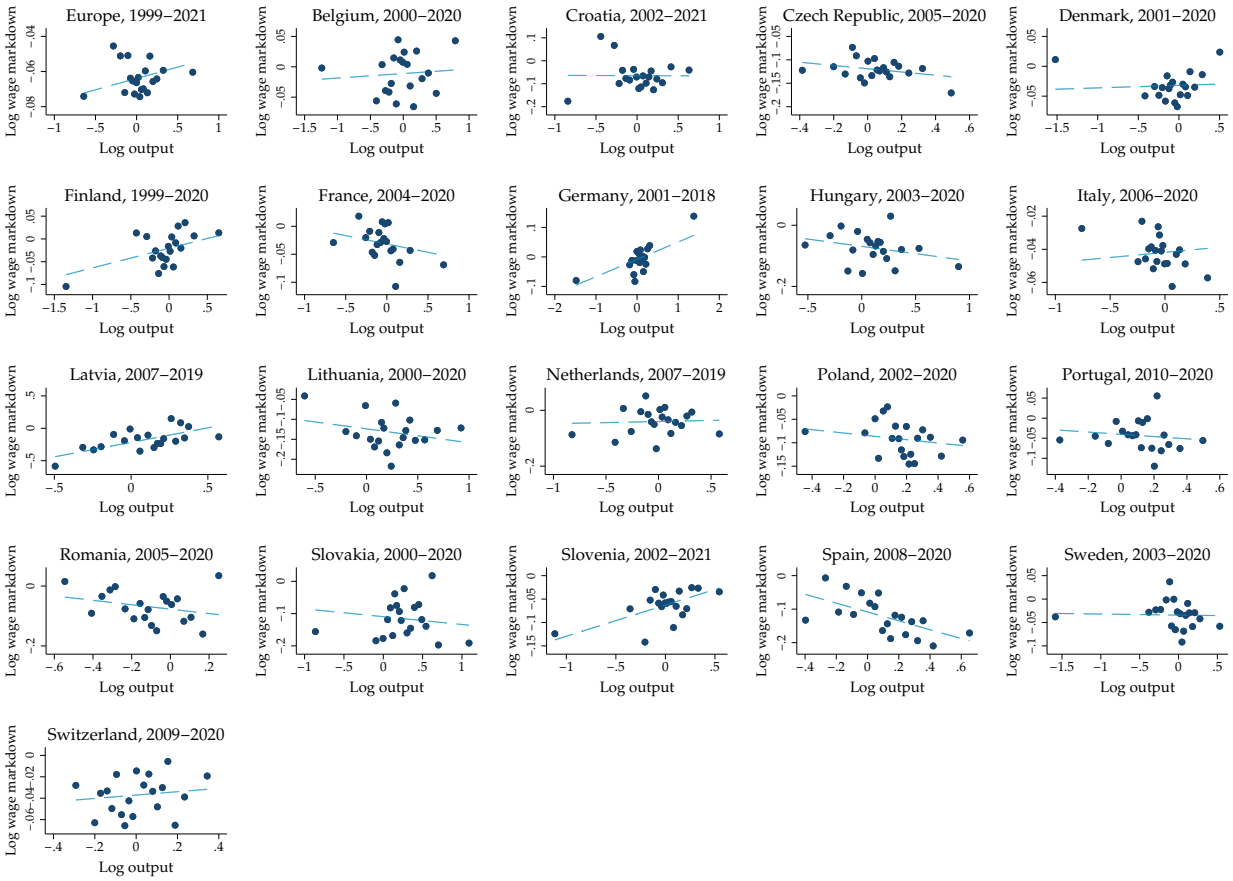
Notes: This figure reports results from separate country-level regressions (except for the "Europe" panel) that project industry-level 4-year changes in log labor output elasticity over returns to scale on 4-year changes in log output while controlling for year fixed effects. For the panel on Europe, we pool all countries and include industry-year fixed effects and country-year fixed effects. 2-digit manufacturing NACE Rev. 2 industries for European countries, 6-digit 2012 NAICS manufacturing industries for the US. CompNet data, firms with at least 20 employees.

Figure A.6: Log markup changes against log output changes, industry-level, 4-year changes



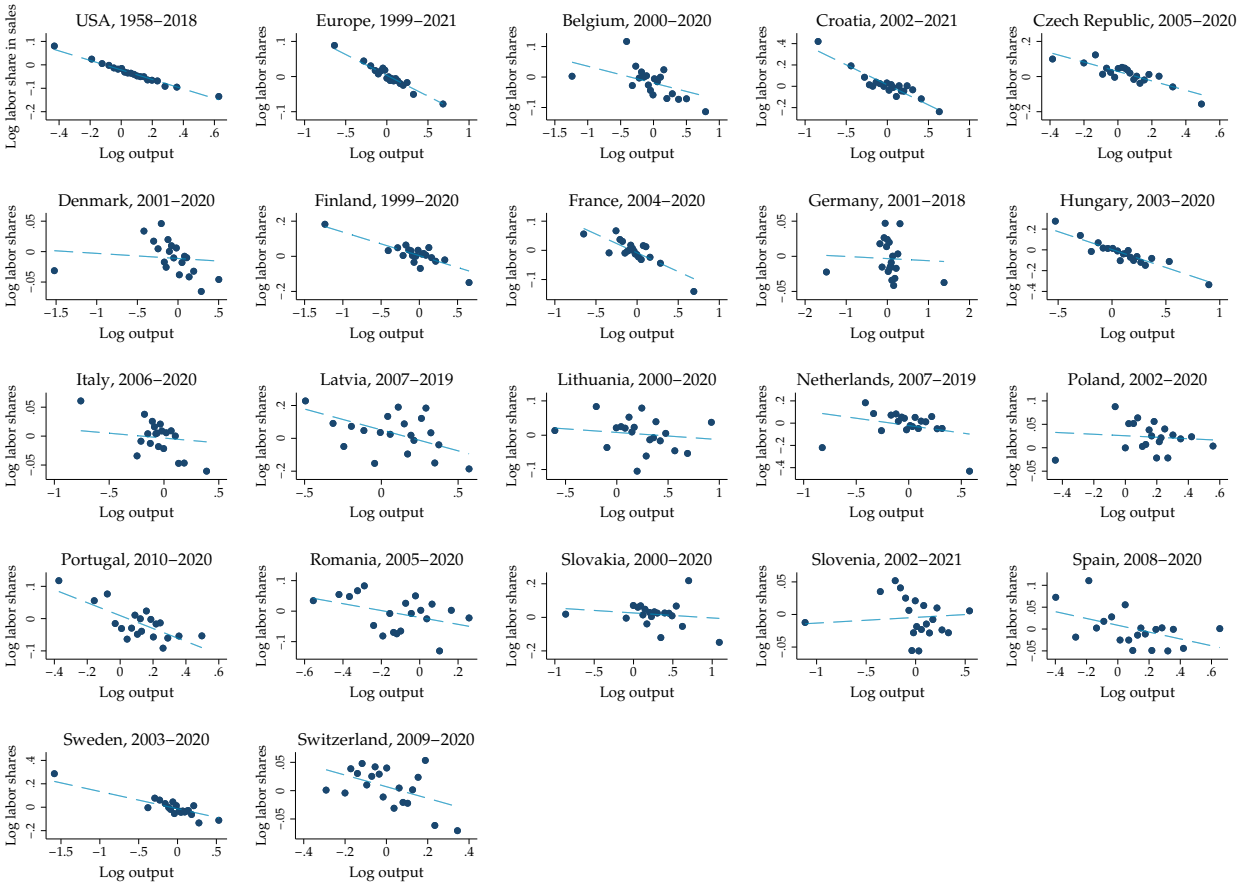
Notes: This figure reports results from separate country-level regressions (except for the "Europe" panel) that project industry-level 4-year changes in log markups on 4-year changes in log output while controlling for year fixed effects. For the panel on Europe, we pool all countries and include industry-year fixed effects and country-year fixed effects. 2-digit manufacturing NACE Rev. 2 industries for European countries, 6-digit 2012 NAICS manufacturing industries for the US. CompNet data, firms with at least 20 employees.

Figure A.7: Log wage markdown changes against log output changes, industry-level, 4-year changes



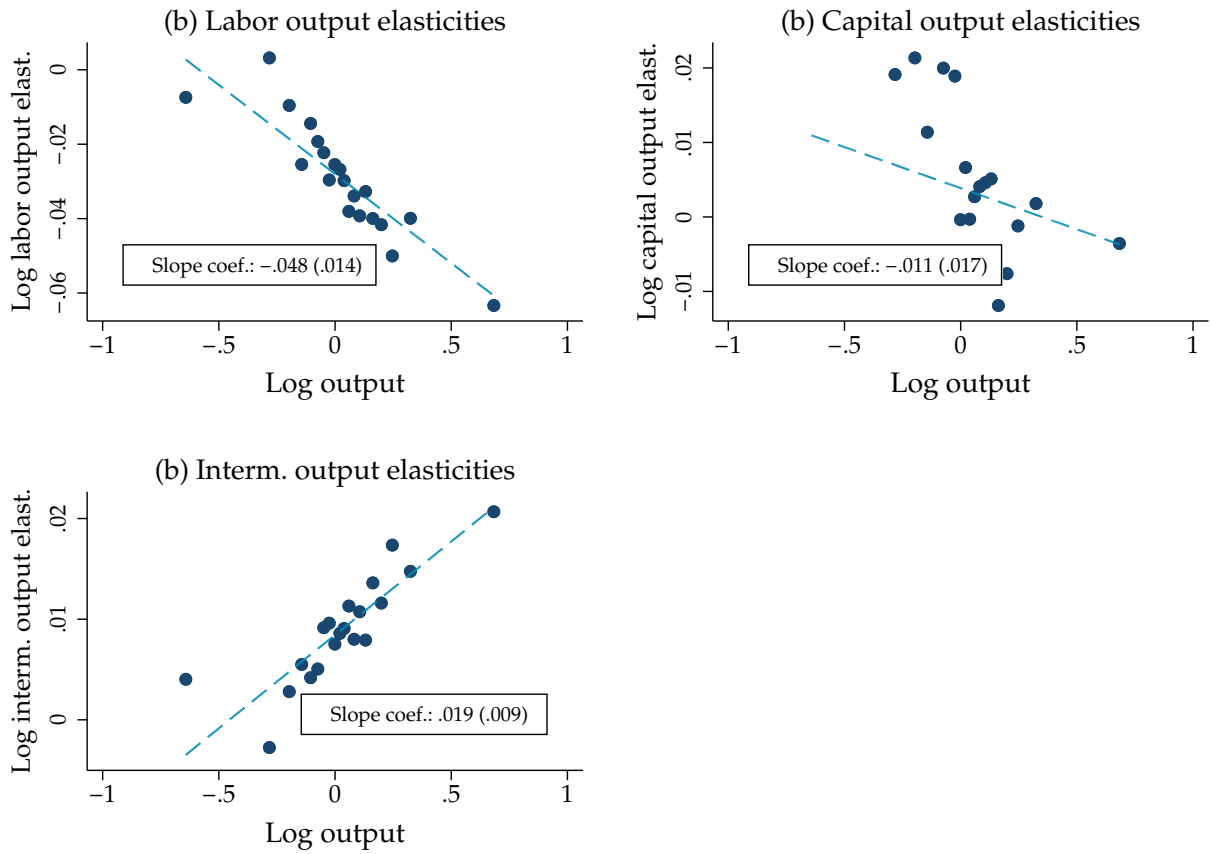
Notes: This figure reports results from separate country-level regressions (except for the "Europe" panel) that project industry-level 4-year changes in log wage markdowns on 4-year changes in log output while controlling for year fixed effects. For the panel on Europe, we pool all countries and include industry-year fixed effects and country-year fixed effects. 2-digit manufacturing NACE Rev. 2 industries for European countries, 6-digit 2012 NAICS manufacturing industries for the US. CompNet data, firms with at least 20 employees.

Figure A.8: Log labor shares in value added changes against log output changes, industry-level, 4-year change



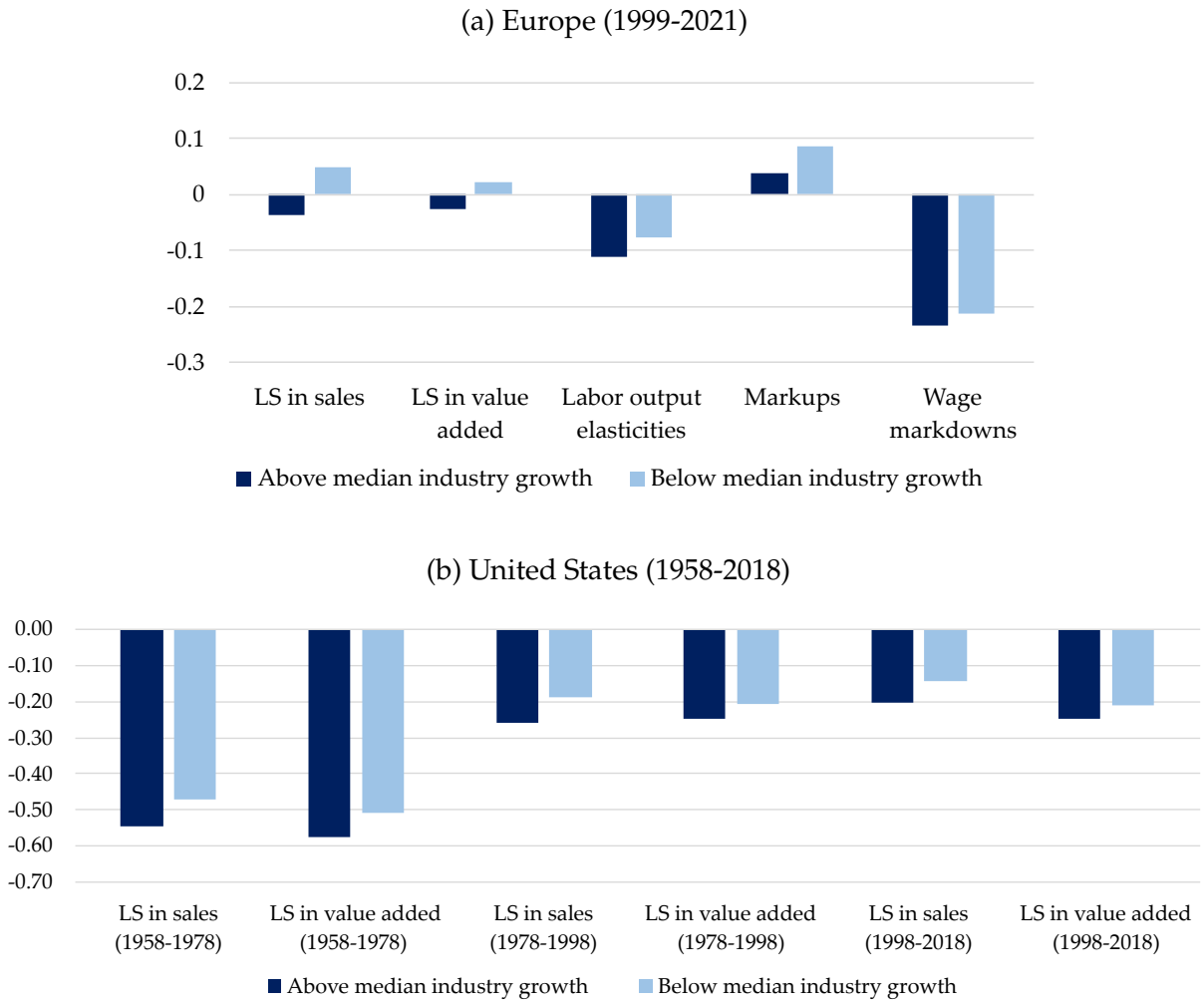
Notes: This figure reports results from separate country-level regressions (except for the "Europe" panel) that project industry-level 4-year changes in log labor costs over value added on 4-year changes in log output while controlling for year fixed effects. For the panel on Europe, we pool all countries and include industry-year fixed effects and country-year fixed effects. 2-digit manufacturing NACE Rev. 2 industries for European countries, 6-digit 2012 NAICS manufacturing industries for the US. CompNet data for Europe (firms with at least 20 employees). NBER-CES Manufacturing Industry Database for the US.

Figure A.9: European industry level 4-year changes in output elasticities (OLS)



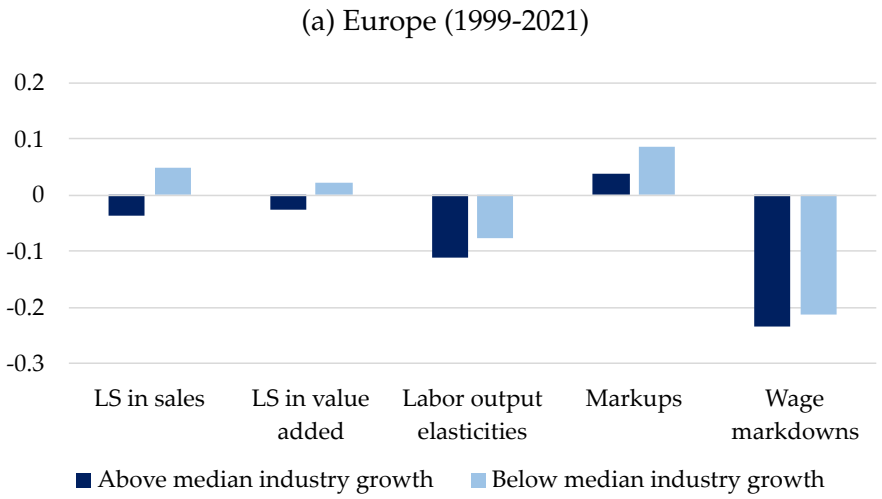
Notes: The figure reports binned scatter plots (OLS) from projecting 4-year industry-level changes in the logs of labor, capital, and intermediate output elasticities over returns to scale against industry-level log output changes (4-year differences). All panels report results that are residualized by country-industry and year fixed effects. CompNet data. Firms with at least 20 employees.

Figure A.10: Changes in labor shares, output elasticities, markups, and wage markdowns in industries with above and below country-median growth rates



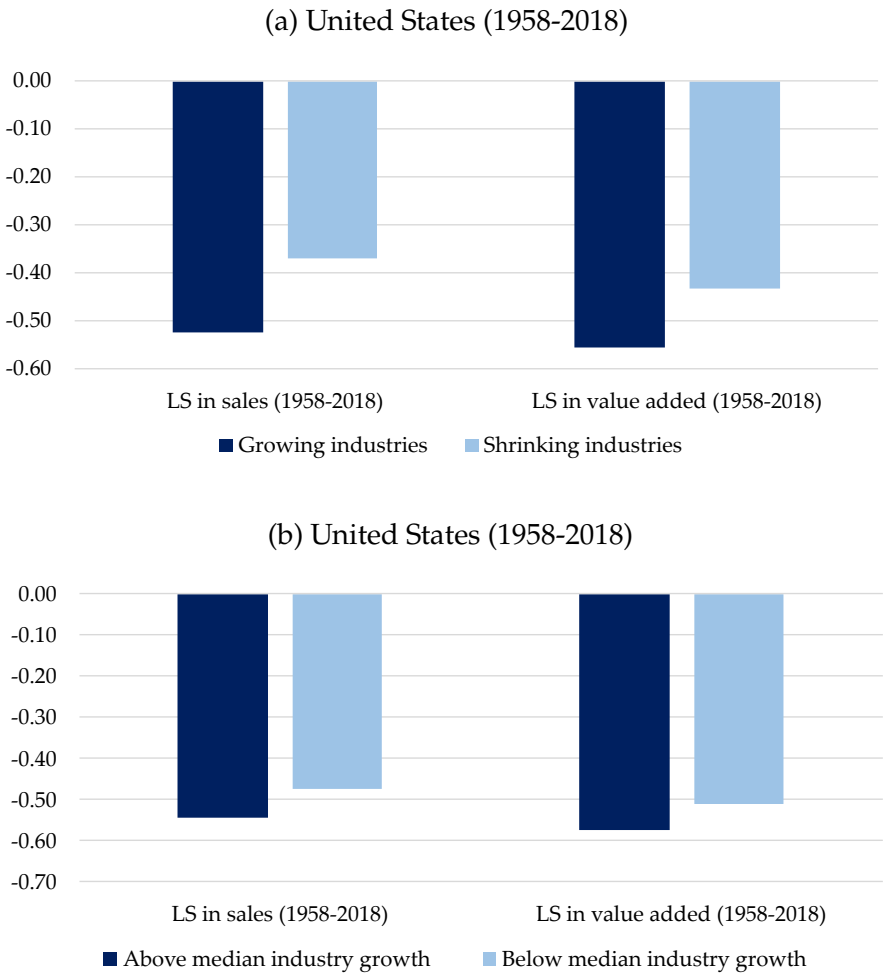
Notes: The figures report average changes in the logs of labor shares, labor output elasticities, markups, and wage markdowns across country-industry pairs for industries with growth rates above and below country-median growth rates. Changes are first computed for each pair and then average across all pairs. Time spans differ across country-industry pairs and not all industries are available for all years within a country. Europe: CompNet data and NACE Rev. 2 two-digit industries. US: NBER-CES Manufacturing Industry Database and NAICS 6-digit industries.

Figure A.11: Europe: changes in labor shares, output elasticities, markups, and wage markdowns in industries with above and below European-median growth rates



Notes: The figure reports average changes in the logs of labor shares, labor output elasticities, markups, and wage markdowns across country-industry pairs for industries with growth rates above and below European-median growth rates. Changes are first computed for each pair and then average across all pairs. Time spans differ across country-industry pairs and not all industries are available for all years within a country. CompNet data and NACE Rev. 2 two-digit industries.

Figure A.12: US: changes in labor shares in growing and shrinking industries and in industries with above and below country-median growth rates, long-run changes



Notes: The figures report average changes in the logs of labor shares across US industries for growing and shrinking industries and for industries with growth rates above and below country-median growth rates. Changes are first computed for each industry over the entire time span and then average across all pairs. NBER-CES Manufacturing Industry Database and NAICS 6-digit industries. There are 36 shrinking and 326 growing industries over the 1958-2018 time span.

B Derivations for the Main Framework

We now show how we derive the equations for firms' markups, wage markdowns, and labor shares from the framework of the main text. Firms manufacture output with the production function $Q_{it} = Q_{it}(.) = F(L_{it}, K_{it}, M_{it}, \Omega_{it})$. Firms minimize costs. The Lagrangian writes:

$$\mathcal{L} = w_{it}(L_{it})L_{it} + z_{it}M_{it} + r_{it}K_{it} - \lambda_{it}(Q_{it} - Q_{it}(.)). \tag{A.1}$$

The first order condition with respect to intermediates writes:

$$z_{it} = \lambda_{it}\frac{\partial Q_{it}}{\partial M_{it}}. \tag{A.2}$$

As λ_{it} is the shadow value of producing one more unit of output, it equals marginal costs: $\lambda_{it} = MC_{it} = \frac{P_{it}}{\mu_{it}}$. Expanding Equation (A.2) with $\frac{M_{it}}{Q_{it}}\frac{Q_{it}}{M_{it}}$ and using the definition of the output elasticity, $\theta_{it}^{M} = \frac{\partial Q_{it}}{\partial M_{it}}\frac{M_{it}}{Q_{it}}$ yields the markup expression from the main text:

$$\mu_{it} = \frac{P_{it}}{MC_{it}} = \theta_{it}^{M}\frac{P_{it}Q_{it}}{z_{it}M_{it}}, \tag{A.3}$$

where μ_{it} is the price-over-marginal-cost markup. The first order condition with respect to labor is:

$$w_{it}\left(1 + \frac{\partial w_{it}}{\partial L_{it}}\frac{L_{it}}{w_{it}}\right) = \lambda_{it}\frac{\partial Q_{it}}{\partial L_{it}} = MRPL_{it} \tag{A.4}$$

$\frac{\partial w_{it}}{\partial L_{it}}\frac{L_{it}}{w_{it}} = \frac{1}{\epsilon_{it}^{L}}$ is the inverse supply elasticity of labor. Expanding Equation (A.4) with $\frac{L_{it}}{Q_{it}}\frac{Q_{it}}{L_{it}}$ and inserting Equation (A.2) yields the wage markdown expression from the main text:

$$\gamma_{it} = \left(1 + \frac{\partial w_{it}}{\partial L_{it}}\frac{L_{it}}{w_{it}}\right) = \frac{\theta_{it}^{L}}{\theta_{it}^{M}}\frac{z_{it}M_{it}}{w_{it}L_{it}}, \tag{A.5}$$

where γ_{it} is the wage markdown, i.e., the wedge between the wage and the marginal revenue product of labor. Finally, expanding Equation (A.5) with $\frac{P_{it}Q_{it}}{P_{it}Q_{it}}$ and combining it with Equation (A.3) yields the labor share equation from the main text:

$$LS = \frac{w_{it}L_{it}}{P_{it}Q_{it}} = \frac{\theta_{it}^{L}}{\mu_{it}\gamma_{it}}. \tag{A.6}$$

C Alternative Labor Market Specification: Rent-Sharing.

For tractability, the main text focuses on a case where firms exert monopsony power. However, a large literature has highlighted that also workers possess labor market power, which can equally drive a wedge between wages and marginal revenue products (e.g., Dobbelaere and Mairesse, 2013, Treuren, 2022, Caselli et al., 2021, Mertens, 2022). We now derive a standard rent-sharing model and show that allowing for worker-side labor market power does not affect any of our derivations and conclusions. Consider that employees maximize utility:

$$U(w_{it}, L_{it}) = w_{it}L_{it} + (\overline{L}_{it} - L_{it})\overline{w}_{it}. \tag{C.1}$$

$\overline{w}_{it} \leq w_{it}$ is the reservation wage. $\overline{L}_{it}$ is the competitive employment level. Firms produce output using the production function of the main text, $Q_{it} = Q_{it}(.) = F(L_{it}, K_{it}, M_{it}, \Omega_{it})$. Firms and workers bargain over wages and employment and solve the following Nash-bargaining problem:

$$\max_{w_{it}, L_{it}, M_{it}, K_{it}} (\zeta_{it} log(L_{it}(w_{it} - \overline{w}_{it})) + (1 - \zeta_{it}) log(P_{it}Q_{it} - w_{it}L_{it} - z_{it}M_{it} - r_{it}K_{it}), \tag{C.2}$$

where $\zeta_{it} \in [0, 1]$ denotes worker's bargaining power.[35] Note that in the event of a breakdown of negotiations, workers receive the reservation wage as outside options, whereas a firm's outside option is the zero-profit outcome. The latter is not essential for our conclusions, but simplifies derivations and follows the literature (e.g., Blanchflower et al., 1996). The first order condition with respect to intermediate inputs yields the same markup expression as in the main text:

$$z_{it} = \frac{P_{it}}{\mu_{it}} \frac{\partial Q_{it}}{\partial M_{it}} \rightarrow \mu_{it} = \theta_{it}^{M} \frac{P_{it}Q_{it}}{z_{it}M_{it}}. \tag{C.3}$$

The first order condition with respect to labor implies:

$$w_{it} \left(1 - \frac{\zeta_{it}}{1 - \zeta_{it}} \frac{P_{it}Q_{it} - w_{it}L_{it} - z_{it}M_{it} - r_{it}K_{it}}{w_{it}L_{it}}\right) = MRPL_{it}, \tag{C.4}$$

where $MRPL_{it}$ denotes the marginal revenue product of labor. Combining Equation (C.3) with Equation (C.4) yields the same equation for the wage markdown as in the main text:

$$\gamma_{it} = \left(1 - \frac{\zeta_{it}}{1 - \zeta_{it}} \frac{P_{it}Q_{it} - w_{it}L_{it} - z_{it}M_{it} - r_{it}K_{it}}{w_{it}L_{it}}\right) = \frac{\theta_{it}^{L}}{\theta_{it}^{M}} \frac{z_{it}M_{it}}{w_{it}L_{it}}, \tag{C.5}$$

where we used $MRPL_{it} = \frac{P_{it}}{\mu_{it}} \frac{\partial Q_{it}}{\partial L_{it}}$. Note that in this setting, $0 \leq \gamma_{it} \leq 1$, which helps explaining why the data features many firms where wages exceed marginal revenue products of labor.[36] Finally, combining Equation (C.5) with Equation (C.3) yields the expression for

[35] We follow the literature and write the rent-sharing model in static terms.

[36] Mertens (2023) generalizes this setting to a case where firms have monopsony power over some workers while they bargain with other workers over rents. This more general model rationalizes why we simultaneously observe firms with wages above and below the marginal revenue product of labor. Even in such a more complex setting, all our derivations and conclusions remain valid.

the labor share from the main text:

$$LS = \frac{w_{it} L_{it}}{P_{it} Q_{it}} = \frac{\theta_{it}^{L}}{\mu_{it} \gamma_{it}}. \tag{C.6}$$

D Further Details on the German Firm-Product Level data

Data access. The data can be accessed at the "Research Data Centres" of the Federal Statistical Office of Germany and the Statistical Offices of the German Länder. Data request can be made at: https://www.forschungsdatenzentrum.de/en/request.

The statistics we used are: "AFiD-Modul Produkte", "AFiD-Panel Industriebetriebe", "AFiD-Panel Industrieunternehmen", "Investitionserhebung im Bereich Verarbeitendes Gewerbe, Bergbau und Gewinnung von Steinen und Erden", "Panel der Kostenstrukturerhebung im Bereich Verarbeitendes Gewerbe, Bergbau und Gewinnung von Steinen und Erden". The data are combined by the statistical offices and provided as a merged dataset.

Variable definitions. The following list presents an overview on the variable definitions of all variables used in this article. This includes variables used in other sections of the online Appendix.

- L_{it}: Labor in headcounts (as of 30th September).
- W_{it}: Firm wage (firm average), defined as gross salary before taxes (including mandatory social costs) + "other social expenses" (including expenditures for company outings, advanced training, and similar costs) divided by the number of employees.
- K_{it}: Capital derived by a perpetual inventory method as described below.
- M_{it}: Deflated total intermediate input expenditures, defined as expenditures for raw materials, energy, intermediate services, goods for resale, renting, temporary agency workers, repairs, and contracted work conducted by other firms. Nominal values are deflated by a 2-digit industry-level deflator supplied by the statistical office of Germany.
- $z_{it}M_{it}$: Nominal values of total intermediate input expenditures.
- $P_{it}Q_{it}$: Nominal total revenue, defined as total gross output, including, among others, sales from own products, sales from intermediate goods, revenue from offered services, and revenue from commissions/brokerage.
- Q_{it}: Quasi-quantity measure of physical output, i.e., $P_{it}Q_{it}$ deflated by a firm-specific price index (denoted by PI_{it}).
- PI_{it}: Firm-specific Törnqvist price index, derived as in Eslava et al., 2004. See online Appendix E.1 for its construction.
- P_{igt}: Price of a product g.
- $share_{igt}$: Revenue share of a product g in total firm revenue.
- ms_{it}: Weighted average of firms' product market shares in terms of revenues. The weights are the sales of each product in firms' total product market sales.
- G_{it}: Headquarter location of the firm. 90% of firms in our sample are single-plant firms.
- D_{it}: A four-digit industry indicator variable. The industry of each firm is defined as the industry in which the firm generates most of its sales.

- E_{it} (or in logs, e_{it}): Deflated expenditures for raw materials and energy inputs. Nominal values are deflated by a 2-digit industry-level deflator for intermediate inputs and which is supplied by the federal statistical office of Germany. E_{it} is part of M_{it}.
- Exp_{it}: Dummy-variable being one, if firms generate export market sales.
- $NumP_{it}$: The number of products a firm produces.

Outlier cleaning. We clean the data from the top and bottom two percent outliers with respect to value-added over revenue and revenue over labor, capital, intermediate input expenditures, and labor costs. We drop quantity and price information for products displaying a price deviation from the average price in the top and bottom one percent tails. We also drop the sectors 16 (tobacco), 23 (mineral oil and coke), and 37 (recycling) as the observation count is insufficient to derive estimates of firms' production functions in these industries.

Capital stock estimation. As capital stocks are not directly observed, we calculate a time series of capital stocks for every firm using the perpetual inventory method of Bräuer et al. (2023):

$$K_{it} = K_{it-1}(1 - \alpha_{jt-1}) + I_{it-1}, \tag{D.1}$$

where K_{it}, α_{jt} and I_{it-1} denote firm i's capital stock, the depreciation rate of capital, and investment. Investment captures firms' total investment in buildings, equipment, machines, and other investment goods. Nominal values are deflated by a two-digit industry-level deflator supplied by the German statistical office.

We derive the industry- and year-specific depreciation rate from official information on the expected lifetime of capital goods (supplied by the statistical offices). To do so, we formulate the lifetime of a capital good LT as a function of its depreciation rate:

$$LT = \alpha \int_0^\infty (1 - \alpha)^t t dt. \tag{D.2}$$

Using partial integration gives:

$$LT = \alpha \left[\frac{(1-\alpha)^t}{\ln(1-\alpha)} t \right]_0^\infty - \alpha \int_0^\infty \frac{(1-\alpha)^t}{\ln(1-\alpha)} dt, \tag{D.3}$$

where the first term on the right-hand side equals zero because $0 < \alpha < 1$. Integrating the remaining expression yields:

$$LT = \frac{\alpha}{\ln(1-\alpha) * \ln(1-\alpha)}, \tag{D.4}$$

which we can numerically solve for α. As the lifetime of capital goods is separately given for years and capital good types (buildings and equipment), we derive a depreciation rate for each year and capital good type separately. To derive a single industry-specific depreciation rate, we weight the depreciation rates for buildings and equipment respectively with the industry-level share of building capital in total capital and equipment capital in total capital (this information is supplied by the statistical offices). For the practical implementation, we

assume that the depreciation rate of a firm's whole capital stock equals the depreciation rate of newly purchased capital.

The initial capital stock for the perpetual inventory method is derived from reported tax depreciation. We do not use the reported tax depreciation when calculating capital stock series as tax depreciation may vary due to state-induced tax incentives and might therefore not reliably reflect the true amount of depreciated capital. Given that firms likely tend to report too high depreciation due to such tax incentives, our first capital values within a capital series are likely overestimated. However, over time, observed investment decisions gradually receive a larger weight in estimated capital stocks, mitigating the impact of the first capital stock. Given that we estimate very reasonable output elasticities (see Table A.1), we are confident that our capital variables reliably reflect firms' true capital stocks.[37]

Deriving a time-consistent industry classification. During our long time-series, the NACE classification of industries (and thus firms into industries) changed twice. Once in 2002 and once in 2008. Because our estimation of the production function requires a time-consistent industry classification at the firm level (as we allow for industry-specific production functions), it is crucial to recover a time-consistent NACE industry classification. Recovering such a time-consistent industry classification from official concordance tables is, however, problematic as they contain many ambiguous sector reclassifications. To address this issue, we follow the procedure described in Mertens (2022) and use information on firms' product mix to classify firms into NACE Rev 1.1 industries based on their main production activities. This procedure exploits that the first four digits of the ten-digit GP product classification reported in the German data are identical to the NACE classification (i.e., they indicate the industry of the product). Applying this method demands a consistent reclassification of all products into the GP2002 scheme (which corresponds to the NACE Rev 1.1 scheme). Reclassifying products is, due to the granularity of the ten-digit classification, less ambiguous than reclassifying industries. In the few ambiguous cases, we can follow the firms' product mix over the reclassification periods and unambiguously reclassify most products (i.e., we observe what firms produce before and after reclassification years). Having constructed a time-consistent product-industry classification according to the GP2002 scheme, we attribute every firm to the NACE Rev 1.1 industry in which it generates most of its revenue. When comparing our classification with the one of the statistical offices for the years 2002-2008 (years in which industries are already reported in NACE Rev 1.1), we find that our two-digit and four-digit classification of firms into industries matches the classification of the statistical offices in 95% and 86% of all cases, respectively.

[37] As firms likely tend to overstate their capital depreciation, our capital stocks are likely a closer approximation of the true capital stock used in firms' production processes than capital measures based on book values.

E Production Function Estimation in the German Data

We follow Mertens (2022) in estimating the production function. This approach yields firm- and time-specific output elasticities by assuming the following translog production function:

$$q_{it} = \boldsymbol{\phi}'_{it}\,\boldsymbol{\beta} + \omega_{it} + \epsilon_{it}\,. \tag{E.1}$$

q_{it} denotes the log of produced quantities and $\boldsymbol{\phi}'_{it}$ captures the production inputs Capital (K_{it}), labor (L_{it}), and intermediates (M_{it}) and its interactions. The production function is specified in logs as

$$\begin{aligned} q_{it} =& \beta_l l_{it} + \beta_m m_{it} + \beta_k k_{it} + \beta_{ll} l_{it}^2 + \beta_{mm} m_{it}^2 + \beta_{kk} k_{it}^2 \\ &+ \beta_{lk} l_{it} k_{it} + \beta_{lm} l_{it} m_{it} + \beta_{km} k_{it} m_{it} + \beta_{lkm} l_{it} k_{it} m_{it} + \omega_{it} + \epsilon_{it}, \end{aligned} \tag{E.2}$$

where smaller letter denote logs. The output elasticity of labor is

$$\frac{\partial q_{it}}{\partial l_{it}} = \beta_l + 2\beta_{ll} l_{it} + \beta_{lm} m_{it} + \beta_{lk} k_{it} + \beta_{lkm} k_{it} m_{it}\,.$$

ϵ_{it} is an i.i.d. error term and ω_{it} denotes Hicks-neutral productivity and follows a Markov process. ω_{it} is unobserved to the econometrician, yet firms know ω_{it} before making input decisions for flexible inputs (intermediates in our case). We assume that only firms' input decision for intermediates depends on productivity shocks. Labor and capital do not respond to contemporary productivity shocks. However, our results are similar when allowing labor to respond to productivity innovations. In fact, the CompNet routine for the production function estimation models labor as flexible and we find consistent results in both datasets. There are three issues preventing us from estimating the production function (E.1) using OLS.

(1) We need to estimate a physical production model to recover the relevant output elasticities. Although we observe product quantities, quantities cannot be aggregated across the various products of multi-product firms. Relying on the standard practice to apply sector-specific output deflators does not solve this issue if output prices vary within industries.

(2) We do not observe firm-specific input prices for capital and intermediate inputs. If input prices are correlated with input decisions and output levels, an endogeneity issue arises.

(3) The fact that productivity is unobserved and that firms' flexible input decisions depend on productivity shocks creates another endogeneity problem.

We now discuss how we solve these three identification problems.

E.1 Solving (1) by Deriving a Firm-specific Output Price Index

As we cannot aggregate output quantities across different products of a firm (a common problem), we follow Eslava et al. (2004) and construct a firm-specific price index from observed output prices. We use this price index to purge observed firm revenue from price variation

by deflating firm revenues with this price index.[38] We construct firm-specific Törnqvist price indices for each firm's composite revenue from its various products in the following way:

$$PI_{it} = \prod_{g=1}^{n} \frac{p_{igt}}{p_{igt-1}}^{1/2(share_{igt}+share_{igt-1})} PI_{it-1} \,. \tag{E.3}$$

PI_{it} is the price index, p_{igt} is the price of good g, and $share_{igt}$ is the share of this good in total product market sales of firm i in period t. The growth of the index value is the product of the individual products' price growths, weighted with the average sales share of that product over the current and the last year. The first year available in the data is the base year (i.e., $PI_{it=1995} = 100$). If firms enter after 1995, we follow Eslava et al. (2004) and use an industry average of the computed firm price indices as a starting value. Similarly, we impute missing product price growth information in other cases with an average of product price changes within the same industry.[39] After deflating firm revenue with this price index, we have a quasi-quantity measure of output, for which, with slightly abusing notation, we keep using q_{it}.[40]

E.2 Solving (2) by Accounting for Unobserved Input Price Variation

While the recent literature stresses the so-called "output-price bias" when estimating production functions, previous work has also highlighted that unobserved input prices introduce another identification problem. To control for input price variation across firms, we use a firm-level analog of De Loecker et al. (2016) and define a price-control function from firm-product-level output price information that we add to the production function (E.1):

$$q_{it} = \boldsymbol{\phi}_{it}'\boldsymbol{\beta} + B_{it}((pi_{it}, ms_{it}, G_{it}, D_{it}) \times \boldsymbol{\phi}_{it}^{c}) + \omega_{it} + \epsilon_{it} \,. \tag{E.4}$$

$B_{it}(.) = B_{it}((pi_{it}, ms_{it}, G_{it}, D_{it}) \times \boldsymbol{\phi}_{it}^{c})$ is the price control function consisting of our logged firm-specific output price index (pi_{it}), a logged sales-weighted average of firms' product market sales shares (ms_{it}), a headquarter location dummy (G_{it}) and a four-digit industry dummy (D_{it}). $\boldsymbol{\phi}_{it}^{c} = [1; \boldsymbol{\phi}_{it}]$, where $\boldsymbol{\phi}_{it}$ includes the production function input terms as specified in (E.2). These are either in monetary terms and deflated by an industry-level deflator (capital and intermediates) or already reported in quantities (labor). The constant entering $\boldsymbol{\phi}_{it}^{c}$ highlights that elements of $B(.)$ enter the price control function linearly and interacted with $\boldsymbol{\phi}_{it}$ (a consequence of the translog production function). The idea behind the price-control function $B(.)$ is that output prices, product market shares, firm location, and firms' industry affiliation are informative about firms' input prices. Particularly, we assume

[38] This approach has also been applied in various other studies, such as Smeets and Warzynski (2013).

[39] For roughly 30% of all product observations in the data, firms do not have to report quantities as the statistical office views them as not being meaningful.

[40] Note that, as discussed in Bond et al. (2021), using an output price index does not fully purge firm-specific price variation. There remains a base year difference in prices. Yet, using a firm-specific price index follows the usual practice of using price indices to deflate nominal values, we are thus following the best practice. Moreover, it is the only available approach when pooling multi- and single-product firms. Estimating the production function separately by single-plant firms requires other strong assumptions like perfect input divisibility of all inputs across all products. Finally, our results are also robust to using cost-share approaches to estimate the production function, which requires other assumptions (constant returns to scale, competitive input markets, and the absence of adjustment costs).

that product prices and market shares contain information about product quality and that producing high-quality products requires expensive high-quality inputs. As De Loecker et al. (2016) discuss, this motivates the addition of a control function containing output price and market share information to the right-hand side of the production function to control for unobserved input price variation emerging from input quality differences across firms. We also include location and four-digit industry dummies into $B(.)$ to further absorb the remaining differences in local and four-digit industry-specific input prices.

Conditional on elements in $B(.)$, we assume that there are no remaining input price differences across firms. Although restrictive, this assumption is more general than the ones employed in most other studies estimating production functions without having access to firm-specific price data and which implicitly assume that firms face identical input and output prices within industries.

A notable difference between the original approach of De Loecker et al. (2016) and our version is that they estimate product-level production functions, whereas we transfer their framework to the firm level. For that, we use firm-product-specific sales shares in firms' total product market sales to aggregate firm-product-level information to the firm-level. This implicitly assumes that i) such firm aggregates of product quality increase in firm aggregates of product prices and input quality, ii) firm-level input costs for inputs entering as deflated expenditures increase in firm-level input quality, and iii) product price elasticities are equal across the various products of a firm. These or even stricter assumptions are always implicitly invoked when estimating firm-level production functions.

Finally, note that even if some of the above assumptions do not hold, including the price control function is still preferable to omitting it. This is because the price control function can nevertheless absorb some of the unobserved price variations and does not require that input prices vary between firms with respect to all elements of $B_{it}(.)$. The estimation can regularly result in coefficients implying that there is no price variation at all. The attractiveness of a price control function lies in its agnostic view about the existence and degree of input price variation.

E.3 Solving (3) by Controlling for Unobserved Productivity

To address the dependence of firms' intermediate input decision on unobserved productivity, we follow Olley and Pakes (1996) and Levinsohn and Petrin (2003) and employ a control function approach. We base our control function on firms' consumption of energy and raw materials, which we denote by e_{it} and which are components of total intermediate inputs. Inverting the demand function for e_{it} defines an expression for productivity:

$$\omega_{it} \equiv g_{it}(.) = g_{it}(e_{it}, k_{it}, l_{it}, \mathbf{\Gamma}_{it}). \tag{E.5}$$

$\mathbf{\Gamma}_{it}$ captures state variables of the firm, that in addition to k_{it} and l_{it} affect firms demand for e_{it}. Ideally, $\mathbf{\Gamma}_{it}$ should include a wide set of variables affecting productivity and demand for e_{it}. We include dummy variables for export (EX_{it}) activities, the log of the number of products a firm produces ($NumP_{it}$), and the average wage a firm pays (W_{it}) into $\mathbf{\Gamma}_{it}$. The latter absorbs unobserved quality and price differences that shift input demand for e_{it}. Remember that productivity follows a first-order Markov process. Firms can shift this Markov process as described in Doraszelski and Jaumandreu (2013) and De Loecker (2013), giving rise to the following law of motion for productivity: $\omega_{it} = h_{it}(\omega_{it-1}, \mathbf{T}_{it-1}) + \xi_{it} = h_{it}(.) + \xi_{it}$, where

ξ_{it} denotes the innovation in productivity and $\mathbf{T}_{it} = (EX_{it}, NumP_{it})$ reflects that we allow for learning effects from export market participation and (dis)economies of scope through adding and dropping products to influence firm productivity.[41] Plugging (E.5) and the law of motion for productivity into (E.4) gives

$$q_{it} = \boldsymbol{\phi}'_{it}\boldsymbol{\beta} + B_{it}(.) + h_{it}(.) + \epsilon_{it} + \xi_{it}\,, \tag{E.6}$$

which constitutes the basis of our estimation.

E.4 Identifying Moments

We estimate Equation (E.6) separately by two-digit NACE Rev. 1.1 industries using a one-step estimator as in Wooldridge (2009).[42] Our estimator uses lagged values of flexible inputs (i.e., intermediates) as instruments for their contemporary values to address the dependence of firms' flexible input decisions on realizations of ξ_{it}. Similarly, we use lagged values of terms including firms' market share and output price index as instruments for their contemporary values as we consider these to be flexible variables.[43] We define identifying moments jointly on ϵ_{it} and ξ_{it}:

$$E[(\epsilon_{it} + \xi_{it})\mathbf{Y}_{it}] = 0. \tag{E.7}$$

$\mathbf{Y}_{it}$ includes lagged interactions of intermediate inputs with labor and capital, contemporary interactions of labor and capital, contemporary location and industry dummies, the lagged output price index, lagged market shares, lagged elements of $h_{it}(.)$, and lagged interactions of the output price index with production inputs. Formally this implies

$$\mathbf{Y}'_{it} = (J_{it}(.), A_{it-1}(.), T_{it-1}(.), \Psi_{it}(.), \boldsymbol{\nu}_{it-1})\ , \tag{E.8}$$

where we defined:

- $J_{it}(.) = (l_{it},\, k_{it},\, l_{it}^2,\, k_{it}^2,\, l_{it}k_{it},\, G_{it},\, D_{it})$,
- $A_{it}(.) = (m_{it},\, m_{it}^2,\, l_{it}m_{it},\, k_{it}m_{it},\, l_{it}k_{it}m_{it},\, ms_{it},\, \pi_{it})$,
- $T_{it}(.) = ((l_{it},\, k_{it},\, l_{it}^2,\, k_{it}^2,\, l_{it}k_{it},\, m_{it},\, m_{it}^2,\, l_{it}m_{it},\, k_{it}m_{it},\, l_{it}k_{it}m_{it}) \times \pi_{it})$,
- $\Psi_{it}(.) = \sum_{n=0}^{3}\sum_{w=0}^{3-b}\sum_{h=0}^{3-n-b} l_{it-1}^n\, k_{it-1}^b\, e_{it-1}^h$, and
- $\boldsymbol{\nu}_{it-1} = (Exp_{it-1},\, NumP_{it-1},\, W_{it-1})$,
- with W_{it} denoting the average wage a firm pays.

[41] Doraszelski and Jaumandreu (2013) also highlight the role of R&D investment in shifting firms' productivity process. We would also like to add this information to the productivity model but do not observe R&D expenditures for the early years in our data.

[42] We approximate $h_{it}(.)$ by a third-order polynomial in all of its elements, except for the variables in $\boldsymbol{\Gamma}_{it}$. Those we add linearly. $B_{it}(.)$ is approximated by a flexible polynomial where we interact the output price index with elements in $\boldsymbol{\phi}_{it}$ and add the vector of market shares, the output price index, and the location and industry dummies linearly. Interacting further elements of $B_{it}(.)$ with $\boldsymbol{\phi}_{it}$ creates too many parameters to be estimated. This implementation is similar to De Loecker et al. (2016).

[43] This also addresses any simultaneity concerns with respect to the price variables entering the right-hand side of our estimation.

F Cost Shares, Technology, and Adjustment Costs

We now discuss how input adjustment costs shape firms' input mix and cost shares (but would not directly affect our estimates of output elasticities).

F.1 Cost Shares with Flexible Inputs

As argued in the main text, cost shares measure firms' output elasticities and thus technology under constant returns to scale, perfect competition in input markets, and perfectly flexible input adjustments. To see this, consider a general production function (notation follows the main text):

$$Q_{it} = F(L_{it}, K_{it}, M_{it}, \Omega_{it}). \tag{F.1}$$

Firms' minimize costs. The Lagrangian writes:

$$\mathcal{L} = w_{it}L_{it} + z_{it}M_{it} + r_{it}K_{it} - \lambda_{it}(Q_{it} - Q_{it}(.)). \tag{F.2}$$

The first order conditions for labor, intermediates, and capital are:

$$w_{it} = \lambda_{it}\frac{\partial Q_{it}}{\partial M_{it}}, \tag{F.3}$$

$$z_{it} = \lambda_{it}\frac{\partial Q_{it}}{\partial L_{it}}, \tag{F.4}$$

$$r_{it} = \lambda_{it}\frac{\partial Q_{it}}{\partial K_{it}}, \tag{F.5}$$

where λ_{it} is the Lagrangian parameter (the shadow value of producing one more unit of output). It thus holds that: $\lambda_{it} = MC_{it} = \frac{P_{it}}{\mu_{it}}$, with MC_{it}, P_{it}, and μ_{it} denoting marginal costs, the output price, and the markup. Rearranging the first order conditions yields three expressions for input expenditures:

$$w_{it}L_{it} = P_{it}Q_{it}\frac{\theta^L_{it}}{\mu_{it}}, \tag{F.6}$$

$$z_{it}M_{it} = P_{it}Q_{it}\frac{\theta^M_{it}}{\mu_{it}}, \tag{F.7}$$

$$r_{it}K_{it} = P_{it}Q_{it}\frac{\theta^K_{it}}{\mu_{it}}, \tag{F.8}$$

where we used the definition of the output elasticity of input $X = \{L, K, M\}$ as $\theta^X_{it} = \frac{\partial Q_{it}}{\partial X_{it}}\frac{X_{it}}{Q_{it}}$.

Combining these cost expressions with each other yields for the labor input:

$$\frac{w_{it}L_{it}}{w_{it}L_{it} + z_{it}M_{it} + r_{it}K_{it}} = \frac{\theta^L_{it}}{\theta^L_{it} + \theta^M_{it} + \theta^K_{it}}. \tag{F.9}$$

The right-hand side of Equation (F.9) is the cost share for the labor input. Under constant returns to scale ($\theta^L_{it} + \theta^M_{it} + \theta^K_{it} = 1$), this cost share equals the labor output elasticity. The same holds for other inputs. Finally, note that in this entire derivation, we did not made

any assumptions on firms' production functions. This derivation holds under any general production function as long as $\theta_{it}^X = \frac{\partial Q_{it}}{\partial X_{it}} \frac{X_{it}}{Q_{it}}$ is true. This means, that cost shares also equal output elasticities in presence of non-Hicks-neutral productivity processes as long as we impose constant returns to scale, competitive input markets, and flexible input adjustments.

F.2 Cost Shares with Input Adjustment Costs

Consider now that inputs face adjustment costs. We use the same setup as in Appendix F.1 but allow for labor adjustments costs. We ignore adjustments costs in other inputs for simplicity, but the following arguments can be extended to other inputs. As will become clear, adjustment costs in only one input will bias the output elasticities for all inputs if measured via cost shares.

If firms face labor adjustment costs, the optimization problems becomes dynamic. In our case with only labor adjustment costs, the first order conditions for materials and capital are unchanged. However, the condition for labor includes a new term that reflects the wedge between the input price and the shadow price of the input under adjustment costs:

$$w_{it} L_{it} \left(1 + \frac{\partial \chi_{it}(L_{it}, L_{it-1})}{\partial L_{it}} \right) = P_{it} Q_{it} \frac{\theta_{it}^L}{\mu_{it}}, \tag{F.10}$$

where $w_{it} L_{it} \frac{\partial \chi_{it}(L_{it}, L_{it-1})}{\partial L_{it}}$ captures firms' adjustment costs as a share of firms' wage bill, which we write as a function of past and present labor.[44] Following the derivations of Appendix F.1, the labor cost share is now given by:

$$\frac{w_{it} L_{it}}{w_{it} L_{it} + z_{it} M_{it} + r_{it} K_{it}} = \frac{\frac{\theta_{it}^L}{1 + \frac{\partial \chi_{it}(L_{it}, L_{it-1})}{\partial L_{it}}}}{\frac{\theta_{it}^L}{1 + \frac{\partial \chi_{it}(L_{it}, L_{it-1})}{\partial L_{it}}} + \theta_{it}^M + \theta_{it}^K}, \tag{F.11}$$

which even under constant returns to scale is not equal to the output elasticity of labor. Note that, as discussed in the main text, the estimation of the input output elasticities using our parametric production function estimation routine is not affected by labor adjustment costs. This is because input adjustments costs as modelled above do not affect the technological relationship between output changes and input changes.

[44] The functional form of χ_{it} can differ, depending on the underlying adjustment cost model. Specifying the precise functional form of χ_{it} is not essential for our argument.

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